Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/15-1.1.1.2a

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**832**]. This is test number [15].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (832)	0.00 (0)
Mathematica	100.00 (832)	0.00 (0)
Sympy	97.48 (811)	2.52 (21)
Maple	83.05 (691)	16.95 (141)
Fricas	83.05 (691)	16.95 (141)
Maxima	82.57 (687)	17.43 (145)
Reduce	79.81 (664)	20.19 (168)
Giac	75.96 (632)	24.04 (200)
Mupad	71.63 (596)	28.37 (236)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

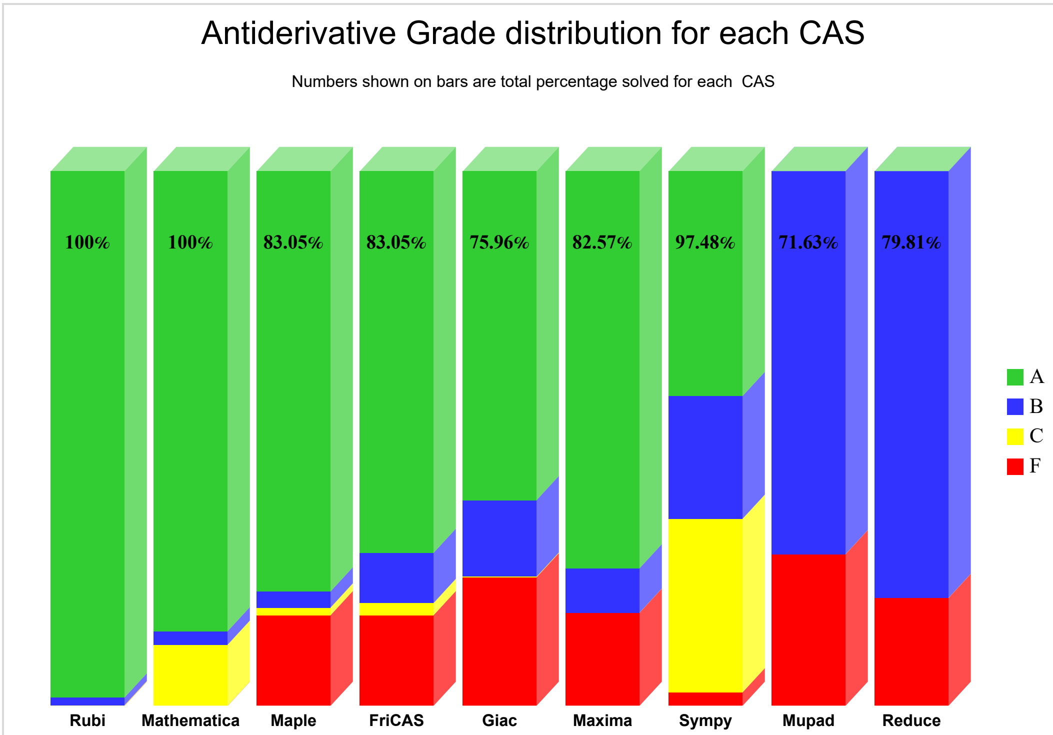
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

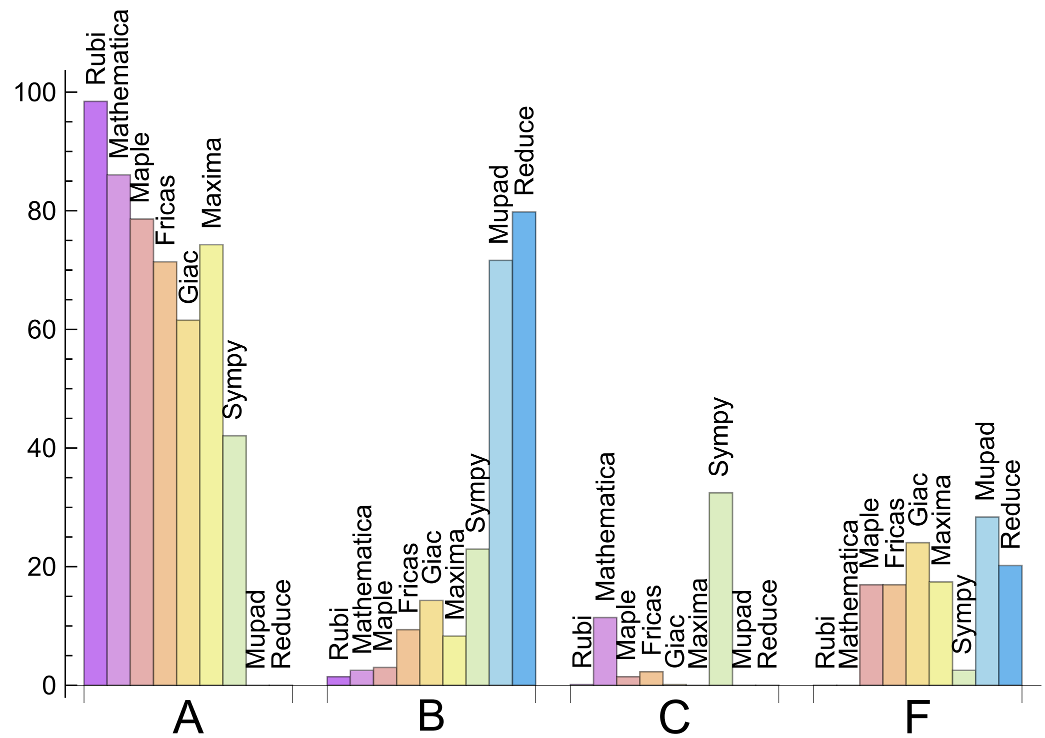
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.438	1.442	0.120	0.000
Mathematica	86.058	2.524	11.418	0.000
Maple	78.606	3.005	1.442	16.947
Maxima	74.279	8.293	0.000	17.428
Fricas	71.394	9.375	2.284	16.947
Giac	61.538	14.303	0.120	24.038
Sympy	42.067	22.957	32.452	2.524
Mupad	0.000	71.635	0.000	28.365
Reduce	0.000	79.808	0.000	20.192

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	21	19.05	80.95	0.00
Fricas	141	99.29	0.00	0.71
Maple	141	100.00	0.00	0.00
Maxima	145	100.00	0.00	0.00
Reduce	168	100.00	0.00	0.00
Giac	200	91.00	6.00	3.00
Mupad	236	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.08
Mupad	0.09
Maple	0.11
Reduce	0.16
Rubi	0.18
Mathematica	1.15
Giac	5.14
Sympy	6.02

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	60.76	1.01	48.00	0.83
Maple	69.44	1.53	46.00	0.82
Mathematica	73.13	1.58	49.00	0.90
Rubi	75.52	1.09	68.00	1.00
Maxima	86.71	1.73	61.00	0.95
Giac	95.80	1.98	66.00	0.95
Reduce	98.22	1.93	65.00	1.00
Fricas	102.54	1.51	81.00	1.27
Sympy	270.52	4.66	85.00	1.25

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

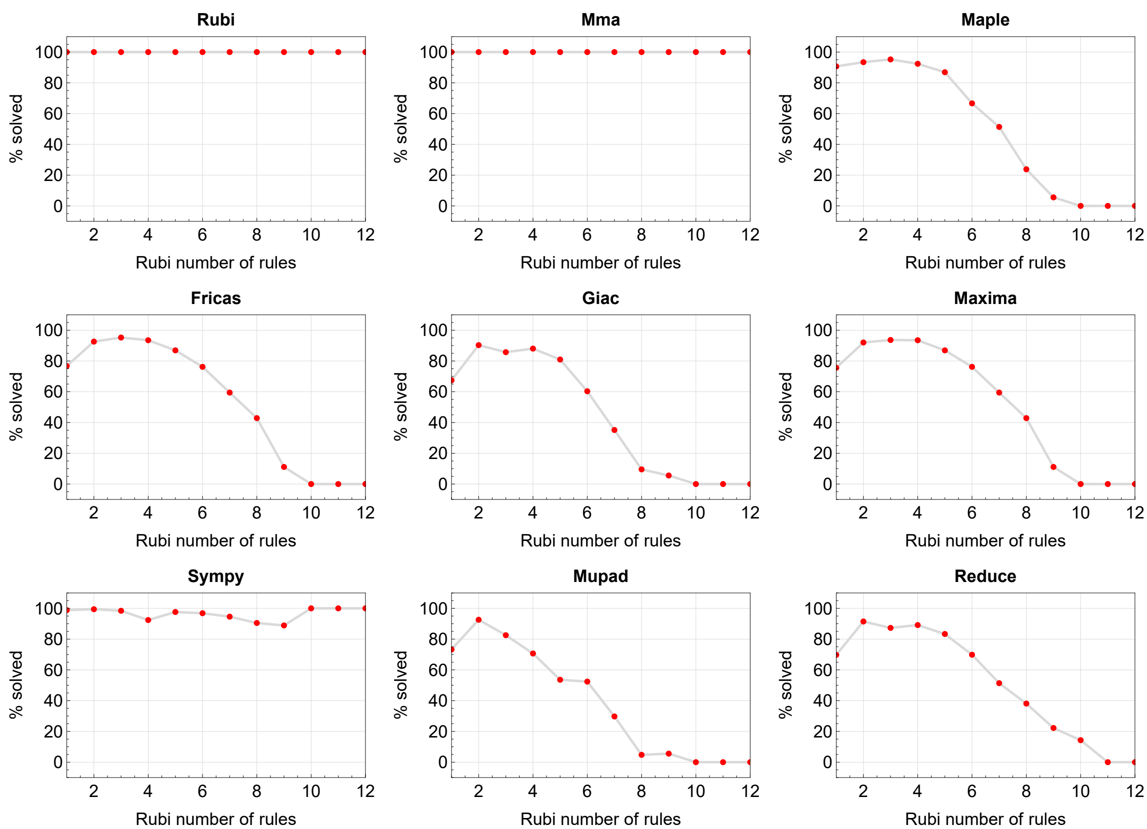


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

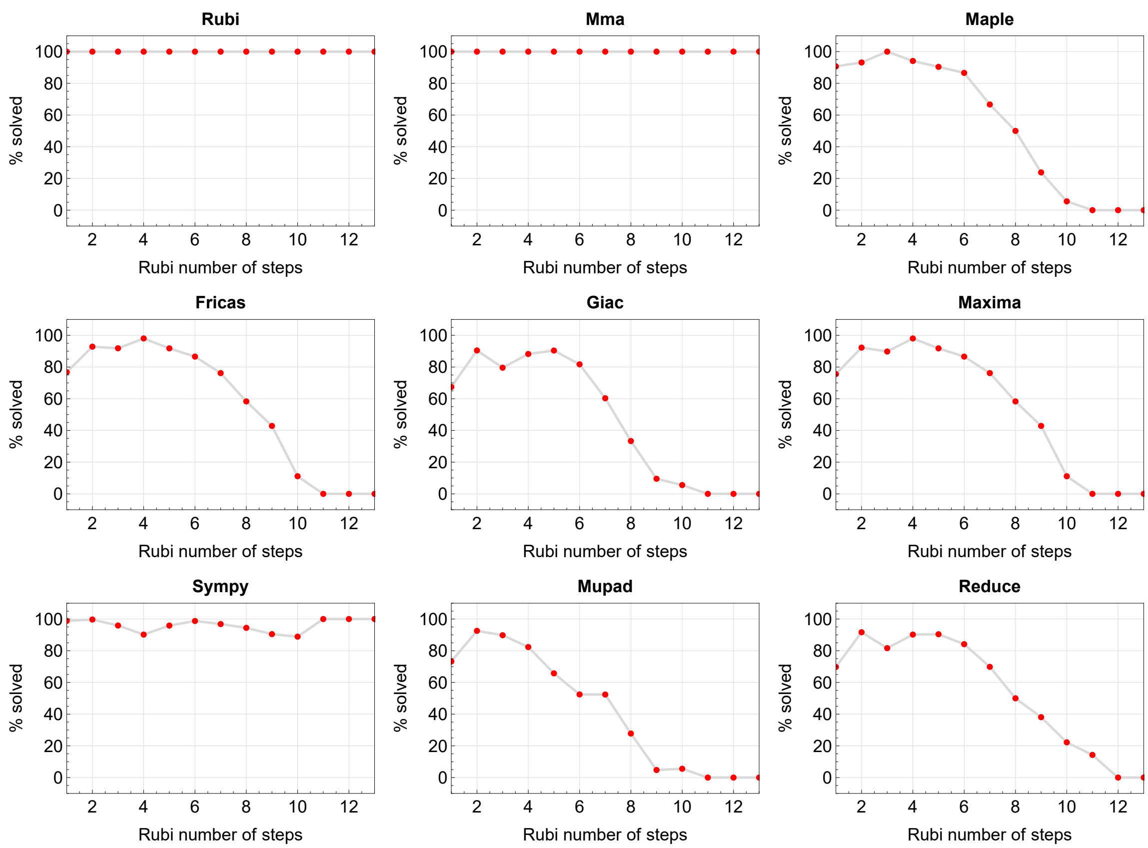


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

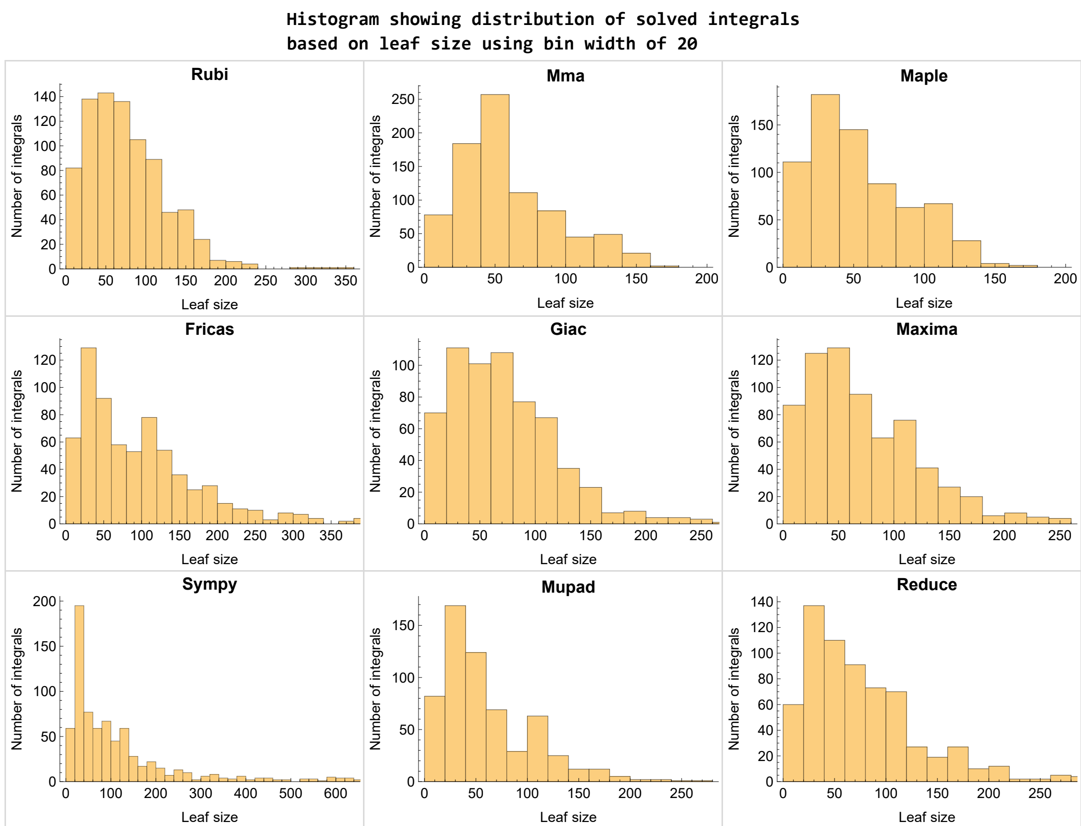


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

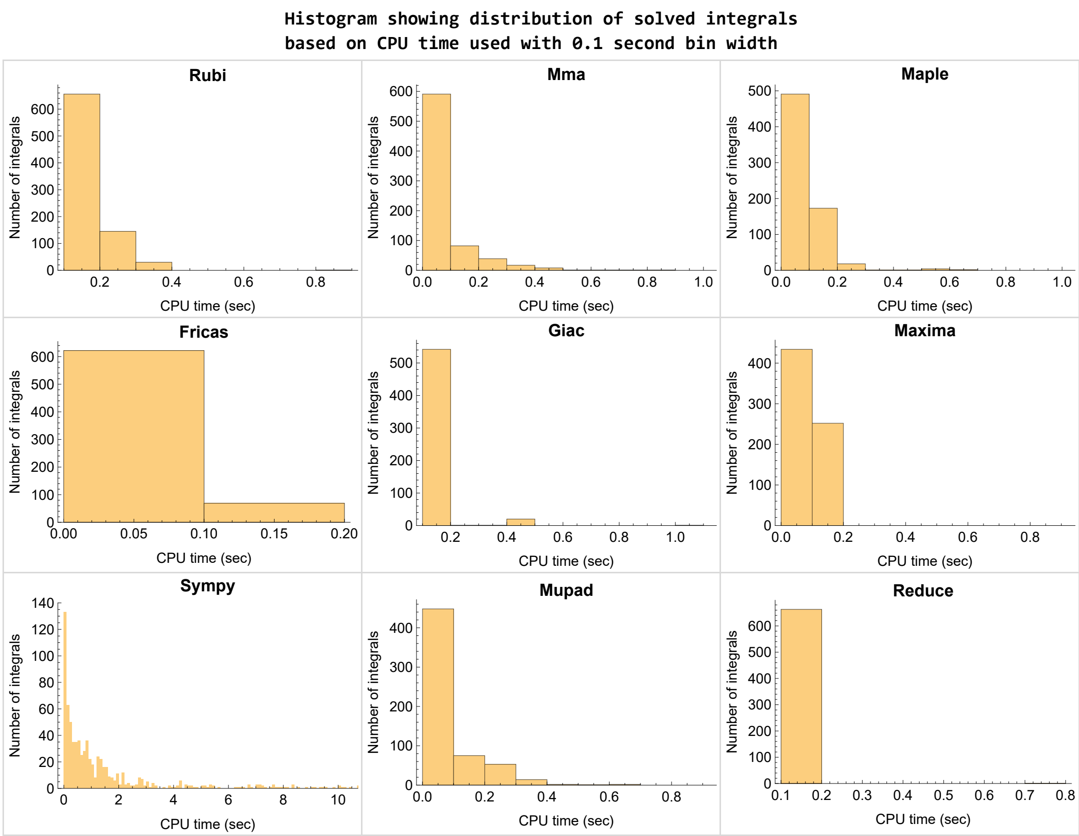


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

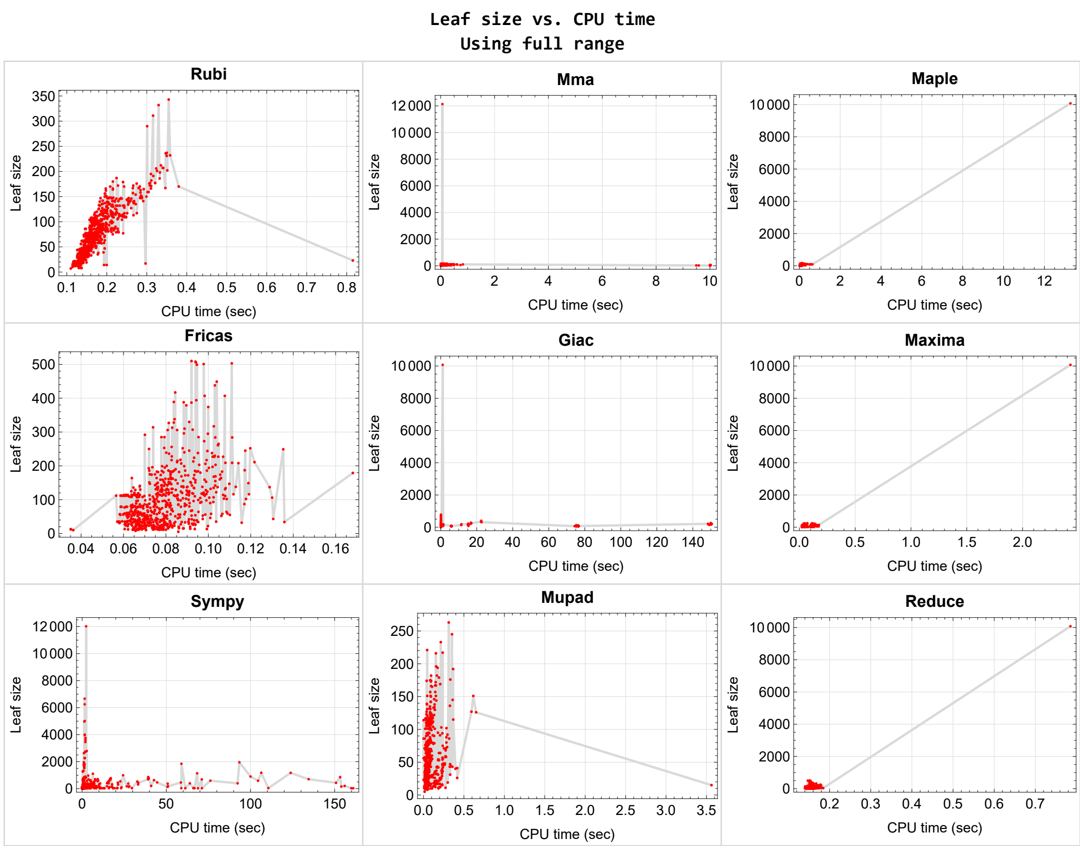


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {691, 692, 693, 694, 695, 696, 697, 706, 707, 708, 709, 710, 711, 720, 721, 722, 723, 724, 725, 726, 734, 735, 736, 737, 738, 739, 740, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 765, 767, 769, 770}

Mathematica {}

Maple $\{5, 527, 763, 765, 766, 790, 795\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

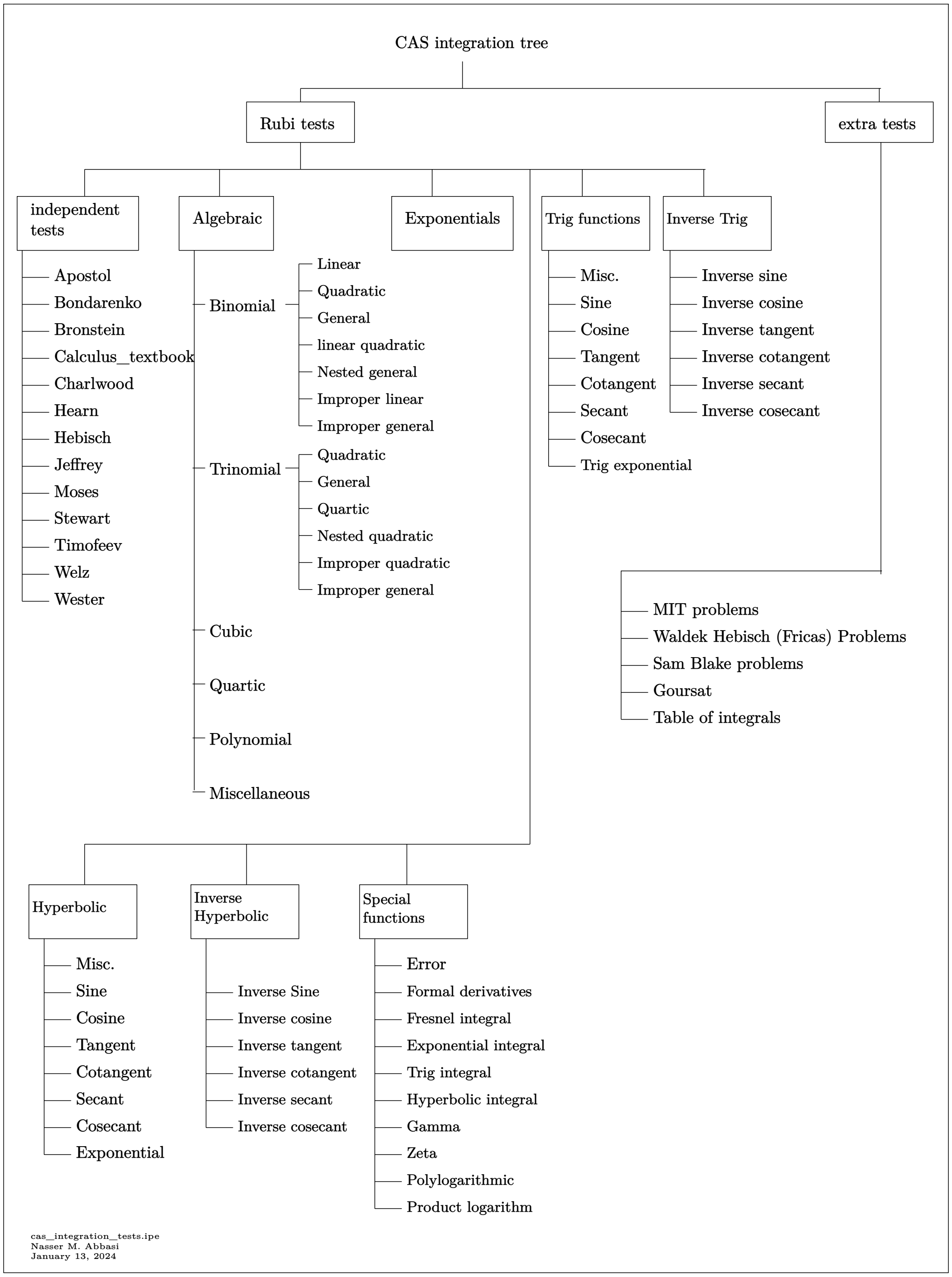
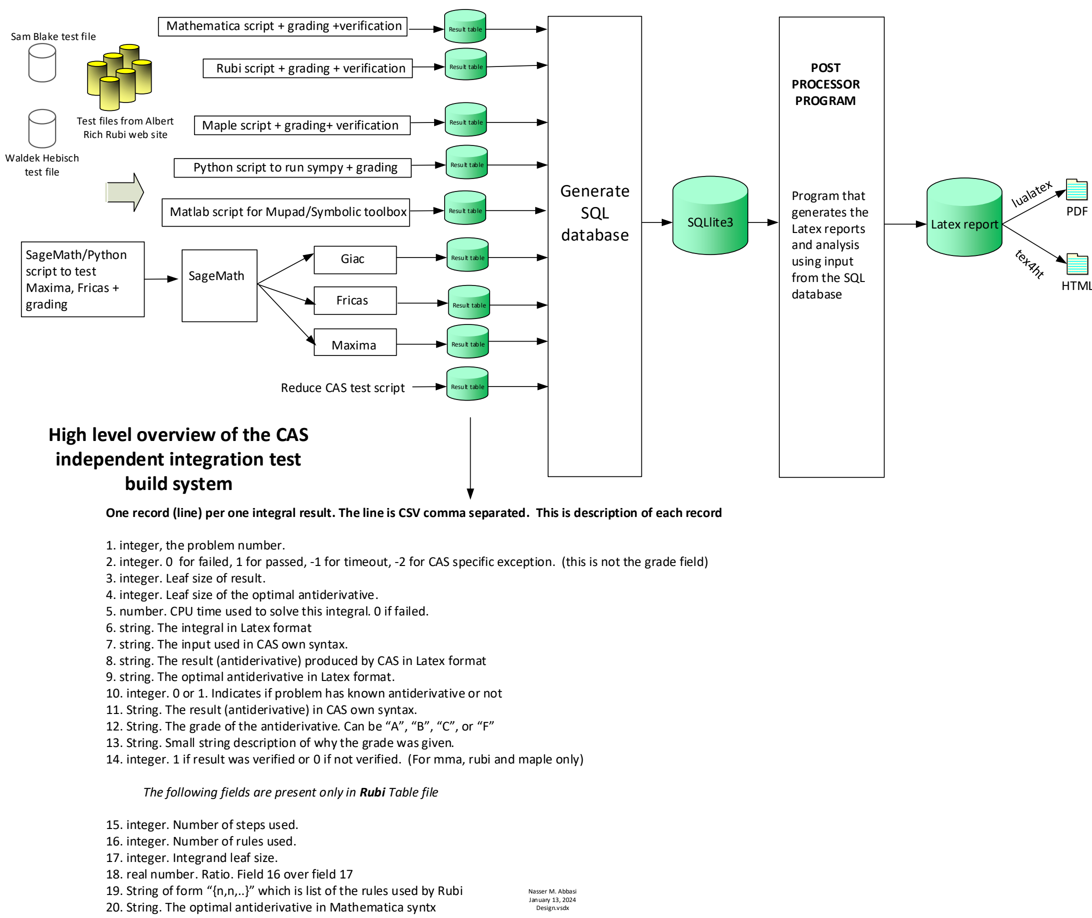


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 675, 676, 679, 680, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 764, 766, 768, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832 }

B grade { 579, 580, 673, 674, 677, 678, 681, 763, 765, 767, 769, 770 }

C grade { 798 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 522, 523, 524, 525, 526, 527, 528, 529, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 576, 577, 578, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 682, 683, 684, 685, 686, 687, 688, 689, 690, 698, 699, 700, 701, 702, 703, 704, 705, 712, 713, 714, 715, 716, 717, 718, 719, 727, 728, 729, 730, 731, 732, 733, 741, 742, 743, 744, 745, 746, 747, 748, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832 }

B grade { 31, 40, 48, 63, 73, 74, 90, 91, 104, 105, 106, 115, 171, 185, 186, 202, 203, 520, 521, 530, 575 }

C grade { 579, 580, 581, 582, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 691, 692, 693, 694, 695, 696, 697, 706, 707, 708, 709, 710, 711, 720, 721, 722, 723, 724, 725, 726, 734, 735, 736, 737, 738, 739, 740, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 488, 489, 490, 491, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 563, 564, 565, 566, 569, 570, 571, 572, 575, 576, 577, 578, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 681, 687, 688, 689, 690, 702, 703, 704, 705, 716, 717, 718, 719, 730, 731, 732, 733, 744, 745, 746, 747, 748, 767, 768, 771, 772, 773, 774, 788, 789, 793, 794, 797, 799, 800, 801, 804, 805, 806, 807, 818, 819, 820, 824, 827, 828, 829, 830, 831, 832 }

B grade { 31, 40, 48, 63, 73, 74, 90, 91, 104, 105, 106, 115, 171, 185, 186, 202, 203, 480, 486, 492, 555, 561, 567, 763, 798 }

C grade { 527, 528, 573, 574, 765, 766, 769, 770, 790, 791, 795, 796 }

F normal fail { 487, 493, 562, 568, 579, 580, 581, 582, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 682, 683, 684, 685, 686, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 764, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 792, 802, 803, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 821, 822, 823, 825, 826 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 163, 164, 165, 166, 167, 168, 169, 170, 181, 182, 189, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 331, 332, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 522, 523, 524, 525, 527, 528, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 576, 577, 578, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 631, 632, 633, 634, 635, 636, 637, 638, 639, 687, 688, 689, 690, 702, 703, 704, 705, 716, 717, 718, 719, 730, 731, 732, 733, 744, 745, 746, 747, 748, 772, 773, 774, 797, 798, 799, 804, 805, 806, 807, 818, 819, 820, 827, 828, 829 }

B grade { 26, 31, 40, 41, 48, 63, 64, 73, 74, 90, 91, 92, 104, 105, 106, 145, 158, 160, 161, 162, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 202, 203, 327, 328, 329, 330, 333, 334, 335, 336, 337, 338, 340, 352, 360, 372, 373, 374, 413, 441, 452, 494, 515, 521, 526, 529, 530, 569, 575, 600, 622, 623, 630, 771 }

C grade { 682, 683, 684, 685, 686, 698, 699, 700, 701, 712, 713, 714, 715, 727, 728, 729, 741, 742, 743 }

F normal fail { 579, 580, 581, 582, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 691, 692, 693, 694, 695, 696, 697, 706, 707, 708, 709, 710, 711, 720, 721, 722, 723, 724, 725, 726, 734, 735, 736, 737, 738, 739, 740, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 800, 801, 802, 803, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 821, 822, 823, 824, 825, 826, 830, 831, 832 }

F(-1) timedout fail { }

F(-2) exception fail { 115 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 430, 431, 432, 433, 434, 439, 440, 441, 442, 443, 444, 445, 449, 450, 451, 452, 453, 454, 455, 460, 461, 462, 463, 464, 469, 470, 475, 476, 477, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 522, 523, 524, 526, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 576, 577, 578, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 682, 683, 684, 685, 686, 687, 688, 689, 690, 698, 699, 700, 701, 702, 703, 704, 705, 712, 713, 714, 715, 716, 717, 718, 719, 727, 728, 729, 730, 731, 732, 733, 741, 742, 743, 744, 745, 746, 747, 748, 771, 772, 773, 774, 797, 798, 799, 804, 805, 806, 807, 827 }

B grade { 26, 31, 40, 41, 48, 63, 73, 74, 90, 91, 104, 105, 106, 115, 145, 158, 171, 172, 173, 174, 175, 185, 186, 187, 188, 190, 191, 192, 202, 203, 426, 427, 428, 429, 435, 436, 437, 438, 446, 447, 448, 456, 457, 458, 459, 465, 466, 467, 468, 471, 472, 473, 474, 478, 479, 480, 498, 499, 500, 501, 519, 520, 521, 525, 527, 529, 573, 574, 575 }

C grade { }

F normal fail { 579, 580, 581, 582, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 691, 692, 693, 694, 695, 696, 697, 706, 707, 708, 709, 710, 711, 720, 721, 722, 723, 724, 725, 726, 734, 735, 736, 737, 738, 739, 740, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 800, 801, 802, 803, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 828, 829, 830, 831, 832 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 347, 348, 353, 354, 355, 356, 361, 362, 363, 364, 365, 366, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 430, 432, 433, 434, 438, 439, 440, 442, 443, 444, 448, 449, 450, 451, 453, 454, 455, 462, 463, 464, 468, 469, 470, 474, 475, 476, 477, 478, 479, 480, 482, 483, 484, 485, 491, 498, 499, 500, 503, 504, 505, 506, 519, 520, 522, 523, 524, 525, 526, 527, 529, 530, 537, 538, 539, 543, 544, 545, 549, 550, 551, 552, 553, 554, 557, 558, 559, 560, 566, 573, 574, 575, 586, 587, 588, 589, 593, 594, 595, 596, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 774, 799, 806, 807 }

B grade { 31, 40, 48, 63, 73, 74, 90, 91, 104, 105, 106, 115, 171, 185, 186, 202, 203, 341, 342, 343, 349, 350, 351, 352, 357, 358, 359, 360, 367, 368, 369, 370, 371, 372, 373, 374, 426, 427, 428, 429, 431, 441, 452, 456, 457, 458, 459, 460, 461, 465, 466, 467, 471, 472, 473, 481, 486, 487, 488, 489, 490, 492, 493, 494, 495, 496, 497, 501, 502, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 521, 531, 532, 533, 534, 535, 536, 555, 556, 561, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 576, 577, 578, 583, 584, 585, 590, 591, 592, 597, 598, 599, 600, 771, 772, 773, 804, 805 }

C grade { 528 }

F normal fail { 579, 580, 581, 582, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 800, 801, 802, 803, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832 }

F(-1) timedout fail { 435, 436, 437, 445, 446, 447, 540, 541, 542, 546, 547, 548 }

F(-2) exception fail { 727, 728, 729, 749, 750, 751 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 428, 429, 431, 432, 433, 434, 441, 442, 443, 444, 452, 453, 454, 455, 458, 459, 461, 462, 463, 464, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 494, 495, 496, 497, 500, 501, 502, 503, 504, 505, 509, 510, 511, 512, 515, 516, 517, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 533, 534, 536, 537, 538, 539, 554, 555, 556, 557, 558, 559, 563, 564, 565, 566, 569, 570, 571, 572, 574, 575, 576, 577, 578, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 687, 688, 689, 690, 716, 717, 718, 719, 744, 745, 746, 747, 748, 768, 771, 772, 773, 774, 797, 799, 804, 805, 806, 807, 818, 819, 820, 827, 828, 829 }

C grade { }

F normal fail { }

F(-1) timedout fail { 426, 427, 430, 435, 436, 437, 438, 439, 440, 445, 446, 447, 448, 449, 450, 451, 456, 457, 460, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 485, 486, 487, 492, 493, 498, 499, 506, 507, 508, 513, 514, 519, 531, 532, 535, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 560, 561, 562, 567, 568, 573, 579, 580, 581, 582, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 769, 770, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 798, 800, 801, 802, 803, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 821, 822, 823, 824, 825, 826, 830, 831, 832 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 177, 178, 179, 180, 181, 182, 183, 184, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 250, 251, 252, 253, 259, 273, 279, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 309, 310, 311, 317, 318, 319, 320, 321, 322, 323, 324, 344, 346, 347, 348, 352, 353, 354, 355, 356, 360, 361, 362, 363, 364, 365, 366, 367, 374, 375, 376, 377, 378, 379, 380, 381, 396, 397, 398, 399, 400, 404, 405, 407, 408, 413, 426, 427, 428, 429, 430, 432, 435, 436, 437, 438, 439, 440, 445, 446, 447, 448, 449, 450, 451, 456, 457, 458, 459, 460, 462, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 485, 486, 487, 488, 489, 498, 499, 500, 501, 502, 503, 506, 507, 508, 509, 510, 515, 582, 586, 593, 600, 607, 614, 621, 627, 628, 702, 703, 716, 717, 730, 731, 744, 745, 799, 807, 820 }

B grade { 13, 26, 31, 40, 41, 48, 49, 62, 63, 64, 73, 74, 89, 90, 91, 92, 104, 105, 106, 115, 145, 146, 158, 160, 171, 172, 173, 174, 175, 176, 185, 186, 187, 188, 189, 190, 191, 192, 193, 202, 203, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 338, 341, 342, 343, 345, 349, 350, 351, 357, 358, 359, 368, 369, 370, 371, 372, 373, 392, 393, 394, 395, 401, 402, 403, 406, 409, 410, 411, 412, 414, 415, 416, 431, 433, 434, 441, 442, 443, 444, 452, 453, 454, 455, 461, 463, 464, 483, 484, 490, 491, 492, 493, 494, 495, 496, 497, 504, 505, 511, 512, 513, 514, 516, 517, 518, 583, 584, 585, 590, 591, 592, 597, 598, 599, 604, 605, 606, 618, 619, 620, 625, 626, 687, 688, 704, 718, 732, 746, 771, 772, 773, 774, 804, 805, 806, 808, 809, 810, 818, 819, 827, 828, 829 }

C grade { 243, 304, 305, 306, 307, 308, 312, 313, 314, 315, 316, 383, 384, 385, 386, 387, 388, 389, 390, 391, 417, 418, 419, 420, 421, 422, 423, 425, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 587, 588, 589, 594, 595, 596, 601, 602, 603, 608, 609, 610, 611, 612, 613, 615, 616, 617, 622, 623, 624, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 720, 721, 722, 723, 724, 725, 727, 728, 729, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 749, 750, 751, 752, 753, 754, 755, 757, 758, 759, 760, 761, 762, 763, 765, 767, 769, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 800, 801, 802, 803, 811, 812, 813, 814, 815, 816, 817, 821, 822, 823, 824, 825, 826, 830, 832 }

F normal fail { 764, 766, 768, 770 }

F(-1) timedout fail { 249, 333, 334, 339, 340, 382, 424, 689, 690, 705, 719, 726, 733, 747, 748, 756, 831 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 687, 688, 689, 690, 710, 711, 724, 725, 726, 738, 739, 740, 753, 754, 755, 756, 771, 772, 773, 774, 797, 798, 799, 804, 805, 806, 807, 828, 829 }

C grade { }

F normal fail { 579, 580, 581, 582, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 800, 801, 802, 803, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 830, 831, 832 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.297	0.029	0.023	0.045	0.076	0.015	0.129	0.162	0.118

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.131	0.001	0.023	0.035	0.067	0.018	0.126	0.153	0.016

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.129	0.001	0.018	0.034	0.035	0.016	0.119	0.152	0.014

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	10	10	8	10	10	10
N.S.	1	1.00	0.86	0.79	0.71	0.71	0.57	0.71	0.71	0.71
time (sec)	N/A	0.115	0.000	0.013	0.034	0.036	0.016	0.121	0.153	0.032

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	9	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00	1.00
time (sec)	N/A	0.137	0.001	0.023	0.025	0.066	0.028	0.127	0.156	0.047

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	7	12	13	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.64	1.09	1.18	1.00
time (sec)	N/A	0.130	0.006	0.026	0.042	0.065	0.036	0.120	0.154	0.038

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	15	12	11	11	12	11	13	11
N.S.	1	1.13	1.00	0.80	0.73	0.73	0.80	0.73	0.87	0.73
time (sec)	N/A	0.119	0.001	0.027	0.041	0.065	0.045	0.118	0.156	0.028

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	14	13	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.82	0.76	0.76	0.76
time (sec)	N/A	0.129	0.002	0.026	0.030	0.061	0.051	0.114	0.153	0.015

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	14	13	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.82	0.76	0.76	0.76
time (sec)	N/A	0.128	0.001	0.028	0.029	0.067	0.051	0.121	0.157	0.015

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.141	0.001	0.032	0.030	0.070	0.018	0.123	0.162	0.040

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.140	0.001	0.029	0.043	0.071	0.025	0.115	0.155	0.018

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.138	0.001	0.025	0.041	0.070	0.018	0.119	0.151	0.018

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	21	20
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.50	1.43
time (sec)	N/A	0.135	0.001	0.022	0.027	0.068	0.015	0.121	0.152	0.018

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	20	21	20	20
N.S.	1	1.00	1.00	0.95	0.91	0.91	0.91	0.95	0.91	0.91
time (sec)	N/A	0.136	0.000	0.030	0.046	0.078	0.034	0.114	0.156	0.032

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	24	17	21	24	20
N.S.	1	1.00	1.00	1.05	1.00	1.20	0.85	1.05	1.20	1.00
time (sec)	N/A	0.136	0.000	0.033	0.062	0.066	0.048	0.114	0.153	0.034

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	21	26	22	22	26	23
N.S.	1	1.00	1.00	0.96	0.88	1.08	0.92	0.92	1.08	0.96
time (sec)	N/A	0.140	0.003	0.035	0.047	0.067	0.061	0.121	0.156	0.040

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	17	26	23	22	22	24	22	24	22
N.S.	1	0.65	1.00	0.88	0.85	0.85	0.92	0.85	0.92	0.85
time (sec)	N/A	0.119	0.004	0.033	0.038	0.062	0.088	0.121	0.149	0.020

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.138	0.002	0.033	0.032	0.061	0.070	0.130	0.152	0.020

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.139	0.004	0.035	0.026	0.068	0.086	0.126	0.156	0.022

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.138	0.003	0.036	0.023	0.066	0.084	0.115	0.152	0.020

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.152	0.005	0.036	0.029	0.070	0.088	0.119	0.153	0.021

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.151	0.002	0.030	0.025	0.057	0.018	0.123	0.151	0.024

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.149	0.001	0.030	0.031	0.057	0.017	0.122	0.151	0.023

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.81
time (sec)	N/A	0.148	0.001	0.031	0.037	0.058	0.018	0.122	0.150	0.028

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	35	34	34	36	34	35	34
N.S.	1	1.00	1.33	1.17	1.13	1.13	1.20	1.13	1.17	1.13
time (sec)	N/A	0.138	0.001	0.026	0.047	0.058	0.018	0.118	0.148	0.022

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	12	32	31
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	0.86	2.29	2.21
time (sec)	N/A	0.115	0.001	0.021	0.046	0.058	0.018	0.120	0.144	0.023

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	34	32	31	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.97	0.91	0.89	0.89
time (sec)	N/A	0.144	0.002	0.032	0.053	0.058	0.037	0.117	0.160	0.021

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	31	33	36	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.91	0.97	1.06	0.94
time (sec)	N/A	0.145	0.003	0.035	0.036	0.059	0.049	0.123	0.184	0.020

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	30	37	32	31	37	32
N.S.	1	1.00	1.00	0.97	0.91	1.12	0.97	0.94	1.12	0.97
time (sec)	N/A	0.161	0.004	0.033	0.109	0.062	0.074	0.124	0.177	0.033

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	37	36	35	37	34
N.S.	1	1.00	1.00	0.92	0.92	1.00	0.97	0.95	1.00	0.92
time (sec)	N/A	0.149	0.003	0.033	0.095	0.066	0.094	0.117	0.160	0.042

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	39	34	33	33	36	33	35	33
N.S.	1	1.00	2.29	2.00	1.94	1.94	2.12	1.94	2.06	1.94
time (sec)	N/A	0.118	0.003	0.035	0.024	0.062	0.089	0.126	0.172	0.017

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	35	35	35	37	35	35	34
N.S.	1	1.00	1.14	0.97	0.97	0.97	1.03	0.97	0.97	0.94
time (sec)	N/A	0.131	0.004	0.034	0.051	0.069	0.099	0.125	0.169	0.016

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.151	0.003	0.034	0.050	0.058	0.103	0.133	0.152	0.015

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.148	0.003	0.036	0.042	0.061	0.126	0.126	0.146	0.018

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	57	56
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.86	0.85
time (sec)	N/A	0.172	0.002	0.036	0.057	0.086	0.018	0.121	0.152	0.015

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83	0.83
time (sec)	N/A	0.181	0.002	0.032	0.069	0.064	0.022	0.117	0.152	0.014

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83	0.83
time (sec)	N/A	0.167	0.002	0.035	0.123	0.062	0.022	0.127	0.150	0.014

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	57	56	56	63	56	57	56
N.S.	1	1.00	1.03	0.89	0.88	0.88	0.98	0.88	0.89	0.88
time (sec)	N/A	0.170	0.002	0.032	0.060	0.060	0.021	0.120	0.147	0.014

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	67	58	57	57	65	57	57	57
N.S.	1	1.00	1.43	1.23	1.21	1.21	1.38	1.21	1.21	1.21
time (sec)	N/A	0.157	0.002	0.033	0.052	0.058	0.021	0.134	0.159	0.016

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	67	58	57	57	65	57	57	57
N.S.	1	1.00	2.23	1.93	1.90	1.90	2.17	1.90	1.90	1.90
time (sec)	N/A	0.140	0.001	0.030	0.065	0.076	0.026	0.120	0.159	0.014

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	53	53	60	12	54	53
N.S.	1	1.00	1.00	0.93	3.79	3.79	4.29	0.86	3.86	3.79
time (sec)	N/A	0.114	0.001	0.027	0.095	0.061	0.029	0.124	0.156	0.014

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	53	53	60	54	53	53
N.S.	1	1.00	1.00	0.92	0.90	0.90	1.02	0.92	0.90	0.90
time (sec)	N/A	0.160	0.003	0.036	0.067	0.069	0.048	0.127	0.157	0.017

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	55	54	59	56	55	59	54
N.S.	1	1.00	1.00	0.95	0.93	1.02	0.97	0.95	1.02	0.93
time (sec)	N/A	0.171	0.003	0.038	0.093	0.069	0.055	0.127	0.155	0.022

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	53	59	60	54	59	55
N.S.	1	1.00	1.00	0.92	0.88	0.98	1.00	0.90	0.98	0.92
time (sec)	N/A	0.167	0.003	0.037	0.038	0.070	0.077	0.117	0.150	0.016

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	55	59	60	56	59	55
N.S.	1	1.00	1.00	0.92	0.92	0.98	1.00	0.93	0.98	0.92
time (sec)	N/A	0.164	0.003	0.038	0.032	0.061	0.118	0.114	0.156	0.027

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	54	54	59	58	55	59	54
N.S.	1	1.00	1.00	0.95	0.95	1.04	1.02	0.96	1.04	0.95
time (sec)	N/A	0.164	0.004	0.039	0.035	0.061	0.127	0.125	0.155	0.025

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	56	59	60	57	59	56
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.98	0.93	0.97	0.92
time (sec)	N/A	0.165	0.003	0.036	0.031	0.109	0.155	0.132	0.155	0.025

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	65	56	55	55	60	55	57	55
N.S.	1	1.00	3.82	3.29	3.24	3.24	3.53	3.24	3.35	3.24
time (sec)	N/A	0.116	0.003	0.039	0.031	0.059	0.165	0.122	0.154	0.041

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	67	57	57	57	61	57	57	57
N.S.	1	1.00	1.86	1.58	1.58	1.58	1.69	1.58	1.58	1.58
time (sec)	N/A	0.128	0.003	0.039	0.031	0.082	0.174	0.128	0.158	0.025

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	67	57	57	57	61	57	57	57
N.S.	1	1.11	1.20	1.02	1.02	1.02	1.09	1.02	1.02	1.02
time (sec)	N/A	0.141	0.003	0.041	0.031	0.069	0.202	0.127	0.154	0.022

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	57	57	57	61	57	57	56
N.S.	1	1.00	1.00	0.85	0.85	0.85	0.91	0.85	0.85	0.84
time (sec)	N/A	0.176	0.004	0.042	0.024	0.065	0.192	0.119	0.158	0.039

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	57	57	57	61	57	57	57
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.88	0.83	0.83	0.83
time (sec)	N/A	0.166	0.003	0.040	0.024	0.062	0.203	0.124	0.155	0.023

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	57	57	57	61	57	57	57
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.88	0.83	0.83	0.83
time (sec)	N/A	0.164	0.003	0.042	0.038	0.062	0.210	0.122	0.158	0.041

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	57	57	57	61	57	57	56
N.S.	1	1.00	1.00	0.85	0.85	0.85	0.91	0.85	0.85	0.84
time (sec)	N/A	0.167	0.003	0.043	0.025	0.058	0.228	0.125	0.157	0.022

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	57	57	57	61	57	57	56
N.S.	1	1.00	1.00	0.85	0.85	0.85	0.91	0.85	0.85	0.84
time (sec)	N/A	0.165	0.003	0.044	0.031	0.060	0.232	0.125	0.158	0.039

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	94	79	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83	0.83
time (sec)	N/A	0.191	0.002	0.041	0.024	0.074	0.022	0.117	0.148	0.134

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	92	79	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.97	0.83	0.83	0.83
time (sec)	N/A	0.201	0.002	0.036	0.032	0.071	0.025	0.121	0.146	0.067

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	94	79	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83	0.83
time (sec)	N/A	0.187	0.002	0.036	0.026	0.074	0.020	0.117	0.145	0.058

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	79	78	78	90	78	79	78
N.S.	1	1.00	0.96	0.82	0.81	0.81	0.94	0.81	0.82	0.81
time (sec)	N/A	0.198	0.002	0.036	0.028	0.082	0.024	0.127	0.163	0.057

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	93	80	79	79	92	79	79	79
N.S.	1	1.00	1.15	0.99	0.98	0.98	1.14	0.98	0.98	0.98
time (sec)	N/A	0.185	0.002	0.036	0.037	0.067	0.023	0.127	0.154	0.036

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	80	79	79	92	79	79	79
N.S.	1	1.00	1.45	1.25	1.23	1.23	1.44	1.23	1.23	1.23
time (sec)	N/A	0.170	0.002	0.034	0.033	0.065	0.026	0.119	0.154	0.040

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	93	80	79	79	92	79	79	31
N.S.	1	1.00	1.98	1.70	1.68	1.68	1.96	1.68	1.68	0.66
time (sec)	N/A	0.157	0.002	0.036	0.028	0.067	0.025	0.117	0.155	0.039

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	91	80	79	79	90	79	79	25
N.S.	1	1.00	3.03	2.67	2.63	2.63	3.00	2.63	2.63	0.83
time (sec)	N/A	0.145	0.002	0.033	0.026	0.059	0.027	0.120	0.155	0.053

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	75	83	12	76	75
N.S.	1	1.00	1.00	0.93	0.86	5.36	5.93	0.86	5.43	5.36
time (sec)	N/A	0.120	0.001	0.028	0.024	0.066	0.023	0.116	0.156	0.053

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	76	75	75	88	76	75	75
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.01	0.87	0.86	0.86
time (sec)	N/A	0.179	0.002	0.036	0.036	0.078	0.058	0.128	0.151	0.036

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	76	81	85	77	81	76
N.S.	1	1.00	1.00	0.90	0.88	0.94	0.99	0.90	0.94	0.88
time (sec)	N/A	0.183	0.003	0.042	0.028	0.069	0.066	0.132	0.148	0.032

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	75	81	85	76	81	77
N.S.	1	1.00	1.00	0.92	0.89	0.96	1.01	0.90	0.96	0.92
time (sec)	N/A	0.185	0.003	0.041	0.041	0.064	0.085	0.124	0.159	0.030

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	77	81	87	78	81	77
N.S.	1	1.00	1.00	0.90	0.90	0.94	1.01	0.91	0.94	0.90
time (sec)	N/A	0.184	0.003	0.042	0.029	0.065	0.123	0.120	0.157	0.051

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	77	81	85	78	81	77
N.S.	1	1.00	1.00	0.90	0.90	0.94	0.99	0.91	0.94	0.90
time (sec)	N/A	0.184	0.003	0.045	0.023	0.089	0.133	0.120	0.155	0.054

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	77	81	83	78	81	77
N.S.	1	1.00	1.00	0.92	0.92	0.96	0.99	0.93	0.96	0.92
time (sec)	N/A	0.201	0.004	0.042	0.032	0.063	0.168	0.128	0.157	0.033

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	76	81	82	77	81	81
N.S.	1	1.00	1.00	0.89	0.89	0.95	0.96	0.91	0.95	0.95
time (sec)	N/A	0.180	0.004	0.043	0.028	0.069	0.200	0.122	0.148	0.071

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	78	78	81	83	79	81	78
N.S.	1	1.00	1.00	0.88	0.88	0.91	0.93	0.89	0.91	0.88
time (sec)	N/A	0.185	0.003	0.042	0.029	0.069	0.235	0.126	0.148	0.038

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	87	78	77	77	83	77	79	77
N.S.	1	1.00	5.12	4.59	4.53	4.53	4.88	4.53	4.65	4.53
time (sec)	N/A	0.118	0.003	0.042	0.029	0.060	0.252	0.128	0.150	0.065

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	91	79	79	79	85	79	79	23
N.S.	1	1.00	2.53	2.19	2.19	2.19	2.36	2.19	2.19	0.64
time (sec)	N/A	0.131	0.003	0.041	0.027	0.059	0.271	0.119	0.157	0.055

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	93	79	79	79	85	79	79	79
N.S.	1	1.11	1.66	1.41	1.41	1.41	1.52	1.41	1.41	1.41
time (sec)	N/A	0.144	0.003	0.043	0.030	0.087	0.298	0.113	0.156	0.042

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	93	79	79	79	85	79	79	79
N.S.	1	1.16	1.22	1.04	1.04	1.04	1.12	1.04	1.04	1.04
time (sec)	N/A	0.157	0.003	0.043	0.034	0.080	0.298	0.120	0.157	0.040

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	114	93	79	79	79	85	79	79	79
N.S.	1	1.19	0.97	0.82	0.82	0.82	0.89	0.82	0.82	0.82
time (sec)	N/A	0.183	0.003	0.045	0.028	0.067	0.310	0.119	0.157	0.040

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	79	79	79	85	79	79	78
N.S.	1	1.00	1.00	0.85	0.85	0.85	0.91	0.85	0.85	0.84
time (sec)	N/A	0.187	0.004	0.043	0.028	0.071	0.345	0.117	0.151	0.063

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	79	79	79	85	79	79	79
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.89	0.83	0.83	0.83
time (sec)	N/A	0.186	0.003	0.044	0.030	0.061	0.342	0.116	0.161	0.039

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	79	79	79	85	79	79	79
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.89	0.83	0.83	0.83
time (sec)	N/A	0.186	0.003	0.046	0.030	0.059	0.344	0.112	0.156	0.071

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	133	112	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	0.85	0.85	0.85
time (sec)	N/A	0.233	0.002	0.046	0.025	0.061	0.025	0.124	0.152	0.090

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	131	112	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85	0.85
time (sec)	N/A	0.223	0.002	0.043	0.023	0.085	0.027	0.122	0.154	0.075

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	133	112	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	0.85	0.85	0.85
time (sec)	N/A	0.244	0.002	0.043	0.024	0.098	0.025	0.126	0.159	0.051

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	112	111	111	126	111	112	111
N.S.	1	1.00	0.85	0.76	0.76	0.76	0.86	0.76	0.76	0.76
time (sec)	N/A	0.235	0.002	0.040	0.025	0.070	0.031	0.122	0.147	0.075

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	130	113	112	112	131	112	112	112
N.S.	1	1.00	0.98	0.86	0.85	0.85	0.99	0.85	0.85	0.85
time (sec)	N/A	0.227	0.002	0.041	0.032	0.061	0.031	0.121	0.154	0.071

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	126	113	112	112	128	112	112	112
N.S.	1	1.00	1.12	1.01	1.00	1.00	1.14	1.00	1.00	1.00
time (sec)	N/A	0.212	0.002	0.040	0.039	0.080	0.027	0.123	0.156	0.050

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	132	113	112	112	133	112	112	112
N.S.	1	1.00	1.35	1.15	1.14	1.14	1.36	1.14	1.14	1.14
time (sec)	N/A	0.198	0.002	0.040	0.032	0.078	0.030	0.124	0.150	0.048

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	130	113	112	112	131	112	112	112
N.S.	1	1.00	1.60	1.40	1.38	1.38	1.62	1.38	1.38	1.38
time (sec)	N/A	0.200	0.002	0.040	0.025	0.057	0.028	0.126	0.157	0.076

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	128	113	112	112	129	112	112	112
N.S.	1	1.00	2.00	1.77	1.75	1.75	2.02	1.75	1.75	1.75
time (sec)	N/A	0.176	0.002	0.040	0.026	0.059	0.034	0.117	0.153	0.056

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	126	113	112	112	128	112	112	31
N.S.	1	1.00	2.68	2.40	2.38	2.38	2.72	2.38	2.38	0.66
time (sec)	N/A	0.160	0.002	0.039	0.028	0.059	0.031	0.124	0.156	0.062

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	128	113	112	112	129	112	112	25
N.S.	1	1.00	4.27	3.77	3.73	3.73	4.30	3.73	3.73	0.83
time (sec)	N/A	0.139	0.002	0.039	0.037	0.059	0.031	0.128	0.159	0.033

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	108	114	12	109	108
N.S.	1	1.00	1.00	0.93	0.86	7.71	8.14	0.86	7.79	7.71
time (sec)	N/A	0.116	0.001	0.033	0.027	0.064	0.034	0.126	0.156	0.066

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	109	108	108	126	109	108	108
N.S.	1	1.00	1.00	0.89	0.89	0.89	1.03	0.89	0.89	0.89
time (sec)	N/A	0.210	0.003	0.099	0.024	0.064	0.065	0.124	0.159	0.072

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	109	114	117	110	114	109
N.S.	1	1.00	1.00	0.96	0.95	0.99	1.02	0.96	0.99	0.95
time (sec)	N/A	0.210	0.006	0.050	0.033	0.068	0.074	0.123	0.157	0.046

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	108	114	122	109	114	110
N.S.	1	1.00	1.00	0.92	0.91	0.96	1.03	0.92	0.96	0.92
time (sec)	N/A	0.224	0.003	0.049	0.030	0.066	0.095	0.122	0.165	0.040

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	108	114	119	109	114	110
N.S.	1	1.00	1.00	0.96	0.94	0.99	1.03	0.95	0.99	0.96
time (sec)	N/A	0.219	0.006	0.051	0.028	0.062	0.125	0.122	0.150	0.062

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	121	111	114	110
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.02	0.93	0.96	0.92
time (sec)	N/A	0.214	0.006	0.052	0.029	0.060	0.151	0.126	0.161	0.061

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	110	110	114	121	111	114	110
N.S.	1	1.00	1.00	0.94	0.94	0.97	1.03	0.95	0.97	0.94
time (sec)	N/A	0.214	0.006	0.049	0.028	0.061	0.187	0.120	0.156	0.054

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	122	111	114	110
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.03	0.93	0.96	0.92
time (sec)	N/A	0.212	0.004	0.052	0.033	0.065	0.226	0.118	0.151	0.035

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	110	114	119	111	114	110
N.S.	1	1.00	1.00	0.96	0.96	0.99	1.03	0.97	0.99	0.96
time (sec)	N/A	0.231	0.007	0.049	0.030	0.061	0.269	0.117	0.158	0.034

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	119	111	114	110
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.00	0.93	0.96	0.92
time (sec)	N/A	0.210	0.003	0.049	0.046	0.064	0.299	0.121	0.160	0.059

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	109	114	117	110	114	114
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	0.96	1.00	1.00
time (sec)	N/A	0.214	0.005	0.049	0.030	0.066	0.343	0.121	0.158	0.067

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	111	111	114	119	112	114	111
N.S.	1	1.00	1.00	0.90	0.90	0.92	0.96	0.90	0.92	0.90
time (sec)	N/A	0.224	0.003	0.047	0.030	0.079	0.405	0.121	0.158	0.042

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	114	111	110	110	119	110	112	110
N.S.	1	1.00	6.71	6.53	6.47	6.47	7.00	6.47	6.59	6.47
time (sec)	N/A	0.121	0.006	0.049	0.050	0.067	0.412	0.126	0.156	0.061

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	128	112	112	112	121	112	112	23
N.S.	1	1.00	3.56	3.11	3.11	3.11	3.36	3.11	3.11	0.64
time (sec)	N/A	0.132	0.003	0.049	0.044	0.061	0.443	0.125	0.153	0.055

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	126	112	112	112	121	112	112	112
N.S.	1	1.11	2.25	2.00	2.00	2.00	2.16	2.00	2.00	2.00
time (sec)	N/A	0.156	0.006	0.050	0.026	0.060	0.455	0.118	0.153	0.080

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	128	112	112	112	121	112	112	112
N.S.	1	1.16	1.68	1.47	1.47	1.47	1.59	1.47	1.47	1.47
time (sec)	N/A	0.160	0.008	0.047	0.029	0.060	0.489	0.119	0.154	0.058

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	114	130	112	112	112	121	112	112	112
N.S.	1	1.19	1.35	1.17	1.17	1.17	1.26	1.17	1.17	1.17
time (sec)	N/A	0.169	0.006	0.051	0.031	0.062	0.501	0.121	0.152	0.057

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	140	132	112	112	112	121	112	112	112
N.S.	1	1.21	1.14	0.97	0.97	0.97	1.04	0.97	0.97	0.97
time (sec)	N/A	0.182	0.003	0.052	0.026	0.060	0.526	0.129	0.156	0.059

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	166	126	112	112	112	121	112	112	112
N.S.	1	1.22	0.93	0.82	0.82	0.82	0.89	0.82	0.82	0.82
time (sec)	N/A	0.197	0.007	0.052	0.032	0.060	0.530	0.120	0.155	0.091

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	112	112	112	121	112	112	112
N.S.	1	1.00	1.00	0.86	0.86	0.86	0.93	0.86	0.86	0.86
time (sec)	N/A	0.216	0.003	0.054	0.030	0.066	0.583	0.126	0.157	0.070

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	126	112	112	112	121	112	112	111
N.S.	1	1.00	1.00	0.89	0.89	0.89	0.96	0.89	0.89	0.88
time (sec)	N/A	0.221	0.005	0.055	0.035	0.066	0.583	0.127	0.144	0.057

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	12	13	12	12	13	11	11
N.S.	1	1.00	0.93	0.80	0.87	0.80	0.80	0.87	0.73	0.73
time (sec)	N/A	0.116	0.000	0.015	0.028	0.035	0.017	0.120	0.154	0.012

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	17	18	27	22	18	23	16
N.S.	1	1.00	0.95	0.85	0.90	1.35	1.10	0.90	1.15	0.80
time (sec)	N/A	0.121	0.001	0.014	0.031	0.076	0.021	0.120	0.140	0.023

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	12138	10076	10076	0	12024	10076	10075	15
N.S.	1	1.00	527.74	438.09	438.09	0.00	522.78	438.09	438.04	0.65
time (sec)	N/A	0.815	0.063	13.275	2.429	0.000	2.372	1.072	0.785	3.562

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	63	64	63	61	65	63	62
N.S.	1	1.00	1.00	0.90	0.91	0.90	0.87	0.93	0.90	0.89
time (sec)	N/A	0.175	0.003	0.061	0.028	0.077	0.054	0.124	0.152	0.051

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	52	51
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.91	0.89
time (sec)	N/A	0.163	0.003	0.100	0.029	0.070	0.051	0.122	0.152	0.037

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	41	40
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.93	0.91
time (sec)	N/A	0.153	0.003	0.053	0.043	0.065	0.048	0.124	0.155	0.022

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94	0.94
time (sec)	N/A	0.154	0.003	0.052	0.033	0.070	0.044	0.121	0.159	0.051

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	17	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	0.94	1.00
time (sec)	N/A	0.137	0.002	0.046	0.030	0.078	0.044	0.121	0.154	0.048

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	1.00
time (sec)	N/A	0.114	0.000	0.044	0.024	0.071	0.018	0.121	0.153	0.012

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	16	10	20	15	15
N.S.	1	1.00	1.00	0.89	1.00	0.89	0.56	1.11	0.83	0.83
time (sec)	N/A	0.141	0.004	0.056	0.038	0.077	0.059	0.124	0.150	0.058

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	26	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.93	0.89
time (sec)	N/A	0.145	0.003	0.066	0.036	0.079	0.076	0.118	0.153	0.033

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	43	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	1.02	0.90
time (sec)	N/A	0.153	0.003	0.065	0.024	0.073	0.088	0.124	0.153	0.072

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	54	48
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.96	0.86
time (sec)	N/A	0.162	0.004	0.068	0.041	0.071	0.097	0.134	0.158	0.042

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	62	65	56	67	65	60
N.S.	1	1.00	1.00	0.93	0.91	0.96	0.82	0.99	0.96	0.88
time (sec)	N/A	0.173	0.003	0.071	0.028	0.067	0.111	0.124	0.160	0.072

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	78	82	96	78	103	94	83
N.S.	1	1.00	0.95	0.96	1.01	1.19	0.96	1.27	1.16	1.02
time (sec)	N/A	0.203	0.014	0.068	0.029	0.066	0.108	0.122	0.160	0.054

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	67	70	85	71	90	83	72
N.S.	1	1.00	0.92	0.93	0.97	1.18	0.99	1.25	1.15	1.00
time (sec)	N/A	0.195	0.010	0.067	0.024	0.064	0.094	0.134	0.164	0.041

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	79	71	62
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.36	1.22	1.07
time (sec)	N/A	0.193	0.012	0.063	0.029	0.083	0.091	0.126	0.153	0.039

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	66	60	50
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.43	1.30	1.09
time (sec)	N/A	0.162	0.008	0.060	0.029	0.063	0.077	0.119	0.158	0.045

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	50	46	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.52	1.39	1.09
time (sec)	N/A	0.150	0.008	0.059	0.032	0.062	0.067	0.121	0.160	0.050

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	42	33	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.83	1.43	1.00
time (sec)	N/A	0.142	0.004	0.056	0.027	0.061	0.060	0.125	0.157	0.021

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00	1.00
time (sec)	N/A	0.119	0.001	0.049	0.035	0.099	0.052	0.119	0.157	0.044

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	38	44	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.31	1.52	0.90
time (sec)	N/A	0.146	0.006	0.066	0.027	0.093	0.091	0.117	0.156	0.047

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	52	70	45
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.67	1.07
time (sec)	N/A	0.160	0.022	0.072	0.025	0.073	0.126	0.121	0.154	0.071

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	74	86	57
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.28	1.48	0.98
time (sec)	N/A	0.183	0.027	0.075	0.029	0.070	0.127	0.122	0.149	0.068

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	90	97	69
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.30	1.41	1.00
time (sec)	N/A	0.181	0.029	0.079	0.029	0.074	0.149	0.124	0.155	0.070

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	104	108	79
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.24	1.29	0.94
time (sec)	N/A	0.191	0.025	0.079	0.024	0.067	0.156	0.124	0.156	0.045

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	89	90	103	129	109	95	131	91
N.S.	1	1.00	0.90	0.91	1.04	1.30	1.10	0.96	1.32	0.92
time (sec)	N/A	0.215	0.014	0.070	0.031	0.081	0.163	0.130	0.153	0.114

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	79	91	117	92	83	119	78
N.S.	1	1.00	0.90	0.92	1.06	1.36	1.07	0.97	1.38	0.91
time (sec)	N/A	0.202	0.014	0.071	0.038	0.099	0.143	0.120	0.149	0.122

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	68	81	107	85	73	109	67
N.S.	1	1.00	0.87	0.88	1.05	1.39	1.10	0.95	1.42	0.87
time (sec)	N/A	0.186	0.012	0.066	0.038	0.066	0.136	0.116	0.155	0.099

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	57	69	95	70	61	97	54
N.S.	1	1.00	0.86	0.89	1.08	1.48	1.09	0.95	1.52	0.84
time (sec)	N/A	0.187	0.011	0.067	0.049	0.076	0.130	0.128	0.149	0.076

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	45	57	83	58	44	85	43
N.S.	1	1.00	0.80	0.90	1.14	1.66	1.16	0.88	1.70	0.86
time (sec)	N/A	0.166	0.023	0.063	0.033	0.064	0.110	0.116	0.150	0.062

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	36	48	61	46	37	71	46
N.S.	1	1.00	0.80	0.88	1.17	1.49	1.12	0.90	1.73	1.12
time (sec)	N/A	0.154	0.008	0.064	0.023	0.062	0.090	0.115	0.145	0.030

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	19	32	32	32	18	26	32
N.S.	1	1.00	1.18	1.12	1.88	1.88	1.88	1.06	1.53	1.88
time (sec)	N/A	0.118	0.003	0.059	0.046	0.058	0.081	0.123	0.148	0.049

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	24	26	12	23	26
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.64	1.86
time (sec)	N/A	0.117	0.002	0.051	0.035	0.083	0.074	0.123	0.143	0.045

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	41	51	80	46	43	97	43
N.S.	1	1.00	0.86	0.95	1.19	1.86	1.07	1.00	2.26	1.00
time (sec)	N/A	0.157	0.015	0.069	0.043	0.091	0.125	0.118	0.150	0.091

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	56	69	109	66	60	119	63
N.S.	1	1.00	0.93	0.98	1.21	1.91	1.16	1.05	2.09	1.11
time (sec)	N/A	0.170	0.029	0.076	0.024	0.074	0.158	0.123	0.150	0.068

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	72	86	130	78	73	138	79
N.S.	1	1.00	0.89	0.95	1.13	1.71	1.03	0.96	1.82	1.04
time (sec)	N/A	0.190	0.026	0.080	0.031	0.068	0.175	0.115	0.149	0.076

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	83	97	141	92	86	149	91
N.S.	1	1.00	0.89	0.93	1.09	1.58	1.03	0.97	1.67	1.02
time (sec)	N/A	0.199	0.035	0.080	0.034	0.079	0.188	0.120	0.153	0.052

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	94	108	152	102	97	160	101
N.S.	1	1.00	0.93	0.97	1.11	1.57	1.05	1.00	1.65	1.04
time (sec)	N/A	0.212	0.029	0.085	0.030	0.092	0.196	0.121	0.148	0.088

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	101	99	125	162	131	106	166	103
N.S.	1	1.00	0.89	0.87	1.10	1.42	1.15	0.93	1.46	0.90
time (sec)	N/A	0.239	0.018	0.077	0.025	0.080	0.215	0.116	0.152	0.221

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	90	88	114	151	119	95	155	90
N.S.	1	1.00	0.86	0.84	1.09	1.44	1.13	0.90	1.48	0.86
time (sec)	N/A	0.227	0.016	0.078	0.026	0.072	0.206	0.117	0.149	0.131

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	77	102	139	107	83	143	79
N.S.	1	1.00	1.00	0.86	1.13	1.54	1.19	0.92	1.59	0.88
time (sec)	N/A	0.222	0.012	0.074	0.031	0.068	0.187	0.123	0.147	0.118

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	66	91	129	94	72	133	66
N.S.	1	1.00	0.84	0.81	1.12	1.59	1.16	0.89	1.64	0.81
time (sec)	N/A	0.197	0.014	0.069	0.033	0.066	0.173	0.118	0.150	0.051

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	54	79	116	82	55	120	55
N.S.	1	1.00	0.78	0.83	1.22	1.78	1.26	0.85	1.85	0.85
time (sec)	N/A	0.187	0.024	0.068	0.031	0.081	0.165	0.121	0.149	0.108

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	44	47	70	94	70	46	108	45
N.S.	1	1.00	0.76	0.81	1.21	1.62	1.21	0.79	1.86	0.78
time (sec)	N/A	0.179	0.009	0.065	0.040	0.077	0.118	0.122	0.148	0.044

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	30	54	54	56	29	37	56
N.S.	1	1.00	1.82	1.76	3.18	3.18	3.29	1.71	2.18	3.29
time (sec)	N/A	0.119	0.006	0.062	0.024	0.065	0.114	0.119	0.148	0.022

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	19	43	43	44	18	42	44
N.S.	1	1.00	0.67	0.63	1.43	1.43	1.47	0.60	1.40	1.47
time (sec)	N/A	0.147	0.004	0.062	0.032	0.131	0.098	0.123	0.150	0.017

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	35	37	12	34	37
N.S.	1	1.00	1.00	0.93	0.86	2.50	2.64	0.86	2.43	2.64
time (sec)	N/A	0.121	0.001	0.058	0.028	0.082	0.100	0.114	0.153	0.044

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	52	73	124	70	54	145	60
N.S.	1	1.00	0.84	0.91	1.28	2.18	1.23	0.95	2.54	1.05
time (sec)	N/A	0.187	0.016	0.075	0.027	0.087	0.165	0.110	0.150	0.105

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	69	91	153	90	71	171	85
N.S.	1	1.00	0.91	0.99	1.30	2.19	1.29	1.01	2.44	1.21
time (sec)	N/A	0.194	0.031	0.083	0.035	0.079	0.195	0.121	0.148	0.075

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	83	108	174	104	86	190	101
N.S.	1	1.00	0.85	0.89	1.16	1.87	1.12	0.92	2.04	1.09
time (sec)	N/A	0.206	0.030	0.089	0.027	0.073	0.213	0.116	0.159	0.080

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	88	93	117	183	114	93	201	113
N.S.	1	1.00	0.86	0.91	1.15	1.79	1.12	0.91	1.97	1.11
time (sec)	N/A	0.212	0.030	0.094	0.035	0.083	0.228	0.127	0.159	0.064

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	105	130	196	128	108	212	123
N.S.	1	1.00	0.86	0.90	1.11	1.68	1.09	0.92	1.81	1.05
time (sec)	N/A	0.224	0.034	0.097	0.028	0.074	0.240	0.124	0.158	0.098

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	121	180	250	190	128	272	126
N.S.	1	1.00	0.93	0.81	1.20	1.67	1.27	0.85	1.81	0.84
time (sec)	N/A	0.285	0.013	0.097	0.038	0.072	0.431	0.128	0.160	0.650

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	128	110	169	239	180	117	261	115
N.S.	1	1.00	0.92	0.79	1.22	1.72	1.29	0.84	1.88	0.83
time (sec)	N/A	0.268	0.016	0.090	0.035	0.080	0.373	0.121	0.155	0.368

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	99	157	228	165	105	250	102
N.S.	1	1.00	0.81	0.77	1.23	1.78	1.29	0.82	1.95	0.80
time (sec)	N/A	0.244	0.024	0.091	0.040	0.083	0.375	0.121	0.156	0.080

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	87	145	215	153	88	237	91
N.S.	1	1.00	0.88	0.74	1.23	1.82	1.30	0.75	2.01	0.77
time (sec)	N/A	0.226	0.015	0.088	0.033	0.085	0.346	0.117	0.159	0.205

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	77	80	136	193	141	79	225	81
N.S.	1	1.00	0.71	0.73	1.25	1.77	1.29	0.72	2.06	0.74
time (sec)	N/A	0.215	0.013	0.082	0.040	0.072	0.267	0.126	0.158	0.094

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	63	120	120	128	62	70	72
N.S.	1	1.00	3.76	3.71	7.06	7.06	7.53	3.65	4.12	4.24
time (sec)	N/A	0.123	0.007	0.079	0.031	0.065	0.248	0.118	0.157	0.042

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	53	52	109	109	116	51	108	22
N.S.	1	1.00	1.51	1.49	3.11	3.11	3.31	1.46	3.09	0.63
time (sec)	N/A	0.137	0.006	0.079	0.033	0.064	0.218	0.122	0.155	0.039

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	64	42	41	98	98	104	40	97	48
N.S.	1	1.23	0.81	0.79	1.88	1.88	2.00	0.77	1.87	0.92
time (sec)	N/A	0.171	0.006	0.078	0.031	0.070	0.210	0.118	0.154	0.074

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	30	87	87	92	29	86	31
N.S.	1	1.00	0.66	0.64	1.85	1.85	1.96	0.62	1.83	0.66
time (sec)	N/A	0.155	0.005	0.082	0.042	0.117	0.181	0.127	0.158	0.045

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	19	76	76	80	18	75	18
N.S.	1	1.00	0.67	0.63	2.53	2.53	2.67	0.60	2.50	0.60
time (sec)	N/A	0.142	0.004	0.078	0.042	0.073	0.170	0.128	0.157	0.064

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	68	73	12	67	70
N.S.	1	1.00	1.00	0.93	0.86	4.86	5.21	0.86	4.79	5.00
time (sec)	N/A	0.115	0.001	0.069	0.032	0.071	0.184	0.118	0.157	0.071

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	85	139	256	141	87	301	102
N.S.	1	1.00	0.82	0.86	1.40	2.59	1.42	0.88	3.04	1.03
time (sec)	N/A	0.209	0.029	0.100	0.038	0.083	0.286	0.131	0.154	0.284

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	104	157	285	162	104	327	151
N.S.	1	1.00	0.83	0.89	1.34	2.44	1.38	0.89	2.79	1.29
time (sec)	N/A	0.244	0.047	0.108	0.059	0.078	0.317	0.117	0.157	0.088

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	112	116	174	306	175	119	346	167
N.S.	1	1.00	0.78	0.81	1.21	2.12	1.22	0.83	2.40	1.16
time (sec)	N/A	0.250	0.035	0.113	0.042	0.081	0.342	0.125	0.159	0.143

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	127	185	317	187	130	357	179
N.S.	1	1.00	0.78	0.81	1.18	2.02	1.19	0.83	2.27	1.14
time (sec)	N/A	0.265	0.047	0.116	0.040	0.103	0.339	0.125	0.159	0.192

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	161	143	234	338	250	149	378	151
N.S.	1	1.00	0.87	0.77	1.26	1.82	1.34	0.80	2.03	0.81
time (sec)	N/A	0.332	0.023	0.119	0.040	0.084	0.616	0.124	0.157	0.616

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	132	223	327	236	138	367	138
N.S.	1	1.00	0.85	0.75	1.26	1.85	1.33	0.78	2.07	0.78
time (sec)	N/A	0.321	0.014	0.118	0.041	0.081	0.598	0.134	0.151	0.138

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	137	120	211	314	224	121	354	127
N.S.	1	1.00	0.86	0.75	1.33	1.97	1.41	0.76	2.23	0.80
time (sec)	N/A	0.302	0.018	0.111	0.044	0.074	0.571	0.127	0.154	0.594

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	111	113	202	292	212	112	342	117
N.S.	1	1.00	0.72	0.73	1.31	1.90	1.38	0.73	2.22	0.76
time (sec)	N/A	0.272	0.018	0.106	0.040	0.070	0.455	0.119	0.158	0.115

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	97	96	186	186	199	95	103	107
N.S.	1	1.00	5.71	5.65	10.94	10.94	11.71	5.59	6.06	6.29
time (sec)	N/A	0.121	0.010	0.102	0.054	0.072	0.411	0.118	0.155	0.092

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	86	85	175	175	187	84	174	22
N.S.	1	1.00	2.46	2.43	5.00	5.00	5.34	2.40	4.97	0.63
time (sec)	N/A	0.130	0.009	0.099	0.071	0.072	0.382	0.121	0.155	0.049

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	60	75	74	164	164	175	73	163	85
N.S.	1	1.15	1.44	1.42	3.15	3.15	3.37	1.40	3.13	1.63
time (sec)	N/A	0.153	0.009	0.099	0.035	0.064	0.353	0.121	0.157	0.086

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	85	64	63	153	153	163	62	152	71
N.S.	1	1.23	0.93	0.91	2.22	2.22	2.36	0.90	2.20	1.03
time (sec)	N/A	0.174	0.009	0.098	0.036	0.079	0.317	0.129	0.155	0.047

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	53	52	142	142	151	51	141	61
N.S.	1	1.00	0.65	0.64	1.75	1.75	1.86	0.63	1.74	0.75
time (sec)	N/A	0.187	0.008	0.096	0.033	0.081	0.296	0.123	0.155	0.045

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	42	41	131	131	139	40	130	48
N.S.	1	1.00	0.66	0.64	2.05	2.05	2.17	0.62	2.03	0.75
time (sec)	N/A	0.172	0.006	0.098	0.032	0.075	0.275	0.114	0.149	0.074

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	30	120	120	128	29	119	31
N.S.	1	1.00	0.66	0.64	2.55	2.55	2.72	0.62	2.53	0.66
time (sec)	N/A	0.156	0.007	0.094	0.032	0.076	0.260	0.116	0.147	0.048

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	19	109	109	116	18	108	18
N.S.	1	1.00	0.67	0.63	3.63	3.63	3.87	0.60	3.60	0.60
time (sec)	N/A	0.146	0.004	0.098	0.030	0.063	0.263	0.119	0.160	0.065

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	101	109	12	100	103
N.S.	1	1.00	1.00	0.93	0.86	7.21	7.79	0.86	7.14	7.36
time (sec)	N/A	0.116	0.001	0.087	0.034	0.073	0.257	0.121	0.157	0.082

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	127	118	205	388	212	120	457	145
N.S.	1	1.00	0.90	0.84	1.45	2.75	1.50	0.85	3.24	1.03
time (sec)	N/A	0.254	0.054	0.133	0.045	0.088	0.403	0.123	0.150	0.361

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	130	137	223	417	233	137	483	217
N.S.	1	1.00	0.82	0.87	1.41	2.64	1.47	0.87	3.06	1.37
time (sec)	N/A	0.284	0.064	0.138	0.058	0.084	0.437	0.126	0.154	0.237

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	145	149	240	438	246	152	502	233
N.S.	1	1.00	0.76	0.78	1.26	2.29	1.29	0.80	2.63	1.22
time (sec)	N/A	0.316	0.055	0.145	0.046	0.103	0.468	0.125	0.147	0.214

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	156	160	251	449	258	163	513	245
N.S.	1	1.00	0.79	0.81	1.27	2.27	1.30	0.82	2.59	1.24
time (sec)	N/A	0.333	0.062	0.145	0.072	0.104	0.484	0.124	0.152	0.352

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	132	132	136	143	133	136	132
N.S.	1	1.00	1.00	0.94	0.94	0.96	1.01	0.94	0.96	0.94
time (sec)	N/A	0.268	0.007	0.061	0.027	0.065	0.365	0.117	0.155	0.077

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	121	121	125	131	122	125	121
N.S.	1	1.00	1.00	0.92	0.92	0.95	0.99	0.92	0.95	0.92
time (sec)	N/A	0.240	0.004	0.057	0.032	0.064	0.351	0.122	0.152	0.048

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	109	114	117	110	114	114
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	0.96	1.00	1.00
time (sec)	N/A	0.222	0.004	0.042	0.030	0.085	0.354	0.127	0.148	0.001

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	100	100	103	107	101	103	100
N.S.	1	1.00	1.00	0.92	0.92	0.94	0.98	0.93	0.94	0.92
time (sec)	N/A	0.217	0.003	0.053	0.032	0.074	0.326	0.124	0.149	0.077

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	96	89	88	88	95	88	90	88
N.S.	1	1.00	5.65	5.24	5.18	5.18	5.59	5.18	5.29	5.18
time (sec)	N/A	0.126	0.006	0.054	0.025	0.075	0.298	0.123	0.155	0.081

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	91	79	79	79	85	79	79	23
N.S.	1	1.00	2.53	2.19	2.19	2.19	2.36	2.19	2.19	0.64
time (sec)	N/A	0.137	0.002	0.043	0.024	0.065	0.269	0.123	0.152	0.001

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	80	68	68	68	73	68	68	68
N.S.	1	1.11	1.43	1.21	1.21	1.21	1.30	1.21	1.21	1.21
time (sec)	N/A	0.158	0.006	0.046	0.025	0.066	0.249	0.121	0.151	0.058

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	57	57	57	61	57	57	56
N.S.	1	1.00	1.00	0.85	0.85	0.85	0.91	0.85	0.85	0.84
time (sec)	N/A	0.169	0.004	0.039	0.032	0.063	0.204	0.120	0.148	0.001

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	46	46	46	49	46	46	46
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.88	0.82	0.82	0.82
time (sec)	N/A	0.161	0.005	0.046	0.024	0.075	0.166	0.115	0.155	0.021

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.153	0.003	0.042	0.024	0.061	0.133	0.118	0.152	0.019

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.142	0.004	0.039	0.028	0.083	0.107	0.120	0.153	0.020

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	14	13	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.82	0.76	0.76	0.76
time (sec)	N/A	0.130	0.002	0.033	0.023	0.074	0.068	0.123	0.154	0.014

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.71
time (sec)	N/A	0.110	0.000	0.020	0.025	0.086	0.033	0.120	0.143	0.014

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	119	117	120	116	122	120	114
N.S.	1	1.00	1.00	0.89	0.87	0.90	0.87	0.91	0.90	0.85
time (sec)	N/A	0.253	0.004	0.085	0.035	0.091	0.161	0.127	0.158	0.075

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	134	135	141	163	139	180	163	135
N.S.	1	1.00	0.92	0.92	0.97	1.12	0.95	1.23	1.12	0.92
time (sec)	N/A	0.261	0.051	0.099	0.037	0.093	0.224	0.125	0.158	0.080

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	145	149	163	207	163	152	215	157
N.S.	1	1.00	0.89	0.91	1.00	1.27	1.00	0.93	1.32	0.96
time (sec)	N/A	0.285	0.057	0.100	0.031	0.087	0.267	0.129	0.157	0.145

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	13	13	12	15	13	10
N.S.	1	1.00	1.00	0.71	0.76	0.76	0.71	0.88	0.76	0.59
time (sec)	N/A	0.121	0.002	0.043	0.029	0.075	0.039	0.117	0.155	0.106

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	13	13	12	15	13	10
N.S.	1	1.00	1.00	0.71	0.76	0.76	0.71	0.88	0.76	0.59
time (sec)	N/A	0.123	0.003	0.039	0.030	0.094	0.043	0.130	0.151	0.091

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	18	21	20	20	21	18
N.S.	1	1.00	1.00	0.79	0.75	0.88	0.83	0.83	0.88	0.75
time (sec)	N/A	0.148	0.002	0.047	0.035	0.081	0.043	0.118	0.152	0.064

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	23	28	26	25	28	18
N.S.	1	1.00	1.00	0.74	0.74	0.90	0.84	0.81	0.90	0.58
time (sec)	N/A	0.147	0.003	0.048	0.028	0.062	0.049	0.121	0.152	0.023

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	28	28	33	31	30	33	24
N.S.	1	1.00	1.00	0.74	0.74	0.87	0.82	0.79	0.87	0.63
time (sec)	N/A	0.148	0.002	0.050	0.024	0.065	0.054	0.126	0.153	0.053

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	33	33	38	36	35	38	28
N.S.	1	1.00	1.00	0.73	0.73	0.84	0.80	0.78	0.84	0.62
time (sec)	N/A	0.154	0.002	0.050	0.044	0.070	0.055	0.121	0.152	0.022

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	22	32	19	25	38	20
N.S.	1	1.00	0.93	0.75	0.79	1.14	0.68	0.89	1.36	0.71
time (sec)	N/A	0.145	0.009	0.066	0.043	0.064	0.045	0.123	0.151	0.062

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	31	48	31	40	51	34
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.89	1.14	1.46	0.97
time (sec)	N/A	0.148	0.009	0.067	0.031	0.087	0.052	0.119	0.148	0.022

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	33	38	59	36	51	58	31
N.S.	1	1.00	0.86	0.79	0.90	1.40	0.86	1.21	1.38	0.74
time (sec)	N/A	0.153	0.008	0.073	0.046	0.068	0.061	0.125	0.147	0.024

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	38	43	64	41	60	63	37
N.S.	1	1.00	0.90	0.78	0.88	1.31	0.84	1.22	1.29	0.76
time (sec)	N/A	0.161	0.018	0.071	0.038	0.072	0.060	0.129	0.154	0.057

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	43	48	69	46	69	68	41
N.S.	1	1.00	1.00	0.77	0.86	1.23	0.82	1.23	1.21	0.73
time (sec)	N/A	0.166	0.007	0.076	0.043	0.066	0.060	0.120	0.152	0.060

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	28	30	50	27	27	64	29
N.S.	1	1.00	0.74	0.72	0.77	1.28	0.69	0.69	1.64	0.74
time (sec)	N/A	0.151	0.013	0.065	0.044	0.062	0.053	0.113	0.149	0.077

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	36	41	68	41	37	77	35
N.S.	1	1.00	0.85	0.78	0.89	1.48	0.89	0.80	1.67	0.76
time (sec)	N/A	0.155	0.012	0.076	0.036	0.063	0.061	0.123	0.150	0.025

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	40	48	79	46	43	86	41
N.S.	1	1.00	0.83	0.75	0.91	1.49	0.87	0.81	1.62	0.77
time (sec)	N/A	0.162	0.015	0.071	0.043	0.062	0.063	0.121	0.149	0.054

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	45	53	84	51	47	91	47
N.S.	1	1.00	0.82	0.75	0.88	1.40	0.85	0.78	1.52	0.78
time (sec)	N/A	0.169	0.011	0.073	0.027	0.075	0.066	0.122	0.152	0.027

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	50	58	89	56	52	96	51
N.S.	1	1.00	0.81	0.75	0.87	1.33	0.84	0.78	1.43	0.76
time (sec)	N/A	0.170	0.012	0.075	0.027	0.066	0.072	0.124	0.148	0.029

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	11	9
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	1.00	0.82
time (sec)	N/A	0.129	0.003	0.059	0.025	0.067	0.046	0.124	0.144	0.060

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	8	13	11	9
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.67	1.08	0.92	0.75
time (sec)	N/A	0.127	0.002	0.049	0.027	0.064	0.050	0.119	0.144	0.022

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	21	14	21	21	16
N.S.	1	1.00	1.00	1.05	1.00	1.11	0.74	1.11	1.11	0.84
time (sec)	N/A	0.141	0.003	0.053	0.037	0.075	0.062	0.111	0.145	0.025

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	17	21	14	19	21	14
N.S.	1	1.00	1.00	1.00	0.94	1.17	0.78	1.06	1.17	0.78
time (sec)	N/A	0.138	0.002	0.053	0.041	0.078	0.061	0.126	0.144	0.016

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	15	10	15	15	20
N.S.	1	1.00	1.00	1.07	1.00	1.07	0.71	1.07	1.07	1.43
time (sec)	N/A	0.134	0.004	0.059	0.033	0.076	0.059	0.124	0.159	0.023

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	15	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.71	0.62
time (sec)	N/A	0.132	0.010	0.095	0.031	0.079	0.175	0.117	0.157	0.057

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	15	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.71	0.62
time (sec)	N/A	0.128	0.009	0.084	0.024	0.068	0.102	0.122	0.154	0.015

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	13	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62	0.62
time (sec)	N/A	0.129	0.008	0.083	0.027	0.068	0.374	0.121	0.153	0.015

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	13	12	17	13	11	12
N.S.	1	1.00	0.84	0.68	0.68	0.63	0.89	0.68	0.58	0.63
time (sec)	N/A	0.141	0.008	0.043	0.030	0.076	0.061	0.116	0.154	0.016

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	13	12	13	12	15	13	13	11
N.S.	1	1.00	0.76	0.71	0.76	0.71	0.88	0.76	0.76	0.65
time (sec)	N/A	0.133	0.010	0.052	0.027	0.074	0.139	0.122	0.153	0.017

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	12	11	11	19	11	17	13
N.S.	1	1.00	0.79	0.63	0.58	0.58	1.00	0.58	0.89	0.68
time (sec)	N/A	0.128	0.011	0.056	0.039	0.077	0.176	0.121	0.157	0.016

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	26	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.72	0.67
time (sec)	N/A	0.135	0.013	0.100	0.027	0.092	0.230	0.124	0.150	0.024

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	26	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.72	0.67
time (sec)	N/A	0.136	0.012	0.092	0.026	0.079	0.146	0.120	0.156	0.022

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	1851	24	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.75	51.42	0.67	0.67	0.67
time (sec)	N/A	0.137	0.011	0.092	0.025	0.081	58.985	0.118	0.152	0.057

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	24	24	24	32	24	23	24
N.S.	1	1.00	0.82	0.71	0.71	0.71	0.94	0.71	0.68	0.71
time (sec)	N/A	0.135	0.012	0.051	0.030	0.082	0.083	0.116	0.158	0.021

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	23	31	24	24	24
N.S.	1	1.00	0.88	0.78	0.75	0.72	0.97	0.75	0.75	0.75
time (sec)	N/A	0.135	0.015	0.059	0.027	0.071	0.149	0.130	0.158	0.023

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	23	23	24	31	23	28	24
N.S.	1	1.00	0.81	0.72	0.72	0.75	0.97	0.72	0.88	0.75
time (sec)	N/A	0.141	0.015	0.063	0.028	0.073	0.182	0.135	0.149	0.018

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	37	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.73	0.69
time (sec)	N/A	0.149	0.014	0.112	0.025	0.075	0.295	0.119	0.152	0.027

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	37	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.73	0.69
time (sec)	N/A	0.146	0.014	0.095	0.038	0.065	0.182	0.115	0.155	0.026

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	0	35	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.00	0.69	0.69	0.69
time (sec)	N/A	0.144	0.014	0.094	0.024	0.067	0.000	0.119	0.154	0.030

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	35	35	35	46	35	34	35
N.S.	1	1.00	0.83	0.74	0.74	0.74	0.98	0.74	0.72	0.74
time (sec)	N/A	0.145	0.013	0.053	0.034	0.063	0.107	0.120	0.147	0.025

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	36	35	34	44	35	35	35
N.S.	1	1.00	0.87	0.80	0.78	0.76	0.98	0.78	0.78	0.78
time (sec)	N/A	0.145	0.016	0.063	0.030	0.069	0.189	0.123	0.150	0.027

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	38	34	34	34	46	34	38	35
N.S.	1	1.00	0.81	0.72	0.72	0.72	0.98	0.72	0.81	0.74
time (sec)	N/A	0.146	0.017	0.068	0.029	0.065	0.205	0.125	0.154	0.023

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	78	61	53	54	132	122	59	55	48
N.S.	1	1.15	0.90	0.78	0.79	1.94	1.79	0.87	0.81	0.71
time (sec)	N/A	0.187	0.048	0.102	0.106	0.083	1.846	0.120	0.160	0.033

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	49	42	42	103	107	45	39	37
N.S.	1	1.11	0.92	0.79	0.79	1.94	2.02	0.85	0.74	0.70
time (sec)	N/A	0.155	0.035	0.080	0.109	0.086	0.552	0.123	0.158	0.061

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	85	88	31	29	28
N.S.	1	1.00	1.00	0.80	0.78	2.12	2.20	0.78	0.72	0.70
time (sec)	N/A	0.138	0.020	0.078	0.105	0.086	0.282	0.121	0.149	0.057

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	73	18	25	19
N.S.	1	1.00	1.00	0.66	0.62	2.34	2.52	0.62	0.86	0.66
time (sec)	N/A	0.126	0.014	0.073	0.109	0.092	0.377	0.126	0.142	0.038

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	87	85	31	33	28
N.S.	1	1.00	1.00	0.80	0.78	2.18	2.12	0.78	0.82	0.70
time (sec)	N/A	0.137	0.023	0.089	0.113	0.075	0.738	0.127	0.153	0.031

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	48	40	41	113	107	41	46	38
N.S.	1	1.11	0.91	0.75	0.77	2.13	2.02	0.77	0.87	0.72
time (sec)	N/A	0.147	0.035	0.094	0.110	0.091	1.783	0.121	0.152	0.034

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	78	61	53	52	139	126	52	61	49
N.S.	1	1.15	0.90	0.78	0.76	2.04	1.85	0.76	0.90	0.72
time (sec)	N/A	0.155	0.045	0.099	0.111	0.088	5.358	0.120	0.152	0.032

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	84	68	56	63	161	389	65	84	58
N.S.	1	1.15	0.93	0.77	0.86	2.21	5.33	0.89	1.15	0.79
time (sec)	N/A	0.159	0.071	0.128	0.103	0.081	7.188	0.125	0.159	0.068

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	65	54	47	49	134	332	46	67	46
N.S.	1	1.14	0.95	0.82	0.86	2.35	5.82	0.81	1.18	0.81
time (sec)	N/A	0.154	0.063	0.107	0.118	0.076	2.866	0.123	0.158	0.072

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	37	115	269	36	60	34
N.S.	1	1.00	1.00	0.80	0.80	2.50	5.85	0.78	1.30	0.74
time (sec)	N/A	0.138	0.048	0.086	0.120	0.090	1.442	0.125	0.160	0.061

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	116	277	35	59	33
N.S.	1	1.00	1.00	0.80	0.78	2.58	6.16	0.78	1.31	0.73
time (sec)	N/A	0.139	0.043	0.081	0.108	0.098	2.162	0.122	0.157	0.058

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	64	54	47	51	143	384	49	71	48
N.S.	1	1.14	0.96	0.84	0.91	2.55	6.86	0.88	1.27	0.86
time (sec)	N/A	0.147	0.063	0.119	0.109	0.078	5.262	0.124	0.152	0.041

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	83	68	53	64	179	452	58	92	58
N.S.	1	1.20	0.99	0.77	0.93	2.59	6.55	0.84	1.33	0.84
time (sec)	N/A	0.158	0.066	0.126	0.113	0.168	15.092	0.126	0.155	0.094

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	111	81	66	86	227	762	77	136	81
N.S.	1	1.12	0.82	0.67	0.87	2.29	7.70	0.78	1.37	0.82
time (sec)	N/A	0.172	0.108	0.167	0.114	0.105	39.619	0.133	0.160	0.038

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	92	70	56	73	200	683	59	120	69
N.S.	1	1.10	0.83	0.67	0.87	2.38	8.13	0.70	1.43	0.82
time (sec)	N/A	0.169	0.101	0.155	0.108	0.080	16.859	0.121	0.159	0.081

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	59	50	61	185	605	47	113	58
N.S.	1	1.03	0.83	0.70	0.86	2.61	8.52	0.66	1.59	0.82
time (sec)	N/A	0.152	0.092	0.097	0.111	0.078	9.555	0.127	0.160	0.080

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	72	60	52	64	186	627	52	110	56
N.S.	1	0.99	0.82	0.71	0.88	2.55	8.59	0.71	1.51	0.77
time (sec)	N/A	0.148	0.093	0.093	0.102	0.081	5.052	0.122	0.152	0.073

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	59	59	60	186	632	47	113	57
N.S.	1	1.03	0.84	0.84	0.86	2.66	9.03	0.67	1.61	0.81
time (sec)	N/A	0.151	0.063	0.087	0.125	0.084	7.862	0.121	0.150	0.081

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	91	70	56	73	209	779	59	124	70
N.S.	1	1.11	0.85	0.68	0.89	2.55	9.50	0.72	1.51	0.85
time (sec)	N/A	0.160	0.096	0.130	0.111	0.108	17.036	0.122	0.157	0.091

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	110	81	64	86	245	869	71	144	80
N.S.	1	1.16	0.85	0.67	0.91	2.58	9.15	0.75	1.52	0.84
time (sec)	N/A	0.173	0.100	0.144	0.126	0.091	39.301	0.125	0.160	0.098

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	77	61	53	70	131	117	61	74	51
N.S.	1	1.13	0.90	0.78	1.03	1.93	1.72	0.90	1.09	0.75
time (sec)	N/A	0.156	0.045	0.102	0.122	0.080	1.878	0.125	0.156	0.053

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	49	41	58	103	102	47	57	37
N.S.	1	1.11	0.92	0.77	1.09	1.94	1.92	0.89	1.08	0.70
time (sec)	N/A	0.157	0.034	0.089	0.107	0.085	0.533	0.123	0.157	0.071

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	47	83	83	33	44	28
N.S.	1	1.00	1.00	0.80	1.18	2.08	2.08	0.82	1.10	0.70
time (sec)	N/A	0.135	0.022	0.085	0.110	0.082	0.267	0.127	0.156	0.073

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	34	67	68	20	37	19
N.S.	1	1.00	1.00	0.66	1.17	2.31	2.34	0.69	1.28	0.66
time (sec)	N/A	0.127	0.016	0.082	0.113	0.073	0.368	0.135	0.156	0.042

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	47	87	76	33	50	28
N.S.	1	1.00	1.00	0.80	1.18	2.18	1.90	0.82	1.25	0.70
time (sec)	N/A	0.135	0.025	0.093	0.123	0.081	0.683	0.117	0.162	0.071

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	48	40	55	109	99	41	66	37
N.S.	1	1.11	0.91	0.75	1.04	2.06	1.87	0.77	1.25	0.70
time (sec)	N/A	0.146	0.041	0.100	0.113	0.076	1.734	0.124	0.154	0.035

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	77	61	53	68	139	117	54	85	48
N.S.	1	1.13	0.90	0.78	1.00	2.04	1.72	0.79	1.25	0.71
time (sec)	N/A	0.157	0.042	0.106	0.113	0.087	5.114	0.119	0.156	0.080

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	83	70	57	81	167	354	69	123	61
N.S.	1	1.14	0.96	0.78	1.11	2.29	4.85	0.95	1.68	0.84
time (sec)	N/A	0.161	0.071	0.131	0.105	0.082	7.216	0.124	0.151	0.042

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	65	56	48	68	138	301	51	104	47
N.S.	1	1.12	0.97	0.83	1.17	2.38	5.19	0.88	1.79	0.81
time (sec)	N/A	0.168	0.056	0.113	0.137	0.076	2.801	0.121	0.153	0.081

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	38	56	123	243	40	97	35
N.S.	1	1.00	1.04	0.81	1.19	2.62	5.17	0.85	2.06	0.74
time (sec)	N/A	0.141	0.053	0.088	0.116	0.090	1.416	0.117	0.157	0.073

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	56	122	252	41	97	34
N.S.	1	1.00	1.00	0.80	1.22	2.65	5.48	0.89	2.11	0.74
time (sec)	N/A	0.139	0.045	0.088	0.105	0.104	2.115	0.120	0.157	0.034

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	65	55	48	69	147	354	52	112	49
N.S.	1	1.14	0.96	0.84	1.21	2.58	6.21	0.91	1.96	0.86
time (sec)	N/A	0.151	0.059	0.118	0.109	0.110	5.087	0.122	0.157	0.082

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	83	69	56	82	183	416	61	138	60
N.S.	1	1.19	0.99	0.80	1.17	2.61	5.94	0.87	1.97	0.86
time (sec)	N/A	0.162	0.075	0.126	0.113	0.094	14.823	0.127	0.159	0.085

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	111	82	67	103	227	695	81	199	83
N.S.	1	1.10	0.81	0.66	1.02	2.25	6.88	0.80	1.97	0.82
time (sec)	N/A	0.173	0.109	0.165	0.116	0.102	39.147	0.119	0.152	0.090

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	93	71	57	90	199	624	63	180	69
N.S.	1	1.08	0.83	0.66	1.05	2.31	7.26	0.73	2.09	0.80
time (sec)	N/A	0.171	0.101	0.155	0.125	0.089	16.350	0.115	0.157	0.088

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	60	51	78	186	552	51	173	58
N.S.	1	1.03	0.82	0.70	1.07	2.55	7.56	0.70	2.37	0.79
time (sec)	N/A	0.153	0.094	0.101	0.102	0.089	9.188	0.127	0.155	0.047

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	74	60	53	80	183	575	55	171	57
N.S.	1	0.99	0.80	0.71	1.07	2.44	7.67	0.73	2.28	0.76
time (sec)	N/A	0.154	0.083	0.097	0.103	0.076	4.885	0.116	0.158	0.082

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	60	61	77	185	580	51	173	58
N.S.	1	1.03	0.83	0.85	1.07	2.57	8.06	0.71	2.40	0.81
time (sec)	N/A	0.151	0.072	0.091	0.125	0.094	7.566	0.125	0.160	0.084

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	71	57	90	209	716	63	190	69
N.S.	1	1.11	0.85	0.68	1.07	2.49	8.52	0.75	2.26	0.82
time (sec)	N/A	0.163	0.095	0.131	0.115	0.111	16.545	0.122	0.154	0.089

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	111	82	66	103	245	799	73	216	80
N.S.	1	1.14	0.85	0.68	1.06	2.53	8.24	0.75	2.23	0.82
time (sec)	N/A	0.176	0.099	0.147	0.122	0.117	39.186	0.123	0.154	0.102

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62	0.62
time (sec)	N/A	0.128	0.009	0.106	0.029	0.065	0.222	0.117	0.150	0.017

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62	0.62
time (sec)	N/A	0.136	0.009	0.099	0.027	0.083	0.171	0.116	0.155	0.015

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	13	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62	0.62
time (sec)	N/A	0.125	0.009	0.098	0.026	0.065	0.104	0.116	0.161	0.015

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	13	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62	0.62
time (sec)	N/A	0.137	0.009	0.099	0.026	0.086	0.395	0.122	0.152	0.016

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	13	13	13	19	13	13	13
N.S.	1	1.00	0.81	0.62	0.62	0.62	0.90	0.62	0.62	0.62
time (sec)	N/A	0.126	0.009	0.046	0.032	0.079	0.354	0.123	0.151	0.015

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	13	12	17	13	12	12
N.S.	1	1.00	0.84	0.68	0.68	0.63	0.89	0.68	0.63	0.63
time (sec)	N/A	0.126	0.010	0.046	0.033	0.067	0.303	0.127	0.151	0.017

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	14	13	12	17	13	12	13
N.S.	1	1.00	0.89	0.74	0.68	0.63	0.89	0.68	0.63	0.68
time (sec)	N/A	0.126	0.011	0.054	0.031	0.077	0.147	0.120	0.153	0.017

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	12	13	13	17	13	13	13
N.S.	1	1.00	0.79	0.63	0.68	0.68	0.89	0.68	0.68	0.68
time (sec)	N/A	0.126	0.011	0.053	0.029	0.074	0.156	0.121	0.152	0.016

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67	0.67
time (sec)	N/A	0.136	0.013	0.107	0.030	0.067	0.316	0.124	0.148	0.062

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67	0.67
time (sec)	N/A	0.155	0.013	0.113	0.027	0.076	0.247	0.119	0.168	0.021

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	34	24	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.75	0.94	0.67	0.67	0.67
time (sec)	N/A	0.139	0.013	0.111	0.028	0.064	0.155	0.116	0.159	0.021

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	2633	24	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.75	73.14	0.67	0.67	0.67
time (sec)	N/A	0.135	0.012	0.108	0.038	0.061	1.081	0.116	0.152	0.022

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	24	24	24	1765	24	24	24
N.S.	1	1.00	0.78	0.67	0.67	0.67	49.03	0.67	0.67	0.67
time (sec)	N/A	0.137	0.012	0.107	0.028	0.064	1.006	0.119	0.157	0.022

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	24	24	24	1741	24	24	24
N.S.	1	1.00	0.82	0.71	0.71	0.71	51.21	0.71	0.71	0.71
time (sec)	N/A	0.137	0.012	0.053	0.033	0.062	1.016	0.122	0.159	0.021

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	23	1826	24	23	24
N.S.	1	1.00	0.88	0.78	0.75	0.72	57.06	0.75	0.72	0.75
time (sec)	N/A	0.137	0.014	0.065	0.033	0.068	1.044	0.127	0.157	0.024

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	23	1957	24	23	24
N.S.	1	1.00	0.82	0.74	0.71	0.68	57.56	0.71	0.68	0.71
time (sec)	N/A	0.137	0.015	0.063	0.043	0.069	1.021	0.129	0.160	0.023

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69	0.69
time (sec)	N/A	0.146	0.015	0.112	0.034	0.067	0.445	0.132	0.149	0.026

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69	0.69
time (sec)	N/A	0.156	0.015	0.112	0.032	0.078	0.348	0.121	0.148	0.026

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	49	35	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.96	0.69	0.69	0.69
time (sec)	N/A	0.144	0.015	0.111	0.028	0.066	0.219	0.119	0.154	0.024

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	5012	35	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.75	98.27	0.69	0.69	0.69
time (sec)	N/A	0.144	0.014	0.110	0.030	0.063	1.532	0.120	0.152	0.027

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	35	35	35	6246	35	35	35
N.S.	1	1.00	0.76	0.69	0.69	0.69	122.47	0.69	0.69	0.69
time (sec)	N/A	0.145	0.014	0.109	0.047	0.076	1.509	0.119	0.164	0.026

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	35	35	35	6667	35	35	35
N.S.	1	1.00	0.80	0.71	0.71	0.71	136.06	0.71	0.71	0.71
time (sec)	N/A	0.145	0.015	0.056	0.036	0.068	1.508	0.117	0.146	0.024

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	4004	35	35	35
N.S.	1	1.00	0.80	0.73	0.71	0.71	81.71	0.71	0.71	0.71
time (sec)	N/A	0.148	0.017	0.066	0.031	0.079	1.551	0.116	0.145	0.031

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	3964	35	35	35
N.S.	1	1.00	0.80	0.73	0.71	0.71	80.90	0.71	0.71	0.71
time (sec)	N/A	0.145	0.017	0.066	0.045	0.066	1.549	0.123	0.150	0.026

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	134	140	121	130	147	180	138	104	151
N.S.	1	1.07	1.12	0.97	1.04	1.18	1.44	1.10	0.83	1.21
time (sec)	N/A	0.214	0.090	0.098	0.108	0.081	14.839	0.130	0.162	0.153

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	132	140	123	128	116	173	136	96	126
N.S.	1	1.07	1.14	1.00	1.04	0.94	1.41	1.11	0.78	1.02
time (sec)	N/A	0.201	0.069	0.155	0.162	0.079	8.005	0.126	0.165	0.082

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	115	127	107	114	128	162	118	87	130
N.S.	1	1.04	1.14	0.96	1.03	1.15	1.46	1.06	0.78	1.17
time (sec)	N/A	0.184	0.061	0.097	0.109	0.079	2.727	0.139	0.165	0.140

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	126	112	115	114	148	119	86	126
N.S.	1	1.04	1.16	1.03	1.06	1.05	1.36	1.09	0.79	1.16
time (sec)	N/A	0.189	0.059	0.140	0.114	0.077	1.820	0.130	0.155	0.084

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	96	103	96	103	313	141	118	71	120
N.S.	1	0.96	1.03	0.96	1.03	3.13	1.41	1.18	0.71	1.20
time (sec)	N/A	0.175	0.048	0.083	0.118	0.083	2.136	0.124	0.150	0.112

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	96	103	95	102	307	141	117	73	110
N.S.	1	0.96	1.03	0.95	1.02	3.07	1.41	1.17	0.73	1.10
time (sec)	N/A	0.190	0.053	0.086	0.107	0.093	3.375	0.128	0.152	0.085

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	127	104	111	113	151	125	96	124
N.S.	1	1.04	1.17	0.95	1.02	1.04	1.39	1.15	0.88	1.14
time (sec)	N/A	0.190	0.077	0.098	0.107	0.072	7.650	0.136	0.160	0.081

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	115	126	112	112	147	155	120	102	138
N.S.	1	1.04	1.14	1.01	1.01	1.32	1.40	1.08	0.92	1.24
time (sec)	N/A	0.191	0.079	0.143	0.105	0.072	10.077	0.131	0.153	0.044

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	140	142	124	133	162	595	135	184	150
N.S.	1	1.09	1.10	0.96	1.03	1.26	4.61	1.05	1.43	1.16
time (sec)	N/A	0.201	0.180	0.121	0.111	0.083	76.181	0.122	0.161	0.092

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	138	142	124	133	147	457	135	182	142
N.S.	1	1.10	1.14	0.99	1.06	1.18	3.66	1.08	1.46	1.14
time (sec)	N/A	0.203	0.170	0.223	0.113	0.076	58.651	0.130	0.154	0.087

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	133	118	120	394	527	136	162	142
N.S.	1	1.05	1.16	1.03	1.04	3.43	4.58	1.18	1.41	1.23
time (sec)	N/A	0.200	0.154	0.095	0.112	0.094	32.771	0.130	0.154	0.092

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	121	134	117	120	389	450	136	177	120
N.S.	1	1.03	1.15	1.00	1.03	3.32	3.85	1.16	1.51	1.03
time (sec)	N/A	0.185	0.143	0.096	0.111	0.084	21.202	0.129	0.149	0.082

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	120	133	116	127	379	544	132	162	144
N.S.	1	1.03	1.15	1.00	1.09	3.27	4.69	1.14	1.40	1.24
time (sec)	N/A	0.188	0.134	0.089	0.122	0.090	22.689	0.131	0.163	0.159

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	120	134	117	127	387	434	132	177	134
N.S.	1	1.06	1.19	1.04	1.12	3.42	3.84	1.17	1.57	1.19
time (sec)	N/A	0.189	0.137	0.087	0.119	0.092	33.374	0.128	0.164	0.136

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	137	142	121	132	156	690	145	197	151
N.S.	1	1.10	1.15	0.98	1.06	1.26	5.56	1.17	1.59	1.22
time (sec)	N/A	0.203	0.183	0.132	0.110	0.088	71.070	0.131	0.161	0.046

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	139	142	124	132	182	590	137	210	166
N.S.	1	1.09	1.11	0.97	1.03	1.42	4.61	1.07	1.64	1.30
time (sec)	N/A	0.204	0.189	0.231	0.107	0.076	104.514	0.129	0.159	0.095

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	148	142	130	143	506	0	146	277	165
N.S.	1	1.05	1.01	0.92	1.01	3.59	0.00	1.04	1.96	1.17
time (sec)	N/A	0.207	0.202	0.105	0.110	0.094	0.000	0.129	0.159	0.106

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	148	142	130	143	503	0	146	290	139
N.S.	1	1.05	1.01	0.92	1.01	3.57	0.00	1.04	2.06	0.99
time (sec)	N/A	0.206	0.195	0.102	0.137	0.111	0.000	0.128	0.159	0.041

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	147	133	132	153	508	1171	149	275	172
N.S.	1	1.03	0.93	0.92	1.07	3.55	8.19	1.04	1.92	1.20
time (sec)	N/A	0.204	0.188	0.103	0.112	0.094	123.845	0.142	0.156	0.150

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	147	136	132	152	501	899	148	290	146
N.S.	1	1.03	0.95	0.92	1.06	3.50	6.29	1.03	2.03	1.02
time (sec)	N/A	0.201	0.182	0.102	0.111	0.098	100.031	0.131	0.161	0.132

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	147	142	139	151	510	1175	143	277	167
N.S.	1	1.05	1.01	0.99	1.08	3.64	8.39	1.02	1.98	1.19
time (sec)	N/A	0.205	0.134	0.095	0.104	0.092	106.402	0.141	0.159	0.154

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	147	142	139	151	499	853	143	290	157
N.S.	1	1.05	1.01	0.99	1.08	3.56	6.09	1.02	2.07	1.12
time (sec)	N/A	0.202	0.132	0.092	0.127	0.095	153.241	0.125	0.161	0.083

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	164	153	133	154	211	0	155	310	174
N.S.	1	1.08	1.01	0.88	1.01	1.39	0.00	1.02	2.04	1.14
time (sec)	N/A	0.215	0.227	0.145	0.123	0.085	0.000	0.127	0.162	0.094

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	166	153	133	154	244	0	150	323	182
N.S.	1	1.09	1.01	0.88	1.01	1.61	0.00	0.99	2.12	1.20
time (sec)	N/A	0.215	0.228	0.161	0.108	0.100	0.000	0.134	0.168	0.094

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	53	1742	116	52	56
N.S.	1	1.00	0.64	0.60	0.78	0.74	24.19	1.61	0.72	0.78
time (sec)	N/A	0.165	0.053	0.144	0.030	0.067	1.242	0.119	0.155	0.031

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	42	666	93	41	37
N.S.	1	1.00	0.66	0.60	0.77	0.79	12.57	1.75	0.77	0.70
time (sec)	N/A	0.161	0.018	0.127	0.039	0.065	0.846	0.127	0.156	0.066

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	21	26	30	202	66	29	25
N.S.	1	1.00	1.00	0.62	0.76	0.88	5.94	1.94	0.85	0.74
time (sec)	N/A	0.145	0.015	0.116	0.037	0.066	0.574	0.112	0.159	0.016

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	16	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	1.00	0.75
time (sec)	N/A	0.115	0.001	0.068	0.035	0.078	0.017	0.116	0.157	0.013

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	42	70	68	32	38	27
N.S.	1	1.00	1.00	0.80	1.20	2.00	1.94	0.91	1.09	0.77
time (sec)	N/A	0.138	0.020	0.097	0.111	0.083	0.690	0.113	0.153	0.028

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	47	90	44	38	51	31
N.S.	1	1.00	1.00	0.82	1.21	2.31	1.13	0.97	1.31	0.79
time (sec)	N/A	0.136	0.048	0.122	0.103	0.091	0.840	0.125	0.156	0.033

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	63	55	44	88	116	97	66	72	48
N.S.	1	0.97	0.85	0.68	1.35	1.78	1.49	1.02	1.11	0.74
time (sec)	N/A	0.151	0.074	0.135	0.121	0.076	1.735	0.123	0.162	0.077

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	91	67	56	121	142	122	72	90	66
N.S.	1	1.05	0.77	0.64	1.39	1.63	1.40	0.83	1.03	0.76
time (sec)	N/A	0.163	0.100	0.149	0.118	0.078	4.221	0.123	0.155	0.074

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	64	1742	193	63	56
N.S.	1	1.00	0.64	0.60	0.78	0.89	24.19	2.68	0.88	0.78
time (sec)	N/A	0.172	0.025	0.134	0.029	0.064	1.318	0.125	0.155	0.025

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	53	733	156	52	37
N.S.	1	1.00	0.66	0.60	0.77	1.00	13.83	2.94	0.98	0.70
time (sec)	N/A	0.158	0.020	0.128	0.029	0.067	0.909	0.123	0.157	0.022

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	26	41	80	119	40	25
N.S.	1	1.00	0.71	0.62	0.76	1.21	2.35	3.50	1.18	0.74
time (sec)	N/A	0.140	0.018	0.116	0.029	0.071	0.177	0.121	0.154	0.016

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	58	27	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	1.69	0.75
time (sec)	N/A	0.115	0.002	0.070	0.045	0.067	0.016	0.120	0.162	0.011

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	51	44	35	52	85	71	44	51	37
N.S.	1	1.04	0.90	0.71	1.06	1.73	1.45	0.90	1.04	0.76
time (sec)	N/A	0.153	0.024	0.104	0.106	0.071	1.047	0.125	0.154	0.024

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	55	45	44	58	99	92	50	59	42
N.S.	1	1.06	0.87	0.85	1.12	1.90	1.77	0.96	1.13	0.81
time (sec)	N/A	0.146	0.054	0.145	0.109	0.081	1.210	0.128	0.148	0.028

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	61	53	42	86	121	76	64	73	46
N.S.	1	0.97	0.84	0.67	1.37	1.92	1.21	1.02	1.16	0.73
time (sec)	N/A	0.142	0.092	0.129	0.109	0.083	1.385	0.124	0.148	0.034

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	67	56	119	142	124	72	90	64
N.S.	1	1.00	0.79	0.66	1.40	1.67	1.46	0.85	1.06	0.75
time (sec)	N/A	0.158	0.106	0.148	0.107	0.078	3.063	0.125	0.156	0.075

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	75	146	281	74	56
N.S.	1	1.00	0.64	0.60	0.78	1.04	2.03	3.90	1.03	0.78
time (sec)	N/A	0.172	0.026	0.141	0.032	0.064	0.340	0.124	0.153	0.026

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	64	124	233	63	37
N.S.	1	1.00	0.66	0.60	0.77	1.21	2.34	4.40	1.19	0.70
time (sec)	N/A	0.153	0.024	0.131	0.030	0.066	0.304	0.122	0.151	0.024

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	26	52	102	182	51	25
N.S.	1	1.00	0.71	0.62	0.76	1.53	3.00	5.35	1.50	0.74
time (sec)	N/A	0.140	0.018	0.122	0.027	0.067	0.269	0.123	0.162	0.016

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	39	12	95	38	12
N.S.	1	1.00	1.00	0.81	0.75	2.44	0.75	5.94	2.38	0.75
time (sec)	N/A	0.116	0.003	0.072	0.030	0.075	0.018	0.132	0.149	0.014

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	56	47	64	111	97	56	72	52
N.S.	1	1.03	0.86	0.72	0.98	1.71	1.49	0.86	1.11	0.80
time (sec)	N/A	0.154	0.044	0.109	0.106	0.085	2.063	0.129	0.150	0.028

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	60	57	71	123	99	65	78	58
N.S.	1	1.03	0.87	0.83	1.03	1.78	1.43	0.94	1.13	0.84
time (sec)	N/A	0.167	0.069	0.166	0.114	0.078	1.903	0.125	0.150	0.076

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	77	63	55	101	130	126	80	84	64
N.S.	1	0.96	0.79	0.69	1.26	1.62	1.58	1.00	1.05	0.80
time (sec)	N/A	0.160	0.093	0.156	0.112	0.076	2.084	0.123	0.152	0.033

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	64	53	115	143	104	67	90	64
N.S.	1	0.98	0.75	0.62	1.35	1.68	1.22	0.79	1.06	0.75
time (sec)	N/A	0.161	0.120	0.147	0.116	0.075	2.263	0.127	0.145	0.069

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	78	67	144	164	155	99	107	79
N.S.	1	1.00	0.73	0.63	1.35	1.53	1.45	0.93	1.00	0.74
time (sec)	N/A	0.172	0.132	0.171	0.105	0.076	6.279	0.127	0.156	0.037

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	135	89	78	175	186	180	96	124	94
N.S.	1	1.05	0.69	0.60	1.36	1.44	1.40	0.74	0.96	0.73
time (sec)	N/A	0.188	0.152	0.186	0.111	0.081	26.536	0.126	0.148	0.042

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	90	87	116	141	279	781	140	116
N.S.	1	1.00	0.62	0.60	0.79	0.97	1.91	5.35	0.96	0.79
time (sec)	N/A	0.213	0.043	0.184	0.033	0.068	1.120	0.124	0.140	0.023

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	76	101	130	257	709	129	101
N.S.	1	1.00	0.62	0.60	0.80	1.02	2.02	5.58	1.02	0.80
time (sec)	N/A	0.203	0.038	0.169	0.026	0.078	1.005	0.130	0.154	0.021

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	68	65	86	119	235	637	118	86
N.S.	1	1.00	0.62	0.59	0.78	1.08	2.14	5.79	1.07	0.78
time (sec)	N/A	0.215	0.031	0.162	0.026	0.067	0.904	0.129	0.157	0.016

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	54	71	108	212	565	107	71
N.S.	1	1.00	0.63	0.59	0.78	1.19	2.33	6.21	1.18	0.78
time (sec)	N/A	0.190	0.027	0.152	0.025	0.069	0.836	0.129	0.156	0.014

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	97	190	493	96	56
N.S.	1	1.00	0.64	0.60	0.78	1.35	2.64	6.85	1.33	0.78
time (sec)	N/A	0.176	0.026	0.148	0.028	0.067	0.743	0.125	0.159	0.025

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	86	168	421	85	36
N.S.	1	1.00	0.66	0.60	0.77	1.62	3.17	7.94	1.60	0.68
time (sec)	N/A	0.161	0.024	0.136	0.029	0.068	0.678	0.123	0.155	0.023

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	26	74	146	347	73	25
N.S.	1	1.00	0.71	0.62	0.76	2.18	4.29	10.21	2.15	0.74
time (sec)	N/A	0.146	0.020	0.128	0.030	0.077	0.589	0.132	0.157	0.019

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	61	12	229	60	12
N.S.	1	1.00	1.00	0.81	0.75	3.81	0.75	14.31	3.75	0.75
time (sec)	N/A	0.122	0.003	0.077	0.024	0.077	0.020	0.120	0.157	0.015

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	99	78	69	88	155	148	80	106	76
N.S.	1	1.02	0.80	0.71	0.91	1.60	1.53	0.82	1.09	0.78
time (sec)	N/A	0.180	0.042	0.121	0.106	0.103	9.433	0.123	0.160	0.022

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	103	82	81	97	169	150	89	116	84
N.S.	1	1.02	0.81	0.80	0.96	1.67	1.49	0.88	1.15	0.83
time (sec)	N/A	0.184	0.080	0.164	0.115	0.103	9.026	0.127	0.157	0.076

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	109	86	79	131	177	184	112	124	117
N.S.	1	0.94	0.74	0.68	1.13	1.53	1.59	0.97	1.07	1.01
time (sec)	N/A	0.198	0.106	0.176	0.111	0.082	8.562	0.122	0.156	0.027

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	115	85	78	145	175	184	94	120	131
N.S.	1	0.95	0.70	0.64	1.20	1.45	1.52	0.78	0.99	1.08
time (sec)	N/A	0.186	0.122	0.186	0.108	0.075	8.130	0.125	0.159	0.084

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	121	86	77	155	174	182	110	118	94
N.S.	1	0.98	0.69	0.62	1.25	1.40	1.47	0.89	0.95	0.76
time (sec)	N/A	0.187	0.141	0.171	0.128	0.106	8.373	0.127	0.159	0.040

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	127	86	75	169	187	158	91	124	94
N.S.	1	0.98	0.67	0.58	1.31	1.45	1.22	0.71	0.96	0.73
time (sec)	N/A	0.189	0.172	0.171	0.110	0.117	8.863	0.120	0.154	0.073

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	100	89	198	208	209	129	141	109
N.S.	1	1.00	0.66	0.59	1.31	1.38	1.38	0.85	0.93	0.72
time (sec)	N/A	0.199	0.189	0.208	0.110	0.114	40.973	0.126	0.155	0.050

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	179	111	100	229	230	0	120	158	124
N.S.	1	1.03	0.64	0.58	1.32	1.33	0.00	0.69	0.91	0.72
time (sec)	N/A	0.241	0.215	0.221	0.111	0.103	0.000	0.126	0.156	0.096

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	31	78	148	31	29	31
N.S.	1	1.00	1.00	0.82	0.79	2.00	3.79	0.79	0.74	0.79
time (sec)	N/A	0.145	0.021	0.303	0.105	0.076	0.757	0.125	0.147	0.025

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	98	117	38	38	34
N.S.	1	1.00	1.00	0.83	0.81	2.33	2.79	0.90	0.90	0.81
time (sec)	N/A	0.145	0.039	0.145	0.132	0.075	0.872	0.124	0.160	0.026

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	68	60	53	83	125	207	66	57	54
N.S.	1	0.96	0.85	0.75	1.17	1.76	2.92	0.93	0.80	0.76
time (sec)	N/A	0.163	0.079	0.158	0.124	0.077	1.765	0.127	0.154	0.029

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	48	40	43	93	187	43	43	43
N.S.	1	1.05	0.87	0.73	0.78	1.69	3.40	0.78	0.78	0.78
time (sec)	N/A	0.155	0.034	0.126	0.158	0.102	1.136	0.126	0.153	0.064

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	48	43	47	103	197	52	47	47
N.S.	1	1.07	0.84	0.75	0.82	1.81	3.46	0.91	0.82	0.82
time (sec)	N/A	0.163	0.049	0.157	0.129	0.107	1.278	0.137	0.154	0.069

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	56	52	80	128	189	66	59	52
N.S.	1	0.96	0.81	0.75	1.16	1.86	2.74	0.96	0.86	0.75
time (sec)	N/A	0.160	0.081	0.155	0.109	0.099	1.390	0.124	0.162	0.025

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	60	51	57	119	240	57	64	57
N.S.	1	1.05	0.82	0.70	0.78	1.63	3.29	0.78	0.88	0.78
time (sec)	N/A	0.172	0.048	0.138	0.129	0.092	2.136	0.129	0.157	0.025

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	64	55	63	132	245	66	69	63
N.S.	1	1.04	0.83	0.71	0.82	1.71	3.18	0.86	0.90	0.82
time (sec)	N/A	0.174	0.060	0.167	0.138	0.081	1.997	0.126	0.160	0.074

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	85	67	62	97	138	267	83	72	69
N.S.	1	0.97	0.76	0.70	1.10	1.57	3.03	0.94	0.82	0.78
time (sec)	N/A	0.165	0.085	0.175	0.115	0.090	2.105	0.124	0.148	0.067

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	54	71	53	3755	61	52	71
N.S.	1	1.00	0.64	0.61	0.80	0.60	42.19	0.69	0.58	0.80
time (sec)	N/A	0.198	0.023	0.108	0.034	0.092	1.988	0.124	0.161	0.015

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	43	56	42	1640	49	41	56
N.S.	1	1.00	0.68	0.63	0.82	0.62	24.12	0.72	0.60	0.82
time (sec)	N/A	0.172	0.022	0.096	0.026	0.072	1.234	0.119	0.147	0.025

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	41	31	600	37	30	37
N.S.	1	1.00	0.69	0.63	0.80	0.61	11.76	0.73	0.59	0.73
time (sec)	N/A	0.164	0.018	0.089	0.026	0.082	0.829	0.121	0.152	0.022

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	26	19	162	23	18	25
N.S.	1	1.00	0.72	0.66	0.81	0.59	5.06	0.72	0.56	0.78
time (sec)	N/A	0.155	0.013	0.080	0.027	0.082	0.534	0.118	0.161	0.016

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	11	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.79	0.86
time (sec)	N/A	0.122	0.001	0.066	0.023	0.066	0.017	0.120	0.162	0.011

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	53	24	21	31	17
N.S.	1	1.00	1.00	0.78	1.39	2.30	1.04	0.91	1.35	0.74
time (sec)	N/A	0.137	0.015	0.089	0.109	0.097	0.462	0.120	0.157	0.025

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	60	90	44	44	51	33
N.S.	1	1.00	1.00	0.83	1.46	2.20	1.07	1.07	1.24	0.80
time (sec)	N/A	0.144	0.045	0.121	0.132	0.082	1.009	0.124	0.155	0.032

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	56	45	92	120	102	69	73	51
N.S.	1	1.01	0.82	0.66	1.35	1.76	1.50	1.01	1.07	0.75
time (sec)	N/A	0.161	0.069	0.118	0.127	0.088	2.144	0.122	0.156	0.074

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	97	67	56	121	142	129	72	90	69
N.S.	1	1.08	0.74	0.62	1.34	1.58	1.43	0.80	1.00	0.77
time (sec)	N/A	0.177	0.074	0.135	0.110	0.092	6.420	0.117	0.157	0.033

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	54	71	63	3606	82	54	71
N.S.	1	1.00	0.67	0.64	0.84	0.74	42.42	0.96	0.64	0.84
time (sec)	N/A	0.181	0.025	0.120	0.030	0.064	1.979	0.121	0.148	0.016

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	42	56	51	1538	61	42	56
N.S.	1	1.00	0.68	0.64	0.85	0.77	23.30	0.92	0.64	0.85
time (sec)	N/A	0.159	0.022	0.112	0.063	0.064	1.277	0.123	0.143	0.028

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	31	41	40	534	51	31	35
N.S.	1	1.00	0.69	0.63	0.84	0.82	10.90	1.04	0.63	0.71
time (sec)	N/A	0.155	0.021	0.108	0.045	0.070	0.780	0.123	0.167	0.024

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	20	26	29	37	29	20	19
N.S.	1	1.00	0.70	0.67	0.87	0.97	1.23	0.97	0.67	0.63
time (sec)	N/A	0.143	0.013	0.099	0.025	0.091	0.154	0.121	0.154	0.022

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	12	13	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.93	0.86
time (sec)	N/A	0.116	0.001	0.074	0.024	0.091	0.018	0.119	0.151	0.012

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	45	107	146	37	57	30
N.S.	1	1.00	1.00	0.82	1.18	2.82	3.84	0.97	1.50	0.79
time (sec)	N/A	0.141	0.030	0.114	0.105	0.102	0.764	0.126	0.148	0.027

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	64	49	50	76	148	73	64	73	60
N.S.	1	1.12	0.86	0.88	1.33	2.60	1.28	1.12	1.28	1.05
time (sec)	N/A	0.155	0.063	0.157	0.157	0.081	1.490	0.135	0.153	0.082

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	92	67	60	108	186	107	80	92	90
N.S.	1	1.06	0.77	0.69	1.24	2.14	1.23	0.92	1.06	1.03
time (sec)	N/A	0.178	0.095	0.158	0.121	0.099	3.256	0.124	0.155	0.034

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	54	71	74	3456	79	61	68
N.S.	1	1.00	0.66	0.62	0.82	0.85	39.72	0.91	0.70	0.78
time (sec)	N/A	0.182	0.028	0.132	0.031	0.069	1.918	0.119	0.159	0.027

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	47	39	56	62	163	59	49	47
N.S.	1	1.00	0.69	0.57	0.82	0.91	2.40	0.87	0.72	0.69
time (sec)	N/A	0.184	0.025	0.135	0.035	0.068	0.278	0.124	0.161	0.072

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	32	41	52	121	43	39	35
N.S.	1	1.00	0.71	0.65	0.84	1.06	2.47	0.88	0.80	0.71
time (sec)	N/A	0.152	0.024	0.117	0.045	0.070	0.274	0.129	0.144	0.020

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	21	26	41	80	20	28	20
N.S.	1	1.00	0.75	0.66	0.81	1.28	2.50	0.62	0.88	0.62
time (sec)	N/A	0.140	0.015	0.098	0.023	0.069	0.268	0.132	0.164	0.020

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	31	14	12	20	12
N.S.	1	1.00	1.00	0.81	0.75	1.94	0.88	0.75	1.25	0.75
time (sec)	N/A	0.133	0.002	0.077	0.029	0.072	0.022	0.124	0.150	0.012

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	49	43	53	174	697	45	121	42
N.S.	1	1.09	0.91	0.80	0.98	3.22	12.91	0.83	2.24	0.78
time (sec)	N/A	0.157	0.051	0.125	0.110	0.082	1.256	0.129	0.164	0.071

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	85	63	60	89	218	818	65	147	73
N.S.	1	1.15	0.85	0.81	1.20	2.95	11.05	0.88	1.99	0.99
time (sec)	N/A	0.175	0.079	0.205	0.133	0.087	2.354	0.124	0.156	0.032

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	113	78	70	123	252	464	93	166	105
N.S.	1	1.07	0.74	0.66	1.16	2.38	4.38	0.88	1.57	0.99
time (sec)	N/A	0.181	0.113	0.194	0.117	0.088	5.660	0.123	0.159	0.037

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	58	54	19	21	19
N.S.	1	1.00	1.00	0.80	0.76	2.32	2.16	0.76	0.84	0.76
time (sec)	N/A	0.130	0.016	0.117	0.116	0.101	0.498	0.122	0.148	0.066

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	46	99	121	40	37	36
N.S.	1	1.00	1.00	0.84	1.05	2.25	2.75	0.91	0.84	0.82
time (sec)	N/A	0.160	0.039	0.154	0.106	0.095	1.071	0.115	0.152	0.069

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	55	86	129	216	68	59	57
N.S.	1	1.00	0.81	0.74	1.16	1.74	2.92	0.92	0.80	0.77
time (sec)	N/A	0.163	0.060	0.153	0.146	0.085	2.196	0.123	0.154	0.068

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	125	437	34	44	34
N.S.	1	1.00	1.00	0.83	0.81	2.98	10.40	0.81	1.05	0.81
time (sec)	N/A	0.153	0.029	0.138	0.129	0.091	0.923	0.126	0.155	0.025

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	69	51	54	67	164	156	64	54	52
N.S.	1	1.11	0.82	0.87	1.08	2.65	2.52	1.03	0.87	0.84
time (sec)	N/A	0.160	0.058	0.183	0.107	0.106	1.553	0.126	0.154	0.079

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	71	72	104	199	226	81	72	101
N.S.	1	1.04	0.75	0.76	1.09	2.09	2.38	0.85	0.76	1.06
time (sec)	N/A	0.173	0.091	0.207	0.107	0.078	3.249	0.120	0.160	0.047

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	66	52	49	42	183	1950	42	89	48
N.S.	1	1.10	0.87	0.82	0.70	3.05	32.50	0.70	1.48	0.80
time (sec)	N/A	0.175	0.054	0.147	0.103	0.083	93.407	0.128	0.161	0.033

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	93	67	68	82	227	0	66	109	70
N.S.	1	1.15	0.83	0.84	1.01	2.80	0.00	0.81	1.35	0.86
time (sec)	N/A	0.183	0.078	0.227	0.106	0.107	0.000	0.120	0.161	0.077

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	123	82	89	121	261	1108	97	124	117
N.S.	1	1.06	0.71	0.77	1.04	2.25	9.55	0.84	1.07	1.01
time (sec)	N/A	0.200	0.130	0.219	0.109	0.105	6.387	0.128	0.159	0.087

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	132	96	98	178	159	153	211	95	0
N.S.	1	1.08	0.79	0.80	1.46	1.30	1.25	1.73	0.78	0.00
time (sec)	N/A	0.183	0.369	0.089	0.108	0.098	17.719	148.406	0.160	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	85	87	146	138	122	171	76	0
N.S.	1	1.04	0.87	0.89	1.49	1.41	1.24	1.74	0.78	0.00
time (sec)	N/A	0.172	0.216	0.076	0.111	0.113	4.478	148.622	0.160	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	71	74	108	111	97	147	56	52
N.S.	1	0.97	0.96	1.00	1.46	1.50	1.31	1.99	0.76	0.70
time (sec)	N/A	0.162	0.161	0.070	0.104	0.084	1.710	149.175	0.161	0.094

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	62	70	90	42	68	37	41
N.S.	1	1.00	1.07	1.41	1.59	2.05	0.95	1.55	0.84	0.93
time (sec)	N/A	0.151	0.049	0.070	0.106	0.101	0.842	74.381	0.160	0.416

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	55	61	54	86	68	65	43	0
N.S.	1	1.00	1.22	1.36	1.20	1.91	1.51	1.44	0.96	0.00
time (sec)	N/A	0.143	0.067	0.069	0.117	0.080	0.740	74.791	0.158	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	15	41	33	40	21
N.S.	1	1.00	1.00	0.76	0.71	0.71	1.95	1.57	1.90	1.00
time (sec)	N/A	0.118	0.018	0.066	0.029	0.088	0.608	0.127	0.159	0.139

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	24	31	34	65	50	61	32
N.S.	1	1.00	0.89	0.55	0.70	0.77	1.48	1.14	1.39	0.73
time (sec)	N/A	0.129	0.064	0.072	0.026	0.084	1.740	0.150	0.157	0.156

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	51	35	46	45	347	68	80	43
N.S.	1	1.09	0.75	0.51	0.68	0.66	5.10	1.00	1.18	0.63
time (sec)	N/A	0.142	0.072	0.073	0.030	0.067	5.415	0.132	0.168	0.165

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	62	46	61	56	559	82	99	54
N.S.	1	1.13	0.67	0.50	0.66	0.61	6.08	0.89	1.08	0.59
time (sec)	N/A	0.157	0.079	0.078	0.025	0.068	15.817	0.161	0.164	0.183

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	156	107	109	212	181	178	0	114	0
N.S.	1	1.08	0.74	0.76	1.47	1.26	1.24	0.00	0.79	0.00
time (sec)	N/A	0.195	0.383	0.077	0.112	0.079	50.978	0.000	0.155	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	96	98	178	160	153	0	95	0
N.S.	1	1.05	0.80	0.82	1.48	1.33	1.28	0.00	0.79	0.00
time (sec)	N/A	0.180	0.286	0.082	0.120	0.105	10.165	0.000	0.159	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	84	87	144	137	124	0	76	0
N.S.	1	1.00	0.88	0.91	1.50	1.43	1.29	0.00	0.79	0.00
time (sec)	N/A	0.177	0.240	0.075	0.175	0.090	3.176	0.000	0.150	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	62	73	107	116	75	85	57	0
N.S.	1	0.94	0.86	1.01	1.49	1.61	1.04	1.18	0.79	0.00
time (sec)	N/A	0.149	0.078	0.074	0.128	0.119	1.462	75.365	0.147	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	62	71	84	106	92	75	59	0
N.S.	1	1.02	0.98	1.13	1.33	1.68	1.46	1.19	0.94	0.00
time (sec)	N/A	0.149	0.138	0.081	0.107	0.130	1.304	75.481	0.155	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	55	67	67	106	71	79	55	0
N.S.	1	0.99	0.82	1.00	1.00	1.58	1.06	1.18	0.82	0.00
time (sec)	N/A	0.148	0.099	0.077	0.108	0.100	1.418	75.738	0.146	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	31	65	33	61	31
N.S.	1	1.00	1.00	0.76	0.71	1.48	3.10	1.57	2.90	1.48
time (sec)	N/A	0.135	0.028	0.069	0.028	0.073	1.662	0.134	0.163	0.181

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	24	31	45	88	52	80	42
N.S.	1	1.00	0.66	0.55	0.70	1.02	2.00	1.18	1.82	0.95
time (sec)	N/A	0.131	0.088	0.073	0.028	0.079	5.344	0.139	0.163	0.172

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	40	35	46	56	406	66	99	53
N.S.	1	1.09	0.59	0.51	0.68	0.82	5.97	0.97	1.46	0.78
time (sec)	N/A	0.162	0.097	0.078	0.031	0.099	15.743	0.136	0.162	0.181

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	51	46	61	67	631	84	118	64
N.S.	1	1.13	0.55	0.50	0.66	0.73	6.86	0.91	1.28	0.70
time (sec)	N/A	0.161	0.108	0.082	0.024	0.092	42.675	0.141	0.171	0.191

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	180	118	120	244	203	207	0	133	0
N.S.	1	1.07	0.70	0.71	1.45	1.21	1.23	0.00	0.79	0.00
time (sec)	N/A	0.215	0.466	0.082	0.114	0.096	155.942	0.000	0.162	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	150	107	109	212	182	180	0	114	0
N.S.	1	1.04	0.74	0.76	1.47	1.26	1.25	0.00	0.79	0.00
time (sec)	N/A	0.195	0.386	0.076	0.118	0.086	28.027	0.000	0.160	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	95	98	176	159	155	0	95	0
N.S.	1	1.00	0.79	0.82	1.47	1.32	1.29	0.00	0.79	0.00
time (sec)	N/A	0.175	0.317	0.076	0.109	0.078	7.009	0.000	0.155	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	92	73	84	141	138	102	101	76	0
N.S.	1	0.96	0.76	0.88	1.47	1.44	1.06	1.05	0.79	0.00
time (sec)	N/A	0.158	0.095	0.076	0.112	0.097	3.027	75.004	0.161	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	88	81	84	125	134	126	90	82	0
N.S.	1	0.94	0.86	0.89	1.33	1.43	1.34	0.96	0.87	0.00
time (sec)	N/A	0.160	0.212	0.076	0.114	0.093	2.864	75.316	0.161	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	88	77	82	100	135	99	94	84	0
N.S.	1	0.99	0.87	0.92	1.12	1.52	1.11	1.06	0.94	0.00
time (sec)	N/A	0.159	0.168	0.081	0.114	0.102	2.703	75.410	0.163	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	87	68	80	82	134	97	96	86	0
N.S.	1	0.96	0.75	0.88	0.90	1.47	1.07	1.05	0.95	0.00
time (sec)	N/A	0.176	0.127	0.083	0.136	0.108	3.900	76.507	0.164	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	42	90	33	80	42
N.S.	1	1.00	1.00	0.76	0.71	2.00	4.29	1.57	3.81	2.00
time (sec)	N/A	0.116	0.031	0.076	0.031	0.091	5.428	0.132	0.160	0.183

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	24	31	56	114	50	99	53
N.S.	1	1.00	0.66	0.55	0.70	1.27	2.59	1.14	2.25	1.20
time (sec)	N/A	0.131	0.107	0.080	0.051	0.087	16.299	0.153	0.160	0.192

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	40	35	46	67	464	68	118	64
N.S.	1	1.09	0.59	0.51	0.68	0.99	6.82	1.00	1.74	0.94
time (sec)	N/A	0.159	0.116	0.080	0.031	0.075	44.574	0.141	0.170	0.207

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	51	46	61	78	702	82	137	75
N.S.	1	1.13	0.55	0.50	0.66	0.85	7.63	0.89	1.49	0.82
time (sec)	N/A	0.167	0.128	0.085	0.027	0.068	134.542	0.161	0.176	0.238

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	124	84	71	163	137	117	184	85	0
N.S.	1	1.15	0.78	0.66	1.51	1.27	1.08	1.70	0.79	0.00
time (sec)	N/A	0.190	0.262	0.099	0.106	0.129	17.730	11.515	0.160	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	95	73	63	134	118	90	148	68	0
N.S.	1	1.13	0.87	0.75	1.60	1.40	1.07	1.76	0.81	0.00
time (sec)	N/A	0.167	0.205	0.063	0.112	0.097	4.161	11.387	0.160	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	65	55	98	98	71	134	51	46
N.S.	1	1.03	1.02	0.86	1.53	1.53	1.11	2.09	0.80	0.72
time (sec)	N/A	0.149	0.149	0.063	0.109	0.099	1.609	11.375	0.156	0.063

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	49	68	83	37	66	37	40
N.S.	1	1.00	1.15	1.22	1.70	2.08	0.92	1.65	0.92	1.00
time (sec)	N/A	0.140	0.044	0.077	0.119	0.082	0.780	5.800	0.159	0.396

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	49	54	84	48	63	43	0
N.S.	1	1.00	1.15	1.20	1.32	2.05	1.17	1.54	1.05	0.00
time (sec)	N/A	0.142	0.050	0.066	0.105	0.081	0.705	5.744	0.161	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	37	29	36	18
N.S.	1	1.00	1.00	0.72	0.67	0.67	2.06	1.61	2.00	1.00
time (sec)	N/A	0.119	0.019	0.061	0.024	0.069	0.580	0.139	0.151	0.134

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	18	26	25	56	42	53	26
N.S.	1	1.00	0.82	0.47	0.68	0.66	1.47	1.11	1.39	0.68
time (sec)	N/A	0.131	0.052	0.073	0.031	0.078	1.719	0.134	0.145	0.140

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	62	40	27	41	34	270	57	70	34
N.S.	1	1.05	0.68	0.46	0.69	0.58	4.58	0.97	1.19	0.58
time (sec)	N/A	0.137	0.059	0.072	0.029	0.095	5.365	0.122	0.149	0.138

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	48	35	56	42	428	68	86	42
N.S.	1	1.08	0.60	0.44	0.70	0.52	5.35	0.85	1.08	0.52
time (sec)	N/A	0.146	0.062	0.071	0.046	0.092	15.220	0.136	0.159	0.163

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	147	92	79	194	153	136	278	101	0
N.S.	1	1.16	0.72	0.62	1.53	1.20	1.07	2.19	0.80	0.00
time (sec)	N/A	0.184	0.332	0.074	0.132	0.099	50.683	16.739	0.157	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	118	83	71	163	134	117	248	84	0
N.S.	1	1.11	0.78	0.67	1.54	1.26	1.10	2.34	0.79	0.00
time (sec)	N/A	0.170	0.266	0.062	0.116	0.078	9.855	16.993	0.158	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	84	74	63	132	121	92	210	69	0
N.S.	1	1.01	0.89	0.76	1.59	1.46	1.11	2.53	0.83	0.00
time (sec)	N/A	0.187	0.224	0.062	0.109	0.081	3.071	16.617	0.147	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	54	54	98	102	76	77	52	0
N.S.	1	1.02	0.87	0.87	1.58	1.65	1.23	1.24	0.84	0.00
time (sec)	N/A	0.150	0.068	0.061	0.106	0.109	1.422	5.859	0.154	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	60	59	55	81	96	73	68	56	0
N.S.	1	1.05	1.04	0.96	1.42	1.68	1.28	1.19	0.98	0.00
time (sec)	N/A	0.151	0.137	0.069	0.129	0.093	1.268	5.869	0.151	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	55	67	105	70	77	54	0
N.S.	1	1.00	0.89	0.89	1.08	1.69	1.13	1.24	0.87	0.00
time (sec)	N/A	0.147	0.086	0.073	0.111	0.096	1.356	6.103	0.152	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	170	100	87	223	169	158	403	117	0
N.S.	1	1.15	0.68	0.59	1.51	1.14	1.07	2.72	0.79	0.00
time (sec)	N/A	0.208	0.417	0.078	0.129	0.087	153.942	22.494	0.161	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	136	92	79	194	152	138	341	101	0
N.S.	1	1.07	0.72	0.62	1.53	1.20	1.09	2.69	0.80	0.00
time (sec)	N/A	0.190	0.359	0.064	0.109	0.081	27.762	22.349	0.164	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	107	84	71	161	137	119	311	85	0
N.S.	1	1.01	0.79	0.67	1.52	1.29	1.12	2.93	0.80	0.00
time (sec)	N/A	0.168	0.291	0.060	0.104	0.104	6.751	22.682	0.161	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	63	63	129	120	97	90	69	0
N.S.	1	1.04	0.76	0.76	1.55	1.45	1.17	1.08	0.83	0.00
time (sec)	N/A	0.169	0.078	0.076	0.105	0.082	2.840	5.820	0.156	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	70	63	113	113	94	77	71	0
N.S.	1	1.02	0.86	0.78	1.40	1.40	1.16	0.95	0.88	0.00
time (sec)	N/A	0.162	0.191	0.066	0.108	0.084	2.702	5.664	0.161	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	63	63	96	120	88	87	78	0
N.S.	1	1.05	0.79	0.79	1.20	1.50	1.10	1.09	0.98	0.00
time (sec)	N/A	0.163	0.100	0.068	0.112	0.080	2.560	5.954	0.158	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	108	84	87	146	137	128	88	76	0
N.S.	1	1.07	0.83	0.86	1.45	1.36	1.27	0.87	0.75	0.00
time (sec)	N/A	0.169	0.213	0.075	0.114	0.076	7.313	75.420	0.159	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	74	76	112	116	100	76	57	0
N.S.	1	1.01	0.96	0.99	1.45	1.51	1.30	0.99	0.74	0.00
time (sec)	N/A	0.164	0.154	0.072	0.111	0.075	2.237	75.068	0.157	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	58	65	73	88	44	63	38	44
N.S.	1	1.00	1.21	1.35	1.52	1.83	0.92	1.31	0.79	0.92
time (sec)	N/A	0.151	0.117	0.069	0.109	0.089	1.038	75.197	0.147	0.338

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	48	41	54	22	38	25	30
N.S.	1	1.00	1.07	1.71	1.46	1.93	0.79	1.36	0.89	1.07
time (sec)	N/A	0.139	0.032	0.067	0.113	0.094	0.491	76.379	0.145	0.019

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	19	33	23	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	1.00	1.74	1.21	0.79
time (sec)	N/A	0.118	0.016	0.066	0.034	0.088	0.406	0.151	0.149	0.234

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	31	23	42	48	40	25
N.S.	1	1.00	0.61	0.50	0.70	0.52	0.95	1.09	0.91	0.57
time (sec)	N/A	0.126	0.055	0.072	0.033	0.084	0.776	0.126	0.155	0.207

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	40	35	46	34	287	66	61	36
N.S.	1	1.09	0.59	0.51	0.68	0.50	4.22	0.97	0.90	0.53
time (sec)	N/A	0.159	0.065	0.072	0.026	0.071	2.421	0.130	0.159	0.238

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	51	46	61	45	488	84	80	47
N.S.	1	1.13	0.55	0.50	0.66	0.49	5.30	0.91	0.87	0.51
time (sec)	N/A	0.161	0.071	0.073	0.025	0.073	6.802	0.131	0.159	0.236

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	84	119	131	172	105	131	85	0
N.S.	1	1.04	0.85	1.20	1.32	1.74	1.06	1.32	0.86	0.00
time (sec)	N/A	0.174	0.224	0.085	0.135	0.089	4.246	15.230	0.157	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	73	65	106	92	142	71	115	68	0
N.S.	1	1.07	0.96	1.56	1.35	2.09	1.04	1.69	1.00	0.00
time (sec)	N/A	0.157	0.176	0.080	0.122	0.103	1.604	15.259	0.162	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	58	0	57	116	46	85	56	0
N.S.	1	1.00	1.21	0.00	1.19	2.42	0.96	1.77	1.17	0.00
time (sec)	N/A	0.138	0.095	0.000	0.130	0.112	0.787	15.393	0.150	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	22	17	45	30	22
N.S.	1	1.00	1.00	0.84	0.79	1.16	0.89	2.37	1.58	1.16
time (sec)	N/A	0.131	0.018	0.067	0.028	0.100	0.402	0.128	0.161	0.195

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	22	32	34	41	82	39	39
N.S.	1	1.00	0.64	0.56	0.82	0.87	1.05	2.10	1.00	1.00
time (sec)	N/A	0.136	0.058	0.084	0.027	0.087	0.654	0.134	0.165	0.230

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	69	38	33	50	49	219	98	55	46
N.S.	1	1.10	0.60	0.52	0.79	0.78	3.48	1.56	0.87	0.73
time (sec)	N/A	0.158	0.079	0.085	0.033	0.071	1.490	0.137	0.158	0.255

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	99	49	44	64	58	348	121	70	58
N.S.	1	1.14	0.56	0.51	0.74	0.67	4.00	1.39	0.80	0.67
time (sec)	N/A	0.166	0.092	0.088	0.031	0.107	4.615	0.140	0.161	0.276

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	102	80	147	109	211	396	195	136	0
N.S.	1	1.09	0.85	1.56	1.16	2.24	4.21	2.07	1.45	0.00
time (sec)	N/A	0.178	0.245	0.098	0.111	0.122	3.273	15.246	0.154	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	68	0	69	183	328	165	94	0
N.S.	1	1.03	0.94	0.00	0.96	2.54	4.56	2.29	1.31	0.00
time (sec)	N/A	0.161	0.185	0.000	0.107	0.106	1.656	15.088	0.153	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	33	42	86	52	36
N.S.	1	1.00	1.00	0.76	0.71	1.57	2.00	4.10	2.48	1.71
time (sec)	N/A	0.126	0.020	0.066	0.027	0.091	0.588	0.142	0.146	0.144

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	29	24	27	43	92	81	61	54
N.S.	1	1.00	0.67	0.56	0.63	1.00	2.14	1.88	1.42	1.26
time (sec)	N/A	0.139	0.061	0.074	0.039	0.097	0.789	0.137	0.153	0.235

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	40	35	46	58	153	159	74	71
N.S.	1	1.06	0.62	0.55	0.72	0.91	2.39	2.48	1.16	1.11
time (sec)	N/A	0.152	0.079	0.088	0.042	0.079	1.501	0.141	0.167	0.254

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	96	49	44	64	71	337	175	92	88
N.S.	1	1.14	0.58	0.52	0.76	0.85	4.01	2.08	1.10	1.05
time (sec)	N/A	0.169	0.088	0.086	0.026	0.077	2.746	0.156	0.154	0.283

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	101	74	63	134	121	95	76	69	0
N.S.	1	1.15	0.84	0.72	1.52	1.38	1.08	0.86	0.78	0.00
time (sec)	N/A	0.180	0.195	0.069	0.113	0.098	7.097	5.827	0.150	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	65	55	102	102	75	67	52	0
N.S.	1	1.07	0.97	0.82	1.52	1.52	1.12	1.00	0.78	0.00
time (sec)	N/A	0.155	0.147	0.059	0.105	0.118	2.140	5.907	0.146	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	49	70	84	54	61	37	43
N.S.	1	1.00	1.33	1.14	1.63	1.95	1.26	1.42	0.86	1.00
time (sec)	N/A	0.153	0.099	0.057	0.104	0.090	0.944	5.785	0.152	0.326

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	18	41	52	24	37	25	30
N.S.	1	1.00	1.25	0.75	1.71	2.17	1.00	1.54	1.04	1.25
time (sec)	N/A	0.129	0.030	0.055	0.102	0.106	0.453	5.972	0.151	0.021

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	29	20	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	1.81	1.25	0.75
time (sec)	N/A	0.124	0.015	0.053	0.024	0.078	0.391	0.130	0.155	0.197

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	18	26	17	34	40	35	17
N.S.	1	1.00	0.61	0.47	0.68	0.45	0.89	1.05	0.92	0.45
time (sec)	N/A	0.135	0.046	0.061	0.024	0.068	0.760	0.133	0.152	0.181

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	62	32	27	41	26	224	55	54	26
N.S.	1	1.05	0.54	0.46	0.69	0.44	3.80	0.93	0.92	0.44
time (sec)	N/A	0.138	0.055	0.059	0.026	0.067	2.291	0.141	0.155	0.206

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	40	35	56	34	374	70	70	33
N.S.	1	1.08	0.50	0.44	0.70	0.42	4.68	0.88	0.88	0.41
time (sec)	N/A	0.148	0.057	0.062	0.032	0.136	6.517	0.137	0.163	0.229

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	97	73	63	119	149	80	119	74	0
N.S.	1	1.13	0.85	0.73	1.38	1.73	0.93	1.38	0.86	0.00
time (sec)	N/A	0.171	0.206	0.074	0.107	0.119	4.066	1.400	0.170	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	68	62	55	90	131	58	106	65	0
N.S.	1	1.10	1.00	0.89	1.45	2.11	0.94	1.71	1.05	0.00
time (sec)	N/A	0.152	0.152	0.092	0.106	0.108	1.513	1.388	0.157	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	50	48	57	114	41	82	56	0
N.S.	1	1.00	1.14	1.09	1.30	2.59	0.93	1.86	1.27	0.00
time (sec)	N/A	0.146	0.058	0.057	0.108	0.107	0.737	1.393	0.154	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	15	44	26	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.00	2.93	1.73	0.73
time (sec)	N/A	0.124	0.016	0.059	0.024	0.079	0.402	0.132	0.162	0.185

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	18	26	28	34	74	34	17
N.S.	1	1.00	0.66	0.56	0.81	0.88	1.06	2.31	1.06	0.53
time (sec)	N/A	0.126	0.050	0.070	0.028	0.081	0.639	0.136	0.149	0.201

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	32	27	41	39	170	86	48	37
N.S.	1	1.06	0.60	0.51	0.77	0.74	3.21	1.62	0.91	0.70
time (sec)	N/A	0.135	0.068	0.072	0.030	0.095	1.449	0.131	0.155	0.223

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	39	35	56	47	269	107	59	46
N.S.	1	1.08	0.53	0.47	0.76	0.64	3.64	1.45	0.80	0.62
time (sec)	N/A	0.170	0.072	0.073	0.027	0.080	4.520	0.136	0.160	0.234

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	97	74	63	105	183	308	179	124	0
N.S.	1	1.14	0.87	0.74	1.24	2.15	3.62	2.11	1.46	0.00
time (sec)	N/A	0.170	0.201	0.084	0.112	0.082	3.064	1.433	0.151	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	70	58	55	69	168	257	154	92	0
N.S.	1	1.04	0.87	0.82	1.03	2.51	3.84	2.30	1.37	0.00
time (sec)	N/A	0.162	0.102	0.060	0.104	0.103	1.538	1.410	0.151	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	27	27	82	49	12
N.S.	1	1.00	1.00	0.72	0.67	1.50	1.50	4.56	2.72	0.67
time (sec)	N/A	0.129	0.017	0.060	0.039	0.079	0.581	0.139	0.148	0.147

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	18	24	32	75	79	55	42
N.S.	1	1.00	0.62	0.49	0.65	0.86	2.03	2.14	1.49	1.14
time (sec)	N/A	0.139	0.052	0.059	0.027	0.070	0.774	0.123	0.148	0.238

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	32	27	40	45	117	145	66	57
N.S.	1	1.00	0.58	0.49	0.73	0.82	2.13	2.64	1.20	1.04
time (sec)	N/A	0.144	0.068	0.072	0.029	0.070	1.448	0.143	0.145	0.235

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	74	40	35	55	55	257	158	81	71
N.S.	1	1.04	0.56	0.49	0.77	0.77	3.62	2.23	1.14	1.00
time (sec)	N/A	0.150	0.069	0.072	0.031	0.110	2.656	0.145	0.153	0.276

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	33	73	34	110	30	28	0
N.S.	1	1.00	0.98	0.77	1.70	0.79	2.56	0.70	0.65	0.00
time (sec)	N/A	0.145	0.043	0.067	0.032	0.072	1.276	0.133	0.144	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	27	49	28	63	22	18	26
N.S.	1	1.00	2.32	1.23	2.23	1.27	2.86	1.00	0.82	1.18
time (sec)	N/A	0.135	0.035	0.050	0.035	0.080	0.802	0.134	0.149	0.417

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	18	7	27	18	26	14	10	14
N.S.	1	1.00	2.25	0.88	3.38	2.25	3.25	1.75	1.25	1.75
time (sec)	N/A	0.136	0.020	0.046	0.033	0.068	0.451	0.126	0.151	0.133

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	16	31	10	16	10
N.S.	1	1.00	1.00	0.79	0.71	1.14	2.21	0.71	1.14	0.71
time (sec)	N/A	0.117	0.013	0.049	0.027	0.077	0.388	0.130	0.155	0.185

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	16	21	25	129	15	28	14
N.S.	1	1.00	0.64	0.48	0.64	0.76	3.91	0.45	0.85	0.42
time (sec)	N/A	0.127	0.040	0.049	0.027	0.080	0.717	0.121	0.159	0.146

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	54	26	21	31	30	301	22	39	20
N.S.	1	1.10	0.53	0.43	0.63	0.61	6.14	0.45	0.80	0.41
time (sec)	N/A	0.142	0.045	0.053	0.024	0.084	2.157	0.122	0.158	0.152

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	17	41	26	53	22	21	27
N.S.	1	1.00	1.50	0.85	2.05	1.30	2.65	1.10	1.05	1.35
time (sec)	N/A	0.132	0.029	0.057	0.105	0.093	0.459	0.122	0.148	0.114

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	17	21	27	42	13	24	27
N.S.	1	1.00	1.90	0.85	1.05	1.35	2.10	0.65	1.20	1.35
time (sec)	N/A	0.139	0.063	0.080	0.110	0.080	0.451	0.123	0.153	0.085

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	C	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	31	41	26	42	23	21	30
N.S.	1	1.00	1.36	1.41	1.86	1.18	1.91	1.05	0.95	1.36
time (sec)	N/A	0.129	0.027	0.071	0.102	0.094	0.458	0.124	0.149	0.101

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	19	21	27	54	22	24	30
N.S.	1	1.00	1.11	0.70	0.78	1.00	2.00	0.81	0.89	1.11
time (sec)	N/A	0.132	0.094	0.059	0.104	0.087	0.460	0.122	0.150	0.039

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	9	27	17	29	14	14	14
N.S.	1	1.00	1.50	0.75	2.25	1.42	2.42	1.17	1.17	1.17
time (sec)	N/A	0.121	0.020	0.056	0.028	0.070	0.458	0.119	0.153	0.084

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	9	14	19	24	8	18	16
N.S.	1	1.00	3.33	0.75	1.17	1.58	2.00	0.67	1.50	1.33
time (sec)	N/A	0.132	0.028	0.075	0.103	0.066	0.442	0.127	0.151	0.029

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	137	98	102	170	173	323	257	102	0
N.S.	1	1.08	0.77	0.80	1.34	1.36	2.54	2.02	0.80	0.00
time (sec)	N/A	0.196	0.286	0.099	0.112	0.093	17.809	149.926	0.147	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	106	87	91	135	151	260	207	82	0
N.S.	1	1.04	0.85	0.89	1.32	1.48	2.55	2.03	0.80	0.00
time (sec)	N/A	0.173	0.229	0.078	0.110	0.094	4.419	150.527	0.151	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	75	78	95	127	207	166	62	58
N.S.	1	0.97	0.97	1.01	1.23	1.65	2.69	2.16	0.81	0.75
time (sec)	N/A	0.152	0.174	0.076	0.109	0.090	1.759	150.141	0.150	0.053

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	57	66	52	103	119	74	42	43
N.S.	1	1.00	1.24	1.43	1.13	2.24	2.59	1.61	0.91	0.93
time (sec)	N/A	0.153	0.102	0.074	0.173	0.089	0.879	76.673	0.152	0.358

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	66	35	100	148	73	48	0
N.S.	1	1.00	1.21	1.40	0.74	2.13	3.15	1.55	1.02	0.00
time (sec)	N/A	0.152	0.081	0.078	0.119	0.091	0.812	75.546	0.154	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	23	88	42	41	21
N.S.	1	1.00	1.00	0.77	0.73	1.05	4.00	1.91	1.86	0.95
time (sec)	N/A	0.127	0.050	0.097	0.028	0.070	0.654	0.129	0.158	0.161

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	25	33	34	241	61	64	32
N.S.	1	1.00	0.89	0.54	0.72	0.74	5.24	1.33	1.39	0.70
time (sec)	N/A	0.143	0.058	0.076	0.025	0.077	1.819	0.132	0.157	0.161

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	52	36	49	46	707	79	85	43
N.S.	1	1.08	0.73	0.51	0.69	0.65	9.96	1.11	1.20	0.61
time (sec)	N/A	0.150	0.065	0.079	0.050	0.101	7.712	0.132	0.157	0.169

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	108	63	47	65	57	1139	97	105	54
N.S.	1	1.12	0.66	0.49	0.68	0.59	11.86	1.01	1.09	0.56
time (sec)	N/A	0.160	0.069	0.082	0.042	0.089	68.305	0.124	0.161	0.182

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	162	108	113	207	194	376	0	122	0
N.S.	1	1.08	0.72	0.75	1.38	1.29	2.51	0.00	0.81	0.00
time (sec)	N/A	0.198	0.383	0.078	0.112	0.100	50.821	0.000	0.163	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	131	99	102	170	172	323	0	102	0
N.S.	1	1.05	0.79	0.82	1.36	1.38	2.58	0.00	0.82	0.00
time (sec)	N/A	0.212	0.312	0.078	0.110	0.090	10.077	0.000	0.159	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	86	91	133	150	264	0	82	0
N.S.	1	1.00	0.86	0.91	1.33	1.50	2.64	0.00	0.82	0.00
time (sec)	N/A	0.174	0.252	0.076	0.152	0.092	3.203	0.000	0.154	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	71	72	77	93	128	190	95	62	0
N.S.	1	0.95	0.96	1.03	1.24	1.71	2.53	1.27	0.83	0.00
time (sec)	N/A	0.164	0.185	0.077	0.122	0.078	1.482	76.011	0.157	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	65	74	68	118	197	82	65	0
N.S.	1	1.00	0.97	1.10	1.01	1.76	2.94	1.22	0.97	0.00
time (sec)	N/A	0.155	0.145	0.079	0.111	0.093	1.367	76.379	0.148	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	71	49	124	187	94	60	0
N.S.	1	1.00	0.96	1.01	0.70	1.77	2.67	1.34	0.86	0.00
time (sec)	N/A	0.163	0.104	0.080	0.107	0.100	1.494	75.973	0.148	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	187	120	124	242	217	435	0	142	0
N.S.	1	1.07	0.69	0.71	1.38	1.24	2.49	0.00	0.81	0.00
time (sec)	N/A	0.224	0.486	0.083	0.114	0.083	150.799	0.000	0.160	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	156	109	113	207	195	379	0	122	0
N.S.	1	1.04	0.73	0.75	1.38	1.30	2.53	0.00	0.81	0.00
time (sec)	N/A	0.195	0.404	0.102	0.138	0.079	27.408	0.000	0.167	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	97	102	168	173	326	0	102	0
N.S.	1	1.00	0.78	0.82	1.34	1.38	2.61	0.00	0.82	0.00
time (sec)	N/A	0.179	0.328	0.077	0.146	0.078	6.796	0.000	0.153	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	96	83	88	130	151	246	112	82	0
N.S.	1	0.96	0.83	0.88	1.30	1.51	2.46	1.12	0.82	0.00
time (sec)	N/A	0.163	0.251	0.074	0.115	0.082	2.936	75.436	0.156	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	92	83	88	112	146	267	100	89	0
N.S.	1	0.94	0.85	0.90	1.14	1.49	2.72	1.02	0.91	0.00
time (sec)	N/A	0.162	0.217	0.078	0.111	0.083	2.822	75.437	0.152	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	92	79	86	84	148	245	112	91	0
N.S.	1	0.99	0.85	0.92	0.90	1.59	2.63	1.20	0.98	0.00
time (sec)	N/A	0.161	0.167	0.081	0.163	0.085	2.722	75.132	0.154	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	112	86	91	135	150	270	116	82	0
N.S.	1	1.07	0.82	0.87	1.29	1.43	2.57	1.10	0.78	0.00
time (sec)	N/A	0.172	0.216	0.100	0.122	0.076	7.097	76.009	0.156	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	75	80	98	128	214	85	62	0
N.S.	1	1.01	0.94	1.00	1.22	1.60	2.68	1.06	0.78	0.00
time (sec)	N/A	0.159	0.169	0.076	0.114	0.080	2.191	75.401	0.150	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	61	70	56	102	121	73	43	47
N.S.	1	1.00	1.22	1.40	1.12	2.04	2.42	1.46	0.86	0.94
time (sec)	N/A	0.144	0.108	0.076	0.114	0.076	1.016	75.628	0.154	0.330

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	39	51	21	66	54	45	28	27
N.S.	1	1.00	1.34	1.76	0.72	2.28	1.86	1.55	0.97	0.93
time (sec)	N/A	0.131	0.056	0.072	0.129	0.083	0.490	75.243	0.155	0.018

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	46	35	25	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	2.30	1.75	1.25	0.80
time (sec)	N/A	0.121	0.018	0.072	0.027	0.074	0.423	0.132	0.156	0.232

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	28	23	32	22	177	54	43	26
N.S.	1	1.00	0.61	0.50	0.70	0.48	3.85	1.17	0.93	0.57
time (sec)	N/A	0.133	0.058	0.073	0.032	0.068	0.801	0.131	0.154	0.217

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	41	36	49	35	586	72	65	37
N.S.	1	1.08	0.58	0.51	0.69	0.49	8.25	1.01	0.92	0.52
time (sec)	N/A	0.147	0.068	0.075	0.034	0.091	2.828	0.131	0.155	0.231

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	108	52	47	65	46	994	90	85	48
N.S.	1	1.12	0.54	0.49	0.68	0.48	10.35	0.94	0.89	0.50
time (sec)	N/A	0.164	0.072	0.075	0.033	0.071	24.336	0.133	0.160	0.247

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	86	127	118	190	224	154	92	0
N.S.	1	1.04	0.83	1.23	1.15	1.84	2.17	1.50	0.89	0.00
time (sec)	N/A	0.172	0.239	0.089	0.115	0.085	4.232	15.241	0.149	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	76	68	114	75	161	155	130	75	0
N.S.	1	1.07	0.96	1.61	1.06	2.27	2.18	1.83	1.06	0.00
time (sec)	N/A	0.158	0.202	0.087	0.110	0.082	1.665	15.242	0.151	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	0	38	137	102	98	62	0
N.S.	1	1.00	1.20	0.00	0.76	2.74	2.04	1.96	1.24	0.00
time (sec)	N/A	0.147	0.127	0.000	0.124	0.088	0.841	15.356	0.160	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	25	44	53	34	24
N.S.	1	1.00	1.00	0.85	0.80	1.25	2.20	2.65	1.70	1.20
time (sec)	N/A	0.122	0.022	0.072	0.038	0.074	0.433	0.123	0.151	0.213

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	26	23	34	38	112	94	42	42
N.S.	1	1.00	0.63	0.56	0.83	0.93	2.73	2.29	1.02	1.02
time (sec)	N/A	0.134	0.088	0.088	0.028	0.078	0.686	0.129	0.144	0.239

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	72	39	34	52	51	452	112	58	48
N.S.	1	1.09	0.59	0.52	0.79	0.77	6.85	1.70	0.88	0.73
time (sec)	N/A	0.148	0.100	0.086	0.041	0.084	1.642	0.131	0.162	0.264

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	103	50	45	68	62	709	137	73	59
N.S.	1	1.13	0.55	0.49	0.75	0.68	7.79	1.51	0.80	0.65
time (sec)	N/A	0.162	0.105	0.090	0.026	0.074	9.304	0.141	0.163	0.272

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	106	82	160	94	224	971	221	150	0
N.S.	1	1.07	0.83	1.62	0.95	2.26	9.81	2.23	1.52	0.00
time (sec)	N/A	0.175	0.260	0.103	0.113	0.093	3.635	15.174	0.165	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	78	70	0	52	197	833	194	104	0
N.S.	1	1.04	0.93	0.00	0.69	2.63	11.11	2.59	1.39	0.00
time (sec)	N/A	0.159	0.192	0.000	0.108	0.102	1.868	15.294	0.160	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	34	95	102	59	37
N.S.	1	1.00	1.00	0.77	0.73	1.55	4.32	4.64	2.68	1.68
time (sec)	N/A	0.119	0.022	0.073	0.030	0.076	0.613	0.153	0.158	0.159

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	25	30	44	197	96	67	56
N.S.	1	1.00	0.67	0.56	0.67	0.98	4.38	2.13	1.49	1.24
time (sec)	N/A	0.131	0.079	0.076	0.026	0.081	0.832	0.134	0.154	0.242

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	41	36	50	59	314	189	80	73
N.S.	1	1.06	0.61	0.54	0.75	0.88	4.69	2.82	1.19	1.09
time (sec)	N/A	0.143	0.147	0.090	0.030	0.074	1.562	0.146	0.154	0.292

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	100	50	45	68	70	688	207	98	92
N.S.	1	1.14	0.57	0.51	0.77	0.80	7.82	2.35	1.11	1.05
time (sec)	N/A	0.160	0.149	0.095	0.027	0.075	3.854	0.163	0.154	0.297

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	50	44	39	63	37	112	24	37	0
N.S.	1	1.06	0.94	0.83	1.34	0.79	2.38	0.51	0.79	0.00
time (sec)	N/A	0.148	0.079	0.092	0.104	0.082	1.952	0.128	0.152	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	35	34	37	32	54	17	26	31
N.S.	1	1.17	1.52	1.48	1.61	1.39	2.35	0.74	1.13	1.35
time (sec)	N/A	0.142	0.052	0.062	0.105	0.073	0.838	0.110	0.150	0.330

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	14	19	20	6	15	16
N.S.	1	1.00	5.00	0.88	1.75	2.38	2.50	0.75	1.88	2.00
time (sec)	N/A	0.130	0.027	0.061	0.105	0.078	0.432	0.117	0.144	0.081

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	32	30	19	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	2.00	1.88	1.19	0.75
time (sec)	N/A	0.123	0.013	0.063	0.025	0.074	0.374	0.126	0.161	0.208

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	18	25	17	60	65	33	17
N.S.	1	1.00	0.62	0.49	0.68	0.46	1.62	1.76	0.89	0.46
time (sec)	N/A	0.132	0.041	0.075	0.026	0.073	0.685	0.128	0.156	0.161

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	28	23	37	22	92	97	46	22
N.S.	1	1.09	0.51	0.42	0.67	0.40	1.67	1.76	0.84	0.40
time (sec)	N/A	0.142	0.044	0.075	0.033	0.071	2.058	0.139	0.181	0.160

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	170	50	0	0	0	42	0	37	0
N.S.	1	3.33	0.98	0.00	0.00	0.00	0.82	0.00	0.73	0.00
time (sec)	N/A	0.380	10.026	0.000	0.000	0.000	1.229	0.000	0.218	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	343	51	0	0	0	48	0	35	0
N.S.	1	6.47	0.96	0.00	0.00	0.00	0.91	0.00	0.66	0.00
time (sec)	N/A	0.355	10.019	0.000	0.000	0.000	1.235	0.000	0.218	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	332	48	0	0	0	44	0	33	0
N.S.	1	1.04	0.15	0.00	0.00	0.00	0.14	0.00	0.10	0.00
time (sec)	N/A	0.329	10.014	0.000	0.000	0.000	1.217	0.000	0.207	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	167	49	0	0	0	46	0	35	0
N.S.	1	1.06	0.31	0.00	0.00	0.00	0.29	0.00	0.22	0.00
time (sec)	N/A	0.347	10.014	0.000	0.000	0.000	1.222	0.000	0.194	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	53	1742	117	53	56
N.S.	1	1.00	0.64	0.60	0.78	0.74	24.19	1.62	0.74	0.78
time (sec)	N/A	0.170	0.022	0.179	0.039	0.073	1.278	0.120	0.150	0.085

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	42	666	92	42	37
N.S.	1	1.00	0.66	0.60	0.77	0.79	12.57	1.74	0.79	0.70
time (sec)	N/A	0.161	0.020	0.172	0.029	0.076	0.847	0.122	0.144	0.026

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	21	26	30	202	67	30	25
N.S.	1	1.00	1.00	0.62	0.76	0.88	5.94	1.97	0.88	0.74
time (sec)	N/A	0.145	0.016	0.159	0.030	0.067	0.560	0.127	0.156	0.018

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.118	0.001	0.075	0.024	0.067	0.016	0.118	0.160	0.012

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	113	87	86	91	180	87	159	107
N.S.	1	0.99	1.24	0.96	0.95	1.00	1.98	0.96	1.75	1.18
time (sec)	N/A	0.181	0.065	0.640	0.120	0.096	0.992	0.419	0.153	0.040

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	96	119	95	93	135	643	96	168	117
N.S.	1	0.99	1.23	0.98	0.96	1.39	6.63	0.99	1.73	1.21
time (sec)	N/A	0.180	0.130	0.227	0.123	0.093	1.076	0.442	0.164	0.040

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	124	129	118	139	184	2266	128	204	196
N.S.	1	0.98	1.02	0.93	1.09	1.45	17.84	1.01	1.61	1.54
time (sec)	N/A	0.195	0.185	0.229	0.124	0.095	1.579	0.426	0.168	0.157

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	53	1742	117	53	56
N.S.	1	1.00	0.64	0.60	0.78	0.74	24.19	1.62	0.74	0.78
time (sec)	N/A	0.172	0.022	0.176	0.027	0.081	1.323	0.123	0.158	0.027

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	42	666	92	42	37
N.S.	1	1.00	0.66	0.60	0.77	0.79	12.57	1.74	0.79	0.70
time (sec)	N/A	0.163	0.020	0.164	0.026	0.064	0.878	0.119	0.158	0.023

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	21	26	31	202	68	31	25
N.S.	1	1.00	1.03	0.62	0.76	0.91	5.94	2.00	0.91	0.74
time (sec)	N/A	0.146	0.016	0.157	0.033	0.067	0.581	0.275	0.155	0.017

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.122	0.002	0.073	0.025	0.067	0.016	0.121	0.156	0.011

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	91	114	86	85	110	182	86	159	117
N.S.	1	0.99	1.24	0.93	0.92	1.20	1.98	0.93	1.73	1.27
time (sec)	N/A	0.185	0.053	0.161	0.107	0.094	1.005	0.449	0.158	0.097

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	120	91	93	252	643	97	168	127
N.S.	1	1.01	1.28	0.97	0.99	2.68	6.84	1.03	1.79	1.35
time (sec)	N/A	0.186	0.117	0.194	0.110	0.120	1.085	0.430	0.161	0.038

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	123	147	107	139	295	2266	129	204	194
N.S.	1	0.97	1.16	0.84	1.09	2.32	17.84	1.02	1.61	1.53
time (sec)	N/A	0.196	0.188	0.198	0.113	0.100	1.601	0.450	0.178	0.175

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	64	1844	193	64	56
N.S.	1	1.00	0.64	0.60	0.78	0.89	25.61	2.68	0.89	0.78
time (sec)	N/A	0.171	0.026	0.184	0.031	0.109	1.397	0.127	0.162	0.025

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	53	733	157	53	37
N.S.	1	1.00	0.66	0.60	0.77	1.00	13.83	2.96	1.00	0.70
time (sec)	N/A	0.160	0.023	0.168	0.025	0.079	0.963	0.125	0.155	0.025

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	26	41	80	118	41	25
N.S.	1	1.00	0.71	0.62	0.76	1.21	2.35	3.47	1.21	0.74
time (sec)	N/A	0.146	0.016	0.164	0.049	0.068	0.230	0.114	0.149	0.018

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	58	28	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	1.75	0.75
time (sec)	N/A	0.122	0.002	0.076	0.035	0.080	0.016	0.120	0.149	0.011

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	106	122	90	96	98	209	97	185	123
N.S.	1	1.01	1.16	0.86	0.91	0.93	1.99	0.92	1.76	1.17
time (sec)	N/A	0.194	0.065	0.174	0.118	0.087	1.260	0.427	0.151	0.037

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	128	97	104	111	719	107	188	131
N.S.	1	1.02	1.19	0.90	0.96	1.03	6.66	0.99	1.74	1.21
time (sec)	N/A	0.195	0.154	0.248	0.111	0.108	1.371	0.461	0.161	0.046

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	118	136	110	136	158	2266	127	206	174
N.S.	1	0.94	1.09	0.88	1.09	1.26	18.13	1.02	1.65	1.39
time (sec)	N/A	0.194	0.201	0.234	0.141	0.094	1.442	0.440	0.180	0.039

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	43	56	42	1640	49	42	56
N.S.	1	1.00	0.64	0.60	0.78	0.58	22.78	0.68	0.58	0.78
time (sec)	N/A	0.170	0.022	0.179	0.040	0.079	1.247	0.122	0.147	0.026

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	41	31	600	37	31	37
N.S.	1	1.00	0.66	0.60	0.77	0.58	11.32	0.70	0.58	0.70
time (sec)	N/A	0.160	0.022	0.168	0.024	0.072	0.808	0.123	0.149	0.023

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	26	20	162	25	20	25
N.S.	1	1.00	0.71	0.62	0.76	0.59	4.76	0.74	0.59	0.74
time (sec)	N/A	0.149	0.014	0.087	0.025	0.066	0.530	0.125	0.147	0.016

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.121	0.001	0.072	0.046	0.062	0.016	0.126	0.145	0.010

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	75	95	75	76	213	155	77	141	99
N.S.	1	0.95	1.20	0.95	0.96	2.70	1.96	0.97	1.78	1.25
time (sec)	N/A	0.172	0.041	0.145	0.133	0.093	0.863	0.415	0.159	0.098

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	120	95	106	306	831	100	166	130
N.S.	1	1.01	1.20	0.95	1.06	3.06	8.31	1.00	1.66	1.30
time (sec)	N/A	0.184	0.127	0.184	0.109	0.085	1.131	0.440	0.168	0.140

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	129	149	109	142	296	2730	130	206	182
N.S.	1	0.99	1.15	0.84	1.09	2.28	21.00	1.00	1.58	1.40
time (sec)	N/A	0.199	0.121	0.199	0.131	0.088	1.910	0.435	0.173	0.166

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	48	45	64	44	4974	57	44	64
N.S.	1	1.00	0.60	0.56	0.80	0.55	62.18	0.71	0.55	0.80
time (sec)	N/A	0.177	0.023	0.181	0.025	0.064	1.309	0.120	0.157	0.030

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	34	47	33	1326	43	33	43
N.S.	1	1.00	0.63	0.58	0.80	0.56	22.47	0.73	0.56	0.73
time (sec)	N/A	0.162	0.022	0.173	0.032	0.080	0.860	0.117	0.159	0.025

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	26	23	30	22	486	29	22	29
N.S.	1	1.00	0.68	0.61	0.79	0.58	12.79	0.76	0.58	0.76
time (sec)	N/A	0.146	0.013	0.088	0.028	0.077	0.575	0.118	0.158	0.018

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	14	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.67	0.78	0.78	0.78
time (sec)	N/A	0.118	0.001	0.076	0.032	0.067	0.016	0.113	0.159	0.013

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	78	100	79	86	285	160	112	136	117
N.S.	1	0.95	1.22	0.96	1.05	3.48	1.95	1.37	1.66	1.43
time (sec)	N/A	0.166	0.059	0.500	0.122	0.079	0.874	0.422	0.157	0.058

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	105	128	103	116	328	838	135	165	133
N.S.	1	1.02	1.24	1.00	1.13	3.18	8.14	1.31	1.60	1.29
time (sec)	N/A	0.185	0.112	0.400	0.108	0.084	1.129	0.401	0.165	0.123

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	135	160	124	159	374	2744	167	203	216
N.S.	1	0.99	1.18	0.91	1.17	2.75	20.18	1.23	1.49	1.59
time (sec)	N/A	0.206	0.108	0.201	0.134	0.100	1.913	0.429	0.174	0.154

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	43	56	42	1640	49	42	56
N.S.	1	1.00	0.66	0.61	0.80	0.60	23.43	0.70	0.60	0.80
time (sec)	N/A	0.171	0.022	0.099	0.031	0.064	1.246	0.119	0.158	0.026

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	41	31	600	37	31	37
N.S.	1	1.00	0.69	0.63	0.80	0.61	11.76	0.73	0.61	0.73
time (sec)	N/A	0.153	0.020	0.091	0.045	0.069	0.817	0.119	0.151	0.022

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	26	19	162	23	19	25
N.S.	1	1.00	0.72	0.66	0.81	0.59	5.06	0.72	0.59	0.78
time (sec)	N/A	0.142	0.013	0.085	0.048	0.076	0.532	0.124	0.158	0.017

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86	0.86
time (sec)	N/A	0.124	0.001	0.073	0.034	0.065	0.016	0.123	0.157	0.010

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	76	93	75	77	115	150	78	141	95
N.S.	1	0.95	1.16	0.94	0.96	1.44	1.88	0.98	1.76	1.19
time (sec)	N/A	0.170	0.048	0.132	0.110	0.098	0.875	0.415	0.162	0.064

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	119	95	106	163	830	99	166	122
N.S.	1	1.04	1.21	0.97	1.08	1.66	8.47	1.01	1.69	1.24
time (sec)	N/A	0.187	0.118	0.218	0.111	0.083	1.143	0.411	0.171	0.131

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	149	122	142	158	2728	130	206	175
N.S.	1	1.00	1.15	0.94	1.09	1.22	20.98	1.00	1.58	1.35
time (sec)	N/A	0.199	0.121	0.233	0.108	0.085	1.938	0.413	0.172	0.082

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	43	56	52	1538	62	42	56
N.S.	1	1.00	0.66	0.61	0.80	0.74	21.97	0.89	0.60	0.80
time (sec)	N/A	0.168	0.023	0.128	0.047	0.071	1.294	0.122	0.155	0.029

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	31	41	40	534	51	30	35
N.S.	1	1.00	0.69	0.63	0.84	0.82	10.90	1.04	0.61	0.71
time (sec)	N/A	0.154	0.023	0.119	0.037	0.079	0.833	0.122	0.153	0.025

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	20	26	29	41	30	19	20
N.S.	1	1.00	0.72	0.62	0.81	0.91	1.28	0.94	0.59	0.62
time (sec)	N/A	0.140	0.014	0.109	0.032	0.093	0.160	0.123	0.163	0.020

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.86	0.86
time (sec)	N/A	0.117	0.001	0.106	0.049	0.074	0.019	0.126	0.150	0.012

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	94	111	92	88	285	184	89	195	114
N.S.	1	1.01	1.19	0.99	0.95	3.06	1.98	0.96	2.10	1.23
time (sec)	N/A	0.180	0.082	0.188	0.154	0.095	0.988	0.397	0.161	0.035

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	120	123	108	122	407	857	120	217	173
N.S.	1	1.06	1.09	0.96	1.08	3.60	7.58	1.06	1.92	1.53
time (sec)	N/A	0.197	0.202	0.228	0.110	0.108	1.325	0.430	0.177	0.092

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	148	142	118	158	407	2793	140	252	221
N.S.	1	0.99	0.95	0.79	1.06	2.73	18.74	0.94	1.69	1.48
time (sec)	N/A	0.211	0.244	0.299	0.110	0.098	2.745	0.400	0.178	0.045

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	97	85	86	88	138	87	165	105
N.S.	1	1.00	1.37	1.20	1.21	1.24	1.94	1.23	2.32	1.48
time (sec)	N/A	0.170	0.052	0.601	0.119	0.072	0.877	0.133	0.159	0.064

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	101	89	90	92	136	91	173	108
N.S.	1	1.00	1.38	1.22	1.23	1.26	1.86	1.25	2.37	1.48
time (sec)	N/A	0.180	0.049	0.519	0.116	0.075	0.875	0.127	0.156	0.093

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	90	94	93	134	95	156	112
N.S.	1	1.00	1.38	1.22	1.27	1.26	1.81	1.28	2.11	1.51
time (sec)	N/A	0.175	0.052	0.520	0.135	0.097	0.885	0.136	0.156	0.118

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	106	94	98	97	139	99	161	115
N.S.	1	1.00	1.39	1.24	1.29	1.28	1.83	1.30	2.12	1.51
time (sec)	N/A	0.178	0.064	0.541	0.117	0.079	0.887	0.132	0.155	0.097

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	95	85	87	86	134	88	165	101
N.S.	1	1.00	1.32	1.18	1.21	1.19	1.86	1.22	2.29	1.40
time (sec)	N/A	0.184	0.043	0.171	0.119	0.086	0.885	0.128	0.155	0.140

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	99	89	91	90	136	92	173	104
N.S.	1	1.00	1.34	1.20	1.23	1.22	1.84	1.24	2.34	1.41
time (sec)	N/A	0.177	0.041	0.167	0.113	0.081	0.888	0.125	0.162	0.118

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	92	93	95	134	94	160	107
N.S.	1	1.00	1.38	1.24	1.26	1.28	1.81	1.27	2.16	1.45
time (sec)	N/A	0.175	0.041	0.167	0.116	0.080	0.895	0.131	0.161	0.061

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	106	96	97	99	133	98	165	110
N.S.	1	1.00	1.39	1.26	1.28	1.30	1.75	1.29	2.17	1.45
time (sec)	N/A	0.179	0.040	0.170	0.107	0.086	0.888	0.129	0.163	0.111

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	172	47	0	0	0	31	0	99	0
N.S.	1	1.17	0.32	0.00	0.00	0.00	0.21	0.00	0.67	0.00
time (sec)	N/A	0.228	10.013	0.000	0.000	0.000	1.565	0.000	0.212	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	142	47	0	0	0	31	0	77	0
N.S.	1	1.15	0.38	0.00	0.00	0.00	0.25	0.00	0.63	0.00
time (sec)	N/A	0.220	10.009	0.000	0.000	0.000	0.792	0.000	0.211	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	112	47	0	0	0	31	0	59	0
N.S.	1	1.13	0.47	0.00	0.00	0.00	0.31	0.00	0.60	0.00
time (sec)	N/A	0.191	10.009	0.000	0.000	0.000	0.700	0.000	0.207	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	85	45	0	0	0	29	0	38	0
N.S.	1	1.10	0.58	0.00	0.00	0.00	0.38	0.00	0.49	0.00
time (sec)	N/A	0.174	10.008	0.000	0.000	0.000	0.630	0.000	0.195	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	45	0	0	0	32	0	45	0
N.S.	1	1.07	0.62	0.00	0.00	0.00	0.44	0.00	0.62	0.00
time (sec)	N/A	0.176	10.010	0.000	0.000	0.000	0.715	0.000	0.201	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	105	47	0	0	0	36	0	50	0
N.S.	1	1.06	0.47	0.00	0.00	0.00	0.36	0.00	0.51	0.00
time (sec)	N/A	0.190	10.027	0.000	0.000	0.000	0.970	0.000	0.211	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	135	47	0	0	0	36	0	52	0
N.S.	1	1.10	0.38	0.00	0.00	0.00	0.29	0.00	0.42	0.00
time (sec)	N/A	0.208	10.009	0.000	0.000	0.000	1.873	0.000	0.227	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	236	47	0	0	0	31	0	99	0
N.S.	1	1.40	0.28	0.00	0.00	0.00	0.18	0.00	0.59	0.00
time (sec)	N/A	0.347	10.011	0.000	0.000	0.000	2.593	0.000	0.212	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	206	47	0	0	0	31	0	79	0
N.S.	1	1.42	0.32	0.00	0.00	0.00	0.21	0.00	0.54	0.00
time (sec)	N/A	0.323	10.009	0.000	0.000	0.000	1.553	0.000	0.218	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	176	47	0	0	0	31	0	59	0
N.S.	1	1.45	0.39	0.00	0.00	0.00	0.26	0.00	0.49	0.00
time (sec)	N/A	0.307	10.010	0.000	0.000	0.000	1.033	0.000	0.202	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	149	45	0	0	0	29	0	38	0
N.S.	1	1.54	0.46	0.00	0.00	0.00	0.30	0.00	0.39	0.00
time (sec)	N/A	0.288	10.008	0.000	0.000	0.000	0.939	0.000	0.200	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	141	45	0	0	0	32	0	45	0
N.S.	1	1.55	0.49	0.00	0.00	0.00	0.35	0.00	0.49	0.00
time (sec)	N/A	0.291	10.011	0.000	0.000	0.000	0.968	0.000	0.217	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	171	47	0	0	0	36	0	50	0
N.S.	1	1.84	0.51	0.00	0.00	0.00	0.39	0.00	0.54	0.00
time (sec)	N/A	0.311	10.009	0.000	0.000	0.000	1.455	0.000	0.221	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	201	47	0	0	0	36	0	52	0
N.S.	1	1.66	0.39	0.00	0.00	0.00	0.30	0.00	0.43	0.00
time (sec)	N/A	0.326	10.010	0.000	0.000	0.000	2.398	0.000	0.259	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	231	47	0	0	0	36	0	52	0
N.S.	1	1.59	0.32	0.00	0.00	0.00	0.25	0.00	0.36	0.00
time (sec)	N/A	0.350	10.010	0.000	0.000	0.000	5.184	0.000	0.254	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	212	47	0	0	0	29	0	15	0
N.S.	1	1.43	0.32	0.00	0.00	0.00	0.20	0.00	0.10	0.00
time (sec)	N/A	0.335	10.009	0.000	0.000	0.000	1.409	0.000	0.177	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	182	47	0	0	0	29	0	13	0
N.S.	1	1.47	0.38	0.00	0.00	0.00	0.23	0.00	0.10	0.00
time (sec)	N/A	0.311	10.011	0.000	0.000	0.000	0.714	0.000	0.174	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	152	47	0	0	0	29	0	12	0
N.S.	1	1.52	0.47	0.00	0.00	0.00	0.29	0.00	0.12	0.00
time (sec)	N/A	0.286	10.009	0.000	0.000	0.000	0.597	0.000	0.166	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	127	45	0	0	0	27	0	64	0
N.S.	1	1.72	0.61	0.00	0.00	0.00	0.36	0.00	0.86	0.00
time (sec)	N/A	0.261	10.010	0.000	0.000	0.000	0.555	0.000	0.213	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	147	45	0	0	0	31	0	70	0
N.S.	1	1.99	0.61	0.00	0.00	0.00	0.42	0.00	0.95	0.00
time (sec)	N/A	0.283	10.011	0.000	0.000	0.000	0.688	0.000	0.231	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	177	47	0	0	0	34	0	73	0
N.S.	1	1.82	0.48	0.00	0.00	0.00	0.35	0.00	0.75	0.00
time (sec)	N/A	0.308	10.010	0.000	0.000	0.000	0.963	0.000	0.243	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	207	47	0	0	0	34	0	73	0
N.S.	1	1.67	0.38	0.00	0.00	0.00	0.27	0.00	0.59	0.00
time (sec)	N/A	0.340	10.011	0.000	0.000	0.000	2.411	0.000	0.268	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	148	47	0	0	0	29	0	15	0
N.S.	1	1.17	0.37	0.00	0.00	0.00	0.23	0.00	0.12	0.00
time (sec)	N/A	0.216	10.011	0.000	0.000	0.000	1.603	0.000	0.186	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	118	47	0	0	0	29	0	13	0
N.S.	1	1.16	0.46	0.00	0.00	0.00	0.28	0.00	0.13	0.00
time (sec)	N/A	0.198	10.008	0.000	0.000	0.000	0.749	0.000	0.171	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	47	0	0	0	29	0	12	0
N.S.	1	1.10	0.59	0.00	0.00	0.00	0.36	0.00	0.15	0.00
time (sec)	N/A	0.178	10.011	0.000	0.000	0.000	0.614	0.000	0.176	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	64	45	0	0	0	27	0	35	0
N.S.	1	1.14	0.80	0.00	0.00	0.00	0.48	0.00	0.62	0.00
time (sec)	N/A	0.168	10.011	0.000	0.000	0.000	0.621	0.000	0.166	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	45	0	0	0	31	0	37	0
N.S.	1	1.07	0.59	0.00	0.00	0.00	0.41	0.00	0.49	0.00
time (sec)	N/A	0.182	10.009	0.000	0.000	0.000	0.852	0.000	0.171	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	111	47	0	0	0	34	0	37	0
N.S.	1	1.09	0.46	0.00	0.00	0.00	0.33	0.00	0.36	0.00
time (sec)	N/A	0.197	10.013	0.000	0.000	0.000	1.561	0.000	0.178	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	141	47	0	0	0	34	0	37	0
N.S.	1	1.12	0.37	0.00	0.00	0.00	0.27	0.00	0.29	0.00
time (sec)	N/A	0.214	10.009	0.000	0.000	0.000	4.550	0.000	0.178	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	237	50	0	0	0	29	0	30	0
N.S.	1	1.57	0.33	0.00	0.00	0.00	0.19	0.00	0.20	0.00
time (sec)	N/A	0.350	10.009	0.000	0.000	0.000	4.617	0.000	0.189	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	207	50	0	0	0	29	0	30	0
N.S.	1	1.63	0.39	0.00	0.00	0.00	0.23	0.00	0.24	0.00
time (sec)	N/A	0.341	10.011	0.000	0.000	0.000	1.636	0.000	0.183	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	177	50	0	0	0	29	0	28	0
N.S.	1	1.72	0.49	0.00	0.00	0.00	0.28	0.00	0.27	0.00
time (sec)	N/A	0.321	10.009	0.000	0.000	0.000	1.038	0.000	0.176	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	147	50	0	0	0	29	0	27	0
N.S.	1	1.91	0.65	0.00	0.00	0.00	0.38	0.00	0.35	0.00
time (sec)	N/A	0.288	10.009	0.000	0.000	0.000	0.770	0.000	0.174	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	150	48	0	0	0	27	0	35	0
N.S.	1	2.63	0.84	0.00	0.00	0.00	0.47	0.00	0.61	0.00
time (sec)	N/A	0.299	10.009	0.000	0.000	0.000	0.823	0.000	0.164	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	172	48	0	0	0	31	0	37	0
N.S.	1	2.23	0.62	0.00	0.00	0.00	0.40	0.00	0.48	0.00
time (sec)	N/A	0.308	10.009	0.000	0.000	0.000	1.257	0.000	0.186	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	202	50	0	0	0	34	0	37	0
N.S.	1	2.00	0.50	0.00	0.00	0.00	0.34	0.00	0.37	0.00
time (sec)	N/A	0.351	10.010	0.000	0.000	0.000	2.700	0.000	0.179	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	232	50	0	0	0	34	0	37	0
N.S.	1	1.83	0.39	0.00	0.00	0.00	0.27	0.00	0.29	0.00
time (sec)	N/A	0.358	10.010	0.000	0.000	0.000	7.690	0.000	0.180	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	159	51	0	0	0	32	0	40	0
N.S.	1	2.30	0.74	0.00	0.00	0.00	0.46	0.00	0.58	0.00
time (sec)	N/A	0.304	10.010	0.000	0.000	0.000	1.371	0.000	0.227	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	311	54	0	0	0	41	0	44	0
N.S.	1	4.32	0.75	0.00	0.00	0.00	0.57	0.00	0.61	0.00
time (sec)	N/A	0.316	10.011	0.000	0.000	0.000	1.378	0.000	0.217	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	166	52	0	0	0	34	0	41	0
N.S.	1	1.73	0.54	0.00	0.00	0.00	0.35	0.00	0.43	0.00
time (sec)	N/A	0.310	10.014	0.000	0.000	0.000	1.375	0.000	0.201	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	290	53	0	0	0	36	0	42	0
N.S.	1	1.21	0.22	0.00	0.00	0.00	0.15	0.00	0.18	0.00
time (sec)	N/A	0.301	10.012	0.000	0.000	0.000	1.384	0.000	0.209	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	142	26	17	0	0	26	0	35	0
N.S.	1	4.44	0.81	0.53	0.00	0.00	0.81	0.00	1.09	0.00
time (sec)	N/A	0.190	9.493	0.078	0.000	0.000	0.511	0.000	0.169	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	170	113	0	207	249	37	0	96	0
N.S.	1	1.10	0.73	0.00	1.34	1.61	0.24	0.00	0.62	0.00
time (sec)	N/A	0.243	0.377	0.000	0.114	0.135	71.137	0.000	0.207	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	102	0	175	238	37	0	78	0
N.S.	1	1.07	0.78	0.00	1.34	1.82	0.28	0.00	0.60	0.00
time (sec)	N/A	0.220	0.306	0.000	0.125	0.095	10.768	0.000	0.209	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	110	89	0	138	217	37	0	60	0
N.S.	1	1.03	0.83	0.00	1.29	2.03	0.35	0.00	0.56	0.00
time (sec)	N/A	0.203	0.269	0.000	0.123	0.099	2.138	0.000	0.204	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	78	0	91	180	37	0	38	0
N.S.	1	1.01	1.00	0.00	1.17	2.31	0.47	0.00	0.49	0.00
time (sec)	N/A	0.181	0.204	0.000	0.109	0.096	1.025	0.000	0.197	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	81	73	0	74	132	41	0	13	0
N.S.	1	1.11	1.00	0.00	1.01	1.81	0.56	0.00	0.18	0.00
time (sec)	N/A	0.181	0.194	0.000	0.107	0.083	1.216	0.000	0.178	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	15	60	0	15	21
N.S.	1	1.00	1.00	0.76	0.71	0.71	2.86	0.00	0.71	1.00
time (sec)	N/A	0.122	0.021	0.079	0.032	0.074	2.903	0.000	0.163	0.177

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	24	31	34	100	0	34	32
N.S.	1	1.00	0.89	0.55	0.70	0.77	2.27	0.00	0.77	0.73
time (sec)	N/A	0.139	0.145	0.084	0.026	0.095	23.397	0.000	0.174	0.168

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	40	35	46	45	0	0	45	43
N.S.	1	1.09	0.59	0.51	0.68	0.66	0.00	0.00	0.66	0.63
time (sec)	N/A	0.149	0.045	0.082	0.033	0.067	0.000	0.000	0.175	0.173

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	51	46	61	56	0	0	56	54
N.S.	1	1.13	0.55	0.50	0.66	0.61	0.00	0.00	0.61	0.59
time (sec)	N/A	0.162	0.050	0.086	0.029	0.069	0.000	0.000	0.183	0.186

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	167	47	0	0	0	37	0	96	0
N.S.	1	1.11	0.31	0.00	0.00	0.00	0.25	0.00	0.64	0.00
time (sec)	N/A	0.265	10.011	0.000	0.000	0.000	15.923	0.000	0.203	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	137	47	0	0	0	37	0	78	0
N.S.	1	1.08	0.37	0.00	0.00	0.00	0.29	0.00	0.61	0.00
time (sec)	N/A	0.254	10.009	0.000	0.000	0.000	2.385	0.000	0.203	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	47	0	0	0	37	0	60	0
N.S.	1	1.04	0.46	0.00	0.00	0.00	0.36	0.00	0.58	0.00
time (sec)	N/A	0.228	10.008	0.000	0.000	0.000	0.597	0.000	0.212	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	45	0	0	0	37	0	39	0
N.S.	1	1.04	0.58	0.00	0.00	0.00	0.48	0.00	0.51	0.00
time (sec)	N/A	0.212	10.009	0.000	0.000	0.000	0.694	0.000	0.195	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	47	0	0	0	31	0	44	0
N.S.	1	1.08	0.60	0.00	0.00	0.00	0.40	0.00	0.56	0.00
time (sec)	N/A	0.210	10.010	0.000	0.000	0.000	1.462	0.000	0.190	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	111	47	0	0	0	32	0	44	0
N.S.	1	1.10	0.47	0.00	0.00	0.00	0.32	0.00	0.44	0.00
time (sec)	N/A	0.231	10.010	0.000	0.000	0.000	8.377	0.000	0.217	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	141	47	0	0	0	44	0	44	0
N.S.	1	1.11	0.37	0.00	0.00	0.00	0.35	0.00	0.35	0.00
time (sec)	N/A	0.255	10.010	0.000	0.000	0.000	66.086	0.000	0.209	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	146	102	0	171	284	36	0	13	0
N.S.	1	1.09	0.76	0.00	1.28	2.12	0.27	0.00	0.10	0.00
time (sec)	N/A	0.224	0.387	0.000	0.108	0.111	23.164	0.000	0.173	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	91	0	137	273	36	0	13	0
N.S.	1	1.05	0.83	0.00	1.25	2.48	0.33	0.00	0.12	0.00
time (sec)	N/A	0.202	0.310	0.000	0.120	0.098	3.832	0.000	0.166	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	81	0	98	236	36	0	13	0
N.S.	1	1.06	1.00	0.00	1.21	2.91	0.44	0.00	0.16	0.00
time (sec)	N/A	0.180	0.253	0.000	0.127	0.081	0.988	0.000	0.171	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	63	50	0	62	147	36	0	13	0
N.S.	1	1.11	0.88	0.00	1.09	2.58	0.63	0.00	0.23	0.00
time (sec)	N/A	0.165	0.193	0.000	0.122	0.080	0.831	0.000	0.166	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	15	27	0	13	0
N.S.	1	1.00	1.00	0.76	0.71	0.71	1.29	0.00	0.62	0.00
time (sec)	N/A	0.129	0.019	0.082	0.031	0.070	1.250	0.000	0.166	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	24	31	23	63	0	13	0
N.S.	1	1.00	0.66	0.55	0.70	0.52	1.43	0.00	0.30	0.00
time (sec)	N/A	0.141	0.176	0.087	0.028	0.072	11.109	0.000	0.173	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	40	35	46	34	396	0	13	0
N.S.	1	1.09	0.59	0.51	0.68	0.50	5.82	0.00	0.19	0.00
time (sec)	N/A	0.156	0.191	0.086	0.025	0.074	92.333	0.000	0.173	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	51	46	61	45	0	0	13	0
N.S.	1	1.13	0.55	0.50	0.66	0.49	0.00	0.00	0.14	0.00
time (sec)	N/A	0.169	0.070	0.090	0.026	0.070	0.000	0.000	0.188	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	142	47	0	0	0	36	0	13	0
N.S.	1	1.27	0.42	0.00	0.00	0.00	0.32	0.00	0.12	0.00
time (sec)	N/A	0.259	10.009	0.000	0.000	0.000	7.300	0.000	0.157	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	112	47	0	0	0	36	0	13	0
N.S.	1	1.27	0.53	0.00	0.00	0.00	0.41	0.00	0.15	0.00
time (sec)	N/A	0.240	10.009	0.000	0.000	0.000	0.955	0.000	0.161	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	86	47	0	0	0	36	0	13	0
N.S.	1	1.37	0.75	0.00	0.00	0.00	0.57	0.00	0.21	0.00
time (sec)	N/A	0.227	10.009	0.000	0.000	0.000	0.563	0.000	0.169	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	110	45	0	0	0	29	0	13	0
N.S.	1	1.49	0.61	0.00	0.00	0.00	0.39	0.00	0.18	0.00
time (sec)	N/A	0.240	10.012	0.000	0.000	0.000	0.999	0.000	0.177	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	140	47	0	0	0	31	0	19	0
N.S.	1	1.35	0.45	0.00	0.00	0.00	0.30	0.00	0.18	0.00
time (sec)	N/A	0.266	10.011	0.000	0.000	0.000	4.249	0.000	0.164	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	170	47	0	0	0	42	0	27	0
N.S.	1	1.31	0.36	0.00	0.00	0.00	0.32	0.00	0.21	0.00
time (sec)	N/A	0.283	10.011	0.000	0.000	0.000	29.574	0.000	0.166	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	146	102	0	175	238	36	0	13	0
N.S.	1	1.09	0.76	0.00	1.31	1.78	0.27	0.00	0.10	0.00
time (sec)	N/A	0.237	0.453	0.000	0.109	0.102	66.753	0.000	0.181	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	91	0	141	227	36	0	13	0
N.S.	1	1.05	0.83	0.00	1.28	2.06	0.33	0.00	0.12	0.00
time (sec)	N/A	0.217	0.360	0.000	0.106	0.103	8.377	0.000	0.174	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	81	0	100	203	36	0	13	0
N.S.	1	1.06	1.00	0.00	1.23	2.51	0.44	0.00	0.16	0.00
time (sec)	N/A	0.194	0.291	0.000	0.107	0.103	1.333	0.000	0.166	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	63	52	0	62	132	36	0	13	0
N.S.	1	1.11	0.91	0.00	1.09	2.32	0.63	0.00	0.23	0.00
time (sec)	N/A	0.180	0.208	0.000	0.112	0.101	0.816	0.000	0.170	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	27	0	13	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	1.42	0.00	0.68	0.79
time (sec)	N/A	0.132	0.028	0.085	0.025	0.081	0.964	0.000	0.169	0.225

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	22	31	23	61	0	13	25
N.S.	1	1.00	0.61	0.50	0.70	0.52	1.39	0.00	0.30	0.57
time (sec)	N/A	0.143	0.233	0.084	0.029	0.068	7.191	0.000	0.166	0.215

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	40	35	46	34	396	0	13	36
N.S.	1	1.09	0.59	0.51	0.68	0.50	5.82	0.00	0.19	0.53
time (sec)	N/A	0.159	0.289	0.082	0.026	0.069	63.974	0.000	0.166	0.227

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	51	46	61	45	0	0	13	47
N.S.	1	1.13	0.55	0.50	0.66	0.49	0.00	0.00	0.14	0.51
time (sec)	N/A	0.171	0.064	0.088	0.025	0.086	0.000	0.000	0.170	0.235

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	143	47	0	0	0	36	0	13	0
N.S.	1	1.10	0.36	0.00	0.00	0.00	0.28	0.00	0.10	0.00
time (sec)	N/A	0.252	10.012	0.000	0.000	0.000	19.129	0.000	0.174	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	113	47	0	0	0	36	0	13	0
N.S.	1	1.07	0.44	0.00	0.00	0.00	0.34	0.00	0.12	0.00
time (sec)	N/A	0.242	10.010	0.000	0.000	0.000	2.774	0.000	0.171	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	83	47	0	0	0	36	0	13	0
N.S.	1	1.04	0.59	0.00	0.00	0.00	0.45	0.00	0.16	0.00
time (sec)	N/A	0.217	10.009	0.000	0.000	0.000	0.740	0.000	0.162	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	45	0	0	0	36	0	13	0
N.S.	1	1.05	0.75	0.00	0.00	0.00	0.60	0.00	0.22	0.00
time (sec)	N/A	0.195	10.011	0.000	0.000	0.000	0.820	0.000	0.176	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	87	47	0	0	0	39	0	19	0
N.S.	1	1.07	0.58	0.00	0.00	0.00	0.48	0.00	0.23	0.00
time (sec)	N/A	0.211	10.016	0.000	0.000	0.000	2.621	0.000	0.164	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	117	47	0	0	0	42	0	27	0
N.S.	1	1.12	0.45	0.00	0.00	0.00	0.40	0.00	0.26	0.00
time (sec)	N/A	0.251	10.010	0.000	0.000	0.000	20.999	0.000	0.163	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	147	47	0	0	0	0	0	38	0
N.S.	1	1.13	0.36	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.255	10.012	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	141	102	0	156	330	36	0	28	0
N.S.	1	1.07	0.77	0.00	1.18	2.50	0.27	0.00	0.21	0.00
time (sec)	N/A	0.229	0.584	0.000	0.111	0.091	20.629	0.000	0.184	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	111	86	0	117	310	36	0	28	0
N.S.	1	1.10	0.85	0.00	1.16	3.07	0.36	0.00	0.28	0.00
time (sec)	N/A	0.205	0.428	0.000	0.113	0.089	3.095	0.000	0.173	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	86	71	0	82	240	36	0	28	0
N.S.	1	1.13	0.93	0.00	1.08	3.16	0.47	0.00	0.37	0.00
time (sec)	N/A	0.186	0.327	0.000	0.106	0.089	1.168	0.000	0.175	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	22	26	0	29	0
N.S.	1	1.00	1.00	0.84	0.79	1.16	1.37	0.00	1.53	0.00
time (sec)	N/A	0.131	0.027	0.086	0.025	0.067	0.979	0.000	0.170	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	22	32	34	61	0	29	0
N.S.	1	1.00	0.66	0.54	0.78	0.83	1.49	0.00	0.71	0.00
time (sec)	N/A	0.143	0.231	0.098	0.029	0.068	4.253	0.000	0.171	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	69	40	35	50	49	308	0	29	0
N.S.	1	1.10	0.63	0.56	0.79	0.78	4.89	0.00	0.46	0.00
time (sec)	N/A	0.156	0.386	0.096	0.026	0.079	33.256	0.000	0.177	0.000

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	99	51	46	65	60	0	0	29	0
N.S.	1	1.14	0.59	0.53	0.75	0.69	0.00	0.00	0.33	0.00
time (sec)	N/A	0.168	0.489	0.097	0.026	0.068	0.000	0.000	0.183	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	167	50	0	0	0	36	0	28	0
N.S.	1	1.25	0.37	0.00	0.00	0.00	0.27	0.00	0.21	0.00
time (sec)	N/A	0.280	10.010	0.000	0.000	0.000	58.908	0.000	0.184	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	137	50	0	0	0	36	0	28	0
N.S.	1	1.27	0.46	0.00	0.00	0.00	0.33	0.00	0.26	0.00
time (sec)	N/A	0.267	10.009	0.000	0.000	0.000	7.183	0.000	0.177	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	109	50	0	0	0	36	0	28	0
N.S.	1	1.30	0.60	0.00	0.00	0.00	0.43	0.00	0.33	0.00
time (sec)	N/A	0.244	10.009	0.000	0.000	0.000	1.522	0.000	0.173	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	109	50	0	0	0	36	0	29	0
N.S.	1	1.68	0.77	0.00	0.00	0.00	0.55	0.00	0.45	0.00
time (sec)	N/A	0.248	10.010	0.000	0.000	0.000	0.938	0.000	0.181	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	135	48	0	0	0	39	0	27	0
N.S.	1	1.75	0.62	0.00	0.00	0.00	0.51	0.00	0.35	0.00
time (sec)	N/A	0.272	10.012	0.000	0.000	0.000	1.966	0.000	0.167	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	165	50	0	0	0	42	0	35	0
N.S.	1	1.34	0.41	0.00	0.00	0.00	0.34	0.00	0.28	0.00
time (sec)	N/A	0.292	10.010	0.000	0.000	0.000	12.932	0.000	0.167	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	195	50	0	0	0	42	0	46	0
N.S.	1	1.31	0.34	0.00	0.00	0.00	0.28	0.00	0.31	0.00
time (sec)	N/A	0.310	10.010	0.000	0.000	0.000	110.622	0.000	0.171	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	145	102	0	160	284	36	0	28	0
N.S.	1	1.08	0.76	0.00	1.19	2.12	0.27	0.00	0.21	0.00
time (sec)	N/A	0.232	0.825	0.000	0.109	0.103	61.306	0.000	0.197	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	115	87	0	120	265	36	0	28	0
N.S.	1	1.12	0.84	0.00	1.17	2.57	0.35	0.00	0.27	0.00
time (sec)	N/A	0.214	0.615	0.000	0.106	0.105	7.939	0.000	0.191	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	88	78	0	82	206	36	0	28	0
N.S.	1	1.13	1.00	0.00	1.05	2.64	0.46	0.00	0.36	0.00
time (sec)	N/A	0.190	0.403	0.000	0.108	0.099	2.819	0.000	0.184	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	22	26	0	29	25
N.S.	1	1.00	1.00	0.76	0.71	1.05	1.24	0.00	1.38	1.19
time (sec)	N/A	0.130	0.021	0.079	0.034	0.067	1.221	0.000	0.177	0.216

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	29	24	32	36	63	0	29	42
N.S.	1	1.00	0.67	0.56	0.74	0.84	1.47	0.00	0.67	0.98
time (sec)	N/A	0.140	0.247	0.089	0.031	0.073	4.430	0.000	0.174	0.239

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	40	35	50	49	308	0	29	46
N.S.	1	1.12	0.62	0.54	0.77	0.75	4.74	0.00	0.45	0.71
time (sec)	N/A	0.165	0.358	0.130	0.051	0.071	21.351	0.000	0.179	0.261

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	101	51	46	65	60	0	0	29	58
N.S.	1	1.13	0.57	0.52	0.73	0.67	0.00	0.00	0.33	0.65
time (sec)	N/A	0.167	0.732	0.096	0.031	0.067	0.000	0.000	0.188	0.279

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	133	62	57	80	71	0	0	29	68
N.S.	1	1.18	0.55	0.50	0.71	0.63	0.00	0.00	0.26	0.60
time (sec)	N/A	0.180	0.066	0.131	0.042	0.077	0.000	0.000	0.178	0.274

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	172	50	0	0	0	36	0	28	0
N.S.	1	1.12	0.32	0.00	0.00	0.00	0.23	0.00	0.18	0.00
time (sec)	N/A	0.266	10.011	0.000	0.000	0.000	159.903	0.000	0.203	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	140	50	0	0	0	36	0	28	0
N.S.	1	1.09	0.39	0.00	0.00	0.00	0.28	0.00	0.22	0.00
time (sec)	N/A	0.257	10.009	0.000	0.000	0.000	18.289	0.000	0.195	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	50	0	0	0	36	0	28	0
N.S.	1	1.08	0.48	0.00	0.00	0.00	0.35	0.00	0.27	0.00
time (sec)	N/A	0.236	10.011	0.000	0.000	0.000	4.164	0.000	0.179	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	87	50	0	0	0	36	0	28	0
N.S.	1	1.04	0.60	0.00	0.00	0.00	0.43	0.00	0.33	0.00
time (sec)	N/A	0.221	10.009	0.000	0.000	0.000	1.534	0.000	0.176	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	87	48	0	0	0	36	0	19	0
N.S.	1	1.04	0.57	0.00	0.00	0.00	0.43	0.00	0.23	0.00
time (sec)	N/A	0.213	10.010	0.000	0.000	0.000	2.698	0.000	0.163	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	114	50	0	0	0	39	0	27	0
N.S.	1	1.12	0.49	0.00	0.00	0.00	0.38	0.00	0.26	0.00
time (sec)	N/A	0.231	10.011	0.000	0.000	0.000	7.646	0.000	0.170	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	146	50	0	0	0	42	0	38	0
N.S.	1	1.17	0.40	0.00	0.00	0.00	0.34	0.00	0.30	0.00
time (sec)	N/A	0.253	10.012	0.000	0.000	0.000	66.211	0.000	0.166	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	176	50	0	0	0	0	0	49	0
N.S.	1	1.17	0.33	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.275	10.010	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	110	45	0	0	0	29	0	13	0
N.S.	1	1.49	0.61	0.00	0.00	0.00	0.39	0.00	0.18	0.00
time (sec)	N/A	0.247	0.005	0.000	0.000	0.000	0.956	0.000	0.177	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	117	46	0	0	0	31	0	14	0
N.S.	1	1.46	0.58	0.00	0.00	0.00	0.39	0.00	0.18	0.00
time (sec)	N/A	0.275	10.014	0.000	0.000	0.000	0.955	0.000	0.174	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	77	48	0	0	0	41	0	17	0
N.S.	1	1.24	0.77	0.00	0.00	0.00	0.66	0.00	0.27	0.00
time (sec)	N/A	0.241	10.011	0.000	0.000	0.000	1.297	0.000	0.179	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	81	50	0	0	0	31	0	22	0
N.S.	1	1.21	0.75	0.00	0.00	0.00	0.46	0.00	0.33	0.00
time (sec)	N/A	0.228	10.015	0.000	0.000	0.000	1.343	0.000	0.174	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	79	49	0	0	0	36	0	18	0
N.S.	1	1.20	0.74	0.00	0.00	0.00	0.55	0.00	0.27	0.00
time (sec)	N/A	0.229	10.010	0.000	0.000	0.000	1.328	0.000	0.174	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	79	49	0	0	0	49	0	21	0
N.S.	1	1.18	0.73	0.00	0.00	0.00	0.73	0.00	0.31	0.00
time (sec)	N/A	0.223	10.009	0.000	0.000	0.000	1.330	0.000	0.194	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	39	24	27	0	0	32	0	11	0
N.S.	1	2.79	1.71	1.93	0.00	0.00	2.29	0.00	0.79	0.00
time (sec)	N/A	0.192	9.585	0.094	0.000	0.000	0.613	0.000	0.159	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	0	0	0	0	0	31	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	2.21	0.00
time (sec)	N/A	0.200	0.006	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	75	26	27	0	0	36	0	11	0
N.S.	1	5.36	1.86	1.93	0.00	0.00	2.57	0.00	0.79	0.00
time (sec)	N/A	0.215	10.006	0.122	0.000	0.000	0.747	0.000	0.187	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	26	61	0	0	0	0	17	0
N.S.	1	1.00	1.86	4.36	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.193	10.007	0.076	0.000	0.000	0.000	0.000	0.183	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	47	32	13	0	0	24	0	13	0
N.S.	1	2.47	1.68	0.68	0.00	0.00	1.26	0.00	0.68	0.00
time (sec)	N/A	0.195	0.008	0.073	0.000	0.000	0.649	0.000	0.167	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	32	13	0	0	0	0	13	25
N.S.	1	1.00	1.68	0.68	0.00	0.00	0.00	0.00	0.68	1.32
time (sec)	N/A	0.149	0.001	0.085	0.000	0.000	0.000	0.000	0.168	0.200

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	87	18	13	0	0	24	0	13	0
N.S.	1	2.42	0.50	0.36	0.00	0.00	0.67	0.00	0.36	0.00
time (sec)	N/A	0.219	10.005	0.095	0.000	0.000	0.762	0.000	0.174	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	113	34	13	0	0	0	0	13	0
N.S.	1	3.14	0.94	0.36	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.267	10.008	0.105	0.000	0.000	0.000	0.000	0.166	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	71	103	100	253	1261	415	282	263
N.S.	1	1.00	0.68	0.99	0.96	2.43	12.12	3.99	2.71	2.53
time (sec)	N/A	0.238	0.056	0.093	0.053	0.079	0.342	0.121	0.151	0.312

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	80	78	159	702	256	171	169
N.S.	1	1.00	0.68	0.99	0.96	1.96	8.67	3.16	2.11	2.09
time (sec)	N/A	0.204	0.042	0.069	0.052	0.077	0.303	0.121	0.146	0.207

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	39	57	56	87	323	135	88	95
N.S.	1	1.00	0.67	0.98	0.97	1.50	5.57	2.33	1.52	1.64
time (sec)	N/A	0.178	0.038	0.062	0.035	0.075	0.230	0.115	0.145	0.178

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	32	34	35	99	51	32	31
N.S.	1	1.00	0.66	0.91	0.97	1.00	2.83	1.46	0.91	0.89
time (sec)	N/A	0.177	0.021	0.039	0.034	0.104	0.173	0.128	0.153	0.122

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	0	0	0	68	0	35	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	2.00	0.00	1.03	0.00
time (sec)	N/A	0.142	0.026	0.000	0.000	0.000	0.377	0.000	0.148	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	0	0	0	282	0	262	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	8.29	0.00	7.71	0.00
time (sec)	N/A	0.142	0.020	0.000	0.000	0.000	0.491	0.000	0.147	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	0	0	0	758	0	555	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	22.29	0.00	16.32	0.00
time (sec)	N/A	0.141	0.020	0.000	0.000	0.000	0.645	0.000	0.157	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	41	0	447	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.82	0.00	8.94	0.00
time (sec)	N/A	0.153	0.114	0.000	0.000	0.000	1.269	0.000	0.170	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	41	0	239	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.82	0.00	4.78	0.00
time (sec)	N/A	0.148	0.060	0.000	0.000	0.000	0.638	0.000	0.165	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	39	0	106	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.81	0.00	2.21	0.00
time (sec)	N/A	0.147	0.056	0.000	0.000	0.000	0.556	0.000	0.162	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	39	0	304	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.81	0.00	6.33	0.00
time (sec)	N/A	0.146	0.079	0.000	0.000	0.000	0.699	0.000	0.161	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	39	0	613	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	12.26	0.00
time (sec)	N/A	0.164	0.082	0.000	0.000	0.000	1.314	0.000	0.165	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	51	51	0	0	0	36	0	630	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.69	0.00	12.12	0.00
time (sec)	N/A	0.154	0.071	0.000	0.000	0.000	0.711	0.000	0.171	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	49	49	0	0	0	36	0	311	0
N.S.	1	0.94	0.94	0.00	0.00	0.00	0.69	0.00	5.98	0.00
time (sec)	N/A	0.150	0.070	0.000	0.000	0.000	0.712	0.000	0.166	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0	103	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	2.24	0.00
time (sec)	N/A	0.149	0.014	0.000	0.000	0.000	0.679	0.000	0.161	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	48	48	0	0	0	37	0	23	0
N.S.	1	0.92	0.92	0.00	0.00	0.00	0.71	0.00	0.44	0.00
time (sec)	N/A	0.154	0.067	0.000	0.000	0.000	0.690	0.000	0.158	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	49	49	0	0	0	32	0	25	0
N.S.	1	0.94	0.94	0.00	0.00	0.00	0.62	0.00	0.48	0.00
time (sec)	N/A	0.152	0.070	0.000	0.000	0.000	0.701	0.000	0.152	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	42	0	94	0
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.35	0.00	3.03	0.00
time (sec)	N/A	0.136	0.037	0.068	0.000	0.000	0.562	0.000	0.154	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	46	0	95	0
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.48	0.00	3.06	0.00
time (sec)	N/A	0.138	0.036	0.108	0.000	0.000	0.590	0.000	0.150	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	43	0	0	36	0	94	0
N.S.	1	1.00	1.00	1.19	0.00	0.00	1.00	0.00	2.61	0.00
time (sec)	N/A	0.144	0.036	0.075	0.000	0.000	0.583	0.000	0.160	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	30	0	0	46	0	95	0
N.S.	1	1.00	0.96	0.60	0.00	0.00	0.92	0.00	1.90	0.00
time (sec)	N/A	0.173	0.048	0.054	0.000	0.000	0.583	0.000	0.163	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	42	0	106	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.88	0.00	2.21	0.00
time (sec)	N/A	0.173	0.013	0.000	0.000	0.000	0.699	0.000	0.162	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	44	0	97	0
N.S.	1	1.00	0.94	0.88	0.00	0.00	1.29	0.00	2.85	0.00
time (sec)	N/A	0.144	0.005	0.051	0.000	0.000	0.578	0.000	0.156	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	53	0	98	0
N.S.	1	1.00	0.94	0.88	0.00	0.00	1.56	0.00	2.88	0.00
time (sec)	N/A	0.144	0.005	0.065	0.000	0.000	0.609	0.000	0.160	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	42	0	97	0
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.86	0.00	1.98	0.00
time (sec)	N/A	0.149	0.007	0.076	0.000	0.000	0.597	0.000	0.157	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	57	31	0	0	48	0	98	0
N.S.	1	1.00	1.54	0.84	0.00	0.00	1.30	0.00	2.65	0.00
time (sec)	N/A	0.135	0.011	0.056	0.000	0.000	0.570	0.000	0.162	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	87	0	12	11
N.S.	1	1.00	1.00	0.92	0.85	1.08	6.69	0.00	0.92	0.85
time (sec)	N/A	0.134	0.157	0.094	0.072	0.091	6.726	0.000	0.159	0.273

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	92	13	53	11	11	80	0	12	0
N.S.	1	7.08	1.00	4.08	0.85	0.85	6.15	0.00	0.92	0.00
time (sec)	N/A	0.197	0.004	0.167	0.077	0.080	1.314	0.000	0.150	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	53	24	21	31	17
N.S.	1	1.00	1.00	0.78	1.39	2.30	1.04	0.91	1.35	0.74
time (sec)	N/A	0.142	0.002	0.085	0.108	0.073	0.467	0.113	0.148	0.093

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	26	23	0	0	26	0	91	0
N.S.	1	1.18	1.18	1.05	0.00	0.00	1.18	0.00	4.14	0.00
time (sec)	N/A	0.133	0.033	0.066	0.000	0.000	0.510	0.000	0.150	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	24	0	0	32	0	94	0
N.S.	1	1.00	1.33	0.89	0.00	0.00	1.19	0.00	3.48	0.00
time (sec)	N/A	0.136	0.005	0.102	0.000	0.000	0.470	0.000	0.158	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	31	0	96	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.03	0.00	3.20	0.00
time (sec)	N/A	0.152	0.106	0.000	0.000	0.000	0.541	0.000	0.149	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	37	0	99	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.92	0.00	2.48	0.00
time (sec)	N/A	0.141	0.007	0.000	0.000	0.000	0.490	0.000	0.157	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	126	101	143	1318	226	142	176
N.S.	1	1.00	0.81	1.52	1.22	1.72	15.88	2.72	1.71	2.12
time (sec)	N/A	0.197	0.056	0.113	0.034	0.081	0.554	0.126	0.152	0.332

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	68	96	597	140	87	192
N.S.	1	1.00	0.95	1.22	1.13	1.60	9.95	2.33	1.45	3.20
time (sec)	N/A	0.184	0.036	0.095	0.035	0.078	0.380	0.118	0.157	0.366

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	42	53	201	76	47	94
N.S.	1	1.00	0.85	0.92	1.08	1.36	5.15	1.95	1.21	2.41
time (sec)	N/A	0.167	0.026	0.125	0.029	0.080	0.263	0.124	0.155	0.234

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	20	18	21	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.11	1.00	1.17	1.00
time (sec)	N/A	0.134	0.002	0.076	0.030	0.086	0.018	0.120	0.153	0.126

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	76	0	36	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.17	0.00	1.03	0.00
time (sec)	N/A	0.144	0.021	0.000	0.000	0.000	0.722	0.000	0.157	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	333	0	39	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	9.51	0.00	1.11	0.00
time (sec)	N/A	0.143	0.019	0.000	0.000	0.000	0.972	0.000	0.156	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	899	0	92	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	23.66	0.00	2.42	0.00
time (sec)	N/A	0.179	0.021	0.000	0.000	0.000	1.946	0.000	0.159	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	45	45	0	0	0	27	0	431	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.59	0.00	9.37	0.00
time (sec)	N/A	0.147	0.081	0.000	0.000	0.000	68.840	0.000	0.159	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	45	45	0	0	0	27	0	238	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.59	0.00	5.17	0.00
time (sec)	N/A	0.148	0.059	0.000	0.000	0.000	3.335	0.000	0.160	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	43	0	0	0	26	0	99	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.59	0.00	2.25	0.00
time (sec)	N/A	0.148	0.055	0.000	0.000	0.000	2.178	0.000	0.164	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	43	0	0	0	29	0	117	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.66	0.00	2.66	0.00
time (sec)	N/A	0.146	0.093	0.000	0.000	0.000	13.321	0.000	0.159	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	45	45	0	0	0	32	0	122	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.70	0.00	2.65	0.00
time (sec)	N/A	0.144	0.068	0.000	0.000	0.000	161.012	0.000	0.156	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	47	47	0	0	0	34	0	498	0
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.87	0.00	12.77	0.00
time (sec)	N/A	0.155	0.019	0.000	0.000	0.000	1.498	0.000	0.169	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	52	48	0	0	0	37	0	501	0
N.S.	1	1.18	1.09	0.00	0.00	0.00	0.84	0.00	11.39	0.00
time (sec)	N/A	0.173	0.006	0.000	0.000	0.000	1.268	0.000	0.171	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	106	64	77	0	104	388	0	18	136
N.S.	1	0.96	0.58	0.70	0.00	0.95	3.53	0.00	0.16	1.24
time (sec)	N/A	0.201	0.035	0.176	0.000	0.088	10.743	0.000	0.149	0.327

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	44	0	64	124	0	18	80
N.S.	1	1.00	0.61	0.69	0.00	1.00	1.94	0.00	0.28	1.25
time (sec)	N/A	0.167	0.028	0.168	0.000	0.095	10.208	0.000	0.152	0.283

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	29	0	33	26	0	18	29
N.S.	1	1.00	0.89	1.04	0.00	1.18	0.93	0.00	0.64	1.04
time (sec)	N/A	0.143	0.022	0.157	0.000	0.089	10.493	0.000	0.149	0.231

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	39	39	0	0	0	27	0	18	0
N.S.	1	1.15	1.15	0.00	0.00	0.00	0.79	0.00	0.53	0.00
time (sec)	N/A	0.153	0.022	0.000	0.000	0.000	4.763	0.000	0.147	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	45	45	0	0	0	32	0	15	0
N.S.	1	1.18	1.18	0.00	0.00	0.00	0.84	0.00	0.39	0.00
time (sec)	N/A	0.147	0.020	0.000	0.000	0.000	11.228	0.000	0.140	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	45	45	0	0	0	32	0	16	0
N.S.	1	1.18	1.18	0.00	0.00	0.00	0.84	0.00	0.42	0.00
time (sec)	N/A	0.148	0.023	0.000	0.000	0.000	12.350	0.000	0.150	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	0	0	37	0	481	0
N.S.	1	1.00	0.89	0.91	0.00	0.00	1.06	0.00	13.74	0.00
time (sec)	N/A	0.142	0.018	0.205	0.000	0.000	1.066	0.000	0.150	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	0	0	0	37	0	497	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.92	0.00	12.42	0.00
time (sec)	N/A	0.151	0.045	0.000	0.000	0.000	1.111	0.000	0.162	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	52	48	0	0	0	37	0	501	0
N.S.	1	1.18	1.09	0.00	0.00	0.00	0.84	0.00	11.39	0.00
time (sec)	N/A	0.170	0.022	0.000	0.000	0.000	1.341	0.000	0.151	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	22	32	31	0	31	19
N.S.	1	1.00	1.00	1.05	1.16	1.68	1.63	0.00	1.63	1.00
time (sec)	N/A	0.137	0.028	0.162	0.037	0.116	0.879	0.000	0.155	0.300

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	64	134	0	54	86
N.S.	1	1.00	0.69	0.71	0.00	1.10	2.31	0.00	0.93	1.48
time (sec)	N/A	0.178	0.029	0.181	0.000	0.089	0.926	0.000	0.156	0.317

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	64	134	0	54	86
N.S.	1	1.00	0.69	0.71	0.00	1.10	2.31	0.00	0.93	1.48
time (sec)	N/A	0.166	0.001	0.135	0.000	0.081	0.933	0.000	0.163	0.001

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	0	0	36	0	478	0
N.S.	1	1.00	1.00	1.03	0.00	0.00	1.24	0.00	16.48	0.00
time (sec)	N/A	0.141	0.016	0.153	0.000	0.000	1.124	0.000	0.155	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	0	0	0	0	575	0
N.S.	1	1.00	1.00	0.76	0.00	0.00	0.00	0.00	19.83	0.00
time (sec)	N/A	0.132	0.052	0.096	0.000	0.000	0.000	0.000	0.151	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	0	0	44	0	615	0
N.S.	1	1.00	1.00	0.91	0.00	0.00	1.38	0.00	19.22	0.00
time (sec)	N/A	0.134	0.047	0.084	0.000	0.000	4.266	0.000	0.160	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [647] had the largest ratio of [.800000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	9	0.222
2	A	2	2	1.00	9	0.222
3	A	2	2	1.00	7	0.286
4	A	1	1	1.00	5	0.200
5	A	2	2	1.00	9	0.222
6	A	2	2	1.00	9	0.222
7	A	1	1	1.13	9	0.111
8	A	2	2	1.00	9	0.222
9	A	2	2	1.00	9	0.222
10	A	2	2	1.00	11	0.182
11	A	2	2	1.00	11	0.182
12	A	2	2	1.00	9	0.222
13	A	1	1	1.00	7	0.143
14	A	2	2	1.00	11	0.182
15	A	2	2	1.00	11	0.182
16	A	2	2	1.00	11	0.182
17	A	1	1	0.65	11	0.091
18	A	2	2	1.00	11	0.182
19	A	2	2	1.00	11	0.182
20	A	2	2	1.00	11	0.182
21	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	11	0.182
23	A	2	2	1.00	11	0.182
24	A	2	2	1.00	11	0.182
25	A	2	2	1.00	9	0.222
26	A	1	1	1.00	7	0.143
27	A	2	2	1.00	11	0.182
28	A	2	2	1.00	11	0.182
29	A	2	2	1.00	11	0.182
30	A	2	2	1.00	11	0.182
31	A	1	1	1.00	11	0.091
32	A	2	2	1.00	11	0.182
33	A	2	2	1.00	11	0.182
34	A	2	2	1.00	11	0.182
35	A	2	2	1.00	11	0.182
36	A	2	2	1.00	11	0.182
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	11	0.182
39	A	2	2	1.00	11	0.182
40	A	2	2	1.00	9	0.222
41	A	1	1	1.00	7	0.143
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	2	2	1.00	11	0.182
45	A	2	2	1.00	11	0.182
46	A	2	2	1.00	11	0.182
47	A	2	2	1.00	11	0.182
48	A	1	1	1.00	11	0.091
49	A	2	2	1.00	11	0.182
50	A	3	3	1.11	11	0.273
51	A	2	2	1.00	11	0.182
52	A	2	2	1.00	11	0.182
53	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	11	0.182
55	A	2	2	1.00	11	0.182
56	A	2	2	1.00	11	0.182
57	A	2	2	1.00	11	0.182
58	A	2	2	1.00	11	0.182
59	A	2	2	1.00	11	0.182
60	A	2	2	1.00	11	0.182
61	A	2	2	1.00	11	0.182
62	A	2	2	1.00	11	0.182
63	A	2	2	1.00	9	0.222
64	A	1	1	1.00	7	0.143
65	A	2	2	1.00	11	0.182
66	A	2	2	1.00	11	0.182
67	A	2	2	1.00	11	0.182
68	A	2	2	1.00	11	0.182
69	A	2	2	1.00	11	0.182
70	A	2	2	1.00	11	0.182
71	A	2	2	1.00	11	0.182
72	A	2	2	1.00	11	0.182
73	A	1	1	1.00	11	0.091
74	A	2	2	1.00	11	0.182
75	A	3	3	1.11	11	0.273
76	A	4	4	1.16	11	0.364
77	A	5	5	1.19	11	0.455
78	A	2	2	1.00	11	0.182
79	A	2	2	1.00	11	0.182
80	A	2	2	1.00	11	0.182
81	A	2	2	1.00	11	0.182
82	A	2	2	1.00	11	0.182
83	A	2	2	1.00	11	0.182
84	A	2	2	1.00	11	0.182
85	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	11	0.182
87	A	2	2	1.00	11	0.182
88	A	2	2	1.00	11	0.182
89	A	2	2	1.00	11	0.182
90	A	2	2	1.00	11	0.182
91	A	2	2	1.00	9	0.222
92	A	1	1	1.00	7	0.143
93	A	2	2	1.00	11	0.182
94	A	2	2	1.00	11	0.182
95	A	2	2	1.00	11	0.182
96	A	2	2	1.00	11	0.182
97	A	2	2	1.00	11	0.182
98	A	2	2	1.00	11	0.182
99	A	2	2	1.00	11	0.182
100	A	2	2	1.00	11	0.182
101	A	2	2	1.00	11	0.182
102	A	2	2	1.00	11	0.182
103	A	2	2	1.00	11	0.182
104	A	1	1	1.00	11	0.091
105	A	2	2	1.00	11	0.182
106	A	3	3	1.11	11	0.273
107	A	4	4	1.16	11	0.364
108	A	5	5	1.19	11	0.455
109	A	6	6	1.21	11	0.545
110	A	7	7	1.22	11	0.636
111	A	2	2	1.00	11	0.182
112	A	2	2	1.00	11	0.182
113	A	1	1	1.00	7	0.143
114	A	1	1	1.00	12	0.083
115	A	2	2	1.00	9	0.222
116	A	2	2	1.00	11	0.182
117	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	11	0.182
119	A	2	2	1.00	11	0.182
120	A	2	2	1.00	9	0.222
121	A	1	1	1.00	7	0.143
122	A	3	3	1.00	11	0.273
123	A	2	2	1.00	11	0.182
124	A	2	2	1.00	11	0.182
125	A	2	2	1.00	11	0.182
126	A	2	2	1.00	11	0.182
127	A	2	2	1.00	11	0.182
128	A	2	2	1.00	11	0.182
129	A	2	2	1.00	11	0.182
130	A	2	2	1.00	11	0.182
131	A	2	2	1.00	11	0.182
132	A	2	2	1.00	9	0.222
133	A	1	1	1.00	7	0.143
134	A	2	2	1.00	11	0.182
135	A	2	2	1.00	11	0.182
136	A	2	2	1.00	11	0.182
137	A	2	2	1.00	11	0.182
138	A	2	2	1.00	11	0.182
139	A	2	2	1.00	11	0.182
140	A	2	2	1.00	11	0.182
141	A	2	2	1.00	11	0.182
142	A	2	2	1.00	11	0.182
143	A	2	2	1.00	11	0.182
144	A	2	2	1.00	11	0.182
145	A	1	1	1.00	9	0.111
146	A	1	1	1.00	7	0.143
147	A	2	2	1.00	11	0.182
148	A	2	2	1.00	11	0.182
149	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.00	11	0.182
151	A	2	2	1.00	11	0.182
152	A	2	2	1.00	11	0.182
153	A	2	2	1.00	11	0.182
154	A	2	2	1.00	11	0.182
155	A	2	2	1.00	11	0.182
156	A	2	2	1.00	11	0.182
157	A	2	2	1.00	11	0.182
158	A	1	1	1.00	11	0.091
159	A	2	2	1.00	9	0.222
160	A	1	1	1.00	7	0.143
161	A	2	2	1.00	11	0.182
162	A	2	2	1.00	11	0.182
163	A	2	2	1.00	11	0.182
164	A	2	2	1.00	11	0.182
165	A	2	2	1.00	11	0.182
166	A	2	2	1.00	11	0.182
167	A	2	2	1.00	11	0.182
168	A	2	2	1.00	11	0.182
169	A	2	2	1.00	11	0.182
170	A	2	2	1.00	11	0.182
171	A	1	1	1.00	11	0.091
172	A	2	2	1.00	11	0.182
173	A	2	2	1.23	11	0.182
174	A	2	2	1.00	11	0.182
175	A	2	2	1.00	9	0.222
176	A	1	1	1.00	7	0.143
177	A	2	2	1.00	11	0.182
178	A	2	2	1.00	11	0.182
179	A	2	2	1.00	11	0.182
180	A	2	2	1.00	11	0.182
181	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.00	11	0.182
183	A	2	2	1.00	11	0.182
184	A	2	2	1.00	11	0.182
185	A	1	1	1.00	11	0.091
186	A	2	2	1.00	11	0.182
187	A	3	3	1.15	11	0.273
188	A	4	4	1.23	11	0.364
189	A	2	2	1.00	11	0.182
190	A	2	2	1.00	11	0.182
191	A	2	2	1.00	11	0.182
192	A	2	2	1.00	9	0.222
193	A	1	1	1.00	7	0.143
194	A	2	2	1.00	11	0.182
195	A	2	2	1.00	11	0.182
196	A	2	2	1.00	11	0.182
197	A	2	2	1.00	11	0.182
198	A	2	2	1.00	11	0.182
199	A	2	2	1.00	11	0.182
200	A	2	2	1.00	11	0.182
201	A	2	2	1.00	11	0.182
202	A	1	1	1.00	11	0.091
203	A	2	2	1.00	11	0.182
204	A	3	3	1.11	11	0.273
205	A	2	2	1.00	11	0.182
206	A	2	2	1.00	11	0.182
207	A	2	2	1.00	11	0.182
208	A	2	2	1.00	11	0.182
209	A	2	2	1.00	9	0.222
210	A	1	1	1.00	3	0.333
211	A	2	2	1.00	11	0.182
212	A	2	2	1.00	11	0.182
213	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	3	3	1.00	11	0.273
215	A	3	3	1.00	11	0.273
216	A	2	2	1.00	11	0.182
217	A	2	2	1.00	11	0.182
218	A	2	2	1.00	11	0.182
219	A	2	2	1.00	11	0.182
220	A	2	2	1.00	11	0.182
221	A	2	2	1.00	11	0.182
222	A	2	2	1.00	11	0.182
223	A	2	2	1.00	11	0.182
224	A	2	2	1.00	11	0.182
225	A	2	2	1.00	11	0.182
226	A	2	2	1.00	11	0.182
227	A	2	2	1.00	11	0.182
228	A	2	2	1.00	11	0.182
229	A	2	2	1.00	11	0.182
230	A	3	3	1.00	11	0.273
231	A	3	3	1.00	11	0.273
232	A	2	2	1.00	11	0.182
233	A	2	2	1.00	11	0.182
234	A	1	1	1.00	17	0.059
235	A	2	2	1.00	11	0.182
236	A	2	2	1.00	11	0.182
237	A	2	2	1.00	11	0.182
238	A	2	2	1.00	11	0.182
239	A	2	2	1.00	11	0.182
240	A	2	2	1.00	11	0.182
241	A	2	2	1.00	13	0.154
242	A	2	2	1.00	13	0.154
243	A	2	2	1.00	13	0.154
244	A	2	2	1.00	13	0.154
245	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
246	A	2	2	1.00	13	0.154
247	A	2	2	1.00	13	0.154
248	A	2	2	1.00	13	0.154
249	A	2	2	1.00	13	0.154
250	A	2	2	1.00	13	0.154
251	A	2	2	1.00	13	0.154
252	A	2	2	1.00	13	0.154
253	A	6	5	1.15	13	0.385
254	A	5	4	1.11	13	0.308
255	A	4	3	1.00	13	0.231
256	A	3	2	1.00	13	0.154
257	A	4	3	1.00	13	0.231
258	A	5	4	1.11	13	0.308
259	A	6	5	1.15	13	0.385
260	A	6	5	1.15	13	0.385
261	A	5	4	1.14	13	0.308
262	A	4	3	1.00	13	0.231
263	A	4	3	1.00	13	0.231
264	A	5	4	1.14	13	0.308
265	A	6	5	1.20	13	0.385
266	A	7	6	1.12	13	0.462
267	A	6	5	1.10	13	0.385
268	A	5	4	1.03	13	0.308
269	A	5	4	0.99	13	0.308
270	A	5	4	1.03	13	0.308
271	A	6	5	1.11	13	0.385
272	A	7	6	1.16	13	0.462
273	A	7	6	1.13	15	0.400
274	A	6	5	1.11	15	0.333
275	A	5	4	1.00	15	0.267
276	A	3	2	1.00	15	0.133
277	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
278	A	6	5	1.11	15	0.333
279	A	7	6	1.13	15	0.400
280	A	7	6	1.14	15	0.400
281	A	6	5	1.12	15	0.333
282	A	5	4	1.00	15	0.267
283	A	5	4	1.00	15	0.267
284	A	6	5	1.14	15	0.333
285	A	7	6	1.19	15	0.400
286	A	7	6	1.10	15	0.400
287	A	6	5	1.08	15	0.333
288	A	5	4	1.03	15	0.267
289	A	5	4	0.99	15	0.267
290	A	5	4	1.03	15	0.267
291	A	6	5	1.11	15	0.333
292	A	7	6	1.14	15	0.400
293	A	2	2	1.00	11	0.182
294	A	2	2	1.00	11	0.182
295	A	2	2	1.00	11	0.182
296	A	2	2	1.00	11	0.182
297	A	2	2	1.00	11	0.182
298	A	2	2	1.00	11	0.182
299	A	2	2	1.00	11	0.182
300	A	2	2	1.00	11	0.182
301	A	2	2	1.00	13	0.154
302	A	2	2	1.00	13	0.154
303	A	2	2	1.00	13	0.154
304	A	2	2	1.00	13	0.154
305	A	2	2	1.00	13	0.154
306	A	2	2	1.00	13	0.154
307	A	2	2	1.00	13	0.154
308	A	2	2	1.00	13	0.154
309	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	2	2	1.00	13	0.154
311	A	2	2	1.00	13	0.154
312	A	2	2	1.00	13	0.154
313	A	2	2	1.00	13	0.154
314	A	2	2	1.00	13	0.154
315	A	2	2	1.00	13	0.154
316	A	2	2	1.00	13	0.154
317	A	7	6	1.07	13	0.462
318	A	7	6	1.07	13	0.462
319	A	6	5	1.04	13	0.385
320	A	6	5	1.04	13	0.385
321	A	5	4	0.96	13	0.308
322	A	5	4	0.96	13	0.308
323	A	6	5	1.04	13	0.385
324	A	6	5	1.04	13	0.385
325	A	7	6	1.09	13	0.462
326	A	7	6	1.10	13	0.462
327	A	6	5	1.05	13	0.385
328	A	6	5	1.03	13	0.385
329	A	6	5	1.03	13	0.385
330	A	6	5	1.06	13	0.385
331	A	7	6	1.10	13	0.462
332	A	7	6	1.09	13	0.462
333	A	7	6	1.05	13	0.462
334	A	7	6	1.05	13	0.462
335	A	7	6	1.03	13	0.462
336	A	7	6	1.03	13	0.462
337	A	7	6	1.05	13	0.462
338	A	7	6	1.05	13	0.462
339	A	8	7	1.08	13	0.538
340	A	8	7	1.09	13	0.538
341	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
342	A	2	2	1.00	13	0.154
343	A	2	2	1.00	11	0.182
344	A	1	1	1.00	9	0.111
345	A	4	3	1.00	13	0.231
346	A	4	3	1.00	13	0.231
347	A	5	4	0.97	13	0.308
348	A	6	5	1.05	13	0.385
349	A	2	2	1.00	13	0.154
350	A	2	2	1.00	13	0.154
351	A	2	2	1.00	11	0.182
352	A	1	1	1.00	9	0.111
353	A	5	4	1.04	13	0.308
354	A	5	4	1.06	13	0.308
355	A	5	4	0.97	13	0.308
356	A	6	5	1.00	13	0.385
357	A	2	2	1.00	13	0.154
358	A	2	2	1.00	13	0.154
359	A	2	2	1.00	11	0.182
360	A	1	1	1.00	9	0.111
361	A	6	5	1.03	13	0.385
362	A	6	5	1.03	13	0.385
363	A	6	5	0.96	13	0.385
364	A	6	5	0.98	13	0.385
365	A	7	6	1.00	13	0.462
366	A	8	7	1.05	13	0.538
367	A	2	2	1.00	13	0.154
368	A	2	2	1.00	13	0.154
369	A	2	2	1.00	13	0.154
370	A	2	2	1.00	13	0.154
371	A	2	2	1.00	13	0.154
372	A	2	2	1.00	13	0.154
373	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	1	1	1.00	9	0.111
375	A	8	7	1.02	13	0.538
376	A	8	7	1.02	13	0.538
377	A	8	7	0.94	13	0.538
378	A	8	7	0.95	13	0.538
379	A	8	7	0.98	13	0.538
380	A	8	7	0.98	13	0.538
381	A	9	8	1.00	13	0.615
382	A	10	9	1.03	13	0.692
383	A	4	3	1.00	15	0.200
384	A	4	3	1.00	15	0.200
385	A	5	4	0.96	15	0.267
386	A	5	4	1.05	15	0.267
387	A	5	4	1.07	15	0.267
388	A	5	4	0.96	15	0.267
389	A	6	5	1.05	15	0.333
390	A	6	5	1.04	15	0.333
391	A	6	5	0.97	15	0.333
392	A	2	2	1.00	13	0.154
393	A	2	2	1.00	13	0.154
394	A	2	2	1.00	13	0.154
395	A	2	2	1.00	11	0.182
396	A	1	1	1.00	9	0.111
397	A	3	2	1.00	13	0.154
398	A	4	3	1.00	13	0.231
399	A	5	4	1.01	13	0.308
400	A	6	5	1.08	13	0.385
401	A	2	2	1.00	13	0.154
402	A	2	2	1.00	13	0.154
403	A	2	2	1.00	13	0.154
404	A	2	2	1.00	11	0.182
405	A	1	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
406	A	4	3	1.00	13	0.231
407	A	5	4	1.12	13	0.308
408	A	6	5	1.06	13	0.385
409	A	2	2	1.00	13	0.154
410	A	2	2	1.00	13	0.154
411	A	2	2	1.00	13	0.154
412	A	2	2	1.00	11	0.182
413	A	1	1	1.00	9	0.111
414	A	5	4	1.09	13	0.308
415	A	6	5	1.15	13	0.385
416	A	7	6	1.07	13	0.462
417	A	3	2	1.00	15	0.133
418	A	4	3	1.00	15	0.200
419	A	5	4	1.00	15	0.267
420	A	4	3	1.00	15	0.200
421	A	5	4	1.11	15	0.267
422	A	6	5	1.04	15	0.333
423	A	5	4	1.10	15	0.267
424	A	6	5	1.15	15	0.333
425	A	7	6	1.06	15	0.400
426	A	7	6	1.08	15	0.400
427	A	6	5	1.04	15	0.333
428	A	5	4	0.97	15	0.267
429	A	4	3	1.00	15	0.200
430	A	4	3	1.00	15	0.200
431	A	1	1	1.00	15	0.067
432	A	2	2	1.00	15	0.133
433	A	3	3	1.09	15	0.200
434	A	4	4	1.13	15	0.267
435	A	8	7	1.08	15	0.467
436	A	7	6	1.05	15	0.400
437	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
438	A	5	4	0.94	15	0.267
439	A	5	4	1.02	15	0.267
440	A	5	4	0.99	15	0.267
441	A	1	1	1.00	15	0.067
442	A	2	2	1.00	15	0.133
443	A	3	3	1.09	15	0.200
444	A	4	4	1.13	15	0.267
445	A	9	8	1.07	15	0.533
446	A	8	7	1.04	15	0.467
447	A	7	6	1.00	15	0.400
448	A	6	5	0.96	15	0.333
449	A	6	5	0.94	15	0.333
450	A	6	5	0.99	15	0.333
451	A	6	5	0.96	15	0.333
452	A	1	1	1.00	15	0.067
453	A	2	2	1.00	15	0.133
454	A	3	3	1.09	15	0.200
455	A	4	4	1.13	15	0.267
456	A	7	6	1.15	15	0.400
457	A	6	5	1.13	15	0.333
458	A	5	4	1.03	15	0.267
459	A	4	3	1.00	15	0.200
460	A	4	3	1.00	15	0.200
461	A	1	1	1.00	15	0.067
462	A	2	2	1.00	15	0.133
463	A	3	3	1.05	15	0.200
464	A	4	4	1.08	15	0.267
465	A	8	7	1.16	15	0.467
466	A	7	6	1.11	15	0.400
467	A	6	5	1.01	15	0.333
468	A	5	4	1.02	15	0.267
469	A	5	4	1.05	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
470	A	5	4	1.00	15	0.267
471	A	9	8	1.15	15	0.533
472	A	8	7	1.07	15	0.467
473	A	7	6	1.01	15	0.400
474	A	6	5	1.04	15	0.333
475	A	6	5	1.02	15	0.333
476	A	6	5	1.05	15	0.333
477	A	6	5	1.07	15	0.333
478	A	5	4	1.01	15	0.267
479	A	4	3	1.00	15	0.200
480	A	3	2	1.00	15	0.133
481	A	1	1	1.00	15	0.067
482	A	2	2	1.00	15	0.133
483	A	3	3	1.09	15	0.200
484	A	4	4	1.13	15	0.267
485	A	6	5	1.04	15	0.333
486	A	5	4	1.07	15	0.267
487	A	4	3	1.00	15	0.200
488	A	1	1	1.00	15	0.067
489	A	2	2	1.00	15	0.133
490	A	3	3	1.10	15	0.200
491	A	4	4	1.14	15	0.267
492	A	6	5	1.09	15	0.333
493	A	5	4	1.03	15	0.267
494	A	1	1	1.00	15	0.067
495	A	2	2	1.00	15	0.133
496	A	3	3	1.06	15	0.200
497	A	4	4	1.14	15	0.267
498	A	6	5	1.15	15	0.333
499	A	5	4	1.07	15	0.267
500	A	4	3	1.00	15	0.200
501	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	1	1	1.00	15	0.067
503	A	2	2	1.00	15	0.133
504	A	3	3	1.05	15	0.200
505	A	4	4	1.08	15	0.267
506	A	6	5	1.13	15	0.333
507	A	5	4	1.10	15	0.267
508	A	4	3	1.00	15	0.200
509	A	1	1	1.00	15	0.067
510	A	2	2	1.00	15	0.133
511	A	3	3	1.06	15	0.200
512	A	4	4	1.08	15	0.267
513	A	6	5	1.14	15	0.333
514	A	5	4	1.04	15	0.267
515	A	1	1	1.00	15	0.067
516	A	2	2	1.00	15	0.133
517	A	3	3	1.00	15	0.200
518	A	4	4	1.04	15	0.267
519	A	5	4	1.00	13	0.308
520	A	4	3	1.00	13	0.231
521	A	3	2	1.00	13	0.154
522	A	1	1	1.00	13	0.077
523	A	2	2	1.00	13	0.154
524	A	3	3	1.10	13	0.231
525	A	3	2	1.00	15	0.133
526	A	3	2	1.00	15	0.133
527	A	3	2	1.00	15	0.133
528	A	3	2	1.00	15	0.133
529	A	3	2	1.00	13	0.154
530	A	4	3	1.00	15	0.200
531	A	7	6	1.08	16	0.375
532	A	6	5	1.04	16	0.312
533	A	5	4	0.97	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	4	3	1.00	16	0.188
535	A	4	3	1.00	16	0.188
536	A	1	1	1.00	16	0.062
537	A	2	2	1.00	16	0.125
538	A	3	3	1.08	16	0.188
539	A	4	4	1.12	16	0.250
540	A	8	7	1.08	16	0.438
541	A	7	6	1.05	16	0.375
542	A	6	5	1.00	16	0.312
543	A	5	4	0.95	16	0.250
544	A	5	4	1.00	16	0.250
545	A	5	4	1.00	16	0.250
546	A	9	8	1.07	16	0.500
547	A	8	7	1.04	16	0.438
548	A	7	6	1.00	16	0.375
549	A	6	5	0.96	16	0.312
550	A	6	5	0.94	16	0.312
551	A	6	5	0.99	16	0.312
552	A	6	5	1.07	16	0.312
553	A	5	4	1.01	16	0.250
554	A	4	3	1.00	16	0.188
555	A	3	2	1.00	16	0.125
556	A	1	1	1.00	16	0.062
557	A	2	2	1.00	16	0.125
558	A	3	3	1.08	16	0.188
559	A	4	4	1.12	16	0.250
560	A	6	5	1.04	16	0.312
561	A	5	4	1.07	16	0.250
562	A	4	3	1.00	16	0.188
563	A	1	1	1.00	16	0.062
564	A	2	2	1.00	16	0.125
565	A	3	3	1.09	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
566	A	4	4	1.13	16	0.250
567	A	6	5	1.07	16	0.312
568	A	5	4	1.04	16	0.250
569	A	1	1	1.00	16	0.062
570	A	2	2	1.00	16	0.125
571	A	3	3	1.06	16	0.188
572	A	4	4	1.14	16	0.250
573	A	6	5	1.06	15	0.333
574	A	5	4	1.17	15	0.267
575	A	4	3	1.00	15	0.200
576	A	1	1	1.00	15	0.067
577	A	2	2	1.00	15	0.133
578	A	3	3	1.09	15	0.200
579	B	10	9	3.33	19	0.474
580	B	7	6	6.47	20	0.300
581	A	7	6	1.04	17	0.353
582	A	10	9	1.06	18	0.500
583	A	2	2	1.00	13	0.154
584	A	2	2	1.00	13	0.154
585	A	2	2	1.00	11	0.182
586	A	1	1	1.00	9	0.111
587	A	6	5	0.99	13	0.385
588	A	6	5	0.99	13	0.385
589	A	7	6	0.98	13	0.462
590	A	2	2	1.00	13	0.154
591	A	2	2	1.00	13	0.154
592	A	2	2	1.00	11	0.182
593	A	1	1	1.00	9	0.111
594	A	6	5	0.99	13	0.385
595	A	6	5	1.01	13	0.385
596	A	7	6	0.97	13	0.462
597	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	2	2	1.00	13	0.154
599	A	2	2	1.00	11	0.182
600	A	1	1	1.00	9	0.111
601	A	7	6	1.01	13	0.462
602	A	7	6	1.02	13	0.462
603	A	7	6	0.94	13	0.462
604	A	2	2	1.00	13	0.154
605	A	2	2	1.00	13	0.154
606	A	2	2	1.00	11	0.182
607	A	1	1	1.00	9	0.111
608	A	5	4	0.95	13	0.308
609	A	6	5	1.01	13	0.385
610	A	7	6	0.99	13	0.462
611	A	2	2	1.00	15	0.133
612	A	2	2	1.00	15	0.133
613	A	2	2	1.00	13	0.154
614	A	1	1	1.00	11	0.091
615	A	5	4	0.95	15	0.267
616	A	6	5	1.02	15	0.333
617	A	7	6	0.99	15	0.400
618	A	2	2	1.00	13	0.154
619	A	2	2	1.00	13	0.154
620	A	2	2	1.00	11	0.182
621	A	1	1	1.00	9	0.111
622	A	5	4	0.95	13	0.308
623	A	6	5	1.04	13	0.385
624	A	7	6	1.00	13	0.462
625	A	2	2	1.00	13	0.154
626	A	2	2	1.00	13	0.154
627	A	2	2	1.00	11	0.182
628	A	1	1	1.00	9	0.111
629	A	6	5	1.01	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	7	6	1.06	13	0.462
631	A	8	7	0.99	13	0.538
632	A	5	4	1.00	17	0.235
633	A	5	4	1.00	18	0.222
634	A	5	4	1.00	19	0.211
635	A	5	4	1.00	20	0.200
636	A	5	4	1.00	17	0.235
637	A	5	4	1.00	18	0.222
638	A	5	4	1.00	19	0.211
639	A	5	4	1.00	20	0.200
640	A	8	7	1.17	15	0.467
641	A	7	6	1.15	15	0.400
642	A	6	5	1.13	15	0.333
643	A	5	4	1.10	15	0.267
644	A	5	4	1.07	15	0.267
645	A	6	5	1.06	15	0.333
646	A	7	6	1.10	15	0.400
647	A	13	12	1.40	15	0.800
648	A	12	11	1.42	15	0.733
649	A	11	10	1.45	15	0.667
650	A	10	9	1.54	15	0.600
651	A	10	9	1.55	15	0.600
652	A	11	10	1.84	15	0.667
653	A	12	11	1.66	15	0.733
654	A	13	12	1.59	15	0.800
655	A	12	11	1.43	15	0.733
656	A	11	10	1.47	15	0.667
657	A	10	9	1.52	15	0.600
658	A	9	8	1.72	15	0.533
659	A	10	9	1.99	15	0.600
660	A	11	10	1.82	15	0.667
661	A	12	11	1.67	15	0.733

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	7	6	1.17	15	0.400
663	A	6	5	1.16	15	0.333
664	A	5	4	1.10	15	0.267
665	A	4	3	1.14	15	0.200
666	A	5	4	1.07	15	0.267
667	A	6	5	1.09	15	0.333
668	A	7	6	1.12	15	0.400
669	A	13	12	1.57	15	0.800
670	A	12	11	1.63	15	0.733
671	A	11	10	1.72	15	0.667
672	A	10	9	1.91	15	0.600
673	B	10	9	2.63	15	0.600
674	B	11	10	2.23	15	0.667
675	A	12	11	2.00	15	0.733
676	A	13	12	1.83	15	0.800
677	B	10	9	2.30	17	0.529
678	B	7	6	4.32	20	0.300
679	A	10	9	1.73	18	0.500
680	A	7	6	1.21	19	0.316
681	B	3	2	4.44	15	0.133
682	A	10	9	1.10	15	0.600
683	A	9	8	1.07	15	0.533
684	A	8	7	1.03	15	0.467
685	A	7	6	1.01	15	0.400
686	A	7	6	1.11	15	0.400
687	A	1	1	1.00	15	0.067
688	A	2	2	1.00	15	0.133
689	A	3	3	1.09	15	0.200
690	A	4	4	1.13	15	0.267
691	A	10	9	1.11	15	0.600
692	A	9	8	1.08	15	0.533
693	A	8	7	1.04	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
694	A	7	6	1.04	15	0.400
695	A	7	6	1.08	15	0.400
696	A	8	7	1.10	15	0.467
697	A	9	8	1.11	15	0.533
698	A	9	8	1.09	15	0.533
699	A	8	7	1.05	15	0.467
700	A	7	6	1.06	15	0.400
701	A	6	5	1.11	15	0.333
702	A	1	1	1.00	15	0.067
703	A	2	2	1.00	15	0.133
704	A	3	3	1.09	15	0.200
705	A	4	4	1.13	15	0.267
706	A	9	8	1.27	15	0.533
707	A	8	7	1.27	15	0.467
708	A	7	6	1.37	15	0.400
709	A	8	7	1.49	15	0.467
710	A	9	8	1.35	15	0.533
711	A	10	9	1.31	15	0.600
712	A	9	8	1.09	15	0.533
713	A	8	7	1.05	15	0.467
714	A	7	6	1.06	15	0.400
715	A	6	5	1.11	15	0.333
716	A	1	1	1.00	15	0.067
717	A	2	2	1.00	15	0.133
718	A	3	3	1.09	15	0.200
719	A	4	4	1.13	15	0.267
720	A	9	8	1.10	15	0.533
721	A	8	7	1.07	15	0.467
722	A	7	6	1.04	15	0.400
723	A	6	5	1.05	15	0.333
724	A	7	6	1.07	15	0.400
725	A	8	7	1.12	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
726	A	9	8	1.13	15	0.533
727	A	9	8	1.07	15	0.533
728	A	8	7	1.10	15	0.467
729	A	7	6	1.13	15	0.400
730	A	1	1	1.00	15	0.067
731	A	2	2	1.00	15	0.133
732	A	3	3	1.10	15	0.200
733	A	4	4	1.14	15	0.267
734	A	10	9	1.25	15	0.600
735	A	9	8	1.27	15	0.533
736	A	8	7	1.30	15	0.467
737	A	8	7	1.68	15	0.467
738	A	9	8	1.75	15	0.533
739	A	10	9	1.34	15	0.600
740	A	11	10	1.31	15	0.667
741	A	9	8	1.08	15	0.533
742	A	8	7	1.12	15	0.467
743	A	7	6	1.13	15	0.400
744	A	1	1	1.00	15	0.067
745	A	2	2	1.00	15	0.133
746	A	3	3	1.12	15	0.200
747	A	4	4	1.13	15	0.267
748	A	5	5	1.18	15	0.333
749	A	10	9	1.12	15	0.600
750	A	9	8	1.09	15	0.533
751	A	8	7	1.08	15	0.467
752	A	7	6	1.04	15	0.400
753	A	7	6	1.04	15	0.400
754	A	8	7	1.12	15	0.467
755	A	9	8	1.17	15	0.533
756	A	10	9	1.17	15	0.600
757	A	8	7	1.49	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
758	A	8	7	1.46	16	0.438
759	A	6	5	1.24	17	0.294
760	A	6	5	1.21	19	0.263
761	A	6	5	1.20	18	0.278
762	A	6	5	1.18	18	0.278
763	B	6	5	2.79	13	0.385
764	A	5	4	1.00	17	0.235
765	B	8	7	5.36	13	0.538
766	A	5	4	1.00	17	0.235
767	B	6	5	2.47	15	0.333
768	A	3	2	1.00	11	0.182
769	B	8	7	2.42	15	0.467
770	B	9	8	3.14	15	0.533
771	A	2	2	1.00	13	0.154
772	A	2	2	1.00	13	0.154
773	A	2	2	1.00	13	0.154
774	A	2	2	1.00	11	0.182
775	A	1	1	1.00	13	0.077
776	A	1	1	1.00	13	0.077
777	A	1	1	1.00	13	0.077
778	A	2	2	1.00	15	0.133
779	A	2	2	1.00	15	0.133
780	A	2	2	1.00	15	0.133
781	A	2	2	1.00	15	0.133
782	A	2	2	1.00	15	0.133
783	A	2	2	0.98	15	0.133
784	A	2	2	0.94	15	0.133
785	A	2	2	1.00	13	0.154
786	A	2	2	0.92	15	0.133
787	A	2	2	0.94	15	0.133
788	A	1	1	1.00	13	0.077
789	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
790	A	1	1	1.00	13	0.077
791	A	3	3	1.00	13	0.231
792	A	2	2	1.00	15	0.133
793	A	1	1	1.00	15	0.067
794	A	1	1	1.00	15	0.067
795	A	3	3	1.00	15	0.200
796	A	1	1	1.00	15	0.067
797	A	2	2	1.00	31	0.065
798	C	1	1	7.08	34	0.029
799	A	4	3	1.00	29	0.103
800	A	1	1	1.18	13	0.077
801	A	1	1	1.00	15	0.067
802	A	1	1	1.00	14	0.071
803	A	2	2	1.00	16	0.125
804	A	2	2	1.00	11	0.182
805	A	2	2	1.00	11	0.182
806	A	2	2	1.00	9	0.222
807	A	1	1	1.00	7	0.143
808	A	1	1	1.00	11	0.091
809	A	1	1	1.00	11	0.091
810	A	1	1	1.00	11	0.091
811	A	2	2	0.98	13	0.154
812	A	2	2	0.98	13	0.154
813	A	2	2	0.98	13	0.154
814	A	2	2	0.98	13	0.154
815	A	2	2	0.98	13	0.154
816	A	2	2	1.21	11	0.182
817	A	2	2	1.18	13	0.154
818	A	3	3	0.96	15	0.200
819	A	2	2	1.00	15	0.133
820	A	1	1	1.00	15	0.067
821	A	2	2	1.15	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
822	A	2	2	1.18	13	0.154
823	A	2	2	1.18	15	0.133
824	A	1	1	1.00	13	0.077
825	A	1	1	1.00	15	0.067
826	A	2	2	1.18	13	0.154
827	A	1	1	1.00	17	0.059
828	A	2	2	1.00	15	0.133
829	A	2	2	1.00	19	0.105
830	A	1	1	1.00	11	0.091
831	A	1	1	1.00	13	0.077
832	A	1	1	1.00	13	0.077

CHAPTER 3

Listing of integrals

3.1 $\int x^3(a+bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int x^3(a+bx)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

output
```
1/4*a*x^4+1/5*b*x^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

input
```
Integrate[x^3*(a + b*x),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(ax^3+bx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

input
```
Int[x^3*(a + b*x),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
parallelrisch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
orering	$\frac{x^4(4bx+5a)}{20}$	14

input
```
int(x^3*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/5*b*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(b*x+a),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^3(a+bx)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

input
```
integrate(x**3*(b*x+a),x)
```

output
```
a*x**4/4 + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(b*x+a),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(b*x+a),x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx)\,dx=\frac{x^4\,(5\,a+4\,b\,x)}{20}$$

input
```
int(x^3*(a + b*x),x)
```

output
```
(x^4*(5*a + 4*b*x))/20
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx)\,dx=\frac{x^4(4bx+5a)}{20}$$

input
```
int(x^3*(b*x+a),x)
```

output
```
(x**4*(5*a + 4*b*x))/20
```

3.2 $\int x^2(a+bx)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int x^2(a+bx)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

output
```
1/3*a*x^3+1/4*b*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

input
```
Integrate[x^2*(a + b*x),x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(ax^2+bx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

input
```
Int[x^2*(a + b*x),x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
orering	$\frac{x^3(3bx+4a)}{12}$	14

input
```
int(x^2*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/4*b*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx)\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(b*x+a),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2(a+bx)\,dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
integrate(x**2*(b*x+a),x)
```

output
```
a*x**3/3 + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx)\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(b*x+a),x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx)\,dx = \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(b*x+a),x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx)\,dx=\frac{x^3\,(4\,a+3\,b\,x)}{12}$$

input
```
int(x^2*(a + b*x),x)
```

output
```
(x^3*(4*a + 3*b*x))/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx)\,dx=\frac{x^3(3bx+4a)}{12}$$

input
```
int(x^2*(b*x+a),x)
```

output
```
(x**3*(4*a + 3*b*x))/12
```

3.3 $\int x(a+bx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int x(a+bx)\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}$$

output
```
1/2*a*x^2+1/3*b*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(a+bx)\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}$$

input
```
Integrate[x*(a + b*x),x]
```

output
```
(a*x^2)/2 + (b*x^3)/3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(ax+bx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

input
```
Int[x*(a + b*x),x]
```

output
```
(a*x^2)/2 + (b*x^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}$	14
default	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}$	14
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}$	14
risch	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}$	14
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{1}{3}b\,x^{3}$	14
orering	$\frac{x^{2}(2bx+3a)}{6}$	14

input
```
int(x*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/3*b*x^3
```

Fricas [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx)\,dx=\frac{1}{3}x^{3}b+\frac{1}{2}x^{2}a$$

input
```
integrate(x*(b*x+a),x, algorithm="fricas")
```

output
```
1/3*x^3*b + 1/2*x^2*a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(a+bx)\,dx = \frac{ax^2}{2} + \frac{bx^3}{3}$$

input
```
integrate(x*(b*x+a),x)
```

output
```
a*x**2/2 + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx)\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x+a),x, algorithm="maxima")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx)\,dx = \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x+a),x, algorithm="giac")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx)\,dx=\frac{x^2\,\left(3\,a+2\,b\,x\right)}{6}$$

input
```
int(x*(a + b*x),x)
```

output
```
(x^2*(3*a + 2*b*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx)\,dx=\frac{x^2(2bx+3a)}{6}$$

input
```
int(x*(b*x+a),x)
```

output
```
(x**2*(3*a + 2*b*x))/6
```

3.4 $\int (a+bx)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 14

$$\int (a+bx)\,dx=\frac{(a+bx)^2}{2b}$$

output
```
1/2*(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)\,dx=ax+\frac{bx^2}{2}$$

input
```
Integrate[a + b*x,x]
```

output
```
a*x + (b*x^2)/2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^2}{2b}$$

input
```
Int[a + b*x,x]
```

output
```
(a + b*x)^2/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{1}{2}b\,x^{2}+ax$	11
default	$\frac{1}{2}b\,x^{2}+ax$	11
norman	$\frac{1}{2}b\,x^{2}+ax$	11
risch	$\frac{1}{2}b\,x^{2}+ax$	11
parallelrisch	$\frac{1}{2}b\,x^{2}+ax$	11
parts	$\frac{1}{2}b\,x^{2}+ax$	11
orering	$\frac{x(bx+2a)}{2}$	11

input
```
int(b*x+a,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+a*x
```

Fricas [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (a+bx)\,dx=\frac{1}{2}x^2b+xa$$

input
```
integrate(b*x+a,x, algorithm="fricas")
```

output
```
1/2*x^2*b + x*a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int (a + bx)\,dx = ax + \frac{bx^2}{2}$$

input
```
integrate(b*x+a,x)
```

output
```
a*x + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (a + bx)\,dx = \frac{1}{2}\,bx^2 + ax$$

input
```
integrate(b*x+a,x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (a + bx)\,dx = \frac{1}{2}\,bx^2 + ax$$

input
```
integrate(b*x+a,x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (a+bx)\,dx=\frac{b\,x^2}{2}+a\,x$$

input
```
int(a + b*x,x)
```

output
```
a*x + (b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (a+bx)\,dx=\frac{x(bx+2a)}{2}$$

input
```
int(b*x+a,x)
```

output
```
(x*(2*a + b*x))/2
```

3.5 $\int \frac{a+bx}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{a+bx}{x} \, dx = bx + a\log(x)$$

output
```
b*x+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x} \, dx = bx + a\log(x)$$

input
```
Integrate[(a + b*x)/x,x]
```

output
```
b*x + a*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a}{x}+b\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)+bx$$

input
```
Int[(a + b*x)/x,x]
```

output
```
b*x + a*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$bx + a\ln(x)$	9
norman	$bx + a\ln(x)$	9
risch	$bx + a\ln(x)$	9
parallelrisch	$bx + a\ln(x)$	9

input
```
int((b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
b*x+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x}\,dx = bx + a\log(x)$$

input
```
integrate((b*x+a)/x,x, algorithm="fricas")
```

output
```
b*x + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{x}\,dx = a\log(x) + bx$$

input
```
integrate((b*x+a)/x,x)
```

output
```
a*log(x) + b*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x}\,dx = bx + a\log(x)$$

input
```
integrate((b*x+a)/x,x, algorithm="maxima")
```

output
```
b*x + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{a+bx}{x}\,dx = bx + a\log(|x|)$$

input
```
integrate((b*x+a)/x,x, algorithm="giac")
```

output
```
b*x + a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x}\,dx = b\,x + a\,\ln(x)$$

input
```
int((a + b*x)/x,x)
```

output
```
b*x + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x} \, dx = \log(x) \, a + bx$$

input
```
int((b*x+a)/x,x)
```

output
```
log(x)*a + b*x
```

3.6 $\int \frac{a+bx}{x^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{a+bx}{x^2} \, dx = -\frac{a}{x} + b\log(x)$$

output
```
-a/x+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^2} \, dx = -\frac{a}{x} + b\log(x)$$

input
```
Integrate[(a + b*x)/x^2,x]
```

output
```
-(a/x) + b*Log[x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^2}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\log(x)-\frac{a}{x}$$

input
```
Int[(a + b*x)/x^2,x]
```

output
```
-(a/x) + b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{a}{x}+b\ln\left(x\right)$	12
norman	$-\frac{a}{x}+b\ln\left(x\right)$	12
risch	$-\frac{a}{x}+b\ln\left(x\right)$	12
parallelrisch	$\frac{b\ln\left(x\right)x-a}{x}$	14

input
```
int((b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{a+bx}{x^2}\,dx=\frac{bx\log\left(x\right)-a}{x}$$

input
```
integrate((b*x+a)/x^2,x, algorithm="fricas")
```

output
```
(b*x*log(x) - a)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{a+bx}{x^2}\,dx=-\frac{a}{x}+b\log\left(x\right)$$

input
```
integrate((b*x+a)/x**2,x)
```

output
```
-a/x + b*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^2}\,dx = b\log(x) - \frac{a}{x}$$

input
```
integrate((b*x+a)/x^2,x, algorithm="maxima")
```

output
```
b*log(x) - a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{a+bx}{x^2}\,dx = b\log(|x|) - \frac{a}{x}$$

input
```
integrate((b*x+a)/x^2,x, algorithm="giac")
```

output
```
b*log(abs(x)) - a/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^2}\,dx = b\,\ln(x) - \frac{a}{x}$$

input
```
int((a + b*x)/x^2,x)
```

output
```
b*log(x) - a/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{a+bx}{x^2}\,dx = \frac{\log(x)\,bx-a}{x}$$

input
```
int((b*x+a)/x^2,x)
```

output
```
(log(x)*b*x - a)/x
```

3.7 $\int \frac{a+bx}{x^3} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{a+bx}{x^3} \, dx = -\frac{a}{2x^2} - \frac{b}{x}$$

output
```
-1/2*a/x^2-b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^3} \, dx = -\frac{a}{2x^2} - \frac{b}{x}$$

input
```
Integrate[(a + b*x)/x^3,x]
```

output
```
-1/2*a/x^2 - b/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^3} \, dx$$

$$\Big\downarrow \ 48$$

$$-\frac{(a+bx)^2}{2ax^2}$$

input
```
Int[(a + b*x)/x^3,x]
```

output
```
-1/2*(a + b*x)^2/(a*x^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{2bx+a}{2x^2}$	12
orering	$-\frac{2bx+a}{2x^2}$	12
norman	$\frac{-bx-\frac{a}{2}}{x^2}$	13
risch	$\frac{-bx-\frac{a}{2}}{x^2}$	13
default	$-\frac{a}{2x^2}-\frac{b}{x}$	14
parallelrisch	$\frac{-2bx-a}{2x^2}$	14

input
```
int((b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*x+a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{a+bx}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate((b*x+a)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{a+bx}{x^3}\,dx = \frac{-a-2bx}{2x^2}$$

input
```
integrate((b*x+a)/x**3,x)
```

output
```
(-a - 2*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{a+bx}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate((b*x+a)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x + a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{a+bx}{x^3}\,dx = -\frac{2\,bx+a}{2\,x^2}$$

input
```
integrate((b*x+a)/x^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{a+bx}{x^3}\,dx=-\frac{a+2\,b\,x}{2\,x^2}$$

input
```
int((a + b*x)/x^3,x)
```

output
```
-(a + 2*b*x)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx}{x^3}\,dx=\frac{-2bx-a}{2x^2}$$

input
```
int((b*x+a)/x^3,x)
```

output
```
( - a - 2*b*x)/(2*x**2)
```

3.8 $\int \frac{a+bx}{x^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{a+bx}{x^4} dx = -\frac{a}{3x^3} - \frac{b}{2x^2}$$

output
```
-1/3*a/x^3-1/2*b/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^4} dx = -\frac{a}{3x^3} - \frac{b}{2x^2}$$

input
```
Integrate[(a + b*x)/x^4,x]
```

output
```
-1/3*a/x^3 - b/(2*x^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^4} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{3x^3}-\frac{b}{2x^2}$$

input
```
Int[(a + b*x)/x^4,x]
```

output
```
-1/3*a/x^3 - b/(2*x^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^3}$	13
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^3}$	13
gosper	$-\frac{3bx+2a}{6x^3}$	14
default	$-\frac{a}{3x^3}-\frac{b}{2x^2}$	14
parallelrisch	$\frac{-3bx-2a}{6x^3}$	14
orering	$-\frac{3bx+2a}{6x^3}$	14

input
```
int((b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/x^3*(-1/2*b*x-1/3*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^4}\,dx = -\frac{3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((b*x+a)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(3*b*x + 2*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx}{x^4}\,dx = \frac{-2a-3bx}{6x^3}$$

input
```
integrate((b*x+a)/x**4,x)
```

output
```
(-2*a - 3*b*x)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^4}\,dx = -\frac{3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((b*x+a)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(3*b*x + 2*a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^4}\,dx = -\frac{3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((b*x+a)/x^4,x, algorithm="giac")
```

output
```
-1/6*(3*b*x + 2*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^4}\,dx=-\frac{2\,a+3\,b\,x}{6\,x^3}$$

input
```
int((a + b*x)/x^4,x)
```

output
```
-(2*a + 3*b*x)/(6*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^4}\,dx=\frac{-3bx-2a}{6x^3}$$

input
```
int((b*x+a)/x^4,x)
```

output
```
( - 2*a - 3*b*x)/(6*x**3)
```

3.9 $\int \frac{a+bx}{x^5} dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{a+bx}{x^5} dx = -\frac{a}{4x^4} - \frac{b}{3x^3}$$

output
```
-1/4*a/x^4-1/3*b/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^5} dx = -\frac{a}{4x^4} - \frac{b}{3x^3}$$

input
```
Integrate[(a + b*x)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(3*x^3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^5} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

input
```
Int[(a + b*x)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(3*x^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^4}$	13
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^4}$	13
gosper	$-\frac{4bx+3a}{12x^4}$	14
default	$-\frac{a}{4x^4}-\frac{b}{3x^3}$	14
parallelrisch	$\frac{-4bx-3a}{12x^4}$	14
orering	$-\frac{4bx+3a}{12x^4}$	14

input
```
int((b*x+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/x^4*(-1/3*b*x-1/4*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^5}\,dx=-\frac{4\,bx+3\,a}{12\,x^4}$$

input
```
integrate((b*x+a)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(4*b*x + 3*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx}{x^5}\,dx=\frac{-3a-4bx}{12x^4}$$

input
```
integrate((b*x+a)/x**5,x)
```

output
```
(-3*a - 4*b*x)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^5}\,dx=-\frac{4\,bx+3\,a}{12\,x^4}$$

input
```
integrate((b*x+a)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(4*b*x + 3*a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^5}\,dx=-\frac{4\,bx+3\,a}{12\,x^4}$$

input
```
integrate((b*x+a)/x^5,x, algorithm="giac")
```

output
```
-1/12*(4*b*x + 3*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^5}\,dx = -\frac{3\,a+4\,b\,x}{12\,x^4}$$

input
```
int((a + b*x)/x^5,x)
```

output
```
-(3*a + 4*b*x)/(12*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^5}\,dx = \frac{-4bx-3a}{12x^4}$$

input
```
int((b*x+a)/x^5,x)
```

output
```
( - 3*a - 4*b*x)/(12*x**4)
```

3.10 $\int x^3(a+bx)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int x^3(a+bx)^2\,dx=\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)^2\,dx=\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

input
```
Integrate[x^3*(a + b*x)^2,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^2 \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2x^3+2abx^4+b^2x^5\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

input
```
Int[x^3*(a + b*x)^2,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
default	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
norman	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
orering	$\frac{x^4(10b^2x^2+24abx+15a^2)}{60}$	25

input
```
int(x^3*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a+bx)^2\,dx=\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^3(a+bx)^2\,dx = \frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

input
```
integrate(x**3*(b*x+a)**2,x)
```

output
```
a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a+bx)^2\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a+bx)^2\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a+bx)^2\,dx = \frac{a^2\,x^4}{4}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^6}{6}$$

input
```
int(x^3*(a + b*x)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a+bx)^2\,dx = \frac{x^4(10b^2x^2+24abx+15a^2)}{60}$$

input
```
int(x^3*(b*x+a)^2,x)
```

output
```
(x**4*(15*a**2 + 24*a*b*x + 10*b**2*x**2))/60
```

3.11 $\int x^2(a+bx)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int x^2(a+bx)^2\,dx=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)^2\,dx=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

input
```
Integrate[x^2*(a + b*x)^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^2\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2x^2+2abx^3+b^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

input
```
Int[x^2*(a + b*x)^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{3}a^2x^3 + \frac{1}{2}ab\,x^4 + \frac{1}{5}b^2x^5$	25
default	$\frac{1}{3}a^2x^3 + \frac{1}{2}ab\,x^4 + \frac{1}{5}b^2x^5$	25
norman	$\frac{1}{3}a^2x^3 + \frac{1}{2}ab\,x^4 + \frac{1}{5}b^2x^5$	25
risch	$\frac{1}{3}a^2x^3 + \frac{1}{2}ab\,x^4 + \frac{1}{5}b^2x^5$	25
parallelrisch	$\frac{1}{3}a^2x^3 + \frac{1}{2}ab\,x^4 + \frac{1}{5}b^2x^5$	25
orering	$\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$	25

input
```
int(x^2*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx)^2\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx)^2\,dx = \frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

input
```
integrate(x**2*(b*x+a)**2,x)
```

output
```
a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx)^2\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx)^2\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx)^2\,dx=\frac{a^2\,x^3}{3}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^5}{5}$$

input
```
int(x^2*(a + b*x)^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx)^2\,dx=\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int(x^2*(b*x+a)^2,x)
```

output
```
(x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.12 $\int x(a+bx)^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int x(a+bx)^2\,dx = \frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

output
```
1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x(a+bx)^2\,dx = \frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

input
```
Integrate[x*(a + b*x)^2,x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^2 \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^2x+2abx^2+b^2x^3\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

input
```
Int[x*(a + b*x)^2,x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
default	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
norman	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
risch	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
parallelrisch	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
orering	$\frac{x^2(3b^2x^2+8abx+6a^2)}{12}$	25

input
```
int(x*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx)^2\,dx=\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(a+bx)^2\,dx = \frac{a^2x^2}{2} + \frac{2abx^3}{3} + \frac{b^4x^4}{4}$$

input
```
integrate(x*(b*x+a)**2,x)
```

output
```
a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx)^2\,dx = \frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx)^2\,dx = \frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx)^2\,dx=\frac{a^2\,x^2}{2}+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^4}{4}$$

input
```
int(x*(a + b*x)^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^4)/4 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx)^2\,dx=\frac{x^2(3b^2x^2+8abx+6a^2)}{12}$$

input
```
int(x*(b*x+a)^2,x)
```

output
```
(x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/12
```

3.13 $\int (a+bx)^2\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (a+bx)^2\,dx=\frac{(a+bx)^3}{3b}$$

output
```
1/3*(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\,dx=\frac{(a+bx)^3}{3b}$$

input
```
Integrate[(a + b*x)^2,x]
```

output
```
(a + b*x)^3/(3*b)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \, dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^3}{3b}$$

input
```
Int[(a + b*x)^2,x]
```

output
```
(a + b*x)^3/(3*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
gosper	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
norman	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
parallelrisch	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
orering	$\frac{x(b^2x^2+3abx+3a^2)}{3}$	22
risch	$\frac{b^2x^3}{3} + ab\,x^2 + a^2x + \frac{a^3}{3b}$	29

input
```
int((b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a+bx)^2\,dx = \frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

input
```
integrate((b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int (a+bx)^2\,dx = a^2x + abx^2 + \frac{b^2x^3}{3}$$

input
```
integrate((b*x+a)**2,x)
```

output
```
a**2*x + a*b*x**2 + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a+bx)^2\,dx = \frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

input
```
integrate((b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)^2\,dx = \frac{(bx+a)^3}{3\,b}$$

input
```
integrate((b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a+bx)^2\,dx = a^2\,x + a\,b\,x^2 + \frac{b^2\,x^3}{3}$$

input
```
int((a + b*x)^2,x)
```

output
```
a^2*x + (b^2*x^3)/3 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int (a+bx)^2\,dx = \frac{x(b^2x^2 + 3abx + 3a^2)}{3}$$

input
```
int((b*x+a)^2,x)
```

output
```
(x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.14 $\int \frac{(a+bx)^2}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{(a+bx)^2}{x} dx = 2abx + \frac{b^2x^2}{2} + a^2 \log(x)$$

output
```
2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x} dx = 2abx + \frac{b^2x^2}{2} + a^2 \log(x)$$

input
```
Integrate[(a + b*x)^2/x,x]
```

output
```
2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^2}{x}+2ab+b^2x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2\log(x)+2abx+\frac{b^2x^2}{2}$$

input
```
Int[(a + b*x)^2/x,x]
```

output
```
2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$2abx+\frac{b^2x^2}{2}+a^2\ln\left(x\right)$	21
norman	$2abx+\frac{b^2x^2}{2}+a^2\ln\left(x\right)$	21
risch	$2abx+\frac{b^2x^2}{2}+a^2\ln\left(x\right)$	21
parallelrisch	$2abx+\frac{b^2x^2}{2}+a^2\ln\left(x\right)$	21

input
```
int((b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2}{x}\,dx=\frac{1}{2}\,b^2x^2+2\,abx+a^2\log\left(x\right)$$

input
```
integrate((b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2}{x}\,dx=a^2\log\left(x\right)+2abx+\frac{b^2x^2}{2}$$

input
```
integrate((b*x+a)**2/x,x)
```

output
```
a**2*log(x) + 2*a*b*x + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(x\right)$$

input
```
integrate((b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^2}{x}\,dx = \frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

input
```
integrate((b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2}{x}\,dx = a^2\,\ln\left(x\right) + \frac{b^2\,x^2}{2} + 2\,a\,b\,x$$

input
```
int((a + b*x)^2/x,x)
```

output
```
a^2*log(x) + (b^2*x^2)/2 + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2}{x}\,dx = \log(x)\,a^2 + 2abx + \frac{b^2x^2}{2}$$

input
```
int((b*x+a)^2/x,x)
```

output
```
(2*log(x)*a**2 + 4*a*b*x + b**2*x**2)/2
```

3.15 $\int \frac{(a+bx)^2}{x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{(a+bx)^2}{x^2} \, dx = -\frac{a^2}{x} + b^2 x + 2ab \log(x)$$

output
```
-a^2/x+b^2*x+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^2} \, dx = -\frac{a^2}{x} + b^2 x + 2ab \log(x)$$

input
```
Integrate[(a + b*x)^2/x^2,x]
```

output
```
-(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^2} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^2}{x^2}+\frac{2ab}{x}+b^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

input
```
Int[(a + b*x)^2/x^2,x]
```

output
```
-(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{a^2}{x}+b^2x+2ab\ln\left(x\right)$	21
risch	$-\frac{a^2}{x}+b^2x+2ab\ln\left(x\right)$	21
norman	$\frac{b^2x^2-a^2}{x}+2ab\ln\left(x\right)$	25
parallelrisch	$\frac{2ab\ln(x)x+b^2x^2-a^2}{x}$	25

input
```
int((b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*x+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int\frac{(a+bx)^2}{x^2}\,dx=\frac{b^2x^2+2\,abx\log\left(x\right)-a^2}{x}$$

input
```
integrate((b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int\frac{(a+bx)^2}{x^2}\,dx=-\frac{a^2}{x}+2ab\log\left(x\right)+b^2x$$

input
```
integrate((b*x+a)**2/x**2,x)
```

output
```
-a**2/x + 2*a*b*log(x) + b**2*x
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^2}\,dx = b^2x + 2\,ab\log(x) - \frac{a^2}{x}$$

input
```
integrate((b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
b^2*x + 2*a*b*log(x) - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^2}{x^2}\,dx = b^2x + 2\,ab\log(|x|) - \frac{a^2}{x}$$

input
```
integrate((b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^2}\,dx = b^2\,x - \frac{a^2}{x} + 2\,a\,b\,\ln(x)$$

input
```
int((a + b*x)^2/x^2,x)
```

output
```
b^2*x - a^2/x + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^2}{x^2}\,dx = \frac{2\log(x)\,abx - a^2 + b^2x^2}{x}$$

input
```
int((b*x+a)^2/x^2,x)
```

output
```
(2*log(x)*a*b*x - a**2 + b**2*x**2)/x
```

3.16 $\int \frac{(a+bx)^2}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{(a+bx)^2}{x^3}\,dx = -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x)$$

output
```
-1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^3}\,dx = -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x)$$

input
```
Integrate[(a + b*x)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^3}\,dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

input
```
Int[(a + b*x)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^{2}}{2x^{2}}-\frac{2ab}{x}+b^{2}\ln\left(x\right)$	23
norman	$\frac{-\frac{1}{2}a^{2}-2abx}{x^{2}}+b^{2}\ln\left(x\right)$	23
risch	$\frac{-\frac{1}{2}a^{2}-2abx}{x^{2}}+b^{2}\ln\left(x\right)$	23
parallelrisch	$\frac{2b^{2}\ln(x)x^{2}-4abx-a^{2}}{2x^{2}}$	27

input
```
int((b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{x^3}\,dx=\frac{2\,b^{2}x^{2}\log\left(x\right)-4\,abx-a^{2}}{2\,x^{2}}$$

input
```
integrate((b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2}{x^3}\,dx=b^{2}\log\left(x\right)+\frac{-a^{2}-4abx}{2x^{2}}$$

input
```
integrate((b*x+a)**2/x**3,x)
```

output
```
b**2*log(x) + (-a**2 - 4*a*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2}{x^3}\,dx = b^2\log(x) - \frac{4\,abx + a^2}{2\,x^2}$$

input
```
integrate((b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
b^2*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2}{x^3}\,dx = b^2\log(|x|) - \frac{4\,abx + a^2}{2\,x^2}$$

input
```
integrate((b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^2}{x^3}\,dx = b^2\,\ln(x) - \frac{\frac{a^2}{2} + 2\,b\,x\,a}{x^2}$$

input
```
int((a + b*x)^2/x^3,x)
```

output
```
b^2*log(x) - (a^2/2 + 2*a*b*x)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{x^3}\,dx = \frac{2\log(x)\,b^2x^2 - a^2 - 4abx}{2x^2}$$

input
```
int((b*x+a)^2/x^3,x)
```

output
```
(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x)/(2*x**2)
```

3.17 $\int \frac{(a+bx)^2}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{(a+bx)^2}{x^4} dx = -\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

output
```
-1/3*a^2/x^3-a*b/x^2-b^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^4} dx = -\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

input
```
Integrate[(a + b*x)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 - (a*b)/x^2 - b^2/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^4} \, dx$$

$$\Big\downarrow \; 48$$

$$-\frac{(a+bx)^3}{3ax^3}$$

input
```
Int[(a + b*x)^2/x^4,x]
```

output
```
-1/3*(a + b*x)^3/(a*x^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{3b^2x^2+3abx+a^2}{3x^3}$	23
orering	$-\frac{3b^2x^2+3abx+a^2}{3x^3}$	23
norman	$\frac{-b^2x^2-abx-\frac{1}{3}a^2}{x^3}$	24
risch	$\frac{-b^2x^2-abx-\frac{1}{3}a^2}{x^3}$	24
default	$-\frac{a^2}{3x^3}-\frac{ab}{x^2}-\frac{b^2}{x}$	25
parallelrisch	$\frac{-3b^2x^2-3abx-a^2}{3x^3}$	25

input
```
int((b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^2}{x^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

input
```
integrate((b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2}{x^4}\,dx = \frac{-a^2-3abx-3b^2x^2}{3x^3}$$

input
```
integrate((b*x+a)**2/x**4,x)
```

output
```
(-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^2}{x^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

input
```
integrate((b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^2}{x^4}\,dx = -\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

input
```
integrate((b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^2}{x^4}\,dx = -\frac{\frac{a^2}{3}+a\,b\,x+b^2\,x^2}{x^3}$$

input
```
int((a + b*x)^2/x^4,x)
```

output
```
-(a^2/3 + b^2*x^2 + a*b*x)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2}{x^4}\,dx = \frac{-3b^2x^2-3abx-a^2}{3x^3}$$

input
```
int((b*x+a)^2/x^4,x)
```

output
```
( - a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

3.18 $\int \frac{(a+bx)^2}{x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{(a+bx)^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

output
```
-1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

input
```
Integrate[(a + b*x)^2/x^5,x]
```

output
```
-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^5} \, dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

input
```
Int[(a + b*x)^2/x^5,x]
```

output
```
-1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{2}b^2x^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^4}$	24
risch	$\frac{-\frac{1}{2}b^2x^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^4}$	24
gosper	$-\frac{6b^2x^2+8abx+3a^2}{12x^4}$	25
default	$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$	25
parallelrisch	$\frac{-6b^2x^2-8abx-3a^2}{12x^4}$	25
orering	$-\frac{6b^2x^2+8abx+3a^2}{12x^4}$	25

input
```
int((b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/x^4*(-1/2*b^2*x^2-2/3*a*b*x-1/4*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^5}\,dx = -\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

input
```
integrate((b*x+a)^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2}{x^5}\,dx = \frac{-3a^2-8abx-6b^2x^2}{12x^4}$$

input
```
integrate((b*x+a)**2/x**5,x)
```

output
```
(-3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^5}\,dx = -\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

input
```
integrate((b*x+a)^2/x^5,x, algorithm="maxima")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^5}\,dx = -\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

input
```
integrate((b*x+a)^2/x^5,x, algorithm="giac")
```

output
```
-1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^5}\,dx = -\frac{\frac{a^2}{4}+\frac{2\,a\,b\,x}{3}+\frac{b^2\,x^2}{2}}{x^4}$$

input
```
int((a + b*x)^2/x^5,x)
```

output
```
-(a^2/4 + (b^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^5}\,dx = \frac{-6b^2x^2-8abx-3a^2}{12x^4}$$

input
```
int((b*x+a)^2/x^5,x)
```

output
```
( - 3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

3.19 $\int \frac{(a+bx)^2}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{(a+bx)^2}{x^6} dx = -\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

output
```
-1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^6} dx = -\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

input
```
Integrate[(a + b*x)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^6}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

input
```
Int[(a + b*x)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{3}b^2x^2-\frac{1}{2}abx-\frac{1}{5}a^2}{x^5}$	24
risch	$\frac{-\frac{1}{3}b^2x^2-\frac{1}{2}abx-\frac{1}{5}a^2}{x^5}$	24
gosper	$-\frac{10b^2x^2+15abx+6a^2}{30x^5}$	25
default	$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$	25
parallelrisch	$\frac{-10b^2x^2-15abx-6a^2}{30x^5}$	25
orering	$-\frac{10b^2x^2+15abx+6a^2}{30x^5}$	25

input
```
int((b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/x^5*(-1/3*b^2*x^2-1/2*a*b*x-1/5*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^6}\,dx = -\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

input
```
integrate((b*x+a)^2/x^6,x, algorithm="fricas")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2}{x^6}\,dx = \frac{-6a^2 - 15abx - 10b^2x^2}{30x^5}$$

input
```
integrate((b*x+a)**2/x**6,x)
```

output
```
(-6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^6}\,dx = -\frac{10\,b^2x^2 + 15\,abx + 6\,a^2}{30\,x^5}$$

input
```
integrate((b*x+a)^2/x^6,x, algorithm="maxima")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^6}\,dx = -\frac{10\,b^2x^2 + 15\,abx + 6\,a^2}{30\,x^5}$$

input
```
integrate((b*x+a)^2/x^6,x, algorithm="giac")
```

output
```
-1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^6}\,dx = -\frac{\frac{a^2}{5}+\frac{a\,b\,x}{2}+\frac{b^2\,x^2}{3}}{x^5}$$

input
```
int((a + b*x)^2/x^6,x)
```

output
```
-(a^2/5 + (b^2*x^2)/3 + (a*b*x)/2)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^6}\,dx = \frac{-10b^2x^2-15abx-6a^2}{30x^5}$$

input
```
int((b*x+a)^2/x^6,x)
```

output
```
( - 6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

3.20 $\int \frac{(a+bx)^2}{x^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{(a+bx)^2}{x^7} dx = -\frac{a^2}{6x^6} - \frac{2ab}{5x^5} - \frac{b^2}{4x^4}$$

output
```
-1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^7} dx = -\frac{a^2}{6x^6} - \frac{2ab}{5x^5} - \frac{b^2}{4x^4}$$

input
```
Integrate[(a + b*x)^2/x^7,x]
```

output
```
-1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^7} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

input
```
Int[(a + b*x)^2/x^7,x]
```

output
```
-1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{4}b^2x^2-\frac{2}{5}abx-\frac{1}{6}a^2}{x^6}$	24
risch	$\frac{-\frac{1}{4}b^2x^2-\frac{2}{5}abx-\frac{1}{6}a^2}{x^6}$	24
gosper	$-\frac{15b^2x^2+24abx+10a^2}{60x^6}$	25
default	$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$	25
parallelrisch	$\frac{-15b^2x^2-24abx-10a^2}{60x^6}$	25
orering	$-\frac{15b^2x^2+24abx+10a^2}{60x^6}$	25

input
```
int((b*x+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/x^6*(-1/4*b^2*x^2-2/5*a*b*x-1/6*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^7}\,dx = -\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

input
```
integrate((b*x+a)^2/x^7,x, algorithm="fricas")
```

output
```
-1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2}{x^7}\,dx = \frac{-10a^2-24abx-15b^2x^2}{60x^6}$$

input
```
integrate((b*x+a)**2/x**7,x)
```

output
```
(-10*a**2 - 24*a*b*x - 15*b**2*x**2)/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^7}\,dx = -\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

input
```
integrate((b*x+a)^2/x^7,x, algorithm="maxima")
```

output
```
-1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^7}\,dx = -\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

input
```
integrate((b*x+a)^2/x^7,x, algorithm="giac")
```

output
```
-1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^7}\,dx = -\frac{\frac{a^2}{6}+\frac{2\,a\,b\,x}{5}+\frac{b^2\,x^2}{4}}{x^6}$$

input
```
int((a + b*x)^2/x^7,x)
```

output
```
-(a^2/6 + (b^2*x^2)/4 + (2*a*b*x)/5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^7}\,dx = \frac{-15b^2x^2-24abx-10a^2}{60x^6}$$

input
```
int((b*x+a)^2/x^7,x)
```

output
```
( - 10*a**2 - 24*a*b*x - 15*b**2*x**2)/(60*x**6)
```

3.21 $\int \frac{(a+bx)^2}{x^8} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{(a+bx)^2}{x^8} \, dx = -\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

output
```
-1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^8} \, dx = -\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

input
```
Integrate[(a + b*x)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^8} \, dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

input
```
Int[(a + b*x)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{5}b^2x^2-\frac{1}{3}abx-\frac{1}{7}a^2}{x^7}$	24
risch	$\frac{-\frac{1}{5}b^2x^2-\frac{1}{3}abx-\frac{1}{7}a^2}{x^7}$	24
gosper	$-\frac{21b^2x^2+35abx+15a^2}{105x^7}$	25
default	$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$	25
parallelrisch	$\frac{-21b^2x^2-35abx-15a^2}{105x^7}$	25
orering	$-\frac{21b^2x^2+35abx+15a^2}{105x^7}$	25

input
```
int((b*x+a)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/x^7*(-1/5*b^2*x^2-1/3*a*b*x-1/7*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^8}\,dx = -\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

input
```
integrate((b*x+a)^2/x^8,x, algorithm="fricas")
```

output
```
-1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2}{x^8}\,dx = \frac{-15a^2 - 35abx - 21b^2x^2}{105x^7}$$

input
```
integrate((b*x+a)**2/x**8,x)
```

output
```
(-15*a**2 - 35*a*b*x - 21*b**2*x**2)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^8}\,dx = -\frac{21\,b^2x^2 + 35\,abx + 15\,a^2}{105\,x^7}$$

input
```
integrate((b*x+a)^2/x^8,x, algorithm="maxima")
```

output
```
-1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^8}\,dx = -\frac{21\,b^2x^2 + 35\,abx + 15\,a^2}{105\,x^7}$$

input
```
integrate((b*x+a)^2/x^8,x, algorithm="giac")
```

output
```
-1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^8}\,dx = -\frac{\frac{a^2}{7}+\frac{a\,b\,x}{3}+\frac{b^2\,x^2}{5}}{x^7}$$

input
```
int((a + b*x)^2/x^8,x)
```

output
```
-(a^2/7 + (b^2*x^2)/5 + (a*b*x)/3)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^8}\,dx = \frac{-21b^2x^2-35abx-15a^2}{105x^7}$$

input
```
int((b*x+a)^2/x^8,x)
```

output
```
( - 15*a**2 - 35*a*b*x - 21*b**2*x**2)/(105*x**7)
```

3.22 $\int x^4(a+bx)^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int x^4(a+bx)^3\,dx=\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

output
```
1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)^3\,dx=\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

input
```
Integrate[x^4*(a + b*x)^3,x]
```

output
```
(a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^3\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

input
```
Int[x^4*(a + b*x)^3,x]
```

output
```
(a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
default	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
norman	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
risch	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
parallelrisch	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
orering	$\frac{x^5(35b^3x^3+120a\,b^2x^2+140a^2bx+56a^3)}{280}$	36

input
```
int(x^4*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx)^3\,dx=\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)^3\,dx=\frac{a^3x^5}{5}+\frac{a^2bx^6}{2}+\frac{3ab^2x^7}{7}+\frac{b^3x^8}{8}$$

input
```
integrate(x**4*(b*x+a)**3,x)
```

output
```
a**3*x**5/5 + a**2*b*x**6/2 + 3*a*b**2*x**7/7 + b**3*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx)^3\,dx=\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx)^3\,dx=\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx)^3\,dx=\frac{a^3\,x^5}{5}+\frac{a^2\,b\,x^6}{2}+\frac{3\,a\,b^2\,x^7}{7}+\frac{b^3\,x^8}{8}$$

input
```
int(x^4*(a + b*x)^3,x)
```

output
```
(a^3*x^5)/5 + (b^3*x^8)/8 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx)^3\,dx=\frac{x^5(35b^3x^3+120a\,b^2x^2+140a^2bx+56a^3)}{280}$$

input
```
int(x^4*(b*x+a)^3,x)
```

output
```
(x**5*(56*a**3 + 140*a**2*b*x + 120*a*b**2*x**2 + 35*b**3*x**3))/280
```

3.23 $\int x^3(a+bx)^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int x^3(a+bx)^3\,dx=\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

output
```
1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)^3\,dx=\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

input
```
Integrate[x^3*(a + b*x)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^3\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

input
```
Int[x^3*(a + b*x)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
default	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
norman	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
risch	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
parallelrisch	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
orering	$\frac{x^4\left(20b^3x^3+70a\,b^2x^2+84a^2bx+35a^3\right)}{140}$	36

input
```
int(x^3*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3(a+bx)^3\,dx=\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)^3\,dx = \frac{a^3x^4}{4} + \frac{3a^2bx^5}{5} + \frac{ab^2x^6}{2} + \frac{b^3x^7}{7}$$

input
```
integrate(x**3*(b*x+a)**3,x)
```

output
```
a**3*x**4/4 + 3*a**2*b*x**5/5 + a*b**2*x**6/2 + b**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3(a+bx)^3\,dx = \frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3(a+bx)^3\,dx = \frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b*x+a)^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3(a+bx)^3\,dx = \frac{a^3\,x^4}{4}+\frac{3\,a^2\,b\,x^5}{5}+\frac{a\,b^2\,x^6}{2}+\frac{b^3\,x^7}{7}$$

input
```
int(x^3*(a + b*x)^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^7)/7 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3(a+bx)^3\,dx = \frac{x^4(20b^3x^3+70a\,b^2x^2+84a^2bx+35a^3)}{140}$$

input
```
int(x^3*(b*x+a)^3,x)
```

output
```
(x**4*(35*a**3 + 84*a**2*b*x + 70*a*b**2*x**2 + 20*b**3*x**3))/140
```

3.24 $\int x^2(a+bx)^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int x^2(a+bx)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

output
```
1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

input
```
Integrate[x^2*(a + b*x)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^3\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

input
```
Int[x^2*(a + b*x)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
default	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
norman	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
risch	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
parallelrisch	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
orering	$\frac{x^3(10b^3x^3+36a\,b^2x^2+45a^2bx+20a^3)}{60}$	36

input
```
int(x^2*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2(a+bx)^3\,dx=\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x^2(a+bx)^3\,dx = \frac{a^3x^3}{3} + \frac{3a^2bx^4}{4} + \frac{3ab^2x^5}{5} + \frac{b^3x^6}{6}$$

input
```
integrate(x**2*(b*x+a)**3,x)
```

output
```
a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*a*b**2*x**5/5 + b**3*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2(a+bx)^3\,dx = \frac{1}{6}\,b^3x^6 + \frac{3}{5}\,ab^2x^5 + \frac{3}{4}\,a^2bx^4 + \frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2(a+bx)^3\,dx = \frac{1}{6}\,b^3x^6 + \frac{3}{5}\,ab^2x^5 + \frac{3}{4}\,a^2bx^4 + \frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b*x+a)^3,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2(a+bx)^3\,dx = \frac{a^3\,x^3}{3} + \frac{3\,a^2\,b\,x^4}{4} + \frac{3\,a\,b^2\,x^5}{5} + \frac{b^3\,x^6}{6}$$

input
```
int(x^2*(a + b*x)^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^6)/6 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2(a+bx)^3\,dx = \frac{x^3(10b^3x^3 + 36a\,b^2x^2 + 45a^2bx + 20a^3)}{60}$$

input
```
int(x^2*(b*x+a)^3,x)
```

output
```
(x**3*(20*a**3 + 45*a**2*b*x + 36*a*b**2*x**2 + 10*b**3*x**3))/60
```

3.25 $\int x(a+bx)^3\,dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int x(a+bx)^3\,dx=-\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}$$

output
```
-1/4*a*(b*x+a)^4/b^2+1/5*(b*x+a)^5/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int x(a+bx)^3\,dx=\frac{a^3x^2}{2}+a^2bx^3+\frac{3}{4}ab^2x^4+\frac{b^3x^5}{5}$$

input
```
Integrate[x*(a + b*x)^3,x]
```

output
```
(a^3*x^2)/2 + a^2*b*x^3 + (3*a*b^2*x^4)/4 + (b^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^3\,dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{(a+bx)^4}{b}-\frac{a(a+bx)^3}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

input
```
Int[x*(a + b*x)^3,x]
```

output
```
-1/4*(a*(a + b*x)^4)/b^2 + (a + b*x)^5/(5*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

method	result	size
gosper	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
default	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
norman	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
risch	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
parallelrisch	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
orering	$\frac{x^2(4b^3x^3+15a\,b^2x^2+20a^2bx+10a^3)}{20}$	36

input
```
int(x*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^3*x^5+3/4*a*b^2*x^4+a^2*b*x^3+1/2*a^3*x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x(a+bx)^3\,dx=\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x(a+bx)^3\,dx = \frac{a^3x^2}{2} + a^2bx^3 + \frac{3ab^2x^4}{4} + \frac{b^3x^5}{5}$$

input
```
integrate(x*(b*x+a)**3,x)
```

output
```
a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x(a+bx)^3\,dx = \frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x(a+bx)^3\,dx = \frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x+a)^3,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x(a+bx)^3\,dx=\frac{a^3\,x^2}{2}+a^2\,b\,x^3+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^5}{5}$$

input
```
int(x*(a + b*x)^3,x)
```

output
```
(a^3*x^2)/2 + (b^3*x^5)/5 + a^2*b*x^3 + (3*a*b^2*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int x(a+bx)^3\,dx=\frac{x^2(4b^3x^3+15a\,b^2x^2+20a^2bx+10a^3)}{20}$$

input
```
int(x*(b*x+a)^3,x)
```

output
```
(x**2*(10*a**3 + 20*a**2*b*x + 15*a*b**2*x**2 + 4*b**3*x**3))/20
```

3.26 $\int (a+bx)^3 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (a+bx)^3 \, dx = \frac{(a+bx)^4}{4b}$$

output
```
1/4*(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+bx)^3 \, dx = \frac{(a+bx)^4}{4b}$$

input
```
Integrate[(a + b*x)^3,x]
```

output
```
(a + b*x)^4/(4*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^4}{4b}$$

input
```
Int[(a + b*x)^3,x]
```

output
```
(a + b*x)^4/(4*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^4}{4b}$	13
gosper	$\frac{1}{4}b^3x^4+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$	32
norman	$\frac{1}{4}b^3x^4+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$	32
parallelrisch	$\frac{1}{4}b^3x^4+a\,b^2x^3+\frac{3}{2}a^2b\,x^2+a^3x$	32
orering	$\frac{x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4}$	33
risch	$\frac{b^3x^4}{4}+a\,b^2x^3+\frac{3a^2b\,x^2}{2}+a^3x+\frac{a^4}{4b}$	40

input
```
int((b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int (a+bx)^3\,dx=\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

input
```
integrate((b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int (a + bx)^3 \, dx = a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

input
```
integrate((b*x+a)**3,x)
```

output
```
a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int (a + bx)^3 \, dx = \frac{1}{4} b^3x^4 + ab^2x^3 + \frac{3}{2} a^2bx^2 + a^3x$$

input
```
integrate((b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a + bx)^3 \, dx = \frac{(bx + a)^4}{4 b}$$

input
```
integrate((b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(b*x + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int (a+bx)^3\,dx = a^3\,x+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x^3+\frac{b^3\,x^4}{4}$$

input
```
int((a + b*x)^3,x)
```

output
```
a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int (a+bx)^3\,dx = \frac{x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4}$$

input
```
int((b*x+a)^3,x)
```

output
```
(x*(4*a**3 + 6*a**2*b*x + 4*a*b**2*x**2 + b**3*x**3))/4
```

3.27 $\int \frac{(a+bx)^3}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{(a+bx)^3}{x} \, dx = 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3\log(x)$$

output
```
3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x} \, dx = 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3\log(x)$$

input
```
Integrate[(a + b*x)^3/x,x]
```

output
```
3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^3}{x}+3a^2b+3ab^2x+b^3x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3\log(x)+3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}$$

input
```
Int[(a + b*x)^3/x,x]
```

output
```
3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$3a^2bx+\frac{3a\,b^2x^2}{2}+\frac{b^3x^3}{3}+a^3\ln\left(x\right)$	32
norman	$3a^2bx+\frac{3a\,b^2x^2}{2}+\frac{b^3x^3}{3}+a^3\ln\left(x\right)$	32
risch	$3a^2bx+\frac{3a\,b^2x^2}{2}+\frac{b^3x^3}{3}+a^3\ln\left(x\right)$	32
parallelrisch	$3a^2bx+\frac{3a\,b^2x^2}{2}+\frac{b^3x^3}{3}+a^3\ln\left(x\right)$	32

input
```
int((b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^3}{x}\,dx=\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log\left(x\right)$$

input
```
integrate((b*x+a)^3/x,x, algorithm="fricas")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x}\,dx=a^3\log\left(x\right)+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3}$$

input
```
integrate((b*x+a)**3/x,x)
```

output
```
a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^3}{x}\,dx = \frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(x\right)$$

input
```
integrate((b*x+a)^3/x,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^3}{x}\,dx = \frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(|x|\right)$$

input
```
integrate((b*x+a)^3/x,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^3}{x}\,dx = a^3\,\ln\left(x\right) + \frac{b^3\,x^3}{3} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,x$$

input
```
int((a + b*x)^3/x,x)
```

output
```
a^3*log(x) + (b^3*x^3)/3 + (3*a*b^2*x^2)/2 + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^3}{x}\,dx = \log(x)\,a^3 + 3a^2bx + \frac{3a\,b^2x^2}{2} + \frac{b^3x^3}{3}$$

input
```
int((b*x+a)^3/x,x)
```

output
```
(6*log(x)*a**3 + 18*a**2*b*x + 9*a*b**2*x**2 + 2*b**3*x**3)/6
```

3.28 $\int \frac{(a+bx)^3}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{(a+bx)^3}{x^2} dx = -\frac{a^3}{x} + 3ab^2x + \frac{b^3x^2}{2} + 3a^2b\log(x)$$

output
```
-a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x^2} dx = -\frac{a^3}{x} + 3ab^2x + \frac{b^3x^2}{2} + 3a^2b\log(x)$$

input
```
Integrate[(a + b*x)^3/x^2,x]
```

output
```
-(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^2} \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^3}{x^2}+\frac{3a^2b}{x}+3ab^2+b^3x\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}$$

input
```
Int[(a + b*x)^3/x^2,x]
```

output
```
-(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{a^3}{x}+3a\,b^2x+\frac{b^3x^2}{2}+3a^2b\ln\left(x\right)$	33
risch	$-\frac{a^3}{x}+3a\,b^2x+\frac{b^3x^2}{2}+3a^2b\ln\left(x\right)$	33
norman	$\frac{-a^3+\frac{1}{2}b^3x^3+3a\,b^2x^2}{x}+3a^2b\ln\left(x\right)$	37
parallelrisch	$\frac{b^3x^3+6a^2b\ln(x)x+6a\,b^2x^2-2a^3}{2x}$	37

input
```
int((b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int\frac{(a+bx)^3}{x^2}\,dx=\frac{b^3x^3+6\,ab^2x^2+6\,a^2bx\log\left(x\right)-2\,a^3}{2\,x}$$

input
```
integrate((b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int\frac{(a+bx)^3}{x^2}\,dx=-\frac{a^3}{x}+3a^2b\log\left(x\right)+3ab^2x+\frac{b^3x^2}{2}$$

input
```
integrate((b*x+a)**3/x**2,x)
```

output
```
-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^3}{x^2}\,dx = \frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log{(x)} - \frac{a^3}{x}$$

input
```
integrate((b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(x) - a^3/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x^2}\,dx = \frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log{(|x|)} - \frac{a^3}{x}$$

input
```
integrate((b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^3}{x^2}\,dx = \frac{b^3\,x^2}{2} - \frac{a^3}{x} + 3\,a^2\,b\,\ln{(x)} + 3\,a\,b^2\,x$$

input
```
int((a + b*x)^3/x^2,x)
```

output
```
(b^3*x^2)/2 - a^3/x + 3*a^2*b*log(x) + 3*a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3}{x^2}\,dx = \frac{6\log(x)\,a^2bx - 2a^3 + 6a\,b^2x^2 + b^3x^3}{2x}$$

input
```
int((b*x+a)^3/x^2,x)
```

output
```
(6*log(x)*a**2*b*x - 2*a**3 + 6*a*b**2*x**2 + b**3*x**3)/(2*x)
```

3.29 $\int \frac{(a+bx)^3}{x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{(a+bx)^3}{x^3}\,dx = -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + b^3x + 3ab^2\log(x)$$

output
```
-1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x^3}\,dx = -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + b^3x + 3ab^2\log(x)$$

input
```
Integrate[(a + b*x)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^3} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}+b^3\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x$$

input
```
Int[(a + b*x)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{a^{3}}{2x^{2}}-\frac{3a^{2}b}{x}+b^{3}x+3a\,b^{2}\ln\left(x\right)$	32
risch	$b^{3}x+\frac{-3a^{2}bx-\frac{1}{2}a^{3}}{x^{2}}+3a\,b^{2}\ln\left(x\right)$	32
norman	$\frac{b^{3}x^{3}-\frac{1}{2}a^{3}-3a^{2}bx}{x^{2}}+3a\,b^{2}\ln\left(x\right)$	34
parallelrisch	$\frac{6a\,b^{2}\ln\left(x\right)x^{2}+2b^{3}x^{3}-6a^{2}bx-a^{3}}{2x^{2}}$	38

input
```
int((b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^3}{x^3}\,dx = \frac{2\,b^3x^3+6\,ab^2x^2\log\left(x\right)-6\,a^2bx-a^3}{2\,x^2}$$

input
```
integrate((b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x^3}\,dx = 3ab^2\log\left(x\right)+b^3x+\frac{-a^3-6a^2bx}{2x^2}$$

input
```
integrate((b*x+a)**3/x**3,x)
```

output
```
3*a*b**2*log(x) + b**3*x + (-a**3 - 6*a**2*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^3}{x^3} \, dx = b^3x + 3\, ab^2 \log{(x)} - \frac{6\, a^2bx + a^3}{2\, x^2}$$

input
```
integrate((b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
b^3*x + 3*a*b^2*log(x) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^3}{x^3} \, dx = b^3x + 3\, ab^2 \log{(|x|)} - \frac{6\, a^2bx + a^3}{2\, x^2}$$

input
```
integrate((b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x^3} \, dx = b^3\, x - \frac{\frac{a^3}{2} + 3\, b\, x\, a^2}{x^2} + 3\, a\, b^2\, \ln{(x)}$$

input
```
int((a + b*x)^3/x^3,x)
```

output
```
b^3*x - (a^3/2 + 3*a^2*b*x)/x^2 + 3*a*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^3}{x^3}\,dx = \frac{6\log(x)\,a\,b^2x^2 - a^3 - 6a^2bx + 2b^3x^3}{2x^2}$$

input
```
int((b*x+a)^3/x^3,x)
```

output
```
(6*log(x)*a*b**2*x**2 - a**3 - 6*a**2*b*x + 2*b**3*x**3)/(2*x**2)
```

3.30 $\int \frac{(a+bx)^3}{x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{(a+bx)^3}{x^4}\,dx = -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x)$$

output
```
-1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x^4}\,dx = -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x)$$

input
```
Integrate[(a + b*x)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^4} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

input
```
Int[(a + b*x)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3a\,b^2}{x}+b^3\ln\left(x\right)$	34
norman	$\frac{-\frac{1}{3}a^3-3a\,b^2x^2-\frac{3}{2}a^2bx}{x^3}+b^3\ln\left(x\right)$	34
risch	$\frac{-\frac{1}{3}a^3-3a\,b^2x^2-\frac{3}{2}a^2bx}{x^3}+b^3\ln\left(x\right)$	34
parallelrisch	$\frac{6b^3\ln(x)x^3-18a\,b^2x^2-9a^2bx-2a^3}{6x^3}$	38

input
```
int((b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int\frac{(a+bx)^3}{x^4}\,dx=\frac{6\,b^3x^3\log\left(x\right)-18\,ab^2x^2-9\,a^2bx-2\,a^3}{6\,x^3}$$

input
```
integrate((b*x+a)^3/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int\frac{(a+bx)^3}{x^4}\,dx=b^3\log\left(x\right)+\frac{-2a^3-9a^2bx-18ab^2x^2}{6x^3}$$

input
```
integrate((b*x+a)**3/x**4,x)
```

output
```
b**3*log(x) + (-2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3}{x^4} dx = b^3 \log(x) - \frac{18 ab^2x^2 + 9 a^2bx + 2 a^3}{6 x^3}$$

input
```
integrate((b*x+a)^3/x^4,x, algorithm="maxima")
```

output
```
b^3*log(x) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^3}{x^4} dx = b^3 \log(|x|) - \frac{18 ab^2x^2 + 9 a^2bx + 2 a^3}{6 x^3}$$

input
```
integrate((b*x+a)^3/x^4,x, algorithm="giac")
```

output
```
b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3}{x^4} dx = b^3 \ln(x) - \frac{\frac{a^3}{3} + \frac{3 a^2 b x}{2} + 3 a b^2 x^2}{x^3}$$

input
```
int((a + b*x)^3/x^4,x)
```

output
```
b^3*log(x) - (a^3/3 + 3*a*b^2*x^2 + (3*a^2*b*x)/2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x^4}\,dx = \frac{6\log(x)\,b^3x^3-2a^3-9a^2bx-18a\,b^2x^2}{6x^3}$$

input
```
int((b*x+a)^3/x^4,x)
```

output
```
(6*log(x)*b**3*x**3 - 2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

3.31 $\int \frac{(a+bx)^3}{x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{(a+bx)^3}{x^5} dx = -\frac{(a+bx)^4}{4ax^4}$$

output
```
-1/4*(b*x+a)^4/a/x^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{(a+bx)^3}{x^5} dx = -\frac{a^3}{4x^4} - \frac{a^2b}{x^3} - \frac{3ab^2}{2x^2} - \frac{b^3}{x}$$

input
```
Integrate[(a + b*x)^3/x^5,x]
```

output
```
-1/4*a^3/x^4 - (a^2*b)/x^3 - (3*a*b^2)/(2*x^2) - b^3/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^5} \, dx$$

$$\Big\downarrow \ 48$$

$$-\frac{(a+bx)^4}{4ax^4}$$

input
```
Int[(a + b*x)^3/x^5,x]
```

output
```
-1/4*(a + b*x)^4/(a*x^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

method	result	size
gosper	$-\frac{4b^3x^3+6a\,b^2x^2+4a^2bx+a^3}{4x^4}$	34
orering	$-\frac{4b^3x^3+6a\,b^2x^2+4a^2bx+a^3}{4x^4}$	34
norman	$\frac{-b^3x^3-\frac{3}{2}a\,b^2x^2-a^2bx-\frac{1}{4}a^3}{x^4}$	35
risch	$\frac{-b^3x^3-\frac{3}{2}a\,b^2x^2-a^2bx-\frac{1}{4}a^3}{x^4}$	35
default	$-\frac{a^2b}{x^3}-\frac{3a\,b^2}{2x^2}-\frac{a^3}{4x^4}-\frac{b^3}{x}$	36
parallelrisch	$\frac{-4b^3x^3-6a\,b^2x^2-4a^2bx-a^3}{4x^4}$	36

input
```
int((b*x+a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b^3*x^3+6*a*b^2*x^2+4*a^2*b*x+a^3)/x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^3}{x^5}\,dx = -\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

input
```
integrate((b*x+a)^3/x^5,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx)^3}{x^5}\,dx = \frac{-a^3 - 4a^2bx - 6ab^2x^2 - 4b^3x^3}{4x^4}$$

input
```
integrate((b*x+a)**3/x**5,x)
```

output
```
(-a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^3}{x^5}\,dx = -\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

input
```
integrate((b*x+a)^3/x^5,x, algorithm="maxima")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^3}{x^5}\,dx = -\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

input
```
integrate((b*x+a)^3/x^5,x, algorithm="giac")
```

output
```
-1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^3}{x^5} \, dx = -\frac{\frac{a^3}{4} + a^2 \, b \, x + \frac{3 \, a \, b^2 \, x^2}{2} + b^3 \, x^3}{x^4}$$

input
```
int((a + b*x)^3/x^5,x)
```

output
```
-(a^3/4 + b^3*x^3 + (3*a*b^2*x^2)/2 + a^2*b*x)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx)^3}{x^5} \, dx = \frac{-4b^3x^3 - 6a \, b^2x^2 - 4a^2bx - a^3}{4x^4}$$

input
```
int((b*x+a)^3/x^5,x)
```

output
```
( - a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

3.32 $\int \frac{(a+bx)^3}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{(a+bx)^3}{x^6}\, dx = -\frac{(a+bx)^4}{5ax^5} + \frac{b(a+bx)^4}{20a^2x^4}$$

output
```
-1/5*(b*x+a)^4/a/x^5+1/20*b*(b*x+a)^4/a^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^3}{x^6}\, dx = -\frac{a^3}{5x^5} - \frac{3a^2b}{4x^4} - \frac{ab^2}{x^3} - \frac{b^3}{2x^2}$$

input
```
Integrate[(a + b*x)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (3*a^2*b)/(4*x^4) - (a*b^2)/x^3 - b^3/(2*x^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^6}\,dx$$

$$\Big\downarrow \; 55$$

$$-\frac{b\int \frac{(a+bx)^3}{x^5}dx}{5a}-\frac{(a+bx)^4}{5ax^5}$$

$$\Big\downarrow \; 48$$

$$\frac{b(a+bx)^4}{20a^2x^4}-\frac{(a+bx)^4}{5ax^5}$$

input
```
Int[(a + b*x)^3/x^6,x]
```

output
```
-1/5*(a + b*x)^4/(a*x^5) + (b*(a + b*x)^4)/(20*a^2*x^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{-\frac{1}{2}b^3x^3-a\,b^2x^2-\frac{3}{4}a^2bx-\frac{1}{5}a^3}{x^5}$	35
risch	$\frac{-\frac{1}{2}b^3x^3-a\,b^2x^2-\frac{3}{4}a^2bx-\frac{1}{5}a^3}{x^5}$	35
gosper	$-\frac{10b^3x^3+20a\,b^2x^2+15a^2bx+4a^3}{20x^5}$	36
default	$-\frac{a\,b^2}{x^3}-\frac{a^3}{5x^5}-\frac{b^3}{2x^2}-\frac{3a^2b}{4x^4}$	36
parallelrisch	$\frac{-10b^3x^3-20a\,b^2x^2-15a^2bx-4a^3}{20x^5}$	36
orering	$-\frac{10b^3x^3+20a\,b^2x^2+15a^2bx+4a^3}{20x^5}$	36

input
```
int((b*x+a)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/x^5*(-1/2*b^3*x^3-a*b^2*x^2-3/4*a^2*b*x-1/5*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x^6}\,dx = -\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

input
```
integrate((b*x+a)^3/x^6,x, algorithm="fricas")
```

output
```
-1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^3}{x^6}\,dx = \frac{-4a^3-15a^2bx-20ab^2x^2-10b^3x^3}{20x^5}$$

input
```
integrate((b*x+a)**3/x**6,x)
```

output
```
(-4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x^6}\,dx = -\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

input
```
integrate((b*x+a)^3/x^6,x, algorithm="maxima")
```

output
```
-1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x^6}\,dx = -\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

input
```
integrate((b*x+a)^3/x^6,x, algorithm="giac")
```

output
```
-1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^3}{x^6}\,dx = -\frac{\frac{a^3}{5}+\frac{3\,a^2\,b\,x}{4}+a\,b^2\,x^2+\frac{b^3\,x^3}{2}}{x^5}$$

input
```
int((a + b*x)^3/x^6,x)
```

output
```
-(a^3/5 + (b^3*x^3)/2 + a*b^2*x^2 + (3*a^2*b*x)/4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3}{x^6}\,dx = \frac{-10b^3x^3-20a\,b^2x^2-15a^2bx-4a^3}{20x^5}$$

input
```
int((b*x+a)^3/x^6,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

3.33 $\int \frac{(a+bx)^3}{x^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{(a+bx)^3}{x^7} dx = -\frac{a^3}{6x^6} - \frac{3a^2b}{5x^5} - \frac{3ab^2}{4x^4} - \frac{b^3}{3x^3}$$

output
```
-1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x^7} dx = -\frac{a^3}{6x^6} - \frac{3a^2b}{5x^5} - \frac{3ab^2}{4x^4} - \frac{b^3}{3x^3}$$

input
```
Integrate[(a + b*x)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^7} \, dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

input
```
Int[(a + b*x)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{-\frac{1}{3}b^{3}x^{3}-\frac{3}{4}a\,b^{2}x^{2}-\frac{3}{5}a^{2}bx-\frac{1}{6}a^{3}}{x^{6}}$	35
risch	$\frac{-\frac{1}{3}b^{3}x^{3}-\frac{3}{4}a\,b^{2}x^{2}-\frac{3}{5}a^{2}bx-\frac{1}{6}a^{3}}{x^{6}}$	35
gosper	$-\frac{20b^{3}x^{3}+45a\,b^{2}x^{2}+36a^{2}bx+10a^{3}}{60x^{6}}$	36
default	$-\frac{a^{3}}{6x^{6}}-\frac{3a^{2}b}{5x^{5}}-\frac{3a\,b^{2}}{4x^{4}}-\frac{b^{3}}{3x^{3}}$	36
parallelrisch	$\frac{-20b^{3}x^{3}-45a\,b^{2}x^{2}-36a^{2}bx-10a^{3}}{60x^{6}}$	36
orering	$-\frac{20b^{3}x^{3}+45a\,b^{2}x^{2}+36a^{2}bx+10a^{3}}{60x^{6}}$	36

input
```
int((b*x+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/x^6*(-1/3*b^3*x^3-3/4*a*b^2*x^2-3/5*a^2*b*x-1/6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^7}\,dx = -\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

input
```
integrate((b*x+a)^3/x^7,x, algorithm="fricas")
```

output
```
-1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^3}{x^7}\,dx = \frac{-10a^3 - 36a^2bx - 45ab^2x^2 - 20b^3x^3}{60x^6}$$

input
```
integrate((b*x+a)**3/x**7,x)
```

output
```
(-10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^7}\,dx = -\frac{20\,b^3x^3 + 45\,ab^2x^2 + 36\,a^2bx + 10\,a^3}{60\,x^6}$$

input
```
integrate((b*x+a)^3/x^7,x, algorithm="maxima")
```

output
```
-1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^7}\,dx = -\frac{20\,b^3x^3 + 45\,ab^2x^2 + 36\,a^2bx + 10\,a^3}{60\,x^6}$$

input
```
integrate((b*x+a)^3/x^7,x, algorithm="giac")
```

output
```
-1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^7}\,dx = -\frac{\frac{a^3}{6}+\frac{3\,a^2\,b\,x}{5}+\frac{3\,a\,b^2\,x^2}{4}+\frac{b^3\,x^3}{3}}{x^6}$$

input
```
int((a + b*x)^3/x^7,x)
```

output
```
-(a^3/6 + (b^3*x^3)/3 + (3*a*b^2*x^2)/4 + (3*a^2*b*x)/5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^7}\,dx = \frac{-20b^3x^3-45a\,b^2x^2-36a^2bx-10a^3}{60x^6}$$

input
```
int((b*x+a)^3/x^7,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

3.34 $\int \frac{(a+bx)^3}{x^8} dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{(a+bx)^3}{x^8}\,dx = -\frac{a^3}{7x^7} - \frac{a^2b}{2x^6} - \frac{3ab^2}{5x^5} - \frac{b^3}{4x^4}$$

output
```
-1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x^8}\,dx = -\frac{a^3}{7x^7} - \frac{a^2b}{2x^6} - \frac{3ab^2}{5x^5} - \frac{b^3}{4x^4}$$

input
```
Integrate[(a + b*x)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^8} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

input
```
Int[(a + b*x)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{-\frac{1}{4}b^3x^3-\frac{3}{5}a\,b^2x^2-\frac{1}{2}a^2bx-\frac{1}{7}a^3}{x^7}$	35
risch	$\frac{-\frac{1}{4}b^3x^3-\frac{3}{5}a\,b^2x^2-\frac{1}{2}a^2bx-\frac{1}{7}a^3}{x^7}$	35
gosper	$-\frac{35b^3x^3+84a\,b^2x^2+70a^2bx+20a^3}{140x^7}$	36
default	$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3a\,b^2}{5x^5}-\frac{b^3}{4x^4}$	36
parallelrisch	$\frac{-35b^3x^3-84a\,b^2x^2-70a^2bx-20a^3}{140x^7}$	36
orering	$-\frac{35b^3x^3+84a\,b^2x^2+70a^2bx+20a^3}{140x^7}$	36

input
```
int((b*x+a)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/x^7*(-1/4*b^3*x^3-3/5*a*b^2*x^2-1/2*a^2*b*x-1/7*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^8}\,dx = -\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

input
```
integrate((b*x+a)^3/x^8,x, algorithm="fricas")
```

output
```
-1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^3}{x^8}\,dx = \frac{-20a^3-70a^2bx-84ab^2x^2-35b^3x^3}{140x^7}$$

input
```
integrate((b*x+a)**3/x**8,x)
```

output
```
(-20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^8}\,dx = -\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

input
```
integrate((b*x+a)^3/x^8,x, algorithm="maxima")
```

output
```
-1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^8}\,dx = -\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

input
```
integrate((b*x+a)^3/x^8,x, algorithm="giac")
```

output
```
-1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^8}\,dx = -\frac{\frac{a^3}{7}+\frac{a^2\,b\,x}{2}+\frac{3\,a\,b^2\,x^2}{5}+\frac{b^3\,x^3}{4}}{x^7}$$

input
```
int((a + b*x)^3/x^8,x)
```

output
```
-(a^3/7 + (b^3*x^3)/4 + (3*a*b^2*x^2)/5 + (a^2*b*x)/2)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^8}\,dx = \frac{-35b^3x^3-84a\,b^2x^2-70a^2bx-20a^3}{140x^7}$$

input
```
int((b*x+a)^3/x^8,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

3.35 $\int x^6(a+bx)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int x^6(a+bx)^5\,dx=\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

output
```
1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/
12*b^5*x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^6(a+bx)^5\,dx=\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

input
```
Integrate[x^6*(a + b*x)^5,x]
```

output
```
(a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b
^4*x^11)/11 + (b^5*x^12)/12
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a+bx)^5\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

input
```
Int[x^6*(a + b*x)^5,x]
```

output
```
(a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b
^4*x^11)/11 + (b^5*x^12)/12
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
default	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
norman	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
risch	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
parallelrisch	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
orering	$\frac{x^7\left(462b^5x^5+2520a\,b^4x^4+5544a^2b^3x^3+6160a^3b^2x^2+3465a^4bx+792a^5\right)}{5544}$	58

input
```
int(x^6*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/
12*b^5*x^12
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^6(a+bx)^5\,dx=\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

input
```
integrate(x^6*(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^
4*b*x^8 + 1/7*a^5*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^6(a+bx)^5\,dx = \frac{a^5x^7}{7} + \frac{5a^4bx^8}{8} + \frac{10a^3b^2x^9}{9} + a^2b^3x^{10} + \frac{5ab^4x^{11}}{11} + \frac{b^5x^{12}}{12}$$

input
```
integrate(x**6*(b*x+a)**5,x)
```

output
```
a**5*x**7/7 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**9/9 + a**2*b**3*x**10 + 5*
a*b**4*x**11/11 + b**5*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^6(a+bx)^5\,dx = \frac{1}{12}\,b^5x^{12} + \frac{5}{11}\,ab^4x^{11} + a^2b^3x^{10} + \frac{10}{9}\,a^3b^2x^9 + \frac{5}{8}\,a^4bx^8 + \frac{1}{7}\,a^5x^7$$

input
```
integrate(x^6*(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^
4*b*x^8 + 1/7*a^5*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^6(a+bx)^5\,dx = \frac{1}{12}\,b^5x^{12} + \frac{5}{11}\,ab^4x^{11} + a^2b^3x^{10} + \frac{10}{9}\,a^3b^2x^9 + \frac{5}{8}\,a^4bx^8 + \frac{1}{7}\,a^5x^7$$

input
```
integrate(x^6*(b*x+a)^5,x, algorithm="giac")
```

output
```
1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^
4*b*x^8 + 1/7*a^5*x^7
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^6(a+bx)^5\,dx=\frac{a^5\,x^7}{7}+\frac{5\,a^4\,b\,x^8}{8}+\frac{10\,a^3\,b^2\,x^9}{9}+a^2\,b^3\,x^{10}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{12}}{12}$$

input
```
int(x^6*(a + b*x)^5,x)
```

output
```
(a^5*x^7)/7 + (b^5*x^12)/12 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^11)/11 + (10*a^
3*b^2*x^9)/9 + a^2*b^3*x^10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int x^6(a+bx)^5\,dx$$
$$=\frac{x^7(462b^5x^5+2520a\,b^4x^4+5544a^2b^3x^3+6160a^3b^2x^2+3465a^4bx+792a^5)}{5544}$$

input
```
int(x^6*(b*x+a)^5,x)
```

output
```
(x**7*(792*a**5 + 3465*a**4*b*x + 6160*a**3*b**2*x**2 + 5544*a**2*b**3*x**
3 + 2520*a*b**4*x**4 + 462*b**5*x**5))/5544
```

3.36 $\int x^5(a+bx)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 69

$$\int x^5(a+bx)^5\,dx=\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

output
```
1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+
1/11*b^5*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^5(a+bx)^5\,dx=\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

input
```
Integrate[x^5*(a + b*x)^5,x]
```

output
```
(a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (
a*b^4*x^10)/2 + (b^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+bx)^5\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

input
```
Int[x^5*(a + b*x)^5,x]
```

output
```
(a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (
a*b^4*x^10)/2 + (b^5*x^11)/11
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
default	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
norman	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
risch	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
parallelrisch	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
orering	$\frac{x^6(252b^5x^5+1386a\,b^4x^4+3080a^2b^3x^3+3465a^3b^2x^2+1980a^4bx+462a^5)}{2772}$	58

input
```
int(x^5*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+
1/11*b^5*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^5(a+bx)^5\,dx=\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*
a^4*b*x^7 + 1/6*a^5*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^5(a+bx)^5\,dx=\frac{a^5x^6}{6}+\frac{5a^4bx^7}{7}+\frac{5a^3b^2x^8}{4}+\frac{10a^2b^3x^9}{9}+\frac{ab^4x^{10}}{2}+\frac{b^5x^{11}}{11}$$

input
```
integrate(x**5*(b*x+a)**5,x)
```

output
```
a**5*x**6/6 + 5*a**4*b*x**7/7 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**9/9 +
 a*b**4*x**10/2 + b**5*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^5(a+bx)^5\,dx=\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*
a^4*b*x^7 + 1/6*a^5*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^5(a+bx)^5\,dx=\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b*x+a)^5,x, algorithm="giac")
```

output
```
1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*
a^4*b*x^7 + 1/6*a^5*x^6
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^5(a+bx)^5\,dx=\frac{a^5\,x^6}{6}+\frac{5\,a^4\,b\,x^7}{7}+\frac{5\,a^3\,b^2\,x^8}{4}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{a\,b^4\,x^{10}}{2}+\frac{b^5\,x^{11}}{11}$$

input
```
int(x^5*(a + b*x)^5,x)
```

output
```
(a^5*x^6)/6 + (b^5*x^11)/11 + (5*a^4*b*x^7)/7 + (a*b^4*x^10)/2 + (5*a^3*b^
2*x^8)/4 + (10*a^2*b^3*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^5(a+bx)^5\,dx=\frac{x^6(252b^5x^5+1386a\,b^4x^4+3080a^2b^3x^3+3465a^3b^2x^2+1980a^4bx+462a^5)}{2772}$$

input
```
int(x^5*(b*x+a)^5,x)
```

output
```
(x**6*(462*a**5 + 1980*a**4*b*x + 3465*a**3*b**2*x**2 + 3080*a**2*b**3*x**
3 + 1386*a*b**4*x**4 + 252*b**5*x**5))/2772
```

3.37 $\int x^4(a+bx)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 69

$$\int x^4(a+bx)^5\,dx=\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

output
```
1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1
/10*b^5*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)^5\,dx=\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

input
```
Integrate[x^4*(a + b*x)^5,x]
```

output
```
(a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (
5*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^5\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

input
```
Int[x^4*(a + b*x)^5,x]
```

output
```
(a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (
5*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
default	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
norman	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
risch	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
parallelrisch	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
orering	$\frac{x^5(126b^5x^5+700a\,b^4x^4+1575a^2b^3x^3+1800a^3b^2x^2+1050a^4bx+252a^5)}{1260}$	58

input
```
int(x^4*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1
/10*b^5*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4(a+bx)^5\,dx=\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a
^4*b*x^6 + 1/5*a^5*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^4(a+bx)^5\,dx=\frac{a^5x^5}{5}+\frac{5a^4bx^6}{6}+\frac{10a^3b^2x^7}{7}+\frac{5a^2b^3x^8}{4}+\frac{5ab^4x^9}{9}+\frac{b^5x^{10}}{10}$$

input
```
integrate(x**4*(b*x+a)**5,x)
```

output
```
a**5*x**5/5 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**7/7 + 5*a**2*b**3*x**8/4 +
 5*a*b**4*x**9/9 + b**5*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4(a+bx)^5\,dx=\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a
^4*b*x^6 + 1/5*a^5*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4(a+bx)^5\,dx=\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b*x+a)^5,x, algorithm="giac")
```

output
```
1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a
^4*b*x^6 + 1/5*a^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4(a+bx)^5\,dx=\frac{a^5\,x^5}{5}+\frac{5\,a^4\,b\,x^6}{6}+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{5\,a^2\,b^3\,x^8}{4}+\frac{5\,a\,b^4\,x^9}{9}+\frac{b^5\,x^{10}}{10}$$

input
```
int(x^4*(a + b*x)^5,x)
```

output
```
(a^5*x^5)/5 + (b^5*x^10)/10 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^9)/9 + (10*a^3*
b^2*x^7)/7 + (5*a^2*b^3*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4(a+bx)^5\,dx$$
$$=\frac{x^5(126b^5x^5+700a\,b^4x^4+1575a^2b^3x^3+1800a^3b^2x^2+1050a^4bx+252a^5)}{1260}$$

input
```
int(x^4*(b*x+a)^5,x)
```

output
```
(x**5*(252*a**5 + 1050*a**4*b*x + 1800*a**3*b**2*x**2 + 1575*a**2*b**3*x**
3 + 700*a*b**4*x**4 + 126*b**5*x**5))/1260
```

3.38 $\int x^3(a+bx)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int x^3(a+bx)^5\,dx=-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}$$

output
```
-1/6*a^3*(b*x+a)^6/b^4+3/7*a^2*(b*x+a)^7/b^4-3/8*a*(b*x+a)^8/b^4+1/9*(b*x+
a)^9/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int x^3(a+bx)^5\,dx=\frac{a^5x^4}{4}+a^4bx^5+\frac{5}{3}a^3b^2x^6+\frac{10}{7}a^2b^3x^7+\frac{5}{8}ab^4x^8+\frac{b^5x^9}{9}$$

input
```
Integrate[x^3*(a + b*x)^5,x]
```

output
```
(a^5*x^4)/4 + a^4*b*x^5 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^7)/7 + (5*a*b^
4*x^8)/8 + (b^5*x^9)/9
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^5\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}+\frac{(a+bx)^8}{b^3}-\frac{3a(a+bx)^7}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

input
```
Int[x^3*(a + b*x)^5,x]
```

output
```
-1/6*(a^3*(a + b*x)^6)/b^4 + (3*a^2*(a + b*x)^7)/(7*b^4) - (3*a*(a + b*x)^
8)/(8*b^4) + (a + b*x)^9/(9*b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
default	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
norman	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
risch	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
parallelrisch	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
orering	$\frac{x^4\left(56b^5x^5+315a\,b^4x^4+720a^2b^3x^3+840a^3b^2x^2+504a^4bx+126a^5\right)}{504}$	58

input
```
int(x^3*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^5*x^9+5/8*a*b^4*x^8+10/7*a^2*b^3*x^7+5/3*a^3*b^2*x^6+a^4*b*x^5+1/4*a
^5*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x^3(a+bx)^5\,dx=\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x
^5 + 1/4*a^5*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int x^3(a+bx)^5\,dx = \frac{a^5x^4}{4} + a^4bx^5 + \frac{5a^3b^2x^6}{3} + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^8}{8} + \frac{b^5x^9}{9}$$

input
```
integrate(x**3*(b*x+a)**5,x)
```

output
```
a**5*x**4/4 + a**4*b*x**5 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**7/7 + 5*a
*b**4*x**8/8 + b**5*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x^3(a+bx)^5\,dx = \frac{1}{9}\,b^5x^9 + \frac{5}{8}\,ab^4x^8 + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{3}\,a^3b^2x^6 + a^4bx^5 + \frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x
^5 + 1/4*a^5*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x^3(a+bx)^5\,dx = \frac{1}{9}\,b^5x^9 + \frac{5}{8}\,ab^4x^8 + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{3}\,a^3b^2x^6 + a^4bx^5 + \frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b*x+a)^5,x, algorithm="giac")
```

output
```
1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x
^5 + 1/4*a^5*x^4
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x^3(a+bx)^5\,dx=\frac{a^5\,x^4}{4}+a^4\,b\,x^5+\frac{5\,a^3\,b^2\,x^6}{3}+\frac{10\,a^2\,b^3\,x^7}{7}+\frac{5\,a\,b^4\,x^8}{8}+\frac{b^5\,x^9}{9}$$

input
```
int(x^3*(a + b*x)^5,x)
```

output
```
(a^5*x^4)/4 + (b^5*x^9)/9 + a^4*b*x^5 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x^6)/
3 + (10*a^2*b^3*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int x^3(a+bx)^5\,dx=\frac{x^4(56b^5x^5+315a\,b^4x^4+720a^2b^3x^3+840a^3b^2x^2+504a^4bx+126a^5)}{504}$$

input
```
int(x^3*(b*x+a)^5,x)
```

output
```
(x**4*(126*a**5 + 504*a**4*b*x + 840*a**3*b**2*x**2 + 720*a**2*b**3*x**3 +
 315*a*b**4*x**4 + 56*b**5*x**5))/504
```

3.39 $\int x^2(a+bx)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int x^2(a+bx)^5\,dx=\frac{a^2(a+bx)^6}{6b^3}-\frac{2a(a+bx)^7}{7b^3}+\frac{(a+bx)^8}{8b^3}$$

output
```
1/6*a^2*(b*x+a)^6/b^3-2/7*a*(b*x+a)^7/b^3+1/8*(b*x+a)^8/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int x^2(a+bx)^5\,dx=\frac{a^5x^3}{3}+\frac{5}{4}a^4bx^4+2a^3b^2x^5+\frac{5}{3}a^2b^3x^6+\frac{5}{7}ab^4x^7+\frac{b^5x^8}{8}$$

input
```
Integrate[x^2*(a + b*x)^5,x]
```

output
```
(a^5*x^3)/3 + (5*a^4*b*x^4)/4 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^6)/3 + (5*a*b
^4*x^7)/7 + (b^5*x^8)/8
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^5\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2(a+bx)^5}{b^2}+\frac{(a+bx)^7}{b^2}-\frac{2a(a+bx)^6}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}$$

input
```
Int[x^2*(a + b*x)^5,x]
```

output
```
(a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)^7)/(7*b^3) + (a + b*x)^8/(8*b^3
)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

method	result	size
gosper	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
default	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
norman	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
risch	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
parallelrisch	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
orering	$\frac{x^3(21b^5x^5+120a\,b^4x^4+280a^2b^3x^3+336a^3b^2x^2+210a^4bx+56a^5)}{168}$	58

input
```
int(x^2*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^5*x^8+5/7*a*b^4*x^7+5/3*a^2*b^3*x^6+2*a^3*b^2*x^5+5/4*a^4*b*x^4+1/3*
a^5*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int x^2(a+bx)^5\,dx=\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

input
```
integrate(x^2*(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*
x^4 + 1/3*a^5*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int x^2(a+bx)^5\,dx = \frac{a^5x^3}{3} + \frac{5a^4bx^4}{4} + 2a^3b^2x^5 + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^7}{7} + \frac{b^5x^8}{8}$$

input
```
integrate(x**2*(b*x+a)**5,x)
```

output
```
a**5*x**3/3 + 5*a**4*b*x**4/4 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**6/3 + 5*
a*b**4*x**7/7 + b**5*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int x^2(a+bx)^5\,dx = \frac{1}{8}\,b^5x^8 + \frac{5}{7}\,ab^4x^7 + \frac{5}{3}\,a^2b^3x^6 + 2\,a^3b^2x^5 + \frac{5}{4}\,a^4bx^4 + \frac{1}{3}\,a^5x^3$$

input
```
integrate(x^2*(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*
x^4 + 1/3*a^5*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int x^2(a+bx)^5\,dx = \frac{1}{8}\,b^5x^8 + \frac{5}{7}\,ab^4x^7 + \frac{5}{3}\,a^2b^3x^6 + 2\,a^3b^2x^5 + \frac{5}{4}\,a^4bx^4 + \frac{1}{3}\,a^5x^3$$

input
```
integrate(x^2*(b*x+a)^5,x, algorithm="giac")
```

output
```
1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*
x^4 + 1/3*a^5*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int x^2(a+bx)^5\,dx=\frac{a^5\,x^3}{3}+\frac{5\,a^4\,b\,x^4}{4}+2\,a^3\,b^2\,x^5+\frac{5\,a^2\,b^3\,x^6}{3}+\frac{5\,a\,b^4\,x^7}{7}+\frac{b^5\,x^8}{8}$$

input
```
int(x^2*(a + b*x)^5,x)
```

output
```
(a^5*x^3)/3 + (b^5*x^8)/8 + (5*a^4*b*x^4)/4 + (5*a*b^4*x^7)/7 + 2*a^3*b^2*
x^5 + (5*a^2*b^3*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int x^2(a+bx)^5\,dx=\frac{x^3(21b^5x^5+120a\,b^4x^4+280a^2b^3x^3+336a^3b^2x^2+210a^4bx+56a^5)}{168}$$

input
```
int(x^2*(b*x+a)^5,x)
```

output
```
(x**3*(56*a**5 + 210*a**4*b*x + 336*a**3*b**2*x**2 + 280*a**2*b**3*x**3 +
120*a*b**4*x**4 + 21*b**5*x**5))/168
```

3.40 $\int x(a+bx)^5\,dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int x(a+bx)^5\,dx=-\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}$$

output
```
-1/6*a*(b*x+a)^6/b^2+1/7*(b*x+a)^7/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.23

$$\int x(a+bx)^5\,dx=\frac{a^5x^2}{2}+\frac{5}{3}a^4bx^3+\frac{5}{2}a^3b^2x^4+2a^2b^3x^5+\frac{5}{6}ab^4x^6+\frac{b^5x^7}{7}$$

input
```
Integrate[x*(a + b*x)^5,x]
```

output
```
(a^5*x^2)/2 + (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^4)/2 + 2*a^2*b^3*x^5 + (5*a*b
^4*x^6)/6 + (b^5*x^7)/7
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^5\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+bx)^6}{b}-\frac{a(a+bx)^5}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

input
```
Int[x*(a + b*x)^5,x]
```

output
```
-1/6*(a*(a + b*x)^6)/b^2 + (a + b*x)^7/(7*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

method	result	size
gosper	$\frac{1}{7}b^5x^7+\frac{5}{6}a\,b^4x^6+2a^2b^3x^5+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^4b\,x^3+\frac{1}{2}a^5x^2$	58
default	$\frac{1}{7}b^5x^7+\frac{5}{6}a\,b^4x^6+2a^2b^3x^5+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^4b\,x^3+\frac{1}{2}a^5x^2$	58
norman	$\frac{1}{7}b^5x^7+\frac{5}{6}a\,b^4x^6+2a^2b^3x^5+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^4b\,x^3+\frac{1}{2}a^5x^2$	58
risch	$\frac{1}{7}b^5x^7+\frac{5}{6}a\,b^4x^6+2a^2b^3x^5+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^4b\,x^3+\frac{1}{2}a^5x^2$	58
parallelrisch	$\frac{1}{7}b^5x^7+\frac{5}{6}a\,b^4x^6+2a^2b^3x^5+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^4b\,x^3+\frac{1}{2}a^5x^2$	58
orering	$\frac{x^2(6b^5x^5+35a\,b^4x^4+84a^2b^3x^3+105a^3b^2x^2+70a^4bx+21a^5)}{42}$	58

input
```
int(x*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^5*x^7+5/6*a*b^4*x^6+2*a^2*b^3*x^5+5/2*a^3*b^2*x^4+5/3*a^4*b*x^3+1/2*
a^5*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int x(a+bx)^5\,dx=\frac{1}{7}\,b^5x^7+\frac{5}{6}\,ab^4x^6+2\,a^2b^3x^5+\frac{5}{2}\,a^3b^2x^4+\frac{5}{3}\,a^4bx^3+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*
x^3 + 1/2*a^5*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int x(a+bx)^5\,dx = \frac{a^5x^2}{2} + \frac{5a^4bx^3}{3} + \frac{5a^3b^2x^4}{2} + 2a^2b^3x^5 + \frac{5ab^4x^6}{6} + \frac{b^5x^7}{7}$$

input
```
integrate(x*(b*x+a)**5,x)
```

output
```
a**5*x**2/2 + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**4/2 + 2*a**2*b**3*x**5 + 5*
a*b**4*x**6/6 + b**5*x**7/7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int x(a+bx)^5\,dx = \frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*
x^3 + 1/2*a^5*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int x(a+bx)^5\,dx = \frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b*x+a)^5,x, algorithm="giac")
```

output
```
1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*
x^3 + 1/2*a^5*x^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int x(a+bx)^5\,dx=\frac{a^5\,x^2}{2}+\frac{5\,a^4\,b\,x^3}{3}+\frac{5\,a^3\,b^2\,x^4}{2}+2\,a^2\,b^3\,x^5+\frac{5\,a\,b^4\,x^6}{6}+\frac{b^5\,x^7}{7}$$

input
```
int(x*(a + b*x)^5,x)
```

output
```
(a^5*x^2)/2 + (b^5*x^7)/7 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^6)/6 + (5*a^3*b^2
*x^4)/2 + 2*a^2*b^3*x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int x(a+bx)^5\,dx=\frac{x^2(6b^5x^5+35a\,b^4x^4+84a^2b^3x^3+105a^3b^2x^2+70a^4bx+21a^5)}{42}$$

input
```
int(x*(b*x+a)^5,x)
```

output
```
(x**2*(21*a**5 + 70*a**4*b*x + 105*a**3*b**2*x**2 + 84*a**2*b**3*x**3 + 35
*a*b**4*x**4 + 6*b**5*x**5))/42
```

3.41 $\int (a+bx)^5\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (a+bx)^5\,dx=\frac{(a+bx)^6}{6b}$$

output
```
1/6*(b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+bx)^5\,dx=\frac{(a+bx)^6}{6b}$$

input
```
Integrate[(a + b*x)^5,x]
```

output
```
(a + b*x)^6/(6*b)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^5 \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^6}{6b}$$

input
```
Int[(a + b*x)^5,x]
```

output
```
(a + b*x)^6/(6*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^6}{6b}$	13
gosper	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4b\,x^2+a^5x$	54
norman	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4b\,x^2+a^5x$	54
parallelrisch	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4b\,x^2+a^5x$	54
orering	$\frac{x\left(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)}{6}$	55
risch	$\frac{b^5x^6}{6}+a\,b^4x^5+\frac{5a^2b^3x^4}{2}+\frac{10a^3b^2x^3}{3}+\frac{5a^4b\,x^2}{2}+a^5x+\frac{a^6}{6b}$	62

input
```
int((b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int (a+bx)^5\,dx=\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

input
```
integrate((b*x+a)^5,x, algorithm="fricas")
```

output
```
1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x
^2 + a^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 4.29

$$\int (a+bx)^5\,dx = a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

input
```
integrate((b*x+a)**5,x)
```

output
```
a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b*
*4*x**5 + b**5*x**6/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int (a+bx)^5\,dx = \frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

input
```
integrate((b*x+a)^5,x, algorithm="maxima")
```

output
```
1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x
^2 + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)^5\,dx = \frac{(bx+a)^6}{6\,b}$$

input
```
integrate((b*x+a)^5,x, algorithm="giac")
```

output
```
1/6*(b*x + a)^6/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int (a+bx)^5\,dx = a^5\,x + \frac{5\,a^4\,b\,x^2}{2} + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^4}{2} + a\,b^4\,x^5 + \frac{b^5\,x^6}{6}$$

input
```
int((a + b*x)^5,x)
```

output
```
a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (
5*a^2*b^3*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.86

$$\int (a+bx)^5\,dx = \frac{x(b^5x^5 + 6a\,b^4x^4 + 15a^2b^3x^3 + 20a^3b^2x^2 + 15a^4bx + 6a^5)}{6}$$

input
```
int((b*x+a)^5,x)
```

output
```
(x*(6*a**5 + 15*a**4*b*x + 20*a**3*b**2*x**2 + 15*a**2*b**3*x**3 + 6*a*b**
4*x**4 + b**5*x**5))/6
```

3.42 $\int \frac{(a+bx)^5}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \frac{(a+bx)^5}{x} \, dx = 5a^4bx + 5a^3b^2x^2 + \frac{10}{3}a^2b^3x^3 + \frac{5}{4}ab^4x^4 + \frac{b^5x^5}{5} + a^5\log(x)$$

output
```
5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(
x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x} \, dx = 5a^4bx + 5a^3b^2x^2 + \frac{10}{3}a^2b^3x^3 + \frac{5}{4}ab^4x^4 + \frac{b^5x^5}{5} + a^5\log(x)$$

input
```
Integrate[(a + b*x)^5/x,x]
```

output
```
5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^
5)/5 + a^5*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x} \, dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{a^5}{x}+5a^4b+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

input
```
Int[(a + b*x)^5/x,x]
```

output
```
5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^
5)/5 + a^5*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	size
default	$5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5a\,b^4x^4}{4} + \frac{b^5x^5}{5} + a^5\ln(x)$	54
norman	$5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5a\,b^4x^4}{4} + \frac{b^5x^5}{5} + a^5\ln(x)$	54
risch	$5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5a\,b^4x^4}{4} + \frac{b^5x^5}{5} + a^5\ln(x)$	54
parallelrisch	$5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5a\,b^4x^4}{4} + \frac{b^5x^5}{5} + a^5\ln(x)$	54

input
```
int((b*x+a)^5/x,x,method=_RETURNVERBOSE)
```

output
```
5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(
x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^5}{x}\,dx = \frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log(x)$$

input
```
integrate((b*x+a)^5/x,x, algorithm="fricas")
```

output
```
1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
 + a^5*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x}\,dx = a^5\log(x) + 5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5ab^4x^4}{4} + \frac{b^5x^5}{5}$$

input
```
integrate((b*x+a)**5/x,x)
```

output
```
a**5*log(x) + 5*a**4*b*x + 5*a**3*b**2*x**2 + 10*a**2*b**3*x**3/3 + 5*a*b*
*4*x**4/4 + b**5*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^5}{x}\,dx = \frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log(x)$$

input
```
integrate((b*x+a)^5/x,x, algorithm="maxima")
```

output
```
1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
 + a^5*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^5}{x}\,dx = \frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log(|x|)$$

input
```
integrate((b*x+a)^5/x,x, algorithm="giac")
```

output
```
1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
 + a^5*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^5}{x}\,dx = a^5\,\ln(x) + \frac{b^5\,x^5}{5} + \frac{5\,a\,b^4\,x^4}{4} + 5\,a^3\,b^2\,x^2 + \frac{10\,a^2\,b^3\,x^3}{3} + 5\,a^4\,b\,x$$

input
```
int((a + b*x)^5/x,x)
```

output
```
a^5*log(x) + (b^5*x^5)/5 + (5*a*b^4*x^4)/4 + 5*a^3*b^2*x^2 + (10*a^2*b^3*x
^3)/3 + 5*a^4*b*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^5}{x}\,dx = \log(x)\,a^5 + 5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5a\,b^4x^4}{4} + \frac{b^5x^5}{5}$$

input
```
int((b*x+a)^5/x,x)
```

output
```
(60*log(x)*a**5 + 300*a**4*b*x + 300*a**3*b**2*x**2 + 200*a**2*b**3*x**3 +
 75*a*b**4*x**4 + 12*b**5*x**5)/60
```

3.43 $\int \frac{(a+bx)^5}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{(a+bx)^5}{x^2} dx = -\frac{a^5}{x} + 10a^3b^2x + 5a^2b^3x^2 + \frac{5}{3}ab^4x^3 + \frac{b^5x^4}{4} + 5a^4b\log(x)$$

output
```
-a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^2} dx = -\frac{a^5}{x} + 10a^3b^2x + 5a^2b^3x^2 + \frac{5}{3}ab^4x^3 + \frac{b^5x^4}{4} + 5a^4b\log(x)$$

input
```
Integrate[(a + b*x)^5/x^2,x]
```

output
```
-(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 +
5*a^4*b*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^2}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^3b^2+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{x}+5a^4b\log(x)+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

input
```
Int[(a + b*x)^5/x^2,x]
```

output
```
-(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 +
5*a^4*b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5a\,b^4x^3}{3}+\frac{b^5x^4}{4}+5a^4b\ln\left(x\right)$	55
risch	$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5a\,b^4x^3}{3}+\frac{b^5x^4}{4}+5a^4b\ln\left(x\right)$	55
norman	$\frac{-a^5+\frac{1}{4}b^5x^5+\frac{5}{3}a\,b^4x^4+5a^2b^3x^3+10a^3b^2x^2}{x}+5a^4b\ln\left(x\right)$	59
parallelrisch	$\frac{3b^5x^5+20a\,b^4x^4+60a^2b^3x^3+60a^4b\ln(x)x+120a^3b^2x^2-12a^5}{12x}$	60

input
```
int((b*x+a)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int\frac{(a+bx)^5}{x^2}\,dx=\frac{3\,b^5x^5+20\,ab^4x^4+60\,a^2b^3x^3+120\,a^3b^2x^2+60\,a^4bx\log\left(x\right)-12\,a^5}{12\,x}$$

input
```
integrate((b*x+a)^5/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4
*b*x*log(x) - 12*a^5)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^5}{x^2}\,dx = -\frac{a^5}{x} + 5a^4b\log(x) + 10a^3b^2x + 5a^2b^3x^2 + \frac{5ab^4x^3}{3} + \frac{b^5x^4}{4}$$

input
```
integrate((b*x+a)**5/x**2,x)
```

output
```
-a**5/x + 5*a**4*b*log(x) + 10*a**3*b**2*x + 5*a**2*b**3*x**2 + 5*a*b**4*x
**3/3 + b**5*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^5}{x^2}\,dx = \frac{1}{4}\,b^5x^4 + \frac{5}{3}\,ab^4x^3 + 5\,a^2b^3x^2 + 10\,a^3b^2x + 5\,a^4b\log(x) - \frac{a^5}{x}$$

input
```
integrate((b*x+a)^5/x^2,x, algorithm="maxima")
```

output
```
1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(x
) - a^5/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^5}{x^2}\,dx = \frac{1}{4}\,b^5x^4 + \frac{5}{3}\,ab^4x^3 + 5\,a^2b^3x^2 + 10\,a^3b^2x + 5\,a^4b\log(|x|) - \frac{a^5}{x}$$

input
```
integrate((b*x+a)^5/x^2,x, algorithm="giac")
```

output
```
1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(a
bs(x)) - a^5/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^5}{x^2}\,dx=\frac{b^5\,x^4}{4}-\frac{a^5}{x}+10\,a^3\,b^2\,x+\frac{5\,a\,b^4\,x^3}{3}+5\,a^4\,b\,\ln\left(x\right)+5\,a^2\,b^3\,x^2$$

input
```
int((a + b*x)^5/x^2,x)
```

output
```
(b^5*x^4)/4 - a^5/x + 10*a^3*b^2*x + (5*a*b^4*x^3)/3 + 5*a^4*b*log(x) + 5*
a^2*b^3*x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x^2}\,dx=\frac{60\log(x)\,a^4bx-12a^5+120a^3b^2x^2+60a^2b^3x^3+20a\,b^4x^4+3b^5x^5}{12x}$$

input
```
int((b*x+a)^5/x^2,x)
```

output
```
(60*log(x)*a**4*b*x - 12*a**5 + 120*a**3*b**2*x**2 + 60*a**2*b**3*x**3 + 2
0*a*b**4*x**4 + 3*b**5*x**5)/(12*x)
```

3.44 $\int \frac{(a+bx)^5}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{(a+bx)^5}{x^3} dx = -\frac{a^5}{2x^2} - \frac{5a^4b}{x} + 10a^2b^3x + \frac{5}{2}ab^4x^2 + \frac{b^5x^3}{3} + 10a^3b^2\log(x)$$

output
```
-1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*l
n(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^3} dx = -\frac{a^5}{2x^2} - \frac{5a^4b}{x} + 10a^2b^3x + \frac{5}{2}ab^4x^2 + \frac{b^5x^3}{3} + 10a^3b^2\log(x)$$

input
```
Integrate[(a + b*x)^5/x^3,x]
```

output
```
-1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3
+ 10*a^3*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^3} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+10a^2b^3+5ab^4x+b^5x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

input
```
Int[(a + b*x)^5/x^3,x]
```

output
```
-1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3
+ 10*a^3*b^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5a\,b^4x^2}{2}+\frac{b^5x^3}{3}+10a^3b^2\ln\left(x\right)$	55
risch	$\frac{b^5x^3}{3}+\frac{5a\,b^4x^2}{2}+10a^2b^3x+\frac{-5a^4bx-\frac{1}{2}a^5}{x^2}+10a^3b^2\ln\left(x\right)$	55
norman	$\frac{-\frac{1}{2}a^5+\frac{1}{3}b^5x^5+\frac{5}{2}a\,b^4x^4+10a^2b^3x^3-5a^4bx}{x^2}+10a^3b^2\ln\left(x\right)$	57
parallelrisch	$\frac{2b^5x^5+15a\,b^4x^4+60a^3b^2\ln(x)x^2+60a^2b^3x^3-30a^4bx-3a^5}{6x^2}$	60

input
```
int((b*x+a)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*l
n(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int\frac{(a+bx)^5}{x^3}\,dx=\frac{2\,b^5x^5+15\,ab^4x^4+60\,a^2b^3x^3+60\,a^3b^2x^2\log\left(x\right)-30\,a^4bx-3\,a^5}{6\,x^2}$$

input
```
integrate((b*x+a)^5/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 3
0*a^4*b*x - 3*a^5)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^3}\,dx = 10a^3b^2\log{(x)} + 10a^2b^3x + \frac{5ab^4x^2}{2} + \frac{b^5x^3}{3} + \frac{-a^5 - 10a^4bx}{2x^2}$$

input
```
integrate((b*x+a)**5/x**3,x)
```

output
```
10*a**3*b**2*log(x) + 10*a**2*b**3*x + 5*a*b**4*x**2/2 + b**5*x**3/3 + (-a
**5 - 10*a**4*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^5}{x^3}\,dx = \frac{1}{3}\,b^5x^3 + \frac{5}{2}\,ab^4x^2 + 10\,a^2b^3x + 10\,a^3b^2\log{(x)} - \frac{10\,a^4bx + a^5}{2\,x^2}$$

input
```
integrate((b*x+a)^5/x^3,x, algorithm="maxima")
```

output
```
1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(x) - 1/2*(10*a
^4*b*x + a^5)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^5}{x^3}\,dx = \frac{1}{3}\,b^5x^3 + \frac{5}{2}\,ab^4x^2 + 10\,a^2b^3x + 10\,a^3b^2\log{(|x|)} - \frac{10\,a^4bx + a^5}{2\,x^2}$$

input
```
integrate((b*x+a)^5/x^3,x, algorithm="giac")
```

output
```
1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(abs(x)) - 1/2*
(10*a^4*b*x + a^5)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^5}{x^3}\,dx = \frac{b^5\,x^3}{3} - \frac{\frac{a^5}{2} + 5\,b\,x\,a^4}{x^2} + 10\,a^2\,b^3\,x + \frac{5\,a\,b^4\,x^2}{2} + 10\,a^3\,b^2\,\ln(x)$$

input
```
int((a + b*x)^5/x^3,x)
```

output
```
(b^5*x^3)/3 - (a^5/2 + 5*a^4*b*x)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + 1
0*a^3*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^5}{x^3}\,dx = \frac{60\log(x)\,a^3b^2x^2 - 3a^5 - 30a^4bx + 60a^2b^3x^3 + 15a\,b^4x^4 + 2b^5x^5}{6x^2}$$

input
```
int((b*x+a)^5/x^3,x)
```

output
```
(60*log(x)*a**3*b**2*x**2 - 3*a**5 - 30*a**4*b*x + 60*a**2*b**3*x**3 + 15*
a*b**4*x**4 + 2*b**5*x**5)/(6*x**2)
```

3.45 $\int \frac{(a+bx)^5}{x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{(a+bx)^5}{x^4}\,dx=-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)$$

output
```
-1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*l
n(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^4}\,dx=-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)$$

input
```
Integrate[(a + b*x)^5/x^4,x]
```

output
```
-1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/
2 + 10*a^2*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^4} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+5ab^4+b^5x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}$$

input
```
Int[(a + b*x)^5/x^4,x]
```

output
```
-1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/
2 + 10*a^2*b^3*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5a\,b^4x+\frac{b^5x^2}{2}+10a^2b^3\ln\left(x\right)$	55
risch	$\frac{b^5x^2}{2}+5a\,b^4x+\frac{-10a^3b^2x^2-\frac{5}{2}a^4bx-\frac{1}{3}a^5}{x^3}+10a^2b^3\ln\left(x\right)$	55
norman	$\frac{-\frac{1}{3}a^5+\frac{1}{2}b^5x^5+5a\,b^4x^4-10a^3b^2x^2-\frac{5}{2}a^4bx}{x^3}+10a^2b^3\ln\left(x\right)$	57
parallelrisch	$\frac{3b^5x^5+60a^2b^3\ln(x)x^3+30a\,b^4x^4-60a^3b^2x^2-15a^4bx-2a^5}{6x^3}$	60

input
```
int((b*x+a)^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*l
n(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^5}{x^4}\,dx=\frac{3\,b^5x^5+30\,ab^4x^4+60\,a^2b^3x^3\log\left(x\right)-60\,a^3b^2x^2-15\,a^4bx-2\,a^5}{6\,x^3}$$

input
```
integrate((b*x+a)^5/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 1
5*a^4*b*x - 2*a^5)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^4}\,dx = 10a^2b^3\log{(x)} + 5ab^4x + \frac{b^5x^2}{2} + \frac{-2a^5 - 15a^4bx - 60a^3b^2x^2}{6x^3}$$

input
```
integrate((b*x+a)**5/x**4,x)
```

output
```
10*a**2*b**3*log(x) + 5*a*b**4*x + b**5*x**2/2 + (-2*a**5 - 15*a**4*b*x -
60*a**3*b**2*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^5}{x^4}\,dx = \frac{1}{2}\,b^5x^2 + 5\,ab^4x + 10\,a^2b^3\log{(x)} - \frac{60\,a^3b^2x^2 + 15\,a^4bx + 2\,a^5}{6\,x^3}$$

input
```
integrate((b*x+a)^5/x^4,x, algorithm="maxima")
```

output
```
1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(x) - 1/6*(60*a^3*b^2*x^2 + 15*a^4
*b*x + 2*a^5)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^5}{x^4}\,dx = \frac{1}{2}\,b^5x^2 + 5\,ab^4x + 10\,a^2b^3\log{(|x|)} - \frac{60\,a^3b^2x^2 + 15\,a^4bx + 2\,a^5}{6\,x^3}$$

input
```
integrate((b*x+a)^5/x^4,x, algorithm="giac")
```

output
```
1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(abs(x)) - 1/6*(60*a^3*b^2*x^2 + 1
5*a^4*b*x + 2*a^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^5}{x^4}\,dx = \frac{b^5\,x^2}{2} - \frac{\frac{a^5}{3} + \frac{5\,a^4\,b\,x}{2} + 10\,a^3\,b^2\,x^2}{x^3} + 10\,a^2\,b^3\,\ln(x) + 5\,a\,b^4\,x$$

input
```
int((a + b*x)^5/x^4,x)
```

output
```
(b^5*x^2)/2 - (a^5/3 + 10*a^3*b^2*x^2 + (5*a^4*b*x)/2)/x^3 + 10*a^2*b^3*lo
g(x) + 5*a*b^4*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^5}{x^4}\,dx = \frac{60\log(x)\,a^2b^3x^3 - 2a^5 - 15a^4bx - 60a^3b^2x^2 + 30a\,b^4x^4 + 3b^5x^5}{6x^3}$$

input
```
int((b*x+a)^5/x^4,x)
```

output
```
(60*log(x)*a**2*b**3*x**3 - 2*a**5 - 15*a**4*b*x - 60*a**3*b**2*x**2 + 30*
a*b**4*x**4 + 3*b**5*x**5)/(6*x**3)
```

3.46 $\int \frac{(a+bx)^5}{x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{(a+bx)^5}{x^5} dx = -\frac{a^5}{4x^4} - \frac{5a^4b}{3x^3} - \frac{5a^3b^2}{x^2} - \frac{10a^2b^3}{x} + b^5x + 5ab^4\log(x)$$

output
```
-1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^5} dx = -\frac{a^5}{4x^4} - \frac{5a^4b}{3x^3} - \frac{5a^3b^2}{x^2} - \frac{10a^2b^3}{x} + b^5x + 5ab^4\log(x)$$

input
```
Integrate[(a + b*x)^5/x^5,x]
```

output
```
-1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*
x + 5*a*b^4*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^5} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}+b^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x$$

input
```
Int[(a + b*x)^5/x^5,x]
```

output
```
-1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*
x + 5*a*b^4*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a^{5}}{4x^{4}}-\frac{5a^{4}b}{3x^{3}}-\frac{5a^{3}b^{2}}{x^{2}}-\frac{10a^{2}b^{3}}{x}+b^{5}x+5a\,b^{4}\ln\left(x\right)$	54
risch	$b^{5}x+\frac{-10a^{2}b^{3}x^{3}-5a^{3}b^{2}x^{2}-\frac{5}{3}a^{4}bx-\frac{1}{4}a^{5}}{x^{4}}+5a\,b^{4}\ln\left(x\right)$	54
norman	$\frac{b^{5}x^{5}-\frac{1}{4}a^{5}-10a^{2}b^{3}x^{3}-5a^{3}b^{2}x^{2}-\frac{5}{3}a^{4}bx}{x^{4}}+5a\,b^{4}\ln\left(x\right)$	56
parallelrisch	$\frac{60a\,b^{4}\ln\left(x\right)x^{4}+12b^{5}x^{5}-120a^{2}b^{3}x^{3}-60a^{3}b^{2}x^{2}-20a^{4}bx-3a^{5}}{12x^{4}}$	60

input
```
int((b*x+a)^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int\frac{(a+bx)^{5}}{x^{5}}\,dx=\frac{12\,b^{5}x^{5}+60\,ab^{4}x^{4}\log\left(x\right)-120\,a^{2}b^{3}x^{3}-60\,a^{3}b^{2}x^{2}-20\,a^{4}bx-3\,a^{5}}{12\,x^{4}}$$

input
```
integrate((b*x+a)^5/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2
- 20*a^4*b*x - 3*a^5)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x^5}\,dx = 5ab^4\log{(x)} + b^5x + \frac{-3a^5 - 20a^4bx - 60a^3b^2x^2 - 120a^2b^3x^3}{12x^4}$$

input
```
integrate((b*x+a)**5/x**5,x)
```

output
```
5*a*b**4*log(x) + b**5*x + (-3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 12
0*a**2*b**3*x**3)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^5}{x^5}\,dx = b^5x + 5\,ab^4\log(x) - \frac{120\,a^2b^3x^3 + 60\,a^3b^2x^2 + 20\,a^4bx + 3\,a^5}{12\,x^4}$$

input
```
integrate((b*x+a)^5/x^5,x, algorithm="maxima")
```

output
```
b^5*x + 5*a*b^4*log(x) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a^4*b
*x + 3*a^5)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^5}{x^5}\,dx = b^5x + 5\,ab^4\log(|x|) - \frac{120\,a^2b^3x^3 + 60\,a^3b^2x^2 + 20\,a^4bx + 3\,a^5}{12\,x^4}$$

input
```
integrate((b*x+a)^5/x^5,x, algorithm="giac")
```

output
```
b^5*x + 5*a*b^4*log(abs(x)) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*
a^4*b*x + 3*a^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^5}{x^5}\,dx = b^5\,x - \frac{\frac{a^5}{4} + \frac{5\,a^4\,b\,x}{3} + 5\,a^3\,b^2\,x^2 + 10\,a^2\,b^3\,x^3}{x^4} + 5\,a\,b^4\,\ln\left(x\right)$$

input
```
int((a + b*x)^5/x^5,x)
```

output
```
b^5*x - (a^5/4 + 5*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + (5*a^4*b*x)/3)/x^4 + 5*a
*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^5}{x^5}\,dx = \frac{60\log(x)\,a\,b^4x^4 - 3a^5 - 20a^4bx - 60a^3b^2x^2 - 120a^2b^3x^3 + 12b^5x^5}{12x^4}$$

input
```
int((b*x+a)^5/x^5,x)
```

output
```
(60*log(x)*a*b**4*x**4 - 3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 120*a*
*2*b**3*x**3 + 12*b**5*x**5)/(12*x**4)
```

3.47 $\int \frac{(a+bx)^5}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 61

$$\int \frac{(a+bx)^5}{x^6} dx = -\frac{a^5}{5x^5} - \frac{5a^4b}{4x^4} - \frac{10a^3b^2}{3x^3} - \frac{5a^2b^3}{x^2} - \frac{5ab^4}{x} + b^5 \log(x)$$

output
```
-1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln
(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^6} dx = -\frac{a^5}{5x^5} - \frac{5a^4b}{4x^4} - \frac{10a^3b^2}{3x^3} - \frac{5a^2b^3}{x^2} - \frac{5ab^4}{x} + b^5 \log(x)$$

input
```
Integrate[(a + b*x)^5/x^6,x]
```

output
```
-1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2
- (5*a*b^4)/x + b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^6} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

input
```
Int[(a + b*x)^5/x^6,x]
```

output
```
-1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2
- (5*a*b^4)/x + b^5*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5a\,b^4}{x}+b^5\ln\left(x\right)$	56
norman	$\frac{-\frac{1}{5}a^5-5a\,b^4x^4-5a^2b^3x^3-\frac{10}{3}a^3b^2x^2-\frac{5}{4}a^4bx}{x^5}+b^5\ln\left(x\right)$	56
risch	$\frac{-\frac{1}{5}a^5-5a\,b^4x^4-5a^2b^3x^3-\frac{10}{3}a^3b^2x^2-\frac{5}{4}a^4bx}{x^5}+b^5\ln\left(x\right)$	56
parallelrisch	$\frac{60b^5\ln(x)x^5-300a\,b^4x^4-300a^2b^3x^3-200a^3b^2x^2-75a^4bx-12a^5}{60x^5}$	60

input
```
int((b*x+a)^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln
(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^5}{x^6}\,dx = \frac{60\,b^5x^5\log\left(x\right)-300\,ab^4x^4-300\,a^2b^3x^3-200\,a^3b^2x^2-75\,a^4bx-12\,a^5}{60\,x^5}$$

input
```
integrate((b*x+a)^5/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^
2 - 75*a^4*b*x - 12*a^5)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^5}{x^6}\,dx = b^5\log(x) + \frac{-12a^5 - 75a^4bx - 200a^3b^2x^2 - 300a^2b^3x^3 - 300ab^4x^4}{60x^5}$$

input
```
integrate((b*x+a)**5/x**6,x)
```

output
```
b**5*log(x) + (-12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a**2*b**3
*x**3 - 300*a*b**4*x**4)/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^5}{x^6}\,dx = b^5\log(x) - \frac{300\,ab^4x^4 + 300\,a^2b^3x^3 + 200\,a^3b^2x^2 + 75\,a^4bx + 12\,a^5}{60\,x^5}$$

input
```
integrate((b*x+a)^5/x^6,x, algorithm="maxima")
```

output
```
b^5*log(x) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 + 75*
a^4*b*x + 12*a^5)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^5}{x^6}\,dx = b^5\log(|x|) - \frac{300\,ab^4x^4 + 300\,a^2b^3x^3 + 200\,a^3b^2x^2 + 75\,a^4bx + 12\,a^5}{60\,x^5}$$

input
```
integrate((b*x+a)^5/x^6,x, algorithm="giac")
```

output
```
b^5*log(abs(x)) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2
+ 75*a^4*b*x + 12*a^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^5}{x^6}\,dx = b^5\ \ln(x) - \frac{\frac{a^5}{5}+\frac{5\,a^4\,b\,x}{4}+\frac{10\,a^3\,b^2\,x^2}{3}+5\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4}{x^5}$$

input
```
int((a + b*x)^5/x^6,x)
```

output
```
b^5*log(x) - (a^5/5 + 5*a*b^4*x^4 + (10*a^3*b^2*x^2)/3 + 5*a^2*b^3*x^3 + (
5*a^4*b*x)/4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^5}{x^6}\,dx = \frac{60\log(x)\,b^5x^5-12a^5-75a^4bx-200a^3b^2x^2-300a^2b^3x^3-300a\,b^4x^4}{60x^5}$$

input
```
int((b*x+a)^5/x^6,x)
```

output
```
(60*log(x)*b**5*x**5 - 12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a*
*2*b**3*x**3 - 300*a*b**4*x**4)/(60*x**5)
```

3.48 $\int \frac{(a+bx)^5}{x^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{(a+bx)^5}{x^7} dx = -\frac{(a+bx)^6}{6ax^6}$$

output
```
-1/6*(b*x+a)^6/a/x^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(17) = 34.

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.82

$$\int \frac{(a+bx)^5}{x^7} dx = -\frac{a^5}{6x^6} - \frac{a^4b}{x^5} - \frac{5a^3b^2}{2x^4} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{2x^2} - \frac{b^5}{x}$$

input
```
Integrate[(a + b*x)^5/x^7,x]
```

output
```
-1/6*a^5/x^6 - (a^4*b)/x^5 - (5*a^3*b^2)/(2*x^4) - (10*a^2*b^3)/(3*x^3) -
(5*a*b^4)/(2*x^2) - b^5/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^7} \, dx$$

$$\Big\downarrow \; 48$$

$$-\frac{(a+bx)^6}{6ax^6}$$

input
```
Int[(a + b*x)^5/x^7,x]
```

output
```
-1/6*(a + b*x)^6/(a*x^6)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

method	result	size
gosper	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6x^6}$	56
orering	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6x^6}$	56
norman	$\frac{-b^5x^5-\frac{5}{2}a\,b^4x^4-\frac{10}{3}a^2b^3x^3-\frac{5}{2}a^3b^2x^2-a^4bx-\frac{1}{6}a^5}{x^6}$	57
risch	$\frac{-b^5x^5-\frac{5}{2}a\,b^4x^4-\frac{10}{3}a^2b^3x^3-\frac{5}{2}a^3b^2x^2-a^4bx-\frac{1}{6}a^5}{x^6}$	57
default	$-\frac{10a^2b^3}{3x^3}-\frac{a^4b}{x^5}-\frac{5a\,b^4}{2x^2}-\frac{5a^3b^2}{2x^4}-\frac{b^5}{x}-\frac{a^5}{6x^6}$	58
parallelrisch	$\frac{-6b^5x^5-15a\,b^4x^4-20a^2b^3x^3-15a^3b^2x^2-6a^4bx-a^5}{6x^6}$	58

input
```
int((b*x+a)^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(6*b^5*x^5+15*a*b^4*x^4+20*a^2*b^3*x^3+15*a^3*b^2*x^2+6*a^4*b*x+a^5)/
x^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(15) = 30.

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx)^5}{x^7}\,dx = -\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

input
```
integrate((b*x+a)^5/x^7,x, algorithm="fricas")
```

output
```
-1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b
*x + a^5)/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(14) = 28.

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.53

$$\int \frac{(a+bx)^5}{x^7}\,dx = \frac{-a^5-6a^4bx-15a^3b^2x^2-20a^2b^3x^3-15ab^4x^4-6b^5x^5}{6x^6}$$

input
```
integrate((b*x+a)**5/x**7,x)
```

output
```
(-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x*
*4 - 6*b**5*x**5)/(6*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(15) = 30.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx)^5}{x^7}\,dx = -\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

input
```
integrate((b*x+a)^5/x^7,x, algorithm="maxima")
```

output
```
-1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b
*x + a^5)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(15) = 30.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx)^5}{x^7}\,dx = -\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

input
```
integrate((b*x+a)^5/x^7,x, algorithm="giac")
```

output
```
-1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b
*x + a^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx)^5}{x^7} \, dx = -\frac{\frac{a^5}{6} + a^4\,b\,x + \frac{5\,a^3\,b^2\,x^2}{2} + \frac{10\,a^2\,b^3\,x^3}{3} + \frac{5\,a\,b^4\,x^4}{2} + b^5\,x^5}{x^6}$$

input
```
int((a + b*x)^5/x^7,x)
```

output
```
-(a^5/6 + b^5*x^5 + (5*a*b^4*x^4)/2 + (5*a^3*b^2*x^2)/2 + (10*a^2*b^3*x^3)
/3 + a^4*b*x)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int \frac{(a+bx)^5}{x^7} \, dx = \frac{-6b^5x^5 - 15a\,b^4x^4 - 20a^2b^3x^3 - 15a^3b^2x^2 - 6a^4bx - a^5}{6x^6}$$

input
```
int((b*x+a)^5/x^7,x)
```

output
```
( - a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*
x**4 - 6*b**5*x**5)/(6*x**6)
```

3.49 $\int \frac{(a+bx)^5}{x^8} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{(a+bx)^5}{x^8} \, dx = -\frac{(a+bx)^6}{7ax^7} + \frac{b(a+bx)^6}{42a^2x^6}$$

output
```
-1/7*(b*x+a)^6/a/x^7+1/42*b*(b*x+a)^6/a^2/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx)^5}{x^8} \, dx = -\frac{a^5}{7x^7} - \frac{5a^4b}{6x^6} - \frac{2a^3b^2}{x^5} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{3x^3} - \frac{b^5}{2x^2}$$

input
```
Integrate[(a + b*x)^5/x^8,x]
```

output
```
-1/7*a^5/x^7 - (5*a^4*b)/(6*x^6) - (2*a^3*b^2)/x^5 - (5*a^2*b^3)/(2*x^4) -
 (5*a*b^4)/(3*x^3) - b^5/(2*x^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^8} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{b \int \frac{(a+bx)^5}{x^7}dx}{7a}-\frac{(a+bx)^6}{7ax^7}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{b(a+bx)^6}{42a^2x^6}-\frac{(a+bx)^6}{7ax^7}$$

input
```
Int[(a + b*x)^5/x^8,x]
```

output
```
-1/7*(a + b*x)^6/(a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

method	result	size
norman	$\frac{-\frac{1}{2}b^5x^5-\frac{5}{3}a\,b^4x^4-\frac{5}{2}a^2b^3x^3-2a^3b^2x^2-\frac{5}{6}a^4bx-\frac{1}{7}a^5}{x^7}$	57
risch	$\frac{-\frac{1}{2}b^5x^5-\frac{5}{3}a\,b^4x^4-\frac{5}{2}a^2b^3x^3-2a^3b^2x^2-\frac{5}{6}a^4bx-\frac{1}{7}a^5}{x^7}$	57
gosper	$-\frac{21b^5x^5+70a\,b^4x^4+105a^2b^3x^3+84a^3b^2x^2+35a^4bx+6a^5}{42x^7}$	58
default	$-\frac{5a\,b^4}{3x^3}-\frac{2a^3b^2}{x^5}-\frac{b^5}{2x^2}-\frac{a^5}{7x^7}-\frac{5a^2b^3}{2x^4}-\frac{5a^4b}{6x^6}$	58
parallelrisch	$\frac{-21b^5x^5-70a\,b^4x^4-105a^2b^3x^3-84a^3b^2x^2-35a^4bx-6a^5}{42x^7}$	58
orering	$-\frac{21b^5x^5+70a\,b^4x^4+105a^2b^3x^3+84a^3b^2x^2+35a^4bx+6a^5}{42x^7}$	58

input
```
int((b*x+a)^5/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/x^7*(-1/2*b^5*x^5-5/3*a*b^4*x^4-5/2*a^2*b^3*x^3-2*a^3*b^2*x^2-5/6*a^4*b*
x-1/7*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^5}{x^8}\,dx = -\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

input
```
integrate((b*x+a)^5/x^8,x, algorithm="fricas")
```

output
```
-1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a
^4*b*x + 6*a^5)/x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx)^5}{x^8}\,dx = \frac{-6a^5 - 35a^4bx - 84a^3b^2x^2 - 105a^2b^3x^3 - 70ab^4x^4 - 21b^5x^5}{42x^7}$$

input
```
integrate((b*x+a)**5/x**8,x)
```

output
```
(-6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b**
4*x**4 - 21*b**5*x**5)/(42*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^5}{x^8}\,dx = -\frac{21\,b^5x^5 + 70\,ab^4x^4 + 105\,a^2b^3x^3 + 84\,a^3b^2x^2 + 35\,a^4bx + 6\,a^5}{42\,x^7}$$

input
```
integrate((b*x+a)^5/x^8,x, algorithm="maxima")
```

output
```
-1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a
^4*b*x + 6*a^5)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^5}{x^8}\,dx = -\frac{21\,b^5x^5 + 70\,ab^4x^4 + 105\,a^2b^3x^3 + 84\,a^3b^2x^2 + 35\,a^4bx + 6\,a^5}{42\,x^7}$$

input
```
integrate((b*x+a)^5/x^8,x, algorithm="giac")
```

output
```
-1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a
^4*b*x + 6*a^5)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^5}{x^8}\,dx = -\frac{\frac{a^5}{7} + \frac{5\,a^4\,b\,x}{6} + 2\,a^3\,b^2\,x^2 + \frac{5\,a^2\,b^3\,x^3}{2} + \frac{5\,a\,b^4\,x^4}{3} + \frac{b^5\,x^5}{2}}{x^7}$$

input
```
int((a + b*x)^5/x^8,x)
```

output
```
-(a^5/7 + (b^5*x^5)/2 + (5*a*b^4*x^4)/3 + 2*a^3*b^2*x^2 + (5*a^2*b^3*x^3)/
2 + (5*a^4*b*x)/6)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^5}{x^8}\,dx = \frac{-21b^5x^5 - 70a\,b^4x^4 - 105a^2b^3x^3 - 84a^3b^2x^2 - 35a^4bx - 6a^5}{42x^7}$$

input
```
int((b*x+a)^5/x^8,x)
```

output
```
( - 6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b
**4*x**4 - 21*b**5*x**5)/(42*x**7)
```

3.50 $\int \frac{(a+bx)^5}{x^9} dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{(a+bx)^5}{x^9} dx = -\frac{(a+bx)^6}{8ax^8} + \frac{b(a+bx)^6}{28a^2x^7} - \frac{b^2(a+bx)^6}{168a^3x^6}$$

output
```
-1/8*(b*x+a)^6/a/x^8+1/28*b*(b*x+a)^6/a^2/x^7-1/168*b^2*(b*x+a)^6/a^3/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^5}{x^9} dx = -\frac{a^5}{8x^8} - \frac{5a^4b}{7x^7} - \frac{5a^3b^2}{3x^6} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{4x^4} - \frac{b^5}{3x^3}$$

input
```
Integrate[(a + b*x)^5/x^9,x]
```

output
```
-1/8*a^5/x^8 - (5*a^4*b)/(7*x^7) - (5*a^3*b^2)/(3*x^6) - (2*a^2*b^3)/x^5 -
 (5*a*b^4)/(4*x^4) - b^5/(3*x^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^9} dx$$

$$\Big\downarrow \ 55$$

$$-\frac{b\int \frac{(a+bx)^5}{x^8}dx}{4a}-\frac{(a+bx)^6}{8ax^8}$$

$$\Big\downarrow \ 55$$

$$-\frac{b\left(-\frac{b\int \frac{(a+bx)^5}{x^7}dx}{7a}-\frac{(a+bx)^6}{7ax^7}\right)}{4a}-\frac{(a+bx)^6}{8ax^8}$$

$$\Big\downarrow \ 48$$

$$-\frac{b\left(\frac{b(a+bx)^6}{42a^2x^6}-\frac{(a+bx)^6}{7ax^7}\right)}{4a}-\frac{(a+bx)^6}{8ax^8}$$

input
```
Int[(a + b*x)^5/x^9,x]
```

output
```
-1/8*(a + b*x)^6/(a*x^8) - (b*(-1/7*(a + b*x)^6/(a*x^7) + (b*(a + b*x)^6)/
(42*a^2*x^6)))/(4*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

method	result	size
norman	$\frac{-\frac{1}{3}b^5x^5-\frac{5}{4}a\,b^4x^4-2a^2b^3x^3-\frac{5}{3}a^3b^2x^2-\frac{5}{7}a^4bx-\frac{1}{8}a^5}{x^8}$	57
risch	$\frac{-\frac{1}{3}b^5x^5-\frac{5}{4}a\,b^4x^4-2a^2b^3x^3-\frac{5}{3}a^3b^2x^2-\frac{5}{7}a^4bx-\frac{1}{8}a^5}{x^8}$	57
gosper	$-\frac{56b^5x^5+210a\,b^4x^4+336a^2b^3x^3+280a^3b^2x^2+120a^4bx+21a^5}{168x^8}$	58
default	$-\frac{b^5}{3x^3}-\frac{2a^2b^3}{x^5}-\frac{5a^4b}{7x^7}-\frac{5a\,b^4}{4x^4}-\frac{a^5}{8x^8}-\frac{5a^3b^2}{3x^6}$	58
parallelrisch	$\frac{-56b^5x^5-210a\,b^4x^4-336a^2b^3x^3-280a^3b^2x^2-120a^4bx-21a^5}{168x^8}$	58
orering	$-\frac{56b^5x^5+210a\,b^4x^4+336a^2b^3x^3+280a^3b^2x^2+120a^4bx+21a^5}{168x^8}$	58

input
```
int((b*x+a)^5/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/x^8*(-1/3*b^5*x^5-5/4*a*b^4*x^4-2*a^2*b^3*x^3-5/3*a^3*b^2*x^2-5/7*a^4*b*
x-1/8*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x^9}\,dx = -\frac{56\,b^5x^5 + 210\,ab^4x^4 + 336\,a^2b^3x^3 + 280\,a^3b^2x^2 + 120\,a^4bx + 21\,a^5}{168\,x^8}$$

input
```
integrate((b*x+a)^5/x^9,x, algorithm="fricas")
```

output
```
-1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 1
20*a^4*b*x + 21*a^5)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^5}{x^9}\,dx = \frac{-21a^5 - 120a^4bx - 280a^3b^2x^2 - 336a^2b^3x^3 - 210ab^4x^4 - 56b^5x^5}{168x^8}$$

input
```
integrate((b*x+a)**5/x**9,x)
```

output
```
(-21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210*a
*b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x^9}\,dx = -\frac{56\,b^5x^5 + 210\,ab^4x^4 + 336\,a^2b^3x^3 + 280\,a^3b^2x^2 + 120\,a^4bx + 21\,a^5}{168\,x^8}$$

input
```
integrate((b*x+a)^5/x^9,x, algorithm="maxima")
```

output
```
-1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 1
20*a^4*b*x + 21*a^5)/x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x^9}\,dx = -\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

input
```
integrate((b*x+a)^5/x^9,x, algorithm="giac")
```

output
```
-1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 1
20*a^4*b*x + 21*a^5)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x^9}\,dx = -\frac{\frac{a^5}{8}+\frac{5\,a^4\,b\,x}{7}+\frac{5\,a^3\,b^2\,x^2}{3}+2\,a^2\,b^3\,x^3+\frac{5\,a\,b^4\,x^4}{4}+\frac{b^5\,x^5}{3}}{x^8}$$

input
```
int((a + b*x)^5/x^9,x)
```

output
```
-(a^5/8 + (b^5*x^5)/3 + (5*a*b^4*x^4)/4 + (5*a^3*b^2*x^2)/3 + 2*a^2*b^3*x^
3 + (5*a^4*b*x)/7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{x^9}\,dx = \frac{-56b^5x^5-210a\,b^4x^4-336a^2b^3x^3-280a^3b^2x^2-120a^4bx-21a^5}{168x^8}$$

input
```
int((b*x+a)^5/x^9,x)
```

output
```
( - 21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210
*a*b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

3.51 $\int \frac{(a+bx)^5}{x^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

output
```
-1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*
b^5/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

input
```
Integrate[(a + b*x)^5/x^10,x]
```

output
```
-1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x
^6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^{10}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

input
```
Int[(a + b*x)^5/x^10,x]
```

output
```
-1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x
^6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result	size
norman	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
risch	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
gosper	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{a\,b^4}{x^5}-\frac{b^5}{4x^4}$	58
parallelrisch	$\frac{-126b^5x^5-504a\,b^4x^4-840a^2b^3x^3-720a^3b^2x^2-315a^4bx-56a^5}{504x^9}$	58
orering	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58

input
```
int((b*x+a)^5/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/4*b^5*x^5-a*b^4*x^4-5/3*a^2*b^3*x^3-10/7*a^3*b^2*x^2-5/8*a^4*b*x
-1/9*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

input
```
integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

output
```
-1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 +
315*a^4*b*x + 56*a^5)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = \frac{-56a^5 - 315a^4bx - 720a^3b^2x^2 - 840a^2b^3x^3 - 504ab^4x^4 - 126b^5x^5}{504x^9}$$

input
```
integrate((b*x+a)**5/x**10,x)
```

output
```
(-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a
*b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{126\,b^5x^5 + 504\,ab^4x^4 + 840\,a^2b^3x^3 + 720\,a^3b^2x^2 + 315\,a^4bx + 56\,a^5}{504\,x^9}$$

input
```
integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

output
```
-1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 +
315*a^4*b*x + 56*a^5)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{126\,b^5x^5 + 504\,ab^4x^4 + 840\,a^2b^3x^3 + 720\,a^3b^2x^2 + 315\,a^4bx + 56\,a^5}{504\,x^9}$$

input
```
integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

output
```
-1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 +
315*a^4*b*x + 56*a^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{\frac{a^5}{9}+\frac{5\,a^4\,b\,x}{8}+\frac{10\,a^3\,b^2\,x^2}{7}+\frac{5\,a^2\,b^3\,x^3}{3}+a\,b^4\,x^4+\frac{b^5\,x^5}{4}}{x^9}$$

input
```
int((a + b*x)^5/x^10,x)
```

output
```
-(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
 + (5*a^4*b*x)/8)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = \frac{-126b^5x^5-504a\,b^4x^4-840a^2b^3x^3-720a^3b^2x^2-315a^4bx-56a^5}{504x^9}$$

input
```
int((b*x+a)^5/x^10,x)
```

output
```
( - 56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504
*a*b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

3.52 $\int \frac{(a+bx)^5}{x^{11}} dx$

Optimal result

Integrand size = 11, antiderivative size = 69

$$\int \frac{(a+bx)^5}{x^{11}} \, dx = -\frac{a^5}{10x^{10}} - \frac{5a^4b}{9x^9} - \frac{5a^3b^2}{4x^8} - \frac{10a^2b^3}{7x^7} - \frac{5ab^4}{6x^6} - \frac{b^5}{5x^5}$$

output
```
-1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^
6-1/5*b^5/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^{11}} \, dx = -\frac{a^5}{10x^{10}} - \frac{5a^4b}{9x^9} - \frac{5a^3b^2}{4x^8} - \frac{10a^2b^3}{7x^7} - \frac{5ab^4}{6x^6} - \frac{b^5}{5x^5}$$

input
```
Integrate[(a + b*x)^5/x^11,x]
```

output
```
-1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7
*x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^{11}}\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

input
```
Int[(a + b*x)^5/x^11,x]
```

output
```
-1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7
*x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{-\frac{1}{5}b^5x^5-\frac{5}{6}a\,b^4x^4-\frac{10}{7}a^2b^3x^3-\frac{5}{4}a^3b^2x^2-\frac{5}{9}a^4bx-\frac{1}{10}a^5}{x^{10}}$	57
risch	$\frac{-\frac{1}{5}b^5x^5-\frac{5}{6}a\,b^4x^4-\frac{10}{7}a^2b^3x^3-\frac{5}{4}a^3b^2x^2-\frac{5}{9}a^4bx-\frac{1}{10}a^5}{x^{10}}$	57
gosper	$-\frac{252b^5x^5+1050a\,b^4x^4+1800a^2b^3x^3+1575a^3b^2x^2+700a^4bx+126a^5}{1260x^{10}}$	58
default	$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a\,b^4}{6x^6}-\frac{b^5}{5x^5}$	58
parallelrisch	$\frac{-252b^5x^5-1050a\,b^4x^4-1800a^2b^3x^3-1575a^3b^2x^2-700a^4bx-126a^5}{1260x^{10}}$	58
orering	$-\frac{252b^5x^5+1050a\,b^4x^4+1800a^2b^3x^3+1575a^3b^2x^2+700a^4bx+126a^5}{1260x^{10}}$	58

input
```
int((b*x+a)^5/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/x^10*(-1/5*b^5*x^5-5/6*a*b^4*x^4-10/7*a^2*b^3*x^3-5/4*a^3*b^2*x^2-5/9*a^
4*b*x-1/10*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{11}}\,dx = -\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

input
```
integrate((b*x+a)^5/x^11,x, algorithm="fricas")
```

output
```
-1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^
2 + 700*a^4*b*x + 126*a^5)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^5}{x^{11}}\,dx = \frac{-126a^5-700a^4bx-1575a^3b^2x^2-1800a^2b^3x^3-1050ab^4x^4-252b^5x^5}{1260x^{10}}$$

input
```
integrate((b*x+a)**5/x**11,x)
```

output
```
(-126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 - 10
50*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{11}}\,dx = -\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

input
```
integrate((b*x+a)^5/x^11,x, algorithm="maxima")
```

output
```
-1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^
2 + 700*a^4*b*x + 126*a^5)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{11}}\,dx = -\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

input
```
integrate((b*x+a)^5/x^11,x, algorithm="giac")
```

output
```
-1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^
2 + 700*a^4*b*x + 126*a^5)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{11}}\,dx = -\frac{\frac{a^5}{10}+\frac{5\,a^4\,b\,x}{9}+\frac{5\,a^3\,b^2\,x^2}{4}+\frac{10\,a^2\,b^3\,x^3}{7}+\frac{5\,a\,b^4\,x^4}{6}+\frac{b^5\,x^5}{5}}{x^{10}}$$

input
```
int((a + b*x)^5/x^11,x)
```

output
```
-(a^5/10 + (b^5*x^5)/5 + (5*a*b^4*x^4)/6 + (5*a^3*b^2*x^2)/4 + (10*a^2*b^3
*x^3)/7 + (5*a^4*b*x)/9)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{11}}\,dx = \frac{-252b^5x^5-1050a\,b^4x^4-1800a^2b^3x^3-1575a^3b^2x^2-700a^4bx-126a^5}{1260x^{10}}$$

input
```
int((b*x+a)^5/x^11,x)
```

output
```
( - 126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 -
1050*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

3.53 $\int \frac{(a+bx)^5}{x^{12}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 69

$$\int \frac{(a+bx)^5}{x^{12}}\,dx=-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

output
```
-1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x
^7-1/6*b^5/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^{12}}\,dx=-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

input
```
Integrate[(a + b*x)^5/x^12,x]
```

output
```
-1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*
x^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^{12}} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{11}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^9}+\frac{5ab^4}{x^8}+\frac{b^5}{x^7}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

input
```
Int[(a + b*x)^5/x^12,x]
```

output
```
-1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*
x^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{-\frac{1}{6}b^5x^5-\frac{5}{7}a\,b^4x^4-\frac{5}{4}a^2b^3x^3-\frac{10}{9}a^3b^2x^2-\frac{1}{2}a^4bx-\frac{1}{11}a^5}{x^{11}}$	57
risch	$\frac{-\frac{1}{6}b^5x^5-\frac{5}{7}a\,b^4x^4-\frac{5}{4}a^2b^3x^3-\frac{10}{9}a^3b^2x^2-\frac{1}{2}a^4bx-\frac{1}{11}a^5}{x^{11}}$	57
gosper	$-\frac{462b^5x^5+1980a\,b^4x^4+3465a^2b^3x^3+3080a^3b^2x^2+1386a^4bx+252a^5}{2772x^{11}}$	58
default	$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5a\,b^4}{7x^7}-\frac{b^5}{6x^6}$	58
parallelrisch	$\frac{-462b^5x^5-1980a\,b^4x^4-3465a^2b^3x^3-3080a^3b^2x^2-1386a^4bx-252a^5}{2772x^{11}}$	58
orering	$-\frac{462b^5x^5+1980a\,b^4x^4+3465a^2b^3x^3+3080a^3b^2x^2+1386a^4bx+252a^5}{2772x^{11}}$	58

input
```
int((b*x+a)^5/x^12,x,method=_RETURNVERBOSE)
```

output
```
1/x^11*(-1/6*b^5*x^5-5/7*a*b^4*x^4-5/4*a^2*b^3*x^3-10/9*a^3*b^2*x^2-1/2*a^
4*b*x-1/11*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{12}}\,dx = -\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

input
```
integrate((b*x+a)^5/x^12,x, algorithm="fricas")
```

output
```
-1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^
2 + 1386*a^4*b*x + 252*a^5)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^5}{x^{12}}\,dx = \frac{-252a^5-1386a^4bx-3080a^3b^2x^2-3465a^2b^3x^3-1980ab^4x^4-462b^5x^5}{2772x^{11}}$$

input
```
integrate((b*x+a)**5/x**12,x)
```

output
```
(-252*a**5 - 1386*a**4*b*x - 3080*a**3*b**2*x**2 - 3465*a**2*b**3*x**3 - 1
980*a*b**4*x**4 - 462*b**5*x**5)/(2772*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{12}}\,dx = -\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

input
```
integrate((b*x+a)^5/x^12,x, algorithm="maxima")
```

output
```
-1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^
2 + 1386*a^4*b*x + 252*a^5)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{12}}\,dx = -\frac{462\,b^5x^5 + 1980\,ab^4x^4 + 3465\,a^2b^3x^3 + 3080\,a^3b^2x^2 + 1386\,a^4bx + 252\,a^5}{2772\,x^{11}}$$

input
```
integrate((b*x+a)^5/x^12,x, algorithm="giac")
```

output
```
-1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^
2 + 1386*a^4*b*x + 252*a^5)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{12}}\,dx = -\frac{\frac{a^5}{11} + \frac{a^4\,b\,x}{2} + \frac{10\,a^3\,b^2\,x^2}{9} + \frac{5\,a^2\,b^3\,x^3}{4} + \frac{5\,a\,b^4\,x^4}{7} + \frac{b^5\,x^5}{6}}{x^{11}}$$

input
```
int((a + b*x)^5/x^12,x)
```

output
```
-(a^5/11 + (b^5*x^5)/6 + (5*a*b^4*x^4)/7 + (10*a^3*b^2*x^2)/9 + (5*a^2*b^3
*x^3)/4 + (a^4*b*x)/2)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^5}{x^{12}}\,dx = \frac{-462b^5x^5 - 1980a\,b^4x^4 - 3465a^2b^3x^3 - 3080a^3b^2x^2 - 1386a^4bx - 252a^5}{2772x^{11}}$$

input
```
int((b*x+a)^5/x^12,x)
```

output
```
( - 252*a**5 - 1386*a**4*b*x - 3080*a**3*b**2*x**2 - 3465*a**2*b**3*x**3 -
 1980*a*b**4*x**4 - 462*b**5*x**5)/(2772*x**11)
```

3.54 $\int \frac{(a+bx)^5}{x^{13}} dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{(a+bx)^5}{x^{13}} dx = -\frac{a^5}{12x^{12}} - \frac{5a^4b}{11x^{11}} - \frac{a^3b^2}{x^{10}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{8x^8} - \frac{b^5}{7x^7}$$

output
```
-1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8
-1/7*b^5/x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^{13}} dx = -\frac{a^5}{12x^{12}} - \frac{5a^4b}{11x^{11}} - \frac{a^3b^2}{x^{10}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{8x^8} - \frac{b^5}{7x^7}$$

input
```
Integrate[(a + b*x)^5/x^13,x]
```

output
```
-1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^
9) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^{13}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

input
```
Int[(a + b*x)^5/x^13,x]
```

output
```
-1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^
9) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result	size
norman	$\frac{-\frac{1}{7}b^5x^5-\frac{5}{8}a\,b^4x^4-\frac{10}{9}a^2b^3x^3-a^3b^2x^2-\frac{5}{11}a^4bx-\frac{1}{12}a^5}{x^{12}}$	57
risch	$\frac{-\frac{1}{7}b^5x^5-\frac{5}{8}a\,b^4x^4-\frac{10}{9}a^2b^3x^3-a^3b^2x^2-\frac{5}{11}a^4bx-\frac{1}{12}a^5}{x^{12}}$	57
gosper	$-\frac{792b^5x^5+3465a\,b^4x^4+6160a^2b^3x^3+5544a^3b^2x^2+2520a^4bx+462a^5}{5544x^{12}}$	58
default	$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a\,b^4}{8x^8}-\frac{b^5}{7x^7}$	58
parallelrisch	$\frac{-792b^5x^5-3465a\,b^4x^4-6160a^2b^3x^3-5544a^3b^2x^2-2520a^4bx-462a^5}{5544x^{12}}$	58
orering	$-\frac{792b^5x^5+3465a\,b^4x^4+6160a^2b^3x^3+5544a^3b^2x^2+2520a^4bx+462a^5}{5544x^{12}}$	58

input
```
int((b*x+a)^5/x^13,x,method=_RETURNVERBOSE)
```

output
```
1/x^12*(-1/7*b^5*x^5-5/8*a*b^4*x^4-10/9*a^2*b^3*x^3-a^3*b^2*x^2-5/11*a^4*b
*x-1/12*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{13}}\,dx$$
$$= -\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

input
```
integrate((b*x+a)^5/x^13,x, algorithm="fricas")
```

output
```
-1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^
2 + 2520*a^4*b*x + 462*a^5)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^5}{x^{13}}\,dx = \frac{-462a^5 - 2520a^4bx - 5544a^3b^2x^2 - 6160a^2b^3x^3 - 3465ab^4x^4 - 792b^5x^5}{5544x^{12}}$$

input
```
integrate((b*x+a)**5/x**13,x)
```

output
```
(-462*a**5 - 2520*a**4*b*x - 5544*a**3*b**2*x**2 - 6160*a**2*b**3*x**3 - 3
465*a*b**4*x**4 - 792*b**5*x**5)/(5544*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{13}}\,dx = -\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

input
```
integrate((b*x+a)^5/x^13,x, algorithm="maxima")
```

output
```
-1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^
2 + 2520*a^4*b*x + 462*a^5)/x^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{13}}\,dx = -\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

input
```
integrate((b*x+a)^5/x^13,x, algorithm="giac")
```

output
```
-1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^
2 + 2520*a^4*b*x + 462*a^5)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^5}{x^{13}}\,dx = -\frac{\frac{a^5}{12} + \frac{5\,a^4\,b\,x}{11} + a^3\,b^2\,x^2 + \frac{10\,a^2\,b^3\,x^3}{9} + \frac{5\,a\,b^4\,x^4}{8} + \frac{b^5\,x^5}{7}}{x^{12}}$$

input
```
int((a + b*x)^5/x^13,x)
```

output
```
-(a^5/12 + (b^5*x^5)/7 + (5*a*b^4*x^4)/8 + a^3*b^2*x^2 + (10*a^2*b^3*x^3)/
9 + (5*a^4*b*x)/11)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{13}}\,dx = \frac{-792b^5x^5 - 3465a\,b^4x^4 - 6160a^2b^3x^3 - 5544a^3b^2x^2 - 2520a^4bx - 462a^5}{5544x^{12}}$$

input
```
int((b*x+a)^5/x^13,x)
```

output
```
( - 462*a**5 - 2520*a**4*b*x - 5544*a**3*b**2*x**2 - 6160*a**2*b**3*x**3 -
 3465*a*b**4*x**4 - 792*b**5*x**5)/(5544*x**12)
```

3.55 $\int \frac{(a+bx)^5}{x^{14}} dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{(a+bx)^5}{x^{14}} dx = -\frac{a^5}{13x^{13}} - \frac{5a^4b}{12x^{12}} - \frac{10a^3b^2}{11x^{11}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{9x^9} - \frac{b^5}{8x^8}$$

output
```
-1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x
^9-1/8*b^5/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^{14}} dx = -\frac{a^5}{13x^{13}} - \frac{5a^4b}{12x^{12}} - \frac{10a^3b^2}{11x^{11}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{9x^9} - \frac{b^5}{8x^8}$$

input
```
Integrate[(a + b*x)^5/x^14,x]
```

output
```
-1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/
x^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^{14}}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{13}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^9}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

input
```
Int[(a + b*x)^5/x^14,x]
```

output
```
-1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/
x^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result	size
norman	$\frac{-\frac{1}{8}b^{5}x^{5}-\frac{5}{9}a\,b^{4}x^{4}-a^{2}b^{3}x^{3}-\frac{10}{11}a^{3}b^{2}x^{2}-\frac{5}{12}a^{4}bx-\frac{1}{13}a^{5}}{x^{13}}$	57
risch	$\frac{-\frac{1}{8}b^{5}x^{5}-\frac{5}{9}a\,b^{4}x^{4}-a^{2}b^{3}x^{3}-\frac{10}{11}a^{3}b^{2}x^{2}-\frac{5}{12}a^{4}bx-\frac{1}{13}a^{5}}{x^{13}}$	57
gosper	$-\frac{1287b^{5}x^{5}+5720a\,b^{4}x^{4}+10296a^{2}b^{3}x^{3}+9360a^{3}b^{2}x^{2}+4290a^{4}bx+792a^{5}}{10296x^{13}}$	58
default	$-\frac{a^{5}}{13x^{13}}-\frac{5a^{4}b}{12x^{12}}-\frac{10a^{3}b^{2}}{11x^{11}}-\frac{a^{2}b^{3}}{x^{10}}-\frac{5a\,b^{4}}{9x^{9}}-\frac{b^{5}}{8x^{8}}$	58
parallelrisch	$\frac{-1287b^{5}x^{5}-5720a\,b^{4}x^{4}-10296a^{2}b^{3}x^{3}-9360a^{3}b^{2}x^{2}-4290a^{4}bx-792a^{5}}{10296x^{13}}$	58
orering	$-\frac{1287b^{5}x^{5}+5720a\,b^{4}x^{4}+10296a^{2}b^{3}x^{3}+9360a^{3}b^{2}x^{2}+4290a^{4}bx+792a^{5}}{10296x^{13}}$	58

input
```
int((b*x+a)^5/x^14,x,method=_RETURNVERBOSE)
```

output
```
1/x^13*(-1/8*b^5*x^5-5/9*a*b^4*x^4-a^2*b^3*x^3-10/11*a^3*b^2*x^2-5/12*a^4*
b*x-1/13*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{14}}\,dx = -\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

input
```
integrate((b*x+a)^5/x^14,x, algorithm="fricas")
```

output
```
-1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2
*x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^5}{x^{14}}\,dx = \frac{-792a^5-4290a^4bx-9360a^3b^2x^2-10296a^2b^3x^3-5720ab^4x^4-1287b^5x^5}{10296x^{13}}$$

input
```
integrate((b*x+a)**5/x**14,x)
```

output
```
(-792*a**5 - 4290*a**4*b*x - 9360*a**3*b**2*x**2 - 10296*a**2*b**3*x**3 -
5720*a*b**4*x**4 - 1287*b**5*x**5)/(10296*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{14}}\,dx = -\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

input
```
integrate((b*x+a)^5/x^14,x, algorithm="maxima")
```

output
```
-1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2
*x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{14}}\,dx = -\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

input
```
integrate((b*x+a)^5/x^14,x, algorithm="giac")
```

output
```
-1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2
*x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^5}{x^{14}}\,dx = -\frac{\frac{a^5}{13}+\frac{5\,a^4\,b\,x}{12}+\frac{10\,a^3\,b^2\,x^2}{11}+a^2\,b^3\,x^3+\frac{5\,a\,b^4\,x^4}{9}+\frac{b^5\,x^5}{8}}{x^{13}}$$

input
```
int((a + b*x)^5/x^14,x)
```

output
```
-(a^5/13 + (b^5*x^5)/8 + (5*a*b^4*x^4)/9 + (10*a^3*b^2*x^2)/11 + a^2*b^3*x
^3 + (5*a^4*b*x)/12)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{14}}\,dx = \frac{-1287b^5x^5-5720a\,b^4x^4-10296a^2b^3x^3-9360a^3b^2x^2-4290a^4bx-792a^5}{10296x^{13}}$$

input
```
int((b*x+a)^5/x^14,x)
```

output
```
( - 792*a**5 - 4290*a**4*b*x - 9360*a**3*b**2*x**2 - 10296*a**2*b**3*x**3
- 5720*a*b**4*x**4 - 1287*b**5*x**5)/(10296*x**13)
```

3.56 $\int x^8(a+bx)^7\,dx$

Optimal result

Integrand size = 11, antiderivative size = 95

$$\int x^8(a+bx)^7\,dx=\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

output
```
1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^
3*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int x^8(a+bx)^7\,dx=\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

input
```
Integrate[x^8*(a + b*x)^7,x]
```

output
```
(a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)
/12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7
*x^16)/16
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8(a+bx)^7\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(a^7x^8+7a^6bx^9+21a^5b^2x^{10}+35a^4b^3x^{11}+35a^3b^4x^{12}+21a^2b^5x^{13}+7ab^6x^{14}+b^7x^{15}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

input
```
Int[x^8*(a + b*x)^7,x]
```

output

```
(a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)
/12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7
*x^16)/16
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$
default	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$
norman	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$
risch	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$
parallelrisch	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$
orering	$\frac{x^9(6435b^7x^7+48048a\,b^6x^6+154440a^2b^5x^5+277200a^3b^4x^4+300300a^4b^3x^3+196560a^5b^2x^2+72072a^6bx+11440a^7)}{102960}$

input
```
int(x^8*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^
3*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^8(a+bx)^7\,dx=\frac{1}{16}\,b^7x^{16}+\frac{7}{15}\,ab^6x^{15}+\frac{3}{2}\,a^2b^5x^{14}+\frac{35}{13}\,a^3b^4x^{13}+\frac{35}{12}\,a^4b^3x^{12}+\frac{21}{11}\,a^5b^2x^{11}+\frac{7}{10}\,a^6bx^{10}+\frac{1}{9}\,a^7x^9$$

input
```
integrate(x^8*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 +
35/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int x^8(a+bx)^7\,dx = \frac{a^7x^9}{9} + \frac{7a^6bx^{10}}{10} + \frac{21a^5b^2x^{11}}{11} + \frac{35a^4b^3x^{12}}{12} + \frac{35a^3b^4x^{13}}{13} + \frac{3a^2b^5x^{14}}{2} + \frac{7ab^6x^{15}}{15} + \frac{b^7x^{16}}{16}$$

input
```
integrate(x**8*(b*x+a)**7,x)
```

output
```
a**7*x**9/9 + 7*a**6*b*x**10/10 + 21*a**5*b**2*x**11/11 + 35*a**4*b**3*x**
12/12 + 35*a**3*b**4*x**13/13 + 3*a**2*b**5*x**14/2 + 7*a*b**6*x**15/15 +
b**7*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^8(a+bx)^7\,dx = \frac{1}{16}\,b^7x^{16} + \frac{7}{15}\,ab^6x^{15} + \frac{3}{2}\,a^2b^5x^{14} + \frac{35}{13}\,a^3b^4x^{13} + \frac{35}{12}\,a^4b^3x^{12} + \frac{21}{11}\,a^5b^2x^{11} + \frac{7}{10}\,a^6bx^{10} + \frac{1}{9}\,a^7x^9$$

input
```
integrate(x^8*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 +
35/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^8(a+bx)^7\,dx = \frac{1}{16}\,b^7x^{16} + \frac{7}{15}\,ab^6x^{15} + \frac{3}{2}\,a^2b^5x^{14} + \frac{35}{13}\,a^3b^4x^{13} + \frac{35}{12}\,a^4b^3x^{12} + \frac{21}{11}\,a^5b^2x^{11} + \frac{7}{10}\,a^6bx^{10} + \frac{1}{9}\,a^7x^9$$

input
```
integrate(x^8*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 +
35/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^8(a+bx)^7\,dx = \frac{a^7\,x^9}{9} + \frac{7\,a^6\,b\,x^{10}}{10} + \frac{21\,a^5\,b^2\,x^{11}}{11} + \frac{35\,a^4\,b^3\,x^{12}}{12} + \frac{35\,a^3\,b^4\,x^{13}}{13} + \frac{3\,a^2\,b^5\,x^{14}}{2} + \frac{7\,a\,b^6\,x^{15}}{15} + \frac{b^7\,x^{16}}{16}$$

input
```
int(x^8*(a + b*x)^7,x)
```

output
```
(a^7*x^9)/9 + (b^7*x^16)/16 + (7*a^6*b*x^10)/10 + (7*a*b^6*x^15)/15 + (21*
a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^
5*x^14)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^8(a+bx)^7\,dx$$
$$=\frac{x^9(6435b^7x^7+48048a\,b^6x^6+154440a^2b^5x^5+277200a^3b^4x^4+300300a^4b^3x^3+196560a^5b^2x^2+72072a^6}{102960}$$

input
```
int(x^8*(b*x+a)^7,x)
```

output
```
(x**9*(11440*a**7 + 72072*a**6*b*x + 196560*a**5*b**2*x**2 + 300300*a**4*b
**3*x**3 + 277200*a**3*b**4*x**4 + 154440*a**2*b**5*x**5 + 48048*a*b**6*x*
*6 + 6435*b**7*x**7))/102960
```

3.57 $\int x^7(a+bx)^7\,dx$

Optimal result

Integrand size = 11, antiderivative size = 95

$$\int x^7(a+bx)^7\,dx = \frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

output
```
1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*
b^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int x^7(a+bx)^7\,dx = \frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

input
```
Integrate[x^7*(a + b*x)^7,x]
```

output
```
(a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/1
1 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^
15)/15
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a+bx)^7 \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^7x^7+7a^6bx^8+21a^5b^2x^9+35a^4b^3x^{10}+35a^3b^4x^{11}+21a^2b^5x^{12}+7ab^6x^{13}+b^7x^{14}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

input
```
Int[x^7*(a + b*x)^7,x]
```

output

```
(a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/1
1 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^
15)/15
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

method	result	s
gosper	$\frac{1}{8}a^{7}x^{8}+\frac{7}{9}a^{6}b\,x^{9}+\frac{21}{10}a^{5}b^{2}x^{10}+\frac{35}{11}a^{4}b^{3}x^{11}+\frac{35}{12}a^{3}b^{4}x^{12}+\frac{21}{13}a^{2}b^{5}x^{13}+\frac{1}{2}a\,b^{6}x^{14}+\frac{1}{15}b^{7}x^{15}$	8
default	$\frac{1}{8}a^{7}x^{8}+\frac{7}{9}a^{6}b\,x^{9}+\frac{21}{10}a^{5}b^{2}x^{10}+\frac{35}{11}a^{4}b^{3}x^{11}+\frac{35}{12}a^{3}b^{4}x^{12}+\frac{21}{13}a^{2}b^{5}x^{13}+\frac{1}{2}a\,b^{6}x^{14}+\frac{1}{15}b^{7}x^{15}$	8
norman	$\frac{1}{8}a^{7}x^{8}+\frac{7}{9}a^{6}b\,x^{9}+\frac{21}{10}a^{5}b^{2}x^{10}+\frac{35}{11}a^{4}b^{3}x^{11}+\frac{35}{12}a^{3}b^{4}x^{12}+\frac{21}{13}a^{2}b^{5}x^{13}+\frac{1}{2}a\,b^{6}x^{14}+\frac{1}{15}b^{7}x^{15}$	8
risch	$\frac{1}{8}a^{7}x^{8}+\frac{7}{9}a^{6}b\,x^{9}+\frac{21}{10}a^{5}b^{2}x^{10}+\frac{35}{11}a^{4}b^{3}x^{11}+\frac{35}{12}a^{3}b^{4}x^{12}+\frac{21}{13}a^{2}b^{5}x^{13}+\frac{1}{2}a\,b^{6}x^{14}+\frac{1}{15}b^{7}x^{15}$	8
parallelrisch	$\frac{1}{8}a^{7}x^{8}+\frac{7}{9}a^{6}b\,x^{9}+\frac{21}{10}a^{5}b^{2}x^{10}+\frac{35}{11}a^{4}b^{3}x^{11}+\frac{35}{12}a^{3}b^{4}x^{12}+\frac{21}{13}a^{2}b^{5}x^{13}+\frac{1}{2}a\,b^{6}x^{14}+\frac{1}{15}b^{7}x^{15}$	8
orering	$\frac{x^{8}\left(3432b^{7}x^{7}+25740a\,b^{6}x^{6}+83160a^{2}b^{5}x^{5}+150150a^{3}b^{4}x^{4}+163800a^{4}b^{3}x^{3}+108108a^{5}b^{2}x^{2}+40040a^{6}bx+6435a^{7}\right)}{51480}$	8

input
```
int(x^7*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*
b^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^7(a+bx)^7\,dx=\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

input
```
integrate(x^7*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
 35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int x^7(a+bx)^7\,dx = \frac{a^7x^8}{8} + \frac{7a^6bx^9}{9} + \frac{21a^5b^2x^{10}}{10} + \frac{35a^4b^3x^{11}}{11} + \frac{35a^3b^4x^{12}}{12} + \frac{21a^2b^5x^{13}}{13} + \frac{ab^6x^{14}}{2} + \frac{b^7x^{15}}{15}$$

input
```
integrate(x**7*(b*x+a)**7,x)
```

output
```
a**7*x**8/8 + 7*a**6*b*x**9/9 + 21*a**5*b**2*x**10/10 + 35*a**4*b**3*x**11
/11 + 35*a**3*b**4*x**12/12 + 21*a**2*b**5*x**13/13 + a*b**6*x**14/2 + b**
7*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^7(a+bx)^7\,dx = \frac{1}{15}\,b^7x^{15} + \frac{1}{2}\,ab^6x^{14} + \frac{21}{13}\,a^2b^5x^{13} + \frac{35}{12}\,a^3b^4x^{12} + \frac{35}{11}\,a^4b^3x^{11} + \frac{21}{10}\,a^5b^2x^{10} + \frac{7}{9}\,a^6bx^9 + \frac{1}{8}\,a^7x^8$$

input
```
integrate(x^7*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
 35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^7(a+bx)^7\,dx = \frac{1}{15}\,b^7x^{15} + \frac{1}{2}\,ab^6x^{14} + \frac{21}{13}\,a^2b^5x^{13} + \frac{35}{12}\,a^3b^4x^{12} + \frac{35}{11}\,a^4b^3x^{11} + \frac{21}{10}\,a^5b^2x^{10} + \frac{7}{9}\,a^6bx^9 + \frac{1}{8}\,a^7x^8$$

input
```
integrate(x^7*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
 35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^7(a+bx)^7\,dx = \frac{a^7\,x^8}{8} + \frac{7\,a^6\,b\,x^9}{9} + \frac{21\,a^5\,b^2\,x^{10}}{10} + \frac{35\,a^4\,b^3\,x^{11}}{11} + \frac{35\,a^3\,b^4\,x^{12}}{12} + \frac{21\,a^2\,b^5\,x^{13}}{13} + \frac{a\,b^6\,x^{14}}{2} + \frac{b^7\,x^{15}}{15}$$

input
```
int(x^7*(a + b*x)^7,x)
```

output
```
(a^7*x^8)/8 + (b^7*x^15)/15 + (7*a^6*b*x^9)/9 + (a*b^6*x^14)/2 + (21*a^5*b
^2*x^10)/10 + (35*a^4*b^3*x^11)/11 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^
13)/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^7(a+bx)^7\,dx$$
$$=\frac{x^8(3432b^7x^7+25740a\,b^6x^6+83160a^2b^5x^5+150150a^3b^4x^4+163800a^4b^3x^3+108108a^5b^2x^2+40040a^6b}{51480}$$

input
```
int(x^7*(b*x+a)^7,x)
```

output
```
(x**8*(6435*a**7 + 40040*a**6*b*x + 108108*a**5*b**2*x**2 + 163800*a**4*b*
*3*x**3 + 150150*a**3*b**4*x**4 + 83160*a**2*b**5*x**5 + 25740*a*b**6*x**6
 + 3432*b**7*x**7))/51480
```

3.58 $\int x^6(a+bx)^7\,dx$

Optimal result

Integrand size = 11, antiderivative size = 95

$$\int x^6(a+bx)^7\,dx=\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

output
```
1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x
^11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int x^6(a+bx)^7\,dx=\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

input
```
Integrate[x^6*(a + b*x)^7,x]
```

output
```
(a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (
35*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/
14
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a+bx)^7\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^7x^6+7a^6bx^7+21a^5b^2x^8+35a^4b^3x^9+35a^3b^4x^{10}+21a^2b^5x^{11}+7ab^6x^{12}+b^7x^{13}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

input
```
Int[x^6*(a + b*x)^7,x]
```

output
```
(a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (
35*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/
14
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
default	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
norman	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
risch	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
parallelrisch	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
orering	$\frac{x^7(1716b^7x^7+12936a\,b^6x^6+42042a^2b^5x^5+76440a^3b^4x^4+84084a^4b^3x^3+56056a^5b^2x^2+21021a^6bx+3432a^7)}{24024}$	80

input
```
int(x^6*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x
^11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^6(a+bx)^7\,dx=\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

input
```
integrate(x^6*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 +
7/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int x^6(a+bx)^7\,dx = \frac{a^7x^7}{7} + \frac{7a^6bx^8}{8} + \frac{7a^5b^2x^9}{3} + \frac{7a^4b^3x^{10}}{2} + \frac{35a^3b^4x^{11}}{11} + \frac{7a^2b^5x^{12}}{4} + \frac{7ab^6x^{13}}{13} + \frac{b^7x^{14}}{14}$$

input
```
integrate(x**6*(b*x+a)**7,x)
```

output
```
a**7*x**7/7 + 7*a**6*b*x**8/8 + 7*a**5*b**2*x**9/3 + 7*a**4*b**3*x**10/2 +
 35*a**3*b**4*x**11/11 + 7*a**2*b**5*x**12/4 + 7*a*b**6*x**13/13 + b**7*x*
*14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^6(a+bx)^7\,dx = \frac{1}{14}\,b^7x^{14} + \frac{7}{13}\,ab^6x^{13} + \frac{7}{4}\,a^2b^5x^{12} + \frac{35}{11}\,a^3b^4x^{11} + \frac{7}{2}\,a^4b^3x^{10} + \frac{7}{3}\,a^5b^2x^9 + \frac{7}{8}\,a^6bx^8 + \frac{1}{7}\,a^7x^7$$

input
```
integrate(x^6*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 +
7/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^6(a+bx)^7\,dx = \frac{1}{14}b^7x^{14} + \frac{7}{13}ab^6x^{13} + \frac{7}{4}a^2b^5x^{12} + \frac{35}{11}a^3b^4x^{11} + \frac{7}{2}a^4b^3x^{10} + \frac{7}{3}a^5b^2x^9 + \frac{7}{8}a^6bx^8 + \frac{1}{7}a^7x^7$$

input
```
integrate(x^6*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 +
7/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^6(a+bx)^7\,dx = \frac{a^7\,x^7}{7} + \frac{7\,a^6\,b\,x^8}{8} + \frac{7\,a^5\,b^2\,x^9}{3} + \frac{7\,a^4\,b^3\,x^{10}}{2} + \frac{35\,a^3\,b^4\,x^{11}}{11} + \frac{7\,a^2\,b^5\,x^{12}}{4} + \frac{7\,a\,b^6\,x^{13}}{13} + \frac{b^7\,x^{14}}{14}$$

input
```
int(x^6*(a + b*x)^7,x)
```

output
```
(a^7*x^7)/7 + (b^7*x^14)/14 + (7*a^6*b*x^8)/8 + (7*a*b^6*x^13)/13 + (7*a^5
*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (35*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)
/4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int x^6(a+bx)^7\,dx$$
$$=\frac{x^7(1716b^7x^7+12936a\,b^6x^6+42042a^2b^5x^5+76440a^3b^4x^4+84084a^4b^3x^3+56056a^5b^2x^2+21021a^6bx+}{24024}$$

input
```
int(x^6*(b*x+a)^7,x)
```

output
```
(x**7*(3432*a**7 + 21021*a**6*b*x + 56056*a**5*b**2*x**2 + 84084*a**4*b**3
*x**3 + 76440*a**3*b**4*x**4 + 42042*a**2*b**5*x**5 + 12936*a*b**6*x**6 +
1716*b**7*x**7))/24024
```

3.59 $\int x^5(a+bx)^7\,dx$

Optimal result

Integrand size = 11, antiderivative size = 96

$$\int x^5(a+bx)^7\,dx = -\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+bx)^{13}}{13b^6}$$

output
```
-1/8*a^5*(b*x+a)^8/b^6+5/9*a^4*(b*x+a)^9/b^6-a^3*(b*x+a)^10/b^6+10/11*a^2*
(b*x+a)^11/b^6-5/12*a*(b*x+a)^12/b^6+1/13*(b*x+a)^13/b^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^5(a+bx)^7\,dx = \frac{a^7x^6}{6}+a^6bx^7+\frac{21}{8}a^5b^2x^8+\frac{35}{9}a^4b^3x^9+\frac{7}{2}a^3b^4x^{10}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{12}ab^6x^{12}+\frac{b^7x^{13}}{13}$$

input
```
Integrate[x^5*(a + b*x)^7,x]
```

output
```
(a^7*x^6)/6 + a^6*b*x^7 + (21*a^5*b^2*x^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3
*b^4*x^10)/2 + (21*a^2*b^5*x^11)/11 + (7*a*b^6*x^12)/12 + (b^7*x^13)/13
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+bx)^7\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^5(a+bx)^7}{b^5}+\frac{5a^4(a+bx)^8}{b^5}-\frac{10a^3(a+bx)^9}{b^5}+\frac{10a^2(a+bx)^{10}}{b^5}+\frac{(a+bx)^{12}}{b^5}-\frac{5a(a+bx)^{11}}{b^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

input
```
Int[x^5*(a + b*x)^7,x]
```

output
```
-1/8*(a^5*(a + b*x)^8)/b^6 + (5*a^4*(a + b*x)^9)/(9*b^6) - (a^3*(a + b*x)^
10)/b^6 + (10*a^2*(a + b*x)^11)/(11*b^6) - (5*a*(a + b*x)^12)/(12*b^6) + (
a + b*x)^13/(13*b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
default	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
norman	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
risch	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
parallelrisch	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
orering	$\frac{x^6(792b^7x^7+6006a\,b^6x^6+19656a^2b^5x^5+36036a^3b^4x^4+40040a^4b^3x^3+27027a^5b^2x^2+10296a^6bx+1716a^7)}{10296}$	80

input
```
int(x^5*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^7*x^13+7/12*a*b^6*x^12+21/11*a^2*b^5*x^11+7/2*a^3*b^4*x^10+35/9*a^4
*b^3*x^9+21/8*a^5*b^2*x^8+a^6*b*x^7+1/6*a^7*x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int x^5(a+bx)^7\,dx=\frac{1}{13}\,b^7x^{13}+\frac{7}{12}\,ab^6x^{12}+\frac{21}{11}\,a^2b^5x^{11}+\frac{7}{2}\,a^3b^4x^{10}+\frac{35}{9}\,a^4b^3x^9+\frac{21}{8}\,a^5b^2x^8+a^6bx^7+\frac{1}{6}\,a^7x^6$$

input
```
integrate(x^5*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 +
35/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int x^5(a+bx)^7\,dx = \frac{a^7x^6}{6} + a^6bx^7 + \frac{21a^5b^2x^8}{8} + \frac{35a^4b^3x^9}{9} + \frac{7a^3b^4x^{10}}{2} + \frac{21a^2b^5x^{11}}{11} + \frac{7ab^6x^{12}}{12} + \frac{b^7x^{13}}{13}$$

input
```
integrate(x**5*(b*x+a)**7,x)
```

output
```
a**7*x**6/6 + a**6*b*x**7 + 21*a**5*b**2*x**8/8 + 35*a**4*b**3*x**9/9 + 7*
a**3*b**4*x**10/2 + 21*a**2*b**5*x**11/11 + 7*a*b**6*x**12/12 + b**7*x**13
/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int x^5(a+bx)^7\,dx = \frac{1}{13}\,b^7x^{13} + \frac{7}{12}\,ab^6x^{12} + \frac{21}{11}\,a^2b^5x^{11} + \frac{7}{2}\,a^3b^4x^{10} + \frac{35}{9}\,a^4b^3x^9 + \frac{21}{8}\,a^5b^2x^8 + a^6bx^7 + \frac{1}{6}\,a^7x^6$$

input
```
integrate(x^5*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 +
35/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int x^5(a+bx)^7\,dx = \frac{1}{13}\,b^7x^{13} + \frac{7}{12}\,ab^6x^{12} + \frac{21}{11}\,a^2b^5x^{11} + \frac{7}{2}\,a^3b^4x^{10} + \frac{35}{9}\,a^4b^3x^9 + \frac{21}{8}\,a^5b^2x^8 + a^6bx^7 + \frac{1}{6}\,a^7x^6$$

input
```
integrate(x^5*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 +
35/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int x^5(a+bx)^7\,dx = \frac{a^7\,x^6}{6} + a^6\,b\,x^7 + \frac{21\,a^5\,b^2\,x^8}{8} + \frac{35\,a^4\,b^3\,x^9}{9} + \frac{7\,a^3\,b^4\,x^{10}}{2} + \frac{21\,a^2\,b^5\,x^{11}}{11} + \frac{7\,a\,b^6\,x^{12}}{12} + \frac{b^7\,x^{13}}{13}$$

input
```
int(x^5*(a + b*x)^7,x)
```

output
```
(a^7*x^6)/6 + (b^7*x^13)/13 + a^6*b*x^7 + (7*a*b^6*x^12)/12 + (21*a^5*b^2*
x^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*b^4*x^10)/2 + (21*a^2*b^5*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int x^5(a+bx)^7\,dx$$
$$=\frac{x^6(792b^7x^7+6006a\,b^6x^6+19656a^2b^5x^5+36036a^3b^4x^4+40040a^4b^3x^3+27027a^5b^2x^2+10296a^6bx+17}{10296}$$

input
```
int(x^5*(b*x+a)^7,x)
```

output
```
(x**6*(1716*a**7 + 10296*a**6*b*x + 27027*a**5*b**2*x**2 + 40040*a**4*b**3
*x**3 + 36036*a**3*b**4*x**4 + 19656*a**2*b**5*x**5 + 6006*a*b**6*x**6 + 7
92*b**7*x**7))/10296
```

3.60 $\int x^4(a+bx)^7\,dx$

Optimal result

Integrand size = 11, antiderivative size = 81

$$\int x^4(a+bx)^7\,dx=\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}$$

output
```
1/8*a^4*(b*x+a)^8/b^5-4/9*a^3*(b*x+a)^9/b^5+3/5*a^2*(b*x+a)^10/b^5-4/11*a*
(b*x+a)^11/b^5+1/12*(b*x+a)^12/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int x^4(a+bx)^7\,dx=\frac{a^7x^5}{5}+\frac{7}{6}a^6bx^6+3a^5b^2x^7+\frac{35}{8}a^4b^3x^8+\frac{35}{9}a^3b^4x^9+\frac{21}{10}a^2b^5x^{10}+\frac{7}{11}ab^6x^{11}+\frac{b^7x^{12}}{12}$$

input
```
Integrate[x^4*(a + b*x)^7,x]
```

output
```
(a^7*x^5)/5 + (7*a^6*b*x^6)/6 + 3*a^5*b^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a
^3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10 + (7*a*b^6*x^11)/11 + (b^7*x^12)/12
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^7\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^4(a+bx)^7}{b^4}-\frac{4a^3(a+bx)^8}{b^4}+\frac{6a^2(a+bx)^9}{b^4}+\frac{(a+bx)^{11}}{b^4}-\frac{4a(a+bx)^{10}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

input
```
Int[x^4*(a + b*x)^7,x]
```

output
```
(a^4*(a + b*x)^8)/(8*b^5) - (4*a^3*(a + b*x)^9)/(9*b^5) + (3*a^2*(a + b*x)
^10)/(5*b^5) - (4*a*(a + b*x)^11)/(11*b^5) + (a + b*x)^12/(12*b^5)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

method	result	size
gosper	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
default	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
norman	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
risch	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
parallelrisch	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
orering	$\frac{x^5(330b^7x^7+2520a\,b^6x^6+8316a^2b^5x^5+15400a^3b^4x^4+17325a^4b^3x^3+11880a^5b^2x^2+4620a^6bx+792a^7)}{3960}$	80

input
```
int(x^4*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^7*x^12+7/11*a*b^6*x^11+21/10*a^2*b^5*x^10+35/9*a^3*b^4*x^9+35/8*a^4
*b^3*x^8+3*a^5*b^2*x^7+7/6*a^6*b*x^6+1/5*a^7*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int x^4(a+bx)^7\,dx=\frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

input
```
integrate(x^4*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 +
35/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int x^4(a+bx)^7\,dx = \frac{a^7x^5}{5} + \frac{7a^6bx^6}{6} + 3a^5b^2x^7 + \frac{35a^4b^3x^8}{8} + \frac{35a^3b^4x^9}{9} + \frac{21a^2b^5x^{10}}{10} + \frac{7ab^6x^{11}}{11} + \frac{b^7x^{12}}{12}$$

input
```
integrate(x**4*(b*x+a)**7,x)
```

output
```
a**7*x**5/5 + 7*a**6*b*x**6/6 + 3*a**5*b**2*x**7 + 35*a**4*b**3*x**8/8 + 3
5*a**3*b**4*x**9/9 + 21*a**2*b**5*x**10/10 + 7*a*b**6*x**11/11 + b**7*x**1
2/12
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int x^4(a+bx)^7\,dx = \frac{1}{12}\,b^7x^{12} + \frac{7}{11}\,ab^6x^{11} + \frac{21}{10}\,a^2b^5x^{10} + \frac{35}{9}\,a^3b^4x^9 + \frac{35}{8}\,a^4b^3x^8 + 3\,a^5b^2x^7 + \frac{7}{6}\,a^6bx^6 + \frac{1}{5}\,a^7x^5$$

input
```
integrate(x^4*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 +
35/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int x^4(a+bx)^7\,dx = \frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

input
```
integrate(x^4*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 +
35/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int x^4(a+bx)^7\,dx = \frac{a^7\,x^5}{5}+\frac{7\,a^6\,b\,x^6}{6}+3\,a^5\,b^2\,x^7+\frac{35\,a^4\,b^3\,x^8}{8}+\frac{35\,a^3\,b^4\,x^9}{9}+\frac{21\,a^2\,b^5\,x^{10}}{10}+\frac{7\,a\,b^6\,x^{11}}{11}+\frac{b^7\,x^{12}}{12}$$

input
```
int(x^4*(a + b*x)^7,x)
```

output
```
(a^7*x^5)/5 + (b^7*x^12)/12 + (7*a^6*b*x^6)/6 + (7*a*b^6*x^11)/11 + 3*a^5*
b^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int x^4(a+bx)^7\,dx$$
$$=\frac{x^5(330b^7x^7+2520a\,b^6x^6+8316a^2b^5x^5+15400a^3b^4x^4+17325a^4b^3x^3+11880a^5b^2x^2+4620a^6bx+792a}{3960}$$

input
```
int(x^4*(b*x+a)^7,x)
```

output
```
(x**5*(792*a**7 + 4620*a**6*b*x + 11880*a**5*b**2*x**2 + 17325*a**4*b**3*x
**3 + 15400*a**3*b**4*x**4 + 8316*a**2*b**5*x**5 + 2520*a*b**6*x**6 + 330*
b**7*x**7))/3960
```

3.61 $\int x^3(a+bx)^7\,dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int x^3(a+bx)^7\,dx=-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}$$

output
```
-1/8*a^3*(b*x+a)^8/b^4+1/3*a^2*(b*x+a)^9/b^4-3/10*a*(b*x+a)^10/b^4+1/11*(b
*x+a)^11/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.45

$$\int x^3(a+bx)^7\,dx=\frac{a^7x^4}{4}+\frac{7}{5}a^6bx^5+\frac{7}{2}a^5b^2x^6+5a^4b^3x^7+\frac{35}{8}a^3b^4x^8+\frac{7}{3}a^2b^5x^9+\frac{7}{10}ab^6x^{10}+\frac{b^7x^{11}}{11}$$

input
```
Integrate[x^3*(a + b*x)^7,x]
```

output
```
(a^7*x^4)/4 + (7*a^6*b*x^5)/5 + (7*a^5*b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^
3*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3 + (7*a*b^6*x^10)/10 + (b^7*x^11)/11
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^7\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^3(a+bx)^7}{b^3}+\frac{3a^2(a+bx)^8}{b^3}+\frac{(a+bx)^{10}}{b^3}-\frac{3a(a+bx)^9}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

input
```
Int[x^3*(a + b*x)^7,x]
```

output
```
-1/8*(a^3*(a + b*x)^8)/b^4 + (a^2*(a + b*x)^9)/(3*b^4) - (3*a*(a + b*x)^10
)/(10*b^4) + (a + b*x)^11/(11*b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

method	result	size
gosper	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
default	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
norman	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
risch	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
parallelrisch	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
orering	$\frac{x^4(120b^7x^7+924a\,b^6x^6+3080a^2b^5x^5+5775a^3b^4x^4+6600a^4b^3x^3+4620a^5b^2x^2+1848a^6bx+330a^7)}{1320}$	80

input
```
int(x^3*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^7*x^11+7/10*a*b^6*x^10+7/3*a^2*b^5*x^9+35/8*a^3*b^4*x^8+5*a^4*b^3*x
^7+7/2*a^5*b^2*x^6+7/5*a^6*b*x^5+1/4*a^7*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int x^3(a+bx)^7\,dx=\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

input
```
integrate(x^3*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a
^4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.44

$$\int x^3(a+bx)^7\,dx=\frac{a^7x^4}{4}+\frac{7a^6bx^5}{5}+\frac{7a^5b^2x^6}{2}+5a^4b^3x^7+\frac{35a^3b^4x^8}{8}+\frac{7a^2b^5x^9}{3}+\frac{7ab^6x^{10}}{10}+\frac{b^7x^{11}}{11}$$

input
```
integrate(x**3*(b*x+a)**7,x)
```

output
```
a**7*x**4/4 + 7*a**6*b*x**5/5 + 7*a**5*b**2*x**6/2 + 5*a**4*b**3*x**7 + 35
*a**3*b**4*x**8/8 + 7*a**2*b**5*x**9/3 + 7*a*b**6*x**10/10 + b**7*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int x^3(a+bx)^7\,dx=\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

input
```
integrate(x^3*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a
^4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int x^3(a+bx)^7\,dx = \frac{1}{11}\,b^7x^{11} + \frac{7}{10}\,ab^6x^{10} + \frac{7}{3}\,a^2b^5x^9 + \frac{35}{8}\,a^3b^4x^8 + 5\,a^4b^3x^7 + \frac{7}{2}\,a^5b^2x^6 + \frac{7}{5}\,a^6bx^5 + \frac{1}{4}\,a^7x^4$$

input
```
integrate(x^3*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a
^4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int x^3(a+bx)^7\,dx = \frac{a^7\,x^4}{4} + \frac{7\,a^6\,b\,x^5}{5} + \frac{7\,a^5\,b^2\,x^6}{2} + 5\,a^4\,b^3\,x^7 + \frac{35\,a^3\,b^4\,x^8}{8} + \frac{7\,a^2\,b^5\,x^9}{3} + \frac{7\,a\,b^6\,x^{10}}{10} + \frac{b^7\,x^{11}}{11}$$

input
```
int(x^3*(a + b*x)^7,x)
```

output
```
(a^7*x^4)/4 + (b^7*x^11)/11 + (7*a^6*b*x^5)/5 + (7*a*b^6*x^10)/10 + (7*a^5
*b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int x^3(a+bx)^7\,dx$$
$$=\frac{x^4(120b^7x^7+924a\,b^6x^6+3080a^2b^5x^5+5775a^3b^4x^4+6600a^4b^3x^3+4620a^5b^2x^2+1848a^6bx+330a^7)}{1320}$$

input
```
int(x^3*(b*x+a)^7,x)
```

output
```
(x**4*(330*a**7 + 1848*a**6*b*x + 4620*a**5*b**2*x**2 + 6600*a**4*b**3*x**
3 + 5775*a**3*b**4*x**4 + 3080*a**2*b**5*x**5 + 924*a*b**6*x**6 + 120*b**7
*x**7))/1320
```

3.62 $\int x^2(a+bx)^7\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int x^2(a+bx)^7\,dx=\frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}$$

output
```
1/8*a^2*(b*x+a)^8/b^3-2/9*a*(b*x+a)^9/b^3+1/10*(b*x+a)^10/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int x^2(a+bx)^7\,dx=\frac{a^7x^3}{3}+\frac{7}{4}a^6bx^4+\frac{21}{5}a^5b^2x^5+\frac{35}{6}a^4b^3x^6+5a^3b^4x^7+\frac{21}{8}a^2b^5x^8+\frac{7}{9}ab^6x^9+\frac{b^7x^{10}}{10}$$

input
```
Integrate[x^2*(a + b*x)^7,x]
```

output
```
(a^7*x^3)/3 + (7*a^6*b*x^4)/4 + (21*a^5*b^2*x^5)/5 + (35*a^4*b^3*x^6)/6 +
5*a^3*b^4*x^7 + (21*a^2*b^5*x^8)/8 + (7*a*b^6*x^9)/9 + (b^7*x^10)/10
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^7 \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2(a+bx)^7}{b^2}+\frac{(a+bx)^9}{b^2}-\frac{2a(a+bx)^8}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

input
```
Int[x^2*(a + b*x)^7,x]
```

output
```
(a^2*(a + b*x)^8)/(8*b^3) - (2*a*(a + b*x)^9)/(9*b^3) + (a + b*x)^10/(10*b
^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

method	result	size
gosper	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
default	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
norman	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
risch	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
parallelrisch	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
orering	$\frac{x^3(36b^7x^7+280a\,b^6x^6+945a^2b^5x^5+1800a^3b^4x^4+2100a^4b^3x^3+1512a^5b^2x^2+630a^6bx+120a^7)}{360}$	80

input
```
int(x^2*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^7*x^10+7/9*a*b^6*x^9+21/8*a^2*b^5*x^8+5*a^3*b^4*x^7+35/6*a^4*b^3*x^
6+21/5*a^5*b^2*x^5+7/4*a^6*b*x^4+1/3*a^7*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int x^2(a+bx)^7\,dx=\frac{1}{10}\,b^7x^{10}+\frac{7}{9}\,ab^6x^9+\frac{21}{8}\,a^2b^5x^8+5\,a^3b^4x^7+\frac{35}{6}\,a^4b^3x^6+\frac{21}{5}\,a^5b^2x^5+\frac{7}{4}\,a^6bx^4+\frac{1}{3}\,a^7x^3$$

input
```
integrate(x^2*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^
4*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

$$\int x^2(a+bx)^7\,dx = \frac{a^7x^3}{3} + \frac{7a^6bx^4}{4} + \frac{21a^5b^2x^5}{5} + \frac{35a^4b^3x^6}{6} + 5a^3b^4x^7 + \frac{21a^2b^5x^8}{8} + \frac{7ab^6x^9}{9} + \frac{b^7x^{10}}{10}$$

input
```
integrate(x**2*(b*x+a)**7,x)
```

output
```
a**7*x**3/3 + 7*a**6*b*x**4/4 + 21*a**5*b**2*x**5/5 + 35*a**4*b**3*x**6/6
+ 5*a**3*b**4*x**7 + 21*a**2*b**5*x**8/8 + 7*a*b**6*x**9/9 + b**7*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int x^2(a+bx)^7\,dx = \frac{1}{10}\,b^7x^{10} + \frac{7}{9}\,ab^6x^9 + \frac{21}{8}\,a^2b^5x^8 + 5\,a^3b^4x^7 + \frac{35}{6}\,a^4b^3x^6 + \frac{21}{5}\,a^5b^2x^5 + \frac{7}{4}\,a^6bx^4 + \frac{1}{3}\,a^7x^3$$

input
```
integrate(x^2*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^
4*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int x^2(a+bx)^7\,dx = \frac{1}{10}\,b^7x^{10} + \frac{7}{9}\,ab^6x^9 + \frac{21}{8}\,a^2b^5x^8 + 5\,a^3b^4x^7 + \frac{35}{6}\,a^4b^3x^6 + \frac{21}{5}\,a^5b^2x^5 + \frac{7}{4}\,a^6bx^4 + \frac{1}{3}\,a^7x^3$$

input
```
integrate(x^2*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^
4*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^7\,dx = \frac{(a+b\,x)^8\,(8\,a^2-64\,a\,b\,x+288\,b^2\,x^2)}{2880\,b^3}$$

input
```
int(x^2*(a + b*x)^7,x)
```

output
```
((a + b*x)^8*(8*a^2 + 288*b^2*x^2 - 64*a*b*x))/(2880*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int x^2(a+bx)^7\,dx = \frac{x^3(36b^7x^7+280a\,b^6x^6+945a^2b^5x^5+1800a^3b^4x^4+2100a^4b^3x^3+1512a^5b^2x^2+630a^6bx+120a^7)}{360}$$

input
```
int(x^2*(b*x+a)^7,x)
```

output
```
(x**3*(120*a**7 + 630*a**6*b*x + 1512*a**5*b**2*x**2 + 2100*a**4*b**3*x**3
 + 1800*a**3*b**4*x**4 + 945*a**2*b**5*x**5 + 280*a*b**6*x**6 + 36*b**7*x*
*7))/360
```

3.63 $\int x(a+bx)^7\,dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int x(a+bx)^7\,dx=-\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}$$

output
```
-1/8*a*(b*x+a)^8/b^2+1/9*(b*x+a)^9/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(30) = 60.

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.03

$$\int x(a+bx)^7\,dx=\frac{a^7x^2}{2}+\frac{7}{3}a^6bx^3+\frac{21}{4}a^5b^2x^4+7a^4b^3x^5+\frac{35}{6}a^3b^4x^6+3a^2b^5x^7+\frac{7}{8}ab^6x^8+\frac{b^7x^9}{9}$$

input
```
Integrate[x*(a + b*x)^7,x]
```

output
```
(a^7*x^2)/2 + (7*a^6*b*x^3)/3 + (21*a^5*b^2*x^4)/4 + 7*a^4*b^3*x^5 + (35*a
^3*b^4*x^6)/6 + 3*a^2*b^5*x^7 + (7*a*b^6*x^8)/8 + (b^7*x^9)/9
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^7\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+bx)^8}{b}-\frac{a(a+bx)^7}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

input
```
Int[x*(a + b*x)^7,x]
```

output
```
-1/8*(a*(a + b*x)^8)/b^2 + (a + b*x)^9/(9*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.67

method	result	size
gosper	$\frac{1}{9}b^7x^9+\frac{7}{8}a\,b^6x^8+3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6b\,x^3+\frac{1}{2}a^7x^2$	80
default	$\frac{1}{9}b^7x^9+\frac{7}{8}a\,b^6x^8+3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6b\,x^3+\frac{1}{2}a^7x^2$	80
norman	$\frac{1}{9}b^7x^9+\frac{7}{8}a\,b^6x^8+3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6b\,x^3+\frac{1}{2}a^7x^2$	80
risch	$\frac{1}{9}b^7x^9+\frac{7}{8}a\,b^6x^8+3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6b\,x^3+\frac{1}{2}a^7x^2$	80
parallelrisch	$\frac{1}{9}b^7x^9+\frac{7}{8}a\,b^6x^8+3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6b\,x^3+\frac{1}{2}a^7x^2$	80
orering	$\frac{x^2(8b^7x^7+63a\,b^6x^6+216a^2b^5x^5+420a^3b^4x^4+504a^4b^3x^3+378a^5b^2x^2+168a^6bx+36a^7)}{72}$	80

input
```
int(x*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^7*x^9+7/8*a*b^6*x^8+3*a^2*b^5*x^7+35/6*a^3*b^4*x^6+7*a^4*b^3*x^5+21/
4*a^5*b^2*x^4+7/3*a^6*b*x^3+1/2*a^7*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.63

$$\int x(a+bx)^7\,dx = \frac{1}{9}\,b^7x^9+\frac{7}{8}\,ab^6x^8+3\,a^2b^5x^7+\frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5+\frac{21}{4}\,a^5b^2x^4+\frac{7}{3}\,a^6bx^3+\frac{1}{2}\,a^7x^2$$

input
```
integrate(x*(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3
*x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.00

$$\int x(a+bx)^7\,dx = \frac{a^7x^2}{2} + \frac{7a^6bx^3}{3} + \frac{21a^5b^2x^4}{4} + 7a^4b^3x^5 + \frac{35a^3b^4x^6}{6} + 3a^2b^5x^7 + \frac{7ab^6x^8}{8} + \frac{b^7x^9}{9}$$

input
```
integrate(x*(b*x+a)**7,x)
```

output
```
a**7*x**2/2 + 7*a**6*b*x**3/3 + 21*a**5*b**2*x**4/4 + 7*a**4*b**3*x**5 + 3
5*a**3*b**4*x**6/6 + 3*a**2*b**5*x**7 + 7*a*b**6*x**8/8 + b**7*x**9/9
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(26) = 52.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.63

$$\int x(a+bx)^7\,dx = \frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

input
```
integrate(x*(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3
*x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.63

$$\int x(a+bx)^7\,dx = \frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

input
```
integrate(x*(b*x+a)^7,x, algorithm="giac")
```

output
```
1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3
*x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int x(a+bx)^7\,dx = -\frac{2\left(\frac{a\,(a+b\,x)^8}{16}-\frac{(a+b\,x)^9}{18}\right)}{b^2}$$

input
```
int(x*(a + b*x)^7,x)
```

output
```
-(2*((a*(a + b*x)^8)/16 - (a + b*x)^9/18))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.63

$$\int x(a+bx)^7\,dx = \frac{x^2(8b^7x^7+63a\,b^6x^6+216a^2b^5x^5+420a^3b^4x^4+504a^4b^3x^3+378a^5b^2x^2+168a^6bx+36a^7)}{72}$$

input
```
int(x*(b*x+a)^7,x)
```

output
```
(x**2*(36*a**7 + 168*a**6*b*x + 378*a**5*b**2*x**2 + 504*a**4*b**3*x**3 +
420*a**3*b**4*x**4 + 216*a**2*b**5*x**5 + 63*a*b**6*x**6 + 8*b**7*x**7))/7
2
```

3.64 $\int (a+bx)^7 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (a+bx)^7 \, dx = \frac{(a+bx)^8}{8b}$$

output
```
1/8*(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+bx)^7 \, dx = \frac{(a+bx)^8}{8b}$$

input
```
Integrate[(a + b*x)^7,x]
```

output
```
(a + b*x)^8/(8*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^7 \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^8}{8b}$$

input
```
Int[(a + b*x)^7,x]
```

output
```
(a + b*x)^8/(8*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^8}{8b}$	13
gosper	$\frac{1}{8}b^7x^8+a\,b^6x^7+\frac{7}{2}a^2b^5x^6+7a^3b^4x^5+\frac{35}{4}a^4b^3x^4+7a^5b^2x^3+\frac{7}{2}a^6b\,x^2+a^7x$	76
norman	$\frac{1}{8}b^7x^8+a\,b^6x^7+\frac{7}{2}a^2b^5x^6+7a^3b^4x^5+\frac{35}{4}a^4b^3x^4+7a^5b^2x^3+\frac{7}{2}a^6b\,x^2+a^7x$	76
parallelrisch	$\frac{1}{8}b^7x^8+a\,b^6x^7+\frac{7}{2}a^2b^5x^6+7a^3b^4x^5+\frac{35}{4}a^4b^3x^4+7a^5b^2x^3+\frac{7}{2}a^6b\,x^2+a^7x$	76
orering	$\frac{x\left(b^7x^7+8a\,b^6x^6+28a^2b^5x^5+56a^3b^4x^4+70a^4b^3x^3+56a^5b^2x^2+28a^6bx+8a^7\right)}{8}$	77
risch	$\frac{b^7x^8}{8}+a\,b^6x^7+\frac{7a^2b^5x^6}{2}+7a^3b^4x^5+\frac{35a^4b^3x^4}{4}+7a^5b^2x^3+\frac{7a^6b\,x^2}{2}+a^7x+\frac{a^8}{8b}$	84

input
```
int((b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*(b*x+a)^8/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 5.36

$$\int (a+bx)^7\,dx = \frac{1}{8}\,b^7x^8+ab^6x^7+\frac{7}{2}\,a^2b^5x^6+7\,a^3b^4x^5+\frac{35}{4}\,a^4b^3x^4+7\,a^5b^2x^3+\frac{7}{2}\,a^6bx^2+a^7x$$

input
```
integrate((b*x+a)^7,x, algorithm="fricas")
```

output
```
1/8*b^7*x^8 + a*b^6*x^7 + 7/2*a^2*b^5*x^6 + 7*a^3*b^4*x^5 + 35/4*a^4*b^3*x
^4 + 7*a^5*b^2*x^3 + 7/2*a^6*b*x^2 + a^7*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 5.93

$$\int (a+bx)^7\,dx = a^7x + \frac{7a^6bx^2}{2} + 7a^5b^2x^3 + \frac{35a^4b^3x^4}{4} + 7a^3b^4x^5 + \frac{7a^2b^5x^6}{2} + ab^6x^7 + \frac{b^7x^8}{8}$$

input
```
integrate((b*x+a)**7,x)
```

output
```
a**7*x + 7*a**6*b*x**2/2 + 7*a**5*b**2*x**3 + 35*a**4*b**3*x**4/4 + 7*a**3
*b**4*x**5 + 7*a**2*b**5*x**6/2 + a*b**6*x**7 + b**7*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)^7\,dx = \frac{(bx+a)^8}{8\,b}$$

input
```
integrate((b*x+a)^7,x, algorithm="maxima")
```

output
```
1/8*(b*x + a)^8/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)^7\,dx = \frac{(bx+a)^8}{8\,b}$$

input
```
integrate((b*x+a)^7,x, algorithm="giac")
```

output
```
1/8*(b*x + a)^8/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 5.36

$$\int (a+bx)^7\,dx = a^7\,x+\frac{7\,a^6\,b\,x^2}{2}+7\,a^5\,b^2\,x^3+\frac{35\,a^4\,b^3\,x^4}{4}+7\,a^3\,b^4\,x^5+\frac{7\,a^2\,b^5\,x^6}{2}+a\,b^6\,x^7+\frac{b^7\,x^8}{8}$$

input
```
int((a + b*x)^7,x)
```

output
```
a^7*x + (b^7*x^8)/8 + (7*a^6*b*x^2)/2 + a*b^6*x^7 + 7*a^5*b^2*x^3 + (35*a^
4*b^3*x^4)/4 + 7*a^3*b^4*x^5 + (7*a^2*b^5*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 5.43

$$\int (a+bx)^7\,dx = \frac{x(b^7x^7+8a\,b^6x^6+28a^2b^5x^5+56a^3b^4x^4+70a^4b^3x^3+56a^5b^2x^2+28a^6bx+8a^7)}{8}$$

input
```
int((b*x+a)^7,x)
```

output
```
(x*(8*a**7 + 28*a**6*b*x + 56*a**5*b**2*x**2 + 70*a**4*b**3*x**3 + 56*a**3
*b**4*x**4 + 28*a**2*b**5*x**5 + 8*a*b**6*x**6 + b**7*x**7))/8
```

3.65 $\int \frac{(a+bx)^7}{x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 87

$$\int \frac{(a+bx)^7}{x}\,dx = 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7} + a^7\log(x)$$

output
```
7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*
x^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x}\,dx = 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7} + a^7\log(x)$$

input
```
Integrate[(a + b*x)^7/x,x]
```

output
```
7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
 (21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^7}{x}+7a^6b+21a^5b^2x+35a^4b^3x^2+35a^3b^4x^3+21a^2b^5x^4+7ab^6x^5+b^7x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^7\log(x)+7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}$$

input
```
Int[(a + b*x)^7/x,x]
```

output
```
7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
 (21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

method	result	size
default	$7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}+a^7\ln\left(x\right)$	76
norman	$7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}+a^7\ln\left(x\right)$	76
risch	$7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}+a^7\ln\left(x\right)$	76
parallelrisch	$7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}+a^7\ln\left(x\right)$	76

input
```
int((b*x+a)^7/x,x,method=_RETURNVERBOSE)
```

output
```
7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*
x^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^7}{x}\,dx=\frac{1}{7}\,b^7x^7+\frac{7}{6}\,ab^6x^6+\frac{21}{5}\,a^2b^5x^5+\frac{35}{4}\,a^3b^4x^4+\frac{35}{3}\,a^4b^3x^3+\frac{21}{2}\,a^5b^2x^2+7\,a^6bx+a^7\log\left(x\right)$$

input
```
integrate((b*x+a)^7/x,x, algorithm="fricas")
```

output
```
1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a
^4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^7}{x}\,dx = a^7\log(x) + 7a^6bx + \frac{21a^5b^2x^2}{2} + \frac{35a^4b^3x^3}{3} + \frac{35a^3b^4x^4}{4} + \frac{21a^2b^5x^5}{5} + \frac{7ab^6x^6}{6} + \frac{b^7x^7}{7}$$

input
```
integrate((b*x+a)**7/x,x)
```

output
```
a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**3/3 + 35*
a**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^7}{x}\,dx = \frac{1}{7}\,b^7x^7 + \frac{7}{6}\,ab^6x^6 + \frac{21}{5}\,a^2b^5x^5 + \frac{35}{4}\,a^3b^4x^4 + \frac{35}{3}\,a^4b^3x^3 + \frac{21}{2}\,a^5b^2x^2 + 7\,a^6bx + a^7\log(x)$$

input
```
integrate((b*x+a)^7/x,x, algorithm="maxima")
```

output
```
1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a
^4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^7}{x}\,dx = \frac{1}{7}\,b^7x^7 + \frac{7}{6}\,ab^6x^6 + \frac{21}{5}\,a^2b^5x^5 + \frac{35}{4}\,a^3b^4x^4 + \frac{35}{3}\,a^4b^3x^3 + \frac{21}{2}\,a^5b^2x^2 + 7\,a^6bx + a^7\log(|x|)$$

input
```
integrate((b*x+a)^7/x,x, algorithm="giac")
```

output
```
1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a
^4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^7}{x}\,dx = a^7\,\ln(x) + \frac{b^7\,x^7}{7} + \frac{7\,a\,b^6\,x^6}{6} + \frac{21\,a^5\,b^2\,x^2}{2} + \frac{35\,a^4\,b^3\,x^3}{3} + \frac{35\,a^3\,b^4\,x^4}{4} + \frac{21\,a^2\,b^5\,x^5}{5} + 7\,a^6\,b\,x$$

input
```
int((a + b*x)^7/x,x)
```

output
```
a^7*log(x) + (b^7*x^7)/7 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/2 + (35*a^4*
b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 + (21*a^2*b^5*x^5)/5 + 7*a^6*b*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^7}{x}\,dx = \log(x)\,a^7 + 7a^6bx + \frac{21a^5b^2x^2}{2} + \frac{35a^4b^3x^3}{3} + \frac{35a^3b^4x^4}{4} + \frac{21a^2b^5x^5}{5} + \frac{7a\,b^6x^6}{6} + \frac{b^7x^7}{7}$$

input
```
int((b*x+a)^7/x,x)
```

output
```
(420*log(x)*a**7 + 2940*a**6*b*x + 4410*a**5*b**2*x**2 + 4900*a**4*b**3*x*
*3 + 3675*a**3*b**4*x**4 + 1764*a**2*b**5*x**5 + 490*a*b**6*x**6 + 60*b**7
*x**7)/420
```

3.66 $\int \frac{(a+bx)^7}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 86

$$\int \frac{(a+bx)^7}{x^2} dx = -\frac{a^7}{x} + 21a^5b^2x + \frac{35}{2}a^4b^3x^2 + \frac{35}{3}a^3b^4x^3 + \frac{21}{4}a^2b^5x^4 + \frac{7}{5}ab^6x^5 + \frac{b^7x^6}{6} + 7a^6b\log(x)$$

output
```
-a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5
*a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^2} dx = -\frac{a^7}{x} + 21a^5b^2x + \frac{35}{2}a^4b^3x^2 + \frac{35}{3}a^3b^4x^3 + \frac{21}{4}a^2b^5x^4 + \frac{7}{5}ab^6x^5 + \frac{b^7x^6}{6} + 7a^6b\log(x)$$

input
```
Integrate[(a + b*x)^7/x^2,x]
```

output
```
-(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^
2*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^2}\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^7}{x^2}+\frac{7a^6b}{x}+21a^5b^2+35a^4b^3x+35a^3b^4x^2+21a^2b^5x^3+7ab^6x^4+b^7x^5\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^7}{x}+7a^6b\log(x)+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

input
```
Int[(a + b*x)^7/x^2,x]
```

output
```
-(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^
2*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```


Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^7}{x}+21a^5b^2x+\frac{35a^4b^3x^2}{2}+\frac{35a^3b^4x^3}{3}+\frac{21a^2b^5x^4}{4}+\frac{7a\,b^6x^5}{5}+\frac{b^7x^6}{6}+7a^6b\ln\left(x\right)$	77
risch	$-\frac{a^7}{x}+21a^5b^2x+\frac{35a^4b^3x^2}{2}+\frac{35a^3b^4x^3}{3}+\frac{21a^2b^5x^4}{4}+\frac{7a\,b^6x^5}{5}+\frac{b^7x^6}{6}+7a^6b\ln\left(x\right)$	77
norman	$\frac{-a^7+\frac{1}{6}b^7x^7+\frac{7}{5}a\,b^6x^6+\frac{21}{4}a^2b^5x^5+\frac{35}{3}a^3b^4x^4+\frac{35}{2}a^4b^3x^3+21a^5b^2x^2}{x}+7a^6b\ln\left(x\right)$	81
parallelrisch	$\frac{10b^7x^7+84a\,b^6x^6+315a^2b^5x^5+700a^3b^4x^4+1050a^4b^3x^3+420a^6b\ln(x)x+1260a^5b^2x^2-60a^7}{60x}$	82

input
```
int((b*x+a)^7/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5
*a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^7}{x^2}\,dx = \frac{10\,b^7x^7+84\,ab^6x^6+315\,a^2b^5x^5+700\,a^3b^4x^4+1050\,a^4b^3x^3+1260\,a^5b^2x^2+420\,a^6bx\log\left(x\right)-60\,a^7}{60\,x}$$

input
```
integrate((b*x+a)^7/x^2,x, algorithm="fricas")
```

output
```
1/60*(10*b^7*x^7 + 84*a*b^6*x^6 + 315*a^2*b^5*x^5 + 700*a^3*b^4*x^4 + 1050
*a^4*b^3*x^3 + 1260*a^5*b^2*x^2 + 420*a^6*b*x*log(x) - 60*a^7)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^7}{x^2}\,dx = -\frac{a^7}{x} + 7a^6b\log(x) + 21a^5b^2x + \frac{35a^4b^3x^2}{2} + \frac{35a^3b^4x^3}{3} + \frac{21a^2b^5x^4}{4} + \frac{7ab^6x^5}{5} + \frac{b^7x^6}{6}$$

input
```
integrate((b*x+a)**7/x**2,x)
```

output
```
-a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*a**3
*b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^7}{x^2}\,dx = \frac{1}{6}\,b^7x^6 + \frac{7}{5}\,ab^6x^5 + \frac{21}{4}\,a^2b^5x^4 + \frac{35}{3}\,a^3b^4x^3 + \frac{35}{2}\,a^4b^3x^2 + 21\,a^5b^2x + 7\,a^6b\log(x) - \frac{a^7}{x}$$

input
```
integrate((b*x+a)^7/x^2,x, algorithm="maxima")
```

output
```
1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a
^4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(x) - a^7/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^7}{x^2}\,dx = \frac{1}{6}\,b^7x^6 + \frac{7}{5}\,ab^6x^5 + \frac{21}{4}\,a^2b^5x^4 + \frac{35}{3}\,a^3b^4x^3 + \frac{35}{2}\,a^4b^3x^2 + 21\,a^5b^2x + 7\,a^6b\log\left(|x|\right) - \frac{a^7}{x}$$

input
```
integrate((b*x+a)^7/x^2,x, algorithm="giac")
```

output
```
1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a
^4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(abs(x)) - a^7/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^7}{x^2}\,dx = \frac{b^7\,x^6}{6} - \frac{a^7}{x} + 21\,a^5\,b^2\,x + \frac{7\,a\,b^6\,x^5}{5} + 7\,a^6\,b\,\ln\left(x\right) + \frac{35\,a^4\,b^3\,x^2}{2} + \frac{35\,a^3\,b^4\,x^3}{3} + \frac{21\,a^2\,b^5\,x^4}{4}$$

input
```
int((a + b*x)^7/x^2,x)
```

output
```
(b^7*x^6)/6 - a^7/x + 21*a^5*b^2*x + (7*a*b^6*x^5)/5 + 7*a^6*b*log(x) + (3
5*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2*b^5*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^7}{x^2}\,dx$$
$$= \frac{420\log(x)\,a^6bx - 60a^7 + 1260a^5b^2x^2 + 1050a^4b^3x^3 + 700a^3b^4x^4 + 315a^2b^5x^5 + 84a\,b^6x^6 + 10b^7x^7}{60x}$$

input
```
int((b*x+a)^7/x^2,x)
```

output
```
(420*log(x)*a**6*b*x - 60*a**7 + 1260*a**5*b**2*x**2 + 1050*a**4*b**3*x**3
 + 700*a**3*b**4*x**4 + 315*a**2*b**5*x**5 + 84*a*b**6*x**6 + 10*b**7*x**7
)/(60*x)
```

3.67 $\int \frac{(a+bx)^7}{x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int \frac{(a+bx)^7}{x^3}\,dx = -\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)$$

output
```
-1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b
^6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^3}\,dx = -\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)$$

input
```
Integrate[(a + b*x)^7/x^3,x]
```

output
```
-1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5
*x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^3} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^7}{x^3}+\frac{7a^6b}{x^2}+\frac{21a^5b^2}{x}+35a^4b^3+35a^3b^4x+21a^2b^5x^2+7ab^6x^3+b^7x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+21a^5b^2\log(x)+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

input
```
Int[(a + b*x)^7/x^3,x]
```

output

```
-1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5
*x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```


Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^{7}}{2x^{2}}-\frac{7a^{6}b}{x}+35a^{4}b^{3}x+\frac{35a^{3}b^{4}x^{2}}{2}+7a^{2}b^{5}x^{3}+\frac{7a\,b^{6}x^{4}}{4}+\frac{b^{7}x^{5}}{5}+21a^{5}b^{2}\ln\left(x\right)$	77
risch	$\frac{b^{7}x^{5}}{5}+\frac{7a\,b^{6}x^{4}}{4}+7a^{2}b^{5}x^{3}+\frac{35a^{3}b^{4}x^{2}}{2}+35a^{4}b^{3}x+\frac{-7a^{6}bx-\frac{1}{2}a^{7}}{x^{2}}+21a^{5}b^{2}\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{2}a^{7}+\frac{1}{5}b^{7}x^{7}+\frac{7}{4}a\,b^{6}x^{6}+7a^{2}b^{5}x^{5}+\frac{35}{2}a^{3}b^{4}x^{4}+35a^{4}b^{3}x^{3}-7a^{6}bx}{x^{2}}+21a^{5}b^{2}\ln\left(x\right)$	79
parallelrisch	$\frac{4b^{7}x^{7}+35a\,b^{6}x^{6}+140a^{2}b^{5}x^{5}+350a^{3}b^{4}x^{4}+420a^{5}b^{2}\ln\left(x\right)x^{2}+700a^{4}b^{3}x^{3}-140a^{6}bx-10a^{7}}{20x^{2}}$	82

input
```
int((b*x+a)^7/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b
^6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int\frac{(a+bx)^{7}}{x^{3}}\,dx$$
$$=\frac{4\,b^{7}x^{7}+35\,ab^{6}x^{6}+140\,a^{2}b^{5}x^{5}+350\,a^{3}b^{4}x^{4}+700\,a^{4}b^{3}x^{3}+420\,a^{5}b^{2}x^{2}\log\left(x\right)-140\,a^{6}bx-10\,a^{7}}{20\,x^{2}}$$

input
```
integrate((b*x+a)^7/x^3,x, algorithm="fricas")
```

output
```
1/20*(4*b^7*x^7 + 35*a*b^6*x^6 + 140*a^2*b^5*x^5 + 350*a^3*b^4*x^4 + 700*a
^4*b^3*x^3 + 420*a^5*b^2*x^2*log(x) - 140*a^6*b*x - 10*a^7)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^7}{x^3}\,dx = 21a^5b^2\log(x) + 35a^4b^3x + \frac{35a^3b^4x^2}{2} + 7a^2b^5x^3 + \frac{7ab^6x^4}{4} + \frac{b^7x^5}{5} + \frac{-a^7 - 14a^6bx}{2x^2}$$

input
```
integrate((b*x+a)**7/x**3,x)
```

output
```
21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a**3*b**4*x**2/2 + 7*a**2*b**5*x
**3 + 7*a*b**6*x**4/4 + b**7*x**5/5 + (-a**7 - 14*a**6*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^7}{x^3}\,dx = \frac{1}{5}\,b^7x^5 + \frac{7}{4}\,ab^6x^4 + 7\,a^2b^5x^3 + \frac{35}{2}\,a^3b^4x^2 + 35\,a^4b^3x + 21\,a^5b^2\log(x) - \frac{14\,a^6bx + a^7}{2\,x^2}$$

input
```
integrate((b*x+a)^7/x^3,x, algorithm="maxima")
```

output
```
1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^
3*x + 21*a^5*b^2*log(x) - 1/2*(14*a^6*b*x + a^7)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^7}{x^3}\,dx = \frac{1}{5}\,b^7x^5 + \frac{7}{4}\,ab^6x^4 + 7\,a^2b^5x^3 + \frac{35}{2}\,a^3b^4x^2 + 35\,a^4b^3x + 21\,a^5b^2\log\left(|x|\right) - \frac{14\,a^6bx + a^7}{2\,x^2}$$

input
```
integrate((b*x+a)^7/x^3,x, algorithm="giac")
```

output
```
1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^
3*x + 21*a^5*b^2*log(abs(x)) - 1/2*(14*a^6*b*x + a^7)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^7}{x^3}\,dx = \frac{b^7\,x^5}{5} - \frac{\frac{a^7}{2} + 7\,b\,x\,a^6}{x^2} + 35\,a^4\,b^3\,x + \frac{7\,a\,b^6\,x^4}{4} + \frac{35\,a^3\,b^4\,x^2}{2} + 7\,a^2\,b^5\,x^3 + 21\,a^5\,b^2\,\ln\left(x\right)$$

input
```
int((a + b*x)^7/x^3,x)
```

output
```
(b^7*x^5)/5 - (a^7/2 + 7*a^6*b*x)/x^2 + 35*a^4*b^3*x + (7*a*b^6*x^4)/4 + (
35*a^3*b^4*x^2)/2 + 7*a^2*b^5*x^3 + 21*a^5*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^7}{x^3}\,dx$$
$$=\frac{420\log(x)\,a^5b^2x^2-10a^7-140a^6bx+700a^4b^3x^3+350a^3b^4x^4+140a^2b^5x^5+35a\,b^6x^6+4b^7x^7}{20x^2}$$

input
```
int((b*x+a)^7/x^3,x)
```

output
```
(420*log(x)*a**5*b**2*x**2 - 10*a**7 - 140*a**6*b*x + 700*a**4*b**3*x**3 +
 350*a**3*b**4*x**4 + 140*a**2*b**5*x**5 + 35*a*b**6*x**6 + 4*b**7*x**7)/(
20*x**2)
```

3.68 $\int \frac{(a+bx)^7}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 86

$$\int \frac{(a+bx)^7}{x^4} dx = -\frac{a^7}{3x^3} - \frac{7a^6b}{2x^2} - \frac{21a^5b^2}{x} + 35a^3b^4x + \frac{21}{2}a^2b^5x^2 + \frac{7}{3}ab^6x^3 + \frac{b^7x^4}{4} + 35a^4b^3\log(x)$$

output
```
-1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*
a*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^4} dx = -\frac{a^7}{3x^3} - \frac{7a^6b}{2x^2} - \frac{21a^5b^2}{x} + 35a^3b^4x + \frac{21}{2}a^2b^5x^2 + \frac{7}{3}ab^6x^3 + \frac{b^7x^4}{4} + 35a^4b^3\log(x)$$

input
```
Integrate[(a + b*x)^7/x^4,x]
```

output
```
-1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2
*b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^4} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^7}{x^4}+\frac{7a^6b}{x^3}+\frac{21a^5b^2}{x^2}+\frac{35a^4b^3}{x}+35a^3b^4+21a^2b^5x+7ab^6x^2+b^7x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^4b^3\log(x)+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

input
```
Int[(a + b*x)^7/x^4,x]
```

output

```
-1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2
*b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```


Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{7}}{3x^{3}}-\frac{7a^{6}b}{2x^{2}}-\frac{21a^{5}b^{2}}{x}+35a^{3}b^{4}x+\frac{21a^{2}b^{5}x^{2}}{2}+\frac{7a\,b^{6}x^{3}}{3}+\frac{b^{7}x^{4}}{4}+35a^{4}b^{3}\ln\left(x\right)$	77
risch	$\frac{b^{7}x^{4}}{4}+\frac{7a\,b^{6}x^{3}}{3}+\frac{21a^{2}b^{5}x^{2}}{2}+35a^{3}b^{4}x+\frac{-21a^{5}b^{2}x^{2}-\frac{7}{2}a^{6}bx-\frac{1}{3}a^{7}}{x^{3}}+35a^{4}b^{3}\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{3}a^{7}+\frac{1}{4}b^{7}x^{7}+\frac{7}{3}a\,b^{6}x^{6}+\frac{21}{2}a^{2}b^{5}x^{5}+35a^{3}b^{4}x^{4}-21a^{5}b^{2}x^{2}-\frac{7}{2}a^{6}bx}{x^{3}}+35a^{4}b^{3}\ln\left(x\right)$	79
parallelrisch	$\frac{3b^{7}x^{7}+28a\,b^{6}x^{6}+126a^{2}b^{5}x^{5}+420a^{4}b^{3}\ln\left(x\right)x^{3}+420a^{3}b^{4}x^{4}-252a^{5}b^{2}x^{2}-42a^{6}bx-4a^{7}}{12x^{3}}$	82

input
```
int((b*x+a)^7/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*
a*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^7}{x^4}\,dx = \frac{3\,b^7x^7+28\,ab^6x^6+126\,a^2b^5x^5+420\,a^3b^4x^4+420\,a^4b^3x^3\log\left(x\right)-252\,a^5b^2x^2-42\,a^6bx-4\,a^7}{12\,x^3}$$

input
```
integrate((b*x+a)^7/x^4,x, algorithm="fricas")
```

output
```
1/12*(3*b^7*x^7 + 28*a*b^6*x^6 + 126*a^2*b^5*x^5 + 420*a^3*b^4*x^4 + 420*a
^4*b^3*x^3*log(x) - 252*a^5*b^2*x^2 - 42*a^6*b*x - 4*a^7)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^7}{x^4}\,dx = 35a^4b^3\log{(x)} + 35a^3b^4x + \frac{21a^2b^5x^2}{2} + \frac{7ab^6x^3}{3} + \frac{b^7x^4}{4} + \frac{-2a^7 - 21a^6bx - 126a^5b^2x^2}{6x^3}$$

input
```
integrate((b*x+a)**7/x**4,x)
```

output
```
35*a**4*b**3*log(x) + 35*a**3*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3
/3 + b**7*x**4/4 + (-2*a**7 - 21*a**6*b*x - 126*a**5*b**2*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^7}{x^4}\,dx = \frac{1}{4}\,b^7x^4 + \frac{7}{3}\,ab^6x^3 + \frac{21}{2}\,a^2b^5x^2 + 35\,a^3b^4x + 35\,a^4b^3\log{(x)} - \frac{126\,a^5b^2x^2 + 21\,a^6bx + 2\,a^7}{6\,x^3}$$

input
```
integrate((b*x+a)^7/x^4,x, algorithm="maxima")
```

output
```
1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3
*log(x) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^7}{x^4}\,dx = \frac{1}{4}\,b^7x^4 + \frac{7}{3}\,ab^6x^3 + \frac{21}{2}\,a^2b^5x^2 + 35\,a^3b^4x + 35\,a^4b^3\log\left(|x|\right) - \frac{126\,a^5b^2x^2 + 21\,a^6bx + 2\,a^7}{6\,x^3}$$

input
```
integrate((b*x+a)^7/x^4,x, algorithm="giac")
```

output
```
1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3
*log(abs(x)) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^7}{x^4}\,dx = \frac{b^7\,x^4}{4} - \frac{\frac{a^7}{3} + \frac{7\,a^6\,b\,x}{2} + 21\,a^5\,b^2\,x^2}{x^3} + 35\,a^3\,b^4\,x + \frac{7\,a\,b^6\,x^3}{3} + \frac{21\,a^2\,b^5\,x^2}{2} + 35\,a^4\,b^3\,\ln\left(x\right)$$

input
```
int((a + b*x)^7/x^4,x)
```

output
```
(b^7*x^4)/4 - (a^7/3 + 21*a^5*b^2*x^2 + (7*a^6*b*x)/2)/x^3 + 35*a^3*b^4*x
+ (7*a*b^6*x^3)/3 + (21*a^2*b^5*x^2)/2 + 35*a^4*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^7}{x^4}\,dx$$
$$= \frac{420\log(x)\,a^4b^3x^3 - 4a^7 - 42a^6bx - 252a^5b^2x^2 + 420a^3b^4x^4 + 126a^2b^5x^5 + 28a\,b^6x^6 + 3b^7x^7}{12x^3}$$

input
```
int((b*x+a)^7/x^4,x)
```

output
```
(420*log(x)*a**4*b**3*x**3 - 4*a**7 - 42*a**6*b*x - 252*a**5*b**2*x**2 + 4
20*a**3*b**4*x**4 + 126*a**2*b**5*x**5 + 28*a*b**6*x**6 + 3*b**7*x**7)/(12
*x**3)
```

3.69 $\int \frac{(a+bx)^7}{x^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 86

$$\int \frac{(a+bx)^7}{x^5}\,dx = -\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)$$

output
```
-1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*
a*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^5}\,dx = -\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)$$

input
```
Integrate[(a + b*x)^7/x^5,x]
```

output
```
-1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
 21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^5} \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^7}{x^5}+\frac{7a^6b}{x^4}+\frac{21a^5b^2}{x^3}+\frac{35a^4b^3}{x^2}+\frac{35a^3b^4}{x}+21a^2b^5+7ab^6x+b^7x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+35a^3b^4\log(x)+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

input
```
Int[(a + b*x)^7/x^5,x]
```

output

```
-1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
 21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```


Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{7}}{4x^{4}}-\frac{7a^{6}b}{3x^{3}}-\frac{21a^{5}b^{2}}{2x^{2}}-\frac{35a^{4}b^{3}}{x}+21a^{2}b^{5}x+\frac{7a\,b^{6}x^{2}}{2}+\frac{b^{7}x^{3}}{3}+35a^{3}b^{4}\ln\left(x\right)$	77
risch	$\frac{b^{7}x^{3}}{3}+\frac{7a\,b^{6}x^{2}}{2}+21a^{2}b^{5}x+\frac{-35a^{4}b^{3}x^{3}-\frac{21}{2}a^{5}b^{2}x^{2}-\frac{7}{3}a^{6}bx-\frac{1}{4}a^{7}}{x^{4}}+35a^{3}b^{4}\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{4}a^{7}+\frac{1}{3}b^{7}x^{7}+\frac{7}{2}a\,b^{6}x^{6}+21a^{2}b^{5}x^{5}-35a^{4}b^{3}x^{3}-\frac{21}{2}a^{5}b^{2}x^{2}-\frac{7}{3}a^{6}bx}{x^{4}}+35a^{3}b^{4}\ln\left(x\right)$	79
parallelrisch	$\frac{4b^{7}x^{7}+42a\,b^{6}x^{6}+420a^{3}b^{4}\ln\left(x\right)x^{4}+252a^{2}b^{5}x^{5}-420a^{4}b^{3}x^{3}-126a^{5}b^{2}x^{2}-28a^{6}bx-3a^{7}}{12x^{4}}$	82

input
```
int((b*x+a)^7/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*
a*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^7}{x^5}\,dx = \frac{4\,b^7x^7+42\,ab^6x^6+252\,a^2b^5x^5+420\,a^3b^4x^4\log\left(x\right)-420\,a^4b^3x^3-126\,a^5b^2x^2-28\,a^6bx-3\,a^7}{12\,x^4}$$

input
```
integrate((b*x+a)^7/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*b^7*x^7 + 42*a*b^6*x^6 + 252*a^2*b^5*x^5 + 420*a^3*b^4*x^4*log(x)
- 420*a^4*b^3*x^3 - 126*a^5*b^2*x^2 - 28*a^6*b*x - 3*a^7)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^7}{x^5}\,dx = 35a^3b^4\log(x) + 21a^2b^5x + \frac{7ab^6x^2}{2} + \frac{b^7x^3}{3} + \frac{-3a^7 - 28a^6bx - 126a^5b^2x^2 - 420a^4b^3x^3}{12x^4}$$

input
```
integrate((b*x+a)**7/x**5,x)
```

output
```
35*a**3*b**4*log(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3 + (-3
*a**7 - 28*a**6*b*x - 126*a**5*b**2*x**2 - 420*a**4*b**3*x**3)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^7}{x^5}\,dx = \frac{1}{3}\,b^7x^3 + \frac{7}{2}\,ab^6x^2 + 21\,a^2b^5x + 35\,a^3b^4\log(x) - \frac{420\,a^4b^3x^3 + 126\,a^5b^2x^2 + 28\,a^6bx + 3\,a^7}{12\,x^4}$$

input
```
integrate((b*x+a)^7/x^5,x, algorithm="maxima")
```

output
```
1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(x) - 1/12*(420
*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^7}{x^5}\,dx = \frac{1}{3}\,b^7x^3 + \frac{7}{2}\,ab^6x^2 + 21\,a^2b^5x + 35\,a^3b^4\log\left(|x|\right) - \frac{420\,a^4b^3x^3 + 126\,a^5b^2x^2 + 28\,a^6bx + 3\,a^7}{12\,x^4}$$

input
```
integrate((b*x+a)^7/x^5,x, algorithm="giac")
```

output
```
1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(abs(x)) - 1/12
*(420*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^7}{x^5}\,dx = \frac{b^7\,x^3}{3} - \frac{\frac{a^7}{4} + \frac{7\,a^6\,b\,x}{3} + \frac{21\,a^5\,b^2\,x^2}{2} + 35\,a^4\,b^3\,x^3}{x^4} + 21\,a^2\,b^5\,x + \frac{7\,a\,b^6\,x^2}{2} + 35\,a^3\,b^4\,\ln\left(x\right)$$

input
```
int((a + b*x)^7/x^5,x)
```

output
```
(b^7*x^3)/3 - (a^7/4 + (21*a^5*b^2*x^2)/2 + 35*a^4*b^3*x^3 + (7*a^6*b*x)/3
)/x^4 + 21*a^2*b^5*x + (7*a*b^6*x^2)/2 + 35*a^3*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^7}{x^5}\,dx$$
$$=\frac{420\log(x)\,a^3b^4x^4-3a^7-28a^6bx-126a^5b^2x^2-420a^4b^3x^3+252a^2b^5x^5+42a\,b^6x^6+4b^7x^7}{12x^4}$$

input
```
int((b*x+a)^7/x^5,x)
```

output
```
(420*log(x)*a**3*b**4*x**4 - 3*a**7 - 28*a**6*b*x - 126*a**5*b**2*x**2 - 4
20*a**4*b**3*x**3 + 252*a**2*b**5*x**5 + 42*a*b**6*x**6 + 4*b**7*x**7)/(12
*x**4)
```

3.70 $\int \frac{(a+bx)^7}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int \frac{(a+bx)^7}{x^6} dx = -\frac{a^7}{5x^5} - \frac{7a^6b}{4x^4} - \frac{7a^5b^2}{x^3} - \frac{35a^4b^3}{2x^2} - \frac{35a^3b^4}{x} + 7ab^6x + \frac{b^7x^2}{2} + 21a^2b^5\log(x)$$

output
```
-1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a
*b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^6} dx = -\frac{a^7}{5x^5} - \frac{7a^6b}{4x^4} - \frac{7a^5b^2}{x^3} - \frac{35a^4b^3}{2x^2} - \frac{35a^3b^4}{x} + 7ab^6x + \frac{b^7x^2}{2} + 21a^2b^5\log(x)$$

input
```
Integrate[(a + b*x)^7/x^6,x]
```

output
```
-1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2)
- (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^6}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^7}{x^6}+\frac{7a^6b}{x^5}+\frac{21a^5b^2}{x^4}+\frac{35a^4b^3}{x^3}+\frac{35a^3b^4}{x^2}+\frac{21a^2b^5}{x}+7ab^6+b^7x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}$$

input
```
Int[(a + b*x)^7/x^6,x]
```

output
```
-1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2)
- (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7a\,b^6x+\frac{b^7x^2}{2}+21a^2b^5\ln\left(x\right)$	77
risch	$\frac{b^7x^2}{2}+7a\,b^6x+\frac{-35a^3b^4x^4-\frac{35}{2}a^4b^3x^3-7a^5b^2x^2-\frac{7}{4}a^6bx-\frac{1}{5}a^7}{x^5}+21a^2b^5\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{5}a^7+\frac{1}{2}b^7x^7+7a\,b^6x^6-35a^3b^4x^4-\frac{35}{2}a^4b^3x^3-7a^5b^2x^2-\frac{7}{4}a^6bx}{x^5}+21a^2b^5\ln\left(x\right)$	79
parallelrisch	$\frac{10b^7x^7+420a^2b^5\ln(x)x^5+140a\,b^6x^6-700a^3b^4x^4-350a^4b^3x^3-140a^5b^2x^2-35a^6bx-4a^7}{20x^5}$	82

input
```
int((b*x+a)^7/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a
*b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int\frac{(a+bx)^7}{x^6}\,dx = \frac{10\,b^7x^7+140\,ab^6x^6+420\,a^2b^5x^5\log\left(x\right)-700\,a^3b^4x^4-350\,a^4b^3x^3-140\,a^5b^2x^2-35\,a^6bx-4\,a^7}{20\,x^5}$$

input
```
integrate((b*x+a)^7/x^6,x, algorithm="fricas")
```

output
```
1/20*(10*b^7*x^7 + 140*a*b^6*x^6 + 420*a^2*b^5*x^5*log(x) - 700*a^3*b^4*x^
4 - 350*a^4*b^3*x^3 - 140*a^5*b^2*x^2 - 35*a^6*b*x - 4*a^7)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^7}{x^6}\,dx = 21a^2b^5\log(x) + 7ab^6x + \frac{b^7x^2}{2} + \frac{-4a^7 - 35a^6bx - 140a^5b^2x^2 - 350a^4b^3x^3 - 700a^3b^4x^4}{20x^5}$$

input
```
integrate((b*x+a)**7/x**6,x)
```

output
```
21*a**2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2 + (-4*a**7 - 35*a**6*b*x -
140*a**5*b**2*x**2 - 350*a**4*b**3*x**3 - 700*a**3*b**4*x**4)/(20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^7}{x^6}\,dx = \frac{1}{2}\,b^7x^2 + 7\,ab^6x + 21\,a^2b^5\log(x) - \frac{700\,a^3b^4x^4 + 350\,a^4b^3x^3 + 140\,a^5b^2x^2 + 35\,a^6bx + 4\,a^7}{20\,x^5}$$

input
```
integrate((b*x+a)^7/x^6,x, algorithm="maxima")
```

output
```
1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(x) - 1/20*(700*a^3*b^4*x^4 + 350*
a^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^7}{x^6}\,dx = \frac{1}{2}\,b^7x^2 + 7\,ab^6x + 21\,a^2b^5\log\left(|x|\right) - \frac{700\,a^3b^4x^4 + 350\,a^4b^3x^3 + 140\,a^5b^2x^2 + 35\,a^6bx + 4\,a^7}{20\,x^5}$$

input
```
integrate((b*x+a)^7/x^6,x, algorithm="giac")
```

output
```
1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(abs(x)) - 1/20*(700*a^3*b^4*x^4 +
 350*a^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^7}{x^6}\,dx = \frac{b^7\,x^2}{2} - \frac{\frac{a^7}{5} + \frac{7\,a^6\,b\,x}{4} + 7\,a^5\,b^2\,x^2 + \frac{35\,a^4\,b^3\,x^3}{2} + 35\,a^3\,b^4\,x^4}{x^5} + 21\,a^2\,b^5\,\ln\left(x\right) + 7\,a\,b^6\,x$$

input
```
int((a + b*x)^7/x^6,x)
```

output
```
(b^7*x^2)/2 - (a^7/5 + 7*a^5*b^2*x^2 + (35*a^4*b^3*x^3)/2 + 35*a^3*b^4*x^4
 + (7*a^6*b*x)/4)/x^5 + 21*a^2*b^5*log(x) + 7*a*b^6*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^7}{x^6}\,dx = \frac{420\log(x)\,a^2b^5x^5 - 4a^7 - 35a^6bx - 140a^5b^2x^2 - 350a^4b^3x^3 - 700a^3b^4x^4 + 140a\,b^6x^6 + 10b^7x^7}{20x^5}$$

input
```
int((b*x+a)^7/x^6,x)
```

output
```
(420*log(x)*a**2*b**5*x**5 - 4*a**7 - 35*a**6*b*x - 140*a**5*b**2*x**2 - 3
50*a**4*b**3*x**3 - 700*a**3*b**4*x**4 + 140*a*b**6*x**6 + 10*b**7*x**7)/(
20*x**5)
```

3.71 $\int \frac{(a+bx)^7}{x^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \frac{(a+bx)^7}{x^7} dx = -\frac{a^7}{6x^6} - \frac{7a^6b}{5x^5} - \frac{21a^5b^2}{4x^4} - \frac{35a^4b^3}{3x^3} - \frac{35a^3b^4}{2x^2} - \frac{21a^2b^5}{x} + b^7x + 7ab^6\log(x)$$

output
```
-1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/
x^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^7} dx = -\frac{a^7}{6x^6} - \frac{7a^6b}{5x^5} - \frac{21a^5b^2}{4x^4} - \frac{35a^4b^3}{3x^3} - \frac{35a^3b^4}{2x^2} - \frac{21a^2b^5}{x} + b^7x + 7ab^6\log(x)$$

input
```
Integrate[(a + b*x)^7/x^7,x]
```

output
```
-1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*
x^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^7} dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^7}{x^7}+\frac{7a^6b}{x^6}+\frac{21a^5b^2}{x^5}+\frac{35a^4b^3}{x^4}+\frac{35a^3b^4}{x^3}+\frac{21a^2b^5}{x^2}+\frac{7ab^6}{x}+b^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+7ab^6\log(x)+b^7x$$

input
```
Int[(a + b*x)^7/x^7,x]
```

output
```
-1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*
x^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7a\,b^6\ln\left(x\right)$	76
risch	$b^7x+\frac{-21a^2b^5x^5-\frac{35}{2}a^3b^4x^4-\frac{35}{3}a^4b^3x^3-\frac{21}{4}a^5b^2x^2-\frac{7}{5}a^6bx-\frac{1}{6}a^7}{x^6}+7a\,b^6\ln\left(x\right)$	76
norman	$\frac{b^7x^7-\frac{1}{6}a^7-21a^2b^5x^5-\frac{35}{2}a^3b^4x^4-\frac{35}{3}a^4b^3x^3-\frac{21}{4}a^5b^2x^2-\frac{7}{5}a^6bx}{x^6}+7a\,b^6\ln\left(x\right)$	78
parallelrisch	$\frac{420a\,b^6\ln(x)x^6+60b^7x^7-1260a^2b^5x^5-1050a^3b^4x^4-700a^4b^3x^3-315a^5b^2x^2-84a^6bx-10a^7}{60x^6}$	82

input
```
int((b*x+a)^7/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/
x^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int\frac{(a+bx)^7}{x^7}\,dx = \frac{60\,b^7x^7+420\,ab^6x^6\log\left(x\right)-1260\,a^2b^5x^5-1050\,a^3b^4x^4-700\,a^4b^3x^3-315\,a^5b^2x^2-84\,a^6bx-10\,a^7}{60\,x^6}$$

input
```
integrate((b*x+a)^7/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*b^7*x^7 + 420*a*b^6*x^6*log(x) - 1260*a^2*b^5*x^5 - 1050*a^3*b^4*
x^4 - 700*a^4*b^3*x^3 - 315*a^5*b^2*x^2 - 84*a^6*b*x - 10*a^7)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^7}{x^7}\,dx$$
$$= 7ab^6\log(x) + b^7x + \frac{-10a^7 - 84a^6bx - 315a^5b^2x^2 - 700a^4b^3x^3 - 1050a^3b^4x^4 - 1260a^2b^5x^5}{60x^6}$$

input
```
integrate((b*x+a)**7/x**7,x)
```

output
```
7*a*b**6*log(x) + b**7*x + (-10*a**7 - 84*a**6*b*x - 315*a**5*b**2*x**2 -
700*a**4*b**3*x**3 - 1050*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^7}{x^7}\,dx$$
$$= b^7x + 7\,ab^6\log(x) - \frac{1260\,a^2b^5x^5 + 1050\,a^3b^4x^4 + 700\,a^4b^3x^3 + 315\,a^5b^2x^2 + 84\,a^6bx + 10\,a^7}{60\,x^6}$$

input
```
integrate((b*x+a)^7/x^7,x, algorithm="maxima")
```

output
```
b^7*x + 7*a*b^6*log(x) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 700*a
^4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^7}{x^7}\,dx = b^7x + 7\,ab^6\log\left(|x|\right) - \frac{1260\,a^2b^5x^5 + 1050\,a^3b^4x^4 + 700\,a^4b^3x^3 + 315\,a^5b^2x^2 + 84\,a^6bx + 10\,a^7}{60\,x^6}$$

input
```
integrate((b*x+a)^7/x^7,x, algorithm="giac")
```

output
```
b^7*x + 7*a*b^6*log(abs(x)) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 +
700*a^4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^7}{x^7}\,dx = -\frac{10\,a^7 - 60\,b^7\,x^7 + 315\,a^5\,b^2\,x^2 + 700\,a^4\,b^3\,x^3 + 1050\,a^3\,b^4\,x^4 + 1260\,a^2\,b^5\,x^5 + 84\,a^6\,b\,x - 420\,a\,b^6\,x^6\,\ln}{60\,x^6}$$

input
```
int((a + b*x)^7/x^7,x)
```

output
```
-(10*a^7 - 60*b^7*x^7 + 315*a^5*b^2*x^2 + 700*a^4*b^3*x^3 + 1050*a^3*b^4*x
^4 + 1260*a^2*b^5*x^5 + 84*a^6*b*x - 420*a*b^6*x^6*log(x))/(60*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^7}{x^7}\,dx$$
$$= \frac{420\log(x)\,a\,b^6x^6 - 10a^7 - 84a^6bx - 315a^5b^2x^2 - 700a^4b^3x^3 - 1050a^3b^4x^4 - 1260a^2b^5x^5 + 60b^7x^7}{60x^6}$$

input
```
int((b*x+a)^7/x^7,x)
```

output
```
(420*log(x)*a*b**6*x**6 - 10*a**7 - 84*a**6*b*x - 315*a**5*b**2*x**2 - 700
*a**4*b**3*x**3 - 1050*a**3*b**4*x**4 - 1260*a**2*b**5*x**5 + 60*b**7*x**7
)/(60*x**6)
```

3.72 $\int \frac{(a+bx)^7}{x^8} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \frac{(a+bx)^7}{x^8} \, dx = -\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

output
```
-1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/
x^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^8} \, dx = -\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

input
```
Integrate[(a + b*x)^7/x^8,x]
```

output
```
-1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*
x^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log
[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^8}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^7}{x^8}+\frac{7a^6b}{x^7}+\frac{21a^5b^2}{x^6}+\frac{35a^4b^3}{x^5}+\frac{35a^3b^4}{x^4}+\frac{21a^2b^5}{x^3}+\frac{7ab^6}{x^2}+\frac{b^7}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

input
```
Int[(a + b*x)^7/x^8,x]
```

output
```
-1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*
x^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log
[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{7}}{7x^{7}}-\frac{7a^{6}b}{6x^{6}}-\frac{21a^{5}b^{2}}{5x^{5}}-\frac{35a^{4}b^{3}}{4x^{4}}-\frac{35a^{3}b^{4}}{3x^{3}}-\frac{21a^{2}b^{5}}{2x^{2}}-\frac{7a\,b^{6}}{x}+b^{7}\ln\left(x\right)$	78
norman	$\frac{-\frac{1}{7}a^{7}-7a\,b^{6}x^{6}-\frac{21}{2}a^{2}b^{5}x^{5}-\frac{35}{3}a^{3}b^{4}x^{4}-\frac{35}{4}a^{4}b^{3}x^{3}-\frac{21}{5}a^{5}b^{2}x^{2}-\frac{7}{6}a^{6}bx}{x^{7}}+b^{7}\ln\left(x\right)$	78
risch	$\frac{-\frac{1}{7}a^{7}-7a\,b^{6}x^{6}-\frac{21}{2}a^{2}b^{5}x^{5}-\frac{35}{3}a^{3}b^{4}x^{4}-\frac{35}{4}a^{4}b^{3}x^{3}-\frac{21}{5}a^{5}b^{2}x^{2}-\frac{7}{6}a^{6}bx}{x^{7}}+b^{7}\ln\left(x\right)$	78
parallelrisch	$\frac{420b^{7}\ln\left(x\right)x^{7}-2940a\,b^{6}x^{6}-4410a^{2}b^{5}x^{5}-4900a^{3}b^{4}x^{4}-3675a^{4}b^{3}x^{3}-1764a^{5}b^{2}x^{2}-490a^{6}bx-60a^{7}}{420x^{7}}$	82

input
```
int((b*x+a)^7/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/
x^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^7}{x^8}\,dx = \frac{420\,b^7x^7\log\left(x\right)-2940\,ab^6x^6-4410\,a^2b^5x^5-4900\,a^3b^4x^4-3675\,a^4b^3x^3-1764\,a^5b^2x^2-490\,a^6bx-60}{420\,x^7}$$

input
```
integrate((b*x+a)^7/x^8,x, algorithm="fricas")
```

output
```
1/420*(420*b^7*x^7*log(x) - 2940*a*b^6*x^6 - 4410*a^2*b^5*x^5 - 4900*a^3*b
^4*x^4 - 3675*a^4*b^3*x^3 - 1764*a^5*b^2*x^2 - 490*a^6*b*x - 60*a^7)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^7}{x^8}\,dx = b^7\log(x) + \frac{-60a^7-490a^6bx-1764a^5b^2x^2-3675a^4b^3x^3-4900a^3b^4x^4-4410a^2b^5x^5-2940ab^6x^6}{420x^7}$$

input
```
integrate((b*x+a)**7/x**8,x)
```

output
```
b**7*log(x) + (-60*a**7 - 490*a**6*b*x - 1764*a**5*b**2*x**2 - 3675*a**4*b
**3*x**3 - 4900*a**3*b**4*x**4 - 4410*a**2*b**5*x**5 - 2940*a*b**6*x**6)/(
420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^7}{x^8}\,dx = b^7\log(x) - \frac{2940\,ab^6x^6+4410\,a^2b^5x^5+4900\,a^3b^4x^4+3675\,a^4b^3x^3+1764\,a^5b^2x^2+490\,a^6bx+60\,a^7}{420\,x^7}$$

input
```
integrate((b*x+a)^7/x^8,x, algorithm="maxima")
```

output
```
b^7*log(x) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x^4 +
 3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^7}{x^8}\,dx = b^7\log(|x|) - \frac{2940\,ab^6x^6 + 4410\,a^2b^5x^5 + 4900\,a^3b^4x^4 + 3675\,a^4b^3x^3 + 1764\,a^5b^2x^2 + 490\,a^6bx + 60\,a^7}{420\,x^7}$$

input
```
integrate((b*x+a)^7/x^8,x, algorithm="giac")
```

output
```
b^7*log(abs(x)) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*
x^4 + 3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^7}{x^8}\,dx = b^7\,\ln(x) - \frac{\frac{a^7}{7} + \frac{7\,a^6\,b\,x}{6} + \frac{21\,a^5\,b^2\,x^2}{5} + \frac{35\,a^4\,b^3\,x^3}{4} + \frac{35\,a^3\,b^4\,x^4}{3} + \frac{21\,a^2\,b^5\,x^5}{2} + 7\,a\,b^6\,x^6}{x^7}$$

input
```
int((a + b*x)^7/x^8,x)
```

output
```
b^7*log(x) - (a^7/7 + 7*a*b^6*x^6 + (21*a^5*b^2*x^2)/5 + (35*a^4*b^3*x^3)/
4 + (35*a^3*b^4*x^4)/3 + (21*a^2*b^5*x^5)/2 + (7*a^6*b*x)/6)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^7}{x^8}\,dx$$
$$= \frac{420\log(x)\,b^7x^7 - 60a^7 - 490a^6bx - 1764a^5b^2x^2 - 3675a^4b^3x^3 - 4900a^3b^4x^4 - 4410a^2b^5x^5 - 2940a\,b^6x^6}{420x^7}$$

input
```
int((b*x+a)^7/x^8,x)
```

output
```
(420*log(x)*b**7*x**7 - 60*a**7 - 490*a**6*b*x - 1764*a**5*b**2*x**2 - 367
5*a**4*b**3*x**3 - 4900*a**3*b**4*x**4 - 4410*a**2*b**5*x**5 - 2940*a*b**6
*x**6)/(420*x**7)
```

3.73 $\int \frac{(a+bx)^7}{x^9} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{(a+bx)^7}{x^9} dx = -\frac{(a+bx)^8}{8ax^8}$$

output
```
-1/8*(b*x+a)^8/a/x^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(17) = 34.

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.12

$$\int \frac{(a+bx)^7}{x^9} dx = -\frac{a^7}{8x^8} - \frac{a^6b}{x^7} - \frac{7a^5b^2}{2x^6} - \frac{7a^4b^3}{x^5} - \frac{35a^3b^4}{4x^4} - \frac{7a^2b^5}{x^3} - \frac{7ab^6}{2x^2} - \frac{b^7}{x}$$

input
```
Integrate[(a + b*x)^7/x^9,x]
```

output
```
-1/8*a^7/x^8 - (a^6*b)/x^7 - (7*a^5*b^2)/(2*x^6) - (7*a^4*b^3)/x^5 - (35*a
^3*b^4)/(4*x^4) - (7*a^2*b^5)/x^3 - (7*a*b^6)/(2*x^2) - b^7/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^9} \, dx$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(a+bx)^8}{8ax^8}$$

input
```
Int[(a + b*x)^7/x^9,x]
```

output
```
-1/8*(a + b*x)^8/(a*x^8)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.59

method	result	size
gosper	$-\frac{8b^7x^7+28a\,b^6x^6+56a^2b^5x^5+70a^3b^4x^4+56a^4b^3x^3+28a^5b^2x^2+8a^6bx+a^7}{8x^8}$	78
orering	$-\frac{8b^7x^7+28a\,b^6x^6+56a^2b^5x^5+70a^3b^4x^4+56a^4b^3x^3+28a^5b^2x^2+8a^6bx+a^7}{8x^8}$	78
norman	$\frac{-b^7x^7-\frac{7}{2}a\,b^6x^6-7a^2b^5x^5-\frac{35}{4}a^3b^4x^4-7a^4b^3x^3-\frac{7}{2}a^5b^2x^2-a^6bx-\frac{1}{8}a^7}{x^8}$	79
risch	$\frac{-b^7x^7-\frac{7}{2}a\,b^6x^6-7a^2b^5x^5-\frac{35}{4}a^3b^4x^4-7a^4b^3x^3-\frac{7}{2}a^5b^2x^2-a^6bx-\frac{1}{8}a^7}{x^8}$	79
default	$-\frac{7a^2b^5}{x^3}-\frac{7a^4b^3}{x^5}-\frac{7a\,b^6}{2x^2}-\frac{a^6b}{x^7}-\frac{35a^3b^4}{4x^4}-\frac{a^7}{8x^8}-\frac{b^7}{x}-\frac{7a^5b^2}{2x^6}$	80
parallelrisch	$\frac{-8b^7x^7-28a\,b^6x^6-56a^2b^5x^5-70a^3b^4x^4-56a^4b^3x^3-28a^5b^2x^2-8a^6bx-a^7}{8x^8}$	80

input
```
int((b*x+a)^7/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(8*b^7*x^7+28*a*b^6*x^6+56*a^2*b^5*x^5+70*a^3*b^4*x^4+56*a^4*b^3*x^3+
28*a^5*b^2*x^2+8*a^6*b*x+a^7)/x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.53

$$\int \frac{(a+bx)^7}{x^9}\,dx = -\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

input
```
integrate((b*x+a)^7/x^9,x, algorithm="fricas")
```

output
```
-1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*
b^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.88

$$\int \frac{(a+bx)^7}{x^9}\,dx = \frac{-a^7 - 8a^6bx - 28a^5b^2x^2 - 56a^4b^3x^3 - 70a^3b^4x^4 - 56a^2b^5x^5 - 28ab^6x^6 - 8b^7x^7}{8x^8}$$

input
```
integrate((b*x+a)**7/x**9,x)
```

output
```
(-a**7 - 8*a**6*b*x - 28*a**5*b**2*x**2 - 56*a**4*b**3*x**3 - 70*a**3*b**4
*x**4 - 56*a**2*b**5*x**5 - 28*a*b**6*x**6 - 8*b**7*x**7)/(8*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.53

$$\int \frac{(a+bx)^7}{x^9}\,dx = -\frac{8\,b^7x^7 + 28\,ab^6x^6 + 56\,a^2b^5x^5 + 70\,a^3b^4x^4 + 56\,a^4b^3x^3 + 28\,a^5b^2x^2 + 8\,a^6bx + a^7}{8\,x^8}$$

input
```
integrate((b*x+a)^7/x^9,x, algorithm="maxima")
```

output
```
-1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*
b^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.53

$$\int \frac{(a+bx)^7}{x^9}\,dx = -\frac{8\,b^7x^7 + 28\,ab^6x^6 + 56\,a^2b^5x^5 + 70\,a^3b^4x^4 + 56\,a^4b^3x^3 + 28\,a^5b^2x^2 + 8\,a^6bx + a^7}{8\,x^8}$$

input
```
integrate((b*x+a)^7/x^9,x, algorithm="giac")
```

output
```
-1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*
b^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.53

$$\int \frac{(a+bx)^7}{x^9}\,dx = -\frac{\frac{a^7}{8} + a^6\,b\,x + \frac{7\,a^5\,b^2\,x^2}{2} + 7\,a^4\,b^3\,x^3 + \frac{35\,a^3\,b^4\,x^4}{4} + 7\,a^2\,b^5\,x^5 + \frac{7\,a\,b^6\,x^6}{2} + b^7\,x^7}{x^8}$$

input
```
int((a + b*x)^7/x^9,x)
```

output
```
-(a^7/8 + b^7*x^7 + (7*a*b^6*x^6)/2 + (7*a^5*b^2*x^2)/2 + 7*a^4*b^3*x^3 +
(35*a^3*b^4*x^4)/4 + 7*a^2*b^5*x^5 + a^6*b*x)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.65

$$\int \frac{(a+bx)^7}{x^9}\,dx = \frac{-8b^7x^7 - 28a\,b^6x^6 - 56a^2b^5x^5 - 70a^3b^4x^4 - 56a^4b^3x^3 - 28a^5b^2x^2 - 8a^6bx - a^7}{8x^8}$$

input
```
int((b*x+a)^7/x^9,x)
```

output
```
( - a**7 - 8*a**6*b*x - 28*a**5*b**2*x**2 - 56*a**4*b**3*x**3 - 70*a**3*b*
*4*x**4 - 56*a**2*b**5*x**5 - 28*a*b**6*x**6 - 8*b**7*x**7)/(8*x**8)
```

3.74 $\int \frac{(a+bx)^7}{x^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{(a+bx)^8}{9ax^9} + \frac{b(a+bx)^8}{72a^2x^8}$$

output
```
-1/9*(b*x+a)^8/a/x^9+1/72*b*(b*x+a)^8/a^2/x^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(36) = 72.

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{a^7}{9x^9} - \frac{7a^6b}{8x^8} - \frac{3a^5b^2}{x^7} - \frac{35a^4b^3}{6x^6} - \frac{7a^3b^4}{x^5} - \frac{21a^2b^5}{4x^4} - \frac{7ab^6}{3x^3} - \frac{b^7}{2x^2}$$

input
```
Integrate[(a + b*x)^7/x^10,x]
```

output
```
-1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6)
- (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{10}} \, dx$$

$$\Big\downarrow \; 55$$

$$-\frac{b \int \frac{(a+bx)^7}{x^9} dx}{9a} - \frac{(a+bx)^8}{9ax^9}$$

$$\Big\downarrow \; 48$$

$$\frac{b(a+bx)^8}{72a^2x^8} - \frac{(a+bx)^8}{9ax^9}$$

input
```
Int[(a + b*x)^7/x^10,x]
```

output
```
-1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

method	result	size
norman	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
risch	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
gosper	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80
default	$-\frac{7a\,b^6}{3x^3}-\frac{7a^3b^4}{x^5}-\frac{b^7}{2x^2}-\frac{3a^5b^2}{x^7}-\frac{21a^2b^5}{4x^4}-\frac{7a^6b}{8x^8}-\frac{35a^4b^3}{6x^6}-\frac{a^7}{9x^9}$	80
parallelrisch	$\frac{-36b^7x^7-168a\,b^6x^6-378a^2b^5x^5-504a^3b^4x^4-420a^4b^3x^3-216a^5b^2x^2-63a^6bx-8a^7}{72x^9}$	80
orering	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80

input
```
int((b*x+a)^7/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/2*b^7*x^7-7/3*a*b^6*x^6-21/4*a^2*b^5*x^5-7*a^3*b^4*x^4-35/6*a^4*
b^3*x^3-3*a^5*b^2*x^2-7/8*a^6*b*x-1/9*a^7)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{36\,b^7x^7 + 168\,ab^6x^6 + 378\,a^2b^5x^5 + 504\,a^3b^4x^4 + 420\,a^4b^3x^3 + 216\,a^5b^2x^2 + 63\,a^6bx + 8\,a^7}{72\,x^9}$$

input
```
integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

output
```
-1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 42
0*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = \frac{-8a^7 - 63a^6bx - 216a^5b^2x^2 - 420a^4b^3x^3 - 504a^3b^4x^4 - 378a^2b^5x^5 - 168ab^6x^6 - 36b^7x^7}{72x^9}$$

input
```
integrate((b*x+a)**7/x**10,x)
```

output
```
(-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**
3*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**
9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{36\,b^7x^7 + 168\,ab^6x^6 + 378\,a^2b^5x^5 + 504\,a^3b^4x^4 + 420\,a^4b^3x^3 + 216\,a^5b^2x^2 + 63\,a^6bx + 8\,a^7}{72\,x^9}$$

input
```
integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

output
```
-1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 42
0*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{36\,b^7x^7 + 168\,ab^6x^6 + 378\,a^2b^5x^5 + 504\,a^3b^4x^4 + 420\,a^4b^3x^3 + 216\,a^5b^2x^2 + 63\,a^6bx + 8\,a^7}{72\,x^9}$$

input
```
integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

output
```
-1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 42
0*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{(8\,a-b\,x)\,\left(a+b\,x\right)^8}{72\,a^2\,x^9}$$

input
```
int((a + b*x)^7/x^10,x)
```

output
```
-((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = \frac{-36b^7x^7-168a\,b^6x^6-378a^2b^5x^5-504a^3b^4x^4-420a^4b^3x^3-216a^5b^2x^2-63a^6bx-8a^7}{72x^9}$$

input
```
int((b*x+a)^7/x^10,x)
```

output
```
( - 8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a
**3*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x
**9)
```

3.75 $\int \frac{(a+bx)^7}{x^{11}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{(a+bx)^8}{10ax^{10}} + \frac{b(a+bx)^8}{45a^2x^9} - \frac{b^2(a+bx)^8}{360a^3x^8}$$

output
```
-1/10*(b*x+a)^8/a/x^10+1/45*b*(b*x+a)^8/a^2/x^9-1/360*b^2*(b*x+a)^8/a^3/x^
8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{a^7}{10x^{10}} - \frac{7a^6b}{9x^9} - \frac{21a^5b^2}{8x^8} - \frac{5a^4b^3}{x^7} - \frac{35a^3b^4}{6x^6} - \frac{21a^2b^5}{5x^5} - \frac{7ab^6}{4x^4} - \frac{b^7}{3x^3}$$

input
```
Integrate[(a + b*x)^7/x^11,x]
```

output
```
-1/10*a^7/x^10 - (7*a^6*b)/(9*x^9) - (21*a^5*b^2)/(8*x^8) - (5*a^4*b^3)/x^
7 - (35*a^3*b^4)/(6*x^6) - (21*a^2*b^5)/(5*x^5) - (7*a*b^6)/(4*x^4) - b^7/
(3*x^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{11}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{b\int \frac{(a+bx)^7}{x^{10}}dx}{5a}-\frac{(a+bx)^8}{10ax^{10}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{b\left(-\frac{b\int \frac{(a+bx)^7}{x^9}dx}{9a}-\frac{(a+bx)^8}{9ax^9}\right)}{5a}-\frac{(a+bx)^8}{10ax^{10}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{b\left(\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}\right)}{5a}-\frac{(a+bx)^8}{10ax^{10}}$$

input
```
Int[(a + b*x)^7/x^11,x]
```

output
```
-1/10*(a + b*x)^8/(a*x^10) - (b*(-1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8
)/(72*a^2*x^8)))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

method	result	size
norman	$\frac{-\frac{1}{3}b^7x^7-\frac{7}{4}a\,b^6x^6-\frac{21}{5}a^2b^5x^5-\frac{35}{6}a^3b^4x^4-5a^4b^3x^3-\frac{21}{8}a^5b^2x^2-\frac{7}{9}a^6bx-\frac{1}{10}a^7}{x^{10}}$	79
risch	$\frac{-\frac{1}{3}b^7x^7-\frac{7}{4}a\,b^6x^6-\frac{21}{5}a^2b^5x^5-\frac{35}{6}a^3b^4x^4-5a^4b^3x^3-\frac{21}{8}a^5b^2x^2-\frac{7}{9}a^6bx-\frac{1}{10}a^7}{x^{10}}$	79
gosper	$-\frac{120b^7x^7+630a\,b^6x^6+1512a^2b^5x^5+2100a^3b^4x^4+1800a^4b^3x^3+945a^5b^2x^2+280a^6bx+36a^7}{360x^{10}}$	80
default	$-\frac{b^7}{3x^3}-\frac{21a^2b^5}{5x^5}-\frac{5a^4b^3}{x^7}-\frac{7a\,b^6}{4x^4}-\frac{21a^5b^2}{8x^8}-\frac{a^7}{10x^{10}}-\frac{35a^3b^4}{6x^6}-\frac{7a^6b}{9x^9}$	80
parallelrisch	$\frac{-120b^7x^7-630a\,b^6x^6-1512a^2b^5x^5-2100a^3b^4x^4-1800a^4b^3x^3-945a^5b^2x^2-280a^6bx-36a^7}{360x^{10}}$	80
orering	$-\frac{120b^7x^7+630a\,b^6x^6+1512a^2b^5x^5+2100a^3b^4x^4+1800a^4b^3x^3+945a^5b^2x^2+280a^6bx+36a^7}{360x^{10}}$	80

input
```
int((b*x+a)^7/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/x^10*(-1/3*b^7*x^7-7/4*a*b^6*x^6-21/5*a^2*b^5*x^5-35/6*a^3*b^4*x^4-5*a^4
*b^3*x^3-21/8*a^5*b^2*x^2-7/9*a^6*b*x-1/10*a^7)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{120\,b^7x^7+630\,ab^6x^6+1512\,a^2b^5x^5+2100\,a^3b^4x^4+1800\,a^4b^3x^3+945\,a^5b^2x^2+280\,a^6bx+36\,a^7}{360\,x^{10}}$$

input
```
integrate((b*x+a)^7/x^11,x, algorithm="fricas")
```

output
```
-1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4
+ 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = \frac{-36a^7-280a^6bx-945a^5b^2x^2-1800a^4b^3x^3-2100a^3b^4x^4-1512a^2b^5x^5-630ab^6x^6-120b^7x^7}{360x^{10}}$$

input
```
integrate((b*x+a)**7/x**11,x)
```

output
```
(-36*a**7 - 280*a**6*b*x - 945*a**5*b**2*x**2 - 1800*a**4*b**3*x**3 - 2100
*a**3*b**4*x**4 - 1512*a**2*b**5*x**5 - 630*a*b**6*x**6 - 120*b**7*x**7)/(
360*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{120\,b^7x^7+630\,ab^6x^6+1512\,a^2b^5x^5+2100\,a^3b^4x^4+1800\,a^4b^3x^3+945\,a^5b^2x^2+280\,a^6bx+36\,a^7}{360\,x^{10}}$$

input
```
integrate((b*x+a)^7/x^11,x, algorithm="maxima")
```

output
```
-1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4
+ 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{120\,b^7x^7+630\,ab^6x^6+1512\,a^2b^5x^5+2100\,a^3b^4x^4+1800\,a^4b^3x^3+945\,a^5b^2x^2+280\,a^6bx+36\,a^7}{360\,x^{10}}$$

input
```
integrate((b*x+a)^7/x^11,x, algorithm="giac")
```

output
```
-1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4
+ 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{\frac{a^7}{10}+\frac{7\,a^6\,b\,x}{9}+\frac{21\,a^5\,b^2\,x^2}{8}+5\,a^4\,b^3\,x^3+\frac{35\,a^3\,b^4\,x^4}{6}+\frac{21\,a^2\,b^5\,x^5}{5}+\frac{7\,a\,b^6\,x^6}{4}+\frac{b^7\,x^7}{3}}{x^{10}}$$

input
```
int((a + b*x)^7/x^11,x)
```

output
```
-(a^7/10 + (b^7*x^7)/3 + (7*a*b^6*x^6)/4 + (21*a^5*b^2*x^2)/8 + 5*a^4*b^3*
x^3 + (35*a^3*b^4*x^4)/6 + (21*a^2*b^5*x^5)/5 + (7*a^6*b*x)/9)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = \frac{-120b^7x^7-630a\,b^6x^6-1512a^2b^5x^5-2100a^3b^4x^4-1800a^4b^3x^3-945a^5b^2x^2-280a^6bx-36a^7}{360x^{10}}$$

input
```
int((b*x+a)^7/x^11,x)
```

output
```
( - 36*a**7 - 280*a**6*b*x - 945*a**5*b**2*x**2 - 1800*a**4*b**3*x**3 - 21
00*a**3*b**4*x**4 - 1512*a**2*b**5*x**5 - 630*a*b**6*x**6 - 120*b**7*x**7)
/(360*x**10)
```

3.76 $\int \frac{(a+bx)^7}{x^{12}} dx$

Optimal result

Integrand size = 11, antiderivative size = 76

$$\int \frac{(a+bx)^7}{x^{12}} dx = -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{b^2(a+bx)^8}{165a^3x^9} + \frac{b^3(a+bx)^8}{1320a^4x^8}$$

output
```
-1/11*(b*x+a)^8/a/x^11+3/110*b*(b*x+a)^8/a^2/x^10-1/165*b^2*(b*x+a)^8/a^3/
x^9+1/1320*b^3*(b*x+a)^8/a^4/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^7}{x^{12}} dx = -\frac{a^7}{11x^{11}} - \frac{7a^6b}{10x^{10}} - \frac{7a^5b^2}{3x^9} - \frac{35a^4b^3}{8x^8} - \frac{5a^3b^4}{x^7} - \frac{7a^2b^5}{2x^6} - \frac{7ab^6}{5x^5} - \frac{b^7}{4x^4}$$

input
```
Integrate[(a + b*x)^7/x^12,x]
```

output
```
-1/11*a^7/x^11 - (7*a^6*b)/(10*x^10) - (7*a^5*b^2)/(3*x^9) - (35*a^4*b^3)/
(8*x^8) - (5*a^3*b^4)/x^7 - (7*a^2*b^5)/(2*x^6) - (7*a*b^6)/(5*x^5) - b^7/
(4*x^4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{12}} \, dx$$

$$\Big\downarrow \ 55$$

$$-\frac{3b\int \frac{(a+bx)^7}{x^{11}}dx}{11a}-\frac{(a+bx)^8}{11ax^{11}}$$

$$\Big\downarrow \ 55$$

$$-\frac{3b\left(-\frac{b\int \frac{(a+bx)^7}{x^{10}}dx}{5a}-\frac{(a+bx)^8}{10ax^{10}}\right)}{11a}-\frac{(a+bx)^8}{11ax^{11}}$$

$$\Big\downarrow \ 55$$

$$-\frac{3b\left(-\frac{b\left(-\frac{b\int \frac{(a+bx)^7}{x^9}dx}{9a}-\frac{(a+bx)^8}{9ax^9}\right)}{5a}-\frac{(a+bx)^8}{10ax^{10}}\right)}{11a}-\frac{(a+bx)^8}{11ax^{11}}$$

$$\Big\downarrow \ 48$$

$$-\frac{3b\left(-\frac{b\left(\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}\right)}{5a}-\frac{(a+bx)^8}{10ax^{10}}\right)}{11a}-\frac{(a+bx)^8}{11ax^{11}}$$

input
```
Int[(a + b*x)^7/x^12,x]
```

output
```
-1/11*(a + b*x)^8/(a*x^11) - (3*b*(-1/10*(a + b*x)^8/(a*x^10) - (b*(-1/9*(
a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)))/(5*a)))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

method	result	size
norman	$\frac{-\frac{1}{4}b^7x^7-\frac{7}{5}a\,b^6x^6-\frac{7}{2}a^2b^5x^5-5a^3b^4x^4-\frac{35}{8}a^4b^3x^3-\frac{7}{3}a^5b^2x^2-\frac{7}{10}a^6bx-\frac{1}{11}a^7}{x^{11}}$	79
risch	$\frac{-\frac{1}{4}b^7x^7-\frac{7}{5}a\,b^6x^6-\frac{7}{2}a^2b^5x^5-5a^3b^4x^4-\frac{35}{8}a^4b^3x^3-\frac{7}{3}a^5b^2x^2-\frac{7}{10}a^6bx-\frac{1}{11}a^7}{x^{11}}$	79
gosper	$-\frac{330b^7x^7+1848a\,b^6x^6+4620a^2b^5x^5+6600a^3b^4x^4+5775a^4b^3x^3+3080a^5b^2x^2+924a^6bx+120a^7}{1320x^{11}}$	80
default	$-\frac{7a\,b^6}{5x^5}-\frac{a^7}{11x^{11}}-\frac{5a^3b^4}{x^7}-\frac{b^7}{4x^4}-\frac{35a^4b^3}{8x^8}-\frac{7a^6b}{10x^{10}}-\frac{7a^2b^5}{2x^6}-\frac{7a^5b^2}{3x^9}$	80
parallelrisch	$\frac{-330b^7x^7-1848a\,b^6x^6-4620a^2b^5x^5-6600a^3b^4x^4-5775a^4b^3x^3-3080a^5b^2x^2-924a^6bx-120a^7}{1320x^{11}}$	80
orering	$-\frac{330b^7x^7+1848a\,b^6x^6+4620a^2b^5x^5+6600a^3b^4x^4+5775a^4b^3x^3+3080a^5b^2x^2+924a^6bx+120a^7}{1320x^{11}}$	80

input
```
int((b*x+a)^7/x^12,x,method=_RETURNVERBOSE)
```

output
```
1/x^11*(-1/4*b^7*x^7-7/5*a*b^6*x^6-7/2*a^2*b^5*x^5-5*a^3*b^4*x^4-35/8*a^4*
b^3*x^3-7/3*a^5*b^2*x^2-7/10*a^6*b*x-1/11*a^7)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = -\frac{330\,b^7x^7 + 1848\,ab^6x^6 + 4620\,a^2b^5x^5 + 6600\,a^3b^4x^4 + 5775\,a^4b^3x^3 + 3080\,a^5b^2x^2 + 924\,a^6bx + 120\,a^7}{1320\,x^{11}}$$

input
```
integrate((b*x+a)^7/x^12,x, algorithm="fricas")
```

output
```
-1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^
4 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = \frac{-120a^7 - 924a^6bx - 3080a^5b^2x^2 - 5775a^4b^3x^3 - 6600a^3b^4x^4 - 4620a^2b^5x^5 - 1848ab^6x^6 - 330b^7x^7}{1320x^{11}}$$

input
```
integrate((b*x+a)**7/x**12,x)
```

output
```
(-120*a**7 - 924*a**6*b*x - 3080*a**5*b**2*x**2 - 5775*a**4*b**3*x**3 - 66
00*a**3*b**4*x**4 - 4620*a**2*b**5*x**5 - 1848*a*b**6*x**6 - 330*b**7*x**7
)/(1320*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = -\frac{330\,b^7x^7 + 1848\,ab^6x^6 + 4620\,a^2b^5x^5 + 6600\,a^3b^4x^4 + 5775\,a^4b^3x^3 + 3080\,a^5b^2x^2 + 924\,a^6bx + 120\,a^7}{1320\,x^{11}}$$

input
```
integrate((b*x+a)^7/x^12,x, algorithm="maxima")
```

output
```
-1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^
4 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = -\frac{330\,b^7x^7 + 1848\,ab^6x^6 + 4620\,a^2b^5x^5 + 6600\,a^3b^4x^4 + 5775\,a^4b^3x^3 + 3080\,a^5b^2x^2 + 924\,a^6bx + 120\,a^7}{1320\,x^{11}}$$

input
```
integrate((b*x+a)^7/x^12,x, algorithm="giac")
```

output
```
-1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^
4 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = -\frac{\frac{a^7}{11}+\frac{7\,a^6\,b\,x}{10}+\frac{7\,a^5\,b^2\,x^2}{3}+\frac{35\,a^4\,b^3\,x^3}{8}+5\,a^3\,b^4\,x^4+\frac{7\,a^2\,b^5\,x^5}{2}+\frac{7\,a\,b^6\,x^6}{5}+\frac{b^7\,x^7}{4}}{x^{11}}$$

input
```
int((a + b*x)^7/x^12,x)
```

output
```
-(a^7/11 + (b^7*x^7)/4 + (7*a*b^6*x^6)/5 + (7*a^5*b^2*x^2)/3 + (35*a^4*b^3
*x^3)/8 + 5*a^3*b^4*x^4 + (7*a^2*b^5*x^5)/2 + (7*a^6*b*x)/10)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = \frac{-330b^7x^7-1848a\,b^6x^6-4620a^2b^5x^5-6600a^3b^4x^4-5775a^4b^3x^3-3080a^5b^2x^2-924a^6bx-120a^7}{1320x^{11}}$$

input
```
int((b*x+a)^7/x^12,x)
```

output
```
( - 120*a**7 - 924*a**6*b*x - 3080*a**5*b**2*x**2 - 5775*a**4*b**3*x**3 -
6600*a**3*b**4*x**4 - 4620*a**2*b**5*x**5 - 1848*a*b**6*x**6 - 330*b**7*x*
*7)/(1320*x**11)
```

3.77 $\int \frac{(a+bx)^7}{x^{13}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 96

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{(a+bx)^8}{12ax^{12}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^4(a+bx)^8}{3960a^5x^8}$$

output
```
-1/12*(b*x+a)^8/a/x^12+1/33*b*(b*x+a)^8/a^2/x^11-1/110*b^2*(b*x+a)^8/a^3/x
^10+1/495*b^3*(b*x+a)^8/a^4/x^9-1/3960*b^4*(b*x+a)^8/a^5/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{a^7}{12x^{12}}-\frac{7a^6b}{11x^{11}}-\frac{21a^5b^2}{10x^{10}}-\frac{35a^4b^3}{9x^9}-\frac{35a^3b^4}{8x^8}-\frac{3a^2b^5}{x^7}-\frac{7ab^6}{6x^6}-\frac{b^7}{5x^5}$$

input
```
Integrate[(a + b*x)^7/x^13,x]
```

output
```
-1/12*a^7/x^12 - (7*a^6*b)/(11*x^11) - (21*a^5*b^2)/(10*x^10) - (35*a^4*b^
3)/(9*x^9) - (35*a^3*b^4)/(8*x^8) - (3*a^2*b^5)/x^7 - (7*a*b^6)/(6*x^6) -
b^7/(5*x^5)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{13}}dx$$

$$\Big\downarrow \ 55$$

$$-\frac{b\int \frac{(a+bx)^7}{x^{12}}dx}{3a}-\frac{(a+bx)^8}{12ax^{12}}$$

$$\Big\downarrow \ 55$$

$$-\frac{b\left(-\frac{3b\int \frac{(a+bx)^7}{x^{11}}dx}{11a}-\frac{(a+bx)^8}{11ax^{11}}\right)}{3a}-\frac{(a+bx)^8}{12ax^{12}}$$

$$\Big\downarrow \ 55$$

$$-\frac{b\left(-\frac{3b\left(-\frac{b\int \frac{(a+bx)^7}{x^{10}}dx}{5a}-\frac{(a+bx)^8}{10ax^{10}}\right)}{11a}-\frac{(a+bx)^8}{11ax^{11}}\right)}{3a}-\frac{(a+bx)^8}{12ax^{12}}$$

$$\Big\downarrow \ 55$$

$$-\frac{b\left(-\frac{3b\left(-\frac{b\left(-\frac{b\int \frac{(a+bx)^7}{x^9}dx}{9a}-\frac{(a+bx)^8}{9ax^9}\right)}{5a}-\frac{(a+bx)^8}{10ax^{10}}\right)}{11a}-\frac{(a+bx)^8}{11ax^{11}}\right)}{3a}-\frac{(a+bx)^8}{12ax^{12}}$$

$$\Big\downarrow \ 48$$

$$-\frac{b\left(-\frac{3b\left(-\frac{b\left(\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}\right)}{5a}-\frac{(a+bx)^8}{10ax^{10}}\right)}{11a}-\frac{(a+bx)^8}{11ax^{11}}\right)}{3a}-\frac{(a+bx)^8}{12ax^{12}}$$

input
```
Int[(a + b*x)^7/x^13,x]
```

output
```
-1/12*(a + b*x)^8/(a*x^12) - (b*(-1/11*(a + b*x)^8/(a*x^11) - (3*b*(-1/10*
(a + b*x)^8/(a*x^10) - (b*(-1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*
a^2*x^8)))/(5*a)))/(11*a)))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

method	result	size
norman	$\frac{-\frac{1}{5}b^7x^7-\frac{7}{6}a\,b^6x^6-3a^2b^5x^5-\frac{35}{8}a^3b^4x^4-\frac{35}{9}a^4b^3x^3-\frac{21}{10}a^5b^2x^2-\frac{7}{11}a^6bx-\frac{1}{12}a^7}{x^{12}}$	79
risch	$\frac{-\frac{1}{5}b^7x^7-\frac{7}{6}a\,b^6x^6-3a^2b^5x^5-\frac{35}{8}a^3b^4x^4-\frac{35}{9}a^4b^3x^3-\frac{21}{10}a^5b^2x^2-\frac{7}{11}a^6bx-\frac{1}{12}a^7}{x^{12}}$	79
gosper	$-\frac{792b^7x^7+4620a\,b^6x^6+11880a^2b^5x^5+17325a^3b^4x^4+15400a^4b^3x^3+8316a^5b^2x^2+2520a^6bx+330a^7}{3960x^{12}}$	80
default	$-\frac{b^7}{5x^5}-\frac{7a^6b}{11x^{11}}-\frac{3a^2b^5}{x^7}-\frac{35a^3b^4}{8x^8}-\frac{21a^5b^2}{10x^{10}}-\frac{7a\,b^6}{6x^6}-\frac{35a^4b^3}{9x^9}-\frac{a^7}{12x^{12}}$	80
parallelrisch	$\frac{-792b^7x^7-4620a\,b^6x^6-11880a^2b^5x^5-17325a^3b^4x^4-15400a^4b^3x^3-8316a^5b^2x^2-2520a^6bx-330a^7}{3960x^{12}}$	80
orering	$-\frac{792b^7x^7+4620a\,b^6x^6+11880a^2b^5x^5+17325a^3b^4x^4+15400a^4b^3x^3+8316a^5b^2x^2+2520a^6bx+330a^7}{3960x^{12}}$	80

input
```
int((b*x+a)^7/x^13,x,method=_RETURNVERBOSE)
```

output
```
1/x^12*(-1/5*b^7*x^7-7/6*a*b^6*x^6-3*a^2*b^5*x^5-35/8*a^3*b^4*x^4-35/9*a^4
*b^3*x^3-21/10*a^5*b^2*x^2-7/11*a^6*b*x-1/12*a^7)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+33}{3960\,x^{12}}$$

input
```
integrate((b*x+a)^7/x^13,x, algorithm="fricas")
```

output
```
-1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*
x^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = \frac{-330a^7-2520a^6bx-8316a^5b^2x^2-15400a^4b^3x^3-17325a^3b^4x^4-11880a^2b^5x^5-4620ab^6x^6-792b^7x^7}{3960x^{12}}$$

input
```
integrate((b*x+a)**7/x**13,x)
```

output
```
(-330*a**7 - 2520*a**6*b*x - 8316*a**5*b**2*x**2 - 15400*a**4*b**3*x**3 -
17325*a**3*b**4*x**4 - 11880*a**2*b**5*x**5 - 4620*a*b**6*x**6 - 792*b**7*
x**7)/(3960*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+33}{3960\,x^{12}}$$

input
```
integrate((b*x+a)^7/x^13,x, algorithm="maxima")
```

output
```
-1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*
x^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+33}{3960\,x^{12}}$$

input
```
integrate((b*x+a)^7/x^13,x, algorithm="giac")
```

output
```
-1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*
x^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{\frac{a^7}{12}+\frac{7\,a^6\,b\,x}{11}+\frac{21\,a^5\,b^2\,x^2}{10}+\frac{35\,a^4\,b^3\,x^3}{9}+\frac{35\,a^3\,b^4\,x^4}{8}+3\,a^2\,b^5\,x^5+\frac{7\,a\,b^6\,x^6}{6}+\frac{b^7\,x^7}{5}}{x^{12}}$$

input
```
int((a + b*x)^7/x^13,x)
```

output
```
-(a^7/12 + (b^7*x^7)/5 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/10 + (35*a^4*b
^3*x^3)/9 + (35*a^3*b^4*x^4)/8 + 3*a^2*b^5*x^5 + (7*a^6*b*x)/11)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = \frac{-792b^7x^7-4620a\,b^6x^6-11880a^2b^5x^5-17325a^3b^4x^4-15400a^4b^3x^3-8316a^5b^2x^2-2520a^6bx-330a^7}{3960x^{12}}$$

input
```
int((b*x+a)^7/x^13,x)
```

output
```
( - 330*a**7 - 2520*a**6*b*x - 8316*a**5*b**2*x**2 - 15400*a**4*b**3*x**3
- 17325*a**3*b**4*x**4 - 11880*a**2*b**5*x**5 - 4620*a*b**6*x**6 - 792*b**
7*x**7)/(3960*x**12)
```

3.78 $\int \frac{(a+bx)^7}{x^{14}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 93

$$\int \frac{(a+bx)^7}{x^{14}}\,dx = -\frac{a^7}{13x^{13}} - \frac{7a^6b}{12x^{12}} - \frac{21a^5b^2}{11x^{11}} - \frac{7a^4b^3}{2x^{10}} - \frac{35a^3b^4}{9x^9} - \frac{21a^2b^5}{8x^8} - \frac{ab^6}{x^7} - \frac{b^7}{6x^6}$$

output
```
-1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^
3*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^{14}}\,dx = -\frac{a^7}{13x^{13}} - \frac{7a^6b}{12x^{12}} - \frac{21a^5b^2}{11x^{11}} - \frac{7a^4b^3}{2x^{10}} - \frac{35a^3b^4}{9x^9} - \frac{21a^2b^5}{8x^8} - \frac{ab^6}{x^7} - \frac{b^7}{6x^6}$$

input
```
Integrate[(a + b*x)^7/x^14,x]
```

output
```
-1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3
)/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b
^7/(6*x^6)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{14}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^7}{x^{14}}+\frac{7a^6b}{x^{13}}+\frac{21a^5b^2}{x^{12}}+\frac{35a^4b^3}{x^{11}}+\frac{35a^3b^4}{x^{10}}+\frac{21a^2b^5}{x^9}+\frac{7ab^6}{x^8}+\frac{b^7}{x^7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

input
```
Int[(a + b*x)^7/x^14,x]
```

output
```
-1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3
)/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b
^7/(6*x^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

method	result	size
norman	$\frac{-\frac{1}{6}b^7x^7-a\,b^6x^6-\frac{21}{8}a^2b^5x^5-\frac{35}{9}a^3b^4x^4-\frac{7}{2}a^4b^3x^3-\frac{21}{11}a^5b^2x^2-\frac{7}{12}a^6bx-\frac{1}{13}a^7}{x^{13}}$	79
risch	$\frac{-\frac{1}{6}b^7x^7-a\,b^6x^6-\frac{21}{8}a^2b^5x^5-\frac{35}{9}a^3b^4x^4-\frac{7}{2}a^4b^3x^3-\frac{21}{11}a^5b^2x^2-\frac{7}{12}a^6bx-\frac{1}{13}a^7}{x^{13}}$	79
gosper	$-\frac{1716b^7x^7+10296a\,b^6x^6+27027a^2b^5x^5+40040a^3b^4x^4+36036a^4b^3x^3+19656a^5b^2x^2+6006a^6bx+792a^7}{10296x^{13}}$	80
default	$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{a\,b^6}{x^7}-\frac{b^7}{6x^6}$	80
parallelrisch	$\frac{-1716b^7x^7-10296a\,b^6x^6-27027a^2b^5x^5-40040a^3b^4x^4-36036a^4b^3x^3-19656a^5b^2x^2-6006a^6bx-792a^7}{10296x^{13}}$	80
orering	$-\frac{1716b^7x^7+10296a\,b^6x^6+27027a^2b^5x^5+40040a^3b^4x^4+36036a^4b^3x^3+19656a^5b^2x^2+6006a^6bx+792a^7}{10296x^{13}}$	80

input
```
int((b*x+a)^7/x^14,x,method=_RETURNVERBOSE)
```

output
```
1/x^13*(-1/6*b^7*x^7-a*b^6*x^6-21/8*a^2*b^5*x^5-35/9*a^3*b^4*x^4-7/2*a^4*b
^3*x^3-21/11*a^5*b^2*x^2-7/12*a^6*b*x-1/13*a^7)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^7}{x^{14}}\,dx = -\frac{1716\,b^7x^7+10296\,ab^6x^6+27027\,a^2b^5x^5+40040\,a^3b^4x^4+36036\,a^4b^3x^3+19656\,a^5b^2x^2+6006\,a^6bx+}{10296\,x^{13}}$$

input
```
integrate((b*x+a)^7/x^14,x, algorithm="fricas")
```

output
```
-1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b
^4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x
^13
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^7}{x^{14}}\,dx = \frac{-792a^7 - 6006a^6bx - 19656a^5b^2x^2 - 36036a^4b^3x^3 - 40040a^3b^4x^4 - 27027a^2b^5x^5 - 10296ab^6x^6 - 1716b^7x^7}{10296x^{13}}$$

input
```
integrate((b*x+a)**7/x**14,x)
```

output
```
(-792*a**7 - 6006*a**6*b*x - 19656*a**5*b**2*x**2 - 36036*a**4*b**3*x**3 -
 40040*a**3*b**4*x**4 - 27027*a**2*b**5*x**5 - 10296*a*b**6*x**6 - 1716*b*
*7*x**7)/(10296*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^7}{x^{14}}\,dx = -\frac{1716\,b^7x^7 + 10296\,ab^6x^6 + 27027\,a^2b^5x^5 + 40040\,a^3b^4x^4 + 36036\,a^4b^3x^3 + 19656\,a^5b^2x^2 + 6006\,a^6bx + 792\,a^7}{10296\,x^{13}}$$

input
```
integrate((b*x+a)^7/x^14,x, algorithm="maxima")
```

output
```
-1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b
^4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x
^13
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^7}{x^{14}}\,dx = -\frac{1716\,b^7x^7 + 10296\,ab^6x^6 + 27027\,a^2b^5x^5 + 40040\,a^3b^4x^4 + 36036\,a^4b^3x^3 + 19656\,a^5b^2x^2 + 6006\,a^6bx +}{10296\,x^{13}}$$

input
```
integrate((b*x+a)^7/x^14,x, algorithm="giac")
```

output
```
-1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b
^4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x
^13
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^7}{x^{14}}\,dx = -\frac{\frac{a^7}{13} + \frac{7\,a^6\,b\,x}{12} + \frac{21\,a^5\,b^2\,x^2}{11} + \frac{7\,a^4\,b^3\,x^3}{2} + \frac{35\,a^3\,b^4\,x^4}{9} + \frac{21\,a^2\,b^5\,x^5}{8} + a\,b^6\,x^6 + \frac{b^7\,x^7}{6}}{x^{13}}$$

input
```
int((a + b*x)^7/x^14,x)
```

output
```
-(a^7/13 + (b^7*x^7)/6 + a*b^6*x^6 + (21*a^5*b^2*x^2)/11 + (7*a^4*b^3*x^3)
/2 + (35*a^3*b^4*x^4)/9 + (21*a^2*b^5*x^5)/8 + (7*a^6*b*x)/12)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^7}{x^{14}}\,dx$$
$$= \frac{-1716b^7x^7 - 10296a\,b^6x^6 - 27027a^2b^5x^5 - 40040a^3b^4x^4 - 36036a^4b^3x^3 - 19656a^5b^2x^2 - 6006a^6bx - 79}{10296x^{13}}$$

input
```
int((b*x+a)^7/x^14,x)
```

output
```
( - 792*a**7 - 6006*a**6*b*x - 19656*a**5*b**2*x**2 - 36036*a**4*b**3*x**3
 - 40040*a**3*b**4*x**4 - 27027*a**2*b**5*x**5 - 10296*a*b**6*x**6 - 1716*
b**7*x**7)/(10296*x**13)
```

3.79 $\int \frac{(a+bx)^7}{x^{15}} dx$

Optimal result

Integrand size = 11, antiderivative size = 95

$$\int \frac{(a+bx)^7}{x^{15}} dx = -\frac{a^7}{14x^{14}} - \frac{7a^6b}{13x^{13}} - \frac{7a^5b^2}{4x^{12}} - \frac{35a^4b^3}{11x^{11}} - \frac{7a^3b^4}{2x^{10}} - \frac{7a^2b^5}{3x^9} - \frac{7ab^6}{8x^8} - \frac{b^7}{7x^7}$$

output
```
-1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3
*b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^{15}} dx = -\frac{a^7}{14x^{14}} - \frac{7a^6b}{13x^{13}} - \frac{7a^5b^2}{4x^{12}} - \frac{35a^4b^3}{11x^{11}} - \frac{7a^3b^4}{2x^{10}} - \frac{7a^2b^5}{3x^9} - \frac{7ab^6}{8x^8} - \frac{b^7}{7x^7}$$

input
```
Integrate[(a + b*x)^7/x^15,x]
```

output
```
-1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)
/(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8
) - b^7/(7*x^7)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{15}} \, dx$$
$$\Big\downarrow \; 53$$
$$\int \left(\frac{a^7}{x^{15}}+\frac{7a^6b}{x^{14}}+\frac{21a^5b^2}{x^{13}}+\frac{35a^4b^3}{x^{12}}+\frac{35a^3b^4}{x^{11}}+\frac{21a^2b^5}{x^{10}}+\frac{7ab^6}{x^9}+\frac{b^7}{x^8}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

input
```
Int[(a + b*x)^7/x^15,x]
```

output
```
-1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)
/(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8
) - b^7/(7*x^7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{-\frac{1}{14}a^7-\frac{7}{13}a^6bx-\frac{7}{4}a^5b^2x^2-\frac{35}{11}a^4b^3x^3-\frac{7}{2}a^3b^4x^4-\frac{7}{3}a^2b^5x^5-\frac{7}{8}a\,b^6x^6-\frac{1}{7}b^7x^7}{x^{14}}$	79
risch	$\frac{-\frac{1}{14}a^7-\frac{7}{13}a^6bx-\frac{7}{4}a^5b^2x^2-\frac{35}{11}a^4b^3x^3-\frac{7}{2}a^3b^4x^4-\frac{7}{3}a^2b^5x^5-\frac{7}{8}a\,b^6x^6-\frac{1}{7}b^7x^7}{x^{14}}$	79
gosper	$-\frac{3432b^7x^7+21021a\,b^6x^6+56056a^2b^5x^5+84084a^3b^4x^4+76440a^4b^3x^3+42042a^5b^2x^2+12936a^6bx+1716a^7}{24024x^{14}}$	80
default	$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a\,b^6}{8x^8}-\frac{b^7}{7x^7}$	80
parallelrisch	$\frac{-3432b^7x^7-21021a\,b^6x^6-56056a^2b^5x^5-84084a^3b^4x^4-76440a^4b^3x^3-42042a^5b^2x^2-12936a^6bx-1716a^7}{24024x^{14}}$	80
orering	$-\frac{3432b^7x^7+21021a\,b^6x^6+56056a^2b^5x^5+84084a^3b^4x^4+76440a^4b^3x^3+42042a^5b^2x^2+12936a^6bx+1716a^7}{24024x^{14}}$	80

input
```
int((b*x+a)^7/x^15,x,method=_RETURNVERBOSE)
```

output
```
1/x^14*(-1/14*a^7-7/13*a^6*b*x-7/4*a^5*b^2*x^2-35/11*a^4*b^3*x^3-7/2*a^3*b
^4*x^4-7/3*a^2*b^5*x^5-7/8*a*b^6*x^6-1/7*b^7*x^7)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{15}}\,dx = -\frac{3432\,b^7x^7+21021\,ab^6x^6+56056\,a^2b^5x^5+84084\,a^3b^4x^4+76440\,a^4b^3x^3+42042\,a^5b^2x^2+12936\,a^6bx}{24024\,x^{14}}$$

input
```
integrate((b*x+a)^7/x^15,x, algorithm="fricas")
```

output
```
-1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b
^4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)
/x^14
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^7}{x^{15}}\,dx$$
$$= \frac{-1716a^7 - 12936a^6bx - 42042a^5b^2x^2 - 76440a^4b^3x^3 - 84084a^3b^4x^4 - 56056a^2b^5x^5 - 21021ab^6x^6 - 343}{24024x^{14}}$$

input
```
integrate((b*x+a)**7/x**15,x)
```

output
```
(-1716*a**7 - 12936*a**6*b*x - 42042*a**5*b**2*x**2 - 76440*a**4*b**3*x**3
 - 84084*a**3*b**4*x**4 - 56056*a**2*b**5*x**5 - 21021*a*b**6*x**6 - 3432*
b**7*x**7)/(24024*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{15}}\,dx =$$
$$-\frac{3432\,b^7x^7 + 21021\,ab^6x^6 + 56056\,a^2b^5x^5 + 84084\,a^3b^4x^4 + 76440\,a^4b^3x^3 + 42042\,a^5b^2x^2 + 12936\,a^6bx}{24024\,x^{14}}$$

input
```
integrate((b*x+a)^7/x^15,x, algorithm="maxima")
```

output
```
-1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b
^4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)
/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{15}}\,dx = -\frac{3432\,b^7x^7+21021\,ab^6x^6+56056\,a^2b^5x^5+84084\,a^3b^4x^4+76440\,a^4b^3x^3+42042\,a^5b^2x^2+12936\,a^6bx}{24024\,x^{14}}$$

input
```
integrate((b*x+a)^7/x^15,x, algorithm="giac")
```

output
```
-1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b
^4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)
/x^14
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{15}}\,dx = -\frac{\frac{a^7}{14}+\frac{7\,a^6\,b\,x}{13}+\frac{7\,a^5\,b^2\,x^2}{4}+\frac{35\,a^4\,b^3\,x^3}{11}+\frac{7\,a^3\,b^4\,x^4}{2}+\frac{7\,a^2\,b^5\,x^5}{3}+\frac{7\,a\,b^6\,x^6}{8}+\frac{b^7\,x^7}{7}}{x^{14}}$$

input
```
int((a + b*x)^7/x^15,x)
```

output
```
-(a^7/14 + (b^7*x^7)/7 + (7*a*b^6*x^6)/8 + (7*a^5*b^2*x^2)/4 + (35*a^4*b^3
*x^3)/11 + (7*a^3*b^4*x^4)/2 + (7*a^2*b^5*x^5)/3 + (7*a^6*b*x)/13)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{15}}\,dx = \frac{-3432b^7x^7-21021a\,b^6x^6-56056a^2b^5x^5-84084a^3b^4x^4-76440a^4b^3x^3-42042a^5b^2x^2-12936a^6bx-1}{24024x^{14}}$$

input
```
int((b*x+a)^7/x^15,x)
```

output
```
( - 1716*a**7 - 12936*a**6*b*x - 42042*a**5*b**2*x**2 - 76440*a**4*b**3*x*
*3 - 84084*a**3*b**4*x**4 - 56056*a**2*b**5*x**5 - 21021*a*b**6*x**6 - 343
2*b**7*x**7)/(24024*x**14)
```

3.80 $\int \frac{(a+bx)^7}{x^{16}} dx$

Optimal result

Integrand size = 11, antiderivative size = 95

$$\int \frac{(a+bx)^7}{x^{16}} dx = -\frac{a^7}{15x^{15}} - \frac{a^6b}{2x^{14}} - \frac{21a^5b^2}{13x^{13}} - \frac{35a^4b^3}{12x^{12}} - \frac{35a^3b^4}{11x^{11}} - \frac{21a^2b^5}{10x^{10}} - \frac{7ab^6}{9x^9} - \frac{b^7}{8x^8}$$

output
```
-1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*
a^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{x^{16}} dx = -\frac{a^7}{15x^{15}} - \frac{a^6b}{2x^{14}} - \frac{21a^5b^2}{13x^{13}} - \frac{35a^4b^3}{12x^{12}} - \frac{35a^3b^4}{11x^{11}} - \frac{21a^2b^5}{10x^{10}} - \frac{7ab^6}{9x^9} - \frac{b^7}{8x^8}$$

input
```
Integrate[(a + b*x)^7/x^16,x]
```

output
```
-1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/
(12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9
*x^9) - b^7/(8*x^8)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{16}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^7}{x^{16}}+\frac{7a^6b}{x^{15}}+\frac{21a^5b^2}{x^{14}}+\frac{35a^4b^3}{x^{13}}+\frac{35a^3b^4}{x^{12}}+\frac{21a^2b^5}{x^{11}}+\frac{7ab^6}{x^{10}}+\frac{b^7}{x^9}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

input
```
Int[(a + b*x)^7/x^16,x]
```

output
```
-1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/
(12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9
*x^9) - b^7/(8*x^8)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{-\frac{1}{15}a^7-\frac{1}{2}a^6bx-\frac{21}{13}a^5b^2x^2-\frac{35}{12}a^4b^3x^3-\frac{35}{11}a^3b^4x^4-\frac{21}{10}a^2b^5x^5-\frac{7}{9}a\,b^6x^6-\frac{1}{8}b^7x^7}{x^{15}}$	79
risch	$\frac{-\frac{1}{15}a^7-\frac{1}{2}a^6bx-\frac{21}{13}a^5b^2x^2-\frac{35}{12}a^4b^3x^3-\frac{35}{11}a^3b^4x^4-\frac{21}{10}a^2b^5x^5-\frac{7}{9}a\,b^6x^6-\frac{1}{8}b^7x^7}{x^{15}}$	79
gosper	$-\frac{6435b^7x^7+40040a\,b^6x^6+108108a^2b^5x^5+163800a^3b^4x^4+150150a^4b^3x^3+83160a^5b^2x^2+25740a^6bx+3432a^7}{51480x^{15}}$	80
default	$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7a\,b^6}{9x^9}-\frac{b^7}{8x^8}$	80
parallelrisch	$\frac{-6435b^7x^7-40040a\,b^6x^6-108108a^2b^5x^5-163800a^3b^4x^4-150150a^4b^3x^3-83160a^5b^2x^2-25740a^6bx-3432a^7}{51480x^{15}}$	80
orering	$-\frac{6435b^7x^7+40040a\,b^6x^6+108108a^2b^5x^5+163800a^3b^4x^4+150150a^4b^3x^3+83160a^5b^2x^2+25740a^6bx+3432a^7}{51480x^{15}}$	80

input
```
int((b*x+a)^7/x^16,x,method=_RETURNVERBOSE)
```

output
```
1/x^15*(-1/15*a^7-1/2*a^6*b*x-21/13*a^5*b^2*x^2-35/12*a^4*b^3*x^3-35/11*a^
3*b^4*x^4-21/10*a^2*b^5*x^5-7/9*a*b^6*x^6-1/8*b^7*x^7)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{16}}\,dx = -\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6}{51480\,x^{15}}$$

input
```
integrate((b*x+a)^7/x^16,x, algorithm="fricas")
```

output
```
-1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3
*b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a
^7)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^7}{x^{16}}\,dx$$
$$= \frac{-3432a^7 - 25740a^6bx - 83160a^5b^2x^2 - 150150a^4b^3x^3 - 163800a^3b^4x^4 - 108108a^2b^5x^5 - 40040ab^6x^6 -}{51480x^{15}}$$

input
```
integrate((b*x+a)**7/x**16,x)
```

output
```
(-3432*a**7 - 25740*a**6*b*x - 83160*a**5*b**2*x**2 - 150150*a**4*b**3*x**
3 - 163800*a**3*b**4*x**4 - 108108*a**2*b**5*x**5 - 40040*a*b**6*x**6 - 64
35*b**7*x**7)/(51480*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{16}}\,dx =$$
$$-\frac{6435\,b^7x^7 + 40040\,ab^6x^6 + 108108\,a^2b^5x^5 + 163800\,a^3b^4x^4 + 150150\,a^4b^3x^3 + 83160\,a^5b^2x^2 + 25740\,a^6}{51480\,x^{15}}$$

input
```
integrate((b*x+a)^7/x^16,x, algorithm="maxima")
```

output
```
-1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3
*b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a
^7)/x^15
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{16}}\,dx = -\frac{6435\,b^7x^7 + 40040\,ab^6x^6 + 108108\,a^2b^5x^5 + 163800\,a^3b^4x^4 + 150150\,a^4b^3x^3 + 83160\,a^5b^2x^2 + 25740\,a^6}{51480\,x^{15}}$$

input
```
integrate((b*x+a)^7/x^16,x, algorithm="giac")
```

output
```
-1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3
*b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a
^7)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{16}}\,dx = -\frac{\frac{a^7}{15} + \frac{a^6\,b\,x}{2} + \frac{21\,a^5\,b^2\,x^2}{13} + \frac{35\,a^4\,b^3\,x^3}{12} + \frac{35\,a^3\,b^4\,x^4}{11} + \frac{21\,a^2\,b^5\,x^5}{10} + \frac{7\,a\,b^6\,x^6}{9} + \frac{b^7\,x^7}{8}}{x^{15}}$$

input
```
int((a + b*x)^7/x^16,x)
```

output
```
-(a^7/15 + (b^7*x^7)/8 + (7*a*b^6*x^6)/9 + (21*a^5*b^2*x^2)/13 + (35*a^4*b
^3*x^3)/12 + (35*a^3*b^4*x^4)/11 + (21*a^2*b^5*x^5)/10 + (a^6*b*x)/2)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^7}{x^{16}}\,dx$$
$$= \frac{-6435b^7x^7 - 40040a\,b^6x^6 - 108108a^2b^5x^5 - 163800a^3b^4x^4 - 150150a^4b^3x^3 - 83160a^5b^2x^2 - 25740a^6bx}{51480x^{15}}$$

input
```
int((b*x+a)^7/x^16,x)
```

output
```
( - 3432*a**7 - 25740*a**6*b*x - 83160*a**5*b**2*x**2 - 150150*a**4*b**3*x
**3 - 163800*a**3*b**4*x**4 - 108108*a**2*b**5*x**5 - 40040*a*b**6*x**6 -
6435*b**7*x**7)/(51480*x**15)
```

3.81 $\int x^{11}(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 132

$$\int x^{11}(a+bx)^{10}\,dx=\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

output
```
1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^
6*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a
^2*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\int x^{11}(a+bx)^{10}\,dx=\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

input
```
Integrate[x^11*(a + b*x)^10,x]
```

output
```
(a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^1
5 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (
120*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^2
2)/22
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}(a+bx)^{10}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int (a^{10}x^{11}+10a^9bx^{12}+45a^8b^2x^{13}+120a^7b^3x^{14}+210a^6b^4x^{15}+252a^5b^5x^{16}+210a^4b^6x^{17}+120a^3b^7x^{18}+45a^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

input
```
Int[x^11*(a + b*x)^10,x]
```

output
```
(a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^1
5 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (
120*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^2
2)/22
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a$
default	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a$
norman	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a$
risch	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a$
parallelrisch	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a$
orering	$\frac{x^{12}(352716b^{10}x^{10}+3695120a\,b^{9}x^{9}+17459442a^{2}b^{8}x^{8}+49008960a^{3}b^{7}x^{7}+90530440a^{4}b^{6}x^{6}+115026912a^{5}b^{5}x^{5}+101846745a^{6}}{7759752}$

input
```
int(x^11*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^
6*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a
^2*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{11}(a+bx)^{10}\,dx=\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^9x^{21}+\frac{9}{4}\,a^2b^8x^{20}+\frac{120}{19}\,a^3b^7x^{19}+\frac{35}{3}\,a^4b^6x^{18}+\frac{252}{17}\,a^5b^5x^{17}+\frac{105}{8}\,a^6b^4x^{16}+8\,a^7b^3x^{15}+\frac{45}{14}\,a^8b^2x^{14}+\frac{10}{13}\,a^9bx^{13}+\frac{1}{12}\,a^{10}x^{12}$$

input
```
integrate(x^11*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
 + 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^
3*x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01

$$\int x^{11}(a+bx)^{10}\,dx=\frac{a^{10}x^{12}}{12}+\frac{10a^9bx^{13}}{13}+\frac{45a^8b^2x^{14}}{14}+8a^7b^3x^{15}+\frac{105a^6b^4x^{16}}{8}+\frac{252a^5b^5x^{17}}{17}+\frac{35a^4b^6x^{18}}{3}+\frac{120a^3b^7x^{19}}{19}+\frac{9a^2b^8x^{20}}{4}+\frac{10ab^9x^{21}}{21}+\frac{b^{10}x^{22}}{22}$$

input
```
integrate(x**11*(b*x+a)**10,x)
```

output
```
a**10*x**12/12 + 10*a**9*b*x**13/13 + 45*a**8*b**2*x**14/14 + 8*a**7*b**3*
x**15 + 105*a**6*b**4*x**16/8 + 252*a**5*b**5*x**17/17 + 35*a**4*b**6*x**1
8/3 + 120*a**3*b**7*x**19/19 + 9*a**2*b**8*x**20/4 + 10*a*b**9*x**21/21 +
b**10*x**22/22
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{11}(a+bx)^{10}\,dx = \frac{1}{22}\,b^{10}x^{22} + \frac{10}{21}\,ab^{9}x^{21} + \frac{9}{4}\,a^{2}b^{8}x^{20} + \frac{120}{19}\,a^{3}b^{7}x^{19} + \frac{35}{3}\,a^{4}b^{6}x^{18} + \frac{252}{17}\,a^{5}b^{5}x^{17} + \frac{105}{8}\,a^{6}b^{4}x^{16} + 8\,a^{7}b^{3}x^{15} + \frac{45}{14}\,a^{8}b^{2}x^{14} + \frac{10}{13}\,a^{9}bx^{13} + \frac{1}{12}\,a^{10}x^{12}$$

input
```
integrate(x^11*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
 + 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^
3*x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{11}(a+bx)^{10}\,dx = \frac{1}{22}\,b^{10}x^{22} + \frac{10}{21}\,ab^{9}x^{21} + \frac{9}{4}\,a^{2}b^{8}x^{20} + \frac{120}{19}\,a^{3}b^{7}x^{19} + \frac{35}{3}\,a^{4}b^{6}x^{18} + \frac{252}{17}\,a^{5}b^{5}x^{17} + \frac{105}{8}\,a^{6}b^{4}x^{16} + 8\,a^{7}b^{3}x^{15} + \frac{45}{14}\,a^{8}b^{2}x^{14} + \frac{10}{13}\,a^{9}bx^{13} + \frac{1}{12}\,a^{10}x^{12}$$

input
```
integrate(x^11*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
 + 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^
3*x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{11}(a+bx)^{10}\,dx=\frac{a^{10}\,x^{12}}{12}+\frac{10\,a^9\,b\,x^{13}}{13}+\frac{45\,a^8\,b^2\,x^{14}}{14}+8\,a^7\,b^3\,x^{15}+\frac{105\,a^6\,b^4\,x^{16}}{8}+\frac{252\,a^5\,b^5\,x^{17}}{17}+\frac{35\,a^4\,b^6\,x^{18}}{3}+\frac{120\,a^3\,b^7\,x^{19}}{19}+\frac{9\,a^2\,b^8\,x^{20}}{4}+\frac{10\,a\,b^9\,x^{21}}{21}+\frac{b^{10}\,x^{22}}{22}$$

input
```
int(x^11*(a + b*x)^10,x)
```

output
```
(a^10*x^12)/12 + (b^10*x^22)/22 + (10*a^9*b*x^13)/13 + (10*a*b^9*x^21)/21
+ (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15 + (105*a^6*b^4*x^16)/8 + (252*a^5*
b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (120*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^
20)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{11}(a+bx)^{10}\,dx=\frac{x^{12}(352716b^{10}x^{10}+3695120a\,b^9x^9+17459442a^2b^8x^8+49008960a^3b^7x^7+90530440a^4b^6x^6+115026912}{7759752}$$

input
```
int(x^11*(b*x+a)^10,x)
```

output
```
(x**12*(646646*a**10 + 5969040*a**9*b*x + 24942060*a**8*b**2*x**2 + 620780
16*a**7*b**3*x**3 + 101846745*a**6*b**4*x**4 + 115026912*a**5*b**5*x**5 +
90530440*a**4*b**6*x**6 + 49008960*a**3*b**7*x**7 + 17459442*a**2*b**8*x**
8 + 3695120*a*b**9*x**9 + 352716*b**10*x**10))/7759752
```

3.82 $\int x^{10}(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 132

$$\int x^{10}(a+bx)^{10}\,dx=\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

output
```
1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*
b^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2
*b^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\int x^{10}(a+bx)^{10}\,dx=\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

input
```
Integrate[x^10*(a + b*x)^10,x]
```

output
```
(a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^1
4)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20
*a^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}(a+bx)^{10}\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left(a^{10}x^{10}+10a^9bx^{11}+45a^8b^2x^{12}+120a^7b^3x^{13}+210a^6b^4x^{14}+252a^5b^5x^{15}+210a^4b^6x^{16}+120a^3b^7x^{17}+45a^2\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

input
```
Int[x^10*(a + b*x)^10,x]
```

output

```
(a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^1
4)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20
*a^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}$
default	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}$
norman	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}$
risch	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}$
parallelrisch	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}$
orering	$\frac{x^{11}(184756b^{10}x^{10}+1939938a\,b^{9}x^{9}+9189180a^{2}b^{8}x^{8}+25865840a^{3}b^{7}x^{7}+47927880a^{4}b^{6}x^{6}+61108047a^{5}b^{5}x^{5}+54318264a^{6}b^{4}x}{3879876}$

input
```
int(x^10*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*
b^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2
*b^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{10}(a+bx)^{10}\,dx = \frac{1}{21}\,b^{10}x^{21} + \frac{1}{2}\,ab^{9}x^{20} + \frac{45}{19}\,a^{2}b^{8}x^{19} + \frac{20}{3}\,a^{3}b^{7}x^{18} + \frac{210}{17}\,a^{4}b^{6}x^{17} + \frac{63}{4}\,a^{5}b^{5}x^{16} + 14\,a^{6}b^{4}x^{15} + \frac{60}{7}\,a^{7}b^{3}x^{14} + \frac{45}{13}\,a^{8}b^{2}x^{13} + \frac{5}{6}\,a^{9}bx^{12} + \frac{1}{11}\,a^{10}x^{11}$$

input
```
integrate(x^10*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
 210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*
x^14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int x^{10}(a+bx)^{10}\,dx = \frac{a^{10}x^{11}}{11} + \frac{5a^{9}bx^{12}}{6} + \frac{45a^{8}b^{2}x^{13}}{13} + \frac{60a^{7}b^{3}x^{14}}{7} + 14a^{6}b^{4}x^{15} + \frac{63a^{5}b^{5}x^{16}}{4} + \frac{210a^{4}b^{6}x^{17}}{17} + \frac{20a^{3}b^{7}x^{18}}{3} + \frac{45a^{2}b^{8}x^{19}}{19} + \frac{ab^{9}x^{20}}{2} + \frac{b^{10}x^{21}}{21}$$

input
```
integrate(x**10*(b*x+a)**10,x)
```

output
```
a**10*x**11/11 + 5*a**9*b*x**12/6 + 45*a**8*b**2*x**13/13 + 60*a**7*b**3*x
**14/7 + 14*a**6*b**4*x**15 + 63*a**5*b**5*x**16/4 + 210*a**4*b**6*x**17/1
7 + 20*a**3*b**7*x**18/3 + 45*a**2*b**8*x**19/19 + a*b**9*x**20/2 + b**10*
x**21/21
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{10}(a+bx)^{10}\,dx = \frac{1}{21}\,b^{10}x^{21} + \frac{1}{2}\,ab^9x^{20} + \frac{45}{19}\,a^2b^8x^{19} + \frac{20}{3}\,a^3b^7x^{18} + \frac{210}{17}\,a^4b^6x^{17} + \frac{63}{4}\,a^5b^5x^{16} + 14\,a^6b^4x^{15} + \frac{60}{7}\,a^7b^3x^{14} + \frac{45}{13}\,a^8b^2x^{13} + \frac{5}{6}\,a^9bx^{12} + \frac{1}{11}\,a^{10}x^{11}$$

input
```
integrate(x^10*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
 210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*
x^14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{10}(a+bx)^{10}\,dx = \frac{1}{21}\,b^{10}x^{21} + \frac{1}{2}\,ab^9x^{20} + \frac{45}{19}\,a^2b^8x^{19} + \frac{20}{3}\,a^3b^7x^{18} + \frac{210}{17}\,a^4b^6x^{17} + \frac{63}{4}\,a^5b^5x^{16} + 14\,a^6b^4x^{15} + \frac{60}{7}\,a^7b^3x^{14} + \frac{45}{13}\,a^8b^2x^{13} + \frac{5}{6}\,a^9bx^{12} + \frac{1}{11}\,a^{10}x^{11}$$

input
```
integrate(x^10*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
 210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*
x^14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{10}(a+bx)^{10}\,dx = \frac{a^{10}\,x^{11}}{11}+\frac{5\,a^9\,b\,x^{12}}{6}+\frac{45\,a^8\,b^2\,x^{13}}{13}+\frac{60\,a^7\,b^3\,x^{14}}{7}+14\,a^6\,b^4\,x^{15}+\frac{63\,a^5\,b^5\,x^{16}}{4}+\frac{210\,a^4\,b^6\,x^{17}}{17}+\frac{20\,a^3\,b^7\,x^{18}}{3}+\frac{45\,a^2\,b^8\,x^{19}}{19}+\frac{a\,b^9\,x^{20}}{2}+\frac{b^{10}\,x^{21}}{21}$$

input
```
int(x^10*(a + b*x)^10,x)
```

output
```
(a^10*x^11)/11 + (b^10*x^21)/21 + (5*a^9*b*x^12)/6 + (a*b^9*x^20)/2 + (45*
a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^1
6)/4 + (210*a^4*b^6*x^17)/17 + (20*a^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^{10}(a+bx)^{10}\,dx = \frac{x^{11}(184756b^{10}x^{10}+1939938a\,b^9x^9+9189180a^2b^8x^8+25865840a^3b^7x^7+47927880a^4b^6x^6+61108047a^5}{3879876}$$

input
```
int(x^10*(b*x+a)^10,x)
```

output
```
(x**11*(352716*a**10 + 3233230*a**9*b*x + 13430340*a**8*b**2*x**2 + 332560
80*a**7*b**3*x**3 + 54318264*a**6*b**4*x**4 + 61108047*a**5*b**5*x**5 + 47
927880*a**4*b**6*x**6 + 25865840*a**3*b**7*x**7 + 9189180*a**2*b**8*x**8 +
 1939938*a*b**9*x**9 + 184756*b**10*x**10))/3879876
```

3.83 $\int x^9(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 132

$$\int x^9(a+bx)^{10}\,dx=\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

output
```
1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a
^6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a
^2*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\int x^9(a+bx)^{10}\,dx=\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

input
```
Integrate[x^9*(a + b*x)^10,x]
```

output
```
(a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x
^13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (
120*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^2
0)/20
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9(a+bx)^{10}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^{10}x^9+10a^9bx^{10}+45a^8b^2x^{11}+120a^7b^3x^{12}+210a^6b^4x^{13}+252a^5b^5x^{14}+210a^4b^6x^{15}+120a^3b^7x^{16}+45a^2b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

input
```
Int[x^9*(a + b*x)^10,x]
```

output
```
(a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x
^13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (
120*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^2
0)/20
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}$
default	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}$
norman	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}$
risch	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}$
parallelrisch	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}$
orering	$\frac{x^{10}(92378b^{10}x^{10}+972400a\,b^9x^9+4618900a^2b^8x^8+13041600a^3b^7x^7+24249225a^4b^6x^6+31039008a^5b^5x^5+27713400a^6b^4x^4+}{1847560}$

input
```
int(x^9*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a
^6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a
^2*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^9(a+bx)^{10}\,dx = \frac{1}{20}\,b^{10}x^{20} + \frac{10}{19}\,ab^9x^{19} + \frac{5}{2}\,a^2b^8x^{18} + \frac{120}{17}\,a^3b^7x^{17} + \frac{105}{8}\,a^4b^6x^{16} + \frac{84}{5}\,a^5b^5x^{15} + 15\,a^6b^4x^{14} + \frac{120}{13}\,a^7b^3x^{13} + \frac{15}{4}\,a^8b^2x^{12} + \frac{10}{11}\,a^9bx^{11} + \frac{1}{10}\,a^{10}x^{10}$$

input
```
integrate(x^9*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
 + 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b
^3*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01

$$\int x^9(a+bx)^{10}\,dx = \frac{a^{10}x^{10}}{10} + \frac{10a^9bx^{11}}{11} + \frac{15a^8b^2x^{12}}{4} + \frac{120a^7b^3x^{13}}{13} + 15a^6b^4x^{14} + \frac{84a^5b^5x^{15}}{5} + \frac{105a^4b^6x^{16}}{8} + \frac{120a^3b^7x^{17}}{17} + \frac{5a^2b^8x^{18}}{2} + \frac{10ab^9x^{19}}{19} + \frac{b^{10}x^{20}}{20}$$

input
```
integrate(x**9*(b*x+a)**10,x)
```

output
```
a**10*x**10/10 + 10*a**9*b*x**11/11 + 15*a**8*b**2*x**12/4 + 120*a**7*b**3
*x**13/13 + 15*a**6*b**4*x**14 + 84*a**5*b**5*x**15/5 + 105*a**4*b**6*x**1
6/8 + 120*a**3*b**7*x**17/17 + 5*a**2*b**8*x**18/2 + 10*a*b**9*x**19/19 +
b**10*x**20/20
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^9(a+bx)^{10}\,dx = \frac{1}{20}\,b^{10}x^{20} + \frac{10}{19}\,ab^9x^{19} + \frac{5}{2}\,a^2b^8x^{18} + \frac{120}{17}\,a^3b^7x^{17} + \frac{105}{8}\,a^4b^6x^{16} + \frac{84}{5}\,a^5b^5x^{15} + 15\,a^6b^4x^{14} + \frac{120}{13}\,a^7b^3x^{13} + \frac{15}{4}\,a^8b^2x^{12} + \frac{10}{11}\,a^9bx^{11} + \frac{1}{10}\,a^{10}x^{10}$$

input
```
integrate(x^9*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
 + 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b
^3*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^9(a+bx)^{10}\,dx = \frac{1}{20}\,b^{10}x^{20} + \frac{10}{19}\,ab^9x^{19} + \frac{5}{2}\,a^2b^8x^{18} + \frac{120}{17}\,a^3b^7x^{17} + \frac{105}{8}\,a^4b^6x^{16} + \frac{84}{5}\,a^5b^5x^{15} + 15\,a^6b^4x^{14} + \frac{120}{13}\,a^7b^3x^{13} + \frac{15}{4}\,a^8b^2x^{12} + \frac{10}{11}\,a^9bx^{11} + \frac{1}{10}\,a^{10}x^{10}$$

input
```
integrate(x^9*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
 + 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b
^3*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^9(a+bx)^{10}\,dx=\frac{a^{10}\,x^{10}}{10}+\frac{10\,a^9\,b\,x^{11}}{11}+\frac{15\,a^8\,b^2\,x^{12}}{4}+\frac{120\,a^7\,b^3\,x^{13}}{13}+15\,a^6\,b^4\,x^{14}+\frac{84\,a^5\,b^5\,x^{15}}{5}+\frac{105\,a^4\,b^6\,x^{16}}{8}+\frac{120\,a^3\,b^7\,x^{17}}{17}+\frac{5\,a^2\,b^8\,x^{18}}{2}+\frac{10\,a\,b^9\,x^{19}}{19}+\frac{b^{10}\,x^{20}}{20}$$

input
```
int(x^9*(a + b*x)^10,x)
```

output
```
(a^10*x^10)/10 + (b^10*x^20)/20 + (10*a^9*b*x^11)/11 + (10*a*b^9*x^19)/19
+ (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^13)/13 + 15*a^6*b^4*x^14 + (84*a^5*
b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (120*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^
18)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^9(a+bx)^{10}\,dx = \frac{x^{10}(92378b^{10}x^{10}+972400a\,b^9x^9+4618900a^2b^8x^8+13041600a^3b^7x^7+24249225a^4b^6x^6+31039008a^5b^5}{1847560}$$

input
```
int(x^9*(b*x+a)^10,x)
```

output
```
(x**10*(184756*a**10 + 1679600*a**9*b*x + 6928350*a**8*b**2*x**2 + 1705440
0*a**7*b**3*x**3 + 27713400*a**6*b**4*x**4 + 31039008*a**5*b**5*x**5 + 242
49225*a**4*b**6*x**6 + 13041600*a**3*b**7*x**7 + 4618900*a**2*b**8*x**8 +
972400*a*b**9*x**9 + 92378*b**10*x**10))/1847560
```

3.84 $\int x^8(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 147

$$\int x^8(a+bx)^{10}\,dx=\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)^{17}}{17b^9}-\frac{4a(a+bx)^{18}}{9b^9}+\frac{(a+bx)^{19}}{19b^9}$$

output
```
1/11*a^8*(b*x+a)^11/b^9-2/3*a^7*(b*x+a)^12/b^9+28/13*a^6*(b*x+a)^13/b^9-4*
a^5*(b*x+a)^14/b^9+14/3*a^4*(b*x+a)^15/b^9-7/2*a^3*(b*x+a)^16/b^9+28/17*a^
2*(b*x+a)^17/b^9-4/9*a*(b*x+a)^18/b^9+1/19*(b*x+a)^19/b^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int x^8(a+bx)^{10}\,dx=\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}a^2b^8x^{17}+\frac{5}{9}ab^9x^{18}+\frac{b^{10}x^{19}}{19}$$

input
```
Integrate[x^8*(a + b*x)^10,x]
```

output
```
(a^10*x^9)/9 + a^9*b*x^10 + (45*a^8*b^2*x^11)/11 + 10*a^7*b^3*x^12 + (210*
a^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2
 + (45*a^2*b^8*x^17)/17 + (5*a*b^9*x^18)/9 + (b^10*x^19)/19
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8(a+bx)^{10}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^8(a+bx)^{10}}{b^8}-\frac{8a^7(a+bx)^{11}}{b^8}+\frac{28a^6(a+bx)^{12}}{b^8}-\frac{56a^5(a+bx)^{13}}{b^8}+\frac{70a^4(a+bx)^{14}}{b^8}-\frac{56a^3(a+bx)^{15}}{b^8}+\frac{2\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)^{17}}{17b^9}+\frac{(a+bx)^{19}}{19b^9}-\frac{4a(a+bx)^{18}}{9b^9}$$

input
```
Int[x^8*(a + b*x)^10,x]
```

output
```
(a^8*(a + b*x)^11)/(11*b^9) - (2*a^7*(a + b*x)^12)/(3*b^9) + (28*a^6*(a +
b*x)^13)/(13*b^9) - (4*a^5*(a + b*x)^14)/b^9 + (14*a^4*(a + b*x)^15)/(3*b^
9) - (7*a^3*(a + b*x)^16)/(2*b^9) + (28*a^2*(a + b*x)^17)/(17*b^9) - (4*a*
(a + b*x)^18)/(9*b^9) + (a + b*x)^19/(19*b^9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{1}{9}a^{10}x^9 + a^9b\,x^{10} + \frac{45}{11}a^8b^2x^{11} + 10a^7b^3x^{12} + \frac{210}{13}a^6b^4x^{13} + 18a^5b^5x^{14} + 14a^4b^6x^{15} + \frac{15}{2}a^3b^7x$
default	$\frac{1}{9}a^{10}x^9 + a^9b\,x^{10} + \frac{45}{11}a^8b^2x^{11} + 10a^7b^3x^{12} + \frac{210}{13}a^6b^4x^{13} + 18a^5b^5x^{14} + 14a^4b^6x^{15} + \frac{15}{2}a^3b^7x$
norman	$\frac{1}{9}a^{10}x^9 + a^9b\,x^{10} + \frac{45}{11}a^8b^2x^{11} + 10a^7b^3x^{12} + \frac{210}{13}a^6b^4x^{13} + 18a^5b^5x^{14} + 14a^4b^6x^{15} + \frac{15}{2}a^3b^7x$
risch	$\frac{1}{9}a^{10}x^9 + a^9b\,x^{10} + \frac{45}{11}a^8b^2x^{11} + 10a^7b^3x^{12} + \frac{210}{13}a^6b^4x^{13} + 18a^5b^5x^{14} + 14a^4b^6x^{15} + \frac{15}{2}a^3b^7x$
parallelrisch	$\frac{1}{9}a^{10}x^9 + a^9b\,x^{10} + \frac{45}{11}a^8b^2x^{11} + 10a^7b^3x^{12} + \frac{210}{13}a^6b^4x^{13} + 18a^5b^5x^{14} + 14a^4b^6x^{15} + \frac{15}{2}a^3b^7x$
orering	$\frac{x^9(43758b^{10}x^{10}+461890a\,b^9x^9+2200770a^2b^8x^8+6235515a^3b^7x^7+11639628a^4b^6x^6+14965236a^5b^5x^5+13430340a^6b^4x^4+8}{831402}$

input
```
int(x^8*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^10*x^9+a^9*b*x^10+45/11*a^8*b^2*x^11+10*a^7*b^3*x^12+210/13*a^6*b^4*
x^13+18*a^5*b^5*x^14+14*a^4*b^6*x^15+15/2*a^3*b^7*x^16+45/17*a^2*b^8*x^17+
5/9*a*b^9*x^18+1/19*b^10*x^19
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int x^8(a+bx)^{10}\,dx=\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^9x^{18}+\frac{45}{17}\,a^2b^8x^{17}+\frac{15}{2}\,a^3b^7x^{16}+14\,a^4b^6x^{15}+18\,a^5b^5x^{14}+\frac{210}{13}\,a^6b^4x^{13}+10\,a^7b^3x^{12}+\frac{45}{11}\,a^8b^2x^{11}+a^9bx^{10}+\frac{1}{9}\,a^{10}x^9$$

input
```
integrate(x^8*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
 14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12
 + 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int x^8(a+bx)^{10}\,dx=\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45a^8b^2x^{11}}{11}+10a^7b^3x^{12}+\frac{210a^6b^4x^{13}}{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15a^3b^7x^{16}}{2}+\frac{45a^2b^8x^{17}}{17}+\frac{5ab^9x^{18}}{9}+\frac{b^{10}x^{19}}{19}$$

input
```
integrate(x**8*(b*x+a)**10,x)
```

output
```
a**10*x**9/9 + a**9*b*x**10 + 45*a**8*b**2*x**11/11 + 10*a**7*b**3*x**12 +
 210*a**6*b**4*x**13/13 + 18*a**5*b**5*x**14 + 14*a**4*b**6*x**15 + 15*a**
3*b**7*x**16/2 + 45*a**2*b**8*x**17/17 + 5*a*b**9*x**18/9 + b**10*x**19/19
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int x^8(a+bx)^{10}\,dx = \frac{1}{19}\,b^{10}x^{19} + \frac{5}{9}\,ab^9x^{18} + \frac{45}{17}\,a^2b^8x^{17} + \frac{15}{2}\,a^3b^7x^{16} + 14\,a^4b^6x^{15} + 18\,a^5b^5x^{14} + \frac{210}{13}\,a^6b^4x^{13} + 10\,a^7b^3x^{12} + \frac{45}{11}\,a^8b^2x^{11} + a^9bx^{10} + \frac{1}{9}\,a^{10}x^9$$

input
```
integrate(x^8*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
 14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12
 + 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int x^8(a+bx)^{10}\,dx = \frac{1}{19}\,b^{10}x^{19} + \frac{5}{9}\,ab^9x^{18} + \frac{45}{17}\,a^2b^8x^{17} + \frac{15}{2}\,a^3b^7x^{16} + 14\,a^4b^6x^{15} + 18\,a^5b^5x^{14} + \frac{210}{13}\,a^6b^4x^{13} + 10\,a^7b^3x^{12} + \frac{45}{11}\,a^8b^2x^{11} + a^9bx^{10} + \frac{1}{9}\,a^{10}x^9$$

input
```
integrate(x^8*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
 14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12
 + 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int x^8(a+bx)^{10}\,dx=\frac{a^{10}\,x^9}{9}+a^9\,b\,x^{10}+\frac{45\,a^8\,b^2\,x^{11}}{11}+10\,a^7\,b^3\,x^{12}+\frac{210\,a^6\,b^4\,x^{13}}{13}+18\,a^5\,b^5\,x^{14}+14\,a^4\,b^6\,x^{15}+\frac{15\,a^3\,b^7\,x^{16}}{2}+\frac{45\,a^2\,b^8\,x^{17}}{17}+\frac{5\,a\,b^9\,x^{18}}{9}+\frac{b^{10}\,x^{19}}{19}$$

input
```
int(x^8*(a + b*x)^10,x)
```

output
```
(a^10*x^9)/9 + (b^10*x^19)/19 + a^9*b*x^10 + (5*a*b^9*x^18)/9 + (45*a^8*b^
2*x^11)/11 + 10*a^7*b^3*x^12 + (210*a^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 1
4*a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 + (45*a^2*b^8*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int x^8(a+bx)^{10}\,dx=\frac{x^9(43758b^{10}x^{10}+461890a\,b^9x^9+2200770a^2b^8x^8+6235515a^3b^7x^7+11639628a^4b^6x^6+14965236a^5b^5x^5}{831402}$$

input
```
int(x^8*(b*x+a)^10,x)
```

output
```
(x**9*(92378*a**10 + 831402*a**9*b*x + 3401190*a**8*b**2*x**2 + 8314020*a*
*7*b**3*x**3 + 13430340*a**6*b**4*x**4 + 14965236*a**5*b**5*x**5 + 1163962
8*a**4*b**6*x**6 + 6235515*a**3*b**7*x**7 + 2200770*a**2*b**8*x**8 + 46189
0*a*b**9*x**9 + 43758*b**10*x**10))/831402
```

3.85 $\int x^7(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 132

$$\int x^7(a+bx)^{10}\,dx=-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}-\frac{7a(a+bx)^{17}}{17b^8}+\frac{(a+bx)^{18}}{18b^8}$$

output
```
-1/11*a^7*(b*x+a)^11/b^8+7/12*a^6*(b*x+a)^12/b^8-21/13*a^5*(b*x+a)^13/b^8+
5/2*a^4*(b*x+a)^14/b^8-7/3*a^3*(b*x+a)^15/b^8+21/16*a^2*(b*x+a)^16/b^8-7/1
7*a*(b*x+a)^17/b^8+1/18*(b*x+a)^18/b^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int x^7(a+bx)^{10}\,dx=\frac{a^{10}x^8}{8}+\frac{10}{9}a^9bx^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}a^2b^8x^{16}+\frac{10}{17}ab^9x^{17}+\frac{b^{10}x^{18}}{18}$$

input
```
Integrate[x^7*(a + b*x)^10,x]
```

output
```
(a^10*x^8)/8 + (10*a^9*b*x^9)/9 + (9*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/
11 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3
*b^7*x^15 + (45*a^2*b^8*x^16)/16 + (10*a*b^9*x^17)/17 + (b^10*x^18)/18
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a+bx)^{10} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^7 (a+bx)^{10}}{b^7}+\frac{7 a^6 (a+bx)^{11}}{b^7}-\frac{21 a^5 (a+bx)^{12}}{b^7}+\frac{35 a^4 (a+bx)^{13}}{b^7}-\frac{35 a^3 (a+bx)^{14}}{b^7}+\frac{21 a^2 (a+bx)^{15}}{b^7}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^7 (a+bx)^{11}}{11 b^8}+\frac{7 a^6 (a+bx)^{12}}{12 b^8}-\frac{21 a^5 (a+bx)^{13}}{13 b^8}+\frac{5 a^4 (a+bx)^{14}}{2 b^8}-\frac{7 a^3 (a+bx)^{15}}{3 b^8}+\frac{21 a^2 (a+bx)^{16}}{16 b^8}+\frac{(a+bx)^{18}}{18 b^8}-\frac{7 a (a+bx)^{17}}{17 b^8}$$

input
```
Int[x^7*(a + b*x)^10,x]
```

output
```
-1/11*(a^7*(a + b*x)^11)/b^8 + (7*a^6*(a + b*x)^12)/(12*b^8) - (21*a^5*(a
+ b*x)^13)/(13*b^8) + (5*a^4*(a + b*x)^14)/(2*b^8) - (7*a^3*(a + b*x)^15)/
(3*b^8) + (21*a^2*(a + b*x)^16)/(16*b^8) - (7*a*(a + b*x)^17)/(17*b^8) + (
a + b*x)^18/(18*b^8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x$
default	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x$
norman	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x$
risch	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x$
parallelrisch	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x$
orering	$\frac{x^8(19448b^{10}x^{10}+205920a\,b^9x^9+984555a^2b^8x^8+2800512a^3b^7x^7+5250960a^4b^6x^6+6785856a^5b^5x^5+6126120a^6b^4x^4+38188}{350064}$

input
```
int(x^7*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^10*x^8+10/9*a^9*b*x^9+9/2*a^8*b^2*x^10+120/11*a^7*b^3*x^11+35/2*a^6*
b^4*x^12+252/13*a^5*b^5*x^13+15*a^4*b^6*x^14+8*a^3*b^7*x^15+45/16*a^2*b^8*
x^16+10/17*a*b^9*x^17+1/18*b^10*x^18
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^7(a+bx)^{10}\,dx = \frac{1}{18}\,b^{10}x^{18} + \frac{10}{17}\,ab^9x^{17} + \frac{45}{16}\,a^2b^8x^{16} + 8\,a^3b^7x^{15} + 15\,a^4b^6x^{14} + \frac{252}{13}\,a^5b^5x^{13} + \frac{35}{2}\,a^6b^4x^{12} + \frac{120}{11}\,a^7b^3x^{11} + \frac{9}{2}\,a^8b^2x^{10} + \frac{10}{9}\,a^9bx^9 + \frac{1}{8}\,a^{10}x^8$$

input
```
integrate(x^7*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 +
15*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3
*x^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int x^7(a+bx)^{10}\,dx = \frac{a^{10}x^8}{8} + \frac{10a^9bx^9}{9} + \frac{9a^8b^2x^{10}}{2} + \frac{120a^7b^3x^{11}}{11} + \frac{35a^6b^4x^{12}}{2} + \frac{252a^5b^5x^{13}}{13} + 15a^4b^6x^{14} + 8a^3b^7x^{15} + \frac{45a^2b^8x^{16}}{16} + \frac{10ab^9x^{17}}{17} + \frac{b^{10}x^{18}}{18}$$

input
```
integrate(x**7*(b*x+a)**10,x)
```

output
```
a**10*x**8/8 + 10*a**9*b*x**9/9 + 9*a**8*b**2*x**10/2 + 120*a**7*b**3*x**1
1/11 + 35*a**6*b**4*x**12/2 + 252*a**5*b**5*x**13/13 + 15*a**4*b**6*x**14
+ 8*a**3*b**7*x**15 + 45*a**2*b**8*x**16/16 + 10*a*b**9*x**17/17 + b**10*x
**18/18
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^7(a+bx)^{10}\,dx = \frac{1}{18}\,b^{10}x^{18} + \frac{10}{17}\,ab^9x^{17} + \frac{45}{16}\,a^2b^8x^{16} + 8\,a^3b^7x^{15} + 15\,a^4b^6x^{14} + \frac{252}{13}\,a^5b^5x^{13} + \frac{35}{2}\,a^6b^4x^{12} + \frac{120}{11}\,a^7b^3x^{11} + \frac{9}{2}\,a^8b^2x^{10} + \frac{10}{9}\,a^9bx^9 + \frac{1}{8}\,a^{10}x^8$$

input
```
integrate(x^7*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 +
15*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3
*x^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^7(a+bx)^{10}\,dx = \frac{1}{18}\,b^{10}x^{18} + \frac{10}{17}\,ab^9x^{17} + \frac{45}{16}\,a^2b^8x^{16} + 8\,a^3b^7x^{15} + 15\,a^4b^6x^{14} + \frac{252}{13}\,a^5b^5x^{13} + \frac{35}{2}\,a^6b^4x^{12} + \frac{120}{11}\,a^7b^3x^{11} + \frac{9}{2}\,a^8b^2x^{10} + \frac{10}{9}\,a^9bx^9 + \frac{1}{8}\,a^{10}x^8$$

input
```
integrate(x^7*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 +
15*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3
*x^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^7(a+bx)^{10}\,dx = \frac{a^{10}\,x^8}{8}+\frac{10\,a^9\,b\,x^9}{9}+\frac{9\,a^8\,b^2\,x^{10}}{2}+\frac{120\,a^7\,b^3\,x^{11}}{11}+\frac{35\,a^6\,b^4\,x^{12}}{2}+\frac{252\,a^5\,b^5\,x^{13}}{13}+15\,a^4\,b^6\,x^{14}+8\,a^3\,b^7\,x^{15}+\frac{45\,a^2\,b^8\,x^{16}}{16}+\frac{10\,a\,b^9\,x^{17}}{17}+\frac{b^{10}\,x^{18}}{18}$$

input
```
int(x^7*(a + b*x)^10,x)
```

output
```
(a^10*x^8)/8 + (b^10*x^18)/18 + (10*a^9*b*x^9)/9 + (10*a*b^9*x^17)/17 + (9
*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/11 + (35*a^6*b^4*x^12)/2 + (252*a^5*
b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b^7*x^15 + (45*a^2*b^8*x^16)/16
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int x^7(a+bx)^{10}\,dx = \frac{x^8(19448b^{10}x^{10}+205920a\,b^9x^9+984555a^2b^8x^8+2800512a^3b^7x^7+5250960a^4b^6x^6+6785856a^5b^5x^5+6}{350064}$$

input
```
int(x^7*(b*x+a)^10,x)
```

output
```
(x**8*(43758*a**10 + 388960*a**9*b*x + 1575288*a**8*b**2*x**2 + 3818880*a*
*7*b**3*x**3 + 6126120*a**6*b**4*x**4 + 6785856*a**5*b**5*x**5 + 5250960*a
**4*b**6*x**6 + 2800512*a**3*b**7*x**7 + 984555*a**2*b**8*x**8 + 205920*a*
b**9*x**9 + 19448*b**10*x**10))/350064
```

3.86 $\int x^6(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 112

$$\int x^6(a+bx)^{10}\,dx=\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-\frac{3a(a+bx)^{16}}{8b^7}+\frac{(a+bx)^{17}}{17b^7}$$

output
```
1/11*a^6*(b*x+a)^11/b^7-1/2*a^5*(b*x+a)^12/b^7+15/13*a^4*(b*x+a)^13/b^7-10
/7*a^3*(b*x+a)^14/b^7+a^2*(b*x+a)^15/b^7-3/8*a*(b*x+a)^16/b^7+1/17*(b*x+a)
^17/b^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int x^6(a+bx)^{10}\,dx=\frac{a^{10}x^7}{7}+\frac{5}{4}a^9bx^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2b^8x^{15}+\frac{5}{8}ab^9x^{16}+\frac{b^{10}x^{17}}{17}$$

input
```
Integrate[x^6*(a + b*x)^10,x]
```

output
```
(a^10*x^7)/7 + (5*a^9*b*x^8)/4 + 5*a^8*b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^
6*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*a^4*b^6*x^13)/13 + (60*a^3*b^7*x^1
4)/7 + 3*a^2*b^8*x^15 + (5*a*b^9*x^16)/8 + (b^10*x^17)/17
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a+bx)^{10}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^6(a+bx)^{10}}{b^6}-\frac{6a^5(a+bx)^{11}}{b^6}+\frac{15a^4(a+bx)^{12}}{b^6}-\frac{20a^3(a+bx)^{13}}{b^6}+\frac{15a^2(a+bx)^{14}}{b^6}+\frac{(a+bx)^{16}}{b^6}-\frac{6a(a+$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

input
```
Int[x^6*(a + b*x)^10,x]
```

output
```
(a^6*(a + b*x)^11)/(11*b^7) - (a^5*(a + b*x)^12)/(2*b^7) + (15*a^4*(a + b*
x)^13)/(13*b^7) - (10*a^3*(a + b*x)^14)/(7*b^7) + (a^2*(a + b*x)^15)/b^7 -
 (3*a*(a + b*x)^16)/(8*b^7) + (a + b*x)^17/(17*b^7)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

method	result
gosper	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x$
default	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x$
norman	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x$
risch	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x$
parallelrisch	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x$
orering	$\frac{x^7(8008b^{10}x^{10}+85085a\,b^9x^9+408408a^2b^8x^8+1166880a^3b^7x^7+2199120a^4b^6x^6+2858856a^5b^5x^5+2598960a^6b^4x^4+1633632}{136136}$

input
```
int(x^6*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^10*x^7+5/4*a^9*b*x^8+5*a^8*b^2*x^9+12*a^7*b^3*x^10+210/11*a^6*b^4*x^
11+21*a^5*b^5*x^12+210/13*a^4*b^6*x^13+60/7*a^3*b^7*x^14+3*a^2*b^8*x^15+5/
8*a*b^9*x^16+1/17*b^10*x^17
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int x^6(a+bx)^{10}\,dx = \frac{1}{17}\,b^{10}x^{17} + \frac{5}{8}\,ab^9x^{16} + 3\,a^2b^8x^{15} + \frac{60}{7}\,a^3b^7x^{14} + \frac{210}{13}\,a^4b^6x^{13} + 21\,a^5b^5x^{12} + \frac{210}{11}\,a^6b^4x^{11} + 12\,a^7b^3x^{10} + 5\,a^8b^2x^9 + \frac{5}{4}\,a^9bx^8 + \frac{1}{7}\,a^{10}x^7$$

input
```
integrate(x^6*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210
/13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10
 + 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int x^6(a+bx)^{10}\,dx = \frac{a^{10}x^7}{7} + \frac{5a^9bx^8}{4} + 5a^8b^2x^9 + 12a^7b^3x^{10} + \frac{210a^6b^4x^{11}}{11} + 21a^5b^5x^{12} + \frac{210a^4b^6x^{13}}{13} + \frac{60a^3b^7x^{14}}{7} + 3a^2b^8x^{15} + \frac{5ab^9x^{16}}{8} + \frac{b^{10}x^{17}}{17}$$

input
```
integrate(x**6*(b*x+a)**10,x)
```

output
```
a**10*x**7/7 + 5*a**9*b*x**8/4 + 5*a**8*b**2*x**9 + 12*a**7*b**3*x**10 + 2
10*a**6*b**4*x**11/11 + 21*a**5*b**5*x**12 + 210*a**4*b**6*x**13/13 + 60*a
**3*b**7*x**14/7 + 3*a**2*b**8*x**15 + 5*a*b**9*x**16/8 + b**10*x**17/17
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int x^6(a+bx)^{10}\,dx = \frac{1}{17}\,b^{10}x^{17} + \frac{5}{8}\,ab^9x^{16} + 3\,a^2b^8x^{15} + \frac{60}{7}\,a^3b^7x^{14} + \frac{210}{13}\,a^4b^6x^{13} + 21\,a^5b^5x^{12} + \frac{210}{11}\,a^6b^4x^{11} + 12\,a^7b^3x^{10} + 5\,a^8b^2x^9 + \frac{5}{4}\,a^9bx^8 + \frac{1}{7}\,a^{10}x^7$$

input
```
integrate(x^6*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210
/13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10
 + 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int x^6(a+bx)^{10}\,dx = \frac{1}{17}\,b^{10}x^{17} + \frac{5}{8}\,ab^9x^{16} + 3\,a^2b^8x^{15} + \frac{60}{7}\,a^3b^7x^{14} + \frac{210}{13}\,a^4b^6x^{13} + 21\,a^5b^5x^{12} + \frac{210}{11}\,a^6b^4x^{11} + 12\,a^7b^3x^{10} + 5\,a^8b^2x^9 + \frac{5}{4}\,a^9bx^8 + \frac{1}{7}\,a^{10}x^7$$

input
```
integrate(x^6*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210
/13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10
 + 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int x^6(a+bx)^{10}\,dx = \frac{a^{10}\,x^7}{7}+\frac{5\,a^9\,b\,x^8}{4}+5\,a^8\,b^2\,x^9+12\,a^7\,b^3\,x^{10}+\frac{210\,a^6\,b^4\,x^{11}}{11}+21\,a^5\,b^5\,x^{12}+\frac{210\,a^4\,b^6\,x^{13}}{13}+\frac{60\,a^3\,b^7\,x^{14}}{7}+3\,a^2\,b^8\,x^{15}+\frac{5\,a\,b^9\,x^{16}}{8}+\frac{b^{10}\,x^{17}}{17}$$

input
```
int(x^6*(a + b*x)^10,x)
```

output
```
(a^10*x^7)/7 + (b^10*x^17)/17 + (5*a^9*b*x^8)/4 + (5*a*b^9*x^16)/8 + 5*a^8
*b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (21
0*a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)/7 + 3*a^2*b^8*x^15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int x^6(a+bx)^{10}\,dx = \frac{x^7(8008b^{10}x^{10}+85085a\,b^9x^9+408408a^2b^8x^8+1166880a^3b^7x^7+2199120a^4b^6x^6+2858856a^5b^5x^5+259}{136136}$$

input
```
int(x^6*(b*x+a)^10,x)
```

output
```
(x**7*(19448*a**10 + 170170*a**9*b*x + 680680*a**8*b**2*x**2 + 1633632*a**
7*b**3*x**3 + 2598960*a**6*b**4*x**4 + 2858856*a**5*b**5*x**5 + 2199120*a*
*4*b**6*x**6 + 1166880*a**3*b**7*x**7 + 408408*a**2*b**8*x**8 + 85085*a*b*
*9*x**9 + 8008*b**10*x**10))/136136
```

3.87 $\int x^5(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 98

$$\int x^5(a+bx)^{10}\,dx=-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+\frac{(a+bx)^{16}}{16b^6}$$

output
```
-1/11*a^5*(b*x+a)^11/b^6+5/12*a^4*(b*x+a)^12/b^6-10/13*a^3*(b*x+a)^13/b^6+
5/7*a^2*(b*x+a)^14/b^6-1/3*a*(b*x+a)^15/b^6+1/16*(b*x+a)^16/b^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int x^5(a+bx)^{10}\,dx=\frac{a^{10}x^6}{6}+\frac{10}{7}a^9bx^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+\frac{45}{14}a^2b^8x^{14}+\frac{2}{3}ab^9x^{15}+\frac{b^{10}x^{16}}{16}$$

input
```
Integrate[x^5*(a + b*x)^10,x]
```

output
```
(a^10*x^6)/6 + (10*a^9*b*x^7)/7 + (45*a^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3
+ 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/11 + (35*a^4*b^6*x^12)/2 + (120*a^3
*b^7*x^13)/13 + (45*a^2*b^8*x^14)/14 + (2*a*b^9*x^15)/3 + (b^10*x^16)/16
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+bx)^{10}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}+\frac{(a+bx)^{15}}{b^5}-\frac{5a(a+bx)^{14}}{b^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

input
```
Int[x^5*(a + b*x)^10,x]
```

output
```
-1/11*(a^5*(a + b*x)^11)/b^6 + (5*a^4*(a + b*x)^12)/(12*b^6) - (10*a^3*(a
+ b*x)^13)/(13*b^6) + (5*a^2*(a + b*x)^14)/(7*b^6) - (a*(a + b*x)^15)/(3*b
^6) + (a + b*x)^16/(16*b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

method	result
gosper	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x$
default	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x$
norman	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x$
risch	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x$
parallelrisch	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x$
orering	$\frac{x^6(3003b^{10}x^{10}+32032a\,b^9x^9+154440a^2b^8x^8+443520a^3b^7x^7+840840a^4b^6x^6+1100736a^5b^5x^5+1009008a^6b^4x^4+640640a^7b}{48048}$

input
```
int(x^5*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^10*x^6+10/7*a^9*b*x^7+45/8*a^8*b^2*x^8+40/3*a^7*b^3*x^9+21*a^6*b^4*x
^10+252/11*a^5*b^5*x^11+35/2*a^4*b^6*x^12+120/13*a^3*b^7*x^13+45/14*a^2*b^
8*x^14+2/3*a*b^9*x^15+1/16*b^10*x^16
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int x^5(a+bx)^{10}\,dx = \frac{1}{16}\,b^{10}x^{16} + \frac{2}{3}\,ab^9x^{15} + \frac{45}{14}\,a^2b^8x^{14} + \frac{120}{13}\,a^3b^7x^{13} + \frac{35}{2}\,a^4b^6x^{12} + \frac{252}{11}\,a^5b^5x^{11} + 21\,a^6b^4x^{10} + \frac{40}{3}\,a^7b^3x^9 + \frac{45}{8}\,a^8b^2x^8 + \frac{10}{7}\,a^9bx^7 + \frac{1}{6}\,a^{10}x^6$$

input
```
integrate(x^5*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
 + 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^
3*x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.36

$$\int x^5(a+bx)^{10}\,dx = \frac{a^{10}x^6}{6} + \frac{10a^9bx^7}{7} + \frac{45a^8b^2x^8}{8} + \frac{40a^7b^3x^9}{3} + 21a^6b^4x^{10} + \frac{252a^5b^5x^{11}}{11} + \frac{35a^4b^6x^{12}}{2} + \frac{120a^3b^7x^{13}}{13} + \frac{45a^2b^8x^{14}}{14} + \frac{2ab^9x^{15}}{3} + \frac{b^{10}x^{16}}{16}$$

input
```
integrate(x**5*(b*x+a)**10,x)
```

output
```
a**10*x**6/6 + 10*a**9*b*x**7/7 + 45*a**8*b**2*x**8/8 + 40*a**7*b**3*x**9/
3 + 21*a**6*b**4*x**10 + 252*a**5*b**5*x**11/11 + 35*a**4*b**6*x**12/2 + 1
20*a**3*b**7*x**13/13 + 45*a**2*b**8*x**14/14 + 2*a*b**9*x**15/3 + b**10*x
**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int x^5(a+bx)^{10}\,dx = \frac{1}{16}\,b^{10}x^{16} + \frac{2}{3}\,ab^9x^{15} + \frac{45}{14}\,a^2b^8x^{14} + \frac{120}{13}\,a^3b^7x^{13} + \frac{35}{2}\,a^4b^6x^{12} + \frac{252}{11}\,a^5b^5x^{11} + 21\,a^6b^4x^{10} + \frac{40}{3}\,a^7b^3x^9 + \frac{45}{8}\,a^8b^2x^8 + \frac{10}{7}\,a^9bx^7 + \frac{1}{6}\,a^{10}x^6$$

input
```
integrate(x^5*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
 + 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^
3*x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int x^5(a+bx)^{10}\,dx = \frac{1}{16}\,b^{10}x^{16} + \frac{2}{3}\,ab^9x^{15} + \frac{45}{14}\,a^2b^8x^{14} + \frac{120}{13}\,a^3b^7x^{13} + \frac{35}{2}\,a^4b^6x^{12} + \frac{252}{11}\,a^5b^5x^{11} + 21\,a^6b^4x^{10} + \frac{40}{3}\,a^7b^3x^9 + \frac{45}{8}\,a^8b^2x^8 + \frac{10}{7}\,a^9bx^7 + \frac{1}{6}\,a^{10}x^6$$

input
```
integrate(x^5*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
 + 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^
3*x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int x^5(a+bx)^{10}\,dx=\frac{a^{10}\,x^6}{6}+\frac{10\,a^9\,b\,x^7}{7}+\frac{45\,a^8\,b^2\,x^8}{8}+\frac{40\,a^7\,b^3\,x^9}{3}+21\,a^6\,b^4\,x^{10}+\frac{252\,a^5\,b^5\,x^{11}}{11}+\frac{35\,a^4\,b^6\,x^{12}}{2}+\frac{120\,a^3\,b^7\,x^{13}}{13}+\frac{45\,a^2\,b^8\,x^{14}}{14}+\frac{2\,a\,b^9\,x^{15}}{3}+\frac{b^{10}\,x^{16}}{16}$$

input
```
int(x^5*(a + b*x)^10,x)
```

output
```
(a^10*x^6)/6 + (b^10*x^16)/16 + (10*a^9*b*x^7)/7 + (2*a*b^9*x^15)/3 + (45*
a^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 + 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)
/11 + (35*a^4*b^6*x^12)/2 + (120*a^3*b^7*x^13)/13 + (45*a^2*b^8*x^14)/14
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int x^5(a+bx)^{10}\,dx$$
$$=\frac{x^6(3003b^{10}x^{10}+32032a\,b^9x^9+154440a^2b^8x^8+443520a^3b^7x^7+840840a^4b^6x^6+1100736a^5b^5x^5+10090}{48048}$$

input
```
int(x^5*(b*x+a)^10,x)
```

output
```
(x**6*(8008*a**10 + 68640*a**9*b*x + 270270*a**8*b**2*x**2 + 640640*a**7*b
**3*x**3 + 1009008*a**6*b**4*x**4 + 1100736*a**5*b**5*x**5 + 840840*a**4*b
**6*x**6 + 443520*a**3*b**7*x**7 + 154440*a**2*b**8*x**8 + 32032*a*b**9*x*
*9 + 3003*b**10*x**10))/48048
```

3.88 $\int x^4(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 81

$$\int x^4(a+bx)^{10}\,dx = \frac{a^4(a+bx)^{11}}{11b^5} - \frac{a^3(a+bx)^{12}}{3b^5} + \frac{6a^2(a+bx)^{13}}{13b^5} - \frac{2a(a+bx)^{14}}{7b^5} + \frac{(a+bx)^{15}}{15b^5}$$

output
```
1/11*a^4*(b*x+a)^11/b^5-1/3*a^3*(b*x+a)^12/b^5+6/13*a^2*(b*x+a)^13/b^5-2/7
*a*(b*x+a)^14/b^5+1/15*(b*x+a)^15/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.60

$$\int x^4(a+bx)^{10}\,dx = \frac{a^{10}x^5}{5} + \frac{5}{3}a^9bx^6 + \frac{45}{7}a^8b^2x^7 + 15a^7b^3x^8 + \frac{70}{3}a^6b^4x^9 + \frac{126}{5}a^5b^5x^{10} + \frac{210}{11}a^4b^6x^{11} + 10a^3b^7x^{12} + \frac{45}{13}a^2b^8x^{13} + \frac{5}{7}ab^9x^{14} + \frac{b^{10}x^{15}}{15}$$

input
```
Integrate[x^4*(a + b*x)^10,x]
```

output
```
(a^10*x^5)/5 + (5*a^9*b*x^6)/3 + (45*a^8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70
*a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 + (210*a^4*b^6*x^11)/11 + 10*a^3*b^
7*x^12 + (45*a^2*b^8*x^13)/13 + (5*a*b^9*x^14)/7 + (b^10*x^15)/15
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^{10}\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^4(a+bx)^{10}}{b^4}-\frac{4a^3(a+bx)^{11}}{b^4}+\frac{6a^2(a+bx)^{12}}{b^4}+\frac{(a+bx)^{14}}{b^4}-\frac{4a(a+bx)^{13}}{b^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

input
```
Int[x^4*(a + b*x)^10,x]
```

output
```
(a^4*(a + b*x)^11)/(11*b^5) - (a^3*(a + b*x)^12)/(3*b^5) + (6*a^2*(a + b*x
)^13)/(13*b^5) - (2*a*(a + b*x)^14)/(7*b^5) + (a + b*x)^15/(15*b^5)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

method	result
gosper	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^1$
default	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^1$
norman	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^1$
risch	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^1$
parallelrisch	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^1$
orering	$\frac{x^5(1001b^{10}x^{10}+10725a\,b^9x^9+51975a^2b^8x^8+150150a^3b^7x^7+286650a^4b^6x^6+378378a^5b^5x^5+350350a^6b^4x^4+225225a^7b^3x^3}{15015}$

input
```
int(x^4*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^10*x^5+5/3*a^9*b*x^6+45/7*a^8*b^2*x^7+15*a^7*b^3*x^8+70/3*a^6*b^4*x^
9+126/5*a^5*b^5*x^10+210/11*a^4*b^6*x^11+10*a^3*b^7*x^12+45/13*a^2*b^8*x^1
3+5/7*a*b^9*x^14+1/15*b^10*x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int x^4(a+bx)^{10}\,dx = \frac{1}{15}\,b^{10}x^{15} + \frac{5}{7}\,ab^9x^{14} + \frac{45}{13}\,a^2b^8x^{13} + 10\,a^3b^7x^{12} + \frac{210}{11}\,a^4b^6x^{11} + \frac{126}{5}\,a^5b^5x^{10} + \frac{70}{3}\,a^6b^4x^9 + 15\,a^7b^3x^8 + \frac{45}{7}\,a^8b^2x^7 + \frac{5}{3}\,a^9bx^6 + \frac{1}{5}\,a^{10}x^5$$

input

```
integrate(x^4*(b*x+a)^10,x, algorithm="fricas")
```

output

```
1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 2
10/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^
8 + 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int x^4(a+bx)^{10}\,dx = \frac{a^{10}x^5}{5} + \frac{5a^9bx^6}{3} + \frac{45a^8b^2x^7}{7} + 15a^7b^3x^8 + \frac{70a^6b^4x^9}{3} + \frac{126a^5b^5x^{10}}{5} + \frac{210a^4b^6x^{11}}{11} + 10a^3b^7x^{12} + \frac{45a^2b^8x^{13}}{13} + \frac{5ab^9x^{14}}{7} + \frac{b^{10}x^{15}}{15}$$

input

```
integrate(x**4*(b*x+a)**10,x)
```

output

```
a**10*x**5/5 + 5*a**9*b*x**6/3 + 45*a**8*b**2*x**7/7 + 15*a**7*b**3*x**8 +
 70*a**6*b**4*x**9/3 + 126*a**5*b**5*x**10/5 + 210*a**4*b**6*x**11/11 + 10
*a**3*b**7*x**12 + 45*a**2*b**8*x**13/13 + 5*a*b**9*x**14/7 + b**10*x**15/
15
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int x^4(a+bx)^{10}\,dx = \frac{1}{15}\,b^{10}x^{15} + \frac{5}{7}\,ab^9x^{14} + \frac{45}{13}\,a^2b^8x^{13} + 10\,a^3b^7x^{12} + \frac{210}{11}\,a^4b^6x^{11} + \frac{126}{5}\,a^5b^5x^{10} + \frac{70}{3}\,a^6b^4x^9 + 15\,a^7b^3x^8 + \frac{45}{7}\,a^8b^2x^7 + \frac{5}{3}\,a^9bx^6 + \frac{1}{5}\,a^{10}x^5$$

input
```
integrate(x^4*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 2
10/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^
8 + 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int x^4(a+bx)^{10}\,dx = \frac{1}{15}\,b^{10}x^{15} + \frac{5}{7}\,ab^9x^{14} + \frac{45}{13}\,a^2b^8x^{13} + 10\,a^3b^7x^{12} + \frac{210}{11}\,a^4b^6x^{11} + \frac{126}{5}\,a^5b^5x^{10} + \frac{70}{3}\,a^6b^4x^9 + 15\,a^7b^3x^8 + \frac{45}{7}\,a^8b^2x^7 + \frac{5}{3}\,a^9bx^6 + \frac{1}{5}\,a^{10}x^5$$

input
```
integrate(x^4*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 2
10/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^
8 + 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int x^4(a+bx)^{10}\,dx = \frac{a^{10}\,x^5}{5}+\frac{5\,a^9\,b\,x^6}{3}+\frac{45\,a^8\,b^2\,x^7}{7}+15\,a^7\,b^3\,x^8+\frac{70\,a^6\,b^4\,x^9}{3}+\frac{126\,a^5\,b^5\,x^{10}}{5}+\frac{210\,a^4\,b^6\,x^{11}}{11}+10\,a^3\,b^7\,x^{12}+\frac{45\,a^2\,b^8\,x^{13}}{13}+\frac{5\,a\,b^9\,x^{14}}{7}+\frac{b^{10}\,x^{15}}{15}$$

input
```
int(x^4*(a + b*x)^10,x)
```

output
```
(a^10*x^5)/5 + (b^10*x^15)/15 + (5*a^9*b*x^6)/3 + (5*a*b^9*x^14)/7 + (45*a
^8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5
 + (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*x^12 + (45*a^2*b^8*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int x^4(a+bx)^{10}\,dx = \frac{x^5(1001b^{10}x^{10}+10725a\,b^9x^9+51975a^2b^8x^8+150150a^3b^7x^7+286650a^4b^6x^6+378378a^5b^5x^5+350350a}{15015}$$

input
```
int(x^4*(b*x+a)^10,x)
```

output
```
(x**5*(3003*a**10 + 25025*a**9*b*x + 96525*a**8*b**2*x**2 + 225225*a**7*b*
*3*x**3 + 350350*a**6*b**4*x**4 + 378378*a**5*b**5*x**5 + 286650*a**4*b**6
*x**6 + 150150*a**3*b**7*x**7 + 51975*a**2*b**8*x**8 + 10725*a*b**9*x**9 +
 1001*b**10*x**10))/15015
```

3.89 $\int x^3(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int x^3(a+bx)^{10}\,dx=-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}$$

output
```
-1/11*a^3*(b*x+a)^11/b^4+1/4*a^2*(b*x+a)^12/b^4-3/13*a*(b*x+a)^13/b^4+1/14
*(b*x+a)^14/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.00

$$\int x^3(a+bx)^{10}\,dx=\frac{a^{10}x^4}{4}+2a^9bx^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2b^8x^{12}+\frac{10}{13}ab^9x^{13}+\frac{b^{10}x^{14}}{14}$$

input
```
Integrate[x^3*(a + b*x)^10,x]
```

output
```
(a^10*x^4)/4 + 2*a^9*b*x^5 + (15*a^8*b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (1
05*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 21*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/
11 + (15*a^2*b^8*x^12)/4 + (10*a*b^9*x^13)/13 + (b^10*x^14)/14
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{10}\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(-\frac{a^3(a+bx)^{10}}{b^3}+\frac{3a^2(a+bx)^{11}}{b^3}+\frac{(a+bx)^{13}}{b^3}-\frac{3a(a+bx)^{12}}{b^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

input
```
Int[x^3*(a + b*x)^10,x]
```

output
```
-1/11*(a^3*(a + b*x)^11)/b^4 + (a^2*(a + b*x)^12)/(4*b^4) - (3*a*(a + b*x)
^13)/(13*b^4) + (a + b*x)^14/(14*b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.77

method	result
gosper	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{1}$
default	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{1}$
norman	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{1}$
risch	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{1}$
parallelrisch	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{1}$
orering	$\frac{x^4(286b^{10}x^{10}+3080a\,b^9x^9+15015a^2b^8x^8+43680a^3b^7x^7+84084a^4b^6x^6+112112a^5b^5x^5+105105a^6b^4x^4+68640a^7b^3x^3+3003}{4004}$

input
```
int(x^3*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^10*x^4+2*a^9*b*x^5+15/2*a^8*b^2*x^6+120/7*a^7*b^3*x^7+105/4*a^6*b^4*
x^8+28*a^5*b^5*x^9+21*a^4*b^6*x^10+120/11*a^3*b^7*x^11+15/4*a^2*b^8*x^12+1
0/13*a*b^9*x^13+1/14*b^10*x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int x^3(a+bx)^{10}\,dx=\frac{1}{14}\,b^{10}x^{14}+\frac{10}{13}\,ab^9x^{13}+\frac{15}{4}\,a^2b^8x^{12}+\frac{120}{11}\,a^3b^7x^{11}+21\,a^4b^6x^{10}+28\,a^5b^5x^9+\frac{105}{4}\,a^6b^4x^8+\frac{120}{7}\,a^7b^3x^7+\frac{15}{2}\,a^8b^2x^6+2\,a^9bx^5+\frac{1}{4}\,a^{10}x^4$$

input
```
integrate(x^3*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^1
1 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x
^7 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(56) = 112$.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.02

$$\int x^3(a+bx)^{10}\,dx = \frac{a^{10}x^4}{4} + 2a^9bx^5 + \frac{15a^8b^2x^6}{2} + \frac{120a^7b^3x^7}{7} + \frac{105a^6b^4x^8}{4} + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120a^3b^7x^{11}}{11} + \frac{15a^2b^8x^{12}}{4} + \frac{10ab^9x^{13}}{13} + \frac{b^{10}x^{14}}{14}$$

input
```
integrate(x**3*(b*x+a)**10,x)
```

output
```
a**10*x**4/4 + 2*a**9*b*x**5 + 15*a**8*b**2*x**6/2 + 120*a**7*b**3*x**7/7
+ 105*a**6*b**4*x**8/4 + 28*a**5*b**5*x**9 + 21*a**4*b**6*x**10 + 120*a**3
*b**7*x**11/11 + 15*a**2*b**8*x**12/4 + 10*a*b**9*x**13/13 + b**10*x**14/1
4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int x^3(a+bx)^{10}\,dx = \frac{1}{14}\,b^{10}x^{14} + \frac{10}{13}\,ab^9x^{13} + \frac{15}{4}\,a^2b^8x^{12} + \frac{120}{11}\,a^3b^7x^{11} + 21\,a^4b^6x^{10} + 28\,a^5b^5x^9 + \frac{105}{4}\,a^6b^4x^8 + \frac{120}{7}\,a^7b^3x^7 + \frac{15}{2}\,a^8b^2x^6 + 2\,a^9bx^5 + \frac{1}{4}\,a^{10}x^4$$

input
```
integrate(x^3*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^1
1 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x
^7 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int x^3(a+bx)^{10}\,dx = \frac{1}{14}\,b^{10}x^{14} + \frac{10}{13}\,ab^9x^{13} + \frac{15}{4}\,a^2b^8x^{12} + \frac{120}{11}\,a^3b^7x^{11} + 21\,a^4b^6x^{10} + 28\,a^5b^5x^9 + \frac{105}{4}\,a^6b^4x^8 + \frac{120}{7}\,a^7b^3x^7 + \frac{15}{2}\,a^8b^2x^6 + 2\,a^9bx^5 + \frac{1}{4}\,a^{10}x^4$$

input
```
integrate(x^3*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^1
1 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x
^7 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int x^3(a+bx)^{10}\,dx = \frac{a^{10}\,x^4}{4} + 2\,a^9\,b\,x^5 + \frac{15\,a^8\,b^2\,x^6}{2} + \frac{120\,a^7\,b^3\,x^7}{7} + \frac{105\,a^6\,b^4\,x^8}{4} + 28\,a^5\,b^5\,x^9 + 21\,a^4\,b^6\,x^{10} + \frac{120\,a^3\,b^7\,x^{11}}{11} + \frac{15\,a^2\,b^8\,x^{12}}{4} + \frac{10\,a\,b^9\,x^{13}}{13} + \frac{b^{10}\,x^{14}}{14}$$

input
```
int(x^3*(a + b*x)^10,x)
```

output
```
(a^10*x^4)/4 + (b^10*x^14)/14 + 2*a^9*b*x^5 + (10*a*b^9*x^13)/13 + (15*a^8
*b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (105*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 +
 21*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11 + (15*a^2*b^8*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int x^3(a+bx)^{10}\,dx$$
$$= \frac{x^4(286b^{10}x^{10}+3080a\,b^9x^9+15015a^2b^8x^8+43680a^3b^7x^7+84084a^4b^6x^6+112112a^5b^5x^5+105105a^6b^4x}{4004}$$

input
```
int(x^3*(b*x+a)^10,x)
```

output
```
(x**4*(1001*a**10 + 8008*a**9*b*x + 30030*a**8*b**2*x**2 + 68640*a**7*b**3
*x**3 + 105105*a**6*b**4*x**4 + 112112*a**5*b**5*x**5 + 84084*a**4*b**6*x*
*6 + 43680*a**3*b**7*x**7 + 15015*a**2*b**8*x**8 + 3080*a*b**9*x**9 + 286*
b**10*x**10))/4004
```

3.90 $\int x^2(a+bx)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int x^2(a+bx)^{10}\,dx = \frac{a^2(a+bx)^{11}}{11b^3} - \frac{a(a+bx)^{12}}{6b^3} + \frac{(a+bx)^{13}}{13b^3}$$

output
```
1/11*a^2*(b*x+a)^11/b^3-1/6*a*(b*x+a)^12/b^3+1/13*(b*x+a)^13/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(47) = 94.

Time = 0.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.68

$$\int x^2(a+bx)^{10}\,dx = \frac{a^{10}x^3}{3} + \frac{5}{2}a^9bx^4 + 9a^8b^2x^5 + 20a^7b^3x^6 + 30a^6b^4x^7 + \frac{63}{2}a^5b^5x^8 + \frac{70}{3}a^4b^6x^9 + 12a^3b^7x^{10} + \frac{45}{11}a^2b^8x^{11} + \frac{5}{6}ab^9x^{12} + \frac{b^{10}x^{13}}{13}$$

input
```
Integrate[x^2*(a + b*x)^10,x]
```

output
```
(a^10*x^3)/3 + (5*a^9*b*x^4)/2 + 9*a^8*b^2*x^5 + 20*a^7*b^3*x^6 + 30*a^6*b
^4*x^7 + (63*a^5*b^5*x^8)/2 + (70*a^4*b^6*x^9)/3 + 12*a^3*b^7*x^10 + (45*a
^2*b^8*x^11)/11 + (5*a*b^9*x^12)/6 + (b^10*x^13)/13
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{10}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2(a+bx)^{10}}{b^2}+\frac{(a+bx)^{12}}{b^2}-\frac{2a(a+bx)^{11}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

input
```
Int[x^2*(a + b*x)^10,x]
```

output
```
(a^2*(a + b*x)^11)/(11*b^3) - (a*(a + b*x)^12)/(6*b^3) + (a + b*x)^13/(13*
b^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.40

method	result
gosper	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x$
default	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x$
norman	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x$
risch	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x$
parallelrisch	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x$
orering	$\frac{x^3(66b^{10}x^{10}+715a\,b^9x^9+3510a^2b^8x^8+10296a^3b^7x^7+20020a^4b^6x^6+27027a^5b^5x^5+25740a^6b^4x^4+17160a^7b^3x^3+7722a^8b^2x}{858}$

input
```
int(x^2*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^10*x^13+5/6*a*b^9*x^12+45/11*a^2*b^8*x^11+12*a^3*b^7*x^10+70/3*a^4*
b^6*x^9+63/2*a^5*b^5*x^8+30*a^6*b^4*x^7+20*a^7*b^3*x^6+9*a^8*b^2*x^5+5/2*a
^9*b*x^4+1/3*a^10*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int x^2(a+bx)^{10}\,dx = \frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^9x^{12} + \frac{45}{11}\,a^2b^8x^{11} + 12\,a^3b^7x^{10} + \frac{70}{3}\,a^4b^6x^9 + \frac{63}{2}\,a^5b^5x^8 + 30\,a^6b^4x^7 + 20\,a^7b^3x^6 + 9\,a^8b^2x^5 + \frac{5}{2}\,a^9bx^4 + \frac{1}{3}\,a^{10}x^3$$

input
```
integrate(x^2*(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 7
0/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a
^8*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int x^2(a+bx)^{10}\,dx = \frac{a^{10}x^3}{3} + \frac{5a^9bx^4}{2} + 9a^8b^2x^5 + 20a^7b^3x^6 + 30a^6b^4x^7 + \frac{63a^5b^5x^8}{2} + \frac{70a^4b^6x^9}{3} + 12a^3b^7x^{10} + \frac{45a^2b^8x^{11}}{11} + \frac{5ab^9x^{12}}{6} + \frac{b^{10}x^{13}}{13}$$

input
```
integrate(x**2*(b*x+a)**10,x)
```

output
```
a**10*x**3/3 + 5*a**9*b*x**4/2 + 9*a**8*b**2*x**5 + 20*a**7*b**3*x**6 + 30
*a**6*b**4*x**7 + 63*a**5*b**5*x**8/2 + 70*a**4*b**6*x**9/3 + 12*a**3*b**7
*x**10 + 45*a**2*b**8*x**11/11 + 5*a*b**9*x**12/6 + b**10*x**13/13
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(41) = 82.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int x^2(a+bx)^{10}\,dx = \frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^9x^{12} + \frac{45}{11}\,a^2b^8x^{11} + 12\,a^3b^7x^{10} + \frac{70}{3}\,a^4b^6x^9 + \frac{63}{2}\,a^5b^5x^8 + 30\,a^6b^4x^7 + 20\,a^7b^3x^6 + 9\,a^8b^2x^5 + \frac{5}{2}\,a^9bx^4 + \frac{1}{3}\,a^{10}x^3$$

input
```
integrate(x^2*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 7
0/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a
^8*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(41) = 82.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int x^2(a+bx)^{10}\,dx = \frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^9x^{12} + \frac{45}{11}\,a^2b^8x^{11} + 12\,a^3b^7x^{10} + \frac{70}{3}\,a^4b^6x^9 + \frac{63}{2}\,a^5b^5x^8 + 30\,a^6b^4x^7 + 20\,a^7b^3x^6 + 9\,a^8b^2x^5 + \frac{5}{2}\,a^9bx^4 + \frac{1}{3}\,a^{10}x^3$$

input
```
integrate(x^2*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 7
0/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a
^8*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^{10}\,dx=\frac{(a+b\,x)^{11}\,\left(8\,a^2-88\,a\,b\,x+528\,b^2\,x^2\right)}{6864\,b^3}$$

input
```
int(x^2*(a + b*x)^10,x)
```

output
```
((a + b*x)^11*(8*a^2 + 528*b^2*x^2 - 88*a*b*x))/(6864*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int x^2(a+bx)^{10}\,dx$$
$$=\frac{x^3(66b^{10}x^{10}+715a\,b^9x^9+3510a^2b^8x^8+10296a^3b^7x^7+20020a^4b^6x^6+27027a^5b^5x^5+25740a^6b^4x^4+17}{858}$$

input
```
int(x^2*(b*x+a)^10,x)
```

output
```
(x**3*(286*a**10 + 2145*a**9*b*x + 7722*a**8*b**2*x**2 + 17160*a**7*b**3*x
**3 + 25740*a**6*b**4*x**4 + 27027*a**5*b**5*x**5 + 20020*a**4*b**6*x**6 +
 10296*a**3*b**7*x**7 + 3510*a**2*b**8*x**8 + 715*a*b**9*x**9 + 66*b**10*x
**10))/858
```

3.91 $\int x(a+bx)^{10}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int x(a+bx)^{10}\,dx=-\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}$$

output
```
-1/11*a*(b*x+a)^11/b^2+1/12*(b*x+a)^12/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(30) = 60.

Time = 0.00 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.27

$$\int x(a+bx)^{10}\,dx=\frac{a^{10}x^2}{2}+\frac{10}{3}a^9bx^3+\frac{45}{4}a^8b^2x^4+24a^7b^3x^5+35a^6b^4x^6+36a^5b^5x^7+\frac{105}{4}a^4b^6x^8+\frac{40}{3}a^3b^7x^9+\frac{9}{2}a^2b^8x^{10}+\frac{10}{11}ab^9x^{11}+\frac{b^{10}x^{12}}{12}$$

input
```
Integrate[x*(a + b*x)^10,x]
```

output
```
(a^10*x^2)/2 + (10*a^9*b*x^3)/3 + (45*a^8*b^2*x^4)/4 + 24*a^7*b^3*x^5 + 35
*a^6*b^4*x^6 + 36*a^5*b^5*x^7 + (105*a^4*b^6*x^8)/4 + (40*a^3*b^7*x^9)/3 +
 (9*a^2*b^8*x^10)/2 + (10*a*b^9*x^11)/11 + (b^10*x^12)/12
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{10}\,dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{(a+bx)^{11}}{b}-\frac{a(a+bx)^{10}}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

input
```
Int[x*(a + b*x)^10,x]
```

output
```
-1/11*(a*(a + b*x)^11)/b^2 + (a + b*x)^12/(12*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(26) = 52.

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.77

method	result
gosper	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x$
default	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x$
norman	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x$
risch	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x$
parallelrisch	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x$
orering	$\frac{x^2(11b^{10}x^{10}+120a\,b^9x^9+594a^2b^8x^8+1760a^3b^7x^7+3465a^4b^6x^6+4752a^5b^5x^5+4620a^6b^4x^4+3168a^7b^3x^3+1485a^8b^2x^2+440}{132}$

input

```
int(x*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output

```
1/12*b^10*x^12+10/11*a*b^9*x^11+9/2*a^2*b^8*x^10+40/3*a^3*b^7*x^9+105/4*a^
4*b^6*x^8+36*a^5*b^5*x^7+35*a^6*b^4*x^6+24*a^7*b^3*x^5+45/4*a^8*b^2*x^4+10
/3*a^9*b*x^3+1/2*a^10*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(26) = 52.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.73

$$\int x(a+bx)^{10}\,dx = \frac{1}{12}\,b^{10}x^{12} + \frac{10}{11}\,ab^9x^{11} + \frac{9}{2}\,a^2b^8x^{10} + \frac{40}{3}\,a^3b^7x^9 + \frac{105}{4}\,a^4b^6x^8 + 36\,a^5b^5x^7 + 35\,a^6b^4x^6 + 24\,a^7b^3x^5 + \frac{45}{4}\,a^8b^2x^4 + \frac{10}{3}\,a^9bx^3 + \frac{1}{2}\,a^{10}x^2$$

input

```
integrate(x*(b*x+a)^10,x, algorithm="fricas")
```

output

```
1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 +
105/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/
4*a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.30

$$\int x(a+bx)^{10}\,dx = \frac{a^{10}x^2}{2} + \frac{10a^9bx^3}{3} + \frac{45a^8b^2x^4}{4} + 24a^7b^3x^5 + 35a^6b^4x^6 + 36a^5b^5x^7 + \frac{105a^4b^6x^8}{4} + \frac{40a^3b^7x^9}{3} + \frac{9a^2b^8x^{10}}{2} + \frac{10ab^9x^{11}}{11} + \frac{b^{10}x^{12}}{12}$$

input
```
integrate(x*(b*x+a)**10,x)
```

output
```
a**10*x**2/2 + 10*a**9*b*x**3/3 + 45*a**8*b**2*x**4/4 + 24*a**7*b**3*x**5
+ 35*a**6*b**4*x**6 + 36*a**5*b**5*x**7 + 105*a**4*b**6*x**8/4 + 40*a**3*b
**7*x**9/3 + 9*a**2*b**8*x**10/2 + 10*a*b**9*x**11/11 + b**10*x**12/12
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.73

$$\int x(a+bx)^{10}\,dx = \frac{1}{12}\,b^{10}x^{12} + \frac{10}{11}\,ab^9x^{11} + \frac{9}{2}\,a^2b^8x^{10} + \frac{40}{3}\,a^3b^7x^9 + \frac{105}{4}\,a^4b^6x^8 + 36\,a^5b^5x^7 + 35\,a^6b^4x^6 + 24\,a^7b^3x^5 + \frac{45}{4}\,a^8b^2x^4 + \frac{10}{3}\,a^9bx^3 + \frac{1}{2}\,a^{10}x^2$$

input
```
integrate(x*(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 +
105/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/
4*a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.73

$$\int x(a+bx)^{10}\,dx = \frac{1}{12}\,b^{10}x^{12} + \frac{10}{11}\,ab^9x^{11} + \frac{9}{2}\,a^2b^8x^{10} + \frac{40}{3}\,a^3b^7x^9 + \frac{105}{4}\,a^4b^6x^8 + 36\,a^5b^5x^7 + 35\,a^6b^4x^6 + 24\,a^7b^3x^5 + \frac{45}{4}\,a^8b^2x^4 + \frac{10}{3}\,a^9bx^3 + \frac{1}{2}\,a^{10}x^2$$

input
```
integrate(x*(b*x+a)^10,x, algorithm="giac")
```

output
```
1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 +
105/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/
4*a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int x(a+bx)^{10}\,dx = -\frac{2\left(\frac{a\,(a+b\,x)^{11}}{22} - \frac{(a+b\,x)^{12}}{24}\right)}{b^2}$$

input
```
int(x*(a + b*x)^10,x)
```

output
```
-(2*((a*(a + b*x)^11)/22 - (a + b*x)^12/24))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.73

$$\int x(a+bx)^{10}\,dx$$
$$=\frac{x^2(11b^{10}x^{10}+120a\,b^9x^9+594a^2b^8x^8+1760a^3b^7x^7+3465a^4b^6x^6+4752a^5b^5x^5+4620a^6b^4x^4+3168a^7b}{132}$$

input
```
int(x*(b*x+a)^10,x)
```

output
```
(x**2*(66*a**10 + 440*a**9*b*x + 1485*a**8*b**2*x**2 + 3168*a**7*b**3*x**3
 + 4620*a**6*b**4*x**4 + 4752*a**5*b**5*x**5 + 3465*a**4*b**6*x**6 + 1760*
a**3*b**7*x**7 + 594*a**2*b**8*x**8 + 120*a*b**9*x**9 + 11*b**10*x**10))/1
32
```

3.92 $\int (a+bx)^{10}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (a+bx)^{10}\,dx=\frac{(a+bx)^{11}}{11b}$$

output
```
1/11*(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+bx)^{10}\,dx=\frac{(a+bx)^{11}}{11b}$$

input
```
Integrate[(a + b*x)^10,x]
```

output
```
(a + b*x)^11/(11*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{10}\,dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a+bx)^{11}}{11b}$$

input
```
Int[(a + b*x)^10,x]
```

output
```
(a + b*x)^11/(11*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
default	$\frac{(bx+a)^{11}}{11b}$
gosper	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+$
norman	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+$
parallelrisch	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+$
orering	$\frac{x\left(b^{10}x^{10}+11a\,b^9x^9+55a^2b^8x^8+165a^3b^7x^7+330a^4b^6x^6+462a^5b^5x^5+462a^6b^4x^4+330a^7b^3x^3+165a^8b^2x^2+55a^9bx+11a^{10}\right)}{11}$
risch	$\frac{b^{10}x^{11}}{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a$

input
```
int((b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/11*(b*x+a)^11/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int (a+bx)^{10}\,dx = \frac{1}{11}\,b^{10}x^{11}+ab^9x^{10}+5\,a^2b^8x^9+15\,a^3b^7x^8+30\,a^4b^6x^7 + 42\,a^5b^5x^6+42\,a^6b^4x^5+30\,a^7b^3x^4+15\,a^8b^2x^3+5\,a^9bx^2+a^{10}x$$

input
```
integrate((b*x+a)^10,x, algorithm="fricas")
```

output
```
1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*
x^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 +
5*a^9*b*x^2 + a^10*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 8.14

$$\int (a+bx)^{10}\,dx = a^{10}x + 5a^9bx^2 + 15a^8b^2x^3 + 30a^7b^3x^4 + 42a^6b^4x^5 + 42a^5b^5x^6 + 30a^4b^6x^7 + 15a^3b^7x^8 + 5a^2b^8x^9 + ab^9x^{10} + \frac{b^{10}x^{11}}{11}$$

input
```
integrate((b*x+a)**10,x)
```

output
```
a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*
b**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*
a**2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)^{10}\,dx = \frac{(bx+a)^{11}}{11\,b}$$

input
```
integrate((b*x+a)^10,x, algorithm="maxima")
```

output
```
1/11*(b*x + a)^11/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)^{10}\,dx=\frac{(bx+a)^{11}}{11\,b}$$

input
```
integrate((b*x+a)^10,x, algorithm="giac")
```

output
```
1/11*(b*x + a)^11/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int (a+bx)^{10}\,dx=a^{10}\,x+5\,a^9\,b\,x^2+15\,a^8\,b^2\,x^3+30\,a^7\,b^3\,x^4+42\,a^6\,b^4\,x^5+42\,a^5\,b^5\,x^6+30\,a^4\,b^6\,x^7+15\,a^3\,b^7\,x^8+5\,a^2\,b^8\,x^9+a\,b^9\,x^{10}+\frac{b^{10}\,x^{11}}{11}$$

input
```
int((a + b*x)^10,x)
```

output
```
a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a
^7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7
*x^8 + 5*a^2*b^8*x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 7.79

$$\int (a+bx)^{10}\,dx=\frac{x(b^{10}x^{10}+11a\,b^9x^9+55a^2b^8x^8+165a^3b^7x^7+330a^4b^6x^6+462a^5b^5x^5+462a^6b^4x^4+330a^7b^3x^3+165a}{11}$$

input
```
int((b*x+a)^10,x)
```

output

```
(x*(11*a**10 + 55*a**9*b*x + 165*a**8*b**2*x**2 + 330*a**7*b**3*x**3 + 462
*a**6*b**4*x**4 + 462*a**5*b**5*x**5 + 330*a**4*b**6*x**6 + 165*a**3*b**7*
x**7 + 55*a**2*b**8*x**8 + 11*a*b**9*x**9 + b**10*x**10))/11
```

3.93 $\int \frac{(a+bx)^{10}}{x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 122

$$\int \frac{(a+bx)^{10}}{x}\,dx = 10a^9bx + \frac{45}{2}a^8b^2x^2 + 40a^7b^3x^3 + \frac{105}{2}a^6b^4x^4 + \frac{252}{5}a^5b^5x^5 + 35a^4b^6x^6 + \frac{120}{7}a^3b^7x^7 + \frac{45}{8}a^2b^8x^8 + \frac{10}{9}ab^9x^9 + \frac{b^{10}x^{10}}{10} + a^{10}\log(x)$$

output
```
10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5
*x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10
*b^10*x^10+a^10*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x}\,dx = 10a^9bx + \frac{45}{2}a^8b^2x^2 + 40a^7b^3x^3 + \frac{105}{2}a^6b^4x^4 + \frac{252}{5}a^5b^5x^5 + 35a^4b^6x^6 + \frac{120}{7}a^3b^7x^7 + \frac{45}{8}a^2b^8x^8 + \frac{10}{9}ab^9x^9 + \frac{b^{10}x^{10}}{10} + a^{10}\log(x)$$

input
```
Integrate[(a + b*x)^10/x,x]
```

output
```
10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (
252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^
8)/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x} \, dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^{10}}{x}+10a^9b+45a^8b^2x+120a^7b^3x^2+210a^6b^4x^3+252a^5b^5x^4+210a^4b^6x^5+120a^3b^7x^6+45a^2b^8x^7+10ab$$

$$\Big\downarrow \ 2009$$

$$a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}$$

input
```
Int[(a + b*x)^10/x,x]
```

output
```
10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (
252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^
8)/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

method	result
default	$10a^9bx + \frac{45a^8b^2x^2}{2} + 40a^7b^3x^3 + \frac{105a^6b^4x^4}{2} + \frac{252a^5b^5x^5}{5} + 35a^4b^6x^6 + \frac{120a^3b^7x^7}{7} + \frac{45a^2b^8x^8}{8} + \frac{10a}{}$
norman	$10a^9bx + \frac{45a^8b^2x^2}{2} + 40a^7b^3x^3 + \frac{105a^6b^4x^4}{2} + \frac{252a^5b^5x^5}{5} + 35a^4b^6x^6 + \frac{120a^3b^7x^7}{7} + \frac{45a^2b^8x^8}{8} + \frac{10a}{}$
risch	$10a^9bx + \frac{45a^8b^2x^2}{2} + 40a^7b^3x^3 + \frac{105a^6b^4x^4}{2} + \frac{252a^5b^5x^5}{5} + 35a^4b^6x^6 + \frac{120a^3b^7x^7}{7} + \frac{45a^2b^8x^8}{8} + \frac{10a}{}$
parallelrisch	$10a^9bx + \frac{45a^8b^2x^2}{2} + 40a^7b^3x^3 + \frac{105a^6b^4x^4}{2} + \frac{252a^5b^5x^5}{5} + 35a^4b^6x^6 + \frac{120a^3b^7x^7}{7} + \frac{45a^2b^8x^8}{8} + \frac{10a}{}$

input
```
int((b*x+a)^10/x,x,method=_RETURNVERBOSE)
```

output
```
10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5
*x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10
*b^10*x^10+a^10*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x}\,dx = \frac{1}{10}\,b^{10}x^{10} + \frac{10}{9}\,ab^9x^9 + \frac{45}{8}\,a^2b^8x^8 + \frac{120}{7}\,a^3b^7x^7 + 35\,a^4b^6x^6 + \frac{252}{5}\,a^5b^5x^5 + \frac{105}{2}\,a^6b^4x^4 + 40\,a^7b^3x^3 + \frac{45}{2}\,a^8b^2x^2 + 10\,a^9bx + a^{10}\log\left(x\right)$$

input
```
integrate((b*x+a)^10/x,x, algorithm="fricas")
```

output
```
1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 3
5*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 4
5/2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x}\,dx = a^{10}\log(x) + 10a^9bx + \frac{45a^8b^2x^2}{2} + 40a^7b^3x^3 + \frac{105a^6b^4x^4}{2} + \frac{252a^5b^5x^5}{5} + 35a^4b^6x^6 + \frac{120a^3b^7x^7}{7} + \frac{45a^2b^8x^8}{8} + \frac{10ab^9x^9}{9} + \frac{b^{10}x^{10}}{10}$$

input
```
integrate((b*x+a)**10/x,x)
```

output
```
a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**3 + 105
*a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a**3*b*
*7*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x}\,dx = \frac{1}{10}\,b^{10}x^{10} + \frac{10}{9}\,ab^9x^9 + \frac{45}{8}\,a^2b^8x^8 + \frac{120}{7}\,a^3b^7x^7 + 35\,a^4b^6x^6 + \frac{252}{5}\,a^5b^5x^5 + \frac{105}{2}\,a^6b^4x^4 + 40\,a^7b^3x^3 + \frac{45}{2}\,a^8b^2x^2 + 10\,a^9bx + a^{10}\log(x)$$

input
```
integrate((b*x+a)^10/x,x, algorithm="maxima")
```

output
```
1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 3
5*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 4
5/2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x}\,dx = \frac{1}{10}b^{10}x^{10} + \frac{10}{9}ab^9x^9 + \frac{45}{8}a^2b^8x^8 + \frac{120}{7}a^3b^7x^7 + 35\,a^4b^6x^6 + \frac{252}{5}a^5b^5x^5 + \frac{105}{2}a^6b^4x^4 + 40\,a^7b^3x^3 + \frac{45}{2}a^8b^2x^2 + 10\,a^9bx + a^{10}\log(|x|)$$

input
```
integrate((b*x+a)^10/x,x, algorithm="giac")
```

output
```
1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 3
5*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 4
5/2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x}\,dx = a^{10}\,\ln(x) + \frac{b^{10}\,x^{10}}{10} + \frac{10\,a\,b^9\,x^9}{9} + \frac{45\,a^8\,b^2\,x^2}{2} + 40\,a^7\,b^3\,x^3 + \frac{105\,a^6\,b^4\,x^4}{2} + \frac{252\,a^5\,b^5\,x^5}{5} + 35\,a^4\,b^6\,x^6 + \frac{120\,a^3\,b^7\,x^7}{7} + \frac{45\,a^2\,b^8\,x^8}{8} + 10\,a^9\,b\,x$$

input
```
int((a + b*x)^10/x,x)
```

output
```
a^10*log(x) + (b^10*x^10)/10 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/2 + 40*
a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 +
 (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)/8 + 10*a^9*b*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x}\,dx = \log(x)\,a^{10} + 10a^9bx + \frac{45a^8b^2x^2}{2} + 40a^7b^3x^3 + \frac{105a^6b^4x^4}{2} + \frac{252a^5b^5x^5}{5} + 35a^4b^6x^6 + \frac{120a^3b^7x^7}{7} + \frac{45a^2b^8x^8}{8} + \frac{10a\,b^9x^9}{9} + \frac{b^{10}x^{10}}{10}$$

input
```
int((b*x+a)^10/x,x)
```

output
```
(2520*log(x)*a**10 + 25200*a**9*b*x + 56700*a**8*b**2*x**2 + 100800*a**7*b
**3*x**3 + 132300*a**6*b**4*x**4 + 127008*a**5*b**5*x**5 + 88200*a**4*b**6
*x**6 + 43200*a**3*b**7*x**7 + 14175*a**2*b**8*x**8 + 2800*a*b**9*x**9 + 2
52*b**10*x**10)/2520
```

3.94 $\int \frac{(a+bx)^{10}}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{(a+bx)^{10}}{x^2}\,dx = -\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}+10a^9b\log(x)$$

output
```
-a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b
^6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b
*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^2}\,dx = -\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}+10a^9b\log(x)$$

input
```
Integrate[(a + b*x)^10/x^2,x]
```

output
```
-(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^
4 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4
 + (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^2} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^{10}}{x^2}+\frac{10a^9b}{x}+45a^8b^2+120a^7b^3x+210a^6b^4x^2+252a^5b^5x^3+210a^4b^6x^4+120a^3b^7x^5+45a^2b^8x^6+10ab^9x\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^{10}}{x}+10a^9b\log(x)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

input
```
Int[(a + b*x)^10/x^2,x]
```

output
```
-(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^
4 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4
 + (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45a^2b^8x^7}{7}+\frac{5a}{}$
risch	$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45a^2b^8x^7}{7}+\frac{5a}{}$
norman	$\frac{-a^{10}+\frac{1}{9}b^{10}x^{10}+\frac{5}{4}a\,b^9x^9+\frac{45}{7}a^2b^8x^8+20a^3b^7x^7+42a^4b^6x^6+63a^5b^5x^5+70a^6b^4x^4+60a^7b^3x^3+45a^8b^2x^2}{x}+10a^9b\ln\left(x\right)$
parallelrisch	$\frac{28b^{10}x^{10}+315a\,b^9x^9+1620a^2b^8x^8+5040a^3b^7x^7+10584a^4b^6x^6+15876a^5b^5x^5+17640a^6b^4x^4+15120a^7b^3x^3+2520a^9b\ln(x)x}{252x}$

input
```
int((b*x+a)^10/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b
^6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b
*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{10}}{x^2}\,dx = \frac{28\,b^{10}x^{10}+315\,ab^9x^9+1620\,a^2b^8x^8+5040\,a^3b^7x^7+10584\,a^4b^6x^6+15876\,a^5b^5x^5+17640\,a^6b^4x^4+151}{252\,x}$$

input
```
integrate((b*x+a)^10/x^2,x, algorithm="fricas")
```

output
```
1/252*(28*b^10*x^10 + 315*a*b^9*x^9 + 1620*a^2*b^8*x^8 + 5040*a^3*b^7*x^7
+ 10584*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 17640*a^6*b^4*x^4 + 15120*a^7*b^
3*x^3 + 11340*a^8*b^2*x^2 + 2520*a^9*b*x*log(x) - 252*a^10)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{10}}{x^2}\,dx = -\frac{a^{10}}{x} + 10a^9b\log(x) + 45a^8b^2x + 60a^7b^3x^2 + 70a^6b^4x^3 + 63a^5b^5x^4 + 42a^4b^6x^5 + 20a^3b^7x^6 + \frac{45a^2b^8x^7}{7} + \frac{5ab^9x^8}{4} + \frac{b^{10}x^9}{9}$$

input
```
integrate((b*x+a)**10/x**2,x)
```

output
```
-a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b**3*x**2 + 70*a**6
*b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a**3*b**7*x**6 + 4
5*a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{10}}{x^2}\,dx = \frac{1}{9}\,b^{10}x^9 + \frac{5}{4}\,ab^9x^8 + \frac{45}{7}\,a^2b^8x^7 + 20\,a^3b^7x^6 + 42\,a^4b^6x^5 + 63\,a^5b^5x^4 + 70\,a^6b^4x^3 + 60\,a^7b^3x^2 + 45\,a^8b^2x + 10\,a^9b\log(x) - \frac{a^{10}}{x}$$

input
```
integrate((b*x+a)^10/x^2,x, algorithm="maxima")
```

output
```
1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*
b^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x
+ 10*a^9*b*log(x) - a^10/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^2}\,dx = \frac{1}{9}b^{10}x^9 + \frac{5}{4}ab^9x^8 + \frac{45}{7}a^2b^8x^7 + 20\,a^3b^7x^6 + 42\,a^4b^6x^5 + 63\,a^5b^5x^4 + 70\,a^6b^4x^3 + 60\,a^7b^3x^2 + 45\,a^8b^2x + 10\,a^9b\log\left(|x|\right) - \frac{a^{10}}{x}$$

input
```
integrate((b*x+a)^10/x^2,x, algorithm="giac")
```

output
```
1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*
b^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x
+ 10*a^9*b*log(abs(x)) - a^10/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{10}}{x^2}\,dx = \frac{b^{10}\,x^9}{9} - \frac{a^{10}}{x} + 45\,a^8\,b^2\,x + \frac{5\,a\,b^9\,x^8}{4} + 10\,a^9\,b\,\ln\left(x\right) + 60\,a^7\,b^3\,x^2 + 70\,a^6\,b^4\,x^3 + 63\,a^5\,b^5\,x^4 + 42\,a^4\,b^6\,x^5 + 20\,a^3\,b^7\,x^6 + \frac{45\,a^2\,b^8\,x^7}{7}$$

input
```
int((a + b*x)^10/x^2,x)
```

output
```
(b^10*x^9)/9 - a^10/x + 45*a^8*b^2*x + (5*a*b^9*x^8)/4 + 10*a^9*b*log(x) +
 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4 + 42*a^4*b^6*x^5 + 20*a^
3*b^7*x^6 + (45*a^2*b^8*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{10}}{x^2}\,dx$$
$$= \frac{2520\log(x)\,a^9bx - 252a^{10} + 11340a^8b^2x^2 + 15120a^7b^3x^3 + 17640a^6b^4x^4 + 15876a^5b^5x^5 + 10584a^4b^6x^6 +}{252x}$$

input
```
int((b*x+a)^10/x^2,x)
```

output
```
(2520*log(x)*a**9*b*x - 252*a**10 + 11340*a**8*b**2*x**2 + 15120*a**7*b**3
*x**3 + 17640*a**6*b**4*x**4 + 15876*a**5*b**5*x**5 + 10584*a**4*b**6*x**6
 + 5040*a**3*b**7*x**7 + 1620*a**2*b**8*x**8 + 315*a*b**9*x**9 + 28*b**10*
x**10)/(252*x)
```

3.95 $\int \frac{(a+bx)^{10}}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 119

$$\int \frac{(a+bx)^{10}}{x^3} dx = -\frac{a^{10}}{2x^2} - \frac{10a^9b}{x} + 120a^7b^3x + 105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105}{2}a^4b^6x^4 + 24a^3b^7x^5 + \frac{15}{2}a^2b^8x^6 + \frac{10}{7}ab^9x^7 + \frac{b^{10}x^8}{8} + 45a^8b^2\log(x)$$

output
```
-1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/
2*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+
45*a^8*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^3} dx = -\frac{a^{10}}{2x^2} - \frac{10a^9b}{x} + 120a^7b^3x + 105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105}{2}a^4b^6x^4 + 24a^3b^7x^5 + \frac{15}{2}a^2b^8x^6 + \frac{10}{7}ab^9x^7 + \frac{b^{10}x^8}{8} + 45a^8b^2\log(x)$$

input
```
Integrate[(a + b*x)^10/x^3,x]
```

output
```
-1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^
5*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*
b^9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^3}\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^{10}}{x^3}+\frac{10a^9b}{x^2}+\frac{45a^8b^2}{x}+120a^7b^3+210a^6b^4x+252a^5b^5x^2+210a^4b^6x^3+120a^3b^7x^4+45a^2b^8x^5+10ab^9x^6\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}$$

input
```
Int[(a + b*x)^10/x^3,x]
```

output

```
-1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^
5*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*
b^9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105a^4b^6x^4}{2}+24a^3b^7x^5+\frac{15a^2b^8x^6}{2}+\frac{10a\,b^9}{7}$
risch	$\frac{b^{10}x^8}{8}+\frac{10a\,b^9x^7}{7}+\frac{15a^2b^8x^6}{2}+24a^3b^7x^5+\frac{105a^4b^6x^4}{2}+84a^5b^5x^3+105a^6b^4x^2+120a^7b^3x+\frac{-10}{}$
norman	$\frac{-\frac{1}{2}a^{10}+\frac{1}{8}b^{10}x^{10}+\frac{10}{7}a\,b^9x^9+\frac{15}{2}a^2b^8x^8+24a^3b^7x^7+\frac{105}{2}a^4b^6x^6+84a^5b^5x^5+105a^6b^4x^4+120a^7b^3x^3-10a^9bx}{x^2}+45a^8b^2\ln$
parallelrisch	$\frac{7b^{10}x^{10}+80a\,b^9x^9+420a^2b^8x^8+1344a^3b^7x^7+2940a^4b^6x^6+4704a^5b^5x^5+5880a^6b^4x^4+2520a^8b^2\ln(x)x^2+6720a^7b^3x^3-560a}{56x^2}$

input
```
int((b*x+a)^10/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/
2*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+
45*a^8*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^3}\,dx = \frac{7\,b^{10}x^{10}+80\,ab^9x^9+420\,a^2b^8x^8+1344\,a^3b^7x^7+2940\,a^4b^6x^6+4704\,a^5b^5x^5+5880\,a^6b^4x^4+6720\,a^7b^3x}{56\,x^2}$$

input
```
integrate((b*x+a)^10/x^3,x, algorithm="fricas")
```

output
```
1/56*(7*b^10*x^10 + 80*a*b^9*x^9 + 420*a^2*b^8*x^8 + 1344*a^3*b^7*x^7 + 29
40*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 5880*a^6*b^4*x^4 + 6720*a^7*b^3*x^3 +
2520*a^8*b^2*x^2*log(x) - 560*a^9*b*x - 28*a^10)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x^3}\,dx = 45a^8b^2\log(x) + 120a^7b^3x + 105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105a^4b^6x^4}{2} + 24a^3b^7x^5 + \frac{15a^2b^8x^6}{2} + \frac{10ab^9x^7}{7} + \frac{b^{10}x^8}{8} + \frac{-a^{10}-20a^9bx}{2x^2}$$

input
```
integrate((b*x+a)**10/x**3,x)
```

output
```
45*a**8*b**2*log(x) + 120*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*
x**3 + 105*a**4*b**6*x**4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10
*a*b**9*x**7/7 + b**10*x**8/8 + (-a**10 - 20*a**9*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{10}}{x^3}\,dx = \frac{1}{8}\,b^{10}x^8 + \frac{10}{7}\,ab^9x^7 + \frac{15}{2}\,a^2b^8x^6 + 24\,a^3b^7x^5 + \frac{105}{2}\,a^4b^6x^4 + 84\,a^5b^5x^3 + 105\,a^6b^4x^2 + 120\,a^7b^3x + 45\,a^8b^2\log(x) - \frac{20\,a^9bx + a^{10}}{2\,x^2}$$

input
```
integrate((b*x+a)^10/x^3,x, algorithm="maxima")
```

output
```
1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*
a^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^
2*log(x) - 1/2*(20*a^9*b*x + a^10)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^3}\,dx = \frac{1}{8}\,b^{10}x^8 + \frac{10}{7}\,ab^9x^7 + \frac{15}{2}\,a^2b^8x^6 + 24\,a^3b^7x^5 + \frac{105}{2}\,a^4b^6x^4 + 84\,a^5b^5x^3 + 105\,a^6b^4x^2 + 120\,a^7b^3x + 45\,a^8b^2\log\left(|x|\right) - \frac{20\,a^9bx + a^{10}}{2\,x^2}$$

input
```
integrate((b*x+a)^10/x^3,x, algorithm="giac")
```

output
```
1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*
a^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^
2*log(abs(x)) - 1/2*(20*a^9*b*x + a^10)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^3}\,dx = \frac{b^{10}\,x^8}{8} - \frac{\frac{a^{10}}{2} + 10\,b\,x\,a^9}{x^2} + 120\,a^7\,b^3\,x + \frac{10\,a\,b^9\,x^7}{7} + 105\,a^6\,b^4\,x^2 + 84\,a^5\,b^5\,x^3 + \frac{105\,a^4\,b^6\,x^4}{2} + 24\,a^3\,b^7\,x^5 + \frac{15\,a^2\,b^8\,x^6}{2} + 45\,a^8\,b^2\,\ln\left(x\right)$$

input
```
int((a + b*x)^10/x^3,x)
```

output
```
(b^10*x^8)/8 - (a^10/2 + 10*a^9*b*x)/x^2 + 120*a^7*b^3*x + (10*a*b^9*x^7)/
7 + 105*a^6*b^4*x^2 + 84*a^5*b^5*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^
5 + (15*a^2*b^8*x^6)/2 + 45*a^8*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^3}\,dx$$
$$= \frac{2520\log(x)\,a^8b^2x^2 - 28a^{10} - 560a^9bx + 6720a^7b^3x^3 + 5880a^6b^4x^4 + 4704a^5b^5x^5 + 2940a^4b^6x^6 + 1344a^3}{56x^2}$$

input
```
int((b*x+a)^10/x^3,x)
```

output
```
(2520*log(x)*a**8*b**2*x**2 - 28*a**10 - 560*a**9*b*x + 6720*a**7*b**3*x**
3 + 5880*a**6*b**4*x**4 + 4704*a**5*b**5*x**5 + 2940*a**4*b**6*x**6 + 1344
*a**3*b**7*x**7 + 420*a**2*b**8*x**8 + 80*a*b**9*x**9 + 7*b**10*x**10)/(56
*x**2)
```

3.96 $\int \frac{(a+bx)^{10}}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{(a+bx)^{10}}{x^4} dx = -\frac{a^{10}}{3x^3} - \frac{5a^9b}{x^2} - \frac{45a^8b^2}{x} + 210a^6b^4x + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + \frac{5}{3}ab^9x^6 + \frac{b^{10}x^7}{7} + 120a^7b^3\log(x)$$

output
```
-1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^
4*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*
b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^4} dx = -\frac{a^{10}}{3x^3} - \frac{5a^9b}{x^2} - \frac{45a^8b^2}{x} + 210a^6b^4x + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + \frac{5}{3}ab^9x^6 + \frac{b^{10}x^7}{7} + 120a^7b^3\log(x)$$

input
```
Integrate[(a + b*x)^10/x^4,x]
```

output
```
-1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b
^5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3
 + (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^4}\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^{10}}{x^4}+\frac{10a^9b}{x^3}+\frac{45a^8b^2}{x^2}+\frac{120a^7b^3}{x}+210a^6b^4+252a^5b^5x+210a^4b^6x^2+120a^3b^7x^3+45a^2b^8x^4+10ab^9x^5+\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+ 9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

input
```
Int[(a + b*x)^10/x^4,x]
```

output

```
-1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b
^5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3
 + (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5a\,b^9x^6}{3}+$
risch	$\frac{b^{10}x^7}{7}+\frac{5a\,b^9x^6}{3}+9a^2b^8x^5+30a^3b^7x^4+70a^4b^6x^3+126a^5b^5x^2+210a^6b^4x+\frac{-45a^8b^2x^2-5a^9bx-}{x^3}$
norman	$\frac{-\frac{1}{3}a^{10}+\frac{1}{7}b^{10}x^{10}+\frac{5}{3}a\,b^9x^9+9a^2b^8x^8+30a^3b^7x^7+70a^4b^6x^6+126a^5b^5x^5+210a^6b^4x^4-45a^8b^2x^2-5a^9bx}{x^3}+120a^7b^3\ln($
parallelrisch	$\frac{3b^{10}x^{10}+35a\,b^9x^9+189a^2b^8x^8+630a^3b^7x^7+1470a^4b^6x^6+2646a^5b^5x^5+2520a^7b^3\ln(x)x^3+4410a^6b^4x^4-945a^8b^2x^2-105a^9}{21x^3}$

input
```
int((b*x+a)^10/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^
4*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*
b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{10}}{x^4}\,dx = \frac{3\,b^{10}x^{10}+35\,ab^9x^9+189\,a^2b^8x^8+630\,a^3b^7x^7+1470\,a^4b^6x^6+2646\,a^5b^5x^5+4410\,a^6b^4x^4+2520\,a^7b^3x^3}{21\,x^3}$$

input
```
integrate((b*x+a)^10/x^4,x, algorithm="fricas")
```

output
```
1/21*(3*b^10*x^10 + 35*a*b^9*x^9 + 189*a^2*b^8*x^8 + 630*a^3*b^7*x^7 + 147
0*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 4410*a^6*b^4*x^4 + 2520*a^7*b^3*x^3*log
(x) - 945*a^8*b^2*x^2 - 105*a^9*b*x - 7*a^10)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x^4}\,dx = 120a^7b^3\log{(x)} + 210a^6b^4x + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + \frac{5ab^9x^6}{3} + \frac{b^{10}x^7}{7} + \frac{-a^{10} - 15a^9bx - 135a^8b^2x^2}{3x^3}$$

input
```
integrate((b*x+a)**10/x**4,x)
```

output
```
120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b**6
*x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x**
7/7 + (-a**10 - 15*a**9*b*x - 135*a**8*b**2*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{10}}{x^4}\,dx = \frac{1}{7}\,b^{10}x^7 + \frac{5}{3}\,ab^9x^6 + 9\,a^2b^8x^5 + 30\,a^3b^7x^4 + 70\,a^4b^6x^3 + 126\,a^5b^5x^2 + 210\,a^6b^4x + 120\,a^7b^3\log\left(x\right) - \frac{135\,a^8b^2x^2 + 15\,a^9bx + a^{10}}{3\,x^3}$$

input
```
integrate((b*x+a)^10/x^4,x, algorithm="maxima")
```

output
```
1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6
*x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(x) - 1/3*(135*a^8
*b^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{10}}{x^4}\,dx = \frac{1}{7}b^{10}x^7 + \frac{5}{3}ab^9x^6 + 9\,a^2b^8x^5 + 30\,a^3b^7x^4 + 70\,a^4b^6x^3 + 126\,a^5b^5x^2 + 210\,a^6b^4x + 120\,a^7b^3\log(|x|) - \frac{135\,a^8b^2x^2 + 15\,a^9bx + a^{10}}{3\,x^3}$$

input
```
integrate((b*x+a)^10/x^4,x, algorithm="giac")
```

output
```
1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6
*x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(abs(x)) - 1/3*(13
5*a^8*b^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^4}\,dx = \frac{b^{10}\,x^7}{7} - \frac{\frac{a^{10}}{3} + 5\,a^9\,b\,x + 45\,a^8\,b^2\,x^2}{x^3} + 210\,a^6\,b^4\,x + \frac{5\,a\,b^9\,x^6}{3} + 126\,a^5\,b^5\,x^2 + 70\,a^4\,b^6\,x^3 + 30\,a^3\,b^7\,x^4 + 9\,a^2\,b^8\,x^5 + 120\,a^7\,b^3\,\ln(x)$$

input
```
int((a + b*x)^10/x^4,x)
```

output
```
(b^10*x^7)/7 - (a^10/3 + 45*a^8*b^2*x^2 + 5*a^9*b*x)/x^3 + 210*a^6*b^4*x +
 (5*a*b^9*x^6)/3 + 126*a^5*b^5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a
^2*b^8*x^5 + 120*a^7*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{10}}{x^4}\,dx$$
$$= \frac{2520\log(x)\,a^7b^3x^3 - 7a^{10} - 105a^9bx - 945a^8b^2x^2 + 4410a^6b^4x^4 + 2646a^5b^5x^5 + 1470a^4b^6x^6 + 630a^3b^7x}{21x^3}$$

input
```
int((b*x+a)^10/x^4,x)
```

output
```
(2520*log(x)*a**7*b**3*x**3 - 7*a**10 - 105*a**9*b*x - 945*a**8*b**2*x**2
+ 4410*a**6*b**4*x**4 + 2646*a**5*b**5*x**5 + 1470*a**4*b**6*x**6 + 630*a*
*3*b**7*x**7 + 189*a**2*b**8*x**8 + 35*a*b**9*x**9 + 3*b**10*x**10)/(21*x*
*3)
```

3.97 $\int \frac{(a+bx)^{10}}{x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 119

$$\int \frac{(a+bx)^{10}}{x^5} dx = -\frac{a^{10}}{4x^4} - \frac{10a^9b}{3x^3} - \frac{45a^8b^2}{2x^2} - \frac{120a^7b^3}{x} + 252a^5b^5x + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45}{4}a^2b^8x^4 + 2ab^9x^5 + \frac{b^{10}x^6}{6} + 210a^6b^4\log(x)$$

output
```
-1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+
105*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+2
10*a^6*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^5} dx = -\frac{a^{10}}{4x^4} - \frac{10a^9b}{3x^3} - \frac{45a^8b^2}{2x^2} - \frac{120a^7b^3}{x} + 252a^5b^5x + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45}{4}a^2b^8x^4 + 2ab^9x^5 + \frac{b^{10}x^6}{6} + 210a^6b^4\log(x)$$

input
```
Integrate[(a + b*x)^10/x^5,x]
```

output
```
-1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/
x + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4
+ 2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^5} \, dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^{10}}{x^5}+\frac{10a^9b}{x^4}+\frac{45a^8b^2}{x^3}+\frac{120a^7b^3}{x^2}+\frac{210a^6b^4}{x}+252a^5b^5+210a^4b^6x+120a^3b^7x^2+45a^2b^8x^3+10ab^9x^4+b\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+210a^6b^4\log(x)+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}$$

input
```
Int[(a + b*x)^10/x^5,x]
```

output

```
-1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/
x + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4
+ 2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45a^2b^8x^4}{4}+2a\,b^9x^5+$
risch	$\frac{b^{10}x^6}{6}+2a\,b^9x^5+\frac{45a^2b^8x^4}{4}+40a^3b^7x^3+105a^4b^6x^2+252a^5b^5x+\frac{-120a^7b^3x^3-\frac{45}{2}a^8b^2x^2-\frac{10}{3}a^9bx-}{x^4}$
norman	$\frac{-\frac{1}{4}a^{10}+\frac{1}{6}b^{10}x^{10}+2a\,b^9x^9+\frac{45}{4}a^2b^8x^8+40a^3b^7x^7+105a^4b^6x^6+252a^5b^5x^5-120a^7b^3x^3-\frac{45}{2}a^8b^2x^2-\frac{10}{3}a^9bx}{x^4}+210a^6b^4\,\mathrm{l}$
parallelrisch	$\frac{2b^{10}x^{10}+24a\,b^9x^9+135a^2b^8x^8+480a^3b^7x^7+1260a^4b^6x^6+2520a^6b^4\ln(x)x^4+3024a^5b^5x^5-1440a^7b^3x^3-270a^8b^2x^2-40a^9b}{12x^4}$

input
```
int((b*x+a)^10/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+
105*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+2
10*a^6*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^5}\,dx = \frac{2\,b^{10}x^{10}+24\,ab^9x^9+135\,a^2b^8x^8+480\,a^3b^7x^7+1260\,a^4b^6x^6+3024\,a^5b^5x^5+2520\,a^6b^4x^4\log(x)-1440}{12\,x^4}$$

input
```
integrate((b*x+a)^10/x^5,x, algorithm="fricas")
```

output
```
1/12*(2*b^10*x^10 + 24*a*b^9*x^9 + 135*a^2*b^8*x^8 + 480*a^3*b^7*x^7 + 126
0*a^4*b^6*x^6 + 3024*a^5*b^5*x^5 + 2520*a^6*b^4*x^4*log(x) - 1440*a^7*b^3*
x^3 - 270*a^8*b^2*x^2 - 40*a^9*b*x - 3*a^10)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{10}}{x^5}\,dx = 210a^6b^4\log{(x)} + 252a^5b^5x + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45a^2b^8x^4}{4} + 2ab^9x^5 + \frac{b^{10}x^6}{6} + \frac{-3a^{10} - 40a^9bx - 270a^8b^2x^2 - 1440a^7b^3x^3}{12x^4}$$

input
```
integrate((b*x+a)**10/x**5,x)
```

output
```
210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105*a**4*b**6*x**2 + 40*a**3*b**7
*x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 + b**10*x**6/6 + (-3*a**10 - 4
0*a**9*b*x - 270*a**8*b**2*x**2 - 1440*a**7*b**3*x**3)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^5}\,dx = \frac{1}{6}\,b^{10}x^6 + 2\,ab^9x^5 + \frac{45}{4}\,a^2b^8x^4 + 40\,a^3b^7x^3 + 105\,a^4b^6x^2 + 252\,a^5b^5x + 210\,a^6b^4\log{(x)} - \frac{1440\,a^7b^3x^3 + 270\,a^8b^2x^2 + 40\,a^9bx + 3\,a^{10}}{12\,x^4}$$

input
```
integrate((b*x+a)^10/x^5,x, algorithm="maxima")
```

output
```
1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b
^6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(x) - 1/12*(1440*a^7*b^3*x^3 + 270
*a^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{10}}{x^5}\,dx = \frac{1}{6}\,b^{10}x^6 + 2\,ab^9x^5 + \frac{45}{4}\,a^2b^8x^4 + 40\,a^3b^7x^3 + 105\,a^4b^6x^2 + 252\,a^5b^5x + 210\,a^6b^4\log\left(|x|\right) - \frac{1440\,a^7b^3x^3 + 270\,a^8b^2x^2 + 40\,a^9bx + 3\,a^{10}}{12\,x^4}$$

input
```
integrate((b*x+a)^10/x^5,x, algorithm="giac")
```

output
```
1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b
^6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(abs(x)) - 1/12*(1440*a^7*b^3*x^3
+ 270*a^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^5}\,dx = \frac{b^{10}\,x^6}{6} - \frac{\frac{a^{10}}{4} + \frac{10\,a^9\,b\,x}{3} + \frac{45\,a^8\,b^2\,x^2}{2} + 120\,a^7\,b^3\,x^3}{x^4} + 252\,a^5\,b^5\,x + 2\,a\,b^9\,x^5 + 105\,a^4\,b^6\,x^2 + 40\,a^3\,b^7\,x^3 + \frac{45\,a^2\,b^8\,x^4}{4} + 210\,a^6\,b^4\,\ln\left(x\right)$$

input
```
int((a + b*x)^10/x^5,x)
```

output
```
(b^10*x^6)/6 - (a^10/4 + (45*a^8*b^2*x^2)/2 + 120*a^7*b^3*x^3 + (10*a^9*b*
x)/3)/x^4 + 252*a^5*b^5*x + 2*a*b^9*x^5 + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3
 + (45*a^2*b^8*x^4)/4 + 210*a^6*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^5}\,dx$$
$$= \frac{2520\log(x)\,a^6b^4x^4 - 3a^{10} - 40a^9bx - 270a^8b^2x^2 - 1440a^7b^3x^3 + 3024a^5b^5x^5 + 1260a^4b^6x^6 + 480a^3b^7x^7}{12x^4}$$

input
```
int((b*x+a)^10/x^5,x)
```

output
```
(2520*log(x)*a**6*b**4*x**4 - 3*a**10 - 40*a**9*b*x - 270*a**8*b**2*x**2 -
 1440*a**7*b**3*x**3 + 3024*a**5*b**5*x**5 + 1260*a**4*b**6*x**6 + 480*a**
3*b**7*x**7 + 135*a**2*b**8*x**8 + 24*a*b**9*x**9 + 2*b**10*x**10)/(12*x**
4)
```

3.98 $\int \frac{(a+bx)^{10}}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 117

$$\int \frac{(a+bx)^{10}}{x^6} dx = -\frac{a^{10}}{5x^5} - \frac{5a^9b}{2x^4} - \frac{15a^8b^2}{x^3} - \frac{60a^7b^3}{x^2} - \frac{210a^6b^4}{x} + 210a^4b^6x + 60a^3b^7x^2 + 15a^2b^8x^3 + \frac{5}{2}ab^9x^4 + \frac{b^{10}x^5}{5} + 252a^5b^5\log(x)$$

output
```
-1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+21
0*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a
^5*b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^6} dx = -\frac{a^{10}}{5x^5} - \frac{5a^9b}{2x^4} - \frac{15a^8b^2}{x^3} - \frac{60a^7b^3}{x^2} - \frac{210a^6b^4}{x} + 210a^4b^6x + 60a^3b^7x^2 + 15a^2b^8x^3 + \frac{5}{2}ab^9x^4 + \frac{b^{10}x^5}{5} + 252a^5b^5\log(x)$$

input
```
Integrate[(a + b*x)^10/x^6,x]
```

output
```
-1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 -
(210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b
^9*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^6} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^{10}}{x^6}+\frac{10a^9b}{x^5}+\frac{45a^8b^2}{x^4}+\frac{120a^7b^3}{x^3}+\frac{210a^6b^4}{x^2}+\frac{252a^5b^5}{x}+210a^4b^6+120a^3b^7x+45a^2b^8x^2+10ab^9x^3+b^{10}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+ 15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

input
```
Int[(a + b*x)^10/x^6,x]
```

output

```
-1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 -
(210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b
^9*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5a\,b^9x^4}{2}+\frac{b^{10}x^5}{5}$
risch	$\frac{b^{10}x^5}{5}+\frac{5a\,b^9x^4}{2}+15a^2b^8x^3+60a^3b^7x^2+210a^4b^6x+\frac{-210a^6b^4x^4-60a^7b^3x^3-15a^8b^2x^2-\frac{5}{2}a^9bx-\frac{1}{5}a^{10}}{x^5}$
norman	$\frac{-\frac{1}{5}a^{10}+\frac{1}{5}b^{10}x^{10}+\frac{5}{2}a\,b^9x^9+15a^2b^8x^8+60a^3b^7x^7+210a^4b^6x^6-210a^6b^4x^4-60a^7b^3x^3-15a^8b^2x^2-\frac{5}{2}a^9bx}{x^5}+252a^5b^5\ln$
parallelrisch	$\frac{2b^{10}x^{10}+25a\,b^9x^9+150a^2b^8x^8+600a^3b^7x^7+2520a^5b^5\ln(x)x^5+2100a^4b^6x^6-2100a^6b^4x^4-600a^7b^3x^3-150a^8b^2x^2-25a^9bx}{10x^5}$

input
```
int((b*x+a)^10/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+21
0*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a
^5*b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int\frac{(a+bx)^{10}}{x^6}\,dx$$
$$=\frac{2\,b^{10}x^{10}+25\,ab^9x^9+150\,a^2b^8x^8+600\,a^3b^7x^7+2100\,a^4b^6x^6+2520\,a^5b^5x^5\log{(x)}-2100\,a^6b^4x^4-600\,a}{10\,x^5}$$

input
```
integrate((b*x+a)^10/x^6,x, algorithm="fricas")
```

output
```
1/10*(2*b^10*x^10 + 25*a*b^9*x^9 + 150*a^2*b^8*x^8 + 600*a^3*b^7*x^7 + 210
0*a^4*b^6*x^6 + 2520*a^5*b^5*x^5*log(x) - 2100*a^6*b^4*x^4 - 600*a^7*b^3*x
^3 - 150*a^8*b^2*x^2 - 25*a^9*b*x - 2*a^10)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x^6}\,dx = 252a^5b^5\log(x) + 210a^4b^6x + 60a^3b^7x^2 + 15a^2b^8x^3 + \frac{5ab^9x^4}{2} + \frac{b^{10}x^5}{5} + \frac{-2a^{10} - 25a^9bx - 150a^8b^2x^2 - 600a^7b^3x^3 - 2100a^6b^4x^4}{10x^5}$$

input
```
integrate((b*x+a)**10/x**6,x)
```

output
```
252*a**5*b**5*log(x) + 210*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*
x**3 + 5*a*b**9*x**4/2 + b**10*x**5/5 + (-2*a**10 - 25*a**9*b*x - 150*a**8
*b**2*x**2 - 600*a**7*b**3*x**3 - 2100*a**6*b**4*x**4)/(10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{10}}{x^6}\,dx = \frac{1}{5}\,b^{10}x^5 + \frac{5}{2}\,ab^9x^4 + 15\,a^2b^8x^3 + 60\,a^3b^7x^2 + 210\,a^4b^6x + 252\,a^5b^5\log(x) - \frac{2100\,a^6b^4x^4 + 600\,a^7b^3x^3 + 150\,a^8b^2x^2 + 25\,a^9bx + 2\,a^{10}}{10\,x^5}$$

input
```
integrate((b*x+a)^10/x^6,x, algorithm="maxima")
```

output
```
1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b
^6*x + 252*a^5*b^5*log(x) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 + 150
*a^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{10}}{x^6}\,dx = \frac{1}{5}\,b^{10}x^5 + \frac{5}{2}\,ab^9x^4 + 15\,a^2b^8x^3 + 60\,a^3b^7x^2 + 210\,a^4b^6x + 252\,a^5b^5\log\left(|x|\right) - \frac{2100\,a^6b^4x^4 + 600\,a^7b^3x^3 + 150\,a^8b^2x^2 + 25\,a^9bx + 2\,a^{10}}{10\,x^5}$$

input
```
integrate((b*x+a)^10/x^6,x, algorithm="giac")
```

output
```
1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b
^6*x + 252*a^5*b^5*log(abs(x)) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3
+ 150*a^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{10}}{x^6}\,dx = \frac{b^{10}\,x^5}{5} - \frac{\frac{a^{10}}{5} + \frac{5\,a^9\,b\,x}{2} + 15\,a^8\,b^2\,x^2 + 60\,a^7\,b^3\,x^3 + 210\,a^6\,b^4\,x^4}{x^5} + 210\,a^4\,b^6\,x + \frac{5\,a\,b^9\,x^4}{2} + 60\,a^3\,b^7\,x^2 + 15\,a^2\,b^8\,x^3 + 252\,a^5\,b^5\,\ln\left(x\right)$$

input
```
int((a + b*x)^10/x^6,x)
```

output
```
(b^10*x^5)/5 - (a^10/5 + 15*a^8*b^2*x^2 + 60*a^7*b^3*x^3 + 210*a^6*b^4*x^4
 + (5*a^9*b*x)/2)/x^5 + 210*a^4*b^6*x + (5*a*b^9*x^4)/2 + 60*a^3*b^7*x^2 +
 15*a^2*b^8*x^3 + 252*a^5*b^5*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}}{x^6}\,dx$$
$$= \frac{2520\log(x)\,a^5b^5x^5 - 2a^{10} - 25a^9bx - 150a^8b^2x^2 - 600a^7b^3x^3 - 2100a^6b^4x^4 + 2100a^4b^6x^6 + 600a^3b^7x^7}{10x^5}$$

input
```
int((b*x+a)^10/x^6,x)
```

output
```
(2520*log(x)*a**5*b**5*x**5 - 2*a**10 - 25*a**9*b*x - 150*a**8*b**2*x**2 -
 600*a**7*b**3*x**3 - 2100*a**6*b**4*x**4 + 2100*a**4*b**6*x**6 + 600*a**3
*b**7*x**7 + 150*a**2*b**8*x**8 + 25*a*b**9*x**9 + 2*b**10*x**10)/(10*x**5
)
```

3.99 $\int \frac{(a+bx)^{10}}{x^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 119

$$\int \frac{(a+bx)^{10}}{x^7} dx = -\frac{a^{10}}{6x^6} - \frac{2a^9b}{x^5} - \frac{45a^8b^2}{4x^4} - \frac{40a^7b^3}{x^3} - \frac{105a^6b^4}{x^2} - \frac{252a^5b^5}{x} + 120a^3b^7x + \frac{45}{2}a^2b^8x^2 + \frac{10}{3}ab^9x^3 + \frac{b^{10}x^4}{4} + 210a^4b^6\log(x)$$

output
```
-1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-
252*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+2
10*a^4*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^7} dx = -\frac{a^{10}}{6x^6} - \frac{2a^9b}{x^5} - \frac{45a^8b^2}{4x^4} - \frac{40a^7b^3}{x^3} - \frac{105a^6b^4}{x^2} - \frac{252a^5b^5}{x} + 120a^3b^7x + \frac{45}{2}a^2b^8x^2 + \frac{10}{3}ab^9x^3 + \frac{b^{10}x^4}{4} + 210a^4b^6\log(x)$$

input
```
Integrate[(a + b*x)^10/x^7,x]
```

output
```
-1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 -
(105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 +
 (10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^7}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a^4b^6}{x}+120a^3b^7+45a^2b^8x+10ab^9x^2+b^{10}x^3\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+210a^4b^6\log(x)+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

input
```
Int[(a + b*x)^10/x^7,x]
```

output

```
-1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 -
(105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 +
 (10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45a^2b^8x^2}{2}+\frac{10a\,b^9x^3}{3}+\frac{b^{10}x^4}{4}+2$
risch	$\frac{b^{10}x^4}{4}+\frac{10a\,b^9x^3}{3}+\frac{45a^2b^8x^2}{2}+120a^3b^7x+\frac{-252a^5b^5x^5-105a^6b^4x^4-40a^7b^3x^3-\frac{45}{4}a^8b^2x^2-2a^9bx-\frac{1}{6}a^{10}}{x^6}+21$
norman	$\frac{-\frac{1}{6}a^{10}+\frac{1}{4}b^{10}x^{10}+\frac{10}{3}a\,b^9x^9+\frac{45}{2}a^2b^8x^8+120a^3b^7x^7-252a^5b^5x^5-105a^6b^4x^4-40a^7b^3x^3-\frac{45}{4}a^8b^2x^2-2a^9bx}{x^6}+210a^4b^6\,\mathrm{l}$
parallelrisch	$\frac{3b^{10}x^{10}+40a\,b^9x^9+270a^2b^8x^8+2520a^4b^6\ln(x)x^6+1440a^3b^7x^7-3024a^5b^5x^5-1260a^6b^4x^4-480a^7b^3x^3-135a^8b^2x^2-24a^9b}{12x^6}$

input
```
int((b*x+a)^10/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-
252*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+2
10*a^4*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^7}\,dx$$
$$=\frac{3\,b^{10}x^{10}+40\,ab^9x^9+270\,a^2b^8x^8+1440\,a^3b^7x^7+2520\,a^4b^6x^6\log(x)-3024\,a^5b^5x^5-1260\,a^6b^4x^4-480}{12\,x^6}$$

input
```
integrate((b*x+a)^10/x^7,x, algorithm="fricas")
```

output
```
1/12*(3*b^10*x^10 + 40*a*b^9*x^9 + 270*a^2*b^8*x^8 + 1440*a^3*b^7*x^7 + 25
20*a^4*b^6*x^6*log(x) - 3024*a^5*b^5*x^5 - 1260*a^6*b^4*x^4 - 480*a^7*b^3*
x^3 - 135*a^8*b^2*x^2 - 24*a^9*b*x - 2*a^10)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x^7}\,dx = 210a^4b^6\log{(x)} + 120a^3b^7x + \frac{45a^2b^8x^2}{2} + \frac{10ab^9x^3}{3} + \frac{b^{10}x^4}{4} + \frac{-2a^{10} - 24a^9bx - 135a^8b^2x^2 - 480a^7b^3x^3 - 1260a^6b^4x^4 - 3024a^5b^5x^5}{12x^6}$$

input
```
integrate((b*x+a)**10/x**7,x)
```

output
```
210*a**4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x
**3/3 + b**10*x**4/4 + (-2*a**10 - 24*a**9*b*x - 135*a**8*b**2*x**2 - 480*
a**7*b**3*x**3 - 1260*a**6*b**4*x**4 - 3024*a**5*b**5*x**5)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^7}\,dx = \frac{1}{4}\,b^{10}x^4 + \frac{10}{3}\,ab^9x^3 + \frac{45}{2}\,a^2b^8x^2 + 120\,a^3b^7x + 210\,a^4b^6\log\left(x\right) - \frac{3024\,a^5b^5x^5 + 1260\,a^6b^4x^4 + 480\,a^7b^3x^3 + 135\,a^8b^2x^2 + 24\,a^9bx + 2\,a^{10}}{12\,x^6}$$

input
```
integrate((b*x+a)^10/x^7,x, algorithm="maxima")
```

output
```
1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4
*b^6*log(x) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x^3
+ 135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{10}}{x^7}\,dx = \frac{1}{4}\,b^{10}x^4 + \frac{10}{3}\,ab^9x^3 + \frac{45}{2}\,a^2b^8x^2 + 120\,a^3b^7x + 210\,a^4b^6\log\left(|x|\right) - \frac{3024\,a^5b^5x^5 + 1260\,a^6b^4x^4 + 480\,a^7b^3x^3 + 135\,a^8b^2x^2 + 24\,a^9bx + 2\,a^{10}}{12\,x^6}$$

input
```
integrate((b*x+a)^10/x^7,x, algorithm="giac")
```

output
```
1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4
*b^6*log(abs(x)) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3
*x^3 + 135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^7}\,dx = \frac{b^{10}\,x^4}{4} - \frac{\frac{a^{10}}{6} + 2\,a^9\,b\,x + \frac{45\,a^8\,b^2\,x^2}{4} + 40\,a^7\,b^3\,x^3 + 105\,a^6\,b^4\,x^4 + 252\,a^5\,b^5\,x^5}{x^6} + 120\,a^3\,b^7\,x + \frac{10\,a\,b^9\,x^3}{3} + \frac{45\,a^2\,b^8\,x^2}{2} + 210\,a^4\,b^6\,\ln\left(x\right)$$

input
```
int((a + b*x)^10/x^7,x)
```

output
```
(b^10*x^4)/4 - (a^10/6 + (45*a^8*b^2*x^2)/4 + 40*a^7*b^3*x^3 + 105*a^6*b^4
*x^4 + 252*a^5*b^5*x^5 + 2*a^9*b*x)/x^6 + 120*a^3*b^7*x + (10*a*b^9*x^3)/3
 + (45*a^2*b^8*x^2)/2 + 210*a^4*b^6*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^7}\,dx = \frac{2520\log(x)\,a^4b^6x^6 - 2a^{10} - 24a^9bx - 135a^8b^2x^2 - 480a^7b^3x^3 - 1260a^6b^4x^4 - 3024a^5b^5x^5 + 1440a^3b^7x^7}{12x^6}$$

input
```
int((b*x+a)^10/x^7,x)
```

output
```
(2520*log(x)*a**4*b**6*x**6 - 2*a**10 - 24*a**9*b*x - 135*a**8*b**2*x**2 -
 480*a**7*b**3*x**3 - 1260*a**6*b**4*x**4 - 3024*a**5*b**5*x**5 + 1440*a**
3*b**7*x**7 + 270*a**2*b**8*x**8 + 40*a*b**9*x**9 + 3*b**10*x**10)/(12*x**
6)
```

3.100 $\int \frac{(a+bx)^{10}}{x^8}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{(a+bx)^{10}}{x^8}\,dx = -\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\log(x)$$

output
```
-1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-12
6*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*
b^7*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^8}\,dx = -\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\log(x)$$

input
```
Integrate[(a + b*x)^10/x^8,x]
```

output
```
-1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (
70*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a
*b^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^8}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^{10}}{x^8}+\frac{10a^9b}{x^7}+\frac{45a^8b^2}{x^6}+\frac{120a^7b^3}{x^5}+\frac{210a^6b^4}{x^4}+\frac{252a^5b^5}{x^3}+\frac{210a^4b^6}{x^2}+\frac{120a^3b^7}{x}+45a^2b^8+10ab^9x+b^{10}x^2\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}$$

input
```
Int[(a + b*x)^10/x^8,x]
```

output
```
-1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (
70*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a
*b^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5a\,b^9x^2+\frac{b^{10}x^3}{3}+120a$
risch	$\frac{b^{10}x^3}{3}+5a\,b^9x^2+45a^2b^8x+\frac{-210a^4b^6x^6-126a^5b^5x^5-70a^6b^4x^4-30a^7b^3x^3-9a^8b^2x^2-\frac{5}{3}a^9bx-\frac{1}{7}a^{10}}{x^7}+120a^3$
norman	$\frac{-\frac{1}{7}a^{10}+\frac{1}{3}b^{10}x^{10}+5a\,b^9x^9+45a^2b^8x^8-210a^4b^6x^6-126a^5b^5x^5-70a^6b^4x^4-30a^7b^3x^3-9a^8b^2x^2-\frac{5}{3}a^9bx}{x^7}+120a^3b^7\ln($
parallelrisch	$\frac{7b^{10}x^{10}+105a\,b^9x^9+2520a^3b^7\ln(x)x^7+945a^2b^8x^8-4410a^4b^6x^6-2646a^5b^5x^5-1470a^6b^4x^4-630a^7b^3x^3-189a^8b^2x^2-35a^9}{21x^7}$

input
```
int((b*x+a)^10/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-12
6*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*
b^7*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{10}}{x^8}\,dx$$
$$=\frac{7\,b^{10}x^{10}+105\,ab^9x^9+945\,a^2b^8x^8+2520\,a^3b^7x^7\log(x)-4410\,a^4b^6x^6-2646\,a^5b^5x^5-1470\,a^6b^4x^4-63}{21\,x^7}$$

input
```
integrate((b*x+a)^10/x^8,x, algorithm="fricas")
```

output
```
1/21*(7*b^10*x^10 + 105*a*b^9*x^9 + 945*a^2*b^8*x^8 + 2520*a^3*b^7*x^7*log
(x) - 4410*a^4*b^6*x^6 - 2646*a^5*b^5*x^5 - 1470*a^6*b^4*x^4 - 630*a^7*b^3
*x^3 - 189*a^8*b^2*x^2 - 35*a^9*b*x - 3*a^10)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x^8}\,dx = 120a^3b^7\log(x) + 45a^2b^8x + 5ab^9x^2 + \frac{b^{10}x^3}{3} + \frac{-3a^{10} - 35a^9bx - 189a^8b^2x^2 - 630a^7b^3x^3 - 1470a^6b^4x^4 - 2646a^5b^5x^5 - 4410a^4b^6x^6}{21x^7}$$

input
```
integrate((b*x+a)**10/x**8,x)
```

output
```
120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 + b**10*x**3/3 + (-3
*a**10 - 35*a**9*b*x - 189*a**8*b**2*x**2 - 630*a**7*b**3*x**3 - 1470*a**6
*b**4*x**4 - 2646*a**5*b**5*x**5 - 4410*a**4*b**6*x**6)/(21*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^8}\,dx = \frac{1}{3}\,b^{10}x^3 + 5\,ab^9x^2 + 45\,a^2b^8x + 120\,a^3b^7\log(x) - \frac{4410\,a^4b^6x^6 + 2646\,a^5b^5x^5 + 1470\,a^6b^4x^4 + 630\,a^7b^3x^3 + 189\,a^8b^2x^2 + 35\,a^9bx + 3\,a^{10}}{21\,x^7}$$

input
```
integrate((b*x+a)^10/x^8,x, algorithm="maxima")
```

output
```
1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(x) - 1/21*(441
0*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 + 18
9*a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}}{x^8}\,dx = \frac{1}{3}\,b^{10}x^3 + 5\,ab^9x^2 + 45\,a^2b^8x + 120\,a^3b^7\log\left(|x|\right) - \frac{4410\,a^4b^6x^6 + 2646\,a^5b^5x^5 + 1470\,a^6b^4x^4 + 630\,a^7b^3x^3 + 189\,a^8b^2x^2 + 35\,a^9bx + 3\,a^{10}}{21\,x^7}$$

input
```
integrate((b*x+a)^10/x^8,x, algorithm="giac")
```

output
```
1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(abs(x)) - 1/21
*(4410*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3
 + 189*a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^8}\,dx = \frac{b^{10}\,x^3}{3} - \frac{\frac{a^{10}}{7} + \frac{5\,a^9\,b\,x}{3} + 9\,a^8\,b^2\,x^2 + 30\,a^7\,b^3\,x^3 + 70\,a^6\,b^4\,x^4 + 126\,a^5\,b^5\,x^5 + 210\,a^4\,b^6\,x^6}{x^7} + 45\,a^2\,b^8\,x + 5\,a\,b^9\,x^2 + 120\,a^3\,b^7\,\ln\left(x\right)$$

input
```
int((a + b*x)^10/x^8,x)
```

output
```
(b^10*x^3)/3 - (a^10/7 + 9*a^8*b^2*x^2 + 30*a^7*b^3*x^3 + 70*a^6*b^4*x^4 +
 126*a^5*b^5*x^5 + 210*a^4*b^6*x^6 + (5*a^9*b*x)/3)/x^7 + 45*a^2*b^8*x + 5
*a*b^9*x^2 + 120*a^3*b^7*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{10}}{x^8}\,dx$$
$$= \frac{2520\log(x)\,a^3b^7x^7 - 3a^{10} - 35a^9bx - 189a^8b^2x^2 - 630a^7b^3x^3 - 1470a^6b^4x^4 - 2646a^5b^5x^5 - 4410a^4b^6x^6}{21x^7}$$

input
```
int((b*x+a)^10/x^8,x)
```

output
```
(2520*log(x)*a**3*b**7*x**7 - 3*a**10 - 35*a**9*b*x - 189*a**8*b**2*x**2 -
 630*a**7*b**3*x**3 - 1470*a**6*b**4*x**4 - 2646*a**5*b**5*x**5 - 4410*a**
4*b**6*x**6 + 945*a**2*b**8*x**8 + 105*a*b**9*x**9 + 7*b**10*x**10)/(21*x*
*7)
```

3.101 $\int \frac{(a+bx)^{10}}{x^9} dx$

Optimal result

Integrand size = 11, antiderivative size = 119

$$\int \frac{(a+bx)^{10}}{x^9} dx = -\frac{a^{10}}{8x^8} - \frac{10a^9b}{7x^7} - \frac{15a^8b^2}{2x^6} - \frac{24a^7b^3}{x^5} - \frac{105a^6b^4}{2x^4} - \frac{84a^5b^5}{x^3} - \frac{105a^4b^6}{x^2} - \frac{120a^3b^7}{x} + 10ab^9x + \frac{b^{10}x^2}{2} + 45a^2b^8\log(x)$$

output
```
-1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4
/x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+
45*a^2*b^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^9} dx = -\frac{a^{10}}{8x^8} - \frac{10a^9b}{7x^7} - \frac{15a^8b^2}{2x^6} - \frac{24a^7b^3}{x^5} - \frac{105a^6b^4}{2x^4} - \frac{84a^5b^5}{x^3} - \frac{105a^4b^6}{x^2} - \frac{120a^3b^7}{x} + 10ab^9x + \frac{b^{10}x^2}{2} + 45a^2b^8\log(x)$$

input
```
Integrate[(a + b*x)^10/x^9,x]
```

output
```
-1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x
^5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a
^3*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^9} \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^{10}}{x^9}+\frac{10a^9b}{x^8}+\frac{45a^8b^2}{x^7}+\frac{120a^7b^3}{x^6}+\frac{210a^6b^4}{x^5}+\frac{252a^5b^5}{x^4}+\frac{210a^4b^6}{x^3}+\frac{120a^3b^7}{x^2}+\frac{45a^2b^8}{x}+10ab^9+b^{10}x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)+10ab^9x+\frac{b^{10}x^2}{2}$$

input
```
Int[(a + b*x)^10/x^9,x]
```

output

```
-1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x
^5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a
^3*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10a\,b^9x+\frac{b^{10}x^2}{2}+45a$
risch	$\frac{b^{10}x^2}{2}+10a\,b^9x+\frac{-120a^3b^7x^7-105a^4b^6x^6-84a^5b^5x^5-\frac{105}{2}a^6b^4x^4-24a^7b^3x^3-\frac{15}{2}a^8b^2x^2-\frac{10}{7}a^9bx-\frac{1}{8}a^{10}}{x^8}+45a^2b$
norman	$\frac{-\frac{1}{8}a^{10}+\frac{1}{2}b^{10}x^{10}+10a\,b^9x^9-120a^3b^7x^7-105a^4b^6x^6-84a^5b^5x^5-\frac{105}{2}a^6b^4x^4-24a^7b^3x^3-\frac{15}{2}a^8b^2x^2-\frac{10}{7}a^9bx}{x^8}+45a^2b^8\ln$
parallelrisch	$\frac{28b^{10}x^{10}+2520a^2b^8\ln(x)x^8+560a\,b^9x^9-6720a^3b^7x^7-5880a^4b^6x^6-4704a^5b^5x^5-2940a^6b^4x^4-1344a^7b^3x^3-420a^8b^2x^2-80}{56x^8}$

input
```
int((b*x+a)^10/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4
/x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+
45*a^2*b^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^9}\,dx$$
$$=\frac{28\,b^{10}x^{10}+560\,ab^9x^9+2520\,a^2b^8x^8\log\left(x\right)-6720\,a^3b^7x^7-5880\,a^4b^6x^6-4704\,a^5b^5x^5-2940\,a^6b^4x^4-}{56\,x^8}$$

input
```
integrate((b*x+a)^10/x^9,x, algorithm="fricas")
```

output
```
1/56*(28*b^10*x^10 + 560*a*b^9*x^9 + 2520*a^2*b^8*x^8*log(x) - 6720*a^3*b^
7*x^7 - 5880*a^4*b^6*x^6 - 4704*a^5*b^5*x^5 - 2940*a^6*b^4*x^4 - 1344*a^7*
b^3*x^3 - 420*a^8*b^2*x^2 - 80*a^9*b*x - 7*a^10)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^9}\,dx = 45a^2b^8\log{(x)} + 10ab^9x + \frac{b^{10}x^2}{2} + \frac{-7a^{10} - 80a^9bx - 420a^8b^2x^2 - 1344a^7b^3x^3 - 2940a^6b^4x^4 - 4704a^5b^5x^5 - 5880a^4b^6x^6 - 6720a^3b^7x^7}{56x^8}$$

input
```
integrate((b*x+a)**10/x**9,x)
```

output
```
45*a**2*b**8*log(x) + 10*a*b**9*x + b**10*x**2/2 + (-7*a**10 - 80*a**9*b*x
 - 420*a**8*b**2*x**2 - 1344*a**7*b**3*x**3 - 2940*a**6*b**4*x**4 - 4704*a
**5*b**5*x**5 - 5880*a**4*b**6*x**6 - 6720*a**3*b**7*x**7)/(56*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^9}\,dx = \frac{1}{2}\,b^{10}x^2 + 10\,ab^9x + 45\,a^2b^8\log{(x)} - \frac{6720\,a^3b^7x^7 + 5880\,a^4b^6x^6 + 4704\,a^5b^5x^5 + 2940\,a^6b^4x^4 + 1344\,a^7b^3x^3 + 420\,a^8b^2x^2 + 80\,a^9bx + 7\,a^{10}}{56\,x^8}$$

input
```
integrate((b*x+a)^10/x^9,x, algorithm="maxima")
```

output
```
1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(x) - 1/56*(6720*a^3*b^7*x^7 + 5
880*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^3 +
 420*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{10}}{x^9}\,dx = \frac{1}{2}\,b^{10}x^2 + 10\,ab^9x + 45\,a^2b^8\log\left(|x|\right) - \frac{6720\,a^3b^7x^7 + 5880\,a^4b^6x^6 + 4704\,a^5b^5x^5 + 2940\,a^6b^4x^4 + 1344\,a^7b^3x^3 + 420\,a^8b^2x^2 + 80\,a^9bx + 7\,a^{10}}{56\,x^8}$$

input
```
integrate((b*x+a)^10/x^9,x, algorithm="giac")
```

output
```
1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(abs(x)) - 1/56*(6720*a^3*b^7*x^
7 + 5880*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*
x^3 + 420*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^9}\,dx = \frac{b^{10}\,x^2}{2} - \frac{\frac{a^{10}}{8} + \frac{10\,a^9\,b\,x}{7} + \frac{15\,a^8\,b^2\,x^2}{2} + 24\,a^7\,b^3\,x^3 + \frac{105\,a^6\,b^4\,x^4}{2} + 84\,a^5\,b^5\,x^5 + 105\,a^4\,b^6\,x^6 + 120\,a^3\,b^7\,x^7}{x^8} + 45\,a^2\,b^8\,\ln\left(x\right) + 10\,a\,b^9\,x$$

input
```
int((a + b*x)^10/x^9,x)
```

output
```
(b^10*x^2)/2 - (a^10/8 + (15*a^8*b^2*x^2)/2 + 24*a^7*b^3*x^3 + (105*a^6*b^
4*x^4)/2 + 84*a^5*b^5*x^5 + 105*a^4*b^6*x^6 + 120*a^3*b^7*x^7 + (10*a^9*b*
x)/7)/x^8 + 45*a^2*b^8*log(x) + 10*a*b^9*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^9}\,dx$$
$$= \frac{2520\log(x)\,a^2b^8x^8 - 7a^{10} - 80a^9bx - 420a^8b^2x^2 - 1344a^7b^3x^3 - 2940a^6b^4x^4 - 4704a^5b^5x^5 - 5880a^4b^6x}{56x^8}$$

input
```
int((b*x+a)^10/x^9,x)
```

output
```
(2520*log(x)*a**2*b**8*x**8 - 7*a**10 - 80*a**9*b*x - 420*a**8*b**2*x**2 -
 1344*a**7*b**3*x**3 - 2940*a**6*b**4*x**4 - 4704*a**5*b**5*x**5 - 5880*a*
*4*b**6*x**6 - 6720*a**3*b**7*x**7 + 560*a*b**9*x**9 + 28*b**10*x**10)/(56
*x**8)
```

3.102 $\int \frac{(a+bx)^{10}}{x^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 114

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

output
```
-1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5
-63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9
*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

input
```
Integrate[(a + b*x)^10/x^10,x]
```

output
```
-1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^
6 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/
x^2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\frac{45a^2b^8}{x^2}+\frac{10ab^9}{x}+b^{10}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

input
```
Int[(a + b*x)^10/x^10,x]
```

output

```
-1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^
6 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/
x^2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10a\,b^9\ln$
risch	$b^{10}x+\frac{-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx-\frac{1}{9}a^{10}}{x^9}+10a\,b^9\ln(x$
norman	$\frac{b^{10}x^{10}-\frac{1}{9}a^{10}-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx}{x^9}+10a\,b^9\ln(x)$
parallelrisch	$\frac{2520a\,b^9\ln(x)x^9+252b^{10}x^{10}-11340a^2b^8x^8-15120a^3b^7x^7-17640a^4b^6x^6-15876a^5b^5x^5-10584a^6b^4x^4-5040a^7b^3x^3-1620a}{252x^9}$

input
```
int((b*x+a)^10/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5
-63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9
*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$
$$=\frac{252\,b^{10}x^{10}+2520\,ab^9x^9\log(x)-11340\,a^2b^8x^8-15120\,a^3b^7x^7-17640\,a^4b^6x^6-15876\,a^5b^5x^5-10584\,a}{252\,x^9}$$

input
```
integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

output
```
1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a
^3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 5
040*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = 10ab^9\log(x) + b^{10}x + \frac{-28a^{10} - 315a^9bx - 1620a^8b^2x^2 - 5040a^7b^3x^3 - 10584a^6b^4x^4 - 15876a^5b^5x^5 - 17640a^4b^6x^6 - 15120}{252x^9}$$

input
```
integrate((b*x+a)**10/x**10,x)
```

output
```
10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x*
*2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 1
7640*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x*
*9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = b^{10}x + 10\,ab^9\log(x) - \frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 1620\,a^8b}{252\,x^9}$$

input
```
integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

output
```
b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 +
17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x
^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = b^{10}x + 10\,ab^9\log\left(|x|\right)$$
$$-\frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 1620\,a^8b}{252\,x^9}$$

input
```
integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

output
```
b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x
^7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*
b^3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx =$$
$$-\frac{\frac{a^{10}}{9} - b^{10}\,x^{10} + \frac{45\,a^8\,b^2\,x^2}{7} + 20\,a^7\,b^3\,x^3 + 42\,a^6\,b^4\,x^4 + 63\,a^5\,b^5\,x^5 + 70\,a^4\,b^6\,x^6 + 60\,a^3\,b^7\,x^7 + 45\,a^2\,b^8\,x^8}{x^9}$$

input
```
int((a + b*x)^10/x^10,x)
```

output
```
-(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^
4 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5
*a^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$
$$= \frac{2520\log(x)\,a\,b^9x^9 - 28a^{10} - 315a^9bx - 1620a^8b^2x^2 - 5040a^7b^3x^3 - 10584a^6b^4x^4 - 15876a^5b^5x^5 - 1764}{252x^9}$$

input
```
int((b*x+a)^10/x^10,x)
```

output
```
(2520*log(x)*a*b**9*x**9 - 28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**2 -
 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 17640
*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8 + 252*b**10*
x**10)/(252*x**9)
```

3.103 $\int \frac{(a+bx)^{10}}{x^{11}} dx$

Optimal result

Integrand size = 11, antiderivative size = 124

$$\int \frac{(a+bx)^{10}}{x^{11}} dx = -\frac{a^{10}}{10x^{10}} - \frac{10a^9b}{9x^9} - \frac{45a^8b^2}{8x^8} - \frac{120a^7b^3}{7x^7} - \frac{35a^6b^4}{x^6} - \frac{252a^5b^5}{5x^5} - \frac{105a^4b^6}{2x^4} - \frac{40a^3b^7}{x^3} - \frac{45a^2b^8}{2x^2} - \frac{10ab^9}{x} + b^{10}\log(x)$$

output
```
-1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b
^4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2
-10*a*b^9/x+b^10*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{11}} dx = -\frac{a^{10}}{10x^{10}} - \frac{10a^9b}{9x^9} - \frac{45a^8b^2}{8x^8} - \frac{120a^7b^3}{7x^7} - \frac{35a^6b^4}{x^6} - \frac{252a^5b^5}{5x^5} - \frac{105a^4b^6}{2x^4} - \frac{40a^3b^7}{x^3} - \frac{45a^2b^8}{2x^2} - \frac{10ab^9}{x} + b^{10}\log(x)$$

input
```
Integrate[(a + b*x)^10/x^11,x]
```

output
```
-1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3
)/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^
4) - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^{10}}{x^{11}}+\frac{10a^9b}{x^{10}}+\frac{45a^8b^2}{x^9}+\frac{120a^7b^3}{x^8}+\frac{210a^6b^4}{x^7}+\frac{252a^5b^5}{x^6}+\frac{210a^4b^6}{x^5}+\frac{120a^3b^7}{x^4}+\frac{45a^2b^8}{x^3}+\frac{10ab^9}{x^2}+\frac{b^{10}}{x}\right)d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

input
```
Int[(a + b*x)^10/x^11,x]
```

output

```
-1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3
)/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^
4) - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a\,b^9}{x}+b^{10}$
norman	$\frac{-\frac{1}{10}a^{10}-10a\,b^9x^9-\frac{45}{2}a^2b^8x^8-40a^3b^7x^7-\frac{105}{2}a^4b^6x^6-\frac{252}{5}a^5b^5x^5-35a^6b^4x^4-\frac{120}{7}a^7b^3x^3-\frac{45}{8}a^8b^2x^2-\frac{10}{9}a^9bx}{x^{10}}+b^{10}\ln($
risch	$\frac{-\frac{1}{10}a^{10}-10a\,b^9x^9-\frac{45}{2}a^2b^8x^8-40a^3b^7x^7-\frac{105}{2}a^4b^6x^6-\frac{252}{5}a^5b^5x^5-35a^6b^4x^4-\frac{120}{7}a^7b^3x^3-\frac{45}{8}a^8b^2x^2-\frac{10}{9}a^9bx}{x^{10}}+b^{10}\ln($
parallelrisch	$\frac{2520b^{10}\ln(x)x^{10}-25200a\,b^9x^9-56700a^2b^8x^8-100800a^3b^7x^7-132300a^4b^6x^6-127008a^5b^5x^5-88200a^6b^4x^4-43200a^7b^3x^3-}{2520x^{10}}$

input
```
int((b*x+a)^10/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b
^4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2
-10*a*b^9/x+b^10*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx$$
$$=\frac{2520\,b^{10}x^{10}\log{(x)}-25200\,ab^9x^9-56700\,a^2b^8x^8-100800\,a^3b^7x^7-132300\,a^4b^6x^6-127008\,a^5b^5x^5-8}{2520\,x^{10}}$$

input
```
integrate((b*x+a)^10/x^11,x, algorithm="fricas")
```

output
```
1/2520*(2520*b^10*x^10*log(x) - 25200*a*b^9*x^9 - 56700*a^2*b^8*x^8 - 1008
00*a^3*b^7*x^7 - 132300*a^4*b^6*x^6 - 127008*a^5*b^5*x^5 - 88200*a^6*b^4*x
^4 - 43200*a^7*b^3*x^3 - 14175*a^8*b^2*x^2 - 2800*a^9*b*x - 252*a^10)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx = b^{10}\log(x) + \frac{-252a^{10} - 2800a^9bx - 14175a^8b^2x^2 - 43200a^7b^3x^3 - 88200a^6b^4x^4 - 127008a^5b^5x^5 - 132300a^4b^6x^6 -}{2520x^{10}}$$

input
```
integrate((b*x+a)**10/x**11,x)
```

output
```
b**10*log(x) + (-252*a**10 - 2800*a**9*b*x - 14175*a**8*b**2*x**2 - 43200*
a**7*b**3*x**3 - 88200*a**6*b**4*x**4 - 127008*a**5*b**5*x**5 - 132300*a**
4*b**6*x**6 - 100800*a**3*b**7*x**7 - 56700*a**2*b**8*x**8 - 25200*a*b**9*
x**9)/(2520*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx = b^{10}\log(x) - \frac{25200\,ab^9x^9 + 56700\,a^2b^8x^8 + 100800\,a^3b^7x^7 + 132300\,a^4b^6x^6 + 127008\,a^5b^5x^5 + 88200\,a^6b^4x^4 + 4320}{2520\,x^{10}}$$

input
```
integrate((b*x+a)^10/x^11,x, algorithm="maxima")
```

output
```
b^10*log(x) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3*b^7
*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43200
*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx = b^{10}\log\left(|x|\right) - \frac{25200\,ab^9x^9 + 56700\,a^2b^8x^8 + 100800\,a^3b^7x^7 + 132300\,a^4b^6x^6 + 127008\,a^5b^5x^5 + 88200\,a^6b^4x^4 + 4320}{2520\,x^{10}}$$

input
```
integrate((b*x+a)^10/x^11,x, algorithm="giac")
```

output
```
b^10*log(abs(x)) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^
3*b^7*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 +
43200*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx = b^{10}\,\ln\left(x\right) - \frac{\frac{a^{10}}{10} + \frac{10\,a^9\,b\,x}{9} + \frac{45\,a^8\,b^2\,x^2}{8} + \frac{120\,a^7\,b^3\,x^3}{7} + 35\,a^6\,b^4\,x^4 + \frac{252\,a^5\,b^5\,x^5}{5} + \frac{105\,a^4\,b^6\,x^6}{2} + 40\,a^3\,b^7\,x^7 + \frac{45\,a^2\,b^8\,x^8}{2} + 10}{x^{10}}$$

input
```
int((a + b*x)^10/x^11,x)
```

output
```
b^10*log(x) - (a^10/10 + 10*a*b^9*x^9 + (45*a^8*b^2*x^2)/8 + (120*a^7*b^3*
x^3)/7 + 35*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/5 + (105*a^4*b^6*x^6)/2 + 40*a
^3*b^7*x^7 + (45*a^2*b^8*x^8)/2 + (10*a^9*b*x)/9)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx$$
$$= \frac{2520\log(x)\,b^{10}x^{10} - 252a^{10} - 2800a^9bx - 14175a^8b^2x^2 - 43200a^7b^3x^3 - 88200a^6b^4x^4 - 127008a^5b^5x^5 -}{2520x^{10}}$$

input
```
int((b*x+a)^10/x^11,x)
```

output
```
(2520*log(x)*b**10*x**10 - 252*a**10 - 2800*a**9*b*x - 14175*a**8*b**2*x**
2 - 43200*a**7*b**3*x**3 - 88200*a**6*b**4*x**4 - 127008*a**5*b**5*x**5 -
132300*a**4*b**6*x**6 - 100800*a**3*b**7*x**7 - 56700*a**2*b**8*x**8 - 252
00*a*b**9*x**9)/(2520*x**10)
```

3.104 $\int \frac{(a+bx)^{10}}{x^{12}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = -\frac{(a+bx)^{11}}{11ax^{11}}$$

output
```
-1/11*(b*x+a)^11/a/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 6.71

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = -\frac{a^{10}}{11x^{11}} - \frac{a^9b}{x^{10}} - \frac{5a^8b^2}{x^9} - \frac{15a^7b^3}{x^8} - \frac{30a^6b^4}{x^7} - \frac{42a^5b^5}{x^6} - \frac{42a^4b^6}{x^5} - \frac{30a^3b^7}{x^4} - \frac{15a^2b^8}{x^3} - \frac{5ab^9}{x^2} - \frac{b^{10}}{x}$$

input
```
Integrate[(a + b*x)^10/x^12,x]
```

output
```
-1/11*a^10/x^11 - (a^9*b)/x^10 - (5*a^8*b^2)/x^9 - (15*a^7*b^3)/x^8 - (30*
a^6*b^4)/x^7 - (42*a^5*b^5)/x^6 - (42*a^4*b^6)/x^5 - (30*a^3*b^7)/x^4 - (1
5*a^2*b^8)/x^3 - (5*a*b^9)/x^2 - b^10/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

input
```
Int[(a + b*x)^10/x^12,x]
```

output
```
-1/11*(a + b*x)^11/(a*x^11)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(15) = 30.

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.53

method	result
gosper	$-\frac{11b^{10}x^{10}+55a\,b^{9}x^{9}+165a^{2}b^{8}x^{8}+330a^{3}b^{7}x^{7}+462a^{4}b^{6}x^{6}+462a^{5}b^{5}x^{5}+330a^{6}b^{4}x^{4}+165a^{7}b^{3}x^{3}+55a^{8}b^{2}x^{2}+11a^{9}bx+a^{10}}{11x^{11}}$
orering	$-\frac{11b^{10}x^{10}+55a\,b^{9}x^{9}+165a^{2}b^{8}x^{8}+330a^{3}b^{7}x^{7}+462a^{4}b^{6}x^{6}+462a^{5}b^{5}x^{5}+330a^{6}b^{4}x^{4}+165a^{7}b^{3}x^{3}+55a^{8}b^{2}x^{2}+11a^{9}bx+a^{10}}{11x^{11}}$
norman	$\frac{-b^{10}x^{10}-5a\,b^{9}x^{9}-15a^{2}b^{8}x^{8}-30a^{3}b^{7}x^{7}-42a^{4}b^{6}x^{6}-42a^{5}b^{5}x^{5}-30a^{6}b^{4}x^{4}-15a^{7}b^{3}x^{3}-5a^{8}b^{2}x^{2}-a^{9}bx-\frac{1}{11}a^{10}}{x^{11}}$
risch	$\frac{-b^{10}x^{10}-5a\,b^{9}x^{9}-15a^{2}b^{8}x^{8}-30a^{3}b^{7}x^{7}-42a^{4}b^{6}x^{6}-42a^{5}b^{5}x^{5}-30a^{6}b^{4}x^{4}-15a^{7}b^{3}x^{3}-5a^{8}b^{2}x^{2}-a^{9}bx-\frac{1}{11}a^{10}}{x^{11}}$
default	$-\frac{15a^{2}b^{8}}{x^{3}}-\frac{42a^{4}b^{6}}{x^{5}}-\frac{a^{10}}{11x^{11}}-\frac{5a\,b^{9}}{x^{2}}-\frac{30a^{6}b^{4}}{x^{7}}-\frac{30a^{3}b^{7}}{x^{4}}-\frac{15a^{7}b^{3}}{x^{8}}-\frac{a^{9}b}{x^{10}}-\frac{b^{10}}{x}-\frac{42a^{5}b^{5}}{x^{6}}-\frac{5a^{8}b^{2}}{x^{9}}$
parallelrisch	$\frac{-11b^{10}x^{10}-55a\,b^{9}x^{9}-165a^{2}b^{8}x^{8}-330a^{3}b^{7}x^{7}-462a^{4}b^{6}x^{6}-462a^{5}b^{5}x^{5}-330a^{6}b^{4}x^{4}-165a^{7}b^{3}x^{3}-55a^{8}b^{2}x^{2}-11a^{9}bx-a^{10}}{11x^{11}}$

input
```
int((b*x+a)^10/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*(11*b^10*x^10+55*a*b^9*x^9+165*a^2*b^8*x^8+330*a^3*b^7*x^7+462*a^4*b
^6*x^6+462*a^5*b^5*x^5+330*a^6*b^4*x^4+165*a^7*b^3*x^3+55*a^8*b^2*x^2+11*a
^9*b*x+a^10)/x^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.47

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = -\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}}{11\,x^{11}}$$

input
```
integrate((b*x+a)^10/x^12,x, algorithm="fricas")
```

output
```
-1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 4
62*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*
a^8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(14) = 28$.

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 7.00

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = \frac{-a^{10}-11a^9bx-55a^8b^2x^2-165a^7b^3x^3-330a^6b^4x^4-462a^5b^5x^5-462a^4b^6x^6-330a^3b^7x^7-165a^2b^8x^8}{11x^{11}}$$

input
```
integrate((b*x+a)**10/x**12,x)
```

output
```
(-a**10 - 11*a**9*b*x - 55*a**8*b**2*x**2 - 165*a**7*b**3*x**3 - 330*a**6*
b**4*x**4 - 462*a**5*b**5*x**5 - 462*a**4*b**6*x**6 - 330*a**3*b**7*x**7 -
 165*a**2*b**8*x**8 - 55*a*b**9*x**9 - 11*b**10*x**10)/(11*x**11)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.47

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = -\frac{11\,b^{10}x^{10}+55\,ab^9x^9+165\,a^2b^8x^8+330\,a^3b^7x^7+462\,a^4b^6x^6+462\,a^5b^5x^5+330\,a^6b^4x^4+165\,a^7b^3x^3}{11\,x^{11}}$$

input
```
integrate((b*x+a)^10/x^12,x, algorithm="maxima")
```

output
```
-1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 4
62*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*
a^8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.47

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx =$$
$$-\frac{11\,b^{10}x^{10} + 55\,ab^9x^9 + 165\,a^2b^8x^8 + 330\,a^3b^7x^7 + 462\,a^4b^6x^6 + 462\,a^5b^5x^5 + 330\,a^6b^4x^4 + 165\,a^7b^3x^3 +}{11\,x^{11}}$$

input
```
integrate((b*x+a)^10/x^12,x, algorithm="giac")
```

output
```
-1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 4
62*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*
a^8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.47

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx =$$
$$-\frac{\frac{a^{10}}{11} + a^9\,b\,x + 5\,a^8\,b^2\,x^2 + 15\,a^7\,b^3\,x^3 + 30\,a^6\,b^4\,x^4 + 42\,a^5\,b^5\,x^5 + 42\,a^4\,b^6\,x^6 + 30\,a^3\,b^7\,x^7 + 15\,a^2\,b^8\,x^8}{x^{11}}$$

input
```
int((a + b*x)^10/x^12,x)
```

output
```
-(a^10/11 + b^10*x^10 + 5*a*b^9*x^9 + 5*a^8*b^2*x^2 + 15*a^7*b^3*x^3 + 30*
a^6*b^4*x^4 + 42*a^5*b^5*x^5 + 42*a^4*b^6*x^6 + 30*a^3*b^7*x^7 + 15*a^2*b^
8*x^8 + a^9*b*x)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 6.59

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx$$
$$= \frac{-11b^{10}x^{10} - 55a\,b^9x^9 - 165a^2b^8x^8 - 330a^3b^7x^7 - 462a^4b^6x^6 - 462a^5b^5x^5 - 330a^6b^4x^4 - 165a^7b^3x^3 - 55}{11x^{11}}$$

input
```
int((b*x+a)^10/x^12,x)
```

output
```
( - a**10 - 11*a**9*b*x - 55*a**8*b**2*x**2 - 165*a**7*b**3*x**3 - 330*a**
6*b**4*x**4 - 462*a**5*b**5*x**5 - 462*a**4*b**6*x**6 - 330*a**3*b**7*x**7
 - 165*a**2*b**8*x**8 - 55*a*b**9*x**9 - 11*b**10*x**10)/(11*x**11)
```

3.105 $\int \frac{(a+bx)^{10}}{x^{13}} dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{(a+bx)^{10}}{x^{13}} dx = -\frac{(a+bx)^{11}}{12ax^{12}} + \frac{b(a+bx)^{11}}{132a^2x^{11}}$$

output
```
-1/12*(b*x+a)^11/a/x^12+1/132*b*(b*x+a)^11/a^2/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(36) = 72.

Time = 0.00 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.56

$$\int \frac{(a+bx)^{10}}{x^{13}} dx = -\frac{a^{10}}{12x^{12}} - \frac{10a^9b}{11x^{11}} - \frac{9a^8b^2}{2x^{10}} - \frac{40a^7b^3}{3x^9} - \frac{105a^6b^4}{4x^8} - \frac{36a^5b^5}{x^7} - \frac{35a^4b^6}{x^6} - \frac{24a^3b^7}{x^5} - \frac{45a^2b^8}{4x^4} - \frac{10ab^9}{3x^3} - \frac{b^{10}}{2x^2}$$

input
```
Integrate[(a + b*x)^10/x^13,x]
```

output
```
-1/12*a^10/x^12 - (10*a^9*b)/(11*x^11) - (9*a^8*b^2)/(2*x^10) - (40*a^7*b^
3)/(3*x^9) - (105*a^6*b^4)/(4*x^8) - (36*a^5*b^5)/x^7 - (35*a^4*b^6)/x^6 -
 (24*a^3*b^7)/x^5 - (45*a^2*b^8)/(4*x^4) - (10*a*b^9)/(3*x^3) - b^10/(2*x^
2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx$$

$$\Big\downarrow \; 55$$

$$-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}$$

$$\Big\downarrow \; 48$$

$$\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}$$

input
```
Int[(a + b*x)^10/x^13,x]
```

output
```
-1/12*(a + b*x)^11/(a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(32) = 64$.

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

method	result
norman	$\frac{-\frac{1}{2}b^{10}x^{10}-\frac{10}{3}a\,b^{9}x^{9}-\frac{45}{4}a^{2}b^{8}x^{8}-24a^{3}b^{7}x^{7}-35a^{4}b^{6}x^{6}-36a^{5}b^{5}x^{5}-\frac{105}{4}a^{6}b^{4}x^{4}-\frac{40}{3}a^{7}b^{3}x^{3}-\frac{9}{2}a^{8}b^{2}x^{2}-\frac{10}{11}a^{9}bx-\frac{1}{12}a^{10}}{x^{12}}$
risch	$\frac{-\frac{1}{2}b^{10}x^{10}-\frac{10}{3}a\,b^{9}x^{9}-\frac{45}{4}a^{2}b^{8}x^{8}-24a^{3}b^{7}x^{7}-35a^{4}b^{6}x^{6}-36a^{5}b^{5}x^{5}-\frac{105}{4}a^{6}b^{4}x^{4}-\frac{40}{3}a^{7}b^{3}x^{3}-\frac{9}{2}a^{8}b^{2}x^{2}-\frac{10}{11}a^{9}bx-\frac{1}{12}a^{10}}{x^{12}}$
gosper	$-\frac{66b^{10}x^{10}+440a\,b^{9}x^{9}+1485a^{2}b^{8}x^{8}+3168a^{3}b^{7}x^{7}+4620a^{4}b^{6}x^{6}+4752a^{5}b^{5}x^{5}+3465a^{6}b^{4}x^{4}+1760a^{7}b^{3}x^{3}+594a^{8}b^{2}x^{2}+120a}{132x^{12}}$
default	$-\frac{10a\,b^{9}}{3x^{3}}-\frac{24a^{3}b^{7}}{x^{5}}-\frac{10a^{9}b}{11x^{11}}-\frac{b^{10}}{2x^{2}}-\frac{36a^{5}b^{5}}{x^{7}}-\frac{45a^{2}b^{8}}{4x^{4}}-\frac{105a^{6}b^{4}}{4x^{8}}-\frac{9a^{8}b^{2}}{2x^{10}}-\frac{35a^{4}b^{6}}{x^{6}}-\frac{40a^{7}b^{3}}{3x^{9}}-\frac{a^{10}}{12x^{12}}$
parallelrisch	$\frac{-66b^{10}x^{10}-440a\,b^{9}x^{9}-1485a^{2}b^{8}x^{8}-3168a^{3}b^{7}x^{7}-4620a^{4}b^{6}x^{6}-4752a^{5}b^{5}x^{5}-3465a^{6}b^{4}x^{4}-1760a^{7}b^{3}x^{3}-594a^{8}b^{2}x^{2}-120a^{9}}{132x^{12}}$
orering	$-\frac{66b^{10}x^{10}+440a\,b^{9}x^{9}+1485a^{2}b^{8}x^{8}+3168a^{3}b^{7}x^{7}+4620a^{4}b^{6}x^{6}+4752a^{5}b^{5}x^{5}+3465a^{6}b^{4}x^{4}+1760a^{7}b^{3}x^{3}+594a^{8}b^{2}x^{2}+120a}{132x^{12}}$

input
```
int((b*x+a)^10/x^13,x,method=_RETURNVERBOSE)
```

output
```
1/x^12*(-1/2*b^10*x^10-10/3*a*b^9*x^9-45/4*a^2*b^8*x^8-24*a^3*b^7*x^7-35*a
^4*b^6*x^6-36*a^5*b^5*x^5-105/4*a^6*b^4*x^4-40/3*a^7*b^3*x^3-9/2*a^8*b^2*x
^2-10/11*a^9*b*x-1/12*a^10)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(32) = 64.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx = -\frac{66\,b^{10}x^{10} + 440\,ab^{9}x^{9} + 1485\,a^{2}b^{8}x^{8} + 3168\,a^{3}b^{7}x^{7} + 4620\,a^{4}b^{6}x^{6} + 4752\,a^{5}b^{5}x^{5} + 3465\,a^{6}b^{4}x^{4} + 1760}{132\,x^{12}}$$

input
```
integrate((b*x+a)^10/x^13,x, algorithm="fricas")
```

output
```
-1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
 + 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x
^3 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(29) = 58.

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.36

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx = \frac{-11a^{10} - 120a^{9}bx - 594a^{8}b^{2}x^{2} - 1760a^{7}b^{3}x^{3} - 3465a^{6}b^{4}x^{4} - 4752a^{5}b^{5}x^{5} - 4620a^{4}b^{6}x^{6} - 3168a^{3}b^{7}x^{7} -}{132x^{12}}$$

input
```
integrate((b*x+a)**10/x**13,x)
```

output
```
(-11*a**10 - 120*a**9*b*x - 594*a**8*b**2*x**2 - 1760*a**7*b**3*x**3 - 346
5*a**6*b**4*x**4 - 4752*a**5*b**5*x**5 - 4620*a**4*b**6*x**6 - 3168*a**3*b
**7*x**7 - 1485*a**2*b**8*x**8 - 440*a*b**9*x**9 - 66*b**10*x**10)/(132*x*
*12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(32) = 64.

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx = -\frac{66\,b^{10}x^{10} + 440\,ab^{9}x^{9} + 1485\,a^{2}b^{8}x^{8} + 3168\,a^{3}b^{7}x^{7} + 4620\,a^{4}b^{6}x^{6} + 4752\,a^{5}b^{5}x^{5} + 3465\,a^{6}b^{4}x^{4} + 1760}{132\,x^{12}}$$

input
```
integrate((b*x+a)^10/x^13,x, algorithm="maxima")
```

output
```
-1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
 + 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x
^3 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(32) = 64.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx = -\frac{66\,b^{10}x^{10} + 440\,ab^{9}x^{9} + 1485\,a^{2}b^{8}x^{8} + 3168\,a^{3}b^{7}x^{7} + 4620\,a^{4}b^{6}x^{6} + 4752\,a^{5}b^{5}x^{5} + 3465\,a^{6}b^{4}x^{4} + 1760}{132\,x^{12}}$$

input
```
integrate((b*x+a)^10/x^13,x, algorithm="giac")
```

output
```
-1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
 + 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x
^3 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx = -\frac{(11\,a-b\,x)\,(a+b\,x)^{11}}{132\,a^2\,x^{12}}$$

input
```
int((a + b*x)^10/x^13,x)
```

output
```
-((11*a - b*x)*(a + b*x)^11)/(132*a^2*x^12)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx = \frac{-66b^{10}x^{10} - 440a\,b^9x^9 - 1485a^2b^8x^8 - 3168a^3b^7x^7 - 4620a^4b^6x^6 - 4752a^5b^5x^5 - 3465a^6b^4x^4 - 1760a^7b}{132x^{12}}$$

input
```
int((b*x+a)^10/x^13,x)
```

output
```
( - 11*a**10 - 120*a**9*b*x - 594*a**8*b**2*x**2 - 1760*a**7*b**3*x**3 - 3
465*a**6*b**4*x**4 - 4752*a**5*b**5*x**5 - 4620*a**4*b**6*x**6 - 3168*a**3
*b**7*x**7 - 1485*a**2*b**8*x**8 - 440*a*b**9*x**9 - 66*b**10*x**10)/(132*
x**12)
```

3.106 $\int \frac{(a+bx)^{10}}{x^{14}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{(a+bx)^{11}}{13ax^{13}} + \frac{b(a+bx)^{11}}{78a^2x^{12}} - \frac{b^2(a+bx)^{11}}{858a^3x^{11}}$$

output
```
-1/13*(b*x+a)^11/a/x^13+1/78*b*(b*x+a)^11/a^2/x^12-1/858*b^2*(b*x+a)^11/a^
3/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(56) = 112.

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.25

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{a^{10}}{13x^{13}} - \frac{5a^9b}{6x^{12}} - \frac{45a^8b^2}{11x^{11}} - \frac{12a^7b^3}{x^{10}} - \frac{70a^6b^4}{3x^9} - \frac{63a^5b^5}{2x^8} - \frac{30a^4b^6}{x^7} - \frac{20a^3b^7}{x^6} - \frac{9a^2b^8}{x^5} - \frac{5ab^9}{2x^4} - \frac{b^{10}}{3x^3}$$

input
```
Integrate[(a + b*x)^10/x^14,x]
```

output
```
-1/13*a^10/x^13 - (5*a^9*b)/(6*x^12) - (45*a^8*b^2)/(11*x^11) - (12*a^7*b^
3)/x^10 - (70*a^6*b^4)/(3*x^9) - (63*a^5*b^5)/(2*x^8) - (30*a^4*b^6)/x^7 -
 (20*a^3*b^7)/x^6 - (9*a^2*b^8)/x^5 - (5*a*b^9)/(2*x^4) - b^10/(3*x^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx$$

$$\Big\downarrow \ 55$$

$$-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}$$

$$\Big\downarrow \ 55$$

$$-\frac{2b\left(-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}$$

$$\Big\downarrow \ 48$$

$$-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}$$

input
```
Int[(a + b*x)^10/x^14,x]
```

output
```
-1/13*(a + b*x)^11/(a*x^13) - (2*b*(-1/12*(a + b*x)^11/(a*x^12) + (b*(a +
b*x)^11)/(132*a^2*x^11)))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(50) = 100$.

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

method	result
norman	$\frac{-\frac{1}{3}b^{10}x^{10}-\frac{5}{2}a\,b^{9}x^{9}-9a^{2}b^{8}x^{8}-20a^{3}b^{7}x^{7}-30a^{4}b^{6}x^{6}-\frac{63}{2}a^{5}b^{5}x^{5}-\frac{70}{3}a^{6}b^{4}x^{4}-12a^{7}b^{3}x^{3}-\frac{45}{11}a^{8}b^{2}x^{2}-\frac{5}{6}a^{9}bx-\frac{1}{13}a^{10}}{x^{13}}$
risch	$\frac{-\frac{1}{3}b^{10}x^{10}-\frac{5}{2}a\,b^{9}x^{9}-9a^{2}b^{8}x^{8}-20a^{3}b^{7}x^{7}-30a^{4}b^{6}x^{6}-\frac{63}{2}a^{5}b^{5}x^{5}-\frac{70}{3}a^{6}b^{4}x^{4}-12a^{7}b^{3}x^{3}-\frac{45}{11}a^{8}b^{2}x^{2}-\frac{5}{6}a^{9}bx-\frac{1}{13}a^{10}}{x^{13}}$
gosper	$-\frac{286b^{10}x^{10}+2145a\,b^{9}x^{9}+7722a^{2}b^{8}x^{8}+17160a^{3}b^{7}x^{7}+25740a^{4}b^{6}x^{6}+27027a^{5}b^{5}x^{5}+20020a^{6}b^{4}x^{4}+10296a^{7}b^{3}x^{3}+3510a^{8}b^{2}}{858x^{13}}$
default	$-\frac{b^{10}}{3x^{3}}-\frac{9a^{2}b^{8}}{x^{5}}-\frac{45a^{8}b^{2}}{11x^{11}}-\frac{30a^{4}b^{6}}{x^{7}}-\frac{5a\,b^{9}}{2x^{4}}-\frac{63a^{5}b^{5}}{2x^{8}}-\frac{a^{10}}{13x^{13}}-\frac{12a^{7}b^{3}}{x^{10}}-\frac{20a^{3}b^{7}}{x^{6}}-\frac{70a^{6}b^{4}}{3x^{9}}-\frac{5a^{9}b}{6x^{12}}$
parallelrisch	$\frac{-286b^{10}x^{10}-2145a\,b^{9}x^{9}-7722a^{2}b^{8}x^{8}-17160a^{3}b^{7}x^{7}-25740a^{4}b^{6}x^{6}-27027a^{5}b^{5}x^{5}-20020a^{6}b^{4}x^{4}-10296a^{7}b^{3}x^{3}-3510a^{8}b^{2}}{858x^{13}}$
orering	$-\frac{286b^{10}x^{10}+2145a\,b^{9}x^{9}+7722a^{2}b^{8}x^{8}+17160a^{3}b^{7}x^{7}+25740a^{4}b^{6}x^{6}+27027a^{5}b^{5}x^{5}+20020a^{6}b^{4}x^{4}+10296a^{7}b^{3}x^{3}+3510a^{8}b^{2}}{858x^{13}}$

input
```
int((b*x+a)^10/x^14,x,method=_RETURNVERBOSE)
```

output
```
1/x^13*(-1/3*b^10*x^10-5/2*a*b^9*x^9-9*a^2*b^8*x^8-20*a^3*b^7*x^7-30*a^4*b
^6*x^6-63/2*a^5*b^5*x^5-70/3*a^6*b^4*x^4-12*a^7*b^3*x^3-45/11*a^8*b^2*x^2-
5/6*a^9*b*x-1/13*a^10)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx =$$
$$-\frac{286\,b^{10}x^{10} + 2145\,ab^9x^9 + 7722\,a^2b^8x^8 + 17160\,a^3b^7x^7 + 25740\,a^4b^6x^6 + 27027\,a^5b^5x^5 + 20020\,a^6b^4x^4}{858\,x^{13}}$$

input
```
integrate((b*x+a)^10/x^14,x, algorithm="fricas")
```

output
```
-1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*
x^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^
7*b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(48) = 96$.

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx$$
$$= \frac{-66a^{10} - 715a^9bx - 3510a^8b^2x^2 - 10296a^7b^3x^3 - 20020a^6b^4x^4 - 27027a^5b^5x^5 - 25740a^4b^6x^6 - 17160a}{858x^{13}}$$

input
```
integrate((b*x+a)**10/x**14,x)
```

output
```
(-66*a**10 - 715*a**9*b*x - 3510*a**8*b**2*x**2 - 10296*a**7*b**3*x**3 - 2
0020*a**6*b**4*x**4 - 27027*a**5*b**5*x**5 - 25740*a**4*b**6*x**6 - 17160*
a**3*b**7*x**7 - 7722*a**2*b**8*x**8 - 2145*a*b**9*x**9 - 286*b**10*x**10)
/(858*x**13)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{286\,b^{10}x^{10} + 2145\,ab^9x^9 + 7722\,a^2b^8x^8 + 17160\,a^3b^7x^7 + 25740\,a^4b^6x^6 + 27027\,a^5b^5x^5 + 20020\,a^6b^4x^4}{858\,x^{13}}$$

input
```
integrate((b*x+a)^10/x^14,x, algorithm="maxima")
```

output
```
-1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*
x^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^
7*b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{286\,b^{10}x^{10} + 2145\,ab^9x^9 + 7722\,a^2b^8x^8 + 17160\,a^3b^7x^7 + 25740\,a^4b^6x^6 + 27027\,a^5b^5x^5 + 20020\,a^6b^4x^4}{858\,x^{13}}$$

input
```
integrate((b*x+a)^10/x^14,x, algorithm="giac")
```

output
```
-1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*
x^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^
7*b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{\frac{a^{10}}{13}+\frac{5\,a^9\,b\,x}{6}+\frac{45\,a^8\,b^2\,x^2}{11}+12\,a^7\,b^3\,x^3+\frac{70\,a^6\,b^4\,x^4}{3}+\frac{63\,a^5\,b^5\,x^5}{2}+30\,a^4\,b^6\,x^6+20\,a^3\,b^7\,x^7+9\,a^2\,b^8\,x^8+\frac{5\,a\,b}{}}{x^{13}}$$

input
```
int((a + b*x)^10/x^14,x)
```

output
```
-(a^10/13 + (b^10*x^10)/3 + (5*a*b^9*x^9)/2 + (45*a^8*b^2*x^2)/11 + 12*a^7
*b^3*x^3 + (70*a^6*b^4*x^4)/3 + (63*a^5*b^5*x^5)/2 + 30*a^4*b^6*x^6 + 20*a
^3*b^7*x^7 + 9*a^2*b^8*x^8 + (5*a^9*b*x)/6)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = \frac{-286b^{10}x^{10}-2145a\,b^9x^9-7722a^2b^8x^8-17160a^3b^7x^7-25740a^4b^6x^6-27027a^5b^5x^5-20020a^6b^4x^4-}{858x^{13}}$$

input
```
int((b*x+a)^10/x^14,x)
```

output
```
( - 66*a**10 - 715*a**9*b*x - 3510*a**8*b**2*x**2 - 10296*a**7*b**3*x**3 -
 20020*a**6*b**4*x**4 - 27027*a**5*b**5*x**5 - 25740*a**4*b**6*x**6 - 1716
0*a**3*b**7*x**7 - 7722*a**2*b**8*x**8 - 2145*a*b**9*x**9 - 286*b**10*x**1
0)/(858*x**13)
```

3.107 $\int \frac{(a+bx)^{10}}{x^{15}} dx$

Optimal result

Integrand size = 11, antiderivative size = 76

$$\int \frac{(a+bx)^{10}}{x^{15}} dx = -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} - \frac{b^2(a+bx)^{11}}{364a^3x^{12}} + \frac{b^3(a+bx)^{11}}{4004a^4x^{11}}$$

output
```
-1/14*(b*x+a)^11/a/x^14+3/182*b*(b*x+a)^11/a^2/x^13-1/364*b^2*(b*x+a)^11/a
^3/x^12+1/4004*b^3*(b*x+a)^11/a^4/x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^{10}}{x^{15}} dx = -\frac{a^{10}}{14x^{14}} - \frac{10a^9b}{13x^{13}} - \frac{15a^8b^2}{4x^{12}} - \frac{120a^7b^3}{11x^{11}} - \frac{21a^6b^4}{x^{10}} - \frac{28a^5b^5}{x^9} - \frac{105a^4b^6}{4x^8} - \frac{120a^3b^7}{7x^7} - \frac{15a^2b^8}{2x^6} - \frac{2ab^9}{x^5} - \frac{b^{10}}{4x^4}$$

input
```
Integrate[(a + b*x)^10/x^15,x]
```

output
```
-1/14*a^10/x^14 - (10*a^9*b)/(13*x^13) - (15*a^8*b^2)/(4*x^12) - (120*a^7*
b^3)/(11*x^11) - (21*a^6*b^4)/x^10 - (28*a^5*b^5)/x^9 - (105*a^4*b^6)/(4*x
^8) - (120*a^3*b^7)/(7*x^7) - (15*a^2*b^8)/(2*x^6) - (2*a*b^9)/x^5 - b^10/
(4*x^4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{15}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{3b\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{3b\left(-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{3b\left(-\frac{2b\left(-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{3b\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}$$

input
```
Int[(a + b*x)^10/x^15,x]
```

output
```
-1/14*(a + b*x)^11/(a*x^14) - (3*b*(-1/13*(a + b*x)^11/(a*x^13) - (2*b*(-1
/12*(a + b*x)^11/(a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)))/(13*a)))/(14
*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

method	result
norman	$\frac{-\frac{1}{14}a^{10}-\frac{10}{13}a^{9}bx-\frac{15}{4}a^{8}b^{2}x^{2}-\frac{120}{11}a^{7}b^{3}x^{3}-21a^{6}b^{4}x^{4}-28a^{5}b^{5}x^{5}-\frac{105}{4}a^{4}b^{6}x^{6}-\frac{120}{7}a^{3}b^{7}x^{7}-\frac{15}{2}a^{2}b^{8}x^{8}-2a\,b^{9}x^{9}-\frac{1}{4}b^{10}x^{10}}{x^{14}}$
risch	$\frac{-\frac{1}{14}a^{10}-\frac{10}{13}a^{9}bx-\frac{15}{4}a^{8}b^{2}x^{2}-\frac{120}{11}a^{7}b^{3}x^{3}-21a^{6}b^{4}x^{4}-28a^{5}b^{5}x^{5}-\frac{105}{4}a^{4}b^{6}x^{6}-\frac{120}{7}a^{3}b^{7}x^{7}-\frac{15}{2}a^{2}b^{8}x^{8}-2a\,b^{9}x^{9}-\frac{1}{4}b^{10}x^{10}}{x^{14}}$
gosper	$-\frac{1001b^{10}x^{10}+8008a\,b^{9}x^{9}+30030a^{2}b^{8}x^{8}+68640a^{3}b^{7}x^{7}+105105a^{4}b^{6}x^{6}+112112a^{5}b^{5}x^{5}+84084a^{6}b^{4}x^{4}+43680a^{7}b^{3}x^{3}+1501}{4004x^{14}}$
default	$-\frac{2a\,b^{9}}{x^{5}}-\frac{120a^{7}b^{3}}{11x^{11}}-\frac{120a^{3}b^{7}}{7x^{7}}-\frac{b^{10}}{4x^{4}}-\frac{105a^{4}b^{6}}{4x^{8}}-\frac{10a^{9}b}{13x^{13}}-\frac{21a^{6}b^{4}}{x^{10}}-\frac{a^{10}}{14x^{14}}-\frac{15a^{2}b^{8}}{2x^{6}}-\frac{28a^{5}b^{5}}{x^{9}}-\frac{15a^{8}b^{2}}{4x^{12}}$
parallelrisch	$\frac{-1001b^{10}x^{10}-8008a\,b^{9}x^{9}-30030a^{2}b^{8}x^{8}-68640a^{3}b^{7}x^{7}-105105a^{4}b^{6}x^{6}-112112a^{5}b^{5}x^{5}-84084a^{6}b^{4}x^{4}-43680a^{7}b^{3}x^{3}-1501}{4004x^{14}}$
orering	$-\frac{1001b^{10}x^{10}+8008a\,b^{9}x^{9}+30030a^{2}b^{8}x^{8}+68640a^{3}b^{7}x^{7}+105105a^{4}b^{6}x^{6}+112112a^{5}b^{5}x^{5}+84084a^{6}b^{4}x^{4}+43680a^{7}b^{3}x^{3}+1501}{4004x^{14}}$

input
```
int((b*x+a)^10/x^15,x,method=_RETURNVERBOSE)
```

output
```
1/x^14*(-1/14*a^10-10/13*a^9*b*x-15/4*a^8*b^2*x^2-120/11*a^7*b^3*x^3-21*a^
6*b^4*x^4-28*a^5*b^5*x^5-105/4*a^4*b^6*x^6-120/7*a^3*b^7*x^7-15/2*a^2*b^8*
x^8-2*a*b^9*x^9-1/4*b^10*x^10)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = -\frac{1001\,b^{10}x^{10}+8008\,ab^9x^9+30030\,a^2b^8x^8+68640\,a^3b^7x^7+105105\,a^4b^6x^6+112112\,a^5b^5x^5+84084\,a^6}{4004\,x^{14}}$$

input
```
integrate((b*x+a)^10/x^15,x, algorithm="fricas")
```

output
```
-1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b
^7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 436
80*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = \frac{-286a^{10}-3080a^9bx-15015a^8b^2x^2-43680a^7b^3x^3-84084a^6b^4x^4-112112a^5b^5x^5-105105a^4b^6x^6-6}{4004x^{14}}$$

input
```
integrate((b*x+a)**10/x**15,x)
```

output
```
(-286*a**10 - 3080*a**9*b*x - 15015*a**8*b**2*x**2 - 43680*a**7*b**3*x**3
- 84084*a**6*b**4*x**4 - 112112*a**5*b**5*x**5 - 105105*a**4*b**6*x**6 - 6
8640*a**3*b**7*x**7 - 30030*a**2*b**8*x**8 - 8008*a*b**9*x**9 - 1001*b**10
*x**10)/(4004*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = -\frac{1001\,b^{10}x^{10}+8008\,ab^9x^9+30030\,a^2b^8x^8+68640\,a^3b^7x^7+105105\,a^4b^6x^6+112112\,a^5b^5x^5+84084\,a^6}{4004\,x^{14}}$$

input
```
integrate((b*x+a)^10/x^15,x, algorithm="maxima")
```

output
```
-1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b
^7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 436
80*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = -\frac{1001\,b^{10}x^{10}+8008\,ab^9x^9+30030\,a^2b^8x^8+68640\,a^3b^7x^7+105105\,a^4b^6x^6+112112\,a^5b^5x^5+84084\,a^6}{4004\,x^{14}}$$

input
```
integrate((b*x+a)^10/x^15,x, algorithm="giac")
```

output
```
-1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b
^7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 436
80*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = -\frac{\frac{a^{10}}{14}+\frac{10\,a^9\,b\,x}{13}+\frac{15\,a^8\,b^2\,x^2}{4}+\frac{120\,a^7\,b^3\,x^3}{11}+21\,a^6\,b^4\,x^4+28\,a^5\,b^5\,x^5+\frac{105\,a^4\,b^6\,x^6}{4}+\frac{120\,a^3\,b^7\,x^7}{7}+\frac{15\,a^2\,b^8\,x^8}{2}+2\,a}{x^{14}}$$

input
```
int((a + b*x)^10/x^15,x)
```

output
```
-(a^10/14 + (b^10*x^10)/4 + 2*a*b^9*x^9 + (15*a^8*b^2*x^2)/4 + (120*a^7*b^
3*x^3)/11 + 21*a^6*b^4*x^4 + 28*a^5*b^5*x^5 + (105*a^4*b^6*x^6)/4 + (120*a
^3*b^7*x^7)/7 + (15*a^2*b^8*x^8)/2 + (10*a^9*b*x)/13)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = \frac{-1001b^{10}x^{10}-8008a\,b^9x^9-30030a^2b^8x^8-68640a^3b^7x^7-105105a^4b^6x^6-112112a^5b^5x^5-84084a^6b^4x}{4004x^{14}}$$

input
```
int((b*x+a)^10/x^15,x)
```

output
```
( - 286*a**10 - 3080*a**9*b*x - 15015*a**8*b**2*x**2 - 43680*a**7*b**3*x**
3 - 84084*a**6*b**4*x**4 - 112112*a**5*b**5*x**5 - 105105*a**4*b**6*x**6 -
 68640*a**3*b**7*x**7 - 30030*a**2*b**8*x**8 - 8008*a*b**9*x**9 - 1001*b**
10*x**10)/(4004*x**14)
```

3.108 $\int \frac{(a+bx)^{10}}{x^{16}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 96

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx = -\frac{(a+bx)^{11}}{15ax^{15}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}$$

output
```
-1/15*(b*x+a)^11/a/x^15+2/105*b*(b*x+a)^11/a^2/x^14-2/455*b^2*(b*x+a)^11/a
^3/x^13+1/1365*b^3*(b*x+a)^11/a^4/x^12-1/15015*b^4*(b*x+a)^11/a^5/x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx = -\frac{a^{10}}{15x^{15}}-\frac{5a^9b}{7x^{14}}-\frac{45a^8b^2}{13x^{13}}-\frac{10a^7b^3}{x^{12}}-\frac{210a^6b^4}{11x^{11}}-\frac{126a^5b^5}{5x^{10}}-\frac{70a^4b^6}{3x^9}-\frac{15a^3b^7}{x^8}-\frac{45a^2b^8}{7x^7}-\frac{5ab^9}{3x^6}-\frac{b^{10}}{5x^5}$$

input
```
Integrate[(a + b*x)^10/x^16,x]
```

output
```
-1/15*a^10/x^15 - (5*a^9*b)/(7*x^14) - (45*a^8*b^2)/(13*x^13) - (10*a^7*b^
3)/x^12 - (210*a^6*b^4)/(11*x^11) - (126*a^5*b^5)/(5*x^10) - (70*a^4*b^6)/
(3*x^9) - (15*a^3*b^7)/x^8 - (45*a^2*b^8)/(7*x^7) - (5*a*b^9)/(3*x^6) - b^
10/(5*x^5)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{4b\int \frac{(a+bx)^{10}}{x^{15}}dx}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{4b\left(-\frac{3b\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{4b\left(-\frac{3b\left(-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}$$

input
```
Int[(a + b*x)^10/x^16,x]
```

output
```
-1/15*(a + b*x)^11/(a*x^15) - (4*b*(-1/14*(a + b*x)^11/(a*x^14) - (3*b*(-1
/13*(a + b*x)^11/(a*x^13) - (2*b*(-1/12*(a + b*x)^11/(a*x^12) + (b*(a + b*
x)^11)/(132*a^2*x^11)))/(13*a)))/(14*a)))/(15*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result
norman	$\frac{-\frac{1}{15}a^{10}-\frac{5}{7}a^9bx-\frac{45}{13}a^8b^2x^2-10a^7b^3x^3-\frac{210}{11}a^6b^4x^4-\frac{126}{5}a^5b^5x^5-\frac{70}{3}a^4b^6x^6-15a^3b^7x^7-\frac{45}{7}a^2b^8x^8-\frac{5}{3}a\,b^9x^9-\frac{1}{5}b^{10}x^{10}}{x^{15}}$
risch	$\frac{-\frac{1}{15}a^{10}-\frac{5}{7}a^9bx-\frac{45}{13}a^8b^2x^2-10a^7b^3x^3-\frac{210}{11}a^6b^4x^4-\frac{126}{5}a^5b^5x^5-\frac{70}{3}a^4b^6x^6-15a^3b^7x^7-\frac{45}{7}a^2b^8x^8-\frac{5}{3}a\,b^9x^9-\frac{1}{5}b^{10}x^{10}}{x^{15}}$
gosper	$-\frac{3003b^{10}x^{10}+25025a\,b^9x^9+96525a^2b^8x^8+225225a^3b^7x^7+350350a^4b^6x^6+378378a^5b^5x^5+286650a^6b^4x^4+150150a^7b^3x^3+}{15015x^{15}}$
default	$-\frac{b^{10}}{5x^5}-\frac{210a^6b^4}{11x^{11}}-\frac{a^{10}}{15x^{15}}-\frac{45a^2b^8}{7x^7}-\frac{15a^3b^7}{x^8}-\frac{45a^8b^2}{13x^{13}}-\frac{126a^5b^5}{5x^{10}}-\frac{5a^9b}{7x^{14}}-\frac{5a\,b^9}{3x^6}-\frac{70a^4b^6}{3x^9}-\frac{10a^7b^3}{x^{12}}$
parallelrisch	$\frac{-3003b^{10}x^{10}-25025a\,b^9x^9-96525a^2b^8x^8-225225a^3b^7x^7-350350a^4b^6x^6-378378a^5b^5x^5-286650a^6b^4x^4-150150a^7b^3x^3-}{15015x^{15}}$
orering	$-\frac{3003b^{10}x^{10}+25025a\,b^9x^9+96525a^2b^8x^8+225225a^3b^7x^7+350350a^4b^6x^6+378378a^5b^5x^5+286650a^6b^4x^4+150150a^7b^3x^3+}{15015x^{15}}$

input
```
int((b*x+a)^10/x^16,x,method=_RETURNVERBOSE)
```

output
```
1/x^15*(-1/15*a^10-5/7*a^9*b*x-45/13*a^8*b^2*x^2-10*a^7*b^3*x^3-210/11*a^6
*b^4*x^4-126/5*a^5*b^5*x^5-70/3*a^4*b^6*x^6-15*a^3*b^7*x^7-45/7*a^2*b^8*x^
8-5/3*a*b^9*x^9-1/5*b^10*x^10)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx = -\frac{3003\,b^{10}x^{10}+25025\,ab^9x^9+96525\,a^2b^8x^8+225225\,a^3b^7x^7+350350\,a^4b^6x^6+378378\,a^5b^5x^5+28665}{15015\,x^{15}}$$

input
```
integrate((b*x+a)^10/x^16,x, algorithm="fricas")
```

output
```
-1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^
3*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 +
 150150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx = \frac{-1001a^{10} - 10725a^9bx - 51975a^8b^2x^2 - 150150a^7b^3x^3 - 286650a^6b^4x^4 - 378378a^5b^5x^5 - 350350a^4b^6x^{6}}{15015x^{15}}$$

input
```
integrate((b*x+a)**10/x**16,x)
```

output
```
(-1001*a**10 - 10725*a**9*b*x - 51975*a**8*b**2*x**2 - 150150*a**7*b**3*x*
*3 - 286650*a**6*b**4*x**4 - 378378*a**5*b**5*x**5 - 350350*a**4*b**6*x**6
 - 225225*a**3*b**7*x**7 - 96525*a**2*b**8*x**8 - 25025*a*b**9*x**9 - 3003
*b**10*x**10)/(15015*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx = -\frac{3003\,b^{10}x^{10} + 25025\,ab^9x^9 + 96525\,a^2b^8x^8 + 225225\,a^3b^7x^7 + 350350\,a^4b^6x^6 + 378378\,a^5b^5x^5 + 28665}{15015\,x^{15}}$$

input
```
integrate((b*x+a)^10/x^16,x, algorithm="maxima")
```

output
```
-1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^
3*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 +
 150150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx =$$
$$-\frac{3003\,b^{10}x^{10}+25025\,ab^9x^9+96525\,a^2b^8x^8+225225\,a^3b^7x^7+350350\,a^4b^6x^6+378378\,a^5b^5x^5+28665}{15015\,x^{15}}$$

input
```
integrate((b*x+a)^10/x^16,x, algorithm="giac")
```

output
```
-1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^
3*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 +
 150150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx =$$
$$-\frac{\frac{a^{10}}{15}+\frac{5\,a^9\,b\,x}{7}+\frac{45\,a^8\,b^2\,x^2}{13}+10\,a^7\,b^3\,x^3+\frac{210\,a^6\,b^4\,x^4}{11}+\frac{126\,a^5\,b^5\,x^5}{5}+\frac{70\,a^4\,b^6\,x^6}{3}+15\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{7}+\frac{5\,a\,b^9}{3}}{x^{15}}$$

input
```
int((a + b*x)^10/x^16,x)
```

output
```
-(a^10/15 + (b^10*x^10)/5 + (5*a*b^9*x^9)/3 + (45*a^8*b^2*x^2)/13 + 10*a^7
*b^3*x^3 + (210*a^6*b^4*x^4)/11 + (126*a^5*b^5*x^5)/5 + (70*a^4*b^6*x^6)/3
 + 15*a^3*b^7*x^7 + (45*a^2*b^8*x^8)/7 + (5*a^9*b*x)/7)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx$$
$$= \frac{-3003b^{10}x^{10} - 25025a\,b^9x^9 - 96525a^2b^8x^8 - 225225a^3b^7x^7 - 350350a^4b^6x^6 - 378378a^5b^5x^5 - 286650a}{15015x^{15}}$$

input
```
int((b*x+a)^10/x^16,x)
```

output
```
( - 1001*a**10 - 10725*a**9*b*x - 51975*a**8*b**2*x**2 - 150150*a**7*b**3*
x**3 - 286650*a**6*b**4*x**4 - 378378*a**5*b**5*x**5 - 350350*a**4*b**6*x*
*6 - 225225*a**3*b**7*x**7 - 96525*a**2*b**8*x**8 - 25025*a*b**9*x**9 - 30
03*b**10*x**10)/(15015*x**15)
```

3.109 $\int \frac{(a+bx)^{10}}{x^{17}} dx$

Optimal result

Integrand size = 11, antiderivative size = 116

$$\int \frac{(a+bx)^{10}}{x^{17}} dx = -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} + \frac{b^5(a+bx)^{11}}{48048a^6x^{11}}$$

output
```
-1/16*(b*x+a)^11/a/x^16+1/48*b*(b*x+a)^11/a^2/x^15-1/168*b^2*(b*x+a)^11/a^
3/x^14+1/728*b^3*(b*x+a)^11/a^4/x^13-1/4368*b^4*(b*x+a)^11/a^5/x^12+1/4804
8*b^5*(b*x+a)^11/a^6/x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{10}}{x^{17}} dx = -\frac{a^{10}}{16x^{16}} - \frac{2a^9b}{3x^{15}} - \frac{45a^8b^2}{14x^{14}} - \frac{120a^7b^3}{13x^{13}} - \frac{35a^6b^4}{2x^{12}} - \frac{252a^5b^5}{11x^{11}} - \frac{21a^4b^6}{x^{10}} - \frac{40a^3b^7}{3x^9} - \frac{45a^2b^8}{8x^8} - \frac{10ab^9}{7x^7} - \frac{b^{10}}{6x^6}$$

input
```
Integrate[(a + b*x)^10/x^17,x]
```

output
```
-1/16*a^10/x^16 - (2*a^9*b)/(3*x^15) - (45*a^8*b^2)/(14*x^14) - (120*a^7*b
^3)/(13*x^13) - (35*a^6*b^4)/(2*x^12) - (252*a^5*b^5)/(11*x^11) - (21*a^4*
b^6)/x^10 - (40*a^3*b^7)/(3*x^9) - (45*a^2*b^8)/(8*x^8) - (10*a*b^9)/(7*x^
7) - b^10/(6*x^6)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{5b\int \frac{(a+bx)^{10}}{x^{16}}dx}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{5b\left(-\frac{4b\int \frac{(a+bx)^{10}}{x^{15}}dx}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{5b\left(-\frac{4b\left(-\frac{3b\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}$$

$\downarrow$ 55

$$-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}$$

$\downarrow$ 48

$$-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}$$

input
```
Int[(a + b*x)^10/x^17,x]
```

output
```
-1/16*(a + b*x)^11/(a*x^16) - (5*b*(-1/15*(a + b*x)^11/(a*x^15) - (4*b*(-1
/14*(a + b*x)^11/(a*x^14) - (3*b*(-1/13*(a + b*x)^11/(a*x^13) - (2*b*(-1/1
2*(a + b*x)^11/(a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)))/(13*a)))/(14*a
)))/(15*a)))/(16*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

method	result
norman	$\frac{-\frac{1}{16}a^{10}-\frac{2}{3}a^{9}bx-\frac{45}{14}a^{8}b^{2}x^{2}-\frac{120}{13}a^{7}b^{3}x^{3}-\frac{35}{2}a^{6}b^{4}x^{4}-\frac{252}{11}a^{5}b^{5}x^{5}-21a^{4}b^{6}x^{6}-\frac{40}{3}a^{3}b^{7}x^{7}-\frac{45}{8}a^{2}b^{8}x^{8}-\frac{10}{7}a\,b^{9}x^{9}-\frac{1}{6}b^{10}x^{10}}{x^{16}}$
risch	$\frac{-\frac{1}{16}a^{10}-\frac{2}{3}a^{9}bx-\frac{45}{14}a^{8}b^{2}x^{2}-\frac{120}{13}a^{7}b^{3}x^{3}-\frac{35}{2}a^{6}b^{4}x^{4}-\frac{252}{11}a^{5}b^{5}x^{5}-21a^{4}b^{6}x^{6}-\frac{40}{3}a^{3}b^{7}x^{7}-\frac{45}{8}a^{2}b^{8}x^{8}-\frac{10}{7}a\,b^{9}x^{9}-\frac{1}{6}b^{10}x^{10}}{x^{16}}$
gosper	$-\frac{8008b^{10}x^{10}+68640a\,b^{9}x^{9}+270270a^{2}b^{8}x^{8}+640640a^{3}b^{7}x^{7}+1009008a^{4}b^{6}x^{6}+1100736a^{5}b^{5}x^{5}+840840a^{6}b^{4}x^{4}+443520a^{7}b^{3}}{48048x^{16}}$
default	$-\frac{252a^{5}b^{5}}{11x^{11}}-\frac{2a^{9}b}{3x^{15}}-\frac{10a\,b^{9}}{7x^{7}}-\frac{45a^{2}b^{8}}{8x^{8}}-\frac{120a^{7}b^{3}}{13x^{13}}-\frac{21a^{4}b^{6}}{x^{10}}-\frac{45a^{8}b^{2}}{14x^{14}}-\frac{b^{10}}{6x^{6}}-\frac{40a^{3}b^{7}}{3x^{9}}-\frac{a^{10}}{16x^{16}}-\frac{35a^{6}b^{4}}{2x^{12}}$
parallelrisch	$\frac{-8008b^{10}x^{10}-68640a\,b^{9}x^{9}-270270a^{2}b^{8}x^{8}-640640a^{3}b^{7}x^{7}-1009008a^{4}b^{6}x^{6}-1100736a^{5}b^{5}x^{5}-840840a^{6}b^{4}x^{4}-443520a^{7}b^{3}}{48048x^{16}}$
orering	$-\frac{8008b^{10}x^{10}+68640a\,b^{9}x^{9}+270270a^{2}b^{8}x^{8}+640640a^{3}b^{7}x^{7}+1009008a^{4}b^{6}x^{6}+1100736a^{5}b^{5}x^{5}+840840a^{6}b^{4}x^{4}+443520a^{7}b^{3}}{48048x^{16}}$

input
```
int((b*x+a)^10/x^17,x,method=_RETURNVERBOSE)
```

output
```
1/x^16*(-1/16*a^10-2/3*a^9*b*x-45/14*a^8*b^2*x^2-120/13*a^7*b^3*x^3-35/2*a
^6*b^4*x^4-252/11*a^5*b^5*x^5-21*a^4*b^6*x^6-40/3*a^3*b^7*x^7-45/8*a^2*b^8
*x^8-10/7*a*b^9*x^9-1/6*b^10*x^10)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx = -\frac{8008\,b^{10}x^{10}+68640\,ab^9x^9+270270\,a^2b^8x^8+640640\,a^3b^7x^7+1009008\,a^4b^6x^6+1100736\,a^5b^5x^5+840}{48048\,x^{16}}$$

input
```
integrate((b*x+a)^10/x^17,x, algorithm="fricas")
```

output
```
-1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a
^3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^
4 + 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x
^16
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx = \frac{-3003a^{10}-32032a^9bx-154440a^8b^2x^2-443520a^7b^3x^3-840840a^6b^4x^4-1100736a^5b^5x^5-1009008a^4}{48048x^{16}}$$

input
```
integrate((b*x+a)**10/x**17,x)
```

output
```
(-3003*a**10 - 32032*a**9*b*x - 154440*a**8*b**2*x**2 - 443520*a**7*b**3*x
**3 - 840840*a**6*b**4*x**4 - 1100736*a**5*b**5*x**5 - 1009008*a**4*b**6*x
**6 - 640640*a**3*b**7*x**7 - 270270*a**2*b**8*x**8 - 68640*a*b**9*x**9 -
8008*b**10*x**10)/(48048*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx = -\frac{8008\,b^{10}x^{10} + 68640\,ab^9x^9 + 270270\,a^2b^8x^8 + 640640\,a^3b^7x^7 + 1009008\,a^4b^6x^6 + 1100736\,a^5b^5x^5 + 840}{48048\,x^{16}}$$

input
```
integrate((b*x+a)^10/x^17,x, algorithm="maxima")
```

output
```
-1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a
^3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^
4 + 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x
^16
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx = -\frac{8008\,b^{10}x^{10} + 68640\,ab^9x^9 + 270270\,a^2b^8x^8 + 640640\,a^3b^7x^7 + 1009008\,a^4b^6x^6 + 1100736\,a^5b^5x^5 + 840}{48048\,x^{16}}$$

input
```
integrate((b*x+a)^10/x^17,x, algorithm="giac")
```

output
```
-1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a
^3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^
4 + 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x
^16
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx = -\frac{\frac{a^{10}}{16}+\frac{2\,a^9\,b\,x}{3}+\frac{45\,a^8\,b^2\,x^2}{14}+\frac{120\,a^7\,b^3\,x^3}{13}+\frac{35\,a^6\,b^4\,x^4}{2}+\frac{252\,a^5\,b^5\,x^5}{11}+21\,a^4\,b^6\,x^6+\frac{40\,a^3\,b^7\,x^7}{3}+\frac{45\,a^2\,b^8\,x^8}{8}+\frac{10\,a\,b^9}{7}}{x^{16}}$$

input
```
int((a + b*x)^10/x^17,x)
```

output
```
-(a^10/16 + (b^10*x^10)/6 + (10*a*b^9*x^9)/7 + (45*a^8*b^2*x^2)/14 + (120*
a^7*b^3*x^3)/13 + (35*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/11 + 21*a^4*b^6*x
^6 + (40*a^3*b^7*x^7)/3 + (45*a^2*b^8*x^8)/8 + (2*a^9*b*x)/3)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx = \frac{-8008b^{10}x^{10}-68640a\,b^9x^9-270270a^2b^8x^8-640640a^3b^7x^7-1009008a^4b^6x^6-1100736a^5b^5x^5-8408}{48048x^{16}}$$

input
```
int((b*x+a)^10/x^17,x)
```

output
```
( - 3003*a**10 - 32032*a**9*b*x - 154440*a**8*b**2*x**2 - 443520*a**7*b**3
*x**3 - 840840*a**6*b**4*x**4 - 1100736*a**5*b**5*x**5 - 1009008*a**4*b**6
*x**6 - 640640*a**3*b**7*x**7 - 270270*a**2*b**8*x**8 - 68640*a*b**9*x**9
- 8008*b**10*x**10)/(48048*x**16)
```

3.110 $\int \frac{(a+bx)^{10}}{x^{18}} dx$

Optimal result

Integrand size = 11, antiderivative size = 136

$$\int \frac{(a+bx)^{10}}{x^{18}} dx = -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)^{11}}{12376a^6x^{12}} - \frac{b^6(a+bx)^{11}}{136136a^7x^{11}}$$

output
```
-1/17*(b*x+a)^11/a/x^17+3/136*b*(b*x+a)^11/a^2/x^16-1/136*b^2*(b*x+a)^11/a
^3/x^15+1/476*b^3*(b*x+a)^11/a^4/x^14-3/6188*b^4*(b*x+a)^11/a^5/x^13+1/123
76*b^5*(b*x+a)^11/a^6/x^12-1/136136*b^6*(b*x+a)^11/a^7/x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{10}}{x^{18}} dx = -\frac{a^{10}}{17x^{17}} - \frac{5a^9b}{8x^{16}} - \frac{3a^8b^2}{x^{15}} - \frac{60a^7b^3}{7x^{14}} - \frac{210a^6b^4}{13x^{13}} - \frac{21a^5b^5}{x^{12}} - \frac{210a^4b^6}{11x^{11}} - \frac{12a^3b^7}{x^{10}} - \frac{5a^2b^8}{x^9} - \frac{5ab^9}{4x^8} - \frac{b^{10}}{7x^7}$$

input
```
Integrate[(a + b*x)^10/x^18,x]
```

output
```
-1/17*a^10/x^17 - (5*a^9*b)/(8*x^16) - (3*a^8*b^2)/x^15 - (60*a^7*b^3)/(7*
x^14) - (210*a^6*b^4)/(13*x^13) - (21*a^5*b^5)/x^12 - (210*a^4*b^6)/(11*x^
11) - (12*a^3*b^7)/x^10 - (5*a^2*b^8)/x^9 - (5*a*b^9)/(4*x^8) - b^10/(7*x^
7)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {55, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{18}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\int \frac{(a+bx)^{10}}{x^{17}}dx}{17a}-\frac{(a+bx)^{11}}{17ax^{17}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\left(-\frac{5b\int \frac{(a+bx)^{10}}{x^{16}}dx}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{(a+bx)^{11}}{17ax^{17}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\left(-\frac{5b\left(-\frac{4b\int \frac{(a+bx)^{10}}{x^{15}}dx}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{(a+bx)^{11}}{17ax^{17}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{(a+bx)^{11}}{17ax^{17}}$$

$$\Big\downarrow \; 55$$

$$-\frac{6b\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{(a+bx)^{11}}{17ax^{17}}$$

$$\Big\downarrow \; 55$$

$$-\frac{6b\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{(a+bx)^{11}}{17ax^{17}}$$

$$\downarrow 48$$

$$-\frac{6b\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{(a+bx)^{11}}{17ax^{17}}$$

input
```
Int[(a + b*x)^10/x^18,x]
```

output
```
-1/17*(a + b*x)^11/(a*x^17) - (6*b*(-1/16*(a + b*x)^11/(a*x^16) - (5*b*(-1
/15*(a + b*x)^11/(a*x^15) - (4*b*(-1/14*(a + b*x)^11/(a*x^14) - (3*b*(-1/1
3*(a + b*x)^11/(a*x^13) - (2*b*(-1/12*(a + b*x)^11/(a*x^12) + (b*(a + b*x)
^11)/(132*a^2*x^11)))/(13*a)))/(14*a)))/(15*a)))/(16*a)))/(17*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

method	result
norman	$\frac{-\frac{1}{17}a^{10}-\frac{5}{8}a^9bx-3a^8b^2x^2-\frac{60}{7}a^7b^3x^3-\frac{210}{13}a^6b^4x^4-21a^5b^5x^5-\frac{210}{11}a^4b^6x^6-12a^3b^7x^7-5a^2b^8x^8-\frac{5}{4}a\,b^9x^9-\frac{1}{7}b^{10}x^{10}}{x^{17}}$
risch	$\frac{-\frac{1}{17}a^{10}-\frac{5}{8}a^9bx-3a^8b^2x^2-\frac{60}{7}a^7b^3x^3-\frac{210}{13}a^6b^4x^4-21a^5b^5x^5-\frac{210}{11}a^4b^6x^6-12a^3b^7x^7-5a^2b^8x^8-\frac{5}{4}a\,b^9x^9-\frac{1}{7}b^{10}x^{10}}{x^{17}}$
gosper	$-\frac{19448b^{10}x^{10}+170170a\,b^9x^9+680680a^2b^8x^8+1633632a^3b^7x^7+2598960a^4b^6x^6+2858856a^5b^5x^5+2199120a^6b^4x^4+116688}{136136x^{17}}$
default	$-\frac{210a^4b^6}{11x^{11}}-\frac{3a^8b^2}{x^{15}}-\frac{b^{10}}{7x^7}-\frac{5a\,b^9}{4x^8}-\frac{210a^6b^4}{13x^{13}}-\frac{12a^3b^7}{x^{10}}-\frac{60a^7b^3}{7x^{14}}-\frac{a^{10}}{17x^{17}}-\frac{5a^2b^8}{x^9}-\frac{5a^9b}{8x^{16}}-\frac{21a^5b^5}{x^{12}}$
parallelrisch	$\frac{-19448b^{10}x^{10}-170170a\,b^9x^9-680680a^2b^8x^8-1633632a^3b^7x^7-2598960a^4b^6x^6-2858856a^5b^5x^5-2199120a^6b^4x^4-116688}{136136x^{17}}$
orering	$-\frac{19448b^{10}x^{10}+170170a\,b^9x^9+680680a^2b^8x^8+1633632a^3b^7x^7+2598960a^4b^6x^6+2858856a^5b^5x^5+2199120a^6b^4x^4+116688}{136136x^{17}}$

input
```
int((b*x+a)^10/x^18,x,method=_RETURNVERBOSE)
```

output
```
1/x^17*(-1/17*a^10-5/8*a^9*b*x-3*a^8*b^2*x^2-60/7*a^7*b^3*x^3-210/13*a^6*b
^4*x^4-21*a^5*b^5*x^5-210/11*a^4*b^6*x^6-12*a^3*b^7*x^7-5*a^2*b^8*x^8-5/4*
a*b^9*x^9-1/7*b^10*x^10)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{19448\,b^{10}x^{10} + 170170\,ab^9x^9 + 680680\,a^2b^8x^8 + 1633632\,a^3b^7x^7 + 2598960\,a^4b^6x^6 + 2858856\,a^5b^5x^5 +}{136136\,x^{17}}$$

input
```
integrate((b*x+a)^10/x^18,x, algorithm="fricas")
```

output
```
-1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 16336
32*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b
^4*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a
^10)/x^17
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = \frac{-8008a^{10} - 85085a^9bx - 408408a^8b^2x^2 - 1166880a^7b^3x^3 - 2199120a^6b^4x^4 - 2858856a^5b^5x^5 - 2598960}{136136x^{17}}$$

input
```
integrate((b*x+a)**10/x**18,x)
```

output
```
(-8008*a**10 - 85085*a**9*b*x - 408408*a**8*b**2*x**2 - 1166880*a**7*b**3*
x**3 - 2199120*a**6*b**4*x**4 - 2858856*a**5*b**5*x**5 - 2598960*a**4*b**6
*x**6 - 1633632*a**3*b**7*x**7 - 680680*a**2*b**8*x**8 - 170170*a*b**9*x**
9 - 19448*b**10*x**10)/(136136*x**17)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{19448\,b^{10}x^{10} + 170170\,ab^9x^9 + 680680\,a^2b^8x^8 + 1633632\,a^3b^7x^7 + 2598960\,a^4b^6x^6 + 2858856\,a^5b^5x^5 + \cdots}{136136\,x^{17}}$$

input
```
integrate((b*x+a)^10/x^18,x, algorithm="maxima")
```

output
```
-1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 16336
32*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b
^4*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a
^10)/x^17
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{19448\,b^{10}x^{10} + 170170\,ab^9x^9 + 680680\,a^2b^8x^8 + 1633632\,a^3b^7x^7 + 2598960\,a^4b^6x^6 + 2858856\,a^5b^5x^5 + \cdots}{136136\,x^{17}}$$

input
```
integrate((b*x+a)^10/x^18,x, algorithm="giac")
```

output
```
-1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 16336
32*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b
^4*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a
^10)/x^17
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{\frac{a^{10}}{17}+\frac{5\,a^9\,b\,x}{8}+3\,a^8\,b^2\,x^2+\frac{60\,a^7\,b^3\,x^3}{7}+\frac{210\,a^6\,b^4\,x^4}{13}+21\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{11}+12\,a^3\,b^7\,x^7+5\,a^2\,b^8\,x^8+\frac{5\,a}{}}{x^{17}}$$

input
```
int((a + b*x)^10/x^18,x)
```

output
```
-(a^10/17 + (b^10*x^10)/7 + (5*a*b^9*x^9)/4 + 3*a^8*b^2*x^2 + (60*a^7*b^3*
x^3)/7 + (210*a^6*b^4*x^4)/13 + 21*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/11 + 12
*a^3*b^7*x^7 + 5*a^2*b^8*x^8 + (5*a^9*b*x)/8)/x^17
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = \frac{-19448b^{10}x^{10}-170170a\,b^9x^9-680680a^2b^8x^8-1633632a^3b^7x^7-2598960a^4b^6x^6-2858856a^5b^5x^5-2}{136136x^{17}}$$

input
```
int((b*x+a)^10/x^18,x)
```

output
```
( - 8008*a**10 - 85085*a**9*b*x - 408408*a**8*b**2*x**2 - 1166880*a**7*b**
3*x**3 - 2199120*a**6*b**4*x**4 - 2858856*a**5*b**5*x**5 - 2598960*a**4*b*
*6*x**6 - 1633632*a**3*b**7*x**7 - 680680*a**2*b**8*x**8 - 170170*a*b**9*x
**9 - 19448*b**10*x**10)/(136136*x**17)
```

3.111 $\int \frac{(a+bx)^{10}}{x^{19}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 130

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = -\frac{a^{10}}{18x^{18}} - \frac{10a^9b}{17x^{17}} - \frac{45a^8b^2}{16x^{16}} - \frac{8a^7b^3}{x^{15}} - \frac{15a^6b^4}{x^{14}} - \frac{252a^5b^5}{13x^{13}} - \frac{35a^4b^6}{2x^{12}} - \frac{120a^3b^7}{11x^{11}} - \frac{9a^2b^8}{2x^{10}} - \frac{10ab^9}{9x^9} - \frac{b^{10}}{8x^8}$$

output
```
-1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*
b^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2
*b^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = -\frac{a^{10}}{18x^{18}} - \frac{10a^9b}{17x^{17}} - \frac{45a^8b^2}{16x^{16}} - \frac{8a^7b^3}{x^{15}} - \frac{15a^6b^4}{x^{14}} - \frac{252a^5b^5}{13x^{13}} - \frac{35a^4b^6}{2x^{12}} - \frac{120a^3b^7}{11x^{11}} - \frac{9a^2b^8}{2x^{10}} - \frac{10ab^9}{9x^9} - \frac{b^{10}}{8x^8}$$

input
```
Integrate[(a + b*x)^10/x^19,x]
```

output
```
-1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b
^3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x
^12) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9)
 - b^10/(8*x^8)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120a^3b^7}{x^{12}}+\frac{45a^2b^8}{x^{11}}+\frac{10ab^9}{x^{10}}+\frac{b^{10}}{x^9}\right)d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

input
```
Int[(a + b*x)^10/x^19,x]
```

output
```
-1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b
^3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x
^12) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9)
 - b^10/(8*x^8)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

method	result
norman	$\frac{-\frac{1}{18}a^{10}-\frac{10}{17}a^9bx-\frac{45}{16}a^8b^2x^2-8a^7b^3x^3-15a^6b^4x^4-\frac{252}{13}a^5b^5x^5-\frac{35}{2}a^4b^6x^6-\frac{120}{11}a^3b^7x^7-\frac{9}{2}a^2b^8x^8-\frac{10}{9}a\,b^9x^9-\frac{1}{8}b^{10}x^{10}}{x^{18}}$
risch	$\frac{-\frac{1}{18}a^{10}-\frac{10}{17}a^9bx-\frac{45}{16}a^8b^2x^2-8a^7b^3x^3-15a^6b^4x^4-\frac{252}{13}a^5b^5x^5-\frac{35}{2}a^4b^6x^6-\frac{120}{11}a^3b^7x^7-\frac{9}{2}a^2b^8x^8-\frac{10}{9}a\,b^9x^9-\frac{1}{8}b^{10}x^{10}}{x^{18}}$
gosper	$-\frac{43758b^{10}x^{10}+388960a\,b^9x^9+1575288a^2b^8x^8+3818880a^3b^7x^7+6126120a^4b^6x^6+6785856a^5b^5x^5+5250960a^6b^4x^4+28005}{350064x^{18}}$
default	$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10a\,b^9}{9x^9}-\frac{b^{10}}{8x^8}$
parallelrisch	$\frac{-43758b^{10}x^{10}-388960a\,b^9x^9-1575288a^2b^8x^8-3818880a^3b^7x^7-6126120a^4b^6x^6-6785856a^5b^5x^5-5250960a^6b^4x^4-280051}{350064x^{18}}$
orering	$-\frac{43758b^{10}x^{10}+388960a\,b^9x^9+1575288a^2b^8x^8+3818880a^3b^7x^7+6126120a^4b^6x^6+6785856a^5b^5x^5+5250960a^6b^4x^4+28005}{350064x^{18}}$

input
```
int((b*x+a)^10/x^19,x,method=_RETURNVERBOSE)
```

output
```
1/x^18*(-1/18*a^10-10/17*a^9*b*x-45/16*a^8*b^2*x^2-8*a^7*b^3*x^3-15*a^6*b^
4*x^4-252/13*a^5*b^5*x^5-35/2*a^4*b^6*x^6-120/11*a^3*b^7*x^7-9/2*a^2*b^8*x
^8-10/9*a*b^9*x^9-1/8*b^10*x^10)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = -\frac{43758\,b^{10}x^{10} + 388960\,ab^{9}x^{9} + 1575288\,a^{2}b^{8}x^{8} + 3818880\,a^{3}b^{7}x^{7} + 6126120\,a^{4}b^{6}x^{6} + 6785856\,a^{5}b^{5}x^{5} +}{350064\,x^{18}}$$

input
```
integrate((b*x+a)^10/x^19,x, algorithm="fricas")
```

output
```
-1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 3818
880*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*
b^4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 1944
8*a^10)/x^18
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = \frac{-19448a^{10} - 205920a^{9}bx - 984555a^{8}b^{2}x^{2} - 2800512a^{7}b^{3}x^{3} - 5250960a^{6}b^{4}x^{4} - 6785856a^{5}b^{5}x^{5} - 61261}{350064x^{18}}$$

input
```
integrate((b*x+a)**10/x**19,x)
```

output
```
(-19448*a**10 - 205920*a**9*b*x - 984555*a**8*b**2*x**2 - 2800512*a**7*b**
3*x**3 - 5250960*a**6*b**4*x**4 - 6785856*a**5*b**5*x**5 - 6126120*a**4*b*
*6*x**6 - 3818880*a**3*b**7*x**7 - 1575288*a**2*b**8*x**8 - 388960*a*b**9*
x**9 - 43758*b**10*x**10)/(350064*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = -\frac{43758\,b^{10}x^{10} + 388960\,ab^9x^9 + 1575288\,a^2b^8x^8 + 3818880\,a^3b^7x^7 + 6126120\,a^4b^6x^6 + 6785856\,a^5b^5x^5 +}{350064\,x^{18}}$$

input
```
integrate((b*x+a)^10/x^19,x, algorithm="maxima")
```

output
```
-1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 3818
880*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*
b^4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 1944
8*a^10)/x^18
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = -\frac{43758\,b^{10}x^{10} + 388960\,ab^9x^9 + 1575288\,a^2b^8x^8 + 3818880\,a^3b^7x^7 + 6126120\,a^4b^6x^6 + 6785856\,a^5b^5x^5 +}{350064\,x^{18}}$$

input
```
integrate((b*x+a)^10/x^19,x, algorithm="giac")
```

output
```
-1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 3818
880*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*
b^4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 1944
8*a^10)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = -\frac{\frac{a^{10}}{18}+\frac{10\,a^9\,b\,x}{17}+\frac{45\,a^8\,b^2\,x^2}{16}+8\,a^7\,b^3\,x^3+15\,a^6\,b^4\,x^4+\frac{252\,a^5\,b^5\,x^5}{13}+\frac{35\,a^4\,b^6\,x^6}{2}+\frac{120\,a^3\,b^7\,x^7}{11}+\frac{9\,a^2\,b^8\,x^8}{2}+\frac{10\,a\,b^9}{9}}{x^{18}}$$

input
```
int((a + b*x)^10/x^19,x)
```

output
```
-(a^10/18 + (b^10*x^10)/8 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/16 + 8*a^7
*b^3*x^3 + 15*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/13 + (35*a^4*b^6*x^6)/2 + (1
20*a^3*b^7*x^7)/11 + (9*a^2*b^8*x^8)/2 + (10*a^9*b*x)/17)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = \frac{-43758b^{10}x^{10}-388960a\,b^9x^9-1575288a^2b^8x^8-3818880a^3b^7x^7-6126120a^4b^6x^6-6785856a^5b^5x^5-}{350064x^{18}}$$

input
```
int((b*x+a)^10/x^19,x)
```

output
```
( - 19448*a**10 - 205920*a**9*b*x - 984555*a**8*b**2*x**2 - 2800512*a**7*b
**3*x**3 - 5250960*a**6*b**4*x**4 - 6785856*a**5*b**5*x**5 - 6126120*a**4*
b**6*x**6 - 3818880*a**3*b**7*x**7 - 1575288*a**2*b**8*x**8 - 388960*a*b**
9*x**9 - 43758*b**10*x**10)/(350064*x**18)
```

3.112 $\int \frac{(a+bx)^{10}}{x^{20}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 126

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx=-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

output
```
-1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6
*b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^
8/x^11-a*b^9/x^10-1/9*b^10/x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx=-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

input
```
Integrate[(a + b*x)^10/x^20,x]
```

output
```
-1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^
3)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^
13) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*
x^9)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{20}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^{10}}{x^{20}}+\frac{10a^9b}{x^{19}}+\frac{45a^8b^2}{x^{18}}+\frac{120a^7b^3}{x^{17}}+\frac{210a^6b^4}{x^{16}}+\frac{252a^5b^5}{x^{15}}+\frac{210a^4b^6}{x^{14}}+\frac{120a^3b^7}{x^{13}}+\frac{45a^2b^8}{x^{12}}+\frac{10ab^9}{x^{11}}+\frac{b^{10}}{x^{10}}\right) d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

input
```
Int[(a + b*x)^10/x^20,x]
```

output
```
-1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^
3)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^
13) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*
x^9)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

method	result
norman	$\frac{-\frac{1}{19}a^{10}-\frac{5}{9}a^9bx-\frac{45}{17}a^8b^2x^2-\frac{15}{2}a^7b^3x^3-14a^6b^4x^4-18a^5b^5x^5-\frac{210}{13}a^4b^6x^6-10a^3b^7x^7-\frac{45}{11}a^2b^8x^8-a\,b^9x^9-\frac{1}{9}b^{10}x^{10}}{x^{19}}$
risch	$\frac{-\frac{1}{19}a^{10}-\frac{5}{9}a^9bx-\frac{45}{17}a^8b^2x^2-\frac{15}{2}a^7b^3x^3-14a^6b^4x^4-18a^5b^5x^5-\frac{210}{13}a^4b^6x^6-10a^3b^7x^7-\frac{45}{11}a^2b^8x^8-a\,b^9x^9-\frac{1}{9}b^{10}x^{10}}{x^{19}}$
gosper	$-\frac{92378b^{10}x^{10}+831402a\,b^9x^9+3401190a^2b^8x^8+8314020a^3b^7x^7+13430340a^4b^6x^6+14965236a^5b^5x^5+11639628a^6b^4x^4+62}{831402x^{19}}$
default	$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{a\,b^9}{x^{10}}-\frac{b^{10}}{9x^9}$
parallelrisch	$\frac{-92378b^{10}x^{10}-831402a\,b^9x^9-3401190a^2b^8x^8-8314020a^3b^7x^7-13430340a^4b^6x^6-14965236a^5b^5x^5-11639628a^6b^4x^4-623}{831402x^{19}}$
orering	$-\frac{92378b^{10}x^{10}+831402a\,b^9x^9+3401190a^2b^8x^8+8314020a^3b^7x^7+13430340a^4b^6x^6+14965236a^5b^5x^5+11639628a^6b^4x^4+62}{831402x^{19}}$

input
```
int((b*x+a)^10/x^20,x,method=_RETURNVERBOSE)
```

output
```
1/x^19*(-1/19*a^10-5/9*a^9*b*x-45/17*a^8*b^2*x^2-15/2*a^7*b^3*x^3-14*a^6*b
^4*x^4-18*a^5*b^5*x^5-210/13*a^4*b^6*x^6-10*a^3*b^7*x^7-45/11*a^2*b^8*x^8-
a*b^9*x^9-1/9*b^10*x^10)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx = -\frac{92378\,b^{10}x^{10} + 831402\,ab^9x^9 + 3401190\,a^2b^8x^8 + 8314020\,a^3b^7x^7 + 13430340\,a^4b^6x^6 + 14965236\,a^5b^5x^5 \ldots}{831402\,x^{19}}$$

input
```
integrate((b*x+a)^10/x^20,x, algorithm="fricas")
```

output
```
-1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 8314
020*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a
^6*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x +
43758*a^10)/x^19
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx = \frac{-43758a^{10} - 461890a^9bx - 2200770a^8b^2x^2 - 6235515a^7b^3x^3 - 11639628a^6b^4x^4 - 14965236a^5b^5x^5 - 13\ldots}{831402x^{19}}$$

input
```
integrate((b*x+a)**10/x**20,x)
```

output
```
(-43758*a**10 - 461890*a**9*b*x - 2200770*a**8*b**2*x**2 - 6235515*a**7*b*
*3*x**3 - 11639628*a**6*b**4*x**4 - 14965236*a**5*b**5*x**5 - 13430340*a**
4*b**6*x**6 - 8314020*a**3*b**7*x**7 - 3401190*a**2*b**8*x**8 - 831402*a*b
**9*x**9 - 92378*b**10*x**10)/(831402*x**19)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx = -\frac{92378\,b^{10}x^{10} + 831402\,ab^9x^9 + 3401190\,a^2b^8x^8 + 8314020\,a^3b^7x^7 + 13430340\,a^4b^6x^6 + 14965236\,a^5b^5x}{831402\,x^{19}}$$

input
```
integrate((b*x+a)^10/x^20,x, algorithm="maxima")
```

output
```
-1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 8314
020*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a
^6*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x +
43758*a^10)/x^19
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx = -\frac{92378\,b^{10}x^{10} + 831402\,ab^9x^9 + 3401190\,a^2b^8x^8 + 8314020\,a^3b^7x^7 + 13430340\,a^4b^6x^6 + 14965236\,a^5b^5x}{831402\,x^{19}}$$

input
```
integrate((b*x+a)^10/x^20,x, algorithm="giac")
```

output
```
-1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 8314
020*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a
^6*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x +
43758*a^10)/x^19
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx = -\frac{\frac{a^{10}}{19}+\frac{5\,a^9\,b\,x}{9}+\frac{45\,a^8\,b^2\,x^2}{17}+\frac{15\,a^7\,b^3\,x^3}{2}+14\,a^6\,b^4\,x^4+18\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{13}+10\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{11}+a\,b^{\dots}}{x^{19}}$$

input
```
int((a + b*x)^10/x^20,x)
```

output
```
-(a^10/19 + (b^10*x^10)/9 + a*b^9*x^9 + (45*a^8*b^2*x^2)/17 + (15*a^7*b^3*
x^3)/2 + 14*a^6*b^4*x^4 + 18*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/13 + 10*a^3*b
^7*x^7 + (45*a^2*b^8*x^8)/11 + (5*a^9*b*x)/9)/x^19
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx = \frac{-92378b^{10}x^{10}-831402a\,b^9x^9-3401190a^2b^8x^8-8314020a^3b^7x^7-13430340a^4b^6x^6-14965236a^5b^5x^5 \dots}{831402x^{19}}$$

input
```
int((b*x+a)^10/x^20,x)
```

output
```
( - 43758*a**10 - 461890*a**9*b*x - 2200770*a**8*b**2*x**2 - 6235515*a**7*
b**3*x**3 - 11639628*a**6*b**4*x**4 - 14965236*a**5*b**5*x**5 - 13430340*a
**4*b**6*x**6 - 8314020*a**3*b**7*x**7 - 3401190*a**2*b**8*x**8 - 831402*a
*b**9*x**9 - 92378*b**10*x**10)/(831402*x**19)
```

3.113 $\int c(a+bx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int c(a+bx)\,dx=\frac{c(a+bx)^2}{2b}$$

output
```
1/2*c*(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int c(a+bx)\,dx=c\left(ax+\frac{bx^2}{2}\right)$$

input
```
Integrate[c*(a + b*x),x]
```

output
```
c*(a*x + (b*x^2)/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int c(a+bx)\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{c(a+bx)^2}{2b}$$

input
```
Int[c*(a + b*x),x]
```

output
```
(c*(a + b*x)^2)/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x(bx+2a)c}{2}$	12
orering	$\frac{x(bx+2a)c}{2}$	12
default	$\left(\frac{1}{2}b\,x^2+ax\right)c$	13
norman	$acx+\frac{1}{2}x^2bc$	13
risch	$acx+\frac{1}{2}x^2bc$	13
parallelrisch	$\left(\frac{1}{2}b\,x^2+ax\right)c$	13

input
```
int(c*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x+2*a)*c
```

Fricas [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int c(a+bx)\,dx=\frac{1}{2}x^2cb+xca$$

input
```
integrate(c*(b*x+a),x, algorithm="fricas")
```

output
```
1/2*x^2*c*b + x*c*a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int c(a+bx)\,dx = acx + \frac{bcx^2}{2}$$

input
```
integrate(c*(b*x+a),x)
```

output
```
a*c*x + b*c*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int c(a+bx)\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)c$$

input
```
integrate(c*(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)*c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int c(a+bx)\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)c$$

input
```
integrate(c*(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*c
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int c(a+bx)\,dx=\frac{c\,x\,(2\,a+b\,x)}{2}$$

input
```
int(c*(a + b*x),x)
```

output
```
(c*x*(2*a + b*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int c(a+bx)\,dx=\frac{cx(bx+2a)}{2}$$

input
```
int(c*(b*x+a),x)
```

output
```
(c*x*(2*a + b*x))/2
```

3.114 $\int \frac{(c+d)(a+bx)}{e} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{(c+d)(a+bx)}{e} \, dx = \frac{(c+d)(a+bx)^2}{2be}$$

output
```
1/2*(c+d)*(b*x+a)^2/b/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+d)(a+bx)}{e} \, dx = \frac{(c+d)\left(ax+\frac{bx^2}{2}\right)}{e}$$

input
```
Integrate[((c + d)*(a + b*x))/e,x]
```

output
```
((c + d)*(a*x + (b*x^2)/2))/e
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d)(a+bx)}{e} \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{(c+d)(a+bx)^2}{2be}$$

input
```
Int[((c + d)*(a + b*x))/e,x]
```

output
```
((c + d)*(a + b*x)^2)/(2*b*e)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{x(bx+2a)(c+d)}{2e}$	17
orering	$\frac{x(bx+2a)(c+d)}{2e}$	17
default	$\frac{\left(\frac{1}{2}b\,x^{2}+ax\right)(c+d)}{e}$	18
parallelrisch	$\frac{\left(\frac{1}{2}b\,x^{2}+ax\right)(c+d)}{e}$	18
norman	$\frac{a(c+d)x}{e}+\frac{(c+d)b\,x^{2}}{2e}$	23
risch	$\frac{axc}{e}+\frac{axd}{e}+\frac{b\,x^{2}c}{2e}+\frac{b\,x^{2}d}{2e}$	36

input
```
int((c+d)*(b*x+a)/e,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x+2*a)*(c+d)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{(c+d)(a+bx)}{e}\,dx=\frac{(bc+bd)x^{2}+2\,(ac+ad)x}{2\,e}$$

input
```
integrate((c+d)*(b*x+a)/e,x, algorithm="fricas")
```

output
```
1/2*((b*c + b*d)*x^2 + 2*(a*c + a*d)*x)/e
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+d)(a+bx)}{e}\,dx = \frac{x^2(bc+bd)}{2e} + \frac{x(ac+ad)}{e}$$

input
```
integrate((c+d)*(b*x+a)/e,x)
```

output
```
x**2*(b*c + b*d)/(2*e) + x*(a*c + a*d)/e
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(c+d)(a+bx)}{e}\,dx = \frac{(bx^2+2\,ax)(c+d)}{2\,e}$$

input
```
integrate((c+d)*(b*x+a)/e,x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)*(c + d)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(c+d)(a+bx)}{e}\,dx = \frac{(bx^2+2\,ax)(c+d)}{2\,e}$$

input
```
integrate((c+d)*(b*x+a)/e,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*(c + d)/e
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{(c+d)(a+bx)}{e} \, dx = \frac{x\,(c+d)\,\left(2\,a+b\,x\right)}{2\,e}$$

input
```
int(((c + d)*(a + b*x))/e,x)
```

output
```
(x*(c + d)*(2*a + b*x))/(2*e)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{(c+d)(a+bx)}{e} \, dx = \frac{x(bcx+bdx+2ac+2ad)}{2e}$$

input
```
int((c+d)*(b*x+a)/e,x)
```

output
```
(x*(2*a*c + 2*a*d + b*c*x + b*d*x))/(2*e)
```

3.115 $\int (1-x)^{2014} x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int (1-x)^{2014} x \, dx = -\frac{(1-x)^{2015}}{2015} + \frac{(1-x)^{2016}}{2016}$$

output
```
-1/2015*(1-x)^2015+1/2016*(1-x)^2016
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12138 vs. 2(23) = 46.

Time = 0.06 (sec) , antiderivative size = 12138, normalized size of antiderivative = 527.74

$$\int (1-x)^{2014} x \, dx = \text{Result too large to show}$$

input
```
Integrate[(1 - x)^2014*x,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{2014} x \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((1-x)^{2014}-(1-x)^{2015}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-x)^{2016}}{2016}-\frac{(1-x)^{2015}}{2015}$$

input
```
Int[(1 - x)^2014*x,x]
```

output
```
-1/2015*(1 - x)^2015 + (1 - x)^2016/2016
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10075 vs. $2(19) = 38$.

Time = 13.28 (sec) , antiderivative size = 10076, normalized size of antiderivative = 438.09

method	result	size
gosper	Expression too large to display	10076
default	Expression too large to display	10077
risch	Expression too large to display	10077
parallelrisch	Expression too large to display	10077
orering	Expression too large to display	10088

input
```
int((1-x)^2014*x,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-2)]

Exception generated.

$$\int (1-x)^{2014}x\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((1-x)^2014*x,x, algorithm="fricas")
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12024 vs. $2(12) = 24$.

Time = 2.37 (sec) , antiderivative size = 12024, normalized size of antiderivative = 522.78

$$\int (1-x)^{2014} x \, dx = \text{Too large to display}$$

input
```
integrate((1-x)**2014*x,x)
```

output
```
x**2016/2016 - 2014*x**2015/2015 + 2013*x**2014/2 - 2026084*x**2013/3 + 13
58826667*x**2012/4 - 136629987582*x**2011 + 137313103532350 9*x**2010/30 -
91953985549170536*x**2009/7 + 26377651026988133103*x**2008/8 - 66174915585
42444915874*x**2007/9 + 29499299483526473166 1117*x**2006/2 - 1344229107990
97740606580164*x**2005/5 + 538766892328182148445244548 23*x**2004/12 - 6917
627027906234894515626 20638*x**2003 + 25719730872661663428500290706 91063*x*
*2002/26 - 39588591248274824267756569403940400*x**2001/3 + 131961937837090
040663501674882660163383*x**2000/80 - 1939645939135052094029929276517339 59
950*x**1999 + 387541161559807075301489477559812273935075*x**1998/18 - 2262
922530915969121458419278661196219696900*x**1997 + 903358447595910292527771
972191922410542659825*x**1996/4 - 2145475773979272745021812337899023630563
0977010*x**1995 + 38891614703171686462810256309466328170629826 60525*x**199
4/2 - 1685021056289444478187051996700441871304935333534 00*x**1993 + 111885
36994114896125248239804523335798487074321266 6075*x**1992/8 - 1113818576786
3128436005535520922869154210608130448652 30*x**1991 + 17049987754591807907 3
43287673317020956598482816691757837 1*x**1990/2 - 73476903655441985945968 33
0316081626303723736209989757096414 0*x**1989/117 + 17835414835995795079540 5
5928245847409394570340210724349640535*x**1988/4 - 305508278474757676047756
4985383679639490542834878909087548 1890*x**1987 + 121347857443982563149967 8
693607792781035287169850475209321463 1735*x**1986/6 - 12950292252359328 0...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10076 vs. $2(15) = 30$.

Time = 2.43 (sec) , antiderivative size = 10076, normalized size of antiderivative = 438.09

$$\int (1-x)^{2014}x\,dx = \text{Too large to display}$$

input
```
integrate((1-x)^2014*x,x, algorithm="maxima")
```

output
```
1/2016*x^2016 - 2014/2015*x^2015 + 2013/2*x^2014 - 2026084/3*x^2013 + 1358
826667/4*x^2012 - 136629987582*x^2011 + 1373131035323509/30*x^2010 - 91953
985549170536/7*x^2009 + 2637765102698813310 3/8*x^2008 - 661749155854244491
5874/9*x^2007 + 294992994835264731661117/2*x^2006 - 134422910799097740 6065
80164/5*x^2005 + 538766892328182148445244548 23/12*x^2004 - 691762702790623
489451562620638*x^2003 + 2571973087266166342850029070691063/26*x^2002 - 39
588591248274824267756569403940400/3*x^2001 + 131961937837090040663501674 88
2660163383/80*x^2000 - 193964593913505209402992927651733959950*x^1999 + 38
754116155980707530148947755981227393 5075/18*x^1998 - 226292253091596912145
84192786611962196969 00*x^1997 + 903358447595910292527771972191922410542659
825/4*x^1996 - 2145475773979272745021812337899023630563097 7010*x^1995 + 38
891614703171686462810256309466328170629826 60525/2*x^1994 - 168502105628944
447818705199670044187130493533353400*x^1993 + 1118853699411489612524823980
452333579848707432126660 75/8*x^1992 - 1113818576786312843600553552092286 91
542106081304486 5230*x^1991 + 1704998775459180790734328767331702095659848 28
166917578371/2*x^1990 - 734769036554419859459683303160816263037237362099 89
7570964140/117*x^1989 + 1783541483599579507954055928245847409394570340 2107
24349640535/4*x^1988 - 305508278474757676047756498538367963949054283487890
90875481890*x^1987 + 121347857443982563149967869360779278103528716985047 52
093214631735/6*x^1986 - 1295029225235932804921369946870905907982444 5040...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10076 vs. $2(15) = 30$.

Time = 1.07 (sec) , antiderivative size = 10076, normalized size of antiderivative = 438.09

$$\int (1-x)^{2014} x \, dx = \text{Too large to display}$$

input
```
integrate((1-x)^2014*x,x, algorithm="giac")
```

output
```
1/2016*x^2016 - 2014/2015*x^2015 + 2013/2*x^2014 - 2026084/3*x^2013 + 1358
826667/4*x^2012 - 136629987582*x^2011 + 137313103532350 9/30*x^2010 - 91953
985549170536/7*x^2009 + 2637765102698813310 3/8*x^2008 - 661749155854244491
5874/9*x^2007 + 2949929948352647316611 17/2*x^2006 - 1344229107990977406065
80164/5*x^2005 + 5387668923281821484452 4454823/12*x^2004 - 691762702790623
489451562620638*x^2003 + 2571973087266166342850029070691063/26*x^2002 - 39
588591248274824267756569403940400/3*x^2001 + 131961937837090040663501674 88
2660163383/80*x^2000 - 1939645939135052094029929276517339 59950*x^1999 + 38
75411615598070753014894775598122739350 75/18*x^1998 - 226292253091596912145
841927866119621969 6900*x^1997 + 903358447595910292527771972191922410542659
825/4*x^1996 - 2145475773979272745021812337899023630563097 7010*x^1995 + 38
891614703171686462810256309466328170629826605 25/2*x^1994 - 168502105628944
447818705199670044187130493533353400*x^1993 + 1118853699411489612524823980
452333579848707432126660 75/8*x^1992 - 111381857678631284360055355209228691
542106081304486 5230*x^1991 + 1704998775459180790734328767331702095659848 28
166917578371/2*x^1990 - 7347690365544198594596833031608162630372373620998 9
7570964140/117*x^1989 + 178354148359957950795405592824584740939457034021 07
24349640535/4*x^1988 - 305508278474757676047756498538367963949054283487890
90875481890*x^1987 + 1213478574439825631499678693607792781035287169850475 2
093214631735/6*x^1986 - 129502922523593280492136994687090590798244450 40...
```

Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int (1-x)^{2014}x\,dx = \frac{(x-1)^{2015}}{2015} + \frac{(x-1)^{2016}}{2016}$$

input
```
int(x*(x - 1)^2014,x)
```

output
```
(x - 1)^2015/2015 + (x - 1)^2016/2016
```

Reduce [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 10075, normalized size of antiderivative = 438.04

$$\int (1-x)^{2014}x\,dx = \text{Too large to display}$$

input
```
int((1-x)^2014*x,x)
```

output

```
(x**2*(2015*x**2014 - 4060224*x**2013 + 4088644560*x**2012 - 2743479822720
*x**2011 + 1379970009938520*x**2010 - 555023800755103680*x**2009 + 1859329
27231085706672*x**2008 - 533627368938946454520*x**2007 + 133940436384840
34227041340*x**2006 - 298687098986371793722888640*x**2005 + 5991661716698
0290177152796104O*x**2004 - 109211625032905361160334841081472*x**2003 + 18
23833683909362208916841844668196O*x**2002 - 281010612178418236378971574006
0509120*x**2001 + 40184507515446582940688854200477168312O*x**2000 - 536061
196374639740444838154984876168320O0*x**1999 + 6700763279491758084811288047
191717776261974*x**1998 - 787930731979197401845213990423979761467288000*x*
*1997 + 874602893408172607540401452956984339816677260O0*x**1996 - 91925344
2198808640395324913054865773150153505600O*x**1995 + 9174147050405026566795
0410407922872325070361187700O*x**1994 - 8715437508089560915737406951506929
7530186380049102400*x**1993 + 78993536455906075808343167795283248473929653
41445538000*x**1992 - 6844959935701232937070570103076402987289760509295156
16000*x**1991 + 5681315314871661954478551207940859451755766598852757956350
0*x**1990 - 452459837536443148578791266145137159930005007718337333191520O*
x**1989 + 34630571128106512876763098459027667605366310418638963178090552O*
x**1988 - 2551118094916945752044020428574354065265288121210844366387494080
0*x**1987 + 18112933890843889650978210384893527950847498547044032205209417
24600*x**1986 - 1241047949151299621948238358622499877872406272555849965...
```

3.116 $\int \frac{x^5}{a+bx} dx$

Optimal result

Integrand size = 11, antiderivative size = 70

$$\int \frac{x^5}{a+bx}\,dx=\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}$$

output
```
a^4*x/b^5-1/2*a^3*x^2/b^4+1/3*a^2*x^3/b^3-1/4*a*x^4/b^2+1/5*x^5/b-a^5*ln(b
*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{a+bx}\,dx=\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}$$

input
```
Integrate[x^5/(a + b*x),x]
```

output
```
(a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^
5/(5*b) - (a^5*Log[a + b*x])/b^6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^5}{b^5(a+bx)}+\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5\log(a+bx)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

input
```
Int[x^5/(a + b*x),x]
```

output
```
(a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^
5/(5*b) - (a^5*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\frac{1}{5}b^4x^5-\frac{1}{4}a\,b^3x^4+\frac{1}{3}a^2b^2x^3-\frac{1}{2}a^3b\,x^2+a^4x}{b^5}-\frac{a^5\ln(bx+a)}{b^6}$	63
norman	$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{a\,x^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\ln(bx+a)}{b^6}$	63
risch	$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{a\,x^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\ln(bx+a)}{b^6}$	63
parallelrisch	$-\frac{-12b^5x^5+15a\,b^4x^4-20a^2b^3x^3+30a^3b^2x^2+60a^5\ln(bx+a)-60a^4bx}{60b^6}$	64

input
```
int(x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*b^4*x^5-1/4*a*b^3*x^4+1/3*a^2*b^2*x^3-1/2*a^3*b*x^2+a^4*x)-a^5*
ln(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{a+bx}\,dx = \frac{12\,b^5x^5-15\,ab^4x^4+20\,a^2b^3x^3-30\,a^3b^2x^2+60\,a^4bx-60\,a^5\log\left(bx+a\right)}{60\,b^6}$$

input
```
integrate(x^5/(b*x+a),x, algorithm="fricas")
```

output
```
1/60*(12*b^5*x^5 - 15*a*b^4*x^4 + 20*a^2*b^3*x^3 - 30*a^3*b^2*x^2 + 60*a^4
*b*x - 60*a^5*log(b*x + a))/b^6
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{a+bx}\,dx = -\frac{a^5\log{(a+bx)}}{b^6} + \frac{a^4x}{b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^4}{4b^2} + \frac{x^5}{5b}$$

input
```
integrate(x**5/(b*x+a),x)
```

output
```
-a**5*log(a + b*x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*
b**3) - a*x**4/(4*b**2) + x**5/(5*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{a+bx}\,dx = -\frac{a^5\log{(bx+a)}}{b^6} + \frac{12\,b^4x^5 - 15\,ab^3x^4 + 20\,a^2b^2x^3 - 30\,a^3bx^2 + 60\,a^4x}{60\,b^5}$$

input
```
integrate(x^5/(b*x+a),x, algorithm="maxima")
```

output
```
-a^5*log(b*x + a)/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x^3 -
 30*a^3*b*x^2 + 60*a^4*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{a+bx}\,dx = -\frac{a^5\log{(|bx+a|)}}{b^6} + \frac{12\,b^4x^5 - 15\,ab^3x^4 + 20\,a^2b^2x^3 - 30\,a^3bx^2 + 60\,a^4x}{60\,b^5}$$

input
```
integrate(x^5/(b*x+a),x, algorithm="giac")
```

output
```
-a^5*log(abs(b*x + a))/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*
x^3 - 30*a^3*b*x^2 + 60*a^4*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{a+bx}\,dx = \frac{x^5}{5\,b} - \frac{a^5\,\ln\left(a+b\,x\right)}{b^6} - \frac{a\,x^4}{4\,b^2} + \frac{a^4\,x}{b^5} + \frac{a^2\,x^3}{3\,b^3} - \frac{a^3\,x^2}{2\,b^4}$$

input
```
int(x^5/(a + b*x),x)
```

output
```
x^5/(5*b) - (a^5*log(a + b*x))/b^6 - (a*x^4)/(4*b^2) + (a^4*x)/b^5 + (a^2*
x^3)/(3*b^3) - (a^3*x^2)/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{a+bx}\,dx = \frac{-60\log(bx+a)\,a^5+60a^4bx-30a^3b^2x^2+20a^2b^3x^3-15a\,b^4x^4+12b^5x^5}{60b^6}$$

input
```
int(x^5/(b*x+a),x)
```

output
```
( - 60*log(a + b*x)*a**5 + 60*a**4*b*x - 30*a**3*b**2*x**2 + 20*a**2*b**3*
x**3 - 15*a*b**4*x**4 + 12*b**5*x**5)/(60*b**6)
```

3.117 $\int \frac{x^4}{a+bx} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{x^4}{a+bx} \, dx = -\frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b} + \frac{a^4\log(a+bx)}{b^5}$$

output
```
-a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+bx} \, dx = -\frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b} + \frac{a^4\log(a+bx)}{b^5}$$

input
```
Integrate[x^4/(a + b*x),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Lo
g[a + b*x])/b^5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx} \, dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{a^4}{b^4(a+bx)}-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

input
```
Int[x^4/(a + b*x),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Lo
g[a + b*x])/b^5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{-\frac{1}{4}b^{3}x^{4}+\frac{1}{3}a\,b^{2}x^{3}-\frac{1}{2}a^{2}b\,x^{2}+a^{3}x}{b^{4}}+\frac{a^{4}\ln\left(bx+a\right)}{b^{5}}$	52
norman	$-\frac{a^{3}x}{b^{4}}+\frac{a^{2}x^{2}}{2b^{3}}-\frac{a\,x^{3}}{3b^{2}}+\frac{x^{4}}{4b}+\frac{a^{4}\ln\left(bx+a\right)}{b^{5}}$	52
risch	$-\frac{a^{3}x}{b^{4}}+\frac{a^{2}x^{2}}{2b^{3}}-\frac{a\,x^{3}}{3b^{2}}+\frac{x^{4}}{4b}+\frac{a^{4}\ln\left(bx+a\right)}{b^{5}}$	52
parallelrisch	$\frac{3b^{4}x^{4}-4a\,x^{3}b^{3}+6a^{2}b^{2}x^{2}+12a^{4}\ln\left(bx+a\right)-12a^{3}bx}{12b^{5}}$	53

input
```
int(x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/4*b^3*x^4+1/3*a*b^2*x^3-1/2*a^2*b*x^2+a^3*x)+a^4*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{a+bx}\,dx=\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

input
```
integrate(x^4/(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*
x + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{a+bx}\,dx = \frac{a^4\log{(a+bx)}}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

input
```
integrate(x**4/(b*x+a),x)
```

output
```
a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2
) + x**4/(4*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{a+bx}\,dx = \frac{a^4\log{(bx+a)}}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

input
```
integrate(x^4/(b*x+a),x, algorithm="maxima")
```

output
```
a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^
3*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{a+bx}\,dx = \frac{a^4\log{(|bx+a|)}}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

input
```
integrate(x^4/(b*x+a),x, algorithm="giac")
```

output
```
a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 -
12*a^3*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{a+bx}\,dx = \frac{x^4}{4\,b} + \frac{a^4\,\ln\left(a+b\,x\right)}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

input
```
int(x^4/(a + b*x),x)
```

output
```
x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*
x^2)/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{a+bx}\,dx = \frac{12\log(bx+a)\,a^4 - 12a^3bx + 6a^2b^2x^2 - 4a\,b^3x^3 + 3b^4x^4}{12b^5}$$

input
```
int(x^4/(b*x+a),x)
```

output
```
(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b**3*x**3 + 3
*b**4*x**4)/(12*b**5)
```

3.118 $\int \frac{x^3}{a+bx} dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{x^3}{a+bx} dx = \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3 \log(a+bx)}{b^4}$$

output
```
a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a+bx} dx = \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b} - \frac{a^3 \log(a+bx)}{b^4}$$

input
```
Integrate[x^3/(a + b*x),x]
```

output
```
(a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx} \, dx$$

$$\Big\downarrow \textbf{49}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)}+\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

input
```
Int[x^3/(a + b*x),x]
```

output
```
(a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x}{b^3}-\frac{a^3\ln(bx+a)}{b^4}$	41
norman	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41
risch	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41
parallelrisch	$-\frac{-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx}{6b^4}$	42

input
```
int(x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2+a^2*x)-a^3*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+bx}\,dx = \frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

input
```
integrate(x^3/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{a+bx}\,dx = -\frac{a^3\log\left(a+bx\right)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

input
```
integrate(x**3/(b*x+a),x)
```

output
```
-a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{a+bx}\,dx = -\frac{a^3 \log{(bx+a)}}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

input
```
integrate(x^3/(b*x+a),x, algorithm="maxima")
```

output
```
-a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{a+bx}\,dx = -\frac{a^3 \log{(|bx+a|)}}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

input
```
integrate(x^3/(b*x+a),x, algorithm="giac")
```

output
```
-a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a+bx}\,dx = \frac{x^3}{3\,b} - \frac{a^3\,\ln{(a+b\,x)}}{b^4} - \frac{a\,x^2}{2\,b^2} + \frac{a^2\,x}{b^3}$$

input
```
int(x^3/(a + b*x),x)
```

output
```
x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+bx}\,dx = \frac{-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3}{6b^4}$$

input
```
int(x^3/(b*x+a),x)
```

output
```
( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**3)/(6*b**
4)
```

3.119 $\int \frac{x^2}{a+bx} dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x^2}{a+bx} dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

output
```
-a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+bx} dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

input
```
Integrate[x^2/(a + b*x),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^2}{b^2(a+bx)}-\frac{a}{b^2}+\frac{x}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

input
```
Int[x^2/(a + b*x),x]
```

output
```
-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{-\frac{1}{2} b\,x^{2}+a x}{b^{2}}+\frac{a^{2} \ln \left(b x +a \right)}{b^{3}}$	30
norman	$-\frac{a x}{b^{2}}+\frac{x^{2}}{2 b}+\frac{a^{2} \ln \left(b x +a \right)}{b^{3}}$	30
risch	$-\frac{a x}{b^{2}}+\frac{x^{2}}{2 b}+\frac{a^{2} \ln \left(b x +a \right)}{b^{3}}$	30
parallelrisch	$\frac{b^{2} x^{2}+2 a^{2} \ln \left(b x +a \right)-2 a b x}{2 b^{3}}$	30

input
```
int(x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*x^2+a*x)+a^2*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx=\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

input
```
integrate(x^2/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+bx}\,dx=\frac{a^2\log\left(a+bx\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

input
```
integrate(x**2/(b*x+a),x)
```

output
```
a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log(bx+a)}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^2/(b*x+a),x, algorithm="maxima")
```

output
```
a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{a+bx}\,dx = \frac{a^2\log(|bx+a|)}{b^3} + \frac{bx^2-2\,ax}{2\,b^2}$$

input
```
integrate(x^2/(b*x+a),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

input
```
int(x^2/(a + b*x),x)
```

output
```
(2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx}\,dx = \frac{2\log(bx+a)\,a^2 - 2abx + b^2x^2}{2b^3}$$

input
```
int(x^2/(b*x+a),x)
```

output
```
(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2)/(2*b**3)
```

3.120 $\int \frac{x}{a+bx}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{x}{a+bx}\,dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

output
```
x/b-a*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx}\,dx = \frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

input
```
Integrate[x/(a + b*x),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx}\,dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

input
```
Int[x/(a + b*x),x]
```

output
```
x/b - (a*Log[a + b*x])/b^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
parallelrisch	$-\frac{a\ln(bx+a)-bx}{b^2}$	19

input
```
int(x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x/b-a*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+bx}\,dx = \frac{bx-a\log{(bx+a)}}{b^2}$$

input
```
integrate(x/(b*x+a),x, algorithm="fricas")
```

output
```
(b*x - a*log(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{a+bx}\,dx = -\frac{a\log{(a+bx)}}{b^2}+\frac{x}{b}$$

input
```
integrate(x/(b*x+a),x)
```

output
```
-a*log(a + b*x)/b**2 + x/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx}\,dx = \frac{x}{b} - \frac{a\log(bx+a)}{b^2}$$

input
```
integrate(x/(b*x+a),x, algorithm="maxima")
```

output
```
x/b - a*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{a+bx}\,dx = \frac{x}{b} - \frac{a\log(|bx+a|)}{b^2}$$

input
```
integrate(x/(b*x+a),x, algorithm="giac")
```

output
```
x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx}\,dx = -\frac{a\ln(a+b\,x)-b\,x}{b^2}$$

input
```
int(x/(a + b*x),x)
```

output
```
-(a*log(a + b*x) - b*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+bx}\,dx=\frac{-\log(bx+a)\,a+bx}{b^2}$$

input
```
int(x/(b*x+a),x)
```

output
```
( - log(a + b*x)*a + b*x)/b**2
```

3.121 $\int \frac{1}{a+bx} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{a+bx} \, dx = \frac{\log(a+bx)}{b}$$

output
```
ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx} \, dx = \frac{\log(a+bx)}{b}$$

input
```
Integrate[(a + b*x)^(-1),x]
```

output
```
Log[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx} \, dx$$
$$\Big\downarrow \; 16$$
$$\frac{\log(a+bx)}{b}$$

input
```
Int[(a + b*x)^(-1),x]
```

output
```
Log[a + b*x]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11
parallelrisch	$\frac{\ln(bx+a)}{b}$	11

input
```
int(1/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log(bx+a)}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="fricas")
```

output
```
log(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{a+bx}\,dx = \frac{\log(a+bx)}{b}$$

input
```
integrate(1/(b*x+a),x)
```

output
```
log(a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx=\frac{\log\left(bx+a\right)}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="maxima")
```

output
```
log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+bx}\,dx=\frac{\log\left(|bx+a|\right)}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx=\frac{\ln\left(a+b\,x\right)}{b}$$

input
```
int(1/(a + b*x),x)
```

output
```
log(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log(bx+a)}{b}$$

input
```
int(1/(b*x+a),x)
```

output
```
log(a + b*x)/b
```

3.122 $\int \frac{1}{x(a+bx)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{x(a+bx)} \, dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

output
```
ln(x)/a-ln(b*x+a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)} \, dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

input
```
Integrate[1/(x*(a + b*x)),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{47, 14, 16\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)} \, dx$$

$$\Big\downarrow \; 47$$

$$\frac{\int \frac{1}{x}dx}{a} - \frac{b\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \; 14$$

$$\frac{\log(x)}{a} - \frac{b\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

input
```
Int[1/(x*(a + b*x)),x]
```

output
```
Log[x]/a - Log[a + b*x]/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(bx+a)}{a}$	16
default	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
risch	$-\frac{\ln(bx+a)}{a}+\frac{\ln(-x)}{a}$	21

input
```
int(1/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)-ln(b*x+a))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log\left(bx+a\right)-\log\left(x\right)}{a}$$

input
```
integrate(1/x/(b*x+a),x, algorithm="fricas")
```

output
```
-(log(b*x + a) - log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{x(a+bx)}\,dx = \frac{\log\left(x\right) - \log\left(\frac{a}{b} + x\right)}{a}$$

input
```
integrate(1/x/(b*x+a),x)
```

output
```
(log(x) - log(a/b + x))/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log\left(bx+a\right)}{a} + \frac{\log\left(x\right)}{a}$$

input
```
integrate(1/x/(b*x+a),x, algorithm="maxima")
```

output
```
-log(b*x + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{\log\left(|bx+a|\right)}{a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(1/x/(b*x+a),x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

input
```
int(1/(x*(a + b*x)),x)
```

output
```
-(2*atanh((2*b*x)/a + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx)}\,dx = \frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int(1/x/(b*x+a),x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.123 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

input
```
Integrate[1/(x^2*(a + b*x)),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x)),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

input
```
int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2(a+bx)}\,dx = -\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

input
```
integrate(1/x**2/(b*x+a),x)
```

output
```
-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{b\log{(bx+a)}}{a^2} - \frac{b\log{(x)}}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{b\log{(|bx+a|)}}{a^2} - \frac{b\log{(|x|)}}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2} - \frac{1}{a\,x}$$

input
```
int(1/(x^2*(a + b*x)),x)
```

output
```
(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a+bx)}\,dx = \frac{\log(bx+a)\,bx - \log(x)\,bx - a}{a^2x}$$

input
```
int(1/x^2/(b*x+a),x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.124 $\int \frac{1}{x^3(a+bx)} dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{x^3(a+bx)} dx = -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a+bx)} dx = -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3}$$

input
```
Integrate[1/(x^3*(a + b*x)),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)} \, dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{b^3}{a^3(a+bx)}+\frac{b^2}{a^3x}-\frac{b}{a^2x^2}+\frac{1}{ax^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

input
```
Int[1/(x^3*(a + b*x)),x]
```

output
```
-1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b}{a^{2}x}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	41
norman	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	41
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln\left(-x\right)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}$	43
parallelrisch	$\frac{2b^{2}\ln\left(x\right)x^{2}-2b^{2}\ln\left(bx+a\right)x^{2}+2abx-a^{2}}{2a^{3}x^{2}}$	44

input
```
int(1/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3(a+bx)}\,dx=-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(a+bx)}\,dx=\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

input
```
integrate(1/x**3/(b*x+a),x)
```

output
```
(-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{b^2\log(bx+a)}{a^3} + \frac{b^2\log(x)}{a^3} + \frac{2\,bx-a}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(b*x+a),x, algorithm="maxima")
```

output
```
-b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{b^2\log(|bx+a|)}{a^3} + \frac{b^2\log(|x|)}{a^3} + \frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(b*x+a),x, algorithm="giac")
```

output
```
-b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^
3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3(a+bx)}\,dx = -\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2} - \frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

input
```
int(1/(x^3*(a + b*x)),x)
```

output
```
- (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3(a+bx)}\,dx = \frac{-2\log(bx+a)\,b^2x^2 + 2\log(x)\,b^2x^2 - a^2 + 2abx}{2a^3x^2}$$

input
```
int(1/x^3/(b*x+a),x)
```

output
```
( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*x)/(2*a**
3*x**2)
```

3.125 $\int \frac{1}{x^4(a+bx)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{1}{x^4(a+bx)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

output
```
-1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a+bx)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

input
```
Integrate[1/(x^4*(a + b*x)),x]
```

output
```
-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log
[a + b*x])/a^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+bx)} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{b^4}{a^4(a+bx)}-\frac{b^3}{a^4x}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^3}+\frac{1}{ax^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x)),x]
```

output
```
-1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log
[a + b*x])/a^4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b}{2a^{2}x^{2}}-\frac{b^{2}}{a^{3}x}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}$	53
norman	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}$	53
parallelrisch	$-\frac{6b^{3}\ln\left(x\right)x^{3}-6b^{3}\ln\left(bx+a\right)x^{3}+6a\,b^{2}x^{2}-3a^{2}bx+2a^{3}}{6a^{4}x^{3}}$	55
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(-bx-a\right)}{a^{4}}$	56

input
```
int(1/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4(a+bx)}\,dx=\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log\left(x\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/x^4/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
 2*a^3)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4(a+bx)}\,dx = \frac{-2a^2+3abx-6b^2x^2}{6a^3x^3} + \frac{b^3\left(-\log{(x)}+\log{\left(\frac{a}{b}+x\right)}\right)}{a^4}$$

input
```
integrate(1/x**4/(b*x+a),x)
```

output
```
(-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b
+ x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4(a+bx)}\,dx = \frac{b^3\log{(bx+a)}}{a^4} - \frac{b^3\log{(x)}}{a^4} - \frac{6\,b^2x^2-3\,abx+2\,a^2}{6\,a^3x^3}$$

input
```
integrate(1/x^4/(b*x+a),x, algorithm="maxima")
```

output
```
b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/
(a^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a+bx)}\,dx = \frac{b^3\log{(|bx+a|)}}{a^4} - \frac{b^3\log{(|x|)}}{a^4} - \frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4x^3}$$

input
```
integrate(1/x^4/(b*x+a),x, algorithm="giac")
```

output
```
b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2
*b*x + 2*a^3)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4(a+bx)}\,dx = \frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4} - \frac{\frac{a^3}{3}-\frac{a^2\,b\,x}{2}+a\,b^2\,x^2}{a^4\,x^3}$$

input
```
int(1/(x^4*(a + b*x)),x)
```

output
```
(2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*
x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4(a+bx)}\,dx = \frac{6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2}{6a^4x^3}$$

input
```
int(1/x^4/(b*x+a),x)
```

output
```
(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**2*b*x - 6*a
*b**2*x**2)/(6*a**4*x**3)
```

3.126 $\int \frac{1}{x^5(a+bx)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 68

$$\int \frac{1}{x^5(a+bx)}\,dx=-\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}$$

output
```
-1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*
x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(a+bx)}\,dx=-\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}$$

input
```
Integrate[1/(x^5*(a + b*x)),x]
```

output
```
-1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[
x])/a^5 - (b^4*Log[a + b*x])/a^5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(a+bx)} \, dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{b^5}{a^5(a+bx)}+\frac{b^4}{a^5x}-\frac{b^3}{a^4x^2}+\frac{b^2}{a^3x^3}-\frac{b}{a^2x^4}+\frac{1}{ax^5}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b^3}{a^4x}-\frac{b^2}{2a^3x^2}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

input
```
Int[1/(x^5*(a + b*x)),x]
```

output
```
-1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[
x])/a^5 - (b^4*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{4a\,x^{4}}+\frac{b}{3a^{2}x^{3}}-\frac{b^{2}}{2a^{3}x^{2}}+\frac{b^{3}}{a^{4}x}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(bx+a\right)}{a^{5}}$	63
norman	$\frac{\frac{b^{3}x^{3}}{a^{4}}-\frac{1}{4a}+\frac{bx}{3a^{2}}-\frac{b^{2}x^{2}}{2a^{3}}}{x^{4}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(bx+a\right)}{a^{5}}$	63
risch	$\frac{\frac{b^{3}x^{3}}{a^{4}}-\frac{1}{4a}+\frac{bx}{3a^{2}}-\frac{b^{2}x^{2}}{2a^{3}}}{x^{4}}+\frac{b^{4}\ln\left(-x\right)}{a^{5}}-\frac{b^{4}\ln\left(bx+a\right)}{a^{5}}$	65
parallelrisch	$\frac{12b^{4}\ln\left(x\right)x^{4}-12b^{4}\ln\left(bx+a\right)x^{4}+12a\,x^{3}b^{3}-6a^{2}b^{2}x^{2}+4a^{3}bx-3a^{4}}{12a^{5}x^{4}}$	66

input
```
int(1/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*
x+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5(a+bx)}\,dx = -\frac{12\,b^4x^4\log\left(bx+a\right)-12\,b^4x^4\log\left(x\right)-12\,ab^3x^3+6\,a^2b^2x^2-4\,a^3bx+3\,a^4}{12\,a^5x^4}$$

input
```
integrate(1/x^5/(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(12*b^4*x^4*log(b*x + a) - 12*b^4*x^4*log(x) - 12*a*b^3*x^3 + 6*a^2*
b^2*x^2 - 4*a^3*b*x + 3*a^4)/(a^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^5(a+bx)}\,dx = \frac{-3a^3+4a^2bx-6ab^2x^2+12b^3x^3}{12a^4x^4}+\frac{b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

input
```
integrate(1/x**5/(b*x+a),x)
```

output
```
(-3*a**3 + 4*a**2*b*x - 6*a*b**2*x**2 + 12*b**3*x**3)/(12*a**4*x**4) + b**
4*(log(x) - log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^5(a+bx)}\,dx = -\frac{b^4\log\left(bx+a\right)}{a^5}+\frac{b^4\log\left(x\right)}{a^5}+\frac{12\,b^3x^3-6\,ab^2x^2+4\,a^2bx-3\,a^3}{12\,a^4x^4}$$

input
```
integrate(1/x^5/(b*x+a),x, algorithm="maxima")
```

output
```
-b^4*log(b*x + a)/a^5 + b^4*log(x)/a^5 + 1/12*(12*b^3*x^3 - 6*a*b^2*x^2 +
4*a^2*b*x - 3*a^3)/(a^4*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^5(a+bx)}\,dx = -\frac{b^4\log\left(|bx+a|\right)}{a^5}+\frac{b^4\log\left(|x|\right)}{a^5}+\frac{12\,ab^3x^3-6\,a^2b^2x^2+4\,a^3bx-3\,a^4}{12\,a^5x^4}$$

input
```
integrate(1/x^5/(b*x+a),x, algorithm="giac")
```

output
```
-b^4*log(abs(b*x + a))/a^5 + b^4*log(abs(x))/a^5 + 1/12*(12*a*b^3*x^3 - 6*
a^2*b^2*x^2 + 4*a^3*b*x - 3*a^4)/(a^5*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^5(a+bx)}\,dx = -\frac{\frac{a^4}{4}-\frac{a^3\,b\,x}{3}+\frac{a^2\,b^2\,x^2}{2}-a\,b^3\,x^3}{a^5\,x^4}-\frac{2\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

input
```
int(1/(x^5*(a + b*x)),x)
```

output
```
- (a^4/4 - a*b^3*x^3 + (a^2*b^2*x^2)/2 - (a^3*b*x)/3)/(a^5*x^4) - (2*b^4*a
tanh((2*b*x)/a + 1))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5(a+bx)}\,dx = \frac{-12\log(bx+a)\,b^4x^4+12\log(x)\,b^4x^4-3a^4+4a^3bx-6a^2b^2x^2+12a\,b^3x^3}{12a^5x^4}$$

input
```
int(1/x^5/(b*x+a),x)
```

output
```
( - 12*log(a + b*x)*b**4*x**4 + 12*log(x)*b**4*x**4 - 3*a**4 + 4*a**3*b*x
- 6*a**2*b**2*x**2 + 12*a*b**3*x**3)/(12*a**5*x**4)
```

3.127 $\int \frac{x^6}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 81

$$\int \frac{x^6}{(a+bx)^2}\,dx=\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}$$

output
```
5*a^4*x/b^6-2*a^3*x^2/b^5+a^2*x^3/b^4-1/2*a*x^4/b^3+1/5*x^5/b^2-a^6/b^7/(b
*x+a)-6*a^5*ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{x^6}{(a+bx)^2}\,dx =\frac{50a^4bx-20a^3b^2x^2+10a^2b^3x^3-5ab^4x^4+2b^5x^5-\frac{10a^6}{a+bx}-60a^5\log(a+bx)}{10b^7}$$

input
```
Integrate[x^6/(a + b*x)^2,x]
```

output
```
(50*a^4*b*x - 20*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 5*a*b^4*x^4 + 2*b^5*x^5 -
(10*a^6)/(a + b*x) - 60*a^5*Log[a + b*x])/(10*b^7)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx)^2} \, dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}+\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}+\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

input
```
Int[x^6/(a + b*x)^2,x]
```

output
```
(5*a^4*x)/b^6 - (2*a^3*x^2)/b^5 + (a^2*x^3)/b^4 - (a*x^4)/(2*b^3) + x^5/(5
*b^2) - a^6/(b^7*(a + b*x)) - (6*a^5*Log[a + b*x])/b^7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\frac{1}{5}b^4x^5-\frac{1}{2}a\,b^3x^4+a^2b^2x^3-2a^3b\,x^2+5a^4x}{b^6}-\frac{a^6}{b^7(bx+a)}-\frac{6a^5\ln(bx+a)}{b^7}$	78
risch	$\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{a\,x^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(bx+a)}-\frac{6a^5\ln(bx+a)}{b^7}$	78
norman	$\frac{\frac{x^6}{5b}-\frac{3a\,x^5}{10b^2}-\frac{6a^6}{b^7}+\frac{a^2x^4}{2b^3}-\frac{a^3x^3}{b^4}+\frac{3a^4x^2}{b^5}}{bx+a}-\frac{6a^5\ln(bx+a)}{b^7}$	83
parallelrisch	$-\frac{-2b^6x^6+3a\,x^5b^5-5a^2x^4b^4+10a^3x^3b^3+60\ln(bx+a)x\,a^5b-30a^4x^2b^2+60\ln(bx+a)a^6+60a^6}{10b^7(bx+a)}$	93

input
```
int(x^6/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/5*b^4*x^5-1/2*a*b^3*x^4+a^2*b^2*x^3-2*a^3*b*x^2+5*a^4*x)-a^6/b^7/
(b*x+a)-6*a^5*ln(b*x+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \frac{x^6}{(a+bx)^2}\,dx = \frac{2\,b^6x^6-3\,ab^5x^5+5\,a^2b^4x^4-10\,a^3b^3x^3+30\,a^4b^2x^2+50\,a^5bx-10\,a^6-60\,(a^5bx+a^6)\log(bx+a)}{10\,(b^8x+ab^7)}$$

input
```
integrate(x^6/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/10*(2*b^6*x^6 - 3*a*b^5*x^5 + 5*a^2*b^4*x^4 - 10*a^3*b^3*x^3 + 30*a^4*b^
2*x^2 + 50*a^5*b*x - 10*a^6 - 60*(a^5*b*x + a^6)*log(b*x + a))/(b^8*x + a*
b^7)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{(a+bx)^2}\,dx = -\frac{a^6}{ab^7+b^8x} - \frac{6a^5\log(a+bx)}{b^7} + \frac{5a^4x}{b^6} - \frac{2a^3x^2}{b^5} + \frac{a^2x^3}{b^4} - \frac{ax^4}{2b^3} + \frac{x^5}{5b^2}$$

input
```
integrate(x**6/(b*x+a)**2,x)
```

output
```
-a**6/(a*b**7 + b**8*x) - 6*a**5*log(a + b*x)/b**7 + 5*a**4*x/b**6 - 2*a**
3*x**2/b**5 + a**2*x**3/b**4 - a*x**4/(2*b**3) + x**5/(5*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{x^6}{(a+bx)^2}\,dx = -\frac{a^6}{b^8x+ab^7} - \frac{6\,a^5\log(bx+a)}{b^7} + \frac{2\,b^4x^5 - 5\,ab^3x^4 + 10\,a^2b^2x^3 - 20\,a^3bx^2 + 50\,a^4x}{10\,b^6}$$

input
```
integrate(x^6/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-a^6/(b^8*x + a*b^7) - 6*a^5*log(b*x + a)/b^7 + 1/10*(2*b^4*x^5 - 5*a*b^3*
x^4 + 10*a^2*b^2*x^3 - 20*a^3*b*x^2 + 50*a^4*x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27

$$\int \frac{x^6}{(a+bx)^2}\,dx = -\frac{(bx+a)^5\left(\frac{15\,a}{bx+a} - \frac{50\,a^2}{(bx+a)^2} + \frac{100\,a^3}{(bx+a)^3} - \frac{150\,a^4}{(bx+a)^4} - 2\right)}{10\,b^7} + \frac{6\,a^5\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^7} - \frac{a^6}{(bx+a)b^7}$$

input
```
integrate(x^6/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/10*(b*x + a)^5*(15*a/(b*x + a) - 50*a^2/(b*x + a)^2 + 100*a^3/(b*x + a)
^3 - 150*a^4/(b*x + a)^4 - 2)/b^7 + 6*a^5*log(abs(b*x + a)/((b*x + a)^2*ab
s(b)))/b^7 - a^6/((b*x + a)*b^7)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{x^6}{(a+bx)^2} \, dx = \frac{x^5}{5\,b^2} - \frac{6\,a^5\,\ln\left(a+b\,x\right)}{b^7} - \frac{a\,x^4}{2\,b^3} + \frac{5\,a^4\,x}{b^6} + \frac{a^2\,x^3}{b^4} - \frac{2\,a^3\,x^2}{b^5} - \frac{a^6}{b\,\left(x\,b^7+a\,b^6\right)}$$

input
```
int(x^6/(a + b*x)^2,x)
```

output
```
x^5/(5*b^2) - (6*a^5*log(a + b*x))/b^7 - (a*x^4)/(2*b^3) + (5*a^4*x)/b^6 +
 (a^2*x^3)/b^4 - (2*a^3*x^2)/b^5 - a^6/(b*(a*b^6 + b^7*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{x^6}{(a+bx)^2} \, dx = \frac{-60 \log(bx+a)\, a^6 - 60 \log(bx+a)\, a^5 bx + 60a^5 bx + 30a^4 b^2 x^2 - 10a^3 b^3 x^3 + 5a^2 b^4 x^4 - 3a\, b^5 x^5 + 2b^6 x^6}{10b^7\,(bx+a)}$$

input
```
int(x^6/(b*x+a)^2,x)
```

output
```
( - 60*log(a + b*x)*a**6 - 60*log(a + b*x)*a**5*b*x + 60*a**5*b*x + 30*a**
4*b**2*x**2 - 10*a**3*b**3*x**3 + 5*a**2*b**4*x**4 - 3*a*b**5*x**5 + 2*b**
6*x**6)/(10*b**7*(a + b*x))
```

3.128 $\int \frac{x^5}{(a+bx)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{x^5}{(a+bx)^2} dx = -\frac{4a^3x}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{2ax^3}{3b^3} + \frac{x^4}{4b^2} + \frac{a^5}{b^6(a+bx)} + \frac{5a^4\log(a+bx)}{b^6}$$

output
```
-4*a^3*x/b^5+3/2*a^2*x^2/b^4-2/3*a*x^3/b^3+1/4*x^4/b^2+a^5/b^6/(b*x+a)+5*a
^4*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a+bx)^2} dx = \frac{-48a^3bx + 18a^2b^2x^2 - 8ab^3x^3 + 3b^4x^4 + \frac{12a^5}{a+bx} + 60a^4\log(a+bx)}{12b^6}$$

input
```
Integrate[x^5/(a + b*x)^2,x]
```

output
```
(-48*a^3*b*x + 18*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4 + (12*a^5)/(a + b*
x) + 60*a^4*Log[a + b*x]])/(12*b^6)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

input
```
Int[x^5/(a + b*x)^2,x]
```

output
```
(-4*a^3*x)/b^5 + (3*a^2*x^2)/(2*b^4) - (2*a*x^3)/(3*b^3) + x^4/(4*b^2) + a
^5/(b^6*(a + b*x)) + (5*a^4*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2a\,x^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(bx+a)}+\frac{5a^4\ln(bx+a)}{b^6}$	67
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{2}{3}a\,b^2x^3-\frac{3}{2}a^2b\,x^2+4a^3x}{b^5}+\frac{a^5}{b^6(bx+a)}+\frac{5a^4\ln(bx+a)}{b^6}$	68
norman	$\frac{\frac{5a^5}{b^6}+\frac{x^5}{4b}-\frac{5a\,x^4}{12b^2}+\frac{5a^2x^3}{6b^3}-\frac{5a^3x^2}{2b^4}}{bx+a}+\frac{5a^4\ln(bx+a)}{b^6}$	72
parallelrisch	$\frac{3b^5x^5-5a\,b^4x^4+10a^2b^3x^3+60\ln(bx+a)x\,a^4b-30a^3b^2x^2+60a^5\ln(bx+a)+60a^5}{12b^6(bx+a)}$	82

input
```
int(x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4*a^3*x/b^5+3/2*a^2*x^2/b^4-2/3*a*x^3/b^3+1/4*x^4/b^2+a^5/b^6/(b*x+a)+5*a
^4*ln(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{x^5}{(a+bx)^2}\,dx = \frac{3\,b^5x^5-5\,ab^4x^4+10\,a^2b^3x^3-30\,a^3b^2x^2-48\,a^4bx+12\,a^5+60\,(a^4bx+a^5)\log(bx+a)}{12\,(b^7x+ab^6)}$$

input
```
integrate(x^5/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^5*x^5 - 5*a*b^4*x^4 + 10*a^2*b^3*x^3 - 30*a^3*b^2*x^2 - 48*a^4*b
*x + 12*a^5 + 60*(a^4*b*x + a^5)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{(a+bx)^2}\,dx = \frac{a^5}{ab^6+b^7x} + \frac{5a^4\log{(a+bx)}}{b^6} - \frac{4a^3x}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{2ax^3}{3b^3} + \frac{x^4}{4b^2}$$

input
```
integrate(x**5/(b*x+a)**2,x)
```

output
```
a**5/(a*b**6 + b**7*x) + 5*a**4*log(a + b*x)/b**6 - 4*a**3*x/b**5 + 3*a**2
*x**2/(2*b**4) - 2*a*x**3/(3*b**3) + x**4/(4*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx)^2}\,dx = \frac{a^5}{b^7x+ab^6} + \frac{5\,a^4\log{(bx+a)}}{b^6} + \frac{3\,b^3x^4 - 8\,ab^2x^3 + 18\,a^2bx^2 - 48\,a^3x}{12\,b^5}$$

input
```
integrate(x^5/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a^5/(b^7*x + a*b^6) + 5*a^4*log(b*x + a)/b^6 + 1/12*(3*b^3*x^4 - 8*a*b^2*x
^3 + 18*a^2*b*x^2 - 48*a^3*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{(a+bx)^2}\,dx = -\frac{(bx+a)^4\left(\frac{20\,a}{bx+a} - \frac{60\,a^2}{(bx+a)^2} + \frac{120\,a^3}{(bx+a)^3} - 3\right)}{12\,b^6} - \frac{5\,a^4\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^6} + \frac{a^5}{(bx+a)b^6}$$

input
```
integrate(x^5/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/12*(b*x + a)^4*(20*a/(b*x + a) - 60*a^2/(b*x + a)^2 + 120*a^3/(b*x + a)
^3 - 3)/b^6 - 5*a^4*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^6 + a^5/((b*x
 + a)*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx)^2}\,dx=\frac{x^4}{4\,b^2}+\frac{5\,a^4\,\ln\left(a+b\,x\right)}{b^6}-\frac{2\,a\,x^3}{3\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{3\,a^2\,x^2}{2\,b^4}+\frac{a^5}{b\,\left(x\,b^6+a\,b^5\right)}$$

input
```
int(x^5/(a + b*x)^2,x)
```

output
```
x^4/(4*b^2) + (5*a^4*log(a + b*x))/b^6 - (2*a*x^3)/(3*b^3) - (4*a^3*x)/b^5
 + (3*a^2*x^2)/(2*b^4) + a^5/(b*(a*b^5 + b^6*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(a+bx)^2}\,dx = \frac{60\log(bx+a)\,a^5+60\log(bx+a)\,a^4bx-60a^4bx-30a^3b^2x^2+10a^2b^3x^3-5a\,b^4x^4+3b^5x^5}{12b^6\,(bx+a)}$$

input
```
int(x^5/(b*x+a)^2,x)
```

output
```
(60*log(a + b*x)*a**5 + 60*log(a + b*x)*a**4*b*x - 60*a**4*b*x - 30*a**3*b
**2*x**2 + 10*a**2*b**3*x**3 - 5*a*b**4*x**4 + 3*b**5*x**5)/(12*b**6*(a +
b*x))
```

3.129 $\int \frac{x^4}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{x^4}{(a+bx)^2} \, dx = \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2} - \frac{a^4}{b^5(a+bx)} - \frac{4a^3\log(a+bx)}{b^5}$$

output
```
3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(a+bx)^2} \, dx = \frac{9a^2bx - 3ab^2x^2 + b^3x^3 - \frac{3a^4}{a+bx} - 12a^3\log(a+bx)}{3b^5}$$

input
```
Integrate[x^4/(a + b*x)^2,x]
```

output
```
(9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*
x])/(3*b^5)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx)^2}\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}+\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

input
```
Int[x^4/(a + b*x)^2,x]
```

output
```
(3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*L
og[a + b*x])/b^5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x}{b^4}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
risch	$\frac{3a^2x}{b^4}-\frac{a\,x^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{x^4}{3b}-\frac{2a\,x^3}{3b^2}-\frac{4a^4}{b^5}+\frac{2a^2x^2}{b^3}}{bx+a}-\frac{4a^3\ln(bx+a)}{b^5}$	61
parallelrisch	$-\frac{-b^4x^4+2a\,x^3b^3+12\ln(bx+a)x\,a^3b-6a^2b^2x^2+12a^4\ln(bx+a)+12a^4}{3b^5(bx+a)}$	71

input
```
int(x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x)-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(a+bx)^2}\,dx = \frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,(a^3bx+a^4)\log(bx+a)}{3\,(b^6x+ab^5)}$$

input
```
integrate(x^4/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b
*x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(a+bx)^2}\,dx = -\frac{a^4}{ab^5+b^6x} - \frac{4a^3\log(a+bx)}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

input
```
integrate(x**4/(b*x+a)**2,x)
```

output
```
-a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**
2/b**3 + x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx)^2}\,dx = -\frac{a^4}{b^6x+ab^5} - \frac{4\,a^3\log(bx+a)}{b^5} + \frac{b^2x^3-3\,abx^2+9\,a^2x}{3\,b^4}$$

input
```
integrate(x^4/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
 9*a^2*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{x^4}{(a+bx)^2}\,dx = -\frac{(bx+a)^3\left(\frac{6\,a}{bx+a}-\frac{18\,a^2}{(bx+a)^2}-1\right)}{3\,b^5} + \frac{4\,a^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5} - \frac{a^4}{(bx+a)b^5}$$

input
```
integrate(x^4/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*(b*x + a)^3*(6*a/(b*x + a) - 18*a^2/(b*x + a)^2 - 1)/b^5 + 4*a^3*log(
abs(b*x + a)/((b*x + a)^2*abs(b)))/b^5 - a^4/((b*x + a)*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{(a+bx)^2}\,dx = \frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln\left(a+b\,x\right)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5+a\,b^4\right)}$$

input
```
int(x^4/(a + b*x)^2,x)
```

output
```
x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4
/(b*(a*b^4 + b^5*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+bx)^2}\,dx = \frac{-12\log(bx+a)\,a^4 - 12\log(bx+a)\,a^3bx + 12a^3bx + 6a^2b^2x^2 - 2a\,b^3x^3 + b^4x^4}{3b^5\,(bx+a)}$$

input
```
int(x^4/(b*x+a)^2,x)
```

output
```
( - 12*log(a + b*x)*a**4 - 12*log(a + b*x)*a**3*b*x + 12*a**3*b*x + 6*a**2
*b**2*x**2 - 2*a*b**3*x**3 + b**4*x**4)/(3*b**5*(a + b*x))
```

3.130 $\int \frac{x^3}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{x^3}{(a+bx)^2}\,dx = -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}$$

output
```
-2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(a+bx)^2}\,dx = \frac{-4abx+b^2x^2+\frac{2a^3}{a+bx}+6a^2\log(a+bx)}{2b^4}$$

input
```
Integrate[x^3/(a + b*x)^2,x]
```

output
```
(-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^2} \, dx$$
$$\Big\downarrow \; 49$$
$$\int \left(-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}-\frac{2a}{b^3}+\frac{x}{b^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

input
```
Int[x^3/(a + b*x)^2,x]
```

output
```
(-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^
4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	46
norman	$\frac{\frac{3a^3}{b^4}+\frac{x^3}{2b}-\frac{3a\,x^2}{2b^2}}{bx+a}+\frac{3a^2\ln(bx+a)}{b^4}$	50
parallelrisch	$\frac{b^3x^3+6\ln(bx+a)x\,a^2b-3a\,b^2x^2+6a^3\ln(bx+a)+6a^3}{2b^4(bx+a)}$	59

input
```
int(x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{(a+bx)^2}\,dx=\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log{(bx+a)}}{2\,(b^5x+ab^4)}$$

input
```
integrate(x^3/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
 + a))/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(a+bx)^2}\,dx = \frac{a^3}{ab^4+b^5x} + \frac{3a^2\log(a+bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

input
```
integrate(x**3/(b*x+a)**2,x)
```

output
```
a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx)^2}\,dx = \frac{a^3}{b^5x+ab^4} + \frac{3\,a^2\log(bx+a)}{b^4} + \frac{bx^2-4\,ax}{2\,b^3}$$

input
```
integrate(x^3/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{x^3}{(a+bx)^2}\,dx = -\frac{(bx+a)^2\left(\frac{6\,a}{bx+a}-1\right)}{2\,b^4} - \frac{3\,a^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4} + \frac{a^3}{(bx+a)b^4}$$

input
```
integrate(x^3/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b*x + a)^2*(6*a/(b*x + a) - 1)/b^4 - 3*a^2*log(abs(b*x + a)/((b*x +
a)^2*abs(b)))/b^4 + a^3/((b*x + a)*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(a+bx)^2} \, dx = \frac{x^2}{2\,b^2} + \frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4} + \frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)} - \frac{2\,a\,x}{b^3}$$

input
```
int(x^3/(a + b*x)^2,x)
```

output
```
x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)
/b^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{(a+bx)^2} \, dx = \frac{6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3}{2b^4\,(bx+a)}$$

input
```
int(x^3/(b*x+a)^2,x)
```

output
```
(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3*a*b**2*x**
2 + b**3*x**3)/(2*b**4*(a + b*x))
```

3.131 $\int \frac{x^2}{(a+bx)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{x^2}{(a+bx)^2} dx = \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}$$

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx)^2} dx = \frac{bx - \frac{a^2}{a+bx} - 2a\log(a+bx)}{b^3}$$

input
```
Integrate[x^2/(a + b*x)^2,x]
```

output
```
(b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}+\frac{1}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

input
```
Int[x^2/(a + b*x)^2,x]
```

output
```
x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{b^{2}}-\frac{a^{2}}{b^{3} \left(b x +a \right)}-\frac{2 a \ln \left(b x +a \right)}{b^{3}}$	34
risch	$\frac{x}{b^{2}}-\frac{a^{2}}{b^{3} \left(b x +a \right)}-\frac{2 a \ln \left(b x +a \right)}{b^{3}}$	34
norman	$\frac{\frac{x^{2}}{b}-\frac{2 a^{2}}{b^{3}}}{b x +a}-\frac{2 a \ln \left(b x +a \right)}{b^{3}}$	38
parallelrisch	$-\frac{2 \ln \left(b x +a \right) x a b -b^{2} x^{2}+2 a^{2} \ln \left(b x +a \right)+2 a^{2}}{b^{3} \left(b x +a \right)}$	49

input
```
int(x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log\left(bx + a\right)}{b^4x + ab^3}$$

input
```
integrate(x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+bx)^2}\,dx = -\frac{a^2}{ab^3+b^4x} - \frac{2a\log{(a+bx)}}{b^3} + \frac{x}{b^2}$$

input
```
integrate(x**2/(b*x+a)**2,x)
```

output
```
-a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx)^2}\,dx = -\frac{a^2}{b^4x+ab^3} + \frac{x}{b^2} - \frac{2\,a\log{(bx+a)}}{b^3}$$

input
```
integrate(x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{2\,a\log{\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}}{b^3} + \frac{bx+a}{b^3} - \frac{a^2}{(bx+a)b^3}$$

input
```
integrate(x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/((b*x
 + a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln{(a+b\,x)}}{b^3}$$

input
```
int(x^2/(a + b*x)^2,x)
```

output
```
x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{(a+bx)^2}\,dx = \frac{-2\log(bx+a)\,a^2 - 2\log(bx+a)\,abx + 2abx + b^2x^2}{b^3\,(bx+a)}$$

input
```
int(x^2/(b*x+a)^2,x)
```

output
```
( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x**2)/(b**
3*(a + b*x))
```

3.132 $\int \frac{x}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \frac{x}{(a+bx)^2}\,dx=\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(a+bx)^2}\,dx=\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

input
```
Integrate[x/(a + b*x)^2,x]
```

output
```
(a/(a + b*x) + Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{1}{b(a+bx)}-\frac{a}{b(a+bx)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

input
```
Int[x/(a + b*x)^2,x]
```

output
```
a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
parallelrisch	$\frac{b\ln(bx+a)x+a\ln(bx+a)+a}{b^2(bx+a)}$	31

input
```
int(x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

input
```
integrate(x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{a}{ab^2+b^3x}+\frac{\log(a+bx)}{b^2}$$

input
```
integrate(x/(b*x+a)**2,x)
```

output
```
a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{a}{b^3x+ab^2} + \frac{\log(bx+a)}{b^2}$$

input
```
integrate(x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{x}{(a+bx)^2}\,dx = -\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx+a)b}}{b}$$

input
```
integrate(x/(b*x+a)^2,x, algorithm="giac")
```

output
```
-(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{\ln(a+b\,x)}{b^2} + \frac{a}{b^2\,(a+b\,x)}$$

input
```
int(x/(a + b*x)^2,x)
```

output
```
log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{x}{(a+bx)^2} \, dx = \frac{\log(bx+a)\, a+\log(bx+a)\, bx-bx}{b^2\, (bx+a)}$$

input
```
int(x/(b*x+a)^2,x)
```

output
```
(log(a + b*x)*a + log(a + b*x)*b*x - b*x)/(b**2*(a + b*x))
```

3.133 $\int \frac{1}{(a+bx)^2} dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

output
```
-1/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

input
```
Integrate[(a + b*x)^(-2),x]
```

output
```
-(1/(b*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{b(a+bx)}$$

input
```
Int[(a + b*x)^(-2),x]
```

output
```
-(1/(b*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13
orering	$-\frac{1}{b(bx+a)}$	13

input
```
int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b^2x+ab}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{ab+b^2x}$$

input
```
integrate(1/(b*x+a)**2,x)
```

output
```
-1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/((b*x + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b\,(a+b\,x)}$$

input
```
int(1/(a + b*x)^2,x)
```

output
```
-1/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = \frac{x}{a\,(bx+a)}$$

input
```
int(1/(b*x+a)^2,x)
```

output
```
x/(a*(a + b*x))
```

3.134 $\int \frac{1}{x(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{1}{x(a+bx)^2}\,dx=\frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}$$

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx)^2}\,dx=\frac{\frac{a}{a+bx}+\log(x)-\log(a+bx)}{a^2}$$

input
```
Integrate[1/(x*(a + b*x)^2),x]
```

output
```
(a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^2} \, dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(-\frac{b}{a^2(a+bx)}+\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

input
```
Int[1/(x*(a + b*x)^2),x]
```

output
```
1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}-\frac{\ln(bx+a)}{a^2}+\frac{\ln(-x)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33
parallelrisch	$\frac{b\ln(x)x-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)-bx}{a^2(bx+a)}$	45

input
```
int(1/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{x(a+bx)^2}\,dx = -\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

input
```
integrate(1/x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(a+bx)^2}\,dx = \frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

input
```
integrate(1/x/(b*x+a)**2,x)
```

output
```
1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{x(a+bx)^2}\,dx = \frac{1}{abx+a^2} - \frac{\log(bx+a)}{a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(1/x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{1}{x(a+bx)^2}\,dx = b\left(\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2b} + \frac{1}{(bx+a)ab}\right)$$

input
```
integrate(1/x/(b*x+a)^2,x, algorithm="giac")
```

output
```
b*(log(abs(-a/(b*x + a) + 1))/(a^2*b) + 1/((b*x + a)*a*b))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+bx)^2}\,dx = \frac{1}{a^2+b\,x\,a} - \frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}$$

input
```
int(1/(x*(a + b*x)^2),x)
```

output
```
1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{1}{x(a+bx)^2} \, dx = \frac{-\log(bx+a)\, a - \log(bx+a)\, bx + \log(x)\, a + \log(x)\, bx - bx}{a^2\, (bx+a)}$$

input
```
int(1/x/(b*x+a)^2,x)
```

output
```
( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b*x)/(a**2
*(a + b*x))
```

3.135 $\int \frac{1}{x^2(a+bx)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{x^2(a+bx)^2}\, dx = -\frac{1}{a^2x} - \frac{b}{a^2(a+bx)} - \frac{2b\log(x)}{a^3} + \frac{2b\log(a+bx)}{a^3}$$

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(a+bx)^2}\, dx = -\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

input
```
Integrate[1/(x^2*(a + b*x)^2),x]
```

output
```
-((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^2} \, dx$$
$$\Big\downarrow \; 54$$
$$\int \left(\frac{2b^2}{a^3(a+bx)}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{1}{a^2x^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

input
```
Int[1/(x^2*(a + b*x)^2),x]
```

output
```
-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a
^3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{a^{2}x}-\frac{b}{a^{2}(bx+a)}-\frac{2b\ln(x)}{a^{3}}+\frac{2b\ln(bx+a)}{a^{3}}$	43
risch	$\frac{-\frac{2bx}{a^{2}}-\frac{1}{a}}{x(bx+a)}+\frac{2b\ln(-bx-a)}{a^{3}}-\frac{2b\ln(x)}{a^{3}}$	49
norman	$\frac{\frac{2b^{2}x^{2}}{a^{3}}-\frac{1}{a}}{x(bx+a)}-\frac{2b\ln(x)}{a^{3}}+\frac{2b\ln(bx+a)}{a^{3}}$	50
parallelrisch	$-\frac{2b^{2}\ln(x)x^{2}-2b^{2}\ln(bx+a)x^{2}+2ab\ln(x)x-2\ln(bx+a)xab-2b^{2}x^{2}+a^{2}}{a^{3}x(bx+a)}$	70

input
```
int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^2(a+bx)^2}\,dx=-\frac{2\,abx+a^2-2\,(b^2x^2+abx)\log(bx+a)+2\,(b^2x^2+abx)\log(x)}{a^3bx^2+a^4x}$$

input
```
integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*l
og(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(a+bx)^2}\,dx = \frac{-a-2bx}{a^3x+a^2bx^2} + \frac{2b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

input
```
integrate(1/x**2/(b*x+a)**2,x)
```

output
```
(-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)^2}\,dx = -\frac{2\,bx+a}{a^2bx^2+a^3x} + \frac{2\,b\log(bx+a)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

input
```
integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(a+bx)^2}\,dx = -\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3} - \frac{b}{(bx+a)a^2} + \frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

input
```
integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
 a) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)^2}\,dx = \frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3} - \frac{1}{a\,x\,(a+b\,x)} - \frac{2\,b}{a^2\,(a+b\,x)}$$

input
```
int(1/(x^2*(a + b*x)^2),x)
```

output
```
(2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^2(a+bx)^2}\,dx = \frac{2\log(bx+a)\,abx + 2\log(bx+a)\,b^2x^2 - 2\log(x)\,abx - 2\log(x)\,b^2x^2 - a^2 + 2b^2x^2}{a^3x\,(bx+a)}$$

input
```
int(1/x^2/(b*x+a)^2,x)
```

output
```
(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x - 2*log(
x)*b**2*x**2 - a**2 + 2*b**2*x**2)/(a**3*x*(a + b*x))
```

3.136 $\int \frac{1}{x^3(a+bx)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{1}{x^3(a+bx)^2} dx = -\frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{b^2}{a^3(a+bx)} + \frac{3b^2\log(x)}{a^4} - \frac{3b^2\log(a+bx)}{a^4}$$

output
```
-1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3(a+bx)^2} dx = \frac{a\left(-\frac{a}{x^2} + \frac{4b}{x} + \frac{2b^2}{a+bx}\right) + 6b^2\log(x) - 6b^2\log(a+bx)}{2a^4}$$

input
```
Integrate[1/(x^3*(a + b*x)^2),x]
```

output
```
(a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
 b*x])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{3b^3}{a^4(a+bx)}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

input
```
Int[1/(x^3*(a + b*x)^2),x]
```

output
```
-1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^
4 - (3*b^2*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	57
norman	$\frac{-\frac{3b^3x^3}{a^4}-\frac{1}{2a}+\frac{3bx}{2a^2}}{x^2(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	61
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)}+\frac{3b^2\ln(-x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	63
parallelrisch	$\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2\ln(x)x^2-6\ln(bx+a)x^2a\,b^2-6b^3x^3+3a^2bx-a^3}{2a^4x^2(bx+a)}$	87

input
```
int(1/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{6\,ab^2x^2+3\,a^2bx-a^3-6\,(b^3x^3+ab^2x^2)\log(bx+a)+6\,(b^3x^3+ab^2x^2)\log(x)}{2\,(a^4bx^3+a^5x^2)}$$

input
```
integrate(1/x^3/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a)
+ 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{-a^2+3abx+6b^2x^2}{2a^4x^2+2a^3bx^3} + \frac{3b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

input
```
integrate(1/x**3/(b*x+a)**2,x)
```

output
```
(-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(lo
g(x) - log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{6\,b^2x^2+3\,abx-a^2}{2\,(a^3bx^3+a^4x^2)} - \frac{3\,b^2\log\left(bx+a\right)}{a^4} + \frac{3\,b^2\log\left(x\right)}{a^4}$$

input
```
integrate(1/x^3/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)
/a^4 + 3*b^2*log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{3\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4} + \frac{b^2}{(bx+a)a^3} - \frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{2\,a^4\left(\frac{a}{bx+a}-1\right)^2}$$

input
```
integrate(1/x^3/(b*x+a)^2,x, algorithm="giac")
```

output
```
3*b^2*log(abs(-a/(b*x + a) + 1))/a^4 + b^2/((b*x + a)*a^3) - 1/2*(6*a*b^2/
(b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{\frac{3\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^2}}{b\,x^3+a\,x^2}-\frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

input
```
int(1/(x^3*(a + b*x)^2),x)
```

output
```
((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*ata
nh((2*b*x)/a + 1))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3(a+bx)^2}\,dx = \frac{-6\log(bx+a)\,a\,b^2x^2-6\log(bx+a)\,b^3x^3+6\log(x)\,a\,b^2x^2+6\log(x)\,b^3x^3-a^3+3a^2bx-6b^3x^3}{2a^4x^2\,(bx+a)}$$

input
```
int(1/x^3/(b*x+a)^2,x)
```

output
```
( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log(x)*a*b**
2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3)/(2*a**4*x**
2*(a + b*x))
```

3.137 $\int \frac{1}{x^4(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 69

$$\int \frac{1}{x^4(a+bx)^2}\,dx = -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}$$

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4(a+bx)^2}\,dx = -\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)}+12b^3\log(x)-12b^3\log(a+bx)}{3a^5}$$

input
```
Integrate[1/(x^4*(a + b*x)^2),x]
```

output
```
-1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 1
2*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+bx)^2} \, dx$$

$$\Big\downarrow \ 54$$

$$\int \left(\frac{4b^4}{a^5(a+bx)} - \frac{4b^3}{a^5x} + \frac{b^4}{a^4(a+bx)^2} + \frac{3b^2}{a^4x^2} - \frac{2b}{a^3x^3} + \frac{1}{a^2x^4}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4b^3\log(x)}{a^5} + \frac{4b^3\log(a+bx)}{a^5} - \frac{b^3}{a^4(a+bx)} - \frac{3b^2}{a^4x} + \frac{b}{a^3x^2} - \frac{1}{3a^2x^3}$$

input
```
Int[1/(x^4*(a + b*x)^2),x]
```

output
```
-1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (
4*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	68
norman	$\frac{\frac{4b^4x^4}{a^5}-\frac{1}{3a}+\frac{2bx}{3a^2}-\frac{2b^2x^2}{a^3}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	72
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{2b^2x^2}{a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(-bx-a)}{a^5}$	75
parallelrisch	$-\frac{12b^4\ln(x)x^4-12b^4\ln(bx+a)x^4+12\ln(x)x^3a\,b^3-12\ln(bx+a)x^3a\,b^3-12b^4x^4+6a^2b^2x^2-2a^3bx+a^4}{3a^5x^3(bx+a)}$	96

input
```
int(1/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^4(a+bx)^2}\,dx = -\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\,(b^4x^4+ab^3x^3)\log(bx+a)+12\,(b^4x^4+ab^3x^3)\log(x)}{3\,(a^5bx^4+a^6x^3)}$$

input
```
integrate(1/x^4/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3
*x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3
)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4(a+bx)^2}\,dx = \frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4}+\frac{4b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

input
```
integrate(1/x**4/(b*x+a)**2,x)
```

output
```
(-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*
b*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4(a+bx)^2}\,dx = -\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\,(a^4bx^4+a^5x^3)}+\frac{4\,b^3\log\left(bx+a\right)}{a^5}-\frac{4\,b^3\log\left(x\right)}{a^5}$$

input
```
integrate(1/x^4/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) +
4*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^4(a+bx)^2}\,dx = -\frac{4\,b^3\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^5}-\frac{b^3}{(bx+a)a^4}-\frac{\frac{30\,ab^3}{bx+a}-\frac{18\,a^2b^3}{(bx+a)^2}-13\,b^3}{3\,a^5\left(\frac{a}{bx+a}-1\right)^3}$$

input
```
integrate(1/x^4/(b*x+a)^2,x, algorithm="giac")
```

output
```
-4*b^3*log(abs(-a/(b*x + a) + 1))/a^5 - b^3/((b*x + a)*a^4) - 1/3*(30*a*b^
3/(b*x + a) - 18*a^2*b^3/(b*x + a)^2 - 13*b^3)/(a^5*(a/(b*x + a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a+bx)^2}\,dx = \frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

input
```
int(1/(x^4*(a + b*x)^2),x)
```

output
```
(8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3
)/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^4(a+bx)^2}\,dx = \frac{12\log(bx+a)\,a\,b^3x^3+12\log(bx+a)\,b^4x^4-12\log(x)\,a\,b^3x^3-12\log(x)\,b^4x^4-a^4+2a^3bx-6a^2b^2x^2+}{3a^5x^3\,(bx+a)}$$

input
```
int(1/x^4/(b*x+a)^2,x)
```

output
```
(12*log(a + b*x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 - 12*log(x)*a*b**
3*x**3 - 12*log(x)*b**4*x**4 - a**4 + 2*a**3*b*x - 6*a**2*b**2*x**2 + 12*b
**4*x**4)/(3*a**5*x**3*(a + b*x))
```

3.138 $\int \frac{1}{x^5(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int \frac{1}{x^5(a+bx)^2}\,dx = -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}$$

output
```
-1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b
^4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5(a+bx)^2}\,dx = \frac{\frac{a(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4)}{x^4(a+bx)}+60b^4\log(x)-60b^4\log(a+bx)}{12a^6}$$

input
```
Integrate[1/(x^5*(a + b*x)^2),x]
```

output
```
((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^
4*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(a+bx)^2} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{5b^5}{a^6(a+bx)}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{4b^3}{a^5x^2}+\frac{3b^2}{a^4x^3}-\frac{2b}{a^3x^4}+\frac{1}{a^2x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

input
```
Int[1/(x^5*(a + b*x)^2),x]
```

output
```
-1/4*1/(a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*
x) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{4a^{2}x^{4}}+\frac{2b}{3a^{3}x^{3}}-\frac{3b^{2}}{2a^{4}x^{2}}+\frac{4b^{3}}{a^{5}x}+\frac{b^{4}}{a^{5}(bx+a)}+\frac{5b^{4}\ln(x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	79
norman	$\frac{-\frac{5b^{5}x^{5}}{a^{6}}-\frac{1}{4a}+\frac{5bx}{12a^{2}}-\frac{5b^{2}x^{2}}{6a^{3}}+\frac{5b^{3}x^{3}}{2a^{4}}}{x^{4}(bx+a)}+\frac{5b^{4}\ln(x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	83
risch	$\frac{\frac{5b^{4}x^{4}}{a^{5}}+\frac{5b^{3}x^{3}}{2a^{4}}-\frac{5b^{2}x^{2}}{6a^{3}}+\frac{5bx}{12a^{2}}-\frac{1}{4a}}{x^{4}(bx+a)}+\frac{5b^{4}\ln(-x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	85
parallelrisch	$\frac{60b^{5}\ln(x)x^{5}-60\ln(bx+a)x^{5}b^{5}+60a\,b^{4}\ln(x)x^{4}-60\ln(bx+a)x^{4}a\,b^{4}-60b^{5}x^{5}+30a^{2}b^{3}x^{3}-10a^{3}b^{2}x^{2}+5a^{4}bx-3a^{5}}{12a^{6}x^{4}(bx+a)}$	109

input
```
int(1/x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b
^4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^5(a+bx)^2}\,dx$$
$$=\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5-60\,(b^5x^5+ab^4x^4)\log{(bx+a)}+60\,(b^5x^5+ab^4x^4)\log}{12\,(a^6bx^5+a^7x^4)}$$

input
```
integrate(1/x^5/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
 60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/
(a^6*b*x^5 + a^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5(a+bx)^2}\,dx = \frac{-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4}{12a^6x^4+12a^5bx^5} + \frac{5b^4\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^6}$$

input
```
integrate(1/x**5/(b*x+a)**2,x)
```

output
```
(-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)
/(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5(a+bx)^2}\,dx = \frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\,(a^5bx^5+a^6x^4)} - \frac{5\,b^4\log{(bx+a)}}{a^6} + \frac{5\,b^4\log{(x)}}{a^6}$$

input
```
integrate(1/x^5/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5
*b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^5(a+bx)^2}\,dx$$
$$= \frac{5\,b^4\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^6} + \frac{b^4}{(bx+a)a^5} - \frac{\frac{260\,ab^4}{bx+a} - \frac{300\,a^2b^4}{(bx+a)^2} + \frac{120\,a^3b^4}{(bx+a)^3} - 77\,b^4}{12\,a^6\left(\frac{a}{bx+a}-1\right)^4}$$

input
```
integrate(1/x^5/(b*x+a)^2,x, algorithm="giac")
```

output
```
5*b^4*log(abs(-a/(b*x + a) + 1))/a^6 + b^4/((b*x + a)*a^5) - 1/12*(260*a*b
^4/(b*x + a) - 300*a^2*b^4/(b*x + a)^2 + 120*a^3*b^4/(b*x + a)^3 - 77*b^4)
/(a^6*(a/(b*x + a) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5(a+bx)^2}\,dx = \frac{\frac{5\,b^3\,x^3}{2\,a^4} - \frac{5\,b^2\,x^2}{6\,a^3} - \frac{1}{4\,a} + \frac{5\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{12\,a^2}}{b\,x^5 + a\,x^4} - \frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

input
```
int(1/(x^5*(a + b*x)^2),x)
```

output
```
((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (
5*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^5(a+bx)^2}\,dx$$
$$= \frac{-60\log(bx+a)\,a\,b^4x^4 - 60\log(bx+a)\,b^5x^5 + 60\log(x)\,a\,b^4x^4 + 60\log(x)\,b^5x^5 - 3a^5 + 5a^4bx - 10a^3b^2}{12a^6x^4\,(bx+a)}$$

input
```
int(1/x^5/(b*x+a)^2,x)
```

output
```
( - 60*log(a + b*x)*a*b**4*x**4 - 60*log(a + b*x)*b**5*x**5 + 60*log(x)*a*
b**4*x**4 + 60*log(x)*b**5*x**5 - 3*a**5 + 5*a**4*b*x - 10*a**3*b**2*x**2
+ 30*a**2*b**3*x**3 - 60*b**5*x**5)/(12*a**6*x**4*(a + b*x))
```

3.139 $\int \frac{x^7}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 99

$$\int \frac{x^7}{(a+bx)^3} \, dx = \frac{15a^4x}{b^7} - \frac{5a^3x^2}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^5}{5b^3} + \frac{a^7}{2b^8(a+bx)^2} - \frac{7a^6}{b^8(a+bx)} - \frac{21a^5\log(a+bx)}{b^8}$$

output
```
15*a^4*x/b^7-5*a^3*x^2/b^6+2*a^2*x^3/b^5-3/4*a*x^4/b^4+1/5*x^5/b^3+1/2*a^7
/b^8/(b*x+a)^2-7*a^6/b^8/(b*x+a)-21*a^5*ln(b*x+a)/b^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \frac{x^7}{(a+bx)^3} \, dx = \frac{300a^4bx - 100a^3b^2x^2 + 40a^2b^3x^3 - 15ab^4x^4 + 4b^5x^5 + \frac{10a^7}{(a+bx)^2} - \frac{140a^6}{a+bx} - 420a^5\log(a+bx)}{20b^8}$$

input
```
Integrate[x^7/(a + b*x)^3,x]
```

output
```
(300*a^4*b*x - 100*a^3*b^2*x^2 + 40*a^2*b^3*x^3 - 15*a*b^4*x^4 + 4*b^5*x^5
 + (10*a^7)/(a + b*x)^2 - (140*a^6)/(a + b*x) - 420*a^5*Log[a + b*x])/(20*
b^8)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}+\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}+\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

input
```
Int[x^7/(a + b*x)^3,x]
```

output
```
(15*a^4*x)/b^7 - (5*a^3*x^2)/b^6 + (2*a^2*x^3)/b^5 - (3*a*x^4)/(4*b^4) + x
^5/(5*b^3) + a^7/(2*b^8*(a + b*x)^2) - (7*a^6)/(b^8*(a + b*x)) - (21*a^5*L
og[a + b*x])/b^8
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x^{5}}{5 b^{3}}-\frac{3 a \,x^{4}}{4 b^{4}}+\frac{2 a^{2} x^{3}}{b^{5}}-\frac{5 a^{3} x^{2}}{b^{6}}+\frac{15 a^{4} x}{b^{7}}+\frac{-7 a^{6} x -\frac{13 a^{7}}{2 b}}{b^{7} \left(b x +a \right)^{2}}-\frac{21 a^{5} \ln \left(b x +a \right)}{b^{8}}$
norman	$\frac{\frac{x^{7}}{5 b}-\frac{7 a \,x^{6}}{20 b^{2}}+\frac{7 a^{2} x^{5}}{10 b^{3}}-\frac{63 a^{7}}{2 b^{8}}-\frac{7 a^{3} x^{4}}{4 b^{4}}+\frac{7 a^{4} x^{3}}{b^{5}}-\frac{42 a^{6} x}{b^{7}}}{\left(b x +a \right)^{2}}-\frac{21 a^{5} \ln \left(b x +a \right)}{b^{8}}$
default	$\frac{\frac{1}{5} b^{4} x^{5}-\frac{3}{4} a \,b^{3} x^{4}+2 a^{2} b^{2} x^{3}-5 a^{3} b \,x^{2}+15 a^{4} x}{b^{7}}+\frac{a^{7}}{2 b^{8} \left(b x +a \right)^{2}}-\frac{7 a^{6}}{b^{8} \left(b x +a \right)}-\frac{21 a^{5} \ln \left(b x +a \right)}{b^{8}}$
parallelrisch	$-\frac{-4 b^{7} x^{7}+7 a \,b^{6} x^{6}-14 a^{2} b^{5} x^{5}+35 a^{3} b^{4} x^{4}+420 \ln \left(b x +a \right) x^{2} a^{5} b^{2}-140 a^{4} b^{3} x^{3}+840 \ln \left(b x +a \right) x \,a^{6} b +420 \ln \left(b x +a \right) a^{7}+840 a^{6} b x}{20 b^{8} \left(b x +a \right)^{2}}$

input
```
int(x^7/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5/b^3-3/4*a*x^4/b^4+2*a^2*x^3/b^5-5*a^3*x^2/b^6+15*a^4*x/b^7+(-7*a^6
*x-13/2*a^7/b)/b^7/(b*x+a)^2-21*a^5*ln(b*x+a)/b^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \frac{x^7}{(a+bx)^3}\,dx = \frac{4\,b^7x^7 - 7\,ab^6x^6 + 14\,a^2b^5x^5 - 35\,a^3b^4x^4 + 140\,a^4b^3x^3 + 500\,a^5b^2x^2 + 160\,a^6bx - 130\,a^7 - 420\,(a^5b^2x^2 +}{20\,(b^{10}x^2 + 2\,ab^9x + a^2b^8)}$$

input
```
integrate(x^7/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/20*(4*b^7*x^7 - 7*a*b^6*x^6 + 14*a^2*b^5*x^5 - 35*a^3*b^4*x^4 + 140*a^4*
b^3*x^3 + 500*a^5*b^2*x^2 + 160*a^6*b*x - 130*a^7 - 420*(a^5*b^2*x^2 + 2*a
^6*b*x + a^7)*log(b*x + a))/(b^10*x^2 + 2*a*b^9*x + a^2*b^8)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \frac{x^7}{(a+bx)^3}\,dx = -\frac{21a^5\log{(a+bx)}}{b^8} + \frac{15a^4x}{b^7} - \frac{5a^3x^2}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^4}{4b^4} + \frac{-13a^7 - 14a^6bx}{2a^2b^8 + 4ab^9x + 2b^{10}x^2} + \frac{x^5}{5b^3}$$

input
```
integrate(x**7/(b*x+a)**3,x)
```

output
```
-21*a**5*log(a + b*x)/b**8 + 15*a**4*x/b**7 - 5*a**3*x**2/b**6 + 2*a**2*x*
*3/b**5 - 3*a*x**4/(4*b**4) + (-13*a**7 - 14*a**6*b*x)/(2*a**2*b**8 + 4*a*
b**9*x + 2*b**10*x**2) + x**5/(5*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{(a+bx)^3}\,dx = -\frac{14\,a^6bx + 13\,a^7}{2\,(b^{10}x^2 + 2\,ab^9x + a^2b^8)} - \frac{21\,a^5\log{(bx+a)}}{b^8} + \frac{4\,b^4x^5 - 15\,ab^3x^4 + 40\,a^2b^2x^3 - 100\,a^3bx^2 + 300\,a^4x}{20\,b^7}$$

input
```
integrate(x^7/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(14*a^6*b*x + 13*a^7)/(b^10*x^2 + 2*a*b^9*x + a^2*b^8) - 21*a^5*log(b
*x + a)/b^8 + 1/20*(4*b^4*x^5 - 15*a*b^3*x^4 + 40*a^2*b^2*x^3 - 100*a^3*b*
x^2 + 300*a^4*x)/b^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx)^3}\,dx = -\frac{21\,a^5\log\left(|bx+a|\right)}{b^8} - \frac{14\,a^6bx+13\,a^7}{2\,(bx+a)^2b^8} + \frac{4\,b^{12}x^5-15\,ab^{11}x^4+40\,a^2b^{10}x^3-100\,a^3b^9x^2+300\,a^4b^8x}{20\,b^{15}}$$

input
```
integrate(x^7/(b*x+a)^3,x, algorithm="giac")
```

output
```
-21*a^5*log(abs(b*x + a))/b^8 - 1/2*(14*a^6*b*x + 13*a^7)/((b*x + a)^2*b^8
) + 1/20*(4*b^12*x^5 - 15*a*b^11*x^4 + 40*a^2*b^10*x^3 - 100*a^3*b^9*x^2 +
 300*a^4*b^8*x)/b^15
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a+bx)^3}\,dx = -\frac{\frac{7\,a\,(a+b\,x)^4}{4}-\frac{(a+b\,x)^5}{5}-7\,a^2\,(a+b\,x)^3+\frac{35\,a^3\,(a+b\,x)^2}{2}+\frac{7\,a^6}{a+b\,x}-\frac{a^7}{2\,(a+b\,x)^2}+21\,a^5\,\ln\left(a+b\,x\right)-35\,a^4\,b\,x}{b^8}$$

input
```
int(x^7/(a + b*x)^3,x)
```

output
```
-((7*a*(a + b*x)^4)/4 - (a + b*x)^5/5 - 7*a^2*(a + b*x)^3 + (35*a^3*(a + b
*x)^2)/2 + (7*a^6)/(a + b*x) - a^7/(2*(a + b*x)^2) + 21*a^5*log(a + b*x) -
 35*a^4*b*x)/b^8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

$$\int \frac{x^7}{(a+bx)^3}\,dx$$
$$= \frac{-420\log(bx+a)\,a^7 - 840\log(bx+a)\,a^6bx - 420\log(bx+a)\,a^5b^2x^2 - 210a^7 + 420a^5b^2x^2 + 140a^4b^3x^3 \cdots}{20b^8\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^7/(b*x+a)^3,x)
```

output
```
( - 420*log(a + b*x)*a**7 - 840*log(a + b*x)*a**6*b*x - 420*log(a + b*x)*a
**5*b**2*x**2 - 210*a**7 + 420*a**5*b**2*x**2 + 140*a**4*b**3*x**3 - 35*a*
*3*b**4*x**4 + 14*a**2*b**5*x**5 - 7*a*b**6*x**6 + 4*b**7*x**7)/(20*b**8*(
a**2 + 2*a*b*x + b**2*x**2))
```

3.140 $\int \frac{x^6}{(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 86

$$\int \frac{x^6}{(a+bx)^3} dx = -\frac{10a^3x}{b^6} + \frac{3a^2x^2}{b^5} - \frac{ax^3}{b^4} + \frac{x^4}{4b^3} - \frac{a^6}{2b^7(a+bx)^2} + \frac{6a^5}{b^7(a+bx)} + \frac{15a^4\log(a+bx)}{b^7}$$

output
```
-10*a^3*x/b^6+3*a^2*x^2/b^5-a*x^3/b^4+1/4*x^4/b^3-1/2*a^6/b^7/(b*x+a)^2+6*
a^5/b^7/(b*x+a)+15*a^4*ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{x^6}{(a+bx)^3} dx = \frac{-40a^3bx + 12a^2b^2x^2 - 4ab^3x^3 + b^4x^4 - \frac{2a^6}{(a+bx)^2} + \frac{24a^5}{a+bx} + 60a^4\log(a+bx)}{4b^7}$$

input
```
Integrate[x^6/(a + b*x)^3,x]
```

output
```
(-40*a^3*b*x + 12*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 - (2*a^6)/(a + b*x)^
2 + (24*a^5)/(a + b*x) + 60*a^4*Log[a + b*x])/(4*b^7)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}$$

input
```
Int[x^6/(a + b*x)^3,x]
```

output
```
(-10*a^3*x)/b^6 + (3*a^2*x^2)/b^5 - (a*x^3)/b^4 + x^4/(4*b^3) - a^6/(2*b^7
*(a + b*x)^2) + (6*a^5)/(b^7*(a + b*x)) + (15*a^4*Log[a + b*x])/b^7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{x^4}{4b^3}-\frac{a\,x^3}{b^4}+\frac{3a^2x^2}{b^5}-\frac{10a^3x}{b^6}+\frac{6a^5x+\frac{11a^6}{2b}}{b^6(bx+a)^2}+\frac{15a^4\ln(bx+a)}{b^7}$	79
norman	$\frac{\frac{x^6}{4b}-\frac{a\,x^5}{2b^2}+\frac{5a^2x^4}{4b^3}+\frac{45a^6}{2b^7}-\frac{5a^3x^3}{b^4}+\frac{30a^5x}{b^6}}{(bx+a)^2}+\frac{15a^4\ln(bx+a)}{b^7}$	81
default	$-\frac{-\frac{1}{4}b^3x^4+a\,b^2x^3-3a^2b\,x^2+10a^3x}{b^6}-\frac{a^6}{2b^7(bx+a)^2}+\frac{6a^5}{b^7(bx+a)}+\frac{15a^4\ln(bx+a)}{b^7}$	83
parallelrisch	$\frac{b^6x^6-2a\,x^5b^5+5a^2x^4b^4+60\ln(bx+a)x^2a^4b^2-20a^3x^3b^3+120\ln(bx+a)x\,a^5b+60\ln(bx+a)a^6+120a^5xb+90a^6}{4b^7(bx+a)^2}$	105

input
```
int(x^6/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/b^3-a*x^3/b^4+3*a^2*x^2/b^5-10*a^3*x/b^6+(6*a^5*x+11/2/b*a^6)/b^6/
(b*x+a)^2+15*a^4*ln(b*x+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int\frac{x^6}{(a+bx)^3}\,dx$$
$$=\frac{b^6x^6-2\,ab^5x^5+5\,a^2b^4x^4-20\,a^3b^3x^3-68\,a^4b^2x^2-16\,a^5bx+22\,a^6+60\,(a^4b^2x^2+2\,a^5bx+a^6)\log\,(bx}{4\,(b^9x^2+2\,ab^8x+a^2b^7)}$$

input
```
integrate(x^6/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(b^6*x^6 - 2*a*b^5*x^5 + 5*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 68*a^4*b^2*x
^2 - 16*a^5*b*x + 22*a^6 + 60*(a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a)
)/(b^9*x^2 + 2*a*b^8*x + a^2*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{x^6}{(a+bx)^3}\,dx = \frac{15a^4\log{(a+bx)}}{b^7} - \frac{10a^3x}{b^6} + \frac{3a^2x^2}{b^5} - \frac{ax^3}{b^4} + \frac{11a^6+12a^5bx}{2a^2b^7+4ab^8x+2b^9x^2} + \frac{x^4}{4b^3}$$

input
```
integrate(x**6/(b*x+a)**3,x)
```

output
```
15*a**4*log(a + b*x)/b**7 - 10*a**3*x/b**6 + 3*a**2*x**2/b**5 - a*x**3/b**
4 + (11*a**6 + 12*a**5*b*x)/(2*a**2*b**7 + 4*a*b**8*x + 2*b**9*x**2) + x**
4/(4*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{x^6}{(a+bx)^3}\,dx = \frac{12\,a^5bx+11\,a^6}{2\,(b^9x^2+2\,ab^8x+a^2b^7)} + \frac{15\,a^4\log{(bx+a)}}{b^7} + \frac{b^3x^4-4\,ab^2x^3+12\,a^2bx^2-40\,a^3x}{4\,b^6}$$

input
```
integrate(x^6/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(12*a^5*b*x + 11*a^6)/(b^9*x^2 + 2*a*b^8*x + a^2*b^7) + 15*a^4*log(b*x
 + a)/b^7 + 1/4*(b^3*x^4 - 4*a*b^2*x^3 + 12*a^2*b*x^2 - 40*a^3*x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^6}{(a+bx)^3}\,dx = \frac{15\,a^4\log\left(|bx+a|\right)}{b^7} + \frac{12\,a^5bx+11\,a^6}{2\,(bx+a)^2b^7} + \frac{b^9x^4-4\,ab^8x^3+12\,a^2b^7x^2-40\,a^3b^6x}{4\,b^{12}}$$

input
```
integrate(x^6/(b*x+a)^3,x, algorithm="giac")
```

output
```
15*a^4*log(abs(b*x + a))/b^7 + 1/2*(12*a^5*b*x + 11*a^6)/((b*x + a)^2*b^7)
 + 1/4*(b^9*x^4 - 4*a*b^8*x^3 + 12*a^2*b^7*x^2 - 40*a^3*b^6*x)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{(a+bx)^3}\,dx = \frac{\frac{(a+b\,x)^4}{4}-2\,a\,(a+b\,x)^3+\frac{15\,a^2\,(a+b\,x)^2}{2}+\frac{6\,a^5}{a+b\,x}-\frac{a^6}{2\,(a+b\,x)^2}+15\,a^4\,\ln\left(a+b\,x\right)-20\,a^3\,b\,x}{b^7}$$

input
```
int(x^6/(a + b*x)^3,x)
```

output
```
((a + b*x)^4/4 - 2*a*(a + b*x)^3 + (15*a^2*(a + b*x)^2)/2 + (6*a^5)/(a + b
*x) - a^6/(2*(a + b*x)^2) + 15*a^4*log(a + b*x) - 20*a^3*b*x)/b^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{x^6}{(a+bx)^3}\,dx = \frac{60\log(bx+a)\,a^6+120\log(bx+a)\,a^5bx+60\log(bx+a)\,a^4b^2x^2+30a^6-60a^4b^2x^2-20a^3b^3x^3+5a^2b^4x}{4b^7\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^6/(b*x+a)^3,x)
```

output
```
(60*log(a + b*x)*a**6 + 120*log(a + b*x)*a**5*b*x + 60*log(a + b*x)*a**4*b
**2*x**2 + 30*a**6 - 60*a**4*b**2*x**2 - 20*a**3*b**3*x**3 + 5*a**2*b**4*x
**4 - 2*a*b**5*x**5 + b**6*x**6)/(4*b**7*(a**2 + 2*a*b*x + b**2*x**2))
```

3.141 $\int \frac{x^5}{(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 77

$$\int \frac{x^5}{(a+bx)^3} dx = \frac{6a^2x}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^3}{3b^3} + \frac{a^5}{2b^6(a+bx)^2} - \frac{5a^4}{b^6(a+bx)} - \frac{10a^3\log(a+bx)}{b^6}$$

output
```
6*a^2*x/b^5-3/2*a*x^2/b^4+1/3*x^3/b^3+1/2*a^5/b^6/(b*x+a)^2-5*a^4/b^6/(b*x
+a)-10*a^3*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(a+bx)^3} dx = \frac{36a^2bx - 9ab^2x^2 + 2b^3x^3 + \frac{3a^5}{(a+bx)^2} - \frac{30a^4}{a+bx} - 60a^3\log(a+bx)}{6b^6}$$

input
```
Integrate[x^5/(a + b*x)^3,x]
```

output
```
(36*a^2*b*x - 9*a*b^2*x^2 + 2*b^3*x^3 + (3*a^5)/(a + b*x)^2 - (30*a^4)/(a
+ b*x) - 60*a^3*Log[a + b*x])/(6*b^6)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}+\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}-\frac{10a^3\log(a+bx)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}$$

input
```
Int[x^5/(a + b*x)^3,x]
```

output
```
(6*a^2*x)/b^5 - (3*a*x^2)/(2*b^4) + x^3/(3*b^3) + a^5/(2*b^6*(a + b*x)^2)
- (5*a^4)/(b^6*(a + b*x)) - (10*a^3*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{x^3}{3b^3}-\frac{3a\,x^2}{2b^4}+\frac{6a^2x}{b^5}+\frac{-5a^4x-\frac{9a^5}{2b}}{b^5(bx+a)^2}-\frac{10a^3\ln(bx+a)}{b^6}$	68
norman	$\frac{\frac{x^5}{3b}-\frac{5a\,x^4}{6b^2}+\frac{10a^2x^3}{3b^3}-\frac{15a^5}{b^6}-\frac{20a^4x}{b^5}}{(bx+a)^2}-\frac{10a^3\ln(bx+a)}{b^6}$	70
default	$\frac{\frac{1}{3}b^2x^3-\frac{3}{2}ab\,x^2+6a^2x}{b^5}+\frac{a^5}{2b^6(bx+a)^2}-\frac{5a^4}{b^6(bx+a)}-\frac{10a^3\ln(bx+a)}{b^6}$	72
parallelrisch	$-\frac{-2b^5x^5+5a\,b^4x^4+60\ln(bx+a)x^2a^3b^2-20a^2b^3x^3+120\ln(bx+a)x\,a^4b+60a^5\ln(bx+a)+120a^4bx+90a^5}{6b^6(bx+a)^2}$	95

input
```
int(x^5/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/b^3-3/2*a*x^2/b^4+6*a^2*x/b^5+(-5*a^4*x-9/2*a^5/b)/b^5/(b*x+a)^2-1
0*a^3*ln(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \frac{x^5}{(a+bx)^3}\,dx = \frac{2\,b^5x^5-5\,ab^4x^4+20\,a^2b^3x^3+63\,a^3b^2x^2+6\,a^4bx-27\,a^5-60\,(a^3b^2x^2+2\,a^4bx+a^5)\log(bx+a)}{6\,(b^8x^2+2\,ab^7x+a^2b^6)}$$

input
```
integrate(x^5/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^5*x^5 - 5*a*b^4*x^4 + 20*a^2*b^3*x^3 + 63*a^3*b^2*x^2 + 6*a^4*b*x
 - 27*a^5 - 60*(a^3*b^2*x^2 + 2*a^4*b*x + a^5)*log(b*x + a))/(b^8*x^2 + 2*
a*b^7*x + a^2*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^5}{(a+bx)^3}\,dx = -\frac{10a^3\log(a+bx)}{b^6} + \frac{6a^2x}{b^5} - \frac{3ax^2}{2b^4} + \frac{-9a^5-10a^4bx}{2a^2b^6+4ab^7x+2b^8x^2} + \frac{x^3}{3b^3}$$

input
```
integrate(x**5/(b*x+a)**3,x)
```

output
```
-10*a**3*log(a + b*x)/b**6 + 6*a**2*x/b**5 - 3*a*x**2/(2*b**4) + (-9*a**5
- 10*a**4*b*x)/(2*a**2*b**6 + 4*a*b**7*x + 2*b**8*x**2) + x**3/(3*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(a+bx)^3}\,dx = -\frac{10\,a^4bx+9\,a^5}{2\,(b^8x^2+2\,ab^7x+a^2b^6)} - \frac{10\,a^3\log(bx+a)}{b^6} + \frac{2\,b^2x^3-9\,abx^2+36\,a^2x}{6\,b^5}$$

input
```
integrate(x^5/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(10*a^4*b*x + 9*a^5)/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) - 10*a^3*log(b*x
 + a)/b^6 + 1/6*(2*b^2*x^3 - 9*a*b*x^2 + 36*a^2*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{(a+bx)^3}\,dx = -\frac{10\,a^3\log(|bx+a|)}{b^6} - \frac{10\,a^4bx+9\,a^5}{2\,(bx+a)^2b^6} + \frac{2\,b^6x^3-9\,ab^5x^2+36\,a^2b^4x}{6\,b^9}$$

input
```
integrate(x^5/(b*x+a)^3,x, algorithm="giac")
```

output
```
-10*a^3*log(abs(b*x + a))/b^6 - 1/2*(10*a^4*b*x + 9*a^5)/((b*x + a)^2*b^6)
 + 1/6*(2*b^6*x^3 - 9*a*b^5*x^2 + 36*a^2*b^4*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(a+bx)^3}\,dx = -\frac{\frac{5\,a\,(a+b\,x)^2}{2}-\frac{(a+b\,x)^3}{3}+\frac{5\,a^4}{a+b\,x}-\frac{a^5}{2\,(a+b\,x)^2}+10\,a^3\,\ln\left(a+b\,x\right)-10\,a^2\,b\,x}{b^6}$$

input
```
int(x^5/(a + b*x)^3,x)
```

output
```
-((5*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (5*a^4)/(a + b*x) - a^5/(2*(a + b*
x)^2) + 10*a^3*log(a + b*x) - 10*a^2*b*x)/b^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

$$\int \frac{x^5}{(a+bx)^3}\,dx = \frac{-60\log(bx+a)\,a^5-120\log(bx+a)\,a^4bx-60\log(bx+a)\,a^3b^2x^2-30a^5+60a^3b^2x^2+20a^2b^3x^3-5a\,b}{6b^6\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(x^5/(b*x+a)^3,x)
```

output
```
( - 60*log(a + b*x)*a**5 - 120*log(a + b*x)*a**4*b*x - 60*log(a + b*x)*a**
3*b**2*x**2 - 30*a**5 + 60*a**3*b**2*x**2 + 20*a**2*b**3*x**3 - 5*a*b**4*x
**4 + 2*b**5*x**5)/(6*b**6*(a**2 + 2*a*b*x + b**2*x**2))
```

3.142 $\int \frac{x^4}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{x^4}{(a+bx)^3} \, dx = -\frac{3ax}{b^4} + \frac{x^2}{2b^3} - \frac{a^4}{2b^5(a+bx)^2} + \frac{4a^3}{b^5(a+bx)} + \frac{6a^2 \log(a+bx)}{b^5}$$

output
```
-3*a*x/b^4+1/2*x^2/b^3-1/2*a^4/b^5/(b*x+a)^2+4*a^3/b^5/(b*x+a)+6*a^2*ln(b*
x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(a+bx)^3} \, dx = \frac{-6abx + b^2x^2 - \frac{a^4}{(a+bx)^2} + \frac{8a^3}{a+bx} + 12a^2 \log(a+bx)}{2b^5}$$

input
```
Integrate[x^4/(a + b*x)^3,x]
```

output
```
(-6*a*b*x + b^2*x^2 - a^4/(a + b*x)^2 + (8*a^3)/(a + b*x) + 12*a^2*Log[a +
 b*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx)^3} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}-\frac{3a}{b^4}+\frac{x}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

input
```
Int[x^4/(a + b*x)^3,x]
```

output
```
(-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)/(b^5*(a + b
*x)) + (6*a^2*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x^{2}}{2b^{3}}-\frac{3ax}{b^{4}}+\frac{4a^{3}x+\frac{7a^{4}}{2b}}{b^{4}\left(bx+a\right)^{2}}+\frac{6a^{2}\ln\left(bx+a\right)}{b^{5}}$	57
norman	$\frac{\frac{9a^{4}}{b^{5}}+\frac{x^{4}}{2b}-\frac{2a\,x^{3}}{b^{2}}+\frac{12a^{3}x}{b^{4}}}{\left(bx+a\right)^{2}}+\frac{6a^{2}\ln\left(bx+a\right)}{b^{5}}$	59
default	$-\frac{-\frac{1}{2}b\,x^{2}+3ax}{b^{4}}-\frac{a^{4}}{2b^{5}\left(bx+a\right)^{2}}+\frac{4a^{3}}{b^{5}\left(bx+a\right)}+\frac{6a^{2}\ln\left(bx+a\right)}{b^{5}}$	62
parallelrisch	$\frac{b^{4}x^{4}+12\ln\left(bx+a\right)x^{2}a^{2}b^{2}-4a\,x^{3}b^{3}+24\ln\left(bx+a\right)x\,a^{3}b+12a^{4}\ln\left(bx+a\right)+24a^{3}bx+18a^{4}}{2b^{5}\left(bx+a\right)^{2}}$	83

input
```
int(x^4/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b^3-3*a*x/b^4+(4*a^3*x+7/2/b*a^4)/b^4/(b*x+a)^2+6*a^2*ln(b*x+a)/b^
5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.48

$$\int \frac{x^4}{(a+bx)^3}\,dx = \frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\,(a^2b^2x^2 + 2\,a^3bx + a^4)\log\left(bx+a\right)}{2\,(b^7x^2 + 2\,ab^6x + a^2b^5)}$$

input
```
integrate(x^4/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*
b^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(a+bx)^3}\,dx = \frac{6a^2\log{(a+bx)}}{b^5} - \frac{3ax}{b^4} + \frac{7a^4+8a^3bx}{2a^2b^5+4ab^6x+2b^7x^2} + \frac{x^2}{2b^3}$$

input
```
integrate(x**4/(b*x+a)**3,x)
```

output
```
6*a**2*log(a + b*x)/b**5 - 3*a*x/b**4 + (7*a**4 + 8*a**3*b*x)/(2*a**2*b**5
 + 4*a*b**6*x + 2*b**7*x**2) + x**2/(2*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(a+bx)^3}\,dx = \frac{8\,a^3bx+7\,a^4}{2\,(b^7x^2+2\,ab^6x+a^2b^5)} + \frac{6\,a^2\log{(bx+a)}}{b^5} + \frac{bx^2-6\,ax}{2\,b^4}$$

input
```
integrate(x^4/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(8*a^3*b*x + 7*a^4)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x +
a)/b^5 + 1/2*(b*x^2 - 6*a*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{(a+bx)^3}\,dx = \frac{6\,a^2\log{(|bx+a|)}}{b^5} + \frac{b^3x^2-6\,ab^2x}{2\,b^6} + \frac{8\,a^3bx+7\,a^4}{2\,(bx+a)^2b^5}$$

input
```
integrate(x^4/(b*x+a)^3,x, algorithm="giac")
```

output
```
6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b
*x + 7*a^4)/((b*x + a)^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(a+bx)^3}\,dx = \frac{\frac{(a+b\,x)^2}{2}+\frac{4\,a^3}{a+b\,x}-\frac{a^4}{2\,(a+b\,x)^2}+6\,a^2\,\ln\left(a+b\,x\right)-4\,a\,b\,x}{b^5}$$

input
```
int(x^4/(a + b*x)^3,x)
```

output
```
((a + b*x)^2/2 + (4*a^3)/(a + b*x) - a^4/(2*(a + b*x)^2) + 6*a^2*log(a + b
*x) - 4*a*b*x)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int \frac{x^4}{(a+bx)^3}\,dx = \frac{12\log(bx+a)\,a^4+24\log(bx+a)\,a^3bx+12\log(bx+a)\,a^2b^2x^2+6a^4-12a^2b^2x^2-4a\,b^3x^3+b^4x^4}{2b^5\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(x^4/(b*x+a)^3,x)
```

output
```
(12*log(a + b*x)*a**4 + 24*log(a + b*x)*a**3*b*x + 12*log(a + b*x)*a**2*b*
*2*x**2 + 6*a**4 - 12*a**2*b**2*x**2 - 4*a*b**3*x**3 + b**4*x**4)/(2*b**5*
(a**2 + 2*a*b*x + b**2*x**2))
```

3.143 $\int \frac{x^3}{(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{x^3}{(a+bx)^3} dx = \frac{x}{b^3} + \frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4}$$

output
```
x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+bx)^3} dx = -\frac{-2bx + \frac{a^2(5a+6bx)}{(a+bx)^2} + 6a\log(a+bx)}{2b^4}$$

input
```
Integrate[x^3/(a + b*x)^3,x]
```

output
```
-1/2*(-2*b*x + (a^2*(5*a + 6*b*x))/(a + b*x)^2 + 6*a*Log[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^3}\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}+\frac{1}{b^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}+\frac{x}{b^3}$$

input
```
Int[x^3/(a + b*x)^3,x]
```

output
```
x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b
*x])/b^4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{x}{b^3}+\frac{-3a^2x-\frac{5a^3}{2b}}{b^3(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	45
norman	$\frac{\frac{x^3}{b}-\frac{9a^3}{2b^4}-\frac{6a^2x}{b^3}}{(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	47
default	$\frac{x}{b^3}+\frac{a^3}{2b^4(bx+a)^2}-\frac{3a^2}{b^4(bx+a)}-\frac{3a\ln(bx+a)}{b^4}$	49
parallelrisch	$-\frac{6\ln(bx+a)x^2a\,b^2-2b^3x^3+12\ln(bx+a)x\,a^2b+6a^3\ln(bx+a)+12a^2bx+9a^3}{2b^4(bx+a)^2}$	73

input
```
int(x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x/b^3+(-3*a^2*x-5/2/b*a^3)/b^3/(b*x+a)^2-3*a*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \frac{x^3}{(a+bx)^3}\,dx=\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3-6\,(ab^2x^2+2\,a^2bx+a^3)\log(bx+a)}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

input
```
integrate(x^3/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*
x + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int \frac{x^3}{(a+bx)^3}\,dx = -\frac{3a\log{(a+bx)}}{b^4} + \frac{-5a^3-6a^2bx}{2a^2b^4+4ab^5x+2b^6x^2} + \frac{x}{b^3}$$

input
```
integrate(x**3/(b*x+a)**3,x)
```

output
```
-3*a*log(a + b*x)/b**4 + (-5*a**3 - 6*a**2*b*x)/(2*a**2*b**4 + 4*a*b**5*x
+ 2*b**6*x**2) + x/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(a+bx)^3}\,dx = -\frac{6\,a^2bx+5\,a^3}{2\,(b^6x^2+2\,ab^5x+a^2b^4)} + \frac{x}{b^3} - \frac{3\,a\log{(bx+a)}}{b^4}$$

input
```
integrate(x^3/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(6*a^2*b*x + 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + x/b^3 - 3*a*log
(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx)^3}\,dx = \frac{x}{b^3} - \frac{3\,a\log{(|bx+a|)}}{b^4} - \frac{6\,a^2bx+5\,a^3}{2\,(bx+a)^2b^4}$$

input
```
integrate(x^3/(b*x+a)^3,x, algorithm="giac")
```

output
```
x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b
^4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(a+bx)^3}\,dx = -\frac{3\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{3\,a^2}{a+b\,x}-\frac{a^3}{2\,{\left(a+b\,x\right)}^2}}{b^4}$$

input
```
int(x^3/(a + b*x)^3,x)
```

output
```
-(3*a*log(a + b*x) - b*x + (3*a^2)/(a + b*x) - a^3/(2*(a + b*x)^2))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.70

$$\int \frac{x^3}{(a+bx)^3}\,dx = \frac{-6\log(bx+a)\,a^3-12\log(bx+a)\,a^2bx-6\log(bx+a)\,a\,b^2x^2-3a^3+6a\,b^2x^2+2b^3x^3}{2b^4\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(x^3/(b*x+a)^3,x)
```

output
```
( - 6*log(a + b*x)*a**3 - 12*log(a + b*x)*a**2*b*x - 6*log(a + b*x)*a*b**2
*x**2 - 3*a**3 + 6*a*b**2*x**2 + 2*b**3*x**3)/(2*b**4*(a**2 + 2*a*b*x + b*
*2*x**2))
```

3.144 $\int \frac{x^2}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{x^2}{(a+bx)^3} \, dx = -\frac{a^2}{2b^3(a+bx)^2} + \frac{2a}{b^3(a+bx)} + \frac{\log(a+bx)}{b^3}$$

output
```
-1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(a+bx)^3} \, dx = \frac{\frac{a(3a+4bx)}{(a+bx)^2} + 2\log(a+bx)}{2b^3}$$

input
```
Integrate[x^2/(a + b*x)^3,x]
```

output
```
((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[a + b*x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^3} \, dx$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

input
```
Int[x^2/(a + b*x)^3,x]
```

output
```
-1/2*a^2/(b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
norman	$\frac{\frac{3a^2}{2b^3}+\frac{2ax}{b^2}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
risch	$\frac{\frac{3a^2}{2b^3}+\frac{2ax}{b^2}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
default	$-\frac{a^2}{2b^3(bx+a)^2}+\frac{2a}{b^3(bx+a)}+\frac{\ln(bx+a)}{b^3}$	40
parallelrisch	$\frac{2b^2\ln(bx+a)x^2+4\ln(bx+a)xab+2a^2\ln(bx+a)+4abx+3a^2}{2b^3(bx+a)^2}$	60

input
```
int(x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/2*a^2/b^3+2*a*x/b^2)/(b*x+a)^2+ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{4\,abx+3\,a^2+2\,(b^2x^2+2\,abx+a^2)\log(bx+a)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(x^2/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{3a^2+4abx}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{\log(a+bx)}{b^3}$$

input
```
integrate(x**2/(b*x+a)**3,x)
```

output
```
(3*a**2 + 4*a*b*x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + log(a + b*x)
/b**3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{4\,abx+3\,a^2}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}+\frac{\log(bx+a)}{b^3}$$

input
```
integrate(x^2/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{\log(|bx+a|)}{b^3}+\frac{4\,ax+\frac{3\,a^2}{b}}{2\,(bx+a)^2b^2}$$

input
```
integrate(x^2/(b*x+a)^3,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{\ln(a+b\,x)}{b^3} + \frac{\frac{3\,a^2}{2\,b^3}+\frac{2\,a\,x}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int(x^2/(a + b*x)^3,x)
```

output
```
log(a + b*x)/b^3 + ((3*a^2)/(2*b^3) + (2*a*x)/b^2)/(a^2 + b^2*x^2 + 2*a*b*
x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(a+bx)^3}\,dx = \frac{2\log(bx+a)\,a^2+4\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2+a^2-2b^2x^2}{2b^3\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^2/(b*x+a)^3,x)
```

output
```
(2*log(a + b*x)*a**2 + 4*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 + a
**2 - 2*b**2*x**2)/(2*b**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.145 $\int \frac{x}{(a+bx)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{x}{(a+bx)^3} dx = \frac{x^2}{2a(a+bx)^2}$$

output
```
1/2*x^2/a/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{x}{(a+bx)^3} dx = -\frac{a+2bx}{2b^2(a+bx)^2}$$

input
```
Integrate[x/(a + b*x)^3,x]
```

output
```
-1/2*(a + 2*b*x)/(b^2*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^3} \, dx$$

$$\Big\downarrow \text{48}$$

$$\frac{x^2}{2a(a+bx)^2}$$

input
```
Int[x/(a + b*x)^3,x]
```

output
```
x^2/(2*a*(a + b*x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

method	result	size
gosper	$-\frac{2bx+a}{2b^2(bx+a)^2}$	19
orering	$-\frac{2bx+a}{2b^2(bx+a)^2}$	19
parallelrisch	$\frac{-2bx-a}{2b^2(bx+a)^2}$	21
norman	$\frac{-\frac{x}{b}-\frac{a}{2b^2}}{(bx+a)^2}$	22
risch	$\frac{-\frac{x}{b}-\frac{a}{2b^2}}{(bx+a)^2}$	22
default	$\frac{a}{2b^2(bx+a)^2}-\frac{1}{(bx+a)b^2}$	27

input
```
int(x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*x+a)/b^2/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate(x/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{x}{(a+bx)^3}\,dx = \frac{-a-2bx}{2a^2b^2+4ab^3x+2b^4x^2}$$

input
```
integrate(x/(b*x+a)**3,x)
```

output
```
(-a - 2*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate(x/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

input
```
integrate(x/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{x}{(a+bx)^3}\,dx = -\frac{\frac{a}{2\,b^2}+\frac{x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int(x/(a + b*x)^3,x)
```

output
```
-(a/(2*b^2) + x/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{x}{(a+bx)^3}\,dx = \frac{x^2}{2a\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(x/(b*x+a)^3,x)
```

output
```
x**2/(2*a*(a**2 + 2*a*b*x + b**2*x**2))
```

3.146 $\int \frac{1}{(a+bx)^3} dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(a+bx)^3} dx = -\frac{1}{2b(a+bx)^2}$$

output
```
-1/2/b/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^3} dx = -\frac{1}{2b(a+bx)^2}$$

input
```
Integrate[(a + b*x)^(-3),x]
```

output
```
-1/2*1/(b*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^3} \, dx$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{2b(a+bx)^2}$$

input
```
Int[(a + b*x)^(-3),x]
```

output
```
-1/2*1/(b*(a + b*x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{2b(bx+a)^2}$	13
default	$-\frac{1}{2b(bx+a)^2}$	13
norman	$-\frac{1}{2b(bx+a)^2}$	13
risch	$-\frac{1}{2b(bx+a)^2}$	13
parallelrisch	$-\frac{1}{2b(bx+a)^2}$	13
orering	$-\frac{1}{2b(bx+a)^2}$	13

input
```
int(1/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a+bx)^3}\,dx=-\frac{1}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate(1/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

input
```
integrate(1/(b*x+a)**3,x)
```

output
```
-1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2\,(bx+a)^2b}$$

input
```
integrate(1/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2/((b*x + a)^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2\,(bx+a)^2b}$$

input
```
integrate(1/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2/((b*x + a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

input
```
int(1/(a + b*x)^3,x)
```

output
```
-1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2b\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/(b*x+a)^3,x)
```

output
```
( - 1)/(2*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.147 $\int \frac{1}{x(a+bx)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{1}{x(a+bx)^3} \, dx = \frac{1}{2a(a+bx)^2} + \frac{1}{a^2(a+bx)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx)}{a^3}$$

output
```
1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(a+bx)^3} \, dx = \frac{\frac{a(3a+2bx)}{(a+bx)^2} + 2\log(x) - 2\log(a+bx)}{2a^3}$$

input
```
Integrate[1/(x*(a + b*x)^3),x]
```

output
```
((a*(3*a + 2*b*x))/(a + b*x)^2 + 2*Log[x] - 2*Log[a + b*x])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^3} \, dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(-\frac{b}{a^3(a+bx)}+\frac{1}{a^3x}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a(a+bx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{a^2(a+bx)}+\frac{1}{2a(a+bx)^2}$$

input
```
Int[1/(x*(a + b*x)^3),x]
```

output
```
1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{\frac{bx}{a^2}+\frac{3}{2a}}{(bx+a)^2}-\frac{\ln(bx+a)}{a^3}+\frac{\ln(-x)}{a^3}$	41
default	$\frac{1}{2a(bx+a)^2}+\frac{1}{a^2(bx+a)}+\frac{\ln(x)}{a^3}-\frac{\ln(bx+a)}{a^3}$	42
norman	$\frac{-\frac{2bx}{a^2}-\frac{3b^2x^2}{2a^3}}{(bx+a)^2}+\frac{\ln(x)}{a^3}-\frac{\ln(bx+a)}{a^3}$	46
parallelrisch	$\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+4ab\ln(x)x-4\ln(bx+a)xab-3b^2x^2+2a^2\ln(x)-2a^2\ln(bx+a)-4abx}{2a^3(bx+a)^2}$	87

input
```
int(1/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(b/a^2*x+3/2/a)/(b*x+a)^2-ln(b*x+a)/a^3+1/a^3*ln(-x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.86

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{2\,abx+3\,a^2-2\,(b^2x^2+2\,abx+a^2)\log(bx+a)+2\,(b^2x^2+2\,abx+a^2)\log(x)}{2\,(a^3b^2x^2+2\,a^4bx+a^5)}$$

input
```
integrate(1/x/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x
^2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{3a+2bx}{2a^4+4a^3bx+2a^2b^2x^2} + \frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^3}$$

input
```
integrate(1/x/(b*x+a)**3,x)
```

output
```
(3*a + 2*b*x)/(2*a**4 + 4*a**3*b*x + 2*a**2*b**2*x**2) + (log(x) - log(a/b
 + x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{2\,bx+3\,a}{2\,(a^2b^2x^2+2\,a^3bx+a^4)} - \frac{\log(bx+a)}{a^3} + \frac{\log(x)}{a^3}$$

input
```
integrate(1/x/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log
(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)^3}\,dx = -\frac{\log(|bx+a|)}{a^3} + \frac{\log(|x|)}{a^3} + \frac{2\,abx+3\,a^2}{2\,(bx+a)^2a^3}$$

input
```
integrate(1/x/(b*x+a)^3,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a
)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,{\left(a+b\,x\right)}^2}$$

input
```
int(1/(x*(a + b*x)^3),x)
```

output
```
(1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \frac{1}{x(a+bx)^3}\,dx = \frac{-2\log(bx+a)\,a^2-4\log(bx+a)\,abx-2\log(bx+a)\,b^2x^2+2\log(x)\,a^2+4\log(x)\,abx+2\log(x)\,b^2x^2+}{2a^3\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(1/x/(b*x+a)^3,x)
```

output
```
( - 2*log(a + b*x)*a**2 - 4*log(a + b*x)*a*b*x - 2*log(a + b*x)*b**2*x**2
+ 2*log(x)*a**2 + 4*log(x)*a*b*x + 2*log(x)*b**2*x**2 + 2*a**2 - b**2*x**2
)/(2*a**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.148 $\int \frac{1}{x^2(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{1}{x^2(a+bx)^3} dx = -\frac{1}{a^3x} - \frac{b}{2a^2(a+bx)^2} - \frac{2b}{a^3(a+bx)} - \frac{3b\log(x)}{a^4} + \frac{3b\log(a+bx)}{a^4}$$

output
```
-1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a
^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a+bx)^3} dx = -\frac{\frac{a(2a^2+9abx+6b^2x^2)}{x(a+bx)^2} + 6b\log(x) - 6b\log(a+bx)}{2a^4}$$

input
```
Integrate[1/(x^2*(a + b*x)^3),x]
```

output
```
-1/2*((a*(2*a^2 + 9*a*b*x + 6*b^2*x^2))/(x*(a + b*x)^2) + 6*b*Log[x] - 6*b
*Log[a + b*x])/a^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^3} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{3b^2}{a^4(a+bx)}-\frac{3b}{a^4x}+\frac{2b^2}{a^3(a+bx)^2}+\frac{1}{a^3x^2}+\frac{b^2}{a^2(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{2b}{a^3(a+bx)}-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}$$

input
```
Int[1/(x^2*(a + b*x)^3),x]
```

output
```
-(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x]
)/a^4 + (3*b*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

method	result
default	$-\frac{1}{a^{3}x}-\frac{b}{2a^{2}(bx+a)^{2}}-\frac{2b}{a^{3}(bx+a)}-\frac{3b\ln(x)}{a^{4}}+\frac{3b\ln(bx+a)}{a^{4}}$
risch	$\frac{-\frac{3b^{2}x^{2}}{a^{3}}-\frac{9bx}{2a^{2}}-\frac{1}{a}}{x(bx+a)^{2}}+\frac{3b\ln(-bx-a)}{a^{4}}-\frac{3b\ln(x)}{a^{4}}$
norman	$\frac{-\frac{1}{a}+\frac{6b^{2}x^{2}}{a^{3}}+\frac{9b^{3}x^{3}}{2a^{4}}}{x(bx+a)^{2}}-\frac{3b\ln(x)}{a^{4}}+\frac{3b\ln(bx+a)}{a^{4}}$
parallelrisch	$-\frac{6b^{3}\ln(x)x^{3}-6b^{3}\ln(bx+a)x^{3}+12a\,b^{2}\ln(x)x^{2}-12\ln(bx+a)x^{2}a\,b^{2}-9b^{3}x^{3}+6a^{2}b\ln(x)x-6\ln(bx+a)x\,a^{2}b-12a\,b^{2}x^{2}+2a^{3}}{2a^{4}x(bx+a)^{2}}$

input
```
int(1/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a
^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

$$\int \frac{1}{x^2(a+bx)^3}\,dx = -\frac{6\,ab^{2}x^{2}+9\,a^{2}bx+2\,a^{3}-6\,(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx)\log(bx+a)+6\,(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx)\log(x)}{2\,(a^{4}b^{2}x^{3}+2\,a^{5}bx^{2}+a^{6}x)}$$

input
```
integrate(1/x^2/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x
)*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3
+ 2*a^5*b*x^2 + a^6*x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2(a+bx)^3}\,dx = \frac{-2a^2-9abx-6b^2x^2}{2a^5x+4a^4bx^2+2a^3b^2x^3} + \frac{3b\left(-\log{(x)}+\log{\left(\frac{a}{b}+x\right)}\right)}{a^4}$$

input
```
integrate(1/x**2/(b*x+a)**3,x)
```

output
```
(-2*a**2 - 9*a*b*x - 6*b**2*x**2)/(2*a**5*x + 4*a**4*b*x**2 + 2*a**3*b**2*
x**3) + 3*b*(-log(x) + log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2(a+bx)^3}\,dx = -\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\,(a^3b^2x^3+2\,a^4bx^2+a^5x)} + \frac{3\,b\log{(bx+a)}}{a^4} - \frac{3\,b\log{(x)}}{a^4}$$

input
```
integrate(1/x^2/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3
*b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2(a+bx)^3}\,dx = \frac{3\,b\log{(|bx+a|)}}{a^4} - \frac{3\,b\log{(|x|)}}{a^4} - \frac{6\,ab^2x^2+9\,a^2bx+2\,a^3}{2\,(bx+a)^2a^4x}$$

input
```
integrate(1/x^2/(b*x+a)^3,x, algorithm="giac")
```

output
```
3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2
*b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2(a+bx)^3}\,dx = \frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4} - \frac{\frac{1}{a}+\frac{3\,b^2\,x^2}{a^3}+\frac{9\,b\,x}{2\,a^2}}{a^2\,x+2\,a\,b\,x^2+b^2\,x^3}$$

input
```
int(1/(x^2*(a + b*x)^3),x)
```

output
```
(6*b*atanh((2*b*x)/a + 1))/a^4 - (1/a + (3*b^2*x^2)/a^3 + (9*b*x)/(2*a^2))
/(a^2*x + b^2*x^3 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^2(a+bx)^3}\,dx$$
$$= \frac{6\log(bx+a)\,a^2bx+12\log(bx+a)\,a\,b^2x^2+6\log(bx+a)\,b^3x^3-6\log(x)\,a^2bx-12\log(x)\,a\,b^2x^2-6\log}{2a^4x\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/x^2/(b*x+a)^3,x)
```

output
```
(6*log(a + b*x)*a**2*b*x + 12*log(a + b*x)*a*b**2*x**2 + 6*log(a + b*x)*b*
*3*x**3 - 6*log(x)*a**2*b*x - 12*log(x)*a*b**2*x**2 - 6*log(x)*b**3*x**3 -
 2*a**3 - 6*a**2*b*x + 3*b**3*x**3)/(2*a**4*x*(a**2 + 2*a*b*x + b**2*x**2)
)
```

3.149 $\int \frac{1}{x^3(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 76

$$\int \frac{1}{x^3(a+bx)^3} dx = -\frac{1}{2a^3x^2} + \frac{3b}{a^4x} + \frac{b^2}{2a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)} + \frac{6b^2\log(x)}{a^5} - \frac{6b^2\log(a+bx)}{a^5}$$

output
```
-1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)
/a^5-6*b^2*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3(a+bx)^3} dx = \frac{\frac{a(-a^3+4a^2bx+18ab^2x^2+12b^3x^3)}{x^2(a+bx)^2} + 12b^2\log(x) - 12b^2\log(a+bx)}{2a^5}$$

input
```
Integrate[1/(x^3*(a + b*x)^3),x]
```

output
```
((a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3))/(x^2*(a + b*x)^2) + 12
*b^2*Log[x] - 12*b^2*Log[a + b*x])/(2*a^5)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^3} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{6b^3}{a^5(a+bx)}+\frac{6b^2}{a^5x}-\frac{3b^3}{a^4(a+bx)^2}-\frac{3b}{a^4x^2}-\frac{b^3}{a^3(a+bx)^3}+\frac{1}{a^3x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b^2}{a^4(a+bx)}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}-\frac{1}{2a^3x^2}$$

input
```
Int[1/(x^3*(a + b*x)^3),x]
```

output
```
-1/2*1/(a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*
(a + b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

method	result
norman	$\frac{-\frac{9b^4x^4}{a^5}-\frac{1}{2a}+\frac{2bx}{a^2}-\frac{12b^3x^3}{a^4}}{x^2(bx+a)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$
default	$-\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(bx+a)^2}+\frac{3b^2}{a^4(bx+a)}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$
risch	$\frac{\frac{6b^3x^3}{a^4}+\frac{9b^2x^2}{a^3}+\frac{2bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^2}+\frac{6b^2\ln(-x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$
parallelrisch	$\frac{12\ln(x)x^4b^6-12\ln(bx+a)x^4b^6+24\ln(x)x^3a\,b^5-24\ln(bx+a)x^3a\,b^5+12\ln(x)x^2a^2b^4-12\ln(bx+a)x^2a^2b^4+12x^3a\,b^5+18x^2a^2}{2a^5b^2x^2(bx+a)^2}$

input
```
int(1/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-9*b^4/a^5*x^4-1/2/a+2*b/a^2*x-12*b^3/a^4*x^3)/x^2/(b*x+a)^2+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\log(bx+a)+12\,(b^4x^4+2\,ab^3x^3+a^2}{2\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)}$$

input
```
integrate(1/x^3/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b
^3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x
^2)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{-a^3+4a^2bx+18ab^2x^2+12b^3x^3}{2a^6x^2+4a^5bx^3+2a^4b^2x^4}+\frac{6b^2\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^5}$$

input
```
integrate(1/x**3/(b*x+a)**3,x)
```

output
```
(-a**3 + 4*a**2*b*x + 18*a*b**2*x**2 + 12*b**3*x**3)/(2*a**6*x**2 + 4*a**5
*b*x**3 + 2*a**4*b**2*x**4) + 6*b**2*(log(x) - log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}-\frac{6\,b^2\log{(bx+a)}}{a^5}+\frac{6\,b^2\log{(x)}}{a^5}$$

input
```
integrate(1/x^3/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x
^3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3(a+bx)^3}\,dx = -\frac{6\,b^2\log{(|bx+a|)}}{a^5}+\frac{6\,b^2\log{(|x|)}}{a^5}+\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,(bx^2+ax)^2a^4}$$

input
```
integrate(1/x^3/(b*x+a)^3,x, algorithm="giac")
```

output
```
-6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 1
8*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3(a+bx)^3}\,dx = \frac{\frac{9\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{6\,b^3\,x^3}{a^4}+\frac{2\,b\,x}{a^2}}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4}-\frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

input
```
int(1/(x^3*(a + b*x)^3),x)
```

output
```
((9*b^2*x^2)/a^3 - 1/(2*a) + (6*b^3*x^3)/a^4 + (2*b*x)/a^2)/(a^2*x^2 + b^2
*x^4 + 2*a*b*x^3) - (12*b^2*atanh((2*b*x)/a + 1))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^3(a+bx)^3}\,dx$$
$$= \frac{-12\log(bx+a)\,a^2b^2x^2-24\log(bx+a)\,a\,b^3x^3-12\log(bx+a)\,b^4x^4+12\log(x)\,a^2b^2x^2+24\log(x)\,a\,b^3x}{2a^5x^2\left(b^2x^2+2abx+a^2\right)}$$

input
```
int(1/x^3/(b*x+a)^3,x)
```

output
```
( - 12*log(a + b*x)*a**2*b**2*x**2 - 24*log(a + b*x)*a*b**3*x**3 - 12*log(
a + b*x)*b**4*x**4 + 12*log(x)*a**2*b**2*x**2 + 24*log(x)*a*b**3*x**3 + 12
*log(x)*b**4*x**4 - a**4 + 4*a**3*b*x + 12*a**2*b**2*x**2 - 6*b**4*x**4)/(
2*a**5*x**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.150 $\int \frac{1}{x^4(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 89

$$\int \frac{1}{x^4(a+bx)^3} dx = -\frac{1}{3a^3x^3} + \frac{3b}{2a^4x^2} - \frac{6b^2}{a^5x} - \frac{b^3}{2a^4(a+bx)^2} - \frac{4b^3}{a^5(a+bx)} - \frac{10b^3\log(x)}{a^6} + \frac{10b^3\log(a+bx)}{a^6}$$

output
```
-1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*
x+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4(a+bx)^3} dx = -\frac{\frac{a(2a^4-5a^3bx+20a^2b^2x^2+90ab^3x^3+60b^4x^4)}{x^3(a+bx)^2} + 60b^3\log(x) - 60b^3\log(a+bx)}{6a^6}$$

input
```
Integrate[1/(x^4*(a + b*x)^3),x]
```

output
```
-1/6*((a*(2*a^4 - 5*a^3*b*x + 20*a^2*b^2*x^2 + 90*a*b^3*x^3 + 60*b^4*x^4))
/(x^3*(a + b*x)^2) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a^6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+bx)^3} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{10b^4}{a^6(a+bx)}-\frac{10b^3}{a^6x}+\frac{4b^4}{a^5(a+bx)^2}+\frac{6b^2}{a^5x^2}+\frac{b^4}{a^4(a+bx)^3}-\frac{3b}{a^4x^3}+\frac{1}{a^3x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}-\frac{4b^3}{a^5(a+bx)}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

input
```
Int[1/(x^4*(a + b*x)^3),x]
```

output
```
-1/3*1/(a^3*x^3) + (3*b)/(2*a^4*x^2) - (6*b^2)/(a^5*x) - b^3/(2*a^4*(a + b
*x)^2) - (4*b^3)/(a^5*(a + b*x)) - (10*b^3*Log[x])/a^6 + (10*b^3*Log[a + b
*x])/a^6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

method	result
norman	$\frac{\frac{15b^5x^5}{a^6}-\frac{1}{3a}+\frac{5bx}{6a^2}-\frac{10b^2x^2}{3a^3}+\frac{20b^4x^4}{a^5}}{x^3(bx+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(bx+a)}{a^6}$
default	$-\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(bx+a)^2}-\frac{4b^3}{a^5(bx+a)}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(bx+a)}{a^6}$
risch	$\frac{-\frac{10b^4x^4}{a^5}-\frac{15b^3x^3}{a^4}-\frac{10b^2x^2}{3a^3}+\frac{5bx}{6a^2}-\frac{1}{3a}}{x^3(bx+a)^2}+\frac{10b^3\ln(-bx-a)}{a^6}-\frac{10b^3\ln(x)}{a^6}$
parallelrisch	$-\frac{60b^5\ln(x)x^5-60\ln(bx+a)x^5b^5+120a\,b^4\ln(x)x^4-120\ln(bx+a)x^4a\,b^4-90b^5x^5+60a^2b^3\ln(x)x^3-60\ln(bx+a)x^3a^2b^3-120}{6a^6x^3(bx+a)^2}$

input
```
int(1/x^4/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(15*b^5/a^6*x^5-1/3/a+5/6*b/a^2*x-10/3*b^2/a^3*x^2+20*b^4/a^5*x^4)/x^3/(b*
x+a)^2-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^4(a+bx)^3}\,dx = -\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5-60\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\log(bx+a)+60\,(b^5x}{6\,(a^6b^2x^5+2\,a^7bx^4+a^8x^3)}$$

input
```
integrate(1/x^4/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(60*a*b^4*x^4 + 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5 -
 60*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*log(b*x + a) + 60*(b^5*x^5 + 2*a
*b^4*x^4 + a^2*b^3*x^3)*log(x))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4(a+bx)^3} dx = \frac{-2a^4 + 5a^3bx - 20a^2b^2x^2 - 90ab^3x^3 - 60b^4x^4}{6a^7x^3 + 12a^6bx^4 + 6a^5b^2x^5} + \frac{10b^3\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

input
```
integrate(1/x**4/(b*x+a)**3,x)
```

output
```
(-2*a**4 + 5*a**3*b*x - 20*a**2*b**2*x**2 - 90*a*b**3*x**3 - 60*b**4*x**4)
/(6*a**7*x**3 + 12*a**6*b*x**4 + 6*a**5*b**2*x**5) + 10*b**3*(-log(x) + lo
g(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4(a+bx)^3} dx = -\frac{60\,b^4x^4 + 90\,ab^3x^3 + 20\,a^2b^2x^2 - 5\,a^3bx + 2\,a^4}{6\,(a^5b^2x^5 + 2\,a^6bx^4 + a^7x^3)} + \frac{10\,b^3\log(bx+a)}{a^6} - \frac{10\,b^3\log(x)}{a^6}$$

input
```
integrate(1/x^4/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(60*b^4*x^4 + 90*a*b^3*x^3 + 20*a^2*b^2*x^2 - 5*a^3*b*x + 2*a^4)/(a^5
*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3) + 10*b^3*log(b*x + a)/a^6 - 10*b^3*log(x
)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4(a+bx)^3}\,dx = \frac{10\,b^3\log\left(|bx+a|\right)}{a^6} - \frac{10\,b^3\log\left(|x|\right)}{a^6} - \frac{60\,ab^4x^4 + 90\,a^2b^3x^3 + 20\,a^3b^2x^2 - 5\,a^4bx + 2\,a^5}{6\,(bx+a)^2a^6x^3}$$

input
```
integrate(1/x^4/(b*x+a)^3,x, algorithm="giac")
```

output
```
10*b^3*log(abs(b*x + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/6*(60*a*b^4*x^4
+ 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5)/((b*x + a)^2*a^6*x^
3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4(a+bx)^3}\,dx = \frac{20\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6} - \frac{\frac{1}{3\,a} + \frac{10\,b^2\,x^2}{3\,a^3} + \frac{15\,b^3\,x^3}{a^4} + \frac{10\,b^4\,x^4}{a^5} - \frac{5\,b\,x}{6\,a^2}}{a^2\,x^3 + 2\,a\,b\,x^4 + b^2\,x^5}$$

input
```
int(1/(x^4*(a + b*x)^3),x)
```

output
```
(20*b^3*atanh((2*b*x)/a + 1))/a^6 - (1/(3*a) + (10*b^2*x^2)/(3*a^3) + (15*
b^3*x^3)/a^4 + (10*b^4*x^4)/a^5 - (5*b*x)/(6*a^2))/(a^2*x^3 + b^2*x^5 + 2*
a*b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^4(a+bx)^3}\,dx$$
$$= \frac{60\log(bx+a)\,a^2b^3x^3 + 120\log(bx+a)\,a\,b^4x^4 + 60\log(bx+a)\,b^5x^5 - 60\log(x)\,a^2b^3x^3 - 120\log(x)\,a\,b^4}{6a^6x^3\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/x^4/(b*x+a)^3,x)
```

output
```
(60*log(a + b*x)*a**2*b**3*x**3 + 120*log(a + b*x)*a*b**4*x**4 + 60*log(a
+ b*x)*b**5*x**5 - 60*log(x)*a**2*b**3*x**3 - 120*log(x)*a*b**4*x**4 - 60*
log(x)*b**5*x**5 - 2*a**5 + 5*a**4*b*x - 20*a**3*b**2*x**2 - 60*a**2*b**3*
x**3 + 30*b**5*x**5)/(6*a**6*x**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.151 $\int \frac{1}{x^5(a+bx)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 97

$$\int \frac{1}{x^5(a+bx)^3} dx = -\frac{1}{4a^3x^4} + \frac{b}{a^4x^3} - \frac{3b^2}{a^5x^2} + \frac{10b^3}{a^6x} + \frac{b^4}{2a^5(a+bx)^2} + \frac{5b^4}{a^6(a+bx)} + \frac{15b^4\log(x)}{a^7} - \frac{15b^4\log(a+bx)}{a^7}$$

output
```
-1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*
b^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5(a+bx)^3} dx = \frac{\frac{a(-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5)}{x^4(a+bx)^2} + 60b^4\log(x) - 60b^4\log(a+bx)}{4a^7}$$

input
```
Integrate[1/(x^5*(a + b*x)^3),x]
```

output
```
((a*(-a^5 + 2*a^4*b*x - 5*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 90*a*b^4*x^4 + 60
*b^5*x^5))/(x^4*(a + b*x)^2) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(4*a^7
)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(a+bx)^3}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{15b^5}{a^7(a+bx)}+\frac{15b^4}{a^7x}-\frac{5b^5}{a^6(a+bx)^2}-\frac{10b^3}{a^6x^2}-\frac{b^5}{a^5(a+bx)^3}+\frac{6b^2}{a^5x^3}-\frac{3b}{a^4x^4}+\frac{1}{a^3x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{5b^4}{a^6(a+bx)}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}-\frac{3b^2}{a^5x^2}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

input
```
Int[1/(x^5*(a + b*x)^3),x]
```

output
```
-1/4*1/(a^3*x^4) + b/(a^4*x^3) - (3*b^2)/(a^5*x^2) + (10*b^3)/(a^6*x) + b^
4/(2*a^5*(a + b*x)^2) + (5*b^4)/(a^6*(a + b*x)) + (15*b^4*Log[x])/a^7 - (1
5*b^4*Log[a + b*x])/a^7
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

method	result
default	$-\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(bx+a)^2}+\frac{5b^4}{a^6(bx+a)}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(bx+a)}{a^7}$
norman	$\frac{-\frac{1}{4a}+\frac{bx}{2a^2}-\frac{5b^2x^2}{4a^3}+\frac{5b^3x^3}{a^4}-\frac{30b^5x^5}{a^6}-\frac{45b^6x^6}{2a^7}}{x^4(bx+a)^2}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(bx+a)}{a^7}$
risch	$\frac{\frac{15b^5x^5}{a^6}+\frac{45b^4x^4}{2a^5}+\frac{5b^3x^3}{a^4}-\frac{5b^2x^2}{4a^3}+\frac{bx}{2a^2}-\frac{1}{4a}}{x^4(bx+a)^2}+\frac{15b^4\ln(-x)}{a^7}-\frac{15b^4\ln(bx+a)}{a^7}$
parallelrisch	$\frac{60\ln(x)x^6b^6-60\ln(bx+a)x^6b^6+120\ln(x)x^5ab^5-120\ln(bx+a)x^5ab^5-90b^6x^6+60\ln(x)x^4a^2b^4-60\ln(bx+a)x^4a^2b^4-120a\,x}{4a^7x^4(bx+a)^2}$

input
```
int(1/x^5/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*
b^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^5(a+bx)^3}\,dx = \frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6-60\,(b^6x^6+2\,ab^5x^5+a^2b^4x^4)\log(bx+a)+}{4\,(a^7b^2x^6+2\,a^8bx^5+a^9x^4)}$$

input
```
integrate(1/x^5/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(60*a*b^5*x^5 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^
5*b*x - a^6 - 60*(b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(b*x + a) + 60*(
b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(x))/(a^7*b^2*x^6 + 2*a^8*b*x^5 +
a^9*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5(a+bx)^3}\,dx = \frac{-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5}{4a^8x^4+8a^7bx^5+4a^6b^2x^6} + \frac{15b^4\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^7}$$

input
```
integrate(1/x**5/(b*x+a)**3,x)
```

output
```
(-a**5 + 2*a**4*b*x - 5*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 90*a*b**4*x**
4 + 60*b**5*x**5)/(4*a**8*x**4 + 8*a**7*b*x**5 + 4*a**6*b**2*x**6) + 15*b*
*4*(log(x) - log(a/b + x))/a**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^5(a+bx)^3}\,dx = \frac{60\,b^5x^5+90\,ab^4x^4+20\,a^2b^3x^3-5\,a^3b^2x^2+2\,a^4bx-a^5}{4\,(a^6b^2x^6+2\,a^7bx^5+a^8x^4)} - \frac{15\,b^4\log{(bx+a)}}{a^7} + \frac{15\,b^4\log{(x)}}{a^7}$$

input
```
integrate(1/x^5/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(60*b^5*x^5 + 90*a*b^4*x^4 + 20*a^2*b^3*x^3 - 5*a^3*b^2*x^2 + 2*a^4*b*
x - a^5)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4) - 15*b^4*log(b*x + a)/a^7 +
 15*b^4*log(x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(a+bx)^3}\,dx = -\frac{15\,b^4\log\left(|bx+a|\right)}{a^7}+\frac{15\,b^4\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6}{4\,(bx+a)^2a^7x^4}$$

input
```
integrate(1/x^5/(b*x+a)^3,x, algorithm="giac")
```

output
```
-15*b^4*log(abs(b*x + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/4*(60*a*b^5*x^5
 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5*b*x - a^6)/((b*
x + a)^2*a^7*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^5(a+bx)^3}\,dx = \frac{\frac{5\,b^3\,x^3}{a^4}-\frac{5\,b^2\,x^2}{4\,a^3}-\frac{1}{4\,a}+\frac{45\,b^4\,x^4}{2\,a^5}+\frac{15\,b^5\,x^5}{a^6}+\frac{b\,x}{2\,a^2}}{a^2\,x^4+2\,a\,b\,x^5+b^2\,x^6}-\frac{30\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}$$

input
```
int(1/(x^5*(a + b*x)^3),x)
```

output
```
((5*b^3*x^3)/a^4 - (5*b^2*x^2)/(4*a^3) - 1/(4*a) + (45*b^4*x^4)/(2*a^5) +
(15*b^5*x^5)/a^6 + (b*x)/(2*a^2))/(a^2*x^4 + b^2*x^6 + 2*a*b*x^5) - (30*b^
4*atanh((2*b*x)/a + 1))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

$$\int \frac{1}{x^5(a+bx)^3}\,dx$$
$$= \frac{-60\log(bx+a)\,a^2b^4x^4 - 120\log(bx+a)\,a\,b^5x^5 - 60\log(bx+a)\,b^6x^6 + 60\log(x)\,a^2b^4x^4 + 120\log(x)\,a\,b}{4a^7x^4\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/x^5/(b*x+a)^3,x)
```

output
```
( - 60*log(a + b*x)*a**2*b**4*x**4 - 120*log(a + b*x)*a*b**5*x**5 - 60*log
(a + b*x)*b**6*x**6 + 60*log(x)*a**2*b**4*x**4 + 120*log(x)*a*b**5*x**5 +
60*log(x)*b**6*x**6 - a**6 + 2*a**5*b*x - 5*a**4*b**2*x**2 + 20*a**3*b**3*
x**3 + 60*a**2*b**4*x**4 - 30*b**6*x**6)/(4*a**7*x**4*(a**2 + 2*a*b*x + b*
*2*x**2))
```

3.152 $\int \frac{x^8}{(a+bx)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 114

$$\int \frac{x^8}{(a+bx)^4} dx = \frac{35a^4x}{b^8} - \frac{10a^3x^2}{b^7} + \frac{10a^2x^3}{3b^6} - \frac{ax^4}{b^5} + \frac{x^5}{5b^4} - \frac{a^8}{3b^9(a+bx)^3} + \frac{4a^7}{b^9(a+bx)^2} - \frac{28a^6}{b^9(a+bx)} - \frac{56a^5\log(a+bx)}{b^9}$$

output
```
35*a^4*x/b^8-10*a^3*x^2/b^7+10/3*a^2*x^3/b^6-a*x^4/b^5+1/5*x^5/b^4-1/3*a^8
/b^9/(b*x+a)^3+4*a^7/b^9/(b*x+a)^2-28*a^6/b^9/(b*x+a)-56*a^5*ln(b*x+a)/b^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{(a+bx)^4} dx = \frac{525a^4bx - 150a^3b^2x^2 + 50a^2b^3x^3 - 15ab^4x^4 + 3b^5x^5 - \frac{5a^8}{(a+bx)^3} + \frac{60a^7}{(a+bx)^2} - \frac{420a^6}{a+bx} - 840a^5\log(a+bx)}{15b^9}$$

input
```
Integrate[x^8/(a + b*x)^4,x]
```

output
```
(525*a^4*b*x - 150*a^3*b^2*x^2 + 50*a^2*b^3*x^3 - 15*a*b^4*x^4 + 3*b^5*x^5
 - (5*a^8)/(a + b*x)^3 + (60*a^7)/(a + b*x)^2 - (420*a^6)/(a + b*x) - 840*
a^5*Log[a + b*x])/(15*b^9)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx)^4}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{a^8}{b^8(a+bx)^4}-\frac{8a^7}{b^8(a+bx)^3}+\frac{28a^6}{b^8(a+bx)^2}-\frac{56a^5}{b^8(a+bx)}+\frac{35a^4}{b^8}-\frac{20a^3x}{b^7}+\frac{10a^2x^2}{b^6}-\frac{4ax^3}{b^5}+\frac{x^4}{b^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

input
```
Int[x^8/(a + b*x)^4,x]
```

output
```
(35*a^4*x)/b^8 - (10*a^3*x^2)/b^7 + (10*a^2*x^3)/(3*b^6) - (a*x^4)/b^5 + x
^5/(5*b^4) - a^8/(3*b^9*(a + b*x)^3) + (4*a^7)/(b^9*(a + b*x)^2) - (28*a^6
)/(b^9*(a + b*x)) - (56*a^5*Log[a + b*x])/b^9
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

method	result
risch	$\frac{x^5}{5b^4}-\frac{a\,x^4}{b^5}+\frac{10a^2x^3}{3b^6}-\frac{10a^3x^2}{b^7}+\frac{35a^4x}{b^8}+\frac{-28a^6b\,x^2-52a^7x-\frac{73a^8}{3b}}{b^8(bx+a)^3}-\frac{56a^5\ln(bx+a)}{b^9}$
norman	$\frac{\frac{x^8}{5b}-\frac{2a\,x^7}{5b^2}+\frac{14a^2x^6}{15b^3}-\frac{14a^3x^5}{5b^4}-\frac{308a^8}{3b^9}+\frac{14a^4x^4}{b^5}-\frac{168a^6x^2}{b^7}-\frac{252a^7x}{b^8}}{(bx+a)^3}-\frac{56a^5\ln(bx+a)}{b^9}$
default	$\frac{\frac{1}{5}b^4x^5-a\,b^3x^4+\frac{10}{3}a^2b^2x^3-10a^3b\,x^2+35a^4x}{b^8}+\frac{4a^7}{b^9(bx+a)^2}-\frac{28a^6}{b^9(bx+a)}-\frac{56a^5\ln(bx+a)}{b^9}-\frac{a^8}{3b^9(bx+a)^3}$
parallelrisch	$-\frac{-3b^8x^8+6a\,x^7b^7-14a^2x^6b^6+42a^3x^5b^5+840\ln(bx+a)x^3a^5b^3-210a^4x^4b^4+2520\ln(bx+a)x^2a^6b^2+2520\ln(bx+a)x\,a^7b+2}{15b^9(bx+a)^3}$

input
```
int(x^8/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5/b^4-a*x^4/b^5+10/3*a^2*x^3/b^6-10*a^3*x^2/b^7+35*a^4*x/b^8+(-28*a^
6*b*x^2-52*a^7*x-73/3/b*a^8)/b^8/(b*x+a)^3-56*a^5*ln(b*x+a)/b^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

$$\int \frac{x^8}{(a+bx)^4}\,dx$$
$$=\frac{3\,b^8x^8-6\,ab^7x^7+14\,a^2b^6x^6-42\,a^3b^5x^5+210\,a^4b^4x^4+1175\,a^5b^3x^3+1005\,a^6b^2x^2-255\,a^7bx-365\,a^8}{15\,(b^{12}x^3+3\,ab^{11}x^2+3\,a^2b^{10}x+a^3b^9)}$$

input
```
integrate(x^8/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/15*(3*b^8*x^8 - 6*a*b^7*x^7 + 14*a^2*b^6*x^6 - 42*a^3*b^5*x^5 + 210*a^4*
b^4*x^4 + 1175*a^5*b^3*x^3 + 1005*a^6*b^2*x^2 - 255*a^7*b*x - 365*a^8 - 84
0*(a^5*b^3*x^3 + 3*a^6*b^2*x^2 + 3*a^7*b*x + a^8)*log(b*x + a))/(b^12*x^3
+ 3*a*b^11*x^2 + 3*a^2*b^10*x + a^3*b^9)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(a+bx)^4}\,dx = -\frac{56a^5\log{(a+bx)}}{b^9} + \frac{35a^4x}{b^8} - \frac{10a^3x^2}{b^7} + \frac{10a^2x^3}{3b^6} - \frac{ax^4}{b^5} + \frac{-73a^8-156a^7bx-84a^6b^2x^2}{3a^3b^9+9a^2b^{10}x+9ab^{11}x^2+3b^{12}x^3} + \frac{x^5}{5b^4}$$

input
```
integrate(x**8/(b*x+a)**4,x)
```

output
```
-56*a**5*log(a + b*x)/b**9 + 35*a**4*x/b**8 - 10*a**3*x**2/b**7 + 10*a**2*
x**3/(3*b**6) - a*x**4/b**5 + (-73*a**8 - 156*a**7*b*x - 84*a**6*b**2*x**2
)/(3*a**3*b**9 + 9*a**2*b**10*x + 9*a*b**11*x**2 + 3*b**12*x**3) + x**5/(5
*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{x^8}{(a+bx)^4}\,dx = -\frac{84\,a^6b^2x^2+156\,a^7bx+73\,a^8}{3\,(b^{12}x^3+3\,ab^{11}x^2+3\,a^2b^{10}x+a^3b^9)} - \frac{56\,a^5\log{(bx+a)}}{b^9} + \frac{3\,b^4x^5-15\,ab^3x^4+50\,a^2b^2x^3-150\,a^3bx^2+525\,a^4x}{15\,b^8}$$

input
```
integrate(x^8/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)/(b^12*x^3 + 3*a*b^11*x^2 + 3*
a^2*b^10*x + a^3*b^9) - 56*a^5*log(b*x + a)/b^9 + 1/15*(3*b^4*x^5 - 15*a*b
^3*x^4 + 50*a^2*b^2*x^3 - 150*a^3*b*x^2 + 525*a^4*x)/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(a+bx)^4}\,dx = -\frac{56\,a^5\log\left(|bx+a|\right)}{b^9} - \frac{84\,a^6b^2x^2 + 156\,a^7bx + 73\,a^8}{3\,(bx+a)^3b^9} + \frac{3\,b^{16}x^5 - 15\,ab^{15}x^4 + 50\,a^2b^{14}x^3 - 150\,a^3b^{13}x^2 + 525\,a^4b^{12}x}{15\,b^{20}}$$

input
```
integrate(x^8/(b*x+a)^4,x, algorithm="giac")
```

output
```
-56*a^5*log(abs(b*x + a))/b^9 - 1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8
)/((b*x + a)^3*b^9) + 1/15*(3*b^16*x^5 - 15*a*b^15*x^4 + 50*a^2*b^14*x^3 -
 150*a^3*b^13*x^2 + 525*a^4*b^12*x)/b^20
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{(a+bx)^4}\,dx = -\frac{2\,a\,(a+b\,x)^4 - \frac{(a+b\,x)^5}{5} - \frac{28\,a^2\,(a+b\,x)^3}{3} + 28\,a^3\,(a+b\,x)^2 + \frac{28\,a^6}{a+b\,x} - \frac{4\,a^7}{(a+b\,x)^2} + \frac{a^8}{3\,(a+b\,x)^3} + 56\,a^5\,\ln(a+b\,x}{b^9}$$

input
```
int(x^8/(a + b*x)^4,x)
```

output
```
-(2*a*(a + b*x)^4 - (a + b*x)^5/5 - (28*a^2*(a + b*x)^3)/3 + 28*a^3*(a + b
*x)^2 + (28*a^6)/(a + b*x) - (4*a^7)/(a + b*x)^2 + a^8/(3*(a + b*x)^3) + 5
6*a^5*log(a + b*x) - 70*a^4*b*x)/b^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.46

$$\int \frac{x^8}{(a+bx)^4}\,dx$$
$$= \frac{-840\log(bx+a)\,a^8 - 2520\log(bx+a)\,a^7bx - 2520\log(bx+a)\,a^6b^2x^2 - 840\log(bx+a)\,a^5b^3x^3 - 700a^8}{15b^9\,(b^3x^3 + 3a\,b^2x^2 + 3a^2bx}$$

input
```
int(x^8/(b*x+a)^4,x)
```

output
```
( - 840*log(a + b*x)*a**8 - 2520*log(a + b*x)*a**7*b*x - 2520*log(a + b*x)
*a**6*b**2*x**2 - 840*log(a + b*x)*a**5*b**3*x**3 - 700*a**8 - 1260*a**7*b
*x + 840*a**5*b**3*x**3 + 210*a**4*b**4*x**4 - 42*a**3*b**5*x**5 + 14*a**2
*b**6*x**6 - 6*a*b**7*x**7 + 3*b**8*x**8)/(15*b**9*(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3))
```

3.153 $\int \frac{x^7}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 105

$$\int \frac{x^7}{(a+bx)^4}\,dx = -\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}$$

output
```
-20*a^3*x/b^7+5*a^2*x^2/b^6-4/3*a*x^3/b^5+1/4*x^4/b^4+1/3*a^7/b^8/(b*x+a)^
3-7/2*a^6/b^8/(b*x+a)^2+21*a^5/b^8/(b*x+a)+35*a^4*ln(b*x+a)/b^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(a+bx)^4}\,dx = \frac{-240a^3bx+60a^2b^2x^2-16ab^3x^3+3b^4x^4+\frac{4a^7}{(a+bx)^3}-\frac{42a^6}{(a+bx)^2}+\frac{252a^5}{a+bx}+420a^4\log(a+bx)}{12b^8}$$

input
```
Integrate[x^7/(a + b*x)^4,x]
```

output
```
(-240*a^3*b*x + 60*a^2*b^2*x^2 - 16*a*b^3*x^3 + 3*b^4*x^4 + (4*a^7)/(a + b
*x)^3 - (42*a^6)/(a + b*x)^2 + (252*a^5)/(a + b*x) + 420*a^4*Log[a + b*x])
/(12*b^8)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx)^4} \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(-\frac{a^7}{b^7(a+bx)^4}+\frac{7a^6}{b^7(a+bx)^3}-\frac{21a^5}{b^7(a+bx)^2}+\frac{35a^4}{b^7(a+bx)}-\frac{20a^3}{b^7}+\frac{10a^2x}{b^6}-\frac{4ax^2}{b^5}+\frac{x^3}{b^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

input
```
Int[x^7/(a + b*x)^4,x]
```

output
```
(-20*a^3*x)/b^7 + (5*a^2*x^2)/b^6 - (4*a*x^3)/(3*b^5) + x^4/(4*b^4) + a^7/
(3*b^8*(a + b*x)^3) - (7*a^6)/(2*b^8*(a + b*x)^2) + (21*a^5)/(b^8*(a + b*x
)) + (35*a^4*Log[a + b*x])/b^8
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

method	result
risch	$\frac{x^4}{4b^4}-\frac{4a\,x^3}{3b^5}+\frac{5a^2x^2}{b^6}-\frac{20a^3x}{b^7}+\frac{21a^5b\,x^2+\frac{77a^6x}{2}+\frac{107a^7}{6b}}{b^7\left(bx+a\right)^3}+\frac{35a^4\ln\left(bx+a\right)}{b^8}$
norman	$\frac{\frac{x^7}{4b}-\frac{7a\,x^6}{12b^2}+\frac{7a^2x^5}{4b^3}-\frac{35a^3x^4}{4b^4}+\frac{385a^7}{6b^8}+\frac{105a^5x^2}{b^6}+\frac{315a^6x}{2b^7}}{\left(bx+a\right)^3}+\frac{35a^4\ln\left(bx+a\right)}{b^8}$
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{4}{3}a\,b^2x^3-5a^2b\,x^2+20a^3x}{b^7}-\frac{7a^6}{2b^8\left(bx+a\right)^2}+\frac{21a^5}{b^8\left(bx+a\right)}+\frac{35a^4\ln\left(bx+a\right)}{b^8}+\frac{a^7}{3b^8\left(bx+a\right)^3}$
parallelrisch	$\frac{3b^7x^7-7a\,b^6x^6+21a^2b^5x^5+420\ln\left(bx+a\right)x^3a^4b^3-105a^3b^4x^4+1260\ln\left(bx+a\right)x^2a^5b^2+1260\ln\left(bx+a\right)x\,a^6b+1260a^5b^2x^2+420}{12b^8\left(bx+a\right)^3}$

input
```
int(x^7/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/b^4-4/3*a*x^3/b^5+5*a^2*x^2/b^6-20*a^3*x/b^7+(21*a^5*b*x^2+77/2*a^
6*x+107/6*a^7/b)/b^7/(b*x+a)^3+35*a^4*ln(b*x+a)/b^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.44

$$\int \frac{x^7}{(a+bx)^4}\,dx$$
$$=\frac{3\,b^7x^7-7\,ab^6x^6+21\,a^2b^5x^5-105\,a^3b^4x^4-556\,a^4b^3x^3-408\,a^5b^2x^2+222\,a^6bx+214\,a^7+420\,(a^4b^3x^3}{12\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)}$$

input
```
integrate(x^7/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/12*(3*b^7*x^7 - 7*a*b^6*x^6 + 21*a^2*b^5*x^5 - 105*a^3*b^4*x^4 - 556*a^4
*b^3*x^3 - 408*a^5*b^2*x^2 + 222*a^6*b*x + 214*a^7 + 420*(a^4*b^3*x^3 + 3*
a^5*b^2*x^2 + 3*a^6*b*x + a^7)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*
a^2*b^9*x + a^3*b^8)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{(a+bx)^4}\,dx = \frac{35a^4\log{(a+bx)}}{b^8} - \frac{20a^3x}{b^7} + \frac{5a^2x^2}{b^6} - \frac{4ax^3}{3b^5} + \frac{107a^7+231a^6bx+126a^5b^2x^2}{6a^3b^8+18a^2b^9x+18ab^{10}x^2+6b^{11}x^3} + \frac{x^4}{4b^4}$$

input
```
integrate(x**7/(b*x+a)**4,x)
```

output
```
35*a**4*log(a + b*x)/b**8 - 20*a**3*x/b**7 + 5*a**2*x**2/b**6 - 4*a*x**3/(
3*b**5) + (107*a**7 + 231*a**6*b*x + 126*a**5*b**2*x**2)/(6*a**3*b**8 + 18
*a**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*x**3) + x**4/(4*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

$$\int \frac{x^7}{(a+bx)^4}\,dx = \frac{126\,a^5b^2x^2+231\,a^6bx+107\,a^7}{6\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)} + \frac{35\,a^4\log{(bx+a)}}{b^8} + \frac{3\,b^3x^4-16\,ab^2x^3+60\,a^2bx^2-240\,a^3x}{12\,b^7}$$

input
```
integrate(x^7/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7)/(b^11*x^3 + 3*a*b^10*x^2 + 3
*a^2*b^9*x + a^3*b^8) + 35*a^4*log(b*x + a)/b^8 + 1/12*(3*b^3*x^4 - 16*a*b
^2*x^3 + 60*a^2*b*x^2 - 240*a^3*x)/b^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{x^7}{(a+bx)^4}\,dx = \frac{35\,a^4\log\left(|bx+a|\right)}{b^8} + \frac{126\,a^5b^2x^2 + 231\,a^6bx + 107\,a^7}{6\,(bx+a)^3b^8} + \frac{3\,b^{12}x^4 - 16\,ab^{11}x^3 + 60\,a^2b^{10}x^2 - 240\,a^3b^9x}{12\,b^{16}}$$

input
```
integrate(x^7/(b*x+a)^4,x, algorithm="giac")
```

output
```
35*a^4*log(abs(b*x + a))/b^8 + 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^
7)/((b*x + a)^3*b^8) + 1/12*(3*b^12*x^4 - 16*a*b^11*x^3 + 60*a^2*b^10*x^2
- 240*a^3*b^9*x)/b^16
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(a+bx)^4}\,dx = \frac{\frac{(a+b\,x)^4}{4} - \frac{7\,a\,(a+b\,x)^3}{3} + \frac{21\,a^2\,(a+b\,x)^2}{2} + \frac{21\,a^5}{a+b\,x} - \frac{7\,a^6}{2\,(a+b\,x)^2} + \frac{a^7}{3\,(a+b\,x)^3} + 35\,a^4\,\ln\left(a+b\,x\right) - 35\,a^3\,b\,x}{b^8}$$

input
```
int(x^7/(a + b*x)^4,x)
```

output
```
((a + b*x)^4/4 - (7*a*(a + b*x)^3)/3 + (21*a^2*(a + b*x)^2)/2 + (21*a^5)/(
a + b*x) - (7*a^6)/(2*(a + b*x)^2) + a^7/(3*(a + b*x)^3) + 35*a^4*log(a +
b*x) - 35*a^3*b*x)/b^8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.48

$$\int \frac{x^7}{(a+bx)^4}\,dx = \frac{420\log(bx+a)\,a^7 + 1260\log(bx+a)\,a^6bx + 1260\log(bx+a)\,a^5b^2x^2 + 420\log(bx+a)\,a^4b^3x^3 + 350a^7 -}{12b^8\,(b^3x^3 + 3a\,b^2x^2 + 3a^2bx + a^3)}$$

input
```
int(x^7/(b*x+a)^4,x)
```

output
```
(420*log(a + b*x)*a**7 + 1260*log(a + b*x)*a**6*b*x + 1260*log(a + b*x)*a*
*5*b**2*x**2 + 420*log(a + b*x)*a**4*b**3*x**3 + 350*a**7 + 630*a**6*b*x -
 420*a**4*b**3*x**3 - 105*a**3*b**4*x**4 + 21*a**2*b**5*x**5 - 7*a*b**6*x*
*6 + 3*b**7*x**7)/(12*b**8*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
)
```

3.154 $\int \frac{x^6}{(a+bx)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 90

$$\int \frac{x^6}{(a+bx)^4} dx = \frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{x^3}{3b^4} - \frac{a^6}{3b^7(a+bx)^3} + \frac{3a^5}{b^7(a+bx)^2} - \frac{15a^4}{b^7(a+bx)} - \frac{20a^3\log(a+bx)}{b^7}$$

output
```
10*a^2*x/b^6-2*a*x^2/b^5+1/3*x^3/b^4-1/3*a^6/b^7/(b*x+a)^3+3*a^5/b^7/(b*x+
a)^2-15*a^4/b^7/(b*x+a)-20*a^3*ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a+bx)^4} dx = \frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{x^3}{3b^4} - \frac{a^6}{3b^7(a+bx)^3} + \frac{3a^5}{b^7(a+bx)^2} - \frac{15a^4}{b^7(a+bx)} - \frac{20a^3\log(a+bx)}{b^7}$$

input
```
Integrate[x^6/(a + b*x)^4,x]
```

output
```
(10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (
3*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x]
)/b^7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx)^4}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^6}{b^6(a+bx)^4}-\frac{6a^5}{b^6(a+bx)^3}+\frac{15a^4}{b^6(a+bx)^2}-\frac{20a^3}{b^6(a+bx)}+\frac{10a^2}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

input
```
Int[x^6/(a + b*x)^4,x]
```

output

```
(10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (
3*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x]
)/b^7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x^3}{3b^4}-\frac{2a\,x^2}{b^5}+\frac{10a^2x}{b^6}+\frac{-15a^4b\,x^2-27a^5x-\frac{37a^6}{3b}}{b^6(bx+a)^3}-\frac{20a^3\ln(bx+a)}{b^7}$
norman	$\frac{\frac{x^6}{3b}-\frac{a\,x^5}{b^2}+\frac{5a^2x^4}{b^3}-\frac{110a^6}{3b^7}-\frac{60a^4x^2}{b^5}-\frac{90a^5x}{b^6}}{(bx+a)^3}-\frac{20a^3\ln(bx+a)}{b^7}$
default	$\frac{\frac{1}{3}b^2x^3-2ab\,x^2+10a^2x}{b^6}+\frac{3a^5}{b^7(bx+a)^2}-\frac{15a^4}{b^7(bx+a)}-\frac{20a^3\ln(bx+a)}{b^7}-\frac{a^6}{3b^7(bx+a)^3}$
parallelrisch	$-\frac{-b^6x^6+3a\,x^5b^5+60\ln(bx+a)x^3a^3b^3-15a^2x^4b^4+180\ln(bx+a)x^2a^4b^2+180\ln(bx+a)x\,a^5b+180a^4x^2b^2+60\ln(bx+a)a^6+2}{3b^7(bx+a)^3}$

input
```
int(x^6/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/b^4-2*a*x^2/b^5+10*a^2*x/b^6+(-15*a^4*b*x^2-27*a^5*x-37/3/b*a^6)/b
^6/(b*x+a)^3-20*a^3*ln(b*x+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.54

$$\int \frac{x^6}{(a+bx)^4}\,dx$$
$$=\frac{b^6x^6-3\,ab^5x^5+15\,a^2b^4x^4+73\,a^3b^3x^3+39\,a^4b^2x^2-51\,a^5bx-37\,a^6-60\,(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+}{3\,(b^{10}x^3+3\,ab^9x^2+3\,a^2b^8x+a^3b^7)}$$

input
```
integrate(x^6/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(b^6*x^6 - 3*a*b^5*x^5 + 15*a^2*b^4*x^4 + 73*a^3*b^3*x^3 + 39*a^4*b^2*
x^2 - 51*a^5*b*x - 37*a^6 - 60*(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*x +
a^6)*log(b*x + a))/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

$$\int \frac{x^6}{(a+bx)^4}\,dx = -\frac{20a^3\log{(a+bx)}}{b^7} + \frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{-37a^6 - 81a^5bx - 45a^4b^2x^2}{3a^3b^7 + 9a^2b^8x + 9ab^9x^2 + 3b^{10}x^3} + \frac{x^3}{3b^4}$$

input
```
integrate(x**6/(b*x+a)**4,x)
```

output
```
-20*a**3*log(a + b*x)/b**7 + 10*a**2*x/b**6 - 2*a*x**2/b**5 + (-37*a**6 -
81*a**5*b*x - 45*a**4*b**2*x**2)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x
**2 + 3*b**10*x**3) + x**3/(3*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \frac{x^6}{(a+bx)^4}\,dx = -\frac{45\,a^4b^2x^2 + 81\,a^5bx + 37\,a^6}{3\,(b^{10}x^3 + 3\,ab^9x^2 + 3\,a^2b^8x + a^3b^7)} - \frac{20\,a^3\log{(bx+a)}}{b^7} + \frac{b^2x^3 - 6\,abx^2 + 30\,a^2x}{3\,b^6}$$

input
```
integrate(x^6/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^
2*b^8*x + a^3*b^7) - 20*a^3*log(b*x + a)/b^7 + 1/3*(b^2*x^3 - 6*a*b*x^2 +
30*a^2*x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(a+bx)^4}\,dx = -\frac{20\,a^3\log\left(|bx+a|\right)}{b^7} - \frac{45\,a^4b^2x^2 + 81\,a^5bx + 37\,a^6}{3\,(bx+a)^3b^7} + \frac{b^8x^3 - 6\,ab^7x^2 + 30\,a^2b^6x}{3\,b^{12}}$$

input
```
integrate(x^6/(b*x+a)^4,x, algorithm="giac")
```

output
```
-20*a^3*log(abs(b*x + a))/b^7 - 1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)
/((b*x + a)^3*b^7) + 1/3*(b^8*x^3 - 6*a*b^7*x^2 + 30*a^2*b^6*x)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{(a+bx)^4}\,dx = -\frac{3\,a\,(a+b\,x)^2 - \frac{(a+b\,x)^3}{3} + \frac{15\,a^4}{a+b\,x} - \frac{3\,a^5}{(a+b\,x)^2} + \frac{a^6}{3\,(a+b\,x)^3} + 20\,a^3\,\ln\left(a+b\,x\right) - 15\,a^2\,b\,x}{b^7}$$

input
```
int(x^6/(a + b*x)^4,x)
```

output
```
-(3*a*(a + b*x)^2 - (a + b*x)^3/3 + (15*a^4)/(a + b*x) - (3*a^5)/(a + b*x)
^2 + a^6/(3*(a + b*x)^3) + 20*a^3*log(a + b*x) - 15*a^2*b*x)/b^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.59

$$\int \frac{x^6}{(a+bx)^4}\,dx = \frac{-60\log(bx+a)\,a^6-180\log(bx+a)\,a^5bx-180\log(bx+a)\,a^4b^2x^2-60\log(bx+a)\,a^3b^3x^3-50a^6-90}{3b^7\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}$$

input
```
int(x^6/(b*x+a)^4,x)
```

output
```
( - 60*log(a + b*x)*a**6 - 180*log(a + b*x)*a**5*b*x - 180*log(a + b*x)*a*
*4*b**2*x**2 - 60*log(a + b*x)*a**3*b**3*x**3 - 50*a**6 - 90*a**5*b*x + 60
*a**3*b**3*x**3 + 15*a**2*b**4*x**4 - 3*a*b**5*x**5 + b**6*x**6)/(3*b**7*(
a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.155 $\int \frac{x^5}{(a+bx)^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 81

$$\int \frac{x^5}{(a+bx)^4} \, dx = -\frac{4ax}{b^5} + \frac{x^2}{2b^4} + \frac{a^5}{3b^6(a+bx)^3} - \frac{5a^4}{2b^6(a+bx)^2} + \frac{10a^3}{b^6(a+bx)} + \frac{10a^2\log(a+bx)}{b^6}$$

output
```
-4*a*x/b^5+1/2*x^2/b^4+1/3*a^5/b^6/(b*x+a)^3-5/2*a^4/b^6/(b*x+a)^2+10*a^3/
b^6/(b*x+a)+10*a^2*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{(a+bx)^4} \, dx = \frac{-24abx + 3b^2x^2 + \frac{2a^5}{(a+bx)^3} - \frac{15a^4}{(a+bx)^2} + \frac{60a^3}{a+bx} + 60a^2\log(a+bx)}{6b^6}$$

input
```
Integrate[x^5/(a + b*x)^4,x]
```

output
```
(-24*a*b*x + 3*b^2*x^2 + (2*a^5)/(a + b*x)^3 - (15*a^4)/(a + b*x)^2 + (60*
a^3)/(a + b*x) + 60*a^2*Log[a + b*x])/(6*b^6)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx)^4} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(-\frac{a^5}{b^5 (a+bx)^4}+\frac{5 a^4}{b^5 (a+bx)^3}-\frac{10 a^3}{b^5 (a+bx)^2}+\frac{10 a^2}{b^5 (a+bx)}-\frac{4 a}{b^5}+\frac{x}{b^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^5}{3 b^6 (a+bx)^3}-\frac{5 a^4}{2 b^6 (a+bx)^2}+\frac{10 a^3}{b^6 (a+bx)}+\frac{10 a^2 \log (a+bx)}{b^6}-\frac{4 a x}{b^5}+\frac{x^2}{2 b^4}$$

input
```
Int[x^5/(a + b*x)^4,x]
```

output
```
(-4*a*x)/b^5 + x^2/(2*b^4) + a^5/(3*b^6*(a + b*x)^3) - (5*a^4)/(2*b^6*(a +
 b*x)^2) + (10*a^3)/(b^6*(a + b*x)) + (10*a^2*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result
risch	$\frac{x^2}{2b^4}-\frac{4ax}{b^5}+\frac{10a^3b\,x^2+\frac{35a^4x}{2}+\frac{47a^5}{6b}}{b^5(bx+a)^3}+\frac{10a^2\ln(bx+a)}{b^6}$
norman	$\frac{\frac{x^5}{2b}-\frac{5a\,x^4}{2b^2}+\frac{55a^5}{3b^6}+\frac{30a^3x^2}{b^4}+\frac{45a^4x}{b^5}}{(bx+a)^3}+\frac{10a^2\ln(bx+a)}{b^6}$
default	$-\frac{-\frac{1}{2}b\,x^2+4ax}{b^5}-\frac{5a^4}{2b^6(bx+a)^2}+\frac{10a^3}{b^6(bx+a)}+\frac{10a^2\ln(bx+a)}{b^6}+\frac{a^5}{3b^6(bx+a)^3}$
parallelrisch	$\frac{3b^5x^5+60\ln(bx+a)x^3a^2b^3-15a\,b^4x^4+180\ln(bx+a)x^2a^3b^2+180\ln(bx+a)x\,a^4b+180a^3b^2x^2+60a^5\ln(bx+a)+270a^4bx+110}{6b^6(bx+a)^3}$

input
```
int(x^5/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b^4-4*a*x/b^5+(10*a^3*b*x^2+35/2*a^4*x+47/6*a^5/b)/b^5/(b*x+a)^3+1
0*a^2*ln(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.59

$$\int \frac{x^5}{(a+bx)^4}\,dx = \frac{3\,b^5x^5-15\,ab^4x^4-63\,a^2b^3x^3-9\,a^3b^2x^2+81\,a^4bx+47\,a^5+60\,(a^2b^3x^3+3\,a^3b^2x^2+3\,a^4bx+a^5)\log(b}{6\,(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6)}$$

input
```
integrate(x^5/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(3*b^5*x^5 - 15*a*b^4*x^4 - 63*a^2*b^3*x^3 - 9*a^3*b^2*x^2 + 81*a^4*b*
x + 47*a^5 + 60*(a^2*b^3*x^3 + 3*a^3*b^2*x^2 + 3*a^4*b*x + a^5)*log(b*x +
a))/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{(a+bx)^4}\,dx = \frac{10a^2\log{(a+bx)}}{b^6} - \frac{4ax}{b^5} + \frac{47a^5+105a^4bx+60a^3b^2x^2}{6a^3b^6+18a^2b^7x+18ab^8x^2+6b^9x^3} + \frac{x^2}{2b^4}$$

input
```
integrate(x**5/(b*x+a)**4,x)
```

output
```
10*a**2*log(a + b*x)/b**6 - 4*a*x/b**5 + (47*a**5 + 105*a**4*b*x + 60*a**3
*b**2*x**2)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
+ x**2/(2*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(a+bx)^4}\,dx = \frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\,(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6)} + \frac{10\,a^2\log{(bx+a)}}{b^6} + \frac{bx^2-8\,ax}{2\,b^5}$$

input
```
integrate(x^5/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(60*a^3*b^2*x^2 + 105*a^4*b*x + 47*a^5)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2
*b^7*x + a^3*b^6) + 10*a^2*log(b*x + a)/b^6 + 1/2*(b*x^2 - 8*a*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(a+bx)^4}\,dx = \frac{10\,a^2\log{(|bx+a|)}}{b^6} + \frac{b^4x^2-8\,ab^3x}{2\,b^8} + \frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\,(bx+a)^3b^6}$$

input
```
integrate(x^5/(b*x+a)^4,x, algorithm="giac")
```

output
```
10*a^2*log(abs(b*x + a))/b^6 + 1/2*(b^4*x^2 - 8*a*b^3*x)/b^8 + 1/6*(60*a^3
*b^2*x^2 + 105*a^4*b*x + 47*a^5)/((b*x + a)^3*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(a+bx)^4}\,dx = \frac{\frac{(a+b\,x)^2}{2}+\frac{10\,a^3}{a+b\,x}-\frac{5\,a^4}{2\,(a+b\,x)^2}+\frac{a^5}{3\,(a+b\,x)^3}+10\,a^2\,\ln\left(a+b\,x\right)-5\,a\,b\,x}{b^6}$$

input
```
int(x^5/(a + b*x)^4,x)
```

output
```
((a + b*x)^2/2 + (10*a^3)/(a + b*x) - (5*a^4)/(2*(a + b*x)^2) + a^5/(3*(a
+ b*x)^3) + 10*a^2*log(a + b*x) - 5*a*b*x)/b^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.64

$$\int \frac{x^5}{(a+bx)^4}\,dx = \frac{60\log(bx+a)\,a^5+180\log(bx+a)\,a^4bx+180\log(bx+a)\,a^3b^2x^2+60\log(bx+a)\,a^2b^3x^3+50a^5+90a^4}{6b^6\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}$$

input
```
int(x^5/(b*x+a)^4,x)
```

output
```
(60*log(a + b*x)*a**5 + 180*log(a + b*x)*a**4*b*x + 180*log(a + b*x)*a**3*
b**2*x**2 + 60*log(a + b*x)*a**2*b**3*x**3 + 50*a**5 + 90*a**4*b*x - 60*a*
*2*b**3*x**3 - 15*a*b**4*x**4 + 3*b**5*x**5)/(6*b**6*(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3))
```

3.156 $\int \frac{x^4}{(a+bx)^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int \frac{x^4}{(a+bx)^4} \, dx = \frac{x}{b^4} - \frac{a^4}{3b^5(a+bx)^3} + \frac{2a^3}{b^5(a+bx)^2} - \frac{6a^2}{b^5(a+bx)} - \frac{4a\log(a+bx)}{b^5}$$

output
```
x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b
*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(a+bx)^4} \, dx = -\frac{-3bx + \frac{a^2(13a^2+30abx+18b^2x^2)}{(a+bx)^3} + 12a\log(a+bx)}{3b^5}$$

input
```
Integrate[x^4/(a + b*x)^4,x]
```

output
```
-1/3*(-3*b*x + (a^2*(13*a^2 + 30*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 12*a*L
og[a + b*x])/b^5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx)^4}\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^4}{b^4(a+bx)^4}-\frac{4a^3}{b^4(a+bx)^3}+\frac{6a^2}{b^4(a+bx)^2}-\frac{4a}{b^4(a+bx)}+\frac{1}{b^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

input
```
Int[x^4/(a + b*x)^4,x]
```

output
```
x/b^4 - a^4/(3*b^5*(a + b*x)^3) + (2*a^3)/(b^5*(a + b*x)^2) - (6*a^2)/(b^5
*(a + b*x)) - (4*a*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x}{b^4}+\frac{-6a^2b\,x^2-10a^3x-\frac{13a^4}{3b}}{b^4(bx+a)^3}-\frac{4a\ln(bx+a)}{b^5}$	54
norman	$\frac{\frac{x^4}{b}-\frac{22a^4}{3b^5}-\frac{12a^2x^2}{b^3}-\frac{18a^3x}{b^4}}{(bx+a)^3}-\frac{4a\ln(bx+a)}{b^5}$	58
default	$\frac{x}{b^4}-\frac{a^4}{3b^5(bx+a)^3}+\frac{2a^3}{b^5(bx+a)^2}-\frac{6a^2}{b^5(bx+a)}-\frac{4a\ln(bx+a)}{b^5}$	64
parallelrisch	$-\frac{12\ln(bx+a)x^3a\,b^3-3b^4x^4+36\ln(bx+a)x^2a^2b^2+36\ln(bx+a)x\,a^3b+36a^2b^2x^2+12a^4\ln(bx+a)+54a^3bx+22a^4}{3b^5(bx+a)^3}$	101

input
```
int(x^4/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
x/b^4+(-6*a^2*b*x^2-10*a^3*x-13/3/b*a^4)/b^4/(b*x+a)^3-4*a*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{x^4}{(a+bx)^4}\,dx = \frac{3\,b^4x^4+9\,ab^3x^3-9\,a^2b^2x^2-27\,a^3bx-13\,a^4-12\,(ab^3x^3+3\,a^2b^2x^2+3\,a^3bx+a^4)\log(bx+a)}{3\,(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5)}$$

input
```
integrate(x^4/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^4*x^4 + 9*a*b^3*x^3 - 9*a^2*b^2*x^2 - 27*a^3*b*x - 13*a^4 - 12*(a
*b^3*x^3 + 3*a^2*b^2*x^2 + 3*a^3*b*x + a^4)*log(b*x + a))/(b^8*x^3 + 3*a*b
^7*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(a+bx)^4}\,dx = -\frac{4a\log{(a+bx)}}{b^5} + \frac{-13a^4-30a^3bx-18a^2b^2x^2}{3a^3b^5+9a^2b^6x+9ab^7x^2+3b^8x^3} + \frac{x}{b^4}$$

input
```
integrate(x**4/(b*x+a)**4,x)
```

output
```
-4*a*log(a + b*x)/b**5 + (-13*a**4 - 30*a**3*b*x - 18*a**2*b**2*x**2)/(3*a
**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + x/b**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+bx)^4}\,dx = -\frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\,(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5)} + \frac{x}{b^4} - \frac{4\,a\log{(bx+a)}}{b^5}$$

input
```
integrate(x^4/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a^4)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2
*b^6*x + a^3*b^5) + x/b^4 - 4*a*log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a+bx)^4}\,dx = \frac{x}{b^4} - \frac{4\,a\log{(|bx+a|)}}{b^5} - \frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\,(bx+a)^3b^5}$$

input
```
integrate(x^4/(b*x+a)^4,x, algorithm="giac")
```

output
```
x/b^4 - 4*a*log(abs(b*x + a))/b^5 - 1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*
a^4)/((b*x + a)^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a+bx)^4}\,dx = -\frac{4\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{6\,a^2}{a+b\,x}-\frac{2\,a^3}{{\left(a+b\,x\right)}^2}+\frac{a^4}{3\,{\left(a+b\,x\right)}^3}}{b^5}$$

input
```
int(x^4/(a + b*x)^4,x)
```

output
```
-(4*a*log(a + b*x) - b*x + (6*a^2)/(a + b*x) - (2*a^3)/(a + b*x)^2 + a^4/(
3*(a + b*x)^3))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{x^4}{(a+bx)^4}\,dx = \frac{-12\log(bx+a)\,a^4-36\log(bx+a)\,a^3bx-36\log(bx+a)\,a^2b^2x^2-12\log(bx+a)\,a\,b^3x^3-10a^4-18a^3b}{3b^5\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}$$

input
```
int(x^4/(b*x+a)^4,x)
```

output
```
( - 12*log(a + b*x)*a**4 - 36*log(a + b*x)*a**3*b*x - 36*log(a + b*x)*a**2
*b**2*x**2 - 12*log(a + b*x)*a*b**3*x**3 - 10*a**4 - 18*a**3*b*x + 12*a*b*
*3*x**3 + 3*b**4*x**4)/(3*b**5*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3))
```

3.157 $\int \frac{x^3}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{x^3}{(a+bx)^4}\,dx=\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}$$

output
```
1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{(a+bx)^4}\,dx=\frac{\frac{a(11a^2+27abx+18b^2x^2)}{(a+bx)^3}+6\log(a+bx)}{6b^4}$$

input
```
Integrate[x^3/(a + b*x)^4,x]
```

output
```
((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^4} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^4}+\frac{3a^2}{b^3(a+bx)^3}-\frac{3a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}$$

input
```
Int[x^3/(a + b*x)^4,x]
```

output
```
a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*
x)) + Log[a + b*x]/b^4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
risch	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
default	$\frac{a^3}{3b^4(bx+a)^3}-\frac{3a^2}{2b^4(bx+a)^2}+\frac{3a}{b^4(bx+a)}+\frac{\ln(bx+a)}{b^4}$	55
parallelrisch	$\frac{6b^3\ln(bx+a)x^3+18\ln(bx+a)x^2a\,b^2+18\ln(bx+a)x\,a^2b+18a\,b^2x^2+6a^3\ln(bx+a)+27a^2bx+11a^3}{6b^4(bx+a)^3}$	88

input
```
int(x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(11/6*a^3/b^4+3*a*x^2/b^2+9/2*a^2*x/b^3)/(b*x+a)^3+ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{18\,ab^2x^2+27\,a^2bx+11\,a^3+6\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log(bx+a)}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}$$

input
```
integrate(x^3/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{11a^3+27a^2bx+18ab^2x^2}{6a^3b^4+18a^2b^5x+18ab^6x^2+6b^7x^3}+\frac{\log{(a+bx)}}{b^4}$$

input
```
integrate(x**3/(b*x+a)**4,x)
```

output
```
(11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 1
8*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{18\,ab^2x^2+27\,a^2bx+11\,a^3}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}+\frac{\log{(bx+a)}}{b^4}$$

input
```
integrate(x^3/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^
5*x + a^3*b^4) + log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{\log{(|bx+a|)}}{b^4}+\frac{18\,abx^2+27\,a^2x+\frac{11\,a^3}{b}}{6\,(bx+a)^3b^3}$$

input
```
integrate(x^3/(b*x+a)^4,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^
3*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{\ln(a+b\,x)+\frac{3\,a}{a+b\,x}-\frac{3\,a^2}{2\,(a+b\,x)^2}+\frac{a^3}{3\,(a+b\,x)^3}}{b^4}$$

input
```
int(x^3/(a + b*x)^4,x)
```

output
```
(log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*(a + b*
x)^3))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.86

$$\int \frac{x^3}{(a+bx)^4}\,dx = \frac{6\log(bx+a)\,a^3+18\log(bx+a)\,a^2bx+18\log(bx+a)\,a\,b^2x^2+6\log(bx+a)\,b^3x^3+5a^3+9a^2bx-6b^3x}{6b^4\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(x^3/(b*x+a)^4,x)
```

output
```
(6*log(a + b*x)*a**3 + 18*log(a + b*x)*a**2*b*x + 18*log(a + b*x)*a*b**2*x
**2 + 6*log(a + b*x)*b**3*x**3 + 5*a**3 + 9*a**2*b*x - 6*b**3*x**3)/(6*b**
4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.158 $\int \frac{x^2}{(a+bx)^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{x^2}{(a+bx)^4} \, dx = \frac{x^3}{3a(a+bx)^3}$$

output
```
1/3*x^3/a/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{x^2}{(a+bx)^4} \, dx = -\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

input
```
Integrate[x^2/(a + b*x)^4,x]
```

output
```
-1/3*(a^2 + 3*a*b*x + 3*b^2*x^2)/(b^3*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^4} \, dx$$
$$\Big\downarrow \; 48$$
$$\frac{x^3}{3a(a+bx)^3}$$

input
```
Int[x^2/(a + b*x)^4,x]
```

output
```
x^3/(3*a*(a + b*x)^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

method	result	size
gosper	$-\frac{3b^{2}x^{2}+3abx+a^{2}}{3\left(bx+a\right)^{3} b^{3}}$	30
orering	$-\frac{3b^{2}x^{2}+3abx+a^{2}}{3\left(bx+a\right)^{3} b^{3}}$	30
parallelrisch	$\frac{-3b^{2}x^{2}-3abx-a^{2}}{3b^{3}\left(bx+a\right)^{3}}$	32
norman	$\frac{-\frac{x^{2}}{b}-\frac{ax}{b^{2}}-\frac{a^{2}}{3b^{3}}}{\left(bx+a\right)^{3}}$	33
risch	$\frac{-\frac{x^{2}}{b}-\frac{ax}{b^{2}}-\frac{a^{2}}{3b^{3}}}{\left(bx+a\right)^{3}}$	33
default	$\frac{a}{b^{3}\left(bx+a\right)^{2}}-\frac{1}{\left(bx+a\right) b^{3}}-\frac{a^{2}}{3b^{3}\left(bx+a\right)^{3}}$	41

input
```
int(x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b^2*x^2+3*a*b*x+a^2)/(b*x+a)^3/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.18

$$\int \frac{x^2}{(a+bx)^4}\,dx=-\frac{3\,b^{2}x^{2}+3\,abx+a^{2}}{3\,\left(b^{6}x^{3}+3\,ab^{5}x^{2}+3\,a^{2}b^{4}x+a^{3}b^{3}\right)}$$

input
```
integrate(x^2/(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^
3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int \frac{x^2}{(a+bx)^4}\,dx = \frac{-a^2 - 3abx - 3b^2x^2}{3a^3b^3 + 9a^2b^4x + 9ab^5x^2 + 3b^6x^3}$$

input
```
integrate(x**2/(b*x+a)**4,x)
```

output
```
(-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x*
*2 + 3*b**6*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(15) = 30$.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.18

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,(b^6x^3 + 3\,ab^5x^2 + 3\,a^2b^4x + a^3b^3)}$$

input
```
integrate(x^2/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^
3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,(bx+a)^3b^3}$$

input
```
integrate(x^2/(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int \frac{x^2}{(a+bx)^4}\,dx = -\frac{a^2+3\,a\,b\,x+3\,b^2\,x^2}{3\,a^3\,b^3+9\,a^2\,b^4\,x+9\,a\,b^5\,x^2+3\,b^6\,x^3}$$

input
```
int(x^2/(a + b*x)^4,x)
```

output
```
-(a^2 + 3*b^2*x^2 + 3*a*b*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^
5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{x^2}{(a+bx)^4}\,dx = \frac{x^3}{3a\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(x^2/(b*x+a)^4,x)
```

output
```
x**3/(3*a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.159 $\int \frac{x}{(a+bx)^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \frac{x}{(a+bx)^4} dx = \frac{a}{3b^2(a+bx)^3} - \frac{1}{2b^2(a+bx)^2}$$

output
```
1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{(a+bx)^4} dx = -\frac{a+3bx}{6b^2(a+bx)^3}$$

input
```
Integrate[x/(a + b*x)^4,x]
```

output
```
-1/6*(a + 3*b*x)/(b^2*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^4} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{1}{b(a+bx)^3}-\frac{a}{b(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

input
```
Int[x/(a + b*x)^4,x]
```

output
```
a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{3bx+a}{6(bx+a)^{3}b^{2}}$	19
orering	$-\frac{3bx+a}{6(bx+a)^{3}b^{2}}$	19
norman	$\frac{-\frac{x}{2b}-\frac{a}{6b^{2}}}{(bx+a)^{3}}$	22
risch	$\frac{-\frac{x}{2b}-\frac{a}{6b^{2}}}{(bx+a)^{3}}$	22
parallelrisch	$\frac{-3b^{2}x-ab}{6b^{3}(bx+a)^{3}}$	24
default	$\frac{a}{3b^{2}(bx+a)^{3}}-\frac{1}{2b^{2}(bx+a)^{2}}$	27

input
```
int(x/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*b*x+a)/(b*x+a)^3/b^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{x}{(a+bx)^4}\,dx=-\frac{3\,bx+a}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

input
```
integrate(x/(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x}{(a+bx)^4}\,dx = \frac{-a-3bx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

input
```
integrate(x/(b*x+a)**4,x)
```

output
```
(-a - 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{x}{(a+bx)^4}\,dx = -\frac{3\,bx+a}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

input
```
integrate(x/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{(a+bx)^4}\,dx = -\frac{3\,bx+a}{6\,(bx+a)^3b^2}$$

input
```
integrate(x/(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x}{(a+bx)^4}\,dx = -\frac{\frac{a}{6\,b^2}+\frac{x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

input
```
int(x/(a + b*x)^4,x)
```

output
```
-(a/(6*b^2) + x/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{x}{(a+bx)^4}\,dx = \frac{-3bx-a}{6b^2\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(x/(b*x+a)^4,x)
```

output
```
( - a - 3*b*x)/(6*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.160 $\int \frac{1}{(a+bx)^4} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(a+bx)^4} \, dx = -\frac{1}{3b(a+bx)^3}$$

output
```
-1/3/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^4} \, dx = -\frac{1}{3b(a+bx)^3}$$

input
```
Integrate[(a + b*x)^(-4),x]
```

output
```
-1/3*1/(b*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^4} \, dx$$
$$\Big\downarrow \; 17$$
$$-\frac{1}{3b(a+bx)^3}$$

input
```
Int[(a + b*x)^(-4),x]
```

output
```
-1/3*1/(b*(a + b*x)^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{3b(bx+a)^{3}}$	13
default	$-\frac{1}{3b(bx+a)^{3}}$	13
norman	$-\frac{1}{3b(bx+a)^{3}}$	13
risch	$-\frac{1}{3b(bx+a)^{3}}$	13
parallelrisch	$-\frac{1}{3b(bx+a)^{3}}$	13
orering	$-\frac{1}{3b(bx+a)^{3}}$	13

input
```
int(1/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

input
```
integrate(1/(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3a^3b + 9a^2b^2x + 9ab^3x^2 + 3b^4x^3}$$

input
```
integrate(1/(b*x+a)**4,x)
```

output
```
-1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3\,(bx+a)^3b}$$

input
```
integrate(1/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3/((b*x + a)^3*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3\,(bx+a)^3b}$$

input
```
integrate(1/(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/3/((b*x + a)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{(a+bx)^4} \, dx = -\frac{1}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3}$$

input
```
int(1/(a + b*x)^4,x)
```

output
```
-1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{1}{(a+bx)^4} \, dx = -\frac{1}{3b\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(1/(b*x+a)^4,x)
```

output
```
( - 1)/(3*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.161 $\int \frac{1}{x(a+bx)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{1}{x(a+bx)^4} dx = \frac{1}{3a(a+bx)^3} + \frac{1}{2a^2(a+bx)^2} + \frac{1}{a^3(a+bx)} + \frac{\log(x)}{a^4} - \frac{\log(a+bx)}{a^4}$$

output
```
1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{x(a+bx)^4} dx = \frac{\frac{a(11a^2+15abx+6b^2x^2)}{(a+bx)^3} + 6\log(x) - 6\log(a+bx)}{6a^4}$$

input
```
Integrate[1/(x*(a + b*x)^4),x]
```

output
```
((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*
x])/(6*a^4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^4} \, dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{b}{a^4(a+bx)}+\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{a^2(a+bx)^3}-\frac{b}{a(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{3a(a+bx)^3}$$

input
```
Int[1/(x*(a + b*x)^4),x]
```

output
```
1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a
^4 - Log[a + b*x]/a^4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\frac{b^2x^2}{a^3}+\frac{5bx}{2a^2}+\frac{11}{6a}}{(bx+a)^3}+\frac{\ln(-x)}{a^4}-\frac{\ln(bx+a)}{a^4}$
default	$\frac{1}{3a(bx+a)^3}+\frac{1}{2a^2(bx+a)^2}+\frac{1}{a^3(bx+a)}+\frac{\ln(x)}{a^4}-\frac{\ln(bx+a)}{a^4}$
norman	$\frac{-\frac{3bx}{a^2}-\frac{9b^2x^2}{2a^3}-\frac{11b^3x^3}{6a^4}}{(bx+a)^3}+\frac{\ln(x)}{a^4}-\frac{\ln(bx+a)}{a^4}$
parallelrisch	$\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+18a\,b^2\ln(x)x^2-18\ln(bx+a)x^2a\,b^2-11b^3x^3+18a^2b\ln(x)x-18\ln(bx+a)x\,a^2b-27a\,b^2x^2+6a^3\ln}{6a^4(bx+a)^3}$

input
```
int(1/x/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(b^2/a^3*x^2+5/2*b/a^2*x+11/6/a)/(b*x+a)^3+1/a^4*ln(-x)-ln(b*x+a)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(53) = 106.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.18

$$\int \frac{1}{x(a+bx)^4}\,dx = \frac{6\,ab^2x^2+15\,a^2bx+11\,a^3-6\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log(bx+a)+6\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+}{6\,(a^4b^3x^3+3\,a^5b^2x^2+3\,a^6bx+a^7)}$$

input
```
integrate(1/x/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(
x))/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{1}{x(a+bx)^4}\,dx = \frac{11a^2+15abx+6b^2x^2}{6a^6+18a^5bx+18a^4b^2x^2+6a^3b^3x^3} + \frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^4}$$

input
```
integrate(1/x/(b*x+a)**4,x)
```

output
```
(11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x*
*2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{1}{x(a+bx)^4}\,dx = \frac{6\,b^2x^2+15\,abx+11\,a^2}{6\,(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+a^6)} - \frac{\log(bx+a)}{a^4} + \frac{\log(x)}{a^4}$$

input
```
integrate(1/x/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b
*x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+bx)^4}\,dx = -\frac{\log(|bx+a|)}{a^4} + \frac{\log(|x|)}{a^4} + \frac{6\,ab^2x^2+15\,a^2bx+11\,a^3}{6\,(bx+a)^3a^4}$$

input
```
integrate(1/x/(b*x+a)^4,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
 11*a^3)/((b*x + a)^3*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(a+bx)^4}\,dx = \frac{\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,(a+b\,x)^2}}{a}+\frac{1}{3\,a\,(a+b\,x)^3}$$

input
```
int(1/(x*(a + b*x)^4),x)
```

output
```
((1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2))/a + 1/(
3*a*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.54

$$\int \frac{1}{x(a+bx)^4}\,dx$$
$$= \frac{-6\log(bx+a)\,a^3-18\log(bx+a)\,a^2bx-18\log(bx+a)\,a\,b^2x^2-6\log(bx+a)\,b^3x^3+6\log(x)\,a^3+18\,\mathrm{lo}}{6a^4\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(1/x/(b*x+a)^4,x)
```

output
```
( - 6*log(a + b*x)*a**3 - 18*log(a + b*x)*a**2*b*x - 18*log(a + b*x)*a*b**
2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log(x)*a**3 + 18*log(x)*a**2*b*x + 1
8*log(x)*a*b**2*x**2 + 6*log(x)*b**3*x**3 + 9*a**3 + 9*a**2*b*x - 2*b**3*x
**3)/(6*a**4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.162 $\int \frac{1}{x^2(a+bx)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 70

$$\int \frac{1}{x^2(a+bx)^4} dx = -\frac{1}{a^4x} - \frac{b}{3a^2(a+bx)^3} - \frac{b}{a^3(a+bx)^2} - \frac{3b}{a^4(a+bx)} - \frac{4b\log(x)}{a^5} + \frac{4b\log(a+bx)}{a^5}$$

output
```
-1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5
+4*b*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2(a+bx)^4} dx = -\frac{\frac{a(3a^3+22a^2bx+30ab^2x^2+12b^3x^3)}{x(a+bx)^3} + 12b\log(x) - 12b\log(a+bx)}{3a^5}$$

input
```
Integrate[1/(x^2*(a + b*x)^4),x]
```

output
```
-1/3*((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3)
 + 12*b*Log[x] - 12*b*Log[a + b*x])/a^5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^4}\,dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(\frac{4b^2}{a^5(a+bx)}-\frac{4b}{a^5x}+\frac{3b^2}{a^4(a+bx)^2}+\frac{1}{a^4x^2}+\frac{2b^2}{a^3(a+bx)^3}+\frac{b^2}{a^2(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{3b}{a^4(a+bx)}-\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}$$

input
```
Int[1/(x^2*(a + b*x)^4),x]
```

output
```
-(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
 + b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

method	result
default	$-\frac{1}{a^{4}x}-\frac{b}{3a^{2}(bx+a)^{3}}-\frac{b}{a^{3}(bx+a)^{2}}-\frac{3b}{a^{4}(bx+a)}-\frac{4b\ln(x)}{a^{5}}+\frac{4b\ln(bx+a)}{a^{5}}$
risch	$\frac{-\frac{4b^{3}x^{3}}{a^{4}}-\frac{10b^{2}x^{2}}{a^{3}}-\frac{22bx}{3a^{2}}-\frac{1}{a}}{x(bx+a)^{3}}-\frac{4b\ln(x)}{a^{5}}+\frac{4b\ln(-bx-a)}{a^{5}}$
norman	$\frac{-\frac{1}{a}+\frac{12b^{2}x^{2}}{a^{3}}+\frac{18b^{3}x^{3}}{a^{4}}+\frac{22b^{4}x^{4}}{3a^{5}}}{x(bx+a)^{3}}-\frac{4b\ln(x)}{a^{5}}+\frac{4b\ln(bx+a)}{a^{5}}$
parallelrisch	$-\frac{12b^{4}\ln(x)x^{4}-12b^{4}\ln(bx+a)x^{4}+36\ln(x)x^{3}a\,b^{3}-36\ln(bx+a)x^{3}a\,b^{3}-22b^{4}x^{4}+36\ln(x)x^{2}a^{2}b^{2}-36\ln(bx+a)x^{2}a^{2}b^{2}-54a\,x^{3}}{3a^{5}x(bx+a)^{3}}$

input
```
int(1/x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5
+4*b*ln(b*x+a)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(68) = 136.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^2(a+bx)^4}\,dx = -\frac{12\,ab^3x^3+30\,a^2b^2x^2+22\,a^3bx+3\,a^4-12\,(b^4x^4+3\,ab^3x^3+3\,a^2b^2x^2+a^3bx)\log(bx+a)+12\,(b^4x^4}{3\,(a^5b^3x^4+3\,a^6b^2x^3+3\,a^7bx^2+a^8x)}$$

input
```
integrate(1/x^2/(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3
*a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3
*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a
^7*b*x^2 + a^8*x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(a+bx)^4}\,dx = \frac{-3a^3-22a^2bx-30ab^2x^2-12b^3x^3}{3a^7x+9a^6bx^2+9a^5b^2x^3+3a^4b^3x^4} + \frac{4b\left(-\log{(x)}+\log{\left(\frac{a}{b}+x\right)}\right)}{a^5}$$

input
```
integrate(1/x**2/(b*x+a)**4,x)
```

output
```
(-3*a**3 - 22*a**2*b*x - 30*a*b**2*x**2 - 12*b**3*x**3)/(3*a**7*x + 9*a**6
*b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b +
x))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^2(a+bx)^4}\,dx = -\frac{12\,b^3x^3+30\,ab^2x^2+22\,a^2bx+3\,a^3}{3\,(a^4b^3x^4+3\,a^5b^2x^3+3\,a^6bx^2+a^7x)} + \frac{4\,b\log{(bx+a)}}{a^5} - \frac{4\,b\log{(x)}}{a^5}$$

input
```
integrate(1/x^2/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5
*b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2(a+bx)^4}\,dx = \frac{4\,b\log{(|bx+a|)}}{a^5} - \frac{4\,b\log{(|x|)}}{a^5} - \frac{12\,ab^3x^3+30\,a^2b^2x^2+22\,a^3bx+3\,a^4}{3\,(bx+a)^3a^5x}$$

input
```
integrate(1/x^2/(b*x+a)^4,x, algorithm="giac")
```

output
```
4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a
^2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2(a+bx)^4}\,dx = \frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{a}+\frac{10\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}+\frac{22\,b\,x}{3\,a^2}}{a^3\,x+3\,a^2\,b\,x^2+3\,a\,b^2\,x^3+b^3\,x^4}$$

input
```
int(1/(x^2*(a + b*x)^4),x)
```

output
```
(8*b*atanh((2*b*x)/a + 1))/a^5 - (1/a + (10*b^2*x^2)/a^3 + (4*b^3*x^3)/a^4
 + (22*b*x)/(3*a^2))/(a^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2(a+bx)^4}\,dx$$
$$= \frac{12\log(bx+a)\,a^3bx+36\log(bx+a)\,a^2b^2x^2+36\log(bx+a)\,a\,b^3x^3+12\log(bx+a)\,b^4x^4-12\log(x)\,a^3b}{3a^5x\,(b^3x^3+3a\,b^2x^2+3}$$

input
```
int(1/x^2/(b*x+a)^4,x)
```

output
```
(12*log(a + b*x)*a**3*b*x + 36*log(a + b*x)*a**2*b**2*x**2 + 36*log(a + b*
x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 - 12*log(x)*a**3*b*x - 36*log(x
)*a**2*b**2*x**2 - 36*log(x)*a*b**3*x**3 - 12*log(x)*b**4*x**4 - 3*a**4 -
18*a**3*b*x - 18*a**2*b**2*x**2 + 4*b**4*x**4)/(3*a**5*x*(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3))
```

3.163 $\int \frac{1}{x^3(a+bx)^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 93

$$\int \frac{1}{x^3(a+bx)^4} dx = -\frac{1}{2a^4x^2} + \frac{4b}{a^5x} + \frac{b^2}{3a^3(a+bx)^3} + \frac{3b^2}{2a^4(a+bx)^2} + \frac{6b^2}{a^5(a+bx)} + \frac{10b^2\log(x)}{a^6} - \frac{10b^2\log(a+bx)}{a^6}$$

output
```
-1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a
^5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(a+bx)^4} dx = \frac{\frac{a(-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4)}{x^2(a+bx)^3} + 60b^2\log(x) - 60b^2\log(a+bx)}{6a^6}$$

input
```
Integrate[1/(x^3*(a + b*x)^4),x]
```

output
```
((a*(-3*a^4 + 15*a^3*b*x + 110*a^2*b^2*x^2 + 150*a*b^3*x^3 + 60*b^4*x^4))/
(x^2*(a + b*x)^3) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(6*a^6)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^4} \, dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{10b^3}{a^6(a+bx)}+\frac{10b^2}{a^6x}-\frac{6b^3}{a^5(a+bx)^2}-\frac{4b}{a^5x^2}-\frac{3b^3}{a^4(a+bx)^3}+\frac{1}{a^4x^3}-\frac{b^3}{a^3(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{6b^2}{a^5(a+bx)}+\frac{4b}{a^5x}+\frac{3b^2}{2a^4(a+bx)^2}-\frac{1}{2a^4x^2}+\frac{b^2}{3a^3(a+bx)^3}$$

input
```
Int[1/(x^3*(a + b*x)^4),x]
```

output
```
-1/2*1/(a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^
4*(a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*L
og[a + b*x])/a^6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

method	result
norman	$\frac{-\frac{1}{2a}+\frac{5bx}{2a^{2}}-\frac{30b^{3}x^{3}}{a^{4}}-\frac{45b^{4}x^{4}}{a^{5}}-\frac{55b^{5}x^{5}}{3a^{6}}}{x^{2}\left(bx+a\right)^{3}}+\frac{10b^{2}\ln\left(x\right)}{a^{6}}-\frac{10b^{2}\ln\left(bx+a\right)}{a^{6}}$
risch	$\frac{\frac{10b^{4}x^{4}}{a^{5}}+\frac{25b^{3}x^{3}}{a^{4}}+\frac{55b^{2}x^{2}}{3a^{3}}+\frac{5bx}{2a^{2}}-\frac{1}{2a}}{x^{2}\left(bx+a\right)^{3}}+\frac{10b^{2}\ln\left(-x\right)}{a^{6}}-\frac{10b^{2}\ln\left(bx+a\right)}{a^{6}}$
default	$-\frac{1}{2a^{4}x^{2}}+\frac{4b}{a^{5}x}+\frac{b^{2}}{3a^{3}\left(bx+a\right)^{3}}+\frac{3b^{2}}{2a^{4}\left(bx+a\right)^{2}}+\frac{6b^{2}}{a^{5}\left(bx+a\right)}+\frac{10b^{2}\ln\left(x\right)}{a^{6}}-\frac{10b^{2}\ln\left(bx+a\right)}{a^{6}}$
parallelrisch	$\frac{60b^{5}\ln\left(x\right)x^{5}-60\ln\left(bx+a\right)x^{5}b^{5}+180a\,b^{4}\ln\left(x\right)x^{4}-180\ln\left(bx+a\right)x^{4}a\,b^{4}-110b^{5}x^{5}+180a^{2}b^{3}\ln\left(x\right)x^{3}-180\ln\left(bx+a\right)x^{3}a^{2}b^{3}-27}{6a^{6}x^{2}\left(bx+a\right)^{3}}$

input
```
int(1/x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a+5/2*b/a^2*x-30*b^3/a^4*x^3-45*b^4/a^5*x^4-55/3*b^5/a^6*x^5)/x^2/(b
*x+a)^3+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^3(a+bx)^4}\,dx$$
$$=\frac{60\,ab^4x^4+150\,a^2b^3x^3+110\,a^3b^2x^2+15\,a^4bx-3\,a^5-60\,(b^5x^5+3\,ab^4x^4+3\,a^2b^3x^3+a^3b^2x^2)\log{(bx+}}{6\,(a^6b^3x^5+3\,a^7b^2x^4+3\,a^8bx^3+a^9x^2)}$$

input
```
integrate(1/x^3/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(60*a*b^4*x^4 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5
 - 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(b*x + a) +
 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(x))/(a^6*b^3
*x^5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4}{6a^8x^2+18a^7bx^3+18a^6b^2x^4+6a^5b^3x^5} + \frac{10b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

input
```
integrate(1/x**3/(b*x+a)**4,x)
```

output
```
(-3*a**4 + 15*a**3*b*x + 110*a**2*b**2*x**2 + 150*a*b**3*x**3 + 60*b**4*x*
*4)/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5)
+ 10*b**2*(log(x) - log(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{60\,b^4x^4+150\,ab^3x^3+110\,a^2b^2x^2+15\,a^3bx-3\,a^4}{6\,(a^5b^3x^5+3\,a^6b^2x^4+3\,a^7bx^3+a^8x^2)} - \frac{10\,b^2\log(bx+a)}{a^6} + \frac{10\,b^2\log(x)}{a^6}$$

input
```
integrate(1/x^3/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(60*b^4*x^4 + 150*a*b^3*x^3 + 110*a^2*b^2*x^2 + 15*a^3*b*x - 3*a^4)/(a
^5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) - 10*b^2*log(b*x + a)/
a^6 + 10*b^2*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3(a+bx)^4}\,dx = -\frac{10\,b^2\log\left(|bx+a|\right)}{a^6} + \frac{10\,b^2\log\left(|x|\right)}{a^6} + \frac{60\,ab^4x^4 + 150\,a^2b^3x^3 + 110\,a^3b^2x^2 + 15\,a^4bx - 3\,a^5}{6\,(bx+a)^3a^6x^2}$$

input
```
integrate(1/x^3/(b*x+a)^4,x, algorithm="giac")
```

output
```
-10*b^2*log(abs(b*x + a))/a^6 + 10*b^2*log(abs(x))/a^6 + 1/6*(60*a*b^4*x^4
 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5)/((b*x + a)^3*a^
6*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(a+bx)^4}\,dx = \frac{\frac{55\,b^2\,x^2}{3\,a^3} - \frac{1}{2\,a} + \frac{25\,b^3\,x^3}{a^4} + \frac{10\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{2\,a^2}}{a^3\,x^2 + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^4 + b^3\,x^5} - \frac{20\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

input
```
int(1/(x^3*(a + b*x)^4),x)
```

output
```
((55*b^2*x^2)/(3*a^3) - 1/(2*a) + (25*b^3*x^3)/a^4 + (10*b^4*x^4)/a^5 + (5
*b*x)/(2*a^2))/(a^3*x^2 + b^3*x^5 + 3*a^2*b*x^3 + 3*a*b^2*x^4) - (20*b^2*a
tanh((2*b*x)/a + 1))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3(a+bx)^4}\,dx$$
$$= \frac{-60\log(bx+a)\,a^3b^2x^2 - 180\log(bx+a)\,a^2b^3x^3 - 180\log(bx+a)\,a\,b^4x^4 - 60\log(bx+a)\,b^5x^5 + 60\log(}{6a^6x^2\,(b^3x^3 -}$$

input
```
int(1/x^3/(b*x+a)^4,x)
```

output
```
( - 60*log(a + b*x)*a**3*b**2*x**2 - 180*log(a + b*x)*a**2*b**3*x**3 - 180
*log(a + b*x)*a*b**4*x**4 - 60*log(a + b*x)*b**5*x**5 + 60*log(x)*a**3*b**
2*x**2 + 180*log(x)*a**2*b**3*x**3 + 180*log(x)*a*b**4*x**4 + 60*log(x)*b*
*5*x**5 - 3*a**5 + 15*a**4*b*x + 90*a**3*b**2*x**2 + 90*a**2*b**3*x**3 - 2
0*b**5*x**5)/(6*a**6*x**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.164 $\int \frac{1}{x^4(a+bx)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 102

$$\int \frac{1}{x^4(a+bx)^4}\,dx=-\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}$$

output
```
-1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x
+a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4(a+bx)^4}\,dx=-\frac{\frac{a(a^5-3a^4bx+15a^3b^2x^2+110a^2b^3x^3+150ab^4x^4+60b^5x^5)}{x^3(a+bx)^3}+60b^3\log(x)-60b^3\log(a+bx)}{3a^7}$$

input
```
Integrate[1/(x^4*(a + b*x)^4),x]
```

output
```
-1/3*((a*(a^5 - 3*a^4*b*x + 15*a^3*b^2*x^2 + 110*a^2*b^3*x^3 + 150*a*b^4*x
^4 + 60*b^5*x^5))/(x^3*(a + b*x)^3) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])
/a^7
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+bx)^4}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{20b^4}{a^7(a+bx)}-\frac{20b^3}{a^7x}+\frac{10b^4}{a^6(a+bx)^2}+\frac{10b^2}{a^6x^2}+\frac{4b^4}{a^5(a+bx)^3}-\frac{4b}{a^5x^3}+\frac{b^4}{a^4(a+bx)^4}+\frac{1}{a^4x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}-\frac{10b^3}{a^6(a+bx)}-\frac{10b^2}{a^6x}-\frac{2b^3}{a^5(a+bx)^2}+\frac{2b}{a^5x^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{1}{3a^4x^3}$$

input
```
Int[1/(x^4*(a + b*x)^4),x]
```

output
```
-1/3*1/(a^4*x^3) + (2*b)/(a^5*x^2) - (10*b^2)/(a^6*x) - b^3/(3*a^4*(a + b*
x)^3) - (2*b^3)/(a^5*(a + b*x)^2) - (10*b^3)/(a^6*(a + b*x)) - (20*b^3*Log
[x])/a^7 + (20*b^3*Log[a + b*x])/a^7
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{bx}{a^2}-\frac{1}{3a}-\frac{5b^2x^2}{a^3}+\frac{60b^4x^4}{a^5}+\frac{90b^5x^5}{a^6}+\frac{110b^6x^6}{3a^7}}{x^3(bx+a)^3}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln(bx+a)}{a^7}$
risch	$\frac{-\frac{20b^5x^5}{a^6}-\frac{50b^4x^4}{a^5}-\frac{110b^3x^3}{3a^4}-\frac{5b^2x^2}{a^3}+\frac{bx}{a^2}-\frac{1}{3a}}{x^3(bx+a)^3}+\frac{20b^3\ln(-bx-a)}{a^7}-\frac{20b^3\ln(x)}{a^7}$
default	$-\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(bx+a)^3}-\frac{2b^3}{a^5(bx+a)^2}-\frac{10b^3}{a^6(bx+a)}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln(bx+a)}{a^7}$
parallelrisch	$-\frac{60\ln(x)x^6b^6-60\ln(bx+a)x^6b^6+180\ln(x)x^5a\,b^5-180\ln(bx+a)x^5a\,b^5-110b^6x^6+180\ln(x)x^4a^2b^4-180\ln(bx+a)x^4a^2b^4-}{3a^7x^3(bx+a)^3}$

input
```
int(1/x^4/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(b/a^2*x-1/3/a-5*b^2/a^3*x^2+60*b^4/a^5*x^4+90*b^5/a^6*x^5+110/3*b^6/a^7*x
^6)/x^3/(b*x+a)^3-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^4(a+bx)^4}\,dx =$$
$$-\frac{60\,ab^5x^5+150\,a^2b^4x^4+110\,a^3b^3x^3+15\,a^4b^2x^2-3\,a^5bx+a^6-60\,(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x}{3\,(a^7b^3x^6+3\,a^8b^2x^5+3\,a^9bx^4+a^{10}x^3)}$$

input
```
integrate(1/x^4/(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(60*a*b^5*x^5 + 150*a^2*b^4*x^4 + 110*a^3*b^3*x^3 + 15*a^4*b^2*x^2 -
3*a^5*b*x + a^6 - 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)
*log(b*x + a) + 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*l
og(x))/(a^7*b^3*x^6 + 3*a^8*b^2*x^5 + 3*a^9*b*x^4 + a^10*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^4(a+bx)^4}\,dx = \frac{-a^5+3a^4bx-15a^3b^2x^2-110a^2b^3x^3-150ab^4x^4-60b^5x^5}{3a^9x^3+9a^8bx^4+9a^7b^2x^5+3a^6b^3x^6} + \frac{20b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

input
```
integrate(1/x**4/(b*x+a)**4,x)
```

output
```
(-a**5 + 3*a**4*b*x - 15*a**3*b**2*x**2 - 110*a**2*b**3*x**3 - 150*a*b**4*
x**4 - 60*b**5*x**5)/(3*a**9*x**3 + 9*a**8*b*x**4 + 9*a**7*b**2*x**5 + 3*a
**6*b**3*x**6) + 20*b**3*(-log(x) + log(a/b + x))/a**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4(a+bx)^4}\,dx = -\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\,(a^6b^3x^6+3\,a^7b^2x^5+3\,a^8bx^4+a^9x^3)} + \frac{20\,b^3\log\left(bx+a\right)}{a^7} - \frac{20\,b^3\log\left(x\right)}{a^7}$$

input
```
integrate(1/x^4/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(60*b^5*x^5 + 150*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^
4*b*x + a^5)/(a^6*b^3*x^6 + 3*a^7*b^2*x^5 + 3*a^8*b*x^4 + a^9*x^3) + 20*b^
3*log(b*x + a)/a^7 - 20*b^3*log(x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4(a+bx)^4}\,dx = \frac{20\,b^3\log\left(|bx+a|\right)}{a^7} - \frac{20\,b^3\log\left(|x|\right)}{a^7} - \frac{60\,b^5x^5 + 150\,ab^4x^4 + 110\,a^2b^3x^3 + 15\,a^3b^2x^2 - 3\,a^4bx + a^5}{3\,(bx^2+ax)^3a^6}$$

input
```
integrate(1/x^4/(b*x+a)^4,x, algorithm="giac")
```

output
```
20*b^3*log(abs(b*x + a))/a^7 - 20*b^3*log(abs(x))/a^7 - 1/3*(60*b^5*x^5 +
150*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4*b*x + a^5)/((b*x^
2 + a*x)^3*a^6)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4(a+bx)^4}\,dx = \frac{40\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7} - \frac{\frac{1}{3\,a}+\frac{5\,b^2\,x^2}{a^3}+\frac{110\,b^3\,x^3}{3\,a^4}+\frac{50\,b^4\,x^4}{a^5}+\frac{20\,b^5\,x^5}{a^6}-\frac{b\,x}{a^2}}{a^3\,x^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^5+b^3\,x^6}$$

input
```
int(1/(x^4*(a + b*x)^4),x)
```

output
```
(40*b^3*atanh((2*b*x)/a + 1))/a^7 - (1/(3*a) + (5*b^2*x^2)/a^3 + (110*b^3*
x^3)/(3*a^4) + (50*b^4*x^4)/a^5 + (20*b^5*x^5)/a^6 - (b*x)/a^2)/(a^3*x^3 +
 b^3*x^6 + 3*a^2*b*x^4 + 3*a*b^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^4(a+bx)^4}\,dx$$
$$= \frac{60\log(bx+a)\,a^3b^3x^3+180\log(bx+a)\,a^2b^4x^4+180\log(bx+a)\,a\,b^5x^5+60\log(bx+a)\,b^6x^6-60\log(x)}{3a^7x^3\,(b^3}$$

input
```
int(1/x^4/(b*x+a)^4,x)
```

output
```
(60*log(a + b*x)*a**3*b**3*x**3 + 180*log(a + b*x)*a**2*b**4*x**4 + 180*lo
g(a + b*x)*a*b**5*x**5 + 60*log(a + b*x)*b**6*x**6 - 60*log(x)*a**3*b**3*x
**3 - 180*log(x)*a**2*b**4*x**4 - 180*log(x)*a*b**5*x**5 - 60*log(x)*b**6*
x**6 - a**6 + 3*a**5*b*x - 15*a**4*b**2*x**2 - 90*a**3*b**3*x**3 - 90*a**2
*b**4*x**4 + 20*b**6*x**6)/(3*a**7*x**3*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
 + b**3*x**3))
```

3.165 $\int \frac{1}{x^5(a+bx)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 117

$$\int \frac{1}{x^5(a+bx)^4}\,dx = -\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(a+bx)^3}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{15b^4}{a^7(a+bx)}+\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}$$

output
```
-1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^
3+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+
a)/a^8
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5(a+bx)^4}\,dx = \frac{\frac{a(-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6)}{x^4(a+bx)^3}+420b^4\log(x)-420b^4\log(a+bx)}{12a^8}$$

input
```
Integrate[1/(x^5*(a + b*x)^4),x]
```

output
```
((a*(-3*a^6 + 7*a^5*b*x - 21*a^4*b^2*x^2 + 105*a^3*b^3*x^3 + 770*a^2*b^4*x
^4 + 1050*a*b^5*x^5 + 420*b^6*x^6))/(x^4*(a + b*x)^3) + 420*b^4*Log[x] - 4
20*b^4*Log[a + b*x])/(12*a^8)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(a+bx)^4} \, dx$$

$$\Big\downarrow \ 54$$

$$\int \left(-\frac{35b^5}{a^8(a+bx)}+\frac{35b^4}{a^8x}-\frac{15b^5}{a^7(a+bx)^2}-\frac{20b^3}{a^7x^2}-\frac{5b^5}{a^6(a+bx)^3}+\frac{10b^2}{a^6x^3}-\frac{b^5}{a^5(a+bx)^4}-\frac{4b}{a^5x^4}+\frac{1}{a^4x^5}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}+\frac{15b^4}{a^7(a+bx)}+\frac{20b^3}{a^7x}+\frac{5b^4}{2a^6(a+bx)^2}-\frac{5b^2}{a^6x^2}+\frac{b^4}{3a^5(a+bx)^3}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

input
```
Int[1/(x^5*(a + b*x)^4),x]
```

output
```
-1/4*1/(a^4*x^4) + (4*b)/(3*a^5*x^3) - (5*b^2)/(a^6*x^2) + (20*b^3)/(a^7*x
) + b^4/(3*a^5*(a + b*x)^3) + (5*b^4)/(2*a^6*(a + b*x)^2) + (15*b^4)/(a^7*
(a + b*x)) + (35*b^4*Log[x])/a^8 - (35*b^4*Log[a + b*x])/a^8
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

method	result
norman	$\frac{-\frac{1}{4a}+\frac{7bx}{12a^2}-\frac{7b^2x^2}{4a^3}+\frac{35b^3x^3}{4a^4}-\frac{105b^5x^5}{a^6}-\frac{315b^6x^6}{2a^7}-\frac{385b^7x^7}{6a^8}}{x^4(bx+a)^3}+\frac{35b^4\ln(x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$
risch	$\frac{\frac{35b^6x^6}{a^7}+\frac{175b^5x^5}{2a^6}+\frac{385b^4x^4}{6a^5}+\frac{35b^3x^3}{4a^4}-\frac{7b^2x^2}{4a^3}+\frac{7bx}{12a^2}-\frac{1}{4a}}{x^4(bx+a)^3}+\frac{35b^4\ln(-x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$
default	$-\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(bx+a)^3}+\frac{5b^4}{2a^6(bx+a)^2}+\frac{15b^4}{a^7(bx+a)}+\frac{35b^4\ln(x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$
parallelrisch	$\frac{420b^7\ln(x)x^7-420\ln(bx+a)x^7b^7+1260a\,b^6\ln(x)x^6-1260\ln(bx+a)x^6a\,b^6-770b^7x^7+1260\ln(x)x^5a^2b^5-1260\ln(bx+a)x^5a^2}{12a^8x^4(bx+a)^3}$

input
```
int(1/x^5/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a+7/12*b/a^2*x-7/4*b^2/a^3*x^2+35/4*b^3/a^4*x^3-105*b^5/a^6*x^5-315/
2*b^6/a^7*x^6-385/6*b^7/a^8*x^7)/x^4/(b*x+a)^3+35*b^4*ln(x)/a^8-35*b^4*ln(
b*x+a)/a^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.68

$$\int \frac{1}{x^5(a+bx)^4}\,dx$$
$$=\frac{420\,ab^6x^6+1050\,a^2b^5x^5+770\,a^3b^4x^4+105\,a^4b^3x^3-21\,a^5b^2x^2+7\,a^6bx-3\,a^7-420\,(b^7x^7+3\,ab^6x^6+}{12\,(a^8b^3x^7+3\,a^9b^2x^6+3\,a^{10}bx^5}$$

input
```
integrate(1/x^5/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/12*(420*a*b^6*x^6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3
 - 21*a^5*b^2*x^2 + 7*a^6*b*x - 3*a^7 - 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2
*b^5*x^5 + a^3*b^4*x^4)*log(b*x + a) + 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*
b^5*x^5 + a^3*b^4*x^4)*log(x))/(a^8*b^3*x^7 + 3*a^9*b^2*x^6 + 3*a^10*b*x^5
 + a^11*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^5(a+bx)^4}\,dx = \frac{-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6}{12a^{10}x^4+36a^9bx^5+36a^8b^2x^6+12a^7b^3x^7} + \frac{35b^4\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

input
```
integrate(1/x**5/(b*x+a)**4,x)
```

output
```
(-3*a**6 + 7*a**5*b*x - 21*a**4*b**2*x**2 + 105*a**3*b**3*x**3 + 770*a**2*
b**4*x**4 + 1050*a*b**5*x**5 + 420*b**6*x**6)/(12*a**10*x**4 + 36*a**9*b*x
**5 + 36*a**8*b**2*x**6 + 12*a**7*b**3*x**7) + 35*b**4*(log(x) - log(a/b +
 x))/a**8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^5(a+bx)^4}\,dx = \frac{420\,b^6x^6+1050\,ab^5x^5+770\,a^2b^4x^4+105\,a^3b^3x^3-21\,a^4b^2x^2+7\,a^5bx-3\,a^6}{12\,(a^7b^3x^7+3\,a^8b^2x^6+3\,a^9bx^5+a^{10}x^4)} - \frac{35\,b^4\log(bx+a)}{a^8} + \frac{35\,b^4\log(x)}{a^8}$$

input
```
integrate(1/x^5/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/12*(420*b^6*x^6 + 1050*a*b^5*x^5 + 770*a^2*b^4*x^4 + 105*a^3*b^3*x^3 - 2
1*a^4*b^2*x^2 + 7*a^5*b*x - 3*a^6)/(a^7*b^3*x^7 + 3*a^8*b^2*x^6 + 3*a^9*b*
x^5 + a^10*x^4) - 35*b^4*log(b*x + a)/a^8 + 35*b^4*log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5(a+bx)^4}\,dx = -\frac{35\,b^4\log\left(|bx+a|\right)}{a^8} + \frac{35\,b^4\log\left(|x|\right)}{a^8} + \frac{420\,ab^6x^6 + 1050\,a^2b^5x^5 + 770\,a^3b^4x^4 + 105\,a^4b^3x^3 - 21\,a^5b^2x^2 + 7\,a^6bx - 3\,a^7}{12\,(bx+a)^3a^8x^4}$$

input
```
integrate(1/x^5/(b*x+a)^4,x, algorithm="giac")
```

output
```
-35*b^4*log(abs(b*x + a))/a^8 + 35*b^4*log(abs(x))/a^8 + 1/12*(420*a*b^6*x
^6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3 - 21*a^5*b^2*x^2
 + 7*a^6*b*x - 3*a^7)/((b*x + a)^3*a^8*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5(a+bx)^4}\,dx = \frac{\frac{35\,b^3\,x^3}{4\,a^4} - \frac{7\,b^2\,x^2}{4\,a^3} - \frac{1}{4\,a} + \frac{385\,b^4\,x^4}{6\,a^5} + \frac{175\,b^5\,x^5}{2\,a^6} + \frac{35\,b^6\,x^6}{a^7} + \frac{7\,b\,x}{12\,a^2}}{a^3\,x^4 + 3\,a^2\,b\,x^5 + 3\,a\,b^2\,x^6 + b^3\,x^7} - \frac{70\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^8}$$

input
```
int(1/(x^5*(a + b*x)^4),x)
```

output
```
((35*b^3*x^3)/(4*a^4) - (7*b^2*x^2)/(4*a^3) - 1/(4*a) + (385*b^4*x^4)/(6*a
^5) + (175*b^5*x^5)/(2*a^6) + (35*b^6*x^6)/a^7 + (7*b*x)/(12*a^2))/(a^3*x^
4 + b^3*x^7 + 3*a^2*b*x^5 + 3*a*b^2*x^6) - (70*b^4*atanh((2*b*x)/a + 1))/a
^8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^5(a+bx)^4}\,dx = \frac{-420\log(bx+a)\,a^3b^4x^4 - 1260\log(bx+a)\,a^2b^5x^5 - 1260\log(bx+a)\,a\,b^6x^6 - 420\log(bx+a)\,b^7x^7 + 42}{}$$

input
```
int(1/x^5/(b*x+a)^4,x)
```

output
```
( - 420*log(a + b*x)*a**3*b**4*x**4 - 1260*log(a + b*x)*a**2*b**5*x**5 - 1
260*log(a + b*x)*a*b**6*x**6 - 420*log(a + b*x)*b**7*x**7 + 420*log(x)*a**
3*b**4*x**4 + 1260*log(x)*a**2*b**5*x**5 + 1260*log(x)*a*b**6*x**6 + 420*l
og(x)*b**7*x**7 - 3*a**7 + 7*a**6*b*x - 21*a**5*b**2*x**2 + 105*a**4*b**3*
x**3 + 630*a**3*b**4*x**4 + 630*a**2*b**5*x**5 - 140*b**7*x**7)/(12*a**8*x
**4*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.166 $\int \frac{x^{10}}{(a+bx)^7} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 150

$$\int \frac{x^{10}}{(a+bx)^7} \, dx = -\frac{84a^3x}{b^{10}} + \frac{14a^2x^2}{b^9} - \frac{7ax^3}{3b^8} + \frac{x^4}{4b^7} - \frac{a^{10}}{6b^{11}(a+bx)^6} + \frac{2a^9}{b^{11}(a+bx)^5} - \frac{45a^8}{4b^{11}(a+bx)^4} + \frac{40a^7}{b^{11}(a+bx)^3} - \frac{105a^6}{b^{11}(a+bx)^2} + \frac{252a^5}{b^{11}(a+bx)} + \frac{210a^4\log(a+bx)}{b^{11}}$$

output
```
-84*a^3*x/b^10+14*a^2*x^2/b^9-7/3*a*x^3/b^8+1/4*x^4/b^7-1/6*a^10/b^11/(b*x
+a)^6+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4+40*a^7/b^11/(b*x+a)^3-1
05*a^6/b^11/(b*x+a)^2+252*a^5/b^11/(b*x+a)+210*a^4*ln(b*x+a)/b^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}}{(a+bx)^7} \, dx = \frac{2131a^{10} + 10266a^9bx + 18105a^8b^2x^2 + 11540a^7b^3x^3 - 3945a^6b^4x^4 - 9138a^5b^5x^5 - 4043a^4b^6x^6 - 360a^3b}{12b^{11}(a+bx)^6}$$

input
```
Integrate[x^10/(a + b*x)^7,x]
```

output
```
(2131*a^10 + 10266*a^9*b*x + 18105*a^8*b^2*x^2 + 11540*a^7*b^3*x^3 - 3945*
a^6*b^4*x^4 - 9138*a^5*b^5*x^5 - 4043*a^4*b^6*x^6 - 360*a^3*b^7*x^7 + 45*a
^2*b^8*x^8 - 10*a*b^9*x^9 + 3*b^10*x^10 + 2520*a^4*(a + b*x)^6*Log[a + b*x
])/(12*b^11*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx)^7} \, dx$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{a^{10}}{b^{10}(a+bx)^7} - \frac{10a^9}{b^{10}(a+bx)^6} + \frac{45a^8}{b^{10}(a+bx)^5} - \frac{120a^7}{b^{10}(a+bx)^4} + \frac{210a^6}{b^{10}(a+bx)^3} - \frac{252a^5}{b^{10}(a+bx)^2} + \frac{210a^4}{b^{10}(a+bx)} - \right.$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^{10}}{6b^{11}(a+bx)^6} + \frac{2a^9}{b^{11}(a+bx)^5} - \frac{45a^8}{4b^{11}(a+bx)^4} + \frac{40a^7}{b^{11}(a+bx)^3} - \frac{105a^6}{b^{11}(a+bx)^2} + \frac{252a^5}{b^{11}(a+bx)} + \frac{210a^4\log(a+bx)}{b^{11}} - \frac{84a^3x}{b^{10}} + \frac{14a^2x^2}{b^9} - \frac{7ax^3}{3b^8} + \frac{x^4}{4b^7}$$

input
```
Int[x^10/(a + b*x)^7,x]
```

output
```
(-84*a^3*x)/b^10 + (14*a^2*x^2)/b^9 - (7*a*x^3)/(3*b^8) + x^4/(4*b^7) - a^
10/(6*b^11*(a + b*x)^6) + (2*a^9)/(b^11*(a + b*x)^5) - (45*a^8)/(4*b^11*(a
 + b*x)^4) + (40*a^7)/(b^11*(a + b*x)^3) - (105*a^6)/(b^11*(a + b*x)^2) +
(252*a^5)/(b^11*(a + b*x)) + (210*a^4*Log[a + b*x])/b^11
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

method	result
risch	$\frac{x^4}{4b^7}-\frac{7a\,x^3}{3b^8}+\frac{14a^2x^2}{b^9}-\frac{84a^3x}{b^{10}}+\frac{252a^5b^4x^5+1155a^6b^3x^4+2140a^7b^2x^3+\frac{7995a^8b\,x^2}{4}+\frac{1879a^9x}{2}+\frac{2131a^{10}}{12b}}{b^{10}(bx+a)^6}+\frac{210a^4\ln}{b^1}$
norman	$\frac{\frac{x^{10}}{4b}-\frac{5a\,x^9}{6b^2}+\frac{15a^2x^8}{4b^3}-\frac{30a^3x^7}{b^4}+\frac{1029a^{10}}{2b^{11}}+\frac{1260a^5x^5}{b^6}+\frac{4725a^6x^4}{b^7}+\frac{7700a^7x^3}{b^8}+\frac{13125a^8x^2}{2b^9}+\frac{2877a^9x}{b^{10}}}{(bx+a)^6}+\frac{210a^4\ln(bx+a)}{b^{11}}$
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{7}{3}a\,b^2x^3-14a^2b\,x^2+84a^3x}{b^{10}}+\frac{2a^9}{b^{11}(bx+a)^5}-\frac{45a^8}{4b^{11}(bx+a)^4}-\frac{105a^6}{b^{11}(bx+a)^2}+\frac{252a^5}{b^{11}(bx+a)}+\frac{210a^4\ln(bx+a}{b^{11}}$
parallelrisch	$3b^{10}x^{10}-10a\,b^9x^9+45a^2b^8x^8+2520\ln(bx+a)x^6a^4b^6-360a^3b^7x^7+15120\ln(bx+a)x^5a^5b^5+37800\ln(bx+a)x^4a^6b^4+15120a^5$

input
```
int(x^10/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/b^7-7/3*a*x^3/b^8+14*a^2*x^2/b^9-84*a^3*x/b^10+(252*a^5*b^4*x^5+11
55*a^6*b^3*x^4+2140*a^7*b^2*x^3+7995/4*a^8*b*x^2+1879/2*a^9*x+2131/12*a^10
/b)/b^10/(b*x+a)^6+210*a^4*ln(b*x+a)/b^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.67

$$\int \frac{x^{10}}{(a+bx)^7}\,dx = \frac{3\,b^{10}x^{10}-10\,ab^9x^9+45\,a^2b^8x^8-360\,a^3b^7x^7-4043\,a^4b^6x^6-9138\,a^5b^5x^5-3945\,a^6b^4x^4+11540\,a^7b^3x^3}{12\,(b^{17}x^6+6\,ab^{16}x^5+15}$$

input
```
integrate(x^10/(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/12*(3*b^10*x^10 - 10*a*b^9*x^9 + 45*a^2*b^8*x^8 - 360*a^3*b^7*x^7 - 4043
*a^4*b^6*x^6 - 9138*a^5*b^5*x^5 - 3945*a^6*b^4*x^4 + 11540*a^7*b^3*x^3 + 1
8105*a^8*b^2*x^2 + 10266*a^9*b*x + 2131*a^10 + 2520*(a^4*b^6*x^6 + 6*a^5*b
^5*x^5 + 15*a^6*b^4*x^4 + 20*a^7*b^3*x^3 + 15*a^8*b^2*x^2 + 6*a^9*b*x + a^
10)*log(b*x + a))/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14
*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.27

$$\int \frac{x^{10}}{(a+bx)^7}\,dx = \frac{210a^4\log(a+bx)}{b^{11}} - \frac{84a^3x}{b^{10}} + \frac{14a^2x^2}{b^9} - \frac{7ax^3}{3b^8} + \frac{2131a^{10} + 11274a^9bx + 23985a^8b^2x^2 + 25680a^7b^3x^3 + 13860a^6b^4x^4 + 3024a^5b^5x^5}{12a^6b^{11} + 72a^5b^{12}x + 180a^4b^{13}x^2 + 240a^3b^{14}x^3 + 180a^2b^{15}x^4 + 72ab^{16}x^5 + 12b^{17}x^6} + \frac{x^4}{4b^7}$$

input
```
integrate(x**10/(b*x+a)**7,x)
```

output
```
210*a**4*log(a + b*x)/b**11 - 84*a**3*x/b**10 + 14*a**2*x**2/b**9 - 7*a*x*
*3/(3*b**8) + (2131*a**10 + 11274*a**9*b*x + 23985*a**8*b**2*x**2 + 25680*
a**7*b**3*x**3 + 13860*a**6*b**4*x**4 + 3024*a**5*b**5*x**5)/(12*a**6*b**1
1 + 72*a**5*b**12*x + 180*a**4*b**13*x**2 + 240*a**3*b**14*x**3 + 180*a**2
*b**15*x**4 + 72*a*b**16*x**5 + 12*b**17*x**6) + x**4/(4*b**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.20

$$\int \frac{x^{10}}{(a+bx)^7}\,dx = \frac{3024\,a^5b^5x^5 + 13860\,a^6b^4x^4 + 25680\,a^7b^3x^3 + 23985\,a^8b^2x^2 + 11274\,a^9bx + 2131\,a^{10}}{12\,(b^{17}x^6 + 6\,ab^{16}x^5 + 15\,a^2b^{15}x^4 + 20\,a^3b^{14}x^3 + 15\,a^4b^{13}x^2 + 6\,a^5b^{12}x + a^6b^{11})} + \frac{210\,a^4\log\left(bx+a\right)}{b^{11}} + \frac{3\,b^3x^4 - 28\,ab^2x^3 + 168\,a^2bx^2 - 1008\,a^3x}{12\,b^{10}}$$

input
```
integrate(x^10/(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4 + 25680*a^7*b^3*x^3 + 23985*a^8
*b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^
15*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 21
0*a^4*log(b*x + a)/b^11 + 1/12*(3*b^3*x^4 - 28*a*b^2*x^3 + 168*a^2*b*x^2 -
 1008*a^3*x)/b^10
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int \frac{x^{10}}{(a+bx)^7}\,dx = \frac{210\,a^4\log\left(|bx+a|\right)}{b^{11}} + \frac{3024\,a^5b^5x^5 + 13860\,a^6b^4x^4 + 25680\,a^7b^3x^3 + 23985\,a^8b^2x^2 + 11274\,a^9bx + 2131\,a^{10}}{12\,(bx+a)^6b^{11}} + \frac{3\,b^{21}x^4 - 28\,ab^{20}x^3 + 168\,a^2b^{19}x^2 - 1008\,a^3b^{18}x}{12\,b^{28}}$$

input
```
integrate(x^10/(b*x+a)^7,x, algorithm="giac")
```

output
```
210*a^4*log(abs(b*x + a))/b^11 + 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^
4 + 25680*a^7*b^3*x^3 + 23985*a^8*b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/((b
*x + a)^6*b^11) + 1/12*(3*b^21*x^4 - 28*a*b^20*x^3 + 168*a^2*b^19*x^2 - 10
08*a^3*b^18*x)/b^28
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{x^{10}}{(a+bx)^7}\,dx = \frac{\frac{(a+b\,x)^4}{4}-\frac{10\,a\,(a+b\,x)^3}{3}+\frac{45\,a^2\,(a+b\,x)^2}{2}+\frac{252\,a^5}{a+b\,x}-\frac{105\,a^6}{(a+b\,x)^2}+\frac{40\,a^7}{(a+b\,x)^3}-\frac{45\,a^8}{4\,(a+b\,x)^4}+\frac{2\,a^9}{(a+b\,x)^5}-\frac{a^{10}}{6\,(a+b\,x)^6}+210\,a^4\,\mathrm{l}}{b^{11}}$$

input
```
int(x^10/(a + b*x)^7,x)
```

output
```
((a + b*x)^4/4 - (10*a*(a + b*x)^3)/3 + (45*a^2*(a + b*x)^2)/2 + (252*a^5)
/(a + b*x) - (105*a^6)/(a + b*x)^2 + (40*a^7)/(a + b*x)^3 - (45*a^8)/(4*(a
 + b*x)^4) + (2*a^9)/(a + b*x)^5 - a^10/(6*(a + b*x)^6) + 210*a^4*log(a +
b*x) - 120*a^3*b*x)/b^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int \frac{x^{10}}{(a+bx)^7}\,dx = \frac{2520\log(bx+a)\,a^{10}+15120\log(bx+a)\,a^9bx+37800\log(bx+a)\,a^8b^2x^2+50400\log(bx+a)\,a^7b^3x^3+3}{}$$

input
```
int(x^10/(b*x+a)^7,x)
```

output
```
(2520*log(a + b*x)*a**10 + 15120*log(a + b*x)*a**9*b*x + 37800*log(a + b*x
)*a**8*b**2*x**2 + 50400*log(a + b*x)*a**7*b**3*x**3 + 37800*log(a + b*x)*
a**6*b**4*x**4 + 15120*log(a + b*x)*a**5*b**5*x**5 + 2520*log(a + b*x)*a**
4*b**6*x**6 + 3654*a**10 + 19404*a**9*b*x + 40950*a**8*b**2*x**2 + 42000*a
**7*b**3*x**3 + 18900*a**6*b**4*x**4 - 2520*a**4*b**6*x**6 - 360*a**3*b**7
*x**7 + 45*a**2*b**8*x**8 - 10*a*b**9*x**9 + 3*b**10*x**10)/(12*b**11*(a**
6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4
 + 6*a*b**5*x**5 + b**6*x**6))
```

3.167 $\int \frac{x^9}{(a+bx)^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 139

$$\int \frac{x^9}{(a+bx)^7} dx = \frac{28a^2x}{b^9} - \frac{7ax^2}{2b^8} + \frac{x^3}{3b^7} + \frac{a^9}{6b^{10}(a+bx)^6} - \frac{9a^8}{5b^{10}(a+bx)^5} + \frac{9a^7}{b^{10}(a+bx)^4} - \frac{28a^6}{b^{10}(a+bx)^3} + \frac{63a^5}{b^{10}(a+bx)^2} - \frac{126a^4}{b^{10}(a+bx)} - \frac{84a^3\log(a+bx)}{b^{10}}$$

output
```
28*a^2*x/b^9-7/2*a*x^2/b^8+1/3*x^3/b^7+1/6*a^9/b^10/(b*x+a)^6-9/5*a^8/b^10
/(b*x+a)^5+9*a^7/b^10/(b*x+a)^4-28*a^6/b^10/(b*x+a)^3+63*a^5/b^10/(b*x+a)^
2-126*a^4/b^10/(b*x+a)-84*a^3*ln(b*x+a)/b^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{(a+bx)^7} dx = -\frac{2509a^9 + 12534a^8bx + 23775a^7b^2x^2 + 19100a^6b^3x^3 + 1725a^5b^4x^4 - 6870a^4b^5x^5 - 3665a^3b^6x^6 - 360a}{30b^{10}(a+bx)^6}$$

input
```
Integrate[x^9/(a + b*x)^7,x]
```

output
```
-1/30*(2509*a^9 + 12534*a^8*b*x + 23775*a^7*b^2*x^2 + 19100*a^6*b^3*x^3 +
1725*a^5*b^4*x^4 - 6870*a^4*b^5*x^5 - 3665*a^3*b^6*x^6 - 360*a^2*b^7*x^7 +
 45*a*b^8*x^8 - 10*b^9*x^9 + 2520*a^3*(a + b*x)^6*Log[a + b*x])/(b^10*(a +
 b*x)^6)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx)^7}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^9}{b^9(a+bx)^7}+\frac{9a^8}{b^9(a+bx)^6}-\frac{36a^7}{b^9(a+bx)^5}+\frac{84a^6}{b^9(a+bx)^4}-\frac{126a^5}{b^9(a+bx)^3}+\frac{126a^4}{b^9(a+bx)^2}-\frac{84a^3}{b^9(a+bx)}+\frac{28a}{b^9}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}-\frac{84a^3\log(a+bx)}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}$$

input
```
Int[x^9/(a + b*x)^7,x]
```

output
```
(28*a^2*x)/b^9 - (7*a*x^2)/(2*b^8) + x^3/(3*b^7) + a^9/(6*b^10*(a + b*x)^6
) - (9*a^8)/(5*b^10*(a + b*x)^5) + (9*a^7)/(b^10*(a + b*x)^4) - (28*a^6)/(
b^10*(a + b*x)^3) + (63*a^5)/(b^10*(a + b*x)^2) - (126*a^4)/(b^10*(a + b*x
)) - (84*a^3*Log[a + b*x])/b^10
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x^3}{3b^7}-\frac{7a\,x^2}{2b^8}+\frac{28a^2x}{b^9}+\frac{-126a^4b^4x^5-567a^5b^3x^4-1036a^6b^2x^3-957a^7b\,x^2-\frac{2229a^8x}{5}-\frac{2509a^9}{30b}}{b^9(bx+a)^6}-\frac{84a^3\ln(bx+a)}{b^{10}}$
norman	$\frac{\frac{x^9}{3b}-\frac{3a\,x^8}{2b^2}+\frac{12a^2x^7}{b^3}-\frac{1029a^9}{5b^{10}}-\frac{504a^4x^5}{b^5}-\frac{1890a^5x^4}{b^6}-\frac{3080a^6x^3}{b^7}-\frac{2625a^7x^2}{b^8}-\frac{5754a^8x}{5b^9}}{(bx+a)^6}-\frac{84a^3\ln(bx+a)}{b^{10}}$
default	$\frac{\frac{1}{3}b^2x^3-\frac{7}{2}ab\,x^2+28a^2x}{b^9}-\frac{9a^8}{5b^{10}(bx+a)^5}+\frac{9a^7}{b^{10}(bx+a)^4}+\frac{63a^5}{b^{10}(bx+a)^2}-\frac{126a^4}{b^{10}(bx+a)}-\frac{84a^3\ln(bx+a)}{b^{10}}-\frac{28a^6}{b^{10}(bx+a)^3}$
parallelrisch	$-\frac{-10b^9x^9+45a\,x^8b^8+2520\ln(bx+a)x^6a^3b^6-360a^2x^7b^7+15120\ln(bx+a)x^5a^4b^5+37800\ln(bx+a)x^4a^5b^4+15120a^4x^5b^5+5}{}$

input
```
int(x^9/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/b^7-7/2*a*x^2/b^8+28*a^2*x/b^9+(-126*a^4*b^4*x^5-567*a^5*b^3*x^4-1
036*a^6*b^2*x^3-957*a^7*b*x^2-2229/5*a^8*x-2509/30*a^9/b)/b^9/(b*x+a)^6-84
*a^3*ln(b*x+a)/b^10
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.72

$$\int \frac{x^9}{(a+bx)^7}\,dx = \frac{10\,b^9x^9-45\,ab^8x^8+360\,a^2b^7x^7+3665\,a^3b^6x^6+6870\,a^4b^5x^5-1725\,a^5b^4x^4-19100\,a^6b^3x^3-23775\,a^7b^2}{30\,(b^{16}x^6+6\,ab^{15}x^5+15\,a^2b^{14}x^4}$$

input
```
integrate(x^9/(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/30*(10*b^9*x^9 - 45*a*b^8*x^8 + 360*a^2*b^7*x^7 + 3665*a^3*b^6*x^6 + 687
0*a^4*b^5*x^5 - 1725*a^5*b^4*x^4 - 19100*a^6*b^3*x^3 - 23775*a^7*b^2*x^2 -
 12534*a^8*b*x - 2509*a^9 - 2520*(a^3*b^6*x^6 + 6*a^4*b^5*x^5 + 15*a^5*b^4
*x^4 + 20*a^6*b^3*x^3 + 15*a^7*b^2*x^2 + 6*a^8*b*x + a^9)*log(b*x + a))/(b
^16*x^6 + 6*a*b^15*x^5 + 15*a^2*b^14*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x
^2 + 6*a^5*b^11*x + a^6*b^10)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.29

$$\int \frac{x^9}{(a+bx)^7}\,dx = -\frac{84a^3\log{(a+bx)}}{b^{10}} + \frac{28a^2x}{b^9} - \frac{7ax^2}{2b^8} + \frac{-2509a^9 - 13374a^8bx - 28710a^7b^2x^2 - 31080a^6b^3x^3 - 17010a^5b^4x^4 - 3780a^4b^5x^5}{30a^6b^{10} + 180a^5b^{11}x + 450a^4b^{12}x^2 + 600a^3b^{13}x^3 + 450a^2b^{14}x^4 + 180ab^{15}x^5 + 30b^{16}x^6} + \frac{x^3}{3b^7}$$

input
```
integrate(x**9/(b*x+a)**7,x)
```

output
```
-84*a**3*log(a + b*x)/b**10 + 28*a**2*x/b**9 - 7*a*x**2/(2*b**8) + (-2509*
a**9 - 13374*a**8*b*x - 28710*a**7*b**2*x**2 - 31080*a**6*b**3*x**3 - 1701
0*a**5*b**4*x**4 - 3780*a**4*b**5*x**5)/(30*a**6*b**10 + 180*a**5*b**11*x
+ 450*a**4*b**12*x**2 + 600*a**3*b**13*x**3 + 450*a**2*b**14*x**4 + 180*a*
b**15*x**5 + 30*b**16*x**6) + x**3/(3*b**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.22

$$\int \frac{x^9}{(a+bx)^7}\,dx = -\frac{3780\,a^4b^5x^5 + 17010\,a^5b^4x^4 + 31080\,a^6b^3x^3 + 28710\,a^7b^2x^2 + 13374\,a^8bx + 2509\,a^9}{30\,(b^{16}x^6 + 6\,ab^{15}x^5 + 15\,a^2b^{14}x^4 + 20\,a^3b^{13}x^3 + 15\,a^4b^{12}x^2 + 6\,a^5b^{11}x + a^6b^{10})} - \frac{84\,a^3\log(bx+a)}{b^{10}} + \frac{2\,b^2x^3 - 21\,abx^2 + 168\,a^2x}{6\,b^9}$$

input
```
integrate(x^9/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4 + 31080*a^6*b^3*x^3 + 28710*a^
7*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/(b^16*x^6 + 6*a*b^15*x^5 + 15*a^2*b^
14*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 + 6*a^5*b^11*x + a^6*b^10) - 84
*a^3*log(b*x + a)/b^10 + 1/6*(2*b^2*x^3 - 21*a*b*x^2 + 168*a^2*x)/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{x^9}{(a+bx)^7}\,dx = -\frac{84\,a^3\log(|bx+a|)}{b^{10}} - \frac{3780\,a^4b^5x^5 + 17010\,a^5b^4x^4 + 31080\,a^6b^3x^3 + 28710\,a^7b^2x^2 + 13374\,a^8bx + 2509\,a^9}{30\,(bx+a)^6b^{10}} + \frac{2\,b^{14}x^3 - 21\,ab^{13}x^2 + 168\,a^2b^{12}x}{6\,b^{21}}$$

input
```
integrate(x^9/(b*x+a)^7,x, algorithm="giac")
```

output
```
-84*a^3*log(abs(b*x + a))/b^10 - 1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^
4 + 31080*a^6*b^3*x^3 + 28710*a^7*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/((b*
x + a)^6*b^10) + 1/6*(2*b^14*x^3 - 21*a*b^13*x^2 + 168*a^2*b^12*x)/b^21
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{(a+bx)^7} \, dx = -\frac{\frac{9\,a\,(a+b\,x)^2}{2}-\frac{(a+b\,x)^3}{3}+\frac{126\,a^4}{a+b\,x}-\frac{63\,a^5}{(a+b\,x)^2}+\frac{28\,a^6}{(a+b\,x)^3}-\frac{9\,a^7}{(a+b\,x)^4}+\frac{9\,a^8}{5\,(a+b\,x)^5}-\frac{a^9}{6\,(a+b\,x)^6}+84\,a^3\,\ln\left(a+b\,x\right)-\dots}{b^{10}}$$

input
```
int(x^9/(a + b*x)^7,x)
```

output
```
-((9*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (126*a^4)/(a + b*x) - (63*a^5)/(a
+ b*x)^2 + (28*a^6)/(a + b*x)^3 - (9*a^7)/(a + b*x)^4 + (9*a^8)/(5*(a + b*
x)^5) - a^9/(6*(a + b*x)^6) + 84*a^3*log(a + b*x) - 36*a^2*b*x)/b^10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.88

$$\int \frac{x^9}{(a+bx)^7} \, dx = \frac{-2520\log(bx+a)\,a^9-15120\log(bx+a)\,a^8bx-37800\log(bx+a)\,a^7b^2x^2-50400\log(bx+a)\,a^6b^3x^3-\dots}{\dots}$$

input
```
int(x^9/(b*x+a)^7,x)
```

output
```
( - 2520*log(a + b*x)*a**9 - 15120*log(a + b*x)*a**8*b*x - 37800*log(a + b
*x)*a**7*b**2*x**2 - 50400*log(a + b*x)*a**6*b**3*x**3 - 37800*log(a + b*x
)*a**5*b**4*x**4 - 15120*log(a + b*x)*a**4*b**5*x**5 - 2520*log(a + b*x)*a
**3*b**6*x**6 - 3654*a**9 - 19404*a**8*b*x - 40950*a**7*b**2*x**2 - 42000*
a**6*b**3*x**3 - 18900*a**5*b**4*x**4 + 2520*a**3*b**6*x**6 + 360*a**2*b**
7*x**7 - 45*a*b**8*x**8 + 10*b**9*x**9)/(30*b**10*(a**6 + 6*a**5*b*x + 15*
a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b
**6*x**6))
```

3.168 $\int \frac{x^8}{(a+bx)^7}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 128

$$\int \frac{x^8}{(a+bx)^7}\,dx = -\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}$$

output
```
-7*a*x/b^8+1/2*x^2/b^7-1/6*a^8/b^9/(b*x+a)^6+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b
^9/(b*x+a)^4+56/3*a^5/b^9/(b*x+a)^3-35*a^4/b^9/(b*x+a)^2+56*a^3/b^9/(b*x+a
)+28*a^2*ln(b*x+a)/b^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int \frac{x^8}{(a+bx)^7}\,dx = \frac{-210abx+15b^2x^2-\frac{5a^8}{(a+bx)^6}+\frac{48a^7}{(a+bx)^5}-\frac{210a^6}{(a+bx)^4}+\frac{560a^5}{(a+bx)^3}-\frac{1050a^4}{(a+bx)^2}+\frac{1680a^3}{a+bx}+840a^2\log(a+bx)}{30b^9}$$

input
```
Integrate[x^8/(a + b*x)^7,x]
```

output
```
(-210*a*b*x + 15*b^2*x^2 - (5*a^8)/(a + b*x)^6 + (48*a^7)/(a + b*x)^5 - (2
10*a^6)/(a + b*x)^4 + (560*a^5)/(a + b*x)^3 - (1050*a^4)/(a + b*x)^2 + (16
80*a^3)/(a + b*x) + 840*a^2*Log[a + b*x])/(30*b^9)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx)^7} \, dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^8}{b^8(a+bx)^7} - \frac{8a^7}{b^8(a+bx)^6} + \frac{28a^6}{b^8(a+bx)^5} - \frac{56a^5}{b^8(a+bx)^4} + \frac{70a^4}{b^8(a+bx)^3} - \frac{56a^3}{b^8(a+bx)^2} + \frac{28a^2}{b^8(a+bx)} - \frac{7a}{b^8} + \right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^8}{6b^9(a+bx)^6} + \frac{8a^7}{5b^9(a+bx)^5} - \frac{7a^6}{b^9(a+bx)^4} + \frac{56a^5}{3b^9(a+bx)^3} - \frac{35a^4}{b^9(a+bx)^2} + \frac{56a^3}{b^9(a+bx)} + \frac{28a^2\log(a+bx)}{b^9} - \frac{7ax}{b^8} + \frac{x^2}{2b^7}$$

input
```
Int[x^8/(a + b*x)^7,x]
```

output
```
(-7*a*x)/b^8 + x^2/(2*b^7) - a^8/(6*b^9*(a + b*x)^6) + (8*a^7)/(5*b^9*(a +
 b*x)^5) - (7*a^6)/(b^9*(a + b*x)^4) + (56*a^5)/(3*b^9*(a + b*x)^3) - (35*
a^4)/(b^9*(a + b*x)^2) + (56*a^3)/(b^9*(a + b*x)) + (28*a^2*Log[a + b*x])/
b^9
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x^2}{2b^7}-\frac{7ax}{b^8}+\frac{56a^3b^4x^5+245a^4b^3x^4+\frac{1316a^5b^2x^3}{3}+399a^6b\,x^2+\frac{918a^7x}{5}+\frac{341a^8}{10b}}{b^8(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}$
norman	$\frac{\frac{x^8}{2b}-\frac{4a\,x^7}{b^2}+\frac{343a^8}{5b^9}+\frac{168a^3x^5}{b^4}+\frac{630a^4x^4}{b^5}+\frac{3080a^5x^3}{3b^6}+\frac{875a^6x^2}{b^7}+\frac{1918a^7x}{5b^8}}{(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}$
default	$-\frac{-\frac{1}{2}b\,x^2+7ax}{b^8}+\frac{8a^7}{5b^9(bx+a)^5}-\frac{7a^6}{b^9(bx+a)^4}-\frac{35a^4}{b^9(bx+a)^2}+\frac{56a^3}{b^9(bx+a)}+\frac{28a^2\ln(bx+a)}{b^9}+\frac{56a^5}{3b^9(bx+a)^3}-\frac{a^8}{6b^9(bx}$
parallelrisch	$\frac{15b^8x^8+840\ln(bx+a)x^6a^2b^6-120a\,x^7b^7+5040\ln(bx+a)x^5a^3b^5+12600\ln(bx+a)x^4a^4b^4+5040a^3x^5b^5+16800\ln(bx+a)x^3a^5}{30}$

input
```
int(x^8/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b^7-7*a*x/b^8+(56*a^3*b^4*x^5+245*a^4*b^3*x^4+1316/3*a^5*b^2*x^3+3
99*a^6*b*x^2+918/5*a^7*x+341/10/b*a^8)/b^8/(b*x+a)^6+28*a^2*ln(b*x+a)/b^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.78

$$\int \frac{x^8}{(a+bx)^7}\,dx$$
$$=\frac{15\,b^8x^8-120\,ab^7x^7-1035\,a^2b^6x^6-1170\,a^3b^5x^5+3375\,a^4b^4x^4+10100\,a^5b^3x^3+10725\,a^6b^2x^2+5298\,a}{30\,(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a}$$

input
```
integrate(x^8/(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/30*(15*b^8*x^8 - 120*a*b^7*x^7 - 1035*a^2*b^6*x^6 - 1170*a^3*b^5*x^5 + 3
375*a^4*b^4*x^4 + 10100*a^5*b^3*x^3 + 10725*a^6*b^2*x^2 + 5298*a^7*b*x + 1
023*a^8 + 840*(a^2*b^6*x^6 + 6*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 20*a^5*b^3*x
^3 + 15*a^6*b^2*x^2 + 6*a^7*b*x + a^8)*log(b*x + a))/(b^15*x^6 + 6*a*b^14*
x^5 + 15*a^2*b^13*x^4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x +
 a^6*b^9)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int \frac{x^8}{(a+bx)^7}\,dx = \frac{28a^2\log{(a+bx)}}{b^9} - \frac{7ax}{b^8} + \frac{1023a^8+5508a^7bx+11970a^6b^2x^2+13160a^5b^3x^3+7350a^4b^4x^4+1680a^3b^5x^5}{30a^6b^9+180a^5b^{10}x+450a^4b^{11}x^2+600a^3b^{12}x^3+450a^2b^{13}x^4+180ab^{14}x^5+30b^{15}x^6} + \frac{x^2}{2b^7}$$

input
```
integrate(x**8/(b*x+a)**7,x)
```

output
```
28*a**2*log(a + b*x)/b**9 - 7*a*x/b**8 + (1023*a**8 + 5508*a**7*b*x + 1197
0*a**6*b**2*x**2 + 13160*a**5*b**3*x**3 + 7350*a**4*b**4*x**4 + 1680*a**3*
b**5*x**5)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a*
*3*b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) +
x**2/(2*b**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23

$$\int \frac{x^8}{(a+bx)^7}\,dx = \frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023\,a^8}{30\,(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a^3b^{12}x^3+15\,a^4b^{11}x^2+6\,a^5b^{10}x+a^6b^9)} + \frac{28\,a^2\log(bx+a)}{b^9} + \frac{bx^2-14\,ax}{2\,b^8}$$

input
```
integrate(x^8/(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/30*(1680*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*
b^2*x^2 + 5508*a^7*b*x + 1023*a^8)/(b^15*x^6 + 6*a*b^14*x^5 + 15*a^2*b^13*
x^4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*b^9) + 28*a^2
*log(b*x + a)/b^9 + 1/2*(b*x^2 - 14*a*x)/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(a+bx)^7}\,dx = \frac{28\,a^2\log(|bx+a|)}{b^9} + \frac{b^7x^2-14\,ab^6x}{2\,b^{14}} + \frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023\,a^8}{30\,(bx+a)^6b^9}$$

input
```
integrate(x^8/(b*x+a)^7,x, algorithm="giac")
```

output
```
28*a^2*log(abs(b*x + a))/b^9 + 1/2*(b^7*x^2 - 14*a*b^6*x)/b^14 + 1/30*(168
0*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b^2*x^2 +
 5508*a^7*b*x + 1023*a^8)/((b*x + a)^6*b^9)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{(a+bx)^7}\,dx = \frac{\frac{(a+b\,x)^2}{2}+\frac{56\,a^3}{a+b\,x}-\frac{35\,a^4}{(a+b\,x)^2}+\frac{56\,a^5}{3\,(a+b\,x)^3}-\frac{7\,a^6}{(a+b\,x)^4}+\frac{8\,a^7}{5\,(a+b\,x)^5}-\frac{a^8}{6\,(a+b\,x)^6}+28\,a^2\,\ln\left(a+b\,x\right)-8\,a\,b\,x}{b^9}$$

input
```
int(x^8/(a + b*x)^7,x)
```

output
```
((a + b*x)^2/2 + (56*a^3)/(a + b*x) - (35*a^4)/(a + b*x)^2 + (56*a^5)/(3*(
a + b*x)^3) - (7*a^6)/(a + b*x)^4 + (8*a^7)/(5*(a + b*x)^5) - a^8/(6*(a +
b*x)^6) + 28*a^2*log(a + b*x) - 8*a*b*x)/b^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.95

$$\int \frac{x^8}{(a+bx)^7}\,dx = \frac{840\log(bx+a)\,a^8+5040\log(bx+a)\,a^7bx+12600\log(bx+a)\,a^6b^2x^2+16800\log(bx+a)\,a^5b^3x^3+126}{}$$

input
```
int(x^8/(b*x+a)^7,x)
```

output
```
(840*log(a + b*x)*a**8 + 5040*log(a + b*x)*a**7*b*x + 12600*log(a + b*x)*a
**6*b**2*x**2 + 16800*log(a + b*x)*a**5*b**3*x**3 + 12600*log(a + b*x)*a**
4*b**4*x**4 + 5040*log(a + b*x)*a**3*b**5*x**5 + 840*log(a + b*x)*a**2*b**
6*x**6 + 1218*a**8 + 6468*a**7*b*x + 13650*a**6*b**2*x**2 + 14000*a**5*b**
3*x**3 + 6300*a**4*b**4*x**4 - 840*a**2*b**6*x**6 - 120*a*b**7*x**7 + 15*b
**8*x**8)/(30*b**9*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x
**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.169 $\int \frac{x^7}{(a+bx)^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 118

$$\int \frac{x^7}{(a+bx)^7} dx = \frac{x}{b^7} + \frac{a^7}{6b^8(a+bx)^6} - \frac{7a^6}{5b^8(a+bx)^5} + \frac{21a^5}{4b^8(a+bx)^4} - \frac{35a^4}{3b^8(a+bx)^3} + \frac{35a^3}{2b^8(a+bx)^2} - \frac{21a^2}{b^8(a+bx)} - \frac{7a\log(a+bx)}{b^8}$$

output
```
x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-3
5/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x
+a)/b^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(a+bx)^7} dx = -\frac{669a^7 + 3594a^6bx + 7725a^5b^2x^2 + 8200a^4b^3x^3 + 4050a^3b^4x^4 + 360a^2b^5x^5 - 360ab^6x^6 - 60b^7x^7 + 420}{60b^8(a+bx)^6}$$

input
```
Integrate[x^7/(a + b*x)^7,x]
```

output
```
-1/60*(669*a^7 + 3594*a^6*b*x + 7725*a^5*b^2*x^2 + 8200*a^4*b^3*x^3 + 4050
*a^3*b^4*x^4 + 360*a^2*b^5*x^5 - 360*a*b^6*x^6 - 60*b^7*x^7 + 420*a*(a + b
*x)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx)^7} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^7}{b^7(a+bx)^7}+\frac{7a^6}{b^7(a+bx)^6}-\frac{21a^5}{b^7(a+bx)^5}+\frac{35a^4}{b^7(a+bx)^4}-\frac{35a^3}{b^7(a+bx)^3}+\frac{21a^2}{b^7(a+bx)^2}-\frac{7a}{b^7(a+bx)}+\frac{1}{b^7}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

input
```
Int[x^7/(a + b*x)^7,x]
```

output
```
x/b^7 + a^7/(6*b^8*(a + b*x)^6) - (7*a^6)/(5*b^8*(a + b*x)^5) + (21*a^5)/(
4*b^8*(a + b*x)^4) - (35*a^4)/(3*b^8*(a + b*x)^3) + (35*a^3)/(2*b^8*(a + b
*x)^2) - (21*a^2)/(b^8*(a + b*x)) - (7*a*Log[a + b*x])/b^8
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x}{b^7}+\frac{-21a^2b^4x^5-\frac{175a^3b^3x^4}{2}-\frac{455a^4b^2x^3}{3}-\frac{539a^5b\,x^2}{4}-\frac{609a^6x}{10}-\frac{223a^7}{20b}}{b^7(bx+a)^6}-\frac{7a\ln(bx+a)}{b^8}$
norman	$\frac{\frac{x^7}{b}-\frac{343a^7}{20b^8}-\frac{42a^2x^5}{b^3}-\frac{315a^3x^4}{2b^4}-\frac{770a^4x^3}{3b^5}-\frac{875a^5x^2}{4b^6}-\frac{959a^6x}{10b^7}}{(bx+a)^6}-\frac{7a\ln(bx+a)}{b^8}$
default	$\frac{x}{b^7}+\frac{a^7}{6b^8(bx+a)^6}-\frac{7a^6}{5b^8(bx+a)^5}+\frac{21a^5}{4b^8(bx+a)^4}-\frac{35a^4}{3b^8(bx+a)^3}+\frac{35a^3}{2b^8(bx+a)^2}-\frac{21a^2}{b^8(bx+a)}-\frac{7a\ln(bx+a)}{b^8}$
parallelrisch	$-\frac{420\ln(bx+a)x^6a\,b^6-60b^7x^7+2520\ln(bx+a)x^5a^2b^5+6300\ln(bx+a)x^4a^3b^4+2520a^2b^5x^5+8400\ln(bx+a)x^3a^4b^3+9450a^3b}{60b^8(bx+}$

input
```
int(x^7/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
x/b^7+(-21*a^2*b^4*x^5-175/2*a^3*b^3*x^4-455/3*a^4*b^2*x^3-539/4*a^5*b*x^2
-609/10*a^6*x-223/20*a^7/b)/b^7/(b*x+a)^6-7*a*ln(b*x+a)/b^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

$$\int \frac{x^7}{(a+bx)^7}\,dx = \frac{60\,b^7x^7+360\,ab^6x^6-360\,a^2b^5x^5-4050\,a^3b^4x^4-8200\,a^4b^3x^3-7725\,a^5b^2x^2-3594\,a^6bx-669\,a^7-420}{60\,(b^{14}x^6+6\,ab^{13}x^5+15\,a^2b^{12}x^4+20\,a^3b^{11}x^3+}$$

input
```
integrate(x^7/(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/60*(60*b^7*x^7 + 360*a*b^6*x^6 - 360*a^2*b^5*x^5 - 4050*a^3*b^4*x^4 - 82
00*a^4*b^3*x^3 - 7725*a^5*b^2*x^2 - 3594*a^6*b*x - 669*a^7 - 420*(a*b^6*x^
6 + 6*a^2*b^5*x^5 + 15*a^3*b^4*x^4 + 20*a^4*b^3*x^3 + 15*a^5*b^2*x^2 + 6*a
^6*b*x + a^7)*log(b*x + a))/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4 + 2
0*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.30

$$\int \frac{x^7}{(a+bx)^7}\,dx = -\frac{7a\log{(a+bx)}}{b^8} + \frac{-669a^7-3654a^6bx-8085a^5b^2x^2-9100a^4b^3x^3-5250a^3b^4x^4-1260a^2b^5x^5}{60a^6b^8+360a^5b^9x+900a^4b^{10}x^2+1200a^3b^{11}x^3+900a^2b^{12}x^4+360ab^{13}x^5+60b^{14}x^6} + \frac{x}{b^7}$$

input
```
integrate(x**7/(b*x+a)**7,x)
```

output
```
-7*a*log(a + b*x)/b**8 + (-669*a**7 - 3654*a**6*b*x - 8085*a**5*b**2*x**2
- 9100*a**4*b**3*x**3 - 5250*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*a**
6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 90
0*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) + x/b**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{(a+bx)^7}\,dx = -\frac{1260\,a^2b^5x^5+5250\,a^3b^4x^4+9100\,a^4b^3x^3+8085\,a^5b^2x^2+3654\,a^6bx+669\,a^7}{60\,(b^{14}x^6+6\,ab^{13}x^5+15\,a^2b^{12}x^4+20\,a^3b^{11}x^3+15\,a^4b^{10}x^2+6\,a^5b^9x+a^6b^8)} + \frac{x}{b^7} - \frac{7\,a\log{(bx+a)}}{b^8}$$

input
```
integrate(x^7/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b
^2*x^2 + 3654*a^6*b*x + 669*a^7)/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^
4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8) + x/b^7 - 7
*a*log(b*x + a)/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{(a+bx)^7}\,dx = \frac{x}{b^7} - \frac{7\,a\log(|bx+a|)}{b^8} - \frac{1260\,a^2b^5x^5 + 5250\,a^3b^4x^4 + 9100\,a^4b^3x^3 + 8085\,a^5b^2x^2 + 3654\,a^6bx + 669\,a^7}{60\,(bx+a)^6b^8}$$

input
```
integrate(x^7/(b*x+a)^7,x, algorithm="giac")
```

output
```
x/b^7 - 7*a*log(abs(b*x + a))/b^8 - 1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*
x^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^2*x^2 + 3654*a^6*b*x + 669*a^7)/((b*x
+ a)^6*b^8)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.77

$$\int \frac{x^7}{(a+bx)^7}\,dx = -\frac{7\,a\,\ln(a+b\,x) - b\,x + \frac{21\,a^2}{a+b\,x} - \frac{35\,a^3}{2\,(a+b\,x)^2} + \frac{35\,a^4}{3\,(a+b\,x)^3} - \frac{21\,a^5}{4\,(a+b\,x)^4} + \frac{7\,a^6}{5\,(a+b\,x)^5} - \frac{a^7}{6\,(a+b\,x)^6}}{b^8}$$

input
```
int(x^7/(a + b*x)^7,x)
```

output
```
-(7*a*log(a + b*x) - b*x + (21*a^2)/(a + b*x) - (35*a^3)/(2*(a + b*x)^2) +
 (35*a^4)/(3*(a + b*x)^3) - (21*a^5)/(4*(a + b*x)^4) + (7*a^6)/(5*(a + b*x
)^5) - a^7/(6*(a + b*x)^6))/b^8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.01

$$\int \frac{x^7}{(a+bx)^7}\,dx = \frac{-420\log(bx+a)\,a^7 - 2520\log(bx+a)\,a^6bx - 6300\log(bx+a)\,a^5b^2x^2 - 8400\log(bx+a)\,a^4b^3x^3 - 630}{60b^8\,(b^6x}$$

input
```
int(x^7/(b*x+a)^7,x)
```

output
```
( - 420*log(a + b*x)*a**7 - 2520*log(a + b*x)*a**6*b*x - 6300*log(a + b*x)
*a**5*b**2*x**2 - 8400*log(a + b*x)*a**4*b**3*x**3 - 6300*log(a + b*x)*a**
3*b**4*x**4 - 2520*log(a + b*x)*a**2*b**5*x**5 - 420*log(a + b*x)*a*b**6*x
**6 - 609*a**7 - 3234*a**6*b*x - 6825*a**5*b**2*x**2 - 7000*a**4*b**3*x**3
 - 3150*a**3*b**4*x**4 + 420*a*b**6*x**6 + 60*b**7*x**7)/(60*b**8*(a**6 +
6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6
*a*b**5*x**5 + b**6*x**6))
```

3.170 $\int \frac{x^6}{(a+bx)^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 109

$$\int \frac{x^6}{(a+bx)^7} dx = -\frac{a^6}{6b^7(a+bx)^6} + \frac{6a^5}{5b^7(a+bx)^5} - \frac{15a^4}{4b^7(a+bx)^4} + \frac{20a^3}{3b^7(a+bx)^3} - \frac{15a^2}{2b^7(a+bx)^2} + \frac{6a}{b^7(a+bx)} + \frac{\log(a+bx)}{b^7}$$

output
```
-1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a
^3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(a+bx)^7} dx = \frac{\frac{a(147a^5+822a^4bx+1875a^3b^2x^2+2200a^2b^3x^3+1350ab^4x^4+360b^5x^5)}{(a+bx)^6} + 60\log(a+bx)}{60b^7}$$

input
```
Integrate[x^6/(a + b*x)^7,x]
```

output
```
((a*(147*a^5 + 822*a^4*b*x + 1875*a^3*b^2*x^2 + 2200*a^2*b^3*x^3 + 1350*a*
b^4*x^4 + 360*b^5*x^5))/(a + b*x)^6 + 60*Log[a + b*x])/(60*b^7)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx)^7}\,dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^6}{b^6(a+bx)^7}-\frac{6a^5}{b^6(a+bx)^6}+\frac{15a^4}{b^6(a+bx)^5}-\frac{20a^3}{b^6(a+bx)^4}+\frac{15a^2}{b^6(a+bx)^3}-\frac{6a}{b^6(a+bx)^2}+\frac{1}{b^6(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

input
```
Int[x^6/(a + b*x)^7,x]
```

output
```
-1/6*a^6/(b^7*(a + b*x)^6) + (6*a^5)/(5*b^7*(a + b*x)^5) - (15*a^4)/(4*b^7
*(a + b*x)^4) + (20*a^3)/(3*b^7*(a + b*x)^3) - (15*a^2)/(2*b^7*(a + b*x)^2
) + (6*a)/(b^7*(a + b*x)) + Log[a + b*x]/b^7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result
norman	$\frac{\frac{49a^6}{20b^7}+\frac{6a\,x^5}{b^2}+\frac{45a^2x^4}{2b^3}+\frac{125a^4x^2}{4b^5}+\frac{137a^5x}{10b^6}+\frac{110a^3x^3}{3b^4}}{(bx+a)^6}+\frac{\ln(bx+a)}{b^7}$
risch	$\frac{\frac{49a^6}{20b^7}+\frac{6a\,x^5}{b^2}+\frac{45a^2x^4}{2b^3}+\frac{125a^4x^2}{4b^5}+\frac{137a^5x}{10b^6}+\frac{110a^3x^3}{3b^4}}{(bx+a)^6}+\frac{\ln(bx+a)}{b^7}$
default	$-\frac{a^6}{6b^7(bx+a)^6}+\frac{6a^5}{5b^7(bx+a)^5}-\frac{15a^4}{4b^7(bx+a)^4}+\frac{20a^3}{3b^7(bx+a)^3}-\frac{15a^2}{2b^7(bx+a)^2}+\frac{6a}{b^7(bx+a)}+\frac{\ln(bx+a)}{b^7}$
parallelrisch	$\frac{60\ln(bx+a)x^6b^6+360\ln(bx+a)x^5a\,b^5+900\ln(bx+a)x^4a^2b^4+360a\,x^5b^5+1200\ln(bx+a)x^3a^3b^3+1350a^2x^4b^4+900\ln(bx+a)}{60b^7(bx+a)^6}$

input
```
int(x^6/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
(49/20*a^6/b^7+6*a/b^2*x^5+45/2*a^2/b^3*x^4+125/4*a^4/b^5*x^2+137/10*a^5/b
^6*x+110/3*a^3/b^4*x^3)/(b*x+a)^6+ln(b*x+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.77

$$\int \frac{x^6}{(a+bx)^7}\,dx = \frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6+60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4}{60\,(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,}$$

input
```
integrate(x^6/(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x
^2 + 822*a^5*b*x + 147*a^6 + 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 +
20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a))/(b^13*x^6
 + 6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a
^5*b^8*x + a^6*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.29

$$\int \frac{x^6}{(a+bx)^7}\,dx = \frac{147a^6+822a^5bx+1875a^4b^2x^2+2200a^3b^3x^3+1350a^2b^4x^4+360ab^5x^5}{60a^6b^7+360a^5b^8x+900a^4b^9x^2+1200a^3b^{10}x^3+900a^2b^{11}x^4+360ab^{12}x^5+60b^{13}x^6} + \frac{\log(a+bx)}{b^7}$$

input
```
integrate(x**6/(b*x+a)**7,x)
```

output
```
(147*a**6 + 822*a**5*b*x + 1875*a**4*b**2*x**2 + 2200*a**3*b**3*x**3 + 135
0*a**2*b**4*x**4 + 360*a*b**5*x**5)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*
a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*
x**5 + 60*b**13*x**6) + log(a + b*x)/b**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.25

$$\int \frac{x^6}{(a+bx)^7}\,dx = \frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6}{60\,(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7)} + \frac{\log(bx+a)}{b^7}$$

input
```
integrate(x^6/(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x
^2 + 822*a^5*b*x + 147*a^6)/(b^13*x^6 + 6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 2
0*a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b^8*x + a^6*b^7) + log(b*x + a)/b^
7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \frac{x^6}{(a+bx)^7}\,dx = \frac{\log(|bx+a|)}{b^7} + \frac{360\,ab^4x^5 + 1350\,a^2b^3x^4 + 2200\,a^3b^2x^3 + 1875\,a^4bx^2 + 822\,a^5x + \frac{147\,a^6}{b}}{60\,(bx+a)^6b^6}$$

input
```
integrate(x^6/(b*x+a)^7,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^7 + 1/60*(360*a*b^4*x^5 + 1350*a^2*b^3*x^4 + 2200*a^3*
b^2*x^3 + 1875*a^4*b*x^2 + 822*a^5*x + 147*a^6/b)/((b*x + a)^6*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \frac{x^6}{(a+bx)^7}\,dx = \frac{\ln(a+b\,x) + \frac{6\,a}{a+b\,x} - \frac{15\,a^2}{2\,(a+b\,x)^2} + \frac{20\,a^3}{3\,(a+b\,x)^3} - \frac{15\,a^4}{4\,(a+b\,x)^4} + \frac{6\,a^5}{5\,(a+b\,x)^5} - \frac{a^6}{6\,(a+b\,x)^6}}{b^7}$$

input
```
int(x^6/(a + b*x)^7,x)
```

output
```
(log(a + b*x) + (6*a)/(a + b*x) - (15*a^2)/(2*(a + b*x)^2) + (20*a^3)/(3*(
a + b*x)^3) - (15*a^4)/(4*(a + b*x)^4) + (6*a^5)/(5*(a + b*x)^5) - a^6/(6*
(a + b*x)^6))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.06

$$\int \frac{x^6}{(a+bx)^7}\,dx$$
$$= \frac{60\log(bx+a)\,a^6 + 360\log(bx+a)\,a^5bx + 900\log(bx+a)\,a^4b^2x^2 + 1200\log(bx+a)\,a^3b^3x^3 + 900\log(b\cdots}{60b^7\,(b^6x^6 + 6a\,b^5x^5 + \cdots}$$

input
```
int(x^6/(b*x+a)^7,x)
```

output
```
(60*log(a + b*x)*a**6 + 360*log(a + b*x)*a**5*b*x + 900*log(a + b*x)*a**4*
b**2*x**2 + 1200*log(a + b*x)*a**3*b**3*x**3 + 900*log(a + b*x)*a**2*b**4*
x**4 + 360*log(a + b*x)*a*b**5*x**5 + 60*log(a + b*x)*b**6*x**6 + 87*a**6
+ 462*a**5*b*x + 975*a**4*b**2*x**2 + 1000*a**3*b**3*x**3 + 450*a**2*b**4*
x**4 - 60*b**6*x**6)/(60*b**7*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*
a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.171 $\int \frac{x^5}{(a+bx)^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{x^5}{(a+bx)^7} dx = \frac{x^6}{6a(a+bx)^6}$$

output
```
1/6*x^6/a/(b*x+a)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int \frac{x^5}{(a+bx)^7} dx = -\frac{a^5+6a^4bx+15a^3b^2x^2+20a^2b^3x^3+15ab^4x^4+6b^5x^5}{6b^6(a+bx)^6}$$

input
```
Integrate[x^5/(a + b*x)^7,x]
```

output
```
-1/6*(a^5 + 6*a^4*b*x + 15*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 15*a*b^4*x^4 + 6
*b^5*x^5)/(b^6*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx)^7} \, dx$$

$$\Big\downarrow \ 48$$

$$\frac{x^6}{6a(a+bx)^6}$$

input
```
Int[x^5/(a + b*x)^7,x]
```

output
```
x^6/(6*a*(a + b*x)^6)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.71

method	result	size
gosper	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6(bx+a)^6b^6}$	63
orering	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6(bx+a)^6b^6}$	63
parallelrisch	$\frac{-6b^5x^5-15a\,b^4x^4-20a^2b^3x^3-15a^3b^2x^2-6a^4bx-a^5}{6b^6(bx+a)^6}$	65
norman	$\frac{-\frac{x^5}{b}-\frac{5a\,x^4}{2b^2}-\frac{10a^2x^3}{3b^3}-\frac{5a^3x^2}{2b^4}-\frac{a^4x}{b^5}-\frac{a^5}{6b^6}}{(bx+a)^6}$	66
risch	$\frac{-\frac{x^5}{b}-\frac{5a\,x^4}{2b^2}-\frac{10a^2x^3}{3b^3}-\frac{5a^3x^2}{2b^4}-\frac{a^4x}{b^5}-\frac{a^5}{6b^6}}{(bx+a)^6}$	66
default	$-\frac{a^4}{b^6(bx+a)^5}+\frac{5a^3}{2b^6(bx+a)^4}+\frac{5a}{2b^6(bx+a)^2}-\frac{1}{(bx+a)b^6}-\frac{10a^2}{3b^6(bx+a)^3}+\frac{a^5}{6b^6(bx+a)^6}$	87

input
```
int(x^5/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(6*b^5*x^5+15*a*b^4*x^4+20*a^2*b^3*x^3+15*a^3*b^2*x^2+6*a^4*b*x+a^5)/
(b*x+a)^6/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.06

$$\int \frac{x^5}{(a+bx)^7}\,dx = -\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,(b^{12}x^6+6\,ab^{11}x^5+15\,a^2b^{10}x^4+20\,a^3b^9x^3+15\,a^4b^8x^2+6\,a^5b^7x+a^6b^6)}$$

input
```
integrate(x^5/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b
*x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15
*a^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \frac{x^5}{(a+bx)^7}\,dx = \frac{-a^5 - 6a^4bx - 15a^3b^2x^2 - 20a^2b^3x^3 - 15ab^4x^4 - 6b^5x^5}{6a^6b^6 + 36a^5b^7x + 90a^4b^8x^2 + 120a^3b^9x^3 + 90a^2b^{10}x^4 + 36ab^{11}x^5 + 6b^{12}x^6}$$

input
```
integrate(x**5/(b*x+a)**7,x)
```

output
```
(-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x*
*4 - 6*b**5*x**5)/(6*a**6*b**6 + 36*a**5*b**7*x + 90*a**4*b**8*x**2 + 120*
a**3*b**9*x**3 + 90*a**2*b**10*x**4 + 36*a*b**11*x**5 + 6*b**12*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.06

$$\int \frac{x^5}{(a+bx)^7}\,dx = -\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,(b^{12}x^6 + 6\,ab^{11}x^5 + 15\,a^2b^{10}x^4 + 20\,a^3b^9x^3 + 15\,a^4b^8x^2 + 6\,a^5b^7x + a^6b^6)}$$

input
```
integrate(x^5/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b
*x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15
*a^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.65

$$\int \frac{x^5}{(a+bx)^7}\,dx = -\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,(bx+a)^6b^6}$$

input
```
integrate(x^5/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b
*x + a^5)/((b*x + a)^6*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.24

$$\int \frac{x^5}{(a+bx)^7}\,dx = \frac{\frac{5\,a}{2\,(a+b\,x)^2} - \frac{1}{a+b\,x} - \frac{10\,a^2}{3\,(a+b\,x)^3} + \frac{5\,a^3}{2\,(a+b\,x)^4} - \frac{a^4}{(a+b\,x)^5} + \frac{a^5}{6\,(a+b\,x)^6}}{b^6}$$

input
```
int(x^5/(a + b*x)^7,x)
```

output
```
((5*a)/(2*(a + b*x)^2) - 1/(a + b*x) - (10*a^2)/(3*(a + b*x)^3) + (5*a^3)/
(2*(a + b*x)^4) - a^4/(a + b*x)^5 + a^5/(6*(a + b*x)^6))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{x^5}{(a+bx)^7}\,dx = \frac{x^6}{6a\left(b^6x^6 + 6a\,b^5x^5 + 15a^2b^4x^4 + 20a^3b^3x^3 + 15a^4b^2x^2 + 6a^5bx + a^6\right)}$$

input
```
int(x^5/(b*x+a)^7,x)
```

output
```
x**6/(6*a*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*
a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.172 $\int \frac{x^4}{(a+bx)^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{x^4}{(a+bx)^7} dx = \frac{x^5}{6a(a+bx)^6} + \frac{x^5}{30a^2(a+bx)^5}$$

output
```
1/6*x^5/a/(b*x+a)^6+1/30*x^5/a^2/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{x^4}{(a+bx)^7} dx = -\frac{a^4+6a^3bx+15a^2b^2x^2+20ab^3x^3+15b^4x^4}{30b^5(a+bx)^6}$$

input
```
Integrate[x^4/(a + b*x)^7,x]
```

output
```
-1/30*(a^4 + 6*a^3*b*x + 15*a^2*b^2*x^2 + 20*a*b^3*x^3 + 15*b^4*x^4)/(b^5*
(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx)^7} \, dx$$

$$\Big\downarrow \; 55$$

$$\frac{\int \frac{x^4}{(a+bx)^6}dx}{6a}+\frac{x^5}{6a(a+bx)^6}$$

$$\Big\downarrow \; 48$$

$$\frac{x^5}{30a^2(a+bx)^5}+\frac{x^5}{6a(a+bx)^6}$$

input
```
Int[x^4/(a + b*x)^7,x]
```

output
```
x^5/(6*a*(a + b*x)^6) + x^5/(30*a^2*(a + b*x)^5)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

method	result	size
gosper	$-\frac{15b^4x^4+20a\,x^3b^3+15a^2b^2x^2+6a^3bx+a^4}{30(bx+a)^6b^5}$	52
orering	$-\frac{15b^4x^4+20a\,x^3b^3+15a^2b^2x^2+6a^3bx+a^4}{30(bx+a)^6b^5}$	52
norman	$\frac{-\frac{x^4}{2b}-\frac{2a\,x^3}{3b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3x}{5b^4}-\frac{a^4}{30b^5}}{(bx+a)^6}$	55
risch	$\frac{-\frac{x^4}{2b}-\frac{2a\,x^3}{3b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3x}{5b^4}-\frac{a^4}{30b^5}}{(bx+a)^6}$	55
parallelrisch	$\frac{-15b^5x^4-20a\,b^4x^3-15a^2b^3x^2-6a^3b^2x-a^4b}{30b^6(bx+a)^6}$	57
default	$\frac{4a^3}{5b^5(bx+a)^5}-\frac{3a^2}{2b^5(bx+a)^4}-\frac{1}{2b^5(bx+a)^2}+\frac{4a}{3b^5(bx+a)^3}-\frac{a^4}{6b^5(bx+a)^6}$	72

input
```
int(x^4/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(15*b^4*x^4+20*a*b^3*x^3+15*a^2*b^2*x^2+6*a^3*b*x+a^4)/(b*x+a)^6/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.11

$$\int \frac{x^4}{(a+bx)^7}\,dx = -\frac{15\,b^4x^4+20\,ab^3x^3+15\,a^2b^2x^2+6\,a^3bx+a^4}{30\,(b^{11}x^6+6\,ab^{10}x^5+15\,a^2b^9x^4+20\,a^3b^8x^3+15\,a^4b^7x^2+6\,a^5b^6x+a^6b^5)}$$

input
```
integrate(x^4/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11
*x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6
*a^5*b^6*x + a^6*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(27) = 54$.

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.31

$$\int \frac{x^4}{(a+bx)^7}\,dx = \frac{-a^4-6a^3bx-15a^2b^2x^2-20ab^3x^3-15b^4x^4}{30a^6b^5+180a^5b^6x+450a^4b^7x^2+600a^3b^8x^3+450a^2b^9x^4+180ab^{10}x^5+30b^{11}x^6}$$

input
```
integrate(x**4/(b*x+a)**7,x)
```

output
```
(-a**4 - 6*a**3*b*x - 15*a**2*b**2*x**2 - 20*a*b**3*x**3 - 15*b**4*x**4)/(
30*a**6*b**5 + 180*a**5*b**6*x + 450*a**4*b**7*x**2 + 600*a**3*b**8*x**3 +
 450*a**2*b**9*x**4 + 180*a*b**10*x**5 + 30*b**11*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.11

$$\int \frac{x^4}{(a+bx)^7}\,dx = -\frac{15\,b^4x^4 + 20\,ab^3x^3 + 15\,a^2b^2x^2 + 6\,a^3bx + a^4}{30\,(b^{11}x^6 + 6\,ab^{10}x^5 + 15\,a^2b^9x^4 + 20\,a^3b^8x^3 + 15\,a^4b^7x^2 + 6\,a^5b^6x + a^6b^5)}$$

input
```
integrate(x^4/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11
*x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6
*a^5*b^6*x + a^6*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{x^4}{(a+bx)^7}\,dx = -\frac{15\,b^4x^4 + 20\,ab^3x^3 + 15\,a^2b^2x^2 + 6\,a^3bx + a^4}{30\,(bx+a)^6b^5}$$

input
```
integrate(x^4/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/((b*x
 + a)^6*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{(a+bx)^7} dx = \frac{x^5 (6 a + b x)}{30 a^2 (a + b x)^6}$$

input
```
int(x^4/(a + b*x)^7,x)
```

output
```
(x^5*(6*a + b*x))/(30*a^2*(a + b*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.09

$$\int \frac{x^4}{(a+bx)^7} dx = \frac{-15b^4x^4 - 20a\,b^3x^3 - 15a^2b^2x^2 - 6a^3bx - a^4}{30b^5 (b^6x^6 + 6a\,b^5x^5 + 15a^2b^4x^4 + 20a^3b^3x^3 + 15a^4b^2x^2 + 6a^5bx + a^6)}$$

input
```
int(x^4/(b*x+a)^7,x)
```

output
```
( - a**4 - 6*a**3*b*x - 15*a**2*b**2*x**2 - 20*a*b**3*x**3 - 15*b**4*x**4)
/(30*b**5*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*
a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.173 $\int \frac{x^3}{(a+bx)^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{x^3}{(a+bx)^7} dx = \frac{x^4}{6a(a+bx)^6} + \frac{x^4}{15a^2(a+bx)^5} + \frac{x^4}{60a^3(a+bx)^4}$$

output
```
1/6*x^4/a/(b*x+a)^6+1/15*x^4/a^2/(b*x+a)^5+1/60*x^4/a^3/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{(a+bx)^7} dx = -\frac{a^3 + 6a^2bx + 15ab^2x^2 + 20b^3x^3}{60b^4(a+bx)^6}$$

input
```
Integrate[x^3/(a + b*x)^7,x]
```

output
```
-1/60*(a^3 + 6*a^2*b*x + 15*a*b^2*x^2 + 20*b^3*x^3)/(b^4*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^7} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^7}+\frac{3a^2}{b^3(a+bx)^6}-\frac{3a}{b^3(a+bx)^5}+\frac{1}{b^3(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}$$

input
```
Int[x^3/(a + b*x)^7,x]
```

output
```
a^3/(6*b^4*(a + b*x)^6) - (3*a^2)/(5*b^4*(a + b*x)^5) + (3*a)/(4*b^4*(a +
b*x)^4) - 1/(3*b^4*(a + b*x)^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{20b^{3}x^{3}+15a\,b^{2}x^{2}+6a^{2}bx+a^{3}}{60(bx+a)^{6}b^{4}}$	41
orering	$-\frac{20b^{3}x^{3}+15a\,b^{2}x^{2}+6a^{2}bx+a^{3}}{60(bx+a)^{6}b^{4}}$	41
norman	$\frac{-\frac{x^{3}}{3b}-\frac{a\,x^{2}}{4b^{2}}-\frac{a^{2}x}{10b^{3}}-\frac{a^{3}}{60b^{4}}}{(bx+a)^{6}}$	44
risch	$\frac{-\frac{x^{3}}{3b}-\frac{a\,x^{2}}{4b^{2}}-\frac{a^{2}x}{10b^{3}}-\frac{a^{3}}{60b^{4}}}{(bx+a)^{6}}$	44
parallelrisch	$\frac{-20b^{5}x^{3}-15a\,b^{4}x^{2}-6a^{2}b^{3}x-a^{3}b^{2}}{60b^{6}(bx+a)^{6}}$	48
default	$-\frac{3a^{2}}{5b^{4}(bx+a)^{5}}+\frac{3a}{4b^{4}(bx+a)^{4}}-\frac{1}{3b^{4}(bx+a)^{3}}+\frac{a^{3}}{6b^{4}(bx+a)^{6}}$	57

input
```
int(x^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(20*b^3*x^3+15*a*b^2*x^2+6*a^2*b*x+a^3)/(b*x+a)^6/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(46) = 92.

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(a+bx)^7}\,dx = -\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\,(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4)}$$

input
```
integrate(x^3/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^
5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b
^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{(a+bx)^7}\,dx = \frac{-a^3 - 6a^2bx - 15ab^2x^2 - 20b^3x^3}{60a^6b^4 + 360a^5b^5x + 900a^4b^6x^2 + 1200a^3b^7x^3 + 900a^2b^8x^4 + 360ab^9x^5 + 60b^{10}x^6}$$

input
```
integrate(x**3/(b*x+a)**7,x)
```

output
```
(-a**3 - 6*a**2*b*x - 15*a*b**2*x**2 - 20*b**3*x**3)/(60*a**6*b**4 + 360*a
**5*b**5*x + 900*a**4*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4
 + 360*a*b**9*x**5 + 60*b**10*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(a+bx)^7}\,dx = -\frac{20\,b^3x^3 + 15\,ab^2x^2 + 6\,a^2bx + a^3}{60\,(b^{10}x^6 + 6\,ab^9x^5 + 15\,a^2b^8x^4 + 20\,a^3b^7x^3 + 15\,a^4b^6x^2 + 6\,a^5b^5x + a^6b^4)}$$

input
```
integrate(x^3/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^
5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b
^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(a+bx)^7}\,dx = -\frac{20\,b^3x^3 + 15\,ab^2x^2 + 6\,a^2bx + a^3}{60\,(bx+a)^6b^4}$$

input
```
integrate(x^3/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/((b*x + a)^6*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a+bx)^7}\,dx = \frac{\frac{3\,a}{4\,(a+b\,x)^4} - \frac{1}{3\,(a+b\,x)^3} - \frac{3\,a^2}{5\,(a+b\,x)^5} + \frac{a^3}{6\,(a+b\,x)^6}}{b^4}$$

input
```
int(x^3/(a + b*x)^7,x)
```

output
```
((3*a)/(4*(a + b*x)^4) - 1/(3*(a + b*x)^3) - (3*a^2)/(5*(a + b*x)^5) + a^3
/(6*(a + b*x)^6))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{(a+bx)^7}\,dx = \frac{-20b^3x^3 - 15a\,b^2x^2 - 6a^2bx - a^3}{60b^4\,(b^6x^6 + 6a\,b^5x^5 + 15a^2b^4x^4 + 20a^3b^3x^3 + 15a^4b^2x^2 + 6a^5bx + a^6)}$$

input
```
int(x^3/(b*x+a)^7,x)
```

output
```
( - a**3 - 6*a**2*b*x - 15*a*b**2*x**2 - 20*b**3*x**3)/(60*b**4*(a**6 + 6*
a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a
*b**5*x**5 + b**6*x**6))
```

3.174 $\int \frac{x^2}{(a+bx)^7}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int \frac{x^2}{(a+bx)^7}\,dx = -\frac{a^2}{6b^3(a+bx)^6} + \frac{2a}{5b^3(a+bx)^5} - \frac{1}{4b^3(a+bx)^4}$$

output
```
-1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(a+bx)^7}\,dx = -\frac{a^2+6abx+15b^2x^2}{60b^3(a+bx)^6}$$

input
```
Integrate[x^2/(a + b*x)^7,x]
```

output
```
-1/60*(a^2 + 6*a*b*x + 15*b^2*x^2)/(b^3*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^7} \, dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2}{b^2(a+bx)^7}-\frac{2a}{b^2(a+bx)^6}+\frac{1}{b^2(a+bx)^5}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

input
```
Int[x^2/(a + b*x)^7,x]
```

output
```
-1/6*a^2/(b^3*(a + b*x)^6) + (2*a)/(5*b^3*(a + b*x)^5) - 1/(4*b^3*(a + b*x
)^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{15b^2x^2+6abx+a^2}{60(bx+a)^6 b^3}$	30
orering	$-\frac{15b^2x^2+6abx+a^2}{60(bx+a)^6 b^3}$	30
norman	$\frac{-\frac{x^2}{4b}-\frac{ax}{10b^2}-\frac{a^2}{60b^3}}{(bx+a)^6}$	33
risch	$\frac{-\frac{x^2}{4b}-\frac{ax}{10b^2}-\frac{a^2}{60b^3}}{(bx+a)^6}$	33
parallelrisch	$\frac{-15b^5x^2-6a\,b^4x-a^2b^3}{60b^6(bx+a)^6}$	37
default	$-\frac{a^2}{6b^3(bx+a)^6}+\frac{2a}{5b^3(bx+a)^5}-\frac{1}{4b^3(bx+a)^4}$	42

input
```
int(x^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(15*b^2*x^2+6*a*b*x+a^2)/(b*x+a)^6/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int \frac{x^2}{(a+bx)^7}\,dx$$
$$= -\frac{15\,b^2x^2+6\,abx+a^2}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

input
```
integrate(x^2/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
 + 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(42) = 84$.

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

$$\int \frac{x^2}{(a+bx)^7}\,dx = \frac{-a^2-6abx-15b^2x^2}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

input
```
integrate(x**2/(b*x+a)**7,x)
```

output
```
(-a**2 - 6*a*b*x - 15*b**2*x**2)/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**
4*b**5*x**2 + 1200*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 +
 60*b**9*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int \frac{x^2}{(a+bx)^7}\,dx = -\frac{15\,b^2x^2+6\,abx+a^2}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

input
```
integrate(x^2/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
 + 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(a+bx)^7}\,dx = -\frac{15\,b^2x^2+6\,abx+a^2}{60\,(bx+a)^6b^3}$$

input
```
integrate(x^2/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/((b*x + a)^6*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(a+bx)^7}\,dx = -\frac{8\,a^2+48\,a\,b\,x+120\,b^2\,x^2}{480\,b^3\,(a+b\,x)^6}$$

input
```
int(x^2/(a + b*x)^7,x)
```

output
```
-(8*a^2 + 120*b^2*x^2 + 48*a*b*x)/(480*b^3*(a + b*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.83

$$\int \frac{x^2}{(a+bx)^7}\,dx = \frac{-15b^2x^2-6abx-a^2}{60b^3\,(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6)}$$

input
```
int(x^2/(b*x+a)^7,x)
```

output
```
( - a**2 - 6*a*b*x - 15*b**2*x**2)/(60*b**3*(a**6 + 6*a**5*b*x + 15*a**4*b
**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x*
*6))
```

3.175 $\int \frac{x}{(a+bx)^7} dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \frac{x}{(a+bx)^7} dx = \frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

output
```
1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{(a+bx)^7} dx = -\frac{a+6bx}{30b^2(a+bx)^6}$$

input
```
Integrate[x/(a + b*x)^7,x]
```

output
```
-1/30*(a + 6*b*x)/(b^2*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^7} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{1}{b(a+bx)^6}-\frac{a}{b(a+bx)^7}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a}{6b^2(a+bx)^6}-\frac{1}{5b^2(a+bx)^5}$$

input
```
Int[x/(a + b*x)^7,x]
```

output
```
a/(6*b^2*(a + b*x)^6) - 1/(5*b^2*(a + b*x)^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{6bx+a}{30(bx+a)^{6} b^{2}}$	19
orering	$-\frac{6bx+a}{30(bx+a)^{6} b^{2}}$	19
norman	$\frac{-\frac{x}{5b}-\frac{a}{30b^{2}}}{(bx+a)^{6}}$	22
risch	$\frac{-\frac{x}{5b}-\frac{a}{30b^{2}}}{(bx+a)^{6}}$	22
parallelrisch	$\frac{-6b^{5}x-a\,b^{4}}{30b^{6}(bx+a)^{6}}$	26
default	$\frac{a}{6b^{2}(bx+a)^{6}}-\frac{1}{5b^{2}(bx+a)^{5}}$	27

input
```
int(x/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(6*b*x+a)/(b*x+a)^6/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.53

$$\int \frac{x}{(a+bx)^7}\,dx = -\frac{6\,bx+a}{30\,(b^8x^6+6\,ab^7x^5+15\,a^2b^6x^4+20\,a^3b^5x^3+15\,a^4b^4x^2+6\,a^5b^3x+a^6b^2)}$$

input
```
integrate(x/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
 + 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(26) = 52$.

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.67

$$\int \frac{x}{(a+bx)^7}\,dx = \frac{-a-6bx}{30a^6b^2+180a^5b^3x+450a^4b^4x^2+600a^3b^5x^3+450a^2b^6x^4+180ab^7x^5+30b^8x^6}$$

input
```
integrate(x/(b*x+a)**7,x)
```

output
```
(-a - 6*b*x)/(30*a**6*b**2 + 180*a**5*b**3*x + 450*a**4*b**4*x**2 + 600*a*
*3*b**5*x**3 + 450*a**2*b**6*x**4 + 180*a*b**7*x**5 + 30*b**8*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.53

$$\int \frac{x}{(a+bx)^7}\,dx = -\frac{6\,bx+a}{30\,(b^8x^6+6\,ab^7x^5+15\,a^2b^6x^4+20\,a^3b^5x^3+15\,a^4b^4x^2+6\,a^5b^3x+a^6b^2)}$$

input
```
integrate(x/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
 + 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{(a+bx)^7}\,dx = -\frac{6\,bx+a}{30\,(bx+a)^6b^2}$$

input
```
integrate(x/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/30*(6*b*x + a)/((b*x + a)^6*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{(a+bx)^7}\,dx = -\frac{a+6\,b\,x}{30\,b^2\,(a+b\,x)^6}$$

input
```
int(x/(a + b*x)^7,x)
```

output
```
-(a + 6*b*x)/(30*b^2*(a + b*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{x}{(a+bx)^7}\,dx = \frac{-6bx-a}{30b^2\,(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6)}$$

input
```
int(x/(b*x+a)^7,x)
```

output
```
( - a - 6*b*x)/(30*b**2*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b
**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.176 $\int \frac{1}{(a+bx)^7} dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(a+bx)^7} dx = -\frac{1}{6b(a+bx)^6}$$

output
```
-1/6/b/(b*x+a)^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^7} dx = -\frac{1}{6b(a+bx)^6}$$

input
```
Integrate[(a + b*x)^(-7),x]
```

output
```
-1/6*1/(b*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^7}\,dx$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{6b(a+bx)^6}$$

input
```
Int[(a + b*x)^(-7),x]
```

output
```
-1/6*1/(b*(a + b*x)^6)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{6b(bx+a)^6}$	13
default	$-\frac{1}{6b(bx+a)^6}$	13
norman	$-\frac{1}{6b(bx+a)^6}$	13
risch	$-\frac{1}{6b(bx+a)^6}$	13
parallelrisch	$-\frac{1}{6b(bx+a)^6}$	13
orering	$-\frac{1}{6b(bx+a)^6}$	13

input
```
int(1/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6/b/(b*x+a)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.86

$$\int \frac{1}{(a+bx)^7}\,dx = -\frac{1}{6\left(b^7x^6+6\,ab^6x^5+15\,a^2b^5x^4+20\,a^3b^4x^3+15\,a^4b^3x^2+6\,a^5b^2x+a^6b\right)}$$

input
```
integrate(1/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/6/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a^3*b^4*x^3 + 15*a^4*b^3
*x^2 + 6*a^5*b^2*x + a^6*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.21

$$\int \frac{1}{(a+bx)^7}\,dx$$
$$= -\frac{1}{6a^6b + 36a^5b^2x + 90a^4b^3x^2 + 120a^3b^4x^3 + 90a^2b^5x^4 + 36ab^6x^5 + 6b^7x^6}$$

input
```
integrate(1/(b*x+a)**7,x)
```

output
```
-1/(6*a**6*b + 36*a**5*b**2*x + 90*a**4*b**3*x**2 + 120*a**3*b**4*x**3 + 9
0*a**2*b**5*x**4 + 36*a*b**6*x**5 + 6*b**7*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^7}\,dx = -\frac{1}{6\,(bx+a)^6b}$$

input
```
integrate(1/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/6/((b*x + a)^6*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^7}\,dx = -\frac{1}{6\,(bx+a)^6b}$$

input
```
integrate(1/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/6/((b*x + a)^6*b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 5.00

$$\int \frac{1}{(a+bx)^7}\,dx$$
$$= -\frac{1}{6\,a^6\,b+36\,a^5\,b^2\,x+90\,a^4\,b^3\,x^2+120\,a^3\,b^4\,x^3+90\,a^2\,b^5\,x^4+36\,a\,b^6\,x^5+6\,b^7\,x^6}$$

input
```
int(1/(a + b*x)^7,x)
```

output
```
-1/(6*a^6*b + 6*b^7*x^6 + 36*a^5*b^2*x + 36*a*b^6*x^5 + 90*a^4*b^3*x^2 + 1
20*a^3*b^4*x^3 + 90*a^2*b^5*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.79

$$\int \frac{1}{(a+bx)^7}\,dx = -\frac{1}{6b\,(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6)}$$

input
```
int(1/(b*x+a)^7,x)
```

output
```
( - 1)/(6*b*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 1
5*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.177 $\int \frac{1}{x(a+bx)^7}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 99

$$\int \frac{1}{x(a+bx)^7}\,dx = \frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx)}+\frac{\log(x)}{a^7}-\frac{\log(a+bx)}{a^7}$$

output
```
1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/
a^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a+bx)^7}\,dx = \frac{\frac{a(147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5)}{(a+bx)^6}+60\log(x)-60\log(a+bx)}{60a^7}$$

input
```
Integrate[1/(x*(a + b*x)^7),x]
```

output
```
((a*(147*a^5 + 522*a^4*b*x + 855*a^3*b^2*x^2 + 740*a^2*b^3*x^3 + 330*a*b^4
*x^4 + 60*b^5*x^5))/(a + b*x)^6 + 60*Log[x] - 60*Log[a + b*x])/(60*a^7)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^7} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{b}{a^7(a+bx)}+\frac{1}{a^7x}-\frac{b}{a^6(a+bx)^2}-\frac{b}{a^5(a+bx)^3}-\frac{b}{a^4(a+bx)^4}-\frac{b}{a^3(a+bx)^5}-\frac{b}{a^2(a+bx)^6}-\frac{b}{a(a+bx)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{6a(a+bx)^6}$$

input
```
Int[1/(x*(a + b*x)^7),x]
```

output
```
1/(6*a*(a + b*x)^6) + 1/(5*a^2*(a + b*x)^5) + 1/(4*a^3*(a + b*x)^4) + 1/(3
*a^4*(a + b*x)^3) + 1/(2*a^5*(a + b*x)^2) + 1/(a^6*(a + b*x)) + Log[x]/a^7
 - Log[a + b*x]/a^7
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\frac{b^5x^5}{a^6}+\frac{11b^4x^4}{2a^5}+\frac{37b^3x^3}{3a^4}+\frac{57b^2x^2}{4a^3}+\frac{87bx}{10a^2}+\frac{49}{20a}}{(bx+a)^6}-\frac{\ln(bx+a)}{a^7}+\frac{\ln(-x)}{a^7}$
default	$\frac{1}{6a(bx+a)^6}+\frac{1}{5a^2(bx+a)^5}+\frac{1}{4a^3(bx+a)^4}+\frac{1}{3a^4(bx+a)^3}+\frac{1}{2a^5(bx+a)^2}+\frac{1}{a^6(bx+a)}+\frac{\ln(x)}{a^7}-\frac{\ln(bx+a)}{a^7}$
norman	$\frac{-\frac{6bx}{a^2}-\frac{45b^2x^2}{2a^3}-\frac{110b^3x^3}{3a^4}-\frac{125b^4x^4}{4a^5}-\frac{137b^5x^5}{10a^6}-\frac{49b^6x^6}{20a^7}}{(bx+a)^6}+\frac{\ln(x)}{a^7}-\frac{\ln(bx+a)}{a^7}$
parallelrisch	$-360a^5xb+1200\ln(x)x^3a^3b^3+360\ln(x)x^5a\,b^5-360\ln(bx+a)x^5a\,b^5+900\ln(x)x^4a^2b^4-900\ln(bx+a)x^4a^2b^4-147b^6x^6-60\ln$

input
```
int(1/x/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
(b^5/a^6*x^5+11/2*b^4/a^5*x^4+37/3*b^3/a^4*x^3+57/4*b^2/a^3*x^2+87/10*b/a^
2*x+49/20/a)/(b*x+a)^6-ln(b*x+a)/a^7+1/a^7*ln(-x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(89) = 178.

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.59

$$\int \frac{1}{x(a+bx)^7}\,dx$$
$$=\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6-60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+}{60\,(a^7b^6x^6+6\,a^8b^5x^5+1}$$

input
```
integrate(1/x/(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/60*(60*a*b^5*x^5 + 330*a^2*b^4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 +
 522*a^5*b*x + 147*a^6 - 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a
^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a) + 60*(b^6*x^6
+ 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b
*x + a^6)*log(x))/(a^7*b^6*x^6 + 6*a^8*b^5*x^5 + 15*a^9*b^4*x^4 + 20*a^10*
b^3*x^3 + 15*a^11*b^2*x^2 + 6*a^12*b*x + a^13)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.42

$$\int \frac{1}{x(a+bx)^7}\,dx = \frac{147a^5 + 522a^4bx + 855a^3b^2x^2 + 740a^2b^3x^3 + 330ab^4x^4 + 60b^5x^5}{60a^{12} + 360a^{11}bx + 900a^{10}b^2x^2 + 1200a^9b^3x^3 + 900a^8b^4x^4 + 360a^7b^5x^5 + 60a^6b^6x^6} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^7}$$

input
```
integrate(1/x/(b*x+a)**7,x)
```

output
```
(147*a**5 + 522*a**4*b*x + 855*a**3*b**2*x**2 + 740*a**2*b**3*x**3 + 330*a
*b**4*x**4 + 60*b**5*x**5)/(60*a**12 + 360*a**11*b*x + 900*a**10*b**2*x**2
 + 1200*a**9*b**3*x**3 + 900*a**8*b**4*x**4 + 360*a**7*b**5*x**5 + 60*a**6
*b**6*x**6) + (log(x) - log(a/b + x))/a**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

$$\int \frac{1}{x(a+bx)^7}\,dx = \frac{60\,b^5x^5 + 330\,ab^4x^4 + 740\,a^2b^3x^3 + 855\,a^3b^2x^2 + 522\,a^4bx + 147\,a^5}{60\,(a^6b^6x^6 + 6\,a^7b^5x^5 + 15\,a^8b^4x^4 + 20\,a^9b^3x^3 + 15\,a^{10}b^2x^2 + 6\,a^{11}bx + a^{12})} - \frac{\log(bx+a)}{a^7} + \frac{\log(x)}{a^7}$$

input
```
integrate(1/x/(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/60*(60*b^5*x^5 + 330*a*b^4*x^4 + 740*a^2*b^3*x^3 + 855*a^3*b^2*x^2 + 522
*a^4*b*x + 147*a^5)/(a^6*b^6*x^6 + 6*a^7*b^5*x^5 + 15*a^8*b^4*x^4 + 20*a^9
*b^3*x^3 + 15*a^10*b^2*x^2 + 6*a^11*b*x + a^12) - log(b*x + a)/a^7 + log(x
)/a^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(a+bx)^7}\,dx = -\frac{\log(|bx+a|)}{a^7} + \frac{\log(|x|)}{a^7} + \frac{60\,ab^5x^5 + 330\,a^2b^4x^4 + 740\,a^3b^3x^3 + 855\,a^4b^2x^2 + 522\,a^5bx + 147\,a^6}{60\,(bx+a)^6a^7}$$

input
```
integrate(1/x/(b*x+a)^7,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^7 + log(abs(x))/a^7 + 1/60*(60*a*b^5*x^5 + 330*a^2*b^
4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 + 522*a^5*b*x + 147*a^6)/((b*x +
 a)^6*a^7)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{1}{x(a+bx)^7}\,dx = -\frac{\ln\left(\frac{a+b\,x}{x}\right) - \frac{15\,b^2\,x^2}{2\,(a+b\,x)^2} + \frac{20\,b^3\,x^3}{3\,(a+b\,x)^3} - \frac{15\,b^4\,x^4}{4\,(a+b\,x)^4} + \frac{6\,b^5\,x^5}{5\,(a+b\,x)^5} - \frac{b^6\,x^6}{6\,(a+b\,x)^6} + \frac{6\,b\,x}{a+b\,x}}{a^7}$$

input
```
int(1/(x*(a + b*x)^7),x)
```

output
```
-(log((a + b*x)/x) - (15*b^2*x^2)/(2*(a + b*x)^2) + (20*b^3*x^3)/(3*(a + b
*x)^3) - (15*b^4*x^4)/(4*(a + b*x)^4) + (6*b^5*x^5)/(5*(a + b*x)^5) - (b^6
*x^6)/(6*(a + b*x)^6) + (6*b*x)/(a + b*x))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.04

$$\int \frac{1}{x(a+bx)^7}\,dx$$
$$= \frac{-60\log(bx+a)\,a^6 - 360\log(bx+a)\,a^5bx - 900\log(bx+a)\,a^4b^2x^2 - 1200\log(bx+a)\,a^3b^3x^3 - 900\log(}{}$$

input
```
int(1/x/(b*x+a)^7,x)
```

output
```
( - 60*log(a + b*x)*a**6 - 360*log(a + b*x)*a**5*b*x - 900*log(a + b*x)*a*
*4*b**2*x**2 - 1200*log(a + b*x)*a**3*b**3*x**3 - 900*log(a + b*x)*a**2*b*
*4*x**4 - 360*log(a + b*x)*a*b**5*x**5 - 60*log(a + b*x)*b**6*x**6 + 60*lo
g(x)*a**6 + 360*log(x)*a**5*b*x + 900*log(x)*a**4*b**2*x**2 + 1200*log(x)*
a**3*b**3*x**3 + 900*log(x)*a**2*b**4*x**4 + 360*log(x)*a*b**5*x**5 + 60*l
og(x)*b**6*x**6 + 137*a**6 + 462*a**5*b*x + 705*a**4*b**2*x**2 + 540*a**3*
b**3*x**3 + 180*a**2*b**4*x**4 - 10*b**6*x**6)/(60*a**7*(a**6 + 6*a**5*b*x
 + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x*
*5 + b**6*x**6))
```

3.178 $\int \frac{1}{x^2(a+bx)^7}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 117

$$\int \frac{1}{x^2(a+bx)^7}\,dx = -\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}-\frac{6b}{a^7(a+bx)}-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}$$

output
```
-1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b
/a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*
x+a)/a^8
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(a+bx)^7}\,dx = -\frac{\frac{a(60a^6+1029a^5bx+3654a^4b^2x^2+5985a^3b^3x^3+5180a^2b^4x^4+2310ab^5x^5+420b^6x^6)}{x(a+bx)^6}+420b\log(x)-420b\log(a+bx)}{60a^8}$$

input
```
Integrate[1/(x^2*(a + b*x)^7),x]
```

output
```
-1/60*((a*(60*a^6 + 1029*a^5*b*x + 3654*a^4*b^2*x^2 + 5985*a^3*b^3*x^3 + 5
180*a^2*b^4*x^4 + 2310*a*b^5*x^5 + 420*b^6*x^6))/(x*(a + b*x)^6) + 420*b*L
og[x] - 420*b*Log[a + b*x])/a^8
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^7} dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{7b^2}{a^8(a+bx)} - \frac{7b}{a^8x} + \frac{6b^2}{a^7(a+bx)^2} + \frac{1}{a^7x^2} + \frac{5b^2}{a^6(a+bx)^3} + \frac{4b^2}{a^5(a+bx)^4} + \frac{3b^2}{a^4(a+bx)^5} + \frac{2b^2}{a^3(a+bx)^6} + \frac{}{a^2(}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7b\log(x)}{a^8} + \frac{7b\log(a+bx)}{a^8} - \frac{6b}{a^7(a+bx)} - \frac{1}{a^7x} - \frac{5b}{2a^6(a+bx)^2} - \frac{4b}{3a^5(a+bx)^3} - \frac{3b}{4a^4(a+bx)^4} - \frac{2b}{5a^3(a+bx)^5} - \frac{b}{6a^2(a+bx)^6}$$

input
```
Int[1/(x^2*(a + b*x)^7),x]
```

output
```
-(1/(a^7*x)) - b/(6*a^2*(a + b*x)^6) - (2*b)/(5*a^3*(a + b*x)^5) - (3*b)/(
4*a^4*(a + b*x)^4) - (4*b)/(3*a^5*(a + b*x)^3) - (5*b)/(2*a^6*(a + b*x)^2)
 - (6*b)/(a^7*(a + b*x)) - (7*b*Log[x])/a^8 + (7*b*Log[a + b*x])/a^8
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
risch	$\frac{-\frac{7b^6x^6}{a^7}-\frac{77b^5x^5}{2a^6}-\frac{259b^4x^4}{3a^5}-\frac{399b^3x^3}{4a^4}-\frac{609b^2x^2}{10a^3}-\frac{343bx}{20a^2}-\frac{1}{a}}{x(bx+a)^6}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(-bx-a)}{a^8}$
norman	$\frac{-\frac{1}{a}+\frac{42b^2x^2}{a^3}+\frac{315b^3x^3}{2a^4}+\frac{770b^4x^4}{3a^5}+\frac{875b^5x^5}{4a^6}+\frac{959b^6x^6}{10a^7}+\frac{343b^7x^7}{20a^8}}{x(bx+a)^6}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(bx+a)}{a^8}$
default	$-\frac{1}{a^7x}-\frac{b}{6a^2(bx+a)^6}-\frac{2b}{5a^3(bx+a)^5}-\frac{3b}{4a^4(bx+a)^4}-\frac{4b}{3a^5(bx+a)^3}-\frac{5b}{2a^6(bx+a)^2}-\frac{6b}{a^7(bx+a)}-\frac{7b\ln(x)}{a^8}+7b$
parallelrisch	$-\frac{420b^7\ln(x)x^7-1029b^7x^7+60a^7-2520a^5b^2x^2-9450a^4b^3x^3-15400a^3b^4x^4-13125a^2b^5x^5-5754a\,b^6x^6-2520\ln(bx+a)x^2a}{}$

input
```
int(1/x^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
(-7*b^6/a^7*x^6-77/2*b^5/a^6*x^5-259/3*b^4/a^5*x^4-399/4*b^3/a^4*x^3-609/1
0*b^2/a^3*x^2-343/20*b/a^2*x-1/a)/x/(b*x+a)^6-7*b*ln(x)/a^8+7/a^8*b*ln(-b*
x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(107) = 214.

Time = 0.08 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2(a+bx)^7}\,dx = -\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60\,a^7-420\,(b^7x^7}{60\,(a^8b^6x}$$

input
```
integrate(1/x^2/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/60*(420*a*b^6*x^6 + 2310*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*
x^3 + 3654*a^5*b^2*x^2 + 1029*a^6*b*x + 60*a^7 - 420*(b^7*x^7 + 6*a*b^6*x^
6 + 15*a^2*b^5*x^5 + 20*a^3*b^4*x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6
*b*x)*log(b*x + a) + 420*(b^7*x^7 + 6*a*b^6*x^6 + 15*a^2*b^5*x^5 + 20*a^3*
b^4*x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*x)*log(x))/(a^8*b^6*x^7 +
 6*a^9*b^5*x^6 + 15*a^10*b^4*x^5 + 20*a^11*b^3*x^4 + 15*a^12*b^2*x^3 + 6*a
^13*b*x^2 + a^14*x)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^2(a+bx)^7}\,dx = \frac{-60a^6-1029a^5bx-3654a^4b^2x^2-5985a^3b^3x^3-5180a^2b^4x^4-2310ab^5x^5-420b^6x^6}{60a^{13}x+360a^{12}bx^2+900a^{11}b^2x^3+1200a^{10}b^3x^4+900a^9b^4x^5+360a^8b^5x^6+60a^7b^6x^7} + \frac{7b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

input
```
integrate(1/x**2/(b*x+a)**7,x)
```

output
```
(-60*a**6 - 1029*a**5*b*x - 3654*a**4*b**2*x**2 - 5985*a**3*b**3*x**3 - 51
80*a**2*b**4*x**4 - 2310*a*b**5*x**5 - 420*b**6*x**6)/(60*a**13*x + 360*a*
*12*b*x**2 + 900*a**11*b**2*x**3 + 1200*a**10*b**3*x**4 + 900*a**9*b**4*x*
*5 + 360*a**8*b**5*x**6 + 60*a**7*b**6*x**7) + 7*b*(-log(x) + log(a/b + x)
)/a**8
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^2(a+bx)^7}\,dx = -\frac{420\,b^6x^6 + 2310\,ab^5x^5 + 5180\,a^2b^4x^4 + 5985\,a^3b^3x^3 + 3654\,a^4b^2x^2 + 1029\,a^5bx + 60\,a^6}{60\,(a^7b^6x^7 + 6\,a^8b^5x^6 + 15\,a^9b^4x^5 + 20\,a^{10}b^3x^4 + 15\,a^{11}b^2x^3 + 6\,a^{12}bx^2 + a^{13}x)} + \frac{7\,b\log(bx+a)}{a^8} - \frac{7\,b\log(x)}{a^8}$$

input
```
integrate(1/x^2/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/60*(420*b^6*x^6 + 2310*a*b^5*x^5 + 5180*a^2*b^4*x^4 + 5985*a^3*b^3*x^3
+ 3654*a^4*b^2*x^2 + 1029*a^5*b*x + 60*a^6)/(a^7*b^6*x^7 + 6*a^8*b^5*x^6 +
 15*a^9*b^4*x^5 + 20*a^10*b^3*x^4 + 15*a^11*b^2*x^3 + 6*a^12*b*x^2 + a^13*
x) + 7*b*log(b*x + a)/a^8 - 7*b*log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(a+bx)^7}\,dx = \frac{7\,b\log(|bx+a|)}{a^8} - \frac{7\,b\log(|x|)}{a^8} - \frac{420\,ab^6x^6 + 2310\,a^2b^5x^5 + 5180\,a^3b^4x^4 + 5985\,a^4b^3x^3 + 3654\,a^5b^2x^2 + 1029\,a^6bx + 60\,a^7}{60\,(bx+a)^6a^8x}$$

input
```
integrate(1/x^2/(b*x+a)^7,x, algorithm="giac")
```

output
```
7*b*log(abs(b*x + a))/a^8 - 7*b*log(abs(x))/a^8 - 1/60*(420*a*b^6*x^6 + 23
10*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x^3 + 3654*a^5*b^2*x^2 +
1029*a^6*b*x + 60*a^7)/((b*x + a)^6*a^8*x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2(a+bx)^7}\,dx$$
$$= \frac{14\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8} - \frac{\frac{1}{a}+\frac{609\,b^2\,x^2}{10\,a^3}+\frac{399\,b^3\,x^3}{4\,a^4}+\frac{259\,b^4\,x^4}{3\,a^5}+\frac{77\,b^5\,x^5}{2\,a^6}+\frac{7\,b^6\,x^6}{a^7}+\frac{343\,b\,x}{20\,a^2}}{a^6\,x+6\,a^5\,b\,x^2+15\,a^4\,b^2\,x^3+20\,a^3\,b^3\,x^4+15\,a^2\,b^4\,x^5+6\,a\,b^5\,x^6+b^6\,x^7}$$

input
```
int(1/(x^2*(a + b*x)^7),x)
```

output
```
(14*b*atanh((2*b*x)/a + 1))/a^8 - (1/a + (609*b^2*x^2)/(10*a^3) + (399*b^3
*x^3)/(4*a^4) + (259*b^4*x^4)/(3*a^5) + (77*b^5*x^5)/(2*a^6) + (7*b^6*x^6)
/a^7 + (343*b*x)/(20*a^2))/(a^6*x + b^6*x^7 + 6*a^5*b*x^2 + 6*a*b^5*x^6 +
15*a^4*b^2*x^3 + 20*a^3*b^3*x^4 + 15*a^2*b^4*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.79

$$\int \frac{1}{x^2(a+bx)^7}\,dx$$
$$= \frac{420\log(bx+a)\,a^6bx+2520\log(bx+a)\,a^5b^2x^2+6300\log(bx+a)\,a^4b^3x^3+8400\log(bx+a)\,a^3b^4x^4+63}{}$$

input
```
int(1/x^2/(b*x+a)^7,x)
```

output

```
(420*log(a + b*x)*a**6*b*x + 2520*log(a + b*x)*a**5*b**2*x**2 + 6300*log(a
 + b*x)*a**4*b**3*x**3 + 8400*log(a + b*x)*a**3*b**4*x**4 + 6300*log(a + b
*x)*a**2*b**5*x**5 + 2520*log(a + b*x)*a*b**6*x**6 + 420*log(a + b*x)*b**7
*x**7 - 420*log(x)*a**6*b*x - 2520*log(x)*a**5*b**2*x**2 - 6300*log(x)*a**
4*b**3*x**3 - 8400*log(x)*a**3*b**4*x**4 - 6300*log(x)*a**2*b**5*x**5 - 25
20*log(x)*a*b**6*x**6 - 420*log(x)*b**7*x**7 - 60*a**7 - 959*a**6*b*x - 32
34*a**5*b**2*x**2 - 4935*a**4*b**3*x**3 - 3780*a**3*b**4*x**4 - 1260*a**2*
b**5*x**5 + 70*b**7*x**7)/(60*a**8*x*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**
2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.179 $\int \frac{1}{x^3(a+bx)^7} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 144

$$\int \frac{1}{x^3(a+bx)^7} \, dx = -\frac{1}{2a^7x^2} + \frac{7b}{a^8x} + \frac{b^2}{6a^3(a+bx)^6} + \frac{3b^2}{5a^4(a+bx)^5} + \frac{3b^2}{2a^5(a+bx)^4} + \frac{10b^2}{3a^6(a+bx)^3} + \frac{15b^2}{2a^7(a+bx)^2} + \frac{21b^2}{a^8(a+bx)} + \frac{28b^2\log(x)}{a^9} - \frac{28b^2\log(a+bx)}{a^9}$$

output
```
-1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2
/a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b
*x+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3(a+bx)^7} \, dx = \frac{\frac{a(-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7)}{x^2(a+bx)^6} + 840b^2\log(x) - 840b^2\log(a+}{30a^9}$$

input
```
Integrate[1/(x^3*(a + b*x)^7),x]
```

output
```
((a*(-15*a^7 + 120*a^6*b*x + 2058*a^5*b^2*x^2 + 7308*a^4*b^3*x^3 + 11970*a
^3*b^4*x^4 + 10360*a^2*b^5*x^5 + 4620*a*b^6*x^6 + 840*b^7*x^7))/(x^2*(a +
b*x)^6) + 840*b^2*Log[x] - 840*b^2*Log[a + b*x])/(30*a^9)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^7}\,dx$$

$$\Big\downarrow \; 54$$

$$\int \left(-\frac{28b^3}{a^9(a+bx)}+\frac{28b^2}{a^9x}-\frac{21b^3}{a^8(a+bx)^2}-\frac{7b}{a^8x^2}-\frac{15b^3}{a^7(a+bx)^3}+\frac{1}{a^7x^3}-\frac{10b^3}{a^6(a+bx)^4}-\frac{6b^3}{a^5(a+bx)^5}-\frac{3b^3}{a^4(a+b$$

$$\Big\downarrow \; 2009$$

$$\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a+bx)}{a^9}+\frac{21b^2}{a^8(a+bx)}+\frac{7b}{a^8x}+\frac{15b^2}{2a^7(a+bx)^2}-\frac{1}{2a^7x^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{b^2}{6a^3(a+bx)^6}$$

input
```
Int[1/(x^3*(a + b*x)^7),x]
```

output
```
-1/2*1/(a^7*x^2) + (7*b)/(a^8*x) + b^2/(6*a^3*(a + b*x)^6) + (3*b^2)/(5*a^
4*(a + b*x)^5) + (3*b^2)/(2*a^5*(a + b*x)^4) + (10*b^2)/(3*a^6*(a + b*x)^3
) + (15*b^2)/(2*a^7*(a + b*x)^2) + (21*b^2)/(a^8*(a + b*x)) + (28*b^2*Log[
x])/a^9 - (28*b^2*Log[a + b*x])/a^9
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

method	result
norman	$\frac{-\frac{1}{2a}+\frac{4bx}{a^2}-\frac{168b^3x^3}{a^4}-\frac{630b^4x^4}{a^5}-\frac{3080b^5x^5}{3a^6}-\frac{875b^6x^6}{a^7}-\frac{1918b^7x^7}{5a^8}-\frac{343b^8x^8}{5a^9}}{x^2(bx+a)^6}+\frac{28b^2\ln(x)}{a^9}-\frac{28b^2\ln(bx+a)}{a^9}$
risch	$\frac{\frac{28b^7x^7}{a^8}+\frac{154b^6x^6}{a^7}+\frac{1036b^5x^5}{3a^6}+\frac{399b^4x^4}{a^5}+\frac{1218b^3x^3}{5a^4}+\frac{343b^2x^2}{5a^3}+\frac{4bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^6}-\frac{28b^2\ln(bx+a)}{a^9}+\frac{28b^2\ln(-x)}{a^9}$
default	$-\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(bx+a)^6}+\frac{3b^2}{5a^4(bx+a)^5}+\frac{3b^2}{2a^5(bx+a)^4}+\frac{10b^2}{3a^6(bx+a)^3}+\frac{15b^2}{2a^7(bx+a)^2}+\frac{21b^2}{a^8(bx+a)}+\frac{28b^2}{\cdots}$
parallelrisch	$120a^7xb-2058b^8x^8-15a^8-5040a^5b^3x^3+840\ln(x)x^8b^8-840\ln(bx+a)x^8b^8-11508a\,x^7b^7-26250a^2x^6b^6-30800a^3x^5b^5-189\cdots$

input
```
int(1/x^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a+4*b/a^2*x-168*b^3/a^4*x^3-630*b^4/a^5*x^4-3080/3*b^5/a^6*x^5-875*b
^6/a^7*x^6-1918/5*b^7/a^8*x^7-343/5*b^8/a^9*x^8)/x^2/(b*x+a)^6+28*b^2*ln(x
)/a^9-28*b^2*ln(b*x+a)/a^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(132) = 264.

Time = 0.08 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^3(a+bx)^7}\,dx$$
$$=\frac{840\,ab^7x^7+4620\,a^2b^6x^6+10360\,a^3b^5x^5+11970\,a^4b^4x^4+7308\,a^5b^3x^3+2058\,a^6b^2x^2+120\,a^7bx-15\,a^8\cdots}{30\,\cdots}$$

input
```
integrate(1/x^3/(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/30*(840*a*b^7*x^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4
*x^4 + 7308*a^5*b^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8 - 840*(b
^8*x^8 + 6*a*b^7*x^7 + 15*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 +
6*a^5*b^3*x^3 + a^6*b^2*x^2)*log(b*x + a) + 840*(b^8*x^8 + 6*a*b^7*x^7 + 1
5*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a^5*b^3*x^3 + a^6*b^2*
x^2)*log(x))/(a^9*b^6*x^8 + 6*a^10*b^5*x^7 + 15*a^11*b^4*x^6 + 20*a^12*b^3
*x^5 + 15*a^13*b^2*x^4 + 6*a^14*b*x^3 + a^15*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3(a+bx)^7}\,dx = \frac{-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7}{30a^{14}x^2+180a^{13}bx^3+450a^{12}b^2x^4+600a^{11}b^3x^5+450a^{10}b^4x^6+180a^9b^5x^7+30a^8b^6x^8} + \frac{28b^2\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^9}$$

input
```
integrate(1/x**3/(b*x+a)**7,x)
```

output
```
(-15*a**7 + 120*a**6*b*x + 2058*a**5*b**2*x**2 + 7308*a**4*b**3*x**3 + 119
70*a**3*b**4*x**4 + 10360*a**2*b**5*x**5 + 4620*a*b**6*x**6 + 840*b**7*x**
7)/(30*a**14*x**2 + 180*a**13*b*x**3 + 450*a**12*b**2*x**4 + 600*a**11*b**
3*x**5 + 450*a**10*b**4*x**6 + 180*a**9*b**5*x**7 + 30*a**8*b**6*x**8) + 2
8*b**2*(log(x) - log(a/b + x))/a**9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3(a+bx)^7}\,dx = \frac{840\,b^7x^7+4620\,ab^6x^6+10360\,a^2b^5x^5+11970\,a^3b^4x^4+7308\,a^4b^3x^3+2058\,a^5b^2x^2+120\,a^6bx-15\,a^7}{30\,(a^8b^6x^8+6\,a^9b^5x^7+15\,a^{10}b^4x^6+20\,a^{11}b^3x^5+15\,a^{12}b^2x^4+6\,a^{13}bx^3+a^{14}x^2)} - \frac{28\,b^2\log(bx+a)}{a^9} + \frac{28\,b^2\log(x)}{a^9}$$

input
```
integrate(1/x^3/(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/30*(840*b^7*x^7 + 4620*a*b^6*x^6 + 10360*a^2*b^5*x^5 + 11970*a^3*b^4*x^4
 + 7308*a^4*b^3*x^3 + 2058*a^5*b^2*x^2 + 120*a^6*b*x - 15*a^7)/(a^8*b^6*x^
8 + 6*a^9*b^5*x^7 + 15*a^10*b^4*x^6 + 20*a^11*b^3*x^5 + 15*a^12*b^2*x^4 +
6*a^13*b*x^3 + a^14*x^2) - 28*b^2*log(b*x + a)/a^9 + 28*b^2*log(x)/a^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3(a+bx)^7}\,dx = -\frac{28\,b^2\log(|bx+a|)}{a^9} + \frac{28\,b^2\log(|x|)}{a^9} + \frac{840\,ab^7x^7+4620\,a^2b^6x^6+10360\,a^3b^5x^5+11970\,a^4b^4x^4+7308\,a^5b^3x^3+2058\,a^6b^2x^2+120\,a^7bx-15}{30\,(bx+a)^6a^9x^2}$$

input
```
integrate(1/x^3/(b*x+a)^7,x, algorithm="giac")
```

output
```
-28*b^2*log(abs(b*x + a))/a^9 + 28*b^2*log(abs(x))/a^9 + 1/30*(840*a*b^7*x
^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*x^4 + 7308*a^5*b
^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8)/((b*x + a)^6*a^9*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3(a+bx)^7}\,dx = \frac{\frac{343\,b^2\,x^2}{5\,a^3}-\frac{1}{2\,a}+\frac{1218\,b^3\,x^3}{5\,a^4}+\frac{399\,b^4\,x^4}{a^5}+\frac{1036\,b^5\,x^5}{3\,a^6}+\frac{154\,b^6\,x^6}{a^7}+\frac{28\,b^7\,x^7}{a^8}+\frac{4\,b\,x}{a^2}}{a^6\,x^2+6\,a^5\,b\,x^3+15\,a^4\,b^2\,x^4+20\,a^3\,b^3\,x^5+15\,a^2\,b^4\,x^6+6\,a\,b^5\,x^7+b^6\,x^8} - \frac{56\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^9}$$

input
```
int(1/(x^3*(a + b*x)^7),x)
```

output
```
((343*b^2*x^2)/(5*a^3) - 1/(2*a) + (1218*b^3*x^3)/(5*a^4) + (399*b^4*x^4)/
a^5 + (1036*b^5*x^5)/(3*a^6) + (154*b^6*x^6)/a^7 + (28*b^7*x^7)/a^8 + (4*b
*x)/a^2)/(a^6*x^2 + b^6*x^8 + 6*a^5*b*x^3 + 6*a*b^5*x^7 + 15*a^4*b^2*x^4 +
 20*a^3*b^3*x^5 + 15*a^2*b^4*x^6) - (56*b^2*atanh((2*b*x)/a + 1))/a^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.40

$$\int \frac{1}{x^3(a+bx)^7}\,dx = \frac{-840\log(bx+a)\,a^6b^2x^2-5040\log(bx+a)\,a^5b^3x^3-12600\log(bx+a)\,a^4b^4x^4-16800\log(bx+a)\,a^3b^5x}{}$$

input
```
int(1/x^3/(b*x+a)^7,x)
```

output
```
( - 840*log(a + b*x)*a**6*b**2*x**2 - 5040*log(a + b*x)*a**5*b**3*x**3 - 1
2600*log(a + b*x)*a**4*b**4*x**4 - 16800*log(a + b*x)*a**3*b**5*x**5 - 126
00*log(a + b*x)*a**2*b**6*x**6 - 5040*log(a + b*x)*a*b**7*x**7 - 840*log(a
 + b*x)*b**8*x**8 + 840*log(x)*a**6*b**2*x**2 + 5040*log(x)*a**5*b**3*x**3
 + 12600*log(x)*a**4*b**4*x**4 + 16800*log(x)*a**3*b**5*x**5 + 12600*log(x
)*a**2*b**6*x**6 + 5040*log(x)*a*b**7*x**7 + 840*log(x)*b**8*x**8 - 15*a**
8 + 120*a**7*b*x + 1918*a**6*b**2*x**2 + 6468*a**5*b**3*x**3 + 9870*a**4*b
**4*x**4 + 7560*a**3*b**5*x**5 + 2520*a**2*b**6*x**6 - 140*b**8*x**8)/(30*
a**9*x**2*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b**3*x**3 + 15*
a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.180 $\int \frac{1}{x^4(a+bx)^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 157

$$\int \frac{1}{x^4(a+bx)^7} dx = -\frac{1}{3a^7x^3} + \frac{7b}{2a^8x^2} - \frac{28b^2}{a^9x} - \frac{b^3}{6a^4(a+bx)^6} - \frac{4b^3}{5a^5(a+bx)^5} - \frac{5b^3}{2a^6(a+bx)^4} - \frac{20b^3}{3a^7(a+bx)^3} - \frac{35b^3}{2a^8(a+bx)^2} - \frac{56b^3}{a^9(a+bx)} - \frac{84b^3\log(x)}{a^{10}} + \frac{84b^3\log(a+bx)}{a^{10}}$$

output
```
-1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/
(b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a
)^2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4(a+bx)^7} dx = -\frac{\frac{a(10a^8-45a^7bx+360a^6b^2x^2+6174a^5b^3x^3+21924a^4b^4x^4+35910a^3b^5x^5+31080a^2b^6x^6+13860ab^7x^7+2520b^8x^8)}{x^3(a+bx)^6} + 2520b^3\log(x) -}{30a^{10}}$$

input
```
Integrate[1/(x^4*(a + b*x)^7),x]
```

output
```
-1/30*((a*(10*a^8 - 45*a^7*b*x + 360*a^6*b^2*x^2 + 6174*a^5*b^3*x^3 + 2192
4*a^4*b^4*x^4 + 35910*a^3*b^5*x^5 + 31080*a^2*b^6*x^6 + 13860*a*b^7*x^7 +
2520*b^8*x^8))/(x^3*(a + b*x)^6) + 2520*b^3*Log[x] - 2520*b^3*Log[a + b*x]
)/a^10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+bx)^7}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{84b^4}{a^{10}(a+bx)}-\frac{84b^3}{a^{10}x}+\frac{56b^4}{a^9(a+bx)^2}+\frac{28b^2}{a^9x^2}+\frac{35b^4}{a^8(a+bx)^3}-\frac{7b}{a^8x^3}+\frac{20b^4}{a^7(a+bx)^4}+\frac{1}{a^7x^4}+\frac{10b^4}{a^6(a+bx)^5}+\ldots\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3\log(a+bx)}{a^{10}}-\frac{56b^3}{a^9(a+bx)}-\frac{28b^2}{a^9x}-\frac{35b^3}{2a^8(a+bx)^2}+\frac{7b}{2a^8x^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{1}{3a^7x^3}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{b^3}{6a^4(a+bx)^6}$$

input
```
Int[1/(x^4*(a + b*x)^7),x]
```

output
```
-1/3*1/(a^7*x^3) + (7*b)/(2*a^8*x^2) - (28*b^2)/(a^9*x) - b^3/(6*a^4*(a +
b*x)^6) - (4*b^3)/(5*a^5*(a + b*x)^5) - (5*b^3)/(2*a^6*(a + b*x)^4) - (20*
b^3)/(3*a^7*(a + b*x)^3) - (35*b^3)/(2*a^8*(a + b*x)^2) - (56*b^3)/(a^9*(a
 + b*x)) - (84*b^3*Log[x])/a^10 + (84*b^3*Log[a + b*x])/a^10
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

method	result
norman	$\frac{-\frac{1}{3a}+\frac{3bx}{2a^2}-\frac{12b^2x^2}{a^3}+\frac{504b^4x^4}{a^5}+\frac{1890b^5x^5}{a^6}+\frac{3080b^6x^6}{a^7}+\frac{2625b^7x^7}{a^8}+\frac{5754b^8x^8}{5a^9}+\frac{1029b^9x^9}{5a^{10}}}{x^3(bx+a)^6}-\frac{84b^3\ln(x)}{a^{10}}+\frac{84b^3\ln(bx+a)}{a^{10}}$
risch	$\frac{-\frac{84b^8x^8}{a^9}-\frac{462b^7x^7}{a^8}-\frac{1036b^6x^6}{a^7}-\frac{1197b^5x^5}{a^6}-\frac{3654b^4x^4}{5a^5}-\frac{1029b^3x^3}{5a^4}-\frac{12b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{3a}}{x^3(bx+a)^6}-\frac{84b^3\ln(x)}{a^{10}}+\frac{84b^3\ln(-bx-a)}{a^{10}}$
default	$-\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(bx+a)^6}-\frac{4b^3}{5a^5(bx+a)^5}-\frac{5b^3}{2a^6(bx+a)^4}-\frac{20b^3}{3a^7(bx+a)^3}-\frac{35b^3}{2a^8(bx+a)^2}-\frac{56b^3}{a^9(bx+}$
parallelrisch	$-\frac{-56700a^4x^5b^5-6174b^9x^9+10a^9-2520\ln(bx+a)x^9b^9-92400x^6a^3b^6+2520\ln(x)x^9b^9-34524a\,x^8b^8-78750a^2x^7b^7-50400}{}$

input
```
int(1/x^4/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/3/a+3/2*b/a^2*x-12*b^2/a^3*x^2+504*b^4/a^5*x^4+1890*b^5/a^6*x^5+3080*b
^6/a^7*x^6+2625*b^7/a^8*x^7+5754/5*b^8/a^9*x^8+1029/5*b^9/a^10*x^9)/x^3/(b
*x+a)^6-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(143) = 286.

Time = 0.10 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^4(a+bx)^7}\,dx =$$
$$-\frac{2520\,ab^8x^8+13860\,a^2b^7x^7+31080\,a^3b^6x^6+35910\,a^4b^5x^5+21924\,a^5b^4x^4+6174\,a^6b^3x^3+360\,a^7b^2x^2}{}$$

input
```
integrate(1/x^4/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/30*(2520*a*b^8*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*
b^5*x^5 + 21924*a^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*
b*x + 10*a^9 - 2520*(b^9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x
^6 + 15*a^4*b^5*x^5 + 6*a^5*b^4*x^4 + a^6*b^3*x^3)*log(b*x + a) + 2520*(b^
9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6 + 15*a^4*b^5*x^5 + 6
*a^5*b^4*x^4 + a^6*b^3*x^3)*log(x))/(a^10*b^6*x^9 + 6*a^11*b^5*x^8 + 15*a^
12*b^4*x^7 + 20*a^13*b^3*x^6 + 15*a^14*b^2*x^5 + 6*a^15*b*x^4 + a^16*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4(a+bx)^7}\,dx = \frac{-10a^8+45a^7bx-360a^6b^2x^2-6174a^5b^3x^3-21924a^4b^4x^4-35910a^3b^5x^5-31080a^2b^6x^6-13860ab^7x^7}{30a^{15}x^3+180a^{14}bx^4+450a^{13}b^2x^5+600a^{12}b^3x^6+450a^{11}b^4x^7+180a^{10}b^5x^8+30a^9b^6x^9} + \frac{84b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

input
```
integrate(1/x**4/(b*x+a)**7,x)
```

output
```
(-10*a**8 + 45*a**7*b*x - 360*a**6*b**2*x**2 - 6174*a**5*b**3*x**3 - 21924
*a**4*b**4*x**4 - 35910*a**3*b**5*x**5 - 31080*a**2*b**6*x**6 - 13860*a*b*
*7*x**7 - 2520*b**8*x**8)/(30*a**15*x**3 + 180*a**14*b*x**4 + 450*a**13*b*
*2*x**5 + 600*a**12*b**3*x**6 + 450*a**11*b**4*x**7 + 180*a**10*b**5*x**8
+ 30*a**9*b**6*x**9) + 84*b**3*(-log(x) + log(a/b + x))/a**10
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^4(a+bx)^7}\,dx = -\frac{2520\,b^8x^8+13860\,ab^7x^7+31080\,a^2b^6x^6+35910\,a^3b^5x^5+21924\,a^4b^4x^4+6174\,a^5b^3x^3+360\,a^6b^2x^2-}{30\,(a^9b^6x^9+6\,a^{10}b^5x^8+15\,a^{11}b^4x^7+20\,a^{12}b^3x^6+15\,a^{13}b^2x^5+6\,a^{14}bx^4+a^{15}x^3)} + \frac{84\,b^3\log(bx+a)}{a^{10}} - \frac{84\,b^3\log(x)}{a^{10}}$$

input
```
integrate(1/x^4/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/30*(2520*b^8*x^8 + 13860*a*b^7*x^7 + 31080*a^2*b^6*x^6 + 35910*a^3*b^5*
x^5 + 21924*a^4*b^4*x^4 + 6174*a^5*b^3*x^3 + 360*a^6*b^2*x^2 - 45*a^7*b*x
+ 10*a^8)/(a^9*b^6*x^9 + 6*a^10*b^5*x^8 + 15*a^11*b^4*x^7 + 20*a^12*b^3*x^
6 + 15*a^13*b^2*x^5 + 6*a^14*b*x^4 + a^15*x^3) + 84*b^3*log(b*x + a)/a^10
- 84*b^3*log(x)/a^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4(a+bx)^7}\,dx = \frac{84\,b^3\log(|bx+a|)}{a^{10}} - \frac{84\,b^3\log(|x|)}{a^{10}} - \frac{2520\,ab^8x^8+13860\,a^2b^7x^7+31080\,a^3b^6x^6+35910\,a^4b^5x^5+21924\,a^5b^4x^4+6174\,a^6b^3x^3+360\,a^7b^2x^2}{30\,(bx+a)^6a^{10}x^3}$$

input
```
integrate(1/x^4/(b*x+a)^7,x, algorithm="giac")
```

output
```
84*b^3*log(abs(b*x + a))/a^10 - 84*b^3*log(abs(x))/a^10 - 1/30*(2520*a*b^8
*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b^5*x^5 + 21924*a
^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*x + 10*a^9)/((b
*x + a)^6*a^10*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4(a+bx)^7}\,dx$$
$$= \frac{168\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{10}} - \frac{\frac{1}{3\,a}+\frac{12\,b^2\,x^2}{a^3}+\frac{1029\,b^3\,x^3}{5\,a^4}+\frac{3654\,b^4\,x^4}{5\,a^5}+\frac{1197\,b^5\,x^5}{a^6}+\frac{1036\,b^6\,x^6}{a^7}+\frac{462\,b^7\,x^7}{a^8}+\frac{84\,b^8\,x^8}{a^9}-\frac{3\,b\,x}{2\,a^2}}{a^6\,x^3+6\,a^5\,b\,x^4+15\,a^4\,b^2\,x^5+20\,a^3\,b^3\,x^6+15\,a^2\,b^4\,x^7+6\,a\,b^5\,x^8+b^6\,x^9}$$

input
```
int(1/(x^4*(a + b*x)^7),x)
```

output
```
(168*b^3*atanh((2*b*x)/a + 1))/a^10 - (1/(3*a) + (12*b^2*x^2)/a^3 + (1029*
b^3*x^3)/(5*a^4) + (3654*b^4*x^4)/(5*a^5) + (1197*b^5*x^5)/a^6 + (1036*b^6
*x^6)/a^7 + (462*b^7*x^7)/a^8 + (84*b^8*x^8)/a^9 - (3*b*x)/(2*a^2))/(a^6*x
^3 + b^6*x^9 + 6*a^5*b*x^4 + 6*a*b^5*x^8 + 15*a^4*b^2*x^5 + 20*a^3*b^3*x^6
 + 15*a^2*b^4*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^4(a+bx)^7}\,dx$$
$$= \frac{2520\log(bx+a)\,a^6b^3x^3+15120\log(bx+a)\,a^5b^4x^4+37800\log(bx+a)\,a^4b^5x^5+50400\log(bx+a)\,a^3b^6}{}$$

input
```
int(1/x^4/(b*x+a)^7,x)
```

output
```
(2520*log(a + b*x)*a**6*b**3*x**3 + 15120*log(a + b*x)*a**5*b**4*x**4 + 37
800*log(a + b*x)*a**4*b**5*x**5 + 50400*log(a + b*x)*a**3*b**6*x**6 + 3780
0*log(a + b*x)*a**2*b**7*x**7 + 15120*log(a + b*x)*a*b**8*x**8 + 2520*log(
a + b*x)*b**9*x**9 - 2520*log(x)*a**6*b**3*x**3 - 15120*log(x)*a**5*b**4*x
**4 - 37800*log(x)*a**4*b**5*x**5 - 50400*log(x)*a**3*b**6*x**6 - 37800*lo
g(x)*a**2*b**7*x**7 - 15120*log(x)*a*b**8*x**8 - 2520*log(x)*b**9*x**9 - 1
0*a**9 + 45*a**8*b*x - 360*a**7*b**2*x**2 - 5754*a**6*b**3*x**3 - 19404*a*
*5*b**4*x**4 - 29610*a**4*b**5*x**5 - 22680*a**3*b**6*x**6 - 7560*a**2*b**
7*x**7 + 420*b**9*x**9)/(30*a**10*x**3*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x
**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.181 $\int \frac{x^{12}}{(a+bx)^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 186

$$\int \frac{x^{12}}{(a+bx)^{10}} dx = \frac{55a^2x}{b^{12}} - \frac{5ax^2}{b^{11}} + \frac{x^3}{3b^{10}} - \frac{a^{12}}{9b^{13}(a+bx)^9} + \frac{3a^{11}}{2b^{13}(a+bx)^8} - \frac{66a^{10}}{7b^{13}(a+bx)^7} + \frac{110a^9}{3b^{13}(a+bx)^6} - \frac{99a^8}{b^{13}(a+bx)^5} + \frac{198a^7}{b^{13}(a+bx)^4} - \frac{308a^6}{b^{13}(a+bx)^3} + \frac{396a^5}{b^{13}(a+bx)^2} - \frac{495a^4}{b^{13}(a+bx)} - \frac{220a^3\log(a+bx)}{b^{13}}$$

output
```
55*a^2*x/b^12-5*a*x^2/b^11+1/3*x^3/b^10-1/9*a^12/b^13/(b*x+a)^9+3/2*a^11/b
^13/(b*x+a)^8-66/7*a^10/b^13/(b*x+a)^7+110/3*a^9/b^13/(b*x+a)^6-99*a^8/b^1
3/(b*x+a)^5+198*a^7/b^13/(b*x+a)^4-308*a^6/b^13/(b*x+a)^3+396*a^5/b^13/(b*
x+a)^2-495*a^4/b^13/(b*x+a)-220*a^3*ln(b*x+a)/b^13
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.87

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx = -\frac{35201a^{12}+289089a^{11}bx+1031616a^{10}b^2x^2+2074464a^9b^3x^3+2529576a^8b^4x^4+1831032a^7b^5x^5+63\ldots}{}$$

input
```
Integrate[x^12/(a + b*x)^10,x]
```

output
```
-1/126*(35201*a^12 + 289089*a^11*b*x + 1031616*a^10*b^2*x^2 + 2074464*a^9*
b^3*x^3 + 2529576*a^8*b^4*x^4 + 1831032*a^7*b^5*x^5 + 638568*a^6*b^6*x^6 -
 58968*a^5*b^7*x^7 - 139482*a^4*b^8*x^8 - 43218*a^3*b^9*x^9 - 2772*a^2*b^1
0*x^10 + 252*a*b^11*x^11 - 42*b^12*x^12 + 27720*a^3*(a + b*x)^9*Log[a + b*
x])/(b^13*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^{12}}{b^{12}(a+bx)^{10}}-\frac{12a^{11}}{b^{12}(a+bx)^9}+\frac{66a^{10}}{b^{12}(a+bx)^8}-\frac{220a^9}{b^{12}(a+bx)^7}+\frac{495a^8}{b^{12}(a+bx)^6}-\frac{792a^7}{b^{12}(a+bx)^5}+\frac{924a^6}{b^{12}(a+bx)^4}\ldots\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{396a^5}{b^{13}(a+bx)^2}-\frac{495a^4}{b^{13}(a+bx)}-\frac{220a^3\log(a+bx)}{b^{13}}+\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}$$

input
```
Int[x^12/(a + b*x)^10,x]
```

output
```
(55*a^2*x)/b^12 - (5*a*x^2)/b^11 + x^3/(3*b^10) - a^12/(9*b^13*(a + b*x)^9
) + (3*a^11)/(2*b^13*(a + b*x)^8) - (66*a^10)/(7*b^13*(a + b*x)^7) + (110*
a^9)/(3*b^13*(a + b*x)^6) - (99*a^8)/(b^13*(a + b*x)^5) + (198*a^7)/(b^13*
(a + b*x)^4) - (308*a^6)/(b^13*(a + b*x)^3) + (396*a^5)/(b^13*(a + b*x)^2)
 - (495*a^4)/(b^13*(a + b*x)) - (220*a^3*Log[a + b*x])/b^13
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x^3}{3b^{10}}-\frac{5a\,x^2}{b^{11}}+\frac{55a^2x}{b^{12}}+\frac{-495a^4b^7x^8-3564a^5b^6x^7-11396a^6b^5x^6-21054a^7b^4x^5-24519b^3a^8x^4-\frac{55198b^2a^9x^3}{3}-\frac{60742a^1}{7}}{b^{12}(bx+a)^9}$
norman	$\frac{\frac{x^{12}}{3b}-\frac{2a\,x^{11}}{b^2}+\frac{22a^2x^{10}}{b^3}-\frac{78419a^{12}}{126b^{13}}-\frac{1980a^4x^8}{b^5}-\frac{11880a^5x^7}{b^6}-\frac{33880a^6x^6}{b^7}-\frac{57750a^7x^5}{b^8}-\frac{63294a^8x^4}{b^9}-\frac{45276a^9x^3}{b^{10}}-\frac{143748a^{10}x^2}{7b^{11}}-\frac{75}{}}{(bx+a)^9}$
default	$\frac{\frac{1}{3}b^2x^3-5ab\,x^2+55a^2x}{b^{12}}-\frac{a^{12}}{9b^{13}(bx+a)^9}-\frac{99a^8}{b^{13}(bx+a)^5}+\frac{198a^7}{b^{13}(bx+a)^4}+\frac{3a^{11}}{2b^{13}(bx+a)^8}+\frac{396a^5}{b^{13}(bx+a)^2}-\frac{495a^4}{b^{13}(bx+a)}-$
parallelrisch	$-\frac{4268880a^6x^6b^6+7276500a^7x^5b^5+7975044a^8x^4b^4+5704776a^9x^3b^3+2587464a^{10}x^2b^2+678051a^{11}xb+249480a^4x^8b^8+149}{}$

input
```
int(x^12/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/b^10-5*a*x^2/b^11+55*a^2*x/b^12+(-495*a^4*b^7*x^8-3564*a^5*b^6*x^7
-11396*a^6*b^5*x^6-21054*a^7*b^4*x^5-24519*b^3*a^8*x^4-55198/3*b^2*a^9*x^3
-60742/7*a^10*b*x^2-32891/14*a^11*x-35201/126*a^12/b)/b^12/(b*x+a)^9-220*a
^3*ln(b*x+a)/b^13
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.82

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx$$
$$= \frac{42\,b^{12}x^{12} - 252\,ab^{11}x^{11} + 2772\,a^2b^{10}x^{10} + 43218\,a^3b^9x^9 + 139482\,a^4b^8x^8 + 58968\,a^5b^7x^7 - 638568\,a^6b^6x^6}{}$$

input
```
integrate(x^12/(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/126*(42*b^12*x^12 - 252*a*b^11*x^11 + 2772*a^2*b^10*x^10 + 43218*a^3*b^9
*x^9 + 139482*a^4*b^8*x^8 + 58968*a^5*b^7*x^7 - 638568*a^6*b^6*x^6 - 18310
32*a^7*b^5*x^5 - 2529576*a^8*b^4*x^4 - 2074464*a^9*b^3*x^3 - 1031616*a^10*
b^2*x^2 - 289089*a^11*b*x - 35201*a^12 - 27720*(a^3*b^9*x^9 + 9*a^4*b^8*x^
8 + 36*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 126*a^7*b^5*x^5 + 126*a^8*b^4*x^4 +
84*a^9*b^3*x^3 + 36*a^10*b^2*x^2 + 9*a^11*b*x + a^12)*log(b*x + a))/(b^22*
x^9 + 9*a*b^21*x^8 + 36*a^2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5
+ 126*a^5*b^17*x^4 + 84*a^6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^
9*b^13)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.34

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx = -\frac{220a^3\log(a+bx)}{b^{13}} + \frac{55a^2x}{b^{12}} - \frac{5ax^2}{b^{11}} + \frac{-35201a^{12} - 296019a^{11}bx - 1093356a^{10}b^2x^2 - 2318316a^9b^3x^3 - 3089394a^8b^4x^4 - 2652804a^7b^5x^5 - 1}{126a^9b^{13} + 1134a^8b^{14}x + 4536a^7b^{15}x^2 + 10584a^6b^{16}x^3 + 15876a^5b^{17}x^4 + 15876a^4b^{18}x^5 + 10584a^3b^{1}} + \frac{x^3}{3b^{10}}$$

input
```
integrate(x**12/(b*x+a)**10,x)
```

output
```
-220*a**3*log(a + b*x)/b**13 + 55*a**2*x/b**12 - 5*a*x**2/b**11 + (-35201*
a**12 - 296019*a**11*b*x - 1093356*a**10*b**2*x**2 - 2318316*a**9*b**3*x**
3 - 3089394*a**8*b**4*x**4 - 2652804*a**7*b**5*x**5 - 1435896*a**6*b**6*x*
*6 - 449064*a**5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*a**9*b**13 + 1134*
a**8*b**14*x + 4536*a**7*b**15*x**2 + 10584*a**6*b**16*x**3 + 15876*a**5*b
**17*x**4 + 15876*a**4*b**18*x**5 + 10584*a**3*b**19*x**6 + 4536*a**2*b**2
0*x**7 + 1134*a*b**21*x**8 + 126*b**22*x**9) + x**3/(3*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.26

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx = -\frac{62370\,a^4b^8x^8 + 449064\,a^5b^7x^7 + 1435896\,a^6b^6x^6 + 2652804\,a^7b^5x^5 + 3089394\,a^8b^4x^4 + 2318316\,a^9b^3x^3}{126\,(b^{22}x^9 + 9\,ab^{21}x^8 + 36\,a^2b^{20}x^7 + 84\,a^3b^{19}x^6 + 126\,a^4b^{18}x^5 + 126\,a^5b^{17}x^4 + 84\,a^6b^{16}x} - \frac{220\,a^3\log(bx+a)}{b^{13}} + \frac{b^2x^3 - 15\,abx^2 + 165\,a^2x}{3\,b^{12}}$$

input
```
integrate(x^12/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7*x^7 + 1435896*a^6*b^6*x^6 + 265
2804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 2318316*a^9*b^3*x^3 + 1093356*a^1
0*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/(b^22*x^9 + 9*a*b^21*x^8 + 36*a^
2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a^5*b^17*x^4 + 84*a^
6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13) - 220*a^3*log(b*x
+ a)/b^13 + 1/3*(b^2*x^3 - 15*a*b*x^2 + 165*a^2*x)/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.80

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx = -\frac{220\,a^3\log\left(|bx+a|\right)}{b^{13}} - \frac{62370\,a^4b^8x^8 + 449064\,a^5b^7x^7 + 1435896\,a^6b^6x^6 + 2652804\,a^7b^5x^5 + 3089394\,a^8b^4x^4 + 2318316\,a^9b^3x^3}{126\,(bx+a)^9b^{13}} + \frac{b^{20}x^3 - 15\,ab^{19}x^2 + 165\,a^2b^{18}x}{3\,b^{30}}$$

input
```
integrate(x^12/(b*x+a)^10,x, algorithm="giac")
```

output
```
-220*a^3*log(abs(b*x + a))/b^13 - 1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^
7*x^7 + 1435896*a^6*b^6*x^6 + 2652804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 +
2318316*a^9*b^3*x^3 + 1093356*a^10*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)
/((b*x + a)^9*b^13) + 1/3*(b^20*x^3 - 15*a*b^19*x^2 + 165*a^2*b^18*x)/b^30
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx = -\frac{6\,a\,(a+b\,x)^2 - \frac{(a+b\,x)^3}{3} + \frac{495\,a^4}{a+b\,x} - \frac{396\,a^5}{(a+b\,x)^2} + \frac{308\,a^6}{(a+b\,x)^3} - \frac{198\,a^7}{(a+b\,x)^4} + \frac{99\,a^8}{(a+b\,x)^5} - \frac{110\,a^9}{3\,(a+b\,x)^6} + \frac{66\,a^{10}}{7\,(a+b\,x)^7} - \frac{3\,a^{11}}{2\,(a+b\,x)}}{b^{13}}$$

input
```
int(x^12/(a + b*x)^10,x)
```

output
```
-(6*a*(a + b*x)^2 - (a + b*x)^3/3 + (495*a^4)/(a + b*x) - (396*a^5)/(a + b
*x)^2 + (308*a^6)/(a + b*x)^3 - (198*a^7)/(a + b*x)^4 + (99*a^8)/(a + b*x)
^5 - (110*a^9)/(3*(a + b*x)^6) + (66*a^10)/(7*(a + b*x)^7) - (3*a^11)/(2*(
a + b*x)^8) + a^12/(9*(a + b*x)^9) + 220*a^3*log(a + b*x) - 66*a^2*b*x)/b^
13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.03

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx = \frac{-27720\log(bx+a)\,a^{12}-249480\log(bx+a)\,a^{11}bx-997920\log(bx+a)\,a^{10}b^2x^2-2328480\log(bx+a)}{}$$

input
```
int(x^12/(b*x+a)^10,x)
```

output
```
( - 27720*log(a + b*x)*a**12 - 249480*log(a + b*x)*a**11*b*x - 997920*log(
a + b*x)*a**10*b**2*x**2 - 2328480*log(a + b*x)*a**9*b**3*x**3 - 3492720*l
og(a + b*x)*a**8*b**4*x**4 - 3492720*log(a + b*x)*a**7*b**5*x**5 - 2328480
*log(a + b*x)*a**6*b**6*x**6 - 997920*log(a + b*x)*a**5*b**7*x**7 - 249480
*log(a + b*x)*a**4*b**8*x**8 - 27720*log(a + b*x)*a**3*b**9*x**9 - 50699*a
**12 - 428571*a**11*b*x - 1589544*a**10*b**2*x**2 - 3376296*a**9*b**3*x**3
 - 4482324*a**8*b**4*x**4 - 3783780*a**7*b**5*x**5 - 1940400*a**6*b**6*x**
6 - 498960*a**5*b**7*x**7 + 27720*a**3*b**9*x**9 + 2772*a**2*b**10*x**10 -
 252*a*b**11*x**11 + 42*b**12*x**12)/(126*b**13*(a**9 + 9*a**8*b*x + 36*a*
*7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x**5
 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.182 $\int \frac{x^{11}}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 177

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx = -\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7} -\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3} -\frac{165a^4}{b^{12}(a+bx)^2}+\frac{165a^3}{b^{12}(a+bx)}+\frac{55a^2\log(a+bx)}{b^{12}}$$

output
```
-10*a*x/b^11+1/2*x^2/b^10+1/9*a^11/b^12/(b*x+a)^9-11/8*a^10/b^12/(b*x+a)^8
+55/7*a^9/b^12/(b*x+a)^7-55/2*a^8/b^12/(b*x+a)^6+66*a^7/b^12/(b*x+a)^5-231
/2*a^6/b^12/(b*x+a)^4+154*a^5/b^12/(b*x+a)^3-165*a^4/b^12/(b*x+a)^2+165*a^
3/b^12/(b*x+a)+55*a^2*ln(b*x+a)/b^12
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx$$
$$= \frac{42131a^{11} + 351459a^{10}bx + 1281096a^9b^2x^2 + 2656584a^8b^3x^3 + 3402756a^7b^4x^4 + 2704212a^6b^5x^5 + 12206}{50}$$

input
```
Integrate[x^11/(a + b*x)^10,x]
```

output
```
(42131*a^11 + 351459*a^10*b*x + 1281096*a^9*b^2*x^2 + 2656584*a^8*b^3*x^3
+ 3402756*a^7*b^4*x^4 + 2704212*a^6*b^5*x^5 + 1220688*a^5*b^6*x^6 + 190512
*a^4*b^7*x^7 - 77112*a^3*b^8*x^8 - 36288*a^2*b^9*x^9 - 2772*a*b^10*x^10 +
252*b^11*x^11 + 27720*a^2*(a + b*x)^9*Log[a + b*x])/(504*b^12*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{a^{11}}{b^{11}(a+bx)^{10}} + \frac{11a^{10}}{b^{11}(a+bx)^9} - \frac{55a^9}{b^{11}(a+bx)^8} + \frac{165a^8}{b^{11}(a+bx)^7} - \frac{330a^7}{b^{11}(a+bx)^6} + \frac{462a^6}{b^{11}(a+bx)^5} - \frac{462a^5}{b^{11}(a+bx}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{11}}{9b^{12}(a+bx)^9} - \frac{11a^{10}}{8b^{12}(a+bx)^8} + \frac{55a^9}{7b^{12}(a+bx)^7} - \frac{55a^8}{2b^{12}(a+bx)^6} + \frac{66a^7}{b^{12}(a+bx)^5} - \frac{231a^6}{2b^{12}(a+bx)^4} + \frac{154a^5}{b^{12}(a+bx)^3} - \frac{165a^4}{b^{12}(a+bx)^2} + \frac{165a^3}{b^{12}(a+bx)} + \frac{55a^2\log(a+bx)}{b^{12}} - \frac{10ax}{b^{11}} + \frac{x^2}{2b^{10}}$$

input
```
Int[x^11/(a + b*x)^10,x]
```

output
```
(-10*a*x)/b^11 + x^2/(2*b^10) + a^11/(9*b^12*(a + b*x)^9) - (11*a^10)/(8*b
^12*(a + b*x)^8) + (55*a^9)/(7*b^12*(a + b*x)^7) - (55*a^8)/(2*b^12*(a + b
*x)^6) + (66*a^7)/(b^12*(a + b*x)^5) - (231*a^6)/(2*b^12*(a + b*x)^4) + (1
54*a^5)/(b^12*(a + b*x)^3) - (165*a^4)/(b^12*(a + b*x)^2) + (165*a^3)/(b^1
2*(a + b*x)) + (55*a^2*Log[a + b*x])/b^12
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x^{2}}{2 b^{10}}-\frac{10 a x}{b^{11}}+\frac{165 a^{3} b^{7} x^{8}+1155 a^{4} b^{6} x^{7}+3619 a^{5} b^{5} x^{6}+\frac{13167 a^{6} b^{4} x^{5}}{2}+\frac{15147 a^{7} b^{3} x^{4}}{2}+\frac{11253 a^{8} b^{2} x^{3}}{2}+\frac{36839 a^{9} b\,x^{2}}{14}+\frac{39611 a^{10}}{56}}{b^{11} \left(b x +a \right)^{9}}$
norman	$\frac{\frac{x^{11}}{2 b}-\frac{11 a\,x^{10}}{2 b^{2}}+\frac{78419 a^{11}}{504 b^{12}}+\frac{495 a^{3} x^{8}}{b^{4}}+\frac{2970 a^{4} x^{7}}{b^{5}}+\frac{8470 a^{5} x^{6}}{b^{6}}+\frac{28875 a^{6} x^{5}}{2 b^{7}}+\frac{31647 a^{7} x^{4}}{2 b^{8}}+\frac{11319 a^{8} x^{3}}{b^{9}}+\frac{35937 a^{9} x^{2}}{7 b^{10}}+\frac{75339 a^{10} x}{56 b^{11}}}{\left(b x +a \right)^{9}}+55$
default	$-\frac{-\frac{1}{2} b\,x^{2}+10 a x}{b^{11}}+\frac{a^{11}}{9 b^{12} \left(b x +a \right)^{9}}+\frac{66 a^{7}}{b^{12} \left(b x +a \right)^{5}}-\frac{231 a^{6}}{2 b^{12} \left(b x +a \right)^{4}}-\frac{11 a^{10}}{8 b^{12} \left(b x +a \right)^{8}}-\frac{165 a^{4}}{b^{12} \left(b x +a \right)^{2}}+\frac{165 a^{3}}{b^{12} \left(b x +a \right)}+55$
parallelrisch	$249480 a^{3} x^{8} b^{8}+27720 \ln \left(b x +a \right) x^{9} a^{2} b^{9}+249480 \ln \left(b x +a \right) x^{8} a^{3} b^{8}+997920 \ln \left(b x +a \right) x^{7} a^{4} b^{7}+2328480 \ln \left(b x +a \right) x^{6} a^{5} b^{6}+252 b^{1}$

input
```
int(x^11/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b^10-10*a*x/b^11+(165*a^3*b^7*x^8+1155*a^4*b^6*x^7+3619*a^5*b^5*x^
6+13167/2*a^6*b^4*x^5+15147/2*a^7*b^3*x^4+11253/2*a^8*b^2*x^3+36839/14*a^9
*b*x^2+39611/56*a^10*x+42131/504/b*a^11)/b^11/(b*x+a)^9+55*a^2*ln(b*x+a)/b
^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.85

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx = \frac{252\,b^{11}x^{11} - 2772\,ab^{10}x^{10} - 36288\,a^2b^9x^9 - 77112\,a^3b^8x^8 + 190512\,a^4b^7x^7 + 1220688\,a^5b^6x^6 + 2704212\,a}{50}$$

input
```
integrate(x^11/(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/504*(252*b^11*x^11 - 2772*a*b^10*x^10 - 36288*a^2*b^9*x^9 - 77112*a^3*b^
8*x^8 + 190512*a^4*b^7*x^7 + 1220688*a^5*b^6*x^6 + 2704212*a^6*b^5*x^5 + 3
402756*a^7*b^4*x^4 + 2656584*a^8*b^3*x^3 + 1281096*a^9*b^2*x^2 + 351459*a^
10*b*x + 42131*a^11 + 27720*(a^2*b^9*x^9 + 9*a^3*b^8*x^8 + 36*a^4*b^7*x^7
+ 84*a^5*b^6*x^6 + 126*a^6*b^5*x^5 + 126*a^7*b^4*x^4 + 84*a^8*b^3*x^3 + 36
*a^9*b^2*x^2 + 9*a^10*b*x + a^11)*log(b*x + a))/(b^21*x^9 + 9*a*b^20*x^8 +
 36*a^2*b^19*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 +
 84*a^6*b^15*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.33

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx = \frac{55a^2\log{(a+bx)}}{b^{12}} - \frac{10ax}{b^{11}} + \frac{42131a^{11} + 356499a^{10}bx + 1326204a^9b^2x^2 + 2835756a^8b^3x^3 + 3817044a^7b^4x^4 + 3318084a^6b^5x^5 + 182}{504a^9b^{12} + 4536a^8b^{13}x + 18144a^7b^{14}x^2 + 42336a^6b^{15}x^3 + 63504a^5b^{16}x^4 + 63504a^4b^{17}x^5 + 42336a^3b^{18}} + \frac{x^2}{2b^{10}}$$

input
```
integrate(x**11/(b*x+a)**10,x)
```

output
```
55*a**2*log(a + b*x)/b**12 - 10*a*x/b**11 + (42131*a**11 + 356499*a**10*b*
x + 1326204*a**9*b**2*x**2 + 2835756*a**8*b**3*x**3 + 3817044*a**7*b**4*x*
*4 + 3318084*a**6*b**5*x**5 + 1823976*a**5*b**6*x**6 + 582120*a**4*b**7*x*
*7 + 83160*a**3*b**8*x**8)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**
7*b**14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*
b**17*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*
x**8 + 504*b**21*x**9) + x**2/(2*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.26

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx = \frac{83160\,a^3b^8x^8 + 582120\,a^4b^7x^7 + 1823976\,a^5b^6x^6 + 3318084\,a^6b^5x^5 + 3817044\,a^7b^4x^4 + 2835756\,a^8b^3x^3 + \cdots}{504\,(b^{21}x^9 + 9\,ab^{20}x^8 + 36\,a^2b^{19}x^7 + 84\,a^3b^{18}x^6 + 126\,a^4b^{17}x^5 + 126\,a^5b^{16}x^4 + 84\,a^6b^{15}x^3 + \cdots} + \frac{55\,a^2\log(bx+a)}{b^{12}} + \frac{bx^2 - 20\,ax}{2\,b^{11}}$$

input
```
integrate(x^11/(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/504*(83160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 3318
084*a^6*b^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*
b^2*x^2 + 356499*a^10*b*x + 42131*a^11)/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^2*
b^19*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6*
b^15*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12) + 55*a^2*log(b*x + a
)/b^12 + 1/2*(b*x^2 - 20*a*x)/b^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx = \frac{55\,a^2\log(|bx+a|)}{b^{12}} + \frac{b^{10}x^2 - 20\,ab^9x}{2\,b^{20}} + \frac{83160\,a^3b^8x^8 + 582120\,a^4b^7x^7 + 1823976\,a^5b^6x^6 + 3318084\,a^6b^5x^5 + 3817044\,a^7b^4x^4 + 2835756\,a^8b^3x^3 \cdots}{504\,(bx+a)^9b^{12}}$$

input
```
integrate(x^11/(b*x+a)^10,x, algorithm="giac")
```

output
```
55*a^2*log(abs(b*x + a))/b^12 + 1/2*(b^10*x^2 - 20*a*b^9*x)/b^20 + 1/504*(
83160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 3318084*a^6
*b^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^2*x^2
 + 356499*a^10*b*x + 42131*a^11)/((b*x + a)^9*b^12)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx = \frac{\frac{(a+b\,x)^2}{2}+\frac{165\,a^3}{a+b\,x}-\frac{165\,a^4}{(a+b\,x)^2}+\frac{154\,a^5}{(a+b\,x)^3}-\frac{231\,a^6}{2\,(a+b\,x)^4}+\frac{66\,a^7}{(a+b\,x)^5}-\frac{55\,a^8}{2\,(a+b\,x)^6}+\frac{55\,a^9}{7\,(a+b\,x)^7}-\frac{11\,a^{10}}{8\,(a+b\,x)^8}+\frac{a^{11}}{9\,(a+b\,x)^9}+55}{b^{12}}$$

input
```
int(x^11/(a + b*x)^10,x)
```

output
```
((a + b*x)^2/2 + (165*a^3)/(a + b*x) - (165*a^4)/(a + b*x)^2 + (154*a^5)/(
a + b*x)^3 - (231*a^6)/(2*(a + b*x)^4) + (66*a^7)/(a + b*x)^5 - (55*a^8)/(
2*(a + b*x)^6) + (55*a^9)/(7*(a + b*x)^7) - (11*a^10)/(8*(a + b*x)^8) + a^
11/(9*(a + b*x)^9) + 55*a^2*log(a + b*x) - 11*a*b*x)/b^12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.07

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx = \frac{27720\log(bx+a)\,a^{11}+249480\log(bx+a)\,a^{10}bx+997920\log(bx+a)\,a^{9}b^{2}x^{2}+2328480\log(bx+a)\,a^{8}b}{}$$

input
```
int(x^11/(b*x+a)^10,x)
```

output

```
(27720*log(a + b*x)*a**11 + 249480*log(a + b*x)*a**10*b*x + 997920*log(a +
 b*x)*a**9*b**2*x**2 + 2328480*log(a + b*x)*a**8*b**3*x**3 + 3492720*log(a
 + b*x)*a**7*b**4*x**4 + 3492720*log(a + b*x)*a**6*b**5*x**5 + 2328480*log
(a + b*x)*a**5*b**6*x**6 + 997920*log(a + b*x)*a**4*b**7*x**7 + 249480*log
(a + b*x)*a**3*b**8*x**8 + 27720*log(a + b*x)*a**2*b**9*x**9 + 50699*a**11
 + 428571*a**10*b*x + 1589544*a**9*b**2*x**2 + 3376296*a**8*b**3*x**3 + 44
82324*a**7*b**4*x**4 + 3783780*a**6*b**5*x**5 + 1940400*a**5*b**6*x**6 + 4
98960*a**4*b**7*x**7 - 27720*a**2*b**9*x**9 - 2772*a*b**10*x**10 + 252*b**
11*x**11)/(504*b**12*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3
*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a
**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.183 $\int \frac{x^{10}}{(a+bx)^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 159

$$\int \frac{x^{10}}{(a+bx)^{10}} dx = \frac{x}{b^{10}} - \frac{a^{10}}{9b^{11}(a+bx)^9} + \frac{5a^9}{4b^{11}(a+bx)^8} - \frac{45a^8}{7b^{11}(a+bx)^7} + \frac{20a^7}{b^{11}(a+bx)^6} - \frac{42a^6}{b^{11}(a+bx)^5} + \frac{63a^5}{b^{11}(a+bx)^4} - \frac{70a^4}{b^{11}(a+bx)^3} + \frac{60a^3}{b^{11}(a+bx)^2} - \frac{45a^2}{b^{11}(a+bx)} - \frac{10a\log(a+bx)}{b^{11}}$$

output
```
x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a
)^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a
^4/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)
/b^11
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = -\frac{4861a^{10}+41229a^9bx+153576a^8b^2x^2+328104a^7b^3x^3+439236a^6b^4x^4+375732a^5b^5x^5+197568a^4b^6}{252b^{11}(a+bx)}$$

input
```
Integrate[x^10/(a + b*x)^10,x]
```

output
```
-1/252*(4861*a^10 + 41229*a^9*b*x + 153576*a^8*b^2*x^2 + 328104*a^7*b^3*x^
3 + 439236*a^6*b^4*x^4 + 375732*a^5*b^5*x^5 + 197568*a^4*b^6*x^6 + 54432*a
^3*b^7*x^7 + 2268*a^2*b^8*x^8 - 2268*a*b^9*x^9 - 252*b^10*x^10 + 2520*a*(a
 + b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210a^6}{b^{10}(a+bx)^6}-\frac{252a^5}{b^{10}(a+bx)^5}+\frac{210a^4}{b^{10}(a+bx)^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{60a^3}{b^{11}(a+bx)^2}-\frac{45a^2}{b^{11}(a+bx)}-\frac{10a\log(a+bx)}{b^{11}}+\frac{x}{b^{10}}$$

input
```
Int[x^10/(a + b*x)^10,x]
```

output
```
x/b^10 - a^10/(9*b^11*(a + b*x)^9) + (5*a^9)/(4*b^11*(a + b*x)^8) - (45*a^
8)/(7*b^11*(a + b*x)^7) + (20*a^7)/(b^11*(a + b*x)^6) - (42*a^6)/(b^11*(a
+ b*x)^5) + (63*a^5)/(b^11*(a + b*x)^4) - (70*a^4)/(b^11*(a + b*x)^3) + (6
0*a^3)/(b^11*(a + b*x)^2) - (45*a^2)/(b^11*(a + b*x)) - (10*a*Log[a + b*x]
)/b^11
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x}{b^{10}}+\frac{-45a^{2}b^{7}x^{8}-300a^{3}b^{6}x^{7}-910a^{4}b^{5}x^{6}-1617a^{5}b^{4}x^{5}-1827a^{6}b^{3}x^{4}-1338a^{7}b^{2}x^{3}-\frac{4329a^{8}b\,x^{2}}{7}-\frac{4609a^{9}x}{28}-\frac{4861a^{10}}{252b}}{b^{10}\left(bx+a\right)^{9}}-10$
norman	$\frac{\frac{x^{10}}{b}-\frac{7129a^{10}}{252b^{11}}-\frac{90a^{2}x^{8}}{b^{3}}-\frac{540a^{3}x^{7}}{b^{4}}-\frac{1540a^{4}x^{6}}{b^{5}}-\frac{2625a^{5}x^{5}}{b^{6}}-\frac{2877a^{6}x^{4}}{b^{7}}-\frac{2058a^{7}x^{3}}{b^{8}}-\frac{6534a^{8}x^{2}}{7b^{9}}-\frac{6849a^{9}x}{28b^{10}}}{\left(bx+a\right)^{9}}-\frac{10a\ln\left(bx+a\right)}{b^{11}}$
default	$\frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}\left(bx+a\right)^{9}}+\frac{5a^{9}}{4b^{11}\left(bx+a\right)^{8}}-\frac{45a^{8}}{7b^{11}\left(bx+a\right)^{7}}+\frac{20a^{7}}{b^{11}\left(bx+a\right)^{6}}-\frac{42a^{6}}{b^{11}\left(bx+a\right)^{5}}+\frac{63a^{5}}{b^{11}\left(bx+a\right)^{4}}-\frac{70a^{4}}{b^{11}\left(bx+a\right)^{3}}+$
parallelrisch	$-\frac{-252b^{10}x^{10}+7129a^{10}+2520\ln\left(bx+a\right)a^{10}+211680\ln\left(bx+a\right)x^{6}a^{4}b^{6}+317520\ln\left(bx+a\right)x^{5}a^{5}b^{5}+317520\ln\left(bx+a\right)x^{4}a^{6}b^{4}+2}{}$

input
```
int(x^10/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
x/b^10+(-45*a^2*b^7*x^8-300*a^3*b^6*x^7-910*a^4*b^5*x^6-1617*a^5*b^4*x^5-1
827*a^6*b^3*x^4-1338*a^7*b^2*x^3-4329/7*a^8*b*x^2-4609/28*a^9*x-4861/252*a
^10/b)/b^10/(b*x+a)^9-10*a*ln(b*x+a)/b^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(153) = 306$.

Time = 0.07 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.97

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = \frac{252\,b^{10}x^{10} + 2268\,ab^9x^9 - 2268\,a^2b^8x^8 - 54432\,a^3b^7x^7 - 197568\,a^4b^6x^6 - 375732\,a^5b^5x^5 - 439236\,a^6b^4x^4}{252\,(b^{20}x^9 + 9\,ab^{19}}$$

input
```
integrate(x^10/(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/252*(252*b^10*x^10 + 2268*a*b^9*x^9 - 2268*a^2*b^8*x^8 - 54432*a^3*b^7*x
^7 - 197568*a^4*b^6*x^6 - 375732*a^5*b^5*x^5 - 439236*a^6*b^4*x^4 - 328104
*a^7*b^3*x^3 - 153576*a^8*b^2*x^2 - 41229*a^9*b*x - 4861*a^10 - 2520*(a*b^
9*x^9 + 9*a^2*b^8*x^8 + 36*a^3*b^7*x^7 + 84*a^4*b^6*x^6 + 126*a^5*b^5*x^5
+ 126*a^6*b^4*x^4 + 84*a^7*b^3*x^3 + 36*a^8*b^2*x^2 + 9*a^9*b*x + a^10)*lo
g(b*x + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 +
 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 +
 9*a^8*b^12*x + a^9*b^11)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.41

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = -\frac{10a\log{(a+bx)}}{b^{11}} + \frac{-4861a^{10} - 41481a^9bx - 155844a^8b^2x^2 - 337176a^7b^3x^3 - 460404a^6b^4x^4 - 407484a^5b^5x^5 - 22932}{252a^9b^{11} + 2268a^8b^{12}x + 9072a^7b^{13}x^2 + 21168a^6b^{14}x^3 + 31752a^5b^{15}x^4 + 31752a^4b^{16}x^5 + 21168a^3b^{17}x^6} + \frac{x}{b^{10}}$$

input
```
integrate(x**10/(b*x+a)**10,x)
```

output
```
-10*a*log(a + b*x)/b**11 + (-4861*a**10 - 41481*a**9*b*x - 155844*a**8*b**
2*x**2 - 337176*a**7*b**3*x**3 - 460404*a**6*b**4*x**4 - 407484*a**5*b**5*
x**5 - 229320*a**4*b**6*x**6 - 75600*a**3*b**7*x**7 - 11340*a**2*b**8*x**8
)/(252*a**9*b**11 + 2268*a**8*b**12*x + 9072*a**7*b**13*x**2 + 21168*a**6*
b**14*x**3 + 31752*a**5*b**15*x**4 + 31752*a**4*b**16*x**5 + 21168*a**3*b*
*17*x**6 + 9072*a**2*b**18*x**7 + 2268*a*b**19*x**8 + 252*b**20*x**9) + x/
b**10
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.33

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = -\frac{11340\,a^2b^8x^8+75600\,a^3b^7x^7+229320\,a^4b^6x^6+407484\,a^5b^5x^5+460404\,a^6b^4x^4+337176\,a^7b^3x^3+15}{252\,(b^{20}x^9+9\,ab^{19}x^8+36\,a^2b^{18}x^7+84\,a^3b^{17}x^6+126\,a^4b^{16}x^5+126\,a^5b^{15}x^4+84\,a^6b^{14}x^3+36} + \frac{x}{b^{10}} - \frac{10\,a\log{(bx+a)}}{b^{11}}$$

input
```
integrate(x^10/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/252*(11340*a^2*b^8*x^8 + 75600*a^3*b^7*x^7 + 229320*a^4*b^6*x^6 + 40748
4*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 337176*a^7*b^3*x^3 + 155844*a^8*b^2*x
^2 + 41481*a^9*b*x + 4861*a^10)/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7
 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3
 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + x/b^10 - 10*a*log(b*x + a)
/b^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = \frac{x}{b^{10}} - \frac{10\,a\log{(|bx+a|)}}{b^{11}} - \frac{11340\,a^2b^8x^8+75600\,a^3b^7x^7+229320\,a^4b^6x^6+407484\,a^5b^5x^5+460404\,a^6b^4x^4+337176\,a^7b^3x^3+15}{252\,(bx+a)^9b^{11}}$$

input
```
integrate(x^10/(b*x+a)^10,x, algorithm="giac")
```

output
```
x/b^10 - 10*a*log(abs(b*x + a))/b^11 - 1/252*(11340*a^2*b^8*x^8 + 75600*a^
3*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 +
 337176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2 + 41481*a^9*b*x + 4861*a^10)/((b*
x + a)^9*b^11)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = -\frac{10\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{45\,a^2}{a+b\,x}-\frac{60\,a^3}{{\left(a+b\,x\right)}^2}+\frac{70\,a^4}{{\left(a+b\,x\right)}^3}-\frac{63\,a^5}{{\left(a+b\,x\right)}^4}+\frac{42\,a^6}{{\left(a+b\,x\right)}^5}-\frac{20\,a^7}{{\left(a+b\,x\right)}^6}+\frac{45\,a^8}{7\,{\left(a+b\,x\right)}^7}-\frac{5\,a^9}{4\,{\left(a+b\,x\right)}^8}}{b^{11}}$$

input
```
int(x^10/(a + b*x)^10,x)
```

output
```
-(10*a*log(a + b*x) - b*x + (45*a^2)/(a + b*x) - (60*a^3)/(a + b*x)^2 + (7
0*a^4)/(a + b*x)^3 - (63*a^5)/(a + b*x)^4 + (42*a^6)/(a + b*x)^5 - (20*a^7
)/(a + b*x)^6 + (45*a^8)/(7*(a + b*x)^7) - (5*a^9)/(4*(a + b*x)^8) + a^10/
(9*(a + b*x)^9))/b^11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.23

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = \frac{-2520\log(bx+a)\,a^{10}-22680\log(bx+a)\,a^9bx-90720\log(bx+a)\,a^8b^2x^2-211680\log(bx+a)\,a^7b^3x^3}{}$$

input
```
int(x^10/(b*x+a)^10,x)
```

output
```
( - 2520*log(a + b*x)*a**10 - 22680*log(a + b*x)*a**9*b*x - 90720*log(a +
b*x)*a**8*b**2*x**2 - 211680*log(a + b*x)*a**7*b**3*x**3 - 317520*log(a +
b*x)*a**6*b**4*x**4 - 317520*log(a + b*x)*a**5*b**5*x**5 - 211680*log(a +
b*x)*a**4*b**6*x**6 - 90720*log(a + b*x)*a**3*b**7*x**7 - 22680*log(a + b*
x)*a**2*b**8*x**8 - 2520*log(a + b*x)*a*b**9*x**9 - 4609*a**10 - 38961*a**
9*b*x - 144504*a**8*b**2*x**2 - 306936*a**7*b**3*x**3 - 407484*a**6*b**4*x
**4 - 343980*a**5*b**5*x**5 - 176400*a**4*b**6*x**6 - 45360*a**3*b**7*x**7
 + 2520*a*b**9*x**9 + 252*b**10*x**10)/(252*b**11*(a**9 + 9*a**8*b*x + 36*
a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x*
*5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.184 $\int \frac{x^9}{(a+bx)^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 154

$$\int \frac{x^9}{(a+bx)^{10}} \, dx = \frac{a^9}{9b^{10}(a+bx)^9} - \frac{9a^8}{8b^{10}(a+bx)^8} + \frac{36a^7}{7b^{10}(a+bx)^7} - \frac{14a^6}{b^{10}(a+bx)^6} + \frac{126a^5}{5b^{10}(a+bx)^5} - \frac{63a^4}{2b^{10}(a+bx)^4} + \frac{28a^3}{b^{10}(a+bx)^3} - \frac{18a^2}{b^{10}(a+bx)^2} + \frac{9a}{b^{10}(a+bx)} + \frac{\log(a+bx)}{b^{10}}$$

output
```
1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a
^6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/
b^10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.72

$$\int \frac{x^9}{(a+bx)^{10}} \, dx = \frac{a(7129a^8 + 61641a^7bx + 235224a^6b^2x^2 + 518616a^5b^3x^3 + 725004a^4b^4x^4 + 661500a^3b^5x^5 + 388080a^2b^6x}{2520b^{10}(a+bx)^9} + \frac{\log(a+bx)}{b^{10}}$$

input
```
Integrate[x^9/(a + b*x)^10,x]
```

output
```
(a*(7129*a^8 + 61641*a^7*b*x + 235224*a^6*b^2*x^2 + 518616*a^5*b^3*x^3 + 7
25004*a^4*b^4*x^4 + 661500*a^3*b^5*x^5 + 388080*a^2*b^6*x^6 + 136080*a*b^7
*x^7 + 22680*b^8*x^8))/(2520*b^10*(a + b*x)^9) + Log[a + b*x]/b^10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx)^{10}}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{126a^4}{b^9(a+bx)^5}-\frac{84a^3}{b^9(a+bx)^4}+\frac{}{b^9}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a^2}{b^{10}(a+bx)^2}+\frac{9a}{b^{10}(a+bx)}+\frac{\log(a+bx)}{b^{10}}$$

input
```
Int[x^9/(a + b*x)^10,x]
```

output
```
a^9/(9*b^10*(a + b*x)^9) - (9*a^8)/(8*b^10*(a + b*x)^8) + (36*a^7)/(7*b^10
*(a + b*x)^7) - (14*a^6)/(b^10*(a + b*x)^6) + (126*a^5)/(5*b^10*(a + b*x)^
5) - (63*a^4)/(2*b^10*(a + b*x)^4) + (28*a^3)/(b^10*(a + b*x)^3) - (18*a^2
)/(b^10*(a + b*x)^2) + (9*a)/(b^10*(a + b*x)) + Log[a + b*x]/b^10
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

method	result
norman	$\frac{\frac{7129a^9}{2520b^{10}}+\frac{9a\,x^8}{b^2}+\frac{54a^2x^7}{b^3}+\frac{154a^3x^6}{b^4}+\frac{525a^4x^5}{2b^5}+\frac{2877a^5x^4}{10b^6}+\frac{1029a^6x^3}{5b^7}+\frac{3267a^7x^2}{35b^8}+\frac{6849a^8x}{280b^9}}{(bx+a)^9}+\frac{\ln(bx+a)}{b^{10}}$
risch	$\frac{\frac{7129a^9}{2520b^{10}}+\frac{9a\,x^8}{b^2}+\frac{54a^2x^7}{b^3}+\frac{154a^3x^6}{b^4}+\frac{525a^4x^5}{2b^5}+\frac{2877a^5x^4}{10b^6}+\frac{1029a^6x^3}{5b^7}+\frac{3267a^7x^2}{35b^8}+\frac{6849a^8x}{280b^9}}{(bx+a)^9}+\frac{\ln(bx+a)}{b^{10}}$
default	$\frac{a^9}{9b^{10}(bx+a)^9}-\frac{9a^8}{8b^{10}(bx+a)^8}+\frac{36a^7}{7b^{10}(bx+a)^7}-\frac{14a^6}{b^{10}(bx+a)^6}+\frac{126a^5}{5b^{10}(bx+a)^5}-\frac{63a^4}{2b^{10}(bx+a)^4}+\frac{28a^3}{b^{10}(bx+a)^3}-\frac{}{b^{10}(}$
parallelrisch	$\frac{661500a^4x^5b^5+7129a^9+2520\ln(bx+a)x^9b^9+388080x^6a^3b^6+22680a\,x^8b^8+2520\ln(bx+a)a^9+136080a^2x^7b^7+211680\ln(bx-}{}$

input
```
int(x^9/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(7129/2520*a^9/b^10+9*a/b^2*x^8+54*a^2/b^3*x^7+154*a^3/b^4*x^6+525/2*a^4/b
^5*x^5+2877/10*a^5/b^6*x^4+1029/5*a^6/b^7*x^3+3267/35*a^7/b^8*x^2+6849/280
*a^8/b^9*x)/(b*x+a)^9+ln(b*x+a)/b^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(144) = 288$.

Time = 0.07 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.90

$$\int \frac{x^9}{(a+bx)^{10}}\,dx$$
$$= \frac{22680\,ab^8x^8+136080\,a^2b^7x^7+388080\,a^3b^6x^6+661500\,a^4b^5x^5+725004\,a^5b^4x^4+518616\,a^6b^3x^3+2352}{2520\,(b^{19}x^9+9\,ab^{18}x^8+36\,a^2b^{17}x}$$

input
```
integrate(x^9/(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500
*a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^
2 + 61641*a^8*b*x + 7129*a^9 + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^
7 + 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 +
36*a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a))/(b^19*x^9 + 9*a*b^18*x^8 +
 36*a^2*b^17*x^7 + 84*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 +
 84*a^6*b^13*x^3 + 36*a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.38

$$\int \frac{x^9}{(a+bx)^{10}}\,dx = \frac{7129a^9 + 61641a^8bx + 235224a^7b^2x^2 + 518616a^6b^3x^3 + 725004a^5b^4x^4 + 661500a^4b^5x^5 + 3880}{2520a^9b^{10} + 22680a^8b^{11}x + 90720a^7b^{12}x^2 + 211680a^6b^{13}x^3 + 317520a^5b^{14}x^4 + 317520a^4b^{15}x^5 + 211680a} + \frac{\log(a+bx)}{b^{10}}$$

input
```
integrate(x**9/(b*x+a)**10,x)
```

output
```
(7129*a**9 + 61641*a**8*b*x + 235224*a**7*b**2*x**2 + 518616*a**6*b**3*x**
3 + 725004*a**5*b**4*x**4 + 661500*a**4*b**5*x**5 + 388080*a**3*b**6*x**6
+ 136080*a**2*b**7*x**7 + 22680*a*b**8*x**8)/(2520*a**9*b**10 + 22680*a**8
*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b*
*14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b*
*17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + log(a + b*x)/b**10
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.31

$$\int \frac{x^9}{(a+bx)^{10}}\,dx$$
$$= \frac{22680\,ab^8x^8 + 136080\,a^2b^7x^7 + 388080\,a^3b^6x^6 + 661500\,a^4b^5x^5 + 725004\,a^5b^4x^4 + 518616\,a^6b^3x^3 + 2352}{2520\,(b^{19}x^9 + 9\,ab^{18}x^8 + 36\,a^2b^{17}x^7 + 84\,a^3b^{16}x^6 + 126\,a^4b^{15}x^5 + 126\,a^5b^{14}x^4 + 84\,a^6b^{13}x^3 + 36\,a}$$
$$+ \frac{\log(bx+a)}{b^{10}}$$

input
```
integrate(x^9/(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500
*a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^
2 + 61641*a^8*b*x + 7129*a^9)/(b^19*x^9 + 9*a*b^18*x^8 + 36*a^2*b^17*x^7 +
 84*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^6*b^13*x^3 +
 36*a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10) + log(b*x + a)/b^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{(a+bx)^{10}}\,dx = \frac{\log(|bx+a|)}{b^{10}}$$
$$+ \frac{22680\,ab^7x^8 + 136080\,a^2b^6x^7 + 388080\,a^3b^5x^6 + 661500\,a^4b^4x^5 + 725004\,a^5b^3x^4 + 518616\,a^6b^2x^3 + 23}{2520\,(bx+a)^9b^9}$$

input
```
integrate(x^9/(b*x+a)^10,x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b^10 + 1/2520*(22680*a*b^7*x^8 + 136080*a^2*b^6*x^7 + 38
8080*a^3*b^5*x^6 + 661500*a^4*b^4*x^5 + 725004*a^5*b^3*x^4 + 518616*a^6*b^
2*x^3 + 235224*a^7*b*x^2 + 61641*a^8*x + 7129*a^9/b)/((b*x + a)^9*b^9)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \frac{x^9}{(a+bx)^{10}}\,dx$$
$$= \frac{\ln(a+b\,x)+\frac{9\,a}{a+b\,x}-\frac{18\,a^2}{(a+b\,x)^2}+\frac{28\,a^3}{(a+b\,x)^3}-\frac{63\,a^4}{2\,(a+b\,x)^4}+\frac{126\,a^5}{5\,(a+b\,x)^5}-\frac{14\,a^6}{(a+b\,x)^6}+\frac{36\,a^7}{7\,(a+b\,x)^7}-\frac{9\,a^8}{8\,(a+b\,x)^8}+\frac{a^9}{9\,(a+b\,x)^9}}{b^{10}}$$

input
```
int(x^9/(a + b*x)^10,x)
```

output
```
(log(a + b*x) + (9*a)/(a + b*x) - (18*a^2)/(a + b*x)^2 + (28*a^3)/(a + b*x
)^3 - (63*a^4)/(2*(a + b*x)^4) + (126*a^5)/(5*(a + b*x)^5) - (14*a^6)/(a +
 b*x)^6 + (36*a^7)/(7*(a + b*x)^7) - (9*a^8)/(8*(a + b*x)^8) + a^9/(9*(a +
 b*x)^9))/b^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.22

$$\int \frac{x^9}{(a+bx)^{10}}\,dx$$
$$= \frac{2520\log(bx+a)\,a^9+22680\log(bx+a)\,a^8bx+90720\log(bx+a)\,a^7b^2x^2+211680\log(bx+a)\,a^6b^3x^3+}{}$$

input
```
int(x^9/(b*x+a)^10,x)
```

output
```
(2520*log(a + b*x)*a**9 + 22680*log(a + b*x)*a**8*b*x + 90720*log(a + b*x)
*a**7*b**2*x**2 + 211680*log(a + b*x)*a**6*b**3*x**3 + 317520*log(a + b*x)
*a**5*b**4*x**4 + 317520*log(a + b*x)*a**4*b**5*x**5 + 211680*log(a + b*x)
*a**3*b**6*x**6 + 90720*log(a + b*x)*a**2*b**7*x**7 + 22680*log(a + b*x)*a
*b**8*x**8 + 2520*log(a + b*x)*b**9*x**9 + 4609*a**9 + 38961*a**8*b*x + 14
4504*a**7*b**2*x**2 + 306936*a**6*b**3*x**3 + 407484*a**5*b**4*x**4 + 3439
80*a**4*b**5*x**5 + 176400*a**3*b**6*x**6 + 45360*a**2*b**7*x**7 - 2520*b*
*9*x**9)/(2520*b**10*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3
*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a
**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.185 $\int \frac{x^8}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = \frac{x^9}{9a(a+bx)^9}$$

output
```
1/9*x^9/a/(b*x+a)^9
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.71

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = -\frac{a^8+9a^7bx+36a^6b^2x^2+84a^5b^3x^3+126a^4b^4x^4+126a^3b^5x^5+84a^2b^6x^6+36ab^7x^7+9b^8x^8}{9b^9(a+bx)^9}$$

input
```
Integrate[x^8/(a + b*x)^10,x]
```

output
```
-1/9*(a^8 + 9*a^7*b*x + 36*a^6*b^2*x^2 + 84*a^5*b^3*x^3 + 126*a^4*b^4*x^4
+ 126*a^3*b^5*x^5 + 84*a^2*b^6*x^6 + 36*a*b^7*x^7 + 9*b^8*x^8)/(b^9*(a + b
*x)^9)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx)^{10}}\,dx$$
$$\Big\downarrow \; 48$$
$$\frac{x^9}{9a(a+bx)^9}$$

input
```
Int[x^8/(a + b*x)^10,x]
```

output
```
x^9/(9*a*(a + b*x)^9)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(15) = 30$.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.65

method	result
gosper	$-\frac{9b^8x^8+36a\,x^7b^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5b^3x^3+36a^6x^2b^2+9a^7xb+a^8}{9(bx+a)^9 b^9}$
orering	$-\frac{9b^8x^8+36a\,x^7b^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5b^3x^3+36a^6x^2b^2+9a^7xb+a^8}{9(bx+a)^9 b^9}$
parallelrisch	$\frac{-9b^8x^8-36a\,x^7b^7-84a^2x^6b^6-126a^3x^5b^5-126a^4x^4b^4-84a^5b^3x^3-36a^6x^2b^2-9a^7xb-a^8}{9b^9(bx+a)^9}$
norman	$\frac{-\frac{x^8}{b}-\frac{4a\,x^7}{b^2}-\frac{28a^2x^6}{3b^3}-\frac{14a^3x^5}{b^4}-\frac{14a^4x^4}{b^5}-\frac{28a^5x^3}{3b^6}-\frac{4a^6x^2}{b^7}-\frac{a^7x}{b^8}-\frac{a^8}{9b^9}}{(bx+a)^9}$
risch	$\frac{-\frac{x^8}{b}-\frac{4a\,x^7}{b^2}-\frac{28a^2x^6}{3b^3}-\frac{14a^3x^5}{b^4}-\frac{14a^4x^4}{b^5}-\frac{28a^5x^3}{3b^6}-\frac{4a^6x^2}{b^7}-\frac{a^7x}{b^8}-\frac{a^8}{9b^9}}{(bx+a)^9}$
default	$-\frac{a^8}{9b^9(bx+a)^9}-\frac{14a^4}{b^9(bx+a)^5}+\frac{14a^3}{b^9(bx+a)^4}-\frac{4a^6}{b^9(bx+a)^7}+\frac{a^7}{b^9(bx+a)^8}+\frac{4a}{b^9(bx+a)^2}-\frac{1}{(bx+a)b^9}-\frac{28a^2}{3b^9(bx+a)^3}+$

input
```
int(x^8/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(9*b^8*x^8+36*a*b^7*x^7+84*a^2*b^6*x^6+126*a^3*b^5*x^5+126*a^4*b^4*x^
4+84*a^5*b^3*x^3+36*a^6*b^2*x^2+9*a^7*b*x+a^8)/(b*x+a)^9/b^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 10.94

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = -\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx}{9\,(b^{18}x^9+9\,ab^{17}x^8+36\,a^2b^{16}x^7+84\,a^3b^{15}x^6+126\,a^4b^{14}x^5+126\,a^5b^{13}x^4+84\,a^6b^{12}x^3+36\,a^7b^{11}x^2+}$$

input
```
integrate(x^8/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^
4*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 +
 9*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126
*a^5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(12) = 24$.

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 11.71

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = \frac{-a^8 - 9a^7bx - 36a^6b^2x^2 - 84a^5b^3x^3 - 126a^4b^4x^4 - 126a^3b^5x^5 - 84a^2b^6x^6 - 36ab^7x^7 - 9}{9a^9b^9 + 81a^8b^{10}x + 324a^7b^{11}x^2 + 756a^6b^{12}x^3 + 1134a^5b^{13}x^4 + 1134a^4b^{14}x^5 + 756a^3b^{15}x^6 + 324a^2b^{16}x^7 +}$$

input
```
integrate(x**8/(b*x+a)**10,x)
```

output
```
(-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**
4*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*
x**8)/(9*a**9*b**9 + 81*a**8*b**10*x + 324*a**7*b**11*x**2 + 756*a**6*b**1
2*x**3 + 1134*a**5*b**13*x**4 + 1134*a**4*b**14*x**5 + 756*a**3*b**15*x**6
 + 324*a**2*b**16*x**7 + 81*a*b**17*x**8 + 9*b**18*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(15) = 30$.

Time = 0.05 (sec) , antiderivative size = 186, normalized size of antiderivative = 10.94

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = -\frac{9\,b^8x^8 + 36\,ab^7x^7 + 84\,a^2b^6x^6 + 126\,a^3b^5x^5 + 126\,a^4b^4x^4 + 84\,a^5b^3x^3 + 36\,a^6b^2x^2 + 9\,a^7bx}{9\,(b^{18}x^9 + 9\,ab^{17}x^8 + 36\,a^2b^{16}x^7 + 84\,a^3b^{15}x^6 + 126\,a^4b^{14}x^5 + 126\,a^5b^{13}x^4 + 84\,a^6b^{12}x^3 + 36\,a^7b^{11}x^2 +}$$

input
```
integrate(x^8/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^
4*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 +
 9*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126
*a^5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(15) = 30.

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.59

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = -\frac{9\,b^8x^8 + 36\,ab^7x^7 + 84\,a^2b^6x^6 + 126\,a^3b^5x^5 + 126\,a^4b^4x^4 + 84\,a^5b^3x^3 + 36\,a^6b^2x^2 + 9\,a^7bx + a^8}{9\,(bx+a)^9b^9}$$

input
```
integrate(x^8/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^
4*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/((b*x + a)^
9*b^9)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 6.29

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = -\frac{\frac{1}{a+b\,x} - \frac{4\,a}{{(a+b\,x)}^2} + \frac{28\,a^2}{3\,{(a+b\,x)}^3} - \frac{14\,a^3}{{(a+b\,x)}^4} + \frac{14\,a^4}{{(a+b\,x)}^5} - \frac{28\,a^5}{3\,{(a+b\,x)}^6} + \frac{4\,a^6}{{(a+b\,x)}^7} - \frac{a^7}{{(a+b\,x)}^8} + \frac{a^8}{9\,{(a+b\,x)}^9}}{b^9}$$

input
```
int(x^8/(a + b*x)^10,x)
```

output
```
-(1/(a + b*x) - (4*a)/(a + b*x)^2 + (28*a^2)/(3*(a + b*x)^3) - (14*a^3)/(a
 + b*x)^4 + (14*a^4)/(a + b*x)^5 - (28*a^5)/(3*(a + b*x)^6) + (4*a^6)/(a +
 b*x)^7 - a^7/(a + b*x)^8 + a^8/(9*(a + b*x)^9))/b^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.06

$$\int \frac{x^8}{(a+bx)^{10}}\,dx$$
$$= \frac{x^9}{9a\left(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx+a\right.}$$

input
```
int(x^8/(b*x+a)^10,x)
```

output
```
x**9/(9*a*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126
*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x*
*7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.186 $\int \frac{x^7}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = \frac{x^8}{9a(a+bx)^9} + \frac{x^8}{72a^2(a+bx)^8}$$

output
```
1/9*x^8/a/(b*x+a)^9+1/72*x^8/a^2/(b*x+a)^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(35) = 70.

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.46

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = -\frac{a^7+9a^6bx+36a^5b^2x^2+84a^4b^3x^3+126a^3b^4x^4+126a^2b^5x^5+84ab^6x^6+36b^7x^7}{72b^8(a+bx)^9}$$

input
```
Integrate[x^7/(a + b*x)^10,x]
```

output
```
-1/72*(a^7 + 9*a^6*b*x + 36*a^5*b^2*x^2 + 84*a^4*b^3*x^3 + 126*a^3*b^4*x^4
 + 126*a^2*b^5*x^5 + 84*a*b^6*x^6 + 36*b^7*x^7)/(b^8*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx)^{10}}\,dx$$

$$\Big\downarrow \; 55$$

$$\frac{\int \frac{x^7}{(a+bx)^9}dx}{9a}+\frac{x^8}{9a(a+bx)^9}$$

$$\Big\downarrow \; 48$$

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

input
```
Int[x^7/(a + b*x)^10,x]
```

output
```
x^8/(9*a*(a + b*x)^9) + x^8/(72*a^2*(a + b*x)^8)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(31) = 62.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.43

method	result
gosper	$-\frac{36b^7x^7+84a\,b^6x^6+126a^2b^5x^5+126a^3b^4x^4+84a^4b^3x^3+36a^5b^2x^2+9a^6bx+a^7}{72(bx+a)^9b^8}$
orering	$-\frac{36b^7x^7+84a\,b^6x^6+126a^2b^5x^5+126a^3b^4x^4+84a^4b^3x^3+36a^5b^2x^2+9a^6bx+a^7}{72(bx+a)^9b^8}$
norman	$\frac{-\frac{x^7}{2b}-\frac{7a\,x^6}{6b^2}-\frac{7a^2x^5}{4b^3}-\frac{7a^3x^4}{4b^4}-\frac{7a^4x^3}{6b^5}-\frac{a^5x^2}{2b^6}-\frac{a^6x}{8b^7}-\frac{a^7}{72b^8}}{(bx+a)^9}$
risch	$\frac{-\frac{x^7}{2b}-\frac{7a\,x^6}{6b^2}-\frac{7a^2x^5}{4b^3}-\frac{7a^3x^4}{4b^4}-\frac{7a^4x^3}{6b^5}-\frac{a^5x^2}{2b^6}-\frac{a^6x}{8b^7}-\frac{a^7}{72b^8}}{(bx+a)^9}$
parallelrisch	$\frac{-36b^8x^7-84a\,b^7x^6-126a^2b^6x^5-126a^3b^5x^4-84a^4b^4x^3-36a^5b^3x^2-9a^6b^2x-a^7b}{72b^9(bx+a)^9}$
default	$\frac{a^7}{9b^8(bx+a)^9}+\frac{7a^3}{b^8(bx+a)^5}+\frac{3a^5}{b^8(bx+a)^7}-\frac{21a^2}{4b^8(bx+a)^4}-\frac{7a^6}{8b^8(bx+a)^8}-\frac{1}{2b^8(bx+a)^2}+\frac{7a}{3b^8(bx+a)^3}-\frac{35a^4}{6b^8(bx+a)}$

input
```
int(x^7/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/72*(36*b^7*x^7+84*a*b^6*x^6+126*a^2*b^5*x^5+126*a^3*b^4*x^4+84*a^4*b^3*
x^3+36*a^5*b^2*x^2+9*a^6*b*x+a^7)/(b*x+a)^9/b^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(31) = 62$.

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.00

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = -\frac{36\,b^7x^7 + 84\,ab^6x^6 + 126\,a^2b^5x^5 + 126\,a^3b^4x^4 + 84\,a^4b^3x^3 + 36\,a^5b^2x^2 + 9\,a^6bx + a^7}{72\,(b^{17}x^9 + 9\,ab^{16}x^8 + 36\,a^2b^{15}x^7 + 84\,a^3b^{14}x^6 + 126\,a^4b^{13}x^5 + 126\,a^5b^{12}x^4 + 84\,a^6b^{11}x^3 + 36\,a^7b^{10}x^2 -}$$

input
```
integrate(x^7/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*
a^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 +
 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 +
 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.

Time = 0.38 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.34

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = \frac{-a^7 - 9a^6bx - 36a^5b^2x^2 - 84a^4b^3x^3 - 126a^3b^4x^4 - 126a^2b^5x^5 - 84ab^6x^6 - 36b}{72a^9b^8 + 648a^8b^9x + 2592a^7b^{10}x^2 + 6048a^6b^{11}x^3 + 9072a^5b^{12}x^4 + 9072a^4b^{13}x^5 + 6048a^3b^{14}x^6 + 2592a^2b}$$

input
```
integrate(x**7/(b*x+a)**10,x)
```

output
```
(-a**7 - 9*a**6*b*x - 36*a**5*b**2*x**2 - 84*a**4*b**3*x**3 - 126*a**3*b**
4*x**4 - 126*a**2*b**5*x**5 - 84*a*b**6*x**6 - 36*b**7*x**7)/(72*a**9*b**8
 + 648*a**8*b**9*x + 2592*a**7*b**10*x**2 + 6048*a**6*b**11*x**3 + 9072*a*
*5*b**12*x**4 + 9072*a**4*b**13*x**5 + 6048*a**3*b**14*x**6 + 2592*a**2*b*
*15*x**7 + 648*a*b**16*x**8 + 72*b**17*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(31) = 62$.

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.00

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = -\frac{36\,b^7x^7 + 84\,ab^6x^6 + 126\,a^2b^5x^5 + 126\,a^3b^4x^4 + 84\,a^4b^3x^3 + 36\,a^5b^2x^2 + 9\,a^6bx + a^7}{72\,(b^{17}x^9 + 9\,ab^{16}x^8 + 36\,a^2b^{15}x^7 + 84\,a^3b^{14}x^6 + 126\,a^4b^{13}x^5 + 126\,a^5b^{12}x^4 + 84\,a^6b^{11}x^3 + 36\,a^7b^{10}x^2 -}$$

input
```
integrate(x^7/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*
a^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 +
 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 +
 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.40

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = -\frac{36\,b^7x^7 + 84\,ab^6x^6 + 126\,a^2b^5x^5 + 126\,a^3b^4x^4 + 84\,a^4b^3x^3 + 36\,a^5b^2x^2 + 9\,a^6bx + a^7}{72\,(bx+a)^9b^8}$$

input
```
integrate(x^7/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*
a^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/((b*x + a)^9*b^8)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = \frac{x^8\,(9\,a+b\,x)}{72\,a^2\,{(a+b\,x)}^9}$$

input
```
int(x^7/(a + b*x)^10,x)
```

output
```
(x^8*(9*a + b*x))/(72*a^2*(a + b*x)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.97

$$\int \frac{x^7}{(a+bx)^{10}}\,dx = \frac{-36b^7x^7-84a\,b^6x^6-126a^2b^5x^5-126a^3b^4x^4-84a^4b^3x^3-36a^5b^2x^2-9a^6bx-a^7}{72b^8\left(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx+\right.}$$

input
```
int(x^7/(b*x+a)^10,x)
```

output
```
( - a**7 - 9*a**6*b*x - 36*a**5*b**2*x**2 - 84*a**4*b**3*x**3 - 126*a**3*b
**4*x**4 - 126*a**2*b**5*x**5 - 84*a*b**6*x**6 - 36*b**7*x**7)/(72*b**8*(a
**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x
**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**
8*x**8 + b**9*x**9))
```

3.187 $\int \frac{x^6}{(a+bx)^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{x^6}{(a+bx)^{10}} \, dx = \frac{x^7}{9a(a+bx)^9} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{252a^3(a+bx)^7}$$

output
```
1/9*x^7/a/(b*x+a)^9+1/36*x^7/a^2/(b*x+a)^8+1/252*x^7/a^3/(b*x+a)^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

$$\int \frac{x^6}{(a+bx)^{10}} \, dx = -\frac{a^6 + 9a^5bx + 36a^4b^2x^2 + 84a^3b^3x^3 + 126a^2b^4x^4 + 126ab^5x^5 + 84b^6x^6}{252b^7(a+bx)^9}$$

input
```
Integrate[x^6/(a + b*x)^10,x]
```

output
```
-1/252*(a^6 + 9*a^5*b*x + 36*a^4*b^2*x^2 + 84*a^3*b^3*x^3 + 126*a^2*b^4*x^
4 + 126*a*b^5*x^5 + 84*b^6*x^6)/(b^7*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx)^{10}} \, dx$$

$$\Big\downarrow \; 55$$

$$\frac{2\int \frac{x^6}{(a+bx)^9}dx}{9a}+\frac{x^7}{9a(a+bx)^9}$$

$$\Big\downarrow \; 55$$

$$\frac{2\left(\frac{\int \frac{x^6}{(a+bx)^8}dx}{8a}+\frac{x^7}{8a(a+bx)^8}\right)}{9a}+\frac{x^7}{9a(a+bx)^9}$$

$$\Big\downarrow \; 48$$

$$\frac{2\left(\frac{x^7}{56a^2(a+bx)^7}+\frac{x^7}{8a(a+bx)^8}\right)}{9a}+\frac{x^7}{9a(a+bx)^9}$$

input
```
Int[x^6/(a + b*x)^10,x]
```

output
```
x^7/(9*a*(a + b*x)^9) + (2*(x^7/(8*a*(a + b*x)^8) + x^7/(56*a^2*(a + b*x)^
7)))/(9*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

method	result	size
gosper	$-\frac{84b^6x^6+126a\,x^5b^5+126a^2x^4b^4+84a^3x^3b^3+36a^4x^2b^2+9a^5xb+a^6}{252(bx+a)^9b^7}$	74
orering	$-\frac{84b^6x^6+126a\,x^5b^5+126a^2x^4b^4+84a^3x^3b^3+36a^4x^2b^2+9a^5xb+a^6}{252(bx+a)^9b^7}$	74
norman	$\frac{-\frac{x^6}{3b}-\frac{a\,x^5}{2b^2}-\frac{a^2x^4}{2b^3}-\frac{a^3x^3}{3b^4}-\frac{a^4x^2}{7b^5}-\frac{a^5x}{28b^6}-\frac{a^6}{252b^7}}{(bx+a)^9}$	77
risch	$\frac{-\frac{x^6}{3b}-\frac{a\,x^5}{2b^2}-\frac{a^2x^4}{2b^3}-\frac{a^3x^3}{3b^4}-\frac{a^4x^2}{7b^5}-\frac{a^5x}{28b^6}-\frac{a^6}{252b^7}}{(bx+a)^9}$	77
parallelrisch	$\frac{-84b^8x^6-126a\,b^7x^5-126a^2b^6x^4-84a^3b^5x^3-36a^4b^4x^2-9a^5b^3x-b^2a^6}{252b^9(bx+a)^9}$	81
default	$-\frac{a^6}{9b^7(bx+a)^9}-\frac{3a^2}{b^7(bx+a)^5}+\frac{3a}{2b^7(bx+a)^4}-\frac{15a^4}{7b^7(bx+a)^7}+\frac{3a^5}{4b^7(bx+a)^8}-\frac{1}{3b^7(bx+a)^3}+\frac{10a^3}{3b^7(bx+a)^6}$	102

input
```
int(x^6/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/252*(84*b^6*x^6+126*a*b^5*x^5+126*a^2*b^4*x^4+84*a^3*b^3*x^3+36*a^4*b^2
*x^2+9*a^5*b*x+a^6)/(b*x+a)^9/b^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(46) = 92$.

Time = 0.06 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.15

$$\int \frac{x^6}{(a+bx)^{10}}\,dx = -\frac{84\,b^6x^6 + 126\,ab^5x^5 + 126\,a^2b^4x^4 + 84\,a^3b^3x^3 + 36\,a^4b^2x^2 + 9\,a^5bx + a^6}{252\,(b^{16}x^9 + 9\,ab^{15}x^8 + 36\,a^2b^{14}x^7 + 84\,a^3b^{13}x^6 + 126\,a^4b^{12}x^5 + 126\,a^5b^{11}x^4 + 84\,a^6b^{10}x^3 + 36\,a^7b^9x^2 }$$

input
```
integrate(x^6/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36
*a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7
 + 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3
 + 36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(42) = 84$.

Time = 0.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.37

$$\int \frac{x^6}{(a+bx)^{10}}\,dx = \frac{-a^6 - 9a^5bx - 36a^4b^2x^2 - 84a^3b^3x^3 - 126a^2b^4x^4 - 126ab^5x^5 - 84b^6}{252a^9b^7 + 2268a^8b^8x + 9072a^7b^9x^2 + 21168a^6b^{10}x^3 + 31752a^5b^{11}x^4 + 31752a^4b^{12}x^5 + 21168a^3b^{13}x^6 + 9}$$

input
```
integrate(x**6/(b*x+a)**10,x)
```

output
```
(-a**6 - 9*a**5*b*x - 36*a**4*b**2*x**2 - 84*a**3*b**3*x**3 - 126*a**2*b**
4*x**4 - 126*a*b**5*x**5 - 84*b**6*x**6)/(252*a**9*b**7 + 2268*a**8*b**8*x
 + 9072*a**7*b**9*x**2 + 21168*a**6*b**10*x**3 + 31752*a**5*b**11*x**4 + 3
1752*a**4*b**12*x**5 + 21168*a**3*b**13*x**6 + 9072*a**2*b**14*x**7 + 2268
*a*b**15*x**8 + 252*b**16*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(46) = 92$.

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.15

$$\int \frac{x^6}{(a+bx)^{10}}\,dx = -\frac{84\,b^6x^6 + 126\,ab^5x^5 + 126\,a^2b^4x^4 + 84\,a^3b^3x^3 + 36\,a^4b^2x^2 + 9\,a^5bx + a^6}{252\,(b^{16}x^9 + 9\,ab^{15}x^8 + 36\,a^2b^{14}x^7 + 84\,a^3b^{13}x^6 + 126\,a^4b^{12}x^5 + 126\,a^5b^{11}x^4 + 84\,a^6b^{10}x^3 + 36\,a^7b^9x^2 +}$$

input
```
integrate(x^6/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36
*a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7
 + 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3
 + 36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{x^6}{(a+bx)^{10}}\,dx = -\frac{84\,b^6x^6 + 126\,ab^5x^5 + 126\,a^2b^4x^4 + 84\,a^3b^3x^3 + 36\,a^4b^2x^2 + 9\,a^5bx + a^6}{252\,(bx+a)^9b^7}$$

input
```
integrate(x^6/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36
*a^4*b^2*x^2 + 9*a^5*b*x + a^6)/((b*x + a)^9*b^7)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{x^6}{(a+bx)^{10}}\,dx = -\frac{\frac{1}{3\,(a+b\,x)^3}-\frac{3\,a}{2\,(a+b\,x)^4}+\frac{3\,a^2}{(a+b\,x)^5}-\frac{10\,a^3}{3\,(a+b\,x)^6}+\frac{15\,a^4}{7\,(a+b\,x)^7}-\frac{3\,a^5}{4\,(a+b\,x)^8}+\frac{a^6}{9\,(a+b\,x)^9}}{b^7}$$

input
```
int(x^6/(a + b*x)^10,x)
```

output
```
-(1/(3*(a + b*x)^3) - (3*a)/(2*(a + b*x)^4) + (3*a^2)/(a + b*x)^5 - (10*a^
3)/(3*(a + b*x)^6) + (15*a^4)/(7*(a + b*x)^7) - (3*a^5)/(4*(a + b*x)^8) +
a^6/(9*(a + b*x)^9))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.13

$$\int \frac{x^6}{(a+bx)^{10}}\,dx = \frac{-84b^6x^6-126a\,b^5x^5-126a^2b^4x^4-84a^3b^3x^3-36a^4b^2x^2-9a^5bx-a^6}{252b^7\left(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx\right.}$$

input
```
int(x^6/(b*x+a)^10,x)
```

output
```
( - a**6 - 9*a**5*b*x - 36*a**4*b**2*x**2 - 84*a**3*b**3*x**3 - 126*a**2*b
**4*x**4 - 126*a*b**5*x**5 - 84*b**6*x**6)/(252*b**7*(a**9 + 9*a**8*b*x +
36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5
*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9)
)
```

3.188 $\int \frac{x^5}{(a+bx)^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 69

$$\int \frac{x^5}{(a+bx)^{10}} \, dx = \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{x^6}{84a^3(a+bx)^7} + \frac{x^6}{504a^4(a+bx)^6}$$

output
```
1/9*x^6/a/(b*x+a)^9+1/24*x^6/a^2/(b*x+a)^8+1/84*x^6/a^3/(b*x+a)^7+1/504*x^
6/a^4/(b*x+a)^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(a+bx)^{10}} \, dx = -\frac{a^5 + 9a^4bx + 36a^3b^2x^2 + 84a^2b^3x^3 + 126ab^4x^4 + 126b^5x^5}{504b^6(a+bx)^9}$$

input
```
Integrate[x^5/(a + b*x)^10,x]
```

output
```
-1/504*(a^5 + 9*a^4*b*x + 36*a^3*b^2*x^2 + 84*a^2*b^3*x^3 + 126*a*b^4*x^4
+ 126*b^5*x^5)/(b^6*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx)^{10}} \, dx$$

$$\Big\downarrow \text{55}$$

$$\frac{\int \frac{x^5}{(a+bx)^9}dx}{3a}+\frac{x^6}{9a(a+bx)^9}$$

$$\Big\downarrow \text{55}$$

$$\frac{\frac{\int \frac{x^5}{(a+bx)^8}dx}{4a}+\frac{x^6}{8a(a+bx)^8}}{3a}+\frac{x^6}{9a(a+bx)^9}$$

$$\Big\downarrow \text{55}$$

$$\frac{\frac{\frac{\int \frac{x^5}{(a+bx)^7}dx}{7a}+\frac{x^6}{7a(a+bx)^7}}{4a}+\frac{x^6}{8a(a+bx)^8}}{3a}+\frac{x^6}{9a(a+bx)^9}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{\frac{x^6}{42a^2(a+bx)^6}+\frac{x^6}{7a(a+bx)^7}}{4a}+\frac{x^6}{8a(a+bx)^8}}{3a}+\frac{x^6}{9a(a+bx)^9}$$

input
```
Int[x^5/(a + b*x)^10,x]
```

output
```
x^6/(9*a*(a + b*x)^9) + (x^6/(8*a*(a + b*x)^8) + (x^6/(7*a*(a + b*x)^7) +
x^6/(42*a^2*(a + b*x)^6))/(4*a))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{126 b^{5} x^{5}+126 a\,b^{4} x^{4}+84 a^{2} b^{3} x^{3}+36 a^{3} b^{2} x^{2}+9 a^{4} b x +a^{5}}{504 \left(b x +a \right)^{9} b^{6}}$	63
orering	$-\frac{126 b^{5} x^{5}+126 a\,b^{4} x^{4}+84 a^{2} b^{3} x^{3}+36 a^{3} b^{2} x^{2}+9 a^{4} b x +a^{5}}{504 \left(b x +a \right)^{9} b^{6}}$	63
norman	$\frac{-\frac{x^{5}}{4 b}-\frac{a\,x^{4}}{4 b^{2}}-\frac{a^{2} x^{3}}{6 b^{3}}-\frac{a^{3} x^{2}}{14 b^{4}}-\frac{a^{4} x}{56 b^{5}}-\frac{a^{5}}{504 b^{6}}}{\left(b x +a \right)^{9}}$	66
risch	$\frac{-\frac{x^{5}}{4 b}-\frac{a\,x^{4}}{4 b^{2}}-\frac{a^{2} x^{3}}{6 b^{3}}-\frac{a^{3} x^{2}}{14 b^{4}}-\frac{a^{4} x}{56 b^{5}}-\frac{a^{5}}{504 b^{6}}}{\left(b x +a \right)^{9}}$	66
parallelrisch	$\frac{-126 b^{8} x^{5}-126 a\,b^{7} x^{4}-84 a^{2} b^{6} x^{3}-36 a^{3} b^{5} x^{2}-9 a^{4} b^{4} x -a^{5} b^{3}}{504 b^{9} \left(b x +a \right)^{9}}$	70
default	$\frac{a^{5}}{9 b^{6} \left(b x +a \right)^{9}}+\frac{a}{b^{6} \left(b x +a \right)^{5}}-\frac{1}{4 b^{6} \left(b x +a \right)^{4}}+\frac{10 a^{3}}{7 b^{6} \left(b x +a \right)^{7}}-\frac{5 a^{4}}{8 b^{6} \left(b x +a \right)^{8}}-\frac{5 a^{2}}{3 b^{6} \left(b x +a \right)^{6}}$	86

input
```
int(x^5/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/504*(126*b^5*x^5+126*a*b^4*x^4+84*a^2*b^3*x^3+36*a^3*b^2*x^2+9*a^4*b*x+
a^5)/(b*x+a)^9/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.22

$$\int \frac{x^5}{(a+bx)^{10}}\,dx =$$

$$-\frac{126\,b^5x^5 + 126\,ab^4x^4 + 84\,a^2b^3x^3 + 36\,a^3b^2x^2 + 9\,a^4bx + a^5}{504\,(b^{15}x^9 + 9\,ab^{14}x^8 + 36\,a^2b^{13}x^7 + 84\,a^3b^{12}x^6 + 126\,a^4b^{11}x^5 + 126\,a^5b^{10}x^4 + 84\,a^6b^9x^3 + 36\,a^7b^8x^2 +}$$

input
```
integrate(x^5/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*
a^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^
6 + 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2
+ 9*a^8*b^7*x + a^9*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.36

$$\int \frac{x^5}{(a+bx)^{10}}\,dx$$

$$= \frac{-a^5 - 9a^4bx - 36a^3b^2x^2 - 84a^2b^3x^3 - 126ab^4x^4 - 126b^5x^5}{504a^9b^6 + 4536a^8b^7x + 18144a^7b^8x^2 + 42336a^6b^9x^3 + 63504a^5b^{10}x^4 + 63504a^4b^{11}x^5 + 42336a^3b^{12}x^6 + 1}$$

input
```
integrate(x**5/(b*x+a)**10,x)
```

output
```
(-a**5 - 9*a**4*b*x - 36*a**3*b**2*x**2 - 84*a**2*b**3*x**3 - 126*a*b**4*x
**4 - 126*b**5*x**5)/(504*a**9*b**6 + 4536*a**8*b**7*x + 18144*a**7*b**8*x
**2 + 42336*a**6*b**9*x**3 + 63504*a**5*b**10*x**4 + 63504*a**4*b**11*x**5
 + 42336*a**3*b**12*x**6 + 18144*a**2*b**13*x**7 + 4536*a*b**14*x**8 + 504
*b**15*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(61) = 122$.

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.22

$$\int \frac{x^5}{(a+bx)^{10}}\,dx =$$
$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(b^{15}x^9+9\,ab^{14}x^8+36\,a^2b^{13}x^7+84\,a^3b^{12}x^6+126\,a^4b^{11}x^5+126\,a^5b^{10}x^4+84\,a^6b^9x^3+36\,a^7b^8x^2+}$$

input
```
integrate(x^5/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*
a^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^
6 + 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2
+ 9*a^8*b^7*x + a^9*b^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{(a+bx)^{10}}\,dx = -\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(bx+a)^9b^6}$$

input
```
integrate(x^5/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*
a^4*b*x + a^5)/((b*x + a)^9*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{(a+bx)^{10}}\,dx = \frac{\frac{a}{(a+b\,x)^5}-\frac{1}{4\,(a+b\,x)^4}-\frac{5\,a^2}{3\,(a+b\,x)^6}+\frac{10\,a^3}{7\,(a+b\,x)^7}-\frac{5\,a^4}{8\,(a+b\,x)^8}+\frac{a^5}{9\,(a+b\,x)^9}}{b^6}$$

input
```
int(x^5/(a + b*x)^10,x)
```

output
```
(a/(a + b*x)^5 - 1/(4*(a + b*x)^4) - (5*a^2)/(3*(a + b*x)^6) + (10*a^3)/(7
*(a + b*x)^7) - (5*a^4)/(8*(a + b*x)^8) + a^5/(9*(a + b*x)^9))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.20

$$\int \frac{x^5}{(a+bx)^{10}}\,dx$$
$$= \frac{-126b^5x^5-126a\,b^4x^4-84a^2b^3x^3-36a^3b^2x^2-9a^4bx-a^5}{504b^6\left(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx\right.}$$

input
```
int(x^5/(b*x+a)^10,x)
```

output
```
( - a**5 - 9*a**4*b*x - 36*a**3*b**2*x**2 - 84*a**2*b**3*x**3 - 126*a*b**4
*x**4 - 126*b**5*x**5)/(504*b**6*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 +
84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6
*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.189 $\int \frac{x^4}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 81

$$\int \frac{x^4}{(a+bx)^{10}}\,dx = -\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

output
```
-1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b
^5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{(a+bx)^{10}}\,dx = -\frac{a^4+9a^3bx+36a^2b^2x^2+84ab^3x^3+126b^4x^4}{630b^5(a+bx)^9}$$

input
```
Integrate[x^4/(a + b*x)^10,x]
```

output
```
-1/630*(a^4 + 9*a^3*b*x + 36*a^2*b^2*x^2 + 84*a*b^3*x^3 + 126*b^4*x^4)/(b^
5*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx)^{10}} \, dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

input
```
Int[x^4/(a + b*x)^10,x]
```

output
```
-1/9*a^4/(b^5*(a + b*x)^9) + a^3/(2*b^5*(a + b*x)^8) - (6*a^2)/(7*b^5*(a +
 b*x)^7) + (2*a)/(3*b^5*(a + b*x)^6) - 1/(5*b^5*(a + b*x)^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{126b^4x^4+84a\,x^3b^3+36a^2b^2x^2+9a^3bx+a^4}{630(bx+a)^9b^5}$	52
orering	$-\frac{126b^4x^4+84a\,x^3b^3+36a^2b^2x^2+9a^3bx+a^4}{630(bx+a)^9b^5}$	52
norman	$\frac{-\frac{x^4}{5b}-\frac{2a\,x^3}{15b^2}-\frac{2a^2x^2}{35b^3}-\frac{a^3x}{70b^4}-\frac{a^4}{630b^5}}{(bx+a)^9}$	55
risch	$\frac{-\frac{x^4}{5b}-\frac{2a\,x^3}{15b^2}-\frac{2a^2x^2}{35b^3}-\frac{a^3x}{70b^4}-\frac{a^4}{630b^5}}{(bx+a)^9}$	55
parallelrisch	$\frac{-126b^8x^4-84a\,b^7x^3-36a^2b^6x^2-9a^3b^5x-a^4b^4}{630b^9(bx+a)^9}$	59
default	$-\frac{a^4}{9b^5(bx+a)^9}+\frac{a^3}{2b^5(bx+a)^8}-\frac{6a^2}{7b^5(bx+a)^7}+\frac{2a}{3b^5(bx+a)^6}-\frac{1}{5b^5(bx+a)^5}$	72

input
```
int(x^4/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/630*(126*b^4*x^4+84*a*b^3*x^3+36*a^2*b^2*x^2+9*a^3*b*x+a^4)/(b*x+a)^9/b
^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.75

$$\int \frac{x^4}{(a+bx)^{10}}\,dx = -\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,(b^{14}x^9+9\,ab^{13}x^8+36\,a^2b^{12}x^7+84\,a^3b^{11}x^6+126\,a^4b^{10}x^5+126\,a^5b^9x^4+84\,a^6b^8x^3+36\,a^7b^7x^2+}$$

input
```
integrate(x^4/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^
14*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x
^5 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9
*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.86

$$\int \frac{x^4}{(a+bx)^{10}}\,dx$$
$$= \frac{-a^4 - 9a^3bx - 36a^2b^2x^2 - 84ab^3x^3 - 126b^4x^4}{630a^9b^5 + 5670a^8b^6x + 22680a^7b^7x^2 + 52920a^6b^8x^3 + 79380a^5b^9x^4 + 79380a^4b^{10}x^5 + 52920a^3b^{11}x^6 + 22}$$

input
```
integrate(x**4/(b*x+a)**10,x)
```

output
```
(-a**4 - 9*a**3*b*x - 36*a**2*b**2*x**2 - 84*a*b**3*x**3 - 126*b**4*x**4)/
(630*a**9*b**5 + 5670*a**8*b**6*x + 22680*a**7*b**7*x**2 + 52920*a**6*b**8
*x**3 + 79380*a**5*b**9*x**4 + 79380*a**4*b**10*x**5 + 52920*a**3*b**11*x*
*6 + 22680*a**2*b**12*x**7 + 5670*a*b**13*x**8 + 630*b**14*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.75

$$\int \frac{x^4}{(a+bx)^{10}}\,dx =$$
$$-\frac{126\,b^4x^4 + 84\,ab^3x^3 + 36\,a^2b^2x^2 + 9\,a^3bx + a^4}{630\,(b^{14}x^9 + 9\,ab^{13}x^8 + 36\,a^2b^{12}x^7 + 84\,a^3b^{11}x^6 + 126\,a^4b^{10}x^5 + 126\,a^5b^9x^4 + 84\,a^6b^8x^3 + 36\,a^7b^7x^2 +}$$

input
```
integrate(x^4/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^
14*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x
^5 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9
*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{(a+bx)^{10}}\,dx = -\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,(bx+a)^9b^5}$$

input
```
integrate(x^4/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/((b
*x + a)^9*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{(a+bx)^{10}}\,dx = -\frac{\frac{1}{5\,(a+b\,x)^5}-\frac{2\,a}{3\,(a+b\,x)^6}+\frac{6\,a^2}{7\,(a+b\,x)^7}-\frac{a^3}{2\,(a+b\,x)^8}+\frac{a^4}{9\,(a+b\,x)^9}}{b^5}$$

input
```
int(x^4/(a + b*x)^10,x)
```

output
```
-(1/(5*(a + b*x)^5) - (2*a)/(3*(a + b*x)^6) + (6*a^2)/(7*(a + b*x)^7) - a^
3/(2*(a + b*x)^8) + a^4/(9*(a + b*x)^9))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{x^4}{(a+bx)^{10}}\,dx = \frac{-126b^4x^4-84a\,b^3x^3-36a^2b^2x^2-9a^3bx-a^4}{630b^5\,(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx}$$

input
```
int(x^4/(b*x+a)^10,x)
```

output
```
( - a**4 - 9*a**3*b*x - 36*a**2*b**2*x**2 - 84*a*b**3*x**3 - 126*b**4*x**4
)/(630*b**5*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 1
26*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*
x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.190 $\int \frac{x^3}{(a+bx)^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{x^3}{(a+bx)^{10}} dx = \frac{a^3}{9b^4(a+bx)^9} - \frac{3a^2}{8b^4(a+bx)^8} + \frac{3a}{7b^4(a+bx)^7} - \frac{1}{6b^4(a+bx)^6}$$

output
```
1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b
*x+a)^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{(a+bx)^{10}} dx = -\frac{a^3 + 9a^2bx + 36ab^2x^2 + 84b^3x^3}{504b^4(a+bx)^9}$$

input
```
Integrate[x^3/(a + b*x)^10,x]
```

output
```
-1/504*(a^3 + 9*a^2*b*x + 36*a*b^2*x^2 + 84*b^3*x^3)/(b^4*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^{10}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}$$

input
```
Int[x^3/(a + b*x)^10,x]
```

output
```
a^3/(9*b^4*(a + b*x)^9) - (3*a^2)/(8*b^4*(a + b*x)^8) + (3*a)/(7*b^4*(a +
b*x)^7) - 1/(6*b^4*(a + b*x)^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{84b^3x^3+36a\,b^2x^2+9a^2bx+a^3}{504(bx+a)^9 b^4}$	41
orering	$-\frac{84b^3x^3+36a\,b^2x^2+9a^2bx+a^3}{504(bx+a)^9 b^4}$	41
norman	$\frac{-\frac{x^3}{6b}-\frac{a\,x^2}{14b^2}-\frac{a^2x}{56b^3}-\frac{a^3}{504b^4}}{(bx+a)^9}$	44
risch	$\frac{-\frac{x^3}{6b}-\frac{a\,x^2}{14b^2}-\frac{a^2x}{56b^3}-\frac{a^3}{504b^4}}{(bx+a)^9}$	44
parallelrisch	$\frac{-84b^8x^3-36a\,b^7x^2-9a^2b^6x-a^3b^5}{504b^9(bx+a)^9}$	48
default	$\frac{a^3}{9b^4(bx+a)^9}-\frac{3a^2}{8b^4(bx+a)^8}+\frac{3a}{7b^4(bx+a)^7}-\frac{1}{6b^4(bx+a)^6}$	57

input
```
int(x^3/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/504*(84*b^3*x^3+36*a*b^2*x^2+9*a^2*b*x+a^3)/(b*x+a)^9/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(56) = 112.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.05

$$\int \frac{x^3}{(a+bx)^{10}}\,dx =$$

$$-\frac{84\,b^3x^3+36\,ab^2x^2+9\,a^2bx+a^3}{504\,(b^{13}x^9+9\,ab^{12}x^8+36\,a^2b^{11}x^7+84\,a^3b^{10}x^6+126\,a^4b^9x^5+126\,a^5b^8x^4+84\,a^6b^7x^3+36\,a^7b^6x^2+9}$$

input
```
integrate(x^3/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*
x^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^
4 + 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.17

$$\int \frac{x^3}{(a+bx)^{10}}\,dx$$
$$= \frac{-a^3 - 9a^2bx - 36ab^2x^2 - 84b^3x^3}{504a^9b^4 + 4536a^8b^5x + 18144a^7b^6x^2 + 42336a^6b^7x^3 + 63504a^5b^8x^4 + 63504a^4b^9x^5 + 42336a^3b^{10}x^6 + 18\cdots}$$

input
```
integrate(x**3/(b*x+a)**10,x)
```

output
```
(-a**3 - 9*a**2*b*x - 36*a*b**2*x**2 - 84*b**3*x**3)/(504*a**9*b**4 + 4536
*a**8*b**5*x + 18144*a**7*b**6*x**2 + 42336*a**6*b**7*x**3 + 63504*a**5*b*
*8*x**4 + 63504*a**4*b**9*x**5 + 42336*a**3*b**10*x**6 + 18144*a**2*b**11*
x**7 + 4536*a*b**12*x**8 + 504*b**13*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(56) = 112$.

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.05

$$\int \frac{x^3}{(a+bx)^{10}}\,dx =$$
$$-\frac{84\,b^3x^3 + 36\,ab^2x^2 + 9\,a^2bx + a^3}{504\,(b^{13}x^9 + 9\,ab^{12}x^8 + 36\,a^2b^{11}x^7 + 84\,a^3b^{10}x^6 + 126\,a^4b^9x^5 + 126\,a^5b^8x^4 + 84\,a^6b^7x^3 + 36\,a^7b^6x^2 + 9\cdots}$$

input
```
integrate(x^3/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*
x^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^
4 + 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{(a+bx)^{10}}\,dx = -\frac{84\,b^3x^3+36\,ab^2x^2+9\,a^2bx+a^3}{504\,(bx+a)^9b^4}$$

input
```
integrate(x^3/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/((b*x + a)^9*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{(a+bx)^{10}}\,dx = \frac{\frac{3\,a}{7\,(a+b\,x)^7}-\frac{1}{6\,(a+b\,x)^6}-\frac{3\,a^2}{8\,(a+b\,x)^8}+\frac{a^3}{9\,(a+b\,x)^9}}{b^4}$$

input
```
int(x^3/(a + b*x)^10,x)
```

output
```
((3*a)/(7*(a + b*x)^7) - 1/(6*(a + b*x)^6) - (3*a^2)/(8*(a + b*x)^8) + a^3
/(9*(a + b*x)^9))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.03

$$\int \frac{x^3}{(a+bx)^{10}}\,dx$$
$$= \frac{-84b^3x^3-36a\,b^2x^2-9a^2bx-a^3}{504b^4\,(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx -}$$

input
```
int(x^3/(b*x+a)^10,x)
```

output
```
( - a**3 - 9*a**2*b*x - 36*a*b**2*x**2 - 84*b**3*x**3)/(504*b**4*(a**9 + 9
*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 1
26*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8
+ b**9*x**9))
```

3.191 $\int \frac{x^2}{(a+bx)^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int \frac{x^2}{(a+bx)^{10}} \, dx = -\frac{a^2}{9b^3(a+bx)^9} + \frac{a}{4b^3(a+bx)^8} - \frac{1}{7b^3(a+bx)^7}$$

output
```
-1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(a+bx)^{10}} \, dx = -\frac{a^2+9abx+36b^2x^2}{252b^3(a+bx)^9}$$

input
```
Integrate[x^2/(a + b*x)^10,x]
```

output
```
-1/252*(a^2 + 9*a*b*x + 36*b^2*x^2)/(b^3*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^{10}} \, dx$$
$$\Big\downarrow \text{ 53}$$
$$\int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

input
```
Int[x^2/(a + b*x)^10,x]
```

output
```
-1/9*a^2/(b^3*(a + b*x)^9) + a/(4*b^3*(a + b*x)^8) - 1/(7*b^3*(a + b*x)^7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{36b^2x^2+9abx+a^2}{252(bx+a)^9 b^3}$	30
orering	$-\frac{36b^2x^2+9abx+a^2}{252(bx+a)^9 b^3}$	30
norman	$\frac{-\frac{x^2}{7b}-\frac{ax}{28b^2}-\frac{a^2}{252b^3}}{(bx+a)^9}$	33
risch	$\frac{-\frac{x^2}{7b}-\frac{ax}{28b^2}-\frac{a^2}{252b^3}}{(bx+a)^9}$	33
parallelrisch	$\frac{-36b^8x^2-9a\,b^7x-a^2b^6}{252b^9(bx+a)^9}$	37
default	$-\frac{a^2}{9b^3(bx+a)^9}+\frac{a}{4b^3(bx+a)^8}-\frac{1}{7b^3(bx+a)^7}$	42

input
```
int(x^2/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/252*(36*b^2*x^2+9*a*b*x+a^2)/(b*x+a)^9/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.55

$$\int \frac{x^2}{(a+bx)^{10}}\,dx =$$

$$-\frac{36\,b^2x^2+9\,abx+a^2}{252\,(b^{12}x^9+9\,ab^{11}x^8+36\,a^2b^{10}x^7+84\,a^3b^9x^6+126\,a^4b^8x^5+126\,a^5b^7x^4+84\,a^6b^6x^3+36\,a^7b^5x^2+9}$$

input
```
integrate(x^2/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10
*x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3
 + 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int \frac{x^2}{(a+bx)^{10}}\,dx = \frac{-a^2 - 9abx - 36b^2x^2}{252a^9b^3 + 2268a^8b^4x + 9072a^7b^5x^2 + 21168a^6b^6x^3 + 31752a^5b^7x^4 + 31752a^4b^8x^5 + 21168a^3b^9x^6 + 9072}$$

input
```
integrate(x**2/(b*x+a)**10,x)
```

output
```
(-a**2 - 9*a*b*x - 36*b**2*x**2)/(252*a**9*b**3 + 2268*a**8*b**4*x + 9072*
a**7*b**5*x**2 + 21168*a**6*b**6*x**3 + 31752*a**5*b**7*x**4 + 31752*a**4*
b**8*x**5 + 21168*a**3*b**9*x**6 + 9072*a**2*b**10*x**7 + 2268*a*b**11*x**
8 + 252*b**12*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(41) = 82$.

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.55

$$\int \frac{x^2}{(a+bx)^{10}}\,dx = -\frac{36\,b^2x^2 + 9\,abx + a^2}{252\,(b^{12}x^9 + 9\,ab^{11}x^8 + 36\,a^2b^{10}x^7 + 84\,a^3b^9x^6 + 126\,a^4b^8x^5 + 126\,a^5b^7x^4 + 84\,a^6b^6x^3 + 36\,a^7b^5x^2 + 9}$$

input
```
integrate(x^2/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10
*x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3
 + 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(a+bx)^{10}}\,dx = -\frac{36\,b^2x^2+9\,abx+a^2}{252\,(bx+a)^9b^3}$$

input
```
integrate(x^2/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/((b*x + a)^9*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(a+bx)^{10}}\,dx = -\frac{8\,a^2+72\,a\,b\,x+288\,b^2\,x^2}{2016\,b^3\,(a+b\,x)^9}$$

input
```
int(x^2/(a + b*x)^10,x)
```

output
```
-(8*a^2 + 288*b^2*x^2 + 72*a*b*x)/(2016*b^3*(a + b*x)^9)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.53

$$\int \frac{x^2}{(a+bx)^{10}}\,dx$$
$$= \frac{-36b^2x^2-9abx-a^2}{252b^3\,(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx}$$

input
```
int(x^2/(b*x+a)^10,x)
```

output

```
( - a**2 - 9*a*b*x - 36*b**2*x**2)/(252*b**3*(a**9 + 9*a**8*b*x + 36*a**7*
b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x**5 +
84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.192 $\int \frac{x}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \frac{x}{(a+bx)^{10}}\,dx = \frac{a}{9b^2(a+bx)^9} - \frac{1}{8b^2(a+bx)^8}$$

output
```
1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{(a+bx)^{10}}\,dx = -\frac{a+9bx}{72b^2(a+bx)^9}$$

input
```
Integrate[x/(a + b*x)^10,x]
```

output
```
-1/72*(a + 9*b*x)/(b^2*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^{10}}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{1}{b(a+bx)^9}-\frac{a}{b(a+bx)^{10}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}$$

input
```
Int[x/(a + b*x)^10,x]
```

output
```
a/(9*b^2*(a + b*x)^9) - 1/(8*b^2*(a + b*x)^8)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{9bx+a}{72(bx+a)^{9} b^{2}}$	19
orering	$-\frac{9bx+a}{72(bx+a)^{9} b^{2}}$	19
norman	$\frac{-\frac{x}{8b}-\frac{a}{72b^{2}}}{(bx+a)^{9}}$	22
risch	$\frac{-\frac{x}{8b}-\frac{a}{72b^{2}}}{(bx+a)^{9}}$	22
parallelrisch	$\frac{-9b^{8}x-a\,b^{7}}{72b^{9}(bx+a)^{9}}$	26
default	$\frac{a}{9b^{2}(bx+a)^{9}}-\frac{1}{8b^{2}(bx+a)^{8}}$	27

input
```
int(x/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/72*(9*b*x+a)/(b*x+a)^9/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.63

$$\int \frac{x}{(a+bx)^{10}}\,dx =$$

$$-\frac{9\,bx+a}{72\,(b^{11}x^{9}+9\,ab^{10}x^{8}+36\,a^{2}b^{9}x^{7}+84\,a^{3}b^{8}x^{6}+126\,a^{4}b^{7}x^{5}+126\,a^{5}b^{6}x^{4}+84\,a^{6}b^{5}x^{3}+36\,a^{7}b^{4}x^{2}+9\,a^{8}}$$

input
```
integrate(x/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x
^6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 +
 9*a^8*b^3*x + a^9*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.87

$$\int \frac{x}{(a+bx)^{10}}\,dx$$
$$= \frac{-a-9bx}{72a^9b^2 + 648a^8b^3x + 2592a^7b^4x^2 + 6048a^6b^5x^3 + 9072a^5b^6x^4 + 9072a^4b^7x^5 + 6048a^3b^8x^6 + 2592a^2b^9x^7}$$

input
```
integrate(x/(b*x+a)**10,x)
```

output
```
(-a - 9*b*x)/(72*a**9*b**2 + 648*a**8*b**3*x + 2592*a**7*b**4*x**2 + 6048*
a**6*b**5*x**3 + 9072*a**5*b**6*x**4 + 9072*a**4*b**7*x**5 + 6048*a**3*b**
8*x**6 + 2592*a**2*b**9*x**7 + 648*a*b**10*x**8 + 72*b**11*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.63

$$\int \frac{x}{(a+bx)^{10}}\,dx =$$
$$-\frac{9\,bx+a}{72\,(b^{11}x^9 + 9\,ab^{10}x^8 + 36\,a^2b^9x^7 + 84\,a^3b^8x^6 + 126\,a^4b^7x^5 + 126\,a^5b^6x^4 + 84\,a^6b^5x^3 + 36\,a^7b^4x^2 + 9\,a^8}$$

input
```
integrate(x/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x
^6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 +
 9*a^8*b^3*x + a^9*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{(a+bx)^{10}}\,dx = -\frac{9\,bx+a}{72\,(bx+a)^9b^2}$$

input
```
integrate(x/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/72*(9*b*x + a)/((b*x + a)^9*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x}{(a+bx)^{10}}\,dx = -\frac{a+9\,b\,x}{72\,b^2\,(a+b\,x)^9}$$

input
```
int(x/(a + b*x)^10,x)
```

output
```
-(a + 9*b*x)/(72*b^2*(a + b*x)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.60

$$\int \frac{x}{(a+bx)^{10}}\,dx$$
$$= \frac{-9bx-a}{72b^2\,(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx+$$

input
```
int(x/(b*x+a)^10,x)
```

output
```
( - a - 9*b*x)/(72*b**2*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b
**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 3
6*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.193 $\int \frac{1}{(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9b(a+bx)^9}$$

output
```
-1/9/b/(b*x+a)^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9b(a+bx)^9}$$

input
```
Integrate[(a + b*x)^(-10),x]
```

output
```
-1/9*1/(b*(a + b*x)^9)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{10}} \, dx$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{9b(a+bx)^9}$$

input
```
Int[(a + b*x)^(-10),x]
```

output
```
-1/9*1/(b*(a + b*x)^9)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{9b(bx+a)^9}$	13
default	$-\frac{1}{9b(bx+a)^9}$	13
norman	$-\frac{1}{9b(bx+a)^9}$	13
risch	$-\frac{1}{9b(bx+a)^9}$	13
parallelrisch	$-\frac{1}{9b(bx+a)^9}$	13
orering	$-\frac{1}{9b(bx+a)^9}$	13

input
```
int(1/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9/b/(b*x+a)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 7.21

$$\int \frac{1}{(a+bx)^{10}}\,dx =$$

$$-\frac{1}{9\,(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2}$$

input
```
integrate(1/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b
^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x +
 a^9*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 7.79

$$\int \frac{1}{(a+bx)^{10}}\,dx =$$
$$-\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81a}$$

input
```
integrate(1/(b*x+a)**10,x)
```

output
```
-1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 +
1134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*
b**8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9\,(bx+a)^9b}$$

input
```
integrate(1/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/9/((b*x + a)^9*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9\,(bx+a)^9b}$$

input
```
integrate(1/(b*x+a)^10,x, algorithm="giac")
```

output
```
-1/9/((b*x + a)^9*b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 7.36

$$\int \frac{1}{(a+bx)^{10}}\,dx =$$

$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8}$$

input
```
int(1/(a + b*x)^10,x)
```

output
```
-1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
 756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 +
 324*a^2*b^8*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 7.14

$$\int \frac{1}{(a+bx)^{10}}\,dx =$$

$$-\frac{1}{9b\,(b^9x^9+9a\,b^8x^8+36a^2b^7x^7+84a^3b^6x^6+126a^4b^5x^5+126a^5b^4x^4+84a^6b^3x^3+36a^7b^2x^2+9a^8bx+}$$

input
```
int(1/(b*x+a)^10,x)
```

output
```
( - 1)/(9*b*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 1
26*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*
x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.194 $\int \frac{1}{x(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 141

$$\int \frac{1}{x(a+bx)^{10}}\,dx = \frac{1}{9a(a+bx)^9} + \frac{1}{8a^2(a+bx)^8} + \frac{1}{7a^3(a+bx)^7} + \frac{1}{6a^4(a+bx)^6} + \frac{1}{5a^5(a+bx)^5} + \frac{1}{4a^6(a+bx)^4} + \frac{1}{3a^7(a+bx)^3} + \frac{1}{2a^8(a+bx)^2} + \frac{1}{a^9(a+bx)} + \frac{\log(x)}{a^{10}} - \frac{\log(a+bx)}{a^{10}}$$

output
```
1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/
a^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/
(b*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a+bx)^{10}}\,dx = \frac{280a^8 + 315a^7(a+bx) + 360a^6(a+bx)^2 + 420a^5(a+bx)^3 + 504a^4(a+bx)^4 + 630a^3(a+bx)^5 + 840a^2(}{2520a^9(a+bx)^9} + \frac{\log(x)}{a^{10}} - \frac{\log(a+bx)}{a^{10}}$$

input
```
Integrate[1/(x*(a + b*x)^10),x]
```

output
```
(280*a^8 + 315*a^7*(a + b*x) + 360*a^6*(a + b*x)^2 + 420*a^5*(a + b*x)^3 +
 504*a^4*(a + b*x)^4 + 630*a^3*(a + b*x)^5 + 840*a^2*(a + b*x)^6 + 1260*a*
(a + b*x)^7 + 2520*(a + b*x)^8)/(2520*a^9*(a + b*x)^9) + Log[x]/a^10 - Log
[a + b*x]/a^10
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^{10}} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{b}{a^{10}(a+bx)}+\frac{1}{a^{10}x}-\frac{b}{a^9(a+bx)^2}-\frac{b}{a^8(a+bx)^3}-\frac{b}{a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{b}{a^5(a+bx)^6}-\frac{b}{a^4(a+b$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{a^9(a+bx)}+\frac{1}{2a^8(a+bx)^2}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{9a(a+bx)^9}$$

input
```
Int[1/(x*(a + b*x)^10),x]
```

output
```
1/(9*a*(a + b*x)^9) + 1/(8*a^2*(a + b*x)^8) + 1/(7*a^3*(a + b*x)^7) + 1/(6
*a^4*(a + b*x)^6) + 1/(5*a^5*(a + b*x)^5) + 1/(4*a^6*(a + b*x)^4) + 1/(3*a
^7*(a + b*x)^3) + 1/(2*a^8*(a + b*x)^2) + 1/(a^9*(a + b*x)) + Log[x]/a^10
- Log[a + b*x]/a^10
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\frac{b^8x^8}{a^9}+\frac{17b^7x^7}{2a^8}+\frac{191b^6x^6}{6a^7}+\frac{275b^5x^5}{4a^6}+\frac{1879b^4x^4}{20a^5}+\frac{2509b^3x^3}{30a^4}+\frac{3349b^2x^2}{70a^3}+\frac{4609bx}{280a^2}+\frac{7129}{2520a}}{(bx+a)^9}+\frac{\ln(-x)}{a^{10}}-\frac{\ln(bx+a)}{a^{10}}$
norman	$\frac{-\frac{9bx}{a^2}-\frac{54b^2x^2}{a^3}-\frac{154b^3x^3}{a^4}-\frac{525b^4x^4}{2a^5}-\frac{2877b^5x^5}{10a^6}-\frac{1029b^6x^6}{5a^7}-\frac{3267b^7x^7}{35a^8}-\frac{6849b^8x^8}{280a^9}-\frac{7129b^9x^9}{2520a^{10}}}{(bx+a)^9}+\frac{\ln(x)}{a^{10}}-\frac{\ln(bx+a)}{a^{10}}$
default	$\frac{1}{9a(bx+a)^9}+\frac{1}{8a^2(bx+a)^8}+\frac{1}{7a^3(bx+a)^7}+\frac{1}{6a^4(bx+a)^6}+\frac{1}{5a^5(bx+a)^5}+\frac{1}{4a^6(bx+a)^4}+\frac{1}{3a^7(bx+a)^3}+\frac{1}{2a^8(bx+}$
parallelrisch	$-725004a^4x^5b^5+90720\ln(x)x^2a^7b^2+22680\ln(x)x\,a^8b+22680\ln(x)x^8a\,b^8+211680\ln(x)x^6a^3b^6-7129b^9x^9-2520\ln(bx+a)x$

input
```
int(1/x/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(b^8/a^9*x^8+17/2*b^7/a^8*x^7+191/6*b^6/a^7*x^6+275/4*b^5/a^6*x^5+1879/20*
b^4/a^5*x^4+2509/30*b^3/a^4*x^3+3349/70*b^2/a^3*x^2+4609/280*b/a^2*x+7129/
2520/a)/(b*x+a)^9+1/a^10*ln(-x)-ln(b*x+a)/a^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(125) = 250.

Time = 0.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.75

$$\int \frac{1}{x(a+bx)^{10}}\,dx$$
$$= \frac{2520\,ab^8x^8+21420\,a^2b^7x^7+80220\,a^3b^6x^6+173250\,a^4b^5x^5+236754\,a^5b^4x^4+210756\,a^6b^3x^3+120564\,a}{}$$

input
```
integrate(1/x/(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/2520*(2520*a*b^8*x^8 + 21420*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^
4*b^5*x^5 + 236754*a^5*b^4*x^4 + 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 +
 41481*a^8*b*x + 7129*a^9 - 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 +
 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*
a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a) + 2520*(b^9*x^9 + 9*a*b^8*x^8
+ 36*a^2*b^7*x^7 + 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84
*a^6*b^3*x^3 + 36*a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(x))/(a^10*b^9*x^9 + 9
*a^11*b^8*x^8 + 36*a^12*b^7*x^7 + 84*a^13*b^6*x^6 + 126*a^14*b^5*x^5 + 126
*a^15*b^4*x^4 + 84*a^16*b^3*x^3 + 36*a^17*b^2*x^2 + 9*a^18*b*x + a^19)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.50

$$\int \frac{1}{x(a+bx)^{10}}\,dx = \frac{7129a^8 + 41481a^7bx + 120564a^6b^2x^2 + 210756a^5b^3x^3 + 236754a^4b^4x^4 + 173250a^3b^5x^5 + 80}{2520a^{18} + 22680a^{17}bx + 90720a^{16}b^2x^2 + 211680a^{15}b^3x^3 + 317520a^{14}b^4x^4 + 317520a^{13}b^5x^5 + 211680a^{12}b} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^{10}}$$

input
```
integrate(1/x/(b*x+a)**10,x)
```

output
```
(7129*a**8 + 41481*a**7*b*x + 120564*a**6*b**2*x**2 + 210756*a**5*b**3*x**
3 + 236754*a**4*b**4*x**4 + 173250*a**3*b**5*x**5 + 80220*a**2*b**6*x**6 +
 21420*a*b**7*x**7 + 2520*b**8*x**8)/(2520*a**18 + 22680*a**17*b*x + 90720
*a**16*b**2*x**2 + 211680*a**15*b**3*x**3 + 317520*a**14*b**4*x**4 + 31752
0*a**13*b**5*x**5 + 211680*a**12*b**6*x**6 + 90720*a**11*b**7*x**7 + 22680
*a**10*b**8*x**8 + 2520*a**9*b**9*x**9) + (log(x) - log(a/b + x))/a**10
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.45

$$\int \frac{1}{x(a+bx)^{10}}\,dx = \frac{2520\,b^8x^8 + 21420\,ab^7x^7 + 80220\,a^2b^6x^6 + 173250\,a^3b^5x^5 + 236754\,a^4b^4x^4 + 210756\,a^5b^3x^3 + 120564\,a^6b}{2520\,(a^9b^9x^9 + 9\,a^{10}b^8x^8 + 36\,a^{11}b^7x^7 + 84\,a^{12}b^6x^6 + 126\,a^{13}b^5x^5 + 126\,a^{14}b^4x^4 + 84\,a^{15}b^3x^3 + 36\,a} - \frac{\log(bx+a)}{a^{10}} + \frac{\log(x)}{a^{10}}$$

input
```
integrate(1/x/(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/2520*(2520*b^8*x^8 + 21420*a*b^7*x^7 + 80220*a^2*b^6*x^6 + 173250*a^3*b^
5*x^5 + 236754*a^4*b^4*x^4 + 210756*a^5*b^3*x^3 + 120564*a^6*b^2*x^2 + 414
81*a^7*b*x + 7129*a^8)/(a^9*b^9*x^9 + 9*a^10*b^8*x^8 + 36*a^11*b^7*x^7 + 8
4*a^12*b^6*x^6 + 126*a^13*b^5*x^5 + 126*a^14*b^4*x^4 + 84*a^15*b^3*x^3 + 3
6*a^16*b^2*x^2 + 9*a^17*b*x + a^18) - log(b*x + a)/a^10 + log(x)/a^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(a+bx)^{10}}\,dx = -\frac{\log(|bx+a|)}{a^{10}} + \frac{\log(|x|)}{a^{10}} + \frac{2520\,ab^8x^8 + 21420\,a^2b^7x^7 + 80220\,a^3b^6x^6 + 173250\,a^4b^5x^5 + 236754\,a^5b^4x^4 + 210756\,a^6b^3x^3 + 12056}{2520\,(bx+a)^9a^{10}}$$

input
```
integrate(1/x/(b*x+a)^10,x, algorithm="giac")
```

output
```
-log(abs(b*x + a))/a^10 + log(abs(x))/a^10 + 1/2520*(2520*a*b^8*x^8 + 2142
0*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4*b^5*x^5 + 236754*a^5*b^4*x^
4 + 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 41481*a^8*b*x + 7129*a^9)/((
b*x + a)^9*a^10)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03

$$\int \frac{1}{x(a+bx)^{10}}\,dx = \frac{1}{9\,a\,{\left(a+b\,x\right)}^9} - \frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{14\,b^2\,x^2}{{\left(a+b\,x\right)}^2}+\frac{56\,b^3\,x^3}{3\,{\left(a+b\,x\right)}^3}-\frac{35\,b^4\,x^4}{2\,{\left(a+b\,x\right)}^4}+\frac{56\,b^5\,x^5}{5\,{\left(a+b\,x\right)}^5}-\frac{14\,b^6\,x^6}{3\,{\left(a+b\,x\right)}^6}+\frac{8\,b^7\,x^7}{7\,{\left(a+b\,x\right)}^7}-\frac{b^8\,x^8}{8\,{\left(a+b\,x\right)}^8}+\frac{8\,b\,x}{a+b\,x}}{a^{10}}$$

input
```
int(1/(x*(a + b*x)^10),x)
```

output
```
1/(9*a*(a + b*x)^9) - (log((a + b*x)/x) - (14*b^2*x^2)/(a + b*x)^2 + (56*b
^3*x^3)/(3*(a + b*x)^3) - (35*b^4*x^4)/(2*(a + b*x)^4) + (56*b^5*x^5)/(5*(
a + b*x)^5) - (14*b^6*x^6)/(3*(a + b*x)^6) + (8*b^7*x^7)/(7*(a + b*x)^7) -
 (b^8*x^8)/(8*(a + b*x)^8) + (8*b*x)/(a + b*x))/a^10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.24

$$\int \frac{1}{x(a+bx)^{10}}\,dx = \frac{317520\log(x)\,a^5b^4x^4+317520\log(x)\,a^4b^5x^5+211680\log(x)\,a^3b^6x^6+90720\log(x)\,a^2b^7x^7+22680\log(x}{}$$

input
```
int(1/x/(b*x+a)^10,x)
```

output

```
( - 2520*log(a + b*x)*a**9 - 22680*log(a + b*x)*a**8*b*x - 90720*log(a + b
*x)*a**7*b**2*x**2 - 211680*log(a + b*x)*a**6*b**3*x**3 - 317520*log(a + b
*x)*a**5*b**4*x**4 - 317520*log(a + b*x)*a**4*b**5*x**5 - 211680*log(a + b
*x)*a**3*b**6*x**6 - 90720*log(a + b*x)*a**2*b**7*x**7 - 22680*log(a + b*x
)*a*b**8*x**8 - 2520*log(a + b*x)*b**9*x**9 + 2520*log(x)*a**9 + 22680*log
(x)*a**8*b*x + 90720*log(x)*a**7*b**2*x**2 + 211680*log(x)*a**6*b**3*x**3
+ 317520*log(x)*a**5*b**4*x**4 + 317520*log(x)*a**4*b**5*x**5 + 211680*log
(x)*a**3*b**6*x**6 + 90720*log(x)*a**2*b**7*x**7 + 22680*log(x)*a*b**8*x**
8 + 2520*log(x)*b**9*x**9 + 6849*a**9 + 38961*a**8*b*x + 110484*a**7*b**2*
x**2 + 187236*a**6*b**3*x**3 + 201474*a**5*b**4*x**4 + 137970*a**4*b**5*x*
*5 + 56700*a**3*b**6*x**6 + 11340*a**2*b**7*x**7 - 280*b**9*x**9)/(2520*a*
*10*(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*
b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9
*a*b**8*x**8 + b**9*x**9))
```

3.195 $\int \frac{1}{x^2(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 158

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = -\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}-\frac{3b}{2a^7(a+bx)^4}-\frac{7b}{3a^8(a+bx)^3}-\frac{4b}{a^9(a+bx)^2}-\frac{9b}{a^{10}(a+bx)}-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}$$

output
```
-1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*
b/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*
b/a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = -\frac{\frac{a\left(252a^9+7129a^8bx+41481a^7b^2x^2+120564a^6b^3x^3+210756a^5b^4x^4+236754a^4b^5x^5+173250a^3b^6x^6+80220a^2b^7x^7+21420ab^8x^8+2520b^9x^9\right)}{x(a+bx)^9}}{252a^{11}}$$

input
```
Integrate[1/(x^2*(a + b*x)^10),x]
```

output
```
-1/252*((a*(252*a^9 + 7129*a^8*b*x + 41481*a^7*b^2*x^2 + 120564*a^6*b^3*x^
3 + 210756*a^5*b^4*x^4 + 236754*a^4*b^5*x^5 + 173250*a^3*b^6*x^6 + 80220*a
^2*b^7*x^7 + 21420*a*b^8*x^8 + 2520*b^9*x^9))/(x*(a + b*x)^9) + 2520*b*Log
[x] - 2520*b*Log[a + b*x])/a^11
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{10b^2}{a^{11}(a+bx)}-\frac{10b}{a^{11}x}+\frac{9b^2}{a^{10}(a+bx)^2}+\frac{1}{a^{10}x^2}+\frac{8b^2}{a^9(a+bx)^3}+\frac{7b^2}{a^8(a+bx)^4}+\frac{6b^2}{a^7(a+bx)^5}+\frac{5b^2}{a^6(a+bx)^6}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}-\frac{9b}{a^{10}(a+bx)}-\frac{1}{a^{10}x}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{2b}{3a^5(a+bx)^6}-\frac{3b}{7a^4(a+bx)^7}-\frac{b}{4a^3(a+bx)^8}-\frac{b}{9a^2(a+bx)^9}$$

input
```
Int[1/(x^2*(a + b*x)^10),x]
```

output
```
-(1/(a^10*x)) - b/(9*a^2*(a + b*x)^9) - b/(4*a^3*(a + b*x)^8) - (3*b)/(7*a
^4*(a + b*x)^7) - (2*b)/(3*a^5*(a + b*x)^6) - b/(a^6*(a + b*x)^5) - (3*b)/
(2*a^7*(a + b*x)^4) - (7*b)/(3*a^8*(a + b*x)^3) - (4*b)/(a^9*(a + b*x)^2)
- (9*b)/(a^10*(a + b*x)) - (10*b*Log[x])/a^11 + (10*b*Log[a + b*x])/a^11
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

method	result
risch	$\frac{-\frac{10b^9x^9}{a^{10}}-\frac{85b^8x^8}{a^9}-\frac{955b^7x^7}{3a^8}-\frac{1375b^6x^6}{2a^7}-\frac{1879b^5x^5}{2a^6}-\frac{2509b^4x^4}{3a^5}-\frac{3349b^3x^3}{7a^4}-\frac{4609b^2x^2}{28a^3}-\frac{7129bx}{252a^2}-\frac{1}{a}}{x(bx+a)^9}+\frac{10b\ln(-bx-a)}{a^{11}}-\frac{10b\ln}{a^1}$
norman	$\frac{-\frac{1}{a}+\frac{90b^2x^2}{a^3}+\frac{540b^3x^3}{a^4}+\frac{1540b^4x^4}{a^5}+\frac{2625b^5x^5}{a^6}+\frac{2877b^6x^6}{a^7}+\frac{2058b^7x^7}{a^8}+\frac{6534b^8x^8}{7a^9}+\frac{6849b^9x^9}{28a^{10}}+\frac{7129b^{10}x^{10}}{252a^{11}}}{x(bx+a)^9}-\frac{10b\ln(x)}{a^{11}}+10b$
default	$-\frac{1}{a^{10}x}-\frac{b}{9a^2(bx+a)^9}-\frac{b}{4a^3(bx+a)^8}-\frac{3b}{7a^4(bx+a)^7}-\frac{2b}{3a^5(bx+a)^6}-\frac{b}{a^6(bx+a)^5}-\frac{3b}{2a^7(bx+a)^4}-\frac{7b}{3a^8(bx+a)^3}$
parallelrisch	$-\frac{-7129b^{10}x^{10}+252a^{10}-2520\ln(bx+a)x^{10}b^{10}+90720a^7b^3\ln(x)x^3+211680a^6b^4\ln(x)x^4+211680a^3b^7\ln(x)x^7-317520\ln(b}{}$

input
```
int(1/x^2/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-10*b^9/a^10*x^9-85*b^8/a^9*x^8-955/3*b^7/a^8*x^7-1375/2*b^6/a^7*x^6-1879
/2*b^5/a^6*x^5-2509/3*b^4/a^5*x^4-3349/7*b^3/a^4*x^3-4609/28*b^2/a^3*x^2-7
129/252*b/a^2*x-1/a)/x/(b*x+a)^9+10/a^11*b*ln(-b*x-a)-10*b*ln(x)/a^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(146) = 292.

Time = 0.08 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.64

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx =$$
$$-\frac{2520\,ab^9x^9+21420\,a^2b^8x^8+80220\,a^3b^7x^7+173250\,a^4b^6x^6+236754\,a^5b^5x^5+210756\,a^6b^4x^4+12056}{}$$

input
```
integrate(1/x^2/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/252*(2520*a*b^9*x^9 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^
4*b^6*x^6 + 236754*a^5*b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 +
 41481*a^8*b^2*x^2 + 7129*a^9*b*x + 252*a^10 - 2520*(b^10*x^10 + 9*a*b^9*x
^9 + 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x^6 + 126*a^5*b^5*x^5 +
 84*a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^9*b*x)*log(b*x + a) +
 2520*(b^10*x^10 + 9*a*b^9*x^9 + 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4
*b^6*x^6 + 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x
^2 + a^9*b*x)*log(x))/(a^11*b^9*x^10 + 9*a^12*b^8*x^9 + 36*a^13*b^7*x^8 +
84*a^14*b^6*x^7 + 126*a^15*b^5*x^6 + 126*a^16*b^4*x^5 + 84*a^17*b^3*x^4 +
36*a^18*b^2*x^3 + 9*a^19*b*x^2 + a^20*x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = \frac{-252a^9 - 7129a^8bx - 41481a^7b^2x^2 - 120564a^6b^3x^3 - 210756a^5b^4x^4 - 236754a^4b^5x^5 - 173250a^3b^6x^6 \cdots}{252a^{19}x + 2268a^{18}bx^2 + 9072a^{17}b^2x^3 + 21168a^{16}b^3x^4 + 31752a^{15}b^4x^5 + 31752a^{14}b^5x^6 + 21168a^{13}b^6x^7 + \cdots} + \frac{10b\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^{11}}$$

input
```
integrate(1/x**2/(b*x+a)**10,x)
```

output
```
(-252*a**9 - 7129*a**8*b*x - 41481*a**7*b**2*x**2 - 120564*a**6*b**3*x**3
- 210756*a**5*b**4*x**4 - 236754*a**4*b**5*x**5 - 173250*a**3*b**6*x**6 -
80220*a**2*b**7*x**7 - 21420*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**19*x +
2268*a**18*b*x**2 + 9072*a**17*b**2*x**3 + 21168*a**16*b**3*x**4 + 31752*a
**15*b**4*x**5 + 31752*a**14*b**5*x**6 + 21168*a**13*b**6*x**7 + 9072*a**1
2*b**7*x**8 + 2268*a**11*b**8*x**9 + 252*a**10*b**9*x**10) + 10*b*(-log(x)
 + log(a/b + x))/a**11
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = -\frac{2520\,b^9x^9 + 21420\,ab^8x^8 + 80220\,a^2b^7x^7 + 173250\,a^3b^6x^6 + 236754\,a^4b^5x^5 + 210756\,a^5b^4x^4 + 120564\,a}{252\,(a^{10}b^9x^{10} + 9\,a^{11}b^8x^9 + 36\,a^{12}b^7x^8 + 84\,a^{13}b^6x^7 + 126\,a^{14}b^5x^6 + 126\,a^{15}b^4x^5 + 84\,a^{16}b^3x} + \frac{10\,b\log(bx+a)}{a^{11}} - \frac{10\,b\log(x)}{a^{11}}$$

input
```
integrate(1/x^2/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/252*(2520*b^9*x^9 + 21420*a*b^8*x^8 + 80220*a^2*b^7*x^7 + 173250*a^3*b^
6*x^6 + 236754*a^4*b^5*x^5 + 210756*a^5*b^4*x^4 + 120564*a^6*b^3*x^3 + 414
81*a^7*b^2*x^2 + 7129*a^8*b*x + 252*a^9)/(a^10*b^9*x^10 + 9*a^11*b^8*x^9 +
 36*a^12*b^7*x^8 + 84*a^13*b^6*x^7 + 126*a^14*b^5*x^6 + 126*a^15*b^4*x^5 +
 84*a^16*b^3*x^4 + 36*a^17*b^2*x^3 + 9*a^18*b*x^2 + a^19*x) + 10*b*log(b*x
 + a)/a^11 - 10*b*log(x)/a^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = \frac{10\,b\log(|bx+a|)}{a^{11}} - \frac{10\,b\log(|x|)}{a^{11}} - \frac{2520\,ab^9x^9 + 21420\,a^2b^8x^8 + 80220\,a^3b^7x^7 + 173250\,a^4b^6x^6 + 236754\,a^5b^5x^5 + 210756\,a^6b^4x^4 + 12056}{252\,(bx+a)^9a^{11}x}$$

input
```
integrate(1/x^2/(b*x+a)^10,x, algorithm="giac")
```

output
```
10*b*log(abs(b*x + a))/a^11 - 10*b*log(abs(x))/a^11 - 1/252*(2520*a*b^9*x^
9 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4*b^6*x^6 + 236754*a^
5*b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 41481*a^8*b^2*x^2 +
7129*a^9*b*x + 252*a^10)/((b*x + a)^9*a^11*x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = \frac{20\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}} - \frac{\frac{1}{a}+\frac{4609\,b^2\,x^2}{28\,a^3}+\frac{3349\,b^3\,x^3}{7\,a^4}+\frac{2509\,b^4\,x^4}{3\,a^5}+\frac{1879\,b^5\,x^5}{2\,a^6}+\frac{1375\,b^6\,x^6}{2\,a^7}+\frac{955\,b^7\,x^7}{3\,a^8}+\frac{85\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}+\frac{7129\,b}{252\,a}}{a^9\,x+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^3+84\,a^6\,b^3\,x^4+126\,a^5\,b^4\,x^5+126\,a^4\,b^5\,x^6+84\,a^3\,b^6\,x^7+36\,a^2\,b^7\,x^8+9\,a\,b}$$

input
```
int(1/(x^2*(a + b*x)^10),x)
```

output
```
(20*b*atanh((2*b*x)/a + 1))/a^11 - (1/a + (4609*b^2*x^2)/(28*a^3) + (3349*
b^3*x^3)/(7*a^4) + (2509*b^4*x^4)/(3*a^5) + (1879*b^5*x^5)/(2*a^6) + (1375
*b^6*x^6)/(2*a^7) + (955*b^7*x^7)/(3*a^8) + (85*b^8*x^8)/a^9 + (10*b^9*x^9
)/a^10 + (7129*b*x)/(252*a^2))/(a^9*x + b^9*x^10 + 9*a^8*b*x^2 + 9*a*b^8*x
^9 + 36*a^7*b^2*x^3 + 84*a^6*b^3*x^4 + 126*a^5*b^4*x^5 + 126*a^4*b^5*x^6 +
 84*a^3*b^6*x^7 + 36*a^2*b^7*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.06

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx = \frac{280b^{10}x^{10} - 2520\log(x)\,b^{10}x^{10} - 22680\log(x)\,a\,b^9x^9 - 211680\log(x)\,a^6b^4x^4 - 90720\log(x)\,a^7b^3x^3 - 252}{}$$

input
```
int(1/x^2/(b*x+a)^10,x)
```

output
```
(2520*log(a + b*x)*a**9*b*x + 22680*log(a + b*x)*a**8*b**2*x**2 + 90720*lo
g(a + b*x)*a**7*b**3*x**3 + 211680*log(a + b*x)*a**6*b**4*x**4 + 317520*lo
g(a + b*x)*a**5*b**5*x**5 + 317520*log(a + b*x)*a**4*b**6*x**6 + 211680*lo
g(a + b*x)*a**3*b**7*x**7 + 90720*log(a + b*x)*a**2*b**8*x**8 + 22680*log(
a + b*x)*a*b**9*x**9 + 2520*log(a + b*x)*b**10*x**10 - 2520*log(x)*a**9*b*
x - 22680*log(x)*a**8*b**2*x**2 - 90720*log(x)*a**7*b**3*x**3 - 211680*log
(x)*a**6*b**4*x**4 - 317520*log(x)*a**5*b**5*x**5 - 317520*log(x)*a**4*b**
6*x**6 - 211680*log(x)*a**3*b**7*x**7 - 90720*log(x)*a**2*b**8*x**8 - 2268
0*log(x)*a*b**9*x**9 - 2520*log(x)*b**10*x**10 - 252*a**10 - 6849*a**9*b*x
 - 38961*a**8*b**2*x**2 - 110484*a**7*b**3*x**3 - 187236*a**6*b**4*x**4 -
201474*a**5*b**5*x**5 - 137970*a**4*b**6*x**6 - 56700*a**3*b**7*x**7 - 113
40*a**2*b**8*x**8 + 280*b**10*x**10)/(252*a**11*x*(a**9 + 9*a**8*b*x + 36*
a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x*
*5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.196 $\int \frac{1}{x^3(a+bx)^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 191

$$\int \frac{1}{x^3(a+bx)^{10}} dx = -\frac{1}{2a^{10}x^2} + \frac{10b}{a^{11}x} + \frac{b^2}{9a^3(a+bx)^9} + \frac{3b^2}{8a^4(a+bx)^8} + \frac{6b^2}{7a^5(a+bx)^7} + \frac{5b^2}{3a^6(a+bx)^6} + \frac{3b^2}{a^7(a+bx)^5} + \frac{21b^2}{4a^8(a+bx)^4} + \frac{28b^2}{3a^9(a+bx)^3} + \frac{18b^2}{a^{10}(a+bx)^2} + \frac{45b^2}{a^{11}(a+bx)} + \frac{55b^2\log(x)}{a^{12}} - \frac{55b^2\log(a+bx)}{a^{12}}$$

output
```
-1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*
b^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(
b*x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+
55*b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx$$
$$= \frac{\frac{a(-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235}{x^2(a+bx)^9}}{504a^{12}}$$

input
```
Integrate[1/(x^3*(a + b*x)^10),x]
```

output
```
((a*(-252*a^10 + 2772*a^9*b*x + 78419*a^8*b^2*x^2 + 456291*a^7*b^3*x^3 + 1
326204*a^6*b^4*x^4 + 2318316*a^5*b^5*x^5 + 2604294*a^4*b^6*x^6 + 1905750*a
^3*b^7*x^7 + 882420*a^2*b^8*x^8 + 235620*a*b^9*x^9 + 27720*b^10*x^10))/(x^
2*(a + b*x)^9) + 27720*b^2*Log[x] - 27720*b^2*Log[a + b*x])/(504*a^12)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(-\frac{55b^3}{a^{12}(a+bx)}+\frac{55b^2}{a^{12}x}-\frac{45b^3}{a^{11}(a+bx)^2}-\frac{10b}{a^{11}x^2}-\frac{36b^3}{a^{10}(a+bx)^3}+\frac{1}{a^{10}x^3}-\frac{28b^3}{a^9(a+bx)^4}-\frac{21b^3}{a^8(a+bx)^5}-\frac{}{a^7(}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{55b^2\log(x)}{a^{12}}-\frac{55b^2\log(a+bx)}{a^{12}}+\frac{45b^2}{a^{11}(a+bx)}+\frac{10b}{a^{11}x}+\frac{18b^2}{a^{10}(a+bx)^2}-\frac{1}{2a^{10}x^2}+\frac{28b^2}{3a^9(a+bx)^3}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{3b^2}{a^7(a+bx)^5}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{b^2}{9a^3(a+bx)^9}$$

input
```
Int[1/(x^3*(a + b*x)^10),x]
```

output
```
-1/2*1/(a^10*x^2) + (10*b)/(a^11*x) + b^2/(9*a^3*(a + b*x)^9) + (3*b^2)/(8
*a^4*(a + b*x)^8) + (6*b^2)/(7*a^5*(a + b*x)^7) + (5*b^2)/(3*a^6*(a + b*x)
^6) + (3*b^2)/(a^7*(a + b*x)^5) + (21*b^2)/(4*a^8*(a + b*x)^4) + (28*b^2)/
(3*a^9*(a + b*x)^3) + (18*b^2)/(a^10*(a + b*x)^2) + (45*b^2)/(a^11*(a + b*
x)) + (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x])/a^12
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

method	result
norman	$\frac{-\frac{1}{2a}+\frac{11bx}{2a^2}-\frac{495b^3x^3}{a^4}-\frac{2970b^4x^4}{a^5}-\frac{8470b^5x^5}{a^6}-\frac{28875b^6x^6}{2a^7}-\frac{31647b^7x^7}{2a^8}-\frac{11319b^8x^8}{a^9}-\frac{35937b^9x^9}{7a^{10}}-\frac{75339b^{10}x^{10}}{56a^{11}}-\frac{78419b^{11}x^{11}}{504a^{12}}}{x^2(bx+a)^9}+$
risch	$\frac{\frac{55b^{10}x^{10}}{a^{11}}+\frac{935b^9x^9}{2a^{10}}+\frac{10505b^8x^8}{6a^9}+\frac{15125b^7x^7}{4a^8}+\frac{20669b^6x^6}{4a^7}+\frac{27599b^5x^5}{6a^6}+\frac{36839b^4x^4}{14a^5}+\frac{50699b^3x^3}{56a^4}+\frac{78419b^2x^2}{504a^3}+\frac{11bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)^9}+\frac{55b^2}{}$
default	$-\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(bx+a)^9}+\frac{3b^2}{8a^4(bx+a)^8}+\frac{6b^2}{7a^5(bx+a)^7}+\frac{5b^2}{3a^6(bx+a)^6}+\frac{3b^2}{a^7(bx+a)^5}+\frac{21b^2}{4a^8(bx+a)^4}+$
parallelrisch	$-5704776a^3x^8b^8-997920\ln(bx+a)x^9a^2b^9-2328480\ln(bx+a)x^8a^3b^8-3492720\ln(bx+a)x^7a^4b^7-3492720\ln(bx+a)x^6a^5b^6-$

input
```
int(1/x^3/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a+11/2*b/a^2*x-495*b^3/a^4*x^3-2970*b^4/a^5*x^4-8470*b^5/a^6*x^5-288
75/2*b^6/a^7*x^6-31647/2*b^7/a^8*x^7-11319*b^8/a^9*x^8-35937/7*b^9/a^10*x^
9-75339/56*b^10/a^11*x^10-78419/504*b^11/a^12*x^11)/x^2/(b*x+a)^9+55*b^2*l
n(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(177) = 354$.

Time = 0.10 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.29

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx = \frac{27720\,ab^{10}x^{10} + 235620\,a^2b^9x^9 + 882420\,a^3b^8x^8 + 1905750\,a^4b^7x^7 + 2604294\,a^5b^6x^6 + 2318316\,a^6b^5x^5 + \cdots}{\cdots}$$

input
```
integrate(1/x^3/(b*x+a)^10,x, algorithm="fricas")
```

output
```
1/504*(27720*a*b^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 19057
50*a^4*b^7*x^7 + 2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b
^4*x^4 + 456291*a^8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11
 - 27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*b^9*x^9 + 84*a^3*b^8*x^8 + 12
6*a^4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x^5 + 36*a^7*b^4*x^4 + 9*a^8*
b^3*x^3 + a^9*b^2*x^2)*log(b*x + a) + 27720*(b^11*x^11 + 9*a*b^10*x^10 + 3
6*a^2*b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^
6*b^5*x^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x^3 + a^9*b^2*x^2)*log(x))/(a^12*b^
9*x^11 + 9*a^13*b^8*x^10 + 36*a^14*b^7*x^9 + 84*a^15*b^6*x^8 + 126*a^16*b^
5*x^7 + 126*a^17*b^4*x^6 + 84*a^18*b^3*x^5 + 36*a^19*b^2*x^4 + 9*a^20*b*x^
3 + a^21*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx$$
$$= \frac{-252a^{10} + 2772a^9bx + 78419a^8b^2x^2 + 456291a^7b^3x^3 + 1326204a^6b^4x^4 + 2318316a^5b^5x^5 + 2604294a^4b^6x}{504a^{20}x^2 + 4536a^{19}bx^3 + 18144a^{18}b^2x^4 + 42336a^{17}b^3x^5 + 63504a^{16}b^4x^6 + 63504a^{15}b^5x^7 + 423}$$
$$+ \frac{55b^2\left(\log{(x)} - \log{\left(\frac{a}{b} + x\right)}\right)}{a^{12}}$$

input
```
integrate(1/x**3/(b*x+a)**10,x)
```

output
```
(-252*a**10 + 2772*a**9*b*x + 78419*a**8*b**2*x**2 + 456291*a**7*b**3*x**3
 + 1326204*a**6*b**4*x**4 + 2318316*a**5*b**5*x**5 + 2604294*a**4*b**6*x**
6 + 1905750*a**3*b**7*x**7 + 882420*a**2*b**8*x**8 + 235620*a*b**9*x**9 +
27720*b**10*x**10)/(504*a**20*x**2 + 4536*a**19*b*x**3 + 18144*a**18*b**2*
x**4 + 42336*a**17*b**3*x**5 + 63504*a**16*b**4*x**6 + 63504*a**15*b**5*x*
*7 + 42336*a**14*b**6*x**8 + 18144*a**13*b**7*x**9 + 4536*a**12*b**8*x**10
 + 504*a**11*b**9*x**11) + 55*b**2*(log(x) - log(a/b + x))/a**12
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx$$
$$= \frac{27720\,b^{10}x^{10} + 235620\,ab^9x^9 + 882420\,a^2b^8x^8 + 1905750\,a^3b^7x^7 + 2604294\,a^4b^6x^6 + 2318316\,a^5b^5x^5 + 13}{504\,(a^{11}b^9x^{11} + 9\,a^{12}b^8x^{10} + 36\,a^{13}b^7x^9 + 84\,a^{14}b^6x^8 + 126\,a^{15}b^5x^7 + 126\,a^{16}b^4x^6}$$
$$- \frac{55\,b^2\log{(bx+a)}}{a^{12}} + \frac{55\,b^2\log{(x)}}{a^{12}}$$

input
```
integrate(1/x^3/(b*x+a)^10,x, algorithm="maxima")
```

output
```
1/504*(27720*b^10*x^10 + 235620*a*b^9*x^9 + 882420*a^2*b^8*x^8 + 1905750*a
^3*b^7*x^7 + 2604294*a^4*b^6*x^6 + 2318316*a^5*b^5*x^5 + 1326204*a^6*b^4*x
^4 + 456291*a^7*b^3*x^3 + 78419*a^8*b^2*x^2 + 2772*a^9*b*x - 252*a^10)/(a^
11*b^9*x^11 + 9*a^12*b^8*x^10 + 36*a^13*b^7*x^9 + 84*a^14*b^6*x^8 + 126*a^
15*b^5*x^7 + 126*a^16*b^4*x^6 + 84*a^17*b^3*x^5 + 36*a^18*b^2*x^4 + 9*a^19
*b*x^3 + a^20*x^2) - 55*b^2*log(b*x + a)/a^12 + 55*b^2*log(x)/a^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx = -\frac{55\,b^2\log\left(|bx+a|\right)}{a^{12}} + \frac{55\,b^2\log\left(|x|\right)}{a^{12}} + \frac{27720\,ab^{10}x^{10} + 235620\,a^2b^9x^9 + 882420\,a^3b^8x^8 + 1905750\,a^4b^7x^7 + 2604294\,a^5b^6x^6 + 2318316\,a^6b^5x^5}{504\,(bx+a)^9a^{12}x^2}$$

input
```
integrate(1/x^3/(b*x+a)^10,x, algorithm="giac")
```

output
```
-55*b^2*log(abs(b*x + a))/a^12 + 55*b^2*log(abs(x))/a^12 + 1/504*(27720*a*
b^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 1905750*a^4*b^7*x^7
+ 2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4*x^4 + 456291
*a^8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11)/((b*x + a)^9*
a^12*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx = \frac{\frac{78419\,b^2\,x^2}{504\,a^3} - \frac{1}{2\,a} + \frac{50699\,b^3\,x^3}{56\,a^4} + \frac{36839\,b^4\,x^4}{14\,a^5} + \frac{27599\,b^5\,x^5}{6\,a^6} + \frac{20669\,b^6\,x^6}{4\,a^7} + \frac{15125\,b^7\,x^7}{4\,a^8} + \frac{10505\,b^8\,x^8}{6\,a^9} + \frac{935\,b^9\,x^9}{2\,a^{10}} + \frac{55\,b^{10}}{a^{11}}}{a^9\,x^2 + 9\,a^8\,b\,x^3 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^5 + 126\,a^5\,b^4\,x^6 + 126\,a^4\,b^5\,x^7 + 84\,a^3\,b^6\,x^8 + 36\,a^2\,b^7\,x^9 + 9\,a\,b^8} - \frac{110\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}}$$

input
```
int(1/(x^3*(a + b*x)^10),x)
```

output
```
((78419*b^2*x^2)/(504*a^3) - 1/(2*a) + (50699*b^3*x^3)/(56*a^4) + (36839*b
^4*x^4)/(14*a^5) + (27599*b^5*x^5)/(6*a^6) + (20669*b^6*x^6)/(4*a^7) + (15
125*b^7*x^7)/(4*a^8) + (10505*b^8*x^8)/(6*a^9) + (935*b^9*x^9)/(2*a^10) +
(55*b^10*x^10)/a^11 + (11*b*x)/(2*a^2))/(a^9*x^2 + b^9*x^11 + 9*a^8*b*x^3
+ 9*a*b^8*x^10 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^5 + 126*a^5*b^4*x^6 + 126*a
^4*b^5*x^7 + 84*a^3*b^6*x^8 + 36*a^2*b^7*x^9) - (110*b^2*atanh((2*b*x)/a +
 1))/a^12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.63

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx = \frac{-27720\log(bx+a)\,a^9b^2x^2 - 249480\log(bx+a)\,a^8b^3x^3 - 997920\log(bx+a)\,a^7b^4x^4 - 2328480\log(bx+}{}$$

input
```
int(1/x^3/(b*x+a)^10,x)
```

output
```
( - 27720*log(a + b*x)*a**9*b**2*x**2 - 249480*log(a + b*x)*a**8*b**3*x**3
 - 997920*log(a + b*x)*a**7*b**4*x**4 - 2328480*log(a + b*x)*a**6*b**5*x**
5 - 3492720*log(a + b*x)*a**5*b**6*x**6 - 3492720*log(a + b*x)*a**4*b**7*x
**7 - 2328480*log(a + b*x)*a**3*b**8*x**8 - 997920*log(a + b*x)*a**2*b**9*
x**9 - 249480*log(a + b*x)*a*b**10*x**10 - 27720*log(a + b*x)*b**11*x**11
+ 27720*log(x)*a**9*b**2*x**2 + 249480*log(x)*a**8*b**3*x**3 + 997920*log(
x)*a**7*b**4*x**4 + 2328480*log(x)*a**6*b**5*x**5 + 3492720*log(x)*a**5*b*
*6*x**6 + 3492720*log(x)*a**4*b**7*x**7 + 2328480*log(x)*a**3*b**8*x**8 +
997920*log(x)*a**2*b**9*x**9 + 249480*log(x)*a*b**10*x**10 + 27720*log(x)*
b**11*x**11 - 252*a**11 + 2772*a**10*b*x + 75339*a**9*b**2*x**2 + 428571*a
**8*b**3*x**3 + 1215324*a**7*b**4*x**4 + 2059596*a**6*b**5*x**5 + 2216214*
a**5*b**6*x**6 + 1517670*a**4*b**7*x**7 + 623700*a**3*b**8*x**8 + 124740*a
**2*b**9*x**9 - 3080*b**11*x**11)/(504*a**12*x**2*(a**9 + 9*a**8*b*x + 36*
a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b**5*x*
*5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b**8*x**8 + b**9*x**9))
```

3.197 $\int \frac{1}{x^4(a+bx)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 198

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^{10}}\,dx = &-\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}\\&-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{7b^3}{a^8(a+bx)^5}\\&-\frac{14b^3}{a^9(a+bx)^4}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{60b^3}{a^{11}(a+bx)^2}\\&-\frac{165b^3}{a^{12}(a+bx)}-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}\end{aligned}$$

output
```
-1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5
/(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)
^5-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^
3/a^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = -\frac{\frac{a(42a^{11}-252a^{10}bx+2772a^9b^2x^2+78419a^8b^3x^3+456291a^7b^4x^4+1326204a^6b^5x^5+2318316a^5b^6x^6+2604294a^4b^7x^7+1905750a^3b^8x^8+88242}{x^3(a+bx)^9}}{126a^{13}}$$

input
```
Integrate[1/(x^4*(a + b*x)^10),x]
```

output
```
-1/126*((a*(42*a^11 - 252*a^10*b*x + 2772*a^9*b^2*x^2 + 78419*a^8*b^3*x^3
+ 456291*a^7*b^4*x^4 + 1326204*a^6*b^5*x^5 + 2318316*a^5*b^6*x^6 + 2604294
*a^4*b^7*x^7 + 1905750*a^3*b^8*x^8 + 882420*a^2*b^9*x^9 + 235620*a*b^10*x^
10 + 27720*b^11*x^11))/(x^3*(a + b*x)^9) + 27720*b^3*Log[x] - 27720*b^3*Lo
g[a + b*x])/a^13
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx$$

$$\Big\downarrow \; 54$$

$$\int \left(\frac{220b^4}{a^{13}(a+bx)} - \frac{220b^3}{a^{13}x} + \frac{165b^4}{a^{12}(a+bx)^2} + \frac{55b^2}{a^{12}x^2} + \frac{120b^4}{a^{11}(a+bx)^3} - \frac{10b}{a^{11}x^3} + \frac{84b^4}{a^{10}(a+bx)^4} + \frac{1}{a^{10}x^4} + \frac{56b^4}{a^9(a+bx\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}-\frac{165b^3}{a^{12}(a+bx)}-\frac{55b^2}{a^{12}x}-\frac{60b^3}{a^{11}(a+bx)^2}+\frac{5b}{a^{11}x^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{1}{3a^{10}x^3}-\frac{14b^3}{a^9(a+bx)^4}-\frac{7b^3}{a^8(a+bx)^5}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{b^3}{2a^5(a+bx)^8}-\frac{b^3}{9a^4(a+bx)^9}$$

input
```
Int[1/(x^4*(a + b*x)^10),x]
```

output
```
-1/3*1/(a^10*x^3) + (5*b)/(a^11*x^2) - (55*b^2)/(a^12*x) - b^3/(9*a^4*(a +
 b*x)^9) - b^3/(2*a^5*(a + b*x)^8) - (10*b^3)/(7*a^6*(a + b*x)^7) - (10*b^
3)/(3*a^7*(a + b*x)^6) - (7*b^3)/(a^8*(a + b*x)^5) - (14*b^3)/(a^9*(a + b*
x)^4) - (28*b^3)/(a^10*(a + b*x)^3) - (60*b^3)/(a^11*(a + b*x)^2) - (165*b
^3)/(a^12*(a + b*x)) - (220*b^3*Log[x])/a^13 + (220*b^3*Log[a + b*x])/a^13
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81

method	result
norman	$\frac{-\frac{1}{3a}+\frac{2bx}{a^2}-\frac{22b^2x^2}{a^3}+\frac{1980b^4x^4}{a^5}+\frac{11880b^5x^5}{a^6}+\frac{33880b^6x^6}{a^7}+\frac{57750b^7x^7}{a^8}+\frac{63294b^8x^8}{a^9}+\frac{45276b^9x^9}{a^{10}}+\frac{143748b^{10}x^{10}}{7a^{11}}+\frac{75339b^{11}x^{11}}{14a^{12}}+\cdots}{x^3(bx+a)^9}$
risch	$\frac{-\frac{220b^{11}x^{11}}{a^{12}}-\frac{1870b^{10}x^{10}}{a^{11}}-\frac{21010b^9x^9}{3a^{10}}-\frac{15125b^8x^8}{a^9}-\frac{20669b^7x^7}{a^8}-\frac{55198b^6x^6}{3a^7}-\frac{73678b^5x^5}{7a^6}-\frac{50699b^4x^4}{14a^5}-\frac{78419b^3x^3}{126a^4}-\frac{22b^2x^2}{a^3}+\frac{2bx}{a^2}\cdots}{x^3(bx+a)^9}$
default	$-\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(bx+a)^9}-\frac{b^3}{2a^5(bx+a)^8}-\frac{10b^3}{7a^6(bx+a)^7}-\frac{10b^3}{3a^7(bx+a)^6}-\frac{7b^3}{a^8(bx+a)^5}-\frac{14b^3}{a^9(bx+\cdots}$
parallelrisch	$-\frac{-4268880a^6x^6b^6-1496880a^7x^5b^5-249480a^8x^4b^4+2772a^{10}x^2b^2-252a^{11}xb-7975044a^4x^8b^8-7276500a^5x^7b^7-78419b^{12}\cdots}{\cdots}$

input
```
int(1/x^4/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/3/a+2*b/a^2*x-22*b^2/a^3*x^2+1980*b^4/a^5*x^4+11880*b^5/a^6*x^5+33880*
b^6/a^7*x^6+57750*b^7/a^8*x^7+63294*b^8/a^9*x^8+45276*b^9/a^10*x^9+143748/
7*b^10/a^11*x^10+75339/14*b^11/a^12*x^11+78419/126*b^12/a^13*x^12)/x^3/(b*
x+a)^9-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(188) = 376$.

Time = 0.10 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = -\frac{27720\,ab^{11}x^{11}+235620\,a^2b^{10}x^{10}+882420\,a^3b^9x^9+1905750\,a^4b^8x^8+2604294\,a^5b^7x^7+2318316\,a^6b^6x^6}{}$$

input
```
integrate(1/x^4/(b*x+a)^10,x, algorithm="fricas")
```

output
```
-1/126*(27720*a*b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 19
05750*a^4*b^8*x^8 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^
7*b^5*x^5 + 456291*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 2
52*a^11*b*x + 42*a^12 - 27720*(b^12*x^12 + 9*a*b^11*x^11 + 36*a^2*b^10*x^1
0 + 84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a^5*b^7*x^7 + 84*a^6*b^6*x^6 +
36*a^7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)*log(b*x + a) + 27720*(b^12*x
^12 + 9*a*b^11*x^11 + 36*a^2*b^10*x^10 + 84*a^3*b^9*x^9 + 126*a^4*b^8*x^8
+ 126*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*
b^3*x^3)*log(x))/(a^13*b^9*x^12 + 9*a^14*b^8*x^11 + 36*a^15*b^7*x^10 + 84*
a^16*b^6*x^9 + 126*a^17*b^5*x^8 + 126*a^18*b^4*x^7 + 84*a^19*b^3*x^6 + 36*
a^20*b^2*x^5 + 9*a^21*b*x^4 + a^22*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = \frac{-42a^{11}+252a^{10}bx-2772a^9b^2x^2-78419a^8b^3x^3-456291a^7b^4x^4-1326204a^6b^5x^5-2318316a^5b^6x^6-}{126a^{21}x^3+1134a^{20}bx^4+4536a^{19}b^2x^5+10584a^{18}b^3x^6+15876a^{17}b^4x^7+15876a^{16}b^5x^8} + \frac{220b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{13}}$$

input
```
integrate(1/x**4/(b*x+a)**10,x)
```

output
```
(-42*a**11 + 252*a**10*b*x - 2772*a**9*b**2*x**2 - 78419*a**8*b**3*x**3 -
456291*a**7*b**4*x**4 - 1326204*a**6*b**5*x**5 - 2318316*a**5*b**6*x**6 -
2604294*a**4*b**7*x**7 - 1905750*a**3*b**8*x**8 - 882420*a**2*b**9*x**9 -
235620*a*b**10*x**10 - 27720*b**11*x**11)/(126*a**21*x**3 + 1134*a**20*b*x
**4 + 4536*a**19*b**2*x**5 + 10584*a**18*b**3*x**6 + 15876*a**17*b**4*x**7
 + 15876*a**16*b**5*x**8 + 10584*a**15*b**6*x**9 + 4536*a**14*b**7*x**10 +
 1134*a**13*b**8*x**11 + 126*a**12*b**9*x**12) + 220*b**3*(-log(x) + log(a
/b + x))/a**13
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = -\frac{27720\,b^{11}x^{11}+235620\,ab^{10}x^{10}+882420\,a^2b^9x^9+1905750\,a^3b^8x^8+2604294\,a^4b^7x^7+2318316\,a^5b^6x^6}{126\left(a^{12}b^9x^{12}+9\,a^{13}b^8x^{11}+36\,a^{14}b^7x^{10}+84\,a^{15}b^6x^9+126\,a^{16}b^5x^8+1\right.} + \frac{220\,b^3\log(bx+a)}{a^{13}} - \frac{220\,b^3\log(x)}{a^{13}}$$

input
```
integrate(1/x^4/(b*x+a)^10,x, algorithm="maxima")
```

output
```
-1/126*(27720*b^11*x^11 + 235620*a*b^10*x^10 + 882420*a^2*b^9*x^9 + 190575
0*a^3*b^8*x^8 + 2604294*a^4*b^7*x^7 + 2318316*a^5*b^6*x^6 + 1326204*a^6*b^
5*x^5 + 456291*a^7*b^4*x^4 + 78419*a^8*b^3*x^3 + 2772*a^9*b^2*x^2 - 252*a^
10*b*x + 42*a^11)/(a^12*b^9*x^12 + 9*a^13*b^8*x^11 + 36*a^14*b^7*x^10 + 84
*a^15*b^6*x^9 + 126*a^16*b^5*x^8 + 126*a^17*b^4*x^7 + 84*a^18*b^3*x^6 + 36
*a^19*b^2*x^5 + 9*a^20*b*x^4 + a^21*x^3) + 220*b^3*log(b*x + a)/a^13 - 220
*b^3*log(x)/a^13
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = \frac{220\,b^3\log\left(|bx+a|\right)}{a^{13}} - \frac{220\,b^3\log\left(|x|\right)}{a^{13}} - \frac{27720\,ab^{11}x^{11} + 235620\,a^2b^{10}x^{10} + 882420\,a^3b^9x^9 + 1905750\,a^4b^8x^8 + 2604294\,a^5b^7x^7 + 2318316\,a^6b^6x}{126\,(bx+a)^9a^{13}}$$

input
```
integrate(1/x^4/(b*x+a)^10,x, algorithm="giac")
```

output
```
220*b^3*log(abs(b*x + a))/a^13 - 220*b^3*log(abs(x))/a^13 - 1/126*(27720*a
*b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 1905750*a^4*b^8*x
^8 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*b^5*x^5 + 456
291*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*a^11*b*x + 4
2*a^12)/((b*x + a)^9*a^13*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = \frac{440\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{13}} - \frac{\frac{1}{3\,a} + \frac{22\,b^2\,x^2}{a^3} + \frac{78419\,b^3\,x^3}{126\,a^4} + \frac{50699\,b^4\,x^4}{14\,a^5} + \frac{73678\,b^5\,x^5}{7\,a^6} + \frac{55198\,b^6\,x^6}{3\,a^7} + \frac{20669\,b^7\,x^7}{a^8} + \frac{15125\,b^8\,x^8}{a^9} + \frac{21010\,b^9\,x^9}{3\,a^{10}} + \frac{1870\,b^{10}}{a^{11}}}{a^9\,x^3 + 9\,a^8\,b\,x^4 + 36\,a^7\,b^2\,x^5 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^7 + 126\,a^4\,b^5\,x^8 + 84\,a^3\,b^6\,x^9 + 36\,a^2\,b^7\,x^{10} +}$$

input
```
int(1/(x^4*(a + b*x)^10),x)
```

output
```
(440*b^3*atanh((2*b*x)/a + 1))/a^13 - (1/(3*a) + (22*b^2*x^2)/a^3 + (78419
*b^3*x^3)/(126*a^4) + (50699*b^4*x^4)/(14*a^5) + (73678*b^5*x^5)/(7*a^6) +
 (55198*b^6*x^6)/(3*a^7) + (20669*b^7*x^7)/a^8 + (15125*b^8*x^8)/a^9 + (21
010*b^9*x^9)/(3*a^10) + (1870*b^10*x^10)/a^11 + (220*b^11*x^11)/a^12 - (2*
b*x)/a^2)/(a^9*x^3 + b^9*x^12 + 9*a^8*b*x^4 + 9*a*b^8*x^11 + 36*a^7*b^2*x^
5 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^7 + 126*a^4*b^5*x^8 + 84*a^3*b^6*x^9 +
36*a^2*b^7*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.59

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = \frac{27720\log(bx+a)\,a^9b^3x^3 + 249480\log(bx+a)\,a^8b^4x^4 + 997920\log(bx+a)\,a^7b^5x^5 + 2328480\log(bx+a}{}$$

input
```
int(1/x^4/(b*x+a)^10,x)
```

output
```
(27720*log(a + b*x)*a**9*b**3*x**3 + 249480*log(a + b*x)*a**8*b**4*x**4 +
997920*log(a + b*x)*a**7*b**5*x**5 + 2328480*log(a + b*x)*a**6*b**6*x**6 +
 3492720*log(a + b*x)*a**5*b**7*x**7 + 3492720*log(a + b*x)*a**4*b**8*x**8
 + 2328480*log(a + b*x)*a**3*b**9*x**9 + 997920*log(a + b*x)*a**2*b**10*x*
*10 + 249480*log(a + b*x)*a*b**11*x**11 + 27720*log(a + b*x)*b**12*x**12 -
 27720*log(x)*a**9*b**3*x**3 - 249480*log(x)*a**8*b**4*x**4 - 997920*log(x
)*a**7*b**5*x**5 - 2328480*log(x)*a**6*b**6*x**6 - 3492720*log(x)*a**5*b**
7*x**7 - 3492720*log(x)*a**4*b**8*x**8 - 2328480*log(x)*a**3*b**9*x**9 - 9
97920*log(x)*a**2*b**10*x**10 - 249480*log(x)*a*b**11*x**11 - 27720*log(x)
*b**12*x**12 - 42*a**12 + 252*a**11*b*x - 2772*a**10*b**2*x**2 - 75339*a**
9*b**3*x**3 - 428571*a**8*b**4*x**4 - 1215324*a**7*b**5*x**5 - 2059596*a**
6*b**6*x**6 - 2216214*a**5*b**7*x**7 - 1517670*a**4*b**8*x**8 - 623700*a**
3*b**9*x**9 - 124740*a**2*b**10*x**10 + 3080*b**12*x**12)/(126*a**13*x**3*
(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4
*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**7*x**7 + 9*a*b
**8*x**8 + b**9*x**9))
```

3.198 $\int \frac{(a+bx)^{12}}{x^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 141

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx = -\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}+220a^3b^9\log(x)$$

output
```
-1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^
4/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2
*b^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx = -\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}+220a^3b^9\log(x)$$

input
```
Integrate[(a + b*x)^12/x^10,x]
```

output
```
-1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)
/(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396
*a^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3
)/3 + 220*a^3*b^9*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^{12}}{x^{10}}+\frac{12a^{11}b}{x^9}+\frac{66a^{10}b^2}{x^8}+\frac{220a^9b^3}{x^7}+\frac{495a^8b^4}{x^6}+\frac{792a^7b^5}{x^5}+\frac{924a^6b^6}{x^4}+\frac{792a^5b^7}{x^3}+\frac{495a^4b^8}{x^2}+\frac{220a^3b^9}{x}+6\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+220a^3b^9\log(x)+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

input
```
Int[(a + b*x)^12/x^10,x]
```

output
```
-1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)
/(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396
*a^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3
)/3 + 220*a^3*b^9*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+$
risch	$\frac{b^{12}x^3}{3}+6a\,b^{11}x^2+66a^2b^{10}x+\frac{-495a^4x^8b^8-396a^5x^7b^7-308a^6x^6b^6-198a^7x^5b^5-99a^8x^4b^4-\frac{110}{3}a^9x^3b^3-\frac{66}{7}a^{10}}{x^9}$
norman	$\frac{-\frac{1}{9}a^{12}+\frac{1}{3}b^{12}x^{12}+6a\,x^{11}b^{11}+66a^2x^{10}b^{10}-495a^4x^8b^8-396a^5x^7b^7-308a^6x^6b^6-198a^7x^5b^5-99a^8x^4b^4-\frac{110}{3}a^9x^3b^3-\frac{66}{7}a^{10}}{x^9}$
parallelrisch	$\frac{42b^{12}x^{12}+756a\,x^{11}b^{11}+27720\ln(x)x^9a^3b^9+8316a^2x^{10}b^{10}-62370a^4x^8b^8-49896a^5x^7b^7-38808a^6x^6b^6-24948a^7x^5b^5}{126x^9}$

input
```
int((b*x+a)^12/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^
4/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2
*b^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx = \frac{42\,b^{12}x^{12}+756\,ab^{11}x^{11}+8316\,a^2b^{10}x^{10}+27720\,a^3b^9x^9\log(x)-62370\,a^4b^8x^8-49896\,a^5b^7x^7-38808}{126\,x^9}$$

input
```
integrate((b*x+a)^12/x^10,x, algorithm="fricas")
```

output
```
1/126*(42*b^12*x^12 + 756*a*b^11*x^11 + 8316*a^2*b^10*x^10 + 27720*a^3*b^9
*x^9*log(x) - 62370*a^4*b^8*x^8 - 49896*a^5*b^7*x^7 - 38808*a^6*b^6*x^6 -
24948*a^7*b^5*x^5 - 12474*a^8*b^4*x^4 - 4620*a^9*b^3*x^3 - 1188*a^10*b^2*x
^2 - 189*a^11*b*x - 14*a^12)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx = 220a^3b^9\log(x) + 66a^2b^{10}x + 6ab^{11}x^2 + \frac{b^{12}x^3}{3} + \frac{-14a^{12} - 189a^{11}bx - 1188a^{10}b^2x^2 - 4620a^9b^3x^3 - 12474a^8b^4x^4 - 24948a^7b^5x^5 - 38808a^6b^6x^6 - 4989}{126x^9}$$

input
```
integrate((b*x+a)**12/x**10,x)
```

output
```
220*a**3*b**9*log(x) + 66*a**2*b**10*x + 6*a*b**11*x**2 + b**12*x**3/3 + (
-14*a**12 - 189*a**11*b*x - 1188*a**10*b**2*x**2 - 4620*a**9*b**3*x**3 - 1
2474*a**8*b**4*x**4 - 24948*a**7*b**5*x**5 - 38808*a**6*b**6*x**6 - 49896*
a**5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx = \frac{1}{3}\,b^{12}x^3 + 6\,ab^{11}x^2 + 66\,a^2b^{10}x + 220\,a^3b^9\log(x) - \frac{62370\,a^4b^8x^8 + 49896\,a^5b^7x^7 + 38808\,a^6b^6x^6 + 24948\,a^7b^5x^5 + 12474\,a^8b^4x^4 + 4620\,a^9b^3x^3 + 1188\,a^{10}}{126\,x^9}$$

input
```
integrate((b*x+a)^12/x^10,x, algorithm="maxima")
```

output
```
1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(x) - 1/126*(
62370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b^5*
x^5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^11*
b*x + 14*a^12)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx = \frac{1}{3}\,b^{12}x^3 + 6\,ab^{11}x^2 + 66\,a^2b^{10}x + 220\,a^3b^9\log\left(|x|\right) - \frac{62370\,a^4b^8x^8 + 49896\,a^5b^7x^7 + 38808\,a^6b^6x^6 + 24948\,a^7b^5x^5 + 12474\,a^8b^4x^4 + 4620\,a^9b^3x^3 + 1188\,a^{10}b}{126\,x^9}$$

input
```
integrate((b*x+a)^12/x^10,x, algorithm="giac")
```

output
```
1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(abs(x)) - 1/
126*(62370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7
*b^5*x^5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*
a^11*b*x + 14*a^12)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx = \frac{b^{12}\,x^3}{3} - \frac{\frac{a^{12}}{9} + \frac{3\,a^{11}\,b\,x}{2} + \frac{66\,a^{10}\,b^2\,x^2}{7} + \frac{110\,a^9\,b^3\,x^3}{3} + 99\,a^8\,b^4\,x^4 + 198\,a^7\,b^5\,x^5 + 308\,a^6\,b^6\,x^6 + 396\,a^5\,b^7\,x^7 + 495\,a^4\,b}{x^9} + 66\,a^2\,b^{10}\,x + 6\,a\,b^{11}\,x^2 + 220\,a^3\,b^9\,\ln\left(x\right)$$

input
```
int((a + b*x)^12/x^10,x)
```

output
```
(b^12*x^3)/3 - (a^12/9 + (66*a^10*b^2*x^2)/7 + (110*a^9*b^3*x^3)/3 + 99*a^
8*b^4*x^4 + 198*a^7*b^5*x^5 + 308*a^6*b^6*x^6 + 396*a^5*b^7*x^7 + 495*a^4*
b^8*x^8 + (3*a^11*b*x)/2)/x^9 + 66*a^2*b^10*x + 6*a*b^11*x^2 + 220*a^3*b^9
*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx$$
$$= \frac{27720\log(x)\,a^3b^9x^9 - 14a^{12} - 189a^{11}bx - 1188a^{10}b^2x^2 - 4620a^9b^3x^3 - 12474a^8b^4x^4 - 24948a^7b^5x^5 - 3}{126x^9}$$

input
```
int((b*x+a)^12/x^10,x)
```

output
```
(27720*log(x)*a**3*b**9*x**9 - 14*a**12 - 189*a**11*b*x - 1188*a**10*b**2*
x**2 - 4620*a**9*b**3*x**3 - 12474*a**8*b**4*x**4 - 24948*a**7*b**5*x**5 -
 38808*a**6*b**6*x**6 - 49896*a**5*b**7*x**7 - 62370*a**4*b**8*x**8 + 8316
*a**2*b**10*x**10 + 756*a*b**11*x**11 + 42*b**12*x**12)/(126*x**9)
```

3.199 $\int \frac{(a+bx)^{11}}{x^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 132

$$\int \frac{(a+bx)^{11}}{x^{10}} dx = -\frac{a^{11}}{9x^9} - \frac{11a^{10}b}{8x^8} - \frac{55a^9b^2}{7x^7} - \frac{55a^8b^3}{2x^6} - \frac{66a^7b^4}{x^5} - \frac{231a^6b^5}{2x^4} - \frac{154a^5b^6}{x^3} - \frac{165a^4b^7}{x^2} - \frac{165a^3b^8}{x} + 11ab^{10}x + \frac{b^{11}x^2}{2} + 55a^2b^9\log(x)$$

output
```
-1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4
/x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*
b^10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{11}}{x^{10}} dx = -\frac{a^{11}}{9x^9} - \frac{11a^{10}b}{8x^8} - \frac{55a^9b^2}{7x^7} - \frac{55a^8b^3}{2x^6} - \frac{66a^7b^4}{x^5} - \frac{231a^6b^5}{2x^4} - \frac{154a^5b^6}{x^3} - \frac{165a^4b^7}{x^2} - \frac{165a^3b^8}{x} + 11ab^{10}x + \frac{b^{11}x^2}{2} + 55a^2b^9\log(x)$$

input
```
Integrate[(a + b*x)^11/x^10,x]
```

output
```
-1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/
(2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (
165*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b
^9*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{49, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{a^{11}}{x^{10}}+\frac{11a^{10}b}{x^9}+\frac{55a^9b^2}{x^8}+\frac{165a^8b^3}{x^7}+\frac{330a^7b^4}{x^6}+\frac{462a^6b^5}{x^5}+\frac{462a^5b^6}{x^4}+\frac{330a^4b^7}{x^3}+\frac{165a^3b^8}{x^2}+\frac{55a^2b^9}{x}+11a$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+ 55a^2b^9\log(x)+11ab^{10}x+\frac{b^{11}x^2}{2}$$

input
```
Int[(a + b*x)^11/x^10,x]
```

output
```
-1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/
(2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (
165*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b
^9*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+11a\,b^{10}x+$
risch	$\frac{b^{11}x^2}{2}+11a\,b^{10}x+\frac{-165a^3x^8b^8-165a^4b^7x^7-154a^5b^6x^6-\frac{231}{2}a^6b^5x^5-66a^7b^4x^4-\frac{55}{2}b^3a^8x^3-\frac{55}{7}b^2a^9x^2-\frac{11}{8}a^{10}bx-\frac{1}{9}}{x^9}$
norman	$\frac{-\frac{1}{9}a^{11}+\frac{1}{2}b^{11}x^{11}+11a\,x^{10}b^{10}-165a^3x^8b^8-165a^4b^7x^7-154a^5b^6x^6-\frac{231}{2}a^6b^5x^5-66a^7b^4x^4-\frac{11}{8}a^{10}bx-\frac{55}{7}b^2a^9x^2-\frac{55}{2}b^3a^8x^3}{x^9}$
parallelrisch	$\frac{252b^{11}x^{11}+27720\ln(x)x^9a^2b^9+5544a\,x^{10}b^{10}-83160a^3x^8b^8-83160a^4b^7x^7-77616a^5b^6x^6-58212a^6b^5x^5-33264a^7b^4x^4-13}{504x^9}$

input
```
int((b*x+a)^11/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4
/x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*
b^10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx$$
$$=\frac{252\,b^{11}x^{11}+5544\,ab^{10}x^{10}+27720\,a^2b^9x^9\log(x)-83160\,a^3b^8x^8-83160\,a^4b^7x^7-77616\,a^5b^6x^6-58212}{504\,x^9}$$

input
```
integrate((b*x+a)^11/x^10,x, algorithm="fricas")
```

output
```
1/504*(252*b^11*x^11 + 5544*a*b^10*x^10 + 27720*a^2*b^9*x^9*log(x) - 83160
*a^3*b^8*x^8 - 83160*a^4*b^7*x^7 - 77616*a^5*b^6*x^6 - 58212*a^6*b^5*x^5 -
 33264*a^7*b^4*x^4 - 13860*a^8*b^3*x^3 - 3960*a^9*b^2*x^2 - 693*a^10*b*x -
 56*a^11)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx = 55a^2b^9\log(x) + 11ab^{10}x + \frac{b^{11}x^2}{2} + \frac{-56a^{11} - 693a^{10}bx - 3960a^9b^2x^2 - 13860a^8b^3x^3 - 33264a^7b^4x^4 - 58212a^6b^5x^5 - 77616a^5b^6x^6 - 831}{504x^9}$$

input
```
integrate((b*x+a)**11/x**10,x)
```

output
```
55*a**2*b**9*log(x) + 11*a*b**10*x + b**11*x**2/2 + (-56*a**11 - 693*a**10
*b*x - 3960*a**9*b**2*x**2 - 13860*a**8*b**3*x**3 - 33264*a**7*b**4*x**4 -
 58212*a**6*b**5*x**5 - 77616*a**5*b**6*x**6 - 83160*a**4*b**7*x**7 - 8316
0*a**3*b**8*x**8)/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx = \frac{1}{2}\,b^{11}x^2 + 11\,ab^{10}x + 55\,a^2b^9\log(x) - \frac{83160\,a^3b^8x^8 + 83160\,a^4b^7x^7 + 77616\,a^5b^6x^6 + 58212\,a^6b^5x^5 + 33264\,a^7b^4x^4 + 13860\,a^8b^3x^3 + 3960\,a^9}{504\,x^9}$$

input
```
integrate((b*x+a)^11/x^10,x, algorithm="maxima")
```

output
```
1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(x) - 1/504*(83160*a^3*b^8*x^8
+ 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7*b^
4*x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx = \frac{1}{2}\,b^{11}x^2 + 11\,ab^{10}x + 55\,a^2b^9\log\left(|x|\right) - \frac{83160\,a^3b^8x^8 + 83160\,a^4b^7x^7 + 77616\,a^5b^6x^6 + 58212\,a^6b^5x^5 + 33264\,a^7b^4x^4 + 13860\,a^8b^3x^3 + 3960\,a^9}{504\,x^9}$$

input
```
integrate((b*x+a)^11/x^10,x, algorithm="giac")
```

output
```
1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(abs(x)) - 1/504*(83160*a^3*b^8
*x^8 + 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a
^7*b^4*x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11
)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx = \frac{b^{11}\,x^2}{2} - \frac{\frac{a^{11}}{9} + \frac{11\,a^{10}\,b\,x}{8} + \frac{55\,a^9\,b^2\,x^2}{7} + \frac{55\,a^8\,b^3\,x^3}{2} + 66\,a^7\,b^4\,x^4 + \frac{231\,a^6\,b^5\,x^5}{2} + 154\,a^5\,b^6\,x^6 + 165\,a^4\,b^7\,x^7 + 165\,a^3\,b^8\,x}{x^9} + 55\,a^2\,b^9\,\ln\left(x\right) + 11\,a\,b^{10}\,x$$

input
```
int((a + b*x)^11/x^10,x)
```

output
```
(b^11*x^2)/2 - (a^11/9 + (55*a^9*b^2*x^2)/7 + (55*a^8*b^3*x^3)/2 + 66*a^7*
b^4*x^4 + (231*a^6*b^5*x^5)/2 + 154*a^5*b^6*x^6 + 165*a^4*b^7*x^7 + 165*a^
3*b^8*x^8 + (11*a^10*b*x)/8)/x^9 + 55*a^2*b^9*log(x) + 11*a*b^10*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx$$
$$= \frac{27720\log(x)\,a^2b^9x^9 - 56a^{11} - 693a^{10}bx - 3960a^9b^2x^2 - 13860a^8b^3x^3 - 33264a^7b^4x^4 - 58212a^6b^5x^5 - 7}{504x^9}$$

input
```
int((b*x+a)^11/x^10,x)
```

output
```
(27720*log(x)*a**2*b**9*x**9 - 56*a**11 - 693*a**10*b*x - 3960*a**9*b**2*x
**2 - 13860*a**8*b**3*x**3 - 33264*a**7*b**4*x**4 - 58212*a**6*b**5*x**5 -
 77616*a**5*b**6*x**6 - 83160*a**4*b**7*x**7 - 83160*a**3*b**8*x**8 + 5544
*a*b**10*x**10 + 252*b**11*x**11)/(504*x**9)
```

3.200 $\int \frac{(a+bx)^{10}}{x^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 114

$$\int \frac{(a+bx)^{10}}{x^{10}} \, dx = -\frac{a^{10}}{9x^9} - \frac{5a^9b}{4x^8} - \frac{45a^8b^2}{7x^7} - \frac{20a^7b^3}{x^6} - \frac{42a^6b^4}{x^5} - \frac{63a^5b^5}{x^4} - \frac{70a^4b^6}{x^3} - \frac{60a^3b^7}{x^2} - \frac{45a^2b^8}{x} + b^{10}x + 10ab^9\log(x)$$

output
```
-1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5
-63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9
*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}} \, dx = -\frac{a^{10}}{9x^9} - \frac{5a^9b}{4x^8} - \frac{45a^8b^2}{7x^7} - \frac{20a^7b^3}{x^6} - \frac{42a^6b^4}{x^5} - \frac{63a^5b^5}{x^4} - \frac{70a^4b^6}{x^3} - \frac{60a^3b^7}{x^2} - \frac{45a^2b^8}{x} + b^{10}x + 10ab^9\log(x)$$

input
```
Integrate[(a + b*x)^10/x^10,x]
```

output
```
-1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^
6 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/
x^2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}}{x^{10}} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\frac{45a^2b^8}{x^2}+\frac{10ab^9}{x}+b^{10}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

input
```
Int[(a + b*x)^10/x^10,x]
```

output
```
-1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^
6 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/
x^2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10a\,b^9\ln$
risch	$b^{10}x+\frac{-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx-\frac{1}{9}a^{10}}{x^9}+10a\,b^9\ln(x$
norman	$\frac{b^{10}x^{10}-\frac{1}{9}a^{10}-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx}{x^9}+10a\,b^9\ln(x)$
parallelrisch	$\frac{2520a\,b^9\ln(x)x^9+252b^{10}x^{10}-11340a^2b^8x^8-15120a^3b^7x^7-17640a^4b^6x^6-15876a^5b^5x^5-10584a^6b^4x^4-5040a^7b^3x^3-1620a}{252x^9}$

input
```
int((b*x+a)^10/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5
-63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9
*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$
$$=\frac{252\,b^{10}x^{10}+2520\,ab^9x^9\log(x)-11340\,a^2b^8x^8-15120\,a^3b^7x^7-17640\,a^4b^6x^6-15876\,a^5b^5x^5-10584\,a}{252\,x^9}$$

input
```
integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

output
```
1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a
^3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 5
040*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = 10ab^9\log(x) + b^{10}x + \frac{-28a^{10} - 315a^9bx - 1620a^8b^2x^2 - 5040a^7b^3x^3 - 10584a^6b^4x^4 - 15876a^5b^5x^5 - 17640a^4b^6x^6 - 15120}{252x^9}$$

input
```
integrate((b*x+a)**10/x**10,x)
```

output
```
10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x*
*2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 1
7640*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x*
*9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = b^{10}x + 10\,ab^9\log(x) - \frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 1620\,a^8b}{252\,x^9}$$

input
```
integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

output
```
b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 +
17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x
^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = b^{10}x + 10\,ab^9\log\left(|x|\right)$$
$$-\frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 1620\,a^8b}{252\,x^9}$$

input
```
integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

output
```
b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x
^7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*
b^3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx =$$
$$-\frac{\frac{a^{10}}{9} - b^{10}\,x^{10} + \frac{45\,a^8\,b^2\,x^2}{7} + 20\,a^7\,b^3\,x^3 + 42\,a^6\,b^4\,x^4 + 63\,a^5\,b^5\,x^5 + 70\,a^4\,b^6\,x^6 + 60\,a^3\,b^7\,x^7 + 45\,a^2\,b^8\,x^8}{x^9}$$

input
```
int((a + b*x)^10/x^10,x)
```

output
```
-(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^
4 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5
*a^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$
$$= \frac{2520\log(x)\,a\,b^9x^9 - 28a^{10} - 315a^9bx - 1620a^8b^2x^2 - 5040a^7b^3x^3 - 10584a^6b^4x^4 - 15876a^5b^5x^5 - 1764}{252x^9}$$

input
```
int((b*x+a)^10/x^10,x)
```

output
```
(2520*log(x)*a*b**9*x**9 - 28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**2 -
 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 17640
*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8 + 252*b**10*
x**10)/(252*x**9)
```

3.201 $\int \frac{(a+bx)^9}{x^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 109

$$\int \frac{(a+bx)^9}{x^{10}}\,dx = -\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

output
```
-1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x
^5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^9}{x^{10}}\,dx = -\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

input
```
Integrate[(a + b*x)^9/x^10,x]
```

output
```
-1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
 - (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a
^2*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^9}{x^{10}}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^9}{x^{10}}+\frac{9a^8b}{x^9}+\frac{36a^7b^2}{x^8}+\frac{84a^6b^3}{x^7}+\frac{126a^5b^4}{x^6}+\frac{126a^4b^5}{x^5}+\frac{84a^3b^6}{x^4}+\frac{36a^2b^7}{x^3}+\frac{9ab^8}{x^2}+\frac{b^9}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

input
```
Int[(a + b*x)^9/x^10,x]
```

output
```
-1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
 - (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a
^2*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9a\,b^8}{x}+b^9\ln\left(x\right)$
norman	$\frac{-\frac{1}{9}a^9-9a\,x^8b^8-18a^2x^7b^7-\frac{63}{2}a^4x^5b^5-\frac{126}{5}a^5b^4x^4-14a^6b^3x^3-\frac{36}{7}a^7b^2x^2-\frac{9}{8}a^8bx-28x^6a^3b^6}{x^9}+b^9\ln\left(x\right)$
risch	$\frac{-\frac{1}{9}a^9-9a\,x^8b^8-18a^2x^7b^7-\frac{63}{2}a^4x^5b^5-\frac{126}{5}a^5b^4x^4-14a^6b^3x^3-\frac{36}{7}a^7b^2x^2-\frac{9}{8}a^8bx-28x^6a^3b^6}{x^9}+b^9\ln\left(x\right)$
parallelrisch	$\frac{2520\ln(x)x^9b^9-22680a\,x^8b^8-45360a^2x^7b^7-70560x^6a^3b^6-79380a^4x^5b^5-63504a^5b^4x^4-35280a^6b^3x^3-12960a^7b^2x^2-2835}{2520x^9}$

input
```
int((b*x+a)^9/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x
^5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^9}{x^{10}}\,dx = \frac{2520\,b^9x^9\log\left(x\right)-22680\,ab^8x^8-45360\,a^2b^7x^7-70560\,a^3b^6x^6-79380\,a^4b^5x^5-63504\,a^5b^4x^4-35280}{2520\,x^9}$$

input
```
integrate((b*x+a)^9/x^10,x, algorithm="fricas")
```

output
```
1/2520*(2520*b^9*x^9*log(x) - 22680*a*b^8*x^8 - 45360*a^2*b^7*x^7 - 70560*
a^3*b^6*x^6 - 79380*a^4*b^5*x^5 - 63504*a^5*b^4*x^4 - 35280*a^6*b^3*x^3 -
12960*a^7*b^2*x^2 - 2835*a^8*b*x - 280*a^9)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^9}{x^{10}}\,dx = b^9\log{(x)} + \frac{-280a^9 - 2835a^8bx - 12960a^7b^2x^2 - 35280a^6b^3x^3 - 63504a^5b^4x^4 - 79380a^4b^5x^5 - 70560a^3b^6x^6 - 45}{2520x^9}$$

input
```
integrate((b*x+a)**9/x**10,x)
```

output
```
b**9*log(x) + (-280*a**9 - 2835*a**8*b*x - 12960*a**7*b**2*x**2 - 35280*a*
*6*b**3*x**3 - 63504*a**5*b**4*x**4 - 79380*a**4*b**5*x**5 - 70560*a**3*b*
*6*x**6 - 45360*a**2*b**7*x**7 - 22680*a*b**8*x**8)/(2520*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^9}{x^{10}}\,dx = b^9\log{(x)} - \frac{22680\,ab^8x^8 + 45360\,a^2b^7x^7 + 70560\,a^3b^6x^6 + 79380\,a^4b^5x^5 + 63504\,a^5b^4x^4 + 35280\,a^6b^3x^3 + 12960\,a^7}{2520\,x^9}$$

input
```
integrate((b*x+a)^9/x^10,x, algorithm="maxima")
```

output
```
b^9*log(x) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b^6*x
^6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*a^7
*b^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^9}{x^{10}}\,dx = b^9\log\left(|x|\right) - \frac{22680\,ab^8x^8 + 45360\,a^2b^7x^7 + 70560\,a^3b^6x^6 + 79380\,a^4b^5x^5 + 63504\,a^5b^4x^4 + 35280\,a^6b^3x^3 + 12960\,a^7}{2520\,x^9}$$

input
```
integrate((b*x+a)^9/x^10,x, algorithm="giac")
```

output
```
b^9*log(abs(x)) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*
b^6*x^6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 1296
0*a^7*b^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^9}{x^{10}}\,dx = b^9\,\ln\left(x\right) - \frac{\frac{a^9}{9} + \frac{9\,a^8\,b\,x}{8} + \frac{36\,a^7\,b^2\,x^2}{7} + 14\,a^6\,b^3\,x^3 + \frac{126\,a^5\,b^4\,x^4}{5} + \frac{63\,a^4\,b^5\,x^5}{2} + 28\,a^3\,b^6\,x^6 + 18\,a^2\,b^7\,x^7 + 9\,a\,b^8\,x^8}{x^9}$$

input
```
int((a + b*x)^9/x^10,x)
```

output
```
b^9*log(x) - (a^9/9 + 9*a*b^8*x^8 + (36*a^7*b^2*x^2)/7 + 14*a^6*b^3*x^3 +
(126*a^5*b^4*x^4)/5 + (63*a^4*b^5*x^5)/2 + 28*a^3*b^6*x^6 + 18*a^2*b^7*x^7
 + (9*a^8*b*x)/8)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^9}{x^{10}}\,dx$$
$$= \frac{2520\log(x)\,b^9x^9 - 280a^9 - 2835a^8bx - 12960a^7b^2x^2 - 35280a^6b^3x^3 - 63504a^5b^4x^4 - 79380a^4b^5x^5 - 70}{2520x^9}$$

input
```
int((b*x+a)^9/x^10,x)
```

output
```
(2520*log(x)*b**9*x**9 - 280*a**9 - 2835*a**8*b*x - 12960*a**7*b**2*x**2 -
 35280*a**6*b**3*x**3 - 63504*a**5*b**4*x**4 - 79380*a**4*b**5*x**5 - 7056
0*a**3*b**6*x**6 - 45360*a**2*b**7*x**7 - 22680*a*b**8*x**8)/(2520*x**9)
```

3.202 $\int \frac{(a+bx)^8}{x^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{(a+bx)^8}{x^{10}} \, dx = -\frac{(a+bx)^9}{9ax^9}$$

output
```
-1/9*(b*x+a)^9/a/x^9
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.65

$$\int \frac{(a+bx)^8}{x^{10}} \, dx = -\frac{a^8}{9x^9} - \frac{a^7b}{x^8} - \frac{4a^6b^2}{x^7} - \frac{28a^5b^3}{3x^6} - \frac{14a^4b^4}{x^5} - \frac{14a^3b^5}{x^4} - \frac{28a^2b^6}{3x^3} - \frac{4ab^7}{x^2} - \frac{b^8}{x}$$

input
```
Integrate[(a + b*x)^8/x^10,x]
```

output
```
-1/9*a^8/x^9 - (a^7*b)/x^8 - (4*a^6*b^2)/x^7 - (28*a^5*b^3)/(3*x^6) - (14*
a^4*b^4)/x^5 - (14*a^3*b^5)/x^4 - (28*a^2*b^6)/(3*x^3) - (4*a*b^7)/x^2 - b
^8/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^8}{x^{10}}\,dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{(a+bx)^9}{9ax^9}$$

input
```
Int[(a + b*x)^8/x^10,x]
```

output
```
-1/9*(a + b*x)^9/(a*x^9)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(15) = 30.

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.24

method	result	size
gosper	$-\frac{9b^8x^8+36a\,x^7b^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5b^3x^3+36a^6x^2b^2+9a^7xb+a^8}{9x^9}$	89
orering	$-\frac{9b^8x^8+36a\,x^7b^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5b^3x^3+36a^6x^2b^2+9a^7xb+a^8}{9x^9}$	89
norman	$\frac{-b^8x^8-4a\,x^7b^7-\frac{28}{3}a^2x^6b^6-14a^3x^5b^5-14a^4x^4b^4-\frac{28}{3}a^5b^3x^3-4a^6x^2b^2-a^7xb-\frac{1}{9}a^8}{x^9}$	90
risch	$\frac{-b^8x^8-4a\,x^7b^7-\frac{28}{3}a^2x^6b^6-14a^3x^5b^5-14a^4x^4b^4-\frac{28}{3}a^5b^3x^3-4a^6x^2b^2-a^7xb-\frac{1}{9}a^8}{x^9}$	90
default	$-\frac{28a^2b^6}{3x^3}-\frac{14a^4b^4}{x^5}-\frac{4a\,b^7}{x^2}-\frac{4b^2a^6}{x^7}-\frac{14a^3b^5}{x^4}-\frac{a^7b}{x^8}-\frac{b^8}{x}-\frac{28a^5b^3}{3x^6}-\frac{a^8}{9x^9}$	91
parallelrisch	$\frac{-9b^8x^8-36a\,x^7b^7-84a^2x^6b^6-126a^3x^5b^5-126a^4x^4b^4-84a^5b^3x^3-36a^6x^2b^2-9a^7xb-a^8}{9x^9}$	91

input
```
int((b*x+a)^8/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(9*b^8*x^8+36*a*b^7*x^7+84*a^2*b^6*x^6+126*a^3*b^5*x^5+126*a^4*b^4*x^
4+84*a^5*b^3*x^3+36*a^6*b^2*x^2+9*a^7*b*x+a^8)/x^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int \frac{(a+bx)^8}{x^{10}}\,dx = -\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

input
```
integrate((b*x+a)^8/x^10,x, algorithm="fricas")
```

output
```
-1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^
4*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(14) = 28$.

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.59

$$\int \frac{(a+bx)^8}{x^{10}}\,dx = \frac{-a^8 - 9a^7bx - 36a^6b^2x^2 - 84a^5b^3x^3 - 126a^4b^4x^4 - 126a^3b^5x^5 - 84a^2b^6x^6 - 36ab^7x^7 - 9b^8x^8}{9x^9}$$

input
```
integrate((b*x+a)**8/x**10,x)
```

output
```
(-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**
4*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*
x**8)/(9*x**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int \frac{(a+bx)^8}{x^{10}}\,dx = -\frac{9\,b^8x^8 + 36\,ab^7x^7 + 84\,a^2b^6x^6 + 126\,a^3b^5x^5 + 126\,a^4b^4x^4 + 84\,a^5b^3x^3 + 36\,a^6b^2x^2 + 9\,a^7bx + a^8}{9\,x^9}$$

input
```
integrate((b*x+a)^8/x^10,x, algorithm="maxima")
```

output
```
-1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^
4*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int \frac{(a+bx)^8}{x^{10}}\,dx = -\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

input
```
integrate((b*x+a)^8/x^10,x, algorithm="giac")
```

output
```
-1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^
4*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int \frac{(a+bx)^8}{x^{10}}\,dx = -\frac{\frac{a^8}{9}+a^7\,b\,x+4\,a^6\,b^2\,x^2+\frac{28\,a^5\,b^3\,x^3}{3}+14\,a^4\,b^4\,x^4+14\,a^3\,b^5\,x^5+\frac{28\,a^2\,b^6\,x^6}{3}+4\,a\,b^7\,x^7+b^8\,x^8}{x^9}$$

input
```
int((a + b*x)^8/x^10,x)
```

output
```
-(a^8/9 + b^8*x^8 + 4*a*b^7*x^7 + 4*a^6*b^2*x^2 + (28*a^5*b^3*x^3)/3 + 14*
a^4*b^4*x^4 + 14*a^3*b^5*x^5 + (28*a^2*b^6*x^6)/3 + a^7*b*x)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.29

$$\int \frac{(a+bx)^8}{x^{10}}\,dx$$
$$= \frac{-9b^8x^8 - 36a\,b^7x^7 - 84a^2b^6x^6 - 126a^3b^5x^5 - 126a^4b^4x^4 - 84a^5b^3x^3 - 36a^6b^2x^2 - 9a^7bx - a^8}{9x^9}$$

input
```
int((b*x+a)^8/x^10,x)
```

output
```
( - a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b
**4*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**
8*x**8)/(9*x**9)
```

3.203 $\int \frac{(a+bx)^7}{x^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{(a+bx)^7}{x^{10}} \, dx = -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}$$

output
```
-1/9*(b*x+a)^8/a/x^9+1/72*b*(b*x+a)^8/a^2/x^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(36) = 72.

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \frac{(a+bx)^7}{x^{10}} \, dx = -\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

input
```
Integrate[(a + b*x)^7/x^10,x]
```

output
```
-1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6)
- (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{x^{10}} \, dx$$

$$\Big\downarrow \; 55$$

$$-\frac{b \int \frac{(a+bx)^7}{x^9}dx}{9a}-\frac{(a+bx)^8}{9ax^9}$$

$$\Big\downarrow \; 48$$

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

input
```
Int[(a + b*x)^7/x^10,x]
```

output
```
-1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

method	result	size
norman	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
risch	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
gosper	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80
default	$-\frac{7a\,b^6}{3x^3}-\frac{7a^3b^4}{x^5}-\frac{b^7}{2x^2}-\frac{3a^5b^2}{x^7}-\frac{21a^2b^5}{4x^4}-\frac{7a^6b}{8x^8}-\frac{35a^4b^3}{6x^6}-\frac{a^7}{9x^9}$	80
parallelrisch	$\frac{-36b^7x^7-168a\,b^6x^6-378a^2b^5x^5-504a^3b^4x^4-420a^4b^3x^3-216a^5b^2x^2-63a^6bx-8a^7}{72x^9}$	80
orering	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80

input
```
int((b*x+a)^7/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/2*b^7*x^7-7/3*a*b^6*x^6-21/4*a^2*b^5*x^5-7*a^3*b^4*x^4-35/6*a^4*
b^3*x^3-3*a^5*b^2*x^2-7/8*a^6*b*x-1/9*a^7)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{36\,b^7x^7 + 168\,ab^6x^6 + 378\,a^2b^5x^5 + 504\,a^3b^4x^4 + 420\,a^4b^3x^3 + 216\,a^5b^2x^2 + 63\,a^6bx + 8\,a^7}{72\,x^9}$$

input
```
integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

output
```
-1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 42
0*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = \frac{-8a^7 - 63a^6bx - 216a^5b^2x^2 - 420a^4b^3x^3 - 504a^3b^4x^4 - 378a^2b^5x^5 - 168ab^6x^6 - 36b^7x^7}{72x^9}$$

input
```
integrate((b*x+a)**7/x**10,x)
```

output
```
(-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**
3*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**
9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{36\,b^7x^7 + 168\,ab^6x^6 + 378\,a^2b^5x^5 + 504\,a^3b^4x^4 + 420\,a^4b^3x^3 + 216\,a^5b^2x^2 + 63\,a^6bx + 8\,a^7}{72\,x^9}$$

input
```
integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

output
```
-1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 42
0*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{36\,b^7x^7 + 168\,ab^6x^6 + 378\,a^2b^5x^5 + 504\,a^3b^4x^4 + 420\,a^4b^3x^3 + 216\,a^5b^2x^2 + 63\,a^6bx + 8\,a^7}{72\,x^9}$$

input
```
integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

output
```
-1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 42
0*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = -\frac{(8\,a-b\,x)\,\left(a+b\,x\right)^8}{72\,a^2\,x^9}$$

input
```
int((a + b*x)^7/x^10,x)
```

output
```
-((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx)^7}{x^{10}}\,dx = \frac{-36b^7x^7-168a\,b^6x^6-378a^2b^5x^5-504a^3b^4x^4-420a^4b^3x^3-216a^5b^2x^2-63a^6bx-8a^7}{72x^9}$$

input
```
int((b*x+a)^7/x^10,x)
```

output
```
( - 8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a
**3*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x
**9)
```

3.204 $\int \frac{(a+bx)^6}{x^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{(a+bx)^6}{x^{10}} dx = -\frac{(a+bx)^7}{9ax^9} + \frac{b(a+bx)^7}{36a^2x^8} - \frac{b^2(a+bx)^7}{252a^3x^7}$$

output
```
-1/9*(b*x+a)^7/a/x^9+1/36*b*(b*x+a)^7/a^2/x^8-1/252*b^2*(b*x+a)^7/a^3/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^6}{x^{10}} dx = -\frac{a^6}{9x^9} - \frac{3a^5b}{4x^8} - \frac{15a^4b^2}{7x^7} - \frac{10a^3b^3}{3x^6} - \frac{3a^2b^4}{x^5} - \frac{3ab^5}{2x^4} - \frac{b^6}{3x^3}$$

input
```
Integrate[(a + b*x)^6/x^10,x]
```

output
```
-1/9*a^6/x^9 - (3*a^5*b)/(4*x^8) - (15*a^4*b^2)/(7*x^7) - (10*a^3*b^3)/(3*
x^6) - (3*a^2*b^4)/x^5 - (3*a*b^5)/(2*x^4) - b^6/(3*x^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^6}{x^{10}} \, dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{2b \int \frac{(a+bx)^6}{x^9}dx}{9a}-\frac{(a+bx)^7}{9ax^9}$$

$$\Big\downarrow \text{55}$$

$$-\frac{2b \left(-\frac{b \int \frac{(a+bx)^6}{x^8}dx}{8a}-\frac{(a+bx)^7}{8ax^8}\right)}{9a}-\frac{(a+bx)^7}{9ax^9}$$

$$\Big\downarrow \text{48}$$

$$-\frac{2b \left(\frac{b(a+bx)^7}{56a^2x^7}-\frac{(a+bx)^7}{8ax^8}\right)}{9a}-\frac{(a+bx)^7}{9ax^9}$$

input
```
Int[(a + b*x)^6/x^10,x]
```

output
```
-1/9*(a + b*x)^7/(a*x^9) - (2*b*(-1/8*(a + b*x)^7/(a*x^8) + (b*(a + b*x)^7
)/(56*a^2*x^7)))/(9*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

method	result	size
norman	$\frac{-\frac{1}{3} b^{6} x^{6}-\frac{3}{2} a\,x^{5} b^{5}-3 a^{2} x^{4} b^{4}-\frac{10}{3} a^{3} x^{3} b^{3}-\frac{15}{7} a^{4} x^{2} b^{2}-\frac{3}{4} a^{5} x b-\frac{1}{9} a^{6}}{x^{9}}$	68
risch	$\frac{-\frac{1}{3} b^{6} x^{6}-\frac{3}{2} a\,x^{5} b^{5}-3 a^{2} x^{4} b^{4}-\frac{10}{3} a^{3} x^{3} b^{3}-\frac{15}{7} a^{4} x^{2} b^{2}-\frac{3}{4} a^{5} x b-\frac{1}{9} a^{6}}{x^{9}}$	68
gosper	$-\frac{84 b^{6} x^{6}+378 a\,x^{5} b^{5}+756 a^{2} x^{4} b^{4}+840 a^{3} x^{3} b^{3}+540 a^{4} x^{2} b^{2}+189 a^{5} x b+28 a^{6}}{252 x^{9}}$	69
default	$-\frac{b^{6}}{3 x^{3}}-\frac{3 a^{2} b^{4}}{x^{5}}-\frac{15 a^{4} b^{2}}{7 x^{7}}-\frac{3 a\,b^{5}}{2 x^{4}}-\frac{3 a^{5} b}{4 x^{8}}-\frac{10 a^{3} b^{3}}{3 x^{6}}-\frac{a^{6}}{9 x^{9}}$	69
parallelrisch	$\frac{-84 b^{6} x^{6}-378 a\,x^{5} b^{5}-756 a^{2} x^{4} b^{4}-840 a^{3} x^{3} b^{3}-540 a^{4} x^{2} b^{2}-189 a^{5} x b-28 a^{6}}{252 x^{9}}$	69
orering	$-\frac{84 b^{6} x^{6}+378 a\,x^{5} b^{5}+756 a^{2} x^{4} b^{4}+840 a^{3} x^{3} b^{3}+540 a^{4} x^{2} b^{2}+189 a^{5} x b+28 a^{6}}{252 x^{9}}$	69

input
```
int((b*x+a)^6/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/3*b^6*x^6-3/2*a*x^5*b^5-3*a^2*x^4*b^4-10/3*a^3*x^3*b^3-15/7*a^4*
x^2*b^2-3/4*a^5*x*b-1/9*a^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = -\frac{84\,b^6x^6 + 378\,ab^5x^5 + 756\,a^2b^4x^4 + 840\,a^3b^3x^3 + 540\,a^4b^2x^2 + 189\,a^5bx + 28\,a^6}{252\,x^9}$$

input
```
integrate((b*x+a)^6/x^10,x, algorithm="fricas")
```

output
```
-1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 5
40*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = \frac{-28a^6 - 189a^5bx - 540a^4b^2x^2 - 840a^3b^3x^3 - 756a^2b^4x^4 - 378ab^5x^5 - 84b^6x^6}{252x^9}$$

input
```
integrate((b*x+a)**6/x**10,x)
```

output
```
(-28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756*a
**2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = -\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

input
```
integrate((b*x+a)^6/x^10,x, algorithm="maxima")
```

output
```
-1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 5
40*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = -\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

input
```
integrate((b*x+a)^6/x^10,x, algorithm="giac")
```

output
```
-1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 5
40*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = -\frac{\frac{a^6}{9}+\frac{3\,a^5\,b\,x}{4}+\frac{15\,a^4\,b^2\,x^2}{7}+\frac{10\,a^3\,b^3\,x^3}{3}+3\,a^2\,b^4\,x^4+\frac{3\,a\,b^5\,x^5}{2}+\frac{b^6\,x^6}{3}}{x^9}$$

input
```
int((a + b*x)^6/x^10,x)
```

output
```
-(a^6/9 + (b^6*x^6)/3 + (3*a*b^5*x^5)/2 + (15*a^4*b^2*x^2)/7 + (10*a^3*b^3
*x^3)/3 + 3*a^2*b^4*x^4 + (3*a^5*b*x)/4)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = \frac{-84b^6x^6-378a\,b^5x^5-756a^2b^4x^4-840a^3b^3x^3-540a^4b^2x^2-189a^5bx-28a^6}{252x^9}$$

input
```
int((b*x+a)^6/x^10,x)
```

output
```
( - 28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756
*a**2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

3.205 $\int \frac{(a+bx)^5}{x^{10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{(a+bx)^5}{x^{10}} \, dx = -\frac{a^5}{9x^9} - \frac{5a^4b}{8x^8} - \frac{10a^3b^2}{7x^7} - \frac{5a^2b^3}{3x^6} - \frac{ab^4}{x^5} - \frac{b^5}{4x^4}$$

output
```
-1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*
b^5/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{x^{10}} \, dx = -\frac{a^5}{9x^9} - \frac{5a^4b}{8x^8} - \frac{10a^3b^2}{7x^7} - \frac{5a^2b^3}{3x^6} - \frac{ab^4}{x^5} - \frac{b^5}{4x^4}$$

input
```
Integrate[(a + b*x)^5/x^10,x]
```

output
```
-1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x
^6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{x^{10}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

input
```
Int[(a + b*x)^5/x^10,x]
```

output
```
-1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x
^6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result	size
norman	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
risch	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
gosper	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{a\,b^4}{x^5}-\frac{b^5}{4x^4}$	58
parallelrisch	$\frac{-126b^5x^5-504a\,b^4x^4-840a^2b^3x^3-720a^3b^2x^2-315a^4bx-56a^5}{504x^9}$	58
orering	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58

input
```
int((b*x+a)^5/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/4*b^5*x^5-a*b^4*x^4-5/3*a^2*b^3*x^3-10/7*a^3*b^2*x^2-5/8*a^4*b*x
-1/9*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

input
```
integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

output
```
-1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 +
315*a^4*b*x + 56*a^5)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = \frac{-56a^5 - 315a^4bx - 720a^3b^2x^2 - 840a^2b^3x^3 - 504ab^4x^4 - 126b^5x^5}{504x^9}$$

input
```
integrate((b*x+a)**5/x**10,x)
```

output
```
(-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a
*b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{126\,b^5x^5 + 504\,ab^4x^4 + 840\,a^2b^3x^3 + 720\,a^3b^2x^2 + 315\,a^4bx + 56\,a^5}{504\,x^9}$$

input
```
integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

output
```
-1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 +
315*a^4*b*x + 56*a^5)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{126\,b^5x^5 + 504\,ab^4x^4 + 840\,a^2b^3x^3 + 720\,a^3b^2x^2 + 315\,a^4bx + 56\,a^5}{504\,x^9}$$

input
```
integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

output
```
-1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 +
315*a^4*b*x + 56*a^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = -\frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x}{8} + \frac{10\,a^3\,b^2\,x^2}{7} + \frac{5\,a^2\,b^3\,x^3}{3} + a\,b^4\,x^4 + \frac{b^5\,x^5}{4}}{x^9}$$

input
```
int((a + b*x)^5/x^10,x)
```

output
```
-(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
 + (5*a^4*b*x)/8)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^5}{x^{10}}\,dx = \frac{-126b^5x^5 - 504a\,b^4x^4 - 840a^2b^3x^3 - 720a^3b^2x^2 - 315a^4bx - 56a^5}{504x^9}$$

input
```
int((b*x+a)^5/x^10,x)
```

output
```
( - 56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504
*a*b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

3.206 $\int \frac{(a+bx)^4}{x^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{(a+bx)^4}{x^{10}} dx = -\frac{a^4}{9x^9} - \frac{a^3 b}{2x^8} - \frac{6a^2b^2}{7x^7} - \frac{2ab^3}{3x^6} - \frac{b^4}{5x^5}$$

output
```
-1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^4}{x^{10}} dx = -\frac{a^4}{9x^9} - \frac{a^3 b}{2x^8} - \frac{6a^2b^2}{7x^7} - \frac{2ab^3}{3x^6} - \frac{b^4}{5x^5}$$

input
```
Integrate[(a + b*x)^4/x^10,x]
```

output
```
-1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
 b^4/(5*x^5)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{x^{10}} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^4}{x^{10}}+\frac{4a^3b}{x^9}+\frac{6a^2b^2}{x^8}+\frac{4ab^3}{x^7}+\frac{b^4}{x^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

input
```
Int[(a + b*x)^4/x^10,x]
```

output
```
-1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
 b^4/(5*x^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
norman	$\frac{-\frac{1}{5}b^4x^4-\frac{2}{3}a\,x^3b^3-\frac{6}{7}a^2b^2x^2-\frac{1}{2}a^3bx-\frac{1}{9}a^4}{x^9}$	46
risch	$\frac{-\frac{1}{5}b^4x^4-\frac{2}{3}a\,x^3b^3-\frac{6}{7}a^2b^2x^2-\frac{1}{2}a^3bx-\frac{1}{9}a^4}{x^9}$	46
gosper	$-\frac{126b^4x^4+420a\,x^3b^3+540a^2b^2x^2+315a^3bx+70a^4}{630x^9}$	47
default	$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2a\,b^3}{3x^6}-\frac{b^4}{5x^5}$	47
parallelrisch	$\frac{-126b^4x^4-420a\,x^3b^3-540a^2b^2x^2-315a^3bx-70a^4}{630x^9}$	47
orering	$-\frac{126b^4x^4+420a\,x^3b^3+540a^2b^2x^2+315a^3bx+70a^4}{630x^9}$	47

input
```
int((b*x+a)^4/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/5*b^4*x^4-2/3*a*x^3*b^3-6/7*a^2*b^2*x^2-1/2*a^3*b*x-1/9*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^4}{x^{10}}\,dx = -\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

input
```
integrate((b*x+a)^4/x^10,x, algorithm="fricas")
```

output
```
-1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a
^4)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^4}{x^{10}}\,dx = \frac{-70a^4 - 315a^3bx - 540a^2b^2x^2 - 420ab^3x^3 - 126b^4x^4}{630x^9}$$

input
```
integrate((b*x+a)**4/x**10,x)
```

output
```
(-70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b**4
*x**4)/(630*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^4}{x^{10}}\,dx = -\frac{126\,b^4x^4 + 420\,ab^3x^3 + 540\,a^2b^2x^2 + 315\,a^3bx + 70\,a^4}{630\,x^9}$$

input
```
integrate((b*x+a)^4/x^10,x, algorithm="maxima")
```

output
```
-1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a
^4)/x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^4}{x^{10}}\,dx = -\frac{126\,b^4x^4 + 420\,ab^3x^3 + 540\,a^2b^2x^2 + 315\,a^3bx + 70\,a^4}{630\,x^9}$$

input
```
integrate((b*x+a)^4/x^10,x, algorithm="giac")
```

output
```
-1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a
^4)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^4}{x^{10}}\,dx = -\frac{\frac{a^4}{9}+\frac{a^3\,b\,x}{2}+\frac{6\,a^2\,b^2\,x^2}{7}+\frac{2\,a\,b^3\,x^3}{3}+\frac{b^4\,x^4}{5}}{x^9}$$

input
```
int((a + b*x)^4/x^10,x)
```

output
```
-(a^4/9 + (b^4*x^4)/5 + (2*a*b^3*x^3)/3 + (6*a^2*b^2*x^2)/7 + (a^3*b*x)/2)
/x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^4}{x^{10}}\,dx = \frac{-126b^4x^4-420a\,b^3x^3-540a^2b^2x^2-315a^3bx-70a^4}{630x^9}$$

input
```
int((b*x+a)^4/x^10,x)
```

output
```
( - 70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b*
*4*x**4)/(630*x**9)
```

3.207 $\int \frac{(a+bx)^3}{x^{10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{(a+bx)^3}{x^{10}} dx = -\frac{a^3}{9x^9} - \frac{3a^2b}{8x^8} - \frac{3ab^2}{7x^7} - \frac{b^3}{6x^6}$$

output
```
-1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{x^{10}} dx = -\frac{a^3}{9x^9} - \frac{3a^2b}{8x^8} - \frac{3ab^2}{7x^7} - \frac{b^3}{6x^6}$$

input
```
Integrate[(a + b*x)^3/x^10,x]
```

output
```
-1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^{10}} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{3ab^2}{x^8}+\frac{b^3}{x^7}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

input
```
Int[(a + b*x)^3/x^10,x]
```

output
```
-1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{-\frac{1}{6}b^{3}x^{3}-\frac{3}{7}a\,b^{2}x^{2}-\frac{3}{8}a^{2}bx-\frac{1}{9}a^{3}}{x^{9}}$	35
risch	$\frac{-\frac{1}{6}b^{3}x^{3}-\frac{3}{7}a\,b^{2}x^{2}-\frac{3}{8}a^{2}bx-\frac{1}{9}a^{3}}{x^{9}}$	35
gosper	$-\frac{84b^{3}x^{3}+216a\,b^{2}x^{2}+189a^{2}bx+56a^{3}}{504x^{9}}$	36
default	$-\frac{a^{3}}{9x^{9}}-\frac{3a^{2}b}{8x^{8}}-\frac{3a\,b^{2}}{7x^{7}}-\frac{b^{3}}{6x^{6}}$	36
parallelrisch	$\frac{-84b^{3}x^{3}-216a\,b^{2}x^{2}-189a^{2}bx-56a^{3}}{504x^{9}}$	36
orering	$-\frac{84b^{3}x^{3}+216a\,b^{2}x^{2}+189a^{2}bx+56a^{3}}{504x^{9}}$	36

input
```
int((b*x+a)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/6*b^3*x^3-3/7*a*b^2*x^2-3/8*a^2*b*x-1/9*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^{10}}\,dx = -\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

input
```
integrate((b*x+a)^3/x^10,x, algorithm="fricas")
```

output
```
-1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^3}{x^{10}}\,dx = \frac{-56a^3-189a^2bx-216ab^2x^2-84b^3x^3}{504x^9}$$

input
```
integrate((b*x+a)**3/x**10,x)
```

output
```
(-56*a**3 - 189*a**2*b*x - 216*a*b**2*x**2 - 84*b**3*x**3)/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^{10}}\,dx = -\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

input
```
integrate((b*x+a)^3/x^10,x, algorithm="maxima")
```

output
```
-1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^{10}}\,dx = -\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

input
```
integrate((b*x+a)^3/x^10,x, algorithm="giac")
```

output
```
-1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^{10}}\,dx=-\frac{\frac{a^3}{9}+\frac{3\,a^2\,b\,x}{8}+\frac{3\,a\,b^2\,x^2}{7}+\frac{b^3\,x^3}{6}}{x^9}$$

input
```
int((a + b*x)^3/x^10,x)
```

output
```
-(a^3/9 + (b^3*x^3)/6 + (3*a*b^2*x^2)/7 + (3*a^2*b*x)/8)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^{10}}\,dx=\frac{-84b^3x^3-216a\,b^2x^2-189a^2bx-56a^3}{504x^9}$$

input
```
int((b*x+a)^3/x^10,x)
```

output
```
( - 56*a**3 - 189*a**2*b*x - 216*a*b**2*x**2 - 84*b**3*x**3)/(504*x**9)
```

3.208 $\int \frac{(a+bx)^2}{x^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = -\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

output
```
-1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = -\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

input
```
Integrate[(a + b*x)^2/x^10,x]
```

output
```
-1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^{10}}\,dx$$
$$\Big\downarrow \ 53$$
$$\int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^9}+\frac{b^2}{x^8}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

input
```
Int[(a + b*x)^2/x^10,x]
```

output
```
-1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{7}b^2x^2-\frac{1}{4}abx-\frac{1}{9}a^2}{x^9}$	24
risch	$\frac{-\frac{1}{7}b^2x^2-\frac{1}{4}abx-\frac{1}{9}a^2}{x^9}$	24
gosper	$-\frac{36b^2x^2+63abx+28a^2}{252x^9}$	25
default	$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$	25
parallelrisch	$\frac{-36b^2x^2-63abx-28a^2}{252x^9}$	25
orering	$-\frac{36b^2x^2+63abx+28a^2}{252x^9}$	25

input
```
int((b*x+a)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/7*b^2*x^2-1/4*a*b*x-1/9*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = -\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

input
```
integrate((b*x+a)^2/x^10,x, algorithm="fricas")
```

output
```
-1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = \frac{-28a^2-63abx-36b^2x^2}{252x^9}$$

input
```
integrate((b*x+a)**2/x**10,x)
```

output
```
(-28*a**2 - 63*a*b*x - 36*b**2*x**2)/(252*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = -\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

input
```
integrate((b*x+a)^2/x^10,x, algorithm="maxima")
```

output
```
-1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = -\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

input
```
integrate((b*x+a)^2/x^10,x, algorithm="giac")
```

output
```
-1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = -\frac{\frac{a^2}{9}+\frac{a\,b\,x}{4}+\frac{b^2\,x^2}{7}}{x^9}$$

input
```
int((a + b*x)^2/x^10,x)
```

output
```
-(a^2/9 + (b^2*x^2)/7 + (a*b*x)/4)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2}{x^{10}}\,dx = \frac{-36b^2x^2-63abx-28a^2}{252x^9}$$

input
```
int((b*x+a)^2/x^10,x)
```

output
```
( - 28*a**2 - 63*a*b*x - 36*b**2*x**2)/(252*x**9)
```

3.209 $\int \frac{a+bx}{x^{10}} dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{a+bx}{x^{10}} dx = -\frac{a}{9x^9} - \frac{b}{8x^8}$$

output
```
-1/9*a/x^9-1/8*b/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^{10}} dx = -\frac{a}{9x^9} - \frac{b}{8x^8}$$

input
```
Integrate[(a + b*x)/x^10,x]
```

output
```
-1/9*a/x^9 - b/(8*x^8)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^{10}}\,dx$$
$$\Big\downarrow \ 53$$
$$\int \left(\frac{a}{x^{10}}+\frac{b}{x^9}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

input
```
Int[(a + b*x)/x^10,x]
```

output
```
-1/9*a/x^9 - b/(8*x^8)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{bx}{8}-\frac{a}{9}}{x^9}$	13
risch	$\frac{-\frac{bx}{8}-\frac{a}{9}}{x^9}$	13
gosper	$-\frac{9bx+8a}{72x^9}$	14
default	$-\frac{a}{9x^9}-\frac{b}{8x^8}$	14
parallelrisch	$\frac{-9bx-8a}{72x^9}$	14
orering	$-\frac{9bx+8a}{72x^9}$	14

input
```
int((b*x+a)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/8*b*x-1/9*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{10}}\,dx=-\frac{9\,bx+8\,a}{72\,x^9}$$

input
```
integrate((b*x+a)/x^10,x, algorithm="fricas")
```

output
```
-1/72*(9*b*x + 8*a)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx}{x^{10}}\,dx = \frac{-8a-9bx}{72x^9}$$

input
```
integrate((b*x+a)/x**10,x)
```

output
```
(-8*a - 9*b*x)/(72*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{10}}\,dx = -\frac{9\,bx+8\,a}{72\,x^9}$$

input
```
integrate((b*x+a)/x^10,x, algorithm="maxima")
```

output
```
-1/72*(9*b*x + 8*a)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{10}}\,dx = -\frac{9\,bx+8\,a}{72\,x^9}$$

input
```
integrate((b*x+a)/x^10,x, algorithm="giac")
```

output
```
-1/72*(9*b*x + 8*a)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{10}}\,dx=-\frac{8\,a+9\,b\,x}{72\,x^9}$$

input
```
int((a + b*x)/x^10,x)
```

output
```
-(8*a + 9*b*x)/(72*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{10}}\,dx=\frac{-9bx-8a}{72x^9}$$

input
```
int((b*x+a)/x^10,x)
```

output
```
( - 8*a - 9*b*x)/(72*x**9)
```

3.210 $\int \frac{1}{x^{10}} \, dx$

Optimal result

Integrand size = 3, antiderivative size = 7

$$\int \frac{1}{x^{10}} \, dx = -\frac{1}{9x^9}$$

output
```
-1/9/x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{10}} \, dx = -\frac{1}{9x^9}$$

input
```
Integrate[x^(-10),x]
```

output
```
-1/9*1/x^9
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}} \, dx$$

$$\Big\downarrow \ 15$$

$$-\frac{1}{9x^9}$$

input
```
Int[x^(-10),x]
```

output
```
-1/9*1/x^9
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{1}{9x^9}$	6
default	$-\frac{1}{9x^9}$	6
norman	$-\frac{1}{9x^9}$	6
risch	$-\frac{1}{9x^9}$	6
parallelrisch	$-\frac{1}{9x^9}$	6
orering	$-\frac{1}{9x^9}$	6

input
```
int(1/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9/x^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{10}}\,dx = -\frac{1}{9\,x^9}$$

input
```
integrate(1/x^10,x, algorithm="fricas")
```

output
```
-1/9/x^9
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{10}}\,dx = -\frac{1}{9x^9}$$

input
```
integrate(1/x**10,x)
```

output
```
-1/(9*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{10}}\,dx = -\frac{1}{9\,x^9}$$

input
```
integrate(1/x^10,x, algorithm="maxima")
```

output
```
-1/9/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{10}}\,dx = -\frac{1}{9\,x^9}$$

input
```
integrate(1/x^10,x, algorithm="giac")
```

output
```
-1/9/x^9
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{10}} \, dx = -\frac{1}{9 \, x^9}$$

input
```
int(1/x^10,x)
```

output
```
-1/(9*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{10}} \, dx = -\frac{1}{9x^9}$$

input
```
int(1/x^10,x)
```

output
```
( - 1)/(9*x**9)
```

3.211 $\int \frac{1}{x^{10}(a+bx)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 134

$$\int \frac{1}{x^{10}(a+bx)}\,dx = -\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}$$

output
```
-1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1
/4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^
9*ln(b*x+a)/a^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{10}(a+bx)}\,dx = -\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}$$

input
```
Integrate[1/(x^10*(a + b*x)),x]
```

output
```
-1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(
5*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^
9*x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}(a+bx)}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{b^{10}}{a^{10}(a+bx)}-\frac{b^9}{a^{10}x}+\frac{b^8}{a^9x^2}-\frac{b^7}{a^8x^3}+\frac{b^6}{a^7x^4}-\frac{b^5}{a^6x^5}+\frac{b^4}{a^5x^6}-\frac{b^3}{a^4x^7}+\frac{b^2}{a^3x^8}-\frac{b}{a^2x^9}+\frac{1}{ax^{10}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

input
```
Int[1/(x^10*(a + b*x)),x]
```

output

```
-1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(
5*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^
9*x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

method	result
default	$-\frac{1}{9a\,x^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\ln(x)}{a^{10}}+\frac{b^9\ln(bx+a)}{a^{10}}$
norman	$\frac{-\frac{1}{9a}+\frac{bx}{8a^2}-\frac{b^2x^2}{7a^3}+\frac{b^3x^3}{6a^4}-\frac{b^4x^4}{5a^5}+\frac{b^5x^5}{4a^6}-\frac{b^6x^6}{3a^7}+\frac{b^7x^7}{2a^8}-\frac{b^8x^8}{a^9}}{x^9}+\frac{b^9\ln(bx+a)}{a^{10}}-\frac{b^9\ln(x)}{a^{10}}$
parallelrisch	$-\frac{2520\ln(x)x^9b^9-2520\ln(bx+a)x^9b^9+2520a\,x^8b^8-1260a^2x^7b^7+840x^6a^3b^6-630a^4x^5b^5+504a^5b^4x^4-420a^6b^3x^3+360a^7b^2}{2520a^{10}x^9}$
risch	$\frac{-\frac{1}{9a}+\frac{bx}{8a^2}-\frac{b^2x^2}{7a^3}+\frac{b^3x^3}{6a^4}-\frac{b^4x^4}{5a^5}+\frac{b^5x^5}{4a^6}-\frac{b^6x^6}{3a^7}+\frac{b^7x^7}{2a^8}-\frac{b^8x^8}{a^9}}{x^9}-\frac{b^9\ln(x)}{a^{10}}+\frac{b^9\ln(-bx-a)}{a^{10}}$

input
```
int(1/x^10/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1
/4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^
9*ln(b*x+a)/a^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{10}(a+bx)}\,dx$$
$$=\frac{2520\,b^9x^9\log(bx+a)-2520\,b^9x^9\log(x)-2520\,ab^8x^8+1260\,a^2b^7x^7-840\,a^3b^6x^6+630\,a^4b^5x^5-504\,a}{2520\,a^{10}x^9}$$

input
```
integrate(1/x^10/(b*x+a),x, algorithm="fricas")
```

output
```
1/2520*(2520*b^9*x^9*log(b*x + a) - 2520*b^9*x^9*log(x) - 2520*a*b^8*x^8 +
 1260*a^2*b^7*x^7 - 840*a^3*b^6*x^6 + 630*a^4*b^5*x^5 - 504*a^5*b^4*x^4 +
420*a^6*b^3*x^3 - 360*a^7*b^2*x^2 + 315*a^8*b*x - 280*a^9)/(a^10*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{10}(a+bx)}\,dx = \frac{-280a^8+315a^7bx-360a^6b^2x^2+420a^5b^3x^3-504a^4b^4x^4+630a^3b^5x^5-840a^2b^6x^6+1260ab^7x^7-2520}{2520a^9x^9} + \frac{b^9\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

input
```
integrate(1/x**10/(b*x+a),x)
```

output
```
(-280*a**8 + 315*a**7*b*x - 360*a**6*b**2*x**2 + 420*a**5*b**3*x**3 - 504*
a**4*b**4*x**4 + 630*a**3*b**5*x**5 - 840*a**2*b**6*x**6 + 1260*a*b**7*x**
7 - 2520*b**8*x**8)/(2520*a**9*x**9) + b**9*(-log(x) + log(a/b + x))/a**10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{10}(a+bx)}\,dx = \frac{b^9\log\left(bx+a\right)}{a^{10}} - \frac{b^9\log\left(x\right)}{a^{10}} - \frac{2520\,b^8x^8-1260\,ab^7x^7+840\,a^2b^6x^6-630\,a^3b^5x^5+504\,a^4b^4x^4-420\,a^5b^3x^3+360\,a^6b^2x^2-315\,a^7bx}{2520\,a^9x^9}$$

input
```
integrate(1/x^10/(b*x+a),x, algorithm="maxima")
```

output
```
b^9*log(b*x + a)/a^10 - b^9*log(x)/a^10 - 1/2520*(2520*b^8*x^8 - 1260*a*b^
7*x^7 + 840*a^2*b^6*x^6 - 630*a^3*b^5*x^5 + 504*a^4*b^4*x^4 - 420*a^5*b^3*
x^3 + 360*a^6*b^2*x^2 - 315*a^7*b*x + 280*a^8)/(a^9*x^9)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{10}(a+bx)}\,dx = \frac{b^9\log\left(|bx+a|\right)}{a^{10}} - \frac{b^9\log\left(|x|\right)}{a^{10}} - \frac{2520\,ab^8x^8 - 1260\,a^2b^7x^7 + 840\,a^3b^6x^6 - 630\,a^4b^5x^5 + 504\,a^5b^4x^4 - 420\,a^6b^3x^3 + 360\,a^7b^2x^2 - 315\,a^8}{2520\,a^{10}x^9}$$

input
```
integrate(1/x^10/(b*x+a),x, algorithm="giac")
```

output
```
b^9*log(abs(b*x + a))/a^10 - b^9*log(abs(x))/a^10 - 1/2520*(2520*a*b^8*x^8
 - 1260*a^2*b^7*x^7 + 840*a^3*b^6*x^6 - 630*a^4*b^5*x^5 + 504*a^5*b^4*x^4
- 420*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 315*a^8*b*x + 280*a^9)/(a^10*x^9)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{10}(a+bx)}\,dx = -\frac{280\,a^9 + 2520\,a\,b^8\,x^8 - 5040\,b^9\,x^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right) + 360\,a^7\,b^2\,x^2 - 420\,a^6\,b^3\,x^3 + 504\,a^5\,b^4\,x^4 - 630\,a^4}{2520\,a^{10}\,x^9}$$

input
```
int(1/(x^10*(a + b*x)),x)
```

output
```
-(280*a^9 + 2520*a*b^8*x^8 - 5040*b^9*x^9*atanh((2*b*x)/a + 1) + 360*a^7*b
^2*x^2 - 420*a^6*b^3*x^3 + 504*a^5*b^4*x^4 - 630*a^4*b^5*x^5 + 840*a^3*b^6
*x^6 - 1260*a^2*b^7*x^7 - 315*a^8*b*x)/(2520*a^10*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{10}(a+bx)}\,dx$$
$$= \frac{2520\log(bx+a)\,b^9x^9 - 2520\log(x)\,b^9x^9 - 280a^9 + 315a^8bx - 360a^7b^2x^2 + 420a^6b^3x^3 - 504a^5b^4x^4 + 63}{2520a^{10}x^9}$$

input
```
int(1/x^10/(b*x+a),x)
```

output
```
(2520*log(a + b*x)*b**9*x**9 - 2520*log(x)*b**9*x**9 - 280*a**9 + 315*a**8
*b*x - 360*a**7*b**2*x**2 + 420*a**6*b**3*x**3 - 504*a**5*b**4*x**4 + 630*
a**4*b**5*x**5 - 840*a**3*b**6*x**6 + 1260*a**2*b**7*x**7 - 2520*a*b**8*x*
*8)/(2520*a**10*x**9)
```

3.212 $\int \frac{1}{x^{10}(a+bx)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 146

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx = -\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(a+bx)}-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}$$

output
```
-1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2
*b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-1
0*b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx = -\frac{\frac{a\left(28a^9-35a^8bx+45a^7b^2x^2-60a^6b^3x^3+84a^5b^4x^4-126a^4b^5x^5+210a^3b^6x^6-420a^2b^7x^7+1260ab^8x^8+2520b^9x^9\right)}{x^9(a+bx)}+2520b^9\log(x)-2}{252a^{11}}$$

input
```
Integrate[1/(x^10*(a + b*x)^2),x]
```

output
```
-1/252*((a*(28*a^9 - 35*a^8*b*x + 45*a^7*b^2*x^2 - 60*a^6*b^3*x^3 + 84*a^5
*b^4*x^4 - 126*a^4*b^5*x^5 + 210*a^3*b^6*x^6 - 420*a^2*b^7*x^7 + 1260*a*b^
8*x^8 + 2520*b^9*x^9))/(x^9*(a + b*x)) + 2520*b^9*Log[x] - 2520*b^9*Log[a
+ b*x])/a^11
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}(a+bx)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{10b^{10}}{a^{11}(a+bx)}-\frac{10b^9}{a^{11}x}+\frac{b^{10}}{a^{10}(a+bx)^2}+\frac{9b^8}{a^{10}x^2}-\frac{8b^7}{a^9x^3}+\frac{7b^6}{a^8x^4}-\frac{6b^5}{a^7x^5}+\frac{5b^4}{a^6x^6}-\frac{4b^3}{a^5x^7}+\frac{3b^2}{a^4x^8}-\frac{2b}{a^3x^9}+\frac{1}{a^2x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}+\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x^9}$$

input
```
Int[1/(x^10*(a + b*x)^2),x]
```

output
```
-1/9*1/(a^2*x^9) + b/(4*a^3*x^8) - (3*b^2)/(7*a^4*x^7) + (2*b^3)/(3*a^5*x^
6) - b^4/(a^6*x^5) + (3*b^5)/(2*a^7*x^4) - (7*b^6)/(3*a^8*x^3) + (4*b^7)/(
a^9*x^2) - (9*b^8)/(a^10*x) - b^9/(a^10*(a + b*x)) - (10*b^9*Log[x])/a^11
+ (10*b^9*Log[a + b*x])/a^11
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

method	result
default	$-\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(bx+a)}-\frac{10b^9\ln(x)}{a^{11}}+$
norman	$\frac{\frac{10b^{10}x^{10}}{a^{11}}-\frac{1}{9a}+\frac{5bx}{36a^2}-\frac{5b^2x^2}{28a^3}+\frac{5b^3x^3}{21a^4}-\frac{b^4x^4}{3a^5}+\frac{b^5x^5}{2a^6}-\frac{5b^6x^6}{6a^7}+\frac{5b^7x^7}{3a^8}-\frac{5b^8x^8}{a^9}}{x^9(bx+a)}-\frac{10b^9\ln(x)}{a^{11}}+\frac{10b^9\ln(bx+a)}{a^{11}}$
risch	$\frac{-\frac{10b^9x^9}{a^{10}}-\frac{5b^8x^8}{a^9}+\frac{5b^7x^7}{3a^8}-\frac{5b^6x^6}{6a^7}+\frac{b^5x^5}{2a^6}-\frac{b^4x^4}{3a^5}+\frac{5b^3x^3}{21a^4}-\frac{5b^2x^2}{28a^3}+\frac{5bx}{36a^2}-\frac{1}{9a}}{x^9(bx+a)}+\frac{10b^9\ln(-bx-a)}{a^{11}}-\frac{10b^9\ln(x)}{a^{11}}$
parallelrisch	$-\frac{2520b^{10}\ln(x)x^{10}-2520\ln(bx+a)x^{10}b^{10}+2520a\,b^9\ln(x)x^9-2520\ln(bx+a)x^9a\,b^9-2520b^{10}x^{10}+1260a^2b^8x^8-420a^3b^7x^7+}{252a^{11}x^9(bx+a)}$

input
```
int(1/x^10/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2
*b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-1
0*b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx = -\frac{2520\,ab^9x^9+1260\,a^2b^8x^8-420\,a^3b^7x^7+210\,a^4b^6x^6-126\,a^5b^5x^5+84\,a^6b^4x^4-60\,a^7b^3x^3+45\,a^8b^2x^2}{252\,(a^{11}bx^{10}+a^{12}}$$

input
```
integrate(1/x^10/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/252*(2520*a*b^9*x^9 + 1260*a^2*b^8*x^8 - 420*a^3*b^7*x^7 + 210*a^4*b^6*
x^6 - 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 - 60*a^7*b^3*x^3 + 45*a^8*b^2*x^2 -
 35*a^9*b*x + 28*a^10 - 2520*(b^10*x^10 + a*b^9*x^9)*log(b*x + a) + 2520*(
b^10*x^10 + a*b^9*x^9)*log(x))/(a^11*b*x^10 + a^12*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx = \frac{-28a^9+35a^8bx-45a^7b^2x^2+60a^6b^3x^3-84a^5b^4x^4+126a^4b^5x^5-210a^3b^6x^6+420a^2b^7x^7-1260ab^8x^8}{252a^{11}x^9+252a^{10}bx^{10}} + \frac{10b^9\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

input
```
integrate(1/x**10/(b*x+a)**2,x)
```

output
```
(-28*a**9 + 35*a**8*b*x - 45*a**7*b**2*x**2 + 60*a**6*b**3*x**3 - 84*a**5*
b**4*x**4 + 126*a**4*b**5*x**5 - 210*a**3*b**6*x**6 + 420*a**2*b**7*x**7 -
 1260*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**11*x**9 + 252*a**10*b*x**10) +
 10*b**9*(-log(x) + log(a/b + x))/a**11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx = -\frac{2520\,b^9x^9+1260\,ab^8x^8-420\,a^2b^7x^7+210\,a^3b^6x^6-126\,a^4b^5x^5+84\,a^5b^4x^4-60\,a^6b^3x^3+45\,a^7b^2x^2-}{252\,(a^{10}bx^{10}+a^{11}x^9)} + \frac{10\,b^9\log(bx+a)}{a^{11}} - \frac{10\,b^9\log(x)}{a^{11}}$$

input
```
integrate(1/x^10/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/252*(2520*b^9*x^9 + 1260*a*b^8*x^8 - 420*a^2*b^7*x^7 + 210*a^3*b^6*x^6
- 126*a^4*b^5*x^5 + 84*a^5*b^4*x^4 - 60*a^6*b^3*x^3 + 45*a^7*b^2*x^2 - 35*
a^8*b*x + 28*a^9)/(a^10*b*x^10 + a^11*x^9) + 10*b^9*log(b*x + a)/a^11 - 10
*b^9*log(x)/a^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx = -\frac{10\,b^9\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^{11}} - \frac{b^9}{(bx+a)a^{10}} - \frac{\frac{41481\,ab^9}{bx+a} - \frac{155844\,a^2b^9}{(bx+a)^2} + \frac{337176\,a^3b^9}{(bx+a)^3} - \frac{460404\,a^4b^9}{(bx+a)^4} + \frac{407484\,a^5b^9}{(bx+a)^5} - \frac{229320\,a^6b^9}{(bx+a)^6} + \frac{75600\,a^7b^9}{(bx+a)^7} - \frac{11340\,a^8b^9}{(bx+a)^8} - 4861\,b^9}{252\,a^{11}\left(\frac{a}{bx+a}-1\right)^9}$$

input
```
integrate(1/x^10/(b*x+a)^2,x, algorithm="giac")
```

output
```
-10*b^9*log(abs(-a/(b*x + a) + 1))/a^11 - b^9/((b*x + a)*a^10) - 1/252*(41
481*a*b^9/(b*x + a) - 155844*a^2*b^9/(b*x + a)^2 + 337176*a^3*b^9/(b*x + a
)^3 - 460404*a^4*b^9/(b*x + a)^4 + 407484*a^5*b^9/(b*x + a)^5 - 229320*a^6
*b^9/(b*x + a)^6 + 75600*a^7*b^9/(b*x + a)^7 - 11340*a^8*b^9/(b*x + a)^8 -
 4861*b^9)/(a^11*(a/(b*x + a) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx = \frac{20\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}} - \frac{\frac{1}{9\,a} + \frac{5\,b^2\,x^2}{28\,a^3} - \frac{5\,b^3\,x^3}{21\,a^4} + \frac{b^4\,x^4}{3\,a^5} - \frac{b^5\,x^5}{2\,a^6} + \frac{5\,b^6\,x^6}{6\,a^7} - \frac{5\,b^7\,x^7}{3\,a^8} + \frac{5\,b^8\,x^8}{a^9} + \frac{10\,b^9\,x^9}{a^{10}} - \frac{5\,b\,x}{36\,a^2}}{b\,x^{10}+a\,x^9}$$

input
```
int(1/(x^10*(a + b*x)^2),x)
```

output
```
(20*b^9*atanh((2*b*x)/a + 1))/a^11 - (1/(9*a) + (5*b^2*x^2)/(28*a^3) - (5*
b^3*x^3)/(21*a^4) + (b^4*x^4)/(3*a^5) - (b^5*x^5)/(2*a^6) + (5*b^6*x^6)/(6
*a^7) - (5*b^7*x^7)/(3*a^8) + (5*b^8*x^8)/a^9 + (10*b^9*x^9)/a^10 - (5*b*x
)/(36*a^2))/(a*x^9 + b*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx$$
$$= \frac{2520\log(bx+a)\,a\,b^9x^9 + 2520\log(bx+a)\,b^{10}x^{10} - 2520\log(x)\,a\,b^9x^9 - 2520\log(x)\,b^{10}x^{10} - 28a^{10} + 35}{252a^{11}x}$$

input
```
int(1/x^10/(b*x+a)^2,x)
```

output
```
(2520*log(a + b*x)*a*b**9*x**9 + 2520*log(a + b*x)*b**10*x**10 - 2520*log(
x)*a*b**9*x**9 - 2520*log(x)*b**10*x**10 - 28*a**10 + 35*a**9*b*x - 45*a**
8*b**2*x**2 + 60*a**7*b**3*x**3 - 84*a**6*b**4*x**4 + 126*a**5*b**5*x**5 -
 210*a**4*b**6*x**6 + 420*a**3*b**7*x**7 - 1260*a**2*b**8*x**8 + 2520*b**1
0*x**10)/(252*a**11*x**9*(a + b*x))
```

3.213 $\int \frac{1}{x^{10}(a+bx)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 163

$$\int \frac{1}{x^{10}(a+bx)^3} \, dx = -\frac{1}{9a^3x^9} + \frac{3b}{8a^4x^8} - \frac{6b^2}{7a^5x^7} + \frac{5b^3}{3a^6x^6} - \frac{3b^4}{a^7x^5} + \frac{21b^5}{4a^8x^4} - \frac{28b^6}{3a^9x^3} + \frac{18b^7}{a^{10}x^2} - \frac{45b^8}{a^{11}x} - \frac{b^9}{2a^{10}(a+bx)^2} - \frac{10b^9}{a^{11}(a+bx)} - \frac{55b^9\log(x)}{a^{12}} + \frac{55b^9\log(a+bx)}{a^{12}}$$

output
```
-1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+2
1/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^1
0/(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{10}(a+bx)^3} \, dx = -\frac{\frac{a\left(56a^{10}-77a^9bx+110a^8b^2x^2-165a^7b^3x^3+264a^6b^4x^4-462a^5b^5x^5+924a^4b^6x^6-2310a^3b^7x^7+9240a^2b^8x^8+41580ab^9x^9+27720b^{10}x^{10}\right)}{x^9(a+bx)^2} +}{504a^{12}}$$

input
```
Integrate[1/(x^10*(a + b*x)^3),x]
```

output
```
-1/504*((a*(56*a^10 - 77*a^9*b*x + 110*a^8*b^2*x^2 - 165*a^7*b^3*x^3 + 264
*a^6*b^4*x^4 - 462*a^5*b^5*x^5 + 924*a^4*b^6*x^6 - 2310*a^3*b^7*x^7 + 9240
*a^2*b^8*x^8 + 41580*a*b^9*x^9 + 27720*b^10*x^10))/(x^9*(a + b*x)^2) + 277
20*b^9*Log[x] - 27720*b^9*Log[a + b*x])/a^12
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}(a+bx)^3} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{55b^{10}}{a^{12}(a+bx)} - \frac{55b^9}{a^{12}x} + \frac{10b^{10}}{a^{11}(a+bx)^2} + \frac{45b^8}{a^{11}x^2} + \frac{b^{10}}{a^{10}(a+bx)^3} - \frac{36b^7}{a^{10}x^3} + \frac{28b^6}{a^9x^4} - \frac{21b^5}{a^8x^5} + \frac{15b^4}{a^7x^6} - \frac{10b^3}{a^6x^7} + \frac{6b}{a^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{55b^9\log(x)}{a^{12}} + \frac{55b^9\log(a+bx)}{a^{12}} - \frac{10b^9}{a^{11}(a+bx)} - \frac{45b^8}{a^{11}x} - \frac{b^9}{2a^{10}(a+bx)^2} + \frac{18b^7}{a^{10}x^2} - \frac{28b^6}{3a^9x^3} + \frac{21b^5}{4a^8x^4} - \frac{3b^4}{a^7x^5} + \frac{5b^3}{3a^6x^6} - \frac{6b^2}{7a^5x^7} + \frac{3b}{8a^4x^8} - \frac{1}{9a^3x^9}$$

input
```
Int[1/(x^10*(a + b*x)^3),x]
```

output
```
-1/9*1/(a^3*x^9) + (3*b)/(8*a^4*x^8) - (6*b^2)/(7*a^5*x^7) + (5*b^3)/(3*a^
6*x^6) - (3*b^4)/(a^7*x^5) + (21*b^5)/(4*a^8*x^4) - (28*b^6)/(3*a^9*x^3) +
 (18*b^7)/(a^10*x^2) - (45*b^8)/(a^11*x) - b^9/(2*a^10*(a + b*x)^2) - (10*
b^9)/(a^11*(a + b*x)) - (55*b^9*Log[x])/a^12 + (55*b^9*Log[a + b*x])/a^12
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

method	result
norman	$\frac{-\frac{1}{9a}+\frac{11bx}{72a^2}-\frac{55b^2x^2}{252a^3}+\frac{55b^3x^3}{168a^4}-\frac{11b^4x^4}{21a^5}+\frac{11b^5x^5}{12a^6}-\frac{11b^6x^6}{6a^7}+\frac{55b^7x^7}{12a^8}-\frac{55b^8x^8}{3a^9}+\frac{110b^{10}x^{10}}{a^{11}}+\frac{165b^{11}x^{11}}{2a^{12}}}{x^9(bx+a)^2}-\frac{55b^9\ln(x)}{a^{12}}+\frac{55b^9\ln}{a}$
default	$-\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(bx+a)^2}-\frac{10b^9}{a^{11}(bx+}$
risch	$\frac{-\frac{55b^{10}x^{10}}{a^{11}}-\frac{165b^9x^9}{2a^{10}}-\frac{55b^8x^8}{3a^9}+\frac{55b^7x^7}{12a^8}-\frac{11b^6x^6}{6a^7}+\frac{11b^5x^5}{12a^6}-\frac{11b^4x^4}{21a^5}+\frac{55b^3x^3}{168a^4}-\frac{55b^2x^2}{252a^3}+\frac{11bx}{72a^2}-\frac{1}{9a}}{x^9(bx+a)^2}-\frac{55b^9\ln(x)}{a^{12}}+\frac{55b^9\ln(-}{a^{12}}$
parallelrisch	$-\frac{27720\ln(x)x^{11}b^{11}-27720\ln(bx+a)x^{11}b^{11}+55440\ln(x)x^{10}a\,b^{10}-55440\ln(bx+a)x^{10}a\,b^{10}-41580b^{11}x^{11}+27720\ln(x)x^9a}{}$

input
```
int(1/x^10/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/9/a+11/72*b/a^2*x-55/252*b^2/a^3*x^2+55/168*b^3/a^4*x^3-11/21*b^4/a^5*
x^4+11/12*b^5/a^6*x^5-11/6*b^6/a^7*x^6+55/12*b^7/a^8*x^7-55/3*b^8/a^9*x^8+
110*b^10/a^11*x^10+165/2*b^11/a^12*x^11)/x^9/(b*x+a)^2-55*b^9*ln(x)/a^12+5
5*b^9*ln(b*x+a)/a^12
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = -\frac{27720\,ab^{10}x^{10}+41580\,a^2b^9x^9+9240\,a^3b^8x^8-2310\,a^4b^7x^7+924\,a^5b^6x^6-462\,a^6b^5x^5+264\,a^7b^4x^4-}{}$$

input
```
integrate(1/x^10/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/504*(27720*a*b^10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^
4*b^7*x^7 + 924*a^5*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*
b^3*x^3 + 110*a^9*b^2*x^2 - 77*a^10*b*x + 56*a^11 - 27720*(b^11*x^11 + 2*a
*b^10*x^10 + a^2*b^9*x^9)*log(b*x + a) + 27720*(b^11*x^11 + 2*a*b^10*x^10
+ a^2*b^9*x^9)*log(x))/(a^12*b^2*x^11 + 2*a^13*b*x^10 + a^14*x^9)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = \frac{-56a^{10}+77a^9bx-110a^8b^2x^2+165a^7b^3x^3-264a^6b^4x^4+462a^5b^5x^5-924a^4b^6x^6+2310a^3b^7x^7-9240}{504a^{13}x^9+1008a^{12}bx^{10}+504a^{11}b^2x^{11}} + \frac{55b^9\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{12}}$$

input
```
integrate(1/x**10/(b*x+a)**3,x)
```

output
```
(-56*a**10 + 77*a**9*b*x - 110*a**8*b**2*x**2 + 165*a**7*b**3*x**3 - 264*a
**6*b**4*x**4 + 462*a**5*b**5*x**5 - 924*a**4*b**6*x**6 + 2310*a**3*b**7*x
**7 - 9240*a**2*b**8*x**8 - 41580*a*b**9*x**9 - 27720*b**10*x**10)/(504*a*
*13*x**9 + 1008*a**12*b*x**10 + 504*a**11*b**2*x**11) + 55*b**9*(-log(x) +
 log(a/b + x))/a**12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = -\frac{27720\,b^{10}x^{10} + 41580\,ab^9x^9 + 9240\,a^2b^8x^8 - 2310\,a^3b^7x^7 + 924\,a^4b^6x^6 - 462\,a^5b^5x^5 + 264\,a^6b^4x^4 - 16}{504\,(a^{11}b^2x^{11} + 2\,a^{12}bx^{10} + a^{13}x^9)} + \frac{55\,b^9\log(bx+a)}{a^{12}} - \frac{55\,b^9\log(x)}{a^{12}}$$

input
```
integrate(1/x^10/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/504*(27720*b^10*x^10 + 41580*a*b^9*x^9 + 9240*a^2*b^8*x^8 - 2310*a^3*b^
7*x^7 + 924*a^4*b^6*x^6 - 462*a^5*b^5*x^5 + 264*a^6*b^4*x^4 - 165*a^7*b^3*
x^3 + 110*a^8*b^2*x^2 - 77*a^9*b*x + 56*a^10)/(a^11*b^2*x^11 + 2*a^12*b*x^
10 + a^13*x^9) + 55*b^9*log(b*x + a)/a^12 - 55*b^9*log(x)/a^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = \frac{55\,b^9\log(|bx+a|)}{a^{12}} - \frac{55\,b^9\log(|x|)}{a^{12}} - \frac{27720\,ab^{10}x^{10} + 41580\,a^2b^9x^9 + 9240\,a^3b^8x^8 - 2310\,a^4b^7x^7 + 924\,a^5b^6x^6 - 462\,a^6b^5x^5 + 264\,a^7b^4x^4 -}{504\,(bx+a)^2a^{12}x^9}$$

input
```
integrate(1/x^10/(b*x+a)^3,x, algorithm="giac")
```

output
```
55*b^9*log(abs(b*x + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/504*(27720*a*b
^10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4*b^7*x^7 + 924*a
^5*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^3*x^3 + 110*a^9
*b^2*x^2 - 77*a^10*b*x + 56*a^11)/((b*x + a)^2*a^12*x^9)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = \frac{110\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}} - \frac{\frac{1}{9\,a}+\frac{55\,b^2\,x^2}{252\,a^3}-\frac{55\,b^3\,x^3}{168\,a^4}+\frac{11\,b^4\,x^4}{21\,a^5}-\frac{11\,b^5\,x^5}{12\,a^6}+\frac{11\,b^6\,x^6}{6\,a^7}-\frac{55\,b^7\,x^7}{12\,a^8}+\frac{55\,b^8\,x^8}{3\,a^9}+\frac{165\,b^9\,x^9}{2\,a^{10}}+\frac{55\,b^{10}\,x^{10}}{a^{11}}-\frac{11\,b\,x}{72\,a^2}}{a^2\,x^9+2\,a\,b\,x^{10}+b^2\,x^{11}}$$

input

```
int(1/(x^10*(a + b*x)^3),x)
```

output

```
(110*b^9*atanh((2*b*x)/a + 1))/a^12 - (1/(9*a) + (55*b^2*x^2)/(252*a^3) -
(55*b^3*x^3)/(168*a^4) + (11*b^4*x^4)/(21*a^5) - (11*b^5*x^5)/(12*a^6) + (
11*b^6*x^6)/(6*a^7) - (55*b^7*x^7)/(12*a^8) + (55*b^8*x^8)/(3*a^9) + (165*
b^9*x^9)/(2*a^10) + (55*b^10*x^10)/a^11 - (11*b*x)/(72*a^2))/(a^2*x^9 + b^
2*x^11 + 2*a*b*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = \frac{27720\log(bx+a)\,a^2b^9x^9+55440\log(bx+a)\,a\,b^{10}x^{10}+27720\log(bx+a)\,b^{11}x^{11}-27720\log(x)\,a^2b^9x^9}{}$$

input

```
int(1/x^10/(b*x+a)^3,x)
```

output

```
(27720*log(a + b*x)*a**2*b**9*x**9 + 55440*log(a + b*x)*a*b**10*x**10 + 27
720*log(a + b*x)*b**11*x**11 - 27720*log(x)*a**2*b**9*x**9 - 55440*log(x)*
a*b**10*x**10 - 27720*log(x)*b**11*x**11 - 56*a**11 + 77*a**10*b*x - 110*a
**9*b**2*x**2 + 165*a**8*b**3*x**3 - 264*a**7*b**4*x**4 + 462*a**6*b**5*x*
*5 - 924*a**5*b**6*x**6 + 2310*a**4*b**7*x**7 - 9240*a**3*b**8*x**8 - 2772
0*a**2*b**9*x**9 + 13860*b**11*x**11)/(504*a**12*x**9*(a**2 + 2*a*b*x + b*
*2*x**2))
```

3.214 $\int \frac{1}{x(2+3x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{x(2+3x)}\,dx = \frac{\log(x)}{2} - \frac{1}{2}\log(2+3x)$$

output
```
1/2*ln(x)-1/2*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(2+3x)}\,dx = \frac{\log(x)}{2} - \frac{1}{2}\log(2+3x)$$

input
```
Integrate[1/(x*(2 + 3*x)),x]
```

output
```
Log[x]/2 - Log[2 + 3*x]/2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(3x+2)}\,dx$$
$$\Big\downarrow \ 47$$
$$\frac{\int \frac{1}{x}dx}{2}-\frac{3}{2}\int \frac{1}{3x+2}dx$$
$$\Big\downarrow \ 14$$
$$\frac{\log(x)}{2}-\frac{3}{2}\int \frac{1}{3x+2}dx$$
$$\Big\downarrow \ 16$$
$$\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)$$

input
```
Int[1/(x*(2 + 3*x)),x]
```

output
```
Log[x]/2 - Log[2 + 3*x]/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{\ln(\frac{2}{3}+x)}{2}$	12
default	$\frac{\ln(x)}{2}-\frac{\ln(2+3x)}{2}$	14
norman	$\frac{\ln(x)}{2}-\frac{\ln(2+3x)}{2}$	14
risch	$\frac{\ln(x)}{2}-\frac{\ln(2+3x)}{2}$	14
meijerg	$\frac{\ln(x)}{2}+\frac{\ln(3)}{2}-\frac{\ln(2)}{2}-\frac{\ln(1+\frac{3x}{2})}{2}$	22

input
```
int(1/x/(2+3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-1/2*ln(2/3+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(2+3x)}\,dx = -\frac{1}{2}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate(1/x/(2+3*x),x, algorithm="fricas")
```

output
```
-1/2*log(3*x + 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(2+3x)}\,dx = \frac{\log(x)}{2} - \frac{\log\left(x+\frac{2}{3}\right)}{2}$$

input
```
integrate(1/x/(2+3*x),x)
```

output
```
log(x)/2 - log(x + 2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(2+3x)}\,dx = -\frac{1}{2}\log(3x+2) + \frac{1}{2}\log(x)$$

input
```
integrate(1/x/(2+3*x),x, algorithm="maxima")
```

output
```
-1/2*log(3*x + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(2+3x)}\,dx = -\frac{1}{2}\log(|3x+2|) + \frac{1}{2}\log(|x|)$$

input
```
integrate(1/x/(2+3*x),x, algorithm="giac")
```

output
```
-1/2*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{x(2+3x)}\,dx = -\frac{\ln\left(\frac{2}{x}+3\right)}{2}$$

input
```
int(1/(x*(3*x + 2)),x)
```

output
```
-log(2/x + 3)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(2+3x)}\,dx = -\frac{\log(3x+2)}{2} + \frac{\log(x)}{2}$$

input
```
int(1/x/(2+3*x),x)
```

output
```
( - log(3*x + 2) + log(x))/2
```

3.215 $\int \frac{1}{x(4+6x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{x(4+6x)}\,dx = \frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)$$

output
```
1/4*ln(x)-1/4*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(4+6x)}\,dx = \frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)$$

input
```
Integrate[1/(x*(4 + 6*x)),x]
```

output
```
Log[x]/4 - Log[2 + 3*x]/4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(6x+4)}dx$$

$$\Big\downarrow \; 47$$

$$\frac{\int \frac{1}{x}dx}{4} - \frac{3}{2}\int \frac{1}{6x+4}dx$$

$$\Big\downarrow \; 14$$

$$\frac{\log(x)}{4} - \frac{3}{2}\int \frac{1}{6x+4}dx$$

$$\Big\downarrow \; 16$$

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

input
```
Int[1/(x*(4 + 6*x)),x]
```

output
```
Log[x]/4 - Log[2 + 3*x]/4
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$\frac{\ln(x)}{4}-\frac{\ln(\frac{2}{3}+x)}{4}$	12
default	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
norman	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
risch	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
meijerg	$\frac{\ln(x)}{4}+\frac{\ln(3)}{4}-\frac{\ln(2)}{4}-\frac{\ln(1+\frac{3x}{2})}{4}$	22

input
```
int(1/x/(4+6*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x)-1/4*ln(2/3+x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(4+6x)}\,dx=-\frac{1}{4}\,\log\left(3\,x+2\right)+\frac{1}{4}\,\log\left(x\right)$$

input
```
integrate(1/x/(4+6*x),x, algorithm="fricas")
```

output
```
-1/4*log(3*x + 2) + 1/4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(4+6x)}\,dx = \frac{\log{(x)}}{4} - \frac{\log{\left(x+\frac{2}{3}\right)}}{4}$$

input
```
integrate(1/x/(4+6*x),x)
```

output
```
log(x)/4 - log(x + 2/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(4+6x)}\,dx = -\frac{1}{4}\log\left(3\,x+2\right) + \frac{1}{4}\log\left(x\right)$$

input
```
integrate(1/x/(4+6*x),x, algorithm="maxima")
```

output
```
-1/4*log(3*x + 2) + 1/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(4+6x)}\,dx = -\frac{1}{4}\log\left(|3\,x+2|\right) + \frac{1}{4}\log\left(|x|\right)$$

input
```
integrate(1/x/(4+6*x),x, algorithm="giac")
```

output
```
-1/4*log(abs(3*x + 2)) + 1/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{x(4+6x)}\,dx = -\frac{\ln\left(\frac{4}{x}+6\right)}{4}$$

input
```
int(1/(x*(6*x + 4)),x)
```

output
```
-log(4/x + 6)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(4+6x)}\,dx = -\frac{\log(3x+2)}{4} + \frac{\log(x)}{4}$$

input
```
int(1/x/(4+6*x),x)
```

output
```
( - log(3*x + 2) + log(x))/4
```

3.216 $\int \frac{1}{x^2(4+6x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{x^2(4+6x)}\,dx = -\frac{1}{4x} - \frac{3\log(x)}{8} + \frac{3}{8}\log(2+3x)$$

output
```
-1/4/x-3/8*ln(x)+3/8*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(4+6x)}\,dx = -\frac{1}{4x} - \frac{3\log(x)}{8} + \frac{3}{8}\log(2+3x)$$

input
```
Integrate[1/(x^2*(4 + 6*x)),x]
```

output
```
-1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(6x+4)}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{4x^2}+\frac{9}{8(3x+2)}-\frac{3}{8x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

input
```
Int[1/(x^2*(4 + 6*x)),x]
```

output
```
-1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
norman	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
risch	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
parallelrisch	$-\frac{3\ln(x)x-3\ln(\frac{2}{3}+x)x+2}{8x}$	20
meijerg	$-\frac{1}{4x}-\frac{3\ln(x)}{8}-\frac{3\ln(3)}{8}+\frac{3\ln(2)}{8}+\frac{3\ln(1+\frac{3x}{2})}{8}$	27

input
```
int(1/x^2/(4+6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x-3/8*ln(x)+3/8*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(4+6x)}\,dx = \frac{3\,x\log\left(3\,x+2\right)-3\,x\log\left(x\right)-2}{8\,x}$$

input
```
integrate(1/x^2/(4+6*x),x, algorithm="fricas")
```

output
```
1/8*(3*x*log(3*x + 2) - 3*x*log(x) - 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(4+6x)}\,dx = -\frac{3\log{(x)}}{8} + \frac{3\log{\left(x+\frac{2}{3}\right)}}{8} - \frac{1}{4x}$$

input
```
integrate(1/x**2/(4+6*x),x)
```

output
```
-3*log(x)/8 + 3*log(x + 2/3)/8 - 1/(4*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2(4+6x)}\,dx = -\frac{1}{4\,x} + \frac{3}{8}\,\log\left(3\,x+2\right) - \frac{3}{8}\,\log\left(x\right)$$

input
```
integrate(1/x^2/(4+6*x),x, algorithm="maxima")
```

output
```
-1/4/x + 3/8*log(3*x + 2) - 3/8*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(4+6x)}\,dx = -\frac{1}{4\,x} + \frac{3}{8}\,\log\left(|3\,x+2|\right) - \frac{3}{8}\,\log\left(|x|\right)$$

input
```
integrate(1/x^2/(4+6*x),x, algorithm="giac")
```

output
```
-1/4/x + 3/8*log(abs(3*x + 2)) - 3/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2(4+6x)}\,dx = -\frac{3\,\ln\left(\frac{x}{6\,x+4}\right)}{8} - \frac{1}{4\,x}$$

input
```
int(1/(x^2*(6*x + 4)),x)
```

output
```
- (3*log(x/(6*x + 4)))/8 - 1/(4*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(4+6x)}\,dx = \frac{3\log(3x+2)\,x - 3\log(x)\,x - 2}{8x}$$

input
```
int(1/x^2/(4+6*x),x)
```

output
```
(3*log(3*x + 2)*x - 3*log(x)*x - 2)/(8*x)
```

3.217 $\int \frac{1}{x^3(4+6x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{1}{x^3(4+6x)}\,dx = -\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)$$

output
```
-1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(4+6x)}\,dx = -\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)$$

input
```
Integrate[1/(x^3*(4 + 6*x)),x]
```

output
```
-1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(6x+4)}\,dx$$

$$\Big\downarrow \; 54$$

$$\int \left(\frac{1}{4x^3}-\frac{3}{8x^2}-\frac{27}{16(3x+2)}+\frac{9}{16x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

input
```
Int[1/(x^3*(4 + 6*x)),x]
```

output
```
-1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{-\frac{1}{8}+\frac{3x}{8}}{x^2}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	23
risch	$\frac{-\frac{1}{8}+\frac{3x}{8}}{x^2}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	23
default	$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	24
parallelrisch	$\frac{9\ln(x)x^2-9\ln\left(\frac{2}{3}+x\right)x^2-2+6x}{16x^2}$	27
meijerg	$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\ln(x)}{16}+\frac{9\ln(3)}{16}-\frac{9\ln(2)}{16}-\frac{9\ln\left(1+\frac{3x}{2}\right)}{16}$	32

input
```
int(1/x^3/(4+6*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/8+3/8*x)/x^2+9/16*ln(x)-9/16*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3(4+6x)}\,dx = -\frac{9\,x^2\log\left(3\,x+2\right)-9\,x^2\log\left(x\right)-6\,x+2}{16\,x^2}$$

input
```
integrate(1/x^3/(4+6*x),x, algorithm="fricas")
```

output
```
-1/16*(9*x^2*log(3*x + 2) - 9*x^2*log(x) - 6*x + 2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3(4+6x)}\,dx = \frac{9\log(x)}{16} - \frac{9\log\left(x+\frac{2}{3}\right)}{16} + \frac{3x-1}{8x^2}$$

input
```
integrate(1/x**3/(4+6*x),x)
```

output
```
9*log(x)/16 - 9*log(x + 2/3)/16 + (3*x - 1)/(8*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(4+6x)}\,dx = \frac{3\,x-1}{8\,x^2} - \frac{9}{16}\log(3\,x+2) + \frac{9}{16}\log(x)$$

input
```
integrate(1/x^3/(4+6*x),x, algorithm="maxima")
```

output
```
1/8*(3*x - 1)/x^2 - 9/16*log(3*x + 2) + 9/16*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(4+6x)}\,dx = \frac{3\,x-1}{8\,x^2} - \frac{9}{16}\log(|3\,x+2|) + \frac{9}{16}\log(|x|)$$

input
```
integrate(1/x^3/(4+6*x),x, algorithm="giac")
```

output
```
1/8*(3*x - 1)/x^2 - 9/16*log(abs(3*x + 2)) + 9/16*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^3(4+6x)}\,dx = \frac{\frac{3\,x}{8}-\frac{1}{8}}{x^2}-\frac{9\,\mathrm{atanh}(3\,x+1)}{8}$$

input
```
int(1/(x^3*(6*x + 4)),x)
```

output
```
((3*x)/8 - 1/8)/x^2 - (9*atanh(3*x + 1))/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3(4+6x)}\,dx = \frac{-9\log(3x+2)\,x^2+9\log(x)\,x^2+6x-2}{16x^2}$$

input
```
int(1/x^3/(4+6*x),x)
```

output
```
( - 9*log(3*x + 2)*x**2 + 9*log(x)*x**2 + 6*x - 2)/(16*x**2)
```

3.218 $\int \frac{1}{x^4(4+6x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{x^4(4+6x)}\,dx = -\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

output
```
-1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(4+6x)}\,dx = -\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

input
```
Integrate[1/(x^4*(4 + 6*x)),x]
```

output
```
-1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/3
2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(6x+4)}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{4x^4}-\frac{3}{8x^3}+\frac{9}{16x^2}+\frac{81}{32(3x+2)}-\frac{27}{32x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

input
```
Int[1/(x^4*(4 + 6*x)),x]
```

output
```
-1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/3
2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
norman	$\frac{-\frac{1}{12}+\frac{3}{16}x-\frac{9}{16}x^{2}}{x^{3}}-\frac{27\ln\left(x\right)}{32}+\frac{27\ln\left(2+3x\right)}{32}$	28
risch	$\frac{-\frac{1}{12}+\frac{3}{16}x-\frac{9}{16}x^{2}}{x^{3}}-\frac{27\ln\left(x\right)}{32}+\frac{27\ln\left(2+3x\right)}{32}$	28
default	$-\frac{1}{12x^{3}}+\frac{3}{16x^{2}}-\frac{9}{16x}-\frac{27\ln\left(x\right)}{32}+\frac{27\ln\left(2+3x\right)}{32}$	29
parallelrisch	$-\frac{81\ln\left(x\right)x^{3}-81\ln\left(\frac{2}{3}+x\right)x^{3}+8+54x^{2}-18x}{96x^{3}}$	32
meijerg	$-\frac{1}{12x^{3}}+\frac{3}{16x^{2}}-\frac{9}{16x}-\frac{27\ln\left(x\right)}{32}-\frac{27\ln\left(3\right)}{32}+\frac{27\ln\left(2\right)}{32}+\frac{27\ln\left(1+\frac{3x}{2}\right)}{32}$	37

input
```
int(1/x^4/(4+6*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/12+3/16*x-9/16*x^2)/x^3-27/32*ln(x)+27/32*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4(4+6x)}\,dx=\frac{81\,x^{3}\log\left(3\,x+2\right)-81\,x^{3}\log\left(x\right)-54\,x^{2}+18\,x-8}{96\,x^{3}}$$

input
```
integrate(1/x^4/(4+6*x),x, algorithm="fricas")
```

output
```
1/96*(81*x^3*log(3*x + 2) - 81*x^3*log(x) - 54*x^2 + 18*x - 8)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4(4+6x)}\,dx = -\frac{27\log(x)}{32} + \frac{27\log\left(x+\frac{2}{3}\right)}{32} + \frac{-27x^2+9x-4}{48x^3}$$

input
```
integrate(1/x**4/(4+6*x),x)
```

output
```
-27*log(x)/32 + 27*log(x + 2/3)/32 + (-27*x**2 + 9*x - 4)/(48*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4(4+6x)}\,dx = -\frac{27\,x^2-9\,x+4}{48\,x^3} + \frac{27}{32}\log\left(3\,x+2\right) - \frac{27}{32}\log\left(x\right)$$

input
```
integrate(1/x^4/(4+6*x),x, algorithm="maxima")
```

output
```
-1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(3*x + 2) - 27/32*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4(4+6x)}\,dx = -\frac{27\,x^2-9\,x+4}{48\,x^3} + \frac{27}{32}\log\left(|3\,x+2|\right) - \frac{27}{32}\log\left(|x|\right)$$

input
```
integrate(1/x^4/(4+6*x),x, algorithm="giac")
```

output
```
-1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(abs(3*x + 2)) - 27/32*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4(4+6x)}\,dx = \frac{27\,\mathrm{atanh}(3\,x+1)}{16} - \frac{\frac{9\,x^2}{16} - \frac{3\,x}{16} + \frac{1}{12}}{x^3}$$

input
```
int(1/(x^4*(6*x + 4)),x)
```

output
```
(27*atanh(3*x + 1))/16 - ((9*x^2)/16 - (3*x)/16 + 1/12)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4(4+6x)}\,dx = \frac{81\log(3x+2)\,x^3 - 81\log(x)\,x^3 - 54x^2 + 18x - 8}{96x^3}$$

input
```
int(1/x^4/(4+6*x),x)
```

output
```
(81*log(3*x + 2)*x**3 - 81*log(x)*x**3 - 54*x**2 + 18*x - 8)/(96*x**3)
```

3.219 $\int \frac{1}{x^5(4+6x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \frac{1}{x^5(4+6x)}\,dx=-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)$$

output
```
-1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(4+6x)}\,dx=-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)$$

input
```
Integrate[1/(x^5*(4 + 6*x)),x]
```

output
```
-1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Lo
g[2 + 3*x])/64
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(6x+4)} \, dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(\frac{1}{4x^5}-\frac{3}{8x^4}+\frac{9}{16x^3}-\frac{27}{32x^2}-\frac{243}{64(3x+2)}+\frac{81}{64x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

input
```
Int[1/(x^5*(4 + 6*x)),x]
```

output
```
-1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Lo
g[2 + 3*x])/64
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

method	result	size
norman	$\frac{-\frac{1}{16}+\frac{1}{8}x-\frac{9}{32}x^2+\frac{27}{32}x^3}{x^4}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	33
risch	$\frac{-\frac{1}{16}+\frac{1}{8}x-\frac{9}{32}x^2+\frac{27}{32}x^3}{x^4}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	33
default	$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	34
parallelrisch	$\frac{81\ln(x)x^4-81\ln\left(\frac{2}{3}+x\right)x^4-4+54x^3-18x^2+8x}{64x^4}$	37
meijerg	$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\ln(x)}{64}+\frac{81\ln(3)}{64}-\frac{81\ln(2)}{64}-\frac{81\ln\left(1+\frac{3x}{2}\right)}{64}$	42

input
```
int(1/x^5/(4+6*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/16+1/8*x-9/32*x^2+27/32*x^3)/x^4+81/64*ln(x)-81/64*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5(4+6x)}\,dx = -\frac{81\,x^4\log\left(3\,x+2\right)-81\,x^4\log\left(x\right)-54\,x^3+18\,x^2-8\,x+4}{64\,x^4}$$

input
```
integrate(1/x^5/(4+6*x),x, algorithm="fricas")
```

output
```
-1/64*(81*x^4*log(3*x + 2) - 81*x^4*log(x) - 54*x^3 + 18*x^2 - 8*x + 4)/x^
4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5(4+6x)}\,dx = \frac{81\log(x)}{64} - \frac{81\log\left(x+\frac{2}{3}\right)}{64} + \frac{27x^3-9x^2+4x-2}{32x^4}$$

input
```
integrate(1/x**5/(4+6*x),x)
```

output
```
81*log(x)/64 - 81*log(x + 2/3)/64 + (27*x**3 - 9*x**2 + 4*x - 2)/(32*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^5(4+6x)}\,dx = \frac{27\,x^3-9\,x^2+4\,x-2}{32\,x^4} - \frac{81}{64}\log(3\,x+2) + \frac{81}{64}\log(x)$$

input
```
integrate(1/x^5/(4+6*x),x, algorithm="maxima")
```

output
```
1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(3*x + 2) + 81/64*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5(4+6x)}\,dx = \frac{27\,x^3-9\,x^2+4\,x-2}{32\,x^4} - \frac{81}{64}\log(|3\,x+2|) + \frac{81}{64}\log(|x|)$$

input
```
integrate(1/x^5/(4+6*x),x, algorithm="giac")
```

output
```
1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(abs(3*x + 2)) + 81/64*log(
abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^5(4+6x)}\,dx = \frac{\frac{27\,x^3}{32}-\frac{9\,x^2}{32}+\frac{x}{8}-\frac{1}{16}}{x^4}-\frac{81\,\mathrm{atanh}(3\,x+1)}{32}$$

input
```
int(1/(x^5*(6*x + 4)),x)
```

output
```
(x/8 - (9*x^2)/32 + (27*x^3)/32 - 1/16)/x^4 - (81*atanh(3*x + 1))/32
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5(4+6x)}\,dx = \frac{-81\log(3x+2)\,x^4+81\log(x)\,x^4+54x^3-18x^2+8x-4}{64x^4}$$

input
```
int(1/x^5/(4+6*x),x)
```

output
```
( - 81*log(3*x + 2)*x**4 + 81*log(x)*x**4 + 54*x**3 - 18*x**2 + 8*x - 4)/(
64*x**4)
```

3.220 $\int \frac{1}{x(4+6x)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{x(4+6x)^2}\,dx=\frac{1}{8(2+3x)}+\frac{\log(x)}{16}-\frac{1}{16}\log(2+3x)$$

output
```
1/(16+24*x)+1/16*ln(x)-1/16*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(4+6x)^2}\,dx=\frac{1}{16}\left(\frac{2}{2+3x}+\log(-6x)-\log(4+6x)\right)$$

input
```
Integrate[1/(x*(4 + 6*x)^2),x]
```

output
```
(2/(2 + 3*x) + Log[-6*x] - Log[4 + 6*x])/16
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(6x+4)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{3}{16(3x+2)}-\frac{3}{8(3x+2)^2}+\frac{1}{16x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8(3x+2)}+\frac{\log(x)}{16}-\frac{1}{16}\log(3x+2)$$

input
```
Int[1/(x*(4 + 6*x)^2),x]
```

output
```
1/(8*(2 + 3*x)) + Log[x]/16 - Log[2 + 3*x]/16
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{1}{16+24x}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	21
default	$\frac{1}{16+24x}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	23
norman	$-\frac{3x}{16(2+3x)}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	24
meijerg	$\frac{1}{16}+\frac{\ln(x)}{16}+\frac{\ln(3)}{16}-\frac{\ln(2)}{16}-\frac{3x}{16(2+3x)}-\frac{\ln(1+\frac{3x}{2})}{16}$	33
parallelrisch	$\frac{3\ln(x)x-3\ln(\frac{2}{3}+x)x+2\ln(x)-2\ln(\frac{2}{3}+x)-3x}{32+48x}$	36

input
```
int(1/x/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/(2/3+x)+1/16*ln(x)-1/16*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(4+6x)^2}\,dx=-\frac{(3\,x+2)\log(3\,x+2)-(3\,x+2)\log(x)-2}{16\,(3\,x+2)}$$

input
```
integrate(1/x/(4+6*x)^2,x, algorithm="fricas")
```

output
```
-1/16*((3*x + 2)*log(3*x + 2) - (3*x + 2)*log(x) - 2)/(3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x(4+6x)^2}\,dx = \frac{\log(x)}{16} - \frac{\log\left(x+\frac{2}{3}\right)}{16} + \frac{1}{24x+16}$$

input
```
integrate(1/x/(4+6*x)**2,x)
```

output
```
log(x)/16 - log(x + 2/3)/16 + 1/(24*x + 16)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x(4+6x)^2}\,dx = \frac{1}{8\,(3\,x+2)} - \frac{1}{16}\,\log\left(3\,x+2\right) + \frac{1}{16}\,\log\left(x\right)$$

input
```
integrate(1/x/(4+6*x)^2,x, algorithm="maxima")
```

output
```
1/8/(3*x + 2) - 1/16*log(3*x + 2) + 1/16*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(4+6x)^2}\,dx = \frac{1}{8\,(3\,x+2)} + \frac{1}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

input
```
integrate(1/x/(4+6*x)^2,x, algorithm="giac")
```

output
```
1/8/(3*x + 2) + 1/16*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(4+6x)^2}\,dx = \frac{1}{8\,(3\,x+2)} - \frac{\ln\left(\frac{6\,x+4}{x}\right)}{16}$$

input
```
int(1/(x*(6*x + 4)^2),x)
```

output
```
1/(8*(3*x + 2)) - log((6*x + 4)/x)/16
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{1}{x(4+6x)^2}\,dx = \frac{-3\log(3x+2)\,x - 2\log(3x+2) + 3\log(x)\,x + 2\log(x) - 3x}{48x+32}$$

input
```
int(1/x/(4+6*x)^2,x)
```

output
```
( - 3*log(3*x + 2)*x - 2*log(3*x + 2) + 3*log(x)*x + 2*log(x) - 3*x)/(16*(
3*x + 2))
```

3.221 $\int \frac{1}{x^2(4+6x)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{1}{x^2(4+6x)^2}\,dx = -\frac{1}{16x} - \frac{3}{16(2+3x)} - \frac{3\log(x)}{16} + \frac{3}{16}\log(2+3x)$$

output
```
-1/16/x-3/(32+48*x)-3/16*ln(x)+3/16*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(4+6x)^2}\,dx = \frac{1}{16}\left(-\frac{1}{x} - \frac{3}{2+3x} - 3\log(x) + 3\log(2+3x)\right)$$

input
```
Integrate[1/(x^2*(4 + 6*x)^2),x]
```

output
```
(-x^(-1) - 3/(2 + 3*x) - 3*Log[x] + 3*Log[2 + 3*x])/16
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(6x+4)^2}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{1}{16x^2}+\frac{9}{16(3x+2)}+\frac{9}{16(3x+2)^2}-\frac{3}{16x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

input
```
Int[1/(x^2*(4 + 6*x)^2),x]
```

output
```
-1/16*1/x - 3/(16*(2 + 3*x)) - (3*Log[x])/16 + (3*Log[2 + 3*x])/16
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{16x}-\frac{3\ln(x)}{16}-\frac{3}{16(2+3x)}+\frac{3\ln(2+3x)}{16}$	28
risch	$\frac{-\frac{3x}{8}-\frac{1}{8}}{x(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	31
norman	$\frac{-\frac{1}{8}+\frac{9x^2}{16}}{x(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	32
meijerg	$-\frac{1}{16x}-\frac{3}{32}-\frac{3\ln(x)}{16}-\frac{3\ln(3)}{16}+\frac{3\ln(2)}{16}+\frac{27x}{64\left(\frac{9x}{2}+3\right)}+\frac{3\ln\left(1+\frac{3x}{2}\right)}{16}$	38
parallelrisch	$-\frac{9\ln(x)x^2-9\ln\left(\frac{2}{3}+x\right)x^2+2+6\ln(x)x-6\ln\left(\frac{2}{3}+x\right)x-9x^2}{16x(2+3x)}$	48

input
```
int(1/x^2/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/x-3/16*ln(x)-3/16/(2+3*x)+3/16*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2(4+6x)^2}\,dx=\frac{3\,(3\,x^2+2\,x)\log\left(3\,x+2\right)-3\,(3\,x^2+2\,x)\log\left(x\right)-6\,x-2}{16\,(3\,x^2+2\,x)}$$

input
```
integrate(1/x^2/(4+6*x)^2,x, algorithm="fricas")
```

output
```
1/16*(3*(3*x^2 + 2*x)*log(3*x + 2) - 3*(3*x^2 + 2*x)*log(x) - 6*x - 2)/(3*
x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(4+6x)^2}\,dx = \frac{-3x-1}{24x^2+16x} - \frac{3\log{(x)}}{16} + \frac{3\log{\left(x+\frac{2}{3}\right)}}{16}$$

input
```
integrate(1/x**2/(4+6*x)**2,x)
```

output
```
(-3*x - 1)/(24*x**2 + 16*x) - 3*log(x)/16 + 3*log(x + 2/3)/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(4+6x)^2}\,dx = -\frac{3\,x+1}{8\,(3\,x^2+2\,x)} + \frac{3}{16}\,\log\left(3\,x+2\right) - \frac{3}{16}\,\log\left(x\right)$$

input
```
integrate(1/x^2/(4+6*x)^2,x, algorithm="maxima")
```

output
```
-1/8*(3*x + 1)/(3*x^2 + 2*x) + 3/16*log(3*x + 2) - 3/16*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(4+6x)^2}\,dx = -\frac{3}{16\,(3\,x+2)} + \frac{3}{32\,\left(\frac{2}{3\,x+2}-1\right)} - \frac{3}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

input
```
integrate(1/x^2/(4+6*x)^2,x, algorithm="giac")
```

output
```
-3/16/(3*x + 2) + 3/32/(2/(3*x + 2) - 1) - 3/16*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(4+6x)^2} \, dx = \frac{3 \ln \left(\frac{6 x+4}{x}\right)}{16} - \frac{3}{4 (6 x+4)} - \frac{1}{4 x (6 x+4)}$$

input
```
int(1/(x^2*(6*x + 4)^2),x)
```

output
```
(3*log((6*x + 4)/x))/16 - 3/(4*(6*x + 4)) - 1/(4*x*(6*x + 4))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2(4+6x)^2} \, dx = \frac{9 \log(3x+2) x^2 + 6 \log(3x+2) x - 9 \log(x) x^2 - 6 \log(x) x + 9x^2 - 2}{16x (3x+2)}$$

input
```
int(1/x^2/(4+6*x)^2,x)
```

output
```
(9*log(3*x + 2)*x**2 + 6*log(3*x + 2)*x - 9*log(x)*x**2 - 6*log(x)*x + 9*x
**2 - 2)/(16*x*(3*x + 2))
```

3.222 $\int \frac{1}{x^3(4+6x)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{x^3(4+6x)^2} dx = -\frac{1}{32x^2} + \frac{3}{16x} + \frac{9}{32(2+3x)} + \frac{27\log(x)}{64} - \frac{27}{64}\log(2+3x)$$

output
```
-1/32/x^2+3/16/x+9/(64+96*x)+27/64*ln(x)-27/64*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(4+6x)^2} dx = \frac{1}{64}\left(-\frac{2}{x^2} + \frac{12}{x} + \frac{18}{2+3x} + 27\log(x) - 27\log(2+3x)\right)$$

input
```
Integrate[1/(x^3*(4 + 6*x)^2),x]
```

output
```
(-2/x^2 + 12/x + 18/(2 + 3*x) + 27*Log[x] - 27*Log[2 + 3*x])/64
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(6x+4)^2} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{16x^3}-\frac{3}{16x^2}-\frac{81}{64(3x+2)}-\frac{27}{32(3x+2)^2}+\frac{27}{64x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

input
```
Int[1/(x^3*(4 + 6*x)^2),x]
```

output
```
-1/32*1/x^2 + 3/(16*x) + 9/(32*(2 + 3*x)) + (27*Log[x])/64 - (27*Log[2 + 3
*x])/64
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{32x^{2}}+\frac{3}{16x}+\frac{27\ln(x)}{64}+\frac{9}{32(2+3x)}-\frac{27\ln(2+3x)}{64}$	33
norman	$\frac{-\frac{1}{16}-\frac{81}{64}x^{3}+\frac{9}{32}x}{x^{2}(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	35
risch	$\frac{\frac{27}{32}x^{2}+\frac{9}{32}x-\frac{1}{16}}{x^{2}(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	36
meijerg	$-\frac{1}{32x^{2}}+\frac{3}{16x}+\frac{9}{64}+\frac{27\ln(x)}{64}+\frac{27\ln(3)}{64}-\frac{27\ln(2)}{64}-\frac{27x}{32(4+6x)}-\frac{27\ln\left(1+\frac{3x}{2}\right)}{64}$	43
parallelrisch	$\frac{81\ln(x)x^{3}-81\ln\left(\frac{2}{3}+x\right)x^{3}-4+54\ln(x)x^{2}-54\ln\left(\frac{2}{3}+x\right)x^{2}-81x^{3}+18x}{64x^{2}(2+3x)}$	55

input
```
int(1/x^3/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32/x^2+3/16/x+27/64*ln(x)+9/32/(2+3*x)-27/64*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^3(4+6x)^2}\,dx = \frac{54\,x^2-27\,(3\,x^3+2\,x^2)\log(3\,x+2)+27\,(3\,x^3+2\,x^2)\log(x)+18\,x-4}{64\,(3\,x^3+2\,x^2)}$$

input
```
integrate(1/x^3/(4+6*x)^2,x, algorithm="fricas")
```

output
```
1/64*(54*x^2 - 27*(3*x^3 + 2*x^2)*log(3*x + 2) + 27*(3*x^3 + 2*x^2)*log(x)
 + 18*x - 4)/(3*x^3 + 2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(4+6x)^2}\,dx = \frac{27\log{(x)}}{64} - \frac{27\log{\left(x+\frac{2}{3}\right)}}{64} + \frac{27x^2+9x-2}{96x^3+64x^2}$$

input
```
integrate(1/x**3/(4+6*x)**2,x)
```

output
```
27*log(x)/64 - 27*log(x + 2/3)/64 + (27*x**2 + 9*x - 2)/(96*x**3 + 64*x**2
)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3(4+6x)^2}\,dx = \frac{27\,x^2+9\,x-2}{32\,(3\,x^3+2\,x^2)} - \frac{27}{64}\log\left(3\,x+2\right) + \frac{27}{64}\log\left(x\right)$$

input
```
integrate(1/x^3/(4+6*x)^2,x, algorithm="maxima")
```

output
```
1/32*(27*x^2 + 9*x - 2)/(3*x^3 + 2*x^2) - 27/64*log(3*x + 2) + 27/64*log(x
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3(4+6x)^2}\,dx = \frac{9}{32\,(3\,x+2)} - \frac{9\left(\frac{12}{3\,x+2}-5\right)}{128\left(\frac{2}{3\,x+2}-1\right)^2} + \frac{27}{64}\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

input
```
integrate(1/x^3/(4+6*x)^2,x, algorithm="giac")
```

output
```
9/32/(3*x + 2) - 9/128*(12/(3*x + 2) - 5)/(2/(3*x + 2) - 1)^2 + 27/64*log(
abs(-2/(3*x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(4+6x)^2}\,dx = \frac{\frac{9\,x^2}{32}+\frac{3\,x}{32}-\frac{1}{48}}{x^3+\frac{2\,x^2}{3}} - \frac{27\,\mathrm{atanh}(3\,x+1)}{32}$$

input
```
int(1/(x^3*(6*x + 4)^2),x)
```

output
```
((3*x)/32 + (9*x^2)/32 - 1/48)/((2*x^2)/3 + x^3) - (27*atanh(3*x + 1))/32
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^3(4+6x)^2}\,dx = \frac{-81\log(3x+2)\,x^3-54\log(3x+2)\,x^2+81\log(x)\,x^3+54\log(x)\,x^2-81x^3+18x-4}{64x^2\,(3x+2)}$$

input
```
int(1/x^3/(4+6*x)^2,x)
```

output
```
( - 81*log(3*x + 2)*x**3 - 54*log(3*x + 2)*x**2 + 81*log(x)*x**3 + 54*log(
x)*x**2 - 81*x**3 + 18*x - 4)/(64*x**2*(3*x + 2))
```

3.223 $\int \frac{1}{x^4(4+6x)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{x^4(4+6x)^2}\,dx = -\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

output
```
-1/48/x^3+3/32/x^2-27/64/x-27/(128+192*x)-27/32*ln(x)+27/32*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4(4+6x)^2}\,dx = \frac{1}{192}\left(-\frac{4(2-6x+27x^2+81x^3)}{x^3(2+3x)}-162\log(x)+162\log(2+3x)\right)$$

input
```
Integrate[1/(x^4*(4 + 6*x)^2),x]
```

output
```
((-4*(2 - 6*x + 27*x^2 + 81*x^3))/(x^3*(2 + 3*x)) - 162*Log[x] + 162*Log[2
 + 3*x])/192
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(6x+4)^2}dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(\frac{1}{16x^4}-\frac{3}{16x^3}+\frac{27}{64x^2}+\frac{81}{32(3x+2)}+\frac{81}{64(3x+2)^2}-\frac{27}{32x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

input
```
Int[1/(x^4*(4 + 6*x)^2),x]
```

output
```
-1/48*1/x^3 + 3/(32*x^2) - 27/(64*x) - 27/(64*(2 + 3*x)) - (27*Log[x])/32
+ (27*Log[2 + 3*x])/32
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27\ln(x)}{32}-\frac{27}{64(2+3x)}+\frac{27\ln(2+3x)}{32}$	38
norman	$\frac{-\frac{1}{24}+\frac{81}{32}x^4+\frac{1}{8}x-\frac{9}{16}x^2}{x^3(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	40
risch	$\frac{-\frac{27}{16}x^3-\frac{9}{16}x^2+\frac{1}{8}x-\frac{1}{24}}{x^3(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	41
meijerg	$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{128}-\frac{27\ln(x)}{32}-\frac{27\ln(3)}{32}+\frac{27\ln(2)}{32}+\frac{405x}{256(5+\frac{15x}{2})}+\frac{27\ln(1+\frac{3x}{2})}{32}$	48
parallelrisch	$-\frac{243\ln(x)x^4-243\ln(\frac{2}{3}+x)x^4+4+162\ln(x)x^3-162\ln(\frac{2}{3}+x)x^3-243x^4+54x^2-12x}{96x^3(2+3x)}$	60

input
```
int(1/x^4/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48/x^3+3/32/x^2-27/64/x-27/32*ln(x)-27/64/(2+3*x)+27/32*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^4(4+6x)^2}\,dx = -\frac{162\,x^3+54\,x^2-81\,(3\,x^4+2\,x^3)\log(3\,x+2)+81\,(3\,x^4+2\,x^3)\log(x)-12\,x+4}{96\,(3\,x^4+2\,x^3)}$$

input
```
integrate(1/x^4/(4+6*x)^2,x, algorithm="fricas")
```

output
```
-1/96*(162*x^3 + 54*x^2 - 81*(3*x^4 + 2*x^3)*log(3*x + 2) + 81*(3*x^4 + 2*
x^3)*log(x) - 12*x + 4)/(3*x^4 + 2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4(4+6x)^2}\,dx = -\frac{27\log(x)}{32} + \frac{27\log\left(x+\frac{2}{3}\right)}{32} + \frac{-81x^3-27x^2+6x-2}{144x^4+96x^3}$$

input
```
integrate(1/x**4/(4+6*x)**2,x)
```

output
```
-27*log(x)/32 + 27*log(x + 2/3)/32 + (-81*x**3 - 27*x**2 + 6*x - 2)/(144*x
**4 + 96*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4(4+6x)^2}\,dx = -\frac{81\,x^3+27\,x^2-6\,x+2}{48\,(3\,x^4+2\,x^3)} + \frac{27}{32}\,\log(3\,x+2) - \frac{27}{32}\,\log(x)$$

input
```
integrate(1/x^4/(4+6*x)^2,x, algorithm="maxima")
```

output
```
-1/48*(81*x^3 + 27*x^2 - 6*x + 2)/(3*x^4 + 2*x^3) + 27/32*log(3*x + 2) - 2
7/32*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4(4+6x)^2}\,dx = -\frac{27}{64\,(3\,x+2)} - \frac{9\left(\frac{60}{3\,x+2}-\frac{72}{(3\,x+2)^2}-13\right)}{128\left(\frac{2}{3\,x+2}-1\right)^3} - \frac{27}{32}\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

input
```
integrate(1/x^4/(4+6*x)^2,x, algorithm="giac")
```

output
```
-27/64/(3*x + 2) - 9/128*(60/(3*x + 2) - 72/(3*x + 2)^2 - 13)/(2/(3*x + 2)
 - 1)^3 - 27/32*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4(4+6x)^2}\,dx = \frac{27\,\mathrm{atanh}(3\,x+1)}{16} - \frac{\frac{9\,x^3}{16}+\frac{3\,x^2}{16}-\frac{x}{24}+\frac{1}{72}}{x^4+\frac{2\,x^3}{3}}$$

input
```
int(1/(x^4*(6*x + 4)^2),x)
```

output
```
(27*atanh(3*x + 1))/16 - ((3*x^2)/16 - x/24 + (9*x^3)/16 + 1/72)/((2*x^3)/
3 + x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^4(4+6x)^2}\,dx = \frac{243\log(3x+2)\,x^4+162\log(3x+2)\,x^3-243\log(x)\,x^4-162\log(x)\,x^3+243x^4-54x^2+12x-4}{96x^3\,(3x+2)}$$

input
```
int(1/x^4/(4+6*x)^2,x)
```

output
```
(243*log(3*x + 2)*x**4 + 162*log(3*x + 2)*x**3 - 243*log(x)*x**4 - 162*log
(x)*x**3 + 243*x**4 - 54*x**2 + 12*x - 4)/(96*x**3*(3*x + 2))
```

3.224 $\int \frac{1}{x^5(4+6x)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{1}{x^5(4+6x)^2}\,dx = -\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)$$

output
```
-1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/(256+384*x)+405/256*ln(x)-405/256
*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(4+6x)^2}\,dx = -\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)$$

input
```
Integrate[1/(x^5*(4 + 6*x)^2),x]
```

output
```
-1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
 (405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(6x+4)^2}dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(\frac{1}{16x^5}-\frac{3}{16x^4}+\frac{27}{64x^3}-\frac{27}{32x^2}-\frac{1215}{256(3x+2)}-\frac{243}{128(3x+2)^2}+\frac{405}{256x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

input
```
Int[1/(x^5*(4 + 6*x)^2),x]
```

output
```
-1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
 (405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	s
default	$-\frac{1}{64x^{4}}+\frac{1}{16x^{3}}-\frac{27}{128x^{2}}+\frac{27}{32x}+\frac{405\ln(x)}{256}+\frac{81}{128(2+3x)}-\frac{405\ln(2+3x)}{256}$	4
norman	$\frac{-\frac{1}{32}-\frac{1215}{256}x^{5}+\frac{5}{64}x-\frac{15}{64}x^{2}+\frac{135}{128}x^{3}}{x^{4}(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	4
risch	$\frac{\frac{405}{128}x^{4}+\frac{135}{128}x^{3}-\frac{15}{64}x^{2}+\frac{5}{64}x-\frac{1}{32}}{x^{4}(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	4
meijerg	$-\frac{1}{64x^{4}}+\frac{1}{16x^{3}}-\frac{27}{128x^{2}}+\frac{27}{32x}+\frac{81}{256}+\frac{405\ln(x)}{256}+\frac{405\ln(3)}{256}-\frac{405\ln(2)}{256}-\frac{729x}{256(9x+6)}-\frac{405\ln\left(1+\frac{3x}{2}\right)}{256}$	5
parallelrisch	$\frac{1215\ln(x)x^{5}-1215\ln\left(\frac{2}{3}+x\right)x^{5}-8+810\ln(x)x^{4}-810\ln\left(\frac{2}{3}+x\right)x^{4}-1215x^{5}+270x^{3}-60x^{2}+20x}{256x^{4}(2+3x)}$	6

input
```
int(1/x^5/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+405/256*ln(x)+81/128/(2+3*x)-405/256
*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5(4+6x)^2}\,dx = \frac{810\,x^{4}+270\,x^{3}-60\,x^{2}-405\,(3\,x^{5}+2\,x^{4})\log(3\,x+2)+405\,(3\,x^{5}+2\,x^{4})\log(x)+20\,x-8}{256\,(3\,x^{5}+2\,x^{4})}$$

input
```
integrate(1/x^5/(4+6*x)^2,x, algorithm="fricas")
```

output
```
1/256*(810*x^4 + 270*x^3 - 60*x^2 - 405*(3*x^5 + 2*x^4)*log(3*x + 2) + 405
*(3*x^5 + 2*x^4)*log(x) + 20*x - 8)/(3*x^5 + 2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^5(4+6x)^2}\,dx = \frac{405\log{(x)}}{256} - \frac{405\log{\left(x+\frac{2}{3}\right)}}{256} + \frac{405x^4+135x^3-30x^2+10x-4}{384x^5+256x^4}$$

input
```
integrate(1/x**5/(4+6*x)**2,x)
```

output
```
405*log(x)/256 - 405*log(x + 2/3)/256 + (405*x**4 + 135*x**3 - 30*x**2 + 1
0*x - 4)/(384*x**5 + 256*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5(4+6x)^2}\,dx = \frac{405\,x^4+135\,x^3-30\,x^2+10\,x-4}{128\,(3\,x^5+2\,x^4)} - \frac{405}{256}\,\log\left(3\,x+2\right) + \frac{405}{256}\,\log\left(x\right)$$

input
```
integrate(1/x^5/(4+6*x)^2,x, algorithm="maxima")
```

output
```
1/128*(405*x^4 + 135*x^3 - 30*x^2 + 10*x - 4)/(3*x^5 + 2*x^4) - 405/256*lo
g(3*x + 2) + 405/256*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5(4+6x)^2}\,dx = \frac{81}{128\,(3\,x+2)} - \frac{27\left(\frac{520}{3\,x+2} - \frac{1200}{(3\,x+2)^2} + \frac{960}{(3\,x+2)^3} - 77\right)}{1024\left(\frac{2}{3\,x+2}-1\right)^4} + \frac{405}{256}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

input
```
integrate(1/x^5/(4+6*x)^2,x, algorithm="giac")
```

output
```
81/128/(3*x + 2) - 27/1024*(520/(3*x + 2) - 1200/(3*x + 2)^2 + 960/(3*x +
2)^3 - 77)/(2/(3*x + 2) - 1)^4 + 405/256*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^5(4+6x)^2}\,dx = \frac{\frac{135\,x^4}{128}+\frac{45\,x^3}{128}-\frac{5\,x^2}{64}+\frac{5\,x}{192}-\frac{1}{96}}{x^5+\frac{2\,x^4}{3}}-\frac{405\,\mathrm{atanh}(3\,x+1)}{128}$$

input
```
int(1/(x^5*(6*x + 4)^2),x)
```

output
```
((5*x)/192 - (5*x^2)/64 + (45*x^3)/128 + (135*x^4)/128 - 1/96)/((2*x^4)/3
+ x^5) - (405*atanh(3*x + 1))/128
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^5(4+6x)^2}\,dx = \frac{-1215\log(3x+2)\,x^5-810\log(3x+2)\,x^4+1215\log(x)\,x^5+810\log(x)\,x^4-1215x^5+270x^3-60x^2+}{256x^4\,(3x+2)}$$

input
```
int(1/x^5/(4+6*x)^2,x)
```

output
```
( - 1215*log(3*x + 2)*x**5 - 810*log(3*x + 2)*x**4 + 1215*log(x)*x**5 + 81
0*log(x)*x**4 - 1215*x**5 + 270*x**3 - 60*x**2 + 20*x - 8)/(256*x**4*(3*x
+ 2))
```

3.225 $\int \frac{1}{x(4+6x)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{x(4+6x)^3}\,dx = \frac{1}{32(2+3x)^2} + \frac{1}{32(2+3x)} + \frac{\log(x)}{64} - \frac{1}{64}\log(2+3x)$$

output
```
1/32/(2+3*x)^2+1/(64+96*x)+1/64*ln(x)-1/64*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(4+6x)^3}\,dx = \frac{1}{64}\left(\frac{6(1+x)}{(2+3x)^2} + \log(-6x) - \log(4+6x)\right)$$

input
```
Integrate[1/(x*(4 + 6*x)^3),x]
```

output
```
((6*(1 + x))/(2 + 3*x)^2 + Log[-6*x] - Log[4 + 6*x])/64
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(6x+4)^3} \, dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(-\frac{3}{64(3x+2)}-\frac{3}{32(3x+2)^2}-\frac{3}{16(3x+2)^3}+\frac{1}{64x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{32(3x+2)}+\frac{1}{32(3x+2)^2}+\frac{\log(x)}{64}-\frac{1}{64}\log(3x+2)$$

input
```
Int[1/(x*(4 + 6*x)^3),x]
```

output
```
1/(32*(2 + 3*x)^2) + 1/(32*(2 + 3*x)) + Log[x]/64 - Log[2 + 3*x]/64
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{\frac{3x}{32}+\frac{3}{32}}{(2+3x)^{2}}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	28
norman	$\frac{-\frac{3}{16}x-\frac{27}{128}x^{2}}{(2+3x)^{2}}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	31
default	$\frac{1}{32(2+3x)^{2}}+\frac{1}{64+96x}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	32
meijerg	$\frac{3}{128}+\frac{\ln(x)}{64}+\frac{\ln(3)}{64}-\frac{\ln(2)}{64}-\frac{3x\left(4+\frac{9x}{2}\right)}{256\left(1+\frac{3x}{2}\right)^{2}}-\frac{\ln\left(1+\frac{3x}{2}\right)}{64}$	38
parallelrisch	$\frac{18\ln(x)x^{2}-18\ln\left(\frac{2}{3}+x\right)x^{2}+24\ln(x)x-24\ln\left(\frac{2}{3}+x\right)x-27x^{2}+8\ln(x)-8\ln\left(\frac{2}{3}+x\right)-24x}{128(2+3x)^{2}}$	57

input
```
int(1/x/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
9*(1/96*x+1/96)/(2+3*x)^2+1/64*ln(x)-1/64*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{1}{x(4+6x)^3}\,dx = -\frac{(9\,x^2+12\,x+4)\log(3\,x+2)-(9\,x^2+12\,x+4)\log(x)-6\,x-6}{64\,(9\,x^2+12\,x+4)}$$

input
```
integrate(1/x/(4+6*x)^3,x, algorithm="fricas")
```

output
```
-1/64*((9*x^2 + 12*x + 4)*log(3*x + 2) - (9*x^2 + 12*x + 4)*log(x) - 6*x -
 6)/(9*x^2 + 12*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{x(4+6x)^3}\,dx = \frac{3x+3}{288x^2+384x+128} + \frac{\log(x)}{64} - \frac{\log\left(x+\frac{2}{3}\right)}{64}$$

input
```
integrate(1/x/(4+6*x)**3,x)
```

output
```
(3*x + 3)/(288*x**2 + 384*x + 128) + log(x)/64 - log(x + 2/3)/64
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x(4+6x)^3}\,dx = \frac{3\,(x+1)}{32\,(9\,x^2+12\,x+4)} - \frac{1}{64}\,\log(3\,x+2) + \frac{1}{64}\,\log(x)$$

input
```
integrate(1/x/(4+6*x)^3,x, algorithm="maxima")
```

output
```
3/32*(x + 1)/(9*x^2 + 12*x + 4) - 1/64*log(3*x + 2) + 1/64*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{x(4+6x)^3}\,dx = \frac{3\,(x+1)}{32\,(3\,x+2)^2} - \frac{1}{64}\,\log(|3\,x+2|) + \frac{1}{64}\,\log(|x|)$$

input
```
integrate(1/x/(4+6*x)^3,x, algorithm="giac")
```

output
```
3/32*(x + 1)/(3*x + 2)^2 - 1/64*log(abs(3*x + 2)) + 1/64*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(4+6x)^3}\,dx = \frac{1}{32\,(3\,x+2)} - \frac{\ln\left(\frac{6\,x+4}{x}\right)}{64} + \frac{1}{8\,{(6\,x+4)}^2}$$

input
```
int(1/(x*(6*x + 4)^3),x)
```

output
```
1/(32*(3*x + 2)) - log((6*x + 4)/x)/64 + 1/(8*(6*x + 4)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int \frac{1}{x(4+6x)^3}\,dx = \frac{-18\log(3x+2)\,x^2 - 24\log(3x+2)\,x - 8\log(3x+2) + 18\log(x)\,x^2 + 24\log(x)\,x + 8\log(x) - 9x^2 + 8}{1152x^2 + 1536x + 512}$$

input
```
int(1/x/(4+6*x)^3,x)
```

output
```
( - 18*log(3*x + 2)*x**2 - 24*log(3*x + 2)*x - 8*log(3*x + 2) + 18*log(x)*
x**2 + 24*log(x)*x + 8*log(x) - 9*x**2 + 8)/(128*(9*x**2 + 12*x + 4))
```

3.226 $\int \frac{1}{x^2(4+6x)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{x^2(4+6x)^3} dx = -\frac{1}{64x} - \frac{3}{64(2+3x)^2} - \frac{3}{32(2+3x)} - \frac{9\log(x)}{128} + \frac{9}{128}\log(2+3x)$$

output
```
-1/64/x-3/64/(2+3*x)^2-3/(64+96*x)-9/128*ln(x)+9/128*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2(4+6x)^3} dx = \frac{1}{128}\left(-\frac{2(4+27x+27x^2)}{x(2+3x)^2} - 9\log(x) + 9\log(2+3x)\right)$$

input
```
Integrate[1/(x^2*(4 + 6*x)^3),x]
```

output
```
((-2*(4 + 27*x + 27*x^2))/(x*(2 + 3*x)^2) - 9*Log[x] + 9*Log[2 + 3*x])/128
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(6x+4)^3} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{64x^2}+\frac{27}{128(3x+2)}+\frac{9}{32(3x+2)^2}+\frac{9}{32(3x+2)^3}-\frac{9}{128x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

input
```
Int[1/(x^2*(4 + 6*x)^3),x]
```

output
```
-1/64*1/x - 3/(64*(2 + 3*x)^2) - 3/(32*(2 + 3*x)) - (9*Log[x])/128 + (9*Lo
g[2 + 3*x])/128
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{-\frac{27}{64}x^2-\frac{27}{64}x-\frac{1}{16}}{x(2+3x)^2}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	36
default	$-\frac{1}{64x}-\frac{9\ln(x)}{128}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}+\frac{9\ln(2+3x)}{128}$	37
norman	$\frac{-\frac{1}{16}+\frac{27}{32}x^2+\frac{243}{256}x^3}{x(2+3x)^2}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	37
meijerg	$-\frac{1}{64x}-\frac{15}{256}-\frac{9\ln(x)}{128}-\frac{9\ln(3)}{128}+\frac{9\ln(2)}{128}+\frac{9x\left(\frac{15x}{2}+6\right)}{512\left(1+\frac{3x}{2}\right)^2}+\frac{9\ln\left(1+\frac{3x}{2}\right)}{128}$	43
parallelrisch	$-\frac{162\ln(x)x^3-162\ln\left(\frac{2}{3}+x\right)x^3+16+216\ln(x)x^2-216\ln\left(\frac{2}{3}+x\right)x^2-243x^3+72\ln(x)x-72\ln\left(\frac{2}{3}+x\right)x-216x^2}{256x(2+3x)^2}$	69

input
```
int(1/x^2/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
9*(-3/64*x^2-3/64*x-1/144)/x/(2+3*x)^2-9/128*ln(x)+9/128*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2(4+6x)^3}\,dx$$
$$= -\frac{54\,x^2-9\,(9\,x^3+12\,x^2+4\,x)\log(3\,x+2)+9\,(9\,x^3+12\,x^2+4\,x)\log(x)+54\,x+8}{128\,(9\,x^3+12\,x^2+4\,x)}$$

input
```
integrate(1/x^2/(4+6*x)^3,x, algorithm="fricas")
```

output
```
-1/128*(54*x^2 - 9*(9*x^3 + 12*x^2 + 4*x)*log(3*x + 2) + 9*(9*x^3 + 12*x^2
 + 4*x)*log(x) + 54*x + 8)/(9*x^3 + 12*x^2 + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(4+6x)^3}\,dx = \frac{-27x^2-27x-4}{576x^3+768x^2+256x} - \frac{9\log{(x)}}{128} + \frac{9\log{\left(x+\frac{2}{3}\right)}}{128}$$

input
```
integrate(1/x**2/(4+6*x)**3,x)
```

output
```
(-27*x**2 - 27*x - 4)/(576*x**3 + 768*x**2 + 256*x) - 9*log(x)/128 + 9*log
(x + 2/3)/128
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(4+6x)^3}\,dx = -\frac{27\,x^2+27\,x+4}{64\,(9\,x^3+12\,x^2+4\,x)} + \frac{9}{128}\,\log{(3\,x+2)} - \frac{9}{128}\,\log{(x)}$$

input
```
integrate(1/x^2/(4+6*x)^3,x, algorithm="maxima")
```

output
```
-1/64*(27*x^2 + 27*x + 4)/(9*x^3 + 12*x^2 + 4*x) + 9/128*log(3*x + 2) - 9/
128*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2(4+6x)^3}\,dx = -\frac{27\,x^2+27\,x+4}{64\,(3\,x+2)^2x} + \frac{9}{128}\,\log{(|3\,x+2|)} - \frac{9}{128}\,\log{(|x|)}$$

input
```
integrate(1/x^2/(4+6*x)^3,x, algorithm="giac")
```

output
```
-1/64*(27*x^2 + 27*x + 4)/((3*x + 2)^2*x) + 9/128*log(abs(3*x + 2)) - 9/12
8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2(4+6x)^3}\,dx = \frac{9\,\mathrm{atanh}(3\,x+1)}{64} - \frac{\frac{3\,x^2}{64}+\frac{3\,x}{64}+\frac{1}{144}}{x^3+\frac{4\,x^2}{3}+\frac{4\,x}{9}}$$

input
```
int(1/(x^2*(6*x + 4)^3),x)
```

output
```
(9*atanh(3*x + 1))/64 - ((3*x)/64 + (3*x^2)/64 + 1/144)/((4*x)/9 + (4*x^2)
/3 + x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^2(4+6x)^3}\,dx = \frac{162\log(3x+2)\,x^3+216\log(3x+2)\,x^2+72\log(3x+2)\,x-162\log(x)\,x^3-216\log(x)\,x^2-72\log(x)\,x}{256x\,(9x^2+12x+4)}$$

input
```
int(1/x^2/(4+6*x)^3,x)
```

output
```
(162*log(3*x + 2)*x**3 + 216*log(3*x + 2)*x**2 + 72*log(3*x + 2)*x - 162*l
og(x)*x**3 - 216*log(x)*x**2 - 72*log(x)*x + 81*x**3 - 72*x - 16)/(256*x*(
9*x**2 + 12*x + 4))
```

3.227 $\int \frac{1}{x^3(4+6x)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{1}{x^3(4+6x)^3}\,dx=-\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)$$

output
```
-1/128/x^2+9/128/x+9/128/(2+3*x)^2+27/(256+384*x)+27/128*ln(x)-27/128*ln(2
+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3(4+6x)^3}\,dx=\frac{1}{128}\left(\frac{2(-2+12x+81x^2+81x^3)}{x^2(2+3x)^2}+27\log(x)-27\log(2+3x)\right)$$

input
```
Integrate[1/(x^3*(4 + 6*x)^3),x]
```

output
```
((2*(-2 + 12*x + 81*x^2 + 81*x^3))/(x^2*(2 + 3*x)^2) + 27*Log[x] - 27*Log[
2 + 3*x])/128
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(6x+4)^3}\,dx$$

$$\Big\downarrow \ 54$$

$$\int \left(\frac{1}{64x^3}-\frac{9}{128x^2}-\frac{81}{128(3x+2)}-\frac{81}{128(3x+2)^2}-\frac{27}{64(3x+2)^3}+\frac{27}{128x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

input
```
Int[1/(x^3*(4 + 6*x)^3),x]
```

output
```
-1/128*1/x^2 + 9/(128*x) + 9/(128*(2 + 3*x)^2) + 27/(128*(2 + 3*x)) + (27*
Log[x])/128 - (27*Log[2 + 3*x])/128
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
norman	$\frac{-\frac{1}{32}-\frac{81}{32}x^{3}-\frac{729}{256}x^{4}+\frac{3}{16}x}{x^{2}\left(2+3x\right)^{2}}+\frac{27\ln\left(x\right)}{128}-\frac{27\ln\left(2+3x\right)}{128}$	40
risch	$\frac{\frac{81}{64}x^{3}+\frac{81}{64}x^{2}+\frac{3}{16}x-\frac{1}{32}}{x^{2}\left(2+3x\right)^{2}}+\frac{27\ln\left(x\right)}{128}-\frac{27\ln\left(2+3x\right)}{128}$	41
default	$-\frac{1}{128x^{2}}+\frac{9}{128x}+\frac{27\ln\left(x\right)}{128}+\frac{9}{128\left(2+3x\right)^{2}}+\frac{27}{128\left(2+3x\right)}-\frac{27\ln\left(2+3x\right)}{128}$	42
meijerg	$-\frac{1}{128x^{2}}+\frac{9}{128x}+\frac{63}{512}+\frac{27\ln\left(x\right)}{128}+\frac{27\ln\left(3\right)}{128}-\frac{27\ln\left(2\right)}{128}-\frac{27x\left(8+\frac{21x}{2}\right)}{1024\left(1+\frac{3x}{2}\right)^{2}}-\frac{27\ln\left(1+\frac{3x}{2}\right)}{128}$	48
parallelrisch	$\frac{486\ln\left(x\right)x^{4}-486\ln\left(\frac{2}{3}+x\right)x^{4}-8+648\ln\left(x\right)x^{3}-648\ln\left(\frac{2}{3}+x\right)x^{3}-729x^{4}+216\ln\left(x\right)x^{2}-216\ln\left(\frac{2}{3}+x\right)x^{2}-648x^{3}+48x}{256x^{2}\left(2+3x\right)^{2}}$	76

input
```
int(1/x^3/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/32-81/32*x^3-729/256*x^4+3/16*x)/x^2/(2+3*x)^2+27/128*ln(x)-27/128*ln(
2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^3(4+6x)^3}\,dx = \frac{162\,x^{3}+162\,x^{2}-27\,\left(9\,x^{4}+12\,x^{3}+4\,x^{2}\right)\log\left(3\,x+2\right)+27\,\left(9\,x^{4}+12\,x^{3}+4\,x^{2}\right)\log\left(x\right)+24\,x-4}{128\,\left(9\,x^{4}+12\,x^{3}+4\,x^{2}\right)}$$

input
```
integrate(1/x^3/(4+6*x)^3,x, algorithm="fricas")
```

output
```
1/128*(162*x^3 + 162*x^2 - 27*(9*x^4 + 12*x^3 + 4*x^2)*log(3*x + 2) + 27*(
9*x^4 + 12*x^3 + 4*x^2)*log(x) + 24*x - 4)/(9*x^4 + 12*x^3 + 4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3(4+6x)^3}\,dx = \frac{27\log{(x)}}{128} - \frac{27\log{\left(x+\frac{2}{3}\right)}}{128} + \frac{81x^3+81x^2+12x-2}{576x^4+768x^3+256x^2}$$

input
```
integrate(1/x**3/(4+6*x)**3,x)
```

output
```
27*log(x)/128 - 27*log(x + 2/3)/128 + (81*x**3 + 81*x**2 + 12*x - 2)/(576*
x**4 + 768*x**3 + 256*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3(4+6x)^3}\,dx = \frac{81\,x^3+81\,x^2+12\,x-2}{64\,(9\,x^4+12\,x^3+4\,x^2)} - \frac{27}{128}\,\log\left(3\,x+2\right) + \frac{27}{128}\,\log\left(x\right)$$

input
```
integrate(1/x^3/(4+6*x)^3,x, algorithm="maxima")
```

output
```
1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(9*x^4 + 12*x^3 + 4*x^2) - 27/128*log(3*
x + 2) + 27/128*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(4+6x)^3}\,dx = \frac{81\,x^3+81\,x^2+12\,x-2}{64\,(3\,x^2+2\,x)^2} - \frac{27}{128}\,\log\left(|3\,x+2|\right) + \frac{27}{128}\,\log\left(|x|\right)$$

input
```
integrate(1/x^3/(4+6*x)^3,x, algorithm="giac")
```

output
```
1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(3*x^2 + 2*x)^2 - 27/128*log(abs(3*x + 2
)) + 27/128*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3(4+6x)^3}\,dx = \frac{\frac{9\,x^3}{64}+\frac{9\,x^2}{64}+\frac{x}{48}-\frac{1}{288}}{x^4+\frac{4\,x^3}{3}+\frac{4\,x^2}{9}} - \frac{27\,\mathrm{atanh}(3\,x+1)}{64}$$

input
```
int(1/(x^3*(6*x + 4)^3),x)
```

output
```
(x/48 + (9*x^2)/64 + (9*x^3)/64 - 1/288)/((4*x^2)/9 + (4*x^3)/3 + x^4) - (
27*atanh(3*x + 1))/64
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^3(4+6x)^3}\,dx$$
$$= \frac{-486\log(3x+2)\,x^4 - 648\log(3x+2)\,x^3 - 216\log(3x+2)\,x^2 + 486\log(x)\,x^4 + 648\log(x)\,x^3 + 216\log}{256x^2\,(9x^2+12x+4)}$$

input
```
int(1/x^3/(4+6*x)^3,x)
```

output
```
( - 486*log(3*x + 2)*x**4 - 648*log(3*x + 2)*x**3 - 216*log(3*x + 2)*x**2
+ 486*log(x)*x**4 + 648*log(x)*x**3 + 216*log(x)*x**2 - 243*x**4 + 216*x**
2 + 48*x - 8)/(256*x**2*(9*x**2 + 12*x + 4))
```

3.228 $\int \frac{1}{x^4(4+6x)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{x^4(4+6x)^3}\,dx = -\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\log(x)}{256}+\frac{135}{256}\log(2+3x)$$

output
```
-1/192/x^3+9/256/x^2-27/128/x-27/256/(2+3*x)^2-27/(128+192*x)-135/256*ln(x
)+135/256*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4(4+6x)^3}\,dx = \frac{1}{768}\left(-\frac{2(8-30x+180x^2+1215x^3+1215x^4)}{x^3(2+3x)^2}-405\log(x)+405\log(2+3x)\right)$$

input
```
Integrate[1/(x^4*(4 + 6*x)^3),x]
```

output
```
((-2*(8 - 30*x + 180*x^2 + 1215*x^3 + 1215*x^4))/(x^3*(2 + 3*x)^2) - 405*L
og[x] + 405*Log[2 + 3*x])/768
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(6x+4)^3}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{64x^4}-\frac{9}{128x^3}+\frac{27}{128x^2}+\frac{405}{256(3x+2)}+\frac{81}{64(3x+2)^2}+\frac{81}{128(3x+2)^3}-\frac{135}{256x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{64(3x+2)}-\frac{27}{256(3x+2)^2}-\frac{135\log(x)}{256}+\frac{135}{256}\log(3x+2)$$

input
```
Int[1/(x^4*(4 + 6*x)^3),x]
```

output
```
-1/192*1/x^3 + 9/(256*x^2) - 27/(128*x) - 27/(256*(2 + 3*x)^2) - 27/(64*(2
 + 3*x)) - (135*Log[x])/256 + (135*Log[2 + 3*x])/256
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result
norman	$\frac{-\frac{1}{48}+\frac{405}{64}x^4+\frac{3645}{512}x^5+\frac{5}{64}x-\frac{15}{32}x^2}{x^3(2+3x)^2}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$
risch	$\frac{-\frac{405}{128}x^4-\frac{405}{128}x^3-\frac{15}{32}x^2+\frac{5}{64}x-\frac{1}{48}}{x^3(2+3x)^2}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$
default	$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{135\ln(x)}{256}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}+\frac{135\ln(2+3x)}{256}$
meijerg	$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{243}{1024}-\frac{135\ln(x)}{256}-\frac{135\ln(3)}{256}+\frac{135\ln(2)}{256}+\frac{81x\left(\frac{27x}{2}+10\right)}{2048\left(1+\frac{3x}{2}\right)^2}+\frac{135\ln\left(1+\frac{3x}{2}\right)}{256}$
parallelrisch	$-\frac{7290\ln(x)x^5-7290\ln\left(\frac{2}{3}+x\right)x^5+32+9720\ln(x)x^4-9720\ln\left(\frac{2}{3}+x\right)x^4-10935x^5+3240\ln(x)x^3-3240\ln\left(\frac{2}{3}+x\right)x^3-9720x^4-}{1536x^3(2+3x)^2}$

input
```
int(1/x^4/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/48+405/64*x^4+3645/512*x^5+5/64*x-15/32*x^2)/x^3/(2+3*x)^2-135/256*ln(
x)+135/256*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^4(4+6x)^3}\,dx = -\frac{2430\,x^4+2430\,x^3+360\,x^2-405\,(9\,x^5+12\,x^4+4\,x^3)\log(3\,x+2)+405\,(9\,x^5+12\,x^4+4\,x^3)\log(x)}{768\,(9\,x^5+12\,x^4+4\,x^3)}$$

input
```
integrate(1/x^4/(4+6*x)^3,x, algorithm="fricas")
```

output
```
-1/768*(2430*x^4 + 2430*x^3 + 360*x^2 - 405*(9*x^5 + 12*x^4 + 4*x^3)*log(3
*x + 2) + 405*(9*x^5 + 12*x^4 + 4*x^3)*log(x) - 60*x + 16)/(9*x^5 + 12*x^4
 + 4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(4+6x)^3}\,dx = -\frac{135\log(x)}{256} + \frac{135\log\left(x+\frac{2}{3}\right)}{256} + \frac{-1215x^4-1215x^3-180x^2+30x-8}{3456x^5+4608x^4+1536x^3}$$

input
```
integrate(1/x**4/(4+6*x)**3,x)
```

output
```
-135*log(x)/256 + 135*log(x + 2/3)/256 + (-1215*x**4 - 1215*x**3 - 180*x**
2 + 30*x - 8)/(3456*x**5 + 4608*x**4 + 1536*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4(4+6x)^3}\,dx = -\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\,(9\,x^5+12\,x^4+4\,x^3)} + \frac{135}{256}\log(3\,x+2) - \frac{135}{256}\log(x)$$

input
```
integrate(1/x^4/(4+6*x)^3,x, algorithm="maxima")
```

output
```
-1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/(9*x^5 + 12*x^4 + 4*x^3)
 + 135/256*log(3*x + 2) - 135/256*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4(4+6x)^3}\,dx = -\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\,(3\,x+2)^2x^3} + \frac{135}{256}\,\log\left(|3\,x+2|\right) - \frac{135}{256}\,\log\left(|x|\right)$$

input
```
integrate(1/x^4/(4+6*x)^3,x, algorithm="giac")
```

output
```
-1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/((3*x + 2)^2*x^3) + 135/
256*log(abs(3*x + 2)) - 135/256*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4(4+6x)^3}\,dx = \frac{135\,\mathrm{atanh}(3\,x+1)}{128} - \frac{\frac{45\,x^4}{128}+\frac{45\,x^3}{128}+\frac{5\,x^2}{96}-\frac{5\,x}{576}+\frac{1}{432}}{x^5+\frac{4\,x^4}{3}+\frac{4\,x^3}{9}}$$

input
```
int(1/(x^4*(6*x + 4)^3),x)
```

output
```
(135*atanh(3*x + 1))/128 - ((5*x^2)/96 - (5*x)/576 + (45*x^3)/128 + (45*x^
4)/128 + 1/432)/((4*x^3)/9 + (4*x^4)/3 + x^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^4(4+6x)^3}\,dx = \frac{7290\log(3x+2)\,x^5+9720\log(3x+2)\,x^4+3240\log(3x+2)\,x^3-7290\log(x)\,x^5-9720\log(x)\,x^4-324\cdots}{1536x^3\,(9x^2+12x+4)}$$

input
```
int(1/x^4/(4+6*x)^3,x)
```

output
```
(7290*log(3*x + 2)*x**5 + 9720*log(3*x + 2)*x**4 + 3240*log(3*x + 2)*x**3
- 7290*log(x)*x**5 - 9720*log(x)*x**4 - 3240*log(x)*x**3 + 3645*x**5 - 324
0*x**3 - 720*x**2 + 120*x - 32)/(1536*x**3*(9*x**2 + 12*x + 4))
```

3.229 $\int \frac{1}{x^5(4+6x)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{1}{x^5(4+6x)^3}\,dx = -\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{1215\log(2+3x)}{1024}$$

output
```
-1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(2+3*x)^2+405/(1024+1536*
x)+1215/1024*ln(x)-1215/1024*ln(2+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \frac{\frac{2(-8+24x-90x^2+540x^3+3645x^4+3645x^5)}{x^4(2+3x)^2}+1215\log(x)-1215\log(2+3x)}{1024}$$

input
```
Integrate[1/(x^5*(4 + 6*x)^3),x]
```

output
```
((2*(-8 + 24*x - 90*x^2 + 540*x^3 + 3645*x^4 + 3645*x^5))/(x^4*(2 + 3*x)^2
) + 1215*Log[x] - 1215*Log[2 + 3*x])/1024
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5(6x+4)^3}\,dx$$
$$\Big\downarrow \ 54$$
$$\int \left(\frac{1}{64x^5}-\frac{9}{128x^4}+\frac{27}{128x^3}-\frac{135}{256x^2}-\frac{3645}{1024(3x+2)}-\frac{1215}{512(3x+2)^2}-\frac{243}{256(3x+2)^3}+\frac{1215}{1024x}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

input
```
Int[1/(x^5*(4 + 6*x)^3),x]
```

output
```
-1/256*1/x^4 + 3/(128*x^3) - 27/(256*x^2) + 135/(256*x) + 81/(512*(2 + 3*x
)^2) + 405/(512*(2 + 3*x)) + (1215*Log[x])/1024 - (1215*Log[2 + 3*x])/1024
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result
norman	$\frac{-\frac{1}{64}-\frac{3645}{256}x^5-\frac{32805}{2048}x^6+\frac{3}{64}x-\frac{45}{256}x^2+\frac{135}{128}x^3}{x^4(2+3x)^2}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
risch	$\frac{\frac{3645}{512}x^5+\frac{3645}{512}x^4+\frac{135}{128}x^3-\frac{45}{256}x^2+\frac{3}{64}x-\frac{1}{64}}{x^4(2+3x)^2}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
default	$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{1215\ln(x)}{1024}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}-\frac{1215\ln(2+3x)}{1024}$
meijerg	$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{891}{2048}+\frac{1215\ln(x)}{1024}+\frac{1215\ln(3)}{1024}-\frac{1215\ln(2)}{1024}-\frac{243x(\frac{33x}{2}+12)}{4096(1+\frac{3x}{2})^2}-$ 1215 l
parallelrisch	$\frac{21870\ln(x)x^6-21870\ln(\frac{2}{3}+x)x^6-32+29160\ln(x)x^5-29160\ln(\frac{2}{3}+x)x^5-32805x^6+9720\ln(x)x^4-9720\ln(\frac{2}{3}+x)x^4-29160}{2048x^4(2+3x)^2}$

input
```
int(1/x^5/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/64-3645/256*x^5-32805/2048*x^6+3/64*x-45/256*x^2+135/128*x^3)/x^4/(2+3
*x)^2+1215/1024*ln(x)-1215/1024*ln(2+3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \frac{7290\,x^5+7290\,x^4+1080\,x^3-180\,x^2-1215\,(9\,x^6+12\,x^5+4\,x^4)\log(3\,x+2)+1215\,(9\,x^6+12\,x^5+4}{1024\,(9\,x^6+12\,x^5+4\,x^4)}$$

input
```
integrate(1/x^5/(4+6*x)^3,x, algorithm="fricas")
```

output
```
1/1024*(7290*x^5 + 7290*x^4 + 1080*x^3 - 180*x^2 - 1215*(9*x^6 + 12*x^5 +
4*x^4)*log(3*x + 2) + 1215*(9*x^6 + 12*x^5 + 4*x^4)*log(x) + 48*x - 16)/(9
*x^6 + 12*x^5 + 4*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \frac{1215\log{(x)}}{1024} - \frac{1215\log{\left(x+\frac{2}{3}\right)}}{1024} + \frac{3645x^5+3645x^4+540x^3-90x^2+24x-8}{4608x^6+6144x^5+2048x^4}$$

input
```
integrate(1/x**5/(4+6*x)**3,x)
```

output
```
1215*log(x)/1024 - 1215*log(x + 2/3)/1024 + (3645*x**5 + 3645*x**4 + 540*x
**3 - 90*x**2 + 24*x - 8)/(4608*x**6 + 6144*x**5 + 2048*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \frac{3645\,x^5+3645\,x^4+540\,x^3-90\,x^2+24\,x-8}{512\,(9\,x^6+12\,x^5+4\,x^4)} - \frac{1215}{1024}\,\log{(3\,x+2)} + \frac{1215}{1024}\,\log{(x)}$$

input
```
integrate(1/x^5/(4+6*x)^3,x, algorithm="maxima")
```

output
```
1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/(9*x^6 + 12*x^5
+ 4*x^4) - 1215/1024*log(3*x + 2) + 1215/1024*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \frac{3645\,x^5+3645\,x^4+540\,x^3-90\,x^2+24\,x-8}{512\,(3\,x+2)^2x^4} - \frac{1215}{1024}\,\log\left(|3\,x+2|\right)+\frac{1215}{1024}\,\log\left(|x|\right)$$

input
```
integrate(1/x^5/(4+6*x)^3,x, algorithm="giac")
```

output
```
1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/((3*x + 2)^2*x^4
) - 1215/1024*log(abs(3*x + 2)) + 1215/1024*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \frac{\frac{405\,x^5}{512}+\frac{405\,x^4}{512}+\frac{15\,x^3}{128}-\frac{5\,x^2}{256}+\frac{x}{192}-\frac{1}{576}}{x^6+\frac{4\,x^5}{3}+\frac{4\,x^4}{9}} - \frac{1215\,\mathrm{atanh}(3\,x+1)}{512}$$

input
```
int(1/(x^5*(6*x + 4)^3),x)
```

output
```
(x/192 - (5*x^2)/256 + (15*x^3)/128 + (405*x^4)/512 + (405*x^5)/512 - 1/57
6)/((4*x^4)/9 + (4*x^5)/3 + x^6) - (1215*atanh(3*x + 1))/512
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \frac{-21870\log(3x+2)\,x^6-29160\log(3x+2)\,x^5-9720\log(3x+2)\,x^4+21870\log(x)\,x^6+29160\log(x)\,x^5}{2048x^4\,(9x^2+12x+4)}$$

input
```
int(1/x^5/(4+6*x)^3,x)
```

output
```
( - 21870*log(3*x + 2)*x**6 - 29160*log(3*x + 2)*x**5 - 9720*log(3*x + 2)*
x**4 + 21870*log(x)*x**6 + 29160*log(x)*x**5 + 9720*log(x)*x**4 - 10935*x*
*6 + 9720*x**4 + 2160*x**3 - 360*x**2 + 96*x - 32)/(2048*x**4*(9*x**2 + 12
*x + 4))
```

3.230 $\int \frac{1}{x(1+bx)} dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{x(1+bx)} dx = \log(x) - \log(1+bx)$$

output
```
ln(x)-ln(b*x+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+bx)} dx = \log(x) - \log(1+bx)$$

input
```
Integrate[1/(x*(1 + b*x)),x]
```

output
```
Log[x] - Log[1 + b*x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(bx+1)}\,dx$$
$$\Big\downarrow \text{47}$$
$$\int \frac{1}{x}dx - b\int \frac{1}{bx+1}dx$$
$$\Big\downarrow \text{14}$$
$$\log(x) - b\int \frac{1}{bx+1}dx$$
$$\Big\downarrow \text{16}$$
$$\log(x) - \log(bx+1)$$

input
```
Int[1/(x*(1 + b*x)),x]
```

output
```
Log[x] - Log[1 + b*x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\ln(x) - \ln(bx + 1)$	12
norman	$\ln(x) - \ln(bx + 1)$	12
parallelrisch	$\ln(x) - \ln(bx + 1)$	12
meijerg	$\ln(x) + \ln(b) - \ln(bx + 1)$	14
risch	$\ln(-x) - \ln(bx + 1)$	14

input
```
int(1/x/(b*x+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-ln(b*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+bx)}\,dx = -\log(bx+1) + \log(x)$$

input
```
integrate(1/x/(b*x+1),x, algorithm="fricas")
```

output
```
-log(b*x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{x(1+bx)}\,dx = \log(x) - \log\left(x + \frac{1}{b}\right)$$

input
```
integrate(1/x/(b*x+1),x)
```

output
```
log(x) - log(x + 1/b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+bx)}\,dx = -\log(bx+1) + \log(x)$$

input
```
integrate(1/x/(b*x+1),x, algorithm="maxima")
```

output
```
-log(b*x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(1+bx)}\,dx = -\log(|bx+1|) + \log(|x|)$$

input
```
integrate(1/x/(b*x+1),x, algorithm="giac")
```

output
```
-log(abs(b*x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(1+bx)}\,dx = -2\,\mathrm{atanh}(2\,b\,x+1)$$

input
```
int(1/(x*(b*x + 1)),x)
```

output
```
-2*atanh(2*b*x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+bx)}\,dx = -\log(bx+1)+\log(x)$$

input
```
int(1/x/(b*x+1),x)
```

output
```
- log(b*x + 1) + log(x)
```

3.231 $\int \frac{1}{x(-1+bx)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{x(-1+bx)}\,dx = -\log(x)+\log(1-bx)$$

output
```
-ln(x)+ln(-b*x+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(-1+bx)}\,dx = -\log(x)+\log(1-bx)$$

input
```
Integrate[1/(x*(-1 + b*x)),x]
```

output
```
-Log[x] + Log[1 - b*x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(bx-1)}dx$$
$$\Big\downarrow \text{47}$$
$$b\int \frac{1}{bx-1}dx-\int \frac{1}{x}dx$$
$$\Big\downarrow \text{14}$$
$$b\int \frac{1}{bx-1}dx-\log(x)$$
$$\Big\downarrow \text{16}$$
$$\log(1-bx)-\log(x)$$

input
```
Int[1/(x*(-1 + b*x)),x]
```

output
```
-Log[x] + Log[1 - b*x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
default	$-\ln(x)+\ln(bx-1)$	12
norman	$-\ln(x)+\ln(bx-1)$	12
parallelrisch	$-\ln(x)+\ln(bx-1)$	12
risch	$-\ln(x)+\ln(-bx+1)$	13
meijerg	$-\ln(x)-\ln(-b)+\ln(-bx+1)$	19

input
```
int(1/x/(b*x-1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(b*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(-1+bx)}\,dx = \log(bx-1) - \log(x)$$

input
```
integrate(1/x/(b*x-1),x, algorithm="fricas")
```

output
```
log(b*x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x(-1+bx)} dx = -\log(x) + \log\left(x - \frac{1}{b}\right)$$

input
```
integrate(1/x/(b*x-1),x)
```

output
```
-log(x) + log(x - 1/b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(-1+bx)} dx = \log(bx-1) - \log(x)$$

input
```
integrate(1/x/(b*x-1),x, algorithm="maxima")
```

output
```
log(b*x - 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(-1+bx)} dx = \log(|bx-1|) - \log(|x|)$$

input
```
integrate(1/x/(b*x-1),x, algorithm="giac")
```

output
```
log(abs(b*x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{x(-1+bx)}\,dx = -2\,\mathrm{atanh}(2\,b\,x-1)$$

input
```
int(1/(x*(b*x - 1)),x)
```

output
```
-2*atanh(2*b*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(-1+bx)}\,dx = \log(bx-1)-\log(x)$$

input
```
int(1/x/(b*x-1),x)
```

output
```
log(b*x - 1) - log(x)
```

3.232 $\int \frac{1}{x^2(1+bx)} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{x^2(1+bx)} dx = -\frac{1}{x} - b\log(x) + b\log(1+bx)$$

output
```
-1/x-b*ln(x)+b*ln(b*x+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(1+bx)} dx = -\frac{1}{x} - b\log(x) + b\log(1+bx)$$

input
```
Integrate[1/(x^2*(1 + b*x)),x]
```

output
```
-x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(bx+1)}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(\frac{b^2}{bx+1}-\frac{b}{x}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-b\log(x)+b\log(bx+1)-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 + b*x)),x]
```

output
```
-x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx+1\right)$	20
norman	$-\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx+1\right)$	20
risch	$-\frac{1}{x}+b\ln\left(-bx-1\right)-b\ln\left(x\right)$	21
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+1)x+1}{x}$	23
meijerg	$b\left(-\frac{1}{xb}-\ln\left(x\right)-\ln\left(b\right)+\ln\left(bx+1\right)\right)$	26

input
```
int(1/x^2/(b*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x-b*ln(x)+b*ln(b*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2(1+bx)}\,dx=\frac{bx\log\left(bx+1\right)-bx\log\left(x\right)-1}{x}$$

input
```
integrate(1/x^2/(b*x+1),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + 1) - b*x*log(x) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2(1+bx)}\,dx = b\left(-\log{(x)} + \log{\left(x + \frac{1}{b}\right)}\right) - \frac{1}{x}$$

input
```
integrate(1/x**2/(b*x+1),x)
```

output
```
b*(-log(x) + log(x + 1/b)) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(1+bx)}\,dx = b\log{(bx+1)} - b\log{(x)} - \frac{1}{x}$$

input
```
integrate(1/x^2/(b*x+1),x, algorithm="maxima")
```

output
```
b*log(b*x + 1) - b*log(x) - 1/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2(1+bx)}\,dx = b\log{(|bx+1|)} - b\log{(|x|)} - \frac{1}{x}$$

input
```
integrate(1/x^2/(b*x+1),x, algorithm="giac")
```

output
```
b*log(abs(b*x + 1)) - b*log(abs(x)) - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(1+bx)}\,dx = 2\,b\,\mathrm{atanh}(2\,b\,x+1) - \frac{1}{x}$$

input
```
int(1/(x^2*(b*x + 1)),x)
```

output
```
2*b*atanh(2*b*x + 1) - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2(1+bx)}\,dx = \frac{\log(bx+1)\,bx - \log(x)\,bx - 1}{x}$$

input
```
int(1/x^2/(b*x+1),x)
```

output
```
(log(b*x + 1)*b*x - log(x)*b*x - 1)/x
```

3.233 $\int \frac{1}{x^2(-1+bx)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{x^2(-1+bx)}\,dx = \frac{1}{x} - b\log(x) + b\log(1-bx)$$

output
```
1/x-b*ln(x)+b*ln(-b*x+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(-1+bx)}\,dx = \frac{1}{x} - b\log(x) + b\log(1-bx)$$

input
```
Integrate[1/(x^2*(-1 + b*x)),x]
```

output
```
x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(bx-1)} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{b^2}{bx-1} - \frac{b}{x} - \frac{1}{x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-b\log(x) + b\log(1-bx) + \frac{1}{x}$$

input
```
Int[1/(x^2*(-1 + b*x)),x]
```

output
```
x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx-1\right)$	18
norman	$\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx-1\right)$	18
risch	$\frac{1}{x}-b\ln\left(x\right)+b\ln\left(-bx+1\right)$	19
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx-1)x-1}{x}$	23
meijerg	$b\left(\frac{1}{xb}-\ln\left(x\right)-\ln\left(-b\right)+\ln\left(-bx+1\right)\right)$	28

input
```
int(1/x^2/(b*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/x-b*ln(x)+b*ln(b*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(-1+bx)}\,dx=\frac{bx\log\left(bx-1\right)-bx\log\left(x\right)+1}{x}$$

input
```
integrate(1/x^2/(b*x-1),x, algorithm="fricas")
```

output
```
(b*x*log(b*x - 1) - b*x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2(-1+bx)}\,dx = b\left(-\log{(x)} + \log{\left(x - \frac{1}{b}\right)}\right) + \frac{1}{x}$$

input
```
integrate(1/x**2/(b*x-1),x)
```

output
```
b*(-log(x) + log(x - 1/b)) + 1/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(-1+bx)}\,dx = b\log{(bx-1)} - b\log{(x)} + \frac{1}{x}$$

input
```
integrate(1/x^2/(b*x-1),x, algorithm="maxima")
```

output
```
b*log(b*x - 1) - b*log(x) + 1/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(-1+bx)}\,dx = b\log{(|bx-1|)} - b\log{(|x|)} + \frac{1}{x}$$

input
```
integrate(1/x^2/(b*x-1),x, algorithm="giac")
```

output
```
b*log(abs(b*x - 1)) - b*log(abs(x)) + 1/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2(-1+bx)}\,dx = \frac{1}{x} - 2\,b\,\text{atanh}(2\,b\,x-1)$$

input
```
int(1/(x^2*(b*x - 1)),x)
```

output
```
1/x - 2*b*atanh(2*b*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2(-1+bx)}\,dx = \frac{\log(bx-1)\,bx - \log(x)\,bx + 1}{x}$$

input
```
int(1/x^2/(b*x-1),x)
```

output
```
(log(b*x - 1)*b*x - log(x)*b*x + 1)/x
```

3.234 $\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right)\,dx=-\frac{1}{x}+b\log(1+bx)$$

output
```
-1/x+b*ln(b*x+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right)\,dx=-\frac{1}{x}+b\log(1+bx)$$

input
```
Integrate[b/x + 1/(x^2*(1 + b*x)),x]
```

output
```
-x^(-1) + b*Log[1 + b*x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{x^2(bx+1)}+\frac{b}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$b\log(bx+1)-\frac{1}{x}$$

input
```
Int[b/x + 1/(x^2*(1 + b*x)),x]
```

output
```
-x^(-1) + b*Log[1 + b*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{1}{x}+b\ln\left(bx+1\right)$	15
norman	$-\frac{1}{x}+b\ln\left(bx+1\right)$	15
risch	$-\frac{1}{x}+b\ln\left(-bx-1\right)$	16
parallelrisch	$\frac{b\ln(bx+1)x-1}{x}$	16

input
```
int(b/x+1/x^2/(b*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+b*ln(b*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx = \frac{bx\log(bx+1)-1}{x}$$

input
```
integrate(b/x+1/x^2/(b*x+1),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + 1) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx = b\log(bx+1)-\frac{1}{x}$$

input
```
integrate(b/x+1/x**2/(b*x+1),x)
```

output
```
b*log(b*x + 1) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(\frac{b}{x} + \frac{1}{x^2(1+bx)}\right) dx = b\log(bx+1) - \frac{1}{x}$$

input
```
integrate(b/x+1/x^2/(b*x+1),x, algorithm="maxima")
```

output
```
b*log(b*x + 1) - 1/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \left(\frac{b}{x} + \frac{1}{x^2(1+bx)}\right) dx = b\log(|bx+1|) - \frac{1}{x}$$

input
```
integrate(b/x+1/x^2/(b*x+1),x, algorithm="giac")
```

output
```
b*log(abs(b*x + 1)) - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(\frac{b}{x} + \frac{1}{x^2(1+bx)}\right) dx = b\ln(x) + 2b\,\mathrm{atanh}(2bx+1) - \frac{1}{x}$$

input
```
int(1/(x^2*(b*x + 1)) + b/x,x)
```

output
```
b*log(x) + 2*b*atanh(2*b*x + 1) - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \left(\frac{b}{x} + \frac{1}{x^2(1+bx)}\right) dx = \frac{\log(bx+1)\,bx - 1}{x}$$

input
```
int(b/x+1/x^2/(b*x+1),x)
```

output
```
(log(b*x + 1)*b*x - 1)/x
```

3.235 $\int x^{5/2}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^{5/2}(a+bx)\,dx = \frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

output
```
2/7*a*x^(7/2)+2/9*b*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx)\,dx = \frac{2}{63}x^{7/2}(9a+7bx)$$

input
```
Integrate[x^(5/2)*(a + b*x),x]
```

output
```
(2*x^(7/2)*(9*a + 7*b*x))/63
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(ax^{5/2}+bx^{7/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

input
```
Int[x^(5/2)*(a + b*x),x]
```

output
```
(2*a*x^(7/2))/7 + (2*b*x^(9/2))/9
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
trager	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14
risch	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14
orering	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14

input
```
int(x^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/63*x^(7/2)*(7*b*x+9*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{5/2}(a+bx)\,dx=\frac{2}{63}\left(7\,bx^4+9\,ax^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x+a),x, algorithm="fricas")
```

output
```
2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{5/2}(a+bx)\,dx=\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{9}{2}}}{9}$$

input
```
integrate(x**(5/2)*(b*x+a),x)
```

output
```
2*a*x**(7/2)/7 + 2*b*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}(a+bx)\,dx=\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a),x, algorithm="maxima")
```

output
```
2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}(a+bx)\,dx=\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a),x, algorithm="giac")
```

output
```
2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}(a+bx)\,dx=\frac{2\,x^{7/2}\,(9\,a+7\,b\,x)}{63}$$

input
```
int(x^(5/2)*(a + b*x),x)
```

output
```
(2*x^(7/2)*(9*a + 7*b*x))/63
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{5/2}(a+bx)\,dx=\frac{2\sqrt{x}\,x^3(7bx+9a)}{63}$$

input
```
int(x^(5/2)*(b*x+a),x)
```

output
```
(2*sqrt(x)*x**3*(9*a + 7*b*x))/63
```

3.236 $\int x^{3/2}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^{3/2}(a+bx)\,dx = \frac{2}{5}ax^{5/2} + \frac{2}{7}bx^{7/2}$$

output
```
2/5*a*x^(5/2)+2/7*b*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx)\,dx = \frac{2}{35}x^{5/2}(7a+5bx)$$

input
```
Integrate[x^(3/2)*(a + b*x),x]
```

output
```
(2*x^(5/2)*(7*a + 5*b*x))/35
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(ax^{3/2}+bx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

input
```
Int[x^(3/2)*(a + b*x),x]
```

output
```
(2*a*x^(5/2))/5 + (2*b*x^(7/2))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}(5bx+7a)}{35}$	14
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
trager	$\frac{2x^{\frac{5}{2}}(5bx+7a)}{35}$	14
risch	$\frac{2x^{\frac{5}{2}}(5bx+7a)}{35}$	14
orering	$\frac{2x^{\frac{5}{2}}(5bx+7a)}{35}$	14

input
```
int(x^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/35*x^(5/2)*(5*b*x+7*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{3/2}(a+bx)\,dx=\frac{2}{35}\left(5\,bx^3+7\,ax^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x+a),x, algorithm="fricas")
```

output
```
2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{3/2}(a+bx)\,dx = \frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{7}{2}}}{7}$$

input
```
integrate(x**(3/2)*(b*x+a),x)
```

output
```
2*a*x**(5/2)/5 + 2*b*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}(a+bx)\,dx = \frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a),x, algorithm="maxima")
```

output
```
2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}(a+bx)\,dx = \frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a),x, algorithm="giac")
```

output
```
2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}(a+bx)\,dx=\frac{2\,x^{5/2}\,(7\,a+5\,b\,x)}{35}$$

input
```
int(x^(3/2)*(a + b*x),x)
```

output
```
(2*x^(5/2)*(7*a + 5*b*x))/35
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{3/2}(a+bx)\,dx=\frac{2\sqrt{x}\,x^2(5bx+7a)}{35}$$

input
```
int(x^(3/2)*(b*x+a),x)
```

output
```
(2*sqrt(x)*x**2*(7*a + 5*b*x))/35
```

3.237 $\int \sqrt{x}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \sqrt{x}(a+bx)\,dx=\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

output
```
2/3*a*x^(3/2)+2/5*b*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx)\,dx=\frac{2}{15}x^{3/2}(5a+3bx)$$

input
```
Integrate[Sqrt[x]*(a + b*x),x]
```

output
```
(2*x^(3/2)*(5*a + 3*b*x))/15
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)\,dx$$
$$\Big\downarrow \ 53$$
$$\int \left(a\sqrt{x}+bx^{3/2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

input
```
Int[Sqrt[x]*(a + b*x),x]
```

output
```
(2*a*x^(3/2))/3 + (2*b*x^(5/2))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14
derivativedivides	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
default	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
trager	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14
risch	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14
orering	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14

input
```
int(x^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/15*x^(3/2)*(3*b*x+5*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \sqrt{x}(a+bx)\,dx=\frac{2}{15}\left(3\,bx^2+5\,ax\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x+a),x, algorithm="fricas")
```

output
```
2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{x}(a+bx)\,dx = \frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{5}{2}}}{5}$$

input
```
integrate(x**(1/2)*(b*x+a),x)
```

output
```
2*a*x**(3/2)/3 + 2*b*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(a+bx)\,dx = \frac{2}{5}\,bx^{\frac{5}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a),x, algorithm="maxima")
```

output
```
2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(a+bx)\,dx = \frac{2}{5}\,bx^{\frac{5}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a),x, algorithm="giac")
```

output
```
2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(a+bx)\,dx = \frac{2\,x^{3/2}\,(5\,a+3\,b\,x)}{15}$$

input
```
int(x^(1/2)*(a + b*x),x)
```

output
```
(2*x^(3/2)*(5*a + 3*b*x))/15
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(a+bx)\,dx = \frac{2\sqrt{x}\,x(3bx+5a)}{15}$$

input
```
int(x^(1/2)*(b*x+a),x)
```

output
```
(2*sqrt(x)*x*(5*a + 3*b*x))/15
```

3.238 $\int \frac{a+bx}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{a+bx}{\sqrt{x}} \, dx = 2a\sqrt{x} + \frac{2}{3}bx^{3/2}$$

output
```
2*a*x^(1/2)+2/3*b*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{a+bx}{\sqrt{x}} \, dx = \frac{2}{3}\sqrt{x}(3a+bx)$$

input
```
Integrate[(a + b*x)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(3*a + b*x))/3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt{x}} \, dx$$
$$\Big\downarrow \text{ 53}$$
$$\int \left(\frac{a}{\sqrt{x}}+b\sqrt{x}\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

input
```
Int[(a + b*x)/Sqrt[x],x]
```

output
```
2*a*Sqrt[x] + (2*b*x^(3/2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{2\sqrt{x}\,(bx+3a)}{3}$	13
trager	$\left(\frac{2bx}{3}+2a\right)\sqrt{x}$	13
risch	$\frac{2\sqrt{x}\,(bx+3a)}{3}$	13
orering	$\frac{2\sqrt{x}\,(bx+3a)}{3}$	13
derivativedivides	$2a\sqrt{x}+\frac{2b\,x^{\frac{3}{2}}}{3}$	14
default	$2a\sqrt{x}+\frac{2b\,x^{\frac{3}{2}}}{3}$	14

input
```
int((b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(1/2)*(b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{\sqrt{x}}\,dx = \frac{2}{3}\,(bx+3\,a)\sqrt{x}$$

input
```
integrate((b*x+a)/x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x + 3*a)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{\sqrt{x}}\,dx = 2a\sqrt{x} + \frac{2bx^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x+a)/x**(1/2),x)
```

output
```
2*a*sqrt(x) + 2*b*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{\sqrt{x}}\,dx = \frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((b*x+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{\sqrt{x}}\,dx = \frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((b*x+a)/x^(1/2),x, algorithm="giac")
```

output
```
2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(3\,a+b\,x)}{3}$$

input
```
int((a + b*x)/x^(1/2),x)
```

output
```
(2*x^(1/2)*(3*a + b*x))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{a+bx}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(bx+3a)}{3}$$

input
```
int((b*x+a)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(3*a + b*x))/3
```

3.239 $\int \frac{a+bx}{x^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx}{x^{3/2}}\,dx=-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$$

output
```
-2*a/x^(1/2)+2*b*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{3/2}}\,dx=-\frac{2(a-bx)}{\sqrt{x}}$$

input
```
Integrate[(a + b*x)/x^(3/2),x]
```

output
```
(-2*(a - b*x))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a}{x^{3/2}}+\frac{b}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

input
```
Int[(a + b*x)/x^(3/2),x]
```

output
```
(-2*a)/Sqrt[x] + 2*b*Sqrt[x]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
trager	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
risch	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
orering	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
derivativedivides	$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$	14
default	$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$	14

input
```
int((b*x+a)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-b*x+a)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{a+bx}{x^{3/2}}\,dx=\frac{2\,(bx-a)}{\sqrt{x}}$$

input
```
integrate((b*x+a)/x^(3/2),x, algorithm="fricas")
```

output
```
2*(b*x - a)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{x^{3/2}}\,dx = -\frac{2a}{\sqrt{x}} + 2b\sqrt{x}$$

input
```
integrate((b*x+a)/x**(3/2),x)
```

output
```
-2*a/sqrt(x) + 2*b*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{3/2}}\,dx = 2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

input
```
integrate((b*x+a)/x^(3/2),x, algorithm="maxima")
```

output
```
2*b*sqrt(x) - 2*a/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{3/2}}\,dx = 2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

input
```
integrate((b*x+a)/x^(3/2),x, algorithm="giac")
```

output
```
2*b*sqrt(x) - 2*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{a+bx}{x^{3/2}}\,dx = -\frac{2\,(a-b\,x)}{\sqrt{x}}$$

input
```
int((a + b*x)/x^(3/2),x)
```

output
```
-(2*(a - b*x))/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx}{x^{3/2}}\,dx = \frac{2bx-2a}{\sqrt{x}}$$

input
```
int((b*x+a)/x^(3/2),x)
```

output
```
(2*( - a + b*x))/sqrt(x)
```

3.240 $\int \frac{a+bx}{x^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{a+bx}{x^{5/2}}\,dx = -\frac{2a}{3x^{3/2}} - \frac{2b}{\sqrt{x}}$$

output
```
-2/3*a/x^(3/2)-2*b/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+bx}{x^{5/2}}\,dx = -\frac{2(a+3bx)}{3x^{3/2}}$$

input
```
Integrate[(a + b*x)/x^(5/2),x]
```

output
```
(-2*(a + 3*b*x))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^{5/2}} \, dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(\frac{a}{x^{5/2}}+\frac{b}{x^{3/2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

input
```
Int[(a + b*x)/x^(5/2),x]
```

output
```
(-2*a)/(3*x^(3/2)) - (2*b)/Sqrt[x]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
trager	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
risch	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
orering	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$	14
default	$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$	14

input
```
int((b*x+a)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*b*x+a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{a+bx}{x^{5/2}}\,dx=-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*b*x + a)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{x^{5/2}}\,dx = -\frac{2a}{3x^{\frac{3}{2}}} - \frac{2b}{\sqrt{x}}$$

input
```
integrate((b*x+a)/x**(5/2),x)
```

output
```
-2*a/(3*x**(3/2)) - 2*b/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{a+bx}{x^{5/2}}\,dx = -\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(3*b*x + a)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{a+bx}{x^{5/2}}\,dx = -\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*b*x + a)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{5/2}}\,dx=-\frac{2\,a+6\,b\,x}{3\,x^{3/2}}$$

input
```
int((a + b*x)/x^(5/2),x)
```

output
```
-(2*a + 6*b*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{x^{5/2}}\,dx=\frac{-2bx-\frac{2a}{3}}{\sqrt{x}\,x}$$

input
```
int((b*x+a)/x^(5/2),x)
```

output
```
(2*( - a - 3*b*x))/(3*sqrt(x)*x)
```

3.241 $\int x^{5/2}(a+bx)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

output
```
2/7*a^2*x^(7/2)+4/9*a*b*x^(9/2)+2/11*b^2*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2}{693}x^{7/2}\left(99a^2+154abx+63b^2x^2\right)$$

input
```
Integrate[x^(5/2)*(a + b*x)^2,x]
```

output
```
(2*x^(7/2)*(99*a^2 + 154*a*b*x + 63*b^2*x^2))/693
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^2\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(a^2x^{5/2}+2abx^{7/2}+b^2x^{9/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

input
```
Int[x^(5/2)*(a + b*x)^2,x]
```

output
```
(2*a^2*x^(7/2))/7 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(11/2))/11
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(63b^{2}x^{2}+154abx+99a^{2}\right)}{693}$	25
derivativedivides	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$	25
default	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$	25
trager	$\frac{2x^{\frac{7}{2}}\left(63b^{2}x^{2}+154abx+99a^{2}\right)}{693}$	25
risch	$\frac{2x^{\frac{7}{2}}\left(63b^{2}x^{2}+154abx+99a^{2}\right)}{693}$	25
orering	$\frac{2x^{\frac{7}{2}}\left(63b^{2}x^{2}+154abx+99a^{2}\right)}{693}$	25

input
```
int(x^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/693*x^(7/2)*(63*b^2*x^2+154*a*b*x+99*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(5/2)*(b*x+a)**2,x)
```

output
```
2*a**2*x**(7/2)/7 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2\,x^{7/2}\,\left(99\,a^2+154\,a\,b\,x+63\,b^2\,x^2\right)}{693}$$

input
```
int(x^(5/2)*(a + b*x)^2,x)
```

output
```
(2*x^(7/2)*(99*a^2 + 63*b^2*x^2 + 154*a*b*x))/693
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^{5/2}(a+bx)^2\,dx=\frac{2\sqrt{x}\,x^3(63b^2x^2+154abx+99a^2)}{693}$$

input
```
int(x^(5/2)*(b*x+a)^2,x)
```

output
```
(2*sqrt(x)*x**3*(99*a**2 + 154*a*b*x + 63*b**2*x**2))/693
```

3.242 $\int x^{3/2}(a+bx)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int x^{3/2}(a+bx)^2\,dx=\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

output
```
2/5*a^2*x^(5/2)+4/7*a*b*x^(7/2)+2/9*b^2*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{3/2}(a+bx)^2\,dx=\frac{2}{315}x^{5/2}\left(63a^2+90abx+35b^2x^2\right)$$

input
```
Integrate[x^(3/2)*(a + b*x)^2,x]
```

output
```
(2*x^(5/2)*(63*a^2 + 90*a*b*x + 35*b^2*x^2))/315
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^2\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2x^{3/2}+2abx^{5/2}+b^2x^{7/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

input
```
Int[x^(3/2)*(a + b*x)^2,x]
```

output
```
(2*a^2*x^(5/2))/5 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(9/2))/9
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}(35b^2x^2+90abx+63a^2)}{315}$	25
derivativedivides	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
default	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
trager	$\frac{2x^{\frac{5}{2}}(35b^2x^2+90abx+63a^2)}{315}$	25
risch	$\frac{2x^{\frac{5}{2}}(35b^2x^2+90abx+63a^2)}{315}$	25
orering	$\frac{2x^{\frac{5}{2}}(35b^2x^2+90abx+63a^2)}{315}$	25

input
```
int(x^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(5/2)*(35*b^2*x^2+90*a*b*x+63*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx)^2\,dx = \frac{2}{315}\left(35\,b^2x^4+90\,abx^3+63\,a^2x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{3/2}(a+bx)^2\,dx = \frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{9}{2}}}{9}$$

input
```
integrate(x**(3/2)*(b*x+a)**2,x)
```

output
```
2*a**2*x**(5/2)/5 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}(a+bx)^2\,dx = \frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}(a+bx)^2\,dx = \frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}(a+bx)^2\,dx = \frac{2\,x^{5/2}\,\left(63\,a^2+90\,a\,b\,x+35\,b^2\,x^2\right)}{315}$$

input
```
int(x^(3/2)*(a + b*x)^2,x)
```

output
```
(2*x^(5/2)*(63*a^2 + 35*b^2*x^2 + 90*a*b*x))/315
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^{3/2}(a+bx)^2\,dx = \frac{2\sqrt{x}\,x^2(35b^2x^2+90abx+63a^2)}{315}$$

input
```
int(x^(3/2)*(b*x+a)^2,x)
```

output
```
(2*sqrt(x)*x**2*(63*a**2 + 90*a*b*x + 35*b**2*x**2))/315
```

3.243 $\int \sqrt{x}(a+bx)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \sqrt{x}(a+bx)^2\,dx=\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

output
```
2/3*a^2*x^(3/2)+4/5*a*b*x^(5/2)+2/7*b^2*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sqrt{x}(a+bx)^2\,dx=\frac{2}{105}x^{3/2}\left(35a^2+42abx+15b^2x^2\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x)^2,x]
```

output
```
(2*x^(3/2)*(35*a^2 + 42*a*b*x + 15*b^2*x^2))/105
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2\sqrt{x}+2abx^{3/2}+b^2x^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

input
```
Int[Sqrt[x]*(a + b*x)^2,x]
```

output
```
(2*a^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*b^2*x^(7/2))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(15b^2x^2+42abx+35a^2)}{105}$	25
derivativedivides	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{7}{2}}}{7}$	25
default	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{7}{2}}}{7}$	25
trager	$\frac{2x^{\frac{3}{2}}(15b^2x^2+42abx+35a^2)}{105}$	25
risch	$\frac{2x^{\frac{3}{2}}(15b^2x^2+42abx+35a^2)}{105}$	25
orering	$\frac{2x^{\frac{3}{2}}(15b^2x^2+42abx+35a^2)}{105}$	25

input
```
int(x^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/105*x^(3/2)*(15*b^2*x^2+42*a*b*x+35*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{x}(a+bx)^2\,dx = \frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.99 (sec) , antiderivative size = 1851, normalized size of antiderivative = 51.42

$$\int \sqrt{x}(a+bx)^2\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(b*x+a)**2,x)
```

output
```
Piecewise((16*a**(23/2)*sqrt(-1 + b*(a/b + x)/a)/(-105*a**8*b**(3/2) + 315
*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9
/2)*(a/b + x)**3) - 16*I*a**(23/2)/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)
*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)*
*3) - 40*a**(21/2)*b*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)/(-105*a**8*b**(3/2
) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**
5*b**(9/2)*(a/b + x)**3) + 48*I*a**(21/2)*b*(a/b + x)/(-105*a**8*b**(3/2)
+ 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*
b**(9/2)*(a/b + x)**3) + 30*a**(19/2)*b**2*sqrt(-1 + b*(a/b + x)/a)*(a/b +
 x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/
2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 48*I*a**(19/2)*b**2*(a
/b + x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b*
*(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 40*a**(17/2)*b**3*
sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(5
/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b +
x)**3) + 16*I*a**(17/2)*b**3*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b
**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/
b + x)**3) + 100*a**(15/2)*b**4*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**4/(-10
5*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x
)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 96*a**(13/2)*b**5*sqrt(-1 + b*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}(a+bx)^2\,dx = \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}(a+bx)^2\,dx = \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}(a+bx)^2\,dx = \frac{2\,x^{3/2}\,\left(35\,a^2+42\,a\,b\,x+15\,b^2\,x^2\right)}{105}$$

input
```
int(x^(1/2)*(a + b*x)^2,x)
```

output
```
(2*x^(3/2)*(35*a^2 + 15*b^2*x^2 + 42*a*b*x))/105
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}(a+bx)^2\,dx = \frac{2\sqrt{x}\,x(15b^2x^2+42abx+35a^2)}{105}$$

input
```
int(x^(1/2)*(b*x+a)^2,x)
```

output
```
(2*sqrt(x)*x*(35*a**2 + 42*a*b*x + 15*b**2*x**2))/105
```

3.244 $\int \frac{(a+bx)^2}{\sqrt{x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(a+bx)^2}{\sqrt{x}} dx = 2a^2\sqrt{x} + \frac{4}{3}abx^{3/2} + \frac{2}{5}b^2x^{5/2}$$

output
```
2*a^2*x^(1/2)+4/3*a*b*x^(3/2)+2/5*b^2*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2}{\sqrt{x}} dx = \frac{2}{15}\sqrt{x}(15a^2 + 10abx + 3b^2x^2)$$

input
```
Integrate[(a + b*x)^2/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(15*a^2 + 10*a*b*x + 3*b^2*x^2))/15
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^2}{\sqrt{x}}+2ab\sqrt{x}+b^2x^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}$$

input
```
Int[(a + b*x)^2/Sqrt[x],x]
```

output
```
2*a^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(5/2))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
trager	$\left(\frac{2}{5}b^2x^2+\frac{4}{3}abx+2a^2\right)\sqrt{x}$	24
gosper	$\frac{2\sqrt{x}\,(3b^2x^2+10abx+15a^2)}{15}$	25
derivativedivides	$2a^2\sqrt{x}+\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{5}{2}}}{5}$	25
default	$2a^2\sqrt{x}+\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{5}{2}}}{5}$	25
risch	$\frac{2\sqrt{x}\,(3b^2x^2+10abx+15a^2)}{15}$	25
orering	$\frac{2\sqrt{x}\,(3b^2x^2+10abx+15a^2)}{15}$	25

input
```
int((b*x+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5*b^2*x^2+4/3*a*b*x+2*a^2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{\sqrt{x}}\,dx=\frac{2}{15}\left(3\,b^2x^2+10\,abx+15\,a^2\right)\sqrt{x}$$

input
```
integrate((b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x+a)**2/x**(1/2),x)
```

output
```
2*a**2*sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{\sqrt{x}}\,dx = \frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

input
```
integrate((b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{\sqrt{x}}\,dx = \frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

input
```
integrate((b*x+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(15\,a^2+10\,a\,b\,x+3\,b^2\,x^2)}{15}$$

input
```
int((a + b*x)^2/x^(1/2),x)
```

output
```
(2*x^(1/2)*(15*a^2 + 3*b^2*x^2 + 10*a*b*x))/15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(3b^2x^2+10abx+15a^2)}{15}$$

input
```
int((b*x+a)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*(15*a**2 + 10*a*b*x + 3*b**2*x**2))/15
```

3.245 $\int \frac{(a+bx)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx = -\frac{2a^2}{\sqrt{x}} + 4ab\sqrt{x} + \frac{2}{3}b^2x^{3/2}$$

output
```
-2*a^2/x^(1/2)+4*a*b*x^(1/2)+2/3*b^2*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx = -\frac{2(3a^2-6abx-b^2x^2)}{3\sqrt{x}}$$

input
```
Integrate[(a + b*x)^2/x^(3/2),x]
```

output
```
(-2*(3*a^2 - 6*a*b*x - b^2*x^2))/(3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx$$
$$\Big\downarrow \ 53$$
$$\int \left(\frac{a^2}{x^{3/2}}+\frac{2ab}{\sqrt{x}}+b^2\sqrt{x}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

input
```
Int[(a + b*x)^2/x^(3/2),x]
```

output
```
(-2*a^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*b^2*x^(3/2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{2\left(-b^{2} x^{2}-6 a b x+3 a^{2}\right)}{3 \sqrt{x}}$	25
derivativedivides	$-\frac{2 a^{2}}{\sqrt{x}}+4 a b \sqrt{x}+\frac{2 b^{2} x^{\frac{3}{2}}}{3}$	25
default	$-\frac{2 a^{2}}{\sqrt{x}}+4 a b \sqrt{x}+\frac{2 b^{2} x^{\frac{3}{2}}}{3}$	25
trager	$-\frac{2\left(-b^{2} x^{2}-6 a b x+3 a^{2}\right)}{3 \sqrt{x}}$	25
risch	$-\frac{2\left(-b^{2} x^{2}-6 a b x+3 a^{2}\right)}{3 \sqrt{x}}$	25
orering	$-\frac{2\left(-b^{2} x^{2}-6 a b x+3 a^{2}\right)}{3 \sqrt{x}}$	25

input
```
int((b*x+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b^2*x^2-6*a*b*x+3*a^2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx=\frac{2\left(b^2x^2+6\,abx-3\,a^2\right)}{3\,\sqrt{x}}$$

input
```
integrate((b*x+a)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx = -\frac{2a^2}{\sqrt{x}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x+a)**2/x**(3/2),x)
```

output
```
-2*a**2/sqrt(x) + 4*a*b*sqrt(x) + 2*b**2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx = \frac{2}{3}\,b^2x^{\frac{3}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((b*x+a)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx = \frac{2}{3}\,b^2x^{\frac{3}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((b*x+a)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx = \frac{-6\,a^2+12\,a\,b\,x+2\,b^2\,x^2}{3\,\sqrt{x}}$$

input
```
int((a + b*x)^2/x^(3/2),x)
```

output
```
(2*b^2*x^2 - 6*a^2 + 12*a*b*x)/(3*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{3/2}}\,dx = \frac{\frac{2}{3}b^2x^2+4abx-2a^2}{\sqrt{x}}$$

input
```
int((b*x+a)^2/x^(3/2),x)
```

output
```
(2*( - 3*a**2 + 6*a*b*x + b**2*x**2))/(3*sqrt(x))
```

3.246 $\int \frac{(a+bx)^2}{x^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{(a+bx)^2}{x^{5/2}} \, dx = -\frac{2a^2}{3x^{3/2}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

output
```
-2/3*a^2/x^(3/2)-4*a*b/x^(1/2)+2*b^2*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^2}{x^{5/2}} \, dx = -\frac{2(a^2+6abx-3b^2x^2)}{3x^{3/2}}$$

input
```
Integrate[(a + b*x)^2/x^(5/2),x]
```

output
```
(-2*(a^2 + 6*a*b*x - 3*b^2*x^2))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{x^{3/2}}+\frac{b^2}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

input
```
Int[(a + b*x)^2/x^(5/2),x]
```

output
```
(-2*a^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*b^2*Sqrt[x]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{2(-3b^2x^2+6abx+a^2)}{3x^{\frac{3}{2}}}$	23
trager	$-\frac{2(-3b^2x^2+6abx+a^2)}{3x^{\frac{3}{2}}}$	23
risch	$-\frac{2(-3b^2x^2+6abx+a^2)}{3x^{\frac{3}{2}}}$	23
orering	$-\frac{2(-3b^2x^2+6abx+a^2)}{3x^{\frac{3}{2}}}$	23
derivativedivides	$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$	25
default	$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$	25

input
```
int((b*x+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*b^2*x^2+6*a*b*x+a^2)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{5/2}}\,dx = \frac{2\,(3\,b^2x^2-6\,abx-a^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^2}{x^{5/2}}\,dx = -\frac{2a^2}{3x^{\frac{3}{2}}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

input
```
integrate((b*x+a)**2/x**(5/2),x)
```

output
```
-2*a**2/(3*x**(3/2)) - 4*a*b/sqrt(x) + 2*b**2*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^2}{x^{5/2}}\,dx = 2\,b^2\sqrt{x} - \frac{2\,(6\,abx+a^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^2}{x^{5/2}}\,dx = 2\,b^2\sqrt{x} - \frac{2\,(6\,abx+a^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^2/x^(5/2),x, algorithm="giac")
```

output
```
2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{5/2}}\,dx = -\frac{2\,a^2+12\,a\,b\,x-6\,b^2\,x^2}{3\,x^{3/2}}$$

input
```
int((a + b*x)^2/x^(5/2),x)
```

output
```
-(2*a^2 - 6*b^2*x^2 + 12*a*b*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2}{x^{5/2}}\,dx = \frac{2b^2x^2-4abx-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((b*x+a)^2/x^(5/2),x)
```

output
```
(2*( - a**2 - 6*a*b*x + 3*b**2*x**2))/(3*sqrt(x)*x)
```

3.247 $\int x^{5/2}(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x^{5/2}(a+bx)^3\,dx=\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

output
```
2/7*a^3*x^(7/2)+2/3*a^2*b*x^(9/2)+6/11*a*b^2*x^(11/2)+2/13*b^3*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{5/2}(a+bx)^3\,dx=\frac{2x^{7/2}(429a^3+1001a^2bx+819ab^2x^2+231b^3x^3)}{3003}$$

input
```
Integrate[x^(5/2)*(a + b*x)^3,x]
```

output
```
(2*x^(7/2)*(429*a^3 + 1001*a^2*b*x + 819*a*b^2*x^2 + 231*b^3*x^3))/3003
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^3\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(a^3x^{5/2}+3a^2bx^{7/2}+3ab^2x^{9/2}+b^3x^{11/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

input
```
Int[x^(5/2)*(a + b*x)^3,x]
```

output
```
(2*a^3*x^(7/2))/7 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x
^(13/2))/13
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)}{3003}$	36
derivativedivides	$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{2a^{2}b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{13}{2}}}{13}$	36
default	$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{2a^{2}b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{13}{2}}}{13}$	36
trager	$\frac{2x^{\frac{7}{2}}\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)}{3003}$	36
risch	$\frac{2x^{\frac{7}{2}}\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)}{3003}$	36
orering	$\frac{2x^{\frac{7}{2}}\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)}{3003}$	36

input
```
int(x^(5/2)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3003*x^(7/2)*(231*b^3*x^3+819*a*b^2*x^2+1001*a^2*b*x+429*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{5/2}(a+bx)^3\,dx=\frac{2}{3003}\left(231\,b^3x^6+819\,ab^2x^5+1001\,a^2bx^4+429\,a^3x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x
)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{5/2}(a+bx)^3\,dx = \frac{2a^3x^{\frac{7}{2}}}{7} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6ab^2x^{\frac{11}{2}}}{11} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(5/2)*(b*x+a)**3,x)
```

output
```
2*a**3*x**(7/2)/7 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x
**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx)^3\,dx = \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7
/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx)^3\,dx = \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^3,x, algorithm="giac")
```

output
```
2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7
/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx)^3\,dx=\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,b^3\,x^{13/2}}{13}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,a\,b^2\,x^{11/2}}{11}$$

input
```
int(x^(5/2)*(a + b*x)^3,x)
```

output
```
(2*a^3*x^(7/2))/7 + (2*b^3*x^(13/2))/13 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x
^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int x^{5/2}(a+bx)^3\,dx=\frac{2\sqrt{x}\,x^3(231b^3x^3+819a\,b^2x^2+1001a^2bx+429a^3)}{3003}$$

input
```
int(x^(5/2)*(b*x+a)^3,x)
```

output
```
(2*sqrt(x)*x**3*(429*a**3 + 1001*a**2*b*x + 819*a*b**2*x**2 + 231*b**3*x**
3))/3003
```

3.248 $\int x^{3/2}(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x^{3/2}(a+bx)^3\,dx=\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

output
```
2/5*a^3*x^(5/2)+6/7*a^2*b*x^(7/2)+2/3*a*b^2*x^(9/2)+2/11*b^3*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{3/2}(a+bx)^3\,dx=\frac{2x^{5/2}(231a^3+495a^2bx+385ab^2x^2+105b^3x^3)}{1155}$$

input
```
Integrate[x^(3/2)*(a + b*x)^3,x]
```

output
```
(2*x^(5/2)*(231*a^3 + 495*a^2*b*x + 385*a*b^2*x^2 + 105*b^3*x^3))/1155
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^3\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^3x^{3/2}+3a^2bx^{5/2}+3ab^2x^{7/2}+b^3x^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

input
```
Int[x^(3/2)*(a + b*x)^3,x]
```

output
```
(2*a^3*x^(5/2))/5 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(
11/2))/11
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(105b^{3}x^{3}+385a\,b^{2}x^{2}+495a^{2}bx+231a^{3}\right)}{1155}$	36
derivativedivides	$\frac{2a^{3}x^{\frac{5}{2}}}{5}+\frac{6a^{2}b\,x^{\frac{7}{2}}}{7}+\frac{2a\,b^{2}x^{\frac{9}{2}}}{3}+\frac{2b^{3}x^{\frac{11}{2}}}{11}$	36
default	$\frac{2a^{3}x^{\frac{5}{2}}}{5}+\frac{6a^{2}b\,x^{\frac{7}{2}}}{7}+\frac{2a\,b^{2}x^{\frac{9}{2}}}{3}+\frac{2b^{3}x^{\frac{11}{2}}}{11}$	36
trager	$\frac{2x^{\frac{5}{2}}\left(105b^{3}x^{3}+385a\,b^{2}x^{2}+495a^{2}bx+231a^{3}\right)}{1155}$	36
risch	$\frac{2x^{\frac{5}{2}}\left(105b^{3}x^{3}+385a\,b^{2}x^{2}+495a^{2}bx+231a^{3}\right)}{1155}$	36
orering	$\frac{2x^{\frac{5}{2}}\left(105b^{3}x^{3}+385a\,b^{2}x^{2}+495a^{2}bx+231a^{3}\right)}{1155}$	36

input
```
int(x^(3/2)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/1155*x^(5/2)*(105*b^3*x^3+385*a*b^2*x^2+495*a^2*b*x+231*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{3/2}(a+bx)^3\,dx=\frac{2}{1155}\left(105\,b^3x^5+385\,ab^2x^4+495\,a^2bx^3+231\,a^3x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{3/2}(a+bx)^3\,dx = \frac{2a^3x^{\frac{5}{2}}}{5} + \frac{6a^2bx^{\frac{7}{2}}}{7} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(3/2)*(b*x+a)**3,x)
```

output
```
2*a**3*x**(5/2)/5 + 6*a**2*b*x**(7/2)/7 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**
(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx)^3\,dx = \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx)^3\,dx = \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^3,x, algorithm="giac")
```

output
```
2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx)^3\,dx=\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,b^3\,x^{11/2}}{11}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{2\,a\,b^2\,x^{9/2}}{3}$$

input
```
int(x^(3/2)*(a + b*x)^3,x)
```

output
```
(2*a^3*x^(5/2))/5 + (2*b^3*x^(11/2))/11 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x
^(9/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int x^{3/2}(a+bx)^3\,dx=\frac{2\sqrt{x}\,x^2(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3)}{1155}$$

input
```
int(x^(3/2)*(b*x+a)^3,x)
```

output
```
(2*sqrt(x)*x**2*(231*a**3 + 495*a**2*b*x + 385*a*b**2*x**2 + 105*b**3*x**3
))/1155
```

3.249 $\int \sqrt{x}(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \sqrt{x}(a+bx)^3\,dx = \frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

output
```
2/3*a^3*x^(3/2)+6/5*a^2*b*x^(5/2)+6/7*a*b^2*x^(7/2)+2/9*b^3*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \sqrt{x}(a+bx)^3\,dx = \frac{2}{315}x^{3/2}\left(105a^3+189a^2bx+135ab^2x^2+35b^3x^3\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x)^3,x]
```

output
```
(2*x^(3/2)*(105*a^3 + 189*a^2*b*x + 135*a*b^2*x^2 + 35*b^3*x^3))/315
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^3 \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(a^3\sqrt{x}+3a^2bx^{3/2}+3ab^2x^{5/2}+b^3x^{7/2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

input
```
Int[Sqrt[x]*(a + b*x)^3,x]
```

output
```
(2*a^3*x^(3/2))/3 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(
9/2))/9
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36
derivativedivides	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
default	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
trager	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36
risch	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36
orering	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36

input
```
int(x^(1/2)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(3/2)*(35*b^3*x^3+135*a*b^2*x^2+189*a^2*b*x+105*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \sqrt{x}(a+bx)^3\,dx = \frac{2}{315}\left(35\,b^3x^4+135\,ab^2x^3+189\,a^2bx^2+105\,a^3x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^3\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx)^3\,dx = \frac{2}{9}\,b^3x^{\frac{9}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + \frac{2}{3}\,a^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx)^3\,dx = \frac{2}{9}\,b^3x^{\frac{9}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + \frac{2}{3}\,a^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^3,x, algorithm="giac")
```

output
```
2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx)^3\,dx = \frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{9/2}}{9}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{6\,a\,b^2\,x^{7/2}}{7}$$

input
```
int(x^(1/2)*(a + b*x)^3,x)
```

output
```
(2*a^3*x^(3/2))/3 + (2*b^3*x^(9/2))/9 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(
7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx)^3\,dx = \frac{2\sqrt{x}\,x(35b^3x^3+135a\,b^2x^2+189a^2bx+105a^3)}{315}$$

input
```
int(x^(1/2)*(b*x+a)^3,x)
```

output
```
(2*sqrt(x)*x*(105*a**3 + 189*a**2*b*x + 135*a*b**2*x**2 + 35*b**3*x**3))/3
15
```

3.250 $\int \frac{(a+bx)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x} + 2a^2bx^{3/2} + \frac{6}{5}ab^2x^{5/2} + \frac{2}{7}b^3x^{7/2}$$

output
```
2*a^3*x^(1/2)+2*a^2*b*x^(3/2)+6/5*a*b^2*x^(5/2)+2/7*b^3*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx = \frac{2}{35}\sqrt{x}\left(35a^3 + 35a^2bx + 21ab^2x^2 + 5b^3x^3\right)$$

input
```
Integrate[(a + b*x)^3/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(35*a^3 + 35*a^2*b*x + 21*a*b^2*x^2 + 5*b^3*x^3))/35
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^3}{\sqrt{x}}+3a^2b\sqrt{x}+3ab^2x^{3/2}+b^3x^{5/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

input
```
Int[(a + b*x)^3/Sqrt[x],x]
```

output
```
2*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(7/2))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
trager	$\left(\frac{2}{7}b^{3}x^{3}+\frac{6}{5}a\,b^{2}x^{2}+2a^{2}bx+2a^{3}\right)\sqrt{x}$	35
gosper	$\frac{2\sqrt{x}\,\left(5b^{3}x^{3}+21a\,b^{2}x^{2}+35a^{2}bx+35a^{3}\right)}{35}$	36
derivativedivides	$2a^{3}\sqrt{x}+2a^{2}b\,x^{\frac{3}{2}}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{7}{2}}}{7}$	36
default	$2a^{3}\sqrt{x}+2a^{2}b\,x^{\frac{3}{2}}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{7}{2}}}{7}$	36
risch	$\frac{2\sqrt{x}\,\left(5b^{3}x^{3}+21a\,b^{2}x^{2}+35a^{2}bx+35a^{3}\right)}{35}$	36
orering	$\frac{2\sqrt{x}\,\left(5b^{3}x^{3}+21a\,b^{2}x^{2}+35a^{2}bx+35a^{3}\right)}{35}$	36

input
```
int((b*x+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7*b^3*x^3+6/5*a*b^2*x^2+2*a^2*b*x+2*a^3)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx=\frac{2}{35}\left(5\,b^{3}x^{3}+21\,ab^{2}x^{2}+35\,a^{2}bx+35\,a^{3}\right)\sqrt{x}$$

input
```
integrate((b*x+a)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x+a)**3/x**(1/2),x)
```

output
```
2*a**3*sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(7/2)
/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx = \frac{2}{7}\,b^3x^{\frac{7}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + 2\,a^2bx^{\frac{3}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate((b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx = \frac{2}{7}\,b^3x^{\frac{7}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + 2\,a^2bx^{\frac{3}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate((b*x+a)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx = 2\,a^3\,\sqrt{x}+\frac{2\,b^3\,x^{7/2}}{7}+2\,a^2\,b\,x^{3/2}+\frac{6\,a\,b^2\,x^{5/2}}{5}$$

input
```
int((a + b*x)^3/x^(1/2),x)
```

output
```
2*a^3*x^(1/2) + (2*b^3*x^(7/2))/7 + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(5b^3x^3+21a\,b^2x^2+35a^2bx+35a^3)}{35}$$

input
```
int((b*x+a)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*(35*a**3 + 35*a**2*b*x + 21*a*b**2*x**2 + 5*b**3*x**3))/35
```

3.251 $\int \frac{(a+bx)^3}{x^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}} + 6a^2b\sqrt{x} + 2ab^2x^{3/2} + \frac{2}{5}b^3x^{5/2}$$

output
```
-2*a^3/x^(1/2)+6*a^2*b*x^(1/2)+2*a*b^2*x^(3/2)+2/5*b^3*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = -\frac{2(5a^3 - 15a^2bx - 5ab^2x^2 - b^3x^3)}{5\sqrt{x}}$$

input
```
Integrate[(a + b*x)^3/x^(3/2),x]
```

output
```
(-2*(5*a^3 - 15*a^2*b*x - 5*a*b^2*x^2 - b^3*x^3))/(5*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^3}{x^{3/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2\sqrt{x}+b^3x^{3/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

input
```
Int[(a + b*x)^3/x^(3/2),x]
```

output
```
(-2*a^3)/Sqrt[x] + 6*a^2*b*Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(5/2))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{2\left(-b^{3} x^{3}-5 a \,b^{2} x^{2}-15 a^{2} b x +5 a^{3}\right)}{5 \sqrt{x}}$	36
derivativedivides	$-\frac{2 a^{3}}{\sqrt{x}}+6 a^{2} b \sqrt{x}+2 a \,b^{2} x^{\frac{3}{2}}+\frac{2 b^{3} x^{\frac{5}{2}}}{5}$	36
default	$-\frac{2 a^{3}}{\sqrt{x}}+6 a^{2} b \sqrt{x}+2 a \,b^{2} x^{\frac{3}{2}}+\frac{2 b^{3} x^{\frac{5}{2}}}{5}$	36
trager	$-\frac{2\left(-b^{3} x^{3}-5 a \,b^{2} x^{2}-15 a^{2} b x +5 a^{3}\right)}{5 \sqrt{x}}$	36
risch	$-\frac{2\left(-b^{3} x^{3}-5 a \,b^{2} x^{2}-15 a^{2} b x +5 a^{3}\right)}{5 \sqrt{x}}$	36
orering	$-\frac{2\left(-b^{3} x^{3}-5 a \,b^{2} x^{2}-15 a^{2} b x +5 a^{3}\right)}{5 \sqrt{x}}$	36

input
```
int((b*x+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-b^3*x^3-5*a*b^2*x^2-15*a^2*b*x+5*a^3)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = \frac{2\left(b^3x^3+5\,ab^2x^2+15\,a^2bx-5\,a^3\right)}{5\,\sqrt{x}}$$

input
```
integrate((b*x+a)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}} + 6a^2b\sqrt{x} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x+a)**3/x**(3/2),x)
```

output
```
-2*a**3/sqrt(x) + 6*a**2*b*sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = \frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((b*x+a)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = \frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((b*x+a)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = \frac{2\,b^3\,x^{5/2}}{5} - \frac{2\,a^3}{\sqrt{x}} + 6\,a^2\,b\,\sqrt{x} + 2\,a\,b^2\,x^{3/2}$$

input
```
int((a + b*x)^3/x^(3/2),x)
```

output
```
(2*b^3*x^(5/2))/5 - (2*a^3)/x^(1/2) + 6*a^2*b*x^(1/2) + 2*a*b^2*x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^3}{x^{3/2}}\,dx = \frac{\frac{2}{5}b^3x^3 + 2a\,b^2x^2 + 6a^2bx - 2a^3}{\sqrt{x}}$$

input
```
int((b*x+a)^3/x^(3/2),x)
```

output
```
(2*( - 5*a**3 + 15*a**2*b*x + 5*a*b**2*x**2 + b**3*x**3))/(5*sqrt(x))
```

3.252 $\int \frac{(a+bx)^3}{x^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{(a+bx)^3}{x^{5/2}} dx = -\frac{2a^3}{3x^{3/2}} - \frac{6a^2b}{\sqrt{x}} + 6ab^2\sqrt{x} + \frac{2}{3}b^3x^{3/2}$$

output
```
-2/3*a^3/x^(3/2)-6*a^2*b/x^(1/2)+6*a*b^2*x^(1/2)+2/3*b^3*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^{5/2}} dx = \frac{2(-a^3 - 9a^2bx + 9ab^2x^2 + b^3x^3)}{3x^{3/2}}$$

input
```
Integrate[(a + b*x)^3/x^(5/2),x]
```

output
```
(2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^{5/2}} \, dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{x^{3/2}}+\frac{3ab^2}{\sqrt{x}}+b^3\sqrt{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

input
```
Int[(a + b*x)^3/x^(5/2),x]
```

output
```
(-2*a^3)/(3*x^(3/2)) - (6*a^2*b)/Sqrt[x] + 6*a*b^2*Sqrt[x] + (2*b^3*x^(3/2
))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{2\left(-b^{3}x^{3}-9a\,b^{2}x^{2}+9a^{2}bx+a^{3}\right)}{3x^{\frac{3}{2}}}$	34
trager	$-\frac{2\left(-b^{3}x^{3}-9a\,b^{2}x^{2}+9a^{2}bx+a^{3}\right)}{3x^{\frac{3}{2}}}$	34
risch	$-\frac{2\left(-b^{3}x^{3}-9a\,b^{2}x^{2}+9a^{2}bx+a^{3}\right)}{3x^{\frac{3}{2}}}$	34
orering	$-\frac{2\left(-b^{3}x^{3}-9a\,b^{2}x^{2}+9a^{2}bx+a^{3}\right)}{3x^{\frac{3}{2}}}$	34
derivativedivides	$-\frac{2a^{3}}{3x^{\frac{3}{2}}}-\frac{6a^{2}b}{\sqrt{x}}+6a\,b^{2}\sqrt{x}+\frac{2b^{3}x^{\frac{3}{2}}}{3}$	36
default	$-\frac{2a^{3}}{3x^{\frac{3}{2}}}-\frac{6a^{2}b}{\sqrt{x}}+6a\,b^{2}\sqrt{x}+\frac{2b^{3}x^{\frac{3}{2}}}{3}$	36

input
```
int((b*x+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b^3*x^3-9*a*b^2*x^2+9*a^2*b*x+a^3)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^3}{x^{5/2}}\,dx = \frac{2\left(b^3x^3+9\,ab^2x^2-9\,a^2bx-a^3\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^3}{x^{5/2}}\,dx = -\frac{2a^3}{3x^{\frac{3}{2}}} - \frac{6a^2b}{\sqrt{x}} + 6ab^2\sqrt{x} + \frac{2b^3x^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x+a)**3/x**(5/2),x)
```

output
```
-2*a**3/(3*x**(3/2)) - 6*a**2*b/sqrt(x) + 6*a*b**2*sqrt(x) + 2*b**3*x**(3/
2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^3}{x^{5/2}}\,dx = \frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\,(9\,a^2bx + a^3)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^3}{x^{5/2}}\,dx = \frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\,(9\,a^2bx + a^3)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^3}{x^{5/2}}\,dx = -\frac{2\,a^3+18\,a^2\,b\,x-18\,a\,b^2\,x^2-2\,b^3\,x^3}{3\,x^{3/2}}$$

input
```
int((a + b*x)^3/x^(5/2),x)
```

output
```
-(2*a^3 - 2*b^3*x^3 - 18*a*b^2*x^2 + 18*a^2*b*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3}{x^{5/2}}\,dx = \frac{\frac{2}{3}b^3x^3+6a\,b^2x^2-6a^2bx-\frac{2}{3}a^3}{\sqrt{x}\,x}$$

input
```
int((b*x+a)^3/x^(5/2),x)
```

output
```
(2*( - a**3 - 9*a**2*b*x + 9*a*b**2*x**2 + b**3*x**3))/(3*sqrt(x)*x)
```

3.253 $\int \frac{x^{5/2}}{a+bx} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{x^{5/2}}{a+bx} \, dx = \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
2*a^2*x^(1/2)/b^3-2/3*a*x^(3/2)/b^2+2/5*x^(5/2)/b-2*a^(5/2)*arctan(b^(1/2)
*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^{5/2}}{a+bx} \, dx = \frac{2\sqrt{x}(15a^2 - 5abx + 3b^2x^2)}{15b^3} - \frac{2a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(a + b*x),x]
```

output
```
(2*Sqrt[x]*(15*a^2 - 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{a+bx}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}$$

input
```
Int[x^(5/2)/(a + b*x),x]
```

output
```
(2*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{2(3b^2x^2-5abx+15a^2)\sqrt{x}}{15b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}-\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}-\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

input
```
int(x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*x^2-5*a*b*x+15*a^2)*x^(1/2)/b^3-2*a^3/b^3/(a*b)^(1/2)*arctan(b
*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.94

$$\int \frac{x^{5/2}}{a+bx}\,dx = \left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}}{15\,b^3}, -\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}\right)}{15\,b^3}\right]$$

input
```
integrate(x^(5/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
 + 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*
arctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^
3]
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

$$\int \frac{x^{5/2}}{a+bx}\,dx = \begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for } a=0\wedge b=0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a=0 \\ -\frac{a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}}+\frac{a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0)
), (2*x**(5/2)/(5*b), Eq(a, 0)), (-a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sq
rt(-a/b)) + a**3*log(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*a**2*sqrt
(x)/b**3 - 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{x^{5/2}}{a+bx}\,dx = -\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,b^2x^{\frac{5}{2}} - 5\,abx^{\frac{3}{2}} + 15\,a^2\sqrt{x}\right)}{15\,b^3}$$

input
```
integrate(x^(5/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
 5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{a+bx}\,dx = -\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,b^4x^{\frac{5}{2}} - 5\,ab^3x^{\frac{3}{2}} + 15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

input
```
integrate(x^(5/2)/(b*x+a),x, algorithm="giac")
```

output
```
-2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{x^{5/2}}{a+bx}\,dx = \frac{2\,x^{5/2}}{5\,b} - \frac{2\,a\,x^{3/2}}{3\,b^2} + \frac{2\,a^2\,\sqrt{x}}{b^3} - \frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
int(x^(5/2)/(a + b*x),x)
```

output
```
(2*x^(5/2))/(5*b) - (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 - (2*a^(5/
2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{x^{5/2}}{a+bx}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2 + 2\sqrt{x}\,a^2b - \frac{2\sqrt{x}\,a\,b^2x}{3} + \frac{2\sqrt{x}\,b^3x^2}{5}}{b^4}$$

input
```
int(x^(5/2)/(b*x+a),x)
```

output
```
(2*( - 15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 15*sq
rt(x)*a**2*b - 5*sqrt(x)*a*b**2*x + 3*sqrt(x)*b**3*x**2))/(15*b**4)
```

3.254 $\int \frac{x^{3/2}}{a+bx} dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^{3/2}}{a+bx} dx = -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{2a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
-2*a*x^(1/2)/b^2+2/3*x^(3/2)/b+2*a^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b
^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{a+bx} dx = \frac{2\sqrt{x}(-3a+bx)}{3b^2} + \frac{2a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(a + b*x),x]
```

output
```
(2*Sqrt[x]*(-3*a + b*x))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{a+bx}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}$$

input
```
Int[x^(3/2)/(a + b*x),x]
```

output
```
(2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{2(-bx+3a)\sqrt{x}}{3b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	42
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43

input
```
int(x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x+3*a)*x^(1/2)/b^2+2*a^2/b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int \frac{x^{3/2}}{a+bx}\,dx = \left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a\right.}{3\,b^2}\right.$$

input
```
integrate(x^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2
*(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a
) + (b*x - 3*a)*sqrt(x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(49) = 98.

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{x^{3/2}}{a+bx}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for } b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0 \\ \frac{a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}}-\frac{a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a), Eq(b, 0)
), (2*x**(3/2)/(3*b), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(b**3*sqr
t(-a/b)) - a**2*log(sqrt(x) + sqrt(-a/b))/(b**3*sqrt(-a/b)) - 2*a*sqrt(x)/
b**2 + 2*x**(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(bx^{\frac{3}{2}} - 3\,a\sqrt{x}\right)}{3\,b^2}$$

input
```
integrate(x^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*s
qrt(x))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(b^2x^{\frac{3}{2}} - 3\,ab\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a
*b*sqrt(x))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\,x^{3/2}}{3\,b} - \frac{2\,a\,\sqrt{x}}{b^2} + \frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
int(x^(3/2)/(a + b*x),x)
```

output
```
(2*x^(3/2))/(3*b) - (2*a*x^(1/2))/b^2 + (2*a^(3/2)*atan((b^(1/2)*x^(1/2))/
a^(1/2)))/b^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{x^{3/2}}{a+bx}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-2\sqrt{x}\,ab+\frac{2\sqrt{x}\,b^2x}{3}}{b^3}$$

input
```
int(x^(3/2)/(b*x+a),x)
```

output
```
(2*(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(x)*a*
b + sqrt(x)*b**2*x))/(3*b**3)
```

3.255 $\int \frac{\sqrt{x}}{a+bx} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\sqrt{x}}{a+bx} dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
2*x^(1/2)/b-2*a^(1/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{a+bx} dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(a + b*x),x]
```

output
```
(2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a+bx} \, dx$$

$$\Big\downarrow \; 60$$

$$\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}$$

$$\Big\downarrow \; 73$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Int[Sqrt[x]/(a + b*x),x]
```

output
```
(2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32

input
```
int(x^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b}, -\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

input
```
integrate(x^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x)
)/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0))
, (2*sqrt(x)/b, Eq(a, 0)), (-a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b))
 + a*log(sqrt(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*sqrt(x)/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}}{a+bx}\,dx = -\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\,\sqrt{x}}{b}$$

input
```
integrate(x^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}}{a+bx}\,dx = -\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\,\sqrt{x}}{b}$$

input
```
integrate(x^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
-2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \frac{2\,\sqrt{x}}{b} - \frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int(x^(1/2)/(a + b*x),x)
```

output
```
(2*x^(1/2))/b - (2*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right) + 2\sqrt{x}\,b}{b^2}$$

input
```
int(x^(1/2)/(b*x+a),x)
```

output
```
(2*( - sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))) + sqrt(x)*b))/b
**2
```

3.256 $\int \frac{1}{\sqrt{x}(a+bx)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{\sqrt{x}(a+bx)} \, dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
2*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(a+bx)} \, dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)),x]
```

output
```
(2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a+bx)} \, dx$$

$$\Big\downarrow \text{73}$$

$$2\int \frac{1}{a+bx}d\sqrt{x}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)),x]
```

output
```
(2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

input
```
int(1/x^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

input
```
integrate(1/x^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqr
t(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ \frac{2\sqrt{x}}{a} & \text{for } b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a=0 \\ \frac{\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x+a),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a, Eq(b, 0)), (-2
/(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - sqrt(-a/b))/(b*sqrt(-a/b)) - log(s
qrt(x) + sqrt(-a/b))/(b*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/x^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/x^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
int(1/(x^(1/2)*(a + b*x)),x)
```

output
```
(2*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{x}(a+bx)}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/x^(1/2)/(b*x+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.257 $\int \frac{1}{x^{3/2}(a+bx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx=-\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-2/a/x^(1/2)-2*b^(1/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx=-\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)),x]
```

output
```
-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx)} \, dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{b \int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2b \int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(a + b*x)),x]
```

output
```
-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{2}{a\sqrt{x}}-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
default	$-\frac{2}{a\sqrt{x}}-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
risch	$-\frac{2}{a\sqrt{x}}-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32

input
```
int(1/x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/a/x^(1/2)-2*b/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = \left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\sqrt{x}}{ax},\right.$$
$$\left.-\frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+\sqrt{x}\right)}{ax}\right]$$

input
```
integrate(1/x^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(
x))/(a*x), -2*(x*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + sqrt(x))/(a*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(37) = 74$.

Time = 0.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}}-\frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(3/2)/(b*x+a),x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0
)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - sqrt(-a/b))/(a*sqrt(-a/b))
 + log(sqrt(x) + sqrt(-a/b))/(a*sqrt(-a/b)) - 2/(a*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = -\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = -\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
-2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = -\frac{2}{a\,\sqrt{x}} - \frac{2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(1/(x^(3/2)*(a + b*x)),x)
```

output
```
- 2/(a*x^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = \frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)-2a}{\sqrt{x}\,a^2}$$

input
```
int(1/x^(3/2)/(b*x+a),x)
```

output
```
( - 2*(sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))) + a))/(
sqrt(x)*a**2)
```

3.258 $\int \frac{1}{x^{5/2}(a+bx)} dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{1}{x^{5/2}(a+bx)} dx = -\frac{2}{3ax^{3/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-2/3/a/x^(3/2)+2*b/a^2/x^(1/2)+2*b^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a
^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}(a+bx)} dx = -\frac{2(a-3bx)}{3a^2x^{3/2}} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)),x]
```

output
```
(-2*(a - 3*b*x))/(3*a^2*x^(3/2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx)}dx$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

input
```
Int[1/(x^(5/2)*(a + b*x)),x]
```

output
```
-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(3/2)))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(-3bx+a)}{3a^2x^{\frac{3}{2}}}+\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	40
derivativedivides	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	43
default	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	43

input
```
int(1/x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*b*x+a)/a^2/x^(3/2)+2*b^2/a^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.13

$$\int \frac{1}{x^{5/2}(a+bx)}\,dx = \left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\ \frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+\right.}{3\,a^2x^2}\right.$$

input
```
integrate(1/x^(5/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
 + 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), 2/3*(3*b*x^2*sqrt(b/a)*arctan(sqrt(x)
*sqrt(b/a)) + (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(49) = 98.

Time = 1.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^{5/2}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\ a=0\wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\ a=0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for}\ b=0 \\ -\frac{2}{3ax^{\frac{3}{2}}}+\frac{b\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}}-\frac{b\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}}+\frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(5/2)/(b*x+a),x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0
)), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + b*log(sqrt(x) - sq
rt(-a/b))/(a**2*sqrt(-a/b)) - b*log(sqrt(x) + sqrt(-a/b))/(a**2*sqrt(-a/b)
) + 2*b/(a**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(a+bx)}\,dx = \frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x
^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(a+bx)}\,dx = \frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+a),x, algorithm="giac")
```

output
```
2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x
^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^{5/2}(a+bx)}\,dx = \frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2}{3\,a}-\frac{2\,b\,x}{a^2}}{x^{3/2}}$$

input
```
int(1/(x^(5/2)*(a + b*x)),x)
```

output
```
(2*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/(3*a) - (2*b*x)/a
^2)/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{5/2}(a+bx)}\,dx = \frac{2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx - \frac{2a^2}{3} + 2abx}{\sqrt{x}\,a^3x}$$

input
```
int(1/x^(5/2)/(b*x+a),x)
```

output
```
(2*(3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - a*
*2 + 3*a*b*x))/(3*sqrt(x)*a**3*x)
```

3.259 $\int \frac{1}{x^{7/2}(a+bx)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{1}{x^{7/2}(a+bx)} \, dx = -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-2/5/a/x^(5/2)+2/3*b/a^2/x^(3/2)-2*b^2/a^3/x^(1/2)-2*b^(5/2)*arctan(b^(1/2
)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{7/2}(a+bx)} \, dx = -\frac{2(3a^2-5abx+15b^2x^2)}{15a^3x^{5/2}} - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x)),x]
```

output
```
(-2*(3*a^2 - 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTan[(
Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(a+bx)}\,dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{b\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

input
```
Int[1/(x^(7/2)*(a + b*x)),x]
```

output
```
-2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{2\left(15 b^{2} x^{2}-5 a b x+3 a^{2}\right)}{15 a^{3} x^{\frac{5}{2}}}-\frac{2 b^{3} \arctan\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{a^{3} \sqrt{a b}}$	53
derivativedivides	$-\frac{2}{5 a\, x^{\frac{5}{2}}}-\frac{2 b^{2}}{a^{3} \sqrt{x}}+\frac{2 b}{3 a^{2} x^{\frac{3}{2}}}-\frac{2 b^{3} \arctan\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{a^{3} \sqrt{a b}}$	54
default	$-\frac{2}{5 a\, x^{\frac{5}{2}}}-\frac{2 b^{2}}{a^{3} \sqrt{x}}+\frac{2 b}{3 a^{2} x^{\frac{3}{2}}}-\frac{2 b^{3} \arctan\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{a^{3} \sqrt{a b}}$	54

input
```
int(1/x^(7/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*b^2*x^2-5*a*b*x+3*a^2)/a^3/x^(5/2)-2*b^3/a^3/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^{7/2}(a+bx)}\,dx = \left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}}{15\,a^3x^3},\right.$$
$$\left.-\frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

input
```
integrate(1/x^(7/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
 a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), -2/15*(15*b^2*
x^3*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (15*b^2*x^2 - 5*a*b*x + 3*a^2)*s
qrt(x))/(a^3*x^3)]
```

Sympy [A] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^{7/2}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a=0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b=0 \\ -\frac{2}{5ax^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}-\frac{b^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}}+\frac{b^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}}-\frac{2b^2}{a^3\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(7/2)/(b*x+a),x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0
)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/
2)) - b**2*log(sqrt(x) - sqrt(-a/b))/(a**3*sqrt(-a/b)) + b**2*log(sqrt(x)
+ sqrt(-a/b))/(a**3*sqrt(-a/b)) - 2*b**2/(a**3*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{7/2}(a+bx)}\,dx = -\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\,(15\,b^2x^2 - 5\,abx + 3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{7/2}(a+bx)}\,dx = -\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\,(15\,b^2x^2 - 5\,abx + 3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+a),x, algorithm="giac")
```

output
```
-2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^{7/2}(a+bx)}\,dx = -\frac{\frac{2}{5\,a}+\frac{2\,b^2\,x^2}{a^3}-\frac{2\,b\,x}{3\,a^2}}{x^{5/2}}-\frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
int(1/(x^(7/2)*(a + b*x)),x)
```

output
```
- (2/(5*a) + (2*b^2*x^2)/a^3 - (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atan(
(b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{7/2}(a+bx)}\,dx = \frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-\frac{2a^3}{5}+\frac{2a^2bx}{3}-2a\,b^2x^2}{\sqrt{x}\,a^4x^2}$$

input
```
int(1/x^(7/2)/(b*x+a),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2
*x**2 - 3*a**3 + 5*a**2*b*x - 15*a*b**2*x**2))/(15*sqrt(x)*a**4*x**2)
```

3.260 $\int \frac{x^{5/2}}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{x^{5/2}}{(a+bx)^2} \, dx = -\frac{4a\sqrt{x}}{b^3} + \frac{2x^{3/2}}{3b^2} - \frac{a^2\sqrt{x}}{b^3(a+bx)} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
-4*a*x^(1/2)/b^3+2/3*x^(3/2)/b^2-a^2*x^(1/2)/b^3/(b*x+a)+5*a^(3/2)*arctan(
b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{x^{5/2}}{(a+bx)^2} \, dx = \frac{\sqrt{x}(-15a^2 - 10abx + 2b^2x^2)}{3b^3(a+bx)} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(a + b*x)^2,x]
```

output
```
(Sqrt[x]*(-15*a^2 - 10*a*b*x + 2*b^2*x^2))/(3*b^3*(a + b*x)) + (5*a^(3/2)*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx$$

$$\Big\downarrow \; 51$$

$$\frac{5\int \frac{x^{3/2}}{a+bx}dx}{2b}-\frac{x^{5/2}}{b(a+bx)}$$

$$\Big\downarrow \; 60$$

$$\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}$$

$$\Big\downarrow \; 60$$

$$\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}$$

$$\Big\downarrow \; 73$$

$$\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}$$

$$\Big\downarrow \; 218$$

$$\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}$$

input
```
Int[x^(5/2)/(a + b*x)^2,x]
```

output
```
-(x^(5/2)/(b*(a + b*x))) + (5*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*
Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/(2*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{2(-bx+6a)\sqrt{x}}{3b^{3}}+\frac{a^{2}\left(-\frac{\sqrt{x}}{bx+a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^{3}}$	56
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	59
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	59

input

```
int(x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-2/3*(-b*x+6*a)*x^(1/2)/b^3+a^2/b^3*(-x^(1/2)/(b*x+a)+5/(a*b)^(1/2)*arctan
(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.21

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \left[\frac{15\,(abx+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,b^2x^2-10\,abx-15\,a^2)\sqrt{x}}{6\,(b^4x+ab^3)}, \frac{15\,(abx+a\right.$$

input

```
integrate(x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

output

```
[1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(
b*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/
3*(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 -
 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(68) = 136$.

Time = 7.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 5.33

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ \frac{2x^{\frac{3}{2}}}{3b^2} \\ \frac{15a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} - \frac{15a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} - \frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} + \frac{15a^2bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}} - \frac{15a^2}{6ab^4} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
 0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (15*a**3*log(sqrt(x) - sqrt(-a/b))/
(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**3*log(sqrt(x) + sqrt(-
a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 30*a**2*b*sqrt(x)*sqrt
(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 15*a**2*b*x*log(sqrt(
x) - sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**2*b*x
*log(sqrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 2
0*a*b**2*x**(3/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) +
 4*b**3*x**(5/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = -\frac{a^2\sqrt{x}}{b^4x+ab^3} + \frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(bx^{\frac{3}{2}}-6\,a\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-a^2*sqrt(x)/(b^4*x + a*b^3) + 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b
)*b^3) + 2/3*(b*x^(3/2) - 6*a*sqrt(x))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{a^2\sqrt{x}}{(bx+a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}} - 6\,ab^3\sqrt{x}\right)}{3\,b^6}$$

input
```
integrate(x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)
*b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \frac{2\,x^{3/2}}{3\,b^2} - \frac{4\,a\,\sqrt{x}}{b^3} - \frac{a^2\,\sqrt{x}}{x\,b^4+a\,b^3} + \frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
int(x^(5/2)/(a + b*x)^2,x)
```

output
```
(2*x^(3/2))/(3*b^2) - (4*a*x^(1/2))/b^3 - (a^2*x^(1/2))/(a*b^3 + b^4*x) +
(5*a^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{x^{5/2}}{(a+bx)^2}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx-15\sqrt{x}\,a^2b-10\sqrt{x}\,a\,b^2x+2\sqrt{x}}{3b^4\,(bx+a)}$$

input
```
int(x^(5/2)/(b*x+a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 15*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x - 15*sqrt(x)*a**2*b - 10
*sqrt(x)*a*b**2*x + 2*sqrt(x)*b**3*x**2)/(3*b**4*(a + b*x))
```

3.261 $\int \frac{x^{3/2}}{(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{x^{3/2}}{(a+bx)^2} dx = \frac{2\sqrt{x}}{b^2} + \frac{a\sqrt{x}}{b^2(a+bx)} - \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
2*x^(1/2)/b^2+a*x^(1/2)/b^2/(b*x+a)-3*a^(1/2)*arctan(b^(1/2)*x^(1/2)/a^(1/
2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^{3/2}}{(a+bx)^2} dx = \frac{\sqrt{x}(3a+2bx)}{b^2(a+bx)} - \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(a + b*x)^2,x]
```

output
```
(Sqrt[x]*(3*a + 2*b*x))/(b^2*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{3\int \frac{\sqrt{x}}{a+bx}dx}{2b}-\frac{x^{3/2}}{b(a+bx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}$$

input
```
Int[x^(3/2)/(a + b*x)^2,x]
```

output
```
-(x^(3/2)/(b*(a + b*x))) + (3*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	47
default	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	47
risch	$\frac{2\sqrt{x}}{b^2}+\frac{a\sqrt{x}}{b^2(bx+a)}-\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	47

input
```
int(x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b^2-2*a/b^2*(-1/2*x^(1/2)/(b*x+a)+3/2/(a*b)^(1/2)*arctan(b*x^(1/
2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.35

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\,(b^3x+ab^2)},\ -\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

input
```
integrate(x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
 a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*a
rctan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(53) = 106.

Time = 2.87 (sec) , antiderivative size = 332, normalized size of antiderivative = 5.82

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ \frac{2\sqrt{x}}{b^2} \\ -\frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}+\frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}+\frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}-\frac{3abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}+\frac{3abx}{2ab^3} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b,
0)), (2*sqrt(x)/b**2, Eq(a, 0)), (-3*a**2*log(sqrt(x) - sqrt(-a/b))/(2*a*b
**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a**2*log(sqrt(x) + sqrt(-a/b))/(
2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 6*a*b*sqrt(x)*sqrt(-a/b)/(2*a
*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*a*b*x*log(sqrt(x) - sqrt(-a/b)
)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a*b*x*log(sqrt(x) + sqrt
(-a/b))/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 4*b**2*x**(3/2)*sqrt
(-a/b)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \frac{a\sqrt{x}}{b^3x+ab^2} - \frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,\sqrt{x}}{b^2}$$

input
```
integrate(x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a*sqrt(x)/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2
) + 2*sqrt(x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = -\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{a\sqrt{x}}{(bx+a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

input
```
integrate(x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^
2) + 2*sqrt(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \frac{2\sqrt{x}}{b^2} + \frac{a\sqrt{x}}{x\,b^3 + a\,b^2} - \frac{3\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
int(x^(3/2)/(a + b*x)^2,x)
```

output
```
(2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 + b^3*x) - (3*a^(1/2)*atan((b^(1/2)*x
^(1/2))/a^(1/2)))/b^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{x^{3/2}}{(a+bx)^2}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx + 3\sqrt{x}\,ab + 2\sqrt{x}\,b^2x}{b^3\,(bx+a)}$$

input
```
int(x^(3/2)/(b*x+a)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x + 3*sqrt(x)*a*b + 2*sqrt(x)*
b**2*x)/(b**3*(a + b*x))
```

3.262 $\int \frac{\sqrt{x}}{(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\sqrt{x}}{(a+bx)^2} dx = -\frac{\sqrt{x}}{b(a+bx)} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
-x^(1/2)/b/(b*x+a)+arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{(a+bx)^2} dx = -\frac{\sqrt{x}}{b(a+bx)} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(a + b*x)^2,x]
```

output
```
-(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3
/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2b}-\frac{\sqrt{x}}{b(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{a+bx}d\sqrt{x}}{b}-\frac{\sqrt{x}}{b(a+bx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

input
```
Int[Sqrt[x]/(a + b*x)^2,x]
```

output
```
-(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3
/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{b(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37
default	$-\frac{\sqrt{x}}{b(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37

input
```
int(x^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)/b/(b*x+a)+1/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(ab^3x+a^2b^2)},\right.$$
$$\left.-\frac{ab\sqrt{x}+\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

input
```
integrate(x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sq
rt(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a
)*arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(37) = 74$.

Time = 1.44 (sec) , antiderivative size = 269, normalized size of antiderivative = 5.85

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{fo} \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{fo} \\ -\frac{2}{b^2\sqrt{x}} & \text{fo} \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} & \text{ot} \end{cases}$$

input
```
integrate(x**(1/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b,
0)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2
*sqrt(-a/b) + 2*b**3*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2
*sqrt(-a/b) + 2*b**3*x*sqrt(-a/b)) - 2*b*sqrt(x)*sqrt(-a/b)/(2*a*b**2*sqrt
(-a/b) + 2*b**3*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2*sq
rt(-a/b) + 2*b**3*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2*
sqrt(-a/b) + 2*b**3*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = -\frac{\sqrt{x}}{b^2x+ab} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate(x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\sqrt{x}}{(bx+a)b}$$

input
```
integrate(x^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sqrt{x}}{b\,(a+b\,x)}$$

input
```
int(x^(1/2)/(a + b*x)^2,x)
```

output
```
atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2)) - x^(1/2)/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-\sqrt{x}\,ab}{a\,b^2\,(bx+a)}$$

input
```
int(x^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*a
tan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - sqrt(x)*a*b)/(a*b**2*(a + b*x))
```

3.263 $\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx=\frac{\sqrt{x}}{a(a+bx)}+\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

output
```
x^(1/2)/a/(b*x+a)+arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx=\frac{\sqrt{x}}{a(a+bx)}+\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)^2),x]
```

output
```
Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a+bx)^2} \, dx$$
$$\Big\downarrow \ 52$$
$$\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}$$
$$\Big\downarrow \ 73$$
$$\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}$$
$$\Big\downarrow \ 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)^2),x]
```

output
```
Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{x}}{a(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36
default	$\frac{\sqrt{x}}{a(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36

input
```
int(1/x^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)/a/(b*x+a)+1/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(a^2b^2x+a^3b)}, \frac{ab\sqrt{x}-\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

input
```
integrate(1/x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(37) = 74$.

Time = 2.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 6.16

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{2\sqrt{x}}{a^2} \\ -\frac{2}{3b^2x^{\frac{3}{2}}} \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
 (-2/(3*b**2*x**(3/2)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b*
sqrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a**2*
b*sqrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) + 2*b*sqrt(x)*sqrt(-a/b)/(2*a**2*b*s
qrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a**2
*b*sqrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*
a**2*b*sqrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{x}}{abx+a^2} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{\sqrt{x}}{(bx+a)a}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{x}}{a\,(a+b\,x)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

input
```
int(1/(x^(1/2)*(a + b*x)^2),x)
```

output
```
x^(1/2)/(a*(a + b*x)) + atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx + \sqrt{x}\,ab}{a^2b\,(bx+a)}$$

input
```
int(1/x^(1/2)/(b*x+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*a
tan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x + sqrt(x)*a*b)/(a**2*b*(a + b*x))
```

3.264 $\int \frac{1}{x^{3/2}(a+bx)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{1}{x^{3/2}(a+bx)^2} \, dx = -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a+bx)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-3/a^2/x^(1/2)+1/a/x^(1/2)/(b*x+a)-3*b^(1/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2
))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}(a+bx)^2} \, dx = \frac{-2a-3bx}{a^2\sqrt{x}(a+bx)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)^2),x]
```

output
```
(-2*a - 3*b*x)/(a^2*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx)^2}dx$$

$$\Big\downarrow \ 52$$

$$\frac{3\int \frac{1}{x^{3/2}(a+bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a+bx)}$$

$$\Big\downarrow \ 61$$

$$\frac{3\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}$$

$$\Big\downarrow \ 73$$

$$\frac{3\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}$$

$$\Big\downarrow \ 218$$

$$\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}$$

input
```
Int[1/(x^(3/2)*(a + b*x)^2),x]
```

output
```
1/(a*Sqrt[x]*(a + b*x)) + (3*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2b\left(\frac{\sqrt{x}}{2bx+2a}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	47
default	$-\frac{2b\left(\frac{\sqrt{x}}{2bx+2a}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	47
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{b\sqrt{x}}{a^{2}(bx+a)}-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$	48

input
```
int(1/x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*b/a^2*(1/2*x^(1/2)/(b*x+a)+3/2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)
))-2/a^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.55

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = \left[\frac{3\,(bx^2+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\,(a^2bx^2+a^3x)},\right.$$
$$\left.-\frac{3\,(bx^2+ax)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(3\,bx+2\,a)\sqrt{x}}{a^2bx^2+a^3x}\right]$$

input
```
integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b
*x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), -(3*(b*x^2 + a*x)
*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 +
 a^3*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(51) = 102$.

Time = 5.26 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.86

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ -\frac{3a\sqrt{x}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} + \frac{3a\sqrt{x}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} - \frac{4a\sqrt{-\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} - \frac{3bx^{\frac{3}{2}}\log}{2a^3\sqrt{x}\sqrt{-}} \end{cases}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0
)), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (-3*a*sqrt(x)*log(sqrt(x) - sqrt(-a/
b))/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) + 3*a*sqrt(
x)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2
)*sqrt(-a/b)) - 4*a*sqrt(-a/b)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3
/2)*sqrt(-a/b)) - 3*b*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(2*a**3*sqrt(x)*s
qrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) + 3*b*x**(3/2)*log(sqrt(x) + sqr
t(-a/b))/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) - 6*b*
x*sqrt(-a/b)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = -\frac{3\,bx+2\,a}{a^2bx^{\frac{3}{2}}+a^3\sqrt{x}} - \frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a
*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = -\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/
2) + a*sqrt(x))*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = -\frac{\frac{2}{a}+\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}+b\,x^{3/2}} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int(1/(x^(3/2)*(a + b*x)^2),x)
```

output
```
- (2/a + (3*b*x)/a^2)/(a*x^(1/2) + b*x^(3/2)) - (3*b^(1/2)*atan((b^(1/2)*x
^(1/2))/a^(1/2)))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = \frac{-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-2a^2-3abx}{\sqrt{x}\,a^3\,(bx+a)}$$

input
```
int(1/x^(3/2)/(b*x+a)^2,x)
```

output
```
( - 3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sq
rt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - 2*a**2 - 3
*a*b*x)/(sqrt(x)*a**3*(a + b*x))
```

3.265 $\int \frac{1}{x^{5/2}(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{x^{5/2}(a+bx)^2} dx = -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-5/3/a^2/x^(3/2)+5*b/a^3/x^(1/2)+1/a/x^(3/2)/(b*x+a)+5*b^(3/2)*arctan(b^(1
/2)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^{5/2}(a+bx)^2} dx = \frac{-2a^2+10abx+15b^2x^2}{3a^3x^{3/2}(a+bx)} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)^2),x]
```

output
```
(-2*a^2 + 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a + b*x)) + (5*b^(3/2)*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{5\int \frac{1}{x^{5/2}(a+bx)}dx}{2a}+\frac{1}{ax^{3/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}$$

input
```
Int[1/(x^(5/2)*(a + b*x)^2),x]
```

output
```
1/(a*x^(3/2)*(a + b*x)) + (5*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*S
qrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{2(-6bx+a)}{3a^3x^{\frac{3}{2}}}+\frac{b^2\left(\frac{\sqrt{x}}{bx+a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^3}$	53
derivativedivides	$\frac{2b^2\left(\frac{\sqrt{x}}{2bx+2a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{2}{3a^2x^{\frac{3}{2}}}+\frac{4b}{a^3\sqrt{x}}$	58
default	$\frac{2b^2\left(\frac{\sqrt{x}}{2bx+2a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{2}{3a^2x^{\frac{3}{2}}}+\frac{4b}{a^3\sqrt{x}}$	58

input
```
int(1/x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-6*b*x+a)/a^3/x^(3/2)+1/a^3*b^2*(x^(1/2)/(b*x+a)+5/(a*b)^(1/2)*arcta
n(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.59

$$\int \frac{1}{x^{5/2}(a+bx)^2}\,dx = \left[\frac{15\,(b^2x^3+abx^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(15\,b^2x^2+10\,abx-2\,a^2)\sqrt{x}}{6\,(a^3bx^3+a^4x^2)}, \frac{15\,(\right.$$

input
```
integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a)
- a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 +
a^4*x^2), 1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a))
+ (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(65) = 130$.

Time = 15.09 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.55

$$\int \frac{1}{x^{5/2}(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{2}{3a^2x^{\frac{3}{2}}} \\ -\frac{2}{7b^2x^{\frac{7}{2}}} \\ -\frac{4a^2\sqrt{-\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{20a}{6a^4x^{\frac{3}{2}}\sqrt{-}} \end{cases}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**2*x**(3/2)), Eq(b
, 0)), (-2/(7*b**2*x**(7/2)), Eq(a, 0)), (-4*a**2*sqrt(-a/b)/(6*a**4*x**(3
/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*a*b*x**(3/2)*log(sqrt(
x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b
)) - 15*a*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b)
 + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 20*a*b*x*sqrt(-a/b)/(6*a**4*x**(3/2)*sq
rt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*b**2*x**(5/2)*log(sqrt(x) -
sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) -
15*b**2*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6
*a**3*b*x**(5/2)*sqrt(-a/b)) + 30*b**2*x**2*sqrt(-a/b)/(6*a**4*x**(3/2)*sq
rt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{5/2}(a+bx)^2}\,dx = \frac{15\,b^2x^2 + 10\,abx - 2\,a^2}{3\left(a^3bx^{\frac{5}{2}} + a^4x^{\frac{3}{2}}\right)} + \frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*
arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^{5/2}(a+bx)^2}\,dx = \frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{b^2\sqrt{x}}{(bx+a)a^3} + \frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)
*a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^{5/2}(a+bx)^2}\,dx = \frac{\frac{5\,b^2\,x^2}{a^3}-\frac{2}{3\,a}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}+b\,x^{5/2}} + \frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
int(1/(x^(5/2)*(a + b*x)^2),x)
```

output
```
((5*b^2*x^2)/a^3 - 2/(3*a) + (10*b*x)/(3*a^2))/(a*x^(3/2) + b*x^(5/2)) + (
5*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^{5/2}(a+bx)^2}\,dx = \frac{15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-2a^3+10a^2bx+1}{3\sqrt{x}\,a^4x\,(bx+a)}$$

input
```
int(1/x^(5/2)/(b*x+a)^2,x)
```

output
```
(15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + 15
*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - 2
*a**3 + 10*a**2*b*x + 15*a*b**2*x**2)/(3*sqrt(x)*a**4*x*(a + b*x))
```

3.266 $\int \frac{x^{7/2}}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = -\frac{6a\sqrt{x}}{b^4}+\frac{2x^{3/2}}{3b^3}+\frac{a^3\sqrt{x}}{2b^4(a+bx)^2}-\frac{13a^2\sqrt{x}}{4b^4(a+bx)}+\frac{35a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

output
```
-6*a*x^(1/2)/b^4+2/3*x^(3/2)/b^3+1/2*a^3*x^(1/2)/b^4/(b*x+a)^2-13/4*a^2*x^
(1/2)/b^4/(b*x+a)+35/4*a^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = \frac{\sqrt{x}(-105a^3-175a^2bx-56ab^2x^2+8b^3x^3)}{12b^4(a+bx)^2}+\frac{35a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

input
```
Integrate[x^(7/2)/(a + b*x)^3,x]
```

output
```
(Sqrt[x]*(-105*a^3 - 175*a^2*b*x - 56*a*b^2*x^2 + 8*b^3*x^3))/(12*b^4*(a +
 b*x)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx)^3} \, dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{7 \int \frac{x^{5/2}}{(a+bx)^2}dx}{4b} - \frac{x^{7/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{7 \left(\frac{5 \int \frac{x^{3/2}}{a+bx}dx}{2b} - \frac{x^{5/2}}{b(a+bx)}\right)}{4b} - \frac{x^{7/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{7 \left(\frac{5 \left(\frac{2x^{3/2}}{3b} - \frac{a \int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2b} - \frac{x^{5/2}}{b(a+bx)}\right)}{4b} - \frac{x^{7/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{7 \left(\frac{5 \left(\frac{2x^{3/2}}{3b} - \frac{a \left(\frac{2\sqrt{x}}{b} - \frac{a \int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{2b} - \frac{x^{5/2}}{b(a+bx)}\right)}{4b} - \frac{x^{7/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}$$

$\Big\downarrow$ 218

$$\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4b}-\frac{x^{7/2}}{2b(a+bx)^2}$$

input
```
Int[x^(7/2)/(a + b*x)^3,x]
```

output
```
-1/2*x^(7/2)/(b*(a + b*x)^2) + (7*(-(x^(5/2)/(b*(a + b*x))) + (5*((2*x^(3/
2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
]])/b^(3/2)))/b))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{2(-bx+9a)\sqrt{x}}{3b^{4}}+\frac{a^{2}\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{4}-\frac{11a\sqrt{x}}{4}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^{4}}$	66
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}\right)}{b^{4}}+\frac{2a^{2}\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}$	68
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}\right)}{b^{4}}+\frac{2a^{2}\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}$	68

input
```
int(x^(7/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x+9*a)*x^(1/2)/b^4+a^2/b^4*(2*(-13/8*b*x^(3/2)-11/8*a*x^(1/2))/(b
*x+a)^2+35/4/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.29

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = \left[\frac{105\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^3x^3-56\,ab^2x^2-175\,a^2bx}{24\,(b^6x^2+2\,ab^5x+a^2b^4)}\right.$$

input
```
integrate(x^(7/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)
*sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x -
105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 +
2*a^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56
*a*b^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^
4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(94) = 188$.

Time = 39.62 (sec) , antiderivative size = 762, normalized size of antiderivative = 7.70

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(7/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a**3), Eq(b,
 0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (105*a**4*log(sqrt(x) - sqrt(-a/b))
/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/
b)) - 105*a**4*log(sqrt(x) + sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b
**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 210*a**3*b*sqrt(x)*sqrt(-a/b
)/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a
/b)) + 210*a**3*b*x*log(sqrt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 4
8*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 210*a**3*b*x*log(sqrt(x
) + sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**
7*x**2*sqrt(-a/b)) - 350*a**2*b**2*x**(3/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt(
-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 105*a**2*b**2*
x**2*log(sqrt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt
(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 105*a**2*b**2*x**2*log(sqrt(x) + sqrt(
-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sq
rt(-a/b)) - 112*a*b**3*x**(5/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt(-a/b) + 48*a
*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 16*b**4*x**(7/2)*sqrt(-a/b
)/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a
/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = -\frac{13\,a^2bx^{\frac{3}{2}}+11\,a^3\sqrt{x}}{4\,(b^6x^2+2\,ab^5x+a^2b^4)}+\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4}+\frac{2\left(bx^{\frac{3}{2}}-9\,a\sqrt{x}\right)}{3\,b^4}$$

input
```
integrate(x^(7/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(13*a^2*b*x^(3/2) + 11*a^3*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(b*x^(3/2) - 9
*a*sqrt(x))/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = \frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4} - \frac{13\,a^2bx^{\frac{3}{2}} + 11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4} + \frac{2\left(b^6x^{\frac{3}{2}} - 9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

input
```
integrate(x^(7/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/
2) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x
))/b^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = \frac{2\,x^{3/2}}{3\,b^3} - \frac{\frac{11\,a^3\,\sqrt{x}}{4} + \frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4 + 2\,a\,b^5\,x + b^6\,x^2} - \frac{6\,a\,\sqrt{x}}{b^4} + \frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{9/2}}$$

input
```
int(x^(7/2)/(a + b*x)^3,x)
```

output
```
(2*x^(3/2))/(3*b^3) - ((11*a^3*x^(1/2))/4 + (13*a^2*b*x^(3/2))/4)/(a^2*b^4
 + b^6*x^2 + 2*a*b^5*x) - (6*a*x^(1/2))/b^4 + (35*a^(3/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/(4*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int \frac{x^{7/2}}{(a+bx)^3}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3 + 210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx + 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2}{12b^5\,(b^2x^2 + 2abx + a^2)}$$

input
```
int(x^(7/2)/(b*x+a)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 + 210*sqrt(b
)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 105*sqrt(b)*sqrt(
a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 - 105*sqrt(x)*a**3*b -
175*sqrt(x)*a**2*b**2*x - 56*sqrt(x)*a*b**3*x**2 + 8*sqrt(x)*b**4*x**3)/(1
2*b**5*(a**2 + 2*a*b*x + b**2*x**2))
```

3.267 $\int \frac{x^{5/2}}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx=\frac{2\sqrt{x}}{b^3}-\frac{a^2\sqrt{x}}{2b^3(a+bx)^2}+\frac{9a\sqrt{x}}{4b^3(a+bx)}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

output
```
2*x^(1/2)/b^3-1/2*a^2*x^(1/2)/b^3/(b*x+a)^2+9/4*a*x^(1/2)/b^3/(b*x+a)-15/4
*a^(1/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx=\frac{\sqrt{x}(15a^2+25abx+8b^2x^2)}{4b^3(a+bx)^2}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

input
```
Integrate[x^(5/2)/(a + b*x)^3,x]
```

output
```
(Sqrt[x]*(15*a^2 + 25*a*b*x + 8*b^2*x^2))/(4*b^3*(a + b*x)^2) - (15*Sqrt[a
]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {51, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5\int \frac{x^{3/2}}{(a+bx)^2}dx}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{x}}{a+bx}dx}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx)^2}$$

input
```
Int[x^(5/2)/(a + b*x)^3,x]
```

output
```
-1/2*x^(5/2)/(b*(a + b*x)^2) + (5*(-(x^(3/2)/(b*(a + b*x))) + (3*((2*Sqrt[
x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/(2*b)))/(4
*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	56
default	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	56
risch	$\frac{2\sqrt{x}}{b^3}-\frac{a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{4}-\frac{7a\sqrt{x}}{4}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^3}$	57

input
```
int(x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b^3-2/b^3*a*((-9/8*b*x^(3/2)-7/8*a*x^(1/2))/(b*x+a)^2+15/8/(a*b)
^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.38

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \left[\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2+2\,ab^4x+a^2b^3)},\right.$$
$$\left.-\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

input
```
integrate(x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(
-a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^
2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5
*x^2 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(78) = 156$.

Time = 16.86 (sec) , antiderivative size = 683, normalized size of antiderivative = 8.13

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{7}{2}}}{7a^3} \\ \frac{2\sqrt{x}}{b^3} \\ -\frac{15a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{15a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{30a^2b\sqrt{x}\sqrt{-\ldots}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\ldots}} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**3), Eq(b,
0)), (2*sqrt(x)/b**3, Eq(a, 0)), (-15*a**3*log(sqrt(x) - sqrt(-a/b))/(8*a*
*2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 15
*a**3*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt
(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b*
*4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 30*a**2
*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(
-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*x*log(sqrt(x) + sqrt(-a/b))/(8
*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) +
 50*a*b**2*x**(3/2)*sqrt(-a/b)/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(
-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 15*a*b**2*x**2*log(sqrt(x) - sqrt(-a/b))
/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)
) + 15*a*b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*
a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 16*b**3*x**(5/2)*sqrt(-a/b
)/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)} - \frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} + \frac{2\,\sqrt{x}}{b^3}$$

input
```
integrate(x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) - 15/4
*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = -\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} + \frac{2\,\sqrt{x}}{b^3} + \frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

input
```
integrate(x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*
(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \frac{\frac{7\,a^2\,\sqrt{x}}{4}+\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2} + \frac{2\,\sqrt{x}}{b^3} - \frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

input
```
int(x^(5/2)/(a + b*x)^3,x)
```

output
```
((7*a^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x) +
(2*x^(1/2))/b^3 - (15*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \frac{x^{5/2}}{(a+bx)^3}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2 - 30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx - 15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2 +}{4b^4\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^(5/2)/(b*x+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 - 30*sqrt(
b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x - 15*sqrt(b)*sqrt(a)*
atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 + 15*sqrt(x)*a**2*b + 25*sqr
t(x)*a*b**2*x + 8*sqrt(x)*b**3*x**2)/(4*b**4*(a**2 + 2*a*b*x + b**2*x**2))
```

3.268 $\int \frac{x^{3/2}}{(a+bx)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{x^{3/2}}{(a+bx)^3} dx = \frac{a\sqrt{x}}{2b^2(a+bx)^2} - \frac{5\sqrt{x}}{4b^2(a+bx)} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

output
```
1/2*a*x^(1/2)/b^2/(b*x+a)^2-5/4*x^(1/2)/b^2/(b*x+a)+3/4*arctan(b^(1/2)*x^(
1/2)/a^(1/2))/a^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{x^{3/2}}{(a+bx)^3} dx = -\frac{\sqrt{x}(3a+5bx)}{4b^2(a+bx)^2} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

input
```
Integrate[x^(3/2)/(a + b*x)^3,x]
```

output
```
-1/4*(Sqrt[x]*(3*a + 5*b*x))/(b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {51, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx)^3}dx$$

$$\Big\downarrow \ 51$$

$$\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}dx}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \ 51$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{3\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4b}-\frac{x^{3/2}}{2b(a+bx)^2}$$

input
```
Int[x^(3/2)/(a + b*x)^3,x]
```

output
```
-1/2*x^(3/2)/(b*(a + b*x)^2) + (3*(-(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^{2}}}{(bx+a)^{2}}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}\sqrt{ab}}$	50
default	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^{2}}}{(bx+a)^{2}}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}\sqrt{ab}}$	50

input
```
int(x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-5/8*x^(3/2)/b-3/8*a*x^(1/2)/b^2)/(b*x+a)^2+3/4/b^2/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.61

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(5\,ab^2x+3\,a^2b)\sqrt{x}}{8\,(ab^5x^2+2\,a^2b^4x+a^3b^3)},\right.$$
$$\left.-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(5\,ab^2x+3\,a^2b)\sqrt{x}}{4\,(ab^5x^2+2\,a^2b^4x+a^3b^3)}\right]$$

input
```
integrate(x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*
sqrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*
b^4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(
a*b)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*
x + a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(65) = 130$.

Time = 9.55 (sec) , antiderivative size = 605, normalized size of antiderivative = 8.52

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \frac{2x^{\frac{5}{2}}}{5a^3} \\ -\frac{2}{b^3\sqrt{x}} \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**3), Eq(b,
0)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a
**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3
*a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt
(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 6*a*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b**3*s
qrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) + 6*a*b*x*log
(sqrt(x) - sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) +
8*b**5*x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*s
qrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 10*b**2*x**
(3/2)*sqrt(-a/b)/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5
*x**2*sqrt(-a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b**3*sqr
t(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3*b**2*x**2*l
og(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b)
+ 8*b**5*x**2*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = -\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^2}$$

input
```
integrate(x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arc
tan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^2}-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

input
```
integrate(x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*s
qrt(x))/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}} - \frac{\frac{5\,x^{3/2}}{4\,b}+\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int(x^(3/2)/(a + b*x)^3,x)
```

output
```
(3*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2)) - ((5*x^(3/2))/(4*
b) + (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \frac{x^{3/2}}{(a+bx)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-3\sqrt{x}}{4a\,b^3\,(b^2x^2+2abx+a^2)}$$

input
```
int(x^(3/2)/(b*x+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + 3*sqrt(b)*sqrt(a)*atan((
sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - 3*sqrt(x)*a**2*b - 5*sqrt(x)*a*b
**2*x)/(4*a*b**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.269 $\int \frac{\sqrt{x}}{(a+bx)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{\sqrt{x}}{(a+bx)^3} dx = -\frac{\sqrt{x}}{2b(a+bx)^2} + \frac{\sqrt{x}}{4ab(a+bx)} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

output
```
-1/2*x^(1/2)/b/(b*x+a)^2+1/4*x^(1/2)/a/b/(b*x+a)+1/4*arctan(b^(1/2)*x^(1/2
)/a^(1/2))/a^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{(a+bx)^3} dx = -\frac{\sqrt{x}(a-bx)}{4ab(a+bx)^2} + \frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(a + b*x)^3,x]
```

output
```
-1/4*(Sqrt[x]*(a - b*x))/(a*b*(a + b*x)^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]]/(4*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {51, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx)^3} \, dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}}{4b}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

input
```
Int[Sqrt[x]/(a + b*x)^3,x]
```

output
```
-1/2*Sqrt[x]/(b*(a + b*x)^2) + (Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]))/(4*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ba\sqrt{ab}}$	52
default	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ba\sqrt{ab}}$	52

input
```
int(x^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8/a*x^(3/2)-1/8*x^(1/2)/b)/(b*x+a)^2+1/4/b/a/(a*b)^(1/2)*arctan(b*x^(
1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.55

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \left[-\frac{(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\,(ab^2x-a^2b)\sqrt{x}}{8\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)}, -\frac{(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(ab^2x-a^2b)\sqrt{x}}{4\,(a^2b^4x^2+2\,a^3b^3x+a^4b^2)}\right]$$

input
```
integrate(x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sq
rt(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*
x + a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(
b*sqrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*
b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(58) = 116.

Time = 5.05 (sec) , antiderivative size = 627, normalized size of antiderivative = 8.59

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^3} \\ -\frac{2}{3b^3x^{\frac{3}{2}}} \\ \frac{a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}} - \frac{a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}} - \frac{2ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}} + \end{cases}$$

input
```
integrate(x**(1/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
 0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(8
*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/
b)) - a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**
3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*sqrt(x)*sqrt(-a/b)/(8*a
**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)
) + 2*a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b*
*3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*x*log(sqrt(x) + sqrt(-
a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*
sqrt(-a/b)) + 2*b**2*x**(3/2)*sqrt(-a/b)/(8*a**3*b**2*sqrt(-a/b) + 16*a**2
*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) + b**2*x**2*log(sqrt(x) - s
qrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*
x**2*sqrt(-a/b)) - b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-
a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\,(ab^3x^2 + 2\,a^2b^2x + a^3b)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab}$$

input
```
integrate(x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan
(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}ab} + \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\,(bx+a)^2ab}$$

input
```
integrate(x^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(
x))/((b*x + a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \frac{\frac{x^{3/2}}{4\,a} - \frac{\sqrt{x}}{4\,b}}{a^2 + 2\,a\,b\,x + b^2\,x^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}$$

input
```
int(x^(1/2)/(a + b*x)^3,x)
```

output
```
(x^(3/2)/(4*a) - x^(1/2)/(4*b))/(a^2 + b^2*x^2 + 2*a*b*x) + atan((b^(1/2)*
x^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2 + 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2 - \sqrt{x}\,a^2b + \sqrt{x}\,a\,b^2x}{4a^2b^2\,(b^2x^2 + 2abx + a^2)}$$

input
```
int(x^(1/2)/(b*x+a)^3,x)
```

output

```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 2*sqrt(b)*sqrt
(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + sqrt(b)*sqrt(a)*atan((sqrt
(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - sqrt(x)*a**2*b + sqrt(x)*a*b**2*x)/(
4*a**2*b**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.270 $\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{\sqrt{x}}{2a(a+bx)^2} + \frac{3\sqrt{x}}{4a^2(a+bx)} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

output
```
1/2*x^(1/2)/a/(b*x+a)^2+3/4*x^(1/2)/a^2/(b*x+a)+3/4*arctan(b^(1/2)*x^(1/2)
/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{\sqrt{x}(5a+3bx)}{4a^2(a+bx)^2} + \frac{3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)^3),x]
```

output
```
(Sqrt[x]*(5*a + 3*b*x))/(4*a^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a+bx)^3} \, dx$$

$$\Big\downarrow \text{52}$$

$$\frac{3 \int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4a} + \frac{\sqrt{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{3 \left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a} + \frac{\sqrt{x}}{a(a+bx)}\right)}{4a} + \frac{\sqrt{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{3 \left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a} + \frac{\sqrt{x}}{a(a+bx)}\right)}{4a} + \frac{\sqrt{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3 \left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{\sqrt{x}}{a(a+bx)}\right)}{4a} + \frac{\sqrt{x}}{2a(a+bx)^2}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)^3),x]
```

output
```
Sqrt[x]/(2*a*(a + b*x)^2) + (3*(Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2a(bx+a)^2}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59
default	$\frac{\sqrt{x}}{2a(bx+a)^2}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59

input
```
int(1/x^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)/a/(b*x+a)^2+3/2/a*(1/2*x^(1/2)/a/(b*x+a)+1/2/a/(a*b)^(1/2)*arc
tan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.66

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\,(3\,ab^2x+5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}, -\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(3\,ab^2x+5\,a^2b)\sqrt{x}}{4\,(a^3b^3x^2+2\,a^4b^2x+a^5b)} \right]$$

input
```
integrate(1/x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*
sqrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^
4*b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(
a*b)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^
2*x + a^5*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(61) = 122.

Time = 7.86 (sec) , antiderivative size = 632, normalized size of antiderivative = 9.03

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{2\sqrt{x}}{a^3} \\ -\frac{2}{5b^3x^{\frac{5}{2}}} \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \frac{10ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**3, Eq(b, 0)),
 (-2/(5*b**3*x**(5/2)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a*
*4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b))
 - 3*a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*
sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 10*a*b*sqrt(x)*sqrt(-a/b)/(8*a
**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)
) + 6*a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*
x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-
a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*
sqrt(-a/b)) + 6*b**2*x**(3/2)*sqrt(-a/b)/(8*a**4*b*sqrt(-a/b) + 16*a**3*b*
*2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 3*b**2*x**2*log(sqrt(x) -
 sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**
3*x**2*sqrt(-a/b)) - 3*b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(
-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2}+\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(bx+a)^2a^2}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*s
qrt(x))/((b*x + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx = \frac{\frac{5\,\sqrt{x}}{4\,a}+\frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

input
```
int(1/(x^(1/2)*(a + b*x)^3),x)
```

output
```
((5*x^(1/2))/(4*a) + (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (3
*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+5\sqrt{x}\,a^2b+3\sqrt{x}\,a\,b^2x}{4a^3b\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/x^(1/2)/(b*x+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + 3*sqrt(b)*sqrt(a)*atan((
sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 + 5*sqrt(x)*a**2*b + 3*sqrt(x)*a*b
**2*x)/(4*a**3*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.271 $\int \frac{1}{x^{3/2}(a+bx)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{1}{x^{3/2}(a+bx)^3} \, dx = -\frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx)^2} + \frac{5}{4a^2\sqrt{x}(a+bx)} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
-15/4/a^3/x^(1/2)+1/2/a/x^(1/2)/(b*x+a)^2+5/4/a^2/x^(1/2)/(b*x+a)-15/4*b^(
1/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3/2}(a+bx)^3} \, dx = \frac{-8a^2 - 25abx - 15b^2x^2}{4a^3\sqrt{x}(a+bx)^2} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)^3),x]
```

output
```
(-8*a^2 - 25*a*b*x - 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a + b*x)^2) - (15*Sqrt[b]
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx)^3} \, dx$$

$$\Big\downarrow \text{52}$$

$$\frac{5 \int \frac{1}{x^{3/2}(a+bx)^2}dx}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{5\left(\frac{3 \int \frac{1}{x^{3/2}(a+bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(\frac{3\left(-\frac{b \int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(\frac{3\left(-\frac{2b \int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx)^2}$$

input
```
Int[1/(x^(3/2)*(a + b*x)^3),x]
```

output
```
1/(2*a*Sqrt[x]*(a + b*x)^2) + (5*(1/(a*Sqrt[x]*(a + b*x)) + (3*(-2/(a*Sqrt
[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*
a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-\frac{2b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}-\frac{2}{a^{3}\sqrt{x}}$	56
default	$-\frac{2b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}-\frac{2}{a^{3}\sqrt{x}}$	56
risch	$-\frac{2}{a^{3}\sqrt{x}}-\frac{b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{4}+\frac{9a\sqrt{x}}{4}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^{3}}$	57

input
```
int(1/x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/a^3*b*((7/8*b*x^(3/2)+9/8*a*x^(1/2))/(b*x+a)^2+15/8/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2)))-2/a^3/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.55

$$\int \frac{1}{x^{3/2}(a+bx)^3}\,dx = \left[\frac{15\,(b^2x^3+2\,abx^2+a^2x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(15\,b^2x^2+25\,abx+8\,a^2)\sqrt{}}{8\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}\right.$$

$$\left.-\frac{15\,(b^2x^3+2\,abx^2+a^2x)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(15\,b^2x^2+25\,abx+8\,a^2)\sqrt{x}}{4\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}\right]$$

input
```
integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*s
qrt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^
3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x), -1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*s
qrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(
x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(75) = 150$.

Time = 17.04 (sec) , antiderivative size = 779, normalized size of antiderivative = 9.50

$$\int \frac{1}{x^{3/2}(a+bx)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**3*sqrt(x)), Eq(b, 0
)), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-15*a**2*sqrt(x)*log(sqrt(x) - sqrt
(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**
3*b**2*x**(5/2)*sqrt(-a/b)) + 15*a**2*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(8
*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**
(5/2)*sqrt(-a/b)) - 16*a**2*sqrt(-a/b)/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**
4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 30*a*b*x**(3/
2)*log(sqrt(x) - sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/
2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) + 30*a*b*x**(3/2)*log(sqr
t(x) + sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a
/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 50*a*b*x*sqrt(-a/b)/(8*a**5*sqrt(
x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(
-a/b)) - 15*b**2*x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-
a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) +
15*b**2*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16
*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 30*b**2*x
**2*sqrt(-a/b)/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b)
+ 8*a**3*b**2*x**(5/2)*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{3/2}(a+bx)^3}\,dx = -\frac{15\,b^2x^2+25\,abx+8\,a^2}{4\left(a^3b^2x^{\frac{5}{2}}+2\,a^4bx^{\frac{3}{2}}+a^5\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) +
a^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^{3/2}(a+bx)^3}\,dx = -\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}-\frac{2}{a^3\sqrt{x}}-\frac{7\,b^2x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(bx+a)^2a^3}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/
4*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3/2}(a+bx)^3}\,dx = -\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}+\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}+2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

input
```
int(1/(x^(3/2)*(a + b*x)^3),x)
```

output
```
- (2/a + (15*b^2*x^2)/(4*a^3) + (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/
2) + 2*a*b*x^(3/2)) - (15*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7
/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^{3/2}(a+bx)^3}\,dx = \frac{-15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2 - 30\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx - 15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan}{4\sqrt{x}\,a^4\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/x^(3/2)/(b*x+a)^3,x)
```

output
```
( - 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 -
30*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x - 15*
sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - 8*
a**3 - 25*a**2*b*x - 15*a*b**2*x**2)/(4*sqrt(x)*a**4*(a**2 + 2*a*b*x + b**
2*x**2))
```

3.272 $\int \frac{1}{x^{5/2}(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = -\frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} + \frac{1}{2ax^{3/2}(a+bx)^2} + \frac{7}{4a^2x^{3/2}(a+bx)} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

output
```
-35/12/a^3/x^(3/2)+35/4*b/a^4/x^(1/2)+1/2/a/x^(3/2)/(b*x+a)^2+7/4/a^2/x^(3
/2)/(b*x+a)+35/4*b^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \frac{-8a^3+56a^2bx+175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a+bx)^2} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)^3),x]
```

output
```
(-8*a^3 + 56*a^2*b*x + 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a + b
*x)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {52, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx$$

$$\Big\downarrow \text{52}$$

$$\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}dx}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{7\left(\frac{5\int \frac{1}{x^{5/2}(a+bx)}dx}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(\frac{5\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

$\downarrow$ 73

$$\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

$\downarrow$ 218

$$\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

input
```
Int[1/(x^(5/2)*(a + b*x)^3),x]
```

output
```
1/(2*a*x^(3/2)*(a + b*x)^2) + (7*(1/(a*x^(3/2)*(a + b*x)) + (5*(-2/(3*a*x^
(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/a^(3/2)))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{2(-9bx+a)}{3a^4x^{\frac{3}{2}}}+\frac{b^2\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{4}+\frac{13a\sqrt{x}}{4}}{(bx+a)^2}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^4}$	64
derivativedivides	$-\frac{2}{3a^3x^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{x}}+\frac{2b^2\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(bx+a)^2}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	67
default	$-\frac{2}{3a^3x^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{x}}+\frac{2b^2\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(bx+a)^2}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	67

input
```
int(1/x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-9*b*x+a)/a^4/x^(3/2)+b^2/a^4*(2*(11/8*b*x^(3/2)+13/8*a*x^(1/2))/(b*
x+a)^2+35/4/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \left[\frac{105\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(105\,b^3x^3+175\,ab^2x^2\right.}{24\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}\right.$$

input
```
integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*s
qrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^
2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), 1/12*(105*(
b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (
105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 +
2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(88) = 176$.

Time = 39.30 (sec) , antiderivative size = 869, normalized size of antiderivative = 9.15

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**3*x**(3/2)), Eq(b
, 0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (-16*a**3*sqrt(-a/b)/(24*a**6*x**
(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*s
qrt(-a/b)) + 105*a**2*b*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3/
2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt
(-a/b)) - 105*a**2*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/2)*
sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a
/b)) + 112*a**2*b*x*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x*
*(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 210*a*b**2*x**(5/2
)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5
/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) - 210*a*b**2*x**(5/2)*l
og(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)
*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 350*a*b**2*x**2*sqrt(-a/
b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*
b**2*x**(7/2)*sqrt(-a/b)) + 105*b**3*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(2
4*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*
x**(7/2)*sqrt(-a/b)) - 105*b**3*x**(7/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**
6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7
/2)*sqrt(-a/b)) + 210*b**3*x**3*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) +
48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)), True...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \frac{105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3}{12\left(a^4b^2x^{\frac{7}{2}}+2\,a^5bx^{\frac{5}{2}}+a^6x^{\frac{3}{2}}\right)}+\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
 2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sq
rt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4}+\frac{2\,(9\,bx-a)}{3\,a^4x^{\frac{3}{2}}}+\frac{11\,b^3x^{\frac{3}{2}}+13\,ab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^
4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \frac{\frac{175\,b^2\,x^2}{12\,a^3}-\frac{2}{3\,a}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}+2\,a\,b\,x^{5/2}}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

input
```
int(1/(x^(5/2)*(a + b*x)^3),x)
```

output
```
((175*b^2*x^2)/(12*a^3) - 2/(3*a) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2
))/(a^2*x^(3/2) + b^2*x^(7/2) + 2*a*b*x^(5/2)) + (35*b^(3/2)*atan((b^(1/2)
*x^(1/2))/a^(1/2)))/(4*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \frac{105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2bx + 210\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2 + 105\sqrt{x}\,\sqrt{b}\,\sqrt{}}{12\sqrt{x}\,a^5x\,(b^2x^2+2abx+}$$

input
```
int(1/x^(5/2)/(b*x+a)^3,x)
```

output
```
(105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x
+ 210*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2*x
**2 + 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3
*x**3 - 8*a**4 + 56*a**3*b*x + 175*a**2*b**2*x**2 + 105*a*b**3*x**3)/(12*s
qrt(x)*a**5*x*(a**2 + 2*a*b*x + b**2*x**2))
```

3.273 $\int \frac{x^{5/2}}{-a+bx} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{x^{5/2}}{-a+bx} dx = \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
2*a^2*x^(1/2)/b^3+2/3*a*x^(3/2)/b^2+2/5*x^(5/2)/b-2*a^(5/2)*arctanh(b^(1/2
)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^{5/2}}{-a+bx} dx = \frac{2\sqrt{x}(15a^2 + 5abx + 3b^2x^2)}{15b^3} - \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(-a + b*x),x]
```

output
```
(2*Sqrt[x]*(15*a^2 + 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {60, 25, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{bx-a}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a\int -\frac{x^{3/2}}{a-bx}dx}{b}+\frac{2x^{5/2}}{5b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a-bx}dx}{b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{a\int \frac{\sqrt{x}}{a-bx}dx}{b}-\frac{2x^{3/2}}{3b}\right)}{b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{a\left(\frac{a\int \frac{1}{\sqrt{x}(a-bx)}dx}{b}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{a\left(\frac{2a\int \frac{1}{a-bx}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2x^{5/2}}{5b} - \frac{a\left(\frac{a\left(\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b}\right)}{b} - \frac{2x^{3/2}}{3b}\right)}{b}$$

input
```
Int[x^(5/2)/(-a + b*x),x]
```

output
```
(2*x^(5/2))/(5*b) - (a*((-2*x^(3/2))/(3*b) + (a*((-2*Sqrt[x])/b + (2*Sqrt[
a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{2(3b^2x^2+5abx+15a^2)\sqrt{x}}{15b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

input
```
int(x^(5/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*x^2+5*a*b*x+15*a^2)*x^(1/2)/b^3-2*a^3/b^3/(a*b)^(1/2)*arctanh(
b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.93

$$\int \frac{x^{5/2}}{-a+bx}\,dx=\left[\frac{15\,a^2\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(3\,b^2x^2+5\,abx+15\,a^2)\sqrt{x}}{15\,b^3},\frac{2\left(15\,a^2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{}}{}\right.\right.}{}\right.$$

input
```
integrate(x^(5/2)/(b*x-a),x, algorithm="fricas")
```

output
```
[1/15*(15*a^2*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) +
 2*(3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3, 2/15*(15*a^2*sqrt(-a/b)*ar
ctan(b*sqrt(x)*sqrt(-a/b)/a) + (3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3
]
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

$$\int \frac{x^{5/2}}{-a+bx}\,dx = \begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for } a=0 \wedge b=0 \\ -\frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a=0 \\ \frac{a^3\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b^4\sqrt{\frac{a}{b}}} - \frac{a^3\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b^4\sqrt{\frac{a}{b}}} + \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{\frac{3}{2}}}{3b^2} + \frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x-a),x)
```

output
```
Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(7/2)/(7*a), Eq(b, 0
)), (2*x**(5/2)/(5*b), Eq(a, 0)), (a**3*log(sqrt(x) - sqrt(a/b))/(b**4*sqr
t(a/b)) - a**3*log(sqrt(x) + sqrt(a/b))/(b**4*sqrt(a/b)) + 2*a**2*sqrt(x)/
b**3 + 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{x^{5/2}}{-a+bx}\,dx = \frac{a^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,b^2x^{\frac{5}{2}} + 5\,abx^{\frac{3}{2}} + 15\,a^2\sqrt{x}\right)}{15\,b^3}$$

input
```
integrate(x^(5/2)/(b*x-a),x, algorithm="maxima")
```

output
```
a^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) +
 2/15*(3*b^2*x^(5/2) + 5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^{5/2}}{-a+bx}\,dx = \frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^3} + \frac{2\left(3\,b^4x^{\frac{5}{2}} + 5\,ab^3x^{\frac{3}{2}} + 15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

input
```
integrate(x^(5/2)/(b*x-a),x, algorithm="giac")
```

output
```
2*a^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2/15*(3*b^4*x^(5/2)
+ 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{x^{5/2}}{-a+bx}\,dx = \frac{2\,x^{5/2}}{5\,b} + \frac{2\,a\,x^{3/2}}{3\,b^2} + \frac{2\,a^2\,\sqrt{x}}{b^3} + \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}}{b^{7/2}}$$

input
```
int(-x^(5/2)/(a - b*x),x)
```

output
```
(2*x^(5/2))/(5*b) + (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 + (a^(5/2)
*atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*2i)/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{x^{5/2}}{-a+bx}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2 - 15\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2 + 30\sqrt{x}\,a^2b + 10\sqrt{}}{15b^4}$$

input
```
int(x^(5/2)/(b*x-a),x)
```

output
```
(15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 - 15*sqrt(b)*
sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 + 30*sqrt(x)*a**2*b + 10*sqr
t(x)*a*b**2*x + 6*sqrt(x)*b**3*x**2)/(15*b**4)
```

3.274 $\int \frac{x^{3/2}}{-a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{x^{3/2}}{-a+bx}\,dx=\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
2*a*x^(1/2)/b^2+2/3*x^(3/2)/b-2*a^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b
^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{-a+bx}\,dx=\frac{2\sqrt{x}(3a+bx)}{3b^2}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(-a + b*x),x]
```

output
```
(2*Sqrt[x]*(3*a + b*x))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 25, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{bx-a}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a\int -\frac{\sqrt{x}}{a-bx}dx}{b}+\frac{2x^{3/2}}{3b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a-bx}dx}{b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{a\int \frac{1}{\sqrt{x}(a-bx)}dx}{b}-\frac{2\sqrt{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2a\int \frac{1}{a-bx}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b}\right)}{b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b}\right)}{b}$$

input
```
Int[x^(3/2)/(-a + b*x),x]
```

output
```
(2*x^(3/2))/(3*b) - (a*((-2*Sqrt[x])/b + (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2(bx+3a)\sqrt{x}}{3b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	41
derivativedivides	$\frac{2a\sqrt{x}+\frac{2b\,x^{\frac{3}{2}}}{3}}{b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43
default	$\frac{2a\sqrt{x}+\frac{2b\,x^{\frac{3}{2}}}{3}}{b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43

input
```
int(x^(3/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+3*a)*x^(1/2)/b^2-2*a^2/b^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int \frac{x^{3/2}}{-a+bx}\,dx = \left[\frac{3\,a\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(bx+3\,a)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,a\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(bx+3\right.}{3\,b^2}\right.$$

input
```
integrate(x^(3/2)/(b*x-a),x, algorithm="fricas")
```

output
```
[1/3*(3*a*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(
b*x + 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a
) + (b*x + 3*a)*sqrt(x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{x^{3/2}}{-a+bx}\,dx = \begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{5}{2}}}{5a} & \text{for}\: b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for}\: a = 0 \\ \frac{a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b^3\sqrt{\frac{a}{b}}} - \frac{a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b^3\sqrt{\frac{a}{b}}} + \frac{2a\sqrt{x}}{b^2} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x-a),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(5/2)/(5*a), Eq(b, 0
)), (2*x**(3/2)/(3*b), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(a/b))/(b**3*sqr
t(a/b)) - a**2*log(sqrt(x) + sqrt(a/b))/(b**3*sqrt(a/b)) + 2*a*sqrt(x)/b**
2 + 2*x**(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{x^{3/2}}{-a+bx}\,dx = \frac{a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(bx^{\frac{3}{2}}+3\,a\sqrt{x}\right)}{3\,b^2}$$

input
```
integrate(x^(3/2)/(b*x-a),x, algorithm="maxima")
```

output
```
a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2) +
 2/3*(b*x^(3/2) + 3*a*sqrt(x))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^{3/2}}{-a+bx}\,dx = \frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^2} + \frac{2\left(b^2x^{\frac{3}{2}}+3\,ab\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(3/2)/(b*x-a),x, algorithm="giac")
```

output
```
2*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) + 2/3*(b^2*x^(3/2) + 3
*a*b*sqrt(x))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{-a+bx}\,dx = \frac{2\,x^{3/2}}{3\,b} + \frac{2\,a\,\sqrt{x}}{b^2} - \frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
int(-x^(3/2)/(a - b*x),x)
```

output
```
(2*x^(3/2))/(3*b) + (2*a*x^(1/2))/b^2 - (2*a^(3/2)*atanh((b^(1/2)*x^(1/2))
/a^(1/2)))/b^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{x^{3/2}}{-a+bx}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a - 3\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a + 6\sqrt{x}\,ab + 2\sqrt{x}\,b^2x}{3b^3}$$

input
```
int(x^(3/2)/(b*x-a),x)
```

output
```
(3*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a - 3*sqrt(b)*sqrt(
a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a + 6*sqrt(x)*a*b + 2*sqrt(x)*b**2*x)/
(3*b**3)
```

3.275 $\int \frac{\sqrt{x}}{-a+bx} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\sqrt{x}}{-a+bx} \, dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
2*x^(1/2)/b-2*a^(1/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{-a+bx} \, dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(-a + b*x),x]
```

output
```
(2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {60, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{bx-a}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a\int -\frac{1}{\sqrt{x}(a-bx)}dx}{b}+\frac{2\sqrt{x}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a-bx)}dx}{b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a-bx}d\sqrt{x}}{b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Int[Sqrt[x]/(-a + b*x),x]
```

output
```
(2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{2a\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{b}-\frac{2a\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{b}-\frac{2a\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32

input
```
int(x^(1/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,\sqrt{x}}{b}, \frac{2\left(\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\sqrt{x}\right)}{b}\right]$$

input
```
integrate(x^(1/2)/(b*x-a),x, algorithm="fricas")
```

output
```
[(sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*sqrt(x))/
b, 2*(sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + sqrt(x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\ a=0\wedge b=0 \\ -\frac{2x^{\frac{3}{2}}}{3a} & \text{for}\ b=0 \\ \frac{2\sqrt{x}}{b} & \text{for}\ a=0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b^2\sqrt{\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b^2\sqrt{\frac{a}{b}}}+\frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(b*x-a),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(3/2)/(3*a), Eq(b, 0)
), (2*sqrt(x)/b, Eq(a, 0)), (a*log(sqrt(x) - sqrt(a/b))/(b**2*sqrt(a/b)) -
 a*log(sqrt(x) + sqrt(a/b))/(b**2*sqrt(a/b)) + 2*sqrt(x)/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \frac{a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2\sqrt{x}}{b}$$

input
```
integrate(x^(1/2)/(b*x-a),x, algorithm="maxima")
```

output
```
a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b) + 2*s
qrt(x)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b} + \frac{2\sqrt{x}}{b}$$

input
```
integrate(x^(1/2)/(b*x-a),x, algorithm="giac")
```

output
```
2*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) + 2*sqrt(x)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int(-x^(1/2)/(a - b*x),x)
```

output
```
(2*x^(1/2))/b - (2*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \frac{\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)-\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)+2\sqrt{x}\,b}{b^2}$$

input
```
int(x^(1/2)/(b*x-a),x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b) - sqrt(b)*sqrt(a)*log
(sqrt(b)*sqrt(a) + sqrt(x)*b) + 2*sqrt(x)*b)/b**2
```

3.276 $\int \frac{1}{\sqrt{x}(-a+bx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
-2*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(-a + b*x)),x]
```

output
```
(-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(bx-a)}dx$$

$$\Big\downarrow \text{73}$$

$$2\int \frac{1}{bx-a}d\sqrt{x}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*(-a + b*x)),x]
```

output
```
(-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

input
```
int(1/x^(1/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = \left[\frac{\sqrt{ab}\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{ab}, \frac{2\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab}\right]$$

input
```
integrate(1/x^(1/2)/(b*x-a),x, algorithm="fricas")
```

output
```
[sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt(x))/(b*x - a))/(a*b), 2*sqrt(-a
*b)*arctan(sqrt(-a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(29) = 58$.

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ -\frac{2\sqrt{x}}{a} & \text{for } b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a=0 \\ \frac{\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b\sqrt{\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x-a),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*sqrt(x)/a, Eq(b, 0)), (-
2/(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - sqrt(a/b))/(b*sqrt(a/b)) - log(sq
rt(x) + sqrt(a/b))/(b*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/x^(1/2)/(b*x-a),x, algorithm="maxima")
```

output
```
log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = \frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

input
```
integrate(1/x^(1/2)/(b*x-a),x, algorithm="giac")
```

output
```
2*arctan(b*sqrt(x)/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
int(-1/(x^(1/2)*(a - b*x)),x)
```

output
```
-(2*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{x}(-a+bx)}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)-\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)\right)}{ab}$$

input
```
int(1/x^(1/2)/(b*x-a),x)
```

output
```
(sqrt(b)*sqrt(a)*(log( - sqrt(b)*sqrt(a) + sqrt(x)*b) - log(sqrt(b)*sqrt(a
) + sqrt(x)*b)))/(a*b)
```

3.277 $\int \frac{1}{x^{3/2}(-a+bx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
2/a/x^(1/2)-2*b^(1/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*(-a + b*x)),x]
```

output
```
2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {61, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(bx-a)}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{b\int -\frac{1}{\sqrt{x}(a-bx)}dx}{a}+\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2}{a\sqrt{x}}-\frac{b\int \frac{1}{\sqrt{x}(a-bx)}dx}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2}{a\sqrt{x}}-\frac{2b\int \frac{1}{a-bx}d\sqrt{x}}{a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Int[1/(x^(3/2)*(-a + b*x)),x]
```

output
```
2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2}{a\sqrt{x}}-\frac{2b\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
default	$\frac{2}{a\sqrt{x}}-\frac{2b\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
risch	$\frac{2}{a\sqrt{x}}-\frac{2b\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32

input
```
int(1/x^(3/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

output
```
2/a/x^(1/2)-2*b/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,\sqrt{x}}{ax}, \frac{2\left(x\sqrt{-\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{-\frac{b}{a}}\right)+\sqrt{x}\right)}{ax}\right]$$

input
```
integrate(1/x^(3/2)/(b*x-a),x, algorithm="fricas")
```

output
```
[(x*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) + 2*sqrt(x)
)/(a*x), 2*(x*sqrt(-b/a)*arctan(sqrt(x)*sqrt(-b/a)) + sqrt(x))/(a*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.68 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a=0 \\ \frac{2}{a\sqrt{x}} & \text{for } b=0 \\ \frac{\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{a\sqrt{\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{a\sqrt{\frac{a}{b}}} + \frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(3/2)/(b*x-a),x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0
)), (2/(a*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - sqrt(a/b))/(a*sqrt(a/b)) - l
og(sqrt(x) + sqrt(a/b))/(a*sqrt(a/b)) + 2/(a*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x-a),x, algorithm="maxima")
```

output
```
b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a) + 2/(
a*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a} + \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x-a),x, algorithm="giac")
```

output
```
2*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) + 2/(a*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2}{a\,\sqrt{x}} - \frac{2\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(-1/(x^(3/2)*(a - b*x)),x)
```

output
```
2/(a*x^(1/2)) - (2*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)-\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)+2a}{\sqrt{x}\,a^2}$$

input
```
int(1/x^(3/2)/(b*x-a),x)
```

output
```
(sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b) - sqrt(x)*sqr
t(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b) + 2*a)/(sqrt(x)*a**2)
```

3.278 $\int \frac{1}{x^{5/2}(-a+bx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \frac{2}{3ax^{3/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
2/3/a/x^(3/2)+2*b/a^2/x^(1/2)-2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a
^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \frac{2(a+3bx)}{3a^2x^{3/2}} - \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^(5/2)*(-a + b*x)),x]
```

output
```
(2*(a + 3*b*x))/(3*a^2*x^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {61, 25, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(bx-a)}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{b\int -\frac{1}{x^{3/2}(a-bx)}dx}{a}+\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3ax^{3/2}}-\frac{b\int \frac{1}{x^{3/2}(a-bx)}dx}{a}$$

$$\Big\downarrow \text{61}$$

$$\frac{2}{3ax^{3/2}}-\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}(a-bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3ax^{3/2}}-\frac{b\left(\frac{2b\int \frac{1}{a-bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{2}{3ax^{3/2}}-\frac{b\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}$$

input
```
Int[1/(x^(5/2)*(-a + b*x)),x]
```

output
```
2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(3/2)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2bx+\frac{2a}{3}}{a^{2}x^{\frac{3}{2}}}-\frac{2b^{2}\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$	40
derivativedivides	$-\frac{2b^{2}\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}+\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^{2}\sqrt{x}}$	43
default	$-\frac{2b^{2}\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}+\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^{2}\sqrt{x}}$	43

input
```
int(1/x^(5/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*b*x+a)/a^2/x^(3/2)-2*b^2/a^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \left[\frac{3\,bx^2\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(3\,bx+a)\sqrt{x}}{3\,a^2x^2},\ \frac{2\left(3\,bx^2\sqrt{-\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{-\frac{b}{a}}\right)\right.}{3\,a^2x^2}\right.$$

input
```
integrate(1/x^(5/2)/(b*x-a),x, algorithm="fricas")
```

output
```
[1/3*(3*b*x^2*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) +
 2*(3*b*x + a)*sqrt(x))/(a^2*x^2), 2/3*(3*b*x^2*sqrt(-b/a)*arctan(sqrt(x)*
sqrt(-b/a)) + (3*b*x + a)*sqrt(x))/(a^2*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 1.73 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0 \\ \frac{2}{3ax^{\frac{3}{2}}} & \text{for } b=0 \\ \frac{2}{3ax^{\frac{3}{2}}} + \frac{b\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{a^2\sqrt{\frac{a}{b}}} - \frac{b\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{a^2\sqrt{\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(5/2)/(b*x-a),x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0
)), (2/(3*a*x**(3/2)), Eq(b, 0)), (2/(3*a*x**(3/2)) + b*log(sqrt(x) - sqrt
(a/b))/(a**2*sqrt(a/b)) - b*log(sqrt(x) + sqrt(a/b))/(a**2*sqrt(a/b)) + 2*
b/(a**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \frac{b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x-a),x, algorithm="maxima")
```

output
```
b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2) +
 2/3*(3*b*x + a)/(a^2*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^2} + \frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x-a),x, algorithm="giac")
```

output
```
2*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 2/3*(3*b*x + a)/(a^2
*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \frac{\frac{2}{3\,a}+\frac{2\,b\,x}{a^2}}{x^{3/2}}-\frac{2\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int(-1/(x^(5/2)*(a - b*x)),x)
```

output
```
(2/(3*a) + (2*b*x)/a^2)/x^(3/2) - (2*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/
2)))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{5/2}(-a+bx)}\,dx = \frac{3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)bx-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)bx+2a^2+}{3\sqrt{x}\,a^3x}$$

input
```
int(1/x^(5/2)/(b*x-a),x)
```

output
```
(3*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x - 3*sqr
t(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x + 2*a**2 + 6*a*b
*x)/(3*sqrt(x)*a**3*x)
```

3.279 $\int \frac{1}{x^{7/2}(-a+bx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} + \frac{2b^2}{a^3\sqrt{x}} - \frac{2b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
2/5/a/x^(5/2)+2/3*b/a^2/x^(3/2)+2*b^2/a^3/x^(1/2)-2*b^(5/2)*arctanh(b^(1/2
)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \frac{2(3a^2+5abx+15b^2x^2)}{15a^3x^{5/2}} - \frac{2b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[1/(x^(7/2)*(-a + b*x)),x]
```

output
```
(2*(3*a^2 + 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {61, 25, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(bx-a)}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{b\int -\frac{1}{x^{5/2}(a-bx)}dx}{a}+\frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2}{5ax^{5/2}}-\frac{b\int \frac{1}{x^{5/2}(a-bx)}dx}{a}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{2}{5ax^{5/2}}-\frac{b\left(\frac{b\int \frac{1}{x^{3/2}(a-bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{2}{5ax^{5/2}}-\frac{b\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}(a-bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2}{5ax^{5/2}}-\frac{b\left(\frac{b\left(\frac{2b\int \frac{1}{a-bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2}{5ax^{5/2}}-\frac{b\left(\frac{b\left(\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}$$

input
```
Int[1/(x^(7/2)*(-a + b*x)),x]
```

output
```
2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2)) + (b*(-2/(a*Sqrt[x]) + (2*Sqrt[b]*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{2b^2x^2+\frac{2}{3}abx+\frac{2}{5}a^2}{a^3x^{\frac{5}{2}}}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	53
derivativedivides	$\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	54
default	$\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	54

input
```
int(1/x^(7/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

output
```
2/15*(15*b^2*x^2+5*a*b*x+3*a^2)/a^3/x^(5/2)-2*b^3/a^3/(a*b)^(1/2)*arctanh(
b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx=\left[\frac{15\,b^2x^3\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(15\,b^2x^2+5\,abx+3\,a^2)\sqrt{x}}{15\,a^3x^3},\frac{2\left(15\,b^2x^3\sqrt{-\frac{b}{a}}\right.}{}\right.$$

input
```
integrate(1/x^(7/2)/(b*x-a),x, algorithm="fricas")
```

output
```
[1/15*(15*b^2*x^3*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a
)) + 2*(15*b^2*x^2 + 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(-b/a)*arctan(sqrt(x)*sqrt(-b/a)) + (15*b^2*x^2 + 5*a*b*x + 3*a^2)*sq
rt(x))/(a^3*x^3)]
```

Sympy [A] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a=0 \\ \frac{2}{5ax^{\frac{5}{2}}} & \text{for } b=0 \\ \frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} + \frac{b^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{a^3\sqrt{\frac{a}{b}}} - \frac{b^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{a^3\sqrt{\frac{a}{b}}} + \frac{2b^2}{a^3\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(7/2)/(b*x-a),x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0
)), (2/(5*a*x**(5/2)), Eq(b, 0)), (2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2)
) + b**2*log(sqrt(x) - sqrt(a/b))/(a**3*sqrt(a/b)) - b**2*log(sqrt(x) + sq
rt(a/b))/(a**3*sqrt(a/b)) + 2*b**2/(a**3*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \frac{b^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{2\left(15\,b^2x^2+5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x-a),x, algorithm="maxima")
```

output
```
b^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3) +
 2/15*(15*b^2*x^2 + 5*a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^3} + \frac{2\,(15\,b^2x^2+5\,abx+3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x-a),x, algorithm="giac")
```

output
```
2*b^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/15*(15*b^2*x^2 + 5
*a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \frac{\frac{2}{5\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{2\,b\,x}{3\,a^2}}{x^{5/2}} - \frac{2\,b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
int(-1/(x^(7/2)*(a - b*x)),x)
```

output
```
(2/(5*a) + (2*b^2*x^2)/a^3 + (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atanh((
b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{7/2}(-a+bx)}\,dx = \frac{15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)b^2x^2 - 15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)b^2x^2 +}{15\sqrt{x}\,a^4x^2}$$

input
```
int(1/x^(7/2)/(b*x-a),x)
```

output

```
(15*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2
- 15*sqrt(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 +
6*a**3 + 10*a**2*b*x + 30*a*b**2*x**2)/(15*sqrt(x)*a**4*x**2)
```

3.280 $\int \frac{x^{5/2}}{(-a+bx)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{x^{5/2}}{(-a+bx)^2} \, dx = \frac{4a\sqrt{x}}{b^3} + \frac{2x^{3/2}}{3b^2} + \frac{a^2\sqrt{x}}{b^3(a-bx)} - \frac{5a^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
4*a*x^(1/2)/b^3+2/3*x^(3/2)/b^2+a^2*x^(1/2)/b^3/(-b*x+a)-5*a^(3/2)*arctanh
(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x^{5/2}}{(-a+bx)^2} \, dx = \frac{\sqrt{x}(-15a^2+10abx+2b^2x^2)}{3b^3(-a+bx)} - \frac{5a^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(-a + b*x)^2,x]
```

output
```
(Sqrt[x]*(-15*a^2 + 10*a*b*x + 2*b^2*x^2))/(3*b^3*(-a + b*x)) - (5*a^(3/2)
*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {51, 25, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(bx-a)^2} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5 \int -\frac{x^{3/2}}{a-bx}dx}{2b}+\frac{x^{5/2}}{b(a-bx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^{5/2}}{b(a-bx)}-\frac{5 \int \frac{x^{3/2}}{a-bx}dx}{2b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}}{b(a-bx)}-\frac{5 \left(\frac{a \int \frac{\sqrt{x}}{a-bx}dx}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}}{b(a-bx)}-\frac{5 \left(\frac{a \left(\frac{a \int \frac{1}{\sqrt{x}(a-bx)}dx}{b}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{5/2}}{b(a-bx)}-\frac{5 \left(\frac{a \left(\frac{2a \int \frac{1}{a-bx}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{x^{5/2}}{b(a-bx)}-\frac{5\left(\frac{a\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}$$

input
```
Int[x^(5/2)/(-a + b*x)^2,x]
```

output
```
x^(5/2)/(b*(a - b*x)) - (5*((-2*x^(3/2))/(3*b) + (a*((-2*Sqrt[x])/b + (2*S
qrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{2(bx+6a)\sqrt{x}}{3b^{3}}+\frac{a^{2}\left(-\frac{\sqrt{x}}{bx-a}-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^{3}}$	57
derivativedivides	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^{3}}-\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	60
default	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^{3}}-\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	60

input
```
int(x^(5/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+6*a)*x^(1/2)/b^3+a^2/b^3*(-x^(1/2)/(b*x-a)-5/(a*b)^(1/2)*arctanh(
b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.29

$$\int \frac{x^{5/2}}{(-a+bx)^2}\,dx = \left[\frac{15\,(abx-a^2)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,b^2x^2+10\,abx-15\,a^2)\sqrt{x}}{6\,(b^4x-ab^3)},\frac{15\,(abx-a^2}{}\right.$$

input
```
integrate(x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

output
```
[1/6*(15*(a*b*x - a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*
x - a)) + 2*(2*b^2*x^2 + 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3), 1/3*
(15*(a*b*x - a^2)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b^2*x^2 +
 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(68) = 136$.

Time = 7.22 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.85

$$\int \frac{x^{5/2}}{(-a+bx)^2}\,dx = \begin{cases} \tilde{\infty} x^{\frac{3}{2}} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ \frac{2x^{\frac{3}{2}}}{3b^2} \\ -\frac{15a^3\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} + \frac{15a^3\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} - \frac{30a^2b\sqrt{x}\sqrt{\frac{a}{b}}}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} + \frac{15a^2bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} - \frac{15a^2bx}{-6ab^4} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x-a)**2,x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
 0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (-15*a**3*log(sqrt(x) - sqrt(a/b))/
(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 15*a**3*log(sqrt(x) + sqrt(a/
b))/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) - 30*a**2*b*sqrt(x)*sqrt(a/
b)/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 15*a**2*b*x*log(sqrt(x) -
sqrt(a/b))/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) - 15*a**2*b*x*log(sq
rt(x) + sqrt(a/b))/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 20*a*b**2*
x**(3/2)*sqrt(a/b)/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 4*b**3*x**
(5/2)*sqrt(a/b)/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{x^{5/2}}{(-a+bx)^2}\,dx = -\frac{a^2\sqrt{x}}{b^4x-ab^3}+\frac{5\,a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}b^3}+\frac{2\left(bx^{\frac{3}{2}}+6\,a\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

output
```
-a^2*sqrt(x)/(b^4*x - a*b^3) + 5/2*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt
(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2/3*(b*x^(3/2) + 6*a*sqrt(x))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{x^{5/2}}{(-a+bx)^2}\,dx = \frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^3}-\frac{a^2\sqrt{x}}{(bx-a)b^3}+\frac{2\left(b^4x^{\frac{3}{2}}+6\,ab^3\sqrt{x}\right)}{3\,b^6}$$

input
```
integrate(x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

output
```
5*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) - a^2*sqrt(x)/((b*x -
a)*b^3) + 2/3*(b^4*x^(3/2) + 6*a*b^3*sqrt(x))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}}{(-a+bx)^2}\,dx = \frac{2\,x^{3/2}}{3\,b^2}+\frac{4\,a\,\sqrt{x}}{b^3}+\frac{a^2\,\sqrt{x}}{a\,b^3-b^4\,x}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{b^{7/2}}$$

input
```
int(x^(5/2)/(a - b*x)^2,x)
```

output
```
(2*x^(3/2))/(3*b^2) + (4*a*x^(1/2))/b^3 + (a^2*x^(1/2))/(a*b^3 - b^4*x) +
(a^(3/2)*atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*5i)/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.68

$$\int \frac{x^{5/2}}{(-a+bx)^2}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2-15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)abx-15\sqrt{b}\,\sqrt{a}\,\mathrm{l}}{6}$$

input
```
int(x^(5/2)/(b*x-a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 - 15*sqrt(b)*
sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x - 15*sqrt(b)*sqrt(a)*log
(sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 + 15*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a
) + sqrt(x)*b)*a*b*x + 30*sqrt(x)*a**2*b - 20*sqrt(x)*a*b**2*x - 4*sqrt(x)
*b**3*x**2)/(6*b**4*(a - b*x))
```

3.281 $\int \frac{x^{3/2}}{(-a+bx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{x^{3/2}}{(-a+bx)^2} dx = \frac{2\sqrt{x}}{b^2} + \frac{a\sqrt{x}}{b^2(a-bx)} - \frac{3\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
2*x^(1/2)/b^2+a*x^(1/2)/b^2/(-b*x+a)-3*a^(1/2)*arctanh(b^(1/2)*x^(1/2)/a^(
1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{x^{3/2}}{(-a+bx)^2} dx = \frac{\sqrt{x}(-3a+2bx)}{b^2(-a+bx)} - \frac{3\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(-a + b*x)^2,x]
```

output
```
(Sqrt[x]*(-3*a + 2*b*x))/(b^2*(-a + b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 25, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(bx-a)^2}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{3\int -\frac{\sqrt{x}}{a-bx}dx}{2b}+\frac{x^{3/2}}{b(a-bx)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x^{3/2}}{b(a-bx)}-\frac{3\int \frac{\sqrt{x}}{a-bx}dx}{2b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{x^{3/2}}{b(a-bx)}-\frac{3\left(\frac{a\int \frac{1}{\sqrt{x}(a-bx)}dx}{b}-\frac{2\sqrt{x}}{b}\right)}{2b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x^{3/2}}{b(a-bx)}-\frac{3\left(\frac{2a\int \frac{1}{a-bx}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b}\right)}{2b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{x^{3/2}}{b(a-bx)}-\frac{3\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b}\right)}{2b}$$

input
```
Int[x^(3/2)/(-a + b*x)^2,x]
```

output
```
x^(3/2)/(b*(a - b*x)) - (3*((-2*Sqrt[x])/b + (2*Sqrt[a]*ArcTanh[(Sqrt[b]*S
qrt[x])/Sqrt[a]])/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^2} - \frac{2a\left(-\frac{\sqrt{x}}{2(-bx+a)} + \frac{3\,\text{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	48
default	$\frac{2\sqrt{x}}{b^2} - \frac{2a\left(-\frac{\sqrt{x}}{2(-bx+a)} + \frac{3\,\text{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	48
risch	$\frac{2\sqrt{x}}{b^2} + \frac{a\left(-\frac{\sqrt{x}}{bx-a} - \frac{3\,\text{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^2}$	48

input
```
int(x^(3/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b^2-2*a/b^2*(-1/2*x^(1/2)/(-b*x+a)+3/2/(a*b)^(1/2)*arctanh(b*x^(
1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.38

$$\int \frac{x^{3/2}}{(-a+bx)^2}\,dx = \left[\frac{3\,(bx-a)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,bx-3\,a)\sqrt{x}}{2\,(b^3x-ab^2)}, \frac{3\,(bx-a)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\ldots}}{a\ldots}\right.}{b^3x-a\ldots}\right.$$

input
```
integrate(x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

output
```
[1/2*(3*(b*x - a)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a
)) + 2*(2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2), (3*(b*x - a)*sqrt(-a/b)*arc
tan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(53) = 106$.

Time = 2.80 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.19

$$\int \frac{x^{3/2}}{(-a+bx)^2}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ \frac{2\sqrt{x}}{b^2} \\ -\frac{3a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}} + \frac{3a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}} - \frac{6ab\sqrt{x}\sqrt{\frac{a}{b}}}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}} + \frac{3abx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}} - \frac{3abx\log}{-2ab^3\sqrt{}} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x-a)**2,x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b,
0)), (2*sqrt(x)/b**2, Eq(a, 0)), (-3*a**2*log(sqrt(x) - sqrt(a/b))/(-2*a*b
**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 3*a**2*log(sqrt(x) + sqrt(a/b))/(-2*
a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) - 6*a*b*sqrt(x)*sqrt(a/b)/(-2*a*b**
3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 3*a*b*x*log(sqrt(x) - sqrt(a/b))/(-2*a
*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) - 3*a*b*x*log(sqrt(x) + sqrt(a/b))/(
-2*a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 4*b**2*x**(3/2)*sqrt(a/b)/(-2*
a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{x^{3/2}}{(-a+bx)^2}\,dx = -\frac{a\sqrt{x}}{b^3x-ab^2} + \frac{3\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{2\,\sqrt{x}}{b^2}$$

input
```
integrate(x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

output
```
-a*sqrt(x)/(b^3*x - a*b^2) + 3/2*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x)
+ sqrt(a*b)))/(sqrt(a*b)*b^2) + 2*sqrt(x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{x^{3/2}}{(-a+bx)^2}\,dx = \frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b^2} - \frac{a\sqrt{x}}{(bx-a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

input
```
integrate(x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

output
```
3*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - a*sqrt(x)/((b*x - a)*b
^2) + 2*sqrt(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{(-a+bx)^2}\,dx = \frac{2\,\sqrt{x}}{b^2} + \frac{a\,\sqrt{x}}{a\,b^2-b^3\,x} - \frac{3\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
int(x^(3/2)/(a - b*x)^2,x)
```

output
```
(2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 - b^3*x) - (3*a^(1/2)*atanh((b^(1/2)*
x^(1/2))/a^(1/2)))/b^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int \frac{x^{3/2}}{(-a+bx)^2}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a - 3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)bx - 3\sqrt{b}\,\sqrt{a}\log\left(\sqrt{\right.}}{2b^3\,(-bx+a)}$$

input
```
int(x^(3/2)/(b*x-a)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a - 3*sqrt(b)*sqrt(
a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x - 3*sqrt(b)*sqrt(a)*log(sqrt(b)
*sqrt(a) + sqrt(x)*b)*a + 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*
b)*b*x + 6*sqrt(x)*a*b - 4*sqrt(x)*b**2*x)/(2*b**3*(a - b*x))
```

3.282 $\int \frac{\sqrt{x}}{(-a+bx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\sqrt{x}}{(-a+bx)^2} dx = \frac{\sqrt{x}}{b(a-bx)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
x^(1/2)/b/(-b*x+a)-arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{x}}{(-a+bx)^2} dx = -\frac{\sqrt{x}}{b(-a+bx)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(-a + b*x)^2,x]
```

output
```
-(Sqrt[x]/(b*(-a + b*x))) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^
(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {51, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(bx-a)^2} \, dx$$

$$\Big\downarrow \; 51$$

$$\frac{\int -\frac{1}{\sqrt{x}(a-bx)}dx}{2b}+\frac{\sqrt{x}}{b(a-bx)}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\int \frac{1}{\sqrt{x}(a-bx)}dx}{2b}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\int \frac{1}{a-bx}d\sqrt{x}}{b}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Int[Sqrt[x]/(-a + b*x)^2,x]
```

output
```
Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\sqrt{x}}{b(-bx+a)}-\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	38
default	$\frac{\sqrt{x}}{b(-bx+a)}-\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	38

input
```
int(x^(1/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)/b/(-b*x+a)-1/b/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx = \left[-\frac{2\,ab\sqrt{x}-\sqrt{ab}(bx-a)\log\left(\frac{bx+a-2\,\sqrt{ab}\sqrt{x}}{bx-a}\right)}{2\,(ab^3x-a^2b^2)},\right.$$
$$\left. -\frac{ab\sqrt{x}-\sqrt{-ab}(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab^3x-a^2b^2}\right]$$

input
```
integrate(x^(1/2)/(b*x-a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)))/(a*b^3*x - a^2*b^2), -(a*b*sqrt(x) - sqrt(-a*b)*(b*x - a)
*arctan(sqrt(-a*b)/(b*sqrt(x))))/(a*b^3*x - a^2*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(37) = 74.

Time = 1.42 (sec) , antiderivative size = 243, normalized size of antiderivative = 5.17

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for } b = \\ -\frac{2}{b^2\sqrt{x}} & \text{for } a = \\ -\frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} + \frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} & \text{otherw} \end{cases}$$

input
```
integrate(x**(1/2)/(b*x-a)**2,x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b,
0)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (-a*log(sqrt(x) - sqrt(a/b))/(-2*a*b**
2*sqrt(a/b) + 2*b**3*x*sqrt(a/b)) + a*log(sqrt(x) + sqrt(a/b))/(-2*a*b**2*
sqrt(a/b) + 2*b**3*x*sqrt(a/b)) - 2*b*sqrt(x)*sqrt(a/b)/(-2*a*b**2*sqrt(a/
b) + 2*b**3*x*sqrt(a/b)) + b*x*log(sqrt(x) - sqrt(a/b))/(-2*a*b**2*sqrt(a/
b) + 2*b**3*x*sqrt(a/b)) - b*x*log(sqrt(x) + sqrt(a/b))/(-2*a*b**2*sqrt(a/
b) + 2*b**3*x*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx = -\frac{\sqrt{x}}{b^2x-ab} + \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}b}$$

input
```
integrate(x^(1/2)/(b*x-a)^2,x, algorithm="maxima")
```

output
```
-sqrt(x)/(b^2*x - a*b) + 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt
(a*b)))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx = \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}b} - \frac{\sqrt{x}}{(bx-a)b}$$

input
```
integrate(x^(1/2)/(b*x-a)^2,x, algorithm="giac")
```

output
```
arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) - sqrt(x)/((b*x - a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx = \frac{\sqrt{x}}{b\,(a-b\,x)} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

input
```
int(x^(1/2)/(a - b*x)^2,x)
```

output
```
x^(1/2)/(b*(a - b*x)) - atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx \\ = \frac{\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a - \sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)bx - \sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a + \sqrt{b}}{2a\,b^2\,(-bx+a)}$$

input
```
int(x^(1/2)/(b*x-a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a - sqrt(b)*sqrt(a)*l
og( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(
a) + sqrt(x)*b)*a + sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x +
 2*sqrt(x)*a*b)/(2*a*b**2*(a - b*x))
```

3.283 $\int \frac{1}{\sqrt{x}(-a+bx)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1}{\sqrt{x}(-a+bx)^2} dx = \frac{\sqrt{x}}{a(a-bx)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

output
```
x^(1/2)/a/(-b*x+a)+arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(-a+bx)^2} dx = \frac{\sqrt{x}}{a(a-bx)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(-a + b*x)^2),x]
```

output
```
Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b
])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(bx-a)^2}dx$$

$$\Big\downarrow \; 52$$

$$\frac{\sqrt{x}}{a(a-bx)}-\frac{\int -\frac{1}{\sqrt{x}(a-bx)}dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{1}{\sqrt{x}(a-bx)}dx}{2a}+\frac{\sqrt{x}}{a(a-bx)}$$

$$\Big\downarrow \; 73$$

$$\frac{\int \frac{1}{a-bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a-bx)}$$

$$\Big\downarrow \; 221$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

input
```
Int[1/(Sqrt[x]*(-a + b*x)^2),x]
```

output
```
Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{x}}{a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37
default	$\frac{\sqrt{x}}{a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37

input
```
int(1/x^(1/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)/a/(-b*x+a)+1/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = \left[-\frac{2\,ab\sqrt{x}-\sqrt{ab}(bx-a)\log\left(\frac{bx+a+2\,\sqrt{ab}\sqrt{x}}{bx-a}\right)}{2\,(a^2b^2x-a^3b)},\ -\frac{ab\sqrt{x}+\sqrt{-ab}(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{a^2b^2x-a^3b}\right]$$

input
```
integrate(1/x^(1/2)/(b*x-a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a + 2*sqrt(a*b)*sqrt
(x))/(b*x - a)))/(a^2*b^2*x - a^3*b), -(a*b*sqrt(x) + sqrt(-a*b)*(b*x - a)
*arctan(sqrt(-a*b)/(b*sqrt(x))))/(a^2*b^2*x - a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(37) = 74$.

Time = 2.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 5.48

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } \\ \frac{2\sqrt{x}}{a^2} & \text{for } \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for } \\ \frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} & \text{othe} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x-a)**2,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
 (-2/(3*b**2*x**(3/2)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(a/b))/(-2*a**2*b*
sqrt(a/b) + 2*a*b**2*x*sqrt(a/b)) - a*log(sqrt(x) + sqrt(a/b))/(-2*a**2*b*
sqrt(a/b) + 2*a*b**2*x*sqrt(a/b)) - 2*b*sqrt(x)*sqrt(a/b)/(-2*a**2*b*sqrt(
a/b) + 2*a*b**2*x*sqrt(a/b)) - b*x*log(sqrt(x) - sqrt(a/b))/(-2*a**2*b*sqr
t(a/b) + 2*a*b**2*x*sqrt(a/b)) + b*x*log(sqrt(x) + sqrt(a/b))/(-2*a**2*b*s
qrt(a/b) + 2*a*b**2*x*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = -\frac{\sqrt{x}}{abx-a^2} - \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate(1/x^(1/2)/(b*x-a)^2,x, algorithm="maxima")
```

output
```
-sqrt(x)/(a*b*x - a^2) - 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt
(a*b)))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = -\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a} - \frac{\sqrt{x}}{(bx-a)a}$$

input
```
integrate(1/x^(1/2)/(b*x-a)^2,x, algorithm="giac")
```

output
```
-arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) - sqrt(x)/((b*x - a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = \frac{\sqrt{x}}{a\,(a-b\,x)} + \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

input
```
int(1/(x^(1/2)*(a - b*x)^2),x)
```

output
```
x^(1/2)/(a*(a - b*x)) + atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a+\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)bx+\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a-}{2a^2b\,(-bx+a)}$$

input
```
int(1/x^(1/2)/(b*x-a)^2,x)
```

output
```
( - sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a + sqrt(b)*sqrt(a
)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x + sqrt(b)*sqrt(a)*log(sqrt(b)*sq
rt(a) + sqrt(x)*b)*a - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b*
x + 2*sqrt(x)*a*b)/(2*a**2*b*(a - b*x))
```

3.284 $\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}(a-bx)} + \frac{3\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-3/a^2/x^(1/2)+1/a/x^(1/2)/(-b*x+a)+3*b^(1/2)*arctanh(b^(1/2)*x^(1/2)/a^(1
/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = \frac{-2a+3bx}{a^2\sqrt{x}(a-bx)} + \frac{3\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^(3/2)*(-a + b*x)^2),x]
```

output
```
(-2*a + 3*b*x)/(a^2*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {52, 25, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(bx-a)^2}dx$$

$$\Big\downarrow \ 52$$

$$\frac{1}{a\sqrt{x}(a-bx)}-\frac{3\int -\frac{1}{x^{3/2}(a-bx)}dx}{2a}$$

$$\Big\downarrow \ 25$$

$$\frac{3\int \frac{1}{x^{3/2}(a-bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a-bx)}$$

$$\Big\downarrow \ 61$$

$$\frac{3\left(\frac{b\int \frac{1}{\sqrt{x}(a-bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a-bx)}$$

$$\Big\downarrow \ 73$$

$$\frac{3\left(\frac{2b\int \frac{1}{a-bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a-bx)}$$

$$\Big\downarrow \ 221$$

$$\frac{3\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a-bx)}$$

input
```
Int[1/(x^(3/2)*(-a + b*x)^2),x]
```

output
```
1/(a*Sqrt[x]*(a - b*x)) + (3*(-2/(a*Sqrt[x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2b\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{3\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	48
default	$\frac{2b\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{3\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	48
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{b\left(\frac{\sqrt{x}}{bx-a}-\frac{3\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^{2}}$	48

input
```
int(1/x^(3/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*b/a^2*(1/2*x^(1/2)/(-b*x+a)+3/2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2
)))-2/a^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = \left[\frac{3\,(bx^2-ax)\sqrt{\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(3\,bx-2\,a)\sqrt{x}}{2\,(a^2bx^2-a^3x)},\right.$$
$$\left.-\frac{3\,(bx^2-ax)\sqrt{-\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{-\frac{b}{a}}\right)+(3\,bx-2\,a)\sqrt{x}}{a^2bx^2-a^3x}\right]$$

input
```
integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

output
```
[1/2*(3*(b*x^2 - a*x)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x
 - a)) - 2*(3*b*x - 2*a)*sqrt(x))/(a^2*b*x^2 - a^3*x), -(3*(b*x^2 - a*x)*s
qrt(-b/a)*arctan(sqrt(x)*sqrt(-b/a)) + (3*b*x - 2*a)*sqrt(x))/(a^2*b*x^2 -
 a^3*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(51) = 102$.

Time = 5.09 (sec) , antiderivative size = 354, normalized size of antiderivative = 6.21

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ -\frac{3a\sqrt{x}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}} + \frac{3a\sqrt{x}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}} - \frac{4a\sqrt{\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}} + \frac{3bx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{\right.}}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx} \end{cases}$$

input
```
integrate(1/x**(3/2)/(b*x-a)**2,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0
)), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (-3*a*sqrt(x)*log(sqrt(x) - sqrt(a/b
))/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)) + 3*a*sqrt(x)*
log(sqrt(x) + sqrt(a/b))/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqr
t(a/b)) - 4*a*sqrt(a/b)/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt
(a/b)) + 3*b*x**(3/2)*log(sqrt(x) - sqrt(a/b))/(2*a**3*sqrt(x)*sqrt(a/b) -
 2*a**2*b*x**(3/2)*sqrt(a/b)) - 3*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(2*a
**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)) + 6*b*x*sqrt(a/b)/(2*
a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = -\frac{3\,bx-2\,a}{a^2bx^{\frac{3}{2}}-a^3\sqrt{x}} - \frac{3\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

input
```
integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

output
```
-(3*b*x - 2*a)/(a^2*b*x^(3/2) - a^3*sqrt(x)) - 3/2*b*log((b*sqrt(x) - sqrt
(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = -\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^2} - \frac{3\,bx-2\,a}{\left(bx^{\frac{3}{2}}-a\sqrt{x}\right)a^2}$$

input
```
integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

output
```
-3*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) - (3*b*x - 2*a)/((b*x^(
3/2) - a*sqrt(x))*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = \frac{3\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2}{a}-\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}-b\,x^{3/2}}$$

input
```
int(1/(x^(3/2)*(a - b*x)^2),x)
```

output
```
(3*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/a - (3*b*x)/a^2)
/(a*x^(1/2) - b*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = \frac{-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a+3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)bx+3}{2\sqrt{x}\,a^3}$$

input
```
int(1/x^(3/2)/(b*x-a)^2,x)
```

output
```
( - 3*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a + 3*sq
rt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x + 3*sqrt(x)*
sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a - 3*sqrt(x)*sqrt(b)*sqr
t(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b*x - 4*a**2 + 6*a*b*x)/(2*sqrt(x)*a
**3*(a - b*x))
```

3.285 $\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} + \frac{5b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-5/3/a^2/x^(3/2)-5*b/a^3/x^(1/2)+1/a/x^(3/2)/(-b*x+a)+5*b^(3/2)*arctanh(b^
(1/2)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = \frac{-2a^2-10abx+15b^2x^2}{3a^3x^{3/2}(a-bx)} + \frac{5b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[1/(x^(5/2)*(-a + b*x)^2),x]
```

output
```
(-2*a^2 - 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a - b*x)) + (5*b^(3/2)*Ar
cTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {52, 25, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(bx-a)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{ax^{3/2}(a-bx)}-\frac{5\int -\frac{1}{x^{5/2}(a-bx)}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\int \frac{1}{x^{5/2}(a-bx)}dx}{2a}+\frac{1}{ax^{3/2}(a-bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(\frac{b\int \frac{1}{x^{3/2}(a-bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}(a-bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(\frac{b\left(\frac{2b\int \frac{1}{a-bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{5\left(\frac{b\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}$$

input
```
Int[1/(x^(5/2)*(-a + b*x)^2),x]
```

output
```
1/(a*x^(3/2)*(a - b*x)) + (5*(-2/(3*a*x^(3/2)) + (b*(-2/(a*Sqrt[x]) + (2*S
qrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{2(6bx+a)}{3a^3x^{\frac{3}{2}}}-\frac{b^2\left(\frac{\sqrt{x}}{bx-a}-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^3}$	56
derivativedivides	$-\frac{2}{3a^2x^{\frac{3}{2}}}-\frac{4b}{a^3\sqrt{x}}+\frac{2b^2\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	59
default	$-\frac{2}{3a^2x^{\frac{3}{2}}}-\frac{4b}{a^3\sqrt{x}}+\frac{2b^2\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	59

input
```
int(1/x^(5/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(6*b*x+a)/a^3/x^(3/2)-1/a^3*b^2*(x^(1/2)/(b*x-a)-5/(a*b)^(1/2)*arctan
h(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.61

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = \left[\frac{15\,(b^2x^3-abx^2)\sqrt{\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(15\,b^2x^2-10\,abx-2\,a^2)\sqrt{x}}{6\,(a^3bx^3-a^4x^2)},\right.$$
$$\left.-\frac{15\,(b^2x^3-abx^2)\sqrt{-\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{-\frac{b}{a}}\right)+(15\,b^2x^2-10\,abx-2\,a^2)\sqrt{x}}{3\,(a^3bx^3-a^4x^2)}\right]$$

input
```
integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

output
```
[1/6*(15*(b^2*x^3 - a*b*x^2)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) +
a)/(b*x - a)) - 2*(15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^
4*x^2), -1/3*(15*(b^2*x^3 - a*b*x^2)*sqrt(-b/a)*arctan(sqrt(x)*sqrt(-b/a))
 + (15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(65) = 130$.

Time = 14.82 (sec) , antiderivative size = 416, normalized size of antiderivative = 5.94

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{2}{3a^2x^{\frac{3}{2}}} \\ -\frac{2}{7b^2x^{\frac{7}{2}}} \\ -\frac{4a^2\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{20abx\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx}\end{cases}$$

input
```
integrate(1/x**(5/2)/(b*x-a)**2,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**2*x**(3/2)), Eq(b
, 0)), (-2/(7*b**2*x**(7/2)), Eq(a, 0)), (-4*a**2*sqrt(a/b)/(6*a**4*x**(3/
2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) - 15*a*b*x**(3/2)*log(sqrt(x)
- sqrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) + 1
5*a*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**
3*b*x**(5/2)*sqrt(a/b)) - 20*a*b*x*sqrt(a/b)/(6*a**4*x**(3/2)*sqrt(a/b) -
6*a**3*b*x**(5/2)*sqrt(a/b)) + 15*b**2*x**(5/2)*log(sqrt(x) - sqrt(a/b))/(
6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) - 15*b**2*x**(5/2
)*log(sqrt(x) + sqrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*
sqrt(a/b)) + 30*b**2*x**2*sqrt(a/b)/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*
x**(5/2)*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = -\frac{15\,b^2x^2-10\,abx-2\,a^2}{3\left(a^3bx^{\frac{5}{2}}-a^4x^{\frac{3}{2}}\right)}-\frac{5\,b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}$$

input
```
integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

output
```
-1/3*(15*b^2*x^2 - 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) - a^4*x^(3/2)) - 5/2*b
^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = -\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}a^3}-\frac{b^2\sqrt{x}}{(bx-a)a^3}-\frac{2\,(6\,bx+a)}{3\,a^3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

output
```
-5*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) - b^2*sqrt(x)/((b*x -
 a)*a^3) - 2/3*(6*b*x + a)/(a^3*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = \frac{5\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2}{3\,a}-\frac{5\,b^2\,x^2}{a^3}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}-b\,x^{5/2}}$$

input
```
int(1/(x^(5/2)*(a - b*x)^2),x)
```

output
```
(5*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2) - (2/(3*a) - (5*b^2*x
^2)/a^3 + (10*b*x)/(3*a^2))/(a*x^(3/2) - b*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = \frac{-15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)abx+15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)b^2}{}$$

input
```
int(1/x^(5/2)/(b*x-a)^2,x)
```

output
```
( - 15*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x +
 15*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2
+ 15*sqrt(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x - 15*s
qrt(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 - 4*a**3
 - 20*a**2*b*x + 30*a*b**2*x**2)/(6*sqrt(x)*a**4*x*(a - b*x))
```

3.286 $\int \frac{x^{7/2}}{(-a+bx)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{x^{7/2}}{(-a+bx)^3} \, dx = \frac{6a\sqrt{x}}{b^4} + \frac{2x^{3/2}}{3b^3} - \frac{a^3\sqrt{x}}{2b^4(a-bx)^2} + \frac{13a^2\sqrt{x}}{4b^4(a-bx)} - \frac{35a^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

output
```
6*a*x^(1/2)/b^4+2/3*x^(3/2)/b^3-1/2*a^3*x^(1/2)/b^4/(-b*x+a)^2+13/4*a^2*x^
(1/2)/b^4/(-b*x+a)-35/4*a^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \frac{x^{7/2}}{(-a+bx)^3} \, dx = \frac{\sqrt{x}(105a^3 - 175a^2bx + 56ab^2x^2 + 8b^3x^3)}{12b^4(a-bx)^2} - \frac{35a^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

input
```
Integrate[x^(7/2)/(-a + b*x)^3,x]
```

output
```
(Sqrt[x]*(105*a^3 - 175*a^2*b*x + 56*a*b^2*x^2 + 8*b^3*x^3))/(12*b^4*(a -
b*x)^2) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(bx-a)^3} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{7\int \frac{x^{5/2}}{(a-bx)^2}dx}{4b}-\frac{x^{7/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{7\left(\frac{x^{5/2}}{b(a-bx)}-\frac{5\int \frac{x^{3/2}}{a-bx}dx}{2b}\right)}{4b}-\frac{x^{7/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{x^{5/2}}{b(a-bx)}-\frac{5\left(\frac{a\int \frac{\sqrt{x}}{a-bx}dx}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}\right)}{4b}-\frac{x^{7/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{x^{5/2}}{b(a-bx)}-\frac{5\left(\frac{a\left(\frac{a\int \frac{1}{\sqrt{x}(a-bx)}dx}{b}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}\right)}{4b}-\frac{x^{7/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{7\left(\frac{x^{5/2}}{b(a-bx)}-\frac{5\left(\frac{a\left(\frac{2a\int\frac{1}{a-bx}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}\right)}{4b}-\frac{x^{7/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{7\left(\frac{x^{5/2}}{b(a-bx)}-\frac{5\left(\frac{a\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{3b}\right)}{2b}\right)}{4b}-\frac{x^{7/2}}{2b(a-bx)^2}$$

input
```
Int[x^(7/2)/(-a + b*x)^3,x]
```

output
```
-1/2*x^(7/2)/(b*(a - b*x)^2) + (7*(x^(5/2)/(b*(a - b*x)) - (5*((-2*x^(3/2)
)/(3*b) + (a*((-2*Sqrt[x])/b + (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
]])/b^(3/2)))/b))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{2(bx+9a)\sqrt{x}}{3b^4}+\frac{a^2\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{4}+\frac{11a\sqrt{x}}{4}}{(bx-a)^2}-\frac{35\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^4}$	67
derivativedivides	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^4}-\frac{2a^2\left(\frac{\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	69
default	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^4}-\frac{2a^2\left(\frac{\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	69

input
```
int(x^(7/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+9*a)*x^(1/2)/b^4+1/b^4*a^2*(2*(-13/8*b*x^(3/2)+11/8*a*x^(1/2))/(b
*x-a)^2-35/4/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.25

$$\int \frac{x^{7/2}}{(-a+bx)^3}\,dx = \left[\frac{105\,(ab^2x^2-2\,a^2bx+a^3)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(8\,b^3x^3+56\,ab^2x^2-175\,a^2bx}{24\,(b^6x^2-2\,ab^5x+a^2b^4)}\right.$$

input
```
integrate(x^(7/2)/(b*x-a)^3,x, algorithm="fricas")
```

output
```
[1/24*(105*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*
sqrt(a/b) + a)/(b*x - a)) + 2*(8*b^3*x^3 + 56*a*b^2*x^2 - 175*a^2*b*x + 10
5*a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 - 2*
a^2*b*x + a^3)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (8*b^3*x^3 + 56
*a*b^2*x^2 - 175*a^2*b*x + 105*a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^
4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(94) = 188.

Time = 39.15 (sec) , antiderivative size = 695, normalized size of antiderivative = 6.88

$$\int \frac{x^{7/2}}{(-a+bx)^3}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} \\ -\frac{2x^{\frac{9}{2}}}{9a^3} \\ \frac{2x^{\frac{3}{2}}}{3b^3} \\ \frac{105a^4\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{105a^4\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{210a^3b\sqrt{x}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x} \end{cases}$$

input
```
integrate(x**(7/2)/(b*x-a)**3,x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(9/2)/(9*a**3), Eq(b
, 0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (105*a**4*log(sqrt(x) - sqrt(a/b))
/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b))
 - 105*a**4*log(sqrt(x) + sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x
*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 210*a**3*b*sqrt(x)*sqrt(a/b)/(24*a*
*2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) - 210*
a**3*b*x*log(sqrt(x) - sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sq
rt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 210*a**3*b*x*log(sqrt(x) + sqrt(a/b))/
(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b))
- 350*a**2*b**2*x**(3/2)*sqrt(a/b)/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*s
qrt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 105*a**2*b**2*x**2*log(sqrt(x) - sqrt
(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt
(a/b)) - 105*a**2*b**2*x**2*log(sqrt(x) + sqrt(a/b))/(24*a**2*b**5*sqrt(a/
b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 112*a*b**3*x**(5/2)
*sqrt(a/b)/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*
sqrt(a/b)) + 16*b**4*x**(7/2)*sqrt(a/b)/(24*a**2*b**5*sqrt(a/b) - 48*a*b**
6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int \frac{x^{7/2}}{(-a+bx)^3}\,dx = -\frac{13\,a^2bx^{\frac{3}{2}}-11\,a^3\sqrt{x}}{4\,(b^6x^2-2\,ab^5x+a^2b^4)}+\frac{35\,a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4}+\frac{2\left(bx^{\frac{3}{2}}+9\,a\sqrt{x}\right)}{3\,b^4}$$

input
```
integrate(x^(7/2)/(b*x-a)^3,x, algorithm="maxima")
```

output
```
-1/4*(13*a^2*b*x^(3/2) - 11*a^3*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4) +
 35/8*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*
b^4) + 2/3*(b*x^(3/2) + 9*a*sqrt(x))/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{x^{7/2}}{(-a+bx)^3}\,dx = \frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}b^4} - \frac{13\,a^2bx^{\frac{3}{2}} - 11\,a^3\sqrt{x}}{4\,(bx-a)^2b^4} + \frac{2\left(b^6x^{\frac{3}{2}} + 9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

input
```
integrate(x^(7/2)/(b*x-a)^3,x, algorithm="giac")
```

output
```
35/4*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^4) - 1/4*(13*a^2*b*x^(
3/2) - 11*a^3*sqrt(x))/((b*x - a)^2*b^4) + 2/3*(b^6*x^(3/2) + 9*a*b^5*sqrt
(x))/b^9
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{x^{7/2}}{(-a+bx)^3}\,dx = \frac{\frac{11\,a^3\sqrt{x}}{4} - \frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4 - 2\,a\,b^5\,x + b^6\,x^2} + \frac{2\,x^{3/2}}{3\,b^3} + \frac{6\,a\,\sqrt{x}}{b^4} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,35\mathrm{i}}{4\,b^{9/2}}$$

input
```
int(-x^(7/2)/(a - b*x)^3,x)
```

output
```
((11*a^3*x^(1/2))/4 - (13*a^2*b*x^(3/2))/4)/(a^2*b^4 + b^6*x^2 - 2*a*b^5*x
) + (2*x^(3/2))/(3*b^3) + (6*a*x^(1/2))/b^4 + (a^(3/2)*atan((b^(1/2)*x^(1/
2)*1i)/a^(1/2))*35i)/(4*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.97

$$\int \frac{x^{7/2}}{(-a+bx)^3}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^3 - 210\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2bx + 105\sqrt{b}}{}$$

input
```
int(x^(7/2)/(b*x-a)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**3 - 210*sqrt(b
)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2*b*x + 105*sqrt(b)*sqrt(
a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b**2*x**2 - 105*sqrt(b)*sqrt(a)*l
og(sqrt(b)*sqrt(a) + sqrt(x)*b)*a**3 + 210*sqrt(b)*sqrt(a)*log(sqrt(b)*sqr
t(a) + sqrt(x)*b)*a**2*b*x - 105*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqr
t(x)*b)*a*b**2*x**2 + 210*sqrt(x)*a**3*b - 350*sqrt(x)*a**2*b**2*x + 112*s
qrt(x)*a*b**3*x**2 + 16*sqrt(x)*b**4*x**3)/(24*b**5*(a**2 - 2*a*b*x + b**2
*x**2))
```

3.287 $\int \frac{x^{5/2}}{(-a+bx)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{x^{5/2}}{(-a+bx)^3} dx = \frac{2\sqrt{x}}{b^3} - \frac{a^2\sqrt{x}}{2b^3(a-bx)^2} + \frac{9a\sqrt{x}}{4b^3(a-bx)} - \frac{15\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

output
```
2*x^(1/2)/b^3-1/2*a^2*x^(1/2)/b^3/(-b*x+a)^2+9/4*a*x^(1/2)/b^3/(-b*x+a)-15
/4*a^(1/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}}{(-a+bx)^3} dx = \frac{\sqrt{x}(15a^2-25abx+8b^2x^2)}{4b^3(a-bx)^2} - \frac{15\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

input
```
Integrate[x^(5/2)/(-a + b*x)^3,x]
```

output
```
(Sqrt[x]*(15*a^2 - 25*a*b*x + 8*b^2*x^2))/(4*b^3*(a - b*x)^2) - (15*Sqrt[a
]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(bx-a)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5\int \frac{x^{3/2}}{(a-bx)^2}dx}{4b}-\frac{x^{5/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{5\left(\frac{x^{3/2}}{b(a-bx)}-\frac{3\int \frac{\sqrt{x}}{a-bx}dx}{2b}\right)}{4b}-\frac{x^{5/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5\left(\frac{x^{3/2}}{b(a-bx)}-\frac{3\left(\frac{a\int \frac{1}{\sqrt{x}(a-bx)}dx}{b}-\frac{2\sqrt{x}}{b}\right)}{2b}\right)}{4b}-\frac{x^{5/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(\frac{x^{3/2}}{b(a-bx)}-\frac{3\left(\frac{2a\int \frac{1}{a-bx}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b}\right)}{2b}\right)}{4b}-\frac{x^{5/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{5\left(\frac{x^{3/2}}{b(a-bx)}-\frac{3\left(\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b}\right)}{2b}\right)}{4b}-\frac{x^{5/2}}{2b(a-bx)^2}$$

input
```
Int[x^(5/2)/(-a + b*x)^3,x]
```

output
```
-1/2*x^(5/2)/(b*(a - b*x)^2) + (5*(x^(3/2)/(b*(a - b*x)) - (3*((-2*Sqrt[x]
)/b + (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*
b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	57
default	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	57
risch	$\frac{2\sqrt{x}}{b^3}+\frac{a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{4}+\frac{7a\sqrt{x}}{4}}{(bx-a)^2}-\frac{15\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^3}$	58

input
```
int(x^(5/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b^3-2/b^3*a*((9/8*b*x^(3/2)-7/8*a*x^(1/2))/(-b*x+a)^2+15/8/(a*b)
^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.31

$$\int \frac{x^{5/2}}{(-a+bx)^3}\,dx = \left[\frac{15\,(b^2x^2-2\,abx+a^2)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(8\,b^2x^2-25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2-2\,ab^4x+a^2b^3)}, \frac{15\,(\right.$$

input
```
integrate(x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

output
```
[1/8*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a
/b) + a)/(b*x - a)) + 2*(8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
- 2*a*b^4*x + a^2*b^3), 1/4*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a/b)*arcta
n(b*sqrt(x)*sqrt(-a/b)/a) + (8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*
x^2 - 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(78) = 156$.

Time = 16.35 (sec) , antiderivative size = 624, normalized size of antiderivative = 7.26

$$\int \frac{x^{5/2}}{(-a+bx)^3}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ -\frac{2x^{\frac{7}{2}}}{7a^3} \\ \frac{2\sqrt{x}}{b^3} \\ \frac{15a^3\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{15a^3\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{30a^2b\sqrt{x}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x-a)**3,x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(7/2)/(7*a**3), Eq(b,
 0)), (2*sqrt(x)/b**3, Eq(a, 0)), (15*a**3*log(sqrt(x) - sqrt(a/b))/(8*a**
2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 15*a**
3*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b)
+ 8*b**6*x**2*sqrt(a/b)) + 30*a**2*b*sqrt(x)*sqrt(a/b)/(8*a**2*b**4*sqrt(a
/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 30*a**2*b*x*log(sqr
t(x) - sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*
x**2*sqrt(a/b)) + 30*a**2*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**4*sqrt(a
/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 50*a*b**2*x**(3/2)*
sqrt(a/b)/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqr
t(a/b)) + 15*a*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) -
 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 15*a*b**2*x**2*log(sqrt(
x) + sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x*
*2*sqrt(a/b)) + 16*b**3*x**(5/2)*sqrt(a/b)/(8*a**2*b**4*sqrt(a/b) - 16*a*b
**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{x^{5/2}}{(-a+bx)^3}\,dx = -\frac{9\,abx^{\frac{3}{2}}-7\,a^2\sqrt{x}}{4\,(b^5x^2-2\,ab^4x+a^2b^3)}+\frac{15\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}+\frac{2\,\sqrt{x}}{b^3}$$

input
```
integrate(x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

output
```
-1/4*(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/(b^5*x^2 - 2*a*b^4*x + a^2*b^3) + 15/
8*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) +
 2*sqrt(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{(-a+bx)^3}\,dx = \frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}b^3}+\frac{2\,\sqrt{x}}{b^3}-\frac{9\,abx^{\frac{3}{2}}-7\,a^2\sqrt{x}}{4\,(bx-a)^2b^3}$$

input
```
integrate(x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

output
```
15/4*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2*sqrt(x)/b^3 - 1/4
*(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/((b*x - a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{x^{5/2}}{(-a+bx)^3}\,dx = \frac{\frac{7\,a^2\,\sqrt{x}}{4}-\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3-2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

input
```
int(-x^(5/2)/(a - b*x)^3,x)
```

output
```
((7*a^2*x^(1/2))/4 - (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 - 2*a*b^4*x) +
(2*x^(1/2))/b^3 - (15*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.09

$$\int \frac{x^{5/2}}{(-a+bx)^3}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2-30\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)abx+15\sqrt{b}\,\sqrt{a}\,\mathrm{l}}{}$$

input
```
int(x^(5/2)/(b*x-a)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 - 30*sqrt(b)*
sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x + 15*sqrt(b)*sqrt(a)*log
( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 - 15*sqrt(b)*sqrt(a)*log(sqrt(b
)*sqrt(a) + sqrt(x)*b)*a**2 + 30*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqr
t(x)*b)*a*b*x - 15*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x
**2 + 30*sqrt(x)*a**2*b - 50*sqrt(x)*a*b**2*x + 16*sqrt(x)*b**3*x**2)/(8*b
**4*(a**2 - 2*a*b*x + b**2*x**2))
```

3.288 $\int \frac{x^{3/2}}{(-a+bx)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = -\frac{a\sqrt{x}}{2b^2(a-bx)^2}+\frac{5\sqrt{x}}{4b^2(a-bx)}-\frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

output
```
-1/2*a*x^(1/2)/b^2/(-b*x+a)^2+5/4*x^(1/2)/b^2/(-b*x+a)-3/4*arctanh(b^(1/2)
*x^(1/2)/a^(1/2))/a^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = \frac{\sqrt{x}(3a-5bx)}{4b^2(a-bx)^2}-\frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{5/2}}$$

input
```
Integrate[x^(3/2)/(-a + b*x)^3,x]
```

output
```
(Sqrt[x]*(3*a - 5*b*x))/(4*b^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])
/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(bx-a)^3} \, dx$$

$$\Big\downarrow \ 51$$

$$\frac{3 \int \frac{\sqrt{x}}{(a-bx)^2}dx}{4b}-\frac{x^{3/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \ 51$$

$$\frac{3 \left(\frac{\sqrt{x}}{b(a-bx)}-\frac{\int \frac{1}{\sqrt{x}(a-bx)}dx}{2b}\right)}{4b}-\frac{x^{3/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{3 \left(\frac{\sqrt{x}}{b(a-bx)}-\frac{\int \frac{1}{a-bx}d\sqrt{x}}{b}\right)}{4b}-\frac{x^{3/2}}{2b(a-bx)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{3 \left(\frac{\sqrt{x}}{b(a-bx)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}\right)}{4b}-\frac{x^{3/2}}{2b(a-bx)^2}$$

input
```
Int[x^(3/2)/(-a + b*x)^3,x]
```

output
```
-1/2*x^(3/2)/(b*(a - b*x)^2) + (3*(Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b
]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{2\left(\frac{5x^{\frac{3}{2}}}{8b}-\frac{3a\sqrt{x}}{8b^{2}}\right)}{(-bx+a)^{2}}-\frac{3\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}\sqrt{ab}}$	51
default	$-\frac{2\left(\frac{5x^{\frac{3}{2}}}{8b}-\frac{3a\sqrt{x}}{8b^{2}}\right)}{(-bx+a)^{2}}-\frac{3\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}\sqrt{ab}}$	51

input
```
int(x^(3/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(5/8*x^(3/2)/b-3/8*a*x^(1/2)/b^2)/(-b*x+a)^2-3/4/b^2/(a*b)^(1/2)*arctan
h(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.55

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = \left[\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{ab}\log\left(\frac{bx+a-2\,\sqrt{ab}\sqrt{x}}{bx-a}\right)-2\,(5\,ab^2x-3\,a^2b)\sqrt{x}}{8\,(ab^5x^2-2\,a^2b^4x+a^3b^3)}, \frac{3\,(b^2x^2-2\,abx\right.$$

input
```
integrate(x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqr
t(x))/(b*x - a)) - 2*(5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4
*x + a^3*b^3), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*
b)/(b*sqrt(x))) - (5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x
+ a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(65) = 130$.

Time = 9.19 (sec) , antiderivative size = 552, normalized size of antiderivative = 7.56

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ -\frac{2x^{\frac{5}{2}}}{5a^3} \\ -\frac{2}{b^3\sqrt{x}} \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}} + \frac{6ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x-a)**3,x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(5/2)/(5*a**3), Eq(b,
 0)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(a/b))/(8*a
**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 3*a*
*2*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b)
 + 8*b**5*x**2*sqrt(a/b)) + 6*a*b*sqrt(x)*sqrt(a/b)/(8*a**2*b**3*sqrt(a/b)
 - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 6*a*b*x*log(sqrt(x) -
sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sq
rt(a/b)) + 6*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*
b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 10*b**2*x**(3/2)*sqrt(a/b)/(8*
a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) + 3*b
**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqr
t(a/b) + 8*b**5*x**2*sqrt(a/b)) - 3*b**2*x**2*log(sqrt(x) + sqrt(a/b))/(8*
a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = -\frac{5\,bx^{\frac{3}{2}} - 3\,a\sqrt{x}}{4\,(b^4x^2 - 2\,ab^3x + a^2b^2)} + \frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2}$$

input
```
integrate(x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

output
```
-1/4*(5*b*x^(3/2) - 3*a*sqrt(x))/(b^4*x^2 - 2*a*b^3*x + a^2*b^2) + 3/8*log
((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}b^2} - \frac{5\,bx^{\frac{3}{2}} - 3\,a\sqrt{x}}{4\,(bx-a)^2b^2}$$

input
```
integrate(x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

output
```
3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - 1/4*(5*b*x^(3/2) - 3*a
*sqrt(x))/((b*x - a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = -\frac{\frac{5\,x^{3/2}}{4\,b}-\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}$$

input
```
int(-x^(3/2)/(a - b*x)^3,x)
```

output
```
- ((5*x^(3/2))/(4*b) - (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 - 2*a*b*x) -
(3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.37

$$\int \frac{x^{3/2}}{(-a+bx)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2-6\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)abx+3\sqrt{b}\,\sqrt{a}\log\left(\right.}{}$$

input
```
int(x^(3/2)/(b*x-a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 - 6*sqrt(b)*sq
rt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x + 3*sqrt(b)*sqrt(a)*log( -
 sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 - 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sq
rt(a) + sqrt(x)*b)*a**2 + 6*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*
b)*a*b*x - 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 +
6*sqrt(x)*a**2*b - 10*sqrt(x)*a*b**2*x)/(8*a*b**3*(a**2 - 2*a*b*x + b**2*x
**2))
```

3.289 $\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

output
```
-1/2*x^(1/2)/b/(-b*x+a)^2+1/4*x^(1/2)/a/b/(-b*x+a)+1/4*arctanh(b^(1/2)*x^(
1/2)/a^(1/2))/a^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = -\frac{\sqrt{x}(a+bx)}{4ab(a-bx)^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(-a + b*x)^3,x]
```

output
```
-1/4*(Sqrt[x]*(a + b*x))/(a*b*(a - b*x)^2) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]]/(4*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(bx-a)^3}\,dx$$

$$\Big\downarrow \ 51$$

$$\frac{\int \frac{1}{\sqrt{x}(a-bx)^2}dx}{4b}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

$$\Big\downarrow \ 52$$

$$\frac{\frac{\int \frac{1}{\sqrt{x}(a-bx)}dx}{2a}+\frac{\sqrt{x}}{a(a-bx)}}{4b}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\int \frac{1}{a-bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a-bx)}}{4b}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}}{4b}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

input
```
Int[Sqrt[x]/(-a + b*x)^3,x]
```

output
```
-1/2*Sqrt[x]/(b*(a - b*x)^2) + (Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*S
qrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]))/(4*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{2\left(\frac{x^{\frac{3}{2}}}{8a}+\frac{\sqrt{x}}{8b}\right)}{\left(-bx+a\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ba\sqrt{ab}}$	53
default	$-\frac{2\left(\frac{x^{\frac{3}{2}}}{8a}+\frac{\sqrt{x}}{8b}\right)}{\left(-bx+a\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ba\sqrt{ab}}$	53

input
```
int(x^(1/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(1/8/a*x^(3/2)+1/8*x^(1/2)/b)/(-b*x+a)^2+1/4/b/a/(a*b)^(1/2)*arctanh(b*
x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = \left[\frac{(b^2x^2-2\,abx+a^2)\sqrt{ab}\log\left(\frac{bx+a+2\,\sqrt{ab}\sqrt{x}}{bx-a}\right)-2\,(ab^2x+a^2b)\sqrt{x}}{8\,(a^2b^4x^2-2\,a^3b^3x+a^4b^2)},\; -\frac{(b^2x^2-2\,abx+a^2)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+(ab^2x+a^2b)\sqrt{x}}{4\,(a^2b^4x^2-2\,a^3b^3x+a^4b^2)}\right]$$

input
```
integrate(x^(1/2)/(b*x-a)^3,x, algorithm="fricas")
```

output
```
[1/8*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a + 2*sqrt(a*b)*sqrt(
x))/(b*x - a)) - 2*(a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x +
 a^4*b^2), -1/4*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/(b
*sqrt(x))) + (a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a^4*b
^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(58) = 116.

Time = 4.89 (sec) , antiderivative size = 575, normalized size of antiderivative = 7.67

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ -\frac{2x^{\frac{3}{2}}}{3a^3} \\ -\frac{2}{3b^3x^{\frac{3}{2}}} \\ -\frac{a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}+\frac{a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}-\frac{2ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}+\frac{2ab}{8a^3b^2\sqrt{\frac{a}{b}}-} \end{cases}$$

input
```
integrate(x**(1/2)/(b*x-a)**3,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(3/2)/(3*a**3), Eq(b
, 0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (-a**2*log(sqrt(x) - sqrt(a/b))/(
8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)
) + a**2*log(sqrt(x) + sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*
sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) - 2*a*b*sqrt(x)*sqrt(a/b)/(8*a**3*b**
2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) + 2*a*b*
x*log(sqrt(x) - sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/
b) + 8*a*b**4*x**2*sqrt(a/b)) - 2*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**3*b
**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) - 2*b*
*2*x**(3/2)*sqrt(a/b)/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) +
8*a*b**4*x**2*sqrt(a/b)) - b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**3*b**2
*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) + b**2*x*
*2*log(sqrt(x) + sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a
/b) + 8*a*b**4*x**2*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = -\frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4\,(ab^3x^2-2\,a^2b^2x+a^3b)} - \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}ab}$$

input
```
integrate(x^(1/2)/(b*x-a)^3,x, algorithm="maxima")
```

output
```
-1/4*(b*x^(3/2) + a*sqrt(x))/(a*b^3*x^2 - 2*a^2*b^2*x + a^3*b) - 1/8*log((
b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = -\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}ab} - \frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4\,(bx-a)^2ab}$$

input
```
integrate(x^(1/2)/(b*x-a)^3,x, algorithm="giac")
```

output
```
-1/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a*b) - 1/4*(b*x^(3/2) + a*sq
rt(x))/((b*x - a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}} - \frac{\frac{x^{3/2}}{4\,a}+\frac{\sqrt{x}}{4\,b}}{a^2-2\,a\,b\,x+b^2\,x^2}$$

input
```
int(-x^(1/2)/(a - b*x)^3,x)
```

output
```
atanh((b^(1/2)*x^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2)) - (x^(3/2)/(4*a) + x^
(1/2)/(4*b))/(a^2 + b^2*x^2 - 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2+2\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)abx-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)}{}$$

input
```
int(x^(1/2)/(b*x-a)^3,x)
```

output
```
( - sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 + 2*sqrt(b)*s
qrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x - sqrt(b)*sqrt(a)*log( -
sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 + sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(
a) + sqrt(x)*b)*a**2 - 2*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*
a*b*x + sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 - 2*sqr
t(x)*a**2*b - 2*sqrt(x)*a*b**2*x)/(8*a**2*b**2*(a**2 - 2*a*b*x + b**2*x**2
))
```

3.290 $\int \frac{1}{\sqrt{x}(-a+bx)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{1}{\sqrt{x}(-a+bx)^3} dx = -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} - \frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

output
```
-1/2*x^(1/2)/a/(-b*x+a)^2-3/4*x^(1/2)/a^2/(-b*x+a)-3/4*arctanh(b^(1/2)*x^(
1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{x}(-a+bx)^3} dx = \frac{\sqrt{x}(-5a+3bx)}{4a^2(a-bx)^2} - \frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(-a + b*x)^3),x]
```

output
```
(Sqrt[x]*(-5*a + 3*b*x))/(4*a^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(bx-a)^3} \, dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{3 \int \frac{1}{\sqrt{x}(a-bx)^2}dx}{4a}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

$$\Big\downarrow \text{52}$$

$$-\frac{3 \left(\frac{\int \frac{1}{\sqrt{x}(a-bx)}dx}{2a}+\frac{\sqrt{x}}{a(a-bx)}\right)}{4a}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{3 \left(\frac{\int \frac{1}{a-bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a-bx)}\right)}{4a}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{3 \left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}\right)}{4a}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

input
```
Int[1/(Sqrt[x]*(-a + b*x)^3),x]
```

output
```
-1/2*Sqrt[x]/(a*(a - b*x)^2) - (3*(Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b
]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{2a(-bx+a)^{2}}-\frac{3\left(\frac{\sqrt{x}}{2a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{2a}$	61
default	$-\frac{\sqrt{x}}{2a(-bx+a)^{2}}-\frac{3\left(\frac{\sqrt{x}}{2a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{2a}$	61

input
```
int(1/x^(1/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)/a/(-b*x+a)^2-3/2/a*(1/2*x^(1/2)/a/(-b*x+a)+1/2/a/(a*b)^(1/2)*
arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.57

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = \left[\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{ab}\log\left(\frac{bx+a-2\,\sqrt{ab}\sqrt{x}}{bx-a}\right)+2\,(3\,ab^2x-5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2-2\,a^4b^2x+a^5b)}, \frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{-ab}\,\mathrm{arcta}}{4\,(a^3b^3x^2-2}\right.$$

input
```
integrate(1/x^(1/2)/(b*x-a)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqr
t(x))/(b*x - a)) + 2*(3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b
^2*x + a^5*b), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*
b)/(b*sqrt(x))) + (3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2*
x + a^5*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(63) = 126$.

Time = 7.57 (sec) , antiderivative size = 580, normalized size of antiderivative = 8.06

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2\sqrt{x}}{a^3} \\ -\frac{2}{5b^3x^{\frac{5}{2}}} \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{10ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{6abx\,\mathrm{l}}{8a^4b\sqrt{\frac{a}{b}}-16a} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x-a)**3,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2*sqrt(x)/a**3, Eq(b, 0))
, (-2/(5*b**3*x**(5/2)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(a/b))/(8*a*
*4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) -
3*a**2*log(sqrt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(
a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 10*a*b*sqrt(x)*sqrt(a/b)/(8*a**4*b*sq
rt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 6*a*b*x
*log(sqrt(x) - sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) +
 8*a**2*b**3*x**2*sqrt(a/b)) + 6*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**4*b*
sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) + 6*b**
2*x**(3/2)*sqrt(a/b)/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a*
*2*b**3*x**2*sqrt(a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**4*b*s
qrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 3*b**2
*x**2*log(sqrt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a
/b) + 8*a**2*b**3*x**2*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = \frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\,(a^2b^2x^2-2\,a^3bx+a^4)}+\frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2}$$

input
```
integrate(1/x^(1/2)/(b*x-a)^3,x, algorithm="maxima")
```

output
```
1/4*(3*b*x^(3/2) - 5*a*sqrt(x))/(a^2*b^2*x^2 - 2*a^3*b*x + a^4) + 3/8*log(
(b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = \frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}a^2}+\frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\,(bx-a)^2a^2}$$

input
```
integrate(1/x^(1/2)/(b*x-a)^3,x, algorithm="giac")
```

output
```
3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 1/4*(3*b*x^(3/2) - 5*a
*sqrt(x))/((b*x - a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = -\frac{\frac{5\sqrt{x}}{4a}-\frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

input
```
int(-1/(x^(1/2)*(a - b*x)^3),x)
```

output
```
- ((5*x^(1/2))/(4*a) - (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 - 2*a*b*x) -
(3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2-6\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)abx+3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)}{}$$

input
```
int(1/x^(1/2)/(b*x-a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 - 6*sqrt(b)*sq
rt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x + 3*sqrt(b)*sqrt(a)*log( -
 sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 - 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sq
rt(a) + sqrt(x)*b)*a**2 + 6*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*
b)*a*b*x - 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 -
10*sqrt(x)*a**2*b + 6*sqrt(x)*a*b**2*x)/(8*a**3*b*(a**2 - 2*a*b*x + b**2*x
**2))
```

3.291 $\int \frac{1}{x^{3/2}(-a+bx)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{1}{x^{3/2}(-a+bx)^3} \, dx = \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}(a-bx)^2} - \frac{5}{4a^2\sqrt{x}(a-bx)} - \frac{15\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
15/4/a^3/x^(1/2)-1/2/a/x^(1/2)/(-b*x+a)^2-5/4/a^2/x^(1/2)/(-b*x+a)-15/4*b^
(1/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3/2}(-a+bx)^3} \, dx = \frac{8a^2-25abx+15b^2x^2}{4a^3\sqrt{x}(a-bx)^2} - \frac{15\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[1/(x^(3/2)*(-a + b*x)^3),x]
```

output
```
(8*a^2 - 25*a*b*x + 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a - b*x)^2) - (15*Sqrt[b]*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(bx-a)^3}\,dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{5\int \frac{1}{x^{3/2}(a-bx)^2}dx}{4a}-\frac{1}{2a\sqrt{x}(a-bx)^2}$$

$$\Big\downarrow \text{52}$$

$$-\frac{5\left(\frac{3\int \frac{1}{x^{3/2}(a-bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a-bx)}\right)}{4a}-\frac{1}{2a\sqrt{x}(a-bx)^2}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{\sqrt{x}(a-bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a-bx)}\right)}{4a}-\frac{1}{2a\sqrt{x}(a-bx)^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5\left(\frac{3\left(\frac{2b\int \frac{1}{a-bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a-bx)}\right)}{4a}-\frac{1}{2a\sqrt{x}(a-bx)^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5\left(\frac{3\left(\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a-bx)}\right)}{4a}-\frac{1}{2a\sqrt{x}(a-bx)^2}$$

input
```
Int[1/(x^(3/2)*(-a + b*x)^3),x]
```

output
```
-1/2*1/(a*Sqrt[x]*(a - b*x)^2) - (5*(1/(a*Sqrt[x]*(a - b*x)) + (3*(-2/(a*S
qrt[x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a)))
/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{2}{a^3\sqrt{x}}-\frac{2b\left(\frac{-\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}$	57
default	$\frac{2}{a^3\sqrt{x}}-\frac{2b\left(\frac{-\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}$	57
risch	$\frac{2}{a^3\sqrt{x}}+\frac{b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{4}-\frac{9a\sqrt{x}}{4}}{(bx-a)^2}-\frac{15\,\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^3}$	58

input
```
int(1/x^(3/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/a^3/x^(1/2)-2/a^3*b*((-7/8*b*x^(3/2)+9/8*a*x^(1/2))/(-b*x+a)^2+15/8/(a*b
)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx = \left[\frac{15\,(b^2x^3-2\,abx^2+a^2x)\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(15\,b^2x^2-25\,abx+8\,a^2)\sqrt{}}{8\,(a^3b^2x^3-2\,a^4bx^2+a^5x)}\right.$$

input
```
integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

output
```
[1/8*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(b/a) + a)/(b*x - a)) + 2*(15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 - 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt
(-b/a)*arctan(sqrt(x)*sqrt(-b/a)) + (15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt(x
))/(a^3*b^2*x^3 - 2*a^4*b*x^2 + a^5*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(75) = 150$.

Time = 16.54 (sec) , antiderivative size = 716, normalized size of antiderivative = 8.52

$$\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(3/2)/(b*x-a)**3,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2/(a**3*sqrt(x)), Eq(b, 0)
), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (15*a**2*sqrt(x)*log(sqrt(x) - sqrt(a
/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**
2*x**(5/2)*sqrt(a/b)) - 15*a**2*sqrt(x)*log(sqrt(x) + sqrt(a/b))/(8*a**5*s
qrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqr
t(a/b)) + 16*a**2*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)
*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)) - 30*a*b*x**(3/2)*log(sqrt(x)
 - sqrt(a/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8
*a**3*b**2*x**(5/2)*sqrt(a/b)) + 30*a*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/
(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**
(5/2)*sqrt(a/b)) - 50*a*b*x*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*
b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)) + 15*b**2*x**(5/2)*
log(sqrt(x) - sqrt(a/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sq
rt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)) - 15*b**2*x**(5/2)*log(sqrt(x) +
 sqrt(a/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a
**3*b**2*x**(5/2)*sqrt(a/b)) + 30*b**2*x**2*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt
(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx = \frac{15\,b^2x^2 - 25\,abx + 8\,a^2}{4\left(a^3b^2x^{\frac{5}{2}} - 2\,a^4bx^{\frac{3}{2}} + a^5\sqrt{x}\right)} + \frac{15\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$

input
```
integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

output
```
1/4*(15*b^2*x^2 - 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) - 2*a^4*b*x^(3/2) + a
^5*sqrt(x)) + 15/8*b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/
(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx=\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}a^3}+\frac{2}{a^3\sqrt{x}}+\frac{7\,b^2x^{\frac{3}{2}}-9\,ab\sqrt{x}}{4\,(bx-a)^2a^3}$$

input
```
integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

output
```
15/4*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/(a^3*sqrt(x)) + 1
/4*(7*b^2*x^(3/2) - 9*a*b*sqrt(x))/((b*x - a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx=\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}-\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}-2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

input
```
int(-1/(x^(3/2)*(a - b*x)^3),x)
```

output
```
(2/a + (15*b^2*x^2)/(4*a^3) - (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2)
 - 2*a*b*x^(3/2)) - (15*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/
2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.26

$$\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx = \frac{15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2-30\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)abx+}{}$$

input
```
int(1/x^(3/2)/(b*x-a)^3,x)
```

output
```
(15*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 - 30*
sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x + 15*sqr
t(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 - 15*sq
rt(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2 + 30*sqrt(x)*s
qrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b*x - 15*sqrt(x)*sqrt(b)
*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**2*x**2 + 16*a**3 - 50*a**2*b*
x + 30*a*b**2*x**2)/(8*sqrt(x)*a**4*(a**2 - 2*a*b*x + b**2*x**2))
```

3.292 $\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{35b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

output
```
35/12/a^3/x^(3/2)+35/4*b/a^4/x^(1/2)-1/2/a/x^(3/2)/(-b*x+a)^2-7/4/a^2/x^(3
/2)/(-b*x+a)-35/4*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \frac{8a^3+56a^2bx-175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a-bx)^2}-\frac{35b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

input
```
Integrate[1/(x^(5/2)*(-a + b*x)^3),x]
```

output
```
(8*a^3 + 56*a^2*b*x - 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a - b*
x)^2) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(bx-a)^3}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{7\int \frac{1}{x^{5/2}(a-bx)^2}dx}{4a}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

$$\Big\downarrow \text{52}$$

$$-\frac{7\left(\frac{5\int \frac{1}{x^{5/2}(a-bx)}dx}{2a}+\frac{1}{ax^{3/2}(a-bx)}\right)}{4a}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

$$\Big\downarrow \text{61}$$

$$-\frac{7\left(\frac{5\left(\frac{b\int \frac{1}{x^{3/2}(a-bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}\right)}{4a}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

$$\Big\downarrow \text{61}$$

$$-\frac{7\left(\frac{5\left(\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}(a-bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}\right)}{4a}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

$\Big\downarrow$ 73

$$-\frac{7\left(\frac{5\left(\frac{b\left(\frac{2b\int \frac{1}{a-bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}\right)}{4a}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

$\Big\downarrow$ 221

$$-\frac{7\left(\frac{5\left(\frac{b\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a-bx)}\right)}{4a}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

input
```
Int[1/(x^(5/2)*(-a + b*x)^3),x]
```

output
```
-1/2*1/(a*x^(3/2)*(a - b*x)^2) - (7*(1/(a*x^(3/2)*(a - b*x)) + (5*(-2/(3*a
*x^(3/2)) + (b*(-2/(a*Sqrt[x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(3/2)))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{6bx+\frac{2a}{3}}{a^4x^{\frac{3}{2}}}+\frac{b^2\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{4}-\frac{13a\sqrt{x}}{4}}{(bx-a)^2}-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^4}$	66
derivativedivides	$\frac{2}{3a^3x^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{x}}-\frac{2b^2\left(\frac{-\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	68
default	$\frac{2}{3a^3x^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{x}}-\frac{2b^2\left(\frac{-\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	68

input
```
int(1/x^(5/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(9*b*x+a)/a^4/x^(3/2)+b^2/a^4*(2*(11/8*b*x^(3/2)-13/8*a*x^(1/2))/(b*x-
a)^2-35/4/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.53

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \left[\frac{105\,(b^3x^4-2\,ab^2x^3+a^2bx^2)\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(105\,b^3x^3-175\,ab^2x^2-}{24\,(a^4b^2x^4-2\,a^5bx^3+a^6x^2)}\right.$$

input
```
integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

output
```
[1/24*(105*(b^3*x^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*log((b*x - 2*a*sq
rt(x)*sqrt(b/a) + a)/(b*x - a)) + 2*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*
b*x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2), 1/12*(105*(b^
3*x^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*arctan(sqrt(x)*sqrt(-b/a)) + (
105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 -
2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(88) = 176$.

Time = 39.19 (sec) , antiderivative size = 799, normalized size of antiderivative = 8.24

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(5/2)/(b*x-a)**3,x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (2/(3*a**3*x**(3/2)), Eq(b,
 0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (16*a**3*sqrt(a/b)/(24*a**6*x**(3/
2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a
/b)) + 105*a**2*b*x**(3/2)*log(sqrt(x) - sqrt(a/b))/(24*a**6*x**(3/2)*sqrt
(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) -
105*a**2*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) -
 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 112*a**
2*b*x*sqrt(a/b)/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b)
 + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) - 210*a*b**2*x**(5/2)*log(sqrt(x) - sq
rt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a
**4*b**2*x**(7/2)*sqrt(a/b)) + 210*a*b**2*x**(5/2)*log(sqrt(x) + sqrt(a/b)
)/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**
2*x**(7/2)*sqrt(a/b)) - 350*a*b**2*x**2*sqrt(a/b)/(24*a**6*x**(3/2)*sqrt(a
/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 10
5*b**3*x**(7/2)*log(sqrt(x) - sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*
a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) - 105*b**3*x*
*(7/2)*log(sqrt(x) + sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x*
*(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 210*b**3*x**3*sqrt(a
/b)/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b
**2*x**(7/2)*sqrt(a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \frac{105\,b^3x^3-175\,ab^2x^2+56\,a^2bx+8\,a^3}{12\left(a^4b^2x^{\frac{7}{2}}-2\,a^5bx^{\frac{5}{2}}+a^6x^{\frac{3}{2}}\right)}+\frac{35\,b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}$$

input
```
integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

output
```
1/12*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)/(a^4*b^2*x^(7/2) -
 2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/8*b^2*log((b*sqrt(x) - sqrt(a*b))/(b*
sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}a^4}+\frac{2\,(9\,bx+a)}{3\,a^4x^{\frac{3}{2}}}+\frac{11\,b^3x^{\frac{3}{2}}-13\,ab^2\sqrt{x}}{4\,(bx-a)^2a^4}$$

input
```
integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

output
```
35/4*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^4) + 2/3*(9*b*x + a)/(
a^4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) - 13*a*b^2*sqrt(x))/((b*x - a)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \frac{\frac{2}{3\,a}-\frac{175\,b^2\,x^2}{12\,a^3}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}-2\,a\,b\,x^{5/2}}-\frac{35\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

input
```
int(-1/(x^(5/2)*(a - b*x)^3),x)
```

output
```
(2/(3*a) - (175*b^2*x^2)/(12*a^3) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2
))/(a^2*x^(3/2) + b^2*x^(7/2) - 2*a*b*x^(5/2)) - (35*b^(3/2)*atanh((b^(1/2
)*x^(1/2))/a^(1/2)))/(4*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = \frac{105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a^2bx - 210\sqrt{x}\,\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+\sqrt{x}\,b\right)a}{}$$

input
```
int(1/x^(5/2)/(b*x-a)^3,x)
```

output
```
(105*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2*b*x
- 210*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b**2*x
**2 + 105*sqrt(x)*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + sqrt(x)*b)*b**3
*x**3 - 105*sqrt(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a**2*
b*x + 210*sqrt(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*a*b**2*
x**2 - 105*sqrt(x)*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + sqrt(x)*b)*b**3*x
**3 + 16*a**4 + 112*a**3*b*x - 350*a**2*b**2*x**2 + 210*a*b**3*x**3)/(24*s
qrt(x)*a**5*x*(a**2 - 2*a*b*x + b**2*x**2))
```

3.293 $\int x^{5/3}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^{5/3}(a+bx)\,dx=\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

output
```
3/8*a*x^(8/3)+3/11*b*x^(11/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{5/3}(a+bx)\,dx=\frac{3}{88}x^{8/3}(11a+8bx)$$

input
```
Integrate[x^(5/3)*(a + b*x),x]
```

output
```
(3*x^(8/3)*(11*a + 8*b*x))/88
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/3}(a+bx)\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(ax^{5/3}+bx^{8/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

input
```
Int[x^(5/3)*(a + b*x),x]
```

output
```
(3*a*x^(8/3))/8 + (3*b*x^(11/3))/11
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$	14
derivativedivides	$\frac{3a\,x^{\frac{8}{3}}}{8}+\frac{3b\,x^{\frac{11}{3}}}{11}$	14
default	$\frac{3a\,x^{\frac{8}{3}}}{8}+\frac{3b\,x^{\frac{11}{3}}}{11}$	14
trager	$\frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$	14
risch	$\frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$	14
orering	$\frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$	14

input
```
int(x^(5/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/88*x^(8/3)*(8*b*x+11*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{5/3}(a+bx)\,dx=\frac{3}{88}\left(8\,bx^3+11\,ax^2\right)x^{\frac{2}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a),x, algorithm="fricas")
```

output
```
3/88*(8*b*x^3 + 11*a*x^2)*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{5/3}(a+bx)\,dx = \frac{3ax^{\frac{8}{3}}}{8} + \frac{3bx^{\frac{11}{3}}}{11}$$

input
```
integrate(x**(5/3)*(b*x+a),x)
```

output
```
3*a*x**(8/3)/8 + 3*b*x**(11/3)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/3}(a+bx)\,dx = \frac{3}{11}\,bx^{\frac{11}{3}} + \frac{3}{8}\,ax^{\frac{8}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a),x, algorithm="maxima")
```

output
```
3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/3}(a+bx)\,dx = \frac{3}{11}\,bx^{\frac{11}{3}} + \frac{3}{8}\,ax^{\frac{8}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a),x, algorithm="giac")
```

output
```
3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/3}(a+bx)\,dx = \frac{3\,x^{8/3}\,(11\,a+8\,b\,x)}{88}$$

input
```
int(x^(5/3)*(a + b*x),x)
```

output
```
(3*x^(8/3)*(11*a + 8*b*x))/88
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/3}(a+bx)\,dx = \frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$$

input
```
int(x^(5/3)*(b*x+a),x)
```

output
```
(3*x**(2/3)*x**2*(11*a + 8*b*x))/88
```

3.294 $\int x^{4/3}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^{4/3}(a+bx)\,dx=\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

output
```
3/7*a*x^(7/3)+3/10*b*x^(10/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{4/3}(a+bx)\,dx=\frac{3}{70}x^{7/3}(10a+7bx)$$

input
```
Integrate[x^(4/3)*(a + b*x),x]
```

output
```
(3*x^(7/3)*(10*a + 7*b*x))/70
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4/3}(a+bx)\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(ax^{4/3}+bx^{7/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

input
```
Int[x^(4/3)*(a + b*x),x]
```

output
```
(3*a*x^(7/3))/7 + (3*b*x^(10/3))/10
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14
derivativedivides	$\frac{3a\,x^{\frac{7}{3}}}{7}+\frac{3b\,x^{\frac{10}{3}}}{10}$	14
default	$\frac{3a\,x^{\frac{7}{3}}}{7}+\frac{3b\,x^{\frac{10}{3}}}{10}$	14
trager	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14
risch	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14
orering	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14

input
```
int(x^(4/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/70*x^(7/3)*(7*b*x+10*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{4/3}(a+bx)\,dx=\frac{3}{70}\left(7\,bx^3+10\,ax^2\right)x^{\frac{1}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a),x, algorithm="fricas")
```

output
```
3/70*(7*b*x^3 + 10*a*x^2)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{4/3}(a+bx)\,dx = \frac{3ax^{\frac{7}{3}}}{7} + \frac{3bx^{\frac{10}{3}}}{10}$$

input
```
integrate(x**(4/3)*(b*x+a),x)
```

output
```
3*a*x**(7/3)/7 + 3*b*x**(10/3)/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{4/3}(a+bx)\,dx = \frac{3}{10}\,bx^{\frac{10}{3}} + \frac{3}{7}\,ax^{\frac{7}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a),x, algorithm="maxima")
```

output
```
3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{4/3}(a+bx)\,dx = \frac{3}{10}\,bx^{\frac{10}{3}} + \frac{3}{7}\,ax^{\frac{7}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a),x, algorithm="giac")
```

output
```
3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{4/3}(a+bx)\,dx=\frac{3\,x^{7/3}\,(10\,a+7\,b\,x)}{70}$$

input
```
int(x^(4/3)*(a + b*x),x)
```

output
```
(3*x^(7/3)*(10*a + 7*b*x))/70
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{4/3}(a+bx)\,dx=\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$$

input
```
int(x^(4/3)*(b*x+a),x)
```

output
```
(3*x**(1/3)*x**2*(10*a + 7*b*x))/70
```

3.295 $\int x^{2/3}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^{2/3}(a+bx)\,dx=\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

output
```
3/5*a*x^(5/3)+3/8*b*x^(8/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{2/3}(a+bx)\,dx=\frac{3}{40}x^{5/3}(8a+5bx)$$

input
```
Integrate[x^(2/3)*(a + b*x),x]
```

output
```
(3*x^(5/3)*(8*a + 5*b*x))/40
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2/3}(a+bx)\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(ax^{2/3}+bx^{5/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

input
```
Int[x^(2/3)*(a + b*x),x]
```

output
```
(3*a*x^(5/3))/5 + (3*b*x^(8/3))/8
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14
derivativedivides	$\frac{3a\,x^{\frac{5}{3}}}{5}+\frac{3b\,x^{\frac{8}{3}}}{8}$	14
default	$\frac{3a\,x^{\frac{5}{3}}}{5}+\frac{3b\,x^{\frac{8}{3}}}{8}$	14
trager	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14
risch	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14
orering	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14

input
```
int(x^(2/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/40*x^(5/3)*(5*b*x+8*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x^{2/3}(a+bx)\,dx=\frac{3}{40}\left(5\,bx^2+8\,ax\right)x^{\frac{2}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a),x, algorithm="fricas")
```

output
```
3/40*(5*b*x^2 + 8*a*x)*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{2/3}(a+bx)\,dx = \frac{3ax^{\frac{5}{3}}}{5} + \frac{3bx^{\frac{8}{3}}}{8}$$

input
```
integrate(x**(2/3)*(b*x+a),x)
```

output
```
3*a*x**(5/3)/5 + 3*b*x**(8/3)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{2/3}(a+bx)\,dx = \frac{3}{8}\,bx^{\frac{8}{3}} + \frac{3}{5}\,ax^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a),x, algorithm="maxima")
```

output
```
3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{2/3}(a+bx)\,dx = \frac{3}{8}\,bx^{\frac{8}{3}} + \frac{3}{5}\,ax^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a),x, algorithm="giac")
```

output
```
3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{2/3}(a+bx)\,dx=\frac{3\,x^{5/3}\,(8\,a+5\,b\,x)}{40}$$

input
```
int(x^(2/3)*(a + b*x),x)
```

output
```
(3*x^(5/3)*(8*a + 5*b*x))/40
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{2/3}(a+bx)\,dx=\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$$

input
```
int(x^(2/3)*(b*x+a),x)
```

output
```
(3*x**(2/3)*x*(8*a + 5*b*x))/40
```

3.296 $\int \sqrt[3]{x}(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \sqrt[3]{x}(a+bx)\,dx = \frac{3}{4}ax^{4/3} + \frac{3}{7}bx^{7/3}$$

output
```
3/4*a*x^(4/3)+3/7*b*x^(7/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sqrt[3]{x}(a+bx)\,dx = \frac{3}{28}x^{4/3}(7a+4bx)$$

input
```
Integrate[x^(1/3)*(a + b*x),x]
```

output
```
(3*x^(4/3)*(7*a + 4*b*x))/28
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x}(a+bx)\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a\sqrt[3]{x}+bx^{4/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

input
```
Int[x^(1/3)*(a + b*x),x]
```

output
```
(3*a*x^(4/3))/4 + (3*b*x^(7/3))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14
derivativedivides	$\frac{3a\,x^{\frac{4}{3}}}{4}+\frac{3b\,x^{\frac{7}{3}}}{7}$	14
default	$\frac{3a\,x^{\frac{4}{3}}}{4}+\frac{3b\,x^{\frac{7}{3}}}{7}$	14
trager	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14
risch	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14
orering	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14

input
```
int(x^(1/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/28*x^(4/3)*(4*b*x+7*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \sqrt[3]{x}(a+bx)\,dx=\frac{3}{28}\left(4\,bx^{2}+7\,ax\right)x^{\frac{1}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a),x, algorithm="fricas")
```

output
```
3/28*(4*b*x^2 + 7*a*x)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt[3]{x}(a+bx)\,dx = \frac{3ax^{\frac{4}{3}}}{4} + \frac{3bx^{\frac{7}{3}}}{7}$$

input
```
integrate(x**(1/3)*(b*x+a),x)
```

output
```
3*a*x**(4/3)/4 + 3*b*x**(7/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}(a+bx)\,dx = \frac{3}{7}\,bx^{\frac{7}{3}} + \frac{3}{4}\,ax^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a),x, algorithm="maxima")
```

output
```
3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}(a+bx)\,dx = \frac{3}{7}\,bx^{\frac{7}{3}} + \frac{3}{4}\,ax^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a),x, algorithm="giac")
```

output
```
3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}(a+bx)\,dx=\frac{3\,x^{4/3}\,(7\,a+4\,b\,x)}{28}$$

input
```
int(x^(1/3)*(a + b*x),x)
```

output
```
(3*x^(4/3)*(7*a + 4*b*x))/28
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{x}(a+bx)\,dx=\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$$

input
```
int(x^(1/3)*(b*x+a),x)
```

output
```
(3*x**(1/3)*x*(7*a + 4*b*x))/28
```

3.297 $\int \frac{a+bx}{\sqrt[3]{x}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{a+bx}{\sqrt[3]{x}} \, dx = \frac{3}{2}ax^{2/3} + \frac{3}{5}bx^{5/3}$$

output
```
3/2*a*x^(2/3)+3/5*b*x^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+bx}{\sqrt[3]{x}} \, dx = \frac{3}{10}x^{2/3}(5a+2bx)$$

input
```
Integrate[(a + b*x)/x^(1/3),x]
```

output
```
(3*x^(2/3)*(5*a + 2*b*x))/10
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{a}{\sqrt[3]{x}}+bx^{2/3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}$$

input
```
Int[(a + b*x)/x^(1/3),x]
```

output
```
(3*a*x^(2/3))/2 + (3*b*x^(5/3))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

method	result	size
trager	$\left(\frac{3bx}{5}+\frac{3a}{2}\right)x^{\frac{2}{3}}$	13
gosper	$\frac{3x^{\frac{2}{3}}(2bx+5a)}{10}$	14
derivativedivides	$\frac{3a\,x^{\frac{2}{3}}}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$	14
default	$\frac{3a\,x^{\frac{2}{3}}}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$	14
risch	$\frac{3x^{\frac{2}{3}}(2bx+5a)}{10}$	14
orering	$\frac{3x^{\frac{2}{3}}(2bx+5a)}{10}$	14

input
```
int((b*x+a)/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(3/5*b*x+3/2*a)*x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+bx}{\sqrt[3]{x}}\,dx=\frac{3}{10}\,(2\,bx+5\,a)x^{\frac{2}{3}}$$

input
```
integrate((b*x+a)/x^(1/3),x, algorithm="fricas")
```

output
```
3/10*(2*b*x + 5*a)*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+bx}{\sqrt[3]{x}}\,dx=\frac{3ax^{\frac{2}{3}}}{2}+\frac{3bx^{\frac{5}{3}}}{5}$$

input
```
integrate((b*x+a)/x**(1/3),x)
```

output
```
3*a*x**(2/3)/2 + 3*b*x**(5/3)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+bx}{\sqrt[3]{x}}\,dx=\frac{3}{5}\,bx^{\frac{5}{3}}+\frac{3}{2}\,ax^{\frac{2}{3}}$$

input
```
integrate((b*x+a)/x^(1/3),x, algorithm="maxima")
```

output
```
3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+bx}{\sqrt[3]{x}}\,dx=\frac{3}{5}\,bx^{\frac{5}{3}}+\frac{3}{2}\,ax^{\frac{2}{3}}$$

input
```
integrate((b*x+a)/x^(1/3),x, algorithm="giac")
```

output
```
3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+bx}{\sqrt[3]{x}}\,dx = \frac{3\,x^{2/3}\,(5\,a+2\,b\,x)}{10}$$

input
```
int((a + b*x)/x^(1/3),x)
```

output
```
(3*x^(2/3)*(5*a + 2*b*x))/10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+bx}{\sqrt[3]{x}}\,dx = \frac{3x^{\frac{2}{3}}(2bx+5a)}{10}$$

input
```
int((b*x+a)/x^(1/3),x)
```

output
```
(3*x**(2/3)*(5*a + 2*b*x))/10
```

3.298 $\int \frac{a+bx}{x^{2/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{a+bx}{x^{2/3}}\,dx = 3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

output
```
3*a*x^(1/3)+3/4*b*x^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{a+bx}{x^{2/3}}\,dx = \frac{3}{4}\sqrt[3]{x}(4a+bx)$$

input
```
Integrate[(a + b*x)/x^(2/3),x]
```

output
```
(3*x^(1/3)*(4*a + b*x))/4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^{2/3}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a}{x^{2/3}}+b\sqrt[3]{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

input
```
Int[(a + b*x)/x^(2/3),x]
```

output
```
3*a*x^(1/3) + (3*b*x^(4/3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{3x^{\frac{1}{3}}(bx+4a)}{4}$	13
trager	$\left(\frac{3bx}{4}+3a\right)x^{\frac{1}{3}}$	13
risch	$\frac{3x^{\frac{1}{3}}(bx+4a)}{4}$	13
orering	$\frac{3x^{\frac{1}{3}}(bx+4a)}{4}$	13
derivativedivides	$3a\,x^{\frac{1}{3}}+\frac{3b\,x^{\frac{4}{3}}}{4}$	14
default	$3a\,x^{\frac{1}{3}}+\frac{3b\,x^{\frac{4}{3}}}{4}$	14

input
```
int((b*x+a)/x^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*x^(1/3)*(b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^{2/3}}\,dx = \frac{3}{4}\,(bx+4\,a)x^{\frac{1}{3}}$$

input
```
integrate((b*x+a)/x^(2/3),x, algorithm="fricas")
```

output
```
3/4*(b*x + 4*a)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{x^{2/3}}\,dx = 3a\sqrt[3]{x} + \frac{3bx^{\frac{4}{3}}}{4}$$

input
```
integrate((b*x+a)/x**(2/3),x)
```

output
```
3*a*x**(1/3) + 3*b*x**(4/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{2/3}}\,dx = \frac{3}{4}\,bx^{\frac{4}{3}} + 3\,ax^{\frac{1}{3}}$$

input
```
integrate((b*x+a)/x^(2/3),x, algorithm="maxima")
```

output
```
3/4*b*x^(4/3) + 3*a*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{2/3}}\,dx = \frac{3}{4}\,bx^{\frac{4}{3}} + 3\,ax^{\frac{1}{3}}$$

input
```
integrate((b*x+a)/x^(2/3),x, algorithm="giac")
```

output
```
3/4*b*x^(4/3) + 3*a*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^{2/3}}\,dx = \frac{3\,x^{1/3}\,(4\,a+b\,x)}{4}$$

input
```
int((a + b*x)/x^(2/3),x)
```

output
```
(3*x^(1/3)*(4*a + b*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^{2/3}}\,dx = \frac{3x^{\frac{1}{3}}(bx+4a)}{4}$$

input
```
int((b*x+a)/x^(2/3),x)
```

output
```
(3*x**(1/3)*(4*a + b*x))/4
```

3.299 $\int \frac{a+bx}{x^{4/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{a+bx}{x^{4/3}}\,dx=-\frac{3a}{\sqrt[3]{x}}+\frac{3}{2}bx^{2/3}$$

output
```
-3*a/x^(1/3)+3/2*b*x^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{x^{4/3}}\,dx=-\frac{3(2a-bx)}{2\sqrt[3]{x}}$$

input
```
Integrate[(a + b*x)/x^(4/3),x]
```

output
```
(-3*(2*a - b*x))/(2*x^(1/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^{4/3}} \, dx$$
$$\Big\downarrow \; 53$$
$$\int \left(\frac{a}{x^{4/3}}+\frac{b}{\sqrt[3]{x}}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{3}{2}bx^{2/3}-\frac{3a}{\sqrt[3]{x}}$$

input
```
Int[(a + b*x)/x^(4/3),x]
```

output
```
(-3*a)/x^(1/3) + (3*b*x^(2/3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14
derivativedivides	$-\frac{3a}{x^{\frac{1}{3}}}+\frac{3b\,x^{\frac{2}{3}}}{2}$	14
default	$-\frac{3a}{x^{\frac{1}{3}}}+\frac{3b\,x^{\frac{2}{3}}}{2}$	14
trager	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14
risch	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14
orering	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14

input
```
int((b*x+a)/x^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(-b*x+2*a)/x^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^{4/3}}\,dx=\frac{3\,(bx-2\,a)}{2\,x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)/x^(4/3),x, algorithm="fricas")
```

output
```
3/2*(b*x - 2*a)/x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{x^{4/3}}\,dx = -\frac{3a}{\sqrt[3]{x}} + \frac{3bx^{\frac{2}{3}}}{2}$$

input
```
integrate((b*x+a)/x**(4/3),x)
```

output
```
-3*a/x**(1/3) + 3*b*x**(2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{4/3}}\,dx = \frac{3}{2}\,bx^{\frac{2}{3}} - \frac{3\,a}{x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)/x^(4/3),x, algorithm="maxima")
```

output
```
3/2*b*x^(2/3) - 3*a/x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{4/3}}\,dx = \frac{3}{2}\,bx^{\frac{2}{3}} - \frac{3\,a}{x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)/x^(4/3),x, algorithm="giac")
```

output
```
3/2*b*x^(2/3) - 3*a/x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{4/3}}\,dx = -\frac{6\,a-3\,b\,x}{2\,x^{1/3}}$$

input
```
int((a + b*x)/x^(4/3),x)
```

output
```
-(6*a - 3*b*x)/(2*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{x^{4/3}}\,dx = \frac{\frac{3bx}{2}-3a}{x^{\frac{1}{3}}}$$

input
```
int((b*x+a)/x^(4/3),x)
```

output
```
(3*( - 2*a + b*x))/(2*x**(1/3))
```

3.300 $\int \frac{a+bx}{x^{5/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{a+bx}{x^{5/3}}\,dx=-\frac{3a}{2x^{2/3}}+3b\sqrt[3]{x}$$

output
```
-3/2*a/x^(2/3)+3*b*x^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+bx}{x^{5/3}}\,dx=-\frac{3(a-2bx)}{2x^{2/3}}$$

input
```
Integrate[(a + b*x)/x^(5/3),x]
```

output
```
(-3*(a - 2*b*x))/(2*x^(2/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{x^{5/3}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a}{x^{5/3}}+\frac{b}{x^{2/3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3b\sqrt[3]{x}-\frac{3a}{2x^{2/3}}$$

input
```
Int[(a + b*x)/x^(5/3),x]
```

output
```
(-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
trager	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
risch	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
orering	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
derivativedivides	$-\frac{3a}{2x^{\frac{2}{3}}}+3b\,x^{\frac{1}{3}}$	14
default	$-\frac{3a}{2x^{\frac{2}{3}}}+3b\,x^{\frac{1}{3}}$	14

input
```
int((b*x+a)/x^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(-2*b*x+a)/x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{5/3}}\,dx=\frac{3\,(2\,bx-a)}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)/x^(5/3),x, algorithm="fricas")
```

output
```
3/2*(2*b*x - a)/x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{x^{5/3}}\,dx = -\frac{3a}{2x^{\frac{2}{3}}} + 3b\sqrt[3]{x}$$

input
```
integrate((b*x+a)/x**(5/3),x)
```

output
```
-3*a/(2*x**(2/3)) + 3*b*x**(1/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{5/3}}\,dx = 3\,bx^{\frac{1}{3}} - \frac{3\,a}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)/x^(5/3),x, algorithm="maxima")
```

output
```
3*b*x^(1/3) - 3/2*a/x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{5/3}}\,dx = 3\,bx^{\frac{1}{3}} - \frac{3\,a}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)/x^(5/3),x, algorithm="giac")
```

output
```
3*b*x^(1/3) - 3/2*a/x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{5/3}}\,dx = -\frac{3\,a-6\,b\,x}{2\,x^{2/3}}$$

input
```
int((a + b*x)/x^(5/3),x)
```

output
```
-(3*a - 6*b*x)/(2*x^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx}{x^{5/3}}\,dx = \frac{3bx-\frac{3a}{2}}{x^{\frac{2}{3}}}$$

input
```
int((b*x+a)/x^(5/3),x)
```

output
```
(3*( - a + 2*b*x))/(2*x**(2/3))
```

3.301 $\int x^{5/3}(a+bx)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int x^{5/3}(a+bx)^2\,dx=\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

output
```
3/8*a^2*x^(8/3)+6/11*a*b*x^(11/3)+3/14*b^2*x^(14/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{5/3}(a+bx)^2\,dx=\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

input
```
Integrate[x^(5/3)*(a + b*x)^2,x]
```

output
```
(3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/3}(a+bx)^2\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2x^{5/3}+2abx^{8/3}+b^2x^{11/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

input
```
Int[x^(5/3)*(a + b*x)^2,x]
```

output
```
(3*a^2*x^(8/3))/8 + (6*a*b*x^(11/3))/11 + (3*b^2*x^(14/3))/14
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}(44b^2x^2+112abx+77a^2)}{616}$	25
derivativedivides	$\frac{3a^2x^{\frac{8}{3}}}{8}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{3b^2x^{\frac{14}{3}}}{14}$	25
default	$\frac{3a^2x^{\frac{8}{3}}}{8}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{3b^2x^{\frac{14}{3}}}{14}$	25
trager	$\frac{3x^{\frac{8}{3}}(44b^2x^2+112abx+77a^2)}{616}$	25
risch	$\frac{3x^{\frac{8}{3}}(44b^2x^2+112abx+77a^2)}{616}$	25
orering	$\frac{3x^{\frac{8}{3}}(44b^2x^2+112abx+77a^2)}{616}$	25

input
```
int(x^(5/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/616*x^(8/3)*(44*b^2*x^2+112*a*b*x+77*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{5/3}(a+bx)^2\,dx = \frac{3}{616}\left(44\,b^2x^4+112\,abx^3+77\,a^2x^2\right)x^{\frac{2}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
3/616*(44*b^2*x^4 + 112*a*b*x^3 + 77*a^2*x^2)*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{5/3}(a+bx)^2\,dx = \frac{3a^2x^{\frac{8}{3}}}{8} + \frac{6abx^{\frac{11}{3}}}{11} + \frac{3b^2x^{\frac{14}{3}}}{14}$$

input
```
integrate(x**(5/3)*(b*x+a)**2,x)
```

output
```
3*a**2*x**(8/3)/8 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(14/3)/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/3}(a+bx)^2\,dx = \frac{3}{14}\,b^2x^{\frac{14}{3}} + \frac{6}{11}\,abx^{\frac{11}{3}} + \frac{3}{8}\,a^2x^{\frac{8}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/3}(a+bx)^2\,dx = \frac{3}{14}\,b^2x^{\frac{14}{3}} + \frac{6}{11}\,abx^{\frac{11}{3}} + \frac{3}{8}\,a^2x^{\frac{8}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a)^2,x, algorithm="giac")
```

output
```
3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/3}(a+bx)^2\,dx=\frac{3\,x^{8/3}\,\left(77\,a^2+112\,a\,b\,x+44\,b^2\,x^2\right)}{616}$$

input
```
int(x^(5/3)*(a + b*x)^2,x)
```

output
```
(3*x^(8/3)*(77*a^2 + 44*b^2*x^2 + 112*a*b*x))/616
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/3}(a+bx)^2\,dx=\frac{3x^{\frac{8}{3}}(44b^2x^2+112abx+77a^2)}{616}$$

input
```
int(x^(5/3)*(b*x+a)^2,x)
```

output
```
(3*x**(2/3)*x**2*(77*a**2 + 112*a*b*x + 44*b**2*x**2))/616
```

3.302 $\int x^{4/3}(a+bx)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int x^{4/3}(a+bx)^2\,dx=\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

output
```
3/7*a^2*x^(7/3)+3/5*a*b*x^(10/3)+3/13*b^2*x^(13/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{4/3}(a+bx)^2\,dx=\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

input
```
Integrate[x^(4/3)*(a + b*x)^2,x]
```

output
```
(3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4/3}(a+bx)^2\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2x^{4/3}+2abx^{7/3}+b^2x^{10/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

input
```
Int[x^(4/3)*(a + b*x)^2,x]
```

output
```
(3*a^2*x^(7/3))/7 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(13/3))/13
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}(35b^2x^2+91abx+65a^2)}{455}$	25
derivativedivides	$\frac{3a^2x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{10}{3}}}{5}+\frac{3b^2x^{\frac{13}{3}}}{13}$	25
default	$\frac{3a^2x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{10}{3}}}{5}+\frac{3b^2x^{\frac{13}{3}}}{13}$	25
trager	$\frac{3x^{\frac{7}{3}}(35b^2x^2+91abx+65a^2)}{455}$	25
risch	$\frac{3x^{\frac{7}{3}}(35b^2x^2+91abx+65a^2)}{455}$	25
orering	$\frac{3x^{\frac{7}{3}}(35b^2x^2+91abx+65a^2)}{455}$	25

input
```
int(x^(4/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/455*x^(7/3)*(35*b^2*x^2+91*a*b*x+65*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{4/3}(a+bx)^2\,dx = \frac{3}{455}\left(35\,b^2x^4+91\,abx^3+65\,a^2x^2\right)x^{\frac{1}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
3/455*(35*b^2*x^4 + 91*a*b*x^3 + 65*a^2*x^2)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{4/3}(a+bx)^2\,dx = \frac{3a^2x^{\frac{7}{3}}}{7} + \frac{3abx^{\frac{10}{3}}}{5} + \frac{3b^2x^{\frac{13}{3}}}{13}$$

input
```
integrate(x**(4/3)*(b*x+a)**2,x)
```

output
```
3*a**2*x**(7/3)/7 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(13/3)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{4/3}(a+bx)^2\,dx = \frac{3}{13}\,b^2x^{\frac{13}{3}} + \frac{3}{5}\,abx^{\frac{10}{3}} + \frac{3}{7}\,a^2x^{\frac{7}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{4/3}(a+bx)^2\,dx = \frac{3}{13}\,b^2x^{\frac{13}{3}} + \frac{3}{5}\,abx^{\frac{10}{3}} + \frac{3}{7}\,a^2x^{\frac{7}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a)^2,x, algorithm="giac")
```

output
```
3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{4/3}(a+bx)^2\,dx=\frac{3\,x^{7/3}\,\left(65\,a^2+91\,a\,b\,x+35\,b^2\,x^2\right)}{455}$$

input
```
int(x^(4/3)*(a + b*x)^2,x)
```

output
```
(3*x^(7/3)*(65*a^2 + 35*b^2*x^2 + 91*a*b*x))/455
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{4/3}(a+bx)^2\,dx=\frac{3x^{\frac{7}{3}}(35b^2x^2+91abx+65a^2)}{455}$$

input
```
int(x^(4/3)*(b*x+a)^2,x)
```

output
```
(3*x**(1/3)*x**2*(65*a**2 + 91*a*b*x + 35*b**2*x**2))/455
```

3.303 $\int x^{2/3}(a+bx)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int x^{2/3}(a+bx)^2\,dx=\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

output
```
3/5*a^2*x^(5/3)+3/4*a*b*x^(8/3)+3/11*b^2*x^(11/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{2/3}(a+bx)^2\,dx=\frac{3}{220}x^{5/3}\left(44a^2+55abx+20b^2x^2\right)$$

input
```
Integrate[x^(2/3)*(a + b*x)^2,x]
```

output
```
(3*x^(5/3)*(44*a^2 + 55*a*b*x + 20*b^2*x^2))/220
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2/3}(a+bx)^2\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2x^{2/3}+2abx^{5/3}+b^2x^{8/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

input
```
Int[x^(2/3)*(a + b*x)^2,x]
```

output
```
(3*a^2*x^(5/3))/5 + (3*a*b*x^(8/3))/4 + (3*b^2*x^(11/3))/11
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}\left(20b^{2}x^{2}+55abx+44a^{2}\right)}{220}$	25
derivativedivides	$\frac{3a^{2}x^{\frac{5}{3}}}{5}+\frac{3ab\,x^{\frac{8}{3}}}{4}+\frac{3b^{2}x^{\frac{11}{3}}}{11}$	25
default	$\frac{3a^{2}x^{\frac{5}{3}}}{5}+\frac{3ab\,x^{\frac{8}{3}}}{4}+\frac{3b^{2}x^{\frac{11}{3}}}{11}$	25
trager	$\frac{3x^{\frac{5}{3}}\left(20b^{2}x^{2}+55abx+44a^{2}\right)}{220}$	25
risch	$\frac{3x^{\frac{5}{3}}\left(20b^{2}x^{2}+55abx+44a^{2}\right)}{220}$	25
orering	$\frac{3x^{\frac{5}{3}}\left(20b^{2}x^{2}+55abx+44a^{2}\right)}{220}$	25

input
```
int(x^(2/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/220*x^(5/3)*(20*b^2*x^2+55*a*b*x+44*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int x^{2/3}(a+bx)^2\,dx=\frac{3}{220}\left(20\,b^2x^3+55\,abx^2+44\,a^2x\right)x^{\frac{2}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
3/220*(20*b^2*x^3 + 55*a*b*x^2 + 44*a^2*x)*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{2/3}(a+bx)^2\,dx = \frac{3a^2x^{\frac{5}{3}}}{5} + \frac{3abx^{\frac{8}{3}}}{4} + \frac{3b^2x^{\frac{11}{3}}}{11}$$

input
```
integrate(x**(2/3)*(b*x+a)**2,x)
```

output
```
3*a**2*x**(5/3)/5 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(11/3)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{2/3}(a+bx)^2\,dx = \frac{3}{11}\,b^2x^{\frac{11}{3}} + \frac{3}{4}\,abx^{\frac{8}{3}} + \frac{3}{5}\,a^2x^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{2/3}(a+bx)^2\,dx = \frac{3}{11}\,b^2x^{\frac{11}{3}} + \frac{3}{4}\,abx^{\frac{8}{3}} + \frac{3}{5}\,a^2x^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a)^2,x, algorithm="giac")
```

output
```
3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{2/3}(a+bx)^2\,dx=\frac{3\,x^{5/3}\,\left(44\,a^2+55\,a\,b\,x+20\,b^2\,x^2\right)}{220}$$

input
```
int(x^(2/3)*(a + b*x)^2,x)
```

output
```
(3*x^(5/3)*(44*a^2 + 20*b^2*x^2 + 55*a*b*x))/220
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{2/3}(a+bx)^2\,dx=\frac{3x^{\frac{5}{3}}(20b^2x^2+55abx+44a^2)}{220}$$

input
```
int(x^(2/3)*(b*x+a)^2,x)
```

output
```
(3*x**(2/3)*x*(44*a**2 + 55*a*b*x + 20*b**2*x**2))/220
```

3.304 $\int \sqrt[3]{x}(a+bx)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \sqrt[3]{x}(a+bx)^2\,dx=\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

output
```
3/4*a^2*x^(4/3)+6/7*a*b*x^(7/3)+3/10*b^2*x^(10/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sqrt[3]{x}(a+bx)^2\,dx=\frac{3}{140}x^{4/3}\left(35a^2+40abx+14b^2x^2\right)$$

input
```
Integrate[x^(1/3)*(a + b*x)^2,x]
```

output
```
(3*x^(4/3)*(35*a^2 + 40*a*b*x + 14*b^2*x^2))/140
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2\sqrt[3]{x}+2abx^{4/3}+b^2x^{7/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

input
```
Int[x^(1/3)*(a + b*x)^2,x]
```

output
```
(3*a^2*x^(4/3))/4 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(10/3))/10
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}(14b^2x^2+40abx+35a^2)}{140}$	25
derivativedivides	$\frac{3a^2x^{\frac{4}{3}}}{4}+\frac{6ab\,x^{\frac{7}{3}}}{7}+\frac{3b^2x^{\frac{10}{3}}}{10}$	25
default	$\frac{3a^2x^{\frac{4}{3}}}{4}+\frac{6ab\,x^{\frac{7}{3}}}{7}+\frac{3b^2x^{\frac{10}{3}}}{10}$	25
trager	$\frac{3x^{\frac{4}{3}}(14b^2x^2+40abx+35a^2)}{140}$	25
risch	$\frac{3x^{\frac{4}{3}}(14b^2x^2+40abx+35a^2)}{140}$	25
orering	$\frac{3x^{\frac{4}{3}}(14b^2x^2+40abx+35a^2)}{140}$	25

input
```
int(x^(1/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/140*x^(4/3)*(14*b^2*x^2+40*a*b*x+35*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{x}(a+bx)^2\,dx = \frac{3}{140}\left(14\,b^2x^3+40\,abx^2+35\,a^2x\right)x^{\frac{1}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
3/140*(14*b^2*x^3 + 40*a*b*x^2 + 35*a^2*x)*x^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 2633, normalized size of antiderivative = 73.14

$$\int \sqrt[3]{x}(a+bx)^2\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/3)*(b*x+a)**2,x)
```

output
```
Piecewise((27*a**(34/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(-140*a*
*8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 42
0*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)
**3*exp(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*
a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*ex
p(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 72*a**(31/3
)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(-140*a**8*b**(4/3
)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**
(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*
I*pi/3)) - 81*a**(31/3)*b*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 42
0*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*
exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(28
/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-140*a**8
*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*
a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**
3*exp(2*I*pi/3)) + 81*a**(28/3)*b**2*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(
2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)
*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3
)) - 60*a**(25/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi
/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt[3]{x}(a+bx)^2\,dx = \frac{3}{10}\,b^2x^{\frac{10}{3}} + \frac{6}{7}\,abx^{\frac{7}{3}} + \frac{3}{4}\,a^2x^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt[3]{x}(a+bx)^2\,dx = \frac{3}{10}\,b^2x^{\frac{10}{3}} + \frac{6}{7}\,abx^{\frac{7}{3}} + \frac{3}{4}\,a^2x^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a)^2,x, algorithm="giac")
```

output
```
3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt[3]{x}(a+bx)^2\,dx = \frac{3\,x^{4/3}\,\left(35\,a^2+40\,a\,b\,x+14\,b^2\,x^2\right)}{140}$$

input
```
int(x^(1/3)*(a + b*x)^2,x)
```

output
```
(3*x^(4/3)*(35*a^2 + 14*b^2*x^2 + 40*a*b*x))/140
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt[3]{x}(a+bx)^2\,dx = \frac{3x^{\frac{4}{3}}(14b^2x^2+40abx+35a^2)}{140}$$

input
```
int(x^(1/3)*(b*x+a)^2,x)
```

output
```
(3*x**(1/3)*x*(35*a**2 + 40*a*b*x + 14*b**2*x**2))/140
```

3.305 $\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx = \frac{3}{2}a^2x^{2/3} + \frac{6}{5}abx^{5/3} + \frac{3}{8}b^2x^{8/3}$$

output
```
3/2*a^2*x^(2/3)+6/5*a*b*x^(5/3)+3/8*b^2*x^(8/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx = \frac{3}{40}x^{2/3}\left(20a^2 + 16abx + 5b^2x^2\right)$$

input
```
Integrate[(a + b*x)^2/x^(1/3),x]
```

output
```
(3*x^(2/3)*(20*a^2 + 16*a*b*x + 5*b^2*x^2))/40
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{a^2}{\sqrt[3]{x}}+2abx^{2/3}+b^2x^{5/3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

input
```
Int[(a + b*x)^2/x^(1/3),x]
```

output
```
(3*a^2*x^(2/3))/2 + (6*a*b*x^(5/3))/5 + (3*b^2*x^(8/3))/8
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
trager	$\left(\frac{3}{8}b^2x^2+\frac{6}{5}abx+\frac{3}{2}a^2\right)x^{\frac{2}{3}}$	24
gosper	$\frac{3x^{\frac{2}{3}}\left(5b^2x^2+16abx+20a^2\right)}{40}$	25
derivativedivides	$\frac{3a^2x^{\frac{2}{3}}}{2}+\frac{6ab\,x^{\frac{5}{3}}}{5}+\frac{3b^2x^{\frac{8}{3}}}{8}$	25
default	$\frac{3a^2x^{\frac{2}{3}}}{2}+\frac{6ab\,x^{\frac{5}{3}}}{5}+\frac{3b^2x^{\frac{8}{3}}}{8}$	25
risch	$\frac{3x^{\frac{2}{3}}\left(5b^2x^2+16abx+20a^2\right)}{40}$	25
orering	$\frac{3x^{\frac{2}{3}}\left(5b^2x^2+16abx+20a^2\right)}{40}$	25

input
```
int((b*x+a)^2/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(3/8*b^2*x^2+6/5*a*b*x+3/2*a^2)*x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx=\frac{3}{40}\left(5\,b^2x^2+16\,abx+20\,a^2\right)x^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^2/x^(1/3),x, algorithm="fricas")
```

output
```
3/40*(5*b^2*x^2 + 16*a*b*x + 20*a^2)*x^(2/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 1765, normalized size of antiderivative = 49.03

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2/x**(1/3),x)
```

output
```
Piecewise((-27*a**(32/3)*(-1 + b*(a/b + x)/a)**(2/3)/(-40*a**8*b**(2/3) +
120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**
(11/3)*(a/b + x)**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120
*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11
/3)*(a/b + x)**3) + 63*a**(29/3)*b*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)/(
-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b +
 x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*
I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/
3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(-1
+ b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3
)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)
**3) + 81*a**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 1
20*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(
11/3)*(a/b + x)**3) + 18*a**(23/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b +
 x)**3/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3
)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a/b
+ x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 1
20*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(2
0/3)*b**4*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(-40*a**8*b**(2/3) + 12
0*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx = \frac{3}{8}\,b^2x^{\frac{8}{3}} + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{3}{2}\,a^2x^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^2/x^(1/3),x, algorithm="maxima")
```

output
```
3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx = \frac{3}{8}\,b^2x^{\frac{8}{3}} + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{3}{2}\,a^2x^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^2/x^(1/3),x, algorithm="giac")
```

output
```
3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx = \frac{3\,x^{2/3}\,\left(20\,a^2+16\,a\,b\,x+5\,b^2\,x^2\right)}{40}$$

input
```
int((a + b*x)^2/x^(1/3),x)
```

output
```
(3*x^(2/3)*(20*a^2 + 5*b^2*x^2 + 16*a*b*x))/40
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx = \frac{3x^{\frac{2}{3}}(5b^2x^2+16abx+20a^2)}{40}$$

input
```
int((b*x+a)^2/x^(1/3),x)
```

output
```
(3*x**(2/3)*(20*a**2 + 16*a*b*x + 5*b**2*x**2))/40
```

3.306 $\int \frac{(a+bx)^2}{x^{2/3}} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(a+bx)^2}{x^{2/3}} dx = 3a^2\sqrt[3]{x} + \frac{3}{2}abx^{4/3} + \frac{3}{7}b^2x^{7/3}$$

output
```
3*a^2*x^(1/3)+3/2*a*b*x^(4/3)+3/7*b^2*x^(7/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2}{x^{2/3}} dx = \frac{3}{14}\sqrt[3]{x}\left(14a^2 + 7abx + 2b^2x^2\right)$$

input
```
Integrate[(a + b*x)^2/x^(2/3),x]
```

output
```
(3*x^(1/3)*(14*a^2 + 7*a*b*x + 2*b^2*x^2))/14
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^{2/3}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2}{x^{2/3}}+2ab\sqrt[3]{x}+b^2x^{4/3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

input
```
Int[(a + b*x)^2/x^(2/3),x]
```

output
```
3*a^2*x^(1/3) + (3*a*b*x^(4/3))/2 + (3*b^2*x^(7/3))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
trager	$\left(\frac{3}{7}b^2x^2+\frac{3}{2}abx+3a^2\right)x^{\frac{1}{3}}$	24
gosper	$\frac{3x^{\frac{1}{3}}\left(2b^2x^2+7abx+14a^2\right)}{14}$	25
derivativedivides	$3a^2x^{\frac{1}{3}}+\frac{3abx^{\frac{4}{3}}}{2}+\frac{3b^2x^{\frac{7}{3}}}{7}$	25
default	$3a^2x^{\frac{1}{3}}+\frac{3abx^{\frac{4}{3}}}{2}+\frac{3b^2x^{\frac{7}{3}}}{7}$	25
risch	$\frac{3x^{\frac{1}{3}}\left(2b^2x^2+7abx+14a^2\right)}{14}$	25
orering	$\frac{3x^{\frac{1}{3}}\left(2b^2x^2+7abx+14a^2\right)}{14}$	25

input
```
int((b*x+a)^2/x^(2/3),x,method=_RETURNVERBOSE)
```

output
```
(3/7*b^2*x^2+3/2*a*b*x+3*a^2)*x^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{2/3}}\,dx=\frac{3}{14}\left(2\,b^2x^2+7\,abx+14\,a^2\right)x^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^2/x^(2/3),x, algorithm="fricas")
```

output
```
3/14*(2*b^2*x^2 + 7*a*b*x + 14*a^2)*x^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 1741, normalized size of antiderivative = 51.21

$$\int \frac{(a+bx)^2}{x^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2/x**(2/3),x)
```

output
```
Piecewise((-27*a**(31/3)*(-1 + b*(a/b + x)/a)**(1/3)/(-14*a**8*b**(1/3) +
42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(1
0/3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7
*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a
/b + x)**3) + 72*a**(28/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)/(-14*a*
*8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 +
 14*a**5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)*b*(a/b + x)*exp(I*pi/3)/(-
14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)
**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25/3)*b**2*(-1 + b*(a/b + x
)/a)**(1/3)*(a/b + x)**2/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(
25/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*
(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3
) + 18*a**(22/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(-14*a**8*b
**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*
a**5*b**(10/3)*(a/b + x)**3) - 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/
(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b +
x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 9*a**(19/3)*b**4*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**4/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x)
- 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{2/3}}\,dx = \frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{3}{2}\,abx^{\frac{4}{3}} + 3\,a^2x^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^2/x^(2/3),x, algorithm="maxima")
```

output
```
3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{2/3}}\,dx = \frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{3}{2}\,abx^{\frac{4}{3}} + 3\,a^2x^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^2/x^(2/3),x, algorithm="giac")
```

output
```
3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{2/3}}\,dx = \frac{3\,x^{1/3}\,(14\,a^2+7\,a\,b\,x+2\,b^2\,x^2)}{14}$$

input
```
int((a + b*x)^2/x^(2/3),x)
```

output
```
(3*x^(1/3)*(14*a^2 + 2*b^2*x^2 + 7*a*b*x))/14
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{2/3}}\,dx = \frac{3x^{\frac{1}{3}}(2b^2x^2+7abx+14a^2)}{14}$$

input
```
int((b*x+a)^2/x^(2/3),x)
```

output
```
(3*x**(1/3)*(14*a**2 + 7*a*b*x + 2*b**2*x**2))/14
```

3.307 $\int \frac{(a+bx)^2}{x^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx = -\frac{3a^2}{\sqrt[3]{x}} + 3abx^{2/3} + \frac{3}{5}b^2x^{5/3}$$

output
```
-3*a^2/x^(1/3)+3*a*b*x^(2/3)+3/5*b^2*x^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx = -\frac{3(5a^2-5abx-b^2x^2)}{5\sqrt[3]{x}}$$

input
```
Integrate[(a + b*x)^2/x^(4/3),x]
```

output
```
(-3*(5*a^2 - 5*a*b*x - b^2*x^2))/(5*x^(1/3))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^{4/3}} \, dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2}{x^{4/3}}+\frac{2ab}{\sqrt[3]{x}}+b^2x^{2/3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

input
```
Int[(a + b*x)^2/x^(4/3),x]
```

output
```
(-3*a^2)/x^(1/3) + 3*a*b*x^(2/3) + (3*b^2*x^(5/3))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{3\left(-b^{2} x^{2}-5 a b x+5 a^{2}\right)}{5 x^{\frac{1}{3}}}$	25
derivativedivides	$-\frac{3 a^{2}}{x^{\frac{1}{3}}}+3 a b\, x^{\frac{2}{3}}+\frac{3 b^{2} x^{\frac{5}{3}}}{5}$	25
default	$-\frac{3 a^{2}}{x^{\frac{1}{3}}}+3 a b\, x^{\frac{2}{3}}+\frac{3 b^{2} x^{\frac{5}{3}}}{5}$	25
trager	$-\frac{3\left(-b^{2} x^{2}-5 a b x+5 a^{2}\right)}{5 x^{\frac{1}{3}}}$	25
risch	$-\frac{3\left(-b^{2} x^{2}-5 a b x+5 a^{2}\right)}{5 x^{\frac{1}{3}}}$	25
orering	$-\frac{3\left(-b^{2} x^{2}-5 a b x+5 a^{2}\right)}{5 x^{\frac{1}{3}}}$	25

input
```
int((b*x+a)^2/x^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/5*(-b^2*x^2-5*a*b*x+5*a^2)/x^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx = \frac{3\,(b^2x^2+5\,abx-5\,a^2)}{5\,x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)^2/x^(4/3),x, algorithm="fricas")
```

output
```
3/5*(b^2*x^2 + 5*a*b*x - 5*a^2)/x^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 1826, normalized size of antiderivative = 57.06

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2/x**(4/3),x)
```

output
```
Piecewise((-27*a**(29/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/
(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b
 + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/3
)*b**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**
6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) +
63*a**(26/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(I*pi/3)/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
 x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*
b**(4/3)*(a/b + x)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3)
- 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*p
i/3)) - 42*a**(23/3)*b**(7/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp
(I*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*
b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81
*a**(23/3)*b**(7/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x
)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b +
 x)**3*exp(I*pi/3)) + 3*a**(20/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**3*exp(I*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi
/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp
(I*pi/3)) + 27*a**(20/3)*b**(10/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*
a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + ...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx=\frac{3}{5}\,b^2x^{\frac{5}{3}}+3\,abx^{\frac{2}{3}}-\frac{3\,a^2}{x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)^2/x^(4/3),x, algorithm="maxima")
```

output
```
3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx=\frac{3}{5}\,b^2x^{\frac{5}{3}}+3\,abx^{\frac{2}{3}}-\frac{3\,a^2}{x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)^2/x^(4/3),x, algorithm="giac")
```

output
```
3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx=\frac{-15\,a^2+15\,a\,b\,x+3\,b^2\,x^2}{5\,x^{1/3}}$$

input
```
int((a + b*x)^2/x^(4/3),x)
```

output
```
(3*b^2*x^2 - 15*a^2 + 15*a*b*x)/(5*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^2}{x^{4/3}}\,dx = \frac{\frac{3}{5}b^2x^2 + 3abx - 3a^2}{x^{\frac{1}{3}}}$$

input
```
int((b*x+a)^2/x^(4/3),x)
```

output
```
(3*( - 5*a**2 + 5*a*b*x + b**2*x**2))/(5*x**(1/3))
```

3.308 $\int \frac{(a+bx)^2}{x^{5/3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(a+bx)^2}{x^{5/3}} \, dx = -\frac{3a^2}{2x^{2/3}} + 6ab\sqrt[3]{x} + \frac{3}{4}b^2x^{4/3}$$

output
```
-3/2*a^2/x^(2/3)+6*a*b*x^(1/3)+3/4*b^2*x^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2}{x^{5/3}} \, dx = -\frac{3(2a^2 - 8abx - b^2x^2)}{4x^{2/3}}$$

input
```
Integrate[(a + b*x)^2/x^(5/3),x]
```

output
```
(-3*(2*a^2 - 8*a*b*x - b^2*x^2))/(4*x^(2/3))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{x^{5/3}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^{5/3}}+\frac{2ab}{x^{2/3}}+b^2\sqrt[3]{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}$$

input
```
Int[(a + b*x)^2/x^(5/3),x]
```

output
```
(-3*a^2)/(2*x^(2/3)) + 6*a*b*x^(1/3) + (3*b^2*x^(4/3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{3(-b^2x^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25
derivativedivides	$-\frac{3a^2}{2x^{\frac{2}{3}}}+6ab\,x^{\frac{1}{3}}+\frac{3b^2x^{\frac{4}{3}}}{4}$	25
default	$-\frac{3a^2}{2x^{\frac{2}{3}}}+6ab\,x^{\frac{1}{3}}+\frac{3b^2x^{\frac{4}{3}}}{4}$	25
trager	$-\frac{3(-b^2x^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25
risch	$-\frac{3(-b^2x^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25
orering	$-\frac{3(-b^2x^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25

input
```
int((b*x+a)^2/x^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-b^2*x^2-8*a*b*x+2*a^2)/x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^2}{x^{5/3}}\,dx = \frac{3\,(b^2x^2+8\,abx-2\,a^2)}{4\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^2/x^(5/3),x, algorithm="fricas")
```

output
```
3/4*(b^2*x^2 + 8*a*b*x - 2*a^2)/x^(2/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 1957, normalized size of antiderivative = 57.56

$$\int \frac{(a+bx)^2}{x^{5/3}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2/x**(5/3),x)
```

output
```
Piecewise((-27*a**(28/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3
)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**
2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 2
7*a**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*
pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3
*exp(2*I*pi/3)) + 72*a**(25/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b +
 x)*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/
3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*ex
p(2*I*pi/3)) + 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12
*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3)
+ 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(22/3)*b**(8/3)*(-1 + b*
(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12
*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3)
+ 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x
)**2/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*
b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3))
+ 12*a**(19/3)*b**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*
pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6
*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3))
 + 27*a**(19/3)*b**(11/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{5/3}}\,dx = \frac{3}{4}\,b^2x^{\frac{4}{3}} + 6\,abx^{\frac{1}{3}} - \frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^2/x^(5/3),x, algorithm="maxima")
```

output
```
3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{5/3}}\,dx = \frac{3}{4}\,b^2x^{\frac{4}{3}} + 6\,abx^{\frac{1}{3}} - \frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^2/x^(5/3),x, algorithm="giac")
```

output
```
3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^2}{x^{5/3}}\,dx = \frac{-6\,a^2+24\,a\,b\,x+3\,b^2\,x^2}{4\,x^{2/3}}$$

input
```
int((a + b*x)^2/x^(5/3),x)
```

output
```
(3*b^2*x^2 - 6*a^2 + 24*a*b*x)/(4*x^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^2}{x^{5/3}}\,dx = \frac{\frac{3}{4}b^2x^2+6abx-\frac{3}{2}a^2}{x^{\frac{2}{3}}}$$

input
```
int((b*x+a)^2/x^(5/3),x)
```

output
```
(3*( - 2*a**2 + 8*a*b*x + b**2*x**2))/(4*x**(2/3))
```

3.309 $\int x^{5/3}(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x^{5/3}(a+bx)^3\,dx=\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

output
```
3/8*a^3*x^(8/3)+9/11*a^2*b*x^(11/3)+9/14*a*b^2*x^(14/3)+3/17*b^3*x^(17/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{5/3}(a+bx)^3\,dx=\frac{3x^{8/3}(1309a^3+2856a^2bx+2244ab^2x^2+616b^3x^3)}{10472}$$

input
```
Integrate[x^(5/3)*(a + b*x)^3,x]
```

output
```
(3*x^(8/3)*(1309*a^3 + 2856*a^2*b*x + 2244*a*b^2*x^2 + 616*b^3*x^3))/10472
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/3}(a+bx)^3\,dx$$
$$\Big\downarrow 53$$
$$\int \left(a^3x^{5/3}+3a^2bx^{8/3}+3ab^2x^{11/3}+b^3x^{14/3}\right)dx$$
$$\Big\downarrow 2009$$
$$\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

input
```
Int[x^(5/3)*(a + b*x)^3,x]
```

output
```
(3*a^3*x^(8/3))/8 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*x^(14/3))/14 + (3*b^3
*x^(17/3))/17
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}\left(616b^{3}x^{3}+2244a\,b^{2}x^{2}+2856a^{2}bx+1309a^{3}\right)}{10472}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{8}{3}}}{8}+\frac{9a^{2}b\,x^{\frac{11}{3}}}{11}+\frac{9a\,b^{2}x^{\frac{14}{3}}}{14}+\frac{3b^{3}x^{\frac{17}{3}}}{17}$	36
default	$\frac{3a^{3}x^{\frac{8}{3}}}{8}+\frac{9a^{2}b\,x^{\frac{11}{3}}}{11}+\frac{9a\,b^{2}x^{\frac{14}{3}}}{14}+\frac{3b^{3}x^{\frac{17}{3}}}{17}$	36
trager	$\frac{3x^{\frac{8}{3}}\left(616b^{3}x^{3}+2244a\,b^{2}x^{2}+2856a^{2}bx+1309a^{3}\right)}{10472}$	36
risch	$\frac{3x^{\frac{8}{3}}\left(616b^{3}x^{3}+2244a\,b^{2}x^{2}+2856a^{2}bx+1309a^{3}\right)}{10472}$	36
orering	$\frac{3x^{\frac{8}{3}}\left(616b^{3}x^{3}+2244a\,b^{2}x^{2}+2856a^{2}bx+1309a^{3}\right)}{10472}$	36

input
```
int(x^(5/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/10472*x^(8/3)*(616*b^3*x^3+2244*a*b^2*x^2+2856*a^2*b*x+1309*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{5/3}(a+bx)^3\,dx=\frac{3}{10472}\left(616\,b^3x^5+2244\,ab^2x^4+2856\,a^2bx^3+1309\,a^3x^2\right)x^{\frac{2}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
3/10472*(616*b^3*x^5 + 2244*a*b^2*x^4 + 2856*a^2*b*x^3 + 1309*a^3*x^2)*x^(
2/3)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{5/3}(a+bx)^3\,dx = \frac{3a^3x^{\frac{8}{3}}}{8} + \frac{9a^2bx^{\frac{11}{3}}}{11} + \frac{9ab^2x^{\frac{14}{3}}}{14} + \frac{3b^3x^{\frac{17}{3}}}{17}$$

input
```
integrate(x**(5/3)*(b*x+a)**3,x)
```

output
```
3*a**3*x**(8/3)/8 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(14/3)/14 + 3*b**3
*x**(17/3)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/3}(a+bx)^3\,dx = \frac{3}{17}\,b^3x^{\frac{17}{3}} + \frac{9}{14}\,ab^2x^{\frac{14}{3}} + \frac{9}{11}\,a^2bx^{\frac{11}{3}} + \frac{3}{8}\,a^3x^{\frac{8}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^
(8/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/3}(a+bx)^3\,dx = \frac{3}{17}\,b^3x^{\frac{17}{3}} + \frac{9}{14}\,ab^2x^{\frac{14}{3}} + \frac{9}{11}\,a^2bx^{\frac{11}{3}} + \frac{3}{8}\,a^3x^{\frac{8}{3}}$$

input
```
integrate(x^(5/3)*(b*x+a)^3,x, algorithm="giac")
```

output
```
3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^
(8/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/3}(a+bx)^3\,dx=\frac{3\,a^3\,x^{8/3}}{8}+\frac{3\,b^3\,x^{17/3}}{17}+\frac{9\,a^2\,b\,x^{11/3}}{11}+\frac{9\,a\,b^2\,x^{14/3}}{14}$$

input
```
int(x^(5/3)*(a + b*x)^3,x)
```

output
```
(3*a^3*x^(8/3))/8 + (3*b^3*x^(17/3))/17 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2
*x^(14/3))/14
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/3}(a+bx)^3\,dx=\frac{3x^{\frac{8}{3}}(616b^3x^3+2244a\,b^2x^2+2856a^2bx+1309a^3)}{10472}$$

input
```
int(x^(5/3)*(b*x+a)^3,x)
```

output
```
(3*x**(2/3)*x**2*(1309*a**3 + 2856*a**2*b*x + 2244*a*b**2*x**2 + 616*b**3*
x**3))/10472
```

3.310 $\int x^{4/3}(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x^{4/3}(a+bx)^3\,dx=\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

output
```
3/7*a^3*x^(7/3)+9/10*a^2*b*x^(10/3)+9/13*a*b^2*x^(13/3)+3/16*b^3*x^(16/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{4/3}(a+bx)^3\,dx=\frac{3x^{7/3}(1040a^3+2184a^2bx+1680ab^2x^2+455b^3x^3)}{7280}$$

input
```
Integrate[x^(4/3)*(a + b*x)^3,x]
```

output
```
(3*x^(7/3)*(1040*a^3 + 2184*a^2*b*x + 1680*a*b^2*x^2 + 455*b^3*x^3))/7280
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4/3}(a+bx)^3\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(a^3x^{4/3}+3a^2bx^{7/3}+3ab^2x^{10/3}+b^3x^{13/3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

input
```
Int[x^(4/3)*(a + b*x)^3,x]
```

output
```
(3*a^3*x^(7/3))/7 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(13/3))/13 + (3*b^3
*x^(16/3))/16
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}\left(455b^{3}x^{3}+1680a\,b^{2}x^{2}+2184a^{2}bx+1040a^{3}\right)}{7280}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{7}{3}}}{7}+\frac{9a^{2}b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^{2}x^{\frac{13}{3}}}{13}+\frac{3b^{3}x^{\frac{16}{3}}}{16}$	36
default	$\frac{3a^{3}x^{\frac{7}{3}}}{7}+\frac{9a^{2}b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^{2}x^{\frac{13}{3}}}{13}+\frac{3b^{3}x^{\frac{16}{3}}}{16}$	36
trager	$\frac{3x^{\frac{7}{3}}\left(455b^{3}x^{3}+1680a\,b^{2}x^{2}+2184a^{2}bx+1040a^{3}\right)}{7280}$	36
risch	$\frac{3x^{\frac{7}{3}}\left(455b^{3}x^{3}+1680a\,b^{2}x^{2}+2184a^{2}bx+1040a^{3}\right)}{7280}$	36
orering	$\frac{3x^{\frac{7}{3}}\left(455b^{3}x^{3}+1680a\,b^{2}x^{2}+2184a^{2}bx+1040a^{3}\right)}{7280}$	36

input
```
int(x^(4/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/7280*x^(7/3)*(455*b^3*x^3+1680*a*b^2*x^2+2184*a^2*b*x+1040*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{4/3}(a+bx)^3\,dx=\frac{3}{7280}\left(455\,b^3x^5+1680\,ab^2x^4+2184\,a^2bx^3+1040\,a^3x^2\right)x^{\frac{1}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
3/7280*(455*b^3*x^5 + 1680*a*b^2*x^4 + 2184*a^2*b*x^3 + 1040*a^3*x^2)*x^(1
/3)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{4/3}(a+bx)^3\,dx = \frac{3a^3x^{\frac{7}{3}}}{7} + \frac{9a^2bx^{\frac{10}{3}}}{10} + \frac{9ab^2x^{\frac{13}{3}}}{13} + \frac{3b^3x^{\frac{16}{3}}}{16}$$

input
```
integrate(x**(4/3)*(b*x+a)**3,x)
```

output
```
3*a**3*x**(7/3)/7 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(13/3)/13 + 3*b**3
*x**(16/3)/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{4/3}(a+bx)^3\,dx = \frac{3}{16}\,b^3x^{\frac{16}{3}} + \frac{9}{13}\,ab^2x^{\frac{13}{3}} + \frac{9}{10}\,a^2bx^{\frac{10}{3}} + \frac{3}{7}\,a^3x^{\frac{7}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^
(7/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{4/3}(a+bx)^3\,dx = \frac{3}{16}\,b^3x^{\frac{16}{3}} + \frac{9}{13}\,ab^2x^{\frac{13}{3}} + \frac{9}{10}\,a^2bx^{\frac{10}{3}} + \frac{3}{7}\,a^3x^{\frac{7}{3}}$$

input
```
integrate(x^(4/3)*(b*x+a)^3,x, algorithm="giac")
```

output
```
3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^
(7/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{4/3}(a+bx)^3\,dx=\frac{3\,a^3\,x^{7/3}}{7}+\frac{3\,b^3\,x^{16/3}}{16}+\frac{9\,a^2\,b\,x^{10/3}}{10}+\frac{9\,a\,b^2\,x^{13/3}}{13}$$

input
```
int(x^(4/3)*(a + b*x)^3,x)
```

output
```
(3*a^3*x^(7/3))/7 + (3*b^3*x^(16/3))/16 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2
*x^(13/3))/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{4/3}(a+bx)^3\,dx=\frac{3x^{\frac{7}{3}}(455b^3x^3+1680a\,b^2x^2+2184a^2bx+1040a^3)}{7280}$$

input
```
int(x^(4/3)*(b*x+a)^3,x)
```

output
```
(3*x**(1/3)*x**2*(1040*a**3 + 2184*a**2*b*x + 1680*a*b**2*x**2 + 455*b**3*
x**3))/7280
```

3.311 $\int x^{2/3}(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

output
```
3/5*a^3*x^(5/3)+9/8*a^2*b*x^(8/3)+9/11*a*b^2*x^(11/3)+3/14*b^3*x^(14/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3x^{5/3}(616a^3+1155a^2bx+840ab^2x^2+220b^3x^3)}{3080}$$

input
```
Integrate[x^(2/3)*(a + b*x)^3,x]
```

output
```
(3*x^(5/3)*(616*a^3 + 1155*a^2*b*x + 840*a*b^2*x^2 + 220*b^3*x^3))/3080
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2/3}(a+bx)^3\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(a^3x^{2/3}+3a^2bx^{5/3}+3ab^2x^{8/3}+b^3x^{11/3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

input
```
Int[x^(2/3)*(a + b*x)^3,x]
```

output
```
(3*a^3*x^(5/3))/5 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^(11/3))/11 + (3*b^3*x
^(14/3))/14
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}\left(220b^{3}x^{3}+840a\,b^{2}x^{2}+1155a^{2}bx+616a^{3}\right)}{3080}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{5}{3}}}{5}+\frac{9a^{2}b\,x^{\frac{8}{3}}}{8}+\frac{9a\,b^{2}x^{\frac{11}{3}}}{11}+\frac{3b^{3}x^{\frac{14}{3}}}{14}$	36
default	$\frac{3a^{3}x^{\frac{5}{3}}}{5}+\frac{9a^{2}b\,x^{\frac{8}{3}}}{8}+\frac{9a\,b^{2}x^{\frac{11}{3}}}{11}+\frac{3b^{3}x^{\frac{14}{3}}}{14}$	36
trager	$\frac{3x^{\frac{5}{3}}\left(220b^{3}x^{3}+840a\,b^{2}x^{2}+1155a^{2}bx+616a^{3}\right)}{3080}$	36
risch	$\frac{3x^{\frac{5}{3}}\left(220b^{3}x^{3}+840a\,b^{2}x^{2}+1155a^{2}bx+616a^{3}\right)}{3080}$	36
orering	$\frac{3x^{\frac{5}{3}}\left(220b^{3}x^{3}+840a\,b^{2}x^{2}+1155a^{2}bx+616a^{3}\right)}{3080}$	36

input
```
int(x^(2/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/3080*x^(5/3)*(220*b^3*x^3+840*a*b^2*x^2+1155*a^2*b*x+616*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3}{3080}\left(220\,b^3x^4+840\,ab^2x^3+1155\,a^2bx^2+616\,a^3x\right)x^{\frac{2}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
3/3080*(220*b^3*x^4 + 840*a*b^2*x^3 + 1155*a^2*b*x^2 + 616*a^3*x)*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3a^3x^{\frac{5}{3}}}{5}+\frac{9a^2bx^{\frac{8}{3}}}{8}+\frac{9ab^2x^{\frac{11}{3}}}{11}+\frac{3b^3x^{\frac{14}{3}}}{14}$$

input
```
integrate(x**(2/3)*(b*x+a)**3,x)
```

output
```
3*a**3*x**(5/3)/5 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(11/3)/11 + 3*b**3*x
**(14/3)/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3}{14}\,b^3x^{\frac{14}{3}}+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{8}\,a^2bx^{\frac{8}{3}}+\frac{3}{5}\,a^3x^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5
/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3}{14}\,b^3x^{\frac{14}{3}}+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{8}\,a^2bx^{\frac{8}{3}}+\frac{3}{5}\,a^3x^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(b*x+a)^3,x, algorithm="giac")
```

output
```
3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5
/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3\,a^3\,x^{5/3}}{5}+\frac{3\,b^3\,x^{14/3}}{14}+\frac{9\,a^2\,b\,x^{8/3}}{8}+\frac{9\,a\,b^2\,x^{11/3}}{11}$$

input
```
int(x^(2/3)*(a + b*x)^3,x)
```

output
```
(3*a^3*x^(5/3))/5 + (3*b^3*x^(14/3))/14 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x
^(11/3))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{2/3}(a+bx)^3\,dx=\frac{3x^{\frac{5}{3}}(220b^3x^3+840a\,b^2x^2+1155a^2bx+616a^3)}{3080}$$

input
```
int(x^(2/3)*(b*x+a)^3,x)
```

output
```
(3*x**(2/3)*x*(616*a**3 + 1155*a**2*b*x + 840*a*b**2*x**2 + 220*b**3*x**3)
)/3080
```

3.312 $\int \sqrt[3]{x}(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \frac{3}{4}a^3x^{4/3} + \frac{9}{7}a^2bx^{7/3} + \frac{9}{10}ab^2x^{10/3} + \frac{3}{13}b^3x^{13/3}$$

output
```
3/4*a^3*x^(4/3)+9/7*a^2*b*x^(7/3)+9/10*a*b^2*x^(10/3)+3/13*b^3*x^(13/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \frac{3x^{4/3}(455a^3 + 780a^2bx + 546ab^2x^2 + 140b^3x^3)}{1820}$$

input
```
Integrate[x^(1/3)*(a + b*x)^3,x]
```

output
```
(3*x^(4/3)*(455*a^3 + 780*a^2*b*x + 546*a*b^2*x^2 + 140*b^3*x^3))/1820
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x}(a+bx)^3 \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(a^3 \sqrt[3]{x}+3 a^2 b x^{4/3}+3 a b^2 x^{7/3}+b^3 x^{10/3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{4} a^3 x^{4/3}+\frac{9}{7} a^2 b x^{7/3}+\frac{9}{10} a b^2 x^{10/3}+\frac{3}{13} b^3 x^{13/3}$$

input
```
Int[x^(1/3)*(a + b*x)^3,x]
```

output
```
(3*a^3*x^(4/3))/4 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(10/3))/10 + (3*b^3*x
^(13/3))/13
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}\left(140b^{3}x^{3}+546a\,b^{2}x^{2}+780a^{2}bx+455a^{3}\right)}{1820}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{4}{3}}}{4}+\frac{9a^{2}b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^{2}x^{\frac{10}{3}}}{10}+\frac{3b^{3}x^{\frac{13}{3}}}{13}$	36
default	$\frac{3a^{3}x^{\frac{4}{3}}}{4}+\frac{9a^{2}b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^{2}x^{\frac{10}{3}}}{10}+\frac{3b^{3}x^{\frac{13}{3}}}{13}$	36
trager	$\frac{3x^{\frac{4}{3}}\left(140b^{3}x^{3}+546a\,b^{2}x^{2}+780a^{2}bx+455a^{3}\right)}{1820}$	36
risch	$\frac{3x^{\frac{4}{3}}\left(140b^{3}x^{3}+546a\,b^{2}x^{2}+780a^{2}bx+455a^{3}\right)}{1820}$	36
orering	$\frac{3x^{\frac{4}{3}}\left(140b^{3}x^{3}+546a\,b^{2}x^{2}+780a^{2}bx+455a^{3}\right)}{1820}$	36

input
```
int(x^(1/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/1820*x^(4/3)*(140*b^3*x^3+546*a*b^2*x^2+780*a^2*b*x+455*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \frac{3}{1820}\left(140\,b^3x^4+546\,ab^2x^3+780\,a^2bx^2+455\,a^3x\right)x^{\frac{1}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
3/1820*(140*b^3*x^4 + 546*a*b^2*x^3 + 780*a^2*b*x^2 + 455*a^3*x)*x^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 5012, normalized size of antiderivative = 98.27

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \text{Too large to display}$$

input

```
integrate(x**(1/3)*(b*x+a)**3,x)
```

output

```
Piecewise((-243*a**(73/3)*(-1 + b*(a/b + x)/a)**(1/3)/(1820*a**20*b**(4/3)
 - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 3
6400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 1
0920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 2
43*a**(73/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x
)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x
)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(-1 + b*(a/b
+ x)/a)**(1/3)*(a/b + x)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x
)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x
)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp
(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**
18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**
16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**1
4*b**(22/3)*(a/b + x)**6) - 3213*a**(67/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3
)*(a/b + x)**2/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 273
00*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 182
0*a**14*b**(22/3)*(a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \frac{3}{13}\,b^3x^{\frac{13}{3}} + \frac{9}{10}\,ab^2x^{\frac{10}{3}} + \frac{9}{7}\,a^2bx^{\frac{7}{3}} + \frac{3}{4}\,a^3x^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4
/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \frac{3}{13}\,b^3x^{\frac{13}{3}} + \frac{9}{10}\,ab^2x^{\frac{10}{3}} + \frac{9}{7}\,a^2bx^{\frac{7}{3}} + \frac{3}{4}\,a^3x^{\frac{4}{3}}$$

input
```
integrate(x^(1/3)*(b*x+a)^3,x, algorithm="giac")
```

output
```
3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4
/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \frac{3\,a^3\,x^{4/3}}{4} + \frac{3\,b^3\,x^{13/3}}{13} + \frac{9\,a^2\,b\,x^{7/3}}{7} + \frac{9\,a\,b^2\,x^{10/3}}{10}$$

input
```
int(x^(1/3)*(a + b*x)^3,x)
```

output
```
(3*a^3*x^(4/3))/4 + (3*b^3*x^(13/3))/13 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x
^(10/3))/10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt[3]{x}(a+bx)^3\,dx = \frac{3x^{\frac{4}{3}}(140b^3x^3+546a\,b^2x^2+780a^2bx+455a^3)}{1820}$$

input
```
int(x^(1/3)*(b*x+a)^3,x)
```

output
```
(3*x**(1/3)*x*(455*a**3 + 780*a**2*b*x + 546*a*b**2*x**2 + 140*b**3*x**3))
/1820
```

3.313 $\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx = \frac{3}{2}a^3x^{2/3} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3}$$

output
```
3/2*a^3*x^(2/3)+9/5*a^2*b*x^(5/3)+9/8*a*b^2*x^(8/3)+3/11*b^3*x^(11/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx = \frac{3}{440}x^{2/3}\left(220a^3 + 264a^2bx + 165ab^2x^2 + 40b^3x^3\right)$$

input
```
Integrate[(a + b*x)^3/x^(1/3),x]
```

output
```
(3*x^(2/3)*(220*a^3 + 264*a^2*b*x + 165*a*b^2*x^2 + 40*b^3*x^3))/440
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^3}{\sqrt[3]{x}} + 3a^2bx^{2/3} + 3ab^2x^{5/3} + b^3x^{8/3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}a^3x^{2/3} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3}$$

input
```
Int[(a + b*x)^3/x^(1/3),x]
```

output
```
(3*a^3*x^(2/3))/2 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^(8/3))/8 + (3*b^3*x^(
11/3))/11
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

method	result	size
trager	$\left(\frac{3}{11}b^{3}x^{3}+\frac{9}{8}a\,b^{2}x^{2}+\frac{9}{5}a^{2}bx+\frac{3}{2}a^{3}\right)x^{\frac{2}{3}}$	35
gosper	$\frac{3x^{\frac{2}{3}}\left(40b^{3}x^{3}+165a\,b^{2}x^{2}+264a^{2}bx+220a^{3}\right)}{440}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{2}{3}}}{2}+\frac{9a^{2}b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^{2}x^{\frac{8}{3}}}{8}+\frac{3b^{3}x^{\frac{11}{3}}}{11}$	36
default	$\frac{3a^{3}x^{\frac{2}{3}}}{2}+\frac{9a^{2}b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^{2}x^{\frac{8}{3}}}{8}+\frac{3b^{3}x^{\frac{11}{3}}}{11}$	36
risch	$\frac{3x^{\frac{2}{3}}\left(40b^{3}x^{3}+165a\,b^{2}x^{2}+264a^{2}bx+220a^{3}\right)}{440}$	36
orering	$\frac{3x^{\frac{2}{3}}\left(40b^{3}x^{3}+165a\,b^{2}x^{2}+264a^{2}bx+220a^{3}\right)}{440}$	36

input
```
int((b*x+a)^3/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(3/11*b^3*x^3+9/8*a*b^2*x^2+9/5*a^2*b*x+3/2*a^3)*x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx=\frac{3}{440}\left(40\,b^3x^3+165\,ab^2x^2+264\,a^2bx+220\,a^3\right)x^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^3/x^(1/3),x, algorithm="fricas")
```

output
```
3/440*(40*b^3*x^3 + 165*a*b^2*x^2 + 264*a^2*b*x + 220*a^3)*x^(2/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 6246, normalized size of antiderivative = 122.47

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3/x**(1/3),x)
```

output
```
Piecewise((243*a**(71/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(440*a**2
0*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*
a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**
3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp
(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**
(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3)
 - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a
/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1296*a**(68/3)*b*(-1 + b*(a
/b + x)/a)**(2/3)*(a/b + x)*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) -
2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)*
*2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**1
6*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*e
xp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3
)*b*(a/b + x)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b +
 x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**1
7*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*e
xp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**
(20/3)*(a/b + x)**6*exp(I*pi/3)) + 2808*a**(65/3)*b**2*(-1 + b*(a/b + x...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx = \frac{3}{11}\,b^3x^{\frac{11}{3}} + \frac{9}{8}\,ab^2x^{\frac{8}{3}} + \frac{9}{5}\,a^2bx^{\frac{5}{3}} + \frac{3}{2}\,a^3x^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^3/x^(1/3),x, algorithm="maxima")
```

output
```
3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx = \frac{3}{11}\,b^3x^{\frac{11}{3}} + \frac{9}{8}\,ab^2x^{\frac{8}{3}} + \frac{9}{5}\,a^2bx^{\frac{5}{3}} + \frac{3}{2}\,a^3x^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^3/x^(1/3),x, algorithm="giac")
```

output
```
3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx = \frac{3\,a^3\,x^{2/3}}{2} + \frac{3\,b^3\,x^{11/3}}{11} + \frac{9\,a^2\,b\,x^{5/3}}{5} + \frac{9\,a\,b^2\,x^{8/3}}{8}$$

input
```
int((a + b*x)^3/x^(1/3),x)
```

output
```
(3*a^3*x^(2/3))/2 + (3*b^3*x^(11/3))/11 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x
^(8/3))/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx = \frac{3x^{\frac{2}{3}}(40b^3x^3+165a\,b^2x^2+264a^2bx+220a^3)}{440}$$

input
```
int((b*x+a)^3/x^(1/3),x)
```

output
```
(3*x**(2/3)*(220*a**3 + 264*a**2*b*x + 165*a*b**2*x**2 + 40*b**3*x**3))/44
0
```

3.314 $\int \frac{(a+bx)^3}{x^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{(a+bx)^3}{x^{2/3}}\,dx = 3a^3\sqrt[3]{x} + \frac{9}{4}a^2bx^{4/3} + \frac{9}{7}ab^2x^{7/3} + \frac{3}{10}b^3x^{10/3}$$

output
```
3*a^3*x^(1/3)+9/4*a^2*b*x^(4/3)+9/7*a*b^2*x^(7/3)+3/10*b^3*x^(10/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^3}{x^{2/3}}\,dx = \frac{3}{140}\sqrt[3]{x}\left(140a^3 + 105a^2bx + 60ab^2x^2 + 14b^3x^3\right)$$

input
```
Integrate[(a + b*x)^3/x^(2/3),x]
```

output
```
(3*x^(1/3)*(140*a^3 + 105*a^2*b*x + 60*a*b^2*x^2 + 14*b^3*x^3))/140
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^{2/3}} \, dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{a^3}{x^{2/3}}+3a^2b\sqrt[3]{x}+3ab^2x^{4/3}+b^3x^{7/3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

input
```
Int[(a + b*x)^3/x^(2/3),x]
```

output
```
3*a^3*x^(1/3) + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3))/7 + (3*b^3*x^(10/3
))/10
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
trager	$\left(\frac{3}{10}b^{3}x^{3}+\frac{9}{7}a\,b^{2}x^{2}+\frac{9}{4}a^{2}bx+3a^{3}\right)x^{\frac{1}{3}}$	35
gosper	$\frac{3x^{\frac{1}{3}}\left(14b^{3}x^{3}+60a\,b^{2}x^{2}+105a^{2}bx+140a^{3}\right)}{140}$	36
derivativedivides	$3a^{3}x^{\frac{1}{3}}+\frac{9a^{2}b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^{2}x^{\frac{7}{3}}}{7}+\frac{3b^{3}x^{\frac{10}{3}}}{10}$	36
default	$3a^{3}x^{\frac{1}{3}}+\frac{9a^{2}b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^{2}x^{\frac{7}{3}}}{7}+\frac{3b^{3}x^{\frac{10}{3}}}{10}$	36
risch	$\frac{3x^{\frac{1}{3}}\left(14b^{3}x^{3}+60a\,b^{2}x^{2}+105a^{2}bx+140a^{3}\right)}{140}$	36
orering	$\frac{3x^{\frac{1}{3}}\left(14b^{3}x^{3}+60a\,b^{2}x^{2}+105a^{2}bx+140a^{3}\right)}{140}$	36

input
```
int((b*x+a)^3/x^(2/3),x,method=_RETURNVERBOSE)
```

output
```
(3/10*b^3*x^3+9/7*a*b^2*x^2+9/4*a^2*b*x+3*a^3)*x^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{2/3}}\,dx=\frac{3}{140}\left(14\,b^3x^3+60\,ab^2x^2+105\,a^2bx+140\,a^3\right)x^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^3/x^(2/3),x, algorithm="fricas")
```

output
```
3/140*(14*b^3*x^3 + 60*a*b^2*x^2 + 105*a^2*b*x + 140*a^3)*x^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 6667, normalized size of antiderivative = 136.06

$$\int \frac{(a+bx)^3}{x^{2/3}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3/x**(2/3),x)
```

output
```
Piecewise((243*a**(70/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(140*a*
*20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
 + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b
 + x)**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
 x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1
377*a**(67/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(140*a
**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/
b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)
- 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/
b + x)**6*exp(2*I*pi/3)) - 1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b*
*(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(
2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b*
*(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*e...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{2/3}}\,dx = \frac{3}{10}\,b^3x^{\frac{10}{3}} + \frac{9}{7}\,ab^2x^{\frac{7}{3}} + \frac{9}{4}\,a^2bx^{\frac{4}{3}} + 3\,a^3x^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^3/x^(2/3),x, algorithm="maxima")
```

output
```
3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{2/3}}\,dx = \frac{3}{10}\,b^3x^{\frac{10}{3}} + \frac{9}{7}\,ab^2x^{\frac{7}{3}} + \frac{9}{4}\,a^2bx^{\frac{4}{3}} + 3\,a^3x^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^3/x^(2/3),x, algorithm="giac")
```

output
```
3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{2/3}}\,dx = 3\,a^3\,x^{1/3} + \frac{3\,b^3\,x^{10/3}}{10} + \frac{9\,a^2\,b\,x^{4/3}}{4} + \frac{9\,a\,b^2\,x^{7/3}}{7}$$

input
```
int((a + b*x)^3/x^(2/3),x)
```

output
```
3*a^3*x^(1/3) + (3*b^3*x^(10/3))/10 + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/
3))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{2/3}}\,dx = \frac{3x^{\frac{1}{3}}(14b^3x^3+60a\,b^2x^2+105a^2bx+140a^3)}{140}$$

input
```
int((b*x+a)^3/x^(2/3),x)
```

output
```
(3*x**(1/3)*(140*a**3 + 105*a**2*b*x + 60*a*b**2*x**2 + 14*b**3*x**3))/140
```

3.315 $\int \frac{(a+bx)^3}{x^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = -\frac{3a^3}{\sqrt[3]{x}} + \frac{9}{2}a^2bx^{2/3} + \frac{9}{5}ab^2x^{5/3} + \frac{3}{8}b^3x^{8/3}$$

output
```
-3*a^3/x^(1/3)+9/2*a^2*b*x^(2/3)+9/5*a*b^2*x^(5/3)+3/8*b^3*x^(8/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = -\frac{3(40a^3 - 60a^2bx - 24ab^2x^2 - 5b^3x^3)}{40\sqrt[3]{x}}$$

input
```
Integrate[(a + b*x)^3/x^(4/3),x]
```

output
```
(-3*(40*a^3 - 60*a^2*b*x - 24*a*b^2*x^2 - 5*b^3*x^3))/(40*x^(1/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^{4/3}} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^3}{x^{4/3}}+\frac{3a^2b}{\sqrt[3]{x}}+3ab^2x^{2/3}+b^3x^{5/3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

input
```
Int[(a + b*x)^3/x^(4/3),x]
```

output
```
(-3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3))/5 + (3*b^3*x^(8
/3))/8
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{3(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3)}{40x^{\frac{1}{3}}}$	36
derivativedivides	$-\frac{3a^3}{x^{\frac{1}{3}}}+\frac{9a^2b\,x^{\frac{2}{3}}}{2}+\frac{9a\,b^2x^{\frac{5}{3}}}{5}+\frac{3b^3x^{\frac{8}{3}}}{8}$	36
default	$-\frac{3a^3}{x^{\frac{1}{3}}}+\frac{9a^2b\,x^{\frac{2}{3}}}{2}+\frac{9a\,b^2x^{\frac{5}{3}}}{5}+\frac{3b^3x^{\frac{8}{3}}}{8}$	36
trager	$-\frac{3(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3)}{40x^{\frac{1}{3}}}$	36
risch	$-\frac{3(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3)}{40x^{\frac{1}{3}}}$	36
orering	$-\frac{3(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3)}{40x^{\frac{1}{3}}}$	36

input
```
int((b*x+a)^3/x^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/40*(-5*b^3*x^3-24*a*b^2*x^2-60*a^2*b*x+40*a^3)/x^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = \frac{3\,(5\,b^3x^3+24\,ab^2x^2+60\,a^2bx-40\,a^3)}{40\,x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)^3/x^(4/3),x, algorithm="fricas")
```

output
```
3/40*(5*b^3*x^3 + 24*a*b^2*x^2 + 60*a^2*b*x - 40*a^3)/x^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 4004, normalized size of antiderivative = 81.71

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3/x**(4/3),x)
```

output
```
Piecewise((243*a**(68/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)/(40*a**20 -
240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b
+ x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a
**14*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2*I*pi/3)/(40*a**20 -
 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b
 + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*
a**14*b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(
2/3)*(a/b + x)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x
)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**
15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(65/3)*b**(4/
3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b
**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)*
*4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 2808*a**(
62/3)*b**(7/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(40*a**20 - 240*a*
*19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**
3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b
**6*(a/b + x)**6) - 3645*a**(62/3)*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40
*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b
**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)*
*5 + 40*a**14*b**6*(a/b + x)**6) - 3120*a**(59/3)*b**(10/3)*(-1 + b*(a/...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = \frac{3}{8}\,b^3x^{\frac{8}{3}} + \frac{9}{5}\,ab^2x^{\frac{5}{3}} + \frac{9}{2}\,a^2bx^{\frac{2}{3}} - \frac{3\,a^3}{x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)^3/x^(4/3),x, algorithm="maxima")
```

output
```
3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = \frac{3}{8}\,b^3x^{\frac{8}{3}} + \frac{9}{5}\,ab^2x^{\frac{5}{3}} + \frac{9}{2}\,a^2bx^{\frac{2}{3}} - \frac{3\,a^3}{x^{\frac{1}{3}}}$$

input
```
integrate((b*x+a)^3/x^(4/3),x, algorithm="giac")
```

output
```
3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = \frac{3\,b^3\,x^{8/3}}{8} - \frac{3\,a^3}{x^{1/3}} + \frac{9\,a^2\,b\,x^{2/3}}{2} + \frac{9\,a\,b^2\,x^{5/3}}{5}$$

input
```
int((a + b*x)^3/x^(4/3),x)
```

output
```
(3*b^3*x^(8/3))/8 - (3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/
3))/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{4/3}}\,dx = \frac{\frac{3}{8}b^3x^3+\frac{9}{5}a\,b^2x^2+\frac{9}{2}a^2bx-3a^3}{x^{\frac{1}{3}}}$$

input
```
int((b*x+a)^3/x^(4/3),x)
```

output
```
(3*( - 40*a**3 + 60*a**2*b*x + 24*a*b**2*x**2 + 5*b**3*x**3))/(40*x**(1/3)
)
```

3.316 $\int \frac{(a+bx)^3}{x^{5/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx = -\frac{3a^3}{2x^{2/3}} + 9a^2b\sqrt[3]{x} + \frac{9}{4}ab^2x^{4/3} + \frac{3}{7}b^3x^{7/3}$$

output
```
-3/2*a^3/x^(2/3)+9*a^2*b*x^(1/3)+9/4*a*b^2*x^(4/3)+3/7*b^3*x^(7/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx = -\frac{3(14a^3 - 84a^2bx - 21ab^2x^2 - 4b^3x^3)}{28x^{2/3}}$$

input
```
Integrate[(a + b*x)^3/x^(5/3),x]
```

output
```
(-3*(14*a^3 - 84*a^2*b*x - 21*a*b^2*x^2 - 4*b^3*x^3))/(28*x^(2/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{x^{5/3}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^3}{x^{5/3}}+\frac{3a^2b}{x^{2/3}}+3ab^2\sqrt[3]{x}+b^3x^{4/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

input
```
Int[(a + b*x)^3/x^(5/3),x]
```

output
```
(-3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3))/4 + (3*b^3*x^(7
/3))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{3\left(-4 b^{3} x^{3}-21 a \,b^{2} x^{2}-84 a^{2} b x +14 a^{3}\right)}{28 x^{\frac{2}{3}}}$	36
derivativedivides	$-\frac{3 a^{3}}{2 x^{\frac{2}{3}}}+9 a^{2} b \,x^{\frac{1}{3}}+\frac{9 a \,b^{2} x^{\frac{4}{3}}}{4}+\frac{3 b^{3} x^{\frac{7}{3}}}{7}$	36
default	$-\frac{3 a^{3}}{2 x^{\frac{2}{3}}}+9 a^{2} b \,x^{\frac{1}{3}}+\frac{9 a \,b^{2} x^{\frac{4}{3}}}{4}+\frac{3 b^{3} x^{\frac{7}{3}}}{7}$	36
trager	$-\frac{3\left(-4 b^{3} x^{3}-21 a \,b^{2} x^{2}-84 a^{2} b x +14 a^{3}\right)}{28 x^{\frac{2}{3}}}$	36
risch	$-\frac{3\left(-4 b^{3} x^{3}-21 a \,b^{2} x^{2}-84 a^{2} b x +14 a^{3}\right)}{28 x^{\frac{2}{3}}}$	36
orering	$-\frac{3\left(-4 b^{3} x^{3}-21 a \,b^{2} x^{2}-84 a^{2} b x +14 a^{3}\right)}{28 x^{\frac{2}{3}}}$	36

input
```
int((b*x+a)^3/x^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-3/28*(-4*b^3*x^3-21*a*b^2*x^2-84*a^2*b*x+14*a^3)/x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx=\frac{3\left(4\,b^3x^3+21\,ab^2x^2+84\,a^2bx-14\,a^3\right)}{28\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^3/x^(5/3),x, algorithm="fricas")
```

output
```
3/28*(4*b^3*x^3 + 21*a*b^2*x^2 + 84*a^2*b*x - 14*a^3)/x^(2/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 3964, normalized size of antiderivative = 80.90

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3/x**(5/3),x)
```

output
```
Piecewise((243*a**(67/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)/(28*a**20 -
168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b
+ x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a
**14*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**20 - 1
68*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b +
 x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a*
*14*b**6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/
3)*(a/b + x)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)*
*2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15
*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3)*b**(5/3)
*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*
(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 -
 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a**(61/3
)*b**(8/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(28*a**20 - 168*a**19*
b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 +
420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*
(a/b + x)**6) - 3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20
 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a
/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 2
8*a**14*b**6*(a/b + x)**6) - 3927*a**(58/3)*b**(11/3)*(-1 + b*(a/b + x)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx=\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^3/x^(5/3),x, algorithm="maxima")
```

output
```
3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx=\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^3/x^(5/3),x, algorithm="giac")
```

output
```
3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx=\frac{3\,b^3\,x^{7/3}}{7}-\frac{3\,a^3}{2\,x^{2/3}}+9\,a^2\,b\,x^{1/3}+\frac{9\,a\,b^2\,x^{4/3}}{4}$$

input
```
int((a + b*x)^3/x^(5/3),x)
```

output
```
(3*b^3*x^(7/3))/7 - (3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/
3))/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^3}{x^{5/3}}\,dx = \frac{\frac{3}{7}b^3x^3 + \frac{9}{4}a\,b^2x^2 + 9a^2bx - \frac{3}{2}a^3}{x^{\frac{2}{3}}}$$

input
```
int((b*x+a)^3/x^(5/3),x)
```

output
```
(3*( - 14*a**3 + 84*a**2*b*x + 21*a*b**2*x**2 + 4*b**3*x**3))/(28*x**(2/3)
)
```

3.317 $\int \frac{x^{5/3}}{a+bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 125

$$\int \frac{x^{5/3}}{a+bx}\,dx=-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}-\frac{\sqrt{3}a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}$$

output
```
-3/2*a*x^(2/3)/b^2+3/5*x^(5/3)/b-3^(1/2)*a^(5/3)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x^(1/3))*3^(1/2)/a^(1/3))/b^(8/3)-3/2*a^(5/3)*ln(a^(1/3)+b^(1/3)*x^(1
/3))/b^(8/3)+1/2*a^(5/3)*ln(b*x+a)/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

$$\int \frac{x^{5/3}}{a+bx}\,dx=\frac{-15ab^{2/3}x^{2/3}+6b^{5/3}x^{5/3}-10\sqrt{3}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+}{10b^{8/3}}$$

input
```
Integrate[x^(5/3)/(a + b*x),x]
```

output
```
(-15*a*b^(2/3)*x^(2/3) + 6*b^(5/3)*x^(5/3) - 10*Sqrt[3]*a^(5/3)*ArcTan[(1
- (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 10*a^(5/3)*Log[a^(1/3) + b^(1/3)
*x^(1/3)] + 5*a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2
/3)])/(10*b^(8/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {60, 60, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/3}}{a+bx}dx$$

$$\Big\downarrow \ 60$$

$$\frac{3x^{5/3}}{5b}-\frac{a\int \frac{x^{2/3}}{a+bx}dx}{b}$$

$$\Big\downarrow \ 60$$

$$\frac{3x^{5/3}}{5b}-\frac{a\left(\frac{3x^{2/3}}{2b}-\frac{a\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \ 68$$

$$\frac{3x^{5/3}}{5b}-\frac{a\left(\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{3x^{5/3}}{5b}-\frac{a\left(\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3x^{5/3}}{5b}-\frac{a\left(\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3x^{5/3}}{5b}-\frac{a\left(\frac{3x^{2/3}}{2b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{b}$$

input
```
Int[x^(5/3)/(a + b*x),x]
```

output
```
(3*x^(5/3))/(5*b) - (a*((3*x^(2/3))/(2*b) - (a*(-((Sqrt[3]*ArcTan[(1 - (2*
b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a^(1/3) +
b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b^(2/3))))
/b))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{3(-2bx+5a)x^{\frac{2}{3}}}{10b^2}-\frac{a^2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$-\frac{3\left(-\frac{b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{\frac{2}{3}}}{2}\right)}{b^2}+\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^2}{b^2}$
default	$-\frac{3\left(-\frac{b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{\frac{2}{3}}}{2}\right)}{b^2}+\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^2}{b^2}$

input
```
int(x^(5/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3/10*(-2*b*x+5*a)*x^(2/3)/b^2-a^2/b^3/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))
+1/2*a^2/b^3/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+a^2/b
^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.18

$$\int \frac{x^{5/3}}{a+bx}\,dx = \frac{10\sqrt{3}a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3\,a}\right)-5\,a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}-a\left(\right.\right.}{10\,b^2}$$

input
```
integrate(x^(5/3)/(b*x+a),x, algorithm="fricas")
```

output
```
1/10*(10*sqrt(3)*a*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a^2/
b^2)^(1/3) + sqrt(3)*a)/a) - 5*a*(-a^2/b^2)^(1/3)*log(-b*x^(1/3)*(-a^2/b^2
)^(2/3) + a*x^(2/3) - a*(-a^2/b^2)^(1/3)) + 10*a*(-a^2/b^2)^(1/3)*log(b*(-
a^2/b^2)^(2/3) + a*x^(1/3)) + 3*(2*b*x - 5*a)*x^(2/3))/b^2
```

Sympy [A] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.44

$$\int \frac{x^{5/3}}{a+bx}\,dx = \begin{cases} \tilde{\infty}x^{\frac{5}{3}} \\ \frac{3x^{\frac{8}{3}}}{8a} \\ \frac{3x^{\frac{5}{3}}}{5b} \\ -\frac{a^3\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b^4\left(-\frac{a}{b}\right)^{\frac{4}{3}}}+\frac{a^3\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^4\left(-\frac{a}{b}\right)^{\frac{4}{3}}}-\frac{\sqrt{3}a^3\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b^4\left(-\frac{a}{b}\right)^{\frac{4}{3}}}-\frac{3ax^{\frac{2}{3}}}{2b^2}+ \end{cases}$$

input
```
integrate(x**(5/3)/(b*x+a),x)
```

output
```
Piecewise((zoo*x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(8/3)/(8*a), Eq(b, 0)
), (3*x**(5/3)/(5*b), Eq(a, 0)), (-a**3*log(x**(1/3) - (-a/b)**(1/3))/(b**
4*(-a/b)**(4/3)) + a**3*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/
b)**(2/3))/(2*b**4*(-a/b)**(4/3)) - sqrt(3)*a**3*atan(2*sqrt(3)*x**(1/3)/(
3*(-a/b)**(1/3)) + sqrt(3)/3)/(b**4*(-a/b)**(4/3)) - 3*a*x**(2/3)/(2*b**2)
 + 3*x**(5/3)/(5*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/3}}{a+bx}\,dx = \frac{\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a^2\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a^2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{3\left(2\,bx^{\frac{5}{3}}-5\,ax^{\frac{2}{3}}\right)}{10\,b^2}$$

input
```
integrate(x^(5/3)/(b*x+a),x, algorithm="maxima")
```

output
```
sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3
*(a/b)^(1/3)) + 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(
b^3*(a/b)^(1/3)) - a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/10
*(2*b*x^(5/3) - 5*a*x^(2/3))/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int \frac{x^{5/3}}{a+bx}\,dx = -\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^2} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^4} + \frac{(-ab^2)^{\frac{2}{3}}\,a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^4} + \frac{3\left(2\,b^4x^{\frac{5}{3}}-5\,ab^3x^{\frac{2}{3}}\right)}{10\,b^5}$$

input
```
integrate(x^(5/3)/(b*x+a),x, algorithm="giac")
```

output
```
-a*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - sqrt(3)*(-a*b^2)^(2
/3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/
2*(-a*b^2)^(2/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4
+ 3/10*(2*b^4*x^(5/3) - 5*a*b^3*x^(2/3))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int \frac{x^{5/3}}{a+bx}\,dx = \frac{3\,x^{5/3}}{5\,b} + \frac{(-a)^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3} - \frac{9\,(-a)^{13/3}}{b^{10/3}}\right)}{b^{8/3}} - \frac{3\,a\,x^{2/3}}{2\,b^2} + \frac{(-a)^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3} - \frac{9\,(-a)^{13/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{10/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{8/3}} - \frac{(-a)^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3} - \frac{9\,(-a)^{13/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{10/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{8/3}}$$

input
```
int(x^(5/3)/(a + b*x),x)
```

output
```
(3*x^(5/3))/(5*b) + ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3))/
b^(10/3)))/b^(8/3) - (3*a*x^(2/3))/(2*b^2) + ((-a)^(5/3)*log((9*a^4*x^(1/3
))/b^3 - (9*(-a)^(13/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/
2 - 1/2))/b^(8/3) - ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3)*(
(3^(1/2)*1i)/2 + 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 + 1/2))/b^(8/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/3}}{a+bx}\,dx = \frac{-10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2 - 15x^{\frac{2}{3}}b^{\frac{2}{3}}a^{\frac{4}{3}} + 6x^{\frac{5}{3}}b^{\frac{5}{3}}a^{\frac{1}{3}} + 5\log\left(a^{\frac{2}{3}} - x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}} + x^{\frac{2}{3}}b^{\frac{2}{3}}\right)a^2 - 10}{10b^{\frac{8}{3}}a^{\frac{1}{3}}}$$

input
```
int(x^(5/3)/(b*x+a),x)
```

output

```
( - 10*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a
**2 - 15*x**(2/3)*b**(2/3)*a**(1/3)*a + 6*x**(2/3)*b**(2/3)*a**(1/3)*b*x +
 5*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a**2 - 1
0*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2)/(10*b**(2/3)*a**(1/3)*b**2)
```

3.318 $\int \frac{x^{4/3}}{a+bx} dx$

Optimal result

Integrand size = 13, antiderivative size = 123

$$\int \frac{x^{4/3}}{a+bx} dx = -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{\sqrt{3}a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}}$$

output
```
-3*a*x^(1/3)/b^2+3/4*x^(4/3)/b-3^(1/2)*a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x^(1/3))*3^(1/2)/a^(1/3))/b^(7/3)+3/2*a^(4/3)*ln(a^(1/3)+b^(1/3)*x^(1/3
))/b^(7/3)-1/2*a^(4/3)*ln(b*x+a)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14

$$\int \frac{x^{4/3}}{a+bx} dx = \frac{-12a\sqrt[3]{b}\sqrt[3]{x} + 3b^{4/3}x^{4/3} - 4\sqrt{3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) - 2a}{4b^{7/3}}$$

input
```
Integrate[x^(4/3)/(a + b*x),x]
```

output
```
(-12*a*b^(1/3)*x^(1/3) + 3*b^(4/3)*x^(4/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 -
 (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) + b^(1/3)*x
^(1/3)] - 2*a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3
)])/(4*b^(7/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {60, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4/3}}{a+bx}dx$$

$$\Big\downarrow \; 60$$

$$\frac{3x^{4/3}}{4b}-\frac{a\int \frac{\sqrt[3]{x}}{a+bx}dx}{b}$$

$$\Big\downarrow \; 60$$

$$\frac{3x^{4/3}}{4b}-\frac{a\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\int \frac{1}{x^{2/3}(a+bx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \; 70$$

$$\frac{3x^{4/3}}{4b}-\frac{a\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 16

$$\frac{3x^{4/3}}{4b}-\frac{a\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 1082

$$\frac{3x^{4/3}}{4b}-\frac{a\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 217

$$\frac{3x^{4/3}}{4b}-\frac{a\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{b}$$

input
```
Int[x^(4/3)/(a + b*x),x]
```

output
```
(3*x^(4/3))/(4*b) - (a*((3*x^(1/3))/b - (a*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1
/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/3) + b^(1
/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3)*b^(1/3))))/b))
/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{3\left(-\frac{b\,x^{\frac{4}{3}}}{4}+a\,x^{\frac{1}{3}}\right)}{b^{2}}+\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^{2}}{b^{2}}$	123
default	$-\frac{3\left(-\frac{b\,x^{\frac{4}{3}}}{4}+a\,x^{\frac{1}{3}}\right)}{b^{2}}+\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^{2}}{b^{2}}$	123

input
```
int(x^(4/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3/b^2*(-1/4*b*x^(4/3)+a*x^(1/3))+3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1
/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{x^{4/3}}{a+bx}\,dx=\frac{4\sqrt{3}a\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log}{4\,b^{2}}$$

input
```
integrate(x^(4/3)/(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(3)*a*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a/b)^(2/3) -
 sqrt(3)*a)/a) - 2*a*(a/b)^(1/3)*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)
^(2/3)) + 4*a*(a/b)^(1/3)*log(x^(1/3) + (a/b)^(1/3)) + 3*(b*x - 4*a)*x^(1/
3))/b^2
```

Sympy [A] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.41

$$\int \frac{x^{4/3}}{a+bx}\,dx = \begin{cases} \tilde{\infty}x^{\frac{4}{3}} \\ \frac{3x^{\frac{7}{3}}}{7a} \\ \frac{3x^{\frac{4}{3}}}{4b} \\ -\frac{3a\sqrt[3]{x}}{b^2} - \frac{a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b^2} + \frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^2} + \frac{\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}}{b} \end{cases}$$

input
```
integrate(x**(4/3)/(b*x+a),x)
```

output
```
Piecewise((zoo*x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a), Eq(b, 0)
), (3*x**(4/3)/(4*b), Eq(a, 0)), (-3*a*x**(1/3)/b**2 - a*(-a/b)**(1/3)*log
(x**(1/3) - (-a/b)**(1/3))/b**2 + a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1
/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b**2) + sqrt(3)*a*(-a/b)**(1/3)*at
an(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/b**2 + 3*x**(4/3)/(4*
b), True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{x^{4/3}}{a+bx}\,dx = \frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a^2\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a^2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3\left(bx^{\frac{4}{3}}-4\,ax^{\frac{1}{3}}\right)}{4\,b^2}$$

input
```
integrate(x^(4/3)/(b*x+a),x, algorithm="maxima")
```

output
```
sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3
*(a/b)^(2/3)) - 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(
b^3*(a/b)^(2/3)) + a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3/4*
(b*x^(4/3) - 4*a*x^(1/3))/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^{4/3}}{a+bx}\,dx = -\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^2} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3} + \frac{(-ab^2)^{\frac{1}{3}}\,a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^3} + \frac{3\left(b^3x^{\frac{4}{3}}-4\,ab^2x^{\frac{1}{3}}\right)}{4\,b^4}$$

input
```
integrate(x^(4/3)/(b*x+a),x, algorithm="giac")
```

output
```
-a*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + sqrt(3)*(-a*b^2)^(1
/3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/
2*(-a*b^2)^(1/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
+ 3/4*(b^3*x^(4/3) - 4*a*b^2*x^(1/3))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int \frac{x^{4/3}}{a+bx}\,dx = \frac{3\,x^{4/3}}{4\,b} - \frac{3\,a\,x^{1/3}}{b^2} + \frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{1/3}} + 9\,a^2\,x^{1/3}\right)}{b^{7/3}} + \frac{a^{4/3}\,\ln\left(9\,a^2\,x^{1/3} + \frac{9\,a^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{7/3}} - \frac{a^{4/3}\,\ln\left(9\,a^2\,x^{1/3} - \frac{9\,a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{7/3}}$$

input
```
int(x^(4/3)/(a + b*x),x)
```

output
```
(3*x^(4/3))/(4*b) - (3*a*x^(1/3))/b^2 + (a^(4/3)*log((9*a^(7/3))/b^(1/3) +
 9*a^2*x^(1/3)))/b^(7/3) + (a^(4/3)*log(9*a^2*x^(1/3) + (9*a^(7/3)*((3^(1/
2)*1i)/2 - 1/2))/b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(7/3) - (a^(4/3)*log(9
*a^2*x^(1/3) - (9*a^(7/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2
 + 1/2))/b^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.78

$$\int \frac{x^{4/3}}{a+bx}\,dx = \frac{-4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right) - 2a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}} - x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}} + x^{\frac{2}{3}}b^{\frac{2}{3}}\right) + 4a^{\frac{4}{3}}\log\left(a^{\frac{1}{3}} + x^{\frac{1}{3}}b^{\frac{1}{3}}\right) - 12x^{\frac{1}{3}}}{4b^{\frac{7}{3}}}$$

input
```
int(x^(4/3)/(b*x+a),x)
```

output
```
( - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqr
t(3)))*a - 2*a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)
*b**(2/3))*a + 4*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a - 12*x**(1/3
)*b**(1/3)*a + 3*x**(1/3)*b**(1/3)*b*x)/(4*b**(1/3)*b**2)
```

3.319 $\int \frac{x^{2/3}}{a+bx} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int \frac{x^{2/3}}{a+bx} \, dx = \frac{3x^{2/3}}{2b} + \frac{\sqrt{3}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}} + \frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

output
```
3/2*x^(2/3)/b+3^(1/2)*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/
2)/a^(1/3))/b^(5/3)+3/2*a^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(5/3)-1/2*a^
(2/3)*ln(b*x+a)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

$$\int \frac{x^{2/3}}{a+bx} \, dx = \frac{3b^{2/3}x^{2/3} + 2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) - a^{2/3}\log\left(a^{2/3} - \sqrt[3]{}\right.}{2b^{5/3}}$$

input
```
Integrate[x^(2/3)/(a + b*x),x]
```

output
```
(3*b^(2/3)*x^(2/3) + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(
1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - a^(2/3)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(5/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = $\{60, 68, 16, 1082, 217\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2/3}}{a+bx}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3x^{2/3}}{2b}-\frac{a\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{b}$$

$$\Big\downarrow \text{68}$$

$$\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3x^{2/3}}{2b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}$$

input
```
Int[x^(2/3)/(a + b*x),x]
```

output
```
(3*x^(2/3))/(2*b) - (a*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3)
)/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(
1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b^(2/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{3x^{\frac{2}{3}}}{2b}+\frac{a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	107
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2b}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	112
default	$\frac{3x^{\frac{2}{3}}}{2b}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	112

input
```
int(x^(2/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/2*x^(2/3)/b+a/b^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2*a/b^2/(a/b)^(1
/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-a/b^2*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{x^{2/3}}{a+bx}\,dx = -\frac{2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\right.\right.}{2b}$$

input
```
integrate(x^(2/3)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) + (a^2/b^2)^(1/3)*log(-b*x^(1/3)*(a^2/b^2)^(2/3) + a
*x^(2/3) + a*(a^2/b^2)^(1/3)) - 2*(a^2/b^2)^(1/3)*log(b*(a^2/b^2)^(2/3) +
a*x^(1/3)) - 3*x^(2/3))/b
```

Sympy [A] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.46

$$\int \frac{x^{2/3}}{a+bx}\,dx = \begin{cases} \tilde{\infty}x^{\frac{2}{3}} & \text{for } a \\ \frac{3x^{\frac{5}{3}}}{5a} & \text{for } b \\ \frac{3x^{\frac{2}{3}}}{2b} & \text{for } a \\ -\frac{a\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b^2\sqrt[3]{-\frac{a}{b}}}+\frac{a\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^2\sqrt[3]{-\frac{a}{b}}}-\frac{\sqrt{3}a\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b^2\sqrt[3]{-\frac{a}{b}}}+\frac{3x^{\frac{2}{3}}}{2b} & \text{othe} \end{cases}$$

input
```
integrate(x**(2/3)/(b*x+a),x)
```

output
```
Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a), Eq(b, 0)
), (3*x**(2/3)/(2*b), Eq(a, 0)), (-a*log(x**(1/3) - (-a/b)**(1/3))/(b**2*(
-a/b)**(1/3)) + a*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2
/3))/(2*b**2*(-a/b)**(1/3)) - sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)*
*(1/3)) + sqrt(3)/3)/(b**2*(-a/b)**(1/3)) + 3*x**(2/3)/(2*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \frac{x^{2/3}}{a+bx}\,dx = -\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\,x^{\frac{2}{3}}}{2\,b} - \frac{a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^(2/3)/(b*x+a),x, algorithm="maxima")
```

output
```
-sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*
(a/b)^(1/3)) + 3/2*x^(2/3)/b - 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (
a/b)^(2/3))/(b^2*(a/b)^(1/3)) + a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(1
/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b}+\frac{3\,x^{\frac{2}{3}}}{2\,b} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3}-\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^3}$$

input
```
integrate(x^(2/3)/(b*x+a),x, algorithm="giac")
```

output
```
(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b + 3/2*x^(2/3)/b + sqrt(3)*
(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))
/b^3 - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3
))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{3\,x^{2/3}}{2\,b} + \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{4/3}} + \frac{9\,a^2\,x^{1/3}}{b}\right)}{b^{5/3}} + \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}} + \frac{9\,a^2\,x^{1/3}}{b}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{5/3}} - \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}} + \frac{9\,a^2\,x^{1/3}}{b}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{5/3}}$$

input
```
int(x^(2/3)/(a + b*x),x)
```

output
```
(3*x^(2/3))/(2*b) + (a^(2/3)*log((9*a^(7/3))/b^(4/3) + (9*a^2*x^(1/3))/b))
/b^(5/3) + (a^(2/3)*log((9*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(4/3) + (9*
a^2*x^(1/3))/b)*((3^(1/2)*1i)/2 - 1/2))/b^(5/3) - (a^(2/3)*log((9*a^(7/3)*
((3^(1/2)*1i)/2 + 1/2)^2)/b^(4/3) + (9*a^2*x^(1/3))/b)*((3^(1/2)*1i)/2 + 1
/2))/b^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a + 3x^{\frac{2}{3}}b^{\frac{2}{3}}a^{\frac{1}{3}} - \log\left(a^{\frac{2}{3}} - x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}} + x^{\frac{2}{3}}b^{\frac{2}{3}}\right)a + 2\log\left(a^{\frac{1}{3}} + x^{\frac{1}{3}}b^{\frac{1}{3}}\right)a}{2b^{\frac{5}{3}}a^{\frac{1}{3}}}$$

input
```
int(x^(2/3)/(b*x+a),x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a + 3
*x**(2/3)*b**(2/3)*a**(1/3) - log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) +
x**(2/3)*b**(2/3))*a + 2*log(a**(1/3) + x**(1/3)*b**(1/3))*a)/(2*b**(2/3)*
a**(1/3)*b)
```

3.320 $\int \frac{\sqrt[3]{x}}{a+bx} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{\sqrt[3]{x}}{a+bx} \, dx = \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}} - \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}$$

output
```
3*x^(1/3)/b+3^(1/2)*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)
/a^(1/3))/b^(4/3)-3/2*a^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(4/3)+1/2*a^(1
/3)*ln(b*x+a)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt[3]{x}}{a+bx} \, dx$$

$$= \frac{6\sqrt[3]{b}\sqrt[3]{x} + 2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + \sqrt[3]{a}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x\right.}{2b^{4/3}}$$

input
```
Integrate[x^(1/3)/(a + b*x),x]
```

output
```
(6*b^(1/3)*x^(1/3) + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(
1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + a^(1/3)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(4/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}}{a+bx} \, dx$$

$$\Big\downarrow \ 60$$

$$\frac{3\sqrt[3]{x}}{b}-\frac{a\int \frac{1}{x^{2/3}(a+bx)}dx}{b}$$

$$\Big\downarrow \ 70$$

$$\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$$\Big\downarrow \ 16$$

$$\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$$\Big\downarrow \ 1082$$

$$\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$$\downarrow 217$$

$$\frac{3\sqrt[3]{x}}{b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}$$

input
```
Int[x^(1/3)/(a + b*x),x]
```

output
```
(3*x^(1/3))/b - (a*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sq
rt[3]])/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)
*b^(1/3)) - Log[a + b*x]/(2*a^(2/3)*b^(1/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{3x^{\frac{1}{3}}}{b}-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	112
default	$\frac{3x^{\frac{1}{3}}}{b}-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	112

input
```
int(x^(1/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)/b-3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(\right.\right.}{2b}$$

input
```
integrate(x^(1/3)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b)^(2/3) -
 sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^
(2/3)) + 2*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^(1/3)) + 6*x^(1/3))/b
```

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \begin{cases} \tilde{\infty}\sqrt[3]{x} & \text{fo} \\ \frac{3x^{\frac{4}{3}}}{4a} & \text{fo} \\ \frac{3\sqrt[3]{x}}{b} & \text{fo} \\ \frac{3\sqrt[3]{x}}{b}+\frac{\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b}-\frac{\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b}-\frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b} & \text{ot} \end{cases}$$

input
```
integrate(x**(1/3)/(b*x+a),x)
```

output
```
Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a), Eq(b, 0)
), (3*x**(1/3)/b, Eq(a, 0)), (3*x**(1/3)/b + (-a/b)**(1/3)*log(x**(1/3) -
(-a/b)**(1/3))/b - (-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3)
 + 4*(-a/b)**(2/3))/(2*b) - sqrt(3)*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/
(3*(-a/b)**(1/3)) + sqrt(3)/3)/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = -\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3\,x^{\frac{1}{3}}}{b} + \frac{a\log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a\log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^(1/3)/(b*x+a),x, algorithm="maxima")
```

output
```
-sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*
(a/b)^(2/3)) + 3*x^(1/3)/b + 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/
b)^(2/3))/(b^2*(a/b)^(2/3)) - a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(2/3
))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2} + \frac{3\,x^{\frac{1}{3}}}{b} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^2}$$

input
```
integrate(x^(1/3)/(b*x+a),x, algorithm="giac")
```

output
```
(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b - sqrt(3)*(-a*b^2)^(1/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 3*x^(1/3)
/b - 1/2*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))
/b^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{3\,x^{1/3}}{b}+\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}+9\,a\,b\,x^{1/3}\right)}{b^{4/3}} + \frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+9\,a\,b\,x^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}} - \frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-9\,a\,b\,x^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}}$$

input
```
int(x^(1/3)/(a + b*x),x)
```

output
```
(3*x^(1/3))/b + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3) + 9*a*b*x^(1/3)))/b^(
4/3) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 - 1/2) + 9*a*b
*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(4/3) - ((-a)^(1/3)*log(9*(-a)^(4/3)*b
^(2/3)*((3^(1/2)*1i)/2 + 1/2) - 9*a*b*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/b^(
4/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)-2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+x^{\frac{1}{3}}b^{\frac{1}{3}}\right)+6x^{\frac{1}{3}}b^{\frac{1}{3}}}{2b^{\frac{4}{3}}}$$

input
```
int(x^(1/3)/(b*x+a),x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3
))) + a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/
3)) - 2*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3)) + 6*x**(1/3)*b**(1/3))/
(2*b**(1/3)*b)
```

3.321 $\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}$$

output
```
-3^(1/2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))/a^(1/3)/b
^(2/3)-3/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(1/3)/b^(2/3)+1/2*ln(b*x+a)/a^(1/
3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx$$

$$= \frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[1/(x^(1/3)*(a + b*x)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x
^(2/3)])/(2*a^(1/3)*b^(2/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}dx$$

$$\Big\downarrow \text{68}$$

$$\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}$$

input
```
Int[1/(x^(1/3)*(a + b*x)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(
2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[a +
b*x]/(2*a^(1/3)*b^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	96
default	$-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	96

input
```
int(1/x^(1/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b
)^(1/3)*x^(1/3)+(a/b)^(2/3))+3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.13

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \left[\frac{\sqrt{3}ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+\sqrt{3}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}\right.}{2\,ab^2}\right.$$

input
```
integrate(1/x^(1/3)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x - a*b + sqrt(3)*(a*b
*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x + a)) + (-a*b^2)^(2/3)*log(b^2*x^(2/
3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x
^(1/3) - (-a*b^2)^(1/3)))/(a*b^2), 1/2*(2*sqrt(3)*a*b*sqrt(-(-a*b^2)^(1/3)
/a)*arctan(1/3*sqrt(3)*(2*b*x^(1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)
/a)/b) + (-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b
^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a=0 \wedge b=0 \\ \frac{3x^{\frac{2}{3}}}{2a} & \text{for } b=0 \\ -\frac{3}{b\sqrt[3]{x}} & \text{for } a=0 \\ \frac{\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b\sqrt[3]{-\frac{a}{b}}} - \frac{\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b\sqrt[3]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/3)/(b*x+a),x)
```

output
```
Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a), Eq(b, 0)
), (-3/(b*x**(1/3)), Eq(a, 0)), (log(x**(1/3) - (-a/b)**(1/3))/(b*(-a/b)**
(1/3)) - log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b
*(-a/b)**(1/3)) + sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt
(3)/3)/(b*(-a/b)**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^(1/3)/(b*x+a),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)
^(1/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1
/3)) - log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab^2}$$

input
```
integrate(1/x^(1/3)/(b*x+a),x, algorithm="giac")
```

output
```
-(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*
arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/2*
(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \frac{\ln\left(9\,b\,x^{1/3}-9\,{\left(-a\right)}^{1/3}\,b^{2/3}\right)}{{\left(-a\right)}^{1/3}\,b^{2/3}} + \frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,{\left(-a\right)}^{1/3}\,b^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{\left(-a\right)}^{1/3}\,b^{2/3}} - \frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,{\left(-a\right)}^{1/3}\,b^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{\left(-a\right)}^{1/3}\,b^{2/3}}$$

input
```
int(1/(x^(1/3)*(a + b*x)),x)
```

output
```
log(9*b*x^(1/3) - 9*(-a)^(1/3)*b^(2/3))/((-a)^(1/3)*b^(2/3)) + (log(9*b*x^
(1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*
(-a)^(1/3)*b^(2/3)) - (log(9*b*x^(1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i
 + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*(-a)^(1/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)-2\log\left(a^{\frac{1}{3}}+x^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{2b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(1/x^(1/3)/(b*x+a),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3))) +
log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3)) - 2*log(a**
(1/3) + x**(1/3)*b**(1/3)))/(2*b**(2/3)*a**(1/3))
```

3.322 $\int \frac{1}{x^{2/3}(a+bx)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{1}{x^{2/3}(a+bx)} \, dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

output
```
-3^(1/2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))/a^(2/3)/b
^(1/3)+3/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(2/3)/b^(1/3)-1/2*ln(b*x+a)/a^(2/
3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{2/3}(a+bx)} \, dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{2/3}\sqrt[3]{b}}$$

input
```
Integrate[1/(x^(2/3)*(a + b*x)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[
a^(1/3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/
3)*x^(2/3)])/(a^(2/3)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{2/3}(a+bx)}dx$$

$$\Big\downarrow \text{70}$$

$$\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

input
```
Int[1/(x^(2/3)*(a + b*x)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(2/3)*b^(
1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[a +
b*x]/(2*a^(2/3)*b^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	95
default	$\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	95

input
```
int(1/x^(2/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)
^(1/3)*x^(1/3)+(a/b)^(2/3))+1/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = \left[\frac{\sqrt{3}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+\sqrt{3}\left(2\,abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-(a}{2\,a^2b}\right.$$

input
```
integrate(1/x^(2/3)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x - a^2 + sqrt(3)*(2*a
*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/
b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x + a)) - (a^2*b)^(2/3)*log(a*b*x^(2/3)
 + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1
/3) + (a^2*b)^(2/3)))/(a^2*b), 1/2*(2*sqrt(3)*a*b*sqrt((a^2*b)^(1/3)/b)*ar
ctan(-1/3*sqrt(3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)
^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)
^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b
)]
```

Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for } a=0 \\ \frac{3\sqrt[3]{x}}{a} & \text{for } b=0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for } a=0 \\ \frac{\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(2/3)/(b*x+a),x)
```

output
```
Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a, Eq(b, 0)), (
-3/(2*b*x**(2/3)), Eq(a, 0)), (log(x**(1/3) - (-a/b)**(1/3))/(b*(-a/b)**(2
/3)) - log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b*(
-a/b)**(2/3)) - sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3
)/3)/(b*(-a/b)**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^(2/3)/(b*x+a),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)
^(2/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2
/3)) + log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab}$$

input
```
integrate(1/x^(2/3)/(b*x+a),x, algorithm="giac")
```

output
```
-(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a + sqrt(3)*(-a*b^2)^(1/3)*
arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/2*(-
a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = \frac{\ln\left(9\,a^{1/3}\,b^{5/3}+9\,b^2\,x^{1/3}\right)}{a^{2/3}\,b^{1/3}} + \frac{\ln\left(9\,b^2\,x^{1/3}+\frac{9\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(9\,b^2\,x^{1/3}-\frac{9\,a^{1/3}\,b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}$$

input
```
int(1/(x^(2/3)*(a + b*x)),x)
```

output
```
log(9*a^(1/3)*b^(5/3) + 9*b^2*x^(1/3))/(a^(2/3)*b^(1/3)) + (log(9*b^2*x^(1
/3) + (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^(2/3)
*b^(1/3)) - (log(9*b^2*x^(1/3) - (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(
3^(1/2)*1i + 1))/(2*a^(2/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)+2\log\left(a^{\frac{1}{3}}+x^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{2a^{\frac{2}{3}}b^{\frac{1}{3}}}$$

input
```
int(1/x^(2/3)/(b*x+a),x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sq
rt(3))) - log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3)) +
 2*log(a**(1/3) + x**(1/3)*b**(1/3))))/(2*b**(1/3)*a)
```

3.323 $\int \frac{1}{x^{4/3}(a+bx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = -\frac{3}{a\sqrt[3]{x}}+\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}+\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}$$

output
```
-3/a/x^(1/3)+3^(1/2)*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2
)/a^(1/3))/a^(4/3)+3/2*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(4/3)-1/2*b^(
1/3)*ln(b*x+a)/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = \frac{-\frac{6\sqrt[3]{a}}{\sqrt[3]{x}}+2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)-\sqrt[3]{b}\log\left(a^{2/3}-\right.}{2a^{4/3}}$$

input
```
Integrate[1/(x^(4/3)*(a + b*x)),x]
```

output
```
((-6*a^(1/3))/x^(1/3) + 2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/
a^(1/3))/Sqrt[3]] + 2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - b^(1/3)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a^(4/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {61, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{4/3}(a+bx)}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{b\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{a}-\frac{3}{a\sqrt[3]{x}}$$

$$\Big\downarrow \text{68}$$

$$-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{b\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}$$

input
```
Int[1/(x^(4/3)*(a + b*x)),x]
```

output
```
-3/(a*x^(1/3)) - (b*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/S
qrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3
)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b^(2/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{3}{a\,x^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	104
derivativedivides	$-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}-\frac{3}{a\,x^{\frac{1}{3}}}$	112
default	$-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}-\frac{3}{a\,x^{\frac{1}{3}}}$	112

input
```
int(1/x^(4/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3/a/x^(1/3)+1/a/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/a/(a/b)^(1/3)*ln(
x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-1/a*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = -\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right.\right.}{2ax}$$

input
```
integrate(1/x^(4/3)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^(1/3) - 1/3
*sqrt(3)) + x*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)
^(1/3)) - 2*x*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 6*x^(2/3))/(a*x
)
```

Sympy [A] (verification not implemented)

Time = 7.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} \\ -\frac{3}{4bx^{\frac{4}{3}}} \\ -\frac{3}{a\sqrt[3]{x}} \\ -\frac{\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{a\sqrt[3]{-\frac{a}{b}}}+\frac{\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\sqrt[3]{-\frac{a}{b}}}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a\sqrt[3]{-\frac{a}{b}}}-\frac{3}{a\sqrt[3]{x}} \end{cases}$$

input
```
integrate(1/x**(4/3)/(b*x+a),x)
```

output
```
Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b*x**(4/3)), Eq(a, 0
)), (-3/(a*x**(1/3)), Eq(b, 0)), (-log(x**(1/3) - (-a/b)**(1/3))/(a*(-a/b)
**(1/3)) + log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2
*a*(-a/b)**(1/3)) - sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sq
rt(3)/3)/(a*(-a/b)**(1/3)) - 3/(a*x**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{3}{ax^{\frac{1}{3}}}$$

input
```
integrate(1/x^(4/3)/(b*x+a),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b
)^(1/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(
1/3)) + log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 3/(a*x^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = \frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b} - \frac{3}{ax^{\frac{1}{3}}} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a^2b}$$

input
```
integrate(1/x^(4/3)/(b*x+a),x, algorithm="giac")
```

output
```
b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + sqrt(3)*(-a*b^2)^(2/
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 3
/(a*x^(1/3)) - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a
/b)^(2/3))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = \frac{b^{1/3}\,\ln\left(9\,a^{4/3}\,b^{8/3}+9\,a\,b^3\,x^{1/3}\right)}{a^{4/3}}-\frac{3}{a\,x^{1/3}} + \frac{b^{1/3}\,\ln\left(9\,a\,b^3\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{4/3}} - \frac{b^{1/3}\,\ln\left(9\,a\,b^3\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{4/3}}$$

input
```
int(1/(x^(4/3)*(a + b*x)),x)
```

output
```
(b^(1/3)*log(9*a^(4/3)*b^(8/3) + 9*a*b^3*x^(1/3)))/a^(4/3) - 3/(a*x^(1/3))
 + (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2)
^2)*((3^(1/2)*1i)/2 - 1/2))/a^(4/3) - (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(
4/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 + 1/2))/a^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = \frac{2x^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b-6b^{\frac{2}{3}}a^{\frac{1}{3}}-x^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)b+2x^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+\right.}{2x^{\frac{1}{3}}b^{\frac{2}{3}}a^{\frac{4}{3}}}$$

input
```
int(1/x^(4/3)/(b*x+a),x)
```

output
```
(2*x**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3
)))*b - 6*b**(2/3)*a**(1/3) - x**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a*
*(1/3) + x**(2/3)*b**(2/3))*b + 2*x**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3
))*b)/(2*x**(1/3)*b**(2/3)*a**(1/3)*a)
```

3.324 $\int \frac{1}{x^{5/3}(a+bx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = -\frac{3}{2ax^{2/3}}+\frac{\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}$$

output
```
-3/2/a/x^(2/3)+3^(1/2)*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1
/2)/a^(1/3))/a^(5/3)-3/2*b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(5/3)+1/2*b
^(2/3)*ln(b*x+a)/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = \frac{-\frac{3a^{2/3}}{x^{2/3}}+2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+b^{2/3}\log\left(a^{2/3}-\right.}{2a^{5/3}}$$

input
```
Integrate[1/(x^(5/3)*(a + b*x)),x]
```

output
```
((-3*a^(2/3))/x^(2/3) + 2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/
a^(1/3))/Sqrt[3]] - 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + b^(2/3)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a^(5/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {61, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/3}(a+bx)}dx$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{b\int \frac{1}{x^{2/3}(a+bx)}dx}{a}-\frac{3}{2ax^{2/3}}$$

$$\Big\downarrow \text{ 70}$$

$$-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{b\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}$$

input
```
Int[1/(x^(5/3)*(a + b*x)),x]
```

output
```
-3/(2*a*x^(2/3)) - (b*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))
/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2
/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3)*b^(1/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}-\frac{3}{2a\,x^{\frac{2}{3}}}$	112
default	$-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}-\frac{3}{2a\,x^{\frac{2}{3}}}$	112

input
```
int(1/x^(5/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)
-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*b/a-3/2/a/x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = \frac{2\sqrt{3}x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+\right.}{2ax}$$

input
```
integrate(1/x^(5/3)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*x*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1/3)*(-b^2/a^
2)^(2/3) - sqrt(3)*b)/b) - x*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3
)*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x*(-b^2/a^2)^(1/3)*log(b*x^
(1/3) - a*(-b^2/a^2)^(1/3)) - 3*x^(1/3))/(a*x)
```

Sympy [A] (verification not implemented)

Time = 10.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} \\ -\frac{3}{5bx^{\frac{5}{3}}} \\ -\frac{3}{2ax^{\frac{2}{3}}} \\ -\frac{\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{3}{2ax^{\frac{2}{3}}} \end{cases}$$

input
```
integrate(1/x**(5/3)/(b*x+a),x)
```

output
```
Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-3/(5*b*x**(5/3)), Eq(a, 0
)), (-3/(2*a*x**(2/3)), Eq(b, 0)), (-log(x**(1/3) - (-a/b)**(1/3))/(a*(-a/
b)**(2/3)) + log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/
(2*a*(-a/b)**(2/3)) + sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) +
sqrt(3)/3)/(a*(-a/b)**(2/3)) - 3/(2*a*x**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{3}{2\,ax^{\frac{2}{3}}}$$

input
```
integrate(1/x^(5/3)/(b*x+a),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b
)^(2/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(
2/3)) - log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 3/2/(a*x^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = \frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a^2} - \frac{3}{2ax^{\frac{2}{3}}}$$

input
```
integrate(1/x^(5/3)/(b*x+a),x, algorithm="giac")
```

output
```
b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 - sqrt(3)*(-a*b^2)^(1/
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/2*(
-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/2
/(a*x^(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = \frac{b^{2/3}\,\ln\left(9\,(-a)^{7/3}\,b^{8/3}-9\,a^2\,b^3\,x^{1/3}\right)}{(-a)^{5/3}}-\frac{3}{2\,a\,x^{2/3}} + \frac{b^{2/3}\,\ln\left(9\,(-a)^{7/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-9\,a^2\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}} - \frac{b^{2/3}\,\ln\left(9\,(-a)^{7/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+9\,a^2\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}}$$

input
```
int(1/(x^(5/3)*(a + b*x)),x)
```

output
```
(b^(2/3)*log(9*(-a)^(7/3)*b^(8/3) - 9*a^2*b^3*x^(1/3)))/(-a)^(5/3) - 3/(2*
a*x^(2/3)) + (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2) - 9*
a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(-a)^(5/3) - (b^(2/3)*log(9*(-a)^
(7/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 9*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2
+ 1/2))/(-a)^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = \frac{2x^{\frac{2}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b+x^{\frac{2}{3}}a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)b-2x^{\frac{2}{3}}a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+x\right.}{2x^{\frac{2}{3}}b^{\frac{1}{3}}a^2}$$

input
```
int(1/x^(5/3)/(b*x+a),x)
```

output
```
(2*x**(2/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/
3)*sqrt(3)))*b + x**(2/3)*a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/
3) + x**(2/3)*b**(2/3))*b - 2*x**(2/3)*a**(1/3)*log(a**(1/3) + x**(1/3)*b*
*(1/3))*b - 3*b**(1/3)*a)/(2*x**(2/3)*b**(1/3)*a**2)
```

3.325 $\int \frac{x^{5/3}}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 129

$$\int \frac{x^{5/3}}{(a+bx)^2} \, dx = \frac{3x^{2/3}}{2b^2} + \frac{ax^{2/3}}{b^2(a+bx)} + \frac{5a^{2/3} \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} + \frac{5a^{2/3} \log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}} - \frac{5a^{2/3} \log(a+bx)}{6b^{8/3}}$$

output
```
3/2*x^(2/3)/b^2+a*x^(2/3)/b^2/(b*x+a)+5/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b^(8/3)+5/2*a^(2/3)*ln(a^(1/3)+b^(
1/3)*x^(1/3))/b^(8/3)-5/6*a^(2/3)*ln(b*x+a)/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{x^{5/3}}{(a+bx)^2} \, dx = \frac{\frac{3b^{2/3}x^{2/3}(5a+3bx)}{a+bx} + 10\sqrt{3}a^{2/3} \arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 10a^{2/3} \log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) - 5a^{2/3} \text{lo}}{6b^{8/3}}$$

input
```
Integrate[x^(5/3)/(a + b*x)^2,x]
```

output
```
((3*b^(2/3)*x^(2/3)*(5*a + 3*b*x))/(a + b*x) + 10*Sqrt[3]*a^(2/3)*ArcTan[(
1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 10*a^(2/3)*Log[a^(1/3) + b^(1/
3)*x^(1/3)] - 5*a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^
(2/3)])/(6*b^(8/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 60, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/3}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5 \int \frac{x^{2/3}}{a+bx}dx}{3b} - \frac{x^{5/3}}{b(a+bx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{5 \left(\frac{3x^{2/3}}{2b} - \frac{a \int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{b}\right)}{3b} - \frac{x^{5/3}}{b(a+bx)}$$

$$\Big\downarrow \text{68}$$

$$\frac{5\left(\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^{5/3}}{b(a+bx)}$$

$$\Big\downarrow \; 16$$

$$\frac{5\left(\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^{5/3}}{b(a+bx)}$$

$$\Big\downarrow \; 1082$$

$$\frac{5\left(\frac{3x^{2/3}}{2b}-\frac{a\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^{5/3}}{b(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{5\left(\frac{3x^{2/3}}{2b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{3b}-\frac{x^{5/3}}{b(a+bx)}$$

input
```
Int[x^(5/3)/(a + b*x)^2,x]
```

output
```
-(x^(5/3)/(b*(a + b*x))) + (5*((3*x^(2/3))/(2*b) - (a*(-((Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a^(
1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b^(
2/3))))/b))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2b^{2}}-\frac{3a\left(-\frac{x^{\frac{2}{3}}}{3(bx+a)}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$	124
default	$\frac{3x^{\frac{2}{3}}}{2b^{2}}-\frac{3a\left(-\frac{x^{\frac{2}{3}}}{3(bx+a)}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$	124
risch	$\frac{3x^{\frac{2}{3}}}{2b^{2}}-\frac{a\left(-\frac{x^{\frac{2}{3}}}{bx+a}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$	124

input
```
int(x^(5/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*x^(2/3)/b^2-3*a/b^2*(-1/3*x^(2/3)/(b*x+a)-5/9/b/(a/b)^(1/3)*ln(x^(1/3)
+(a/b)^(1/3))+5/18/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3
))+5/9*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.26

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = -\frac{10\sqrt{3}(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+5(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right.}{6(b^3x+ab^2)}$$

input
```
integrate(x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(10*sqrt(3)*(b*x + a)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)
*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(b*x + a)*(a^2/b^2)^(1/3)*log(-b*x^(1
/3)*(a^2/b^2)^(2/3) + a*x^(2/3) + a*(a^2/b^2)^(1/3)) - 10*(b*x + a)*(a^2/b
^2)^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x^(1/3)) - 3*(3*b*x + 5*a)*x^(2/3))/(b
^3*x + a*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(128) = 256.

Time = 76.18 (sec) , antiderivative size = 595, normalized size of antiderivative = 4.61

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{2}{3}} \\ \frac{3x^{\frac{8}{3}}}{8a^2} \\ \frac{3x^{\frac{2}{3}}}{2b^2} \\ -\frac{10a^2\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}+\frac{5a^2\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}-\frac{10\sqrt{3}a^2\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}- \end{cases}$$

input
```
integrate(x**(5/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(8/3)/(8*a**2), Eq(b,
 0)), (3*x**(2/3)/(2*b**2), Eq(a, 0)), (-10*a**2*log(x**(1/3) - (-a/b)**(1
/3))/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) + 5*a**2*log(4*x**(
2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a*b**3*(-a/b)**(1/3)
 + 6*b**4*x*(-a/b)**(1/3)) - 10*sqrt(3)*a**2*atan(2*sqrt(3)*x**(1/3)/(3*(-
a/b)**(1/3)) + sqrt(3)/3)/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)
) - 10*a**2*log(2)/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) + 15*
a*b*x**(2/3)*(-a/b)**(1/3)/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3
)) - 10*a*b*x*log(x**(1/3) - (-a/b)**(1/3))/(6*a*b**3*(-a/b)**(1/3) + 6*b*
*4*x*(-a/b)**(1/3)) + 5*a*b*x*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) +
4*(-a/b)**(2/3))/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) - 10*sq
rt(3)*a*b*x*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a*b*
*3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) - 10*a*b*x*log(2)/(6*a*b**3*(-a
/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) + 9*b**2*x**(5/3)*(-a/b)**(1/3)/(6*a*
b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = \frac{ax^{\frac{2}{3}}}{b^3x+ab^2} - \frac{5\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{3x^{\frac{2}{3}}}{2b^2} - \frac{5a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a*x^(2/3)/(b^3*x + a*b^2) - 5/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) -
(a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/2*x^(2/3)/b^2 - 5/6*a*log(
x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) + 5/3*a*log
(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = \frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b^2}+\frac{ax^{\frac{2}{3}}}{(bx+a)b^2}+\frac{3\,x^{\frac{2}{3}}}{2\,b^2}+\frac{5\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4}$$

input
```
integrate(x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
5/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + a*x^(2/3)/((b*x +
a)*b^2) + 3/2*x^(2/3)/b^2 + 5/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*
(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 5/6*(-a*b^2)^(2/3)*log(x^(2
/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.16

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = \frac{3\,x^{2/3}}{2\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)}{3\,b^{8/3}}+\frac{a\,x^{2/3}}{x\,b^3+a\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{8/3}}-\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{8/3}}$$

input
```
int(x^(5/3)/(a + b*x)^2,x)
```

output
```
(3*x^(2/3))/(2*b^2) + (5*a^(2/3)*log((25*a^(7/3))/b^(10/3) + (25*a^2*x^(1/
3))/b^3))/(3*b^(8/3)) + (a*x^(2/3))/(a*b^2 + b^3*x) + (5*a^(2/3)*log((25*a
^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2
)*1i)/2 - 1/2))/(3*b^(8/3)) - (5*a^(2/3)*log((25*a^(7/3)*((3^(1/2)*1i)/2 +
 1/2)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(8/
3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.43

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = \frac{10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2+10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)abx+15x^{\frac{2}{3}}b^{\frac{2}{3}}a^{\frac{4}{3}}+9x^{\frac{5}{3}}b^{\frac{5}{3}}a^{\frac{1}{3}}-5\log(}{}$$

input
```
int(x^(5/3)/(b*x+a)^2,x)
```

output
```
(10*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a**2
 + 10*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a*
b*x + 15*x**(2/3)*b**(2/3)*a**(1/3)*a + 9*x**(2/3)*b**(2/3)*a**(1/3)*b*x -
 5*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a**2 - 5
*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a*b*x + 10
*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2 + 10*log(a**(1/3) + x**(1/3)*b**(1
/3))*a*b*x)/(6*b**(2/3)*a**(1/3)*b**2*(a + b*x))
```

3.326 $\int \frac{x^{4/3}}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 125

$$\int \frac{x^{4/3}}{(a+bx)^2} \, dx = \frac{3\sqrt[3]{x}}{b^2} + \frac{a\sqrt[3]{x}}{b^2(a+bx)} + \frac{4\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}$$

output
```
3*x^(1/3)/b^2+a*x^(1/3)/b^2/(b*x+a)+4/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/b^(7/3)-2*a^(1/3)*ln(a^(1/3)+b^(1/3)
*x^(1/3))/b^(7/3)+2/3*a^(1/3)*ln(b*x+a)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \frac{x^{4/3}}{(a+bx)^2} \, dx = \frac{\frac{3\sqrt[3]{b}\sqrt[3]{x}(4a+3bx)}{a+bx} + 4\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + 2\sqrt[3]{a}\log\left(\right.}{3b^{7/3}}$$

input
```
Integrate[x^(4/3)/(a + b*x)^2,x]
```

output
```
((3*b^(1/3)*x^(1/3)*(4*a + 3*b*x))/(a + b*x) + 4*Sqrt[3]*a^(1/3)*ArcTan[(1
 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 4*a^(1/3)*Log[a^(1/3) + b^(1/3)
*x^(1/3)] + 2*a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2
/3)])/(3*b^(7/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4/3}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{4 \int \frac{\sqrt[3]{x}}{a+bx}dx}{3b}-\frac{x^{4/3}}{b(a+bx)}$$

$$\Big\downarrow \text{60}$$

$$\frac{4 \left(\frac{3 \sqrt[3]{x}}{b}-\frac{a \int \frac{1}{x^{2/3}(a+bx)}dx}{b}\right)}{3b}-\frac{x^{4/3}}{b(a+bx)}$$

$$\Big\downarrow \text{70}$$

$$\frac{4\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^{4/3}}{b(a+bx)}$$

$$\Big\downarrow \; 16$$

$$\frac{4\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^{4/3}}{b(a+bx)}$$

$$\Big\downarrow \; 1082$$

$$\frac{4\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^{4/3}}{b(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{4\left(\frac{3\sqrt[3]{x}}{b}-\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^{4/3}}{b(a+bx)}$$

input
```
Int[x^(4/3)/(a + b*x)^2,x]
```

output
```
-(x^(4/3)/(b*(a + b*x))) + (4*((3*x^(1/3))/b - (a*(-((Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/3)
 + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3)*b^(1/3)
)))/b))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$\frac{3x^{\frac{1}{3}}}{b^{2}}-\frac{3a\left(-\frac{x^{\frac{1}{3}}}{3(bx+a)}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^{2}}$	124
default	$\frac{3x^{\frac{1}{3}}}{b^{2}}-\frac{3a\left(-\frac{x^{\frac{1}{3}}}{3(bx+a)}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^{2}}$	124

input
```
int(x^(4/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)/b^2-3*a/b^2*(-1/3*x^(1/3)/(b*x+a)+4/9/b/(a/b)^(2/3)*ln(x^(1/3)+(
a/b)^(1/3))-2/9/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+
4/9/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.18

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx=\frac{4\sqrt{3}(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-2\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3\,(b^3x+ab^2)}$$

input
```
integrate(x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(4*sqrt(3)*(b*x + a)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/
b)^(2/3) - sqrt(3)*a)/a) - 2*(b*x + a)*(-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*
(-a/b)^(1/3) + (-a/b)^(2/3)) + 4*(b*x + a)*(-a/b)^(1/3)*log(x^(1/3) - (-a/
b)^(1/3)) + 3*(3*b*x + 4*a)*x^(1/3))/(b^3*x + a*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(124) = 248$.

Time = 58.65 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.66

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = \begin{cases} \tilde{\infty}\sqrt[3]{x} \\ \frac{3x^{\frac{7}{3}}}{7a^2} \\ \frac{3\sqrt[3]{x}}{b^2} \\ \frac{12a\sqrt[3]{x}}{3ab^2+3b^3x} + \frac{4a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x} - \frac{4\sqrt{3}a\sqrt[3]{-\cdots}}{\cdots} \end{cases}$$

input
```
integrate(x**(4/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a**2), Eq(b,
 0)), (3*x**(1/3)/b**2, Eq(a, 0)), (12*a*x**(1/3)/(3*a*b**2 + 3*b**3*x) +
4*a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x) - 2*
a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3
))/(3*a*b**2 + 3*b**3*x) - 4*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/
3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a*b**2 + 3*b**3*x) + 4*a*(-a/b)**(1/3
)*log(2)/(3*a*b**2 + 3*b**3*x) + 9*b*x**(4/3)/(3*a*b**2 + 3*b**3*x) + 4*b*
x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x) - 2*b*
x*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3
))/(3*a*b**2 + 3*b**3*x) - 4*sqrt(3)*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(
1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a*b**2 + 3*b**3*x) + 4*b*x*(-a/b)**
(1/3)*log(2)/(3*a*b**2 + 3*b**3*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = \frac{ax^{\frac{1}{3}}}{b^3x+ab^2} - \frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3x^{\frac{1}{3}}}{b^2} + \frac{2a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{4a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
a*x^(1/3)/(b^3*x + a*b^2) - 4/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) -
(a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3*x^(1/3)/b^2 + 2/3*a*log(x^
(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 4/3*a*log(x
^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = \frac{4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3b^2} - \frac{4\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3} + \frac{ax^{\frac{1}{3}}}{(bx+a)b^2} + \frac{3x^{\frac{1}{3}}}{b^2} - \frac{2(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b^3}$$

input
```
integrate(x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
4/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - 4/3*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 +
 a*x^(1/3)/((b*x + a)*b^2) + 3*x^(1/3)/b^2 - 2/3*(-a*b^2)^(1/3)*log(x^(2/3
) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = \frac{3\,x^{1/3}}{b^2}+\frac{a\,x^{1/3}}{x\,b^3+a\,b^2}+\frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{12\,{\left(-a\right)}^{4/3}}{b^{1/3}}+12\,a\,x^{1/3}\right)}{3\,b^{7/3}} - \frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}-\frac{12\,{\left(-a\right)}^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{7/3}} + \frac{{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}+\frac{9\,{\left(-a\right)}^{4/3}\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{1/3}}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{7/3}}$$

input
```
int(x^(4/3)/(a + b*x)^2,x)
```

output
```
(3*x^(1/3))/b^2 + (a*x^(1/3))/(a*b^2 + b^3*x) + (4*(-a)^(1/3)*log((12*(-a)
^(4/3))/b^(1/3) + 12*a*x^(1/3)))/(3*b^(7/3)) - (4*(-a)^(1/3)*log(12*a*x^(1
/3) - (12*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1/
2))/(3*b^(7/3)) + ((-a)^(1/3)*log(12*a*x^(1/3) + (9*(-a)^(4/3)*((3^(1/2)*2
i)/3 - 2/3))/b^(1/3))*((3^(1/2)*2i)/3 - 2/3))/b^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.46

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = \frac{4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)+4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx+2a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)}{}$$

input
```
int(x^(4/3)/(b*x+a)^2,x)
```

output
```
(4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3
)))*a + 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)
*sqrt(3)))*b*x + 2*a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x*
*(2/3)*b**(2/3))*a + 2*a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3)
+ x**(2/3)*b**(2/3))*b*x - 4*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a
- 4*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*b*x + 12*x**(1/3)*b**(1/3)*
a + 9*x**(1/3)*b**(1/3)*b*x)/(3*b**(1/3)*b**2*(a + b*x))
```

3.327 $\int \frac{x^{2/3}}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{x^{2/3}}{(a+bx)^2} \, dx = -\frac{x^{2/3}}{b(a+bx)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}b^{5/3}}$$

output
```
-x^(2/3)/b/(b*x+a)-2/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1
/3))*3^(1/2)/a^(1/3)/b^(5/3)-ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(1/3)/b^(5/3)+1
/3*ln(b*x+a)/a^(1/3)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

$$\int \frac{x^{2/3}}{(a+bx)^2} \, dx = \frac{-\frac{3b^{2/3}x^{2/3}}{a+bx} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{a}}}{3b^{5/3}}$$

input
```
Integrate[x^(2/3)/(a + b*x)^2,x]
```

output
```
((-3*b^(2/3)*x^(2/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3
))/a^(1/3))/Sqrt[3]])/a^(1/3) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(1/3)
 + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/a^(1/3))/(3*b^
(5/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {51, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2/3}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{3b}-\frac{x^{2/3}}{b(a+bx)}$$

$$\Big\downarrow \text{68}$$

$$\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}$$

input
```
Int[x^(2/3)/(a + b*x)^2,x]
```

output
```
-(x^(2/3)/(b*(a + b*x))) + (2*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/
a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])
/(2*a^(1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b^(2/3))))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{x^{\frac{2}{3}}}{b(bx+a)}+\frac{-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b}$	118
default	$-\frac{x^{\frac{2}{3}}}{b(bx+a)}+\frac{-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b}$	118

input
```
int(x^(2/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x^(2/3)/b/(b*x+a)+2/b*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(
a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)
^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.43

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = \left[-\frac{3\,ab^2x^{\frac{2}{3}} - 3\,\sqrt{\frac{1}{3}}(ab^2x+a^2b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+3\,\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\cdots}}{bx+a}\right.\cdots}{\cdots}\right.$$

$$-\frac{3\,ab^2x^{\frac{2}{3}} - 6\,\sqrt{\frac{1}{3}}(ab^2x+a^2b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right) - (-ab^2)^{\frac{2}{3}}\,(bx+a)\log\left(\cdots\right.}{3\,(ab^4x+a^2b^3)}$$

input
```
integrate(x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/3*(3*a*b^2*x^(2/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)
/a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(
-a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/
(b*x + a)) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x
^(1/3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*
b^2)^(1/3)))/(a*b^4*x + a^2*b^3), -1/3*(3*a*b^2*x^(2/3) - 6*sqrt(1/3)*(a*b
^2*x + a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*
b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*
x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b
*x + a)*log(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^4*x + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(107) = 214$.

Time = 32.77 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.58

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} \\ \frac{3x^{\frac{5}{3}}}{5a^2} \\ -\frac{3}{b^2\sqrt[3]{x}} \\ \frac{2a\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} - \frac{a\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}a\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{}{3ab^2\sqrt[3]{}} \end{cases}$$

input
```
integrate(x**(2/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a**2), Eq(b,
 0)), (-3/(b**2*x**(1/3)), Eq(a, 0)), (2*a*log(x**(1/3) - (-a/b)**(1/3))/(
3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - a*log(4*x**(2/3) + 4*x*
*(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x
*(-a/b)**(1/3)) + 2*sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) +
sqrt(3)/3)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) + 2*a*log(2)/
(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - 3*b*x**(2/3)*(-a/b)**(
1/3)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) + 2*b*x*log(x**(1/3
) - (-a/b)**(1/3))/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - b*x
*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2*(-
a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) + 2*sqrt(3)*b*x*atan(2*sqrt(3)*x**(1
/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/
b)**(1/3)) + 2*b*x*log(2)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = -\frac{x^{\frac{2}{3}}}{b^2x+ab} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-x^(2/3)/(b^2*x + a*b) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)
^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) + 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(
1/3) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) - 2/3*log(x^(1/3) + (a/b)^(1/3))/(b^
2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.18

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} - \frac{x^{\frac{2}{3}}}{(bx+a)b} - \frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^3}$$

input
```
integrate(x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) - x^(2/3)/((b*x +
 a)*b) - 2/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)
^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/3*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = \frac{2\,\ln\left(\frac{4\,x^{1/3}}{b}-\frac{4\,(-a)^{1/3}}{b^{4/3}}\right)}{3\,(-a)^{1/3}\,b^{5/3}} - \frac{x^{2/3}}{b\,(a+b\,x)} + \frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{(-a)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{b^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3\,(-a)^{1/3}\,b^{5/3}} - \frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{(-a)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{b^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3\,(-a)^{1/3}\,b^{5/3}}$$

input
```
int(x^(2/3)/(a + b*x)^2,x)
```

output
```
(2*log((4*x^(1/3))/b - (4*(-a)^(1/3))/b^(4/3)))/(3*(-a)^(1/3)*b^(5/3)) - x
^(2/3)/(b*(a + b*x)) + (log((4*x^(1/3))/b - ((-a)^(1/3)*(3^(1/2)*1i - 1)^2
)/b^(4/3))*(3^(1/2)*1i - 1))/(3*(-a)^(1/3)*b^(5/3)) - (log((4*x^(1/3))/b -
 ((-a)^(1/3)*(3^(1/2)*1i + 1)^2)/b^(4/3))*(3^(1/2)*1i + 1))/(3*(-a)^(1/3)*
b^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.41

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx-3x^{\frac{2}{3}}b^{\frac{2}{3}}a^{\frac{1}{3}}+\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+\cdots\right.}{3b^{\frac{5}{3}}a^{\frac{1}{3}}\,(b\cdots}$$

input
```
int(x^(2/3)/(b*x+a)^2,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a
- 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*b*x
- 3*x**(2/3)*b**(2/3)*a**(1/3) + log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3)
 + x**(2/3)*b**(2/3))*a + log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(
2/3)*b**(2/3))*b*x - 2*log(a**(1/3) + x**(1/3)*b**(1/3))*a - 2*log(a**(1/3
) + x**(1/3)*b**(1/3))*b*x)/(3*b**(2/3)*a**(1/3)*b*(a + b*x))
```

3.328 $\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 117

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = -\frac{\sqrt[3]{x}}{b(a+bx)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}}$$

output
```
-x^(1/3)/b/(b*x+a)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1
/3))*3^(1/2)/a^(2/3)/b^(4/3)+1/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(2/3)/b^(4/
3)-1/6*ln(b*x+a)/a^(2/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx$$
$$= \frac{-\frac{6\sqrt[3]{b}\sqrt[3]{x}}{a+bx} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{2/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{a^{2/3}}}{6b^{4/3}}$$

input
```
Integrate[x^(1/3)/(a + b*x)^2,x]
```

output
```
((-6*b^(1/3)*x^(1/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3
))/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(2/3)
 - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/a^(2/3))/(6*b^
(4/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {51, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx$$
$$\Big\downarrow \ 51$$
$$\frac{\int \frac{1}{x^{2/3}(a+bx)}dx}{3b} - \frac{\sqrt[3]{x}}{b(a+bx)}$$
$$\Big\downarrow \ 70$$

$$\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

input
```
Int[x^(1/3)/(a + b*x)^2,x]
```

output
```
-(x^(1/3)/(b*(a + b*x))) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(
1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2
*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3)*b^(1/3)))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{x^{\frac{1}{3}}}{b(bx+a)}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b}$	117
default	$-\frac{x^{\frac{1}{3}}}{b(bx+a)}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b}$	117

input
```
int(x^(1/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x^(1/3)/b/(b*x+a)+1/b*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a
/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(84) = 168.

Time = 0.08 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx$$

$$= \left[-\frac{6\,a^2bx^{\frac{1}{3}} - 3\,\sqrt{\frac{1}{3}}(ab^2x + a^2b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}} - (a^2b)^{\frac{1}{3}}a + (a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}} - 3\,(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right.}{6\,(a^2b^3x +}\right.$$

$$-\frac{6\,a^2bx^{\frac{1}{3}} - 6\,\sqrt{\frac{1}{3}}(ab^2x + a^2b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^2b)^{\frac{1}{3}}a - 2\,(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right) + (a^2b)^{\frac{2}{3}}\,(bx + a)\log}{6\,(a^2b^3x + a^3b^2)}$$

input
```
integrate(x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(6*a^2*b*x^(1/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)
/b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a
^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(
b*x + a)) + (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a
^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)
^(2/3)))/(a^2*b^3*x + a^3*b^2), -1/6*(6*a^2*b*x^(1/3) - 6*sqrt(1/3)*(a*b^2
*x + a^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(
a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*(b*x + a)
*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2
/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(107) = 214$.

Time = 21.20 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.85

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} \\ \frac{3x^{\frac{4}{3}}}{4a^2} \\ -\frac{3}{2b^2x^{\frac{2}{3}}} \\ -\frac{6a\sqrt[3]{x}}{6a^2b+6ab^2x} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^2b+6ab^2x} + \frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b+6ab^2x} + \frac{2\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}\right.}{6a^2b+6ab^2x} \end{cases}$$

input
```
integrate(x**(1/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**2), Eq(b,
 0)), (-3/(2*b**2*x**(2/3)), Eq(a, 0)), (-6*a*x**(1/3)/(6*a**2*b + 6*a*b**
2*x) - 2*a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(6*a**2*b + 6*a*b**
2*x) + a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b
)**(2/3))/(6*a**2*b + 6*a*b**2*x) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(
3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**2*b + 6*a*b**2*x) - 2*a*(
-a/b)**(1/3)*log(2)/(6*a**2*b + 6*a*b**2*x) - 2*b*x*(-a/b)**(1/3)*log(x**(
1/3) - (-a/b)**(1/3))/(6*a**2*b + 6*a*b**2*x) + b*x*(-a/b)**(1/3)*log(4*x*
*(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b + 6*a*b**2*
x) + 2*sqrt(3)*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3))
 + sqrt(3)/3)/(6*a**2*b + 6*a*b**2*x) - 2*b*x*(-a/b)**(1/3)*log(2)/(6*a**2
*b + 6*a*b**2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = -\frac{x^{\frac{1}{3}}}{b^2x+ab} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-x^(1/3)/(b^2*x + a*b) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)
^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(
1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*log(x^(1/3) + (a/b)^(1/3))/(b^
2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} - \frac{x^{\frac{1}{3}}}{(bx+a)b} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$

input
```
integrate(x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^2) - x^(1/3)/((b*x + a)*b) + 1/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = \frac{\ln\left(3\,b\,x^{1/3}+3\,a^{1/3}\,b^{2/3}\right)}{3\,a^{2/3}\,b^{4/3}} - \frac{x^{1/3}}{b\,(a+b\,x)} + \frac{\ln\left(3\,b\,x^{1/3}+\frac{3\,a^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(3\,b\,x^{1/3}-\frac{3\,a^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}$$

input
```
int(x^(1/3)/(a + b*x)^2,x)
```

output
```
log(3*b*x^(1/3) + 3*a^(1/3)*b^(2/3))/(3*a^(2/3)*b^(4/3)) - x^(1/3)/(b*(a +
 b*x)) + (log(3*b*x^(1/3) + (3*a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/
2)*1i - 1))/(6*a^(2/3)*b^(4/3)) - (log(3*b*x^(1/3) - (3*a^(1/3)*b^(2/3)*(3
^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = \frac{-2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx-a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)-a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-\right.}{6b^{\frac{4}{3}}a\,(bx+a)}$$

input
```
int(x^(1/3)/(b*x+a)^2,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqr
t(3)))*a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1
/3)*sqrt(3)))*b*x - a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x
**(2/3)*b**(2/3))*a - a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) +
 x**(2/3)*b**(2/3))*b*x + 2*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a +
 2*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*b*x - 6*x**(1/3)*b**(1/3)*a)
/(6*b**(1/3)*a*b*(a + b*x))
```

3.329 $\int \frac{1}{\sqrt[3]{x}(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 116

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2} dx = \frac{x^{2/3}}{a(a+bx)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}}$$

output
```
x^(2/3)/a/(b*x+a)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/
3))*3^(1/2)/a^(4/3)/b^(2/3)-1/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(4/3)/b^(2/3
)+1/6*ln(b*x+a)/a^(4/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2} dx$$
$$= \frac{\frac{6\sqrt[3]{a}x^{2/3}}{a+bx} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{b^{2/3}}}{6a^{4/3}}$$

input
```
Integrate[1/(x^(1/3)*(a + b*x)^2),x]
```

output
```
((6*a^(1/3)*x^(2/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3)
)/a^(1/3))/Sqrt[3]])/b^(2/3) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(2/3)
+ Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/b^(2/3))/(6*a^(
4/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2} dx$$
$$\downarrow 52$$
$$\frac{\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{3a} + \frac{x^{2/3}}{a(a+bx)}$$
$$\downarrow 68$$

$$\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}$$

input
```
Int[1/(x^(1/3)*(a + b*x)^2),x]
```

output
```
x^(2/3)/(a*(a + b*x)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3
))/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^
(1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b^(2/3)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{x^{\frac{2}{3}}}{a(bx+a)}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{a}$	116
default	$\frac{x^{\frac{2}{3}}}{a(bx+a)}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{a}$	116

input
```
int(1/x^(1/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^(2/3)/a/(b*x+a)+1/a*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a
/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(83) = 166.

Time = 0.09 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.27

$$\int\frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx$$

$$=\left[\frac{6\,ab^2x^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}(ab^2x+a^2b)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}-(ab^2)^{\frac{1}{3}}a+2\,(ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}-3\,(ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)}{6\,(a^2b^3x+a^3b}\right.$$

input
```
integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(6*a*b^2*x^(2/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a*b^2)^(1/3)/
a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) - (a*b^2)^(1/3)*a + 2*(a*
b^2)^(2/3)*x^(2/3))*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + (a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) - (a*b^2)^(1/3)*b*x^(1/3)
 + (a*b^2)^(2/3)) - 2*(a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) + (a*b^2)^(1/3
)))/(a^2*b^3*x + a^3*b^2), 1/6*(6*a*b^2*x^(2/3) + 6*sqrt(1/3)*(a*b^2*x + a
^2*b)*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) - (a*b^2)^(1/3))
*sqrt((a*b^2)^(1/3)/a)/b) + (a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) - (a*b
^2)^(1/3)*b*x^(1/3) + (a*b^2)^(2/3)) - 2*(a*b^2)^(2/3)*(b*x + a)*log(b*x^(
1/3) + (a*b^2)^(1/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(107) = 214$.

Time = 22.69 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.69

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} \\ \frac{3x^{\frac{2}{3}}}{2a^2} \\ -\frac{3}{4b^2x^{\frac{4}{3}}} \\ \frac{2a\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}}+6ab^2x\sqrt[3]{-\frac{a}{b}}} - \frac{a\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}}+6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}a\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}}+6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2a\log(2)}{6a^2b\sqrt[3]{-\frac{a}{b}}+6ab^2x\sqrt[3]{-}} \end{cases}$$

input
```
integrate(1/x**(1/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a**2), Eq(b,
 0)), (-3/(4*b**2*x**(4/3)), Eq(a, 0)), (2*a*log(x**(1/3) - (-a/b)**(1/3))
/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) - a*log(4*x**(2/3) +
4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b*(-a/b)**(1/3) + 6*a*
b**2*x*(-a/b)**(1/3)) + 2*sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/
3)) + sqrt(3)/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*a
*log(2)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 6*b*x**(2/3)
*(-a/b)**(1/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*b*x
*log(x**(1/3) - (-a/b)**(1/3))/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)
**(1/3)) - b*x*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3)
)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*sqrt(3)*b*x*atan
(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**2*b*(-a/b)**(1/3)
 + 6*a*b**2*x*(-a/b)**(1/3)) + 2*b*x*log(2)/(6*a**2*b*(-a/b)**(1/3) + 6*a*
b**2*x*(-a/b)**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx = \frac{x^{\frac{2}{3}}}{abx+a^2} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
x^(2/3)/(a*b*x + a^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1
/3) + (a/b)^(2/3))/(a*b*(a/b)^(1/3)) - 1/3*log(x^(1/3) + (a/b)^(1/3))/(a*b
*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}+\frac{x^{\frac{2}{3}}}{(bx+a)a} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2}$$

input
```
integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + x^(2/3)/((b*x + a
)*a) - 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/6*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx = \frac{x^{2/3}}{a\,(a+b\,x)}+\frac{(-1)^{1/3}\,\ln\left(\frac{(-1)^{2/3}\,b^{2/3}}{a^{5/3}}+\frac{b\,x^{1/3}}{a^2}\right)}{3\,a^{4/3}\,b^{2/3}} - \frac{(-1)^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{(-1)^{2/3}\,b^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{a^{5/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{4/3}\,b^{2/3}} + \frac{(-1)^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{9\,(-1)^{2/3}\,b^{2/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2}{a^{5/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{4/3}\,b^{2/3}}$$

input
```
int(1/(x^(1/3)*(a + b*x)^2),x)
```

output
```
x^(2/3)/(a*(a + b*x)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/a^(5/3) + (b*
x^(1/3))/a^2))/(3*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((b*x^(1/3))/a^2 + ((-
1)^(2/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2)^2)/a^(5/3))*((3^(1/2)*1i)/2 + 1/2)
)/(3*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*log((b*x^(1/3))/a^2 + (9*(-1)^(2/3)*b^
(2/3)*((3^(1/2)*1i)/6 - 1/6)^2)/a^(5/3))*((3^(1/2)*1i)/6 - 1/6))/(a^(4/3)*
b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx+6x^{\frac{2}{3}}b^{\frac{2}{3}}a^{\frac{1}{3}}+\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{2}{3}}\right)a+\log\left(a\right.}{6b^{\frac{2}{3}}a^{\frac{4}{3}}\left(bx+a\right)}$$

input
```
int(1/x^(1/3)/(b*x+a)^2,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a
- 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*b*x
+ 6*x**(2/3)*b**(2/3)*a**(1/3) + log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3)
 + x**(2/3)*b**(2/3))*a + log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(
2/3)*b**(2/3))*b*x - 2*log(a**(1/3) + x**(1/3)*b**(1/3))*a - 2*log(a**(1/3
) + x**(1/3)*b**(1/3))*b*x)/(6*b**(2/3)*a**(1/3)*a*(a + b*x))
```

3.330 $\int \frac{1}{x^{2/3}(a+bx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \frac{\sqrt[3]{x}}{a(a+bx)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}$$

output
```
x^(1/3)/a/(b*x+a)-2/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/
3))*3^(1/2)/a^(5/3)/b^(1/3)+ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(5/3)/b^(1/3)-1/
3*ln(b*x+a)/a^(5/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \frac{\frac{3a^{2/3}\sqrt[3]{x}}{a+bx} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right.}{\sqrt[3]{b}}}{3a^{5/3}}$$

input
```
Integrate[1/(x^(2/3)*(a + b*x)^2),x]
```

output
```
((3*a^(2/3)*x^(1/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3)
)/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(1/3)
- Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/b^(1/3))/(3*a^(
5/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{2/3}(a+bx)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{2\int \frac{1}{x^{2/3}(a+bx)}dx}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

$$\Big\downarrow \text{70}$$

$$\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{3\int\frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

input
```
Int[1/(x^(2/3)*(a + b*x)^2),x]
```

output
```
x^(1/3)/(a*(a + b*x)) + (2*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(
1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2
*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3)*b^(1/3))))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{x^{\frac{1}{3}}}{a(bx+a)}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	117
default	$\frac{x^{\frac{1}{3}}}{a(bx+a)}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	117

input
```
int(1/x^(2/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x^(1/3)/a/(b*x+a)+2/a*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(82) = 164.

Time = 0.09 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.42

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \left[\frac{3\,a^2bx^{\frac{1}{3}} + 3\,\sqrt{\frac{1}{3}}(ab^2x + a^2b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}} - (a^2b)^{\frac{1}{3}}a + (a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\cdots}}{bx+a}\cdots\right.}{\cdots}\right.$$

input
```
integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/3*(3*a^2*b*x^(1/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/
b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^
2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b
*x + a)) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^
2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^
(2/3)))/(a^3*b^2*x + a^4*b), 1/3*(3*a^2*b*x^(1/3) + 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*
b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*(b*x + a)*log
(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*
(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^3*b^2*x + a^4*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(107) = 214$.

Time = 33.37 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.84

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} \\ \frac{3\sqrt[3]{x}}{a^2} \\ -\frac{3}{5b^2x^{\frac{5}{3}}} \\ \frac{3a\sqrt[3]{x}}{3a^3+3a^2bx} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{3a^3+3a^2bx} + \frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3a^3+3a^2bx} + \frac{2\sqrt{3}a}{} \end{cases}$$

input
```
integrate(1/x**(2/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a**2, Eq(b, 0))
, (-3/(5*b**2*x**(5/3)), Eq(a, 0)), (3*a*x**(1/3)/(3*a**3 + 3*a**2*b*x) -
2*a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a**3 + 3*a**2*b*x) + a*
(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))
/(3*a**3 + 3*a**2*b*x) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)
/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a**3 + 3*a**2*b*x) - 2*a*(-a/b)**(1/3)*
log(2)/(3*a**3 + 3*a**2*b*x) - 2*b*x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**
(1/3))/(3*a**3 + 3*a**2*b*x) + b*x*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/
3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a**3 + 3*a**2*b*x) + 2*sqrt(3)*b*x*
(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a*
*3 + 3*a**2*b*x) - 2*b*x*(-a/b)**(1/3)*log(2)/(3*a**3 + 3*a**2*b*x), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \frac{x^{\frac{1}{3}}}{abx+a^2} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
x^(1/3)/(a*b*x + a^2) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1
/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) + 2/3*log(x^(1/3) + (a/b)^(1/3))/(a*b
*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} + \frac{x^{\frac{1}{3}}}{(bx+a)a} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^2b}$$

input
```
integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + 2/3*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2
*b) + x^(1/3)/((b*x + a)*a) + 1/3*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a
/b)^(1/3) + (-a/b)^(2/3))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \frac{2\,\ln\left(\frac{6\,b^{5/3}}{a^{2/3}}+\frac{6\,b^2\,x^{1/3}}{a}\right)}{3\,a^{5/3}\,b^{1/3}}+\frac{x^{1/3}}{a\,\left(a+b\,x\right)} + \frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}+\frac{3\,b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3\,a^{5/3}\,b^{1/3}} - \frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}-\frac{3\,b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3\,a^{5/3}\,b^{1/3}}$$

input
```
int(1/(x^(2/3)*(a + b*x)^2),x)
```

output
```
(2*log((6*b^(5/3))/a^(2/3) + (6*b^2*x^(1/3))/a))/(3*a^(5/3)*b^(1/3)) + x^(
1/3)/(a*(a + b*x)) + (log((6*b^2*x^(1/3))/a + (3*b^(5/3)*(3^(1/2)*1i - 1))
/a^(2/3))*(3^(1/2)*1i - 1))/(3*a^(5/3)*b^(1/3)) - (log((6*b^2*x^(1/3))/a -
 (3*b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(3*a^(5/3)*b^(1/3
))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \frac{-2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx-a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{1}{3}}+x^{\frac{2}{3}}\right.}{}$$

input
```
int(1/x^(2/3)/(b*x+a)^2,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqr
t(3)))*a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1
/3)*sqrt(3)))*b*x - a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x
**(2/3)*b**(2/3))*a - a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) +
 x**(2/3)*b**(2/3))*b*x + 2*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a +
 2*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*b*x + 3*x**(1/3)*b**(1/3)*a)
/(3*b**(1/3)*a**2*(a + b*x))
```

3.331 $\int \frac{1}{x^{4/3}(a+bx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = -\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}(a+bx)} + \frac{4\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}+\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}$$

output
```
-4/a^2/x^(1/3)+1/a/x^(1/3)/(b*x+a)+4/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)+2*b^(1/3)*ln(a^(1/3)+b^(1/3)*
x^(1/3))/a^(7/3)-2/3*b^(1/3)*ln(b*x+a)/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{-\frac{3\sqrt[3]{a}(3a+4bx)}{\sqrt[3]{x}(a+bx)}+4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)-2\sqrt[3]{b}\log}{3a^{7/3}}$$

input
```
Integrate[1/(x^(4/3)*(a + b*x)^2),x]
```

output
```
((-3*a^(1/3)*(3*a + 4*b*x))/(x^(1/3)*(a + b*x)) + 4*Sqrt[3]*b^(1/3)*ArcTan
[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*Log[a^(1/3) + b^(1
/3)*x^(1/3)] - 2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x
^(2/3)])/(3*a^(7/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {52, 61, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{4/3}(a+bx)^2} \, dx$$

$$\Big\downarrow \text{52}$$

$$\frac{4\int \frac{1}{x^{4/3}(a+bx)}dx}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{4\left(-\frac{b\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

$$\Big\downarrow \text{68}$$

$$\frac{4\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

$\downarrow$ 16

$$\frac{4\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

$\downarrow$ 1082

$$\frac{4\left(-\frac{b\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

$\downarrow$ 217

$$\frac{4\left(-\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

input
```
Int[1/(x^(4/3)*(a + b*x)^2),x]
```

output
```
1/(a*x^(1/3)*(a + b*x)) + (4*(-3/(a*x^(1/3)) - (b*(-((Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a^(1/3)
 + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b^(2/3)
)))/a))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{3}{a^{2} x^{\frac{1}{3}}}-\frac{b\,x^{\frac{2}{3}}}{a^{2} \left(b x +a \right)}+\frac{4 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 a^{2} \left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3 a^{2} \left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{4 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 a^{2} \left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$-\frac{3 b \left(\frac{x^{\frac{2}{3}}}{3 b x +3 a}-\frac{4 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}-\frac{3}{a^{2} x^{\frac{1}{3}}}$
default	$-\frac{3 b \left(\frac{x^{\frac{2}{3}}}{3 b x +3 a}-\frac{4 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}-\frac{3}{a^{2} x^{\frac{1}{3}}}$

input
```
int(1/x^(4/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/a^2/x^(1/3)-b/a^2*x^(2/3)/(b*x+a)+4/3/a^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^
(1/3))-2/3/a^2/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-4/3
/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx =$$
$$-\frac{4\sqrt{3}(bx^2+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\,(bx^2+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,(a^2bx^2+a^3x)}$$

input
```
integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(3)*(b*x^2 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)
^(1/3) - 1/3*sqrt(3)) + 2*(b*x^2 + a*x)*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(
2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^2 + a*x)*(b/a)^(1/3)*log(a*(b/a
)^(2/3) + b*x^(1/3)) + 3*(4*b*x + 3*a)*x^(2/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(122) = 244$.

Time = 71.07 (sec) , antiderivative size = 690, normalized size of antiderivative = 5.56

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(4/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (-3/(a**2*x**(1/3)), Eq(b,
0)), (-3/(7*b**2*x**(7/3)), Eq(a, 0)), (-4*a*x**(1/3)*log(x**(1/3) - (-a/b
)**(1/3))/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)
) + 2*a*x**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/
3))/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*
sqrt(3)*a*x**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/
(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*a*x*
*(1/3)*log(2)/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(
1/3)) - 9*a*(-a/b)**(1/3)/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/
3)*(-a/b)**(1/3)) - 4*b*x**(4/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a**3*x**
(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) + 2*b*x**(4/3)*log(
4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a**3*x**(1/3)*
(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*sqrt(3)*b*x**(4/3)*at
an(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a**3*x**(1/3)*(-a/
b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*b*x**(4/3)*log(2)/(3*a**3
*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 12*b*x*(-a/b)
**(1/3)/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = -\frac{4\,bx+3\,a}{a^2bx^{\frac{4}{3}}+a^3x^{\frac{1}{3}}} - \frac{4\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{4\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(4*b*x + 3*a)/(a^2*b*x^(4/3) + a^3*x^(1/3)) - 4/3*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/3*log(x^(
2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/3*log(x^(1
/3) + (a/b)^(1/3))/(a^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3} + \frac{4\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b} - \frac{4\,bx+3\,a}{\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)a^2} - \frac{2\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^3b}$$

input
```
integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
4/3*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 4/3*sqrt(3)*(-a*
b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
3*b) - (4*b*x + 3*a)/((b*x^(4/3) + a*x^(1/3))*a^2) - 2/3*(-a*b^2)^(2/3)*lo
g(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}+16\,a^2\,b^3\,x^{1/3}\right)}{3\,a^{7/3}} - \frac{\frac{3}{a}+\frac{4\,b\,x}{a^2}}{a\,x^{1/3}+b\,x^{4/3}} - \frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+16\,a^2\,b^3\,x^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{7/3}} + \frac{b^{1/3}\,\ln\left(9\,a^{7/3}\,b^{8/3}\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16\,a^2\,b^3\,x^{1/3}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)}{a^{7/3}}$$

input
```
int(1/(x^(4/3)*(a + b*x)^2),x)
```

output
```
(4*b^(1/3)*log(16*a^(7/3)*b^(8/3) + 16*a^2*b^3*x^(1/3)))/(3*a^(7/3)) - (3/
a + (4*b*x)/a^2)/(a*x^(1/3) + b*x^(4/3)) - (4*b^(1/3)*log(16*a^(7/3)*b^(8/
3)*((3^(1/2)*1i)/2 + 1/2)^2 + 16*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/
(3*a^(7/3)) + (b^(1/3)*log(9*a^(7/3)*b^(8/3)*((3^(1/2)*2i)/3 - 2/3)^2 + 16
*a^2*b^3*x^(1/3))*((3^(1/2)*2i)/3 - 2/3))/a^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{4x^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)ab+4x^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2-9b^{\frac{2}{3}}a^{\frac{4}{3}}-12b^{\frac{5}{3}}a^{\frac{1}{3}}x-2x^{\frac{1}{3}}}{}$$

input
```
int(1/x^(4/3)/(b*x+a)^2,x)
```

output
```
(4*x**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3
)))*a*b + 4*x**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/
3)*sqrt(3)))*b**2*x - 9*b**(2/3)*a**(1/3)*a - 12*b**(2/3)*a**(1/3)*b*x - 2
*x**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a
*b - 2*x**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2
/3))*b**2*x + 4*x**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a*b + 4*x**(1/3
)*log(a**(1/3) + x**(1/3)*b**(1/3))*b**2*x)/(3*x**(1/3)*b**(2/3)*a**(1/3)*
a**2*(a + b*x))
```

3.332 $\int \frac{1}{x^{5/3}(a+bx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 128

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}$$

output
```
-5/2/a^2/x^(2/3)+1/a/x^(2/3)/(b*x+a)+5/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)-5/2*b^(2/3)*ln(a^(1/3)+b^(1
/3)*x^(1/3))/a^(8/3)+5/6*b^(2/3)*ln(b*x+a)/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{-\frac{3a^{2/3}(3a+5bx)}{x^{2/3}(a+bx)} + 10\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + 5b^{2/3}}{6a^{8/3}}$$

input
```
Integrate[1/(x^(5/3)*(a + b*x)^2),x]
```

output
```
((-3*a^(2/3)*(3*a + 5*b*x))/(x^(2/3)*(a + b*x)) + 10*Sqrt[3]*b^(2/3)*ArcTa
n[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 10*b^(2/3)*Log[a^(1/3) + b^
(1/3)*x^(1/3)] + 5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)
*x^(2/3)])/(6*a^(8/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {52, 61, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/3}(a+bx)^2} \, dx$$

$$\Big\downarrow \text{52}$$

$$\frac{5\int \frac{1}{x^{5/3}(a+bx)}dx}{3a}+\frac{1}{ax^{2/3}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(-\frac{b\int \frac{1}{x^{2/3}(a+bx)}dx}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}$$

$$\Big\downarrow \text{70}$$

$$\frac{5\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{5\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{5\left(-\frac{b\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{5\left(-\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}$$

input
```
Int[1/(x^(5/3)*(a + b*x)^2),x]
```

output
```
1/(a*x^(2/3)*(a + b*x)) + (5*(-3/(2*a*x^(2/3)) - (b*(-((Sqrt[3]*ArcTan[(1
- (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3)*b^(1/
3))))/a))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{3b\left(\frac{x^{\frac{1}{3}}}{3bx+3a}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^2}-\frac{3}{2a^2x^{\frac{2}{3}}}$	124
default	$-\frac{3b\left(\frac{x^{\frac{1}{3}}}{3bx+3a}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^2}-\frac{3}{2a^2x^{\frac{2}{3}}}$	124

input
```
int(1/x^(5/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3*b/a^2*(1/3*x^(1/3)/(b*x+a)+5/9/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-5/
18/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+5/9/b/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))-3/2/a^2/x^(2/3
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx =$$

$$-\frac{10\sqrt{3}(bx^2+ax)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-5\left(bx^2+ax\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}-abx^{\frac{1}{3}}\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\right.\right.}{6\left(a^2bx^2+a^3x\right)}$$

input
```
integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(10*sqrt(3)*(b*x^2 + a*x)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(
1/3)*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(b*x^2 + a*x)*(b^2/a^2)^(1/3)*log
(b^2*x^(2/3) - a*b*x^(1/3)*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(b*
x^2 + a*x)*(b^2/a^2)^(1/3)*log(b*x^(1/3) + a*(b^2/a^2)^(1/3)) + 3*(5*b*x +
 3*a)*x^(1/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(126) = 252$.

Time = 104.51 (sec) , antiderivative size = 590, normalized size of antiderivative = 4.61

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{8}{3}}} \\ -\frac{3}{2a^2x^{\frac{2}{3}}} \\ -\frac{3}{8b^2x^{\frac{8}{3}}} \\ -\frac{9a^2}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} + \frac{10abx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} - \frac{5abx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4(-\right.}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} \end{cases}$$

input
```
integrate(1/x**(5/3)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (-3/(2*a**2*x**(2/3)), Eq(b
, 0)), (-3/(8*b**2*x**(8/3)), Eq(a, 0)), (-9*a**2/(6*a**4*x**(2/3) + 6*a**
3*b*x**(5/3)) + 10*a*b*x**(2/3)*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3)
)/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) - 5*a*b*x**(2/3)*(-a/b)**(1/3)*log
(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**4*x**(2/3)
 + 6*a**3*b*x**(5/3)) - 10*sqrt(3)*a*b*x**(2/3)*(-a/b)**(1/3)*atan(2*sqrt(
3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**4*x**(2/3) + 6*a**3*b*x**
(5/3)) + 10*a*b*x**(2/3)*(-a/b)**(1/3)*log(2)/(6*a**4*x**(2/3) + 6*a**3*b*
x**(5/3)) - 15*a*b*x/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) + 10*b**2*x**(5
/3)*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(6*a**4*x**(2/3) + 6*a**3*
b*x**(5/3)) - 5*b**2*x**(5/3)*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-
a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) - 10*
sqrt(3)*b**2*x**(5/3)*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/
3)) + sqrt(3)/3)/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) + 10*b**2*x**(5/3)*
(-a/b)**(1/3)*log(2)/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = -\frac{5\,bx+3\,a}{2\left(a^2bx^{\frac{5}{3}}+a^3x^{\frac{2}{3}}\right)} - \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(5*b*x + 3*a)/(a^2*b*x^(5/3) + a^3*x^(2/3)) - 5/3*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/6*log
(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/3*log(
x^(1/3) + (a/b)^(1/3))/(a^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{5\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3} - \frac{5\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3} - \frac{bx^{\frac{1}{3}}}{(bx+a)a^2} - \frac{5\,(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3} - \frac{3}{2\,a^2x^{\frac{2}{3}}}$$

input
```
integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
5/3*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 5/3*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3
 - b*x^(1/3)/((b*x + a)*a^2) - 5/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-
a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 3/2/(a^2*x^(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{5\,{(-1)}^{1/3}\,b^{2/3}\,\ln\left(15\,{(-1)}^{1/3}\,a^{13/3}\,b^{8/3}-15\,a^4\,b^3\,x^{1/3}\right)}{3\,a^{8/3}} - \frac{\frac{3}{2\,a}+\frac{5\,b\,x}{2\,a^2}}{a\,x^{2/3}+b\,x^{5/3}} + \frac{5\,{(-1)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}-15\,{(-1)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}} - \frac{5\,{(-1)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}+15\,{(-1)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}$$

input
```
int(1/(x^(5/3)*(a + b*x)^2),x)
```

output
```
(5*(-1)^(1/3)*b^(2/3)*log(15*(-1)^(1/3)*a^(13/3)*b^(8/3) - 15*a^4*b^3*x^(1
/3)))/(3*a^(8/3)) - (3/(2*a) + (5*b*x)/(2*a^2))/(a*x^(2/3) + b*x^(5/3)) +
(5*(-1)^(1/3)*b^(2/3)*log(15*a^4*b^3*x^(1/3) - 15*(-1)^(1/3)*a^(13/3)*b^(8
/3)*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(8/3)) - (5*(-1)^
(1/3)*b^(2/3)*log(15*a^4*b^3*x^(1/3) + 15*(-1)^(1/3)*a^(13/3)*b^(8/3)*((3^
(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.64

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{10x^{\frac{2}{3}}a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b+10x^{\frac{5}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2+5x^{\frac{2}{3}}a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-x^{\frac{1}{3}}\right.}{}$$

input
```
int(1/x^(5/3)/(b*x+a)^2,x)
```

output
```
(10*x**(2/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1
/3)*sqrt(3)))*a*b + 10*x**(2/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/
3)*b**(1/3))/(a**(1/3)*sqrt(3)))*b**2*x + 5*x**(2/3)*a**(1/3)*log(a**(2/3)
 - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a*b + 5*x**(2/3)*a**(1/
3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*b**2*x -
 10*x**(2/3)*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a*b - 10*x**(2/3)*
a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*b**2*x - 9*b**(1/3)*a**2 - 15*b
**(1/3)*a*b*x)/(6*x**(2/3)*b**(1/3)*a**3*(a + b*x))
```

3.333 $\int \frac{x^{5/3}}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 141

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = \frac{ax^{2/3}}{2b^2(a+bx)^2} - \frac{4x^{2/3}}{3b^2(a+bx)} - \frac{5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}b^{8/3}} + \frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}}$$

output
```
1/2*a*x^(2/3)/b^2/(b*x+a)^2-4/3*x^(2/3)/b^2/(b*x+a)-5/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b^(8/3)-5/6*ln(a^(1/
3)+b^(1/3)*x^(1/3))/a^(1/3)/b^(8/3)+5/18*ln(b*x+a)/a^(1/3)/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = \frac{-\frac{3b^{2/3}x^{2/3}(5a+8bx)}{(a+bx)^2} - \frac{10\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}-\right.}{\sqrt[3]{a}}}{18b^{8/3}}$$

input
```
Integrate[x^(5/3)/(a + b*x)^3,x]
```

output
```
((-3*b^(2/3)*x^(2/3)*(5*a + 8*b*x))/(a + b*x)^2 - (10*Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - (10*Log[a^(1/3) + b^(1/3)
*x^(1/3)])/a^(1/3) + (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^
(2/3)])/a^(1/3))/(18*b^(8/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 51, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/3}}{(a+bx)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5\int \frac{x^{2/3}}{(a+bx)^2}dx}{6b}-\frac{x^{5/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{5\left(\frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{3b}-\frac{x^{2/3}}{b(a+bx)}\right)}{6b}-\frac{x^{5/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{68}$$

$$\frac{5\left(\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}\right)}{6b}-\frac{x^{5/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{5\left(\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}\right)}{6b}-\frac{x^{5/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{5\left(\frac{2\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}\right)}{6b}-\frac{x^{5/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{5\left(\frac{2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^{2/3}}{b(a+bx)}\right)}{6b}-\frac{x^{5/3}}{2b(a+bx)^2}$$

input
```
Int[x^(5/3)/(a + b*x)^3,x]
```

output
```
-1/2*x^(5/3)/(b*(a + b*x)^2) + (5*(-(x^(2/3)/(b*(a + b*x))) + (2*(-((Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(2/3))) -
 (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[a + b*x]/(2*
a^(1/3)*b^(2/3))))/(3*b)))/(6*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^{2}}}{(bx+a)^{2}}+\frac{-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b^{2}}$	130
default	$\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^{2}}}{(bx+a)^{2}}+\frac{-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b^{2}}$	130

input
```
int(x^(5/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3*(-4/9*x^(5/3)/b-5/18*a*x^(2/3)/b^2)/(b*x+a)^2+5/3/b^2*(-1/3/b/(a/b)^(1/3
)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)
+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x^(1/3)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.

Time = 0.09 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.59

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = \left[\frac{15\sqrt{\frac{1}{3}}(ab^3x^2+2a^2b^2x+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{}}{bx+a}\right.}{} \right.$$

input
```
integrate(x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-
a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(
b*x + a)) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (
-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 10*(b^2*x^2 + 2*a*b*x + a^2)*(
-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) - 3*(8*a*b^3*x + 5*a^2*b^2)*
x^(2/3))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/18*(30*sqrt(1/3)*(a*b^3*x^
2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(
1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(b^2*x^2 + 2*a*b*x +
 a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)
^(2/3)) - 10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*
b^2)^(1/3)) - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3))/(a*b^6*x^2 + 2*a^2*b^5*x
 + a^3*b^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/3)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = -\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 5/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^
(1/3)) + 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^
(1/3)) - 5/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = -\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} - \frac{5\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4} - \frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\,(bx+a)^2b^2} + \frac{5\,(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4}$$

input
```
integrate(x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-5/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) - 5/9*sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a*b^4) - 1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/((b*x + a)^2*b^2) + 5/18*(-a*b^2
)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.17

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = \frac{5\,\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{25\,(-a)^{1/3}}{9\,b^{10/3}}\right)}{9\,(-a)^{1/3}\,b^{8/3}} - \frac{\frac{4\,x^{5/3}}{3\,b}+\frac{5\,a\,x^{2/3}}{6\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2} + \frac{\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{(-a)^{1/3}\left(-5+\sqrt{3}\,5\mathrm{i}\right)^2}{36\,b^{10/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{18\,(-a)^{1/3}\,b^{8/3}} - \frac{\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{(-a)^{1/3}\left(5+\sqrt{3}\,5\mathrm{i}\right)^2}{36\,b^{10/3}}\right)\left(5+\sqrt{3}\,5\mathrm{i}\right)}{18\,(-a)^{1/3}\,b^{8/3}}$$

input
```
int(x^(5/3)/(a + b*x)^3,x)
```

output
```
(5*log((25*x^(1/3))/(9*b^3) - (25*(-a)^(1/3))/(9*b^(10/3))))/(9*(-a)^(1/3)
*b^(8/3)) - ((4*x^(5/3))/(3*b) + (5*a*x^(2/3))/(6*b^2))/(a^2 + b^2*x^2 + 2
*a*b*x) + (log((25*x^(1/3))/(9*b^3) - ((-a)^(1/3)*(3^(1/2)*5i - 5)^2)/(36*
b^(10/3)))*(3^(1/2)*5i - 5))/(18*(-a)^(1/3)*b^(8/3)) - (log((25*x^(1/3))/(
9*b^3) - ((-a)^(1/3)*(3^(1/2)*5i + 5)^2)/(36*b^(10/3)))*(3^(1/2)*5i + 5))/
(18*(-a)^(1/3)*b^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.96

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = \frac{-10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2-20\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)abx-10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^2}{}$$

input
```
int(x^(5/3)/(b*x+a)^3,x)
```

output
```
( - 10*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a
**2 - 20*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))
*a*b*x - 10*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3
)))*b**2*x**2 - 15*x**(2/3)*b**(2/3)*a**(1/3)*a - 24*x**(2/3)*b**(2/3)*a**
(1/3)*b*x + 5*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3
))*a**2 + 10*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3)
)*a*b*x + 5*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))
*b**2*x**2 - 10*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2 - 20*log(a**(1/3) +
 x**(1/3)*b**(1/3))*a*b*x - 10*log(a**(1/3) + x**(1/3)*b**(1/3))*b**2*x**2
)/(18*b**(2/3)*a**(1/3)*b**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.334 $\int \frac{x^{4/3}}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 141

$$\int \frac{x^{4/3}}{(a+bx)^3} \, dx = \frac{a\sqrt[3]{x}}{2b^2(a+bx)^2} - \frac{7\sqrt[3]{x}}{6b^2(a+bx)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}} - \frac{\log(a+bx)}{9a^{2/3}b^{7/3}}$$

output
```
1/2*a*x^(1/3)/b^2/(b*x+a)^2-7/6*x^(1/3)/b^2/(b*x+a)-2/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(7/3)+1/3*ln(a^(1/
3)+b^(1/3)*x^(1/3))/a^(2/3)/b^(7/3)-1/9*ln(b*x+a)/a^(2/3)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{x^{4/3}}{(a+bx)^3} \, dx = \frac{-\frac{3\sqrt[3]{b}\sqrt[3]{x}(4a+7bx)}{(a+bx)^2} - \frac{4\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{2/3}} - \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+\right.}{a^{2/3}}}{18b^{7/3}}$$

input
```
Integrate[x^(4/3)/(a + b*x)^3,x]
```

output
```
((-3*b^(1/3)*x^(1/3)*(4*a + 7*b*x))/(a + b*x)^2 - (4*Sqrt[3]*ArcTan[(1 - (
2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*Log[a^(1/3) + b^(1/3)*x
^(1/3)])/a^(2/3) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2
/3)])/a^(2/3))/(18*b^(7/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 51, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{51}$$

$$\frac{2\int \frac{\sqrt[3]{x}}{(a+bx)^2}dx}{3b}-\frac{x^{4/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{2\left(\frac{\int \frac{1}{x^{2/3}(a+bx)}dx}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}\right)}{3b}-\frac{x^{4/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{70}$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}\right)}{3b}-\frac{x^{4/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}\right)}{3b}-\frac{x^{4/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}\right)}{3b}-\frac{x^{4/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}}{3b}-\frac{\sqrt[3]{x}}{b(a+bx)}\right)}{3b}-\frac{x^{4/3}}{2b(a+bx)^2}$$

input
```
Int[x^(4/3)/(a + b*x)^3,x]
```

output
```
-1/2*x^(4/3)/(b*(a + b*x)^2) + (2*(-(x^(1/3)/(b*(a + b*x))) + (-((Sqrt[3]*
ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3
*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(
2/3)*b^(1/3)))/(3*b)))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^2}}{(bx+a)^2}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b^2}$	130
default	$\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^2}}{(bx+a)^2}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b^2}$	130

input
```
int(x^(4/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3*(-7/18*x^(4/3)/b-2/9*a*x^(1/3)/b^2)/(b*x+a)^2+2/3/b^2*(1/3/b/(a/b)^(2/3)
*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+
(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
^(1/3)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(100) = 200$.

Time = 0.11 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.57

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a}{}}}{bx+a}\right.}{}\right.$$

input
```
integrate(x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2
*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*
x + a)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2
*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)
^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3
))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 +
2*a^2*b^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3
)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*x^2 + 2
*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2
/3)*x^(1/3)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) +
 (a^2*b)^(2/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^2*b^5*x^2 + 2*a^3*
b^4*x + a^4*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(4/3)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = -\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 2/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^
(2/3)) - 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(
2/3)) + 2/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^3} - \frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\,(bx+a)^2b^2}$$

input
```
integrate(x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-2/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) + 2/9*sqrt(3)*(
-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a*b^3) + 1/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a*b^3) - 1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = \frac{2\ln\left(2\,x^{1/3}+\frac{2\,a^{1/3}}{b^{1/3}}\right)}{9\,a^{2/3}\,b^{7/3}} - \frac{\frac{7\,x^{4/3}}{6\,b}+\frac{2\,a\,x^{1/3}}{3\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2} + \frac{\ln\left(2\,x^{1/3}+\frac{a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}} - \frac{\ln\left(2\,x^{1/3}-\frac{a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}$$

input
```
int(x^(4/3)/(a + b*x)^3,x)
```

output
```
(2*log(2*x^(1/3) + (2*a^(1/3))/b^(1/3)))/(9*a^(2/3)*b^(7/3)) - ((7*x^(4/3)
)/(6*b) + (2*a*x^(1/3))/(3*b^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (log(2*x^(1/3
) + (a^(1/3)*(3^(1/2)*1i - 1))/b^(1/3))*(3^(1/2)*1i - 1))/(9*a^(2/3)*b^(7/
3)) - (log(2*x^(1/3) - (a^(1/3)*(3^(1/2)*1i + 1))/b^(1/3))*(3^(1/2)*1i + 1
))/(9*a^(2/3)*b^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.06

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = \frac{-4a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right) - 8a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx - 4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^2}{}$$

input
```
int(x^(4/3)/(b*x+a)^3,x)
```

output
```
( - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqr
t(3)))*a**2 - 8*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a*
*(1/3)*sqrt(3)))*a*b*x - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b*
*(1/3))/(a**(1/3)*sqrt(3)))*b**2*x**2 - 2*a**(1/3)*log(a**(2/3) - x**(1/3)
*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a**2 - 4*a**(1/3)*log(a**(2/3) - x
**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a*b*x - 2*a**(1/3)*log(a**(
2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*b**2*x**2 + 4*a**(1
/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2 + 8*a**(1/3)*log(a**(1/3) + x**
(1/3)*b**(1/3))*a*b*x + 4*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*b**2*
x**2 - 12*x**(1/3)*b**(1/3)*a**2 - 21*x**(1/3)*b**(1/3)*a*b*x)/(18*b**(1/3
)*a*b**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.335 $\int \frac{x^{2/3}}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \frac{x^{2/3}}{(a+bx)^3} \, dx = -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}}$$

output
```
-1/2*x^(2/3)/b/(b*x+a)^2+1/3*x^(2/3)/a/b/(b*x+a)-1/9*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/b^(5/3)-1/6*ln(a^(1/3)+
b^(1/3)*x^(1/3))/a^(4/3)/b^(5/3)+1/18*ln(b*x+a)/a^(4/3)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \frac{x^{2/3}}{(a+bx)^3} \, dx = \frac{-\frac{3\sqrt[3]{a}b^{2/3}x^{2/3}(a-2bx)}{(a+bx)^2} - 2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + \log\left(a^{2/3} - \right.}{18a^{4/3}b^{5/3}}$$

input
```
Integrate[x^(2/3)/(a + b*x)^3,x]
```

output
```
((-3*a^(1/3)*b^(2/3)*x^(2/3)*(a - 2*b*x))/(a + b*x)^2 - 2*Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b^(1/3)*x^(1/3
)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(4/3)
*b^(5/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 52, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2/3}}{(a+bx)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{\int \frac{1}{\sqrt[3]{x}(a+bx)^2}dx}{3b}-\frac{x^{2/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{3a}+\frac{x^{2/3}}{a(a+bx)}}{3b}-\frac{x^{2/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{68}$$

$$\frac{\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}}{3b}-\frac{x^{2/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}}{3b}-\frac{x^{2/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}}{3b}-\frac{x^{2/3}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}}{3b}-\frac{x^{2/3}}{2b(a+bx)^2}$$

input
```
Int[x^(2/3)/(a + b*x)^3,x]
```

output
```
-1/2*x^(2/3)/(b*(a + b*x)^2) + (x^(2/3)/(a*(a + b*x)) + (-((Sqrt[3]*ArcTan
[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b
^(2/3)))/(3*a))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{(bx+a)^{2}}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3ba}$	132
default	$\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{(bx+a)^{2}}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3ba}$	132

input
```
int(x^(2/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3*(1/9/a*x^(5/3)-1/18*x^(2/3)/b)/(b*x+a)^2+1/3/b/a*(-1/3/b/(a/b)^(1/3)*ln(
x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b
)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/
3)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(102) = 204$.

Time = 0.09 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.55

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+3\,\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{(}}{bx+a}\right.}{}\right.$$

input
```
integrate(x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a
*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b
*x + a)) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*
b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b
^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*a*b^3*x - a^2*b^2)*x^(2/3
))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2
*a^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3)
+ (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^2 + 2*a*b*x + a^2)*(
-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3))
 - 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/
3)) + 3*(2*a*b^3*x - a^2*b^2)*x^(2/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^
3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1171 vs. $2(126) = 252$.

Time = 123.85 (sec) , antiderivative size = 1171, normalized size of antiderivative = 8.19

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(2/3)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a**3), Eq(b,
 0)), (-3/(4*b**3*x**(4/3)), Eq(a, 0)), (2*a**2*log(x**(1/3) - (-a/b)**(1/
3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4
*x**2*(-a/b)**(1/3)) - a**2*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*
(-a/b)**(2/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3)
+ 18*a*b**4*x**2*(-a/b)**(1/3)) + 2*sqrt(3)*a**2*atan(2*sqrt(3)*x**(1/3)/(
3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x
*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 2*a**2*log(2)/(18*a**3*b*
*2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(
1/3)) - 3*a*b*x**(2/3)*(-a/b)**(1/3)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2
*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 4*a*b*x*log(x**(1/
3) - (-a/b)**(1/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(
1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) - 2*a*b*x*log(4*x**(2/3) + 4*x**(1/3)
*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b*
*3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 4*sqrt(3)*a*b*x*atan(
2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**3*b**2*(-a/b)**(1
/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 4*a*b
*x*log(2)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*
a*b**4*x**2*(-a/b)**(1/3)) + 6*b**2*x**(5/3)*(-a/b)**(1/3)/(18*a**3*b**2*(
-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = \frac{2\,bx^{\frac{5}{3}} - ax^{\frac{2}{3}}}{6\,(ab^3x^2 + 2\,a^2b^2x + a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*b*x^(5/3) - a*x^(2/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)
^(1/3)) + 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/
b)^(1/3)) - 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.04

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3} + \frac{2\,bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6\,(bx+a)^2ab} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3}$$

input
```
integrate(x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) - 1/9*sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a^2*b^3) + 1/6*(2*b*x^(5/3) - a*x^(2/3))/((b*x + a)^2*a*b) + 1/18*(-a*b^2
)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.20

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = \frac{\frac{x^{5/3}}{3\,a}-\frac{x^{2/3}}{6\,b}}{a^2+2\,a\,b\,x+b^2\,x^2} + \frac{\ln\left(\frac{1}{9\,a^{5/3}\,(-b)^{4/3}}+\frac{x^{1/3}}{9\,a^2\,b}\right)}{9\,a^{4/3}\,(-b)^{5/3}} + \frac{\ln\left(\frac{x^{1/3}}{9\,a^2\,b}+\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{36\,a^{5/3}\,(-b)^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,(-b)^{5/3}} - \frac{\ln\left(\frac{x^{1/3}}{9\,a^2\,b}+\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{36\,a^{5/3}\,(-b)^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,(-b)^{5/3}}$$

input
```
int(x^(2/3)/(a + b*x)^3,x)
```

output
```
(x^(5/3)/(3*a) - x^(2/3)/(6*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(1/(9*a^(5/
3)*(-b)^(4/3)) + x^(1/3)/(9*a^2*b))/(9*a^(4/3)*(-b)^(5/3)) + (log(x^(1/3)/
(9*a^2*b) + (3^(1/2)*1i - 1)^2/(36*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i - 1))/
(18*a^(4/3)*(-b)^(5/3)) - (log(x^(1/3)/(9*a^2*b) + (3^(1/2)*1i + 1)^2/(36*
a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i + 1))/(18*a^(4/3)*(-b)^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.92

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)abx-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^2-3}{}$$

input
```
int(x^(2/3)/(b*x+a)^3,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a*
*2 - 4*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a
*b*x - 2*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))
*b**2*x**2 - 3*x**(2/3)*b**(2/3)*a**(1/3)*a + 6*x**(2/3)*b**(2/3)*a**(1/3)
*b*x + log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a**2
 + 2*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a*b*x
+ log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*b**2*x**2
 - 2*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2 - 4*log(a**(1/3) + x**(1/3)*b*
*(1/3))*a*b*x - 2*log(a**(1/3) + x**(1/3)*b**(1/3))*b**2*x**2)/(18*b**(2/3
)*a**(1/3)*a*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.336 $\int \frac{\sqrt[3]{x}}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3} \, dx = -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}}$$

output
```
-1/2*x^(1/3)/b/(b*x+a)^2+1/6*x^(1/3)/a/b/(b*x+a)-1/9*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(4/3)+1/6*ln(a^(1/3)+
b^(1/3)*x^(1/3))/a^(5/3)/b^(4/3)-1/18*ln(b*x+a)/a^(5/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3} \, dx = \frac{\frac{3a^{2/3}\sqrt[3]{b}\sqrt[3]{x}(-2a+bx)}{(a+bx)^2} - 2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) - \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}\right.}{18a^{5/3}b^{4/3}}$$

input
```
Integrate[x^(1/3)/(a + b*x)^3,x]
```

output
```
((3*a^(2/3)*b^(1/3)*x^(1/3)*(-2*a + b*x))/(a + b*x)^2 - 2*Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + b^(1/3)*x^(1/3
)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(5/3)
*b^(4/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 52, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx$$

$$\Big\downarrow \ 51$$

$$\frac{\int \frac{1}{x^{2/3}(a+bx)^2}dx}{6b}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \ 52$$

$$\frac{\frac{2\int \frac{1}{x^{2/3}(a+bx)}dx}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}}{6b}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \ 70$$

$$\frac{\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}}{6b}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}}{6b}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}}{6b}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}}{6b}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

input
```
Int[x^(1/3)/(a + b*x)^3,x]
```

output
```
-1/2*x^(1/3)/(b*(a + b*x)^2) + (x^(1/3)/(a*(a + b*x)) + (2*(-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Lo
g[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3
)*b^(1/3))))/(3*a))/(6*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{4}{3}}}{6a}-\frac{x^{\frac{1}{3}}}{3b}}{(bx+a)^{2}}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3ba}$	132
default	$\frac{\frac{x^{\frac{4}{3}}}{6a}-\frac{x^{\frac{1}{3}}}{3b}}{(bx+a)^{2}}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3ba}$	132

input
```
int(x^(1/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3*(1/18/a*x^(4/3)-1/9*x^(1/3)/b)/(b*x+a)^2+1/3/b/a*(1/3/b/(a/b)^(2/3)*ln(x
^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)
^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3
)-1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(102) = 204.

Time = 0.10 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(ab^3x^2+2a^2b^2x+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right.}{}\right.$$

input
```
integrate(x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2
*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*
x + a)) - (b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b
)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(
2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(
a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2
*b^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a -
 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^2 + 2*a*b*x
+ a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(
1/3)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b
)^(2/3)) + 3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a
^5*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(126) = 252.

Time = 100.03 (sec) , antiderivative size = 899, normalized size of antiderivative = 6.29

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/3)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**3), Eq(b,
 0)), (-3/(5*b**3*x**(5/3)), Eq(a, 0)), (-6*a**2*x**(1/3)/(18*a**4*b + 36*
a**3*b**2*x + 18*a**2*b**3*x**2) - 2*a**2*(-a/b)**(1/3)*log(x**(1/3) - (-a
/b)**(1/3))/(18*a**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + a**2*(-a/b)
**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a
**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + 2*sqrt(3)*a**2*(-a/b)**(1/3)
*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b + 36*a*
*3*b**2*x + 18*a**2*b**3*x**2) - 2*a**2*(-a/b)**(1/3)*log(2)/(18*a**4*b +
36*a**3*b**2*x + 18*a**2*b**3*x**2) + 3*a*b*x**(4/3)/(18*a**4*b + 36*a**3*
b**2*x + 18*a**2*b**3*x**2) - 4*a*b*x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)*
*(1/3))/(18*a**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + 2*a*b*x*(-a/b)*
*(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a*
*4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + 4*sqrt(3)*a*b*x*(-a/b)**(1/3)
*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b + 36*a*
*3*b**2*x + 18*a**2*b**3*x**2) - 4*a*b*x*(-a/b)**(1/3)*log(2)/(18*a**4*b +
 36*a**3*b**2*x + 18*a**2*b**3*x**2) - 2*b**2*x**2*(-a/b)**(1/3)*log(x**(1
/3) - (-a/b)**(1/3))/(18*a**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + b*
*2*x**2*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)
**(2/3))/(18*a**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + 2*sqrt(3)*b**2
*x**2*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = \frac{bx^{\frac{4}{3}} - 2\,ax^{\frac{1}{3}}}{6\,(ab^3x^2 + 2\,a^2b^2x + a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/6*(b*x^(4/3) - 2*a*x^(1/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)
^(2/3)) - 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/
b)^(2/3)) + 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2} + \frac{bx^{\frac{4}{3}}-2\,ax^{\frac{1}{3}}}{6\,(bx+a)^2ab}$$

input
```
integrate(x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) + 1/9*sqrt(3)*(
-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/
(a^2*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a^2*b^2) + 1/6*(b*x^(4/3) - 2*a*x^(1/3))/((b*x + a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = \frac{\frac{x^{4/3}}{6\,a}-\frac{x^{1/3}}{3\,b}}{a^2+2\,a\,b\,x+b^2\,x^2} + \frac{\ln\left(\frac{b^{2/3}}{a^{2/3}}+\frac{b\,x^{1/3}}{a}\right)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(\frac{b\,x^{1/3}}{a}+\frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(\frac{b\,x^{1/3}}{a}-\frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}$$

input
```
int(x^(1/3)/(a + b*x)^3,x)
```

output
```
(x^(4/3)/(6*a) - x^(1/3)/(3*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(b^(2/3)/a^
(2/3) + (b*x^(1/3))/a)/(9*a^(5/3)*b^(4/3)) + (log((b*x^(1/3))/a + (b^(2/3)
*(3^(1/2)*1i - 1))/(2*a^(2/3)))*(3^(1/2)*1i - 1))/(18*a^(5/3)*b^(4/3)) - (
log((b*x^(1/3))/a - (b^(2/3)*(3^(1/2)*1i + 1))/(2*a^(2/3)))*(3^(1/2)*1i +
1))/(18*a^(5/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.03

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = \frac{-2a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)-4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^2-a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}-x\right.}{}$$

input
```
int(x^(1/3)/(b*x+a)^3,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqr
t(3)))*a**2 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a*
*(1/3)*sqrt(3)))*a*b*x - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b*
*(1/3))/(a**(1/3)*sqrt(3)))*b**2*x**2 - a**(1/3)*log(a**(2/3) - x**(1/3)*b
**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a**2 - 2*a**(1/3)*log(a**(2/3) - x**
(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a*b*x - a**(1/3)*log(a**(2/3)
 - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*b**2*x**2 + 2*a**(1/3)*
log(a**(1/3) + x**(1/3)*b**(1/3))*a**2 + 4*a**(1/3)*log(a**(1/3) + x**(1/3
)*b**(1/3))*a*b*x + 2*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*b**2*x**2
 - 6*x**(1/3)*b**(1/3)*a**2 + 3*x**(1/3)*b**(1/3)*a*b*x)/(18*b**(1/3)*a**2
*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.337 $\int \frac{1}{\sqrt[3]{x}(a+bx)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3} \, dx = \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}}$$

output
```
1/2*x^(2/3)/a/(b*x+a)^2+2/3*x^(2/3)/a^2/(b*x+a)-2/9*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/b^(2/3)-1/3*ln(a^(1/3)+b
^(1/3)*x^(1/3))/a^(7/3)/b^(2/3)+1/9*ln(b*x+a)/a^(7/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx$$
$$=\frac{\frac{3\sqrt[3]{a}x^{2/3}(7a+4bx)}{(a+bx)^2}-\frac{4\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{2/3}}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{b^{2/3}}}{18a^{7/3}}$$

input
```
Integrate[1/(x^(1/3)*(a + b*x)^3),x]
```

output
```
((3*a^(1/3)*x^(2/3)*(7*a + 4*b*x))/(a + b*x)^2 - (4*Sqrt[3]*ArcTan[(1 - (2
*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/b^(2/3) - (4*Log[a^(1/3) + b^(1/3)*x^
(1/3)])/b^(2/3) + (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/
3)])/b^(2/3))/(18*a^(7/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {52, 52, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx$$
$$\Big\downarrow \; 52$$
$$\frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)^2}dx}{3a}+\frac{x^{2/3}}{2a(a+bx)^2}$$
$$\Big\downarrow \; 52$$

$$\frac{2\left(\frac{\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{3a}+\frac{x^{2/3}}{a(a+bx)}\right)}{3a}+\frac{x^{2/3}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{68}$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}\right)}{3a}+\frac{x^{2/3}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}\right)}{3a}+\frac{x^{2/3}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}\right)}{3a}+\frac{x^{2/3}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^{2/3}}{a(a+bx)}\right)}{3a}+\frac{x^{2/3}}{2a(a+bx)^2}$$

input
```
Int[1/(x^(1/3)*(a + b*x)^3),x]
```

output
```
x^(2/3)/(2*a*(a + b*x)^2) + (2*(x^(2/3)/(a*(a + b*x)) + (-((Sqrt[3]*ArcTan
[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(1/3)*b^(2/3))) - (3*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[a + b*x]/(2*a^(1/3)*b
^(2/3)))/(3*a)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$\frac{x^{\frac{2}{3}}}{2a(bx+a)^{2}}+\frac{\frac{2x^{\frac{2}{3}}}{3a(bx+a)}+\frac{2\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a}}{a}$	139
default	$\frac{x^{\frac{2}{3}}}{2a(bx+a)^{2}}+\frac{\frac{2x^{\frac{2}{3}}}{3a(bx+a)}+\frac{2\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a}}{a}$	139

input
```
int(1/x^(1/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(2/3)/a/(b*x+a)^2+2/a*(1/3*x^(2/3)/a/(b*x+a)+1/3/a*(-1/3/b/(a/b)^(1/
3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3
)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x^(1/3)-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(99) = 198$.

Time = 0.09 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.64

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+3\,\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx+a}\right.}{}\right.$$

input
```
integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a
*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b
*x + a)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-
a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 4*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(4*a*b^3*x + 7*a^2*b^2)*x^
(2/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(12*sqrt(1/3)*(a*b^3*x^
2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(
1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^2 + 2*a*b*x +
 a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)
^(2/3)) - 4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b
^2)^(1/3)) + 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x
 + a^5*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(129) = 258$.

Time = 106.40 (sec) , antiderivative size = 1175, normalized size of antiderivative = 8.39

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(1/3)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a**3), Eq(b,
 0)), (-3/(7*b**3*x**(7/3)), Eq(a, 0)), (4*a**2*log(x**(1/3) - (-a/b)**(1/
3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3
*x**2*(-a/b)**(1/3)) - 2*a**2*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) +
4*(-a/b)**(2/3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) +
 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 4*sqrt(3)*a**2*atan(2*sqrt(3)*x**(1/3)
/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*
(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 4*a**2*log(2)/(18*a**4*
b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)*
*(1/3)) + 21*a*b*x**(2/3)*(-a/b)**(1/3)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3
*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 8*a*b*x*log(x**
(1/3) - (-a/b)**(1/3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(
1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) - 4*a*b*x*log(4*x**(2/3) + 4*x**(1
/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b*
*2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 8*sqrt(3)*a*b*x*at
an(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b*(-a/b)**(1
/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 8*
a*b*x*log(2)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*
a**2*b**3*x**2*(-a/b)**(1/3)) + 12*b**2*x**(5/3)*(-a/b)**(1/3)/(18*a**4*b*
(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)...
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = \frac{4\,bx^{\frac{5}{3}} + 7\,ax^{\frac{2}{3}}}{6\,(a^2b^2x^2 + 2\,a^3bx + a^4)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 2/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)
^(1/3)) + 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b
)^(1/3)) - 2/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} - \frac{2\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2} + \frac{4\,bx^{\frac{5}{3}} + 7\,ax^{\frac{2}{3}}}{6\,(bx+a)^2a^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b^2}$$

input
```
integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-2/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 2/9*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3
*b^2) + 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/((b*x + a)^2*a^2) + 1/9*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = \frac{\frac{7\,x^{2/3}}{6\,a}+\frac{2\,b\,x^{5/3}}{3\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{2\,\ln\left(\frac{4\,b\,x^{1/3}}{9\,a^4}-\frac{4\,b^{2/3}}{9\,{\left(-a\right)}^{11/3}}\right)}{9\,{\left(-a\right)}^{7/3}\,b^{2/3}} + \frac{\ln\left(\frac{4\,b\,x^{1/3}}{9\,a^4}-\frac{b^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{9\,{\left(-a\right)}^{11/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,{\left(-a\right)}^{7/3}\,b^{2/3}} - \frac{\ln\left(\frac{4\,b\,x^{1/3}}{9\,a^4}-\frac{b^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{9\,{\left(-a\right)}^{11/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,{\left(-a\right)}^{7/3}\,b^{2/3}}$$

input
```
int(1/(x^(1/3)*(a + b*x)^3),x)
```

output
```
((7*x^(2/3))/(6*a) + (2*b*x^(5/3))/(3*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (2
*log((4*b*x^(1/3))/(9*a^4) - (4*b^(2/3))/(9*(-a)^(11/3))))/(9*(-a)^(7/3)*b
^(2/3)) + (log((4*b*x^(1/3))/(9*a^4) - (b^(2/3)*(3^(1/2)*1i - 1)^2)/(9*(-a
)^(11/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(7/3)*b^(2/3)) - (log((4*b*x^(1/3))/(
9*a^4) - (b^(2/3)*(3^(1/2)*1i + 1)^2)/(9*(-a)^(11/3)))*(3^(1/2)*1i + 1))/(
9*(-a)^(7/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.98

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx$$
$$= \frac{-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2-8\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)abx-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^2+21x^{\frac{2}{3}}b^{\frac{2}{3}}a^{\frac{4}{3}}+12x^{\frac{5}{3}}}{}$$

input
```
int(1/x^(1/3)/(b*x+a)^3,x)
```

output
```
( - 4*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a*
*2 - 8*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a
*b*x - 4*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))
*b**2*x**2 + 21*x**(2/3)*b**(2/3)*a**(1/3)*a + 12*x**(2/3)*b**(2/3)*a**(1/
3)*b*x + 2*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*
a**2 + 4*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a*
b*x + 2*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*b**
2*x**2 - 4*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2 - 8*log(a**(1/3) + x**(1
/3)*b**(1/3))*a*b*x - 4*log(a**(1/3) + x**(1/3)*b**(1/3))*b**2*x**2)/(18*b
**(2/3)*a**(1/3)*a**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.338 $\int \frac{1}{x^{2/3}(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)} - \frac{5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}$$

output
```
1/2*x^(1/3)/a/(b*x+a)^2+5/6*x^(1/3)/a^2/(b*x+a)-5/9*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/b^(1/3)+5/6*ln(a^(1/3)+b
^(1/3)*x^(1/3))/a^(8/3)/b^(1/3)-5/18*ln(b*x+a)/a^(8/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \frac{\frac{3a^{2/3}\sqrt[3]{x}(8a+5bx)}{(a+bx)^2}-\frac{10\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{b}}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{\sqrt[3]{b}}}{18a^{8/3}}$$

input
```
Integrate[1/(x^(2/3)*(a + b*x)^3),x]
```

output
```
((3*a^(2/3)*x^(1/3)*(8*a + 5*b*x))/(a + b*x)^2 - (10*Sqrt[3]*ArcTan[(1 - (
2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*Log[a^(1/3) + b^(1/3)*
x^(1/3)])/b^(1/3) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(
2/3)])/b^(1/3))/(18*a^(8/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {52, 52, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{2/3}(a+bx)^3}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{5\int \frac{1}{x^{2/3}(a+bx)^2}dx}{6a}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{5\left(\frac{2\int \frac{1}{x^{2/3}(a+bx)}dx}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}\right)}{6a}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \text{70}$$

$$\frac{5\left(\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}\right)}{6a}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{5\left(\frac{2\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}\right)}{6a}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{5\left(\frac{2\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}\right)}{6a}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{5\left(\frac{2\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sqrt[3]{x}}{a(a+bx)}\right)}{6a}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

input
```
Int[1/(x^(2/3)*(a + b*x)^3),x]
```

output
```
x^(1/3)/(2*a*(a + b*x)^2) + (5*(x^(1/3)/(a*(a + b*x)) + (2*(-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(a^(2/3)*b^(1/3))) + (3*Lo
g[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[a + b*x]/(2*a^(2/3
)*b^(1/3))))/(3*a)))/(6*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$\frac{x^{\frac{1}{3}}}{2a(bx+a)^{2}}+\frac{\frac{5x^{\frac{1}{3}}}{6a(bx+a)}+\frac{5\left(\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2a}}{a}$	139
default	$\frac{x^{\frac{1}{3}}}{2a(bx+a)^{2}}+\frac{\frac{5x^{\frac{1}{3}}}{6a(bx+a)}+\frac{5\left(\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2a}}{a}$	139

input
```
int(1/x^(2/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/3)/a/(b*x+a)^2+5/2/a*(1/3*x^(1/3)/a/(b*x+a)+2/3/a*(1/3/b/(a/b)^(2
/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/
3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*x^(1/3)-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(99) = 198$.

Time = 0.09 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.56

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \left[\frac{15\sqrt{\frac{1}{3}}(ab^3x^2+2a^2b^2x+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)}{bx+a}\right.}{}\right.$$

input
```
integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/
b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^
2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b
*x + a)) - 5*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^
2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*
b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1
/3))/(a^4*b^3*x^2 + 2*a^5*b^2*x + a^6*b), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 +
2*a^2*b^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3
)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^2 + 2
*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2
/3)*x^(1/3)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3)
+ (a^2*b)^(2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(a^4*b^3*x^2 + 2*a^5
*b^2*x + a^6*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(133) = 266$.

Time = 153.24 (sec) , antiderivative size = 853, normalized size of antiderivative = 6.09

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(2/3)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a**3, Eq(b, 0))
, (-3/(8*b**3*x**(8/3)), Eq(a, 0)), (24*a**2*x**(1/3)/(18*a**5 + 36*a**4*b
*x + 18*a**3*b**2*x**2) - 10*a**2*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/
3))/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) + 5*a**2*(-a/b)**(1/3)*log
(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**5 + 36*a*
*4*b*x + 18*a**3*b**2*x**2) + 10*sqrt(3)*a**2*(-a/b)**(1/3)*atan(2*sqrt(3)
*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*
b**2*x**2) - 10*a**2*(-a/b)**(1/3)*log(2)/(18*a**5 + 36*a**4*b*x + 18*a**3
*b**2*x**2) + 15*a*b*x**(4/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2)
- 20*a*b*x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(18*a**5 + 36*a**4*
b*x + 18*a**3*b**2*x**2) + 10*a*b*x*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1
/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2
*x**2) + 20*sqrt(3)*a*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)*
*(1/3)) + sqrt(3)/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) - 20*a*b*
x*(-a/b)**(1/3)*log(2)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) - 10*b*
*2*x**2*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(18*a**5 + 36*a**4*b*x
 + 18*a**3*b**2*x**2) + 5*b**2*x**2*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1
/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2
*x**2) + 10*sqrt(3)*b**2*x**2*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a
/b)**(1/3)) + sqrt(3)/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) - ...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 5/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)
^(2/3)) - 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/
b)^(2/3)) + 5/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = -\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{5\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} + \frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} + \frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\,(bx+a)^2a^2}$$

input
```
integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-5/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 5/9*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3
*b) + 5/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a^3*b) + 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/((b*x + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \frac{\frac{4\,x^{1/3}}{3\,a}+\frac{5\,b\,x^{4/3}}{6\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2} + \frac{5\,\ln\left(\frac{5\,b^{5/3}}{a^{5/3}}+\frac{5\,b^2\,x^{1/3}}{a^2}\right)}{9\,a^{8/3}\,b^{1/3}} + \frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}+\frac{b^{5/3}\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{2\,a^{5/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}} - \frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}-\frac{b^{5/3}\left(5+\sqrt{3}\,5\mathrm{i}\right)}{2\,a^{5/3}}\right)\left(5+\sqrt{3}\,5\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}$$

input
```
int(1/(x^(2/3)*(a + b*x)^3),x)
```

output
```
((4*x^(1/3))/(3*a) + (5*b*x^(4/3))/(6*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (5
*log((5*b^(5/3))/a^(5/3) + (5*b^2*x^(1/3))/a^2))/(9*a^(8/3)*b^(1/3)) + (lo
g((5*b^2*x^(1/3))/a^2 + (b^(5/3)*(3^(1/2)*5i - 5))/(2*a^(5/3)))*(3^(1/2)*5
i - 5))/(18*a^(8/3)*b^(1/3)) - (log((5*b^2*x^(1/3))/a^2 - (b^(5/3)*(3^(1/2
)*5i + 5))/(2*a^(5/3)))*(3^(1/2)*5i + 5))/(18*a^(8/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.07

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \frac{-10a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)-20a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)bx-10a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b}{a^{\frac{1}{3}}\sqrt{3}}\right.}{}$$

input
```
int(1/x^(2/3)/(b*x+a)^3,x)
```

output
```
( - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sq
rt(3)))*a**2 - 20*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(
a**(1/3)*sqrt(3)))*a*b*x - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)
*b**(1/3))/(a**(1/3)*sqrt(3)))*b**2*x**2 - 5*a**(1/3)*log(a**(2/3) - x**(1
/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a**2 - 10*a**(1/3)*log(a**(2/3)
 - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a*b*x - 5*a**(1/3)*log(
a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*b**2*x**2 + 10*
a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2 + 20*a**(1/3)*log(a**(1/3)
 + x**(1/3)*b**(1/3))*a*b*x + 10*a**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3)
)*b**2*x**2 + 24*x**(1/3)*b**(1/3)*a**2 + 15*x**(1/3)*b**(1/3)*a*b*x)/(18*
b**(1/3)*a**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.339 $\int \frac{1}{x^{4/3}(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 152

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = -\frac{14}{3a^3\sqrt[3]{x}}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}+\frac{7}{6a^2\sqrt[3]{x}(a+bx)}+\frac{14\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}+\frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}}$$

output

```
-14/3/a^3/x^(1/3)+1/2/a/x^(1/3)/(b*x+a)^2+7/6/a^2/x^(1/3)/(b*x+a)+14/9*b^(
1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(10
/3)+7/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(10/3)-7/9*b^(1/3)*ln(b*x+a)
/a^(10/3)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{-\frac{3\sqrt[3]{a}(18a^2+49abx+28b^2x^2)}{\sqrt[3]{x}(a+bx)^2}+28\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right. \cdots}{18a^{10/3}}$$

input
```
Integrate[1/(x^(4/3)*(a + b*x)^3),x]
```

output
```
((-3*a^(1/3)*(18*a^2 + 49*a*b*x + 28*b^2*x^2))/(x^(1/3)*(a + b*x)^2) + 28*
Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^(
1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(10/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {52, 52, 61, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx$$

$$\Big\downarrow \text{52}$$

$$\frac{7\int \frac{1}{x^{4/3}(a+bx)^2}dx}{6a}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{7\left(\frac{4\int \frac{1}{x^{4/3}(a+bx)}dx}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}\right)}{6a}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(\frac{4\left(-\frac{b\int \frac{1}{\sqrt[3]{x}(a+bx)}dx}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}\right)}{6a}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

$$\Big\downarrow \text{68}$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}\right)}{6a}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2b}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}\right)}{6a}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{3\int\frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}\right)}{6a}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{3}{a\sqrt[3]{x}}\right)}{3a}+\frac{1}{a\sqrt[3]{x}(a+bx)}\right)}{6a}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

input
```
Int[1/(x^(4/3)*(a + b*x)^3),x]
```

output
```
1/(2*a*x^(1/3)*(a + b*x)^2) + (7*(1/(a*x^(1/3)*(a + b*x)) + (4*(-3/(a*x^(1
/3)) - (b*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/(
a^(1/3)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3))
 + Log[a + b*x]/(2*a^(1/3)*b^(2/3))))/a))/(3*a)))/(6*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{3}{a^{3} x^{\frac{1}{3}}}-\frac{3 b \left(\frac{\frac{5 b \,x^{\frac{5}{3}}}{9}+\frac{13 a \,x^{\frac{2}{3}}}{18}}{\left(b x +a \right)^{2}}-\frac{14 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{3}}$
default	$-\frac{3}{a^{3} x^{\frac{1}{3}}}-\frac{3 b \left(\frac{\frac{5 b \,x^{\frac{5}{3}}}{9}+\frac{13 a \,x^{\frac{2}{3}}}{18}}{\left(b x +a \right)^{2}}-\frac{14 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{3}}$
risch	$-\frac{3}{a^{3} x^{\frac{1}{3}}}-\frac{b \left(\frac{\frac{5 b \,x^{\frac{5}{3}}}{3}+\frac{13 a \,x^{\frac{2}{3}}}{6}}{\left(b x +a \right)^{2}}-\frac{14 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{3}}$

input
```
int(1/x^(4/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/a^3/x^(1/3)-3/a^3*b*((5/9*b*x^(5/3)+13/18*a*x^(2/3))/(b*x+a)^2-14/27/b/
(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+7/27/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1
/3)*x^(1/3)+(a/b)^(2/3))+14/27*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x^(1/3)-1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx =$$

$$-\frac{28\sqrt{3}(b^2x^3+2\,abx^2+a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\,(b^2x^3+2\,abx^2+a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-a\right.}{18\,(a^3b^2x^3}$$

input
```
integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/18*(28*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*arctan(2/3*sqr
t(3)*x^(1/3)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^3 + 2*a*b*x^2 + a^2*x)
*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 28*
(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) +
 3*(28*b^2*x^2 + 49*a*b*x + 18*a^2)*x^(2/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 +
a^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(4/3)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = -\frac{28\,b^2x^2+49\,abx+18\,a^2}{6\left(a^3b^2x^{\frac{7}{3}}+2\,a^4bx^{\frac{4}{3}}+a^5x^{\frac{1}{3}}\right)} - \frac{14\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{14\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(28*b^2*x^2 + 49*a*b*x + 18*a^2)/(a^3*b^2*x^(7/3) + 2*a^4*b*x^(4/3) +
 a^5*x^(1/3)) - 14/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/
(a/b)^(1/3))/(a^3*(a/b)^(1/3)) - 7/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (
a/b)^(2/3))/(a^3*(a/b)^(1/3)) + 14/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)
^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{14\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4} + \frac{14\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b} - \frac{3}{a^3x^{\frac{1}{3}}} - \frac{7\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^4b} - \frac{10\,b^2x^{\frac{5}{3}}+13\,abx^{\frac{2}{3}}}{6\left(bx+a\right)^2a^3}$$

input
```
integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
14/9*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 + 14/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a^4*b) - 3/(a^3*x^(1/3)) - 7/9*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/6*(10*b^2*x^(5/3) + 13*a*b*x^(2/3))/((b
*x + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}+588\,a^3\,b^3\,x^{1/3}\right)}{9\,a^{10/3}} - \frac{\frac{3}{a}+\frac{14\,b^2\,x^2}{3\,a^3}+\frac{49\,b\,x}{6\,a^2}}{a^2\,x^{1/3}+b^2\,x^{7/3}+2\,a\,b\,x^{4/3}} + \frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+588\,a^3\,b^3\,x^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}} - \frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+588\,a^3\,b^3\,x^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

input
```
int(1/(x^(4/3)*(a + b*x)^3),x)
```

output
```
(14*b^(1/3)*log(588*a^(10/3)*b^(8/3) + 588*a^3*b^3*x^(1/3)))/(9*a^(10/3))
- (3/a + (14*b^2*x^2)/(3*a^3) + (49*b*x)/(6*a^2))/(a^2*x^(1/3) + b^2*x^(7/
3) + 2*a*b*x^(4/3)) + (14*b^(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2
 - 1/2)^2 + 588*a^3*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (1
4*b^(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2 + 588*a^3*b^3*
x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{28x^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2b+56x^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)a\,b^2+28x^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2\cdots}{a^{\frac{1}{3}}\cdots}\right.}{}$$

input
```
int(1/x^(4/3)/(b*x+a)^3,x)
```

output
```
(28*x**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(
3)))*a**2*b + 56*x**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a
**(1/3)*sqrt(3)))*a*b**2*x + 28*x**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/
3)*b**(1/3))/(a**(1/3)*sqrt(3)))*b**3*x**2 - 54*b**(2/3)*a**(1/3)*a**2 - 1
47*b**(2/3)*a**(1/3)*a*b*x - 84*b**(2/3)*a**(1/3)*b**2*x**2 - 14*x**(1/3)*
log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a**2*b - 28
*x**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*a
*b**2*x - 14*x**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)
*b**(2/3))*b**3*x**2 + 28*x**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2*
b + 56*x**(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a*b**2*x + 28*x**(1/3)*l
og(a**(1/3) + x**(1/3)*b**(1/3))*b**3*x**2)/(18*x**(1/3)*b**(2/3)*a**(1/3)
*a**3*(a**2 + 2*a*b*x + b**2*x**2))
```

3.340 $\int \frac{1}{x^{5/3}(a+bx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 152

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)} + \frac{20b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}$$

output
```
-10/3/a^3/x^(2/3)+1/2/a/x^(2/3)/(b*x+a)^2+4/3/a^2/x^(2/3)/(b*x+a)+20/9*b^(
2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(11
/3)-10/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(11/3)+10/9*b^(2/3)*ln(b*x+
a)/a^(11/3)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \frac{-\frac{3a^{2/3}(9a^2+32abx+20b^2x^2)}{x^{2/3}(a+bx)^2}+40\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-40b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right.}{18a^{11/3}}$$

input
```
Integrate[1/(x^(5/3)*(a + b*x)^3),x]
```

output
```
((-3*a^(2/3)*(9*a^2 + 32*a*b*x + 20*b^2*x^2))/(x^(2/3)*(a + b*x)^2) + 40*S
qrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 40*b^(2
/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + 20*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(11/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {52, 52, 61, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/3}(a+bx)^3}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{4\int \frac{1}{x^{5/3}(a+bx)^2}dx}{3a}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{4\left(\frac{5\int \frac{1}{x^{5/3}(a+bx)}dx}{3a}+\frac{1}{ax^{2/3}(a+bx)}\right)}{3a}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{4\left(\frac{5\left(-\frac{b\int \frac{1}{x^{2/3}(a+bx)}dx}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}\right)}{3a}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

$$\Big\downarrow \text{70}$$

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\sqrt[3]{x}}d\sqrt[3]{x}}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}\right)}{3a}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

$\downarrow$ 16

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{3\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{x}\sqrt[3]{a}}{\sqrt[3]{b}}+x^{2/3}}d\sqrt[3]{x}}{2\sqrt[3]{a}b^{2/3}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}\right)}{3a}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{3\int \frac{1}{-x^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}\right)}{3a}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{4\left(\frac{5\left(-\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{3}{2ax^{2/3}}\right)}{3a}+\frac{1}{ax^{2/3}(a+bx)}\right)}{3a}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

input
```
Int[1/(x^(5/3)*(a + b*x)^3),x]
```

output
```
1/(2*a*x^(2/3)*(a + b*x)^2) + (4*(1/(a*x^(2/3)*(a + b*x)) + (5*(-3/(2*a*x^
(2/3)) - (b*(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])
/(a^(2/3)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3
)) - Log[a + b*x]/(2*a^(2/3)*b^(1/3))))/a))/(3*a)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{3b\left(\frac{\frac{11b\,x^{\frac{4}{3}}}{18}+\frac{7a\,x^{\frac{1}{3}}}{9}}{(bx+a)^{2}}+\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{3}}-\frac{3}{2a^{3}x}$
default	$-\frac{3b\left(\frac{\frac{11b\,x^{\frac{4}{3}}}{18}+\frac{7a\,x^{\frac{1}{3}}}{9}}{(bx+a)^{2}}+\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{3}}-\frac{3}{2a^{3}x}$

input
```
int(1/x^(5/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/a^3*b*((11/18*b*x^(4/3)+7/9*a*x^(1/3))/(b*x+a)^2+20/27/b/(a/b)^(2/3)*ln
(x^(1/3)+(a/b)^(1/3))-10/27/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(
a/b)^(2/3))+20/27/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x^(1/3)-1)))-3/2/a^3/x^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(107) = 214$.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \frac{40\sqrt{3}(b^2x^3+2\,abx^2+a^2x)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-20\,(b^2x^3+2\,abx}{}$$

input
```
integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/18*(40*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*arctan(1/3
*(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(b^2*x^3 + 2*a
*b*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^
(1/3) + a^2*(-b^2/a^2)^(2/3)) + 40*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2
)^(1/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(20*b^2*x^2 + 32*a*b*x + 9
*a^2)*x^(1/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(5/3)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = -\frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(a^3b^2x^{\frac{8}{3}}+2\,a^4bx^{\frac{5}{3}}+a^5x^{\frac{2}{3}}\right)} - \frac{20\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{10\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{20\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/(a^3*b^2*x^(8/3) + 2*a^4*b*x^(5/3) +
a^5*x^(2/3)) - 20/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(
a/b)^(1/3))/(a^3*(a/b)^(2/3)) + 10/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (
a/b)^(2/3))/(a^3*(a/b)^(2/3)) - 20/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)
^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4} - \frac{20\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4} - \frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^4} - \frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)^2a^3}$$

input
```
integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

output
```
20/9*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 - 20/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a
^4 - 10/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3)
)/a^4 - 1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/((b*x^(4/3) + a*x^(1/3))^2*a^3
)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \frac{20\,b^{2/3}\,\ln\left(540\,{(-a)}^{19/3}\,b^{8/3}-540\,a^6\,b^3\,x^{1/3}\right)}{9\,{(-a)}^{11/3}} - \frac{\frac{3}{2\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{16\,b\,x}{3\,a^2}}{a^2\,x^{2/3}+b^2\,x^{8/3}+2\,a\,b\,x^{5/3}} + \frac{20\,b^{2/3}\,\ln\left(540\,{(-a)}^{19/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-540\,a^6\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{(-a)}^{11/3}} - \frac{20\,b^{2/3}\,\ln\left(540\,{(-a)}^{19/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)+540\,a^6\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{(-a)}^{11/3}}$$

input
```
int(1/(x^(5/3)*(a + b*x)^3),x)
```

output
```
(20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3) - 540*a^6*b^3*x^(1/3)))/(9*(-a)^(1
1/3)) - (3/(2*a) + (10*b^2*x^2)/(3*a^3) + (16*b*x)/(3*a^2))/(a^2*x^(2/3) +
 b^2*x^(8/3) + 2*a*b*x^(5/3)) + (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((
3^(1/2)*1i)/2 - 1/2) - 540*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*(-a
)^(11/3)) - (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)
 + 540*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*(-a)^(11/3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \frac{40x^{\frac{2}{3}}a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b+80x^{\frac{5}{3}}a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2+40x^{\frac{8}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a}{}\right.}{}$$

input
```
int(1/x^(5/3)/(b*x+a)^3,x)
```

output
```
(40*x**(2/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1
/3)*sqrt(3)))*a**2*b + 80*x**(2/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*x**
(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*a*b**2*x + 40*x**(2/3)*a**(1/3)*sqrt(3
)*atan((a**(1/3) - 2*x**(1/3)*b**(1/3))/(a**(1/3)*sqrt(3)))*b**3*x**2 + 20
*x**(2/3)*a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b*
*(2/3))*a**2*b + 40*x**(2/3)*a**(1/3)*log(a**(2/3) - x**(1/3)*b**(1/3)*a**
(1/3) + x**(2/3)*b**(2/3))*a*b**2*x + 20*x**(2/3)*a**(1/3)*log(a**(2/3) -
x**(1/3)*b**(1/3)*a**(1/3) + x**(2/3)*b**(2/3))*b**3*x**2 - 40*x**(2/3)*a*
*(1/3)*log(a**(1/3) + x**(1/3)*b**(1/3))*a**2*b - 80*x**(2/3)*a**(1/3)*log
(a**(1/3) + x**(1/3)*b**(1/3))*a*b**2*x - 40*x**(2/3)*a**(1/3)*log(a**(1/3
) + x**(1/3)*b**(1/3))*b**3*x**2 - 27*b**(1/3)*a**3 - 96*b**(1/3)*a**2*b*x
 - 60*b**(1/3)*a*b**2*x**2)/(18*x**(2/3)*b**(1/3)*a**4*(a**2 + 2*a*b*x + b
**2*x**2))
```

3.341 $\int x^3\sqrt{a+bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3\sqrt{a+bx}\,dx=-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}$$

output
```
-2/3*a^3*(b*x+a)^(3/2)/b^4+6/5*a^2*(b*x+a)^(5/2)/b^4-6/7*a*(b*x+a)^(7/2)/b
^4+2/9*(b*x+a)^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^3\sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4}$$

input
```
Integrate[x^3*Sqrt[a + b*x],x]
```

output
```
(2*(a + b*x)^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3))/(31
5*b^4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx}\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

input
```
Int[x^3*Sqrt[a + b*x],x]
```

output
```
(-2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a + b*x)^(5/2))/(5*b^4) - (6*a*
(a + b*x)^(7/2))/(7*b^4) + (2*(a + b*x)^(9/2))/(9*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)}{315b^{4}}$	43
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)}{315b^{4}}$	43
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)}{315b^{4}}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{6a(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^{2}(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^{3}(bx+a)^{\frac{3}{2}}}{3}}{b^{4}}$	50
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{6a(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^{2}(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^{3}(bx+a)^{\frac{3}{2}}}{3}}{b^{4}}$	50
trager	$-\frac{2\left(-35b^{4}x^{4}-5a\,x^{3}b^{3}+6a^{2}b^{2}x^{2}-8a^{3}bx+16a^{4}\right)\sqrt{bx+a}}{315b^{4}}$	54
risch	$-\frac{2\left(-35b^{4}x^{4}-5a\,x^{3}b^{3}+6a^{2}b^{2}x^{2}-8a^{3}bx+16a^{4}\right)\sqrt{bx+a}}{315b^{4}}$	54

input
```
int(x^3*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(b*x+a)^(3/2)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{a+bx}\,dx = \frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(68) = 136.

Time = 1.24 (sec) , antiderivative size = 1742, normalized size of antiderivative = 24.19

$$\int x^3\sqrt{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(b*x+a)**(1/2),x)
```

output
```
-32*a**(49/2)*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a
**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) + 32*a**(49/2)/(315*a**20*b**4 + 1890*a*
*19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8
*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 176*a**(47/2)*b*x*s
qrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2
+ 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315
*a**14*b**10*x**6) + 192*a**(47/2)*b*x/(315*a**20*b**4 + 1890*a**19*b**5*x
 + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 18
90*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 396*a**(45/2)*b**2*x**2*sqrt(
1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 63
00*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**
14*b**10*x**6) + 480*a**(45/2)*b**2*x**2/(315*a**20*b**4 + 1890*a**19*b**5
*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 +
1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 462*a**(43/2)*b**3*x**3*sqr
t(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 +
6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a
**14*b**10*x**6) + 640*a**(43/2)*b**3*x**3/(315*a**20*b**4 + 1890*a**19*b*
*5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4
+ 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 210*a**(41/2)*b**4*x**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^4} - \frac{6\,(bx+a)^{\frac{7}{2}}a}{7\,b^4} + \frac{6\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^4} - \frac{2\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x + a)^(9/2)/b^4 - 6/7*(b*x + a)^(7/2)*a/b^4 + 6/5*(b*x + a)^(5/2)*
a^2/b^4 - 2/3*(b*x + a)^(3/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int x^3\sqrt{a+bx}\,dx$$
$$= \frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a}{b^3} + \frac{35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{b}}{b^3}}{315\,b}$$

input
```
integrate(x^3*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)*a/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2
)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a
)*a^4)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{a+bx}\,dx=\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}$$

input
```
int(x^3*(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(9/2))/(9*b^4) - (2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a
+ b*x)^(5/2))/(5*b^4) - (6*a*(a + b*x)^(7/2))/(7*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int x^3\sqrt{a+bx}\,dx=\frac{2\sqrt{bx+a}\,(35b^4x^4+5a\,b^3x^3-6a^2b^2x^2+8a^3bx-16a^4)}{315b^4}$$

input
```
int(x^3*(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**4 + 8*a**3*b*x - 6*a**2*b**2*x**2 + 5*a*b**3*x*
*3 + 35*b**4*x**4))/(315*b**4)
```

3.342 $\int x^2\sqrt{a+bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2\sqrt{a+bx}\,dx=\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}$$

output
```
2/3*a^2*(b*x+a)^(3/2)/b^3-4/5*a*(b*x+a)^(5/2)/b^3+2/7*(b*x+a)^(7/2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

input
```
Integrate[x^2*Sqrt[a + b*x],x]
```

output
```
(2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

input
```
Int[x^2*Sqrt[a + b*x],x]
```

output
```
(2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a +
b*x)^(7/2))/(7*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}\left(15b^{2}x^{2}-12abx+8a^{2}\right)}{105b^{3}}$	32
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}\left(15b^{2}x^{2}-12abx+8a^{2}\right)}{105b^{3}}$	32
orering	$\frac{2(bx+a)^{\frac{3}{2}}\left(15b^{2}x^{2}-12abx+8a^{2}\right)}{105b^{3}}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	38
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	38
trager	$\frac{2\left(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3}\right)\sqrt{bx+a}}{105b^{3}}$	43
risch	$\frac{2\left(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3}\right)\sqrt{bx+a}}{105b^{3}}$	43

input
```
int(x^2*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x^2\sqrt{a+bx}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.

Time = 0.85 (sec) , antiderivative size = 666, normalized size of antiderivative = 12.57

$$\int x^2\sqrt{a+bx}\,dx = \frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

input
```
integrate(x**2*(b*x+a)**(1/2),x)
```

output
```
16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b
**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b*
*4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1
+ b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*
b**6*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*
b**5*x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(
105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
 - 48*a**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5
*x**2 + 105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*
a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 1
6*a**(17/2)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**
2 + 105*a**5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**
8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a
**(13/2)*b**5*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*
a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x
/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*
x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^3} - \frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^3} + \frac{2\,(bx+a)^{\frac{3}{2}}a^2}{3\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*
a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(41) = 82$.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int x^2\sqrt{a+bx}\,dx = \frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)}{b^2}\right)}{105\,b}$$

input
```
integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{a+bx}\,dx = \frac{30\,{\left(a+b\,x\right)}^{7/2}-84\,a\,{\left(a+b\,x\right)}^{5/2}+70\,a^2\,{\left(a+b\,x\right)}^{3/2}}{105\,b^3}$$

input
```
int(x^2*(a + b*x)^(1/2),x)
```

output
```
(30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*
b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2\sqrt{a+bx}\,dx=\frac{2\sqrt{bx+a}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105b^3}$$

input
```
int(x^2*(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**3 - 4*a**2*b*x + 3*a*b**2*x**2 + 15*b**3*x**3))/(10
5*b**3)
```

3.343 $\int x\sqrt{a+bx}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\sqrt{a+bx}\,dx = -\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}$$

output
```
-2/3*a*(b*x+a)^(3/2)/b^2+2/5*(b*x+a)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\sqrt{a+bx}\,dx = \frac{2\sqrt{a+bx}(-2a^2+abx+3b^2x^2)}{15b^2}$$

input
```
Integrate[x*Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{(a+bx)^{3/2}}{b}-\frac{a\sqrt{a+bx}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

input
```
Int[x*Sqrt[a + b*x],x]
```

output
```
(-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
orering	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
trager	$-\frac{2(-3b^2x^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32
risch	$-\frac{2(-3b^2x^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32

input
```
int(x*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\sqrt{a+bx}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

input
```
integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int x\sqrt{a+bx}\,dx = -\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x} - \frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x} + \frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

input
```
integrate(x*(b*x+a)**(1/2),x)
```

output
```
-4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*
a**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 +
15*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b*
*2*x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**
3*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x\sqrt{a+bx}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2} - \frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

input
```
integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int x\sqrt{a+bx}\,dx = \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a}{b}+\frac{3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2}{b}\right)}{15b}$$

input
```
integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 1
0*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x\sqrt{a+bx}\,dx = -\frac{10\,a\,(a+b\,x)^{3/2}-6\,(a+b\,x)^{5/2}}{15\,b^2}$$

input
```
int(x*(a + b*x)^(1/2),x)
```

output
```
-(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x\sqrt{a+bx}\,dx = \frac{2\sqrt{bx+a}\,(3b^2x^2+abx-2a^2)}{15b^2}$$

input
```
int(x*(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**2 + a*b*x + 3*b**2*x**2))/(15*b**2)
```

3.344 $\int \sqrt{a+bx}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}}{3b}$$

output
```
2/3*(b*x+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}}{3b}$$

input
```
Integrate[Sqrt[a + b*x],x]
```

output
```
(2*(a + b*x)^(3/2))/(3*b)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\,dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{2(a+bx)^{3/2}}{3b}$$

input
```
Int[Sqrt[a + b*x],x]
```

output
```
(2*(a + b*x)^(3/2))/(3*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
trager	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
risch	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
orering	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13

input
```
int((b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx=\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx=\frac{2(a+bx)^{\frac{3}{2}}}{3b}$$

input
```
integrate((b*x+a)**(1/2),x)
```

output
```
2*(a + b*x)**(3/2)/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx=\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx=\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*(b*x + a)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a+bx}\,dx = \frac{2\,{\left(a+b\,x\right)}^{3/2}}{3\,b}$$

input
```
int((a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(3/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{a+bx}\,dx = \frac{2\sqrt{bx+a}\,(bx+a)}{3b}$$

input
```
int((b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(a + b*x))/(3*b)
```

3.345 $\int \frac{\sqrt{a+bx}}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{a+bx} - 2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{a+bx} - 2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*x]/x,x]
```

output
```
2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x} \, dx$$

$$\Big\downarrow \; 60$$

$$a \int \frac{1}{x\sqrt{a+bx}}dx + 2\sqrt{a+bx}$$

$$\Big\downarrow \; 73$$

$$\frac{2a \int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b} + 2\sqrt{a+bx}$$

$$\Big\downarrow \; 221$$

$$2\sqrt{a+bx} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Int[Sqrt[a + b*x]/x,x]
```

output
```
2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28
default	$2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28
pseudoelliptic	$2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28

input
```
int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+2\sqrt{bx+a}\right]$$

input
```
integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2
*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) + 2*sqrt(b*x + a)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.

Time = 0.69 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{a+bx}}{x}\,dx = -2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((b*x+a)**(1/2)/x,x)
```

output
```
-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/
(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + 2\sqrt{bx+a}$$

input
```
integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(
b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx}}{x}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\,\sqrt{bx+a}$$

input
```
integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\,\sqrt{a+b\,x} - 2\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

input
```
int((a + b*x)^(1/2)/x,x)
```

output
```
2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{bx+a} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right) - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)$$

input
```
int((b*x+a)^(1/2)/x,x)
```

output
```
2*sqrt(a + b*x) + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))
```

3.346 $\int \frac{\sqrt{a+bx}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-(b*x+a)^(1/2)/x-b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b*x]/x^2,x]
```

output
```
-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx - \frac{\sqrt{a+bx}}{x}$$

$$\Big\downarrow \ 73$$

$$\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx} - \frac{\sqrt{a+bx}}{x}$$

$$\Big\downarrow \ 221$$

$$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{a+bx}}{x}$$

input
```
Int[Sqrt[a + b*x]/x^2,x]
```

output
```
-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{\sqrt{bx+a}}{x}-\frac{b \,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	32
pseudoelliptic	$b \left(-\frac{\sqrt{bx+a}}{bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$	36
derivativedivides	$2b \left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37
default	$2b \left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37

input
```
int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)^(1/2)/x-b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \left[\frac{\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2ax}, \frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)-\sqrt{bx+a}a}{ax}\right]$$

input
```
integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*
x + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(-a)/sqrt(b*x + a)) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate((b*x+a)**(1/2)/x**2,x)
```

output
```
-sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sq
rt(a)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$$

input
```
integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - s
qrt(b*x + a)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = b\left(\frac{\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+a}}{bx}\right)$$

input
```
integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*(arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)/(b*x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+b\,x}}{x} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int((a + b*x)^(1/2)/x^2,x)
```

output
```
- (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = \frac{-2\sqrt{bx+a}\,a+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2ax}$$

input
```
int((b*x+a)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(a + b*x)*a + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x - sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a*x)
```

3.347 $\int \frac{\sqrt{a+bx}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{\sqrt{a+bx}}{x^3} dx = -\frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax} + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
-1/2*(b*x+a)^(1/2)/x^2-1/4*b*(b*x+a)^(1/2)/a/x+1/4*b^2*arctanh((b*x+a)^(1/
2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+bx}}{x^3} dx = -\frac{\sqrt{a+bx}(2a+bx)}{4ax^2} + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x]/x^3,x]
```

output
```
-1/4*(Sqrt[a + b*x]*(2*a + b*x))/(a*x^2) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^3} \, dx$$

$$\Big\downarrow \textbf{51}$$

$$\frac{1}{4} b \int \frac{1}{x^2 \sqrt{a+bx}} dx - \frac{\sqrt{a+bx}}{2x^2}$$

$$\Big\downarrow \textbf{52}$$

$$\frac{1}{4} b \left(-\frac{b \int \frac{1}{x\sqrt{a+bx}} dx}{2a} - \frac{\sqrt{a+bx}}{ax} \right) - \frac{\sqrt{a+bx}}{2x^2}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{4} b \left(-\frac{\int \frac{1}{\frac{a+bx}{b} - \frac{a}{b}} d\sqrt{a+bx}}{a} - \frac{\sqrt{a+bx}}{ax} \right) - \frac{\sqrt{a+bx}}{2x^2}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{4} b \left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{ax} \right) - \frac{\sqrt{a+bx}}{2x^2}$$

input
```
Int[Sqrt[a + b*x]/x^3,x]
```

output
```
-1/2*Sqrt[a + b*x]/x^2 + (b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/a^(3/2)))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(bx+2a)}{4x^2a}+\frac{b^2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	44
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-\left(2a^{\frac{3}{2}}+bx\sqrt{a}\right)\sqrt{bx+a}}{4a^{\frac{3}{2}}x^2}$	50
derivativedivides	$2b^2\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54
default	$2b^2\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54

input
```
int((b*x+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(b*x+2*a)/x^2/a+1/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/
a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = \left[\frac{\sqrt{a}b^2x^2\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(abx+2\,a^2)\sqrt{bx+a}}{8\,a^2x^2},\ -\frac{\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(abx+2\,a^2)\sqrt{bx+a}}{4\,a^2x^2}\right]$$

input
```
integrate((b*x+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*
b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(
-a)/sqrt(b*x + a)) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{a}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(1/2)/x**3,x)
```

output
```
-a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(a/(
b*x) + 1)) - b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**2*asinh(sqrt(a)
/(sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}} - \frac{(bx+a)^{\frac{3}{2}}b^2 + \sqrt{bx+a}ab^2}{4\left((bx+a)^2a - 2\,(bx+a)a^2 + a^3\right)}$$

input
```
integrate((b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
- 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a - 2*(b*x
+ a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{(bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}ab^3}{ab^2x^2}}{4\,b}$$

input
```
integrate((b*x+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b
^3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{3/2}} - \frac{(a+b\,x)^{3/2}}{4\,a\,x^2} - \frac{\sqrt{a+b\,x}}{4\,x^2}$$

input
```
int((a + b*x)^(1/2)/x^3,x)
```

output
```
(b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x)^(3/2)/(4*a*x^
2) - (a + b*x)^(1/2)/(4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = \frac{-4\sqrt{bx+a}\,a^2-2\sqrt{bx+a}\,abx-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2+\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8a^2x^2}$$

input
```
int((b*x+a)^(1/2)/x^3,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 2*sqrt(a + b*x)*a*b*x - sqrt(a)*log(sqrt(a + b*
x) - sqrt(a))*b**2*x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2)/
(8*a**2*x**2)
```

3.348 $\int \frac{\sqrt{a+bx}}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx=-\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/3*(b*x+a)^(1/2)/x^3-1/12*b*(b*x+a)^(1/2)/a/x^2+1/8*b^2*(b*x+a)^(1/2)/a^
2/x-1/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx=-\frac{\sqrt{a+bx}(8a^2+2abx-3b^2x^2)}{24a^2x^3}-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x]/x^4,x]
```

output
```
-1/24*(Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2*x^2))/(a^2*x^3) - (b^3*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^4}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{3x^3}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}$$

input
```
Int[Sqrt[a + b*x]/x^4,x]
```

output
```
-1/3*Sqrt[a + b*x]/x^3 + (b*(-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a +
 b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/6
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3b^2x^2+2abx+8a^2)}{24x^3a^2}-\frac{b^3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$	56
pseudoelliptic	$-\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^3x^3-\left(\sqrt{a}\,b^2x^2-\frac{2a^{\frac{3}{2}}bx}{3}-\frac{8a^{\frac{5}{2}}}{3}\right)\sqrt{bx+a}}{8a^{\frac{5}{2}}x^3}$	61
derivativedivides	$2b^3\left(-\frac{-\frac{(bx+a)^{\frac{5}{2}}}{16a^2}+\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{\sqrt{bx+a}}{16}}{b^3x^3}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	66
default	$2b^3\left(-\frac{-\frac{(bx+a)^{\frac{5}{2}}}{16a^2}+\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{\sqrt{bx+a}}{16}}{b^3x^3}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	66

input
```
int((b*x+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)^(1/2)*(-3*b^2*x^2+2*a*b*x+8*a^2)/x^3/a^2-1/8*b^3*arctanh((b*
x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx$$
$$=\left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(3\,ab^2x^2-2\,a^2bx-8\,a^3)\sqrt{bx+a}}{48\,a^3x^3},\frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(3}{24\,a}\right.$$

input
```
integrate((b*x+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(-a)/sqrt(b*x + a)) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a
^3)*sqrt(b*x + a))/(a^3*x^3)]
```

Sympy [A] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx = -\frac{a}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{b^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{b^{\frac{5}{2}}}{8a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**(1/2)/x**4,x)
```

output
```
-a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*sqrt(b)/(12*x**(5/2)*sqrt(a/
(b*x) + 1)) + b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + b**(5/2)/(8*a**
2*sqrt(x)*sqrt(a/(b*x) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a*
*(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx = \frac{b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}} + \frac{3\,(bx+a)^{\frac{5}{2}}b^3 - 8\,(bx+a)^{\frac{3}{2}}ab^3 - 3\,\sqrt{bx+a}a^2b^3}{24\,\left((bx+a)^3a^2 - 3\,(bx+a)^2a^3 + 3\,(bx+a)a^4 - a^5\right)}$$

input
```
integrate((b*x+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
+ 1/24*(3*(b*x + a)^(5/2)*b^3 - 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*
a^2*b^3)/((b*x + a)^3*a^2 - 3*(b*x + a)^2*a^3 + 3*(b*x + a)*a^4 - a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx = \frac{1}{24}\,b^3\left(\frac{3\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}-8\,(bx+a)^{\frac{3}{2}}a-3\,\sqrt{bx+a}a^2}{a^2b^3x^3}\right)$$

input
```
integrate((b*x+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/24*b^3*(3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(
5/2) - 8*(b*x + a)^(3/2)*a - 3*sqrt(b*x + a)*a^2)/(a^2*b^3*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx = \frac{(a+b\,x)^{5/2}}{8\,a^2\,x^3}-\frac{(a+b\,x)^{3/2}}{3\,a\,x^3}-\frac{\sqrt{a+b\,x}}{8\,x^3}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{5/2}}$$

input
```
int((a + b*x)^(1/2)/x^4,x)
```

output
```
(a + b*x)^(5/2)/(8*a^2*x^3) - (a + b*x)^(3/2)/(3*a*x^3) - (a + b*x)^(1/2)/
(8*x^3) + (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx}}{x^4}\,dx$$
$$= \frac{-16\sqrt{bx+a}\,a^3-4\sqrt{bx+a}\,a^2bx+6\sqrt{bx+a}\,a\,b^2x^2+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3-3\sqrt{a}\log\left(\sqrt{bx+\right.}{48a^3x^3}$$

input
```
int((b*x+a)^(1/2)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 4*sqrt(a + b*x)*a**2*b*x + 6*sqrt(a + b*x)*a*b
**2*x**2 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 3*sqrt(a)*lo
g(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**3*x**3)
```

3.349 $\int x^3(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3(a+bx)^{3/2}\,dx = -\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}$$

output
```
-2/5*a^3*(b*x+a)^(5/2)/b^4+6/7*a^2*(b*x+a)^(7/2)/b^4-2/3*a*(b*x+a)^(9/2)/b
^4+2/11*(b*x+a)^(11/2)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^3(a+bx)^{3/2}\,dx = \frac{2(a+bx)^{5/2}\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4}$$

input
```
Integrate[x^3*(a + b*x)^(3/2),x]
```

output
```
(2*(a + b*x)^(5/2)*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(1
155*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{3/2}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

input
```
Int[x^3*(a + b*x)^(3/2),x]
```

output
```
(-2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(7/2))/(7*b^4) - (2*a*
(a + b*x)^(9/2))/(3*b^4) + (2*(a + b*x)^(11/2))/(11*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)}{1155b^{4}}$	43
pseudoelliptic	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)}{1155b^{4}}$	43
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)}{1155b^{4}}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{2a(bx+a)^{\frac{9}{2}}}{3}+\frac{6a^{2}(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^{3}(bx+a)^{\frac{5}{2}}}{5}}{b^{4}}$	50
default	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{2a(bx+a)^{\frac{9}{2}}}{3}+\frac{6a^{2}(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^{3}(bx+a)^{\frac{5}{2}}}{5}}{b^{4}}$	50
trager	$-\frac{2\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)\sqrt{bx+a}}{1155b^{4}}$	65
risch	$-\frac{2\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)\sqrt{bx+a}}{1155b^{4}}$	65

input
```
int(x^3*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(b*x+a)^(5/2)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int x^3(a+bx)^{3/2}\,dx=\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(68) = 136$.

Time = 1.32 (sec) , antiderivative size = 1742, normalized size of antiderivative = 24.19

$$\int x^3(a+bx)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(b*x+a)**(3/2),x)
```

output
```
-32*a**(51/2)*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325
*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a*
*15*b**9*x**5 + 1155*a**14*b**10*x**6) + 32*a**(51/2)/(1155*a**20*b**4 + 6
930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a
**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 176*a**(4
9/2)*b*x*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**1
8*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b
**9*x**5 + 1155*a**14*b**10*x**6) + 192*a**(49/2)*b*x/(1155*a**20*b**4 + 6
930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a
**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 396*a**(4
7/2)*b**2*x**2*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 1732
5*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a
**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 480*a**(47/2)*b**2*x**2/(1155*a*
*20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x*
*3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
 - 462*a**(45/2)*b**3*x**3*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b
**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x
**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 640*a**(45/2)*b**3*x
**3/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a
**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{3/2}\,dx = \frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^4} - \frac{2\,(bx+a)^{\frac{9}{2}}a}{3\,b^4} + \frac{6\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^4} - \frac{2\,(bx+a)^{\frac{5}{2}}a^3}{5\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/11*(b*x + a)^(11/2)/b^4 - 2/3*(b*x + a)^(9/2)*a/b^4 + 6/7*(b*x + a)^(7/2
)*a^2/b^4 - 2/5*(b*x + a)^(5/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.68

$$\int x^3(a+bx)^{3/2}\,dx = \frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^2}{b^3}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\right.}{b^3}\right.}{}$$

input
```
integrate(x^3*(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*(35*(b*x + a)^(9/2) - 180*(b*x +
a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*a/b^3 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990
*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4
 - 693*sqrt(b*x + a)*a^5)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{3/2}\,dx = \frac{2\,(a+b\,x)^{11/2}}{11\,b^4}-\frac{2\,a^3\,(a+b\,x)^{5/2}}{5\,b^4}+\frac{6\,a^2\,(a+b\,x)^{7/2}}{7\,b^4}-\frac{2\,a\,(a+b\,x)^{9/2}}{3\,b^4}$$

input
```
int(x^3*(a + b*x)^(3/2),x)
```

output
```
(2*(a + b*x)^(11/2))/(11*b^4) - (2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(
a + b*x)^(7/2))/(7*b^4) - (2*a*(a + b*x)^(9/2))/(3*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int x^3(a+bx)^{3/2}\,dx = \frac{2\sqrt{bx+a}\,(105b^5x^5+140a\,b^4x^4+5a^2b^3x^3-6a^3b^2x^2+8a^4bx-16a^5)}{1155b^4}$$

input
```
int(x^3*(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**5 + 8*a**4*b*x - 6*a**3*b**2*x**2 + 5*a**2*b**3
*x**3 + 140*a*b**4*x**4 + 105*b**5*x**5))/(1155*b**4)
```

3.350 $\int x^2(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2(a+bx)^{3/2}\,dx=\frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}$$

output
```
2/5*a^2*(b*x+a)^(5/2)/b^3-4/7*a*(b*x+a)^(7/2)/b^3+2/9*(b*x+a)^(9/2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^{3/2}\,dx=\frac{2(a+bx)^{5/2}\,(8a^2-20abx+35b^2x^2)}{315b^3}$$

input
```
Integrate[x^2*(a + b*x)^(3/2),x]
```

output
```
(2*(a + b*x)^(5/2)*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{3/2}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

input
```
Int[x^2*(a + b*x)^(3/2),x]
```

output
```
(2*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a +
b*x)^(9/2))/(9*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2(bx+a)^{\frac{5}{2}}(35b^2x^2-20abx+8a^2)}{315b^3}$	32
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}(35b^2x^2-20abx+8a^2)}{315b^3}$	32
orering	$\frac{2(bx+a)^{\frac{5}{2}}(35b^2x^2-20abx+8a^2)}{315b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{4a(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{4a(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}}{b^3}$	38
trager	$\frac{2(35b^4x^4+50a\,x^3b^3+3a^2b^2x^2-4a^3bx+8a^4)\sqrt{bx+a}}{315b^3}$	54
risch	$\frac{2(35b^4x^4+50a\,x^3b^3+3a^2b^2x^2-4a^3bx+8a^4)\sqrt{bx+a}}{315b^3}$	54

input
```
int(x^2*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)^{3/2}\,dx=\frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx+a}}{315\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(49) = 98$.

Time = 0.91 (sec) , antiderivative size = 733, normalized size of antiderivative = 13.83

$$\int x^2(a+bx)^{3/2}\,dx = \frac{16a^{\frac{25}{2}}\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} - \frac{16a^{\frac{25}{2}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} + \frac{40a^{\frac{23}{2}}bx\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} - \frac{48a^{\frac{23}{2}}bx}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} + \frac{30a^{\frac{21}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} - \frac{48a^{\frac{21}{2}}b^2x^2}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} + \frac{110a^{\frac{19}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} - \frac{16a^{\frac{19}{2}}b^3x^3}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} + \frac{380a^{\frac{17}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} + \frac{516a^{\frac{15}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} + \frac{310a^{\frac{13}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3} + \frac{70a^{\frac{11}{2}}b^7x^7\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}$$

input
```
integrate(x**2*(b*x+a)**(3/2),x)
```

output
```
16*a**(25/2)*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b
**5*x**2 + 315*a**5*b**6*x**3) - 16*a**(25/2)/(315*a**8*b**3 + 945*a**7*b*
*4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 40*a**(23/2)*b*x*sqrt(1
+ b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*
b**6*x**3) - 48*a**(23/2)*b*x/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*
b**5*x**2 + 315*a**5*b**6*x**3) + 30*a**(21/2)*b**2*x**2*sqrt(1 + b*x/a)/(
315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3)
 - 48*a**(21/2)*b**2*x**2/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5
*x**2 + 315*a**5*b**6*x**3) + 110*a**(19/2)*b**3*x**3*sqrt(1 + b*x/a)/(315
*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) -
16*a**(19/2)*b**3*x**3/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x*
*2 + 315*a**5*b**6*x**3) + 380*a**(17/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*a*
*8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 516
*a**(15/2)*b**5*x**5*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 94
5*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 310*a**(13/2)*b**6*x**6*sqrt(1 +
b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b*
*6*x**3) + 70*a**(11/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**
7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2(a+bx)^{3/2}\,dx = \frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^3} - \frac{4\,(bx+a)^{\frac{7}{2}}a}{7\,b^3} + \frac{2\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/9*(b*x + a)^(9/2)/b^3 - 4/7*(b*x + a)^(7/2)*a/b^3 + 2/5*(b*x + a)^(5/2)*
a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.94

$$\int x^2(a+bx)^{3/2}\,dx = \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^2}{b^2}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a}{b^2}+35\,(\cdots\right.}{315\,b}$$

input
```
integrate(x^2*(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*a^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a
)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b
*x + a)*a^4)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2(a+bx)^{3/2}\,dx = \frac{70\,{\left(a+b\,x\right)}^{9/2}-180\,a\,{\left(a+b\,x\right)}^{7/2}+126\,a^2\,{\left(a+b\,x\right)}^{5/2}}{315\,b^3}$$

input
```
int(x^2*(a + b*x)^(3/2),x)
```

output
```
(70*(a + b*x)^(9/2) - 180*a*(a + b*x)^(7/2) + 126*a^2*(a + b*x)^(5/2))/(31
5*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int x^2(a+bx)^{3/2}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315b^3}$$

input
```
int(x^2*(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**4 - 4*a**3*b*x + 3*a**2*b**2*x**2 + 50*a*b**3*x**3
+ 35*b**4*x**4))/(315*b**3)
```

3.351 $\int x(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x(a+bx)^{3/2}\,dx=-\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}$$

output
```
-2/5*a*(b*x+a)^(5/2)/b^2+2/7*(b*x+a)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x(a+bx)^{3/2}\,dx=\frac{2(a+bx)^{5/2}(-2a+5bx)}{35b^2}$$

input
```
Integrate[x*(a + b*x)^(3/2),x]
```

output
```
(2*(a + b*x)^(5/2)*(-2*a + 5*b*x))/(35*b^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{3/2}\,dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(\frac{(a+bx)^{5/2}}{b}-\frac{a(a+bx)^{3/2}}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

input
```
Int[x*(a + b*x)^(3/2),x]
```

output
```
(-2*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + b*x)^(7/2))/(7*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^{2}}$	21
pseudoelliptic	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^{2}}$	21
orering	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^{2}}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}}{b^{2}}$	26
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}}{b^{2}}$	26
trager	$-\frac{2(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3})\sqrt{bx+a}}{35b^{2}}$	43
risch	$-\frac{2(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3})\sqrt{bx+a}}{35b^{2}}$	43

input
```
int(x*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x(a+bx)^{3/2}\,dx=\frac{2\left(5\,b^{3}x^{3}+8\,ab^{2}x^{2}+a^{2}bx-2\,a^{3}\right)\sqrt{bx+a}}{35\,b^{2}}$$

input
```
integrate(x*(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(31) = 62.

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int x(a+bx)^{3/2}\,dx = \begin{cases} -\frac{4a^3\sqrt{a+bx}}{35b^2} + \frac{2a^2x\sqrt{a+bx}}{35b} + \frac{16ax^2\sqrt{a+bx}}{35} + \frac{2bx^3\sqrt{a+bx}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**(3/2),x)
```

output
```
Piecewise((-4*a**3*sqrt(a + b*x)/(35*b**2) + 2*a**2*x*sqrt(a + b*x)/(35*b)
 + 16*a*x**2*sqrt(a + b*x)/35 + 2*b*x**3*sqrt(a + b*x)/7, Ne(b, 0)), (a**(
3/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(a+bx)^{3/2}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^2} - \frac{2\,(bx+a)^{\frac{5}{2}}a}{5\,b^2}$$

input
```
integrate(x*(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/7*(b*x + a)^(7/2)/b^2 - 2/5*(b*x + a)^(5/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

$$\int x(a+bx)^{3/2}\,dx = \frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^2}{b} + \frac{14\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a}{b} + \frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+\right.}{b}}{105\,b}$$

input
```
integrate(x*(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a/b + 3*(5*(b*x + a)^(7
/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3
)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x(a+bx)^{3/2}\,dx=-\frac{14\,a\,{\left(a+b\,x\right)}^{5/2}-10\,{\left(a+b\,x\right)}^{7/2}}{35\,b^2}$$

input
```
int(x*(a + b*x)^(3/2),x)
```

output
```
-(14*a*(a + b*x)^(5/2) - 10*(a + b*x)^(7/2))/(35*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int x(a+bx)^{3/2}\,dx=\frac{2\sqrt{bx+a}\,(5b^3x^3+8a\,b^2x^2+a^2bx-2a^3)}{35b^2}$$

input
```
int(x*(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**3 + a**2*b*x + 8*a*b**2*x**2 + 5*b**3*x**3))/(35
*b**2)
```

3.352 $\int (a+bx)^{3/2} dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (a+bx)^{3/2} dx = \frac{2(a+bx)^{5/2}}{5b}$$

output
```
2/5*(b*x+a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+bx)^{3/2} dx = \frac{2(a+bx)^{5/2}}{5b}$$

input
```
Integrate[(a + b*x)^(3/2),x]
```

output
```
(2*(a + b*x)^(5/2))/(5*b)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2} \, dx$$

$$\Big\downarrow \ 17$$

$$\frac{2(a+bx)^{5/2}}{5b}$$

input
```
Int[(a + b*x)^(3/2),x]
```

output
```
(2*(a + b*x)^(5/2))/(5*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
default	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
orering	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
trager	$\frac{2(b^2x^2+2abx+a^2)\sqrt{bx+a}}{5b}$	29
risch	$\frac{2(b^2x^2+2abx+a^2)\sqrt{bx+a}}{5b}$	29

input
```
int((b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (a+bx)^{3/2} \, dx = \frac{2 \left(b^2x^2 + 2 \, abx + a^2\right)\sqrt{bx+a}}{5 \, b}$$

input
```
integrate((b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{3/2}\,dx=\frac{2(a+bx)^{\frac{5}{2}}}{5b}$$

input
```
integrate((b*x+a)**(3/2),x)
```

output
```
2*(a + b*x)**(5/2)/(5*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{3/2}\,dx=\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate((b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(12) = 24.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int (a+bx)^{3/2}\,dx=\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+30\,\sqrt{bx+a}a^2+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a\right)}{15\,b}$$

input
```
integrate((b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 30*sqrt(b*x + a)*a^2 + 10
*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{3/2}\,dx=\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5\,b}$$

input
```
int((a + b*x)^(3/2),x)
```

output
```
(2*(a + b*x)^(5/2))/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int (a+bx)^{3/2}\,dx=\frac{2\sqrt{bx+a}\,(b^2x^2+2abx+a^2)}{5b}$$

input
```
int((b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b)
```

3.353 $\int \frac{(a+bx)^{3/2}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{(a+bx)^{3/2}}{x} dx = 2a\sqrt{a+bx} + \frac{2}{3}(a+bx)^{3/2} - 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*a*(b*x+a)^(1/2)+2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2
))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{3/2}}{x} dx = \frac{2}{3}\sqrt{a+bx}(4a+bx) - 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(3/2)/x,x]
```

output
```
(2*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{60}$$

$$a \int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$a \left(a \int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}$$

$$\Big\downarrow \text{73}$$

$$a \left(\frac{2a \int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}$$

$$\Big\downarrow \text{221}$$

$$a \left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}$$

input
```
Int[(a + b*x)^(3/2)/x,x]
```

output
```
(2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(bx+4a)}{3}$	35
derivativedivides	$2a\sqrt{bx+a}+\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	38
default	$2a\sqrt{bx+a}+\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	38

input
```
int((b*x+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/3*(b*x+a)^(1/2)*(b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx)^{3/2}}{x}\,dx = \left[a^{\frac{3}{2}}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right) + \frac{2}{3}\,(bx+4\,a)\sqrt{bx+a}, 2\,\sqrt{-a}a\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right) + \frac{2}{3}\,(bx+4\,a)\sqrt{bx+a}\right]$$

input
```
integrate((b*x+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sq
rt(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*x + a)) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^{3/2}}{x}\,dx = \frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{3} + a^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) - 2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right) + \frac{2\sqrt{a}bx\sqrt{1+\frac{bx}{a}}}{3}$$

input
```
integrate((b*x+a)**(3/2)/x,x)
```

output
```
8*a**(3/2)*sqrt(1 + b*x/a)/3 + a**(3/2)*log(b*x/a) - 2*a**(3/2)*log(sqrt(1
 + b*x/a) + 1) + 2*sqrt(a)*b*x*sqrt(1 + b*x/a)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{3/2}}{x}\,dx = a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}+2\,\sqrt{bx+a}a$$

input
```
integrate((b*x+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
a^(3/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(b*
x + a)^(3/2) + 2*sqrt(b*x + a)*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{3/2}}{x}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}+2\,\sqrt{bx+a}a$$

input
```
integrate((b*x+a)^(3/2)/x,x, algorithm="giac")
```

output
```
2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(b*x + a)^(3/2) + 2*sq
rt(b*x + a)*a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^{3/2}}{x}\,dx = 2\,a\,\sqrt{a+b\,x}+\frac{2\,(a+b\,x)^{3/2}}{3}-2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

input
```
int((a + b*x)^(3/2)/x,x)
```

output
```
2*a*(a + b*x)^(1/2) + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*atanh((a + b*x)^(1
/2)/a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^{3/2}}{x}\,dx = \frac{8\sqrt{bx+a}\,a}{3} + \frac{2\sqrt{bx+a}\,bx}{3} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a$$

input
```
int((b*x+a)^(3/2)/x,x)
```

output
```
(8*sqrt(a + b*x)*a + 2*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x) - s
qrt(a))*a - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a)/3
```

3.354 $\int \frac{(a+bx)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{(a+bx)^{3/2}}{x^2} dx = 2b\sqrt{a+bx} - \frac{a\sqrt{a+bx}}{x} - 3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*b*(b*x+a)^(1/2)-a*(b*x+a)^(1/2)/x-3*a^(1/2)*b*arctanh((b*x+a)^(1/2)/a^(1
/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^{3/2}}{x^2} dx = -\frac{(a-2bx)\sqrt{a+bx}}{x} - 3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(3/2)/x^2,x]
```

output
```
-(((a - 2*b*x)*Sqrt[a + b*x])/x) - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{3}{2}b\int \frac{\sqrt{a+bx}}{x}dx-\frac{(a+bx)^{3/2}}{x}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{2}b\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{3}{2}b\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)-\frac{(a+bx)^{3/2}}{x}$$

input
```
Int[(a + b*x)^(3/2)/x^2,x]
```

output
```
-((a + b*x)^(3/2)/x) + (3*b*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$-\frac{3 \,\mathrm{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) a b x+\left(-2 b x+a \right) \sqrt{b x+a}\, \sqrt{a}}{\sqrt{a}\, x}$	44
risch	$-\frac{a \sqrt{b x+a}}{x}+\frac{b \left(4 \sqrt{b x+a}-6 \sqrt{a}\, \mathrm{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)\right)}{2}$	45
derivativedivides	$2 b \left(\sqrt{b x+a}-a \left(\frac{\sqrt{b x+a}}{2 b x}+\frac{3 \,\mathrm{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{2 \sqrt{a}}\right)\right)$	48
default	$2 b \left(\sqrt{b x+a}-a \left(\frac{\sqrt{b x+a}}{2 b x}+\frac{3 \,\mathrm{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{2 \sqrt{a}}\right)\right)$	48

input
```
int((b*x+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*(3*arctanh((b*x+a)^(1/2)/a^(1/2))*a*b*x+(-2*b*x+a)*(b*x+a)^(1/2
)*a^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx)^{3/2}}{x^2}\,dx = \left[\frac{3\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(2bx-a)\sqrt{bx+a}}{2x}, \frac{3\sqrt{-a}bx\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(2\right.}{x}$$

input
```
integrate((b*x+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*
x - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(-a)/sqrt(b*x + a)) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)^{3/2}}{x^2}\,dx = -3\sqrt{a}b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{a^2}{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{a\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((b*x+a)**(3/2)/x**2,x)
```

output
```
-3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqr
t(a/(b*x) + 1)) + a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(3/2)*sqrt(
x)/sqrt(a/(b*x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{3/2}}{x^2}\,dx = \frac{3}{2}\sqrt{a}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}b-\frac{\sqrt{bx+a}a}{x}$$

input
```
integrate((b*x+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
3/2*sqrt(a)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2
*sqrt(b*x + a)*b - sqrt(b*x + a)*a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{3/2}}{x^2}\,dx = \left(\frac{3\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}-\frac{\sqrt{bx+a}a}{bx}\right)b$$

input
```
integrate((b*x+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
(3*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a) - sqrt(b*x
+ a)*a/(b*x))*b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^{3/2}}{x^2}\,dx = 2\,b\sqrt{a+b\,x}-3\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)-\frac{a\sqrt{a+b\,x}}{x}$$

input
```
int((a + b*x)^(3/2)/x^2,x)
```

output
```
2*b*(a + b*x)^(1/2) - 3*a^(1/2)*b*atanh((a + b*x)^(1/2)/a^(1/2)) - (a*(a +
 b*x)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{3/2}}{x^2}\,dx = \frac{-2\sqrt{bx+a}\,a+4\sqrt{bx+a}\,bx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-3\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{2x}$$

input
```
int((b*x+a)^(3/2)/x^2,x)
```

output
```
( - 2*sqrt(a + b*x)*a + 4*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x)
- sqrt(a))*b*x - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*x)
```

3.355 $\int \frac{(a+bx)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{(a+bx)^{3/2}}{x^3} dx = -\frac{a\sqrt{a+bx}}{2x^2} - \frac{5b\sqrt{a+bx}}{4x} - \frac{3b^2 \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
-1/2*a*(b*x+a)^(1/2)/x^2-5/4*b*(b*x+a)^(1/2)/x-3/4*b^2*arctanh((b*x+a)^(1/
2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^{3/2}}{x^3} dx = -\frac{\sqrt{a+bx}(2a+5bx)}{4x^2} - \frac{3b^2 \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
Integrate[(a + b*x)^(3/2)/x^3,x]
```

output
```
-1/4*(Sqrt[a + b*x]*(2*a + 5*b*x))/x^2 - (3*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/(4*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \ 51$$

$$\frac{3}{4}b\int \frac{\sqrt{a+bx}}{x^2}dx-\frac{(a+bx)^{3/2}}{2x^2}$$

$$\Big\downarrow \ 51$$

$$\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}$$

$$\Big\downarrow \ 73$$

$$\frac{3}{4}b\left(\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}$$

$$\Big\downarrow \ 221$$

$$\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}$$

input
```
Int[(a + b*x)^(3/2)/x^3,x]
```

output
```
-1/2*(a + b*x)^(3/2)/x^2 + (3*b*(-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/Sqrt[a]))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(5bx+2a)}{4x^{2}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4\sqrt{a}}$	42
derivativedivides	$2b^{2}\left(-\frac{\frac{5(bx+a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx+a}}{8}}{b^{2}x^{2}}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	52
default	$2b^{2}\left(-\frac{\frac{5(bx+a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx+a}}{8}}{b^{2}x^{2}}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	52
pseudoelliptic	$\frac{-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-5bx\sqrt{a}\,\sqrt{bx+a}-2a^{\frac{3}{2}}\sqrt{bx+a}}{4x^{2}\sqrt{a}}$	56

input
```
int((b*x+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(5*b*x+2*a)/x^2-3/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/
a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx)^{3/2}}{x^3}\,dx = \left[\frac{3\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\,(5\,abx+2\,a^2)\sqrt{bx+a}}{8\,ax^2}, \frac{3\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+}}\right.}{4}\right.$$

input
```
integrate((b*x+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(-a)/sqrt(b*x + a)) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{3/2}}{x^3}\,dx = -\frac{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{2x^{\frac{3}{2}}} - \frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{4\sqrt{x}} - \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}}$$

input
```
integrate((b*x+a)**(3/2)/x**3,x)
```

output
```
-a*sqrt(b)*sqrt(a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(a/(b*x) + 1)/(
4*sqrt(x)) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx)^{3/2}}{x^3}\,dx = \frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,\sqrt{a}} - \frac{5\,(bx+a)^{\frac{3}{2}}b^2-3\,\sqrt{bx+a}ab^2}{4\,\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

input
```
integrate((b*x+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
 1/4*(5*(b*x + a)^(3/2)*b^2 - 3*sqrt(b*x + a)*a*b^2)/((b*x + a)^2 - 2*(b*x
 + a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{3/2}}{x^3}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{5\,(bx+a)^{\frac{3}{2}}b^3-3\,\sqrt{bx+a}ab^3}{b^2x^2}}{4\,b}$$

input
```
integrate((b*x+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^
3 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^{3/2}}{x^3}\,dx = \frac{3\,a\,\sqrt{a+b\,x}}{4\,x^2} - \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,\sqrt{a}} - \frac{5\,(a+b\,x)^{3/2}}{4\,x^2}$$

input
```
int((a + b*x)^(3/2)/x^3,x)
```

output
```
(3*a*(a + b*x)^(1/2))/(4*x^2) - (3*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*
a^(1/2)) - (5*(a + b*x)^(3/2))/(4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{3/2}}{x^3}\,dx = \frac{-4\sqrt{bx+a}\,a^2-10\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}+\right.}{8a\,x^2}$$

input
```
int((b*x+a)^(3/2)/x^3,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 10*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
 b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x
**2)/(8*a*x**2)
```

3.356 $\int \frac{(a+bx)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{(a+bx)^{3/2}}{x^4} dx = -\frac{a\sqrt{a+bx}}{3x^3} - \frac{7b\sqrt{a+bx}}{12x^2} - \frac{b^2\sqrt{a+bx}}{8ax} + \frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
-1/3*a*(b*x+a)^(1/2)/x^3-7/12*b*(b*x+a)^(1/2)/x^2-1/8*b^2*(b*x+a)^(1/2)/a/
x+1/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{3/2}}{x^4} dx = -\frac{\sqrt{a+bx}(8a^2+14abx+3b^2x^2)}{24ax^3} + \frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

input
```
Integrate[(a + b*x)^(3/2)/x^4,x]
```

output
```
-1/24*(Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^2))/(a*x^3) + (b^3*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} b \int \frac{\sqrt{a+bx}}{x^3} dx - \frac{(a+bx)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} b \left(\frac{1}{4} b \int \frac{1}{x^2 \sqrt{a+bx}} dx - \frac{\sqrt{a+bx}}{2x^2} \right) - \frac{(a+bx)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} b \left(\frac{1}{4} b \left(-\frac{b \int \frac{1}{x\sqrt{a+bx}} dx}{2a} - \frac{\sqrt{a+bx}}{ax} \right) - \frac{\sqrt{a+bx}}{2x^2} \right) - \frac{(a+bx)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} b \left(\frac{1}{4} b \left(-\frac{\int \frac{1}{\frac{a+bx}{b} - \frac{a}{b}} d\sqrt{a+bx}}{a} - \frac{\sqrt{a+bx}}{ax} \right) - \frac{\sqrt{a+bx}}{2x^2} \right) - \frac{(a+bx)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2} b \left(\frac{1}{4} b \left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{ax} \right) - \frac{\sqrt{a+bx}}{2x^2} \right) - \frac{(a+bx)^{3/2}}{3x^3}$$

input
```
Int[(a + b*x)^(3/2)/x^4,x]
```

output
```
-1/3*(a + b*x)^(3/2)/x^3 + (b*(-1/2*Sqrt[a + b*x]/x^2 + (b*(-(Sqrt[a + b*x
]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/4))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(3b^{2}x^{2}+14abx+8a^{2})}{24x^{3}a}+\frac{b^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}$	56
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}-\left(\sqrt{a}\,b^{2}x^{2}+\frac{14a^{\frac{3}{2}}bx}{3}+\frac{8a^{\frac{5}{2}}}{3}\right)\sqrt{bx+a}}{8a^{\frac{3}{2}}x^{3}}$	61
derivativedivides	$2b^{3}\left(-\frac{\frac{(bx+a)^{\frac{5}{2}}}{16a}+\frac{(bx+a)^{\frac{3}{2}}}{6}-\frac{a\sqrt{bx+a}}{16}}{b^{3}x^{3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	64
default	$2b^{3}\left(-\frac{\frac{(bx+a)^{\frac{5}{2}}}{16a}+\frac{(bx+a)^{\frac{3}{2}}}{6}-\frac{a\sqrt{bx+a}}{16}}{b^{3}x^{3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	64

input
```
int((b*x+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)^(1/2)*(3*b^2*x^2+14*a*b*x+8*a^2)/x^3/a+1/8*b^3*arctanh((b*x+
a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^{3/2}}{x^4}\,dx = \left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,a^2x^3}, -\frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,a^2x^3}\right]$$

input
```
integrate((b*x+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt
(-a)*b^3*x^3*arctan(sqrt(-a)/sqrt(b*x + a)) + (3*a*b^2*x^2 + 14*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^{3/2}}{x^4}\,dx = -\frac{a^2}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} -\frac{17b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(3/2)/x**4,x)
```

output
```
-a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*a*sqrt(b)/(12*x**(5/2)*s
qrt(a/(b*x) + 1)) - 17*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) - b**(5/2)
/(8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(
8*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{3/2}}{x^4}\,dx = -\frac{b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}} - \frac{3\,(bx+a)^{\frac{5}{2}}b^3 + 8\,(bx+a)^{\frac{3}{2}}ab^3 - 3\,\sqrt{bx+a}a^2b^3}{24\,\left((bx+a)^3a - 3\,(bx+a)^2a^2 + 3\,(bx+a)a^3 - a^4\right)}$$

input
```
integrate((b*x+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
 - 1/24*(3*(b*x + a)^(5/2)*b^3 + 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)
*a^2*b^3)/((b*x + a)^3*a - 3*(b*x + a)^2*a^2 + 3*(b*x + a)*a^3 - a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^{3/2}}{x^4}\,dx = -\frac{1}{24}\,b^3\left(\frac{3\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{3\,(bx+a)^{\frac{5}{2}} + 8\,(bx+a)^{\frac{3}{2}}a - 3\,\sqrt{bx+a}a^2}{ab^3x^3}\right)$$

input
```
integrate((b*x+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/24*b^3*(3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x + a)^(5
/2) + 8*(b*x + a)^(3/2)*a - 3*sqrt(b*x + a)*a^2)/(a*b^3*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^{3/2}}{x^4}\,dx = \frac{a\sqrt{a+b\,x}}{8\,x^3} - \frac{(a+b\,x)^{5/2}}{8\,a\,x^3} - \frac{(a+b\,x)^{3/2}}{3\,x^3} - \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{3/2}}$$

input
```
int((a + b*x)^(3/2)/x^4,x)
```

output
```
(a*(a + b*x)^(1/2))/(8*x^3) - (a + b*x)^(5/2)/(8*a*x^3) - (b^3*atan(((a +
b*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(3/2)) - (a + b*x)^(3/2)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{3/2}}{x^4}\,dx = \frac{-16\sqrt{bx+a}\,a^3 - 28\sqrt{bx+a}\,a^2bx - 6\sqrt{bx+a}\,a\,b^2x^2 - 3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3}{48a^2x^3}$$

input
```
int((b*x+a)^(3/2)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 28*sqrt(a + b*x)*a**2*b*x - 6*sqrt(a + b*x)*a*
b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 3*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**2*x**3)
```

3.357 $\int x^3(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3(a+bx)^{5/2}\,dx = -\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}$$

output
```
-2/7*a^3*(b*x+a)^(7/2)/b^4+2/3*a^2*(b*x+a)^(9/2)/b^4-6/11*a*(b*x+a)^(11/2)
/b^4+2/13*(b*x+a)^(13/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^3(a+bx)^{5/2}\,dx = \frac{2(a+bx)^{7/2}\left(-16a^3+56a^2bx-126ab^2x^2+231b^3x^3\right)}{3003b^4}$$

input
```
Integrate[x^3*(a + b*x)^(5/2),x]
```

output
```
(2*(a + b*x)^(7/2)*(-16*a^3 + 56*a^2*b*x - 126*a*b^2*x^2 + 231*b^3*x^3))/(
3003*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

input
```
Int[x^3*(a + b*x)^(5/2),x]
```

output
```
(-2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a + b*x)^(9/2))/(3*b^4) - (6*a*
(a + b*x)^(11/2))/(11*b^4) + (2*(a + b*x)^(13/2))/(13*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-231b^{3}x^{3}+126a\,b^{2}x^{2}-56a^{2}bx+16a^{3}\right)}{3003b^{4}}$	43
pseudoelliptic	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-231b^{3}x^{3}+126a\,b^{2}x^{2}-56a^{2}bx+16a^{3}\right)}{3003b^{4}}$	43
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-231b^{3}x^{3}+126a\,b^{2}x^{2}-56a^{2}bx+16a^{3}\right)}{3003b^{4}}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{6a(bx+a)^{\frac{11}{2}}}{11}+\frac{2a^{2}(bx+a)^{\frac{9}{2}}}{3}-\frac{2a^{3}(bx+a)^{\frac{7}{2}}}{7}}{b^{4}}$	50
default	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{6a(bx+a)^{\frac{11}{2}}}{11}+\frac{2a^{2}(bx+a)^{\frac{9}{2}}}{3}-\frac{2a^{3}(bx+a)^{\frac{7}{2}}}{7}}{b^{4}}$	50
trager	$-\frac{2\left(-231b^{6}x^{6}-567a\,x^{5}b^{5}-371a^{2}x^{4}b^{4}-5a^{3}x^{3}b^{3}+6a^{4}x^{2}b^{2}-8a^{5}xb+16a^{6}\right)\sqrt{bx+a}}{3003b^{4}}$	76
risch	$-\frac{2\left(-231b^{6}x^{6}-567a\,x^{5}b^{5}-371a^{2}x^{4}b^{4}-5a^{3}x^{3}b^{3}+6a^{4}x^{2}b^{2}-8a^{5}xb+16a^{6}\right)\sqrt{bx+a}}{3003b^{4}}$	76

input
```
int(x^3*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3003*(b*x+a)^(7/2)*(-231*b^3*x^3+126*a*b^2*x^2-56*a^2*b*x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int x^3(a+bx)^{5/2}\,dx=\frac{2\,(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6)\sqrt{bx+a}}{3003\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*
a^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.03

$$\int x^3(a+bx)^{5/2}\,dx = \begin{cases} -\frac{32a^6\sqrt{a+bx}}{3003b^4} + \frac{16a^5x\sqrt{a+bx}}{3003b^3} - \frac{4a^4x^2\sqrt{a+bx}}{1001b^2} + \frac{10a^3x^3\sqrt{a+bx}}{3003b} + \frac{106a^2x^4\sqrt{a+bx}}{429} + \frac{54abx^5\sqrt{a+bx}}{143} + \frac{2b^2x^6\sqrt{a+bx}}{13} \\ \frac{a^{\frac{5}{2}}x^4}{4} \end{cases}$$

input
```
integrate(x**3*(b*x+a)**(5/2),x)
```

output
```
Piecewise((-32*a**6*sqrt(a + b*x)/(3003*b**4) + 16*a**5*x*sqrt(a + b*x)/(3
003*b**3) - 4*a**4*x**2*sqrt(a + b*x)/(1001*b**2) + 10*a**3*x**3*sqrt(a +
b*x)/(3003*b) + 106*a**2*x**4*sqrt(a + b*x)/429 + 54*a*b*x**5*sqrt(a + b*x
)/143 + 2*b**2*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{5/2}\,dx = \frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^4} - \frac{6\,(bx+a)^{\frac{11}{2}}a}{11\,b^4} + \frac{2\,(bx+a)^{\frac{9}{2}}a^2}{3\,b^4} - \frac{2\,(bx+a)^{\frac{7}{2}}a^3}{7\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/13*(b*x + a)^(13/2)/b^4 - 6/11*(b*x + a)^(11/2)*a/b^4 + 2/3*(b*x + a)^(9
/2)*a^2/b^4 - 2/7*(b*x + a)^(7/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.90

$$\int x^3(a+bx)^{5/2}\,dx = \frac{2\left(\frac{429\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^3}{b^3}+\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-4\right.}{b^3}\right.}{}$$

input
```
integrate(x^3*(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/15015*(429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2
)*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x
 + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*a^2/b^3 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a
 + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/
2)*a^4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*(231*(b*x + a)^(13/2) - 1638*(b*
x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 90
09*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6
)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{5/2}\,dx = \frac{2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{9/2}}{3\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}$$

input
```
int(x^3*(a + b*x)^(5/2),x)
```

output
```
(2*(a + b*x)^(13/2))/(13*b^4) - (2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(
a + b*x)^(9/2))/(3*b^4) - (6*a*(a + b*x)^(11/2))/(11*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int x^3(a+bx)^{5/2}\,dx = \frac{2\sqrt{bx+a}\,(231b^6x^6+567a\,b^5x^5+371a^2b^4x^4+5a^3b^3x^3-6a^4b^2x^2+8a^5bx-16a^6)}{3003b^4}$$

input
```
int(x^3*(b*x+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**6 + 8*a**5*b*x - 6*a**4*b**2*x**2 + 5*a**3*b**3
*x**3 + 371*a**2*b**4*x**4 + 567*a*b**5*x**5 + 231*b**6*x**6))/(3003*b**4)
```

3.358 $\int x^2(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2(a+bx)^{5/2}\,dx=\frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}$$

output
```
2/7*a^2*(b*x+a)^(7/2)/b^3-4/9*a*(b*x+a)^(9/2)/b^3+2/11*(b*x+a)^(11/2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^{5/2}\,dx=\frac{2(a+bx)^{7/2}\left(8a^2-28abx+63b^2x^2\right)}{693b^3}$$

input
```
Integrate[x^2*(a + b*x)^(5/2),x]
```

output
```
(2*(a + b*x)^(7/2)*(8*a^2 - 28*a*b*x + 63*b^2*x^2))/(693*b^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^2(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

input
```
Int[x^2*(a + b*x)^(5/2),x]
```

output
```
(2*a^2*(a + b*x)^(7/2))/(7*b^3) - (4*a*(a + b*x)^(9/2))/(9*b^3) + (2*(a +
b*x)^(11/2))/(11*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2(bx+a)^{\frac{7}{2}}(63b^2x^2-28abx+8a^2)}{693b^3}$	32
pseudoelliptic	$\frac{2(bx+a)^{\frac{7}{2}}(63b^2x^2-28abx+8a^2)}{693b^3}$	32
orering	$\frac{2(bx+a)^{\frac{7}{2}}(63b^2x^2-28abx+8a^2)}{693b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{4a(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^2(bx+a)^{\frac{7}{2}}}{7}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{4a(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^2(bx+a)^{\frac{7}{2}}}{7}}{b^3}$	38
trager	$\frac{2(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5)\sqrt{bx+a}}{693b^3}$	65
risch	$\frac{2(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5)\sqrt{bx+a}}{693b^3}$	65

input
```
int(x^2*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/693*(b*x+a)^(7/2)*(63*b^2*x^2-28*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int x^2(a+bx)^{5/2}\,dx=\frac{2\,(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5)\sqrt{bx+a}}{693\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^
4*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(49) = 98$.

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int x^2(a+bx)^{5/2}\,dx = \begin{cases} \frac{16a^5\sqrt{a+bx}}{693b^3} - \frac{8a^4x\sqrt{a+bx}}{693b^2} + \frac{2a^3x^2\sqrt{a+bx}}{231b} + \frac{226a^2x^3\sqrt{a+bx}}{693} + \frac{46abx^4\sqrt{a+bx}}{99} + \frac{2b^2x^5\sqrt{a+bx}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**(5/2),x)
```

output
```
Piecewise((16*a**5*sqrt(a + b*x)/(693*b**3) - 8*a**4*x*sqrt(a + b*x)/(693*
b**2) + 2*a**3*x**2*sqrt(a + b*x)/(231*b) + 226*a**2*x**3*sqrt(a + b*x)/69
3 + 46*a*b*x**4*sqrt(a + b*x)/99 + 2*b**2*x**5*sqrt(a + b*x)/11, Ne(b, 0))
, (a**(5/2)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2(a+bx)^{5/2}\,dx = \frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^3} - \frac{4\,(bx+a)^{\frac{9}{2}}a}{9\,b^3} + \frac{2\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/11*(b*x + a)^(11/2)/b^3 - 4/9*(b*x + a)^(9/2)*a/b^3 + 2/7*(b*x + a)^(7/2
)*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.40

$$\int x^2(a+bx)^{5/2}\,dx = \frac{2\left(\frac{231\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a^3}{b^2}+\frac{297\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}a^3\right)a^2}{b^2}+\cdots\right.}{}$$

input
```
integrate(x^2*(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*a^3/b^2 + 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)
^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*(35*(b*x + a)^(9/2) - 180*
(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 31
5*sqrt(b*x + a)*a^4)*a/b^2 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*
a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3
/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2(a+bx)^{5/2}\,dx = \frac{126\,(a+b\,x)^{11/2}-308\,a\,(a+b\,x)^{9/2}+198\,a^2\,(a+b\,x)^{7/2}}{693\,b^3}$$

input
```
int(x^2*(a + b*x)^(5/2),x)
```

output
```
(126*(a + b*x)^(11/2) - 308*a*(a + b*x)^(9/2) + 198*a^2*(a + b*x)^(7/2))/(
693*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int x^2(a+bx)^{5/2}\,dx = \frac{2\sqrt{bx+a}\,(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5)}{693b^3}$$

input
```
int(x^2*(b*x+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**5 - 4*a**4*b*x + 3*a**3*b**2*x**2 + 113*a**2*b**3*x
**3 + 161*a*b**4*x**4 + 63*b**5*x**5))/(693*b**3)
```

3.359 $\int x(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x(a+bx)^{5/2}\,dx=-\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}$$

output
```
-2/7*a*(b*x+a)^(7/2)/b^2+2/9*(b*x+a)^(9/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x(a+bx)^{5/2}\,dx=\frac{2(a+bx)^{7/2}(-2a+7bx)}{63b^2}$$

input
```
Integrate[x*(a + b*x)^(5/2),x]
```

output
```
(2*(a + b*x)^(7/2)*(-2*a + 7*b*x))/(63*b^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{(a+bx)^{7/2}}{b}-\frac{a(a+bx)^{5/2}}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

input
```
Int[x*(a + b*x)^(5/2),x]
```

output
```
(-2*a*(a + b*x)^(7/2))/(7*b^2) + (2*(a + b*x)^(9/2))/(9*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}(-7bx+2a)}{63b^{2}}$	21
pseudoelliptic	$-\frac{2(bx+a)^{\frac{7}{2}}(-7bx+2a)}{63b^{2}}$	21
orering	$-\frac{2(bx+a)^{\frac{7}{2}}(-7bx+2a)}{63b^{2}}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(bx+a)^{\frac{7}{2}}}{7}}{b^{2}}$	26
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(bx+a)^{\frac{7}{2}}}{7}}{b^{2}}$	26
trager	$-\frac{2(-7b^{4}x^{4}-19a\,x^{3}b^{3}-15a^{2}b^{2}x^{2}-a^{3}bx+2a^{4})\sqrt{bx+a}}{63b^{2}}$	54
risch	$-\frac{2(-7b^{4}x^{4}-19a\,x^{3}b^{3}-15a^{2}b^{2}x^{2}-a^{3}bx+2a^{4})\sqrt{bx+a}}{63b^{2}}$	54

input
```
int(x*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*(b*x+a)^(7/2)*(-7*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int x(a+bx)^{5/2}\,dx=\frac{2\left(7\,b^{4}x^{4}+19\,ab^{3}x^{3}+15\,a^{2}b^{2}x^{2}+a^{3}bx-2\,a^{4}\right)\sqrt{bx+a}}{63\,b^{2}}$$

input
```
integrate(x*(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*
x + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(31) = 62$.

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.00

$$\int x(a+bx)^{5/2}\,dx = \begin{cases} -\frac{4a^4\sqrt{a+bx}}{63b^2} + \frac{2a^3x\sqrt{a+bx}}{63b} + \frac{10a^2x^2\sqrt{a+bx}}{21} + \frac{38abx^3\sqrt{a+bx}}{63} + \frac{2b^2x^4\sqrt{a+bx}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**(5/2),x)
```

output
```
Piecewise((-4*a**4*sqrt(a + b*x)/(63*b**2) + 2*a**3*x*sqrt(a + b*x)/(63*b)
 + 10*a**2*x**2*sqrt(a + b*x)/21 + 38*a*b*x**3*sqrt(a + b*x)/63 + 2*b**2*x
**4*sqrt(a + b*x)/9, Ne(b, 0)), (a**(5/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(a+bx)^{5/2}\,dx = \frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^2} - \frac{2\,(bx+a)^{\frac{7}{2}}a}{7\,b^2}$$

input
```
integrate(x*(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/9*(b*x + a)^(9/2)/b^2 - 2/7*(b*x + a)^(7/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.35

$$\int x(a+bx)^{5/2}\,dx = \frac{2\left(\frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^3}{b}+\frac{63\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a^2}{b}+\frac{27\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(\cdots\right.}{b}\cdots\right.}{315\,b}$$

input
```
integrate(x*(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 63*(3*(b*x + a)^(
5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2/b + 27*(5*(b*x + a
)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)
*a^3)*a/b + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5
/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x(a+bx)^{5/2}\,dx = -\frac{18\,a\,(a+b\,x)^{7/2}-14\,(a+b\,x)^{9/2}}{63\,b^2}$$

input
```
int(x*(a + b*x)^(5/2),x)
```

output
```
-(18*a*(a + b*x)^(7/2) - 14*(a + b*x)^(9/2))/(63*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int x(a+bx)^{5/2}\,dx=\frac{2\sqrt{bx+a}\,(7b^4x^4+19a\,b^3x^3+15a^2b^2x^2+a^3bx-2a^4)}{63b^2}$$

input
```
int(x*(b*x+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**4 + a**3*b*x + 15*a**2*b**2*x**2 + 19*a*b**3*x**
3 + 7*b**4*x**4))/(63*b**2)
```

3.360 $\int (a+bx)^{5/2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (a+bx)^{5/2} \, dx = \frac{2(a+bx)^{7/2}}{7b}$$

output
```
2/7*(b*x+a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+bx)^{5/2} \, dx = \frac{2(a+bx)^{7/2}}{7b}$$

input
```
Integrate[(a + b*x)^(5/2),x]
```

output
```
(2*(a + b*x)^(7/2))/(7*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{5/2} \, dx$$

$$\Big\downarrow \ 17$$

$$\frac{2(a+bx)^{7/2}}{7b}$$

input
```
Int[(a + b*x)^(5/2),x]
```

output
```
(2*(a + b*x)^(7/2))/(7*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
default	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
pseudoelliptic	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
orering	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
trager	$\frac{2\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3}\right)\sqrt{bx+a}}{7b}$	40
risch	$\frac{2\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3}\right)\sqrt{bx+a}}{7b}$	40

input
```
int((b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(b*x+a)^(7/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int (a+bx)^{5/2}\,dx=\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}}{7\,b}$$

input
```
integrate((b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{5/2}\,dx = \frac{2(a+bx)^{\frac{7}{2}}}{7b}$$

input
```
integrate((b*x+a)**(5/2),x)
```

output
```
2*(a + b*x)**(7/2)/(7*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{5/2}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

input
```
integrate((b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/7*(b*x + a)^(7/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.94

$$\int (a+bx)^{5/2}\,dx = \frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 + 35\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}\,a\right)a^2 + 7\left(3\,(b\right.\right.}{35\,b}$$

input
```
integrate((b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 +
35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2 + 7*(3*(b*x + a)^(5/2) - 10*(
b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{5/2}\,dx=\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b}$$

input
```
int((a + b*x)^(5/2),x)
```

output
```
(2*(a + b*x)^(7/2))/(7*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int (a+bx)^{5/2}\,dx=\frac{2\sqrt{bx+a}\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}{7b}$$

input
```
int((b*x+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/(7*b)
```

3.361 $\int \frac{(a+bx)^{5/2}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx)^{5/2}}{x} dx = 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}-2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*a^2*(b*x+a)^(1/2)+2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctan
h((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{5/2}}{x} dx = \frac{2}{15}\sqrt{a+bx}\left(23a^2+11abx+3b^2x^2\right)-2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(5/2)/x,x]
```

output
```
(2*Sqrt[a + b*x]*(23*a^2 + 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{60}$$

$$a \int \frac{(a+bx)^{3/2}}{x}dx + \frac{2}{5}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$a \left(a \int \frac{\sqrt{a+bx}}{x}dx + \frac{2}{3}(a+bx)^{3/2}\right) + \frac{2}{5}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$a \left(a \left(a \int \frac{1}{x\sqrt{a+bx}}dx + 2\sqrt{a+bx}\right) + \frac{2}{3}(a+bx)^{3/2}\right) + \frac{2}{5}(a+bx)^{5/2}$$

$$\Big\downarrow \text{73}$$

$$a \left(a \left(\frac{2a \int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b} + 2\sqrt{a+bx}\right) + \frac{2}{3}(a+bx)^{3/2}\right) + \frac{2}{5}(a+bx)^{5/2}$$

$$\Big\downarrow \text{221}$$

$$a \left(a \left(2\sqrt{a+bx} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right) + \frac{2}{3}(a+bx)^{3/2}\right) + \frac{2}{5}(a+bx)^{5/2}$$

input
```
Int[(a + b*x)^(5/2)/x,x]
```

output
```
(2*(a + b*x)^(5/2))/5 + a*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*
Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$-2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(3b^2x^2+11abx+23a^2)}{15}$	47
derivativedivides	$2a^2\sqrt{bx+a}+\frac{2a(bx+a)^{\frac{3}{2}}}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	50
default	$2a^2\sqrt{bx+a}+\frac{2a(bx+a)^{\frac{3}{2}}}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	50

input
```
int((b*x+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/15*(b*x+a)^(1/2)*(3*b^2*x^2+11
*a*b*x+23*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)^{5/2}}{x}\,dx = \left[a^{\frac{5}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right) + \frac{2}{15}\left(3\,b^2x^2+11\,abx+23\,a^2\right)\sqrt{bx+a}, 2\,\sqrt{-a}a^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right) + \frac{2}{15}\left(3\,b^2x^2+11\,abx+23\,a^2\right)\sqrt{bx+a}\right]$$

input
```
integrate((b*x+a)^(5/2)/x,x, algorithm="fricas")
```

output
```
[a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 +
11*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(-a)/sqrt(b*x
+ a)) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^{5/2}}{x}\,dx = \frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{15} + a^{\frac{5}{2}}\log\left(\frac{bx}{a}\right) - 2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right) + \frac{22a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}{15} + \frac{2\sqrt{a}b^2x^2\sqrt{1+\frac{bx}{a}}}{5}$$

input
```
integrate((b*x+a)**(5/2)/x,x)
```

output
```
46*a**(5/2)*sqrt(1 + b*x/a)/15 + a**(5/2)*log(b*x/a) - 2*a**(5/2)*log(sqrt
(1 + b*x/a) + 1) + 22*a**(3/2)*b*x*sqrt(1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**
2*sqrt(1 + b*x/a)/5
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^{5/2}}{x}\,dx = a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a+2\,\sqrt{bx+a}a^2$$

input
```
integrate((b*x+a)^(5/2)/x,x, algorithm="maxima")
```

output
```
a^(5/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/5*(b*
x + a)^(5/2) + 2/3*(b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{5/2}}{x}\,dx = \frac{2\,a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a+2\,\sqrt{bx+a}a^2$$

input
```
integrate((b*x+a)^(5/2)/x,x, algorithm="giac")
```

output
```
2*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/5*(b*x + a)^(5/2) + 2/3*
(b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{5/2}}{x}\,dx = \frac{2\,a\,(a+b\,x)^{3/2}}{3}+\frac{2\,(a+b\,x)^{5/2}}{5} + 2\,a^2\,\sqrt{a+b\,x}+a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int((a + b*x)^(5/2)/x,x)
```

output
```
(2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 + 2*a^2*(a + b*x)^(1/2) +
a^(5/2)*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{5/2}}{x}\,dx = \frac{46\sqrt{bx+a}\,a^2}{15} + \frac{22\sqrt{bx+a}\,abx}{15} + \frac{2\sqrt{bx+a}\,b^2x^2}{5} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2 - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a^2$$

input
```
int((b*x+a)^(5/2)/x,x)
```

output
```
(46*sqrt(a + b*x)*a**2 + 22*sqrt(a + b*x)*a*b*x + 6*sqrt(a + b*x)*b**2*x**
2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2 - 15*sqrt(a)*log(sqrt(a +
 b*x) + sqrt(a))*a**2)/15
```

3.362 $\int \frac{(a+bx)^{5/2}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx)^{5/2}}{x^2} dx = 4ab\sqrt{a+bx} - \frac{a^2\sqrt{a+bx}}{x} + \frac{2}{3}b(a+bx)^{3/2} - 5a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
4*a*b*(b*x+a)^(1/2)-a^2*(b*x+a)^(1/2)/x+2/3*b*(b*x+a)^(3/2)-5*a^(3/2)*b*ar
ctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^{5/2}}{x^2} dx = \frac{\sqrt{a+bx}(-3a^2+14abx+2b^2x^2)}{3x} - 5a^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(5/2)/x^2,x]
```

output
```
(Sqrt[a + b*x]*(-3*a^2 + 14*a*b*x + 2*b^2*x^2))/(3*x) - 5*a^(3/2)*b*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{2}b\int \frac{(a+bx)^{3/2}}{x}dx-\frac{(a+bx)^{5/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{2}b\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{2}b\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{5}{2}b\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}$$

$$\Big\downarrow \text{221}$$

$$\frac{5}{2}b\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}$$

input
```
Int[(a + b*x)^(5/2)/x^2,x]
```

output
```
-((a + b*x)^(5/2)/x) + (5*b*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] -
2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{a^{2}\sqrt{bx+a}}{x}+\frac{b\left(\frac{4(bx+a)^{\frac{3}{2}}}{3}+8a\sqrt{bx+a}-10a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{2}$	57
pseudoelliptic	$-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}bx+\left(-\frac{2\sqrt{a}\,b^{2}x^{2}}{3}+\left(a-\frac{14bx}{3}\right)a^{\frac{3}{2}}\right)\sqrt{bx+a}}{\sqrt{a}\,x}$	59
derivativedivides	$2b\left(\frac{(bx+a)^{\frac{3}{2}}}{3}+2a\sqrt{bx+a}-a^{2}\left(\frac{\sqrt{bx+a}}{2bx}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	62
default	$2b\left(\frac{(bx+a)^{\frac{3}{2}}}{3}+2a\sqrt{bx+a}-a^{2}\left(\frac{\sqrt{bx+a}}{2bx}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	62

input
```
int((b*x+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*(b*x+a)^(1/2)/x+1/2*b*(4/3*(b*x+a)^(3/2)+8*a*(b*x+a)^(1/2)-10*a^(3/2)
*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)^{5/2}}{x^2}\,dx=\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(2\,b^2x^2+14\,abx-3\,a^2)\sqrt{bx+a}}{6\,x},\ \frac{15\,\sqrt{-a}abx\,\mathrm{arct}}{}\right.$$

input
```
integrate((b*x+a)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b
^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan
(sqrt(-a)/sqrt(b*x + a)) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x
]
```

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^{5/2}}{x^2}\,dx = -\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{x} + \frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{bx}{a}}}{3} + \frac{5a^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2} - 5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right) + \frac{2\sqrt{a}b^2x\sqrt{1+\frac{bx}{a}}}{3}$$

input
```
integrate((b*x+a)**(5/2)/x**2,x)
```

output
```
-a**(5/2)*sqrt(1 + b*x/a)/x + 14*a**(3/2)*b*sqrt(1 + b*x/a)/3 + 5*a**(3/2)
*b*log(b*x/a)/2 - 5*a**(3/2)*b*log(sqrt(1 + b*x/a) + 1) + 2*sqrt(a)*b**2*x
*sqrt(1 + b*x/a)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{5/2}}{x^2}\,dx = \frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2}{3}\,(bx+a)^{\frac{3}{2}}b + 4\,\sqrt{bx+a}ab - \frac{\sqrt{bx+a}a^2}{x}$$

input
```
integrate((b*x+a)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
5/2*a^(3/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2
/3*(b*x + a)^(3/2)*b + 4*sqrt(b*x + a)*a*b - sqrt(b*x + a)*a^2/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{5/2}}{x^2}\,dx = \frac{1}{3}\left(\frac{15\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\,(bx+a)^{\frac{3}{2}} + 12\,\sqrt{bx+a}a - \frac{3\,\sqrt{bx+a}a^2}{bx}\right)b$$

input
```
integrate((b*x+a)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/3*(15*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*(b*x + a)^(3/2) +
12*sqrt(b*x + a)*a - 3*sqrt(b*x + a)*a^2/(b*x))*b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^{5/2}}{x^2}\,dx = \frac{2\,b\,(a+b\,x)^{3/2}}{3} - \frac{a^2\,\sqrt{a+b\,x}}{x} + 4\,a\,b\,\sqrt{a+b\,x} + a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}$$

input
```
int((a + b*x)^(5/2)/x^2,x)
```

output
```
(2*b*(a + b*x)^(3/2))/3 - (a^2*(a + b*x)^(1/2))/x + a^(3/2)*b*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*5i + 4*a*b*(a + b*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{5/2}}{x^2}\,dx = \frac{-6\sqrt{bx+a}\,a^2 + 28\sqrt{bx+a}\,abx + 4\sqrt{bx+a}\,b^2x^2 + 15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abx - 1}{6x}$$

input
```
int((b*x+a)^(5/2)/x^2,x)
```

output
```
( - 6*sqrt(a + b*x)*a**2 + 28*sqrt(a + b*x)*a*b*x + 4*sqrt(a + b*x)*b**2*x
**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*x - 15*sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))*a*b*x)/(6*x)
```

3.363 $\int \frac{(a+bx)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx = 2b^2\sqrt{a+bx} - \frac{a^2\sqrt{a+bx}}{2x^2} - \frac{9ab\sqrt{a+bx}}{4x} - \frac{15}{4}\sqrt{a}b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*b^2*(b*x+a)^(1/2)-1/2*a^2*(b*x+a)^(1/2)/x^2-9/4*a*b*(b*x+a)^(1/2)/x-15/4
*a^(1/2)*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx = \frac{1}{4}\left(\frac{\sqrt{a+bx}(-2a^2-9abx+8b^2x^2)}{x^2} - 15\sqrt{a}b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[(a + b*x)^(5/2)/x^3,x]
```

output
```
((Sqrt[a + b*x]*(-2*a^2 - 9*a*b*x + 8*b^2*x^2))/x^2 - 15*Sqrt[a]*b^2*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^3} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{4} b \int \frac{(a+bx)^{3/2}}{x^2}dx-\frac{(a+bx)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{4} b \left(\frac{3}{2} b \int \frac{\sqrt{a+bx}}{x}dx-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{4} b \left(\frac{3}{2} b \left(a \int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{5}{4} b \left(\frac{3}{2} b \left(\frac{2a \int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{5}{4} b \left(\frac{3}{2} b \left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}$$

input
```
Int[(a + b*x)^(5/2)/x^3,x]
```

output
```
-1/2*(a + b*x)^(5/2)/x^2 + (5*b*(-((a + b*x)^(3/2)/x) + (3*b*(2*Sqrt[a + b
*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/2))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{a\sqrt{bx+a}\,(9bx+2a)}{4x^{2}}+\frac{b^{2}\left(16\sqrt{bx+a}-30\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{8}$	55
pseudoelliptic	$-\frac{\frac{15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{2}x^{2}}{2}+\sqrt{a}\,\left(a^{2}+\frac{9}{2}abx-4b^{2}x^{2}\right)\sqrt{bx+a}}{2\sqrt{a}\,x^{2}}$	59
derivativedivides	$2b^{2}\left(\sqrt{bx+a}-a\left(\frac{\frac{9(bx+a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	62
default	$2b^{2}\left(\sqrt{bx+a}-a\left(\frac{\frac{9(bx+a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	62

input
```
int((b*x+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*(b*x+a)^(1/2)*(9*b*x+2*a)/x^2+1/8*b^2*(16*(b*x+a)^(1/2)-30*a^(1/2)*
arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx=\left[\frac{15\,\sqrt{a}b^{2}x^{2}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(8\,b^{2}x^{2}-9\,abx-2\,a^{2})\sqrt{bx+a}}{8\,x^{2}},\frac{15\,\sqrt{-a}b^{2}x^{2}\,\text{ar}}{}\right.$$

input
```
integrate((b*x+a)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2
*arctan(sqrt(-a)/sqrt(b*x + a)) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
 a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx = -\frac{15\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4} - \frac{a^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((b*x+a)**(5/2)/x**3,x)
```

output
```
-15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**3/(2*sqrt(b)*x**(
5/2)*sqrt(a/(b*x) + 1)) - 11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) -
 a*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(a/(b*x
) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx = \frac{15}{8}\sqrt{a}b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + 2\sqrt{bx+a}b^2 - \frac{9(bx+a)^{\frac{3}{2}}ab^2 - 7\sqrt{bx+a}a^2b^2}{4\left((bx+a)^2 - 2(bx+a)a + a^2\right)}$$

input
```
integrate((b*x+a)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
15/8*sqrt(a)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))
+ 2*sqrt(b*x + a)*b^2 - 1/4*(9*(b*x + a)^(3/2)*a*b^2 - 7*sqrt(b*x + a)*a^2
*b^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx = \frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8\,\sqrt{bx+a}b^3 - \frac{9\,(bx+a)^{\frac{3}{2}}ab^3-7\,\sqrt{bx+a}a^2b^3}{b^2x^2}}{4\,b}$$

input
```
integrate((b*x+a)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(15*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x + a)*b^
3 - (9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x + a)*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx = 2\,b^2\,\sqrt{a+b\,x} + \frac{7\,a^2\,\sqrt{a+b\,x}}{4\,x^2} - \frac{9\,a\,(a+b\,x)^{3/2}}{4\,x^2} + \frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{4}$$

input
```
int((a + b*x)^(5/2)/x^3,x)
```

output
```
2*b^2*(a + b*x)^(1/2) + (7*a^2*(a + b*x)^(1/2))/(4*x^2) + (a^(1/2)*b^2*ata
n(((a + b*x)^(1/2)*1i)/a^(1/2))*15i)/4 - (9*a*(a + b*x)^(3/2))/(4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^{5/2}}{x^3}\,dx = \frac{-4\sqrt{bx+a}\,a^2 - 18\sqrt{bx+a}\,abx + 16\sqrt{bx+a}\,b^2x^2 + 15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2 -}{8x^2}$$

input
```
int((b*x+a)^(5/2)/x^3,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 18*sqrt(a + b*x)*a*b*x + 16*sqrt(a + b*x)*b**2*
x**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt(a)*log(
sqrt(a + b*x) + sqrt(a))*b**2*x**2)/(8*x**2)
```

3.364 $\int \frac{(a+bx)^{5/2}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{(a+bx)^{5/2}}{x^4} dx = -\frac{a^2\sqrt{a+bx}}{3x^3} - \frac{13ab\sqrt{a+bx}}{12x^2} - \frac{11b^2\sqrt{a+bx}}{8x} - \frac{5b^3 \text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
-1/3*a^2*(b*x+a)^(1/2)/x^3-13/12*a*b*(b*x+a)^(1/2)/x^2-11/8*b^2*(b*x+a)^(1
/2)/x-5/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^{5/2}}{x^4} dx = -\frac{\sqrt{a+bx}(8a^2+26abx+33b^2x^2)}{24x^3} - \frac{5b^3 \text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

input
```
Integrate[(a + b*x)^(5/2)/x^4,x]
```

output
```
-1/24*(Sqrt[a + b*x]*(8*a^2 + 26*a*b*x + 33*b^2*x^2))/x^3 - (5*b^3*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{6}b\int \frac{(a+bx)^{3/2}}{x^3}dx-\frac{(a+bx)^{5/2}}{3x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\int \frac{\sqrt{a+bx}}{x^2}dx-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}$$

input
```
Int[(a + b*x)^(5/2)/x^4,x]
```

output
```
-1/3*(a + b*x)^(5/2)/x^3 + (5*b*(-1/2*(a + b*x)^(3/2)/x^2 + (3*b*(-(Sqrt[a
 + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]))/4))/6
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{\sqrt{bx+a}\,\left(33b^{2}x^{2}+26abx+8a^{2}\right)}{24x^{3}}-\frac{5b^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}$	53
derivativedivides	$2b^{3}\left(-\frac{\frac{11(bx+a)^{\frac{5}{2}}}{16}-\frac{5a(bx+a)^{\frac{3}{2}}}{6}+\frac{5a^{2}\sqrt{bx+a}}{16}}{b^{3}x^{3}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}\right)$	64
default	$2b^{3}\left(-\frac{\frac{11(bx+a)^{\frac{5}{2}}}{16}-\frac{5a(bx+a)^{\frac{3}{2}}}{6}+\frac{5a^{2}\sqrt{bx+a}}{16}}{b^{3}x^{3}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}\right)$	64
pseudoelliptic	$\frac{-15\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}x^{3}-33b^{2}x^{2}\sqrt{a}\,\sqrt{bx+a}-26a^{\frac{3}{2}}bx\sqrt{bx+a}-8a^{\frac{5}{2}}\sqrt{bx+a}}{24x^{3}\sqrt{a}}$	74

input
```
int((b*x+a)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)^(1/2)*(33*b^2*x^2+26*a*b*x+8*a^2)/x^3-5/8*b^3*arctanh((b*x+a
)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^{5/2}}{x^4}\,dx = \left[\frac{15\,\sqrt{a}b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,ax^3}, \frac{15\,\sqrt{-a}b^3\ldots}{\ldots}\right.$$

input
```
integrate((b*x+a)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt
(-a)*b^3*x^3*arctan(sqrt(-a)/sqrt(b*x + a)) - (33*a*b^2*x^2 + 26*a^2*b*x +
 8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^{5/2}}{x^4}\,dx = -\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x^{\frac{5}{2}}}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{12x^{\frac{3}{2}}}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{8\sqrt{x}}-\frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{a}}$$

input
```
integrate((b*x+a)**(5/2)/x**4,x)
```

output
```
-a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x**(5/2)) - 13*a*b**(3/2)*sqrt(a/(b*x)
+ 1)/(12*x**(3/2)) - 11*b**(5/2)*sqrt(a/(b*x) + 1)/(8*sqrt(x)) - 5*b**3*as
inh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^{5/2}}{x^4}\,dx = \frac{5\,b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,\sqrt{a}} - \frac{33\,(bx+a)^{\frac{5}{2}}b^3 - 40\,(bx+a)^{\frac{3}{2}}ab^3 + 15\,\sqrt{bx+a}a^2b^3}{24\left((bx+a)^3 - 3\,(bx+a)^2a + 3\,(bx+a)a^2 - a^3\right)}$$

input
```
integrate((b*x+a)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
- 1/24*(33*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 15*sqrt(b*x +
a)*a^2*b^3)/((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{5/2}}{x^4}\,dx = \frac{1}{24}\,b^3\left(\frac{15\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{33\,(bx+a)^{\frac{5}{2}} - 40\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2}{b^3x^3}\right)$$

input
```
integrate((b*x+a)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/24*b^3*(15*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (33*(b*x + a)^(5/2)
 - 40*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/(b^3*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^{5/2}}{x^4}\,dx = \frac{5\,a\,(a+b\,x)^{3/2}}{3\,x^3} - \frac{5\,a^2\,\sqrt{a+b\,x}}{8\,x^3} - \frac{11\,(a+b\,x)^{5/2}}{8\,x^3} + \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{8\,\sqrt{a}}$$

input
```
int((a + b*x)^(5/2)/x^4,x)
```

output
```
(b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^(1/2)) - (5*a^2*(a + b*x)
^(1/2))/(8*x^3) - (11*(a + b*x)^(5/2))/(8*x^3) + (5*a*(a + b*x)^(3/2))/(3*
x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{5/2}}{x^4}\,dx = \frac{-16\sqrt{bx+a}\,a^3 - 52\sqrt{bx+a}\,a^2bx - 66\sqrt{bx+a}\,a\,b^2x^2 + 15\sqrt{a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x}{48a\,x^3}$$

input
```
int((b*x+a)^(5/2)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 52*sqrt(a + b*x)*a**2*b*x - 66*sqrt(a + b*x)*a
*b**2*x**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 15*sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a*x**3)
```

3.365 $\int \frac{(a+bx)^{5/2}}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int \frac{(a+bx)^{5/2}}{x^5} dx = -\frac{a^2\sqrt{a+bx}}{4x^4} - \frac{17ab\sqrt{a+bx}}{24x^3} - \frac{59b^2\sqrt{a+bx}}{96x^2} - \frac{5b^3\sqrt{a+bx}}{64ax} + \frac{5b^4\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

output
```
-1/4*a^2*(b*x+a)^(1/2)/x^4-17/24*a*b*(b*x+a)^(1/2)/x^3-59/96*b^2*(b*x+a)^(
1/2)/x^2-5/64*b^3*(b*x+a)^(1/2)/a/x+5/64*b^4*arctanh((b*x+a)^(1/2)/a^(1/2)
)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^{5/2}}{x^5} dx = -\frac{\sqrt{a+bx}(48a^3+136a^2bx+118ab^2x^2+15b^3x^3)}{192ax^4} + \frac{5b^4\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

input
```
Integrate[(a + b*x)^(5/2)/x^5,x]
```

output
```
-1/192*(Sqrt[a + b*x]*(48*a^3 + 136*a^2*b*x + 118*a*b^2*x^2 + 15*b^3*x^3))
/(a*x^4) + (5*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^5} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{8} b \int \frac{(a+bx)^{3/2}}{x^4} dx - \frac{(a+bx)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{8} b \left(\frac{1}{2} b \int \frac{\sqrt{a+bx}}{x^3} dx - \frac{(a+bx)^{3/2}}{3x^3}\right) - \frac{(a+bx)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{8} b \left(\frac{1}{2} b \left(\frac{1}{4} b \int \frac{1}{x^2\sqrt{a+bx}} dx - \frac{\sqrt{a+bx}}{2x^2}\right) - \frac{(a+bx)^{3/2}}{3x^3}\right) - \frac{(a+bx)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{5}{8} b \left(\frac{1}{2} b \left(\frac{1}{4} b \left(-\frac{b \int \frac{1}{x\sqrt{a+bx}} dx}{2a} - \frac{\sqrt{a+bx}}{ax}\right) - \frac{\sqrt{a+bx}}{2x^2}\right) - \frac{(a+bx)^{3/2}}{3x^3}\right) - \frac{(a+bx)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{73}$$

$$\frac{5}{8} b \left(\frac{1}{2} b \left(\frac{1}{4} b \left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}} d\sqrt{a+bx}}{a} - \frac{\sqrt{a+bx}}{ax}\right) - \frac{\sqrt{a+bx}}{2x^2}\right) - \frac{(a+bx)^{3/2}}{3x^3}\right) - \frac{(a+bx)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{221}$$

$$\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}$$

input
```
Int[(a + b*x)^(5/2)/x^5,x]
```

output
```
-1/4*(a + b*x)^(5/2)/x^4 + (5*b*(-1/3*(a + b*x)^(3/2)/x^3 + (b*(-1/2*Sqrt[
a + b*x]/x^2 + (b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/a^(3/2)))/4))/2))/8
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(15b^3x^3+118a\,b^2x^2+136a^2bx+48a^3)}{192x^4a}+\frac{5b^4\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{64a^{\frac{3}{2}}}$	67
pseudoelliptic	$-\frac{5\left(-\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4+\sqrt{bx+a}\left(\sqrt{a}\,b^3x^3+\frac{118a^{\frac{3}{2}}b^2x^2}{15}+\frac{136a^{\frac{5}{2}}bx}{15}+\frac{16a^{\frac{7}{2}}}{5}\right)\right)}{64a^{\frac{3}{2}}x^4}$	72
derivativedivides	$2b^4\left(-\frac{\frac{5(bx+a)^{\frac{7}{2}}}{128a}+\frac{73(bx+a)^{\frac{5}{2}}}{384}-\frac{55a(bx+a)^{\frac{3}{2}}}{384}+\frac{5a^2\sqrt{bx+a}}{128}}{b^4x^4}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}\right)$	76
default	$2b^4\left(-\frac{\frac{5(bx+a)^{\frac{7}{2}}}{128a}+\frac{73(bx+a)^{\frac{5}{2}}}{384}-\frac{55a(bx+a)^{\frac{3}{2}}}{384}+\frac{5a^2\sqrt{bx+a}}{128}}{b^4x^4}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}\right)$	76

input
```
int((b*x+a)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(b*x+a)^(1/2)*(15*b^3*x^3+118*a*b^2*x^2+136*a^2*b*x+48*a^3)/x^4/a+5
/64*b^4*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^{5/2}}{x^5}\,dx=\left[\frac{15\,\sqrt{a}b^4x^4\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx}}{384\,a^2x^4}\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^4x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx+a}}{192\,a^2x^4}\right]$$

input
```
integrate((b*x+a)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
[1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
2*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(
a^2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(-a)/sqrt(b*x + a)) + (15
*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x
^4)]
```

Sympy [A] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^{5/2}}{x^5}\,dx = -\frac{a^3}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{23a^2\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{127ab^{\frac{3}{2}}}{96x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{133b^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5b^{\frac{7}{2}}}{64a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(5/2)/x**5,x)
```

output
```
-a**3/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 23*a**2*sqrt(b)/(24*x**(7/2
)*sqrt(a/(b*x) + 1)) - 127*a*b**(3/2)/(96*x**(5/2)*sqrt(a/(b*x) + 1)) - 13
3*b**(5/2)/(192*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(7/2)/(64*a*sqrt(x)*sqr
t(a/(b*x) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^{5/2}}{x^5}\,dx = -\frac{5\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{128\,a^{\frac{3}{2}}} - \frac{15\,(bx+a)^{\frac{7}{2}}b^4 + 73\,(bx+a)^{\frac{5}{2}}ab^4 - 55\,(bx+a)^{\frac{3}{2}}a^2b^4 + 15\,\sqrt{bx+a}a^3b^4}{192\,\left((bx+a)^4a - 4\,(bx+a)^3a^2 + 6\,(bx+a)^2a^3 - 4\,(bx+a)a^4 + a^5\right)}$$

input
```
integrate((b*x+a)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
-5/128*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2
) - 1/192*(15*(b*x + a)^(7/2)*b^4 + 73*(b*x + a)^(5/2)*a*b^4 - 55*(b*x + a
)^(3/2)*a^2*b^4 + 15*sqrt(b*x + a)*a^3*b^4)/((b*x + a)^4*a - 4*(b*x + a)^3
*a^2 + 6*(b*x + a)^2*a^3 - 4*(b*x + a)*a^4 + a^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{5/2}}{x^5}\,dx = -\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5+73\,(bx+a)^{\frac{5}{2}}ab^5-55\,(bx+a)^{\frac{3}{2}}a^2b^5+15\,\sqrt{bx+a}a^3b^5}{ab^4x^4}}{192\,b}$$

input
```
integrate((b*x+a)^(5/2)/x^5,x, algorithm="giac")
```

output
```
-1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x + a)
^(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)^(3/2)*a^2*b^5 + 15*sq
rt(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^{5/2}}{x^5}\,dx = \frac{55\,a\,(a+b\,x)^{3/2}}{192\,x^4}-\frac{5\,a^2\,\sqrt{a+b\,x}}{64\,x^4} -\frac{5\,(a+b\,x)^{7/2}}{64\,a\,x^4}-\frac{73\,(a+b\,x)^{5/2}}{192\,x^4}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{64\,a^{3/2}}$$

input
```
int((a + b*x)^(5/2)/x^5,x)
```

output
```
(55*a*(a + b*x)^(3/2))/(192*x^4) - (5*a^2*(a + b*x)^(1/2))/(64*x^4) - (5*(
a + b*x)^(7/2))/(64*a*x^4) - (b^4*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(
64*a^(3/2)) - (73*(a + b*x)^(5/2))/(192*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{5/2}}{x^5}\,dx = \frac{-96\sqrt{bx+a}\,a^4-272\sqrt{bx+a}\,a^3bx-236\sqrt{bx+a}\,a^2b^2x^2-30\sqrt{bx+a}\,a\,b^3x^3-15\sqrt{a}}{384a^2x^4}$$

input
```
int((b*x+a)^(5/2)/x^5,x)
```

output
```
( - 96*sqrt(a + b*x)*a**4 - 272*sqrt(a + b*x)*a**3*b*x - 236*sqrt(a + b*x)
*a**2*b**2*x**2 - 30*sqrt(a + b*x)*a*b**3*x**3 - 15*sqrt(a)*log(sqrt(a + b
*x) - sqrt(a))*b**4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x*
*4)/(384*a**2*x**4)
```

3.366 $\int \frac{(a+bx)^{5/2}}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 129

$$\int \frac{(a+bx)^{5/2}}{x^6} dx = -\frac{a^2\sqrt{a+bx}}{5x^5} - \frac{21ab\sqrt{a+bx}}{40x^4} - \frac{31b^2\sqrt{a+bx}}{80x^3} - \frac{b^3\sqrt{a+bx}}{64ax^2} + \frac{3b^4\sqrt{a+bx}}{128a^2x} - \frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128a^{5/2}}$$

output
```
-1/5*a^2*(b*x+a)^(1/2)/x^5-21/40*a*b*(b*x+a)^(1/2)/x^4-31/80*b^2*(b*x+a)^(
1/2)/x^3-1/64*b^3*(b*x+a)^(1/2)/a/x^2+3/128*b^4*(b*x+a)^(1/2)/a^2/x-3/128*
b^5*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^{5/2}}{x^6} dx = -\frac{\sqrt{a+bx}(128a^4+336a^3bx+248a^2b^2x^2+10ab^3x^3-15b^4x^4)}{640a^2x^5} - \frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128a^{5/2}}$$

input
```
Integrate[(a + b*x)^(5/2)/x^6,x]
```

output
```
-1/640*(Sqrt[a + b*x]*(128*a^4 + 336*a^3*b*x + 248*a^2*b^2*x^2 + 10*a*b^3*
x^3 - 15*b^4*x^4))/(a^2*x^5) - (3*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128
*a^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {51, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^6} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}b\int \frac{(a+bx)^{3/2}}{x^5}dx-\frac{(a+bx)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\int \frac{\sqrt{a+bx}}{x^4}dx-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}$$

input
```
Int[(a + b*x)^(5/2)/x^6,x]
```

output
```
-1/5*(a + b*x)^(5/2)/x^5 + (b*(-1/4*(a + b*x)^(3/2)/x^4 + (3*b*(-1/3*Sqrt[
a + b*x]/x^3 + (b*(-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a + b*x]/(a*x
)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/6))/8))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

method	result	size
risch	$-\frac{\sqrt{bx+a}\,\left(-15b^{4}x^{4}+10a\,x^{3}b^{3}+248a^{2}b^{2}x^{2}+336a^{3}bx+128a^{4}\right)}{640x^{5}a^{2}}-\frac{3b^{5}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{5}{2}}}$	78
pseudoelliptic	$-\frac{\frac{15\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{5}x^{5}}{128}+\sqrt{bx+a}\,\left(-\frac{15\sqrt{a}\,b^{4}x^{4}}{128}+\frac{5a^{\frac{3}{2}}b^{3}x^{3}}{64}+\frac{31a^{\frac{5}{2}}b^{2}x^{2}}{16}+\frac{21a^{\frac{7}{2}}bx}{8}+a^{\frac{9}{2}}\right)}{5a^{\frac{5}{2}}x^{5}}$	82
derivativedivides	$2b^{5}\left(-\frac{-\frac{3(bx+a)^{\frac{9}{2}}}{256a^{2}}+\frac{7(bx+a)^{\frac{7}{2}}}{128a}+\frac{(bx+a)^{\frac{5}{2}}}{10}-\frac{7a(bx+a)^{\frac{3}{2}}}{128}+\frac{3a^{2}\sqrt{bx+a}}{256}}{b^{5}x^{5}}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256a^{\frac{5}{2}}}\right)$	88
default	$2b^{5}\left(-\frac{-\frac{3(bx+a)^{\frac{9}{2}}}{256a^{2}}+\frac{7(bx+a)^{\frac{7}{2}}}{128a}+\frac{(bx+a)^{\frac{5}{2}}}{10}-\frac{7a(bx+a)^{\frac{3}{2}}}{128}+\frac{3a^{2}\sqrt{bx+a}}{256}}{b^{5}x^{5}}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256a^{\frac{5}{2}}}\right)$	88

input
```
int((b*x+a)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(b*x+a)^(1/2)*(-15*b^4*x^4+10*a*b^3*x^3+248*a^2*b^2*x^2+336*a^3*b*x
+128*a^4)/x^5/a^2-3/128*b^5*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx)^{5/2}}{x^6}\,dx = \left[\frac{15\sqrt{a}b^5x^5\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15ab^4x^4-10a^2b^3x^3-248a^3b^2x^2-336a^4bx\right.}{1280a^3x^5}\right.$$

input
```
integrate((b*x+a)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/1280*(15*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*x - 128*a^
5)*sqrt(b*x + a))/(a^3*x^5), 1/640*(15*sqrt(-a)*b^5*x^5*arctan(sqrt(-a)/sq
rt(b*x + a)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*
b*x - 128*a^5)*sqrt(b*x + a))/(a^3*x^5)]
```

Sympy [A] (verification not implemented)

Time = 26.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{5/2}}{x^6}\,dx = -\frac{a^3}{5\sqrt{b}x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{29a^2\sqrt{b}}{40x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{73ab^{\frac{3}{2}}}{80x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{129b^{\frac{5}{2}}}{320x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{b^{\frac{7}{2}}}{128ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{3b^{\frac{9}{2}}}{128a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{3b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{128a^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**(5/2)/x**6,x)
```

output
```
-a**3/(5*sqrt(b)*x**(11/2)*sqrt(a/(b*x) + 1)) - 29*a**2*sqrt(b)/(40*x**(9/
2)*sqrt(a/(b*x) + 1)) - 73*a*b**(3/2)/(80*x**(7/2)*sqrt(a/(b*x) + 1)) - 12
9*b**(5/2)/(320*x**(5/2)*sqrt(a/(b*x) + 1)) + b**(7/2)/(128*a*x**(3/2)*sqr
t(a/(b*x) + 1)) + 3*b**(9/2)/(128*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**5
*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(128*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^{5/2}}{x^6}\,dx = \frac{3\,b^5 \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{256\,a^{\frac{5}{2}}} + \frac{15\,(bx+a)^{\frac{9}{2}}b^5 - 70\,(bx+a)^{\frac{7}{2}}ab^5 - 128\,(bx+a)^{\frac{5}{2}}a^2b^5 + 70\,(bx+a)^{\frac{3}{2}}a^3b^5 - 15\,\sqrt{bx+a}a^4b^5}{640\,\left((bx+a)^5a^2 - 5\,(bx+a)^4a^3 + 10\,(bx+a)^3a^4 - 10\,(bx+a)^2a^5 + 5\,(bx+a)a^6 - a^7\right)}$$

input
```
integrate((b*x+a)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
3/256*b^5*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
 + 1/640*(15*(b*x + a)^(9/2)*b^5 - 70*(b*x + a)^(7/2)*a*b^5 - 128*(b*x + a
)^(5/2)*a^2*b^5 + 70*(b*x + a)^(3/2)*a^3*b^5 - 15*sqrt(b*x + a)*a^4*b^5)/(
(b*x + a)^5*a^2 - 5*(b*x + a)^4*a^3 + 10*(b*x + a)^3*a^4 - 10*(b*x + a)^2*
a^5 + 5*(b*x + a)*a^6 - a^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^{5/2}}{x^6}\,dx = \frac{1}{640}\,b^5\left(\frac{15\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{15\,(bx+a)^{\frac{9}{2}} - 70\,(bx+a)^{\frac{7}{2}}a - 128\,(bx+a)^{\frac{5}{2}}a^2 + 70\,(b}{a^2b^5x^5}\right.$$

input
```
integrate((b*x+a)^(5/2)/x^6,x, algorithm="giac")
```

output
```
1/640*b^5*(15*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (15*(b*x + a
)^(9/2) - 70*(b*x + a)^(7/2)*a - 128*(b*x + a)^(5/2)*a^2 + 70*(b*x + a)^(3
/2)*a^3 - 15*sqrt(b*x + a)*a^4)/(a^2*b^5*x^5))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^{5/2}}{x^6}\,dx = \frac{3\,(a+b\,x)^{9/2}}{128\,a^2\,x^5} - \frac{3\,a^2\,\sqrt{a+b\,x}}{128\,x^5} - \frac{7\,(a+b\,x)^{7/2}}{64\,a\,x^5} - \frac{(a+b\,x)^{5/2}}{5\,x^5} + \frac{7\,a\,(a+b\,x)^{3/2}}{64\,x^5} + \frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{128\,a^{5/2}}$$

input
```
int((a + b*x)^(5/2)/x^6,x)
```

output
```
(3*(a + b*x)^(9/2))/(128*a^2*x^5) - (3*a^2*(a + b*x)^(1/2))/(128*x^5) - (7
*(a + b*x)^(7/2))/(64*a*x^5) - (a + b*x)^(5/2)/(5*x^5) + (b^5*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) + (7*a*(a + b*x)^(3/2))/(64*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{5/2}}{x^6}\,dx = \frac{-256\sqrt{bx+a}\,a^5 - 672\sqrt{bx+a}\,a^4bx - 496\sqrt{bx+a}\,a^3b^2x^2 - 20\sqrt{bx+a}\,a^2b^3x^3 + 30\sqrt{}}{1280a^3}$$

input
```
int((b*x+a)^(5/2)/x^6,x)
```

output
```
( - 256*sqrt(a + b*x)*a**5 - 672*sqrt(a + b*x)*a**4*b*x - 496*sqrt(a + b*x
)*a**3*b**2*x**2 - 20*sqrt(a + b*x)*a**2*b**3*x**3 + 30*sqrt(a + b*x)*a*b*
*4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**5*x**5 - 15*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**5*x**5)/(1280*a**3*x**5)
```

3.367 $\int x^7(a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 146

$$\int x^7(a+bx)^{9/2}\,dx=-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a^2(a+bx)^{21/2}}{b^8}-\frac{14a(a+bx)^{23/2}}{23b^8}+\frac{2(a+bx)^{25/2}}{25b^8}$$

output
```
-2/11*a^7*(b*x+a)^(11/2)/b^8+14/13*a^6*(b*x+a)^(13/2)/b^8-14/5*a^5*(b*x+a)
^(15/2)/b^8+70/17*a^4*(b*x+a)^(17/2)/b^8-70/19*a^3*(b*x+a)^(19/2)/b^8+2*a^
2*(b*x+a)^(21/2)/b^8-14/23*a*(b*x+a)^(23/2)/b^8+2/25*(b*x+a)^(25/2)/b^8
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int x^7(a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}\left(-2048a^7+11264a^6bx-36608a^5b^2x^2+91520a^4b^3x^3-194480a^3b^4x^4+369512\right.}{26558675b^8}$$

input
```
Integrate[x^7*(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2)*(-2048*a^7 + 11264*a^6*b*x - 36608*a^5*b^2*x^2 + 91520
*a^4*b^3*x^3 - 194480*a^3*b^4*x^4 + 369512*a^2*b^5*x^5 - 646646*a*b^6*x^6
+ 1062347*b^7*x^7))/(26558675*b^8)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a+bx)^{9/2} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^7 (a+bx)^{9/2}}{b^7}+\frac{7 a^6 (a+bx)^{11/2}}{b^7}-\frac{21 a^5 (a+bx)^{13/2}}{b^7}+\frac{35 a^4 (a+bx)^{15/2}}{b^7}-\frac{35 a^3 (a+bx)^{17/2}}{b^7}+\frac{21 a^2 (a+}{b^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2 a^7 (a+bx)^{11/2}}{11 b^8}+\frac{14 a^6 (a+bx)^{13/2}}{13 b^8}-\frac{14 a^5 (a+bx)^{15/2}}{5 b^8}+\frac{70 a^4 (a+bx)^{17/2}}{17 b^8}-\frac{70 a^3 (a+bx)^{19/2}}{19 b^8}+\frac{2 a^2 (a+bx)^{21/2}}{b^8}+\frac{2 (a+bx)^{25/2}}{25 b^8}-\frac{14 a (a+bx)^{23/2}}{23 b^8}$$

input
```
Int[x^7*(a + b*x)^(9/2),x]
```

output
```
(-2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6*(a + b*x)^(13/2))/(13*b^8) -
(14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(a + b*x)^(17/2))/(17*b^8) - (
70*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a + b*x)^(21/2))/b^8 - (14*a*(
a + b*x)^(23/2))/(23*b^8) + (2*(a + b*x)^(25/2))/(25*b^8)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.60

method	result
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}(-1062347b^7x^7+646646a\,b^6x^6-369512a^2b^5x^5+194480a^3b^4x^4-91520a^4b^3x^3+36608a^5b^2x^2-11264a^6b}{26558675b^8}$
pseudoelliptic	$-\frac{2(bx+a)^{\frac{11}{2}}(-1062347b^7x^7+646646a\,b^6x^6-369512a^2b^5x^5+194480a^3b^4x^4-91520a^4b^3x^3+36608a^5b^2x^2-11264a^6b}{26558675b^8}$
orering	$-\frac{2(bx+a)^{\frac{11}{2}}(-1062347b^7x^7+646646a\,b^6x^6-369512a^2b^5x^5+194480a^3b^4x^4-91520a^4b^3x^3+36608a^5b^2x^2-11264a^6b}{26558675b^8}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{25}{2}}}{25}-\frac{14a(bx+a)^{\frac{23}{2}}}{23}+2a^2(bx+a)^{\frac{21}{2}}-\frac{70a^3(bx+a)^{\frac{19}{2}}}{19}+\frac{70a^4(bx+a)^{\frac{17}{2}}}{17}-\frac{14a^5(bx+a)^{\frac{15}{2}}}{5}+\frac{14a^6(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^7(bx+a)}{11}}{b^8}$
default	$\frac{\frac{2(bx+a)^{\frac{25}{2}}}{25}-\frac{14a(bx+a)^{\frac{23}{2}}}{23}+2a^2(bx+a)^{\frac{21}{2}}-\frac{70a^3(bx+a)^{\frac{19}{2}}}{19}+\frac{70a^4(bx+a)^{\frac{17}{2}}}{17}-\frac{14a^5(bx+a)^{\frac{15}{2}}}{5}+\frac{14a^6(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^7(bx+a)}{11}}{b^8}$
trager	$-\frac{2(-1062347b^{12}x^{12}-4665089a\,x^{11}b^{11}-7759752a^2x^{10}b^{10}-5810090a^3b^9x^9-1659515a^4x^8b^8-429a^5x^7b^7+462a^6x^6b}{26558675b^8}$
risch	$-\frac{2(-1062347b^{12}x^{12}-4665089a\,x^{11}b^{11}-7759752a^2x^{10}b^{10}-5810090a^3b^9x^9-1659515a^4x^8b^8-429a^5x^7b^7+462a^6x^6b}{26558675b^8}$

input
```
int(x^7*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/26558675*(b*x+a)^(11/2)*(-1062347*b^7*x^7+646646*a*b^6*x^6-369512*a^2*b
^5*x^5+194480*a^3*b^4*x^4-91520*a^4*b^3*x^3+36608*a^5*b^2*x^2-11264*a^6*b*
x+2048*a^7)/b^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int x^7(a+bx)^{9/2}\,dx = \frac{2\,(1062347\,b^{12}x^{12}+4665089\,ab^{11}x^{11}+7759752\,a^2b^{10}x^{10}+5810090\,a^3b^9x^9+1659515\,a^4b^8x^8 -$$

input
```
integrate(x^7*(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/26558675*(1062347*b^12*x^12 + 4665089*a*b^11*x^11 + 7759752*a^2*b^10*x^1
0 + 5810090*a^3*b^9*x^9 + 1659515*a^4*b^8*x^8 + 429*a^5*b^7*x^7 - 462*a^6*
b^6*x^6 + 504*a^7*b^5*x^5 - 560*a^8*b^4*x^4 + 640*a^9*b^3*x^3 - 768*a^10*b
^2*x^2 + 1024*a^11*b*x - 2048*a^12)*sqrt(b*x + a)/b^8
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.91

$$\int x^7(a+bx)^{9/2}\,dx = \begin{cases} -\frac{4096a^{12}\sqrt{a+bx}}{26558675b^8} + \frac{2048a^{11}x\sqrt{a+bx}}{26558675b^7} - \frac{1536a^{10}x^2\sqrt{a+bx}}{26558675b^6} + \frac{256a^9x^3\sqrt{a+bx}}{5311735b^5} - \frac{224a^8x^4\sqrt{a+bx}}{5311735b^4} + \frac{1008a^7x^5\sqrt{a+bx}}{26558675b^3} \\ \frac{a^{\frac{9}{2}}x^8}{8} \end{cases}$$

input
```
integrate(x**7*(b*x+a)**(9/2),x)
```

output
```
Piecewise((-4096*a**12*sqrt(a + b*x)/(26558675*b**8) + 2048*a**11*x*sqrt(a
 + b*x)/(26558675*b**7) - 1536*a**10*x**2*sqrt(a + b*x)/(26558675*b**6) +
256*a**9*x**3*sqrt(a + b*x)/(5311735*b**5) - 224*a**8*x**4*sqrt(a + b*x)/(
5311735*b**4) + 1008*a**7*x**5*sqrt(a + b*x)/(26558675*b**3) - 84*a**6*x**
6*sqrt(a + b*x)/(2414425*b**2) + 6*a**5*x**7*sqrt(a + b*x)/(185725*b) + 46
42*a**4*x**8*sqrt(a + b*x)/37145 + 956*a**3*b*x**9*sqrt(a + b*x)/2185 + 33
6*a**2*b**2*x**10*sqrt(a + b*x)/575 + 202*a*b**3*x**11*sqrt(a + b*x)/575 +
 2*b**4*x**12*sqrt(a + b*x)/25, Ne(b, 0)), (a**(9/2)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int x^7(a+bx)^{9/2}\,dx = \frac{2\,(bx+a)^{\frac{25}{2}}}{25\,b^8} - \frac{14\,(bx+a)^{\frac{23}{2}}a}{23\,b^8} + \frac{2\,(bx+a)^{\frac{21}{2}}a^2}{b^8} - \frac{70\,(bx+a)^{\frac{19}{2}}a^3}{19\,b^8} + \frac{70\,(bx+a)^{\frac{17}{2}}a^4}{17\,b^8} - \frac{14\,(bx+a)^{\frac{15}{2}}a^5}{5\,b^8} + \frac{14\,(bx+a)^{\frac{13}{2}}a^6}{13\,b^8} - \frac{2\,(bx+a)^{\frac{11}{2}}a^7}{11\,b^8}$$

input
```
integrate(x^7*(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/25*(b*x + a)^(25/2)/b^8 - 14/23*(b*x + a)^(23/2)*a/b^8 + 2*(b*x + a)^(21
/2)*a^2/b^8 - 70/19*(b*x + a)^(19/2)*a^3/b^8 + 70/17*(b*x + a)^(17/2)*a^4/
b^8 - 14/5*(b*x + a)^(15/2)*a^5/b^8 + 14/13*(b*x + a)^(13/2)*a^6/b^8 - 2/1
1*(b*x + a)^(11/2)*a^7/b^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(116) = 232.

Time = 0.12 (sec) , antiderivative size = 781, normalized size of antiderivative = 5.35

$$\int x^7(a+bx)^{9/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/1673196525*(260015*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 122
85*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2
)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(
b*x + a)*a^7)*a^5/b^7 + 76475*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15
/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850
*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)
*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^4/b^7 + 72
450*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)
^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4
- 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x
 + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a
^3/b^7 + 17250*(46189*(b*x + a)^(21/2) - 510510*(b*x + a)^(19/2)*a + 25675
65*(b*x + a)^(17/2)*a^2 - 7759752*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a
)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^
6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(
b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^10)*a^2/b^7 + 4125*(88179*(b*x
 + a)^(23/2) - 1062347*(b*x + a)^(21/2)*a + 5870865*(b*x + a)^(19/2)*a^2 -
 19684665*(b*x + a)^(17/2)*a^3 + 44618574*(b*x + a)^(15/2)*a^4 - 72076158*
(b*x + a)^(13/2)*a^5 + 85180914*(b*x + a)^(11/2)*a^6 - 74364290*(b*x + a)^
(9/2)*a^7 + 47805615*(b*x + a)^(7/2)*a^8 - 22309287*(b*x + a)^(5/2)*a^9...
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int x^7(a+bx)^{9/2}\,dx=\frac{2\,{\left(a+b\,x\right)}^{25/2}}{25\,b^8}-\frac{2\,a^7\,{\left(a+b\,x\right)}^{11/2}}{11\,b^8}+\frac{14\,a^6\,{\left(a+b\,x\right)}^{13/2}}{13\,b^8}-\frac{14\,a^5\,{\left(a+b\,x\right)}^{15/2}}{5\,b^8}+\frac{70\,a^4\,{\left(a+b\,x\right)}^{17/2}}{17\,b^8}-\frac{70\,a^3\,{\left(a+b\,x\right)}^{19/2}}{19\,b^8}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{21/2}}{b^8}-\frac{14\,a\,{\left(a+b\,x\right)}^{23/2}}{23\,b^8}$$

input
```
int(x^7*(a + b*x)^(9/2),x)
```

output
```
(2*(a + b*x)^(25/2))/(25*b^8) - (2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^
6*(a + b*x)^(13/2))/(13*b^8) - (14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4
*(a + b*x)^(17/2))/(17*b^8) - (70*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*
(a + b*x)^(21/2))/b^8 - (14*a*(a + b*x)^(23/2))/(23*b^8)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.96

$$\int x^7(a+bx)^{9/2}\,dx = \frac{2\sqrt{bx+a}\,(1062347b^{12}x^{12}+4665089a\,b^{11}x^{11}+7759752a^2b^{10}x^{10}+5810090a^3b^9x^9+1659515a$$

input
```
int(x^7*(b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2048*a**12 + 1024*a**11*b*x - 768*a**10*b**2*x**2 + 6
40*a**9*b**3*x**3 - 560*a**8*b**4*x**4 + 504*a**7*b**5*x**5 - 462*a**6*b**
6*x**6 + 429*a**5*b**7*x**7 + 1659515*a**4*b**8*x**8 + 5810090*a**3*b**9*x
**9 + 7759752*a**2*b**10*x**10 + 4665089*a*b**11*x**11 + 1062347*b**12*x**
12))/(26558675*b**8)
```

3.368 $\int x^6(a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 127

$$\int x^6(a+bx)^{9/2}\,dx=\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}+\frac{2(a+bx)^{23/2}}{23b^7}$$

output
```
2/11*a^6*(b*x+a)^(11/2)/b^7-12/13*a^5*(b*x+a)^(13/2)/b^7+2*a^4*(b*x+a)^(15
/2)/b^7-40/17*a^3*(b*x+a)^(17/2)/b^7+30/19*a^2*(b*x+a)^(19/2)/b^7-4/7*a*(b
*x+a)^(21/2)/b^7+2/23*(b*x+a)^(23/2)/b^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int x^6(a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}\left(1024a^6-5632a^5bx+18304a^4b^2x^2-45760a^3b^3x^3+97240a^2b^4x^4-184756ab^5\right.}{7436429b^7}$$

input
```
Integrate[x^6*(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2)*(1024*a^6 - 5632*a^5*b*x + 18304*a^4*b^2*x^2 - 45760*a
^3*b^3*x^3 + 97240*a^2*b^4*x^4 - 184756*a*b^5*x^5 + 323323*b^6*x^6))/(7436
429*b^7)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a+bx)^{9/2}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/2}}{b^6}+\frac{15a^2(a+bx)^{17/2}}{b^6}+\frac{(a+bx)^{21/2}}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}$$

input
```
Int[x^6*(a + b*x)^(9/2),x]
```

output
```
(2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5*(a + b*x)^(13/2))/(13*b^7) + (
2*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b*x)^(17/2))/(17*b^7) + (30*a^2
*(a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^(21/2))/(7*b^7) + (2*(a + b*x
)^(23/2))/(23*b^7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

method	result
gosper	$\frac{2(bx+a)^{\frac{11}{2}}(323323b^6x^6-184756a\,x^5b^5+97240a^2x^4b^4-45760a^3x^3b^3+18304a^4x^2b^2-5632a^5xb+1024a^6)}{7436429b^7}$
pseudoelliptic	$\frac{2(bx+a)^{\frac{11}{2}}(323323b^6x^6-184756a\,x^5b^5+97240a^2x^4b^4-45760a^3x^3b^3+18304a^4x^2b^2-5632a^5xb+1024a^6)}{7436429b^7}$
orering	$\frac{2(bx+a)^{\frac{11}{2}}(323323b^6x^6-184756a\,x^5b^5+97240a^2x^4b^4-45760a^3x^3b^3+18304a^4x^2b^2-5632a^5xb+1024a^6)}{7436429b^7}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{23}{2}}}{23}-\frac{4a(bx+a)^{\frac{21}{2}}}{7}+\frac{30a^2(bx+a)^{\frac{19}{2}}}{19}-\frac{40a^3(bx+a)^{\frac{17}{2}}}{17}+2a^4(bx+a)^{\frac{15}{2}}-\frac{12a^5(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^6(bx+a)^{\frac{11}{2}}}{11}}{b^7}$
default	$\frac{\frac{2(bx+a)^{\frac{23}{2}}}{23}-\frac{4a(bx+a)^{\frac{21}{2}}}{7}+\frac{30a^2(bx+a)^{\frac{19}{2}}}{19}-\frac{40a^3(bx+a)^{\frac{17}{2}}}{17}+2a^4(bx+a)^{\frac{15}{2}}-\frac{12a^5(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^6(bx+a)^{\frac{11}{2}}}{11}}{b^7}$
trager	$\frac{2(323323b^{11}x^{11}+1431859a\,x^{10}b^{10}+2406690x^9a^2b^9+1826110a^3x^8b^8+530959a^4b^7x^7+231a^5b^6x^6-252a^6b^5x^5+280a^7}{7436429b^7}$
risch	$\frac{2(323323b^{11}x^{11}+1431859a\,x^{10}b^{10}+2406690x^9a^2b^9+1826110a^3x^8b^8+530959a^4b^7x^7+231a^5b^6x^6-252a^6b^5x^5+280a^7}{7436429b^7}$

input
```
int(x^6*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/7436429*(b*x+a)^(11/2)*(323323*b^6*x^6-184756*a*b^5*x^5+97240*a^2*b^4*x^
4-45760*a^3*b^3*x^3+18304*a^4*b^2*x^2-5632*a^5*b*x+1024*a^6)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02

$$\int x^6(a+bx)^{9/2}\,dx = \frac{2\,(323323\,b^{11}x^{11}+1431859\,ab^{10}x^{10}+2406690\,a^2b^9x^9+1826110\,a^3b^8x^8+530959\,a^4b^7x^7+23}{7436}$$

input
```
integrate(x^6*(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/7436429*(323323*b^11*x^11 + 1431859*a*b^10*x^10 + 2406690*a^2*b^9*x^9 +
1826110*a^3*b^8*x^8 + 530959*a^4*b^7*x^7 + 231*a^5*b^6*x^6 - 252*a^6*b^5*x
^5 + 280*a^7*b^4*x^4 - 320*a^8*b^3*x^3 + 384*a^9*b^2*x^2 - 512*a^10*b*x +
1024*a^11)*sqrt(b*x + a)/b^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(122) = 244.

Time = 1.00 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.02

$$\int x^6(a+bx)^{9/2}\,dx = \begin{cases} \frac{2048a^{11}\sqrt{a+bx}}{7436429b^7} - \frac{1024a^{10}x\sqrt{a+bx}}{7436429b^6} + \frac{768a^9x^2\sqrt{a+bx}}{7436429b^5} - \frac{640a^8x^3\sqrt{a+bx}}{7436429b^4} + \frac{80a^7x^4\sqrt{a+bx}}{1062347b^3} - \frac{72a^6x^5\sqrt{a+bx}}{1062347b^2} + \frac{6a^5}{9} \\ \frac{a^{\frac{9}{2}}x^7}{7} \end{cases}$$

input
```
integrate(x**6*(b*x+a)**(9/2),x)
```

output
```
Piecewise((2048*a**11*sqrt(a + b*x)/(7436429*b**7) - 1024*a**10*x*sqrt(a +
 b*x)/(7436429*b**6) + 768*a**9*x**2*sqrt(a + b*x)/(7436429*b**5) - 640*a*
*8*x**3*sqrt(a + b*x)/(7436429*b**4) + 80*a**7*x**4*sqrt(a + b*x)/(1062347
*b**3) - 72*a**6*x**5*sqrt(a + b*x)/(1062347*b**2) + 6*a**5*x**6*sqrt(a +
b*x)/(96577*b) + 7426*a**4*x**7*sqrt(a + b*x)/52003 + 25540*a**3*b*x**8*sq
rt(a + b*x)/52003 + 1980*a**2*b**2*x**9*sqrt(a + b*x)/3059 + 62*a*b**3*x**
10*sqrt(a + b*x)/161 + 2*b**4*x**11*sqrt(a + b*x)/23, Ne(b, 0)), (a**(9/2)
*x**7/7, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x^6(a+bx)^{9/2}\,dx=\frac{2\,(bx+a)^{\frac{23}{2}}}{23\,b^7}-\frac{4\,(bx+a)^{\frac{21}{2}}\,a}{7\,b^7}+\frac{30\,(bx+a)^{\frac{19}{2}}\,a^2}{19\,b^7}-\frac{40\,(bx+a)^{\frac{17}{2}}\,a^3}{17\,b^7}+\frac{2\,(bx+a)^{\frac{15}{2}}\,a^4}{b^7}-\frac{12\,(bx+a)^{\frac{13}{2}}\,a^5}{13\,b^7}+\frac{2\,(bx+a)^{\frac{11}{2}}\,a^6}{11\,b^7}$$

input
```
integrate(x^6*(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/23*(b*x + a)^(23/2)/b^7 - 4/7*(b*x + a)^(21/2)*a/b^7 + 30/19*(b*x + a)^(
19/2)*a^2/b^7 - 40/17*(b*x + a)^(17/2)*a^3/b^7 + 2*(b*x + a)^(15/2)*a^4/b^
7 - 12/13*(b*x + a)^(13/2)*a^5/b^7 + 2/11*(b*x + a)^(11/2)*a^6/b^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(101) = 202.

Time = 0.13 (sec) , antiderivative size = 709, normalized size of antiderivative = 5.58

$$\int x^6(a+bx)^{9/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^6*(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/66927861*(22287*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(
b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 -
 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^5/b^6 + 52003*(429*(
b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 2
5025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/
2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^4/b^6 + 611
8*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13
/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 87516
0*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2
)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^3/b^6 + 2898*(12155*(b*x + a)^(19/2) -
 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x +
a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5
 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x
 + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a^2/b^6 + 345*(46189*(b*x + a)
^(21/2) - 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 77597
52*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x +
a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^
7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqr
t(b*x + a)*a^10)*a/b^6 + 33*(88179*(b*x + a)^(23/2) - 1062347*(b*x + a)^(2
1/2)*a + 5870865*(b*x + a)^(19/2)*a^2 - 19684665*(b*x + a)^(17/2)*a^3 +...
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x^6(a+bx)^{9/2}\,dx = \frac{2\,(a+b\,x)^{23/2}}{23\,b^7}+\frac{2\,a^6\,(a+b\,x)^{11/2}}{11\,b^7}-\frac{12\,a^5\,(a+b\,x)^{13/2}}{13\,b^7}+\frac{2\,a^4\,(a+b\,x)^{15/2}}{b^7}-\frac{40\,a^3\,(a+b\,x)^{17/2}}{17\,b^7}+\frac{30\,a^2\,(a+b\,x)^{19/2}}{19\,b^7}-\frac{4\,a\,(a+b\,x)^{21/2}}{7\,b^7}$$

input
```
int(x^6*(a + b*x)^(9/2),x)
```

output
```
(2*(a + b*x)^(23/2))/(23*b^7) + (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^
5*(a + b*x)^(13/2))/(13*b^7) + (2*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a +
 b*x)^(17/2))/(17*b^7) + (30*a^2*(a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*
x)^(21/2))/(7*b^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int x^6(a+bx)^{9/2}\,dx = \frac{2\sqrt{bx+a}\,(323323b^{11}x^{11}+1431859a\,b^{10}x^{10}+2406690a^2b^9x^9+1826110a^3b^8x^8+530959a^4b^7x$$

743642

input
```
int(x^6*(b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*(1024*a**11 - 512*a**10*b*x + 384*a**9*b**2*x**2 - 320*a*
*8*b**3*x**3 + 280*a**7*b**4*x**4 - 252*a**6*b**5*x**5 + 231*a**5*b**6*x**
6 + 530959*a**4*b**7*x**7 + 1826110*a**3*b**8*x**8 + 2406690*a**2*b**9*x**
9 + 1431859*a*b**10*x**10 + 323323*b**11*x**11))/(7436429*b**7)
```

3.369 $\int x^5(a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\int x^5(a+bx)^{9/2}\,dx=-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}+\frac{2(a+bx)^{21/2}}{21b^6}$$

output
```
-2/11*a^5*(b*x+a)^(11/2)/b^6+10/13*a^4*(b*x+a)^(13/2)/b^6-4/3*a^3*(b*x+a)^
(15/2)/b^6+20/17*a^2*(b*x+a)^(17/2)/b^6-10/19*a*(b*x+a)^(19/2)/b^6+2/21*(b
*x+a)^(21/2)/b^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int x^5(a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}\left(-256a^5+1408a^4bx-4576a^3b^2x^2+11440a^2b^3x^3-24310ab^4x^4+46189b^5x^5\right)}{969969b^6}$$

input
```
Integrate[x^5*(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2)*(-256*a^5 + 1408*a^4*b*x - 4576*a^3*b^2*x^2 + 11440*a^
2*b^3*x^3 - 24310*a*b^4*x^4 + 46189*b^5*x^5))/(969969*b^6)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+bx)^{9/2}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}+\frac{(a+bx)^{19/2}}{b^5}-\frac{5a(a+bx)^{17/2}}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

input
```
Int[x^5*(a + b*x)^(9/2),x]
```

output
```
(-2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) -
(4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (1
0*a*(a + b*x)^(19/2))/(19*b^6) + (2*(a + b*x)^(21/2))/(21*b^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

method	result
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-46189b^{5}x^{5}+24310a\,b^{4}x^{4}-11440a^{2}b^{3}x^{3}+4576a^{3}b^{2}x^{2}-1408a^{4}bx+256a^{5}\right)}{969969b^{6}}$
pseudoelliptic	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-46189b^{5}x^{5}+24310a\,b^{4}x^{4}-11440a^{2}b^{3}x^{3}+4576a^{3}b^{2}x^{2}-1408a^{4}bx+256a^{5}\right)}{969969b^{6}}$
orering	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-46189b^{5}x^{5}+24310a\,b^{4}x^{4}-11440a^{2}b^{3}x^{3}+4576a^{3}b^{2}x^{2}-1408a^{4}bx+256a^{5}\right)}{969969b^{6}}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{21}{2}}}{21}-\frac{10a(bx+a)^{\frac{19}{2}}}{19}+\frac{20a^{2}(bx+a)^{\frac{17}{2}}}{17}-\frac{4a^{3}(bx+a)^{\frac{15}{2}}}{3}+\frac{10a^{4}(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^{5}(bx+a)^{\frac{11}{2}}}{11}}{b^{6}}$
default	$\frac{\frac{2(bx+a)^{\frac{21}{2}}}{21}-\frac{10a(bx+a)^{\frac{19}{2}}}{19}+\frac{20a^{2}(bx+a)^{\frac{17}{2}}}{17}-\frac{4a^{3}(bx+a)^{\frac{15}{2}}}{3}+\frac{10a^{4}(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^{5}(bx+a)^{\frac{11}{2}}}{11}}{b^{6}}$
trager	$-\frac{2\left(-46189b^{10}x^{10}-206635a\,b^{9}x^{9}-351780a^{2}b^{8}x^{8}-271414a^{3}b^{7}x^{7}-80773a^{4}b^{6}x^{6}-63a^{5}b^{5}x^{5}+70a^{6}b^{4}x^{4}-80a^{7}b^{3}x^{3}+\right.}{969969b^{6}}$
risch	$-\frac{2\left(-46189b^{10}x^{10}-206635a\,b^{9}x^{9}-351780a^{2}b^{8}x^{8}-271414a^{3}b^{7}x^{7}-80773a^{4}b^{6}x^{6}-63a^{5}b^{5}x^{5}+70a^{6}b^{4}x^{4}-80a^{7}b^{3}x^{3}+\right.}{969969b^{6}}$

input
```
int(x^5*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/969969*(b*x+a)^(11/2)*(-46189*b^5*x^5+24310*a*b^4*x^4-11440*a^2*b^3*x^3
+4576*a^3*b^2*x^2-1408*a^4*b*x+256*a^5)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int x^5(a+bx)^{9/2}\,dx = \frac{2\,(46189\,b^{10}x^{10}+206635\,ab^9x^9+351780\,a^2b^8x^8+271414\,a^3b^7x^7+80773\,a^4b^6x^6+63\,a^5b^5x^5}{969969\,b^6}$$

input
```
integrate(x^5*(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/969969*(46189*b^10*x^10 + 206635*a*b^9*x^9 + 351780*a^2*b^8*x^8 + 271414
*a^3*b^7*x^7 + 80773*a^4*b^6*x^6 + 63*a^5*b^5*x^5 - 70*a^6*b^4*x^4 + 80*a^
7*b^3*x^3 - 96*a^8*b^2*x^2 + 128*a^9*b*x - 256*a^10)*sqrt(b*x + a)/b^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(105) = 210.

Time = 0.90 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.14

$$\int x^5(a+bx)^{9/2}\,dx = \begin{cases} -\frac{512a^{10}\sqrt{a+bx}}{969969b^6}+\frac{256a^9x\sqrt{a+bx}}{969969b^5}-\frac{64a^8x^2\sqrt{a+bx}}{323323b^4}+\frac{160a^7x^3\sqrt{a+bx}}{969969b^3}-\frac{20a^6x^4\sqrt{a+bx}}{138567b^2}+\frac{6a^5x^5\sqrt{a+bx}}{46189b}+\frac{2098a^4}{1} \\ \frac{a^{\frac{9}{2}}x^6}{6} \end{cases}$$

input
```
integrate(x**5*(b*x+a)**(9/2),x)
```

output
```
Piecewise((-512*a**10*sqrt(a + b*x)/(969969*b**6) + 256*a**9*x*sqrt(a + b*
x)/(969969*b**5) - 64*a**8*x**2*sqrt(a + b*x)/(323323*b**4) + 160*a**7*x**
3*sqrt(a + b*x)/(969969*b**3) - 20*a**6*x**4*sqrt(a + b*x)/(138567*b**2) +
 6*a**5*x**5*sqrt(a + b*x)/(46189*b) + 2098*a**4*x**6*sqrt(a + b*x)/12597
+ 3796*a**3*b*x**7*sqrt(a + b*x)/6783 + 1640*a**2*b**2*x**8*sqrt(a + b*x)/
2261 + 170*a*b**3*x**9*sqrt(a + b*x)/399 + 2*b**4*x**10*sqrt(a + b*x)/21,
Ne(b, 0)), (a**(9/2)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int x^5(a+bx)^{9/2}\,dx = \frac{2\,(bx+a)^{\frac{21}{2}}}{21\,b^6} - \frac{10\,(bx+a)^{\frac{19}{2}}\,a}{19\,b^6} + \frac{20\,(bx+a)^{\frac{17}{2}}\,a^2}{17\,b^6} - \frac{4\,(bx+a)^{\frac{15}{2}}\,a^3}{3\,b^6} + \frac{10\,(bx+a)^{\frac{13}{2}}\,a^4}{13\,b^6} - \frac{2\,(bx+a)^{\frac{11}{2}}\,a^5}{11\,b^6}$$

input
```
integrate(x^5*(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/21*(b*x + a)^(21/2)/b^6 - 10/19*(b*x + a)^(19/2)*a/b^6 + 20/17*(b*x + a)
^(17/2)*a^2/b^6 - 4/3*(b*x + a)^(15/2)*a^3/b^6 + 10/13*(b*x + a)^(13/2)*a^
4/b^6 - 2/11*(b*x + a)^(11/2)*a^5/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(86) = 172$.

Time = 0.13 (sec) , antiderivative size = 637, normalized size of antiderivative = 5.79

$$\int x^5(a+bx)^{9/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/2909907*(4199*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x +
a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*s
qrt(b*x + a)*a^5)*a^5/b^5 + 4845*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(1
1/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x +
 a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^4/b^5
 + 4522*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^
(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027
*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)
*a^3/b^5 + 266*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*
(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2
)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(
b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^2/b^5 + 63*(12155*(b*x +
a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492
260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a
)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 +
692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a/b^5 + 3*(46189*(b
*x + a)^(21/2) - 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2
- 7759752*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*
(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(
7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + ...
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int x^5(a+bx)^{9/2}\,dx = \frac{2\,(a+b\,x)^{21/2}}{21\,b^6} - \frac{2\,a^5\,(a+b\,x)^{11/2}}{11\,b^6} + \frac{10\,a^4\,(a+b\,x)^{13/2}}{13\,b^6} - \frac{4\,a^3\,(a+b\,x)^{15/2}}{3\,b^6} + \frac{20\,a^2\,(a+b\,x)^{17/2}}{17\,b^6} - \frac{10\,a\,(a+b\,x)^{19/2}}{19\,b^6}$$

input
```
int(x^5*(a + b*x)^(9/2),x)
```

output
```
(2*(a + b*x)^(21/2))/(21*b^6) - (2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^
4*(a + b*x)^(13/2))/(13*b^6) - (4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*
(a + b*x)^(17/2))/(17*b^6) - (10*a*(a + b*x)^(19/2))/(19*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int x^5(a+bx)^{9/2}\,dx = \frac{2\sqrt{bx+a}\,(46189b^{10}x^{10}+206635a\,b^9x^9+351780a^2b^8x^8+271414a^3b^7x^7+80773a^4b^6x^6+63a}{969969b^6}$$

input
```
int(x^5*(b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 256*a**10 + 128*a**9*b*x - 96*a**8*b**2*x**2 + 80*a**
7*b**3*x**3 - 70*a**6*b**4*x**4 + 63*a**5*b**5*x**5 + 80773*a**4*b**6*x**6
 + 271414*a**3*b**7*x**7 + 351780*a**2*b**8*x**8 + 206635*a*b**9*x**9 + 46
189*b**10*x**10))/(969969*b**6)
```

3.370 $\int x^4(a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int x^4(a+bx)^{9/2}\,dx=\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19/2}}{19b^5}$$

output
```
2/11*a^4*(b*x+a)^(11/2)/b^5-8/13*a^3*(b*x+a)^(13/2)/b^5+4/5*a^2*(b*x+a)^(1
5/2)/b^5-8/17*a*(b*x+a)^(17/2)/b^5+2/19*(b*x+a)^(19/2)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int x^4(a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}\left(128a^4-704a^3bx+2288a^2b^2x^2-5720ab^3x^3+12155b^4x^4\right)}{230945b^5}$$

input
```
Integrate[x^4*(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2)*(128*a^4 - 704*a^3*b*x + 2288*a^2*b^2*x^2 - 5720*a*b^3
*x^3 + 12155*b^4*x^4))/(230945*b^5)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^{9/2}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}+\frac{(a+bx)^{17/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

input
```
Int[x^4*(a + b*x)^(9/2),x]
```

output
```
(2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*(a + b*x)^(13/2))/(13*b^5) + (4
*a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b*x)^(17/2))/(17*b^5) + (2*(a +
 b*x)^(19/2))/(19*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

method	result
gosper	$\frac{2(bx+a)^{\frac{11}{2}}(12155b^4x^4-5720a\,x^3b^3+2288a^2b^2x^2-704a^3bx+128a^4)}{230945b^5}$
pseudoelliptic	$\frac{2(bx+a)^{\frac{11}{2}}(12155b^4x^4-5720a\,x^3b^3+2288a^2b^2x^2-704a^3bx+128a^4)}{230945b^5}$
orering	$\frac{2(bx+a)^{\frac{11}{2}}(12155b^4x^4-5720a\,x^3b^3+2288a^2b^2x^2-704a^3bx+128a^4)}{230945b^5}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{19}{2}}}{19}-\frac{8a(bx+a)^{\frac{17}{2}}}{17}+\frac{4a^2(bx+a)^{\frac{15}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^4(bx+a)^{\frac{11}{2}}}{11}}{b^5}$
default	$\frac{\frac{2(bx+a)^{\frac{19}{2}}}{19}-\frac{8a(bx+a)^{\frac{17}{2}}}{17}+\frac{4a^2(bx+a)^{\frac{15}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^4(bx+a)^{\frac{11}{2}}}{11}}{b^5}$
trager	$\frac{2(12155b^9x^9+55055a\,x^8b^8+95238a^2x^7b^7+75086x^6a^3b^6+23063a^4x^5b^5+35a^5b^4x^4-40a^6b^3x^3+48a^7b^2x^2-64a^8bx+1}{230945b^5}$
risch	$\frac{2(12155b^9x^9+55055a\,x^8b^8+95238a^2x^7b^7+75086x^6a^3b^6+23063a^4x^5b^5+35a^5b^4x^4-40a^6b^3x^3+48a^7b^2x^2-64a^8bx+1}{230945b^5}$

input
```
int(x^4*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/230945*(b*x+a)^(11/2)*(12155*b^4*x^4-5720*a*b^3*x^3+2288*a^2*b^2*x^2-704
*a^3*b*x+128*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int x^4(a+bx)^{9/2}\,dx = \frac{2\,(12155\,b^9x^9+55055\,ab^8x^8+95238\,a^2b^7x^7+75086\,a^3b^6x^6+23063\,a^4b^5x^5+35\,a^5b^4x^4-40\,\ldots}{230945\,b^5}$$

input
```
integrate(x^4*(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/230945*(12155*b^9*x^9 + 55055*a*b^8*x^8 + 95238*a^2*b^7*x^7 + 75086*a^3*
b^6*x^6 + 23063*a^4*b^5*x^5 + 35*a^5*b^4*x^4 - 40*a^6*b^3*x^3 + 48*a^7*b^2
*x^2 - 64*a^8*b*x + 128*a^9)*sqrt(b*x + a)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.84 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.33

$$\int x^4(a+bx)^{9/2}\,dx = \begin{cases} \frac{256a^9\sqrt{a+bx}}{230945b^5} - \frac{128a^8x\sqrt{a+bx}}{230945b^4} + \frac{96a^7x^2\sqrt{a+bx}}{230945b^3} - \frac{16a^6x^3\sqrt{a+bx}}{46189b^2} + \frac{14a^5x^4\sqrt{a+bx}}{46189b} + \frac{46126a^4x^5\sqrt{a+bx}}{230945} + \frac{13652a\ldots}{\ldots} \\ \frac{a^{\frac{9}{2}}x^5}{5} \end{cases}$$

input
```
integrate(x**4*(b*x+a)**(9/2),x)
```

output
```
Piecewise((256*a**9*sqrt(a + b*x)/(230945*b**5) - 128*a**8*x*sqrt(a + b*x)
/(230945*b**4) + 96*a**7*x**2*sqrt(a + b*x)/(230945*b**3) - 16*a**6*x**3*s
qrt(a + b*x)/(46189*b**2) + 14*a**5*x**4*sqrt(a + b*x)/(46189*b) + 46126*a
**4*x**5*sqrt(a + b*x)/230945 + 13652*a**3*b*x**6*sqrt(a + b*x)/20995 + 13
32*a**2*b**2*x**7*sqrt(a + b*x)/1615 + 154*a*b**3*x**8*sqrt(a + b*x)/323 +
 2*b**4*x**9*sqrt(a + b*x)/19, Ne(b, 0)), (a**(9/2)*x**5/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int x^4(a+bx)^{9/2}\,dx = \frac{2\,(bx+a)^{\frac{19}{2}}}{19\,b^5} - \frac{8\,(bx+a)^{\frac{17}{2}}a}{17\,b^5} + \frac{4\,(bx+a)^{\frac{15}{2}}a^2}{5\,b^5} - \frac{8\,(bx+a)^{\frac{13}{2}}a^3}{13\,b^5} + \frac{2\,(bx+a)^{\frac{11}{2}}a^4}{11\,b^5}$$

input
```
integrate(x^4*(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/19*(b*x + a)^(19/2)/b^5 - 8/17*(b*x + a)^(17/2)*a/b^5 + 4/5*(b*x + a)^(1
5/2)*a^2/b^5 - 8/13*(b*x + a)^(13/2)*a^3/b^5 + 2/11*(b*x + a)^(11/2)*a^4/b
^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(71) = 142.

Time = 0.13 (sec) , antiderivative size = 565, normalized size of antiderivative = 6.21

$$\int x^4(a+bx)^{9/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/14549535*(46189*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
 a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^5/b^4 +
 104975*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)
*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x
+ a)*a^5)*a^4/b^4 + 48450*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a
+ 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/
2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^3/b^4 + 2261
0*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)
*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x
+ a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^2/b
^4 + 665*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x +
 a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4
- 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x +
a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a/b^4 + 63*(12155*(b*x + a)^(19/2
) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x
 + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*
a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(
b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int x^4(a+bx)^{9/2}\,dx = \frac{2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^5}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{11/2}}{11\,b^5}-\frac{8\,a^3\,{\left(a+b\,x\right)}^{13/2}}{13\,b^5}+\frac{4\,a^2\,{\left(a+b\,x\right)}^{15/2}}{5\,b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{17/2}}{17\,b^5}$$

input
```
int(x^4*(a + b*x)^(9/2),x)
```

output
```
(2*(a + b*x)^(19/2))/(19*b^5) + (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3
*(a + b*x)^(13/2))/(13*b^5) + (4*a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a +
 b*x)^(17/2))/(17*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int x^4(a+bx)^{9/2}\,dx = \frac{2\sqrt{bx+a}\,(12155b^9x^9+55055a\,b^8x^8+95238a^2b^7x^7+75086a^3b^6x^6+23063a^4b^5x^5+35a^5b^4x^4}{230945b^5}$$

input
```
int(x^4*(b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**9 - 64*a**8*b*x + 48*a**7*b**2*x**2 - 40*a**6*b**
3*x**3 + 35*a**5*b**4*x**4 + 23063*a**4*b**5*x**5 + 75086*a**3*b**6*x**6 +
 95238*a**2*b**7*x**7 + 55055*a*b**8*x**8 + 12155*b**9*x**9))/(230945*b**5
)
```

3.371 $\int x^3(a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3(a+bx)^{9/2}\,dx=-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}$$

output
```
-2/11*a^3*(b*x+a)^(11/2)/b^4+6/13*a^2*(b*x+a)^(13/2)/b^4-2/5*a*(b*x+a)^(15
/2)/b^4+2/17*(b*x+a)^(17/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^3(a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}(-16a^3+88a^2bx-286ab^2x^2+715b^3x^3)}{12155b^4}$$

input
```
Integrate[x^3*(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2)*(-16*a^3 + 88*a^2*b*x - 286*a*b^2*x^2 + 715*b^3*x^3))/
(12155*b^4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{9/2}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

input
```
Int[x^3*(a + b*x)^(9/2),x]
```

output
```
(-2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*(a + b*x)^(13/2))/(13*b^4) - (
2*a*(a + b*x)^(15/2))/(5*b^4) + (2*(a + b*x)^(17/2))/(17*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	si
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-715b^3x^3+286a\,b^2x^2-88a^2bx+16a^3\right)}{12155b^4}$	4
pseudoelliptic	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-715b^3x^3+286a\,b^2x^2-88a^2bx+16a^3\right)}{12155b^4}$	4
orering	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-715b^3x^3+286a\,b^2x^2-88a^2bx+16a^3\right)}{12155b^4}$	4
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{17}{2}}}{17}-\frac{2a(bx+a)^{\frac{15}{2}}}{5}+\frac{6a^2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^3(bx+a)^{\frac{11}{2}}}{11}}{b^4}$	5
default	$\frac{\frac{2(bx+a)^{\frac{17}{2}}}{17}-\frac{2a(bx+a)^{\frac{15}{2}}}{5}+\frac{6a^2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^3(bx+a)^{\frac{11}{2}}}{11}}{b^4}$	5
trager	$-\frac{2\left(-715b^8x^8-3289a\,x^7b^7-5808a^2x^6b^6-4714a^3x^5b^5-1515a^4x^4b^4-5a^5b^3x^3+6a^6x^2b^2-8a^7xb+16a^8\right)\sqrt{bx+a}}{12155b^4}$	9
risch	$-\frac{2\left(-715b^8x^8-3289a\,x^7b^7-5808a^2x^6b^6-4714a^3x^5b^5-1515a^4x^4b^4-5a^5b^3x^3+6a^6x^2b^2-8a^7xb+16a^8\right)\sqrt{bx+a}}{12155b^4}$	9

input
```
int(x^3*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/12155*(b*x+a)^(11/2)*(-715*b^3*x^3+286*a*b^2*x^2-88*a^2*b*x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int x^3(a+bx)^{9/2}\,dx = \frac{2\left(715\,b^8x^8+3289\,ab^7x^7+5808\,a^2b^6x^6+4714\,a^3b^5x^5+1515\,a^4b^4x^4+5\,a^5b^3x^3-6\,a^6b^2x^2+8\right.}{12155\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/12155*(715*b^8*x^8 + 3289*a*b^7*x^7 + 5808*a^2*b^6*x^6 + 4714*a^3*b^5*x^
5 + 1515*a^4*b^4*x^4 + 5*a^5*b^3*x^3 - 6*a^6*b^2*x^2 + 8*a^7*b*x - 16*a^8)
*sqrt(b*x + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(68) = 136$.

Time = 0.74 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.64

$$\int x^3(a+bx)^{9/2}\,dx = \begin{cases} -\frac{32a^8\sqrt{a+bx}}{12155b^4} + \frac{16a^7x\sqrt{a+bx}}{12155b^3} - \frac{12a^6x^2\sqrt{a+bx}}{12155b^2} + \frac{2a^5x^3\sqrt{a+bx}}{2431b} + \frac{606a^4x^4\sqrt{a+bx}}{2431} + \frac{9428a^3bx^5\sqrt{a+bx}}{12155} + \frac{1056a^2}{} \\ \frac{a^{\frac{9}{2}}x^4}{4} \end{cases}$$

input
```
integrate(x**3*(b*x+a)**(9/2),x)
```

output
```
Piecewise((-32*a**8*sqrt(a + b*x)/(12155*b**4) + 16*a**7*x*sqrt(a + b*x)/(
12155*b**3) - 12*a**6*x**2*sqrt(a + b*x)/(12155*b**2) + 2*a**5*x**3*sqrt(a
 + b*x)/(2431*b) + 606*a**4*x**4*sqrt(a + b*x)/2431 + 9428*a**3*b*x**5*sqr
t(a + b*x)/12155 + 1056*a**2*b**2*x**6*sqrt(a + b*x)/1105 + 46*a*b**3*x**7
*sqrt(a + b*x)/85 + 2*b**4*x**8*sqrt(a + b*x)/17, Ne(b, 0)), (a**(9/2)*x**
4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{9/2}\,dx = \frac{2\,(bx+a)^{\frac{17}{2}}}{17\,b^4} - \frac{2\,(bx+a)^{\frac{15}{2}}a}{5\,b^4} + \frac{6\,(bx+a)^{\frac{13}{2}}a^2}{13\,b^4} - \frac{2\,(bx+a)^{\frac{11}{2}}a^3}{11\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/17*(b*x + a)^(17/2)/b^4 - 2/5*(b*x + a)^(15/2)*a/b^4 + 6/13*(b*x + a)^(1
3/2)*a^2/b^4 - 2/11*(b*x + a)^(11/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 493, normalized size of antiderivative = 6.85

$$\int x^3(a+bx)^{9/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/765765*(21879*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^5/b^3 + 12155*(35*(b*x + a)^(9/2) - 180
*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 3
15*sqrt(b*x + a)*a^4)*a^4/b^3 + 11050*(63*(b*x + a)^(11/2) - 385*(b*x + a)
^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x
+ a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3/b^3 + 2550*(231*(b*x + a)^(13/
2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(
7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt
(b*x + a)*a^6)*a^2/b^3 + 595*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)
*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x +
 a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 64
35*sqrt(b*x + a)*a^7)*a/b^3 + 7*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(
15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 8508
50*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/
2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{9/2}\,dx = \frac{2\,{\left(a+b\,x\right)}^{17/2}}{17\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a\,{\left(a+b\,x\right)}^{15/2}}{5\,b^4}$$

input
```
int(x^3*(a + b*x)^(9/2),x)
```

output
```
(2*(a + b*x)^(17/2))/(17*b^4) - (2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2
*(a + b*x)^(13/2))/(13*b^4) - (2*a*(a + b*x)^(15/2))/(5*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int x^3(a+bx)^{9/2}\,dx = \frac{2\sqrt{bx+a}\,(715b^8x^8+3289a\,b^7x^7+5808a^2b^6x^6+4714a^3b^5x^5+1515a^4b^4x^4+5a^5b^3x^3-6a^6b^2}{12155b^4}$$

input
```
int(x^3*(b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**8 + 8*a**7*b*x - 6*a**6*b**2*x**2 + 5*a**5*b**3
*x**3 + 1515*a**4*b**4*x**4 + 4714*a**3*b**5*x**5 + 5808*a**2*b**6*x**6 +
3289*a*b**7*x**7 + 715*b**8*x**8))/(12155*b**4)
```

3.372 $\int x^2(a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2(a+bx)^{9/2}\,dx=\frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}$$

output
```
2/11*a^2*(b*x+a)^(11/2)/b^3-4/13*a*(b*x+a)^(13/2)/b^3+2/15*(b*x+a)^(15/2)/
b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}\left(8a^2-44abx+143b^2x^2\right)}{2145b^3}$$

input
```
Integrate[x^2*(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2)*(8*a^2 - 44*a*b*x + 143*b^2*x^2))/(2145*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{9/2}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2(a+bx)^{9/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

input
```
Int[x^2*(a + b*x)^(9/2),x]
```

output
```
(2*a^2*(a + b*x)^(11/2))/(11*b^3) - (4*a*(a + b*x)^(13/2))/(13*b^3) + (2*(
a + b*x)^(15/2))/(15*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2(bx+a)^{\frac{11}{2}}(143b^2x^2-44abx+8a^2)}{2145b^3}$	32
pseudoelliptic	$\frac{2(bx+a)^{\frac{11}{2}}(143b^2x^2-44abx+8a^2)}{2145b^3}$	32
orering	$\frac{2(bx+a)^{\frac{11}{2}}(143b^2x^2-44abx+8a^2)}{2145b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{15}{2}}}{15}-\frac{4a(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^2(bx+a)^{\frac{11}{2}}}{11}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{15}{2}}}{15}-\frac{4a(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^2(bx+a)^{\frac{11}{2}}}{11}}{b^3}$	38
trager	$\frac{2(143b^7x^7+671a\,b^6x^6+1218a^2b^5x^5+1030a^3b^4x^4+355a^4b^3x^3+3a^5b^2x^2-4a^6bx+8a^7)\sqrt{bx+a}}{2145b^3}$	87
risch	$\frac{2(143b^7x^7+671a\,b^6x^6+1218a^2b^5x^5+1030a^3b^4x^4+355a^4b^3x^3+3a^5b^2x^2-4a^6bx+8a^7)\sqrt{bx+a}}{2145b^3}$	87

input
```
int(x^2*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/2145*(b*x+a)^(11/2)*(143*b^2*x^2-44*a*b*x+8*a^2)/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int x^2(a+bx)^{9/2}\,dx = \frac{2\,(143\,b^7x^7+671\,ab^6x^6+1218\,a^2b^5x^5+1030\,a^3b^4x^4+355\,a^4b^3x^3+3\,a^5b^2x^2-4\,a^6bx+8\,a^7)}{2145\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/2145*(143*b^7*x^7 + 671*a*b^6*x^6 + 1218*a^2*b^5*x^5 + 1030*a^3*b^4*x^4
+ 355*a^4*b^3*x^3 + 3*a^5*b^2*x^2 - 4*a^6*b*x + 8*a^7)*sqrt(b*x + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(49) = 98$.

Time = 0.68 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.17

$$\int x^2(a+bx)^{9/2}\,dx = \begin{cases} \frac{16a^7\sqrt{a+bx}}{2145b^3} - \frac{8a^6x\sqrt{a+bx}}{2145b^2} + \frac{2a^5x^2\sqrt{a+bx}}{715b} + \frac{142a^4x^3\sqrt{a+bx}}{429} + \frac{412a^3bx^4\sqrt{a+bx}}{429} + \frac{812a^2b^2x^5\sqrt{a+bx}}{715} + \cdots \\ \frac{a^{\frac{9}{2}}x^3}{3} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**(9/2),x)
```

output
```
Piecewise((16*a**7*sqrt(a + b*x)/(2145*b**3) - 8*a**6*x*sqrt(a + b*x)/(214
5*b**2) + 2*a**5*x**2*sqrt(a + b*x)/(715*b) + 142*a**4*x**3*sqrt(a + b*x)/
429 + 412*a**3*b*x**4*sqrt(a + b*x)/429 + 812*a**2*b**2*x**5*sqrt(a + b*x)
/715 + 122*a*b**3*x**6*sqrt(a + b*x)/195 + 2*b**4*x**7*sqrt(a + b*x)/15, N
e(b, 0)), (a**(9/2)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2(a+bx)^{9/2}\,dx = \frac{2\,(bx+a)^{\frac{15}{2}}}{15\,b^3} - \frac{4\,(bx+a)^{\frac{13}{2}}a}{13\,b^3} + \frac{2\,(bx+a)^{\frac{11}{2}}a^2}{11\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/15*(b*x + a)^(15/2)/b^3 - 4/13*(b*x + a)^(13/2)*a/b^3 + 2/11*(b*x + a)^(
11/2)*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 421, normalized size of antiderivative = 7.94

$$\int x^2(a+bx)^{9/2}\,dx = \frac{2\left(\frac{3003\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^5}{b^2}+\frac{6435\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^4}{b^2}\right.}{}$$

input
```
integrate(x^2*(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/45045*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)
*a^2)*a^5/b^2 + 6435*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x +
 a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^4/b^2 + 1430*(35*(b*x + a)^(9/2) -
 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3
 + 315*sqrt(b*x + a)*a^4)*a^3/b^2 + 650*(63*(b*x + a)^(11/2) - 385*(b*x +
a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*
x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2/b^2 + 75*(231*(b*x + a)^(13/
2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(
7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt
(b*x + a)*a^6)*a/b^2 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a +
 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^
(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*s
qrt(b*x + a)*a^7)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int x^2(a+bx)^{9/2}\,dx = \frac{\frac{2\,(a+b\,x)^{15/2}}{15}-\frac{4\,a\,(a+b\,x)^{13/2}}{13}+\frac{2\,a^2\,(a+b\,x)^{11/2}}{11}}{b^3}$$

input
```
int(x^2*(a + b*x)^(9/2),x)
```

output
```
((2*(a + b*x)^(15/2))/15 - (4*a*(a + b*x)^(13/2))/13 + (2*a^2*(a + b*x)^(1
1/2))/11)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.60

$$\int x^2(a+bx)^{9/2}\,dx = \frac{2\sqrt{bx+a}\,(143b^7x^7+671a\,b^6x^6+1218a^2b^5x^5+1030a^3b^4x^4+355a^4b^3x^3+3a^5b^2x^2-4a^6bx+}{2145b^3}$$

input
```
int(x^2*(b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**7 - 4*a**6*b*x + 3*a**5*b**2*x**2 + 355*a**4*b**3*x
**3 + 1030*a**3*b**4*x**4 + 1218*a**2*b**5*x**5 + 671*a*b**6*x**6 + 143*b*
*7*x**7))/(2145*b**3)
```

3.373 $\int x(a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x(a+bx)^{9/2}\,dx=-\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}$$

output
```
-2/11*a*(b*x+a)^(11/2)/b^2+2/13*(b*x+a)^(13/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x(a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}(-2a+11bx)}{143b^2}$$

input
```
Integrate[x*(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2)*(-2*a + 11*b*x))/(143*b^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{9/2}\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{(a+bx)^{11/2}}{b}-\frac{a(a+bx)^{9/2}}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

input
```
Int[x*(a + b*x)^(9/2),x]
```

output
```
(-2*a*(a + b*x)^(11/2))/(11*b^2) + (2*(a + b*x)^(13/2))/(13*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}(-11bx+2a)}{143b^2}$	21
pseudoelliptic	$-\frac{2(bx+a)^{\frac{11}{2}}(-11bx+2a)}{143b^2}$	21
orering	$-\frac{2(bx+a)^{\frac{11}{2}}(-11bx+2a)}{143b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a(bx+a)^{\frac{11}{2}}}{11}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a(bx+a)^{\frac{11}{2}}}{11}}{b^2}$	26
trager	$-\frac{2(-11b^6x^6-53a\,x^5b^5-100a^2x^4b^4-90a^3x^3b^3-35a^4x^2b^2-a^5xb+2a^6)\sqrt{bx+a}}{143b^2}$	76
risch	$-\frac{2(-11b^6x^6-53a\,x^5b^5-100a^2x^4b^4-90a^3x^3b^3-35a^4x^2b^2-a^5xb+2a^6)\sqrt{bx+a}}{143b^2}$	76

input
```
int(x*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/143*(b*x+a)^(11/2)*(-11*b*x+2*a)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int x(a+bx)^{9/2}\,dx = \frac{2\,(11\,b^6x^6+53\,ab^5x^5+100\,a^2b^4x^4+90\,a^3b^3x^3+35\,a^4b^2x^2+a^5bx-2\,a^6)\sqrt{bx+a}}{143\,b^2}$$

input
```
integrate(x*(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/143*(11*b^6*x^6 + 53*a*b^5*x^5 + 100*a^2*b^4*x^4 + 90*a^3*b^3*x^3 + 35*a
^4*b^2*x^2 + a^5*b*x - 2*a^6)*sqrt(b*x + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(31) = 62$.

Time = 0.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.29

$$\int x(a+bx)^{9/2}\,dx = \begin{cases} -\frac{4a^6\sqrt{a+bx}}{143b^2} + \frac{2a^5x\sqrt{a+bx}}{143b} + \frac{70a^4x^2\sqrt{a+bx}}{143} + \frac{180a^3bx^3\sqrt{a+bx}}{143} + \frac{200a^2b^2x^4\sqrt{a+bx}}{143} + \frac{106ab^3x^5\sqrt{a+bx}}{143} + 2b^4x \\ \frac{a^{\frac{9}{2}}x^2}{2} \end{cases}$$

input
```
integrate(x*(b*x+a)**(9/2),x)
```

output
```
Piecewise((-4*a**6*sqrt(a + b*x)/(143*b**2) + 2*a**5*x*sqrt(a + b*x)/(143*
b) + 70*a**4*x**2*sqrt(a + b*x)/143 + 180*a**3*b*x**3*sqrt(a + b*x)/143 +
200*a**2*b**2*x**4*sqrt(a + b*x)/143 + 106*a*b**3*x**5*sqrt(a + b*x)/143 +
 2*b**4*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(9/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(a+bx)^{9/2}\,dx = \frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^2} - \frac{2\,(bx+a)^{\frac{11}{2}}a}{11\,b^2}$$

input
```
integrate(x*(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/13*(b*x + a)^(13/2)/b^2 - 2/11*(b*x + a)^(11/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 347, normalized size of antiderivative = 10.21

$$\int x(a+bx)^{9/2}\,dx = \frac{2\left(\frac{3003\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^5}{b}+\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a^4}{b}+\frac{2574\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a\right.}{}\right.}{}$$

input
```
integrate(x*(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/9009*(3003*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^5/b + 3003*(3*(b*x +
a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^4/b + 2574*(5*(b
*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*
x + a)*a^3)*a^3/b + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*
(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2
/b + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)
*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x
+ a)*a^5)*a/b + 3*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(
b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 -
 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x(a+bx)^{9/2}\,dx = -\frac{26\,a\,(a+b\,x)^{11/2}-22\,(a+b\,x)^{13/2}}{143\,b^2}$$

input
```
int(x*(a + b*x)^(9/2),x)
```

output
```
-(26*a*(a + b*x)^(11/2) - 22*(a + b*x)^(13/2))/(143*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int x(a+bx)^{9/2}\,dx = \frac{2\sqrt{bx+a}\,(11b^6x^6+53a\,b^5x^5+100a^2b^4x^4+90a^3b^3x^3+35a^4b^2x^2+a^5bx-2a^6)}{143b^2}$$

input
```
int(x*(b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**6 + a**5*b*x + 35*a**4*b**2*x**2 + 90*a**3*b**3*
x**3 + 100*a**2*b**4*x**4 + 53*a*b**5*x**5 + 11*b**6*x**6))/(143*b**2)
```

3.374 $\int (a+bx)^{9/2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}}{11b}$$

output
```
2/11*(b*x+a)^(11/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}}{11b}$$

input
```
Integrate[(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2))/(11*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{9/2}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{2(a+bx)^{11/2}}{11b}$$

input
```
Int[(a + b*x)^(9/2),x]
```

output
```
(2*(a + b*x)^(11/2))/(11*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
default	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
pseudoelliptic	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
orering	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
trager	$\frac{2\left(b^{5} x^{5}+5 a\,b^{4} x^{4}+10 a^{2} b^{3} x^{3}+10 a^{3} b^{2} x^{2}+5 a^{4} b x+a^{5}\right) \sqrt{b x+a}}{11 b}$	62
risch	$\frac{2\left(b^{5} x^{5}+5 a\,b^{4} x^{4}+10 a^{2} b^{3} x^{3}+10 a^{3} b^{2} x^{2}+5 a^{4} b x+a^{5}\right) \sqrt{b x+a}}{11 b}$	62

input
```
int((b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*(b*x+a)^(11/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.81

$$\int (a+bx)^{9/2}\,dx = \frac{2\,(b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5)\sqrt{bx+a}}{11\,b}$$

input
```
integrate((b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5)*sqrt(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{9/2}\,dx=\frac{2(a+bx)^{\frac{11}{2}}}{11b}$$

input
```
integrate((b*x+a)**(9/2),x)
```

output
```
2*(a + b*x)**(11/2)/(11*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{9/2}\,dx=\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b}$$

input
```
integrate((b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
2/11*(b*x + a)^(11/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(12) = 24.

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 14.31

$$\int (a+bx)^{9/2}\,dx=\frac{2\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4\right.}{}$$

input
```
integrate((b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/693*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a
^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 + 1155*((b*x + a)
^(3/2) - 3*sqrt(b*x + a)*a)*a^4 + 462*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a^3 + 198*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2 + 11*(35*(b*x
 + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{9/2}\,dx=\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b}$$

input
```
int((a + b*x)^(9/2),x)
```

output
```
(2*(a + b*x)^(11/2))/(11*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.75

$$\int (a+bx)^{9/2}\,dx=\frac{2\sqrt{bx+a}\,(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}{11b}$$

input
```
int((b*x+a)^(9/2),x)
```

output
```
(2*sqrt(a + b*x)*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**
3 + 5*a*b**4*x**4 + b**5*x**5))/(11*b)
```

3.375 $\int \frac{(a+bx)^{9/2}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{(a+bx)^{9/2}}{x} \, dx = 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} - 2a^{9/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*a^4*(b*x+a)^(1/2)+2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x
+a)^(7/2)+2/9*(b*x+a)^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{9/2}}{x} \, dx = \frac{2}{315}\sqrt{a+bx}\left(563a^4 + 506a^3bx + 408a^2b^2x^2 + 185ab^3x^3 + 35b^4x^4\right) - 2a^{9/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(9/2)/x,x]
```

output
```
(2*Sqrt[a + b*x]*(563*a^4 + 506*a^3*b*x + 408*a^2*b^2*x^2 + 185*a*b^3*x^3
+ 35*b^4*x^4))/315 - 2*a^(9/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {60, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x} \, dx$$

$$\Big\downarrow \text{60}$$

$$a\int \frac{(a+bx)^{7/2}}{x}dx+\frac{2}{9}(a+bx)^{9/2}$$

$$\Big\downarrow \text{60}$$

$$a\left(a\int \frac{(a+bx)^{5/2}}{x}dx+\frac{2}{7}(a+bx)^{7/2}\right)+\frac{2}{9}(a+bx)^{9/2}$$

$$\Big\downarrow \text{60}$$

$$a\left(a\left(a\int \frac{(a+bx)^{3/2}}{x}dx+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)+\frac{2}{9}(a+bx)^{9/2}$$

$$\Big\downarrow \text{60}$$

$$a\left(a\left(a\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)+\frac{2}{9}(a+bx)^{9/2}$$

$$\Big\downarrow \text{60}$$

$$a\left(a\left(a\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)+\frac{2}{9}(a+bx)^{9/2}$$

$$\Big\downarrow \text{73}$$

$$a\left(a\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)+\frac{2}{9}(a+bx)^{9/2}$$

$$\Big\downarrow \text{ 221}$$

$$a\left(a\left(a\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)+\frac{2}{9}(a+bx)^{9/2}$$

input
```
Int[(a + b*x)^(9/2)/x,x]
```

output
```
(2*(a + b*x)^(9/2))/9 + a*((2*(a + b*x)^(7/2))/7 + a*((2*(a + b*x)^(5/2))/
5 + a*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]))))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-2a^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(35b^4x^4+185a\,x^3b^3+408a^2b^2x^2+506a^3bx+563a^4)}{315}$
derivativedivides	$2a^4\sqrt{bx+a}+\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}+\frac{2a(bx+a)^{\frac{7}{2}}}{7}+\frac{2(bx+a)^{\frac{9}{2}}}{9}-2a^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$
default	$2a^4\sqrt{bx+a}+\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}+\frac{2a(bx+a)^{\frac{7}{2}}}{7}+\frac{2(bx+a)^{\frac{9}{2}}}{9}-2a^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$

input
```
int((b*x+a)^(9/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/315*(b*x+a)^(1/2)*(35*b^4*x^4+
185*a*b^3*x^3+408*a^2*b^2*x^2+506*a^3*b*x+563*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = \left[a^{\frac{9}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + \frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a},\ 2\sqrt{-a}a^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right) + \frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a}\right]$$

input
```
integrate((b*x+a)^(9/2)/x,x, algorithm="fricas")
```

output
```
[a^(9/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/315*(35*b^4*x^4
+ 185*a*b^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a),
2*sqrt(-a)*a^4*arctan(sqrt(-a)/sqrt(b*x + a)) + 2/315*(35*b^4*x^4 + 185*a*
b^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a)]
```

Sympy [A] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = \frac{1126a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{315} + a^{\frac{9}{2}}\log\left(\frac{bx}{a}\right) - 2a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right) + \frac{1012a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{315} + \frac{272a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105} + \frac{74a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{63} + \frac{2\sqrt{a}b^4x^4\sqrt{1+\frac{bx}{a}}}{9}$$

input
```
integrate((b*x+a)**(9/2)/x,x)
```

output
```
1126*a**(9/2)*sqrt(1 + b*x/a)/315 + a**(9/2)*log(b*x/a) - 2*a**(9/2)*log(s
qrt(1 + b*x/a) + 1) + 1012*a**(7/2)*b*x*sqrt(1 + b*x/a)/315 + 272*a**(5/2)
*b**2*x**2*sqrt(1 + b*x/a)/105 + 74*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/63
+ 2*sqrt(a)*b**4*x**4*sqrt(1 + b*x/a)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = a^{\frac{9}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2}{9}(bx+a)^{\frac{9}{2}} + \frac{2}{7}(bx+a)^{\frac{7}{2}}a + \frac{2}{5}(bx+a)^{\frac{5}{2}}a^2 + \frac{2}{3}(bx+a)^{\frac{3}{2}}a^3 + 2\sqrt{bx+a}a^4$$

input
```
integrate((b*x+a)^(9/2)/x,x, algorithm="maxima")
```

output
```
a^(9/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/9*(b*
x + a)^(9/2) + 2/7*(b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x
+ a)^(3/2)*a^3 + 2*sqrt(b*x + a)*a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = \frac{2\,a^5 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2}{9}\,(bx+a)^{\frac{9}{2}} + \frac{2}{7}\,(bx+a)^{\frac{7}{2}}a + \frac{2}{5}\,(bx+a)^{\frac{5}{2}}a^2 + \frac{2}{3}\,(bx+a)^{\frac{3}{2}}a^3 + 2\,\sqrt{bx+a}a^4$$

input
```
integrate((b*x+a)^(9/2)/x,x, algorithm="giac")
```

output
```
2*a^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x + a)^(9/2) + 2/7*
(b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x + a)^(3/2)*a^3 + 2*
sqrt(b*x + a)*a^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = \frac{2\,a\,(a+b\,x)^{7/2}}{7} + \frac{2\,(a+b\,x)^{9/2}}{9} + 2\,a^4\,\sqrt{a+b\,x} + \frac{2\,a^3\,(a+b\,x)^{3/2}}{3} + \frac{2\,a^2\,(a+b\,x)^{5/2}}{5} + a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int((a + b*x)^(9/2)/x,x)
```

output
```
(2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 + 2*a^4*(a + b*x)^(1/2) +
(2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5 + a^(9/2)*atan(((a +
 b*x)^(1/2)*1i)/a^(1/2))*2i
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = \frac{1126\sqrt{bx+a}\,a^4}{315} + \frac{1012\sqrt{bx+a}\,a^3bx}{315} + \frac{272\sqrt{bx+a}\,a^2b^2x^2}{105} + \frac{74\sqrt{bx+a}\,a\,b^3x^3}{63} + \frac{2\sqrt{bx+a}\,b^4x^4}{9} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^4 - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a^4$$

input
```
int((b*x+a)^(9/2)/x,x)
```

output
```
(1126*sqrt(a + b*x)*a**4 + 1012*sqrt(a + b*x)*a**3*b*x + 816*sqrt(a + b*x)
*a**2*b**2*x**2 + 370*sqrt(a + b*x)*a*b**3*x**3 + 70*sqrt(a + b*x)*b**4*x*
*4 + 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**4 - 315*sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))*a**4)/315
```

3.376 $\int \frac{(a+bx)^{9/2}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = 8a^3b\sqrt{a+bx} - \frac{a^4\sqrt{a+bx}}{x} + 2a^2b(a+bx)^{3/2} + \frac{4}{5}ab(a+bx)^{5/2} + \frac{2}{7}b(a+bx)^{7/2} - 9a^{7/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
8*a^3*b*(b*x+a)^(1/2)-a^4*(b*x+a)^(1/2)/x+2*a^2*b*(b*x+a)^(3/2)+4/5*a*b*(b
*x+a)^(5/2)+2/7*b*(b*x+a)^(7/2)-9*a^(7/2)*b*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = \frac{\sqrt{a+bx}(-35a^4+388a^3bx+156a^2b^2x^2+58ab^3x^3+10b^4x^4)}{35x} - 9a^{7/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(9/2)/x^2,x]
```

output
```
(Sqrt[a + b*x]*(-35*a^4 + 388*a^3*b*x + 156*a^2*b^2*x^2 + 58*a*b^3*x^3 + 1
0*b^4*x^4))/(35*x) - 9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {51, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x^2} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{2}b\int \frac{(a+bx)^{7/2}}{x}dx-\frac{(a+bx)^{9/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{2}b\left(a\int \frac{(a+bx)^{5/2}}{x}dx+\frac{2}{7}(a+bx)^{7/2}\right)-\frac{(a+bx)^{9/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{2}b\left(a\left(a\int \frac{(a+bx)^{3/2}}{x}dx+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)-\frac{(a+bx)^{9/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{2}b\left(a\left(a\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)-\frac{(a+bx)^{9/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{2}b\left(a\left(a\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)-\frac{(a+bx)^{9/2}}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{9}{2}b\left(a\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)-\frac{(a+bx)^{9/2}}{x}$$

$$\Big\downarrow \ 221$$

$$\frac{9}{2}b\left(a\left(a\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2}{7}(a+bx)^{7/2}\right)-\frac{(a+bx)^{9/2}}{x}$$

input
```
Int[(a + b*x)^(9/2)/x^2,x]
```

output
```
-((a + b*x)^(9/2)/x) + (9*b*((2*(a + b*x)^(7/2))/7 + a*((2*(a + b*x)^(5/2)
)/5 + a*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]])))))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{a^4\sqrt{bx+a}}{x}+\frac{b\left(\frac{4(bx+a)^{\frac{7}{2}}}{7}+\frac{8a(bx+a)^{\frac{5}{2}}}{5}+4a^2(bx+a)^{\frac{3}{2}}+16a^3\sqrt{bx+a}-18a^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{2}$
pseudoelliptic	$-\frac{9\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^4bx-\frac{2\sqrt{bx+a}\left(\sqrt{a}\,b^4x^4+\frac{29a^{\frac{3}{2}}b^3x^3}{5}+\frac{78a^{\frac{5}{2}}b^2x^2}{5}+\frac{194a^{\frac{7}{2}}bx}{5}-\frac{7a^{\frac{9}{2}}}{2}\right)}{63}\right)}{\sqrt{a}\,x}$
derivativedivides	$2b\left(\frac{(bx+a)^{\frac{7}{2}}}{7}+\frac{2a(bx+a)^{\frac{5}{2}}}{5}+a^2(bx+a)^{\frac{3}{2}}+4a^3\sqrt{bx+a}-a^4\left(\frac{\sqrt{bx+a}}{2bx}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$
default	$2b\left(\frac{(bx+a)^{\frac{7}{2}}}{7}+\frac{2a(bx+a)^{\frac{5}{2}}}{5}+a^2(bx+a)^{\frac{3}{2}}+4a^3\sqrt{bx+a}-a^4\left(\frac{\sqrt{bx+a}}{2bx}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$

input
```
int((b*x+a)^(9/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^4*(b*x+a)^(1/2)/x+1/2*b*(4/7*(b*x+a)^(7/2)+8/5*a*(b*x+a)^(5/2)+4*a^2*(b
*x+a)^(3/2)+16*a^3*(b*x+a)^(1/2)-18*a^(7/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = \left[\frac{315\,a^{\frac{7}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35}{70\,x}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^2,x, algorithm="fricas")
```

output
```
[1/70*(315*a^(7/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(1
0*b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*
x + a))/x, 1/35*(315*sqrt(-a)*a^3*b*x*arctan(sqrt(-a)/sqrt(b*x + a)) + (10
*b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x
 + a))/x]
```

Sympy [A] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^2}\,dx = &-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{388a^{\frac{7}{2}}b\sqrt{1+\frac{bx}{a}}}{35}\\&+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx}{a}\right)}{2}-9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)\\&+\frac{156a^{\frac{5}{2}}b^2x\sqrt{1+\frac{bx}{a}}}{35}+\frac{58a^{\frac{3}{2}}b^3x^2\sqrt{1+\frac{bx}{a}}}{35}+\frac{2\sqrt{a}b^4x^3\sqrt{1+\frac{bx}{a}}}{7}\end{aligned}$$

input
```
integrate((b*x+a)**(9/2)/x**2,x)
```

output
```
-a**(9/2)*sqrt(1 + b*x/a)/x + 388*a**(7/2)*b*sqrt(1 + b*x/a)/35 + 9*a**(7/
2)*b*log(b*x/a)/2 - 9*a**(7/2)*b*log(sqrt(1 + b*x/a) + 1) + 156*a**(5/2)*b
**2*x*sqrt(1 + b*x/a)/35 + 58*a**(3/2)*b**3*x**2*sqrt(1 + b*x/a)/35 + 2*sq
rt(a)*b**4*x**3*sqrt(1 + b*x/a)/7
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = \frac{9}{2}\,a^{\frac{7}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{7}\,(bx+a)^{\frac{7}{2}}b + \frac{4}{5}\,(bx+a)^{\frac{5}{2}}ab+2\,(bx+a)^{\frac{3}{2}}a^2b+8\,\sqrt{bx+a}a^3b-\frac{\sqrt{bx+a}a^4}{x}$$

input
```
integrate((b*x+a)^(9/2)/x^2,x, algorithm="maxima")
```

output
```
9/2*a^(7/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2
/7*(b*x + a)^(7/2)*b + 4/5*(b*x + a)^(5/2)*a*b + 2*(b*x + a)^(3/2)*a^2*b +
 8*sqrt(b*x + a)*a^3*b - sqrt(b*x + a)*a^4/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = \frac{1}{35}\left(\frac{315\,a^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\,(bx+a)^{\frac{7}{2}}+28\,(bx+a)^{\frac{5}{2}}a+70\,(bx+a)^{\frac{3}{2}}a^2+280\,\sqrt{}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^2,x, algorithm="giac")
```

output
```
1/35*(315*a^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 10*(b*x + a)^(7/2)
 + 28*(b*x + a)^(5/2)*a + 70*(b*x + a)^(3/2)*a^2 + 280*sqrt(b*x + a)*a^3 -
 35*sqrt(b*x + a)*a^4/(b*x))*b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = \frac{2\,b\,(a+b\,x)^{7/2}}{7} - \frac{a^4\,\sqrt{a+b\,x}}{x} + \frac{4\,a\,b\,(a+b\,x)^{5/2}}{5} + 8\,a^3\,b\,\sqrt{a+b\,x} + 2\,a^2\,b\,(a+b\,x)^{3/2} + a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}$$

input
```
int((a + b*x)^(9/2)/x^2,x)
```

output
```
(2*b*(a + b*x)^(7/2))/7 - (a^4*(a + b*x)^(1/2))/x + a^(7/2)*b*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*9i + (4*a*b*(a + b*x)^(5/2))/5 + 8*a^3*b*(a + b*x)^
(1/2) + 2*a^2*b*(a + b*x)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = \frac{-70\sqrt{bx+a}\,a^4 + 776\sqrt{bx+a}\,a^3bx + 312\sqrt{bx+a}\,a^2b^2x^2 + 116\sqrt{bx+a}\,a\,b^3x^3 + 20\sqrt{}}{70x}$$

input
```
int((b*x+a)^(9/2)/x^2,x)
```

output
```
( - 70*sqrt(a + b*x)*a**4 + 776*sqrt(a + b*x)*a**3*b*x + 312*sqrt(a + b*x)
*a**2*b**2*x**2 + 116*sqrt(a + b*x)*a*b**3*x**3 + 20*sqrt(a + b*x)*b**4*x*
*4 + 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**3*b*x - 315*sqrt(a)*log(s
qrt(a + b*x) + sqrt(a))*a**3*b*x)/(70*x)
```

3.377 $\int \frac{(a+bx)^{9/2}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 116

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = 12a^2b^2\sqrt{a+bx}-\frac{a^4\sqrt{a+bx}}{2x^2}-\frac{17a^3b\sqrt{a+bx}}{4x}+2ab^2(a+bx)^{3/2}+\frac{2}{5}b^2(a+bx)^{5/2}-\frac{63}{4}a^{5/2}b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
12*a^2*b^2*(b*x+a)^(1/2)-1/2*a^4*(b*x+a)^(1/2)/x^2-17/4*a^3*b*(b*x+a)^(1/2
)/x+2*a*b^2*(b*x+a)^(3/2)+2/5*b^2*(b*x+a)^(5/2)-63/4*a^(5/2)*b^2*arctanh((
b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = \frac{\sqrt{a+bx}(-10a^4-85a^3bx+288a^2b^2x^2+56ab^3x^3+8b^4x^4)}{20x^2}-\frac{63}{4}a^{5/2}b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(9/2)/x^3,x]
```

output
```
(Sqrt[a + b*x]*(-10*a^4 - 85*a^3*b*x + 288*a^2*b^2*x^2 + 56*a*b^3*x^3 + 8*
b^4*x^4))/(20*x^2) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {51, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{4}b\int \frac{(a+bx)^{7/2}}{x^2}dx-\frac{(a+bx)^{9/2}}{2x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{4}b\left(\frac{7}{2}b\int \frac{(a+bx)^{5/2}}{x}dx-\frac{(a+bx)^{7/2}}{x}\right)-\frac{(a+bx)^{9/2}}{2x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{4}b\left(\frac{7}{2}b\left(a\int \frac{(a+bx)^{3/2}}{x}dx+\frac{2}{5}(a+bx)^{5/2}\right)-\frac{(a+bx)^{7/2}}{x}\right)-\frac{(a+bx)^{9/2}}{2x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)-\frac{(a+bx)^{7/2}}{x}\right)-\frac{(a+bx)^{9/2}}{2x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)-\frac{(a+bx)^{7/2}}{x}\right)-\frac{(a+bx)^{9/2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)-\frac{(a+bx)^{7/2}}{x}\right)-\frac{(a+bx)^{9/2}}{2x^2}$$

$$\Big\downarrow \ 221$$

$$\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)-\frac{(a+bx)^{7/2}}{x}\right)-\frac{(a+bx)^{9/2}}{2x^2}$$

input
```
Int[(a + b*x)^(9/2)/x^3,x]
```

output
```
-1/2*(a + b*x)^(9/2)/x^2 + (9*b*(-((a + b*x)^(7/2)/x) + (7*b*((2*(a + b*x)
^(5/2))/5 + a*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]]))))/2))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{a^{3}\sqrt{bx+a}\,(17bx+2a)}{4x^{2}}+\frac{b^{2}\left(\frac{16(bx+a)^{\frac{5}{2}}}{5}+16a(bx+a)^{\frac{3}{2}}+96a^{2}\sqrt{bx+a}-126a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{8}$
pseudoelliptic	$-\frac{63\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{3}b^{2}x^{2}-\frac{8\sqrt{bx+a}\left(\sqrt{a}\,b^{4}x^{4}+7a^{\frac{3}{2}}b^{3}x^{3}+36a^{\frac{5}{2}}b^{2}x^{2}-\frac{85a^{\frac{7}{2}}bx}{8}-\frac{5a^{\frac{9}{2}}}{4}\right)}{315}\right)}{4\sqrt{a}\,x^{2}}$
derivativedivides	$2b^{2}\left(\frac{(bx+a)^{\frac{5}{2}}}{5}+a(bx+a)^{\frac{3}{2}}+6a^{2}\sqrt{bx+a}-a^{3}\left(\frac{\frac{17(bx+a)^{\frac{3}{2}}}{8}-\frac{15a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{63\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$
default	$2b^{2}\left(\frac{(bx+a)^{\frac{5}{2}}}{5}+a(bx+a)^{\frac{3}{2}}+6a^{2}\sqrt{bx+a}-a^{3}\left(\frac{\frac{17(bx+a)^{\frac{3}{2}}}{8}-\frac{15a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{63\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$

input
```
int((b*x+a)^(9/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3*(b*x+a)^(1/2)*(17*b*x+2*a)/x^2+1/8*b^2*(16/5*(b*x+a)^(5/2)+16*a*(
b*x+a)^(3/2)+96*a^2*(b*x+a)^(1/2)-126*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = \left[\frac{315\,a^{\frac{5}{2}}b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10}{40\,x^2}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^3,x, algorithm="fricas")
```

output
```
[1/40*(315*a^(5/2)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt(
b*x + a))/x^2, 1/20*(315*sqrt(-a)*a^2*b^2*x^2*arctan(sqrt(-a)/sqrt(b*x + a
)) + (8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sq
rt(b*x + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = -\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4} - \frac{a^5}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{19a^4\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{203a^3b^{\frac{3}{2}}}{20\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{86a^2b^{\frac{5}{2}}\sqrt{x}}{5\sqrt{\frac{a}{bx}+1}} + \frac{16ab^{\frac{7}{2}}x^{\frac{3}{2}}}{5\sqrt{\frac{a}{bx}+1}} + \frac{2b^{\frac{9}{2}}x^{\frac{5}{2}}}{5\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((b*x+a)**(9/2)/x**3,x)
```

output
```
-63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**5/(2*sqrt(b)*x**
(5/2)*sqrt(a/(b*x) + 1)) - 19*a**4*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1))
+ 203*a**3*b**(3/2)/(20*sqrt(x)*sqrt(a/(b*x) + 1)) + 86*a**2*b**(5/2)*sqrt
(x)/(5*sqrt(a/(b*x) + 1)) + 16*a*b**(7/2)*x**(3/2)/(5*sqrt(a/(b*x) + 1)) +
 2*b**(9/2)*x**(5/2)/(5*sqrt(a/(b*x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = \frac{63}{8}\,a^{\frac{5}{2}}b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}b^2 + 2\,(bx+a)^{\frac{3}{2}}ab^2+12\,\sqrt{bx+a}a^2b^2-\frac{17\,(bx+a)^{\frac{3}{2}}a^3b^2-15\,\sqrt{bx+a}a^4b^2}{4\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

input
```
integrate((b*x+a)^(9/2)/x^3,x, algorithm="maxima")
```

output
```
63/8*a^(5/2)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))
+ 2/5*(b*x + a)^(5/2)*b^2 + 2*(b*x + a)^(3/2)*a*b^2 + 12*sqrt(b*x + a)*a^2
*b^2 - 1/4*(17*(b*x + a)^(3/2)*a^3*b^2 - 15*sqrt(b*x + a)*a^4*b^2)/((b*x +
 a)^2 - 2*(b*x + a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = \frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,(bx+a)^{\frac{5}{2}}b^3+40\,(bx+a)^{\frac{3}{2}}ab^3+240\,\sqrt{bx+a}a^2b^3-\frac{5\left(17\,(bx+a)\right.}{}}{20\,b}$$

input
```
integrate((b*x+a)^(9/2)/x^3,x, algorithm="giac")
```

output
```
1/20*(315*a^3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*(b*x + a)^(5
/2)*b^3 + 40*(b*x + a)^(3/2)*a*b^3 + 240*sqrt(b*x + a)*a^2*b^3 - 5*(17*(b*
x + a)^(3/2)*a^3*b^3 - 15*sqrt(b*x + a)*a^4*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = \frac{2\,b^2\,(a+b\,x)^{5/2}}{5} + \frac{\frac{15\,a^4\,b^2\,\sqrt{a+b\,x}}{4} - \frac{17\,a^3\,b^2\,(a+b\,x)^{3/2}}{4}}{(a+b\,x)^2 - 2\,a\,(a+b\,x) + a^2} + 12\,a^2\,b^2\,\sqrt{a+b\,x} + 2\,a\,b^2\,(a+b\,x)^{3/2} + \frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{4}$$

input

```
int((a + b*x)^(9/2)/x^3,x)
```

output

```
(2*b^2*(a + b*x)^(5/2))/5 + ((15*a^4*b^2*(a + b*x)^(1/2))/4 - (17*a^3*b^2*
(a + b*x)^(3/2))/4)/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + 12*a^2*b^2*(a +
b*x)^(1/2) + (a^(5/2)*b^2*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/4 + 2*a*
b^2*(a + b*x)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = \frac{-20\sqrt{bx+a}\,a^4 - 170\sqrt{bx+a}\,a^3bx + 576\sqrt{bx+a}\,a^2b^2x^2 + 112\sqrt{bx+a}\,ab^3x^3 + 16\sqrt{}}{40x}$$

input

```
int((b*x+a)^(9/2)/x^3,x)
```

output

```
( - 20*sqrt(a + b*x)*a**4 - 170*sqrt(a + b*x)*a**3*b*x + 576*sqrt(a + b*x)
*a**2*b**2*x**2 + 112*sqrt(a + b*x)*a*b**3*x**3 + 16*sqrt(a + b*x)*b**4*x*
*4 + 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2*b**2*x**2 - 315*sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*a**2*b**2*x**2)/(40*x**2)
```

3.378 $\int \frac{(a+bx)^{9/2}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 121

$$\int \frac{(a+bx)^{9/2}}{x^4} dx = 8ab^3\sqrt{a+bx} - \frac{a^4\sqrt{a+bx}}{3x^3} - \frac{25a^3b\sqrt{a+bx}}{12x^2} - \frac{55a^2b^2\sqrt{a+bx}}{8x} + \frac{2}{3}b^3(a+bx)^{3/2} - \frac{105}{8}a^{3/2}b^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
8*a*b^3*(b*x+a)^(1/2)-1/3*a^4*(b*x+a)^(1/2)/x^3-25/12*a^3*b*(b*x+a)^(1/2)/
x^2-55/8*a^2*b^2*(b*x+a)^(1/2)/x+2/3*b^3*(b*x+a)^(3/2)-105/8*a^(3/2)*b^3*a
rctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^{9/2}}{x^4} dx = \frac{1}{24}\left(\frac{\sqrt{a+bx}(-8a^4 - 50a^3bx - 165a^2b^2x^2 + 208ab^3x^3 + 16b^4x^4)}{x^3} - 315a^{3/2}b^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[(a + b*x)^(9/2)/x^4,x]
```

output
```
((Sqrt[a + b*x]*(-8*a^4 - 50*a^3*b*x - 165*a^2*b^2*x^2 + 208*a*b^3*x^3 + 1
6*b^4*x^4))/x^3 - 315*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/24
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {51, 51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x^4}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{3}{2}b\int \frac{(a+bx)^{7/2}}{x^3}dx-\frac{(a+bx)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{3}{2}b\left(\frac{7}{4}b\int \frac{(a+bx)^{5/2}}{x^2}dx-\frac{(a+bx)^{7/2}}{2x^2}\right)-\frac{(a+bx)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\int \frac{(a+bx)^{3/2}}{x}dx-\frac{(a+bx)^{5/2}}{x}\right)-\frac{(a+bx)^{7/2}}{2x^2}\right)-\frac{(a+bx)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)-\frac{(a+bx)^{7/2}}{2x^2}\right)-\frac{(a+bx)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)-\frac{(a+bx)^{7/2}}{2x^2}\right)-\frac{(a+bx)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\left(\frac{2a\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)-\frac{(a+bx)^{7/2}}{2x^2}\right)-\frac{(a+bx)^{9/2}}{3x^3}$$

$\downarrow$ 221

$$\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)-\frac{(a+bx)^{7/2}}{2x^2}\right)-\frac{(a+bx)^{9/2}}{3x^3}$$

input
```
Int[(a + b*x)^(9/2)/x^4,x]
```

output
```
-1/3*(a + b*x)^(9/2)/x^3 + (3*b*(-1/2*(a + b*x)^(7/2)/x^2 + (7*b*(-((a + b
*x)^(5/2)/x) + (5*b*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a
]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/2))/4))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{a^{2} \sqrt{b x +a}\, \left(165 b^{2} x^{2}+50 a b x +8 a^{2}\right)}{24 x^{3}}+\frac{b^{3} \left(\frac{32 \left(b x +a \right)^{\frac{3}{2}}}{3}+128 a \sqrt{b x +a}-210 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)\right)}{16}$
pseudoelliptic	$-\frac{105 \left(\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{2} b^{3} x^{3}-\frac{16 \sqrt{b x +a}\, \left(\sqrt{a}\, b^{4} x^{4}+13 a^{\frac{3}{2}} b^{3} x^{3}-\frac{165 a^{\frac{5}{2}} b^{2} x^{2}}{16}-\frac{25 a^{\frac{7}{2}} b x}{8}-\frac{a^{\frac{9}{2}}}{2}\right)}{315}\right)}{8 \sqrt{a}\, x^{3}}$
derivativedivides	$2 b^{3} \left(\frac{\left(b x +a \right)^{\frac{3}{2}}}{3}+4 a \sqrt{b x +a}-a^{2} \left(-\frac{-\frac{55 \left(b x +a \right)^{\frac{5}{2}}}{16}+\frac{35 a \left(b x +a \right)^{\frac{3}{2}}}{6}-\frac{41 a^{2} \sqrt{b x +a}}{16}}{b^{3} x^{3}}+\frac{105 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)}{16 \sqrt{a}}\right)\right)$
default	$2 b^{3} \left(\frac{\left(b x +a \right)^{\frac{3}{2}}}{3}+4 a \sqrt{b x +a}-a^{2} \left(-\frac{-\frac{55 \left(b x +a \right)^{\frac{5}{2}}}{16}+\frac{35 a \left(b x +a \right)^{\frac{3}{2}}}{6}-\frac{41 a^{2} \sqrt{b x +a}}{16}}{b^{3} x^{3}}+\frac{105 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)}{16 \sqrt{a}}\right)\right)$

input
```
int((b*x+a)^(9/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*a^2*(b*x+a)^(1/2)*(165*b^2*x^2+50*a*b*x+8*a^2)/x^3+1/16*b^3*(32/3*(b
*x+a)^(3/2)+128*a*(b*x+a)^(1/2)-210*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^{9/2}}{x^4}\,dx = \left[\frac{315\,a^{\frac{3}{2}}b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(16\,b^4x^4+208\,ab^3x^3-165\,a^2b^2x^2-50\,a^3bx-\right.}{48\,x^3}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(315*a^(3/2)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt
(b*x + a))/x^3, 1/24*(315*sqrt(-a)*a*b^3*x^3*arctan(sqrt(-a)/sqrt(b*x + a)
) + (16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sq
rt(b*x + a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 8.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^{9/2}}{x^4}\,dx = -\frac{105a^{\frac{3}{2}}b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8}-\frac{a^5}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{29a^4\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{215a^3b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{43a^2b^{\frac{5}{2}}}{24\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{28ab^{\frac{7}{2}}\sqrt{x}}{3\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((b*x+a)**(9/2)/x**4,x)
```

output
```
-105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/8 - a**5/(3*sqrt(b)*x*
*(7/2)*sqrt(a/(b*x) + 1)) - 29*a**4*sqrt(b)/(12*x**(5/2)*sqrt(a/(b*x) + 1)
) - 215*a**3*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) + 43*a**2*b**(5/2)/(
24*sqrt(x)*sqrt(a/(b*x) + 1)) + 28*a*b**(7/2)*sqrt(x)/(3*sqrt(a/(b*x) + 1)
) + 2*b**(9/2)*x**(3/2)/(3*sqrt(a/(b*x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^{9/2}}{x^4}\,dx = \frac{105}{16}\,a^{\frac{3}{2}}b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}b^3 + 8\sqrt{bx+a}ab^3 - \frac{165\,(bx+a)^{\frac{5}{2}}a^2b^3-280\,(bx+a)^{\frac{3}{2}}a^3b^3+123\sqrt{bx+a}a^4b^3}{24\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2-a^3\right)}$$

input
```
integrate((b*x+a)^(9/2)/x^4,x, algorithm="maxima")
```

output
```
105/16*a^(3/2)*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))
) + 2/3*(b*x + a)^(3/2)*b^3 + 8*sqrt(b*x + a)*a*b^3 - 1/24*(165*(b*x + a)^
(5/2)*a^2*b^3 - 280*(b*x + a)^(3/2)*a^3*b^3 + 123*sqrt(b*x + a)*a^4*b^3)/(
(b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^{9/2}}{x^4}\,dx = \frac{1}{24}\left(\frac{315\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\,(bx+a)^{\frac{3}{2}}+192\sqrt{bx+a}a-\frac{165\,(bx+a)^{\frac{5}{2}}a^2-280\,(\right.$$

input
```
integrate((b*x+a)^(9/2)/x^4,x, algorithm="giac")
```

output
```
1/24*(315*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 16*(b*x + a)^(3/2)
 + 192*sqrt(b*x + a)*a - (165*(b*x + a)^(5/2)*a^2 - 280*(b*x + a)^(3/2)*a^
3 + 123*sqrt(b*x + a)*a^4)/(b^3*x^3))*b^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^{9/2}}{x^4}\,dx = \frac{2\,b^3\,(a+b\,x)^{3/2}}{3} + \frac{\frac{41\,a^4\,b^3\,\sqrt{a+b\,x}}{8} - \frac{35\,a^3\,b^3\,(a+b\,x)^{3/2}}{3} + \frac{55\,a^2\,b^3\,(a+b\,x)^{5/2}}{8}}{3\,a\,(a+b\,x)^2 - 3\,a^2\,(a+b\,x) - (a+b\,x)^3 + a^3} + 8\,a\,b^3\,\sqrt{a+b\,x} + \frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,105\mathrm{i}}{8}$$

input
```
int((a + b*x)^(9/2)/x^4,x)
```

output
```
(2*b^3*(a + b*x)^(3/2))/3 + ((41*a^4*b^3*(a + b*x)^(1/2))/8 - (35*a^3*b^3*
(a + b*x)^(3/2))/3 + (55*a^2*b^3*(a + b*x)^(5/2))/8)/(3*a*(a + b*x)^2 - 3*
a^2*(a + b*x) - (a + b*x)^3 + a^3) + (a^(3/2)*b^3*atan(((a + b*x)^(1/2)*1i
)/a^(1/2))*105i)/8 + 8*a*b^3*(a + b*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{9/2}}{x^4}\,dx = \frac{-16\sqrt{bx+a}\,a^4 - 100\sqrt{bx+a}\,a^3bx - 330\sqrt{bx+a}\,a^2b^2x^2 + 416\sqrt{bx+a}\,a\,b^3x^3 + 32\sqrt{}}{48x}$$

input
```
int((b*x+a)^(9/2)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x)*a**4 - 100*sqrt(a + b*x)*a**3*b*x - 330*sqrt(a + b*x)
*a**2*b**2*x**2 + 416*sqrt(a + b*x)*a*b**3*x**3 + 32*sqrt(a + b*x)*b**4*x*
*4 + 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*x**3 - 315*sqrt(a)*lo
g(sqrt(a + b*x) + sqrt(a))*a*b**3*x**3)/(48*x**3)
```

3.379 $\int \frac{(a+bx)^{9/2}}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{(a+bx)^{9/2}}{x^5} dx = 2b^4\sqrt{a+bx} - \frac{a^4\sqrt{a+bx}}{4x^4} - \frac{11a^3b\sqrt{a+bx}}{8x^3} - \frac{105a^2b^2\sqrt{a+bx}}{32x^2} - \frac{325ab^3\sqrt{a+bx}}{64x} - \frac{315}{64}\sqrt{a}b^4\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*b^4*(b*x+a)^(1/2)-1/4*a^4*(b*x+a)^(1/2)/x^4-11/8*a^3*b*(b*x+a)^(1/2)/x^3
-105/32*a^2*b^2*(b*x+a)^(1/2)/x^2-325/64*a*b^3*(b*x+a)^(1/2)/x-315/64*a^(1
/2)*b^4*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^{9/2}}{x^5} dx = \frac{1}{64}\left(-\frac{\sqrt{a+bx}(16a^4+88a^3bx+210a^2b^2x^2+325ab^3x^3-128b^4x^4)}{x^4} - 315\sqrt{a}b^4\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[(a + b*x)^(9/2)/x^5,x]
```

output
```
(-((Sqrt[a + b*x]*(16*a^4 + 88*a^3*b*x + 210*a^2*b^2*x^2 + 325*a*b^3*x^3 -
 128*b^4*x^4))/x^4) - 315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {51, 51, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x^5} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{8}b\int \frac{(a+bx)^{7/2}}{x^4}dx-\frac{(a+bx)^{9/2}}{4x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{8}b\left(\frac{7}{6}b\int \frac{(a+bx)^{5/2}}{x^3}dx-\frac{(a+bx)^{7/2}}{3x^3}\right)-\frac{(a+bx)^{9/2}}{4x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\int \frac{(a+bx)^{3/2}}{x^2}dx-\frac{(a+bx)^{5/2}}{2x^2}\right)-\frac{(a+bx)^{7/2}}{3x^3}\right)-\frac{(a+bx)^{9/2}}{4x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\int \frac{\sqrt{a+bx}}{x}dx-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)-\frac{(a+bx)^{7/2}}{3x^3}\right)-\frac{(a+bx)^{9/2}}{4x^4}$$

$$\Big\downarrow \text{60}$$

$$\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)-\frac{(a+bx)^{7/2}}{3x^3}\right)-\frac{(a+bx)^{9/2}}{4x^4}$$

$$\Big\downarrow \text{73}$$

$$\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2a\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)-\frac{(a+bx)^{7/2}}{3x^3}\right)-\frac{(a+bx)^{9/2}}{4x^4}$$

$\Big\downarrow$ 221

$$\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)-\frac{(a+bx)^{7/2}}{3x^3}\right)-\frac{(a+bx)^{9/2}}{4x^4}$$

input
```
Int[(a + b*x)^(9/2)/x^5,x]
```

output
```
-1/4*(a + b*x)^(9/2)/x^4 + (9*b*(-1/3*(a + b*x)^(7/2)/x^3 + (7*b*(-1/2*(a
+ b*x)^(5/2)/x^2 + (5*b*(-((a + b*x)^(3/2)/x) + (3*b*(2*Sqrt[a + b*x] - 2*
Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/2))/4))/6))/8
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{a\sqrt{bx+a}\,(325b^3x^3+210a\,b^2x^2+88a^2bx+16a^3)}{64x^4}+\frac{b^4\left(256\sqrt{bx+a}-630\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{128}$
derivativedivides	$2b^4\left(\sqrt{bx+a}-a\left(\frac{\frac{325(bx+a)^{\frac{7}{2}}}{128}-\frac{765a(bx+a)^{\frac{5}{2}}}{128}+\frac{643a^2(bx+a)^{\frac{3}{2}}}{128}-\frac{187a^3\sqrt{bx+a}}{128}}{b^4x^4}+\frac{315\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}\right)\right)$
default	$2b^4\left(\sqrt{bx+a}-a\left(\frac{\frac{325(bx+a)^{\frac{7}{2}}}{128}-\frac{765a(bx+a)^{\frac{5}{2}}}{128}+\frac{643a^2(bx+a)^{\frac{3}{2}}}{128}-\frac{187a^3\sqrt{bx+a}}{128}}{b^4x^4}+\frac{315\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}\right)\right)$
pseudoelliptic	$\frac{-315\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^4x^4+128b^4x^4\sqrt{a}\,\sqrt{bx+a}-325a^{\frac{3}{2}}b^3x^3\sqrt{bx+a}-210a^{\frac{5}{2}}b^2x^2\sqrt{bx+a}-88a^{\frac{7}{2}}bx\sqrt{bx+a}-16a^{\frac{9}{2}}\sqrt{}}{64x^4\sqrt{a}}$

input
```
int((b*x+a)^(9/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/64*a*(b*x+a)^(1/2)*(325*b^3*x^3+210*a*b^2*x^2+88*a^2*b*x+16*a^3)/x^4+1/
128*b^4*(256*(b*x+a)^(1/2)-630*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{9/2}}{x^5}\,dx = \left[\frac{315\,\sqrt{a}b^4x^4\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx\right.$$

$$128\,x^4$$

input
```
integrate((b*x+a)^(9/2)/x^5,x, algorithm="fricas")
```

output
```
[1/128*(315*sqrt(a)*b^4*x^4*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
 2*(128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*s
qrt(b*x + a))/x^4, 1/64*(315*sqrt(-a)*b^4*x^4*arctan(sqrt(-a)/sqrt(b*x + a
)) + (128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)
*sqrt(b*x + a))/x^4]
```

Sympy [A] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{9/2}}{x^5}\,dx = -\frac{315\sqrt{a}b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64} - \frac{a^5}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{13a^4\sqrt{b}}{8x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{149a^3b^{\frac{3}{2}}}{32x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{535a^2b^{\frac{5}{2}}}{64x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{197ab^{\frac{7}{2}}}{64\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2b^{\frac{9}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((b*x+a)**(9/2)/x**5,x)
```

output
```
-315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/64 - a**5/(4*sqrt(b)*x*
*(9/2)*sqrt(a/(b*x) + 1)) - 13*a**4*sqrt(b)/(8*x**(7/2)*sqrt(a/(b*x) + 1))
 - 149*a**3*b**(3/2)/(32*x**(5/2)*sqrt(a/(b*x) + 1)) - 535*a**2*b**(5/2)/(
64*x**(3/2)*sqrt(a/(b*x) + 1)) - 197*a*b**(7/2)/(64*sqrt(x)*sqrt(a/(b*x) +
 1)) + 2*b**(9/2)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)^{9/2}}{x^5}\,dx = \frac{315}{128}\sqrt{a}b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}b^4 - \frac{325\,(bx+a)^{\frac{7}{2}}ab^4-765\,(bx+a)^{\frac{5}{2}}a^2b^4+643\,(bx+a)^{\frac{3}{2}}a^3b^4-187\sqrt{bx+a}a^4b^4}{64\left((bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2-4\,(bx+a)a^3+a^4\right)}$$

input
```
integrate((b*x+a)^(9/2)/x^5,x, algorithm="maxima")
```

output
```
315/128*sqrt(a)*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)
)) + 2*sqrt(b*x + a)*b^4 - 1/64*(325*(b*x + a)^(7/2)*a*b^4 - 765*(b*x + a)
^(5/2)*a^2*b^4 + 643*(b*x + a)^(3/2)*a^3*b^4 - 187*sqrt(b*x + a)*a^4*b^4)/
((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2 - 4*(b*x + a)*a^3 + a^4
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{9/2}}{x^5}\,dx = \frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\sqrt{bx+a}b^5-\frac{325\,(bx+a)^{\frac{7}{2}}ab^5-765\,(bx+a)^{\frac{5}{2}}a^2b^5+643\,(bx+a)^{\frac{3}{2}}a^3b^5-187\sqrt{bx}}{b^4x^4}}{64\,b}$$

input
```
integrate((b*x+a)^(9/2)/x^5,x, algorithm="giac")
```

output
```
1/64*(315*a*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x + a
)*b^5 - (325*(b*x + a)^(7/2)*a*b^5 - 765*(b*x + a)^(5/2)*a^2*b^5 + 643*(b*
x + a)^(3/2)*a^3*b^5 - 187*sqrt(b*x + a)*a^4*b^5)/(b^4*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^{9/2}}{x^5}\,dx = 2\,b^4\,\sqrt{a+b\,x} + \frac{187\,a^4\,\sqrt{a+b\,x}}{64\,x^4} - \frac{643\,a^3\,{(a+b\,x)}^{3/2}}{64\,x^4} + \frac{765\,a^2\,{(a+b\,x)}^{5/2}}{64\,x^4} - \frac{325\,a\,{(a+b\,x)}^{7/2}}{64\,x^4} + \frac{\sqrt{a}\,b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,315\mathrm{i}}{64}$$

input
```
int((a + b*x)^(9/2)/x^5,x)
```

output
```
2*b^4*(a + b*x)^(1/2) + (187*a^4*(a + b*x)^(1/2))/(64*x^4) - (643*a^3*(a +
 b*x)^(3/2))/(64*x^4) + (765*a^2*(a + b*x)^(5/2))/(64*x^4) + (a^(1/2)*b^4*
atan(((a + b*x)^(1/2)*1i)/a^(1/2))*315i)/64 - (325*a*(a + b*x)^(7/2))/(64*
x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{9/2}}{x^5}\,dx = \frac{-32\sqrt{bx+a}\,a^4 - 176\sqrt{bx+a}\,a^3bx - 420\sqrt{bx+a}\,a^2b^2x^2 - 650\sqrt{bx+a}\,a\,b^3x^3 + 256\sqrt{}}{128x}$$

input
```
int((b*x+a)^(9/2)/x^5,x)
```

output
```
( - 32*sqrt(a + b*x)*a**4 - 176*sqrt(a + b*x)*a**3*b*x - 420*sqrt(a + b*x)
*a**2*b**2*x**2 - 650*sqrt(a + b*x)*a*b**3*x**3 + 256*sqrt(a + b*x)*b**4*x
**4 + 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*x**4 - 315*sqrt(a)*log
(sqrt(a + b*x) + sqrt(a))*b**4*x**4)/(128*x**4)
```

3.380 $\int \frac{(a+bx)^{9/2}}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 129

$$\int \frac{(a+bx)^{9/2}}{x^6} dx = -\frac{a^4\sqrt{a+bx}}{5x^5} - \frac{41a^3b\sqrt{a+bx}}{40x^4} - \frac{171a^2b^2\sqrt{a+bx}}{80x^3} - \frac{149ab^3\sqrt{a+bx}}{64x^2} - \frac{193b^4\sqrt{a+bx}}{128x} - \frac{63b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}$$

output
```
-1/5*a^4*(b*x+a)^(1/2)/x^5-41/40*a^3*b*(b*x+a)^(1/2)/x^4-171/80*a^2*b^2*(b
*x+a)^(1/2)/x^3-149/64*a*b^3*(b*x+a)^(1/2)/x^2-193/128*b^4*(b*x+a)^(1/2)/x
-63/128*b^5*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^{9/2}}{x^6} dx = \frac{1}{640}\left(-\frac{\sqrt{a+bx}(128a^4+656a^3bx+1368a^2b^2x^2+1490ab^3x^3+965b^4x^4)}{x^5} - \frac{315b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x)^(9/2)/x^6,x]
```

output
```
(-((Sqrt[a + b*x]*(128*a^4 + 656*a^3*b*x + 1368*a^2*b^2*x^2 + 1490*a*b^3*x
^3 + 965*b^4*x^4))/x^5) - (315*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
)/640
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {51, 51, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x^6} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{10} b \int \frac{(a+bx)^{7/2}}{x^5} dx - \frac{(a+bx)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{10} b \left(\frac{7}{8} b \int \frac{(a+bx)^{5/2}}{x^4} dx - \frac{(a+bx)^{7/2}}{4x^4}\right) - \frac{(a+bx)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{10} b \left(\frac{7}{8} b \left(\frac{5}{6} b \int \frac{(a+bx)^{3/2}}{x^3} dx - \frac{(a+bx)^{5/2}}{3x^3}\right) - \frac{(a+bx)^{7/2}}{4x^4}\right) - \frac{(a+bx)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{10} b \left(\frac{7}{8} b \left(\frac{5}{6} b \left(\frac{3}{4} b \int \frac{\sqrt{a+bx}}{x^2} dx - \frac{(a+bx)^{3/2}}{2x^2}\right) - \frac{(a+bx)^{5/2}}{3x^3}\right) - \frac{(a+bx)^{7/2}}{4x^4}\right) - \frac{(a+bx)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}\right)-\frac{(a+bx)^{7/2}}{4x^4}\right)-\frac{(a+bx)^{9/2}}{5x^5}$$

$$\Big\downarrow \; 73$$

$$\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}\right)-\frac{(a+bx)^{7/2}}{4x^4}\right)-\frac{(a+bx)^{9/2}}{5x^5}$$

$$\Big\downarrow \; 221$$

$$\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}\right)-\frac{(a+bx)^{7/2}}{4x^4}\right)-\frac{(a+bx)^{9/2}}{5x^5}$$

input
```
Int[(a + b*x)^(9/2)/x^6,x]
```

output
```
-1/5*(a + b*x)^(9/2)/x^5 + (9*b*(-1/4*(a + b*x)^(7/2)/x^4 + (7*b*(-1/3*(a
+ b*x)^(5/2)/x^3 + (5*b*(-1/2*(a + b*x)^(3/2)/x^2 + (3*b*(-(Sqrt[a + b*x]/
x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]))/4))/6))/8))/10
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{\sqrt{bx+a}\,(965b^4x^4+1490a\,x^3b^3+1368a^2b^2x^2+656a^3bx+128a^4)}{640x^5}-\frac{63b^5\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}$
derivativedivides	$2b^5\left(-\frac{\frac{193(bx+a)^{\frac{9}{2}}}{256}-\frac{237a(bx+a)^{\frac{7}{2}}}{128}+\frac{21a^2(bx+a)^{\frac{5}{2}}}{10}-\frac{147a^3(bx+a)^{\frac{3}{2}}}{128}+\frac{63a^4\sqrt{bx+a}}{256}}{b^5x^5}-\frac{63\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}\right)$
default	$2b^5\left(-\frac{\frac{193(bx+a)^{\frac{9}{2}}}{256}-\frac{237a(bx+a)^{\frac{7}{2}}}{128}+\frac{21a^2(bx+a)^{\frac{5}{2}}}{10}-\frac{147a^3(bx+a)^{\frac{3}{2}}}{128}+\frac{63a^4\sqrt{bx+a}}{256}}{b^5x^5}-\frac{63\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}\right)$
pseudoelliptic	$\frac{-315\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^5x^5-965b^4x^4\sqrt{a}\,\sqrt{bx+a}-1490a^{\frac{3}{2}}b^3x^3\sqrt{bx+a}-1368a^{\frac{5}{2}}b^2x^2\sqrt{bx+a}-656a^{\frac{7}{2}}bx\sqrt{bx+a}-128a}{640x^5\sqrt{a}}$

input
```
int((b*x+a)^(9/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(b*x+a)^(1/2)*(965*b^4*x^4+1490*a*b^3*x^3+1368*a^2*b^2*x^2+656*a^3*
b*x+128*a^4)/x^5-63/128*b^5*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = \left[\frac{315\sqrt{a}b^5x^5\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\left(965\,ab^4x^4 + 1490\,a^2b^3x^3 + 1368\,a^3b^2x^2 + 656\right.}{1280\,ax^5}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^6,x, algorithm="fricas")
```

output
```
[1/1280*(315*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
- 2*(965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 656*a^4*b*x + 1
28*a^5)*sqrt(b*x + a))/(a*x^5), 1/640*(315*sqrt(-a)*b^5*x^5*arctan(sqrt(-a
)/sqrt(b*x + a)) - (965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 +
656*a^4*b*x + 128*a^5)*sqrt(b*x + a))/(a*x^5)]
```

Sympy [A] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = -\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^{\frac{9}{2}}} - \frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{40x^{\frac{7}{2}}} - \frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{80x^{\frac{5}{2}}} - \frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{64x^{\frac{3}{2}}} - \frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{128\sqrt{x}} - \frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{128\sqrt{a}}$$

input
```
integrate((b*x+a)**(9/2)/x**6,x)
```

output
```
-a**4*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**(9/2)) - 41*a**3*b**(3/2)*sqrt(a/(b*
x) + 1)/(40*x**(7/2)) - 171*a**2*b**(5/2)*sqrt(a/(b*x) + 1)/(80*x**(5/2))
- 149*a*b**(7/2)*sqrt(a/(b*x) + 1)/(64*x**(3/2)) - 193*b**(9/2)*sqrt(a/(b*
x) + 1)/(128*sqrt(x)) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(128*sqrt
(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = \frac{63\,b^5\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{256\,\sqrt{a}} - \frac{965\,(bx+a)^{\frac{9}{2}}b^5 - 2370\,(bx+a)^{\frac{7}{2}}ab^5 + 2688\,(bx+a)^{\frac{5}{2}}a^2b^5 - 1470\,(bx+a)^{\frac{3}{2}}a^3b^5 + 315\,\sqrt{bx+a}a^4b^5}{640\,\left((bx+a)^5 - 5\,(bx+a)^4a + 10\,(bx+a)^3a^2 - 10\,(bx+a)^2a^3 + 5\,(bx+a)a^4 - a^5\right)}$$

input
```
integrate((b*x+a)^(9/2)/x^6,x, algorithm="maxima")
```

output
```
63/256*b^5*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a
) - 1/640*(965*(b*x + a)^(9/2)*b^5 - 2370*(b*x + a)^(7/2)*a*b^5 + 2688*(b*
x + a)^(5/2)*a^2*b^5 - 1470*(b*x + a)^(3/2)*a^3*b^5 + 315*sqrt(b*x + a)*a^
4*b^5)/((b*x + a)^5 - 5*(b*x + a)^4*a + 10*(b*x + a)^3*a^2 - 10*(b*x + a)^
2*a^3 + 5*(b*x + a)*a^4 - a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = \frac{1}{640}\,b^5\left(\frac{315\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{965\,(bx+a)^{\frac{9}{2}} - 2370\,(bx+a)^{\frac{7}{2}}a + 2688\,(bx+a)^{\frac{5}{2}}a^2 -}{b^5x^5}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^6,x, algorithm="giac")
```

output
```
1/640*b^5*(315*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (965*(b*x + a)^(9
/2) - 2370*(b*x + a)^(7/2)*a + 2688*(b*x + a)^(5/2)*a^2 - 1470*(b*x + a)^(
3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/(b^5*x^5))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = \frac{147\,a^3\,(a+b\,x)^{3/2}}{64\,x^5} - \frac{63\,a^4\,\sqrt{a+b\,x}}{128\,x^5} - \frac{193\,(a+b\,x)^{9/2}}{128\,x^5} - \frac{21\,a^2\,(a+b\,x)^{5/2}}{5\,x^5} + \frac{237\,a\,(a+b\,x)^{7/2}}{64\,x^5} + \frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{128\,\sqrt{a}}$$

input
```
int((a + b*x)^(9/2)/x^6,x)
```

output
```
(147*a^3*(a + b*x)^(3/2))/(64*x^5) - (63*a^4*(a + b*x)^(1/2))/(128*x^5) -
(193*(a + b*x)^(9/2))/(128*x^5) - (21*a^2*(a + b*x)^(5/2))/(5*x^5) + (b^5*
atan(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/(128*a^(1/2)) + (237*a*(a + b*x)^(
7/2))/(64*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = \frac{-256\sqrt{bx+a}\,a^5 - 1312\sqrt{bx+a}\,a^4bx - 2736\sqrt{bx+a}\,a^3b^2x^2 - 2980\sqrt{bx+a}\,a^2b^3x^3 -}{12}$$

input
```
int((b*x+a)^(9/2)/x^6,x)
```

output
```
( - 256*sqrt(a + b*x)*a**5 - 1312*sqrt(a + b*x)*a**4*b*x - 2736*sqrt(a + b
*x)*a**3*b**2*x**2 - 2980*sqrt(a + b*x)*a**2*b**3*x**3 - 1930*sqrt(a + b*x
)*a*b**4*x**4 + 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**5*x**5 - 315*s
qrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**5*x**5)/(1280*a*x**5)
```

3.381 $\int \frac{(a+bx)^{9/2}}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 151

$$\int \frac{(a+bx)^{9/2}}{x^7} dx = -\frac{a^4\sqrt{a+bx}}{6x^6} - \frac{49a^3b\sqrt{a+bx}}{60x^5} - \frac{253a^2b^2\sqrt{a+bx}}{160x^4} - \frac{1429ab^3\sqrt{a+bx}}{960x^3} - \frac{491b^4\sqrt{a+bx}}{768x^2} - \frac{21b^5\sqrt{a+bx}}{512ax} + \frac{21b^6\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}$$

output
```
-1/6*a^4*(b*x+a)^(1/2)/x^6-49/60*a^3*b*(b*x+a)^(1/2)/x^5-253/160*a^2*b^2*(
b*x+a)^(1/2)/x^4-1429/960*a*b^3*(b*x+a)^(1/2)/x^3-491/768*b^4*(b*x+a)^(1/2
)/x^2-21/512*b^5*(b*x+a)^(1/2)/a/x+21/512*b^6*arctanh((b*x+a)^(1/2)/a^(1/2
))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)^{9/2}}{x^7} dx = -\frac{\sqrt{a+bx}(1280a^5 + 6272a^4bx + 12144a^3b^2x^2 + 11432a^2b^3x^3 + 4910ab^4x^4 + 315b^5x^5)}{7680ax^6} + \frac{21b^6\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}$$

input
```
Integrate[(a + b*x)^(9/2)/x^7,x]
```

output
```
-1/7680*(Sqrt[a + b*x]*(1280*a^5 + 6272*a^4*b*x + 12144*a^3*b^2*x^2 + 1143
2*a^2*b^3*x^3 + 4910*a*b^4*x^4 + 315*b^5*x^5))/(a*x^6) + (21*b^6*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(512*a^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {51, 51, 51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x^7}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{4}b\int \frac{(a+bx)^{7/2}}{x^6}dx-\frac{(a+bx)^{9/2}}{6x^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{4}b\left(\frac{7}{10}b\int \frac{(a+bx)^{5/2}}{x^5}dx-\frac{(a+bx)^{7/2}}{5x^5}\right)-\frac{(a+bx)^{9/2}}{6x^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\int \frac{(a+bx)^{3/2}}{x^4}dx-\frac{(a+bx)^{5/2}}{4x^4}\right)-\frac{(a+bx)^{7/2}}{5x^5}\right)-\frac{(a+bx)^{9/2}}{6x^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}dx-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)-\frac{(a+bx)^{7/2}}{5x^5}\right)-\frac{(a+bx)^{9/2}}{6x^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int\frac{1}{x^2\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)-\frac{(a+bx)^{7/2}}{5x^5}\right)-$$
$$\frac{(a+bx)^{9/2}}{6x^6}$$

$$\Big\downarrow \; 52$$

$$\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int\frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)-\frac{(a+bx)^{7}}{5x^5}$$
$$\frac{(a+bx)^{9/2}}{6x^6}$$

$$\Big\downarrow \; 73$$

$$\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)-($$
$$\frac{(a+bx)^{9/2}}{6x^6}$$

$$\Big\downarrow \; 221$$

$$\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)-\frac{(a+}{5}$$
$$\frac{(a+bx)^{9/2}}{6x^6}$$

input
```
Int[(a + b*x)^(9/2)/x^7,x]
```

output
```
-1/6*(a + b*x)^(9/2)/x^6 + (3*b*(-1/5*(a + b*x)^(7/2)/x^5 + (7*b*(-1/4*(a
+ b*x)^(5/2)/x^4 + (5*b*(-1/3*(a + b*x)^(3/2)/x^3 + (b*(-1/2*Sqrt[a + b*x]
/x^2 + (b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(
3/2)))/4))/2))/8))/10))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{\sqrt{bx+a}\,(315b^5x^5+4910a\,b^4x^4+11432a^2b^3x^3+12144a^3b^2x^2+6272a^4bx+1280a^5)}{7680x^6a}+\frac{21b^6\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{512a^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{49\left(-\frac{45\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^6x^6}{896}+\sqrt{bx+a}\left(\frac{45\sqrt{a}\,b^5x^5}{896}+\frac{2455a^{\frac{3}{2}}b^4x^4}{3136}+\frac{1429a^{\frac{5}{2}}b^3x^3}{784}+\frac{759a^{\frac{7}{2}}b^2x^2}{392}+a^{\frac{9}{2}}bx+\frac{10a^{\frac{11}{2}}}{49}\right)\right)}{60a^{\frac{3}{2}}x^6}$
derivativedivides	$2b^6\left(-\frac{\frac{21(bx+a)^{\frac{11}{2}}}{1024a}+\frac{667(bx+a)^{\frac{9}{2}}}{3072}-\frac{843a(bx+a)^{\frac{7}{2}}}{2560}+\frac{693a^2(bx+a)^{\frac{5}{2}}}{2560}-\frac{119a^3(bx+a)^{\frac{3}{2}}}{1024}+\frac{21a^4\sqrt{bx+a}}{1024}}{b^6x^6}+\frac{21\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{1024a^{\frac{3}{2}}}\right)$
default	$2b^6\left(-\frac{\frac{21(bx+a)^{\frac{11}{2}}}{1024a}+\frac{667(bx+a)^{\frac{9}{2}}}{3072}-\frac{843a(bx+a)^{\frac{7}{2}}}{2560}+\frac{693a^2(bx+a)^{\frac{5}{2}}}{2560}-\frac{119a^3(bx+a)^{\frac{3}{2}}}{1024}+\frac{21a^4\sqrt{bx+a}}{1024}}{b^6x^6}+\frac{21\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{1024a^{\frac{3}{2}}}\right)$

input
```
int((b*x+a)^(9/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(b*x+a)^(1/2)*(315*b^5*x^5+4910*a*b^4*x^4+11432*a^2*b^3*x^3+12144*
a^3*b^2*x^2+6272*a^4*b*x+1280*a^5)/x^6/a+21/512*b^6*arctanh((b*x+a)^(1/2)/
a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = \left[\frac{315\,\sqrt{a}b^6x^6\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12\right.}{15360\,a^2x^6}$$
$$-\frac{315\,\sqrt{-a}b^6x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12144\,a^4b^2x^2+6272\,a^5bx+}{7680\,a^2x^6}$$

input
```
integrate((b*x+a)^(9/2)/x^7,x, algorithm="fricas")
```

output
```
[1/15360*(315*sqrt(a)*b^6*x^6*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 - 2*(315*a*b^5*x^5 + 4910*a^2*b^4*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2
*x^2 + 6272*a^5*b*x + 1280*a^6)*sqrt(b*x + a))/(a^2*x^6), -1/7680*(315*sqr
t(-a)*b^6*x^6*arctan(sqrt(-a)/sqrt(b*x + a)) + (315*a*b^5*x^5 + 4910*a^2*b
^4*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x^2 + 6272*a^5*b*x + 1280*a^6)*
sqrt(b*x + a))/(a^2*x^6)]
```

Sympy [A] (verification not implemented)

Time = 40.97 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = -\frac{a^5}{6\sqrt{b}x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{59a^4\sqrt{b}}{60x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{1151a^3b^{\frac{3}{2}}}{480x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{2947a^2b^{\frac{5}{2}}}{960x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{8171ab^{\frac{7}{2}}}{3840x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{1045b^{\frac{9}{2}}}{1536x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{21b^{\frac{11}{2}}}{512a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{21b^6\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{512a^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(9/2)/x**7,x)
```

output
```
-a**5/(6*sqrt(b)*x**(13/2)*sqrt(a/(b*x) + 1)) - 59*a**4*sqrt(b)/(60*x**(11
/2)*sqrt(a/(b*x) + 1)) - 1151*a**3*b**(3/2)/(480*x**(9/2)*sqrt(a/(b*x) + 1
)) - 2947*a**2*b**(5/2)/(960*x**(7/2)*sqrt(a/(b*x) + 1)) - 8171*a*b**(7/2)
/(3840*x**(5/2)*sqrt(a/(b*x) + 1)) - 1045*b**(9/2)/(1536*x**(3/2)*sqrt(a/(
b*x) + 1)) - 21*b**(11/2)/(512*a*sqrt(x)*sqrt(a/(b*x) + 1)) + 21*b**6*asin
h(sqrt(a)/(sqrt(b)*sqrt(x)))/(512*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = -\frac{21\,b^6\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{1024\,a^{\frac{3}{2}}} - \frac{315\,(bx+a)^{\frac{11}{2}}b^6 + 3335\,(bx+a)^{\frac{9}{2}}ab^6 - 5058\,(bx+a)^{\frac{7}{2}}a^2b^6 + 4158\,(bx+a)^{\frac{5}{2}}a^3b^6 - 1785\,(bx+a)^{\frac{3}{2}}a^4b^6 \cdots}{7680\,\left((bx+a)^6a - 6\,(bx+a)^5a^2 + 15\,(bx+a)^4a^3 - 20\,(bx+a)^3a^4 + 15\,(bx+a)^2a^5 - 6\,(bx+ \cdots\right.}$$

input
```
integrate((b*x+a)^(9/2)/x^7,x, algorithm="maxima")
```

output
```
-21/1024*b^6*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3
/2) - 1/7680*(315*(b*x + a)^(11/2)*b^6 + 3335*(b*x + a)^(9/2)*a*b^6 - 5058
*(b*x + a)^(7/2)*a^2*b^6 + 4158*(b*x + a)^(5/2)*a^3*b^6 - 1785*(b*x + a)^(
3/2)*a^4*b^6 + 315*sqrt(b*x + a)*a^5*b^6)/((b*x + a)^6*a - 6*(b*x + a)^5*a
^2 + 15*(b*x + a)^4*a^3 - 20*(b*x + a)^3*a^4 + 15*(b*x + a)^2*a^5 - 6*(b*x
 + a)*a^6 + a^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = -\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{315\,(bx+a)^{\frac{11}{2}}b^7+3335\,(bx+a)^{\frac{9}{2}}ab^7-5058\,(bx+a)^{\frac{7}{2}}a^2b^7+4158\,(bx+a)^{\frac{5}{2}}a^3b^7-1785\,(bx+a)^{\frac{3}{2}}a^4b^7+315\,\sqrt{bx+a}a^5b^7}{ab^6x^6}}{7680\,b}$$

input
```
integrate((b*x+a)^(9/2)/x^7,x, algorithm="giac")
```

output
```
-1/7680*(315*b^7*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x +
 a)^(11/2)*b^7 + 3335*(b*x + a)^(9/2)*a*b^7 - 5058*(b*x + a)^(7/2)*a^2*b^7
 + 4158*(b*x + a)^(5/2)*a^3*b^7 - 1785*(b*x + a)^(3/2)*a^4*b^7 + 315*sqrt(
b*x + a)*a^5*b^7)/(a*b^6*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = \frac{119\,a^3\,(a+b\,x)^{3/2}}{512\,x^6} - \frac{21\,a^4\,\sqrt{a+b\,x}}{512\,x^6} - \frac{667\,(a+b\,x)^{9/2}}{1536\,x^6} - \frac{693\,a^2\,(a+b\,x)^{5/2}}{1280\,x^6} - \frac{21\,(a+b\,x)^{11/2}}{512\,a\,x^6} + \frac{843\,a\,(a+b\,x)^{7/2}}{1280\,x^6} - \frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,21\mathrm{i}}{512\,a^{3/2}}$$

input
```
int((a + b*x)^(9/2)/x^7,x)
```

output
```
(119*a^3*(a + b*x)^(3/2))/(512*x^6) - (21*a^4*(a + b*x)^(1/2))/(512*x^6) -
 (667*(a + b*x)^(9/2))/(1536*x^6) - (693*a^2*(a + b*x)^(5/2))/(1280*x^6) -
 (21*(a + b*x)^(11/2))/(512*a*x^6) - (b^6*atan(((a + b*x)^(1/2)*1i)/a^(1/2
))*21i)/(512*a^(3/2)) + (843*a*(a + b*x)^(7/2))/(1280*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = \frac{-2560\sqrt{bx+a}\,a^6 - 12544\sqrt{bx+a}\,a^5bx - 24288\sqrt{bx+a}\,a^4b^2x^2 - 22864\sqrt{bx+a}\,a^3b^3}{}$$

input
```
int((b*x+a)^(9/2)/x^7,x)
```

output
```
( - 2560*sqrt(a + b*x)*a**6 - 12544*sqrt(a + b*x)*a**5*b*x - 24288*sqrt(a
+ b*x)*a**4*b**2*x**2 - 22864*sqrt(a + b*x)*a**3*b**3*x**3 - 9820*sqrt(a +
 b*x)*a**2*b**4*x**4 - 630*sqrt(a + b*x)*a*b**5*x**5 - 315*sqrt(a)*log(sqr
t(a + b*x) - sqrt(a))*b**6*x**6 + 315*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))
*b**6*x**6)/(15360*a**2*x**6)
```

3.382 $\int \frac{(a+bx)^{9/2}}{x^8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 173

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = -\frac{a^4\sqrt{a+bx}}{7x^7} - \frac{19a^3b\sqrt{a+bx}}{28x^6} - \frac{351a^2b^2\sqrt{a+bx}}{280x^5} - \frac{2441ab^3\sqrt{a+bx}}{2240x^4} - \frac{253b^4\sqrt{a+bx}}{640x^3} - \frac{3b^5\sqrt{a+bx}}{512ax^2} + \frac{9b^6\sqrt{a+bx}}{1024a^2x} - \frac{9b^7\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}$$

output
```
-1/7*a^4*(b*x+a)^(1/2)/x^7-19/28*a^3*b*(b*x+a)^(1/2)/x^6-351/280*a^2*b^2*(
b*x+a)^(1/2)/x^5-2441/2240*a*b^3*(b*x+a)^(1/2)/x^4-253/640*b^4*(b*x+a)^(1/
2)/x^3-3/512*b^5*(b*x+a)^(1/2)/a/x^2+9/1024*b^6*(b*x+a)^(1/2)/a^2/x-9/1024
*b^7*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^{9/2}}{x^8} dx = -\frac{\sqrt{a+bx}(5120a^6+24320a^5bx+44928a^4b^2x^2+39056a^3b^3x^3+14168a^2b^4x^4+210ab^5x^5-315b^6x^6)}{35840a^2x^7} - \frac{9b^7\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}$$

input
```
Integrate[(a + b*x)^(9/2)/x^8,x]
```

output
```
-1/35840*(Sqrt[a + b*x]*(5120*a^6 + 24320*a^5*b*x + 44928*a^4*b^2*x^2 + 39
056*a^3*b^3*x^3 + 14168*a^2*b^4*x^4 + 210*a*b^5*x^5 - 315*b^6*x^6))/(a^2*x
^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {51, 51, 51, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{9/2}}{x^8} dx$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{14}b\int \frac{(a+bx)^{7/2}}{x^7}dx - \frac{(a+bx)^{9/2}}{7x^7}$$

$$\Big\downarrow \text{51}$$

$$\frac{9}{14}b\left(\frac{7}{12}b\int \frac{(a+bx)^{5/2}}{x^6}dx - \frac{(a+bx)^{7/2}}{6x^6}\right) - \frac{(a+bx)^{9/2}}{7x^7}$$

$\Big\downarrow$ 51

$$\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\int\frac{(a+bx)^{3/2}}{x^5}dx-\frac{(a+bx)^{5/2}}{5x^5}\right)-\frac{(a+bx)^{7/2}}{6x^6}\right)-\frac{(a+bx)^{9/2}}{7x^7}$$

$\Big\downarrow$ 51

$$\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\int\frac{\sqrt{a+bx}}{x^4}dx-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)-\frac{(a+bx)^{7/2}}{6x^6}\right)-\frac{(a+bx)^{9/2}}{7x^7}$$

$\Big\downarrow$ 51

$$\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int\frac{1}{x^3\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)-\frac{(a+bx)^{7/2}}{6x^6}\right)-\frac{(a+bx)^{9/2}}{7x^7}$$

$\Big\downarrow$ 52

$$\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int\frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)-\cdots\right)-\frac{(a+bx)^{9/2}}{7x^7}$$

$\Big\downarrow$ 52

$$\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int\frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\cdots\right)\right)-\frac{(a+bx)^{9/2}}{7x^7}$$

$\Big\downarrow$ 73

$$\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\cdots\right)\right)-\frac{(a+bx)^{9/2}}{7x^7}$$

$$\Big\downarrow \; 221$$

$$\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+\cdots$$

$$\frac{(a+bx)^{9/2}}{7x^7}$$

input
```
Int[(a + b*x)^(9/2)/x^8,x]
```

output
```
-1/7*(a + b*x)^(9/2)/x^7 + (9*b*(-1/6*(a + b*x)^(7/2)/x^6 + (7*b*(-1/5*(a
+ b*x)^(5/2)/x^5 + (b*(-1/4*(a + b*x)^(3/2)/x^4 + (3*b*(-1/3*Sqrt[a + b*x]
/x^3 + (b*(-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a + b*x]/(a*x)) + (b*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/6))/8))/2))/12))/14
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{\sqrt{bx+a}\,(-315b^6x^6+210a\,x^5b^5+14168a^2x^4b^4+39056a^3x^3b^3+44928a^4x^2b^2+24320a^5xb+5120a^6)}{35840x^7a^2}-\frac{9b^7\operatorname{arctanh}(}{1024a}$
pseudoelliptic	$-\frac{351\left(\frac{35\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^7b^7}{4992}+\sqrt{bx+a}\left(-\frac{35\sqrt{a}\,b^6x^6}{4992}+\frac{35a^{\frac{3}{2}}b^5x^5}{7488}+\frac{1771a^{\frac{5}{2}}b^4x^4}{5616}+\frac{2441a^{\frac{7}{2}}b^3x^3}{2808}+a^{\frac{9}{2}}b^2x^2+\frac{190a^{\frac{11}{2}}bx}{351}\right.\right.}{280a^{\frac{5}{2}}x^7}$
derivativedivides	$2b^7\left(-\frac{-\frac{9(bx+a)^{\frac{13}{2}}}{2048a^2}+\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{1199(bx+a)^{\frac{9}{2}}}{10240}-\frac{9a(bx+a)^{\frac{7}{2}}}{70}+\frac{849a^2(bx+a)^{\frac{5}{2}}}{10240}-\frac{15a^3(bx+a)^{\frac{3}{2}}}{512}+\frac{9a^4\sqrt{bx+a}}{2048}}{b^7x^7}-\frac{9\operatorname{arct}}{}\right.$
default	$2b^7\left(-\frac{-\frac{9(bx+a)^{\frac{13}{2}}}{2048a^2}+\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{1199(bx+a)^{\frac{9}{2}}}{10240}-\frac{9a(bx+a)^{\frac{7}{2}}}{70}+\frac{849a^2(bx+a)^{\frac{5}{2}}}{10240}-\frac{15a^3(bx+a)^{\frac{3}{2}}}{512}+\frac{9a^4\sqrt{bx+a}}{2048}}{b^7x^7}-\frac{9\operatorname{arct}}{}\right.$

input
```
int((b*x+a)^(9/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/35840*(b*x+a)^(1/2)*(-315*b^6*x^6+210*a*b^5*x^5+14168*a^2*b^4*x^4+39056
*a^3*b^3*x^3+44928*a^4*b^2*x^2+24320*a^5*b*x+5120*a^6)/x^7/a^2-9/1024*b^7*
arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx=\left[\frac{315\sqrt{a}b^7x^7\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(315\,ab^6x^6-210\,a^2b^5x^5-14168\,a^3b^4x^4-390\right.}{71680\,a^3x^7}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^8,x, algorithm="fricas")
```

output
```
[1/71680*(315*sqrt(a)*b^7*x^7*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 + 2*(315*a*b^6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*
x^3 - 44928*a^5*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^
7), 1/35840*(315*sqrt(-a)*b^7*x^7*arctan(sqrt(-a)/sqrt(b*x + a)) + (315*a*
b^6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^3 - 44928*
a^5*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(9/2)/x**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = \frac{9\,b^7\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2048\,a^{\frac{5}{2}}} + \frac{315\,(bx+a)^{\frac{13}{2}}b^7 - 2100\,(bx+a)^{\frac{11}{2}}ab^7 - 8393\,(bx+a)^{\frac{9}{2}}a^2b^7 + 9216\,(bx+a)^{\frac{7}{2}}a^3b^7 - 5943\,(bx+a)^{\frac{5}{2}}a^4b^7 +}{35840\,\left((bx+a)^7a^2 - 7\,(bx+a)^6a^3 + 21\,(bx+a)^5a^4 - 35\,(bx+a)^4a^5 + 35\,(bx+a)^3a^6 - 21\,(b\right.}$$

input
```
integrate((b*x+a)^(9/2)/x^8,x, algorithm="maxima")
```

output
```
9/2048*b^7*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2
) + 1/35840*(315*(b*x + a)^(13/2)*b^7 - 2100*(b*x + a)^(11/2)*a*b^7 - 8393
*(b*x + a)^(9/2)*a^2*b^7 + 9216*(b*x + a)^(7/2)*a^3*b^7 - 5943*(b*x + a)^(
5/2)*a^4*b^7 + 2100*(b*x + a)^(3/2)*a^5*b^7 - 315*sqrt(b*x + a)*a^6*b^7)/(
(b*x + a)^7*a^2 - 7*(b*x + a)^6*a^3 + 21*(b*x + a)^5*a^4 - 35*(b*x + a)^4*
a^5 + 35*(b*x + a)^3*a^6 - 21*(b*x + a)^2*a^7 + 7*(b*x + a)*a^8 - a^9)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = \frac{1}{35840}\,b^7\left(\frac{315\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{315\,(bx+a)^{\frac{13}{2}} - 2100\,(bx+a)^{\frac{11}{2}}a - 8393\,(bx+a)^{\frac{9}{2}}\right.$$

input
```
integrate((b*x+a)^(9/2)/x^8,x, algorithm="giac")
```

output
```
1/35840*b^7*(315*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*x
 + a)^(13/2) - 2100*(b*x + a)^(11/2)*a - 8393*(b*x + a)^(9/2)*a^2 + 9216*(
b*x + a)^(7/2)*a^3 - 5943*(b*x + a)^(5/2)*a^4 + 2100*(b*x + a)^(3/2)*a^5 -
 315*sqrt(b*x + a)*a^6)/(a^2*b^7*x^7))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^8}\,dx &= \frac{15\,a^3\,(a+b\,x)^{3/2}}{256\,x^7} - \frac{9\,a^4\,\sqrt{a+b\,x}}{1024\,x^7}\\ &- \frac{1199\,(a+b\,x)^{9/2}}{5120\,x^7} - \frac{849\,a^2\,(a+b\,x)^{5/2}}{5120\,x^7} - \frac{15\,(a+b\,x)^{11/2}}{256\,a\,x^7}\\ &+ \frac{9\,(a+b\,x)^{13/2}}{1024\,a^2\,x^7} + \frac{9\,a\,(a+b\,x)^{7/2}}{35\,x^7} + \frac{b^7\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}}{1024\,a^{5/2}}\end{aligned}$$

input
```
int((a + b*x)^(9/2)/x^8,x)
```

output
```
(15*a^3*(a + b*x)^(3/2))/(256*x^7) - (9*a^4*(a + b*x)^(1/2))/(1024*x^7) -
(1199*(a + b*x)^(9/2))/(5120*x^7) - (849*a^2*(a + b*x)^(5/2))/(5120*x^7) -
 (15*(a + b*x)^(11/2))/(256*a*x^7) + (9*(a + b*x)^(13/2))/(1024*a^2*x^7) +
 (b^7*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*9i)/(1024*a^(5/2)) + (9*a*(a + b*
x)^(7/2))/(35*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = \frac{-10240\sqrt{bx+a}\,a^7 - 48640\sqrt{bx+a}\,a^6bx - 89856\sqrt{bx+a}\,a^5b^2x^2 - 78112\sqrt{bx+a}\,a^4b$$

input
```
int((b*x+a)^(9/2)/x^8,x)
```

output
```
( - 10240*sqrt(a + b*x)*a**7 - 48640*sqrt(a + b*x)*a**6*b*x - 89856*sqrt(a
 + b*x)*a**5*b**2*x**2 - 78112*sqrt(a + b*x)*a**4*b**3*x**3 - 28336*sqrt(a
 + b*x)*a**3*b**4*x**4 - 420*sqrt(a + b*x)*a**2*b**5*x**5 + 630*sqrt(a + b
*x)*a*b**6*x**6 + 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**7*x**7 - 315
*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**7*x**7)/(71680*a**3*x**7)
```

3.383 $\int \frac{\sqrt{-a+bx}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sqrt{-a+bx}}{x} dx = 2\sqrt{-a+bx} - 2\sqrt{a} \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

output
```
2*(b*x-a)^(1/2)-2*a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a+bx}}{x} dx = 2\sqrt{-a+bx} - 2\sqrt{a} \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[-a + b*x]/x,x]
```

output
```
2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx-a}}{x}\,dx$$

$$\Big\downarrow \text{60}$$

$$2\sqrt{bx-a}-a\int \frac{1}{x\sqrt{bx-a}}dx$$

$$\Big\downarrow \text{73}$$

$$2\sqrt{bx-a}-\frac{2a\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{b}$$

$$\Big\downarrow \text{218}$$

$$2\sqrt{bx-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

input
```
Int[Sqrt[-a + b*x]/x,x]
```

output
```
2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\sqrt{bx-a}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	32
default	$2\sqrt{bx-a}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	32
pseudoelliptic	$2\sqrt{bx-a}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	32

input

```
int((b*x-a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output

```
2*(b*x-a)^(1/2)-2*a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = \left[\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)+2\,\sqrt{bx-a},2\,\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)+2\,\sqrt{bx-a}\right]$$

input

```
integrate((b*x-a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(b*x - a),
 2*sqrt(a)*arctan(sqrt(a)/sqrt(b*x - a)) + 2*sqrt(b*x - a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.79

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = \begin{cases} -2i\sqrt{a}\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2ia}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{2i\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ 2\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x-a)**(1/2)/x,x)
```

output
```
Piecewise((-2*I*sqrt(a)*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*I*a/(sqrt(b)*
sqrt(x)*sqrt(a/(b*x) - 1)) - 2*I*sqrt(b)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/
(b*x)) > 1), (2*sqrt(a)*asin(sqrt(a)/(sqrt(b)*sqrt(x))) - 2*a/(sqrt(b)*sqr
t(x)*sqrt(-a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = -2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\sqrt{bx-a}$$

input
```
integrate((b*x-a)^(1/2)/x,x, algorithm="maxima")
```

output
```
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = -2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\sqrt{bx-a}$$

input
```
integrate((b*x-a)^(1/2)/x,x, algorithm="giac")
```

output
```
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = 2\sqrt{b\,x-a} - 2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

input
```
int((b*x - a)^(1/2)/x,x)
```

output
```
2*(b*x - a)^(1/2) - 2*a^(1/2)*atan((b*x - a)^(1/2)/a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = -2\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\sqrt{bx-a}$$

input
```
int((b*x-a)^(1/2)/x,x)
```

output
```
2*( - sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a)) + sqrt( - a + b*x))
```

3.384 $\int \frac{\sqrt{-a+bx}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{\sqrt{-a+bx}}{x^2} \, dx = -\frac{\sqrt{-a+bx}}{x} + \frac{b \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-(b*x-a)^(1/2)/x+b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a+bx}}{x^2} \, dx = -\frac{\sqrt{-a+bx}}{x} + \frac{b \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[-a + b*x]/x^2,x]
```

output
```
-(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx-a}}{x^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}b\int \frac{1}{x\sqrt{bx-a}}dx-\frac{\sqrt{bx-a}}{x}$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}-\frac{\sqrt{bx-a}}{x}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

input
```
Int[Sqrt[-a + b*x]/x^2,x]
```

output
```
-(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{\sqrt{bx-a}}{x}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	35
risch	$\frac{-bx+a}{x\sqrt{bx-a}}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	40
derivativedivides	$2b\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	41
default	$2b\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	41

input
```
int((b*x-a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*x-a)^(1/2)/x+b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = \left[-\frac{\sqrt{-a}bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}a}{2ax},\ -\frac{\sqrt{a}bx\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)+\sqrt{bx-a}a}{ax}\right]$$

input
```
integrate((b*x-a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt
(b*x - a)*a)/(a*x), -(sqrt(a)*b*x*arctan(sqrt(a)/sqrt(b*x - a)) + sqrt(b*x
 - a)*a)/(a*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = \begin{cases} -\frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{\sqrt{x}}+\frac{ib\,\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{a}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b\,\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x-a)**(1/2)/x**2,x)
```

output
```
Piecewise((-I*sqrt(b)*sqrt(a/(b*x) - 1)/sqrt(x) + I*b*acosh(sqrt(a)/(sqrt(
b)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1), (a/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x
) + 1)) - sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sqrt(b)*s
qrt(x)))/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = \frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}}{x}$$

input
```
integrate((b*x-a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
b*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = b\left(\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}}{bx}\right)$$

input
```
integrate((b*x-a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*(arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)/(b*x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = \frac{b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{b\,x-a}}{x}$$

input
```
int((b*x - a)^(1/2)/x^2,x)
```

output
```
(b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2) - (b*x - a)^(1/2)/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = \frac{\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)bx-\sqrt{bx-a}\,a}{ax}$$

input
```
int((b*x-a)^(1/2)/x^2,x)
```

output
```
(sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*b*x - sqrt( - a + b*x)*a)/(a*x)
```

3.385 $\int \frac{\sqrt{-a+bx}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\sqrt{-a+bx}}{x^3} dx = -\frac{\sqrt{-a+bx}}{2x^2} + \frac{b\sqrt{-a+bx}}{4ax} + \frac{b^2 \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
-1/2*(b*x-a)^(1/2)/x^2+1/4*b*(b*x-a)^(1/2)/a/x+1/4*b^2*arctan((b*x-a)^(1/2
)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{-a+bx}}{x^3} dx = -\frac{(2a-bx)\sqrt{-a+bx}}{4ax^2} + \frac{b^2 \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[Sqrt[-a + b*x]/x^3,x]
```

output
```
-1/4*((2*a - b*x)*Sqrt[-a + b*x])/(a*x^2) + (b^2*ArcTan[Sqrt[-a + b*x]/Sqr
t[a]])/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {51, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx-a}}{x^3}dx$$

$$\Big\downarrow \ 51$$

$$\frac{1}{4}b\int \frac{1}{x^2\sqrt{bx-a}}dx-\frac{\sqrt{bx-a}}{2x^2}$$

$$\Big\downarrow \ 52$$

$$\frac{1}{4}b\left(\frac{b\int \frac{1}{x\sqrt{bx-a}}dx}{2a}+\frac{\sqrt{bx-a}}{ax}\right)-\frac{\sqrt{bx-a}}{2x^2}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}b\left(\frac{\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{a}+\frac{\sqrt{bx-a}}{ax}\right)-\frac{\sqrt{bx-a}}{2x^2}$$

$$\Big\downarrow \ 218$$

$$\frac{1}{4}b\left(\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}\right)-\frac{\sqrt{bx-a}}{2x^2}$$

input
```
Int[Sqrt[-a + b*x]/x^3,x]
```

output
```
-1/2*Sqrt[-a + b*x]/x^2 + (b*(Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]])/a^(3/2)))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{\left(-2a^{\frac{3}{2}}+\sqrt{a}\,bx\right)\sqrt{bx-a}+\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^{2}x^{2}}{4a^{\frac{3}{2}}x^{2}}$	53
risch	$\frac{(-bx+a)(-bx+2a)}{4x^{2}\sqrt{bx-a}\,a}+\frac{b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	55
derivativedivides	$2b^{2}\left(\frac{\frac{(bx-a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	59
default	$2b^{2}\left(\frac{\frac{(bx-a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	59

input
```
int((b*x-a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((-2*a^(3/2)+a^(1/2)*b*x)*(b*x-a)^(1/2)+arctan((b*x-a)^(1/2)/a^(1/2))*
b^2*x^2)/a^(3/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{-a+bx}}{x^3}\,dx = \left[-\frac{\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\left(abx-2a^2\right)\sqrt{bx-a}}{8a^2x^2}, -\frac{\sqrt{a}b^2x^2\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)-\left(abx-2a^2\right)\sqrt{bx-a}}{4a^2x^2}\right]$$

input
```
integrate((b*x-a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*
(a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2), -1/4*(sqrt(a)*b^2*x^2*arctan(sqr
t(a)/sqrt(b*x - a)) - (a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.92

$$\int \frac{\sqrt{-a+bx}}{x^3}\,dx = \begin{cases} -\frac{ia}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{3i\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{ib^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{a}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x-a)**(1/2)/x**3,x)
```

output
```
Piecewise((-I*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(4*x*
*(3/2)*sqrt(a/(b*x) - 1)) - I*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) - 1)) + I
*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), Abs(a/(b*x)) > 1), (a
/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(-a/(
b*x) + 1)) + b**(3/2)/(4*a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b**2*asin(sqrt(a)
/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-a+bx}}{x^3}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}} + \frac{(bx-a)^{\frac{3}{2}}b^2 - \sqrt{bx-a}ab^2}{4\left((bx-a)^2a + 2\,(bx-a)a^2 + a^3\right)}$$

input
```
integrate((b*x-a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + 1/4*((b*x - a)^(3/2)*b^2 -
 sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a + 2*(b*x - a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-a+bx}}{x^3}\,dx = \frac{\frac{b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{(bx-a)^{\frac{3}{2}}b^3 - \sqrt{bx-a}ab^3}{ab^2x^2}}{4\,b}$$

input
```
integrate((b*x-a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + ((b*x - a)^(3/2)*b^3 - sq
rt(b*x - a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{-a+bx}}{x^3}\,dx = \frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{3/2}} - \frac{\sqrt{b\,x-a}}{4\,x^2} + \frac{(b\,x-a)^{3/2}}{4\,a\,x^2}$$

input
```
int((b*x - a)^(1/2)/x^3,x)
```

output
```
(b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (b*x - a)^(1/2)/(4*x^2)
+ (b*x - a)^(3/2)/(4*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-a+bx}}{x^3}\,dx = \frac{\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^2x^2 - 2\sqrt{bx-a}\,a^2 + \sqrt{bx-a}\,abx}{4a^2x^2}$$

input
```
int((b*x-a)^(1/2)/x^3,x)
```

output
```
(sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*b**2*x**2 - 2*sqrt( - a + b*x)*a**
2 + sqrt( - a + b*x)*a*b*x)/(4*a**2*x**2)
```

3.386 $\int \frac{(-a+bx)^{3/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{(-a+bx)^{3/2}}{x} dx = -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + 2a^{3/2} \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

output
```
-2*a*(b*x-a)^(1/2)+2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2
))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{(-a+bx)^{3/2}}{x} dx = \frac{2}{3}(-4a+bx)\sqrt{-a+bx} + 2a^{3/2} \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(-a + b*x)^(3/2)/x,x]
```

output
```
(2*(-4*a + b*x)*Sqrt[-a + b*x])/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a
]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx-a)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{3}(bx-a)^{3/2}-a\int \frac{\sqrt{bx-a}}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{3}(bx-a)^{3/2}-a\left(2\sqrt{bx-a}-a\int \frac{1}{x\sqrt{bx-a}}dx\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}(bx-a)^{3/2}-a\left(2\sqrt{bx-a}-\frac{2a\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{2}{3}(bx-a)^{3/2}-a\left(2\sqrt{bx-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)$$

input
```
Int[(-a + b*x)^(3/2)/x,x]
```

output
```
(2*(-a + b*x)^(3/2))/3 - a*(2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a +
b*x]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$2a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - \frac{2\sqrt{bx-a}\,(-bx+4a)}{3}$	40
derivativedivides	$-2a\sqrt{bx-a} + \frac{2(bx-a)^{\frac{3}{2}}}{3} + 2a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	44
default	$-2a\sqrt{bx-a} + \frac{2(bx-a)^{\frac{3}{2}}}{3} + 2a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	44

input
```
int((b*x-a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2/3*(b*x-a)^(1/2)*(-b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \frac{(-a+bx)^{3/2}}{x}\,dx = \left[\sqrt{-a}a\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right) + \frac{2}{3}\sqrt{bx-a}(bx-4\,a), -2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)+\frac{2}{3}\sqrt{bx-a}(bx-4\,a)\right]$$

input
```
integrate((b*x-a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(-a)*a*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/3*sqrt(b*x -
 a)*(b*x - 4*a), -2*a^(3/2)*arctan(sqrt(a)/sqrt(b*x - a)) + 2/3*sqrt(b*x -
 a)*(b*x - 4*a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.40

$$\int \frac{(-a+bx)^{3/2}}{x}\,dx = \begin{cases} -\frac{8a^{\frac{3}{2}}\sqrt{-1+\frac{bx}{a}}}{3} - ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) + 2ia^{\frac{3}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - 2a^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2\sqrt{a}bx\sqrt{-1+\frac{bx}{a}}}{3} \\ -\frac{8ia^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{3} - ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) + 2ia^{\frac{3}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right) + \frac{2i\sqrt{a}bx\sqrt{1-\frac{bx}{a}}}{3} \end{cases}$$

input
```
integrate((b*x-a)**(3/2)/x,x)
```

output
```
Piecewise((-8*a**(3/2)*sqrt(-1 + b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a*
*(3/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) - 2*a**(3/2)*asin(sqrt(a)/(sqrt(b)*sqr
t(x))) + 2*sqrt(a)*b*x*sqrt(-1 + b*x/a)/3, Abs(b*x/a) > 1), (-8*I*a**(3/2)
*sqrt(1 - b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**(3/2)*log(sqrt(1 - b*x
/a) + 1) + 2*I*sqrt(a)*b*x*sqrt(1 - b*x/a)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{3/2}}{x}\,dx = 2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\sqrt{bx-a}a$$

input
```
integrate((b*x-a)^(3/2)/x,x, algorithm="maxima")
```

output
```
2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
 - a)*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{3/2}}{x}\,dx = 2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\sqrt{bx-a}a$$

input
```
integrate((b*x-a)^(3/2)/x,x, algorithm="giac")
```

output
```
2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
 - a)*a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{3/2}}{x}\,dx = 2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-2\,a\,\sqrt{b\,x-a}+\frac{2\,{\left(b\,x-a\right)}^{3/2}}{3}$$

input
```
int((b*x - a)^(3/2)/x,x)
```

output
```
2*a^(3/2)*atan((b*x - a)^(1/2)/a^(1/2)) - 2*a*(b*x - a)^(1/2) + (2*(b*x -
a)^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{3/2}}{x}\,dx = 2\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)a - \frac{8\sqrt{bx-a}\,a}{3} + \frac{2\sqrt{bx-a}\,bx}{3}$$

input
```
int((b*x-a)^(3/2)/x,x)
```

output
```
(2*(3*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*a - 4*sqrt( - a + b*x)*a + sq
rt( - a + b*x)*b*x))/3
```

3.387 $\int \frac{(-a+bx)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{(-a+bx)^{3/2}}{x^2} dx = 2b\sqrt{-a+bx} + \frac{a\sqrt{-a+bx}}{x} - 3\sqrt{a}b \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

output
```
2*b*(b*x-a)^(1/2)+a*(b*x-a)^(1/2)/x-3*a^(1/2)*b*arctan((b*x-a)^(1/2)/a^(1/
2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{(-a+bx)^{3/2}}{x^2} dx = \frac{\sqrt{-a+bx}(a+2bx)}{x} - 3\sqrt{a}b \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(-a + b*x)^(3/2)/x^2,x]
```

output
```
(Sqrt[-a + b*x]*(a + 2*b*x))/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]
]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx-a)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{2}b\int \frac{\sqrt{bx-a}}{x}dx-\frac{(bx-a)^{3/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}b\left(2\sqrt{bx-a}-a\int \frac{1}{x\sqrt{bx-a}}dx\right)-\frac{(bx-a)^{3/2}}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{3}{2}b\left(2\sqrt{bx-a}-\frac{2a\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{b}\right)-\frac{(bx-a)^{3/2}}{x}$$

$$\Big\downarrow \text{218}$$

$$\frac{3}{2}b\left(2\sqrt{bx-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)-\frac{(bx-a)^{3/2}}{x}$$

input
```
Int[(-a + b*x)^(3/2)/x^2,x]
```

output
```
-((-a + b*x)^(3/2)/x) + (3*b*(2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]]))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{(2bx+a)\sqrt{bx-a}-3\sqrt{a}\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)x}{x}$	43
derivativedivides	$2b\left(\sqrt{bx-a}-a\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	54
default	$2b\left(\sqrt{bx-a}-a\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	54
risch	$-\frac{a(-bx+a)}{x\sqrt{bx-a}}-3\sqrt{a}\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2b\sqrt{bx-a}$	55

input
```
int((b*x-a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((2*b*x+a)*(b*x-a)^(1/2)-3*a^(1/2)*b*arctan((b*x-a)^(1/2)/a^(1/2))*x)/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.81

$$\int \frac{(-a+bx)^{3/2}}{x^2}\,dx = \left[\frac{3\sqrt{-a}bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2(2bx+a)\sqrt{bx-a}}{2x}, \frac{3\sqrt{a}bx\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)+ \right.$$

input
```
integrate((b*x-a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(3*sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(2*
b*x + a)*sqrt(b*x - a))/x, (3*sqrt(a)*b*x*arctan(sqrt(a)/sqrt(b*x - a)) +
(2*b*x + a)*sqrt(b*x - a))/x]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.46

$$\int \frac{(-a+bx)^{3/2}}{x^2}\,dx = \begin{cases} -3i\sqrt{a}b\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{ia^2}{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{ia\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ 3\sqrt{a}b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{a\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x-a)**(3/2)/x**2,x)
```

output
```
Piecewise((-3*I*sqrt(a)*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + I*a**2/(sqrt(
b)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) - 1)) -
 2*I*b**(3/2)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (3*sqrt(a)*b*a
sin(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)
) - a*sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)/sqrt(-a/(b
*x) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{(-a+bx)^{3/2}}{x^2}\,dx = -3\sqrt{a}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}b+\frac{\sqrt{bx-a}a}{x}$$

input
```
integrate((b*x-a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-3*sqrt(a)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b + sqrt(b*x
- a)*a/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{(-a+bx)^{3/2}}{x^2}\,dx = -\left(3\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\sqrt{bx-a}-\frac{\sqrt{bx-a}a}{bx}\right)b$$

input
```
integrate((b*x-a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
-(3*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) - 2*sqrt(b*x - a) - sqrt(b*x - a
)*a/(b*x))*b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{(-a+bx)^{3/2}}{x^2}\,dx = 2\,b\,\sqrt{b\,x-a}+\frac{a\,\sqrt{b\,x-a}}{x}-3\,\sqrt{a}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

input
```
int((b*x - a)^(3/2)/x^2,x)
```

output
```
2*b*(b*x - a)^(1/2) + (a*(b*x - a)^(1/2))/x - 3*a^(1/2)*b*atan((b*x - a)^(
1/2)/a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{(-a+bx)^{3/2}}{x^2}\,dx = \frac{-3\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)bx+\sqrt{bx-a}\,a+2\sqrt{bx-a}\,bx}{x}$$

input
```
int((b*x-a)^(3/2)/x^2,x)
```

output
```
( - 3*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*b*x + sqrt( - a + b*x)*a + 2*
sqrt( - a + b*x)*b*x)/x
```

3.388 $\int \frac{(-a+bx)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{(-a+bx)^{3/2}}{x^3} dx = \frac{a\sqrt{-a+bx}}{2x^2} - \frac{5b\sqrt{-a+bx}}{4x} + \frac{3b^2 \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
1/2*a*(b*x-a)^(1/2)/x^2-5/4*b*(b*x-a)^(1/2)/x+3/4*b^2*arctan((b*x-a)^(1/2)
/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{(-a+bx)^{3/2}}{x^3} dx = \frac{1}{4}\left(\frac{(2a-5bx)\sqrt{-a+bx}}{x^2} + \frac{3b^2 \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Integrate[(-a + b*x)^(3/2)/x^3,x]
```

output
```
(((2*a - 5*b*x)*Sqrt[-a + b*x])/x^2 + (3*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]
])/Sqrt[a])/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {51, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx-a)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{4}b\int \frac{\sqrt{bx-a}}{x^2}dx-\frac{(bx-a)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x\sqrt{bx-a}}dx-\frac{\sqrt{bx-a}}{x}\right)-\frac{(bx-a)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{3}{4}b\left(\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}-\frac{\sqrt{bx-a}}{x}\right)-\frac{(bx-a)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3}{4}b\left(\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}\right)-\frac{(bx-a)^{3/2}}{2x^2}$$

input
```
Int[(-a + b*x)^(3/2)/x^3,x]
```

output
```
-1/2*(-a + b*x)^(3/2)/x^2 + (3*b*(-(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/Sqrt[a]))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{(-bx+a)(-5bx+2a)}{4x^2\sqrt{bx-a}}+\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}$	52
derivativedivides	$2b^2\left(\frac{-\frac{5(bx-a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	57
default	$2b^2\left(\frac{-\frac{5(bx-a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	57
pseudoelliptic	$\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^2x^2-5bx\sqrt{a}\,\sqrt{bx-a}+2a^{\frac{3}{2}}\sqrt{bx-a}}{4x^2\sqrt{a}}$	62

input
```
int((b*x-a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b*x+a)*(-5*b*x+2*a)/x^2/(b*x-a)^(1/2)+3/4*b^2*arctan((b*x-a)^(1/2)/
a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.86

$$\int \frac{(-a+bx)^{3/2}}{x^3}\,dx = \left[-\frac{3\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(5abx-2a^2\right)\sqrt{bx-a}}{8ax^2}, -\frac{3\sqrt{a}b^2x^2\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)+\left(5abx-2a^2\right)\sqrt{bx-a}}{4ax^2}\right]$$

input
```
integrate((b*x-a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) +
2*(5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2), -1/4*(3*sqrt(a)*b^2*x^2*arctan
(sqrt(a)/sqrt(b*x - a)) + (5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.74

$$\int \frac{(-a+bx)^{3/2}}{x^3}\,dx = \begin{cases} \frac{ia\sqrt{b}\sqrt{\frac{a}{bx}-1}}{2x^{\frac{3}{2}}} - \frac{5ib^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{4\sqrt{x}} + \frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{a^2}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{7a\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{5b^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x-a)**(3/2)/x**3,x)
```

output
```
Piecewise((I*a*sqrt(b)*sqrt(a/(b*x) - 1)/(2*x**(3/2)) - 5*I*b**(3/2)*sqrt(
a/(b*x) - 1)/(4*sqrt(x)) + 3*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sq
rt(a)), Abs(a/(b*x)) > 1), (-a**2/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1))
+ 7*a*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) - 5*b**(3/2)/(4*sqrt(x)*sqrt
(-a/(b*x) + 1)) - 3*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{(-a+bx)^{3/2}}{x^3}\,dx = \frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}} - \frac{5\,(bx-a)^{\frac{3}{2}}b^2 + 3\,\sqrt{bx-a}ab^2}{4\,\left((bx-a)^2 + 2\,(bx-a)a + a^2\right)}$$

input
```
integrate((b*x-a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - 1/4*(5*(b*x - a)^(3/2)*b^2
 + 3*sqrt(b*x - a)*a*b^2)/((b*x - a)^2 + 2*(b*x - a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{(-a+bx)^{3/2}}{x^3}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{5\,(bx-a)^{\frac{3}{2}}b^3 + 3\,\sqrt{bx-a}ab^3}{b^2x^2}}{4\,b}$$

input
```
integrate((b*x-a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - (5*(b*x - a)^(3/2)*b^3
+ 3*sqrt(b*x - a)*a*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{(-a+bx)^{3/2}}{x^3}\,dx = \frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}} - \frac{5\,(b\,x-a)^{3/2}}{4\,x^2} - \frac{3\,a\,\sqrt{b\,x-a}}{4\,x^2}$$

input
```
int((b*x - a)^(3/2)/x^3,x)
```

output
```
(3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(1/2)) - (5*(b*x - a)^(3/2))/(4
*x^2) - (3*a*(b*x - a)^(1/2))/(4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(-a+bx)^{3/2}}{x^3}\,dx = \frac{3\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^2x^2+2\sqrt{bx-a}\,a^2-5\sqrt{bx-a}\,abx}{4a\,x^2}$$

input
```
int((b*x-a)^(3/2)/x^3,x)
```

output
```
(3*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*b**2*x**2 + 2*sqrt( - a + b*x)*a
**2 - 5*sqrt( - a + b*x)*a*b*x)/(4*a*x**2)
```

3.389 $\int \frac{(-a+bx)^{5/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{(-a+bx)^{5/2}}{x} dx = 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - 2a^{5/2}\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

output
```
2*a^2*(b*x-a)^(1/2)-2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan
((b*x-a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(-a+bx)^{5/2}}{x} dx = \frac{2}{15}\sqrt{-a+bx}(23a^2 - 11abx + 3b^2x^2) - 2a^{5/2}\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(-a + b*x)^(5/2)/x,x]
```

output
```
(2*Sqrt[-a + b*x]*(23*a^2 - 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTan[S
qrt[-a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx-a)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{5}(bx-a)^{5/2}-a\int \frac{(bx-a)^{3/2}}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{5}(bx-a)^{5/2}-a\left(\frac{2}{3}(bx-a)^{3/2}-a\int \frac{\sqrt{bx-a}}{x}dx\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{5}(bx-a)^{5/2}-a\left(\frac{2}{3}(bx-a)^{3/2}-a\left(2\sqrt{bx-a}-a\int \frac{1}{x\sqrt{bx-a}}dx\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{5}(bx-a)^{5/2}-a\left(\frac{2}{3}(bx-a)^{3/2}-a\left(2\sqrt{bx-a}-\frac{2a\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{b}\right)\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{2}{5}(bx-a)^{5/2}-a\left(\frac{2}{3}(bx-a)^{3/2}-a\left(2\sqrt{bx-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)\right)$$

input
```
Int[(-a + b*x)^(5/2)/x,x]
```

output
```
(2*(-a + b*x)^(5/2))/5 - a*((2*(-a + b*x)^(3/2))/3 - a*(2*Sqrt[-a + b*x] -
 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$-2a^{\frac{5}{2}} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx-a}\,(3b^2x^2-11abx+23a^2)}{15}$	51
derivativedivides	$2a^2\sqrt{bx-a}-\frac{2a(bx-a)^{\frac{3}{2}}}{3}+\frac{2(bx-a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	58
default	$2a^2\sqrt{bx-a}-\frac{2a(bx-a)^{\frac{3}{2}}}{3}+\frac{2(bx-a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	58

input
```
int((b*x-a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/2))+2/15*(b*x-a)^(1/2)*(3*b^2*x^2-11*
a*b*x+23*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{(-a+bx)^{5/2}}{x}\,dx = \left[\sqrt{-a}a^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right) + \frac{2}{15}\left(3b^2x^2-11abx+23a^2\right)\sqrt{bx-a}, 2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right) + \frac{2}{15}\left(3b^2x^2-11abx+23a^2\right)\sqrt{bx-a}\right]$$

input
```
integrate((b*x-a)^(5/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(-a)*a^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/15*(3*b^2*
x^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a), 2*a^(5/2)*arctan(sqrt(a)/sqrt(b*x
- a)) + 2/15*(3*b^2*x^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.29

$$\int \frac{(-a+bx)^{5/2}}{x}\,dx = \begin{cases} \frac{46a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{15} + ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right) - 2ia^{\frac{5}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + 2a^{\frac{5}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{22a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}}{15} \\ \frac{46ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{15} + ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right) - 2ia^{\frac{5}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right) - \frac{22ia^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}{15} + \frac{2i\sqrt{a}b^2x^2\sqrt{1-}}{5} \end{cases}$$

input
```
integrate((b*x-a)**(5/2)/x,x)
```

output
```
Piecewise((46*a**(5/2)*sqrt(-1 + b*x/a)/15 + I*a**(5/2)*log(b*x/a) - 2*I*a
**(5/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) + 2*a**(5/2)*asin(sqrt(a)/(sqrt(b)*sq
rt(x))) - 22*a**(3/2)*b*x*sqrt(-1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*sqrt(-
1 + b*x/a)/5, Abs(b*x/a) > 1), (46*I*a**(5/2)*sqrt(1 - b*x/a)/15 + I*a**(5
/2)*log(b*x/a) - 2*I*a**(5/2)*log(sqrt(1 - b*x/a) + 1) - 22*I*a**(3/2)*b*x
*sqrt(1 - b*x/a)/15 + 2*I*sqrt(a)*b**2*x**2*sqrt(1 - b*x/a)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{5/2}}{x}\,dx = -2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}a^2$$

input
```
integrate((b*x-a)^(5/2)/x,x, algorithm="maxima")
```

output
```
-2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x
- a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{5/2}}{x}\,dx = -2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}a^2$$

input
```
integrate((b*x-a)^(5/2)/x,x, algorithm="giac")
```

output
```
-2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x
- a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{5/2}}{x}\,dx = \frac{2\,(b\,x-a)^{5/2}}{5} - \frac{2\,a\,(b\,x-a)^{3/2}}{3} - 2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right) + 2\,a^2\,\sqrt{b\,x-a}$$

input
```
int((b*x - a)^(5/2)/x,x)
```

output
```
(2*(b*x - a)^(5/2))/5 - (2*a*(b*x - a)^(3/2))/3 - 2*a^(5/2)*atan((b*x - a)
^(1/2)/a^(1/2)) + 2*a^2*(b*x - a)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{(-a+bx)^{5/2}}{x}\,dx = -2\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)a^2 + \frac{46\sqrt{bx-a}\,a^2}{15} - \frac{22\sqrt{bx-a}\,abx}{15} + \frac{2\sqrt{bx-a}\,b^2x^2}{5}$$

input
```
int((b*x-a)^(5/2)/x,x)
```

output
```
(2*( - 15*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*a**2 + 23*sqrt( - a + b*x
)*a**2 - 11*sqrt( - a + b*x)*a*b*x + 3*sqrt( - a + b*x)*b**2*x**2))/15
```

3.390 $\int \frac{(-a+bx)^{5/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{(-a+bx)^{5/2}}{x^2} dx = -4ab\sqrt{-a+bx} - \frac{a^2\sqrt{-a+bx}}{x} + \frac{2}{3}b(-a+bx)^{3/2} + 5a^{3/2}b \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

output
```
-4*a*b*(b*x-a)^(1/2)-a^2*(b*x-a)^(1/2)/x+2/3*b*(b*x-a)^(3/2)+5*a^(3/2)*b*a
rctan((b*x-a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{(-a+bx)^{5/2}}{x^2} dx = -\frac{\sqrt{-a+bx}(3a^2+14abx-2b^2x^2)}{3x} + 5a^{3/2}b \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(-a + b*x)^(5/2)/x^2,x]
```

output
```
-1/3*(Sqrt[-a + b*x]*(3*a^2 + 14*a*b*x - 2*b^2*x^2))/x + 5*a^(3/2)*b*ArcTa
n[Sqrt[-a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx-a)^{5/2}}{x^2} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{2} b \int \frac{(bx-a)^{3/2}}{x} dx - \frac{(bx-a)^{5/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{2} b \left(\frac{2}{3} (bx-a)^{3/2} - a \int \frac{\sqrt{bx-a}}{x} dx\right) - \frac{(bx-a)^{5/2}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{2} b \left(\frac{2}{3} (bx-a)^{3/2} - a \left(2\sqrt{bx-a} - a \int \frac{1}{x\sqrt{bx-a}} dx\right)\right) - \frac{(bx-a)^{5/2}}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{5}{2} b \left(\frac{2}{3} (bx-a)^{3/2} - a \left(2\sqrt{bx-a} - \frac{2a \int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}} d\sqrt{bx-a}}{b}\right)\right) - \frac{(bx-a)^{5/2}}{x}$$

$$\Big\downarrow \text{218}$$

$$\frac{5}{2} b \left(\frac{2}{3} (bx-a)^{3/2} - a \left(2\sqrt{bx-a} - 2\sqrt{a} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)\right) - \frac{(bx-a)^{5/2}}{x}$$

input
```
Int[(-a + b*x)^(5/2)/x^2,x]
```

output
```
-((-a + b*x)^(5/2)/x) + (5*b*((2*(-a + b*x)^(3/2))/3 - a*(2*Sqrt[-a + b*x]
 - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$\frac{5a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)x-\left(-\frac{2}{3}b^{2}x^{2}+\frac{14}{3}abx+a^{2}\right)\sqrt{bx-a}}{x}$	55
derivativedivides	$2b\left(\frac{(bx-a)^{\frac{3}{2}}}{3}-2a\sqrt{bx-a}+a^{2}\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	69
default	$2b\left(\frac{(bx-a)^{\frac{3}{2}}}{3}-2a\sqrt{bx-a}+a^{2}\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	69
risch	$\frac{a^{2}(-bx+a)}{x\sqrt{bx-a}}+\frac{2b(bx-a)^{\frac{3}{2}}}{3}-4ab\sqrt{bx-a}+5a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	69

input
```
int((b*x-a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(5*a^(3/2)*b*arctan((b*x-a)^(1/2)/a^(1/2))*x-(-2/3*b^2*x^2+14/3*a*b*x+a^2)
*(b*x-a)^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

$$\int \frac{(-a+bx)^{5/2}}{x^2}\,dx = \left[\frac{15\sqrt{-a}abx\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{6x},\right.$$
$$\left.-\frac{15a^{\frac{3}{2}}bx\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)-\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{3x}\right]$$

input
```
integrate((b*x-a)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[1/6*(15*sqrt(-a)*a*b*x*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*
(2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x, -1/3*(15*a^(3/2)*b*x*arct
an(sqrt(a)/sqrt(b*x - a)) - (2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/
x]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.18

$$\int \frac{(-a+bx)^{5/2}}{x^2}\,dx = \begin{cases} -\frac{a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{x}-\frac{14a^{\frac{3}{2}}b\sqrt{-1+\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-5a^{\frac{3}{2}}b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) \\ -\frac{ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{x}-\frac{14ia^{\frac{3}{2}}b\sqrt{1-\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\sqrt{1-\frac{bx}{a}}+1\right)+\frac{2i\sqrt{a}b^2x\sqrt{1-}}{3} \end{cases}$$

input
```
integrate((b*x-a)**(5/2)/x**2,x)
```

output
```
Piecewise((-a**(5/2)*sqrt(-1 + b*x/a)/x - 14*a**(3/2)*b*sqrt(-1 + b*x/a)/3
 - 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqrt(b)*sqrt(x)/sqrt(a
)) - 5*a**(3/2)*b*asin(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*sqrt(a)*b**2*x*sqrt(
-1 + b*x/a)/3, Abs(b*x/a) > 1), (-I*a**(5/2)*sqrt(1 - b*x/a)/x - 14*I*a**(
3/2)*b*sqrt(1 - b*x/a)/3 - 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*lo
g(sqrt(1 - b*x/a) + 1) + 2*I*sqrt(a)*b**2*x*sqrt(1 - b*x/a)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{(-a+bx)^{5/2}}{x^2}\,dx = 5\,a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}b-4\,\sqrt{bx-a}ab-\frac{\sqrt{bx-a}a^2}{x}$$

input
```
integrate((b*x-a)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
5*a^(3/2)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2)*b - 4*sqrt
(b*x - a)*a*b - sqrt(b*x - a)*a^2/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{(-a+bx)^{5/2}}{x^2}\,dx = \frac{1}{3}\left(15\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\,(bx-a)^{\frac{3}{2}}-12\,\sqrt{bx-a}a-\frac{3\,\sqrt{bx-a}a^2}{bx}\right)b$$

input
```
integrate((b*x-a)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/3*(15*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*(b*x - a)^(3/2) - 12*sqr
t(b*x - a)*a - 3*sqrt(b*x - a)*a^2/(b*x))*b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{(-a+bx)^{5/2}}{x^2}\,dx = \frac{2\,b\,(b\,x-a)^{3/2}}{3}-\frac{a^2\,\sqrt{b\,x-a}}{x}+5\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-4\,a\,b\,\sqrt{b\,x-a}$$

input
```
int((b*x - a)^(5/2)/x^2,x)
```

output
```
(2*b*(b*x - a)^(3/2))/3 - (a^2*(b*x - a)^(1/2))/x + 5*a^(3/2)*b*atan((b*x
- a)^(1/2)/a^(1/2)) - 4*a*b*(b*x - a)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{(-a+bx)^{5/2}}{x^2}\,dx = \frac{15\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)abx-3\sqrt{bx-a}\,a^2-14\sqrt{bx-a}\,abx+2\sqrt{bx-a}\,b^2x^2}{3x}$$

input
```
int((b*x-a)^(5/2)/x^2,x)
```

output
```
(15*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*a*b*x - 3*sqrt( - a + b*x)*a**2
 - 14*sqrt( - a + b*x)*a*b*x + 2*sqrt( - a + b*x)*b**2*x**2)/(3*x)
```

3.391 $\int \frac{(-a+bx)^{5/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{(-a+bx)^{5/2}}{x^3} dx = 2b^2\sqrt{-a+bx} - \frac{a^2\sqrt{-a+bx}}{2x^2} + \frac{9ab\sqrt{-a+bx}}{4x} - \frac{15}{4}\sqrt{a}b^2 \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

output
```
2*b^2*(b*x-a)^(1/2)-1/2*a^2*(b*x-a)^(1/2)/x^2+9/4*a*b*(b*x-a)^(1/2)/x-15/4
*a^(1/2)*b^2*arctan((b*x-a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \frac{(-a+bx)^{5/2}}{x^3} dx = \frac{1}{4}\left(\frac{\sqrt{-a+bx}(-2a^2+9abx+8b^2x^2)}{x^2} - 15\sqrt{a}b^2 \arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[(-a + b*x)^(5/2)/x^3,x]
```

output
```
((Sqrt[-a + b*x]*(-2*a^2 + 9*a*b*x + 8*b^2*x^2))/x^2 - 15*Sqrt[a]*b^2*ArcT
an[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {51, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx-a)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{4}b\int \frac{(bx-a)^{3/2}}{x^2}dx-\frac{(bx-a)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\int \frac{\sqrt{bx-a}}{x}dx-\frac{(bx-a)^{3/2}}{x}\right)-\frac{(bx-a)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{bx-a}-a\int \frac{1}{x\sqrt{bx-a}}dx\right)-\frac{(bx-a)^{3/2}}{x}\right)-\frac{(bx-a)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{bx-a}-\frac{2a\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{b}\right)-\frac{(bx-a)^{3/2}}{x}\right)-\frac{(bx-a)^{5/2}}{2x^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{bx-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)-\frac{(bx-a)^{3/2}}{x}\right)-\frac{(bx-a)^{5/2}}{2x^2}$$

input
```
Int[(-a + b*x)^(5/2)/x^3,x]
```

output
```
-1/2*(-a + b*x)^(5/2)/x^2 + (5*b*(-((-a + b*x)^(3/2)/x) + (3*b*(2*Sqrt[-a
+ b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]))/2))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{-15\sqrt{a}\,b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)x^{2}-\left(-8b^{2}x^{2}-9abx+2a^{2}\right)\sqrt{bx-a}}{4x^{2}}$	62
risch	$\frac{a(-bx+a)(-9bx+2a)}{4x^{2}\sqrt{bx-a}}-\frac{15\sqrt{a}\,b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4}+2b^{2}\sqrt{bx-a}$	67
derivativedivides	$2b^{2}\left(\sqrt{bx-a}-a\left(\frac{-\frac{9(bx-a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	70
default	$2b^{2}\left(\sqrt{bx-a}-a\left(\frac{-\frac{9(bx-a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	70

input
```
int((b*x-a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-15*a^(1/2)*b^2*arctan((b*x-a)^(1/2)/a^(1/2))*x^2-(-8*b^2*x^2-9*a*b*x
+2*a^2)*(b*x-a)^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.57

$$\int \frac{(-a+bx)^{5/2}}{x^3}\,dx = \left[\frac{15\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(8b^2x^2+9abx-2a^2\right)\sqrt{bx-a}}{8x^2}, \frac{15\sqrt{a}b^2x^2}{}\right.$$

input
```
integrate((b*x-a)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(15*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) +
2*(8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a))/x^2, 1/4*(15*sqrt(a)*b^2*x^
2*arctan(sqrt(a)/sqrt(b*x - a)) + (8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x -
 a))/x^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.03

$$\int \frac{(-a+bx)^{5/2}}{x^3}\,dx = \begin{cases} -\frac{15i\sqrt{a}b^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4} - \frac{ia^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{11ia^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{iab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{2ib^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > \\ \frac{15\sqrt{a}b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4} + \frac{a^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x-a)**(5/2)/x**3,x)
```

output
```
Piecewise((-15*I*sqrt(a)*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - I*a**3/
(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 11*I*a**2*sqrt(b)/(4*x**(3/2)*sqr
t(a/(b*x) - 1)) - I*a*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) - 2*I*b**(5/2
)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (15*sqrt(a)*b**2*asin(sqrt
(a)/(sqrt(b)*sqrt(x)))/4 + a**3/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) -
11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) + a*b**(3/2)/(4*sqrt(x)*sq
rt(-a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{(-a+bx)^{5/2}}{x^3}\,dx = -\frac{15}{4}\sqrt{a}b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\sqrt{bx-a}b^2 + \frac{9\,(bx-a)^{\frac{3}{2}}ab^2 + 7\sqrt{bx-a}a^2b^2}{4\left((bx-a)^2 + 2\,(bx-a)a + a^2\right)}$$

input
```
integrate((b*x-a)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
-15/4*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b^2 + 1/
4*(9*(b*x - a)^(3/2)*a*b^2 + 7*sqrt(b*x - a)*a^2*b^2)/((b*x - a)^2 + 2*(b*
x - a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{(-a+bx)^{5/2}}{x^3}\,dx = -\frac{15\sqrt{a}b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-8\sqrt{bx-a}b^3-\frac{9(bx-a)^{\frac{3}{2}}ab^3+7\sqrt{bx-a}a^2b^3}{b^2x^2}}{4b}$$

input
```
integrate((b*x-a)^(5/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*(15*sqrt(a)*b^3*arctan(sqrt(b*x - a)/sqrt(a)) - 8*sqrt(b*x - a)*b^3 -
 (9*(b*x - a)^(3/2)*a*b^3 + 7*sqrt(b*x - a)*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{(-a+bx)^{5/2}}{x^3}\,dx = 2b^2\sqrt{bx-a} - \frac{15\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4} + \frac{9a(bx-a)^{3/2}}{4x^2} + \frac{7a^2\sqrt{bx-a}}{4x^2}$$

input
```
int((b*x - a)^(5/2)/x^3,x)
```

output
```
2*b^2*(b*x - a)^(1/2) - (15*a^(1/2)*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/4 +
 (9*a*(b*x - a)^(3/2))/(4*x^2) + (7*a^2*(b*x - a)^(1/2))/(4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{(-a+bx)^{5/2}}{x^3}\,dx = \frac{-15\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^2x^2-2\sqrt{bx-a}\,a^2+9\sqrt{bx-a}\,abx+8\sqrt{bx-a}\,b^2x^2}{4x^2}$$

input
```
int((b*x-a)^(5/2)/x^3,x)
```

output
```
( - 15*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*b**2*x**2 - 2*sqrt( - a + b*
x)*a**2 + 9*sqrt( - a + b*x)*a*b*x + 8*sqrt( - a + b*x)*b**2*x**2)/(4*x**2
)
```

3.392 $\int \frac{x^4}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{x^4}{\sqrt{a+bx}} dx = \frac{2a^4\sqrt{a+bx}}{b^5} - \frac{8a^3(a+bx)^{3/2}}{3b^5} + \frac{12a^2(a+bx)^{5/2}}{5b^5} - \frac{8a(a+bx)^{7/2}}{7b^5} + \frac{2(a+bx)^{9/2}}{9b^5}$$

output
```
2*a^4*(b*x+a)^(1/2)/b^5-8/3*a^3*(b*x+a)^(3/2)/b^5+12/5*a^2*(b*x+a)^(5/2)/b
^5-8/7*a*(b*x+a)^(7/2)/b^5+2/9*(b*x+a)^(9/2)/b^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}(128a^4 - 64a^3bx + 48a^2b^2x^2 - 40ab^3x^3 + 35b^4x^4)}{315b^5}$$

input
```
Integrate[x^4/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(128*a^4 - 64*a^3*b*x + 48*a^2*b^2*x^2 - 40*a*b^3*x^3 + 3
5*b^4*x^4))/(315*b^5)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^4}{b^4\sqrt{a+bx}}-\frac{4a^3\sqrt{a+bx}}{b^4}+\frac{6a^2(a+bx)^{3/2}}{b^4}-\frac{4a(a+bx)^{5/2}}{b^4}+\frac{(a+bx)^{7/2}}{b^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{9/2}}{9b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}$$

input
```
Int[x^4/Sqrt[a + b*x],x]
```

output
```
(2*a^4*Sqrt[a + b*x])/b^5 - (8*a^3*(a + b*x)^(3/2))/(3*b^5) + (12*a^2*(a +
 b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))/(7*b^5) + (2*(a + b*x)^(9/2))
/(9*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,x^3b^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
trager	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,x^3b^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
risch	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,x^3b^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,x^3b^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
orering	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,x^3b^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{8a(bx+a)^{\frac{7}{2}}}{7}+\frac{12a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{3}{2}}}{3}+2a^4\sqrt{bx+a}}{b^5}$	61
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{8a(bx+a)^{\frac{7}{2}}}{7}+\frac{12a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{3}{2}}}{3}+2a^4\sqrt{bx+a}}{b^5}$	61

input
```
int(x^4/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(b*x+a)^(1/2)*(35*b^4*x^4-40*a*b^3*x^3+48*a^2*b^2*x^2-64*a^3*b*x+128
*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt{a+bx}}\,dx = \frac{2\,(35\,b^4x^4-40\,ab^3x^3+48\,a^2b^2x^2-64\,a^3bx+128\,a^4)\sqrt{bx+a}}{315\,b^5}$$

input
```
integrate(x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*
sqrt(b*x + a)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3755 vs. 2(85) = 170.

Time = 1.99 (sec) , antiderivative size = 3755, normalized size of antiderivative = 42.19

$$\int \frac{x^4}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(b*x+a)**(1/2),x)
```

output
```
256*a**(89/2)*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*
a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a*
*35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a
**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**
(89/2)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800
*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*
a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*
a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 2432*a**(87/2)*b*x*sqrt(1 + b*
x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a
**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a*
*34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a*
*31*b**14*x**9 + 315*a**30*b**15*x**10) - 2560*a**(87/2)*b*x/(315*a**40*b*
*5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 6
6150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 3
7800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 3
15*a**30*b**15*x**10) + 10336*a**(85/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a**
40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**
3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**
6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**
9 + 315*a**30*b**15*x**10) - 11520*a**(85/2)*b**2*x**2/(315*a**40*b**5 ...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^5} - \frac{8\,(bx+a)^{\frac{7}{2}}a}{7\,b^5} + \frac{12\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^5} - \frac{8\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^5} + \frac{2\,\sqrt{bx+a}a^4}{b^5}$$

input
```
integrate(x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x + a)^(9/2)/b^5 - 8/7*(b*x + a)^(7/2)*a/b^5 + 12/5*(b*x + a)^(5/2)
*a^2/b^5 - 8/3*(b*x + a)^(3/2)*a^3/b^5 + 2*sqrt(b*x + a)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{\sqrt{a+bx}}\,dx = \frac{2\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx+a}a^4\right)}{315\,b^5}$$

input
```
integrate(x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^
2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{\sqrt{a+bx}}\,dx = \frac{2\,(a+b\,x)^{9/2}}{9\,b^5}+\frac{2\,a^4\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a^3\,(a+b\,x)^{3/2}}{3\,b^5}+\frac{12\,a^2\,(a+b\,x)^{5/2}}{5\,b^5}-\frac{8\,a\,(a+b\,x)^{7/2}}{7\,b^5}$$

input
```
int(x^4/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(9/2))/(9*b^5) + (2*a^4*(a + b*x)^(1/2))/b^5 - (8*a^3*(a + b*
x)^(3/2))/(3*b^5) + (12*a^2*(a + b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2
))/(7*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{x^4}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$$

input
```
int(x^4/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**4 - 64*a**3*b*x + 48*a**2*b**2*x**2 - 40*a*b**3*x
**3 + 35*b**4*x**4))/(315*b**5)
```

3.393 $\int \frac{x^3}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx=-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{7/2}}{7b^4}$$

output
```
-2*a^3*(b*x+a)^(1/2)/b^4+2*a^2*(b*x+a)^(3/2)/b^4-6/5*a*(b*x+a)^(5/2)/b^4+2
/7*(b*x+a)^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx=\frac{2\sqrt{a+bx}(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4}$$

input
```
Integrate[x^3/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{7/2}}{7b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}$$

input
```
Int[x^3/Sqrt[a + b*x],x]
```

output
```
(-2*a^3*Sqrt[a + b*x])/b^4 + (2*a^2*(a + b*x)^(3/2))/b^4 - (6*a*(a + b*x)^
(5/2))/(5*b^4) + (2*(a + b*x)^(7/2))/(7*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)}{35b^4}$	43
trager	$-\frac{2\sqrt{bx+a}\,(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)}{35b^4}$	43
risch	$-\frac{2\sqrt{bx+a}\,(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)}{35b^4}$	43
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)}{35b^4}$	43
orering	$-\frac{2\sqrt{bx+a}\,(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3)}{35b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{6a(bx+a)^{\frac{5}{2}}}{5}+2a^2(bx+a)^{\frac{3}{2}}-2a^3\sqrt{bx+a}}{b^4}$	49
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{6a(bx+a)^{\frac{5}{2}}}{5}+2a^2(bx+a)^{\frac{3}{2}}-2a^3\sqrt{bx+a}}{b^4}$	49

input
```
int(x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(1/2)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx = \frac{2\,(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3)\sqrt{bx+a}}{35\,b^4}$$

input
```
integrate(x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. 2(65) = 130.

Time = 1.23 (sec) , antiderivative size = 1640, normalized size of antiderivative = 24.12

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(b*x+a)**(1/2),x)
```

output
```
-32*a**(47/2)*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x
**5 + 35*a**14*b**10*x**6) + 32*a**(47/2)/(35*a**20*b**4 + 210*a**19*b**5*
x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*
a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 176*a**(45/2)*b*x*sqrt(1 + b*x/a)
/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*
x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) +
192*a**(45/2)*b*x/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2
+ 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**
14*b**10*x**6) - 396*a**(43/2)*b**2*x**2*sqrt(1 + b*x/a)/(35*a**20*b**4 +
210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b
**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 480*a**(43/2)*b**2
*x**2/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*
b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**
6) - 462*a**(41/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b*
*5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 2
10*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 640*a**(41/2)*b**3*x**3/(35*a*
*20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 280*a**
(39/2)*b**4*x**4*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 52...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^4} - \frac{6\,(bx+a)^{\frac{5}{2}}a}{5\,b^4} + \frac{2\,(bx+a)^{\frac{3}{2}}a^2}{b^4} - \frac{2\,\sqrt{bx+a}a^3}{b^4}$$

input
```
integrate(x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/7*(b*x + a)^(7/2)/b^4 - 6/5*(b*x + a)^(5/2)*a/b^4 + 2*(b*x + a)^(3/2)*a^
2/b^4 - 2*sqrt(b*x + a)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx = \frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}a^3\right)}{35\,b^4}$$

input
```
integrate(x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 -
35*sqrt(b*x + a)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx = \frac{2\,(a+b\,x)^{7/2}}{7\,b^4} - \frac{2\,a^3\,\sqrt{a+b\,x}}{b^4} + \frac{2\,a^2\,(a+b\,x)^{3/2}}{b^4} - \frac{6\,a\,(a+b\,x)^{5/2}}{5\,b^4}$$

input
```
int(x^3/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(7/2))/(7*b^4) - (2*a^3*(a + b*x)^(1/2))/b^4 + (2*a^2*(a + b*
x)^(3/2))/b^4 - (6*a*(a + b*x)^(5/2))/(5*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int \frac{x^3}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(5b^3x^3-6a\,b^2x^2+8a^2bx-16a^3)}{35b^4}$$

input
```
int(x^3/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**3 + 8*a**2*b*x - 6*a*b**2*x**2 + 5*b**3*x**3))/
(35*b**4)
```

3.394 $\int \frac{x^2}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3}$$

output
```
2*a^2*(b*x+a)^(1/2)/b^3-4/3*a*(b*x+a)^(3/2)/b^3+2/5*(b*x+a)^(5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(8a^2-4abx+3b^2x^2)}{15b^3}$$

input
```
Integrate[x^2/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

input
```
Int[x^2/Sqrt[a + b*x],x]
```

output
```
(2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(
5/2))/(5*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$	32
trager	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$	32
risch	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$	32
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$	32
orering	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37

input
```
int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx+a}}{15\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(48) = 96$.

Time = 0.83 (sec) , antiderivative size = 600, normalized size of antiderivative = 11.76

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

input
```
integrate(x**2/(b*x+a)**(1/2),x)
```

output
```
16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5
*x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x +
 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)
/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) -
 48*a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*
a**5*b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 4
5*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2
*x**2/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x*
*3) + 10*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*
x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a*
*8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**
(13/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*
b**5*x**2 + 15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15
*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3} - \frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3} + \frac{2\,\sqrt{bx+a}a^2}{b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/
b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{6\,{\left(a+b\,x\right)}^{5/2}-20\,a\,{\left(a+b\,x\right)}^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

input
```
int(x^2/(a + b*x)^(1/2),x)
```

output
```
(6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^
3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$$

input
```
int(x^2/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**2 - 4*a*b*x + 3*b**2*x**2))/(15*b**3)
```

3.395 $\int \frac{x}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x}{\sqrt{a+bx}} dx = -\frac{2a\sqrt{a+bx}}{b^2} + \frac{2(a+bx)^{3/2}}{3b^2}$$

output
```
-2*a*(b*x+a)^(1/2)/b^2+2/3*(b*x+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{a+bx}} dx = \frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

input
```
Integrate[x/Sqrt[a + b*x],x]
```

output
```
(2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(\frac{\sqrt{a+bx}}{b}-\frac{a}{b\sqrt{a+bx}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

input
```
Int[x/Sqrt[a + b*x],x]
```

output
```
(-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
trager	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
risch	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
orering	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26

input
```
int(x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\,\sqrt{bx+a}(bx-2\,a)}{3\,b^2}$$

input
```
integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.06

$$\int \frac{x}{\sqrt{a+bx}}\,dx = -\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x} - \frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x} + \frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

input
```
integrate(x/(b*x+a)**(1/2),x)
```

output
```
-4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**
2*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b
**3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*
sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2} - \frac{2\,\sqrt{bx+a}a}{b^2}$$

input
```
integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)}{3b^2}$$

input
```
integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{a+bx}}\,dx = -\frac{6\,a\,\sqrt{a+b\,x} - 2\,(a+b\,x)^{3/2}}{3\,b^2}$$

input
```
int(x/(a + b*x)^(1/2),x)
```

output
```
-(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.56

$$\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(bx-2a)}{3b^2}$$

input
```
int(x/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a + b*x))/(3*b**2)
```

3.396 $\int \frac{1}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

output
```
2*(b*x+a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

input
```
Integrate[1/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \ 17$$
$$\frac{2\sqrt{a+bx}}{b}$$

input
```
Int[1/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x])/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2\sqrt{bx+a}}{b}$	13
derivativedivides	$\frac{2\sqrt{bx+a}}{b}$	13
default	$\frac{2\sqrt{bx+a}}{b}$	13
trager	$\frac{2\sqrt{bx+a}}{b}$	13
risch	$\frac{2\sqrt{bx+a}}{b}$	13
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13
orering	$\frac{2\sqrt{bx+a}}{b}$	13

input
```
int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

input
```
integrate(1/(b*x+a)**(1/2),x)
```

output
```
2*sqrt(a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}}\, dx = \frac{2\,\sqrt{a+b\,x}}{b}$$

input
```
int(1/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(1/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+bx}}\, dx = \frac{2\sqrt{bx+a}}{b}$$

input
```
int(1/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x))/b
```

3.397 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
pseudoelliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

input
```
int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)}{a}\right]$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(s
qrt(-a)/sqrt(b*x + a))/a]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x)^(1/2)),x)
```

output
```
-(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\sqrt{a}\,\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int(1/x/(b*x+a)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.398 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx=-\frac{\sqrt{a+bx}}{ax}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-(b*x+a)^(1/2)/a/x+b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx=-\frac{\sqrt{a+bx}}{ax}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

output
```
-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx}}dx$$

$$\Big\downarrow \ 52$$

$$-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}$$

$$\Big\downarrow \ 73$$

$$-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}$$

$$\Big\downarrow \ 221$$

$$\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x]),x]
```

output
```
-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{\sqrt{bx+a}}{ax}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	34
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-\sqrt{bx+a}\,\sqrt{a}}{a^{\frac{3}{2}}x}$	36
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40
default	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40

input
```
int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)^(1/2)/a/x+b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = \left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2a^2x},\ -\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

input
```
integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*
x + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(-a)/sqrt(b*x + a)) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/x**2/(b*x+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))
)/a**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{bx+a}b}{(bx+a)a-a^2} - \frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))
/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -b\left(\frac{\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx+a}}{abx}\right)$$

input
```
integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-b*(arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)/(a*b*x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+b\,x}}{a\,x}$$

input
```
int(1/(x^2*(a + b*x)^(1/2)),x)
```

output
```
(b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx$$
$$= \frac{-2\sqrt{bx+a}\,a-\sqrt{a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+\sqrt{a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2a^2x}$$

input
```
int(1/x^2/(b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x)*a - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x + sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a**2*x)
```

3.399 $\int \frac{1}{x^3\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
-1/2*(b*x+a)^(1/2)/a/x^2+3/4*b*(b*x+a)^(1/2)/a^2/x-3/4*b^2*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}(-2a+3bx)}{4a^2x^2}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x]),x]
```

output
```
(Sqrt[a + b*x]*(-2*a + 3*b*x))/(4*a^2*x^2) - (3*b^2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{3b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{3b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}$$

input
```
Int[1/(x^3*Sqrt[a + b*x]),x]
```

output
```
-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3bx+2a)}{4a^{2}x^{2}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	45
pseudoelliptic	$\frac{-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}+3bx\sqrt{a}\,\sqrt{bx+a}-2a^{\frac{3}{2}}\sqrt{bx+a}}{4a^{\frac{5}{2}}x^{2}}$	56
derivativedivides	$2b^{2}\left(-\frac{\sqrt{bx+a}}{4a\,b^{2}x^{2}}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66
default	$2b^{2}\left(-\frac{\sqrt{bx+a}}{4a\,b^{2}x^{2}}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66

input
```
int(1/x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(-3*b*x+2*a)/a^2/x^2-3/4*b^2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \left[\frac{3\,\sqrt{a}b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(3\,abx-2\,a^2)\sqrt{bx+a}}{8\,a^3x^2}, \frac{3\,\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(3\,abx-2\,a^2)\sqrt{bx+a}}{4\,a^3x^2}\right]$$

input
```
integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(
sqrt(-a)/sqrt(b*x + a)) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = -\frac{1}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

input
```
integrate(1/x**3/(b*x+a)**(1/2),x)
```

output
```
-1/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(4*a*x**(3/2)*sqrt(a/(
b*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**2*asinh(
sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}} + \frac{3\,(bx+a)^{\frac{3}{2}}b^2 - 5\,\sqrt{bx+a}ab^2}{4\,\left((bx+a)^2a^2 - 2\,(bx+a)a^3 + a^4\right)}$$

input
```
integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
 1/4*(3*(b*x + a)^(3/2)*b^2 - 5*sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a^2 - 2*
(b*x + a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{3}{2}}b^3 - 5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b}$$

input
```
integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3
/2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{3\,(a+b\,x)^{3/2}}{4\,a^2\,x^2} - \frac{5\,\sqrt{a+b\,x}}{4\,a\,x^2} - \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

input
```
int(1/(x^3*(a + b*x)^(1/2)),x)
```

output
```
(3*(a + b*x)^(3/2))/(4*a^2*x^2) - (5*(a + b*x)^(1/2))/(4*a*x^2) - (3*b^2*a
tanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = \frac{-4\sqrt{bx+a}\,a^2+6\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8a^3x^2}$$

input
```
int(1/x^3/(b*x+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 + 6*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x*
*2)/(8*a**3*x**2)
```

3.400 $\int \frac{1}{x^4\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{3ax^3}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
-1/3*(b*x+a)^(1/2)/a/x^3+5/12*b*(b*x+a)^(1/2)/a^2/x^2-5/8*b^2*(b*x+a)^(1/2
)/a^3/x+5/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}(8a^2-10abx+15b^2x^2)}{24a^3x^3}+\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x]),x]
```

output
```
-1/24*(Sqrt[a + b*x]*(8*a^2 - 10*a*b*x + 15*b^2*x^2))/(a^3*x^3) + (5*b^3*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\int \frac{1}{x^3\sqrt{a+bx}}dx}{6a}-\frac{\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\left(-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + b*x]),x]
```

output
```
-1/3*Sqrt[a + b*x]/(a*x^3) - (5*b*(-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sq
rt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/
(6*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(15b^2x^2-10abx+8a^2)}{24a^3x^3}+\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{7}{2}}}$	56
pseudoelliptic	$\frac{\frac{5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^3x^3}{8}-\frac{5\left(\sqrt{a}\,b^2x^2-\frac{2a^{\frac{3}{2}}bx}{3}+\frac{8a^{\frac{5}{2}}}{15}\right)\sqrt{bx+a}}{8}}{a^{\frac{7}{2}}x^3}$	61
derivativedivides	$2b^3\left(-\frac{\sqrt{bx+a}}{6a\,b^3x^3}+\frac{\frac{5\sqrt{bx+a}}{24a\,b^2x^2}+\frac{5\left(-\frac{3\sqrt{bx+a}}{8abx}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}}{a}\right)$	90
default	$2b^3\left(-\frac{\sqrt{bx+a}}{6a\,b^3x^3}+\frac{\frac{5\sqrt{bx+a}}{24a\,b^2x^2}+\frac{5\left(-\frac{3\sqrt{bx+a}}{8abx}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}}{a}\right)$	90

input
```
int(1/x^4/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)^(1/2)*(15*b^2*x^2-10*a*b*x+8*a^2)/a^3/x^3+5/8*b^3*arctanh((b
*x+a)^(1/2)/a^(1/2))/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx = \left[\frac{15\,\sqrt{a}b^3x^3\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,a^4x^3},\; -\frac{15\,\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,a^4x^3}\right]$$

input
```
integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*s
qrt(-a)*b^3*x^3*arctan(sqrt(-a)/sqrt(b*x + a)) + (15*a*b^2*x^2 - 10*a^2*b*
x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx = -\frac{1}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5b^{\frac{3}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5b^{\frac{5}{2}}}{8a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{7}{2}}}$$

input
```
integrate(1/x**4/(b*x+a)**(1/2),x)
```

output
```
-1/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(12*a*x**(5/2)*sqrt(a/
(b*x) + 1)) - 5*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(5/2)
/(8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x
)))/(8*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx = -\frac{5\,b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}} - \frac{15\,(bx+a)^{\frac{5}{2}}b^3 - 40\,(bx+a)^{\frac{3}{2}}ab^3 + 33\,\sqrt{bx+a}a^2b^3}{24\,\left((bx+a)^3a^3 - 3\,(bx+a)^2a^4 + 3\,(bx+a)a^5 - a^6\right)}$$

input
```
integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
 - 1/24*(15*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 33*sqrt(b*x +
 a)*a^2*b^3)/((b*x + a)^3*a^3 - 3*(b*x + a)^2*a^4 + 3*(b*x + a)*a^5 - a^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx$$
$$= -\frac{1}{24}\,b^3\left(\frac{15\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}-40\,(bx+a)^{\frac{3}{2}}a+33\,\sqrt{bx+a}a^2}{a^3b^3x^3}\right)$$

input
```
integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/24*b^3*(15*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a
)^(5/2) - 40*(b*x + a)^(3/2)*a + 33*sqrt(b*x + a)*a^2)/(a^3*b^3*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx = \frac{5\,{\left(a+b\,x\right)}^{3/2}}{3\,a^2\,x^3}-\frac{11\,\sqrt{a+b\,x}}{8\,a\,x^3}-\frac{5\,{\left(a+b\,x\right)}^{5/2}}{8\,a^3\,x^3}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{8\,a^{7/2}}$$

input
```
int(1/(x^4*(a + b*x)^(1/2)),x)
```

output
```
(5*(a + b*x)^(3/2))/(3*a^2*x^3) - (11*(a + b*x)^(1/2))/(8*a*x^3) - (5*(a +
 b*x)^(5/2))/(8*a^3*x^3) - (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*
a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx$$
$$= \frac{-16\sqrt{bx+a}\,a^3+20\sqrt{bx+a}\,a^2bx-30\sqrt{bx+a}\,a\,b^2x^2-15\sqrt{a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3+15\sqrt{a}\,\log\bigl(\sqrt{b}}{48a^4x^3}$$

input
```
int(1/x^4/(b*x+a)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 + 20*sqrt(a + b*x)*a**2*b*x - 30*sqrt(a + b*x)*a
*b**2*x**2 - 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 15*sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**4*x**3)
```

3.401 $\int \frac{x^4}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx = -\frac{2a^4}{b^5\sqrt{a+bx}} - \frac{8a^3\sqrt{a+bx}}{b^5} + \frac{4a^2(a+bx)^{3/2}}{b^5} - \frac{8a(a+bx)^{5/2}}{5b^5} + \frac{2(a+bx)^{7/2}}{7b^5}$$

output
```
-2*a^4/b^5/(b*x+a)^(1/2)-8*a^3*(b*x+a)^(1/2)/b^5+4*a^2*(b*x+a)^(3/2)/b^5-8
/5*a*(b*x+a)^(5/2)/b^5+2/7*(b*x+a)^(7/2)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx = \frac{2(-128a^4 - 64a^3bx + 16a^2b^2x^2 - 8ab^3x^3 + 5b^4x^4)}{35b^5\sqrt{a+bx}}$$

input
```
Integrate[x^4/(a + b*x)^(3/2),x]
```

output
```
(2*(-128*a^4 - 64*a^3*b*x + 16*a^2*b^2*x^2 - 8*a*b^3*x^3 + 5*b^4*x^4))/(35
*b^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^4}{b^4(a+bx)^{3/2}}-\frac{4a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{4a(a+bx)^{3/2}}{b^4}+\frac{(a+bx)^{5/2}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

input
```
Int[x^4/(a + b*x)^(3/2),x]
```

output
```
(-2*a^4)/(b^5*Sqrt[a + b*x]) - (8*a^3*Sqrt[a + b*x])/b^5 + (4*a^2*(a + b*x
)^(3/2))/b^5 - (8*a*(a + b*x)^(5/2))/(5*b^5) + (2*(a + b*x)^(7/2))/(7*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2\left(-5b^{4}x^{4}+8a\,x^{3}b^{3}-16a^{2}b^{2}x^{2}+64a^{3}bx+128a^{4}\right)}{35\sqrt{bx+a}\,b^{5}}$	54
trager	$-\frac{2\left(-5b^{4}x^{4}+8a\,x^{3}b^{3}-16a^{2}b^{2}x^{2}+64a^{3}bx+128a^{4}\right)}{35\sqrt{bx+a}\,b^{5}}$	54
pseudoelliptic	$\frac{\frac{2}{7}b^{4}x^{4}-\frac{16}{35}a\,x^{3}b^{3}+\frac{32}{35}a^{2}b^{2}x^{2}-\frac{128}{35}a^{3}bx-\frac{256}{35}a^{4}}{\sqrt{bx+a}\,b^{5}}$	54
orering	$-\frac{2\left(-5b^{4}x^{4}+8a\,x^{3}b^{3}-16a^{2}b^{2}x^{2}+64a^{3}bx+128a^{4}\right)}{35\sqrt{bx+a}\,b^{5}}$	54
risch	$-\frac{2\left(-5b^{3}x^{3}+13a\,b^{2}x^{2}-29a^{2}bx+93a^{3}\right)\sqrt{bx+a}}{35b^{5}}-\frac{2a^{4}}{b^{5}\sqrt{bx+a}}$	59
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{8a(bx+a)^{\frac{5}{2}}}{5}+4a^{2}(bx+a)^{\frac{3}{2}}-8a^{3}\sqrt{bx+a}-\frac{2a^{4}}{\sqrt{bx+a}}}{b^{5}}$	62
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{8a(bx+a)^{\frac{5}{2}}}{5}+4a^{2}(bx+a)^{\frac{3}{2}}-8a^{3}\sqrt{bx+a}-\frac{2a^{4}}{\sqrt{bx+a}}}{b^{5}}$	62

input
```
int(x^4/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/(b*x+a)^(1/2)*(-5*b^4*x^4+8*a*b^3*x^3-16*a^2*b^2*x^2+64*a^3*b*x+128*
a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx=\frac{2\left(5\,b^4x^4-8\,ab^3x^3+16\,a^2b^2x^2-64\,a^3bx-128\,a^4\right)\sqrt{bx+a}}{35\left(b^6x+ab^5\right)}$$

input
```
integrate(x^4/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqr
t(b*x + a)/(b^6*x + a*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3606 vs. 2(82) = 164.

Time = 1.98 (sec) , antiderivative size = 3606, normalized size of antiderivative = 42.42

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(b*x+a)**(3/2),x)
```

output
```
-256*a**(87/2)*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a*
*38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b
**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**
13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(87/2)/(35
*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x*
*3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6
 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 3
5*a**30*b**15*x**10) - 2432*a**(85/2)*b*x*sqrt(1 + b*x/a)/(35*a**40*b**5 +
 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**
36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*
b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15
*x**10) + 2560*a**(85/2)*b*x/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**3
8*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**
10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13
*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 10336*a**(83/2)*b**
2*x**2*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7
*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**
5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 11520*a**(83/2)*b**2*x**2
/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^5} - \frac{8\,(bx+a)^{\frac{5}{2}}a}{5\,b^5} + \frac{4\,(bx+a)^{\frac{3}{2}}a^2}{b^5} - \frac{8\,\sqrt{bx+a}a^3}{b^5} - \frac{2\,a^4}{\sqrt{bx+a}b^5}$$

input
```
integrate(x^4/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/7*(b*x + a)^(7/2)/b^5 - 8/5*(b*x + a)^(5/2)*a/b^5 + 4*(b*x + a)^(3/2)*a^
2/b^5 - 8*sqrt(b*x + a)*a^3/b^5 - 2*a^4/(sqrt(b*x + a)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx = -\frac{2\left(\frac{35\,a^4}{\sqrt{bx+a}b} - \frac{5\,(bx+a)^{\frac{7}{2}}b^6 - 28\,(bx+a)^{\frac{5}{2}}ab^6 + 70\,(bx+a)^{\frac{3}{2}}a^2b^6 - 140\,\sqrt{bx+a}a^3b^6}{b^7}\right)}{35\,b^4}$$

input
```
integrate(x^4/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2/35*(35*a^4/(sqrt(b*x + a)*b) - (5*(b*x + a)^(7/2)*b^6 - 28*(b*x + a)^(5
/2)*a*b^6 + 70*(b*x + a)^(3/2)*a^2*b^6 - 140*sqrt(b*x + a)*a^3*b^6)/b^7)/b
^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx = \frac{2\,(a+b\,x)^{7/2}}{7\,b^5} - \frac{8\,a^3\,\sqrt{a+b\,x}}{b^5} + \frac{4\,a^2\,(a+b\,x)^{3/2}}{b^5} - \frac{2\,a^4}{b^5\,\sqrt{a+b\,x}} - \frac{8\,a\,(a+b\,x)^{5/2}}{5\,b^5}$$

input
```
int(x^4/(a + b*x)^(3/2),x)
```

output
```
(2*(a + b*x)^(7/2))/(7*b^5) - (8*a^3*(a + b*x)^(1/2))/b^5 + (4*a^2*(a + b*
x)^(3/2))/b^5 - (2*a^4)/(b^5*(a + b*x)^(1/2)) - (8*a*(a + b*x)^(5/2))/(5*b
^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{(a+bx)^{3/2}}\,dx = \frac{\frac{2}{7}b^4x^4 - \frac{16}{35}a\,b^3x^3 + \frac{32}{35}a^2b^2x^2 - \frac{128}{35}a^3bx - \frac{256}{35}a^4}{\sqrt{bx+a}\,b^5}$$

input
```
int(x^4/(b*x+a)^(3/2),x)
```

output
```
(2*( - 128*a**4 - 64*a**3*b*x + 16*a**2*b**2*x**2 - 8*a*b**3*x**3 + 5*b**4
*x**4))/(35*sqrt(a + b*x)*b**5)
```

3.402 $\int \frac{x^3}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx = \frac{2a^3}{b^4\sqrt{a+bx}} + \frac{6a^2\sqrt{a+bx}}{b^4} - \frac{2a(a+bx)^{3/2}}{b^4} + \frac{2(a+bx)^{5/2}}{5b^4}$$

output
```
2*a^3/b^4/(b*x+a)^(1/2)+6*a^2*(b*x+a)^(1/2)/b^4-2*a*(b*x+a)^(3/2)/b^4+2/5*
(b*x+a)^(5/2)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx = \frac{2(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{a+bx}}$$

input
```
Integrate[x^3/(a + b*x)^(3/2),x]
```

output
```
(2*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

input
```
Int[x^3/(a + b*x)^(3/2),x]
```

output
```
(2*a^3)/(b^4*Sqrt[a + b*x]) + (6*a^2*Sqrt[a + b*x])/b^4 - (2*a*(a + b*x)^(
3/2))/b^4 + (2*(a + b*x)^(5/2))/(5*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\frac{2}{5} b^3 x^3-\frac{4}{5} a\, b^2 x^2+\frac{16}{5} a^2 b x+\frac{32}{5} a^3}{b^4 \sqrt{b x+a}}$	42
trager	$\frac{\frac{2}{5} b^3 x^3-\frac{4}{5} a\, b^2 x^2+\frac{16}{5} a^2 b x+\frac{32}{5} a^3}{b^4 \sqrt{b x+a}}$	42
pseudoelliptic	$\frac{\frac{2}{5} b^3 x^3-\frac{4}{5} a\, b^2 x^2+\frac{16}{5} a^2 b x+\frac{32}{5} a^3}{b^4 \sqrt{b x+a}}$	42
orering	$\frac{\frac{2}{5} b^3 x^3-\frac{4}{5} a\, b^2 x^2+\frac{16}{5} a^2 b x+\frac{32}{5} a^3}{b^4 \sqrt{b x+a}}$	42
risch	$\frac{2\left(b^2 x^2-3 a b x+11 a^2\right) \sqrt{b x+a}}{5 b^4}+\frac{2 a^3}{b^4 \sqrt{b x+a}}$	47
derivativedivides	$\frac{\frac{2(b x+a)^{\frac{5}{2}}}{5}-2 a(b x+a)^{\frac{3}{2}}+6 a^2 \sqrt{b x+a}+\frac{2 a^3}{\sqrt{b x+a}}}{b^4}$	49
default	$\frac{\frac{2(b x+a)^{\frac{5}{2}}}{5}-2 a(b x+a)^{\frac{3}{2}}+6 a^2 \sqrt{b x+a}+\frac{2 a^3}{\sqrt{b x+a}}}{b^4}$	49

input
```
int(x^3/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/(b*x+a)^(1/2)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx=\frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)\sqrt{bx+a}}{5\,(b^5x+ab^4)}$$

input
```
integrate(x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*
b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(63) = 126$.

Time = 1.28 (sec) , antiderivative size = 1538, normalized size of antiderivative = 23.30

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(b*x+a)**(3/2),x)
```

output
```
32*a**(45/2)*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b*
*6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 +
5*a**14*b**10*x**6) - 32*a**(45/2)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a*
*18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x
**5 + 5*a**14*b**10*x**6) + 176*a**(43/2)*b*x*sqrt(1 + b*x/a)/(5*a**20*b**
4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*
b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 192*a**(43/2)*b*x/(
5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3
+ 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 396*a**(
41/2)*b**2*x**2*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18
*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5
 + 5*a**14*b**10*x**6) - 480*a**(41/2)*b**2*x**2/(5*a**20*b**4 + 30*a**19*
b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 3
0*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 462*a**(39/2)*b**3*x**3*sqrt(1 +
 b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b
**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) -
 640*a**(39/2)*b**3*x**3/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x
**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a*
*14*b**10*x**6) + 290*a**(37/2)*b**4*x**4*sqrt(1 + b*x/a)/(5*a**20*b**4 +
30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^4} - \frac{2\,(bx+a)^{\frac{3}{2}}a}{b^4} + \frac{6\,\sqrt{bx+a}a^2}{b^4} + \frac{2\,a^3}{\sqrt{bx+a}b^4}$$

input
```
integrate(x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b^4 - 2*(b*x + a)^(3/2)*a/b^4 + 6*sqrt(b*x + a)*a^2/b^
4 + 2*a^3/(sqrt(b*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx = \frac{2\,a^3}{\sqrt{bx+a}b^4} + \frac{2\,\left((bx+a)^{\frac{5}{2}}b^{16} - 5\,(bx+a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}}$$

input
```
integrate(x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*
a*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx = \frac{2\,(a+b\,x)^{5/2}}{5\,b^4} + \frac{6\,a^2\,\sqrt{a+b\,x}}{b^4} + \frac{2\,a^3}{b^4\,\sqrt{a+b\,x}} - \frac{2\,a\,(a+b\,x)^{3/2}}{b^4}$$

input
```
int(x^3/(a + b*x)^(3/2),x)
```

output
```
(2*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(1/2))/b^4 + (2*a^3)/(b^4*(
a + b*x)^(1/2)) - (2*a*(a + b*x)^(3/2))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(a+bx)^{3/2}}\,dx = \frac{\frac{2}{5}b^3x^3 - \frac{4}{5}a\,b^2x^2 + \frac{16}{5}a^2bx + \frac{32}{5}a^3}{\sqrt{bx+a}\,b^4}$$

input
```
int(x^3/(b*x+a)^(3/2),x)
```

output
```
(2*(16*a**3 + 8*a**2*b*x - 2*a*b**2*x**2 + b**3*x**3))/(5*sqrt(a + b*x)*b*
*4)
```

3.403 $\int \frac{x^2}{(a+bx)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{x^2}{(a+bx)^{3/2}} dx = -\frac{2a^2}{b^3\sqrt{a+bx}} - \frac{4a\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{3/2}}{3b^3}$$

output
```
-2*a^2/b^3/(b*x+a)^(1/2)-4*a*(b*x+a)^(1/2)/b^3+2/3*(b*x+a)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx)^{3/2}} dx = \frac{2(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{a+bx}}$$

input
```
Integrate[x^2/(a + b*x)^(3/2),x]
```

output
```
(2*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^{3/2}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

input
```
Int[x^2/(a + b*x)^(3/2),x]
```

output
```
(-2*a^2)/(b^3*Sqrt[a + b*x]) - (4*a*Sqrt[a + b*x])/b^3 + (2*(a + b*x)^(3/2
))/(3*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{\frac{2}{3}b^2x^2-\frac{8}{3}abx-\frac{16}{3}a^2}{\sqrt{bx+a}\,b^3}$	31
gosper	$-\frac{2(-b^2x^2+4abx+8a^2)}{3\sqrt{bx+a}\,b^3}$	32
trager	$-\frac{2(-b^2x^2+4abx+8a^2)}{3\sqrt{bx+a}\,b^3}$	32
orering	$-\frac{2(-b^2x^2+4abx+8a^2)}{3\sqrt{bx+a}\,b^3}$	32
risch	$-\frac{2(-bx+5a)\sqrt{bx+a}}{3b^3}-\frac{2a^2}{b^3\sqrt{bx+a}}$	37
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-4a\sqrt{bx+a}-\frac{2a^2}{\sqrt{bx+a}}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-4a\sqrt{bx+a}-\frac{2a^2}{\sqrt{bx+a}}}{b^3}$	38

input
```
int(x^2/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b^2*x^2-4*a*b*x-8*a^2)/(b*x+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(a+bx)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)\sqrt{bx+a}}{3\,(b^4x+ab^3)}$$

input
```
integrate(x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(46) = 92$.

Time = 0.78 (sec) , antiderivative size = 534, normalized size of antiderivative = 10.90

$$\int \frac{x^2}{(a+bx)^{3/2}}\,dx = -\frac{16a^{\frac{19}{2}}\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{16a^{\frac{19}{2}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} - \frac{40a^{\frac{17}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{48a^{\frac{17}{2}}bx}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} - \frac{30a^{\frac{15}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{48a^{\frac{15}{2}}b^2x^2}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} - \frac{4a^{\frac{13}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{16a^{\frac{13}{2}}b^3x^3}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{2a^{\frac{11}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}$$

input
```
integrate(x**2/(b*x+a)**(3/2),x)
```

output
```
-16*a**(19/2)*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x
**2 + 3*a**5*b**6*x**3) + 16*a**(19/2)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a*
*6*b**5*x**2 + 3*a**5*b**6*x**3) - 40*a**(17/2)*b*x*sqrt(1 + b*x/a)/(3*a**
8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(17/
2)*b*x/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3)
 - 30*a**(15/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9
*a**6*b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(15/2)*b**2*x**2/(3*a**8*b**3
+ 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 4*a**(13/2)*b**3*
x**3*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a
**5*b**6*x**3) + 16*a**(13/2)*b**3*x**3/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a
**6*b**5*x**2 + 3*a**5*b**6*x**3) + 2*a**(11/2)*b**4*x**4*sqrt(1 + b*x/a)/
(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(a+bx)^{3/2}}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^3} - \frac{4\,\sqrt{bx+a}a}{b^3} - \frac{2\,a^2}{\sqrt{bx+a}b^3}$$

input
```
integrate(x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b^3 - 4*sqrt(b*x + a)*a/b^3 - 2*a^2/(sqrt(b*x + a)*b^3
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(a+bx)^{3/2}}\,dx = -\frac{2\left(\frac{3\,a^2}{\sqrt{bx+a}b} - \frac{(bx+a)^{\frac{3}{2}}b^2 - 6\,\sqrt{bx+a}ab^2}{b^3}\right)}{3\,b^2}$$

input
```
integrate(x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(3*a^2/(sqrt(b*x + a)*b) - ((b*x + a)^(3/2)*b^2 - 6*sqrt(b*x + a)*a*b
^2)/b^3)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(a+bx)^{3/2}}\,dx = -\frac{12\,a\,(a+b\,x) - 2\,(a+b\,x)^2 + 6\,a^2}{3\,b^3\,\sqrt{a+b\,x}}$$

input
```
int(x^2/(a + b*x)^(3/2),x)
```

output
```
-(12*a*(a + b*x) - 2*(a + b*x)^2 + 6*a^2)/(3*b^3*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a+bx)^{3/2}}\,dx = \frac{\frac{2}{3}b^2x^2-\frac{8}{3}abx-\frac{16}{3}a^2}{\sqrt{bx+a}\,b^3}$$

input
```
int(x^2/(b*x+a)^(3/2),x)
```

output
```
(2*( - 8*a**2 - 4*a*b*x + b**2*x**2))/(3*sqrt(a + b*x)*b**3)
```

3.404 $\int \frac{x}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{x}{(a+bx)^{3/2}}\,dx=\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

output
```
2*a/b^2/(b*x+a)^(1/2)+2*(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{x}{(a+bx)^{3/2}}\,dx=\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

input
```
Integrate[x/(a + b*x)^(3/2),x]
```

output
```
(2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{1}{b\sqrt{a+bx}}-\frac{a}{b(a+bx)^{3/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

input
```
Int[x/(a + b*x)^(3/2),x]
```

output
```
(2*a)/(b^2*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2bx+4a}{b^{2}\sqrt{bx+a}}$	20
trager	$\frac{2bx+4a}{b^{2}\sqrt{bx+a}}$	20
pseudoelliptic	$\frac{2bx+4a}{b^{2}\sqrt{bx+a}}$	20
orering	$\frac{2bx+4a}{b^{2}\sqrt{bx+a}}$	20
derivativedivides	$\frac{2\sqrt{bx+a}+\frac{2a}{\sqrt{bx+a}}}{b^{2}}$	23
default	$\frac{2\sqrt{bx+a}+\frac{2a}{\sqrt{bx+a}}}{b^{2}}$	23
risch	$\frac{2a}{b^{2}\sqrt{bx+a}}+\frac{2\sqrt{bx+a}}{b^{2}}$	27

input
```
int(x/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/(b*x+a)^(1/2)*(b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{x}{(a+bx)^{3/2}}\,dx = \frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^{3}x+ab^{2}}$$

input
```
integrate(x/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{x}{(a+bx)^{3/2}}\,dx = \begin{cases} \frac{4a}{b^2\sqrt{a+bx}} + \frac{2x}{b\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x+a)**(3/2),x)
```

output
```
Piecewise((4*a/(b**2*sqrt(a + b*x)) + 2*x/(b*sqrt(a + b*x)), Ne(b, 0)), (x
**2/(2*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{x}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}}{b^2} + \frac{2a}{\sqrt{bx+a}b^2}$$

input
```
integrate(x/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)/b^2 + 2*a/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{x}{(a+bx)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{bx+a}}{b} + \frac{a}{\sqrt{bx+a}b}\right)}{b}$$

input
```
integrate(x/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{x}{(a+bx)^{3/2}}\,dx=\frac{4\,a+2\,b\,x}{b^2\,\sqrt{a+b\,x}}$$

input
```
int(x/(a + b*x)^(3/2),x)
```

output
```
(4*a + 2*b*x)/(b^2*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{(a+bx)^{3/2}}\,dx=\frac{2bx+4a}{\sqrt{bx+a}\,b^2}$$

input
```
int(x/(b*x+a)^(3/2),x)
```

output
```
(2*(2*a + b*x))/(sqrt(a + b*x)*b**2)
```

3.405 $\int \frac{1}{(a+bx)^{3/2}} dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{(a+bx)^{3/2}} dx = -\frac{2}{b\sqrt{a+bx}}$$

output
```
-2/b/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^{3/2}} dx = -\frac{2}{b\sqrt{a+bx}}$$

input
```
Integrate[(a + b*x)^(-3/2),x]
```

output
```
-2/(b*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{3/2}}\,dx$$
$$\Big\downarrow \text{17}$$
$$-\frac{2}{b\sqrt{a+bx}}$$

input
```
Int[(a + b*x)^(-3/2),x]
```

output
```
-2/(b*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2}{b\sqrt{bx+a}}$	13
derivativedivides	$-\frac{2}{b\sqrt{bx+a}}$	13
default	$-\frac{2}{b\sqrt{bx+a}}$	13
trager	$-\frac{2}{b\sqrt{bx+a}}$	13
pseudoelliptic	$-\frac{2}{b\sqrt{bx+a}}$	13
orering	$-\frac{2}{b\sqrt{bx+a}}$	13

input
```
int(1/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt{bx+a}}{b^2x+ab}$$

input
```
integrate(1/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x + a)/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2}{b\sqrt{a+bx}}$$

input
```
integrate(1/(b*x+a)**(3/2),x)
```

output
```
-2/(b*sqrt(a + b*x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

input
```
integrate(1/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(b*x + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

input
```
integrate(1/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2/(sqrt(b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2}{b\,\sqrt{a+b\,x}}$$

input
```
int(1/(a + b*x)^(3/2),x)
```

output
```
-2/(b*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}\,b}$$

input
```
int(1/(b*x+a)^(3/2),x)
```

output
```
( - 2)/(sqrt(a + b*x)*b)
```

3.406 $\int \frac{1}{x(a+bx)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{1}{x(a+bx)^{3/2}} \, dx = \frac{2}{a\sqrt{a+bx}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
2/a/(b*x+a)^(1/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)^{3/2}} \, dx = \frac{2}{a\sqrt{a+bx}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x)^(3/2)),x]
```

output
```
2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Int[1/(x*(a + b*x)^(3/2)),x]
```

output
```
2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2}{a\sqrt{bx+a}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	31
default	$\frac{2}{a\sqrt{bx+a}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	31
pseudoelliptic	$\frac{2}{a\sqrt{bx+a}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	31

input
```
int(1/x/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/a/(b*x+a)^(1/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.82

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \left[\frac{(bx+a)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}a}{a^2bx+a^3}, \frac{2\left((bx+a)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)\right.}{a^2bx+a^3}\right.$$

input
```
integrate(1/x/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(
b*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*
x + a)) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.

Time = 0.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.84

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{2a^3\sqrt{1+\frac{bx}{a}}}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{a^3\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2a^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{a^2bx\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2a^2bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}$$

input
```
integrate(1/x/(b*x+a)**(3/2),x)
```

output
```
2*a**3*sqrt(1 + b*x/a)/(a**(9/2) + a**(7/2)*b*x) + a**3*log(b*x/a)/(a**(9/
2) + a**(7/2)*b*x) - 2*a**3*log(sqrt(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*
b*x) + a**2*b*x*log(b*x/a)/(a**(9/2) + a**(7/2)*b*x) - 2*a**2*b*x*log(sqrt
(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx+a}a}$$

input
```
integrate(1/x/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) + 2/(sqrt
(b*x + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2}{\sqrt{bx+a}a}$$

input
```
integrate(1/x/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{2}{a\sqrt{a+b\,x}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(1/(x*(a + b*x)^(3/2)),x)
```

output
```
2/(a*(a + b*x)^(1/2)) - (2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right) - \sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right) + 2a}{\sqrt{bx+a}\,a^2}$$

input
```
int(1/x/(b*x+a)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*sqrt(a + b*x
)*log(sqrt(a + b*x) + sqrt(a)) + 2*a)/(sqrt(a + b*x)*a**2)
```

3.407 $\int \frac{1}{x^2(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = -\frac{3b}{a^2\sqrt{a+bx}}-\frac{1}{ax\sqrt{a+bx}}+\frac{3b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-3*b/a^2/(b*x+a)^(1/2)-1/a/x/(b*x+a)^(1/2)+3*b*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = \frac{-a-3bx}{a^2x\sqrt{a+bx}}+\frac{3b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^2*(a + b*x)^(3/2)),x]
```

output
```
(-a - 3*b*x)/(a^2*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/
a^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \; 52$$

$$-\frac{3b\int \frac{1}{x(a+bx)^{3/2}}dx}{2a}-\frac{1}{ax\sqrt{a+bx}}$$

$$\Big\downarrow \; 61$$

$$-\frac{3b\left(\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}$$

$$\Big\downarrow \; 73$$

$$-\frac{3b\left(\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}$$

$$\Big\downarrow \; 221$$

$$-\frac{3b\left(\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}$$

input
```
Int[1/(x^2*(a + b*x)^(3/2)),x]
```

output
```
-(1/(a*x*Sqrt[a + b*x])) - (3*b*(2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{\sqrt{bx+a}}{a^2x}-\frac{b\left(\frac{4}{\sqrt{bx+a}}-\frac{6\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{2a^2}$	50
pseudoelliptic	$-\frac{-3\sqrt{bx+a}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx+\sqrt{a}\,(3bx+a)}{\sqrt{bx+a}\,x\,a^{\frac{5}{2}}}$	50
derivativedivides	$2b\left(\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^2}-\frac{1}{a^2\sqrt{bx+a}}\right)$	54
default	$2b\left(\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^2}-\frac{1}{a^2\sqrt{bx+a}}\right)$	54

input
```
int(1/x^2/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(b*x+a)^(1/2)/x-1/2*b/a^2*(4/(b*x+a)^(1/2)-6*arctanh((b*x+a)^(1/2)/
a^(1/2))/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = \left[\frac{3\,(b^2x^2+abx)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(3\,abx+a^2)\sqrt{bx+a}}{2\,(a^3bx^2+a^4x)},\right.$$
$$\left.-\frac{3\,(b^2x^2+abx)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(3\,abx+a^2)\sqrt{bx+a}}{a^3bx^2+a^4x}\right]$$

input
```
integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a
)/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2
+ a*b*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) + (3*a*b*x + a^2)*sqrt(b*
x + a))/(a^3*b*x^2 + a^4*x)]
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = -\frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}}$$

input
```
integrate(1/x**2/(b*x+a)**(3/2),x)
```

output
```
-1/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqrt(a
/(b*x) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = -\frac{3\,(bx+a)b-2\,ab}{(bx+a)^{\frac{3}{2}}a^2-\sqrt{bx+a}a^3} - \frac{3\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

input
```
integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-(3*(b*x + a)*b - 2*a*b)/((b*x + a)^(3/2)*a^2 - sqrt(b*x + a)*a^3) - 3/2*b
*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2}$$

input
```
integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*
b)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx=\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,b}{a}-\frac{3\,b\,(a+b\,x)}{a^2}}{a\,\sqrt{a+b\,x}-(a+b\,x)^{3/2}}$$

input
```
int(1/(x^2*(a + b*x)^(3/2)),x)
```

output
```
(3*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*b)/a - (3*b*(a + b*x))/
a^2)/(a*(a + b*x)^(1/2) - (a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx=\frac{-3\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+3\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx-2a^2}{2\sqrt{bx+a}\,a^3x}$$

input
```
int(1/x^2/(b*x+a)^(3/2),x)
```

output
```
( - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x + 3*sqrt(a)*s
qrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b*x - 2*a**2 - 6*a*b*x)/(2*sqrt(
a + b*x)*a**3*x)
```

3.408 $\int \frac{1}{x^3(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx=\frac{15b^2}{4a^3\sqrt{a+bx}}-\frac{1}{2ax^2\sqrt{a+bx}}+\frac{5b}{4a^2x\sqrt{a+bx}}-\frac{15b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
15/4*b^2/a^3/(b*x+a)^(1/2)-1/2/a/x^2/(b*x+a)^(1/2)+5/4*b/a^2/x/(b*x+a)^(1/
2)-15/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx=\frac{-2a^2+5abx+15b^2x^2}{4a^3x^2\sqrt{a+bx}}-\frac{15b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[1/(x^3*(a + b*x)^(3/2)),x]
```

output
```
(-2*a^2 + 5*a*b*x + 15*b^2*x^2)/(4*a^3*x^2*Sqrt[a + b*x]) - (15*b^2*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\int \frac{1}{x^2(a+bx)^{3/2}}dx}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\left(-\frac{3b\int \frac{1}{x(a+bx)^{3/2}}dx}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}$$

input
```
Int[1/(x^3*(a + b*x)^(3/2)),x]
```

output
```
-1/2*1/(a*x^2*Sqrt[a + b*x]) - (5*b*(-(1/(a*x*Sqrt[a + b*x])) - (3*b*(2/(a
*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*
a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-7bx+2a)}{4a^{3}x^{2}}+\frac{b^{2}\left(\frac{16}{\sqrt{bx+a}}-\frac{30\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{8a^{3}}$	60
pseudoelliptic	$-\frac{15\left(\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,b^{2}x^{2}-\sqrt{a}\,b^{2}x^{2}-\frac{a^{\frac{3}{2}}bx}{3}+\frac{2a^{\frac{5}{2}}}{15}\right)}{4a^{\frac{7}{2}}\sqrt{bx+a}\,x^{2}}$	66
derivativedivides	$2b^{2}\left(\frac{1}{a^{3}\sqrt{bx+a}}-\frac{\frac{-\frac{7(bx+a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{3}}\right)$	68
default	$2b^{2}\left(\frac{1}{a^{3}\sqrt{bx+a}}-\frac{\frac{-\frac{7(bx+a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{3}}\right)$	68

input
```
int(1/x^3/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(-7*b*x+2*a)/a^3/x^2+1/8*b^2/a^3*(16/(b*x+a)^(1/2)-30*a
rctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.14

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx=\left[\frac{15\,(b^3x^3+ab^2x^2)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(15\,ab^2x^2+5\,a^2bx-2\,a^3)\sqrt{bx+a}}{8\,(a^4bx^3+a^5x^2)}\right.$$

input
```
integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3
 + a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b
*x + a)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
a^5*x^2)]
```

Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx = -\frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}}$$

input
```
integrate(1/x**3/(b*x+a)**(3/2),x)
```

output
```
-1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x**(3/2)*s
qrt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15*b*
*2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx = \frac{15\,(bx+a)^2b^2 - 25\,(bx+a)ab^2 + 8\,a^2b^2}{4\left((bx+a)^{\frac{5}{2}}a^3 - 2\,(bx+a)^{\frac{3}{2}}a^4 + \sqrt{bx+a}aa^5\right)} + \frac{15\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

input
```
integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(15*(b*x + a)^2*b^2 - 25*(b*x + a)*a*b^2 + 8*a^2*b^2)/((b*x + a)^(5/2)
*a^3 - 2*(b*x + a)^(3/2)*a^4 + sqrt(b*x + a)*a^5) + 15/8*b^2*log((sqrt(b*x
 + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3}+\frac{2\,b^2}{\sqrt{bx+a}a^3}+\frac{7\,(bx+a)^{\frac{3}{2}}b^2-9\,\sqrt{bx+a}ab^2}{4\,a^3b^2x^2}$$

input
```
integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
 a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^
2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx = \frac{\frac{2\,b^2}{a}+\frac{15\,b^2\,(a+b\,x)^2}{4\,a^3}-\frac{25\,b^2\,(a+b\,x)}{4\,a^2}}{(a+b\,x)^{5/2}-2\,a\,(a+b\,x)^{3/2}+a^2\,\sqrt{a+b\,x}}-\frac{15\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

input
```
int(1/(x^3*(a + b*x)^(3/2)),x)
```

output
```
((2*b^2)/a + (15*b^2*(a + b*x)^2)/(4*a^3) - (25*b^2*(a + b*x))/(4*a^2))/((
a + b*x)^(5/2) - 2*a*(a + b*x)^(3/2) + a^2*(a + b*x)^(1/2)) - (15*b^2*atan
h((a + b*x)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx = \frac{15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2 - 15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2 -}{8\sqrt{bx+a}\,a^4x^2}$$

input
```
int(1/x^3/(b*x+a)^(3/2),x)
```

output
```
(15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 4*a**3 + 10*a**
2*b*x + 30*a*b**2*x**2)/(8*sqrt(a + b*x)*a**4*x**2)
```

3.409 $\int \frac{x^4}{(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = -\frac{2a^4}{3b^5(a+bx)^{3/2}} + \frac{8a^3}{b^5\sqrt{a+bx}} + \frac{12a^2\sqrt{a+bx}}{b^5} - \frac{8a(a+bx)^{3/2}}{3b^5} + \frac{2(a+bx)^{5/2}}{5b^5}$$

output
```
-2/3*a^4/b^5/(b*x+a)^(3/2)+8*a^3/b^5/(b*x+a)^(1/2)+12*a^2*(b*x+a)^(1/2)/b^
5-8/3*a*(b*x+a)^(3/2)/b^5+2/5*(b*x+a)^(5/2)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = \frac{2(128a^4 + 192a^3bx + 48a^2b^2x^2 - 8ab^3x^3 + 3b^4x^4)}{15b^5(a+bx)^{3/2}}$$

input
```
Integrate[x^4/(a + b*x)^(5/2),x]
```

output
```
(2*(128*a^4 + 192*a^3*b*x + 48*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4))/(15
*b^5*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx)^{5/2}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^4}{b^4(a+bx)^{5/2}}-\frac{4a^3}{b^4(a+bx)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^4}+\frac{(a+bx)^{3/2}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

input
```
Int[x^4/(a + b*x)^(5/2),x]
```

output
```
(-2*a^4)/(3*b^5*(a + b*x)^(3/2)) + (8*a^3)/(b^5*Sqrt[a + b*x]) + (12*a^2*S
qrt[a + b*x])/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5) + (2*(a + b*x)^(5/2))/(5
*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,x^3b^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}b^5}$	54
trager	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,x^3b^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}b^5}$	54
pseudoelliptic	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,x^3b^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}b^5}$	54
orering	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,x^3b^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}b^5}$	54
risch	$\frac{2(3b^2x^2-14abx+73a^2)\sqrt{bx+a}}{15b^5}+\frac{2a^3(12bx+11a)}{3b^5(bx+a)^{\frac{3}{2}}}$	56
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a(bx+a)^{\frac{3}{2}}}{3}+12a^2\sqrt{bx+a}+\frac{8a^3}{\sqrt{bx+a}}-\frac{2a^4}{3(bx+a)^{\frac{3}{2}}}}{b^5}$	62
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a(bx+a)^{\frac{3}{2}}}{3}+12a^2\sqrt{bx+a}+\frac{8a^3}{\sqrt{bx+a}}-\frac{2a^4}{3(bx+a)^{\frac{3}{2}}}}{b^5}$	62

input
```
int(x^4/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/(b*x+a)^(3/2)*(3*b^4*x^4-8*a*b^3*x^3+48*a^2*b^2*x^2+192*a^3*b*x+128*a
^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = \frac{2\,(3\,b^4x^4-8\,ab^3x^3+48\,a^2b^2x^2+192\,a^3bx+128\,a^4)\sqrt{bx+a}}{15\,(b^7x^2+2\,ab^6x+a^2b^5)}$$

input
```
integrate(x^4/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sq
rt(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3456 vs. $2(83) = 166$.

Time = 1.92 (sec) , antiderivative size = 3456, normalized size of antiderivative = 39.72

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(b*x+a)**(5/2),x)
```

output
```
256*a**(85/2)*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**3
8*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**
10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*
x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256*a**(85/2)/(15*a*
*40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 +
 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**
30*b**15*x**10) + 2432*a**(83/2)*b*x*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*
a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**
9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*
x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10)
 - 2560*a**(83/2)*b*x/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x
**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5
+ 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 1
50*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 10336*a**(81/2)*b**2*x**2*sq
rt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 18
00*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a
**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31
*b**14*x**9 + 15*a**30*b**15*x**10) - 11520*a**(81/2)*b**2*x**2/(15*a**40*
b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + ...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^5} - \frac{8\,(bx+a)^{\frac{3}{2}}a}{3\,b^5} + \frac{12\,\sqrt{bx+a}a^2}{b^5} + \frac{8\,a^3}{\sqrt{bx+a}b^5} - \frac{2\,a^4}{3\,(bx+a)^{\frac{3}{2}}b^5}$$

input
```
integrate(x^4/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)/b^5 - 8/3*(b*x + a)^(3/2)*a/b^5 + 12*sqrt(b*x + a)*a^2
/b^5 + 8*a^3/(sqrt(b*x + a)*b^5) - 2/3*a^4/((b*x + a)^(3/2)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = \frac{2\left(\frac{5\,(12\,(bx+a)a^3-a^4)}{(bx+a)^{\frac{3}{2}}b} + \frac{3\,(bx+a)^{\frac{5}{2}}b^4-20\,(bx+a)^{\frac{3}{2}}ab^4+90\,\sqrt{bx+a}a^2b^4}{b^5}\right)}{15\,b^4}$$

input
```
integrate(x^4/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/15*(5*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b) + (3*(b*x + a)^(5/2)*
b^4 - 20*(b*x + a)^(3/2)*a*b^4 + 90*sqrt(b*x + a)*a^2*b^4)/b^5)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = \frac{2\,(a+b\,x)^{5/2}}{5\,b^5} + \frac{8\,a^3\,(a+b\,x) - \frac{2\,a^4}{3}}{b^5\,(a+b\,x)^{3/2}} + \frac{12\,a^2\,\sqrt{a+b\,x}}{b^5} - \frac{8\,a\,(a+b\,x)^{3/2}}{3\,b^5}$$

input
```
int(x^4/(a + b*x)^(5/2),x)
```

output
```
(2*(a + b*x)^(5/2))/(5*b^5) + (8*a^3*(a + b*x) - (2*a^4)/3)/(b^5*(a + b*x)
^(3/2)) + (12*a^2*(a + b*x)^(1/2))/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{(a+bx)^{5/2}}\,dx = \frac{\frac{2}{5}b^4x^4 - \frac{16}{15}a\,b^3x^3 + \frac{32}{5}a^2b^2x^2 + \frac{128}{5}a^3bx + \frac{256}{15}a^4}{\sqrt{bx+a}\,b^5\,(bx+a)}$$

input
```
int(x^4/(b*x+a)^(5/2),x)
```

output
```
(2*(128*a**4 + 192*a**3*b*x + 48*a**2*b**2*x**2 - 8*a*b**3*x**3 + 3*b**4*x
**4))/(15*sqrt(a + b*x)*b**5*(a + b*x))
```

3.410 $\int \frac{x^3}{(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{x^3}{(a+bx)^{5/2}} dx = \frac{2a^3}{3b^4(a+bx)^{3/2}} - \frac{6a^2}{b^4\sqrt{a+bx}} - \frac{6a\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{3/2}}{3b^4}$$

output
```
2/3*a^3/b^4/(b*x+a)^(3/2)-6*a^2/b^4/(b*x+a)^(1/2)-6*a*(b*x+a)^(1/2)/b^4+2/
3*(b*x+a)^(3/2)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx)^{5/2}} dx = \frac{2(a^3 - 9a^2(a+bx) - 9a(a+bx)^2 + (a+bx)^3)}{3b^4(a+bx)^{3/2}}$$

input
```
Integrate[x^3/(a + b*x)^(5/2),x]
```

output
```
(2*(a^3 - 9*a^2*(a + b*x) - 9*a*(a + b*x)^2 + (a + b*x)^3))/(3*b^4*(a + b*
x)^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^{5/2}}+\frac{3a^2}{b^3(a+bx)^{3/2}}-\frac{3a}{b^3\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}$$

input
```
Int[x^3/(a + b*x)^(5/2),x]
```

output
```
(2*a^3)/(3*b^4*(a + b*x)^(3/2)) - (6*a^2)/(b^4*Sqrt[a + b*x]) - (6*a*Sqrt[
a + b*x])/b^4 + (2*(a + b*x)^(3/2))/(3*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
trager	$-\frac{2(bx+2a)\left(-b^{2}x^{2}+8abx+8a^{2}\right)}{3b^{4}\left(bx+a\right)^{\frac{3}{2}}}$	39
pseudoelliptic	$\frac{\frac{2}{3}b^{3}x^{3}-4a\,b^{2}x^{2}-16a^{2}bx-\frac{32}{3}a^{3}}{\left(bx+a\right)^{\frac{3}{2}}b^{4}}$	42
gosper	$-\frac{2\left(-b^{3}x^{3}+6a\,b^{2}x^{2}+24a^{2}bx+16a^{3}\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^{4}}$	43
orering	$-\frac{2\left(-b^{3}x^{3}+6a\,b^{2}x^{2}+24a^{2}bx+16a^{3}\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^{4}}$	43
risch	$-\frac{2(-bx+8a)\sqrt{bx+a}}{3b^{4}}-\frac{2a^{2}(9bx+8a)}{3b^{4}\left(bx+a\right)^{\frac{3}{2}}}$	45
derivativedivides	$\frac{\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3}-6a\sqrt{bx+a}-\frac{6a^{2}}{\sqrt{bx+a}}+\frac{2a^{3}}{3\left(bx+a\right)^{\frac{3}{2}}}}{b^{4}}$	50
default	$\frac{\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3}-6a\sqrt{bx+a}-\frac{6a^{2}}{\sqrt{bx+a}}+\frac{2a^{3}}{3\left(bx+a\right)^{\frac{3}{2}}}}{b^{4}}$	50

input
```
int(x^3/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+2*a)*(-b^2*x^2+8*a*b*x+8*a^2)/b^4/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(a+bx)^{5/2}}\,dx=\frac{2\left(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3\right)\sqrt{bx+a}}{3\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

input
```
integrate(x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
 2*a*b^5*x + a^2*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.40

$$\int \frac{x^3}{(a+bx)^{5/2}}\,dx = \begin{cases} -\frac{32a^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} - \frac{48a^2bx}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} - \frac{12ab^2x^2}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} + \frac{2b^3x^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} \\ \frac{x^4}{4a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**3/(b*x+a)**(5/2),x)
```

output
```
Piecewise((-32*a**3/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 48
*a**2*b*x/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 12*a*b**2*x*
*2/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) + 2*b**3*x**3/(3*a*b*
*4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)), Ne(b, 0)), (x**4/(4*a**(5/2)),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{(a+bx)^{5/2}}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^4} - \frac{6\,\sqrt{bx+a}a}{b^4} - \frac{6\,a^2}{\sqrt{bx+a}b^4} + \frac{2\,a^3}{3\,(bx+a)^{\frac{3}{2}}b^4}$$

input
```
integrate(x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)/b^4 - 6*sqrt(b*x + a)*a/b^4 - 6*a^2/(sqrt(b*x + a)*b^4
) + 2/3*a^3/((b*x + a)^(3/2)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(a+bx)^{5/2}}\,dx = -\frac{2\,(9\,(bx+a)a^2-a^3)}{3\,(bx+a)^{\frac{3}{2}}b^4}+\frac{2\left((bx+a)^{\frac{3}{2}}b^8-9\,\sqrt{bx+a}ab^8\right)}{3\,b^{12}}$$

input
```
integrate(x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*
b^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx)^{5/2}}\,dx = -\frac{18\,a\,(a+b\,x)^2+18\,a^2\,(a+b\,x)-2\,(a+b\,x)^3-2\,a^3}{3\,b^4\,(a+b\,x)^{3/2}}$$

input
```
int(x^3/(a + b*x)^(5/2),x)
```

output
```
-(18*a*(a + b*x)^2 + 18*a^2*(a + b*x) - 2*(a + b*x)^3 - 2*a^3)/(3*b^4*(a +
 b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(a+bx)^{5/2}}\,dx = \frac{\frac{2}{3}b^3x^3-4a\,b^2x^2-16a^2bx-\frac{32}{3}a^3}{\sqrt{bx+a}\,b^4\,(bx+a)}$$

input
```
int(x^3/(b*x+a)^(5/2),x)
```

output
```
(2*( - 16*a**3 - 24*a**2*b*x - 6*a*b**2*x**2 + b**3*x**3))/(3*sqrt(a + b*x
)*b**4*(a + b*x))
```

3.411 $\int \frac{x^2}{(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{x^2}{(a+bx)^{5/2}} dx = -\frac{2a^2}{3b^3(a+bx)^{3/2}} + \frac{4a}{b^3\sqrt{a+bx}} + \frac{2\sqrt{a+bx}}{b^3}$$

output
```
-2/3*a^2/b^3/(b*x+a)^(3/2)+4*a/b^3/(b*x+a)^(1/2)+2*(b*x+a)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(a+bx)^{5/2}} dx = \frac{2(8a^2+12abx+3b^2x^2)}{3b^3(a+bx)^{3/2}}$$

input
```
Integrate[x^2/(a + b*x)^(5/2),x]
```

output
```
(2*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

input
```
Int[x^2/(a + b*x)^(5/2),x]
```

output
```
(-2*a^2)/(3*b^3*(a + b*x)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x]) + (2*Sqrt[a +
 b*x])/b^3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2b^2x^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
trager	$\frac{2b^2x^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
pseudoelliptic	$\frac{2b^2x^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
orering	$\frac{2b^2x^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
risch	$\frac{2\sqrt{bx+a}}{b^3}+\frac{2a(6bx+5a)}{3b^3(bx+a)^{\frac{3}{2}}}$	35
derivativedivides	$\frac{2\sqrt{bx+a}+\frac{4a}{\sqrt{bx+a}}-\frac{2a^2}{3(bx+a)^{\frac{3}{2}}}}{b^3}$	36
default	$\frac{2\sqrt{bx+a}+\frac{4a}{\sqrt{bx+a}}-\frac{2a^2}{3(bx+a)^{\frac{3}{2}}}}{b^3}$	36

input
```
int(x^2/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x+a)^(3/2)*(3*b^2*x^2+12*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a+bx)^{5/2}}\,dx = \frac{2\,(3\,b^2x^2+12\,abx+8\,a^2)\sqrt{bx+a}}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^
2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.47

$$\int \frac{x^2}{(a+bx)^{5/2}}\,dx = \begin{cases} \frac{16a^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{24abx}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{6b^2x^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x+a)**(5/2),x)
```

output
```
Piecewise((16*a**2/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 24*
a*b*x/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 6*b**2*x**2/(3*a
*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)), Ne(b, 0)), (x**3/(3*a**(5/2
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(a+bx)^{5/2}}\,dx = \frac{2\sqrt{bx+a}}{b^3} + \frac{4\,a}{\sqrt{bx+a}b^3} - \frac{2\,a^2}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)/b^3 + 4*a/(sqrt(b*x + a)*b^3) - 2/3*a^2/((b*x + a)^(3/2)*b
^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx)^{5/2}}\,dx = \frac{2\left(\frac{3\sqrt{bx+a}}{b}+\frac{6\,(bx+a)a-a^2}{(bx+a)^{\frac{3}{2}}b}\right)}{3\,b^2}$$

input
```
integrate(x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(b*x + a)/b + (6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(a+bx)^{5/2}}\,dx = \frac{6\,(a+b\,x)^2+12\,a\,(a+b\,x)-2\,a^2}{3\,b^3\,(a+b\,x)^{3/2}}$$

input
```
int(x^2/(a + b*x)^(5/2),x)
```

output
```
(6*(a + b*x)^2 + 12*a*(a + b*x) - 2*a^2)/(3*b^3*(a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(a+bx)^{5/2}}\,dx = \frac{2b^2x^2+8abx+\frac{16}{3}a^2}{\sqrt{bx+a}\,b^3\,(bx+a)}$$

input
```
int(x^2/(b*x+a)^(5/2),x)
```

output
```
(2*(8*a**2 + 12*a*b*x + 3*b**2*x**2))/(3*sqrt(a + b*x)*b**3*(a + b*x))
```

3.412 $\int \frac{x}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = \frac{2a}{3b^2(a+bx)^{3/2}} - \frac{2}{b^2\sqrt{a+bx}}$$

output
```
2/3*a/b^2/(b*x+a)^(3/2)-2/b^2/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = -\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

input
```
Integrate[x/(a + b*x)^(5/2),x]
```

output
```
(-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{b(a+bx)^{3/2}}-\frac{a}{b(a+bx)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

input
```
Int[x/(a + b*x)^(5/2),x]
```

output
```
(2*a)/(3*b^2*(a + b*x)^(3/2)) - 2/(b^2*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2(3bx+2a)}{3(bx+a)^{\frac{3}{2}} b^{2}}$	21
trager	$-\frac{2(3bx+2a)}{3(bx+a)^{\frac{3}{2}} b^{2}}$	21
pseudoelliptic	$\frac{-6bx-4a}{3(bx+a)^{\frac{3}{2}} b^{2}}$	21
orering	$-\frac{2(3bx+2a)}{3(bx+a)^{\frac{3}{2}} b^{2}}$	21
derivativedivides	$\frac{-\frac{2}{\sqrt{bx+a}}+\frac{2a}{3(bx+a)^{\frac{3}{2}}}}{b^{2}}$	26
default	$\frac{-\frac{2}{\sqrt{bx+a}}+\frac{2a}{3(bx+a)^{\frac{3}{2}}}}{b^{2}}$	26

input
```
int(x/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x+a)^(3/2)*(3*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = -\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate(x/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = \begin{cases} -\frac{4a}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} - \frac{6bx}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x+a)**(5/2),x)
```

output
```
Piecewise((-4*a/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 6*b*x/
(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)), Ne(b, 0)), (x**2/(2*a**
(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = -\frac{2}{\sqrt{bx+a}b^2} + \frac{2\,a}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(b*x + a)*b^2) + 2/3*a/((b*x + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = -\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = -\frac{4\,a+6\,b\,x}{3\,b^2\,{\left(a+b\,x\right)}^{3/2}}$$

input
```
int(x/(a + b*x)^(5/2),x)
```

output
```
-(4*a + 6*b*x)/(3*b^2*(a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx)^{5/2}}\,dx = \frac{-2bx-\frac{4a}{3}}{\sqrt{bx+a}\,b^2\,(bx+a)}$$

input
```
int(x/(b*x+a)^(5/2),x)
```

output
```
(2*( - 2*a - 3*b*x))/(3*sqrt(a + b*x)*b**2*(a + b*x))
```

3.413 $\int \frac{1}{(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{1}{(a+bx)^{5/2}} dx = -\frac{2}{3b(a+bx)^{3/2}}$$

output
```
-2/3/b/(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^{5/2}} dx = -\frac{2}{3b(a+bx)^{3/2}}$$

input
```
Integrate[(a + b*x)^(-5/2),x]
```

output
```
-2/(3*b*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{2}{3b(a+bx)^{3/2}}$$

input
```
Int[(a + b*x)^(-5/2),x]
```

output
```
-2/(3*b*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
derivativedivides	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
default	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
trager	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
pseudoelliptic	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
orering	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13

input
```
int(1/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b/(b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt{bx+a}}{3\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate(1/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3b\,(a+bx)^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x+a)**(5/2),x)
```

output
```
-2/(3*b*(a + b*x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

input
```
integrate(1/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/3/((b*x + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

input
```
integrate(1/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3/((b*x + a)^(3/2)*b)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3\,b\,(a+b\,x)^{3/2}}$$

input
```
int(1/(a + b*x)^(5/2),x)
```

output
```
-2/(3*b*(a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3\sqrt{bx+a}\,b\,(bx+a)}$$

input
```
int(1/(b*x+a)^(5/2),x)
```

output
```
( - 2)/(3*sqrt(a + b*x)*b*(a + b*x))
```

3.414 $\int \frac{1}{x(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{2}{3a(a+bx)^{3/2}} + \frac{2}{a^2\sqrt{a+bx}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
2/3/a/(b*x+a)^(3/2)+2/a^2/(b*x+a)^(1/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a
^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{2(a+3(a+bx))}{3a^2(a+bx)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x*(a + b*x)^(5/2)),x]
```

output
```
(2*(a + 3*(a + b*x)))/(3*a^2*(a + b*x)^(3/2)) - (2*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^{5/2}} \, dx$$

$$\Big\downarrow \ 61$$

$$\frac{\int \frac{1}{x(a+bx)^{3/2}}dx}{a}+\frac{2}{3a(a+bx)^{3/2}}$$

$$\Big\downarrow \ 61$$

$$\frac{\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx)^{3/2}}$$

input
```
Int[1/(x*(a + b*x)^(5/2)),x]
```

output
```
2/(3*a*(a + b*x)^(3/2)) + (2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/a^(3/2))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2}{3a(bx+a)^{\frac{3}{2}}}+\frac{2}{a^{2}\sqrt{bx+a}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	43
default	$\frac{2}{3a(bx+a)^{\frac{3}{2}}}+\frac{2}{a^{2}\sqrt{bx+a}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	43
pseudoelliptic	$-\frac{2\left((bx+a)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\sqrt{a}\,bx-\frac{4a^{\frac{3}{2}}}{3}\right)}{(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}}$	46

input
```
int(1/x/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/a/(b*x+a)^(3/2)+2/a^2/(b*x+a)^(1/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a
^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.22

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(3\,abx+4\,a^2)\sqrt{bx+a}}{3\,(a^3b^2x^2+2\,a^4bx+a^5)}, \frac{2\left(3\,(b^2x\right.}{}\right.$$

input
```
integrate(1/x/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(
a) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x
 + a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*
x + a)) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(48) = 96.

Time = 1.26 (sec) , antiderivative size = 697, normalized size of antiderivative = 12.91

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{8a^7\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$+\frac{3a^7\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$-\frac{6a^7\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$+\frac{14a^6bx\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$+\frac{9a^6bx\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$-\frac{18a^6bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$+\frac{6a^5b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$+\frac{9a^5b^2x^2\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$-\frac{18a^5b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$+\frac{3a^4b^3x^3\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$
$$-\frac{6a^4b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$

input
```
integrate(1/x/(b*x+a)**(5/2),x)
```

output
```
8*a**7*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x
**2 + 3*a**(13/2)*b**3*x**3) + 3*a**7*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2
)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**7*log(sqrt(1
 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*
a**(13/2)*b**3*x**3) + 14*a**6*b*x*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17
/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*a**6*b*x*log(
b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2
)*b**3*x**3) - 18*a**6*b*x*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(1
7/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x*
*2*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 9*a**5*b**2*x**2*log(b*x/a)/(3*a**(19/2) + 9*a*
*(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**5*b**
2*x**2*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/
2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*a**4*b**3*x**3*log(b*x/a)/(3*a**
(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3)
- 6*a**4*b**3*x**3*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) + 2/3*(3*
b*x + 4*a)/((b*x + a)^(3/2)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x
+ a)^(3/2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{\frac{2\,(a+b\,x)}{a^2}+\frac{2}{3\,a}}{(a+b\,x)^{3/2}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int(1/(x*(a + b*x)^(5/2)),x)
```

output
```
((2*(a + b*x))/a^2 + 2/(3*a))/(a + b*x)^(3/2) - (2*atanh((a + b*x)^(1/2)/a
^(1/2)))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.24

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{3\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)a+3\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-3\sqrt{a}\,\sqrt{}}{3\sqrt{bx+a}\,a^{3}}$$

input
```
int(1/x/(b*x+a)^(5/2),x)
```

output
```
(3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a + 3*sqrt(a)*sqrt(a
 + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x - 3*sqrt(a)*sqrt(a + b*x)*log(sqr
t(a + b*x) + sqrt(a))*a - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt
(a))*b*x + 8*a**2 + 6*a*b*x)/(3*sqrt(a + b*x)*a**3*(a + b*x))
```

3.415 $\int \frac{1}{x^2(a+bx)^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{1}{x^2(a+bx)^{5/2}} \, dx = -\frac{5b}{3a^2(a+bx)^{3/2}} - \frac{1}{ax(a+bx)^{3/2}} - \frac{5b}{a^3\sqrt{a+bx}} + \frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-5/3*b/a^2/(b*x+a)^(3/2)-1/a/x/(b*x+a)^(3/2)-5*b/a^3/(b*x+a)^(1/2)+5*b*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2(a+bx)^{5/2}} \, dx = \frac{-3a^2 - 20abx - 15b^2x^2}{3a^3x(a+bx)^{3/2}} + \frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[1/(x^2*(a + b*x)^(5/2)),x]
```

output
```
(-3*a^2 - 20*a*b*x - 15*b^2*x^2)/(3*a^3*x*(a + b*x)^(3/2)) + (5*b*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\int \frac{1}{x(a+bx)^{5/2}}dx}{2a}-\frac{1}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5b\left(\frac{\int \frac{1}{x(a+bx)^{3/2}}dx}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5b\left(\frac{\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}$$

input
```
Int[1/(x^2*(a + b*x)^(5/2)),x]
```

output
```
-(1/(a*x*(a + b*x)^(3/2))) - (5*b*(2/(3*a*(a + b*x)^(3/2)) + (2/(a*Sqrt[a
+ b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2))/a))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{\sqrt{bx+a}}{a^{3}x}-\frac{b\left(-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{8}{\sqrt{bx+a}}+\frac{4a}{3(bx+a)^{\frac{3}{2}}}\right)}{2a^{3}}$	60
pseudoelliptic	$\frac{5(bx+a)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-a^{\frac{5}{2}}-\frac{20a^{\frac{3}{2}}bx}{3}-5\sqrt{a}\,b^{2}x^{2}}{x\,a^{\frac{7}{2}}(bx+a)^{\frac{3}{2}}}$	62
derivativedivides	$2b\left(\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}-\frac{1}{3a^{2}(bx+a)^{\frac{3}{2}}}-\frac{2}{a^{3}\sqrt{bx+a}}\right)$	66
default	$2b\left(\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}-\frac{1}{3a^{2}(bx+a)^{\frac{3}{2}}}-\frac{2}{a^{3}\sqrt{bx+a}}\right)$	66

input
```
int(1/x^2/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(b*x+a)^(1/2)/x-1/2/a^3*b*(-10*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/
2)+8/(b*x+a)^(1/2)+4/3*a/(b*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.95

$$\int \frac{1}{x^{2}(a+bx)^{5/2}}\,dx=\left[\frac{15\,(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^{2}x^{2}+20\,a^{2}bx+3\,a^{3}}{6\,(a^{4}b^{2}x^{3}+2\,a^{5}bx^{2}+a^{6}x)}\right.$$

$$\left.-\frac{15\,(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(15\,ab^{2}x^{2}+20\,a^{2}bx+3\,a^{3})\sqrt{bx+a}}{3\,(a^{4}b^{2}x^{3}+2\,a^{5}bx^{2}+a^{6}x)}\right]$$

input
```
integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
 a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a
))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 +
a^2*b*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) + (15*a*b^2*x^2 + 20*a^2*
b*x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(68) = 136$.

Time = 2.35 (sec) , antiderivative size = 818, normalized size of antiderivative = 11.05

$$\int \frac{1}{x^2(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(b*x+a)**(5/2),x)
```

output
```
-6*a**17*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6*a*
*(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**
3*x**4) - 15*a**16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 1
8*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(1 +
 b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3
 + 6*a**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(39/2
)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4
) - 45*a**15*b**2*x**2*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 1
8*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**15*b**2*x**2*log(sq
rt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x**3 + 6*a**(33/2)*b**3*x**4) - 30*a**14*b**3*x**3*sqrt(1 + b*x/a)/(6*a*
*(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**
3*x**4) - 45*a**14*b**3*x**3*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x*
*2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x**3*
log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 15*a**13*b**4*x**4*log(b*x/a)/(6*a
**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b*
*3*x**4) + 30*a**13*b**4*x**4*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18
*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2(a+bx)^{5/2}}\,dx = -\frac{15\,(bx+a)^2b - 10\,(bx+a)ab - 2\,a^2b}{3\left((bx+a)^{\frac{5}{2}}a^3 - (bx+a)^{\frac{3}{2}}a^4\right)} - \frac{5\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

input
```
integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(15*(b*x + a)^2*b - 10*(b*x + a)*a*b - 2*a^2*b)/((b*x + a)^(5/2)*a^3
- (b*x + a)^(3/2)*a^4) - 5/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a
) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(a+bx)^{5/2}}\,dx = -\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} - \frac{2\,(6\,(bx+a)b+ab)}{3\,(bx+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx+a}}{a^3x}$$

input
```
integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b +
a*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(a+bx)^{5/2}}\,dx = \frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{\frac{2\,b}{3\,a} + \frac{10\,b\,(a+b\,x)}{3\,a^2} - \frac{5\,b\,(a+b\,x)^2}{a^3}}{a\,(a+b\,x)^{3/2} - (a+b\,x)^{5/2}}$$

input
```
int(1/(x^2*(a + b*x)^(5/2)),x)
```

output
```
(5*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(7/2) - ((2*b)/(3*a) + (10*b*(a + b
*x))/(3*a^2) - (5*b*(a + b*x)^2)/a^3)/(a*(a + b*x)^(3/2) - (a + b*x)^(5/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

$$\int \frac{1}{x^2(a+bx)^{5/2}}\,dx = \frac{-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abx-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2}{}$$

input
```
int(1/x^2/(b*x+a)^(5/2),x)
```

output
```
( - 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*x - 15*sqrt(
a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 + 15*sqrt(a)*sqrt(
a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b*x + 15*sqrt(a)*sqrt(a + b*x)*log
(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 6*a**3 - 40*a**2*b*x - 30*a*b**2*x**
2)/(6*sqrt(a + b*x)*a**4*x*(a + b*x))
```

3.416 $\int \frac{1}{x^3(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{35b^2}{12a^3(a+bx)^{3/2}} - \frac{1}{2ax^2(a+bx)^{3/2}} + \frac{7b}{4a^2x(a+bx)^{3/2}} + \frac{35b^2}{4a^4\sqrt{a+bx}} - \frac{35b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

output
```
35/12*b^2/a^3/(b*x+a)^(3/2)-1/2/a/x^2/(b*x+a)^(3/2)+7/4*b/a^2/x/(b*x+a)^(3
/2)+35/4*b^2/a^4/(b*x+a)^(1/2)-35/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(
9/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{-6a^3+21a^2bx+140ab^2x^2+105b^3x^3}{12a^4x^2(a+bx)^{3/2}} - \frac{35b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

input
```
Integrate[1/(x^3*(a + b*x)^(5/2)),x]
```

output
```
(-6*a^3 + 21*a^2*b*x + 140*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^2*(a + b*x)^
(3/2)) - (35*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{7b\int \frac{1}{x^2(a+bx)^{5/2}}dx}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{7b\left(-\frac{5b\int \frac{1}{x(a+bx)^{5/2}}dx}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{7b\left(-\frac{5b\left(\frac{\int \frac{1}{x(a+bx)^{3/2}}dx}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{7b\left(-\frac{5b\left(\frac{\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{7b\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{7b\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

input
```
Int[1/(x^3*(a + b*x)^(5/2)),x]
```

output
```
-1/2*1/(a*x^2*(a + b*x)^(3/2)) - (7*b*(-(1/(a*x*(a + b*x)^(3/2))) - (5*b*(
2/(3*a*(a + b*x)^(3/2)) + (2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/a^(3/2))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-11bx+2a)}{4a^4x^2}+\frac{b^2\left(-\frac{70\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{48}{\sqrt{bx+a}}+\frac{16a}{3(bx+a)^{\frac{3}{2}}}\right)}{8a^4}$	70
pseudoelliptic	$-\frac{35\left((bx+a)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-\sqrt{a}\,b^3x^3-\frac{4a^{\frac{3}{2}}b^2x^2}{3}-\frac{a^{\frac{5}{2}}bx}{5}+\frac{2a^{\frac{7}{2}}}{35}\right)}{4a^{\frac{9}{2}}(bx+a)^{\frac{3}{2}}x^2}$	77
derivativedivides	$2b^2\left(\frac{3}{a^4\sqrt{bx+a}}+\frac{1}{3a^3(bx+a)^{\frac{3}{2}}}-\frac{\frac{-\frac{11(bx+a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{35\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}\right)$	81
default	$2b^2\left(\frac{3}{a^4\sqrt{bx+a}}+\frac{1}{3a^3(bx+a)^{\frac{3}{2}}}-\frac{\frac{-\frac{11(bx+a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{35\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}\right)$	81

input
```
int(1/x^3/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(-11*b*x+2*a)/a^4/x^2+1/8/a^4*b^2*(-70*arctanh((b*x+a)^
(1/2)/a^(1/2))/a^(1/2)+48/(b*x+a)^(1/2)+16/3*a/(b*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.38

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \left[\frac{105\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(105\,ab^3x^3+140\,a^2b^2x}{24\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)}\right.$$

input
```
integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt
(b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*
b*x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(1
05*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x
 + a)) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x +
 a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(99) = 198$.

Time = 5.66 (sec) , antiderivative size = 464, normalized size of antiderivative = 4.38

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = -\frac{6a^{\frac{89}{2}}b^{75}x^{75}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{21a^{\frac{87}{2}}b^{76}x^{76}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{140a^{\frac{85}{2}}b^{77}x^{77}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{105a^{\frac{83}{2}}b^{78}x^{78}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{105a^{42}b^{\frac{155}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{105a^{41}b^{\frac{157}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate(1/x**3/(b*x+a)**(5/2),x)
```

output
```
-6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x)
+ 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(87/2
)*b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a
**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77*x*
*77/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b
**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 105*a**(83/2)*b**78*x**78/(12*a*
*(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*
x**(157/2)*sqrt(a/(b*x) + 1)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*
x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/
2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1
)) - 105*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(sqrt
(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a
**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{105\,(bx+a)^3b^2 - 175\,(bx+a)^2ab^2 + 56\,(bx+a)a^2b^2 + 8\,a^3b^2}{12\left((bx+a)^{\frac{7}{2}}a^4 - 2\,(bx+a)^{\frac{5}{2}}a^5 + (bx+a)^{\frac{3}{2}}a^6\right)} + \frac{35\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{9}{2}}}$$

input
```
integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(105*(b*x + a)^3*b^2 - 175*(b*x + a)^2*a*b^2 + 56*(b*x + a)*a^2*b^2 +
 8*a^3*b^2)/((b*x + a)^(7/2)*a^4 - 2*(b*x + a)^(5/2)*a^5 + (b*x + a)^(3/2)
*a^6) + 35/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/
a^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^4} + \frac{2\,(9\,(bx+a)b^2 + ab^2)}{3\,(bx+a)^{\frac{3}{2}}a^4} + \frac{11\,(bx+a)^{\frac{3}{2}}b^2 - 13\,\sqrt{bx+a}ab^2}{4\,a^4b^2x^2}$$

input
```
integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*
b^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt
(b*x + a)*a*b^2)/(a^4*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,{(a+b\,x)}^2}{12\,a^3}+\frac{35\,b^2\,{(a+b\,x)}^3}{4\,a^4}+\frac{14\,b^2\,(a+b\,x)}{3\,a^2}}{{(a+b\,x)}^{7/2}-2\,a\,{(a+b\,x)}^{5/2}+a^2\,{(a+b\,x)}^{3/2}}-\frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

input
```
int(1/(x^3*(a + b*x)^(5/2)),x)
```

output
```
((2*b^2)/(3*a) - (175*b^2*(a + b*x)^2)/(12*a^3) + (35*b^2*(a + b*x)^3)/(4*
a^4) + (14*b^2*(a + b*x))/(3*a^2))/((a + b*x)^(7/2) - 2*a*(a + b*x)^(5/2)
+ a^2*(a + b*x)^(3/2)) - (35*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(9/2
))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{105\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)a\,b^2x^2+105\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3}{}$$

input
```
int(1/x^3/(b*x+a)^(5/2),x)
```

output
```
(105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*x**2 + 105*
sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 105*sqrt(a)
*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*x**2 - 105*sqrt(a)*sqrt
(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3 - 12*a**4 + 42*a**3*b*x +
 280*a**2*b**2*x**2 + 210*a*b**3*x**3)/(24*sqrt(a + b*x)*a**5*x**2*(a + b*
x))
```

3.417 $\int \frac{1}{x\sqrt{-a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[-a + b*x]),x]
```

output
```
(2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{bx-a}}dx$$

$$\Big\downarrow \; 73$$

$$\frac{2\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[-a + b*x]),x]
```

output
```
(2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20
default	$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20

input
```
int(1/x/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx = \left[-\frac{\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)}{a}, -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)}{\sqrt{a}}\right]$$

input
```
integrate(1/x/(b*x-a)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x)/a, -2*arctan(sqrt
(a)/sqrt(b*x - a))/sqrt(a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx = \begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x-a)**(1/2),x)
```

output
```
Piecewise((2*I*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1)
, (-2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x-a)^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x-a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(b*x - a)^(1/2)),x)
```

output
```
(2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\sqrt{-a+bx}}\,dx = \frac{2\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a}$$

input
```
int(1/x/(b*x-a)^(1/2),x)
```

output
```
(2*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a)))/a
```

3.418 $\int \frac{1}{x^2\sqrt{-a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx=\frac{\sqrt{-a+bx}}{ax}+\frac{b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(b*x-a)^(1/2)/a/x+b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx=\frac{\sqrt{-a+bx}}{ax}+\frac{b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x^2*Sqrt[-a + b*x]),x]
```

output
```
Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{bx-a}}dx$$

$$\Big\downarrow \; 52$$

$$\frac{b\int \frac{1}{x\sqrt{bx-a}}dx}{2a}+\frac{\sqrt{bx-a}}{ax}$$

$$\Big\downarrow \; 73$$

$$\frac{\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{a}+\frac{\sqrt{bx-a}}{ax}$$

$$\Big\downarrow \; 218$$

$$\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

input
```
Int[1/(x^2*Sqrt[-a + b*x]),x]
```

output
```
Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\sqrt{bx-a}}{ax}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	37
derivativedivides	$2b\left(\frac{\sqrt{bx-a}}{2abx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	44
default	$2b\left(\frac{\sqrt{bx-a}}{2abx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	44
risch	$-\frac{-bx+a}{ax\sqrt{bx-a}}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	44

input
```
int(1/x^2/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x-a)^(1/2)/a/x+b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \left[-\frac{\sqrt{-a}bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\sqrt{bx-a}a}{2a^2x}, -\frac{\sqrt{a}bx\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)-\sqrt{bx-a}a}{a^2x}\right]$$

input
```
integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*sqrt
(b*x - a)*a)/(a^2*x), -(sqrt(a)*b*x*arctan(sqrt(a)/sqrt(b*x - a)) - sqrt(b
*x - a)*a)/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \begin{cases} \frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a\sqrt{x}}+\frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{1}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{\sqrt{b}}{a\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**2/(b*x-a)**(1/2),x)
```

output
```
Piecewise((I*sqrt(b)*sqrt(a/(b*x) - 1)/(a*sqrt(x)) + I*b*acosh(sqrt(a)/(sq
rt(b)*sqrt(x)))/a**(3/2), Abs(a/(b*x)) > 1), (-1/(sqrt(b)*x**(3/2)*sqrt(-a
/(b*x) + 1)) + sqrt(b)/(a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sq
rt(b)*sqrt(x)))/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \frac{\sqrt{bx-a}\,b}{(bx-a)a+a^2} + \frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b*x - a)*b/((b*x - a)*a + a^2) + b*arctan(sqrt(b*x - a)/sqrt(a))/a^(3
/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = b\left(\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{bx-a}}{abx}\right)$$

input
```
integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="giac")
```

output
```
b*(arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + sqrt(b*x - a)/(a*b*x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \frac{\sqrt{b\,x-a}}{a\,x} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(1/(x^2*(b*x - a)^(1/2)),x)
```

output
```
(b*x - a)^(1/2)/(a*x) + (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \frac{\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)bx+\sqrt{bx-a}\,a}{a^2x}$$

input
```
int(1/x^2/(b*x-a)^(1/2),x)
```

output
```
(sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*b*x + sqrt( - a + b*x)*a)/(a**2*x)
```

3.419 $\int \frac{1}{x^3\sqrt{-a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \frac{\sqrt{-a+bx}}{2ax^2} + \frac{3b\sqrt{-a+bx}}{4a^2x} + \frac{3b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
1/2*(b*x-a)^(1/2)/a/x^2+3/4*b*(b*x-a)^(1/2)/a^2/x+3/4*b^2*arctan((b*x-a)^(
1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \frac{\sqrt{-a+bx}(2a+3bx)}{4a^2x^2} + \frac{3b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[1/(x^3*Sqrt[-a + b*x]),x]
```

output
```
(Sqrt[-a + b*x]*(2*a + 3*b*x))/(4*a^2*x^2) + (3*b^2*ArcTan[Sqrt[-a + b*x]/
Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{bx-a}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\int \frac{1}{x^2\sqrt{bx-a}}dx}{4a}+\frac{\sqrt{bx-a}}{2ax^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\left(\frac{b\int \frac{1}{x\sqrt{bx-a}}dx}{2a}+\frac{\sqrt{bx-a}}{ax}\right)}{4a}+\frac{\sqrt{bx-a}}{2ax^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{3b\left(\frac{\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{a}+\frac{\sqrt{bx-a}}{ax}\right)}{4a}+\frac{\sqrt{bx-a}}{2ax^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3b\left(\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}\right)}{4a}+\frac{\sqrt{bx-a}}{2ax^2}$$

input
```
Int[1/(x^3*Sqrt[-a + b*x]),x]
```

output
```
Sqrt[-a + b*x]/(2*a*x^2) + (3*b*(Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/a^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{(-bx+a)(3bx+2a)}{4a^{2}x^{2}\sqrt{bx-a}}+\frac{3b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	55
pseudoelliptic	$\frac{3b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)a^{2}x^{2}+2a^{\frac{5}{2}}\sqrt{bx-a}\left(\frac{3bx}{2}+a\right)}{4a^{\frac{9}{2}}x^{2}}$	55
derivativedivides	$2b^{2}\left(\frac{\sqrt{bx-a}}{4a\,b^{2}x^{2}}+\frac{\frac{3\sqrt{bx-a}}{8abx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}}{a}\right)$	72
default	$2b^{2}\left(\frac{\sqrt{bx-a}}{4a\,b^{2}x^{2}}+\frac{\frac{3\sqrt{bx-a}}{8abx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}}{a}\right)$	72

input
```
int(1/x^3/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b*x+a)*(3*b*x+2*a)/a^2/x^2/(b*x-a)^(1/2)+3/4*b^2*arctan((b*x-a)^(1/
2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \left[-\frac{3\sqrt{-a}b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\,(3\,abx+2\,a^2)\sqrt{bx-a}}{8\,a^3x^2},\ -\frac{3\sqrt{a}b^2x^2\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)-(3\,abx+2\,a^2)\sqrt{bx-a}}{4\,a^3x^2}\right]$$

input
```
integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) -
2*(3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2), -1/4*(3*sqrt(a)*b^2*x^2*arct
an(sqrt(a)/sqrt(b*x - a)) - (3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \begin{cases} \frac{i}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{i\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{3ib^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{1}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(b*x-a)**(1/2),x)
```

output
```
Piecewise((I/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(4*a*x**(3
/2)*sqrt(a/(b*x) - 1)) - 3*I*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) - 1)) +
 3*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), Abs(a/(b*x)) > 1)
, (-1/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - sqrt(b)/(4*a*x**(3/2)*sqrt
(-a/(b*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) - 3*b**2*
asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}} + \frac{3\,(bx-a)^{\frac{3}{2}}b^2 + 5\,\sqrt{bx-a}ab^2}{4\left((bx-a)^2a^2 + 2\,(bx-a)a^3 + a^4\right)}$$

input
```
integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="maxima")
```

output
```
3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 1/4*(3*(b*x - a)^(3/2)*b^2
 + 5*sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{3\,(bx-a)^{\frac{3}{2}}b^3+5\,\sqrt{bx-a}ab^3}{a^2b^2x^2}}{4\,b}$$

input
```
integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + (3*(b*x - a)^(3/2)*b^3
+ 5*sqrt(b*x - a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{5/2}} + \frac{5\,\sqrt{b\,x-a}}{4\,a\,x^2} + \frac{3\,(b\,x-a)^{3/2}}{4\,a^2\,x^2}$$

input
```
int(1/(x^3*(b*x - a)^(1/2)),x)
```

output
```
(3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(5/2)) + (5*(b*x - a)^(1/2))/(4
*a*x^2) + (3*(b*x - a)^(3/2))/(4*a^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3\sqrt{-a+bx}}\,dx = \frac{3\sqrt{a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^2x^2 + 2\sqrt{bx-a}\,a^2 + 3\sqrt{bx-a}\,abx}{4a^3x^2}$$

input
```
int(1/x^3/(b*x-a)^(1/2),x)
```

output
```
(3*sqrt(a)*atan(sqrt( - a + b*x)/sqrt(a))*b**2*x**2 + 2*sqrt( - a + b*x)*a
**2 + 3*sqrt( - a + b*x)*a*b*x)/(4*a**3*x**2)
```

3.420 $\int \frac{1}{x(-a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2}{a\sqrt{-a+bx}} - \frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-2/a/(b*x-a)^(1/2)-2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2}{a\sqrt{-a+bx}} - \frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(-a + b*x)^(3/2)),x]
```

output
```
-2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(bx-a)^{3/2}} \, dx$$
$$\Big\downarrow \; 61$$
$$-\frac{\int \frac{1}{x\sqrt{bx-a}}dx}{a}-\frac{2}{a\sqrt{bx-a}}$$
$$\Big\downarrow \; 73$$
$$-\frac{2\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{ab}-\frac{2}{a\sqrt{bx-a}}$$
$$\Big\downarrow \; 218$$
$$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

input
```
Int[1/(x*(-a + b*x)^(3/2)),x]
```

output
```
-2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2}{a\sqrt{bx-a}}-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	35
default	$-\frac{2}{a\sqrt{bx-a}}-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	35
pseudoelliptic	$-\frac{2}{a\sqrt{bx-a}}-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	35

input
```
int(1/x/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(b*x-a)^(1/2)-2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.98

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = \left[-\frac{(bx-a)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}a}{a^2bx-a^3}, \frac{2\left((bx-a)\sqrt{a}\arctan\left(\frac{\sqrt{}}{\sqrt{b}}\right.\right.}{a^2bx-a}\right.$$

input
```
integrate(1/x/(b*x-a)^(3/2),x, algorithm="fricas")
```

output
```
[-((b*x - a)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sq
rt(b*x - a)*a)/(a^2*b*x - a^3), 2*((b*x - a)*sqrt(a)*arctan(sqrt(a)/sqrt(b
*x - a)) - sqrt(b*x - a)*a)/(a^2*b*x - a^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 437, normalized size of antiderivative = 10.40

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = \begin{cases} -\frac{2a^3\sqrt{-1+\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{2ia^3\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2a^3\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2ia^2bx\log\cdots}{-a^{\frac{9}{2}}+\cdots} \\ -\frac{2ia^3\sqrt{1-\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{2ia^3\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{\pi a^3}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2ia^2bx\log\cdots}{-a\cdots} \end{cases}$$

input
```
integrate(1/x/(b*x-a)**(3/2),x)
```

output
```
Piecewise((-2*a**3*sqrt(-1 + b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**3*lo
g(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(b)*sqrt(x)/sqrt(a)
)/(-a**(9/2) + a**(7/2)*b*x) - 2*a**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a*
*(9/2) + a**(7/2)*b*x) + I*a**2*b*x*log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x)
- 2*I*a**2*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-a**(9/2) + a**(7/2)*b*x) + 2
*a**2*b*x*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a**(9/2) + a**(7/2)*b*x), Abs(
b*x/a) > 1), (-2*I*a**3*sqrt(1 - b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**
3*log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(1 - b*x/a) + 1
)/(-a**(9/2) + a**(7/2)*b*x) - pi*a**3/(-a**(9/2) + a**(7/2)*b*x) + I*a**2
*b*x*log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - 2*I*a**2*b*x*log(sqrt(1 - b*x
/a) + 1)/(-a**(9/2) + a**(7/2)*b*x) + pi*a**2*b*x/(-a**(9/2) + a**(7/2)*b*
x), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{bx-a}a}$$

input
```
integrate(1/x/(b*x-a)^(3/2),x, algorithm="maxima")
```

output
```
-2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{bx-a}a}$$

input
```
integrate(1/x/(b*x-a)^(3/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2}{a\,\sqrt{b\,x-a}}$$

input
```
int(1/(x*(b*x - a)^(3/2)),x)
```

output
```
- (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2) - 2/(a*(b*x - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = \frac{-2\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a}{\sqrt{bx-a}\,a^2}$$

input
```
int(1/x/(b*x-a)^(3/2),x)
```

output
```
( - 2*(sqrt(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a)) + a))/(sqrt
( - a + b*x)*a**2)
```

3.421 $\int \frac{1}{x^2(-a+bx)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{1}{x^2(-a+bx)^{3/2}} \, dx = -\frac{3b}{a^2\sqrt{-a+bx}}+\frac{1}{ax\sqrt{-a+bx}}-\frac{3b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-3*b/a^2/(b*x-a)^(1/2)+1/a/x/(b*x-a)^(1/2)-3*b*arctan((b*x-a)^(1/2)/a^(1/2
))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(-a+bx)^{3/2}} \, dx = \frac{a-3bx}{a^2x\sqrt{-a+bx}}-\frac{3b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^2*(-a + b*x)^(3/2)),x]
```

output
```
(a - 3*b*x)/(a^2*x*Sqrt[-a + b*x]) - (3*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/
a^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(bx-a)^{3/2}} \, dx$$

$$\Big\downarrow \ 52$$

$$\frac{3b\int \frac{1}{x(bx-a)^{3/2}}dx}{2a}+\frac{1}{ax\sqrt{bx-a}}$$

$$\Big\downarrow \ 61$$

$$\frac{3b\left(-\frac{\int \frac{1}{x\sqrt{bx-a}}dx}{a}-\frac{2}{a\sqrt{bx-a}}\right)}{2a}+\frac{1}{ax\sqrt{bx-a}}$$

$$\Big\downarrow \ 73$$

$$\frac{3b\left(-\frac{2\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{ab}-\frac{2}{a\sqrt{bx-a}}\right)}{2a}+\frac{1}{ax\sqrt{bx-a}}$$

$$\Big\downarrow \ 218$$

$$\frac{3b\left(-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}\right)}{2a}+\frac{1}{ax\sqrt{bx-a}}$$

input
```
Int[1/(x^2*(-a + b*x)^(3/2)),x]
```

output
```
1/(a*x*Sqrt[-a + b*x]) + (3*b*(-2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a +
 b*x]/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{\sqrt{bx-a}}{a^{2}x}-\frac{3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2b}{a^{2}\sqrt{bx-a}}$	54
risch	$\frac{-bx+a}{a^{2}x\sqrt{bx-a}}-\frac{2b}{a^{2}\sqrt{bx-a}}-\frac{3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	59
derivativedivides	$2b\left(-\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}-\frac{1}{a^{2}\sqrt{bx-a}}\right)$	61
default	$2b\left(-\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}-\frac{1}{a^{2}\sqrt{bx-a}}\right)$	61

input
```
int(1/x^2/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(b*x-a)^(1/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)-2*b/a^2/(
b*x-a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.65

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = \left[-\frac{3\,(b^2x^2-abx)\sqrt{-a}\log\left(\frac{bx+2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)+2\,(3\,abx-a^2)\sqrt{bx-a}}{2\,(a^3bx^2-a^4x)},\ \frac{3\,(b^2x^2-\right.$$

input
```
integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(3*(b^2*x^2 - a*b*x)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) -
2*a)/x) + 2*(3*a*b*x - a^2)*sqrt(b*x - a))/(a^3*b*x^2 - a^4*x), (3*(b^2*x^
2 - a*b*x)*sqrt(a)*arctan(sqrt(a)/sqrt(b*x - a)) - (3*a*b*x - a^2)*sqrt(b*
x - a))/(a^3*b*x^2 - a^4*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = \begin{cases} -\frac{i}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{3i\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{3ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{3b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**2/(b*x-a)**(3/2),x)
```

output
```
Piecewise((-I/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(a**2*s
qrt(x)*sqrt(a/(b*x) - 1)) - 3*I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2
), Abs(a/(b*x)) > 1), (1/(a*sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(
b)/(a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) + 3*b*asin(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = -\frac{3\,(bx-a)b+2\,ab}{(bx-a)^{\frac{3}{2}}a^2+\sqrt{bx-a}a^3} - \frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

input
```
integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="maxima")
```

output
```
-(3*(b*x - a)*b + 2*a*b)/((b*x - a)^(3/2)*a^2 + sqrt(b*x - a)*a^3) - 3*b*a
rctan(sqrt(b*x - a)/sqrt(a))/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{3\,(bx-a)b+2\,ab}{\left((bx-a)^{\frac{3}{2}}+\sqrt{bx-a}a\right)a^2}$$

input
```
integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="giac")
```

output
```
-3*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) - (3*(b*x - a)*b + 2*a*b)/(((b*
x - a)^(3/2) + sqrt(b*x - a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = \frac{1}{a\,x\,\sqrt{b\,x-a}} - \frac{3\,b}{a^2\,\sqrt{b\,x-a}} - \frac{3\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int(1/(x^2*(b*x - a)^(3/2)),x)
```

output
```
1/(a*x*(b*x - a)^(1/2)) - (3*b)/(a^2*(b*x - a)^(1/2)) - (3*b*atan((b*x - a
)^(1/2)/a^(1/2)))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = \frac{-3\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)bx+a^2-3abx}{\sqrt{bx-a}\,a^3x}$$

input
```
int(1/x^2/(b*x-a)^(3/2),x)
```

output
```
( - 3*sqrt(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*b*x + a**2 -
 3*a*b*x)/(sqrt( - a + b*x)*a**3*x)
```

3.422 $\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = -\frac{15b^2}{4a^3\sqrt{-a+bx}}+\frac{1}{2ax^2\sqrt{-a+bx}}+\frac{5b}{4a^2x\sqrt{-a+bx}}-\frac{15b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
-15/4*b^2/a^3/(b*x-a)^(1/2)+1/2/a/x^2/(b*x-a)^(1/2)+5/4*b/a^2/x/(b*x-a)^(1
/2)-15/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = \frac{2a^2+5abx-15b^2x^2}{4a^3x^2\sqrt{-a+bx}}-\frac{15b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[1/(x^3*(-a + b*x)^(3/2)),x]
```

output
```
(2*a^2 + 5*a*b*x - 15*b^2*x^2)/(4*a^3*x^2*Sqrt[-a + b*x]) - (15*b^2*ArcTan
[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {52, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(bx-a)^{3/2}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\int \frac{1}{x^2(bx-a)^{3/2}}dx}{4a}+\frac{1}{2ax^2\sqrt{bx-a}}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\left(\frac{3b\int \frac{1}{x(bx-a)^{3/2}}dx}{2a}+\frac{1}{ax\sqrt{bx-a}}\right)}{4a}+\frac{1}{2ax^2\sqrt{bx-a}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(\frac{3b\left(-\frac{\int \frac{1}{x\sqrt{bx-a}}dx}{a}-\frac{2}{a\sqrt{bx-a}}\right)}{2a}+\frac{1}{ax\sqrt{bx-a}}\right)}{4a}+\frac{1}{2ax^2\sqrt{bx-a}}$$

$$\Big\downarrow \text{73}$$

$$\frac{5b\left(\frac{3b\left(-\frac{2\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{ab}-\frac{2}{a\sqrt{bx-a}}\right)}{2a}+\frac{1}{ax\sqrt{bx-a}}\right)}{4a}+\frac{1}{2ax^2\sqrt{bx-a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{5b\left(\frac{3b\left(-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}\right)}{2a}+\frac{1}{ax\sqrt{bx-a}}\right)}{4a}+\frac{1}{2ax^2\sqrt{bx-a}}$$

input
```
Int[1/(x^3*(-a + b*x)^(3/2)),x]
```

output
```
1/(2*a*x^2*Sqrt[-a + b*x]) + (5*b*(1/(a*x*Sqrt[-a + b*x]) + (3*b*(-2/(a*Sq
rt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(-bx+a)(7bx+2a)}{4a^3x^2\sqrt{bx-a}}-\frac{2b^2}{a^3\sqrt{bx-a}}-\frac{15b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$	72
pseudoelliptic	$\frac{-15b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)a^3x^2\sqrt{bx-a}+2a^{\frac{7}{2}}\left(a^2+\frac{5}{2}abx-\frac{15}{2}b^2x^2\right)}{4\sqrt{bx-a}\,a^{\frac{13}{2}}x^2}$	75
derivativedivides	$2b^2\left(-\frac{\frac{\frac{7(bx-a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}-\frac{1}{a^3\sqrt{bx-a}}\right)$	77
default	$2b^2\left(-\frac{\frac{\frac{7(bx-a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}-\frac{1}{a^3\sqrt{bx-a}}\right)$	77

input
```
int(1/x^3/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-b*x+a)*(7*b*x+2*a)/a^3/x^2/(b*x-a)^(1/2)-2*b^2/a^3/(b*x-a)^(1/2)-15/
4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = \left[-\frac{15\left(b^3x^3-ab^2x^2\right)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(15ab^2x^2-5a^2bx-2a^3\right)\sqrt{}}{8\left(a^4bx^3-a^5x^2\right)}\right.$$

input
```
integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-
a) - 2*a)/x) + 2*(15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*
x^3 - a^5*x^2), 1/4*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(a)*arctan(sqrt(a)/sqrt(
b*x - a)) - (15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^3 -
 a^5*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.38

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = \begin{cases} -\frac{i}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{5i\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{15ib^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{15ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{15b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(b*x-a)**(3/2),x)
```

output
```
Piecewise((-I/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 5*I*sqrt(b)/(4*a*
*2*x**(3/2)*sqrt(a/(b*x) - 1)) + 15*I*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x
) - 1)) - 15*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), Abs(a/(
b*x)) > 1), (1/(2*a*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 5*sqrt(b)/(4*a*
*2*x**(3/2)*sqrt(-a/(b*x) + 1)) - 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(-a/(b*x
) + 1)) + 15*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = -\frac{15\,(bx-a)^2b^2 + 25\,(bx-a)ab^2 + 8\,a^2b^2}{4\left((bx-a)^{\frac{5}{2}}a^3 + 2\,(bx-a)^{\frac{3}{2}}a^4 + \sqrt{bx-a}a^5\right)} - \frac{15\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}$$

input
```
integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(15*(b*x - a)^2*b^2 + 25*(b*x - a)*a*b^2 + 8*a^2*b^2)/((b*x - a)^(5/2
)*a^3 + 2*(b*x - a)^(3/2)*a^4 + sqrt(b*x - a)*a^5) - 15/4*b^2*arctan(sqrt(
b*x - a)/sqrt(a))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = -\frac{15\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}} - \frac{2\,b^2}{\sqrt{bx-a}a^3} - \frac{7\,(bx-a)^{\frac{3}{2}}b^2 + 9\,\sqrt{bx-a}ab^2}{4\,a^3b^2x^2}$$

input
```
integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="giac")
```

output
```
-15/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) - 2*b^2/(sqrt(b*x - a)*a^3
) - 1/4*(7*(b*x - a)^(3/2)*b^2 + 9*sqrt(b*x - a)*a*b^2)/(a^3*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = -\frac{\frac{2\,b^2}{a} + \frac{15\,b^2\,(a-b\,x)^2}{4\,a^3} - \frac{25\,b^2\,(a-b\,x)}{4\,a^2}}{2\,a\,(b\,x-a)^{3/2} + (b\,x-a)^{5/2} + a^2\,\sqrt{b\,x-a}} - \frac{15\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

input
```
int(1/(x^3*(b*x - a)^(3/2)),x)
```

output
```
- ((2*b^2)/a + (15*b^2*(a - b*x)^2)/(4*a^3) - (25*b^2*(a - b*x))/(4*a^2))/
(2*a*(b*x - a)^(3/2) + (b*x - a)^(5/2) + a^2*(b*x - a)^(1/2)) - (15*b^2*at
an((b*x - a)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = \frac{-15\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^2x^2+2a^3+5a^2bx-15a\,b^2x^2}{4\sqrt{bx-a}\,a^4x^2}$$

input
```
int(1/x^3/(b*x-a)^(3/2),x)
```

output
```
( - 15*sqrt(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*b**2*x**2 +
 2*a**3 + 5*a**2*b*x - 15*a*b**2*x**2)/(4*sqrt( - a + b*x)*a**4*x**2)
```

3.423 $\int \frac{1}{x(-a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-2/3/a/(b*x-a)^(3/2)+2/a^2/(b*x-a)^(1/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a
^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = -\frac{8a-6bx}{3a^2(-a+bx)^{3/2}}+\frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x*(-a + b*x)^(5/2)),x]
```

output
```
-1/3*(8*a - 6*b*x)/(a^2*(-a + b*x)^(3/2)) + (2*ArcTan[Sqrt[-a + b*x]/Sqrt[
a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(bx-a)^{5/2}}\,dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{\int \frac{1}{x(bx-a)^{3/2}}dx}{a}-\frac{2}{3a(bx-a)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{-\frac{\int \frac{1}{x\sqrt{bx-a}}dx}{a}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{2\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{ab}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}$$

input
```
Int[1/(x*(-a + b*x)^(5/2)),x]
```

output
```
-2/(3*a*(-a + b*x)^(3/2)) - (-2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]])/a^(3/2))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2}{3a(bx-a)^{\frac{3}{2}}}+\frac{2}{a^{2}\sqrt{bx-a}}+\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	49
default	$-\frac{2}{3a(bx-a)^{\frac{3}{2}}}+\frac{2}{a^{2}\sqrt{bx-a}}+\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	49
pseudoelliptic	$-\frac{2\left(\sqrt{bx-a}\,(-bx+a)\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{a}\,bx+\frac{4a^{\frac{3}{2}}}{3}\right)}{(bx-a)^{\frac{3}{2}}a^{\frac{5}{2}}}$	58

input
```
int(1/x/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/(b*x-a)^(3/2)+2/a^2/(b*x-a)^(1/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a
^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.05

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = \left[-\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\,(3\,abx-4\,a^2)\sqrt{bx-a}}{3\,(a^3b^2x^2-2\,a^4bx+a^5)},\right.$$
$$\left.-\frac{2\left(3\,(b^2x^2-2\,abx+a^2)\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)-(3\,abx-4\,a^2)\sqrt{bx-a}\right)}{3\,(a^3b^2x^2-2\,a^4bx+a^5)}\right]$$

input
```
integrate(1/x/(b*x-a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqr
t(-a) - 2*a)/x) - 2*(3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*
b*x + a^5), -2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a)*arctan(sqrt(a)/sqrt(
b*x - a)) - (3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*x + a^
5)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 93.41 (sec) , antiderivative size = 1950, normalized size of antiderivative = 32.50

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x-a)**(5/2),x)
```

output
```
Piecewise((8*a**7*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**
(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*I*a**7*log(b*x/a)/(-3*a**(19
/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6
*I*a**7*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a
**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**7*asin(sqrt(a)/(sqrt(b)
*sqrt(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**
(13/2)*b**3*x**3) - 14*a**6*b*x*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/
2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*I*a**6*b*x*log
(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13
/2)*b**3*x**3) + 18*I*a**6*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2)
+ 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a*
*6*b*x*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9
*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(-1 +
 b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13
/2)*b**3*x**3) + 9*I*a**5*b**2*x**2*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)
*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*I*a**5*b**2*x**
2*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/
2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 18*a**5*b**2*x**2*asin(sqrt(a)/(sq
rt(b)*sqrt(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 +
3*a**(13/2)*b**3*x**3) - 3*I*a**4*b**3*x**3*log(b*x/a)/(-3*a**(19/2) + ...
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x/(b*x-a)^(5/2),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/
2)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x/(b*x-a)^(5/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/
2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2\,(a-b\,x)}{a^2}+\frac{2}{3\,a}}{(b\,x-a)^{3/2}}$$

input
```
int(1/(x*(b*x - a)^(5/2)),x)
```

output
```
(2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(5/2) - ((2*(a - b*x))/a^2 + 2/(3*a))/
(b*x - a)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = \frac{2\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)a - 2\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)bx + \frac{8a^2}{3} - 2abx}{\sqrt{bx-a}\,a^3\,(-bx+a)}$$

input
```
int(1/x/(b*x-a)^(5/2),x)
```

output
```
(2*(3*sqrt(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*a - 3*sqrt(a
)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*b*x + 4*a**2 - 3*a*b*x))
/(3*sqrt( - a + b*x)*a**3*(a - b*x))
```

3.424 $\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = -\frac{5b}{3a^2(-a+bx)^{3/2}} + \frac{1}{ax(-a+bx)^{3/2}} + \frac{5b}{a^3\sqrt{-a+bx}} + \frac{5b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-5/3*b/a^2/(b*x-a)^(3/2)+1/a/x/(b*x-a)^(3/2)+5*b/a^3/(b*x-a)^(1/2)+5*b*arc
tan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = \frac{3a^2-20abx+15b^2x^2}{3a^3x(-a+bx)^{3/2}} + \frac{5b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[1/(x^2*(-a + b*x)^(5/2)),x]
```

output
```
(3*a^2 - 20*a*b*x + 15*b^2*x^2)/(3*a^3*x*(-a + b*x)^(3/2)) + (5*b*ArcTan[S
qrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(bx-a)^{5/2}}dx$$
$$\Big\downarrow \; 52$$
$$\frac{5b\int \frac{1}{x(bx-a)^{5/2}}dx}{2a}+\frac{1}{ax(bx-a)^{3/2}}$$
$$\Big\downarrow \; 61$$
$$\frac{5b\left(-\frac{\int \frac{1}{x(bx-a)^{3/2}}dx}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}$$
$$\Big\downarrow \; 61$$
$$\frac{5b\left(-\frac{-\frac{\int \frac{1}{x\sqrt{bx-a}}dx}{a}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}$$
$$\Big\downarrow \; 73$$
$$\frac{5b\left(-\frac{-\frac{2\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{ab}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}$$
$$\Big\downarrow \; 218$$

$$\frac{5b\left(-\frac{-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}$$

input
```
Int[1/(x^2*(-a + b*x)^(5/2)),x]
```

output
```
1/(a*x*(-a + b*x)^(3/2)) + (5*b*(-2/(3*a*(-a + b*x)^(3/2)) - (-2/(a*Sqrt[-
a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2))/a))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\sqrt{bx-a}}{a^{3}x}+\frac{5b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}-\frac{2b}{3a^{2}\left(bx-a\right)^{\frac{3}{2}}}+\frac{4b}{a^{3}\sqrt{bx-a}}$	68
derivativedivides	$2b\left(-\frac{1}{3a^{2}\left(bx-a\right)^{\frac{3}{2}}}+\frac{2}{a^{3}\sqrt{bx-a}}+\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}\right)$	74
default	$2b\left(-\frac{1}{3a^{2}\left(bx-a\right)^{\frac{3}{2}}}+\frac{2}{a^{3}\sqrt{bx-a}}+\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}\right)$	74
risch	$-\frac{-bx+a}{a^{3}x\sqrt{bx-a}}+\frac{5b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{4b}{a^{3}\sqrt{bx-a}}-\frac{2b}{3a^{2}\left(bx-a\right)^{\frac{3}{2}}}$	75

input
```
int(1/x^2/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(b*x-a)^(1/2)/x+5*b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)-2/3*b/a^2/
(b*x-a)^(3/2)+4*b/a^3/(b*x-a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.80

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = \left[-\frac{15\,(b^3x^3-2\,ab^2x^2+a^2bx)\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\,(15\,ab^2x^2-20\,a^2b\cdots}{6\,(a^4b^2x^3-2\,a^5bx^2+a^6x)}\right.$$
$$\left.-\frac{15\,(b^3x^3-2\,ab^2x^2+a^2bx)\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)-(15\,ab^2x^2-20\,a^2bx+3\,a^3)\sqrt{bx-a}}{3\,(a^4b^2x^3-2\,a^5bx^2+a^6x)}\right]$$

input
```
integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(15*(b^3*x^3 - 2*a*b^2*x^2 + a^2*b*x)*sqrt(-a)*log((b*x - 2*sqrt(b*x
 - a)*sqrt(-a) - 2*a)/x) - 2*(15*a*b^2*x^2 - 20*a^2*b*x + 3*a^3)*sqrt(b*x
- a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 - 2*a*b^2*x^2
 + a^2*b*x)*sqrt(a)*arctan(sqrt(a)/sqrt(b*x - a)) - (15*a*b^2*x^2 - 20*a^2
*b*x + 3*a^3)*sqrt(b*x - a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x-a)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = \frac{15\,(bx-a)^2b+10\,(bx-a)ab-2\,a^2b}{3\left((bx-a)^{\frac{5}{2}}a^3+(bx-a)^{\frac{3}{2}}a^4\right)}+\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

input
```
integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(15*(b*x - a)^2*b + 10*(b*x - a)*a*b - 2*a^2*b)/((b*x - a)^(5/2)*a^3 +
 (b*x - a)^(3/2)*a^4) + 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = \frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}} + \frac{2\,(6\,(bx-a)b-ab)}{3\,(bx-a)^{\frac{3}{2}}a^3} + \frac{\sqrt{bx-a}}{a^3x}$$

input
```
integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="giac")
```

output
```
5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) + 2/3*(6*(b*x - a)*b - a*b)/((b*
x - a)^(3/2)*a^3) + sqrt(b*x - a)/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = \frac{1}{a\,x\,(b\,x-a)^{3/2}} - \frac{20\,b}{3\,a^2\,(b\,x-a)^{3/2}} + \frac{5\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{5\,b^2\,x}{a^3\,(b\,x-a)^{3/2}}$$

input
```
int(1/(x^2*(b*x - a)^(5/2)),x)
```

output
```
1/(a*x*(b*x - a)^(3/2)) - (20*b)/(3*a^2*(b*x - a)^(3/2)) + (5*b*atan((b*x
- a)^(1/2)/a^(1/2)))/a^(7/2) + (5*b^2*x)/(a^3*(b*x - a)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = \frac{15\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)abx-15\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^2x^2-3a^3+20a^2}{3\sqrt{bx-a}\,a^4x\,(-bx+a)}$$

input

```
int(1/x^2/(b*x-a)^(5/2),x)
```

output

```
(15*sqrt(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*a*b*x - 15*sqr
t(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*b**2*x**2 - 3*a**3 +
20*a**2*b*x - 15*a*b**2*x**2)/(3*sqrt( - a + b*x)*a**4*x*(a - b*x))
```

3.425 $\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = -\frac{35b^2}{12a^3(-a+bx)^{3/2}}+\frac{1}{2ax^2(-a+bx)^{3/2}}+\frac{7b}{4a^2x(-a+bx)^{3/2}}+\frac{35b^2}{4a^4\sqrt{-a+bx}}+\frac{35b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

output
```
-35/12*b^2/a^3/(b*x-a)^(3/2)+1/2/a/x^2/(b*x-a)^(3/2)+7/4*b/a^2/x/(b*x-a)^(
3/2)+35/4*b^2/a^4/(b*x-a)^(1/2)+35/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(
9/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = \frac{6a^3+21a^2bx-140ab^2x^2+105b^3x^3}{12a^4x^2(-a+bx)^{3/2}}+\frac{35b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

input
```
Integrate[1/(x^3*(-a + b*x)^(5/2)),x]
```

output
```
(6*a^3 + 21*a^2*b*x - 140*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^2*(-a + b*x)^
(3/2)) + (35*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {52, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(bx-a)^{5/2}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{7b\int \frac{1}{x^2(bx-a)^{5/2}}dx}{4a}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{7b\left(\frac{5b\int \frac{1}{x(bx-a)^{5/2}}dx}{2a}+\frac{1}{ax(bx-a)^{3/2}}\right)}{4a}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{7b\left(\frac{5b\left(-\frac{\int \frac{1}{x(bx-a)^{3/2}}dx}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}\right)}{4a}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{7b\left(\frac{5b\left(-\frac{-\frac{\int \frac{1}{x\sqrt{bx-a}}dx}{a}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}\right)}{4a}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{7b\left(\frac{5b\left(-\frac{-\frac{2\int \frac{1}{\frac{a}{b}+\frac{bx-a}{b}}d\sqrt{bx-a}}{ab}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}\right)}{4a}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{7b\left(\frac{5b\left(-\frac{-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}}{a}-\frac{2}{3a(bx-a)^{3/2}}\right)}{2a}+\frac{1}{ax(bx-a)^{3/2}}\right)}{4a}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

input

```
Int[1/(x^3*(-a + b*x)^(5/2)),x]
```

output

```
1/(2*a*x^2*(-a + b*x)^(3/2)) + (7*b*(1/(a*x*(-a + b*x)^(3/2)) + (5*b*(-2/(
3*a*(-a + b*x)^(3/2)) - (-2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/
Sqrt[a]])/a^(3/2))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{(-bx+a)(11bx+2a)}{4a^4x^2\sqrt{bx-a}}+\frac{35b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{9}{2}}}+\frac{6b^2}{a^4\sqrt{bx-a}}-\frac{2b^2}{3a^3(bx-a)^{\frac{3}{2}}}$	89
pseudoelliptic	$-\frac{35\left(\sqrt{bx-a}\,b^2x^2(-bx+a)\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{a}\,b^3x^3+\frac{4a^{\frac{3}{2}}b^2x^2}{3}-\frac{a^{\frac{5}{2}}bx}{5}-\frac{2a^{\frac{7}{2}}}{35}\right)}{4a^{\frac{9}{2}}(bx-a)^{\frac{3}{2}}x^2}$	89
derivativedivides	$2b^2\left(\frac{\frac{\frac{11(bx-a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{35\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}-\frac{1}{3a^3(bx-a)^{\frac{3}{2}}}+\frac{3}{a^4\sqrt{bx-a}}\right)$	90
default	$2b^2\left(\frac{\frac{\frac{11(bx-a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{35\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}-\frac{1}{3a^3(bx-a)^{\frac{3}{2}}}+\frac{3}{a^4\sqrt{bx-a}}\right)$	90

input
```
int(1/x^3/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b*x+a)*(11*b*x+2*a)/a^4/x^2/(b*x-a)^(1/2)+35/4*b^2*arctan((b*x-a)^(
1/2)/a^(1/2))/a^(9/2)+6*b^2/a^4/(b*x-a)^(1/2)-2/3*b^2/a^3/(b*x-a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.25

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = \left[-\frac{105\,(b^4x^4-2\,ab^3x^3+a^2b^2x^2)\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\,(105\,ab^3x^3-14}{24\,(a^5b^2x^4-2\,a^6bx^3+a^7x^2)}\right.$$

$$\left.-\frac{105\,(b^4x^4-2\,ab^3x^3+a^2b^2x^2)\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{bx-a}}\right)-(105\,ab^3x^3-140\,a^2b^2x^2+21\,a^3bx+6\,a^4)\sqrt{bx-a}}{12\,(a^5b^2x^4-2\,a^6bx^3+a^7x^2)}\right]$$

input
```
integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(105*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*log((b*x - 2*sq
rt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a
^3*b*x + 6*a^4)*sqrt(b*x - a))/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2), -1/1
2*(105*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*arctan(sqrt(a)/sqrt(b
*x - a)) - (105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3*b*x + 6*a^4)*sqrt(b*x
 - a))/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 1108, normalized size of antiderivative = 9.55

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x-a)**(5/2),x)
```

output
```
Piecewise((12*I*a**(89/2)*b**75*x**75/(24*a**(93/2)*b**(151/2)*x**(155/2)*
sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1))
+ 42*I*a**(87/2)*b**76*x**76/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b
*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) - 280*I*a
**(85/2)*b**77*x**77/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1)
 - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**(83/2)
*b**78*x**78/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a*
*(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**42*b**(155/2)*
x**(155/2)*sqrt(a/(b*x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2
)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(15
7/2)*sqrt(a/(b*x) - 1)) - 105*pi*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x)
- 1)/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*
b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) - 210*I*a**41*b**(157/2)*x**(157/
2)*sqrt(a/(b*x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(15
1/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqr
t(a/(b*x) - 1)) + 105*pi*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) - 1)/(24
*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/
2)*x**(157/2)*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-6*a**(89/2)*b**75*x
**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)
*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) - 21*a**(87/2)*b**76*x**76/(...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = \frac{105\,(bx-a)^3b^2 + 175\,(bx-a)^2ab^2 + 56\,(bx-a)a^2b^2 - 8\,a^3b^2}{12\left((bx-a)^{\frac{7}{2}}a^4 + 2\,(bx-a)^{\frac{5}{2}}a^5 + (bx-a)^{\frac{3}{2}}a^6\right)} + \frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}$$

input
```
integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(105*(b*x - a)^3*b^2 + 175*(b*x - a)^2*a*b^2 + 56*(b*x - a)*a^2*b^2 -
 8*a^3*b^2)/((b*x - a)^(7/2)*a^4 + 2*(b*x - a)^(5/2)*a^5 + (b*x - a)^(3/2)
*a^6) + 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = \frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}} + \frac{2\,(9\,(bx-a)b^2-ab^2)}{3\,(bx-a)^{\frac{3}{2}}a^4} + \frac{11\,(bx-a)^{\frac{3}{2}}b^2+13\,\sqrt{bx-a}ab^2}{4\,a^4b^2x^2}$$

input
```
integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="giac")
```

output
```
35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2) + 2/3*(9*(b*x - a)*b^2 - a*
b^2)/((b*x - a)^(3/2)*a^4) + 1/4*(11*(b*x - a)^(3/2)*b^2 + 13*sqrt(b*x - a
)*a*b^2)/(a^4*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = \frac{35\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{9/2}} - \frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,(a-b\,x)^2}{12\,a^3}+\frac{35\,b^2\,(a-b\,x)^3}{4\,a^4}+\frac{14\,b^2\,(a-b\,x)}{3\,a^2}}{2\,a\,(b\,x-a)^{5/2}+(b\,x-a)^{7/2}+a^2\,(b\,x-a)^{3/2}}$$

input
```
int(1/(x^3*(b*x - a)^(5/2)),x)
```

output
```
(35*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(9/2)) - ((2*b^2)/(3*a) - (175
*b^2*(a - b*x)^2)/(12*a^3) + (35*b^2*(a - b*x)^3)/(4*a^4) + (14*b^2*(a - b
*x))/(3*a^2))/(2*a*(b*x - a)^(5/2) + (b*x - a)^(7/2) + a^2*(b*x - a)^(3/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = \frac{105\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)a\,b^2x^2 - 105\sqrt{a}\,\sqrt{bx-a}\,atan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)b^3x^3 - 6a^4 -}{12\sqrt{bx-a}\,a^5x^2\,(-bx+a)}$$

input
```
int(1/x^3/(b*x-a)^(5/2),x)
```

output
```
(105*sqrt(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*a*b**2*x**2 -
 105*sqrt(a)*sqrt( - a + b*x)*atan(sqrt( - a + b*x)/sqrt(a))*b**3*x**3 - 6
*a**4 - 21*a**3*b*x + 140*a**2*b**2*x**2 - 105*a*b**3*x**3)/(12*sqrt( - a
+ b*x)*a**5*x**2*(a - b*x))
```

3.426 $\int x^{5/2}\sqrt{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int x^{5/2}\sqrt{a+bx}\,dx=\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{5a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}$$

output
```
5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^3-5/96*a^2*x^(3/2)*(b*x+a)^(1/2)/b^2+1/24
*a*x^(5/2)*(b*x+a)^(1/2)/b+1/4*x^(7/2)*(b*x+a)^(1/2)-5/64*a^4*arctanh(b^(1
/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int x^{5/2}\sqrt{a+bx}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^3-10a^2bx+8ab^2x^2+48b^3x^3)+30a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{192b^{7/2}}$$

input
```
Integrate[x^(5/2)*Sqrt[a + b*x],x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^3 - 10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3
*x^3) + 30*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(192*
b^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a+bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

$\downarrow$ 65

$$\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

$\downarrow$ 219

$$\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

input
```
Int[x^(5/2)*Sqrt[a + b*x],x]
```

output
```
(x^(7/2)*Sqrt[a + b*x])/4 + (a*((x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a*((x^(
3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/(6*b)))/8
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\left(48 b^{3} x^{3}+8 a \,b^{2} x^{2}-10 a^{2} b x +15 a^{3}\right) \sqrt{x}\, \sqrt{b x +a}}{192 b^{3}}-\frac{5 a^{4} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right) \sqrt{x \left(b x +a \right)}}{128 b^{\frac{7}{2}} \sqrt{x}\, \sqrt{b x +a}}$	98
default	$\frac{x^{\frac{5}{2}} \left(b x +a \right)^{\frac{3}{2}}}{4 b}-\frac{5 a \left(\frac{x^{\frac{3}{2}} \left(b x +a \right)^{\frac{3}{2}}}{3 b}-\frac{a \left(\frac{\sqrt{x}\, \left(b x +a \right)^{\frac{3}{2}}}{2 b}-\frac{a \left(\sqrt{x}\, \sqrt{b x +a}+\frac{a \sqrt{x \left(b x +a \right)}\, \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{2 \sqrt{b x +a}\, \sqrt{x}\, \sqrt{b}}\right)}{4 b}\right)}{2 b}\right)}{8 b}$	128

input
```
int(x^(5/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*b^3*x^3+8*a*b^2*x^2-10*a^2*b*x+15*a^3)*x^(1/2)*(b*x+a)^(1/2)/b^3
-5/128*a^4/b^(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(
1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.30

$$\int x^{5/2}\sqrt{a+bx}\,dx = \left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3\right.}{384\,b^4}\right.$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(
x))/b^4, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (
48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x))
/b^4]
```

Sympy [A] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1+\frac{bx}{a}}} - \frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)*(b*x+a)**(1/2),x)
```

output
```
5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*a**(5/2)*x**(3/2)/(192*b*
*2*sqrt(1 + b*x/a)) - a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x/a)) + 7*sqrt(a)
*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64
*b**(7/2)) + b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.46

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{7}{2}}} + \frac{\frac{15\,\sqrt{bx+a}a^4b^3}{\sqrt{x}} + \frac{73\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}} - \frac{55\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}} + \frac{15\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^7 - \frac{4\,(bx+a)b^6}{x} + \frac{6\,(bx+a)^2b^5}{x^2} - \frac{4\,(bx+a)^3b^4}{x^3} + \frac{(bx+a)^4b^3}{x^4}\right)}$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(7/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) + 73*(b*x + a)
^(3/2)*a^4*b^2/x^(3/2) - 55*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(
7/2)*a^4/x^(7/2))/(b^7 - 4*(b*x + a)*b^6/x + 6*(b*x + a)^2*b^5/x^2 - 4*(b*
x + a)^3*b^4/x^3 + (b*x + a)^4*b^3/x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(88) = 176$.

Time = 148.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.73

$$\int x^{5/2}\sqrt{a+bx}\,dx = -\frac{\frac{\left(\frac{105\,a^4\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{5}{2}}} - \left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)}{b^3} - \frac{25\,a}{b^3}\right) + \frac{163\,a^2}{b^3}\right) - \frac{279\,a^3}{b^3}\right)\sqrt{(bx+a)b-ab}\sqrt{bx+a}\right)|b|}{b} - \frac{8\left(15\,a^3\sqrt{b}\log\right.}{}}{192\,b}$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/192*((105*a^4*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))
)/b^(5/2) - (2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 - 25*a/b^3) + 163*a
^2/b^3) - 279*a^3/b^3)*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a))*abs(b)/b - 8
*(15*a^3*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))
) + (2*(4*b*x - 9*a)*(b*x + a) + 33*a^2)*sqrt((b*x + a)*b - a*b)*sqrt(b*x
+ a))*a*abs(b)/b^4)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\sqrt{a+bx}\,dx = \int x^{5/2}\,\sqrt{a+b\,x}\,dx$$

input
```
int(x^(5/2)*(a + b*x)^(1/2),x)
```

output
```
int(x^(5/2)*(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.78

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b-10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,b^4x}{192b^4}$$

input
```
int(x^(5/2)*(b*x+a)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b - 10*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x**3 -
 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(192*b**4
)
```

3.427 $\int x^{3/2}\sqrt{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int x^{3/2}\sqrt{a+bx}\,dx = -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}$$

output
```
-1/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b^2+1/12*a*x^(3/2)*(b*x+a)^(1/2)/b+1/3*x^(5
/2)*(b*x+a)^(1/2)+1/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int x^{3/2}\sqrt{a+bx}\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-3a^2+2abx+8b^2x^2)+6a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{24b^{5/2}}$$

input
```
Integrate[x^(3/2)*Sqrt[a + b*x],x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a^2 + 2*a*b*x + 8*b^2*x^2) + 6*a^3*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(24*b^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{65}$$

$$\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

input
```
Int[x^(3/2)*Sqrt[a + b*x],x]
```

output
```
(x^(5/2)*Sqrt[a + b*x])/3 + (a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqr
t[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/
2)))/(4*b)))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{\left(-8 b^{2} x^{2}-2 a b x+3 a^{2}\right) \sqrt{x}\, \sqrt{b x+a}}{24 b^{2}}+\frac{a^{3} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right) \sqrt{x \left(b x+a\right)}}{16 b^{\frac{5}{2}} \sqrt{x}\, \sqrt{b x+a}}$	87
default	$\frac{x^{\frac{3}{2}} \left(b x+a\right)^{\frac{3}{2}}}{3 b}-\frac{a \left(\frac{\sqrt{x}\, \left(b x+a\right)^{\frac{3}{2}}}{2 b}-\frac{a \left(\sqrt{x}\, \sqrt{b x+a}+\frac{a \sqrt{x \left(b x+a\right)}\, \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\, x^{2}+a x}\right)}{2 \sqrt{b x+a}\, \sqrt{x}\, \sqrt{b}}\right)}{4 b}\right)}{2 b}$	106

input
```
int(x^(3/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*b^2*x^2-2*a*b*x+3*a^2)*x^(1/2)*(b*x+a)^(1/2)/b^2+1/16*a^3/b^(5/2
)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x
+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41

$$\int x^{3/2}\sqrt{a+bx}\,dx = \left[\frac{3\,a^{3}\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b)\sqrt{bx+a}\sqrt{x}}{48\,b^{3}},\right.$$
$$\left.-\frac{3\,a^{3}\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b)\sqrt{bx+a}\sqrt{x}}{24\,b^{3}}\right]$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3
*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (8*b^3*x^2 + 2*a*b^2*x
- 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int x^{3/2}\sqrt{a+bx}\,dx = -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1+\frac{bx}{a}}} - \frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1+\frac{bx}{a}}} + \frac{5\sqrt{a}x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}} + \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}} + \frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)*(b*x+a)**(1/2),x)
```

output
```
-a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 + b*x/a)) - a**(3/2)*x**(3/2)/(24*b*sqrt(
1 + b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*sqrt(1 + b*x/a)) + a**3*asinh(sqrt(b)
*sqrt(x)/sqrt(a))/(8*b**(5/2)) + b*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(70) = 140.

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int x^{3/2}\sqrt{a+bx}\,dx = -\frac{a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{5}{2}}} - \frac{\frac{3\sqrt{bx+a}a^3b^2}{\sqrt{x}} + \frac{8\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}} - \frac{3\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^5 - \frac{3\,(bx+a)b^4}{x} + \frac{3\,(bx+a)^2b^3}{x^2} - \frac{(bx+a)^3b^2}{x^3}\right)}$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) + 8*(b*x + a)^(3
/2)*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x + a)*b^4/
x + 3*(b*x + a)^2*b^3/x^2 - (b*x + a)^3*b^2/x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(70) = 140$.

Time = 148.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\int x^{3/2}\sqrt{a+bx}\,dx = -\frac{\frac{6\left(3\,a^2\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)-\sqrt{(bx+a)b-ab}(2\,bx-3\,a)\sqrt{bx+a}\right)a|b|}{b^3}-\frac{\left(15\,a^3\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+(2\,(4\cdots\right.}{b^3}}{24\,b}$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/24*(6*(3*a^2*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b
- a*b))) - sqrt((b*x + a)*b - a*b)*(2*b*x - 3*a)*sqrt(b*x + a))*a*abs(b)/b
^3 - (15*a^3*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a
*b))) + (2*(4*b*x - 9*a)*(b*x + a) + 33*a^2)*sqrt((b*x + a)*b - a*b)*sqrt(
b*x + a))*abs(b)/b^3)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{a+bx}\,dx = \int x^{3/2}\,\sqrt{a+b\,x}\,dx$$

input
```
int(x^(3/2)*(a + b*x)^(1/2),x)
```

output
```
int(x^(3/2)*(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int x^{3/2}\sqrt{a+bx}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^2b+2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2+3\sqrt{b}\log\Big(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}}{24b^3}$$

input
```
int(x^(3/2)*(b*x+a)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a**2*b + 2*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*
sqrt(x)*sqrt(a + b*x)*b**3*x**2 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**3)
```

3.428 $\int \sqrt{x}\sqrt{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \sqrt{x}\sqrt{a+bx}\,dx=\frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}$$

output
```
1/4*a*x^(1/2)*(b*x+a)^(1/2)/b+1/2*x^(3/2)*(b*x+a)^(1/2)-1/4*a^2*arctanh(b^
(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \sqrt{x}\sqrt{a+bx}\,dx=\frac{\sqrt{x}\sqrt{a+bx}(a+2bx)}{4b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{2b^{3/2}}$$

input
```
Integrate[Sqrt[x]*Sqrt[a + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(a + 2*b*x))/(4*b) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[x])
/(-Sqrt[a] + Sqrt[a + b*x])])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{65}$$

$$\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}$$

input
```
Int[Sqrt[x]*Sqrt[a + b*x],x]
```

output
```
(x^(3/2)*Sqrt[a + b*x])/2 + (a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{(2bx+a)\sqrt{x}\,\sqrt{bx+a}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	74
default	$\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2b}-\frac{a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}$	84

input
```
int(x^(1/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*x+a)*x^(1/2)*(b*x+a)^(1/2)/b-1/8/b^(3/2)*a^2*ln((1/2*a+b*x)/b^(1/
2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \sqrt{x}\sqrt{a+bx}\,dx = \left[\frac{a^2\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x+ab)\sqrt{bx+a}\sqrt{x}}{8\,b^2}, \frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)+(2\,b^2x}{4\,b^2}\right.$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*
b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(-b)
*sqrt(x)/sqrt(b*x + a)) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \sqrt{x}\sqrt{a+bx}\,dx = \frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3\sqrt{a}x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(1/2)*(b*x+a)**(1/2),x)
```

output
```
a**(3/2)*sqrt(x)/(4*b*sqrt(1 + b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 + b*
x/a)) - a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) + b*x**(5/2)/(2*s
qrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \sqrt{x}\sqrt{a+bx}\,dx = \frac{a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}} + \frac{\frac{\sqrt{bx+a}a^2b}{\sqrt{x}} + \frac{(bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^3 - \frac{2\,(bx+a)b^2}{x} + \frac{(bx+a)^2b}{x^2}\right)}$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(3/2) + 1/4*(sqrt(b*x + a)*a^2*b/sqrt(x) + (b*x + a)^(3/2)*a^2/x
^(3/2))/(b^3 - 2*(b*x + a)*b^2/x + (b*x + a)^2*b/x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(52) = 104$.

Time = 149.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

$$\int \sqrt{x}\sqrt{a+bx}\,dx$$
$$= \frac{\frac{4\left(a\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\sqrt{(bx+a)b-ab}\sqrt{bx+a}\right)a|b|}{b^2} - \frac{\left(3\,a^2\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)-\sqrt{(bx+a)b-ab}(2\,b}{b^2}}{4\,b}$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(4*(a*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)
)) + sqrt((b*x + a)*b - a*b)*sqrt(b*x + a))*a*abs(b)/b^2 - (3*a^2*sqrt(b)*
log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))) - sqrt((b*x + a
)*b - a*b)*(2*b*x - 3*a)*sqrt(b*x + a))*abs(b)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\sqrt{a+bx}\,dx = \sqrt{x}\left(\frac{x}{2}+\frac{a}{4\,b}\right)\sqrt{a+b\,x}-\frac{a^2\,\ln\left(a+2\,b\,x+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{a+b\,x}\right)}{8\,b^{3/2}}$$

input
```
int(x^(1/2)*(a + b*x)^(1/2),x)
```

output
```
x^(1/2)*(x/2 + a/(4*b))*(a + b*x)^(1/2) - (a^2*log(a + 2*b*x + 2*b^(1/2)*x
^(1/2)*(a + b*x)^(1/2)))/(8*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \sqrt{x}\sqrt{a+bx}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^2}$$

input
```
int(x^(1/2)*(b*x+a)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x - sqrt(b)*log(
(sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**2)
```

3.429 $\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{a+bx} + \frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

output
```
x^(1/2)*(b*x+a)^(1/2)+a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{a+bx} - \frac{a\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}$$

input
```
Integrate[Sqrt[a + b*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[a + b*x] - (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b
]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2} a \int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}$$

$$\Big\downarrow \text{65}$$

$$a \int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}$$

input
```
Int[Sqrt[a + b*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[
b]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

method	result	size
default	$\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}$	62
risch	$\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}$	62

input
```
int((b*x+a)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/
2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \left[\frac{a\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}b\sqrt{x}}{2\,b},\right.$$
$$\left.-\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-\sqrt{bx+a}b\sqrt{x}}{b}\right]$$

input
```
integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(
b*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a))
 - sqrt(b*x + a)*b*sqrt(x))/b]
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \sqrt{a}\sqrt{x}\sqrt{1+\frac{bx}{a}} + \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate((b*x+a)**(1/2)/x**(1/2),x)
```

output
```
sqrt(a)*sqrt(x)*sqrt(1 + b*x/a) + a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(32) = 64.

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = -\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\sqrt{b}} - \frac{\sqrt{bx+a}\,a}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

input
```
integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqr
t(x)))/sqrt(b) - sqrt(b*x + a)*a/((b - (b*x + a)/x)*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(32) = 64$.

Time = 74.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = -\frac{\left(\frac{a\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \frac{\sqrt{(bx+a)b-ab}\sqrt{bx+a}}{b}\right)b}{|b|}$$

input
```
integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
-(a*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt(b) - s
qrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b)*b/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{a+b\,x} + \frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
int((a + b*x)^(1/2)/x^(1/2),x)
```

output
```
x^(1/2)*(a + b*x)^(1/2) + (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2) -
a^(1/2))))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,b+\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b}$$

input
```
int((b*x+a)^(1/2)/x^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*b + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b
```

3.430 $\int \frac{\sqrt{a+bx}}{x^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{\sqrt{a+bx}}{x^{3/2}} \, dx = -\frac{2\sqrt{a+bx}}{\sqrt{x}} + 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2*(b*x+a)^(1/2)/x^(1/2)+2*b^(1/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+bx}}{x^{3/2}} \, dx = -\frac{2\sqrt{a+bx}}{\sqrt{x}} + 4\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)$$

input
```
Integrate[Sqrt[a + b*x]/x^(3/2),x]
```

output
```
(-2*Sqrt[a + b*x])/Sqrt[x] + 4*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a]
 + Sqrt[a + b*x])]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^{3/2}}dx$$

$$\Big\downarrow \; 57$$

$$b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

$$\Big\downarrow \; 65$$

$$2b\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

$$\Big\downarrow \; 219$$

$$2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

input
```
Int[Sqrt[a + b*x]/x^(3/2),x]
```

output
```
(-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
b*x]]
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

method	result	size
risch	$-\frac{2\sqrt{bx+a}}{\sqrt{x}}+\frac{\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	61

input
```
int((b*x+a)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x+a)^(1/2)/x^(1/2)+b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx = \left[\frac{\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}\sqrt{x}}{x},\right.$$
$$\left.-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)+\sqrt{bx+a}\sqrt{x}\right)}{x}\right]$$

input
```
integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x
+ a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + s
qrt(b*x + a)*sqrt(x))/x]
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx = -\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} + 2\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(1/2)/x**(3/2),x)
```

output
```
-2*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqr
t(a)) - 2*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx = -\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right) - \frac{2\sqrt{bx+a}}{\sqrt{x}}$$

input
```
integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x))) - 2*sqrt(b*x + a)/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 74.79 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx = -\frac{2\,b^2\left(\frac{\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx+a}}{\sqrt{(bx+a)b-ab}}\right)}{|b|}$$

input
```
integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
-2*b^2*(log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt(b)
 + sqrt(b*x + a)/sqrt((b*x + a)*b - a*b))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx = \int \frac{\sqrt{a+b\,x}}{x^{3/2}}\,dx$$

input
```
int((a + b*x)^(1/2)/x^(3/2),x)
```

output
```
int((a + b*x)^(1/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)x-2\sqrt{b}\,x}{x}$$

input
```
int((b*x+a)^(1/2)/x^(3/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*x - sqrt(b)*x))/x
```

3.431 $\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

output
```
-2/3*(b*x+a)^(3/2)/a/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x]/x^(5/2),x]
```

output
```
(-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

input
```
Int[Sqrt[a + b*x]/x^(5/2),x]
```

output
```
(-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	16
risch	$-\frac{2(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	16
orering	$-\frac{2(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	16
default	$-\frac{\sqrt{bx+a}}{x^{\frac{3}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{2}$	49

input
```
int((b*x+a)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(3/2)/a/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x} - \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}$$

input
```
integrate((b*x+a)**(1/2)/x**(5/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}b^4}{3\,((bx+a)b-ab)^{\frac{3}{2}}a|b|}$$

input
```
integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*(b*x + a)^(3/2)*b^4/(((b*x + a)*b - a*b)^(3/2)*a*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{\left(\frac{2\,b\,x}{3\,a}+\frac{2}{3}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

input
```
int((a + b*x)^(1/2)/x^(5/2),x)
```

output
```
-(((2*b*x)/(3*a) + 2/3)*(a + b*x)^(1/2))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}\,a-2\sqrt{x}\,\sqrt{bx+a}\,bx-2\sqrt{b}\,b\,x^2}{3a\,x^2}$$

input
```
int((b*x+a)^(1/2)/x^(5/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a + b*x)*a + sqrt(x)*sqrt(a + b*x)*b*x + sqrt(b)*b*x**
2))/(3*a*x**2)
```

3.432 $\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}+\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}$$

output
```
-2/5*(b*x+a)^(3/2)/a/x^(5/2)+4/15*b*(b*x+a)^(3/2)/a^2/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = -\frac{2\sqrt{a+bx}(3a^2+abx-2b^2x^2)}{15a^2x^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x]/x^(7/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(3*a^2 + a*b*x - 2*b^2*x^2))/(15*a^2*x^(5/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2b\int \frac{\sqrt{a+bx}}{x^{5/2}}dx}{5a}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

input
```
Int[Sqrt[a + b*x]/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2)
)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-2bx+3a)}{15x^{\frac{5}{2}}a^{2}}$	24
orering	$-\frac{2(bx+a)^{\frac{3}{2}}(-2bx+3a)}{15x^{\frac{5}{2}}a^{2}}$	24
risch	$-\frac{2\sqrt{bx+a}\,(-2b^{2}x^{2}+abx+3a^{2})}{15x^{\frac{5}{2}}a^{2}}$	34
default	$-\frac{\sqrt{bx+a}}{2x^{\frac{5}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{4}$	71

input
```
int((b*x+a)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(3/2)*(-2*b*x+3*a)/x^(5/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = \frac{2\,(2\,b^2x^2-abx-3\,a^2)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15ax} + \frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15a^2}$$

input
```
integrate((b*x+a)**(1/2)/x**(7/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(15*a
*x) + 4*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = \frac{2\left(\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} - \frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^2}$$

input
```
integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
2/15*(5*(b*x + a)^(3/2)*b/x^(3/2) - 3*(b*x + a)^(5/2)/x^(5/2))/a^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = \frac{2\left(\frac{2\,(bx+a)b^5}{a^2} - \frac{5\,b^5}{a}\right)(bx+a)^{\frac{3}{2}}b}{15\left((bx+a)b-ab\right)^{\frac{5}{2}}|b|}$$

input
```
integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
2/15*(2*(b*x + a)*b^5/a^2 - 5*b^5/a)*(b*x + a)^(3/2)*b/((((b*x + a)*b - a*b
)^(5/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,b\,x}{15\,a} - \frac{4\,b^2\,x^2}{15\,a^2} + \frac{2}{5}\right)}{x^{5/2}}$$

input
```
int((a + b*x)^(1/2)/x^(7/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*b*x)/(15*a) - (4*b^2*x^2)/(15*a^2) + 2/5))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,abx}{15} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15} - \frac{4\sqrt{b}\,b^2x^3}{15}}{a^2x^3}$$

input
```
int((b*x+a)^(1/2)/x^(7/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 - sqrt(x)*sqrt(a + b*x)*a*b*x + 2*sqrt
(x)*sqrt(a + b*x)*b**2*x**2 - 2*sqrt(b)*b**2*x**3))/(15*a**2*x**3)
```

3.433 $\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx=-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}$$

output
```
-2/7*(b*x+a)^(3/2)/a/x^(7/2)+8/35*b*(b*x+a)^(3/2)/a^2/x^(5/2)-16/105*b^2*(
b*x+a)^(3/2)/a^3/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx=-\frac{2\sqrt{a+bx}(15a^3+3a^2bx-4ab^2x^2+8b^3x^3)}{105a^3x^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x]/x^(9/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(15*a^3 + 3*a^2*b*x - 4*a*b^2*x^2 + 8*b^3*x^3))/(105*a^3
*x^(7/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\int \frac{\sqrt{a+bx}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\left(-\frac{2b\int \frac{\sqrt{a+bx}}{x^{5/2}}dx}{5a}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{4b\left(\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

input
```
Int[Sqrt[a + b*x]/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) - (4*b*((-2*(a + b*x)^(3/2))/(5*a*x^(5/
2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(8b^{2}x^{2}-12abx+15a^{2}\right)}{105x^{\frac{7}{2}}a^{3}}$	35
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(8b^{2}x^{2}-12abx+15a^{2}\right)}{105x^{\frac{7}{2}}a^{3}}$	35
risch	$-\frac{2\sqrt{bx+a}\,\left(8b^{3}x^{3}-4a\,b^{2}x^{2}+3a^{2}bx+15a^{3}\right)}{105x^{\frac{7}{2}}a^{3}}$	46
default	$-\frac{\sqrt{bx+a}}{3x^{\frac{7}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}$	93

input
```
int((b*x+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(b*x+a)^(3/2)*(8*b^2*x^2-12*a*b*x+15*a^2)/x^(7/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{2\,(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

input
```
integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x
^(7/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(63) = 126$.

Time = 5.42 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.10

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{6a^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{24ab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{16b^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}$$

input
```
integrate((b*x+a)**(1/2)/x**(9/2),x)
```

output
```
-30*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x*
*4 + 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*a**5
*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b**(13/2)*
x**2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3
*b**6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3
+ 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt(a/(b
*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) -
16*b**(19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x*
*4 + 105*a**3*b**6*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{2\left(\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}} - \frac{42\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}} + \frac{15\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

input
```
integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/105*(35*(b*x + a)^(3/2)*b^2/x^(3/2) - 42*(b*x + a)^(5/2)*b/x^(5/2) + 15
*(b*x + a)^(7/2)/x^(7/2))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{2\left(\frac{35\,b^3}{a} + 4\,(bx+a)\left(\frac{2\,(bx+a)b^3}{a^3} - \frac{7\,b^3}{a^2}\right)\right)(bx+a)^{\frac{3}{2}}b^5}{105\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

input
```
integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
-2/105*(35*b^3/a + 4*(b*x + a)*(2*(b*x + a)*b^3/a^3 - 7*b^3/a^2))*(b*x + a
)^(3/2)*b^5/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{16\,b^3\,x^3}{105\,a^3}-\frac{8\,b^2\,x^2}{105\,a^2}+\frac{2\,b\,x}{35\,a}+\frac{2}{7}\right)}{x^{7/2}}$$

input
```
int((a + b*x)^(1/2)/x^(9/2),x)
```

output
```
-((a + b*x)^(1/2)*((16*b^3*x^3)/(105*a^3) - (8*b^2*x^2)/(105*a^2) + (2*b*x
)/(35*a) + 2/7))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{105}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{105}+\frac{16\sqrt{b}\,b^3x^4}{105}}{a^3x^4}$$

input
```
int((b*x+a)^(1/2)/x^(9/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*x +
4*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*b**3*x**3 +
8*sqrt(b)*b**3*x**4))/(105*a**3*x**4)
```

3.434 $\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = -\frac{2(a+bx)^{3/2}}{9ax^{9/2}} + \frac{4b(a+bx)^{3/2}}{21a^2x^{7/2}} - \frac{16b^2(a+bx)^{3/2}}{105a^3x^{5/2}} + \frac{32b^3(a+bx)^{3/2}}{315a^4x^{3/2}}$$

output
```
-2/9*(b*x+a)^(3/2)/a/x^(9/2)+4/21*b*(b*x+a)^(3/2)/a^2/x^(7/2)-16/105*b^2*(
b*x+a)^(3/2)/a^3/x^(5/2)+32/315*b^3*(b*x+a)^(3/2)/a^4/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = -\frac{2\sqrt{a+bx}(35a^4+5a^3bx-6a^2b^2x^2+8ab^3x^3-16b^4x^4)}{315a^4x^{9/2}}$$

input
```
Integrate[Sqrt[a + b*x]/x^(11/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(35*a^4 + 5*a^3*b*x - 6*a^2*b^2*x^2 + 8*a*b^3*x^3 - 16*b
^4*x^4))/(315*a^4*x^(9/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2b\int \frac{\sqrt{a+bx}}{x^{9/2}}dx}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2b\left(-\frac{4b\int \frac{\sqrt{a+bx}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{a+bx}}{x^{5/2}}dx}{5a}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{2b\left(-\frac{4b\left(\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}$$

input
```
Int[Sqrt[a + b*x]/x^(11/2),x]
```

output
```
(-2*(a + b*x)^(3/2))/(9*a*x^(9/2)) - (2*b*((-2*(a + b*x)^(3/2))/(7*a*x^(7/
2)) - (4*b*((-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15
*a^2*x^(3/2))))/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16b^{3}x^{3}+24a\,b^{2}x^{2}-30a^{2}bx+35a^{3}\right)}{315x^{\frac{9}{2}}a^{4}}$	46
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16b^{3}x^{3}+24a\,b^{2}x^{2}-30a^{2}bx+35a^{3}\right)}{315x^{\frac{9}{2}}a^{4}}$	46
risch	$-\frac{2\sqrt{bx+a}\left(-16b^{4}x^{4}+8a\,x^{3}b^{3}-6a^{2}b^{2}x^{2}+5a^{3}bx+35a^{4}\right)}{315x^{\frac{9}{2}}a^{4}}$	57
default	$-\frac{\sqrt{bx+a}}{4x^{\frac{9}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{9a\,x^{\frac{9}{2}}}-\frac{8b\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{8}$	115

input
```
int((b*x+a)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(b*x+a)^(3/2)*(-16*b^3*x^3+24*a*b^2*x^2-30*a^2*b*x+35*a^3)/x^(9/2)/
a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = \frac{2\,(16\,b^4x^4-8\,ab^3x^3+6\,a^2b^2x^2-5\,a^3bx-35\,a^4)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

input
```
integrate((b*x+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt
(b*x + a)/(a^4*x^(9/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(87) = 174$.

Time = 15.82 (sec) , antiderivative size = 559, normalized size of antiderivative = 6.08

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = -\frac{70a^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7} - \frac{220a^6b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7} - \frac{228a^5b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7} - \frac{80a^4b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7} + \frac{10a^3b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7} + \frac{60a^2b^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7} + \frac{80ab^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7} + \frac{32b^{\frac{33}{2}}x^7\sqrt{\frac{a}{bx}+1}}{315a^7b^9x^4+945a^6b^{10}x^5+945a^5b^{11}x^6+315a^4b^{12}x^7}$$

input
```
integrate((b*x+a)**(1/2)/x**(11/2),x)
```

output
```
-70*a**7*b**(19/2)*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*
x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 220*a**6*b**(21/2)*x*s
qrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**1
1*x**6 + 315*a**4*b**12*x**7) - 228*a**5*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/
(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4
*b**12*x**7) - 80*a**4*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**
4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) + 10*
a**3*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10
*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) + 60*a**2*b**(29/2)*x**
5*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b
**11*x**6 + 315*a**4*b**12*x**7) + 80*a*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(
315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*
b**12*x**7) + 32*b**(33/2)*x**7*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 94
5*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = \frac{2\left(\frac{105\,(bx+a)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}} - \frac{189\,(bx+a)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}} + \frac{135\,(bx+a)^{\frac{7}{2}}b}{x^{\frac{7}{2}}} - \frac{35\,(bx+a)^{\frac{9}{2}}}{x^{\frac{9}{2}}}\right)}{315\,a^4}$$

input
```
integrate((b*x+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
2/315*(105*(b*x + a)^(3/2)*b^3/x^(3/2) - 189*(b*x + a)^(5/2)*b^2/x^(5/2) +
 135*(b*x + a)^(7/2)*b/x^(7/2) - 35*(b*x + a)^(9/2)/x^(9/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = -\frac{2\left(\frac{105\,b^9}{a} - 2\left(\frac{63\,b^9}{a^2} + 4\left(\frac{2\,(bx+a)b^9}{a^4} - \frac{9\,b^9}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{3}{2}}b}{315\,((bx+a)b-ab)^{\frac{9}{2}}|b|}$$

input
```
integrate((b*x+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
-2/315*(105*b^9/a - 2*(63*b^9/a^2 + 4*(2*(b*x + a)*b^9/a^4 - 9*b^9/a^3)*(b
*x + a))*(b*x + a))*(b*x + a)^(3/2)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{16\,b^3\,x^3}{315\,a^3} - \frac{4\,b^2\,x^2}{105\,a^2} - \frac{32\,b^4\,x^4}{315\,a^4} + \frac{2\,b\,x}{63\,a} + \frac{2}{9}\right)}{x^{9/2}}$$

input
```
int((a + b*x)^(1/2)/x^(11/2),x)
```

output
```
-((a + b*x)^(1/2)*((16*b^3*x^3)/(315*a^3) - (4*b^2*x^2)/(105*a^2) - (32*b^
4*x^4)/(315*a^4) + (2*b*x)/(63*a) + 2/9))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx}}{x^{11/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{105} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{315} + \frac{32\sqrt{x}\,\sqrt{bx+a}\,b^4x^4}{315} - \frac{32\sqrt{b}}{3}}{a^4x^5}$$

input
```
int((b*x+a)^(1/2)/x^(11/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b*x +
6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*a*b**3*x*
*3 + 16*sqrt(x)*sqrt(a + b*x)*b**4*x**4 - 16*sqrt(b)*b**4*x**5))/(315*a**4
*x**5)
```

3.435 $\int x^{5/2}(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int x^{5/2}(a+bx)^{3/2}\,dx=\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{11}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}bx^{9/2}\sqrt{a+bx}-\frac{3a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}$$

output
```
3/128*a^4*x^(1/2)*(b*x+a)^(1/2)/b^3-1/64*a^3*x^(3/2)*(b*x+a)^(1/2)/b^2+1/8
0*a^2*x^(5/2)*(b*x+a)^(1/2)/b+11/40*a*x^(7/2)*(b*x+a)^(1/2)+1/5*b*x^(9/2)*
(b*x+a)^(1/2)-3/128*a^5*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.74

$$\int x^{5/2}(a+bx)^{3/2}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^4-10a^3bx+8a^2b^2x^2+176ab^3x^3+128b^4x^4)+30a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{640b^{7/2}}$$

input
```
Integrate[x^(5/2)*(a + b*x)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^4 - 10*a^3*b*x + 8*a^2*b^2*x^2 + 176*
a*b^3*x^3 + 128*b^4*x^4) + 30*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqr
t[a + b*x])])/(640*b^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {60, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^{3/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\int x^{5/2}\sqrt{a+bx}dx+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a+bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int\frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int\frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

$\downarrow$ 65

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

$\downarrow$ 219

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

input
```
Int[x^(5/2)*(a + b*x)^(3/2),x]
```

output
```
(x^(7/2)*(a + b*x)^(3/2))/5 + (3*a*((x^(7/2)*Sqrt[a + b*x])/4 + (a*((x^(5/
2)*Sqrt[a + b*x])/(3*b) - (5*a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqr
t[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/
2)))/(4*b)))/(6*b)))/8))/10
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

method	result	s
risch	$\frac{(128b^4x^4+176a\,x^3b^3+8a^2b^2x^2-10a^3bx+15a^4)\sqrt{x}\,\sqrt{bx+a}}{640b^3}-\frac{3a^5\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{256b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$	1
default	$\frac{x^{\frac{5}{2}}(bx+a)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)}{2b}$	1

input
```
int(x^(5/2)*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/640*(128*b^4*x^4+176*a*b^3*x^3+8*a^2*b^2*x^2-10*a^3*b*x+15*a^4)*x^(1/2)*
(b*x+a)^(1/2)/b^3-3/256*a^5/b^(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/
2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26

$$\int x^{5/2}(a+bx)^{3/2}\,dx = \left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+\right.}{1280\,b^4}$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/1280*(15*a^5*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
 2*(128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2*x + 15*a^4*b)
*sqrt(b*x + a)*sqrt(x))/b^4, 1/640*(15*a^5*sqrt(-b)*arctan(sqrt(-b)*sqrt(x
)/sqrt(b*x + a)) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b
^2*x + 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 50.98 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int x^{5/2}(a+bx)^{3/2}\,dx = \frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1+\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{19\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}-\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)*(b*x+a)**(3/2),x)
```

output
```
3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 + b*x/a)) + a**(7/2)*x**(3/2)/(128*b**
2*sqrt(1 + b*x/a)) - a**(5/2)*x**(5/2)/(320*b*sqrt(1 + b*x/a)) + 23*a**(3/
2)*x**(7/2)/(80*sqrt(1 + b*x/a)) + 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 + b*x/
a)) - 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2
)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(104) = 208$.

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.47

$$\int x^{5/2}(a+bx)^{3/2}\,dx = \frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{7}{2}}} + \frac{\frac{15\,\sqrt{bx+a}a^5b^4}{\sqrt{x}} - \frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}} - \frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}} + \frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}} - \frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^8 - \frac{5\,(bx+a)b^7}{x} + \frac{10\,(bx+a)^2b^6}{x^2} - \frac{10\,(bx+a)^3b^5}{x^3} + \frac{5\,(bx+a)^4b^4}{x^4} - \frac{(bx+a)^5b^3}{x^5}\right)}$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(7/2) + 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)
^(3/2)*a^5*b^3/x^(3/2) - 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a
)^(7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x + a)
*b^7/x + 10*(b*x + a)^2*b^6/x^2 - 10*(b*x + a)^3*b^5/x^3 + 5*(b*x + a)^4*b
^4/x^4 - (b*x + a)^5*b^3/x^5)
```

Giac [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{3/2}\,dx=\int x^{5/2}\,(a+b\,x)^{3/2}\,dx$$

input
```
int(x^(5/2)*(a + b*x)^(3/2),x)
```

output
```
int(x^(5/2)*(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int x^{5/2}(a+bx)^{3/2}\,dx=\frac{15\sqrt{x}\,\sqrt{bx+a}\,a^4b-10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+176\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^3+}{640b^4}$$

input
```
int(x^(5/2)*(b*x+a)^(3/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**4*b - 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 176*sqrt(x)*sqrt(a + b*x)*a*b**4*
x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*x***4 - 15*sqrt(b)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(640*b**4)
```

3.436 $\int x^{3/2}(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int x^{3/2}(a+bx)^{3/2}\,dx=-\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{3}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}bx^{7/2}\sqrt{a+bx}+\frac{3a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}$$

output
```
-3/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^2+1/32*a^2*x^(3/2)*(b*x+a)^(1/2)/b+3/8*a
*x^(5/2)*(b*x+a)^(1/2)+1/4*b*x^(7/2)*(b*x+a)^(1/2)+3/64*a^4*arctanh(b^(1/2
)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int x^{3/2}(a+bx)^{3/2}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-3a^3+2a^2bx+24ab^2x^2+16b^3x^3)+6a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{64b^{5/2}}$$

input
```
Integrate[x^(3/2)*(a + b*x)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a^3 + 2*a^2*b*x + 24*a*b^2*x^2 + 16*b^3
*x^3) + 6*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(64*b
^(5/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^{3/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\int x^{3/2}\sqrt{a+bx}dx+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

$$\Big\downarrow \text{65}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+$$
$$\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

$$\Big\downarrow \ 219$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+$$
$$\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

input
```
Int[x^(3/2)*(a + b*x)^(3/2),x]
```

output
```
(x^(5/2)*(a + b*x)^(3/2))/4 + (3*a*((x^(5/2)*Sqrt[a + b*x])/3 + (a*((x^(3/
2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/6))/8
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{(-16b^3x^3-24a\,b^2x^2-2a^2bx+3a^3)\sqrt{x}\,\sqrt{bx+a}}{64b^2}+\frac{3a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{128b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$	98
default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}$	122

input
```
int(x^(3/2)*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(-16*b^3*x^3-24*a*b^2*x^2-2*a^2*b*x+3*a^3)*x^(1/2)*(b*x+a)^(1/2)/b^2
+3/128*a^4/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(
1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.33

$$\int x^{3/2}(a+bx)^{3/2}\,dx = \left[\frac{3\,a^4\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b)\sqrt{bx+a}}{128\,b^3}\right.$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x)
)/b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (16*
b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

Sympy [A] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28

$$\int x^{3/2}(a+bx)^{3/2}\,dx = -\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1+\frac{bx}{a}}} - \frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1+\frac{bx}{a}}} + \frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1+\frac{bx}{a}}} + \frac{5\sqrt{a}bx^{\frac{7}{2}}}{8\sqrt{1+\frac{bx}{a}}} + \frac{3a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}} + \frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)*(b*x+a)**(3/2),x)
```

output
```
-3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 + b*x/a)) - a**(5/2)*x**(3/2)/(64*b*sq
rt(1 + b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 + b*x/a)) + 5*sqrt(a)*b*x
**(7/2)/(8*sqrt(1 + b*x/a)) + 3*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b*
*(5/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(86) = 172$.

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.48

$$\int x^{3/2}(a+bx)^{3/2}\,dx = -\frac{3\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{5}{2}}} - \frac{\frac{3\,\sqrt{bx+a}a^4b^3}{\sqrt{x}} - \frac{11\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}} - \frac{11\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}} + \frac{3\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6 - \frac{4\,(bx+a)b^5}{x} + \frac{6\,(bx+a)^2b^4}{x^2} - \frac{4\,(bx+a)^3b^3}{x^3} + \frac{(bx+a)^4b^2}{x^4}\right)}$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-3/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)
/sqrt(x)))/b^(5/2) - 1/64*(3*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 11*(b*x + a)^
(3/2)*a^4*b^2/x^(3/2) - 11*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 3*(b*x + a)^(7/
2)*a^4/x^(7/2))/(b^6 - 4*(b*x + a)*b^5/x + 6*(b*x + a)^2*b^4/x^2 - 4*(b*x
+ a)^3*b^3/x^3 + (b*x + a)^4*b^2/x^4)
```

Giac [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{3/2}\,dx=\int x^{3/2}\,(a+b\,x)^{3/2}\,dx$$

input
```
int(x^(3/2)*(a + b*x)^(3/2),x)
```

output
```
int(x^(3/2)*(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int x^{3/2}(a+bx)^{3/2}\,dx=\frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^3b+2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+24\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+16\sqrt{x}\,\sqrt{bx+a}\,b^4x^3+3\sqrt{}}{64b^3}$$

input
```
int(x^(3/2)*(b*x+a)^(3/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a**3*b + 2*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
 24*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**4*x**3
 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(64*b**3
)
```

3.437 $\int \sqrt{x}(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \sqrt{x}(a+bx)^{3/2}\,dx=\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{7}{12}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}bx^{5/2}\sqrt{a+bx}-\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}$$

output
```
1/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b+7/12*a*x^(3/2)*(b*x+a)^(1/2)+1/3*b*x^(5/2)
*(b*x+a)^(1/2)-1/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \sqrt{x}(a+bx)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx}(3a^2+14abx+8b^2x^2)}{24b}-\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{3/2}}$$

input
```
Integrate[Sqrt[x]*(a + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(3*a^2 + 14*a*b*x + 8*b^2*x^2))/(24*b) - (a^3*ArcTa
nh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^{3/2}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}a\int \sqrt{x}\sqrt{a+bx}dx+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

$$\Big\downarrow \; 65$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

input
```
Int[Sqrt[x]*(a + b*x)^(3/2),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/2))/3 + (a*((x^(3/2)*Sqrt[a + b*x])/2 + (a*((Sqrt[x]
*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))
/4))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{(8b^2x^2+14abx+3a^2)\sqrt{x}\,\sqrt{bx+a}}{24b}-\frac{a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6b}$	100

input
```
int(x^(1/2)*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^2*x^2+14*a*b*x+3*a^2)*x^(1/2)*(b*x+a)^(1/2)/b-1/16/b^(3/2)*a^3*l
n((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.43

$$\int \sqrt{x}(a+bx)^{3/2}\,dx = \left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^3x^2+14\,ab^2x+3\,a^2b)\sqrt{bx+a}\sqrt{x}}{48\,b^2}, \frac{3\,a^3\sqrt{-}\cdots}{}\right.$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3
*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (8*b^3*x^2 + 14*a*b^2*x
 + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

$$\int \sqrt{x}(a+bx)^{3/2}\,dx = \frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1+\frac{bx}{a}}} + \frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}} + \frac{11\sqrt{a}bx^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}} - \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(1/2)*(b*x+a)**(3/2),x)
```

output
```
a**(5/2)*sqrt(x)/(8*b*sqrt(1 + b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 +
 b*x/a)) + 11*sqrt(a)*b*x**(5/2)/(12*sqrt(1 + b*x/a)) - a**3*asinh(sqrt(b)
*sqrt(x)/sqrt(a))/(8*b**(3/2)) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(68) = 136$.

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.50

$$\int \sqrt{x}(a+bx)^{3/2}\,dx = \frac{a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{3}{2}}} + \frac{\frac{3\sqrt{bx+a}a^3b^2}{\sqrt{x}} - \frac{8\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}} - \frac{3\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^4 - \frac{3\,(bx+a)b^3}{x} + \frac{3\,(bx+a)^2b^2}{x^2} - \frac{(bx+a)^3b}{x^3}\right)}$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(3/2) + 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 8*(b*x + a)^(3/
2)*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x + a)*b^3/x
 + 3*(b*x + a)^2*b^2/x^2 - (b*x + a)^3*b/x^3)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{3/2}\,dx = \int \sqrt{x}\,(a+b\,x)^{3/2}\,dx$$

input
```
int(x^(1/2)*(a + b*x)^(3/2),x)
```

output
```
int(x^(1/2)*(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(a+bx)^{3/2}\,dx = \frac{3\sqrt{x}\,\sqrt{bx+a}\,a^2b+14\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3}{24b^2}$$

input
```
int(x^(1/2)*(b*x+a)^(3/2),x)
```

output
```
(3*sqrt(x)*sqrt(a + b*x)*a**2*b + 14*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*sq
rt(x)*sqrt(a + b*x)*b**3*x**2 - 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqr
t(b))/sqrt(a))*a**3)/(24*b**2)
```

3.438 $\int \frac{(a+bx)^{3/2}}{\sqrt{x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}} dx = \frac{5}{4}a\sqrt{x}\sqrt{a+bx} + \frac{1}{2}bx^{3/2}\sqrt{a+bx} + \frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}$$

output
```
5/4*a*x^(1/2)*(b*x+a)^(1/2)+1/2*b*x^(3/2)*(b*x+a)^(1/2)+3/4*a^2*arctanh(b^
(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}} dx = \frac{1}{4}\sqrt{x}\sqrt{a+bx}(5a+2bx) - \frac{3a^2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{4\sqrt{b}}$$

input
```
Integrate[(a + b*x)^(3/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(5*a + 2*b*x))/4 - (3*a^2*Log[-(Sqrt[b]*Sqrt[x]) +
Sqrt[a + b*x]])/(4*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{4}a\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

input
```
Int[(a + b*x)^(3/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a*(Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{(2bx+5a)\sqrt{x}\,\sqrt{bx+a}}{4}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{8\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$	73
default	$\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}$	78

input
```
int((b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*x+5*a)*x^(1/2)*(b*x+a)^(1/2)+3/8*a^2*ln((1/2*a+b*x)/b^(1/2)+(b*x^
2+a*x)^(1/2))/b^(1/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x+5\,ab)\sqrt{bx+a}\sqrt{x}}{8\,b},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(2\,b^2x+5\,ab)\sqrt{bx+a}\sqrt{x}}{4\,b}\right]$$

input
```
integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(-b)*sqrt(x)/sqrt(b*x + a)) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b]
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx = \frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{4}+\frac{\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{2}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}$$

input
```
integrate((b*x+a)**(3/2)/x**(1/2),x)
```

output
```
5*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a)/4 + sqrt(a)*b*x**(3/2)*sqrt(1 + b*x/a)/
2 + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx = -\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,\sqrt{b}} - \frac{\frac{3\,\sqrt{bx+a}a^2b}{\sqrt{x}} - \frac{5\,(bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^2 - \frac{2\,(bx+a)b}{x} + \frac{(bx+a)^2}{x^2}\right)}$$

input
```
integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/sqrt(b) - 1/4*(3*sqrt(b*x + a)*a^2*b/sqrt(x) - 5*(b*x + a)^(3/2)*
a^2/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x + a)^2/x^2)
```

Giac [A] (verification not implemented)

Time = 75.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx = -\frac{\left(\frac{3\,a^2\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b}+\frac{3\,a}{b}\right)\right)b}{4\,|b|}$$

input
```
integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
-1/4*(3*a^2*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqr
t(b) - sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)/b + 3*a/b))*b/ab
s(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(a+b\,x)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int((a + b*x)^(3/2)/x^(1/2),x)
```

output
```
int((a + b*x)^(3/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx = \frac{5\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b}$$

input
```
int((b*x+a)^(3/2)/x^(1/2),x)
```

output
```
(5*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(b)*
log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b)
```

3.439 $\int \frac{(a+bx)^{3/2}}{x^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}} dx = -\frac{2a\sqrt{a+bx}}{\sqrt{x}} + b\sqrt{x}\sqrt{a+bx} + 3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2*a*(b*x+a)^(1/2)/x^(1/2)+b*x^(1/2)*(b*x+a)^(1/2)+3*a*b^(1/2)*arctanh(b^(
1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}} dx = \frac{(-2a+bx)\sqrt{a+bx}}{\sqrt{x}} + 6a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)$$

input
```
Integrate[(a + b*x)^(3/2)/x^(3/2),x]
```

output
```
((-2*a + b*x)*Sqrt[a + b*x])/Sqrt[x] + 6*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x
])/(-Sqrt[a] + Sqrt[a + b*x])]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {57, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$3b\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx-\frac{2(a+bx)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{65}$$

$$3b\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{219}$$

$$3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}$$

input
```
Int[(a + b*x)^(3/2)/x^(3/2),x]
```

output
```
(-2*(a + b*x)^(3/2))/Sqrt[x] + 3*b*(Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-bx+2a)}{\sqrt{x}}+\frac{3a\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$	71

input
```
int((b*x+a)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)^(1/2)*(-b*x+2*a)/x^(1/2)+3/2*a*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*
x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx = \left[\frac{3\,a\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}(bx-2\,a)\sqrt{x}}{2\,x}, -\frac{3\,a\sqrt{-b}x\,\mathrm{arct}}{}\right.$$

input
```
integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*s
qrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(-b)*sqrt
(x)/sqrt(b*x + a)) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx = -\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} - \frac{\sqrt{a}b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}} + 3a\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + \frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(3/2)/x**(3/2),x)
```

output
```
-2*a**(3/2)/(sqrt(x)*sqrt(1 + b*x/a)) - sqrt(a)*b*sqrt(x)/sqrt(1 + b*x/a)
+ 3*a*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt
(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx=-\frac{3}{2}\,a\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+a}a}{\sqrt{x}}-\frac{\sqrt{bx+a}ab}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

input
```
integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-3/2*a*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
+ a)/sqrt(x))) - 2*sqrt(b*x + a)*a/sqrt(x) - sqrt(b*x + a)*a*b/((b - (b*x
+ a)/x)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 75.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx=-\frac{\left(\frac{3\,a\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}}-\frac{\sqrt{bx+a}(bx-2\,a)}{\sqrt{(bx+a)b-ab}}\right)b^2}{|b|}$$

input
```
integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
-(3*a*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt(b) -
 sqrt(b*x + a)*(b*x - 2*a)/sqrt((b*x + a)*b - a*b))*b^2/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx=\int \frac{{\left(a+b\,x\right)}^{3/2}}{x^{3/2}}\,dx$$

input
```
int((a + b*x)^(3/2)/x^(3/2),x)
```

output
```
int((a + b*x)^(3/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx = \frac{-8\sqrt{x}\,\sqrt{bx+a}\,a+4\sqrt{x}\,\sqrt{bx+a}\,bx+12\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ax-9\sqrt{b}\,ax}{4x}$$

input
```
int((b*x+a)^(3/2)/x^(3/2),x)
```

output
```
( - 8*sqrt(x)*sqrt(a + b*x)*a + 4*sqrt(x)*sqrt(a + b*x)*b*x + 12*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*x - 9*sqrt(b)*a*x)/(4*x)
```

3.440 $\int \frac{(a+bx)^{3/2}}{x^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}} dx = -\frac{2a\sqrt{a+bx}}{3x^{3/2}} - \frac{8b\sqrt{a+bx}}{3\sqrt{x}} + 2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2/3*a*(b*x+a)^(1/2)/x^(3/2)-8/3*b*(b*x+a)^(1/2)/x^(1/2)+2*b^(3/2)*arctanh
(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}} dx = -\frac{2\sqrt{a+bx}(a+4bx)}{3x^{3/2}} - 2b^{3/2} \log \left(-\sqrt{b}\sqrt{x} + \sqrt{a+bx}\right)$$

input
```
Integrate[(a + b*x)^(3/2)/x^(5/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(a + 4*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[a + b*x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {57, 57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}} \, dx$$

$$\Big\downarrow \; 57$$

$$b\int \frac{\sqrt{a+bx}}{x^{3/2}}dx-\frac{2(a+bx)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \; 57$$

$$b\left(b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \; 65$$

$$b\left(2b\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \; 219$$

$$b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}$$

input
```
Int[(a + b*x)^(3/2)/x^(5/2),x]
```

output
```
(-2*(a + b*x)^(3/2))/(3*x^(3/2)) + b*((-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[
b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{2\sqrt{bx+a}\,(4bx+a)}{3x^{\frac{3}{2}}}+\frac{b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	67

input
```
int((b*x+a)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(4*b*x+a)/x^(3/2)+b^(3/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2
+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx = \left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\sqrt{x}}{3\,x^2}, -\frac{2\left(3\,\sqrt{-b}bx^2\,\mathrm{a}\right.}{}\right.$$

input
```
integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(
4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(
-b)*sqrt(x)/sqrt(b*x + a)) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3} - b^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)$$

input
```
integrate((b*x+a)**(3/2)/x**(5/2),x)
```

output
```
-2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 8*b**(3/2)*sqrt(a/(b*x) + 1)/3 - b*
*(3/2)*log(a/(b*x)) + 2*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx = -b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right) - \frac{2\sqrt{bx+a}b}{\sqrt{x}} - \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x))) - 2*sqrt(b*x + a)*b/sqrt(x) - 2/3*(b*x + a)^(3/2)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 75.74 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2\,b^3\left(\frac{3\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} + \frac{(4\,(bx+a)b-3\,ab)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{3}{2}}}\right)}{3\,|b|}$$

input
```
integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*b^3*(3*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqr
t(b) + (4*(b*x + a)*b - 3*a*b)*sqrt(b*x + a)/((b*x + a)*b - a*b)^(3/2))/ab
s(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx = \int \frac{(a+b\,x)^{3/2}}{x^{5/2}}\,dx$$

input
```
int((a + b*x)^(3/2)/x^(5/2),x)
```

output
```
int((a + b*x)^(3/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,bx}{3}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)b\,x^2}{x^2}$$

input
```
int((b*x+a)^(3/2)/x^(5/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a - 4*sqrt(x)*sqrt(a + b*x)*b*x + 3*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*x**2))/(3*x**2)
```

3.441 $\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx = -\frac{2(a+bx)^{5/2}}{5ax^{5/2}}$$

output
```
-2/5*(b*x+a)^(5/2)/a/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx = -\frac{2(a+bx)^{5/2}}{5ax^{5/2}}$$

input
```
Integrate[(a + b*x)^(3/2)/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/(5*a*x^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{2(a+bx)^{5/2}}{5ax^{5/2}}$$

input
```
Int[(a + b*x)^(3/2)/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/(5*a*x^(5/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}}{5a\,x^{\frac{5}{2}}}$	16
orering	$-\frac{2(bx+a)^{\frac{5}{2}}}{5a\,x^{\frac{5}{2}}}$	16
risch	$-\frac{2\sqrt{bx+a}\,(b^{2}x^{2}+2abx+a^{2})}{5x^{\frac{5}{2}}\,a}$	32
default	$-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{5}{2}}}-\frac{3a\left(-\frac{\sqrt{bx+a}}{2x^{\frac{5}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{4}\right)}{2}$	87

input
```
int((b*x+a)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x+a)^(5/2)/a/x^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx = -\frac{2\,(b^{2}x^{2}+2\,abx+a^{2})\sqrt{bx+a}}{5\,ax^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^(3/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(a*x^(5/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(19) = 38$.

Time = 1.66 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.10

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx = -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{5x} - \frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{5a}$$

input
```
integrate((b*x+a)**(3/2)/x**(7/2),x)
```

output
```
-2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 4*b**(3/2)*sqrt(a/(b*x) + 1)/(5*
x) - 2*b**(5/2)*sqrt(a/(b*x) + 1)/(5*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx = -\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,ax^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^(3/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-2/5*(b*x + a)^(5/2)/(a*x^(5/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx = -\frac{2\,(bx+a)^{\frac{5}{2}}b^6}{5\,((bx+a)b-ab)^{\frac{5}{2}}a|b|}$$

input
```
integrate((b*x+a)^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
-2/5*(b*x + a)^(5/2)*b^6/((((b*x + a)*b - a*b)^(5/2)*a*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx=-\frac{\sqrt{a+b\,x}\,\left(\frac{2\,a}{5}+\frac{4\,b\,x}{5}+\frac{2\,b^2\,x^2}{5\,a}\right)}{x^{5/2}}$$

input
```
int((a + b*x)^(3/2)/x^(7/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a)/5 + (4*b*x)/5 + (2*b^2*x^2)/(5*a)))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.90

$$\int \frac{(a+bx)^{3/2}}{x^{7/2}}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,abx}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{5}-\frac{2\sqrt{b}\,b^2x^3}{5}}{a\,x^3}$$

input
```
int((b*x+a)^(3/2)/x^(7/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a**2 - 2*sqrt(x)*sqrt(a + b*x)*a*b*x - sqrt(x
)*sqrt(a + b*x)*b**2*x**2 - sqrt(b)*b**2*x**3))/(5*a*x**3)
```

3.442 $\int \frac{(a+bx)^{3/2}}{x^{9/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}} dx = -\frac{2(a+bx)^{5/2}}{7ax^{7/2}} + \frac{4b(a+bx)^{5/2}}{35a^2x^{5/2}}$$

output
```
-2/7*(b*x+a)^(5/2)/a/x^(7/2)+4/35*b*(b*x+a)^(5/2)/a^2/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}} dx = -\frac{2(5a-2bx)(a+bx)^{5/2}}{35a^2x^{7/2}}$$

input
```
Integrate[(a + b*x)^(3/2)/x^(9/2),x]
```

output
```
(-2*(5*a - 2*b*x)*(a + b*x)^(5/2))/(35*a^2*x^(7/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{55, 48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}} \, dx$$

$$\Big\downarrow \ 55$$

$$-\frac{2b\int \frac{(a+bx)^{3/2}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}$$

$$\Big\downarrow \ 48$$

$$\frac{4b(a+bx)^{5/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}$$

input
```
Int[(a + b*x)^(3/2)/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/(7*a*x^(7/2)) + (4*b*(a + b*x)^(5/2))/(35*a^2*x^(5/2)
)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}(-2bx+5a)}{35x^{\frac{7}{2}}a^{2}}$	24
orering	$-\frac{2(bx+a)^{\frac{5}{2}}(-2bx+5a)}{35x^{\frac{7}{2}}a^{2}}$	24
risch	$-\frac{2\sqrt{bx+a}\,(-2b^{3}x^{3}+a\,b^{2}x^{2}+8a^{2}bx+5a^{3})}{35x^{\frac{7}{2}}a^{2}}$	45
default	$-\frac{(bx+a)^{\frac{3}{2}}}{2x^{\frac{7}{2}}}-\frac{3a\left(-\frac{\sqrt{bx+a}}{3x^{\frac{7}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}\right)}{4}$	109

input
```
int((b*x+a)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(5/2)*(-2*b*x+5*a)/x^(7/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}}\,dx = \frac{2\,(2\,b^3x^3 - ab^2x^2 - 8\,a^2bx - 5\,a^3)\sqrt{bx+a}}{35\,a^2x^{\frac{7}{2}}}$$

input
```
integrate((b*x+a)^(3/2)/x^(9/2),x, algorithm="fricas")
```

output
```
2/35*(2*b^3*x^3 - a*b^2*x^2 - 8*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^2*x^(7/2
))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(39) = 78$.

Time = 5.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}}\,dx = -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{7x^3} - \frac{16b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{35x^2} - \frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{35ax} + \frac{4b^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{35a^2}$$

input
```
integrate((b*x+a)**(3/2)/x**(9/2),x)
```

output
```
-2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(7*x**3) - 16*b**(3/2)*sqrt(a/(b*x) + 1)/(3
5*x**2) - 2*b**(5/2)*sqrt(a/(b*x) + 1)/(35*a*x) + 4*b**(7/2)*sqrt(a/(b*x)
+ 1)/(35*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}}\,dx = \frac{2\left(\frac{7\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{5\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^2}$$

input
```
integrate((b*x+a)^(3/2)/x^(9/2),x, algorithm="maxima")
```

output
```
2/35*(7*(b*x + a)^(5/2)*b/x^(5/2) - 5*(b*x + a)^(7/2)/x^(7/2))/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}}\,dx = \frac{2\left(\frac{2\,(bx+a)b^3}{a^2}-\frac{7\,b^3}{a}\right)(bx+a)^{\frac{5}{2}}b^5}{35\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

input
```
integrate((b*x+a)^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
2/35*(2*(b*x + a)*b^3/a^2 - 7*b^3/a)*(b*x + a)^(5/2)*b^5/(((b*x + a)*b - a
*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,a}{7}+\frac{16\,b\,x}{35}+\frac{2\,b^2\,x^2}{35\,a}-\frac{4\,b^3\,x^3}{35\,a^2}\right)}{x^{7/2}}$$

input
```
int((a + b*x)^(3/2)/x^(9/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a)/7 + (16*b*x)/35 + (2*b^2*x^2)/(35*a) - (4*b^3*x^3
)/(35*a^2)))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^{3/2}}{x^{9/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{4\sqrt{b}\,b^3x^4}{35}}{a^2x^4}$$

input
```
int((b*x+a)^(3/2)/x^(9/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 - 8*sqrt(x)*sqrt(a + b*x)*a**2*b*x - s
qrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 2*sqrt(x)*sqrt(a + b*x)*b**3*x**3 - 2*s
qrt(b)*b**3*x**4))/(35*a**2*x**4)
```

3.443 $\int \frac{(a+bx)^{3/2}}{x^{11/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}} dx = -\frac{2(a+bx)^{5/2}}{9ax^{9/2}} + \frac{8b(a+bx)^{5/2}}{63a^2x^{7/2}} - \frac{16b^2(a+bx)^{5/2}}{315a^3x^{5/2}}$$

output
```
-2/9*(b*x+a)^(5/2)/a/x^(9/2)+8/63*b*(b*x+a)^(5/2)/a^2/x^(7/2)-16/315*b^2*(
b*x+a)^(5/2)/a^3/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}} dx = -\frac{2(a+bx)^{5/2}(35a^2-20abx+8b^2x^2)}{315a^3x^{9/2}}$$

input
```
Integrate[(a + b*x)^(3/2)/x^(11/2),x]
```

output
```
(-2*(a + b*x)^(5/2)*(35*a^2 - 20*a*b*x + 8*b^2*x^2))/(315*a^3*x^(9/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}}\,dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{4b\int \frac{(a+bx)^{3/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{3/2}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{4b\left(\frac{4b(a+bx)^{5/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}$$

input
```
Int[(a + b*x)^(3/2)/x^(11/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/(9*a*x^(9/2)) - (4*b*((-2*(a + b*x)^(5/2))/(7*a*x^(7/
2)) + (4*b*(a + b*x)^(5/2))/(35*a^2*x^(5/2))))/(9*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(8b^2x^2-20abx+35a^2\right)}{315x^{\frac{9}{2}}a^3}$	35
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(8b^2x^2-20abx+35a^2\right)}{315x^{\frac{9}{2}}a^3}$	35
risch	$-\frac{2\sqrt{bx+a}\left(8b^4x^4-4a\,x^3b^3+3a^2b^2x^2+50a^3bx+35a^4\right)}{315x^{\frac{9}{2}}a^3}$	57
default	$-\frac{(bx+a)^{\frac{3}{2}}}{3x^{\frac{9}{2}}}-\frac{a\left(-\frac{\sqrt{bx+a}}{4x^{\frac{9}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{9a\,x^{\frac{9}{2}}}-\frac{8b\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{8}\right)}{2}$	131

input
```
int((b*x+a)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(b*x+a)^(5/2)*(8*b^2*x^2-20*a*b*x+35*a^2)/x^(9/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}}\,dx = -\frac{2\,(8\,b^4x^4-4\,ab^3x^3+3\,a^2b^2x^2+50\,a^3bx+35\,a^4)\sqrt{bx+a}}{315\,a^3x^{\frac{9}{2}}}$$

input
```
integrate((b*x+a)^(3/2)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/315*(8*b^4*x^4 - 4*a*b^3*x^3 + 3*a^2*b^2*x^2 + 50*a^3*b*x + 35*a^4)*sqr
t(b*x + a)/(a^3*x^(9/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(63) = 126.

Time = 15.74 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.97

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^{11/2}}\,dx = &-\frac{70a^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{315a^5b^4x^4+630a^4b^5x^5+315a^3b^6x^6} \\ &-\frac{240a^5b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{315a^5b^4x^4+630a^4b^5x^5+315a^3b^6x^6}-\frac{276a^4b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{315a^5b^4x^4+630a^4b^5x^5+315a^3b^6x^6} \\ &-\frac{104a^3b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{315a^5b^4x^4+630a^4b^5x^5+315a^3b^6x^6}-\frac{6a^2b^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{315a^5b^4x^4+630a^4b^5x^5+315a^3b^6x^6} \\ &-\frac{24ab^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{315a^5b^4x^4+630a^4b^5x^5+315a^3b^6x^6}-\frac{16b^{\frac{21}{2}}x^6\sqrt{\frac{a}{bx}+1}}{315a^5b^4x^4+630a^4b^5x^5+315a^3b^6x^6}\end{aligned}$$

input
```
integrate((b*x+a)**(3/2)/x**(11/2),x)
```

output
```
-70*a**6*b**(9/2)*sqrt(a/(b*x) + 1)/(315*a**5*b**4*x**4 + 630*a**4*b**5*x*
*5 + 315*a**3*b**6*x**6) - 240*a**5*b**(11/2)*x*sqrt(a/(b*x) + 1)/(315*a**
5*b**4*x**4 + 630*a**4*b**5*x**5 + 315*a**3*b**6*x**6) - 276*a**4*b**(13/2
)*x**2*sqrt(a/(b*x) + 1)/(315*a**5*b**4*x**4 + 630*a**4*b**5*x**5 + 315*a*
*3*b**6*x**6) - 104*a**3*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(315*a**5*b**4*x
**4 + 630*a**4*b**5*x**5 + 315*a**3*b**6*x**6) - 6*a**2*b**(17/2)*x**4*sqr
t(a/(b*x) + 1)/(315*a**5*b**4*x**4 + 630*a**4*b**5*x**5 + 315*a**3*b**6*x*
*6) - 24*a*b**(19/2)*x**5*sqrt(a/(b*x) + 1)/(315*a**5*b**4*x**4 + 630*a**4
*b**5*x**5 + 315*a**3*b**6*x**6) - 16*b**(21/2)*x**6*sqrt(a/(b*x) + 1)/(31
5*a**5*b**4*x**4 + 630*a**4*b**5*x**5 + 315*a**3*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}}\,dx = -\frac{2\left(\frac{63\,(bx+a)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}} - \frac{90\,(bx+a)^{\frac{7}{2}}b}{x^{\frac{7}{2}}} + \frac{35\,(bx+a)^{\frac{9}{2}}}{x^{\frac{9}{2}}}\right)}{315\,a^3}$$

input
```
integrate((b*x+a)^(3/2)/x^(11/2),x, algorithm="maxima")
```

output
```
-2/315*(63*(b*x + a)^(5/2)*b^2/x^(5/2) - 90*(b*x + a)^(7/2)*b/x^(7/2) + 35
*(b*x + a)^(9/2)/x^(9/2))/a^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}}\,dx = -\frac{2\left(\frac{63\,b^9}{a} + 4\left(\frac{2\,(bx+a)b^9}{a^3} - \frac{9\,b^9}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{315\left((bx+a)b-ab\right)^{\frac{9}{2}}|b|}$$

input
```
integrate((b*x+a)^(3/2)/x^(11/2),x, algorithm="giac")
```

output
```
-2/315*(63*b^9/a + 4*(2*(b*x + a)*b^9/a^3 - 9*b^9/a^2)*(b*x + a))*(b*x + a
)^(5/2)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,a}{9}+\frac{20\,b\,x}{63}+\frac{2\,b^2\,x^2}{105\,a}-\frac{8\,b^3\,x^3}{315\,a^2}+\frac{16\,b^4\,x^4}{315\,a^3}\right)}{x^{9/2}}$$

input
```
int((a + b*x)^(3/2)/x^(11/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a)/9 + (20*b*x)/63 + (2*b^2*x^2)/(105*a) - (8*b^3*x^
3)/(315*a^2) + (16*b^4*x^4)/(315*a^3)))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^{3/2}}{x^{11/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9}-\frac{20\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{105}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{315}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^4x^4}{315}+1}{a^3x^5}$$

input
```
int((b*x+a)^(3/2)/x^(11/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 50*sqrt(x)*sqrt(a + b*x)*a**3*b*x -
 3*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b**3*x
**3 - 8*sqrt(x)*sqrt(a + b*x)*b**4*x**4 + 8*sqrt(b)*b**4*x**5))/(315*a**3*
x**5)
```

3.444 $\int \frac{(a+bx)^{3/2}}{x^{13/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}} dx = -\frac{2(a+bx)^{5/2}}{11ax^{11/2}} + \frac{4b(a+bx)^{5/2}}{33a^2x^{9/2}} - \frac{16b^2(a+bx)^{5/2}}{231a^3x^{7/2}} + \frac{32b^3(a+bx)^{5/2}}{1155a^4x^{5/2}}$$

output
```
-2/11*(b*x+a)^(5/2)/a/x^(11/2)+4/33*b*(b*x+a)^(5/2)/a^2/x^(9/2)-16/231*b^2
*(b*x+a)^(5/2)/a^3/x^(7/2)+32/1155*b^3*(b*x+a)^(5/2)/a^4/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}} dx = -\frac{2(a+bx)^{5/2} (105a^3 - 70a^2bx + 40ab^2x^2 - 16b^3x^3)}{1155a^4x^{11/2}}$$

input
```
Integrate[(a + b*x)^(3/2)/x^(13/2),x]
```

output
```
(-2*(a + b*x)^(5/2)*(105*a^3 - 70*a^2*b*x + 40*a*b^2*x^2 - 16*b^3*x^3))/(1
155*a^4*x^(11/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\int \frac{(a+bx)^{3/2}}{x^{11/2}}dx}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\left(-\frac{4b\int \frac{(a+bx)^{3/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{3/2}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b(a+bx)^{5/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}$$

input
```
Int[(a + b*x)^(3/2)/x^(13/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/(11*a*x^(11/2)) - (6*b*((-2*(a + b*x)^(5/2))/(9*a*x^(
9/2)) - (4*b*((-2*(a + b*x)^(5/2))/(7*a*x^(7/2)) + (4*b*(a + b*x)^(5/2))/(
35*a^2*x^(5/2))))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

method	result
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-16b^{3}x^{3}+40a\,b^{2}x^{2}-70a^{2}bx+105a^{3}\right)}{1155x^{\frac{11}{2}}\,a^{4}}$
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-16b^{3}x^{3}+40a\,b^{2}x^{2}-70a^{2}bx+105a^{3}\right)}{1155x^{\frac{11}{2}}\,a^{4}}$
risch	$-\frac{2\sqrt{bx+a}\,\left(-16b^{5}x^{5}+8a\,b^{4}x^{4}-6a^{2}b^{3}x^{3}+5a^{3}b^{2}x^{2}+140a^{4}bx+105a^{5}\right)}{1155x^{\frac{11}{2}}\,a^{4}}$
	$3a\left(-\frac{\sqrt{bx+a}}{5x^{\frac{11}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{11a\,x^{\frac{11}{2}}}-\frac{10b\left(-\frac{2\sqrt{bx+a}}{9a\,x^{\frac{9}{2}}}-\frac{8b\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{11a}\right)}{10}$

input
```
int((b*x+a)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(b*x+a)^(5/2)*(-16*b^3*x^3+40*a*b^2*x^2-70*a^2*b*x+105*a^3)/x^(11/
2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}}\,dx = \frac{2\,(16\,b^5x^5-8\,ab^4x^4+6\,a^2b^3x^3-5\,a^3b^2x^2-140\,a^4bx-105\,a^5)\sqrt{bx+a}}{1155\,a^4x^{\frac{11}{2}}}$$

input
```
integrate((b*x+a)^(3/2)/x^(13/2),x, algorithm="fricas")
```

output
```
2/1155*(16*b^5*x^5 - 8*a*b^4*x^4 + 6*a^2*b^3*x^3 - 5*a^3*b^2*x^2 - 140*a^4
*b*x - 105*a^5)*sqrt(b*x + a)/(a^4*x^(11/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(87) = 174$.

Time = 42.68 (sec) , antiderivative size = 631, normalized size of antiderivative = 6.86

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}}\,dx = -\frac{210a^8b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} - \frac{910a^7b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} - \frac{1480a^6b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} - \frac{1068a^5b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} - \frac{290a^4b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} + \frac{10a^3b^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} + \frac{60a^2b^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} + \frac{80ab^{\frac{33}{2}}x^7\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8} + \frac{32b^{\frac{35}{2}}x^8\sqrt{\frac{a}{bx}+1}}{1155a^7b^9x^5+3465a^6b^{10}x^6+3465a^5b^{11}x^7+1155a^4b^{12}x^8}$$

input
```
integrate((b*x+a)**(3/2)/x**(13/2),x)
```

output
```
-210*a**8*b**(19/2)*sqrt(a/(b*x) + 1)/(1155*a**7*b**9*x**5 + 3465*a**6*b**
10*x**6 + 3465*a**5*b**11*x**7 + 1155*a**4*b**12*x**8) - 910*a**7*b**(21/2
)*x*sqrt(a/(b*x) + 1)/(1155*a**7*b**9*x**5 + 3465*a**6*b**10*x**6 + 3465*a
**5*b**11*x**7 + 1155*a**4*b**12*x**8) - 1480*a**6*b**(23/2)*x**2*sqrt(a/(
b*x) + 1)/(1155*a**7*b**9*x**5 + 3465*a**6*b**10*x**6 + 3465*a**5*b**11*x*
*7 + 1155*a**4*b**12*x**8) - 1068*a**5*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(1
155*a**7*b**9*x**5 + 3465*a**6*b**10*x**6 + 3465*a**5*b**11*x**7 + 1155*a*
*4*b**12*x**8) - 290*a**4*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(1155*a**7*b**9
*x**5 + 3465*a**6*b**10*x**6 + 3465*a**5*b**11*x**7 + 1155*a**4*b**12*x**8
) + 10*a**3*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(1155*a**7*b**9*x**5 + 3465*a
**6*b**10*x**6 + 3465*a**5*b**11*x**7 + 1155*a**4*b**12*x**8) + 60*a**2*b*
*(31/2)*x**6*sqrt(a/(b*x) + 1)/(1155*a**7*b**9*x**5 + 3465*a**6*b**10*x**6
 + 3465*a**5*b**11*x**7 + 1155*a**4*b**12*x**8) + 80*a*b**(33/2)*x**7*sqrt
(a/(b*x) + 1)/(1155*a**7*b**9*x**5 + 3465*a**6*b**10*x**6 + 3465*a**5*b**1
1*x**7 + 1155*a**4*b**12*x**8) + 32*b**(35/2)*x**8*sqrt(a/(b*x) + 1)/(1155
*a**7*b**9*x**5 + 3465*a**6*b**10*x**6 + 3465*a**5*b**11*x**7 + 1155*a**4*
b**12*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}}\,dx = \frac{2\left(\frac{231\,(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}} - \frac{495\,(bx+a)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}} + \frac{385\,(bx+a)^{\frac{9}{2}}b}{x^{\frac{9}{2}}} - \frac{105\,(bx+a)^{\frac{11}{2}}}{x^{\frac{11}{2}}}\right)}{1155\,a^4}$$

input
```
integrate((b*x+a)^(3/2)/x^(13/2),x, algorithm="maxima")
```

output
```
2/1155*(231*(b*x + a)^(5/2)*b^3/x^(5/2) - 495*(b*x + a)^(7/2)*b^2/x^(7/2)
+ 385*(b*x + a)^(9/2)*b/x^(9/2) - 105*(b*x + a)^(11/2)/x^(11/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}}\,dx = -\frac{2\left(\frac{231\,b^5}{a}-2\left(\frac{99\,b^5}{a^2}+4\left(\frac{2\,(bx+a)b^5}{a^4}-\frac{11\,b^5}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b^7}{1155\left((bx+a)b-ab\right)^{\frac{11}{2}}|b|}$$

input
```
integrate((b*x+a)^(3/2)/x^(13/2),x, algorithm="giac")
```

output
```
-2/1155*(231*b^5/a - 2*(99*b^5/a^2 + 4*(2*(b*x + a)*b^5/a^4 - 11*b^5/a^3)*
(b*x + a))*(b*x + a))*(b*x + a)^(5/2)*b^7/(((b*x + a)*b - a*b)^(11/2)*abs(
b))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,a}{11}+\frac{8\,b\,x}{33}+\frac{2\,b^2\,x^2}{231\,a}-\frac{4\,b^3\,x^3}{385\,a^2}+\frac{16\,b^4\,x^4}{1155\,a^3}-\frac{32\,b^5\,x^5}{1155\,a^4}\right)}{x^{11/2}}$$

input
```
int((a + b*x)^(3/2)/x^(13/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a)/11 + (8*b*x)/33 + (2*b^2*x^2)/(231*a) - (4*b^3*x^
3)/(385*a^2) + (16*b^4*x^4)/(1155*a^3) - (32*b^5*x^5)/(1155*a^4)))/x^(11/2
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)^{3/2}}{x^{13/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5}{11}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,a^4bx}{33}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2}{231}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^3}{385}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^4}{1155}+}{a^4x^6}$$

input
```
int((b*x+a)^(3/2)/x^(13/2),x)
```

output
```
(2*( - 105*sqrt(x)*sqrt(a + b*x)*a**5 - 140*sqrt(x)*sqrt(a + b*x)*a**4*b*x
 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b
**3*x**3 - 8*sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 + 16*sqrt(x)*sqrt(a + b*x)*
b**5*x**5 - 16*sqrt(b)*b**5*x**6))/(1155*a**4*x**6)
```

3.445 $\int x^{5/2}(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 168

$$\int x^{5/2}(a+bx)^{5/2}\,dx=\frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{9}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{5}{12}abx^{9/2}\sqrt{a+bx}+\frac{1}{6}b^2x^{11/2}\sqrt{a+bx}-\frac{5a^6\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}$$

output
```
5/512*a^5*x^(1/2)*(b*x+a)^(1/2)/b^3-5/768*a^4*x^(3/2)*(b*x+a)^(1/2)/b^2+1/
192*a^3*x^(5/2)*(b*x+a)^(1/2)/b+9/32*a^2*x^(7/2)*(b*x+a)^(1/2)+5/12*a*b*x^
(9/2)*(b*x+a)^(1/2)+1/6*b^2*x^(11/2)*(b*x+a)^(1/2)-5/512*a^6*arctanh(b^(1/
2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70

$$\int x^{5/2}(a+bx)^{5/2}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^5-10a^4bx+8a^3b^2x^2+432a^2b^3x^3+640ab^4x^4+256b^5x^5)+30a^6\text{arctanh}}{1536b^{7/2}}$$

input
```
Integrate[x^(5/2)*(a + b*x)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*
a^2*b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) + 30*a^6*ArcTanh[(Sqrt[b]*Sqrt[
x])/(Sqrt[a] - Sqrt[a + b*x])])/(1536*b^(7/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {60, 60, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\int x^{5/2}(a+bx)^{3/2}dx+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\int x^{5/2}\sqrt{a+bx}dx+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a+bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int\frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

$\downarrow$ 60

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a\right.$$

$$\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

$\downarrow$ 65

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7\right.$$

$$\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

$\downarrow$ 219

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x\right.$$

$$\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

input
```
Int[x^(5/2)*(a + b*x)^(5/2),x]
```

output
```
(x^(7/2)*(a + b*x)^(5/2))/6 + (5*a*((x^(7/2)*(a + b*x)^(3/2))/5 + (3*a*((x
^(7/2)*Sqrt[a + b*x])/4 + (a*((x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a*((x^(3/
2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/(6*b)))/8))/10))/12
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(256b^5x^5+640a\,b^4x^4+432a^2b^3x^3+8a^3b^2x^2-10a^4bx+15a^5)\sqrt{x}\,\sqrt{bx+a}}{1536b^3}-\frac{5a^6\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{1024b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{5}{2}}(bx+a)^{\frac{7}{2}}}{6b}-\frac{5a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{7}{2}}}{5b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{7}{2}}}{4b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3}+\frac{5a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)}{10b}\right)}{12b}$

input
```
int(x^(5/2)*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(256*b^5*x^5+640*a*b^4*x^4+432*a^2*b^3*x^3+8*a^3*b^2*x^2-10*a^4*b*x
+15*a^5)*x^(1/2)*(b*x+a)^(1/2)/b^3-5/1024*a^6/b^(7/2)*ln((1/2*a+b*x)/b^(1/
2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \left[\frac{15\,a^6\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(256\,b^6x^5+640\,ab^5x^4+432\,a^2b^4x^3+8\,a^3b^3x}{3072\,b^4}\right.$$

input
```
integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/3072*(15*a^6*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
 2*(256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4
*b^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arc
tan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a
^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x
))/b^4]
```

Sympy [A] (verification not implemented)

Time = 155.94 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.23

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1+\frac{bx}{a}}} + \frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1+\frac{bx}{a}}} - \frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1+\frac{bx}{a}}} + \frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1+\frac{bx}{a}}} + \frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1+\frac{bx}{a}}} + \frac{7\sqrt{a}b^2x^{\frac{11}{2}}}{12\sqrt{1+\frac{bx}{a}}} - \frac{5a^6\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} + \frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)*(b*x+a)**(5/2),x)
```

output
```
5*a**(11/2)*sqrt(x)/(512*b**3*sqrt(1 + b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536
*b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(5/2)/(768*b*sqrt(1 + b*x/a)) + 55*a*
*(5/2)*x**(7/2)/(192*sqrt(1 + b*x/a)) + 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1
+ b*x/a)) + 7*sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 + b*x/a)) - 5*a**6*asinh(s
qrt(b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + b**3*x**(13/2)/(6*sqrt(a)*sqrt(1
+ b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.45

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \frac{5\,a^6\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{1024\,b^{\frac{7}{2}}} + \frac{\frac{15\,\sqrt{bx+a}a^6b^5}{\sqrt{x}} - \frac{85\,(bx+a)^{\frac{3}{2}}a^6b^4}{x^{\frac{3}{2}}} + \frac{198\,(bx+a)^{\frac{5}{2}}a^6b^3}{x^{\frac{5}{2}}} + \frac{198\,(bx+a)^{\frac{7}{2}}a^6b^2}{x^{\frac{7}{2}}} - \frac{85\,(bx+a)^{\frac{9}{2}}a^6b}{x^{\frac{9}{2}}} + \frac{15\,(bx+a)^{\frac{11}{2}}a^6}{x^{\frac{11}{2}}}}{1536\left(b^9 - \frac{6\,(bx+a)b^8}{x} + \frac{15\,(bx+a)^2b^7}{x^2} - \frac{20\,(bx+a)^3b^6}{x^3} + \frac{15\,(bx+a)^4b^5}{x^4} - \frac{6\,(bx+a)^5b^4}{x^5} + \frac{(bx+a)^6b^3}{x^6}\right)}$$

input
```
integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
5/1024*a^6*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)
/sqrt(x)))/b^(7/2) + 1/1536*(15*sqrt(b*x + a)*a^6*b^5/sqrt(x) - 85*(b*x +
a)^(3/2)*a^6*b^4/x^(3/2) + 198*(b*x + a)^(5/2)*a^6*b^3/x^(5/2) + 198*(b*x
+ a)^(7/2)*a^6*b^2/x^(7/2) - 85*(b*x + a)^(9/2)*a^6*b/x^(9/2) + 15*(b*x +
a)^(11/2)*a^6/x^(11/2))/(b^9 - 6*(b*x + a)*b^8/x + 15*(b*x + a)^2*b^7/x^2
- 20*(b*x + a)^3*b^6/x^3 + 15*(b*x + a)^4*b^5/x^4 - 6*(b*x + a)^5*b^4/x^5
+ (b*x + a)^6*b^3/x^6)
```

Giac [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \int x^{5/2}\,{\left(a+b\,x\right)}^{5/2}\,dx$$

input
```
int(x^(5/2)*(a + b*x)^(5/2),x)
```

output
```
int(x^(5/2)*(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^5b-10\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^2+432\sqrt{x}\,\sqrt{bx+a}\,a^2b^4x^3+}{1536b^4}$$

input
```
int(x^(5/2)*(b*x+a)^(5/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**5*b - 10*sqrt(x)*sqrt(a + b*x)*a**4*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a**3*b**3*x**2 + 432*sqrt(x)*sqrt(a + b*x)*a**2*b*
*4*x**3 + 640*sqrt(x)*sqrt(a + b*x)*a*b**5*x**4 + 256*sqrt(x)*sqrt(a + b*x
)*b**6*x**5 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*
*6)/(1536*b**4)
```

3.446 $\int x^{3/2}(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int x^{3/2}(a+bx)^{5/2}\,dx=-\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{31}{80}a^2x^{5/2}\sqrt{a+bx}+\frac{21}{40}abx^{7/2}\sqrt{a+bx}+\frac{1}{5}b^2x^{9/2}\sqrt{a+bx}+\frac{3a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}$$

output
```
-3/128*a^4*x^(1/2)*(b*x+a)^(1/2)/b^2+1/64*a^3*x^(3/2)*(b*x+a)^(1/2)/b+31/8
0*a^2*x^(5/2)*(b*x+a)^(1/2)+21/40*a*b*x^(7/2)*(b*x+a)^(1/2)+1/5*b^2*x^(9/2
)*(b*x+a)^(1/2)+3/128*a^5*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.74

$$\int x^{3/2}(a+bx)^{5/2}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-15a^4+10a^3bx+248a^2b^2x^2+336ab^3x^3+128b^4x^4)+30a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a}}\right.}{640b^{5/2}}$$

input
```
Integrate[x^(3/2)*(a + b*x)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 3
36*a*b^3*x^3 + 128*b^4*x^4) + 30*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
 Sqrt[a + b*x])])/(640*b^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {60, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\int x^{3/2}(a+bx)^{3/2}dx+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\int x^{3/2}\sqrt{a+bx}dx+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

$$\Big\downarrow \; 65$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

input
```
Int[x^(3/2)*(a + b*x)^(5/2),x]
```

output
```
(x^(5/2)*(a + b*x)^(5/2))/5 + (a*((x^(5/2)*(a + b*x)^(3/2))/4 + (3*a*((x^(
5/2)*Sqrt[a + b*x])/3 + (a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]
*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))
/(4*b)))/6))/8))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(-128 b^{4} x^{4}-336 a\,x^{3} b^{3}-248 a^{2} b^{2} x^{2}-10 a^{3} b x+15 a^{4}\right) \sqrt{x}\,\sqrt{b x+a}}{640 b^{2}}+\frac{3 a^{5} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right) \sqrt{x \left(b x+a\right)}}{256 b^{\frac{5}{2}} \sqrt{x}\,\sqrt{b x+a}}$
default	$\frac{x^{\frac{3}{2}} \left(b x+a\right)^{\frac{7}{2}}}{5 b}-\frac{3 a \left(\frac{\sqrt{x}\,\left(b x+a\right)^{\frac{7}{2}}}{4 b}-\frac{a \left(\frac{\sqrt{x}\,\left(b x+a\right)^{\frac{5}{2}}}{3}+\frac{5 a \left(\frac{\sqrt{x}\,\left(b x+a\right)^{\frac{3}{2}}}{2}+\frac{3 a \left(\sqrt{x}\,\sqrt{b x+a}+\frac{a \sqrt{x \left(b x+a\right)}\,\ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right)}{2 \sqrt{b x+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}\right)}{8 b}\right)}{10 b}$

input
```
int(x^(3/2)*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/640*(-128*b^4*x^4-336*a*b^3*x^3-248*a^2*b^2*x^2-10*a^3*b*x+15*a^4)*x^(1
/2)*(b*x+a)^(1/2)/b^2+3/256*a^5/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)
^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.26

$$\int x^{3/2}(a+bx)^{5/2}\,dx = \left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2}{1280\,b^3}\right.$$

input
```
integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/1280*(15*a^5*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
 2*(128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*
b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(-b)*sqr
t(x)/sqrt(b*x + a)) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*
a^3*b^2*x - 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 28.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$\int x^{3/2}(a+bx)^{5/2}\,dx = -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1+\frac{bx}{a}}} - \frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1+\frac{bx}{a}}} + \frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1+\frac{bx}{a}}} + \frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}} + \frac{29\sqrt{a}b^2x^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}} + \frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}} + \frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)*(b*x+a)**(5/2),x)
```

output
```
-3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(3/2)/(128*b*
sqrt(1 + b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 + b*x/a)) + 73*a**(3/
2)*b*x**(7/2)/(80*sqrt(1 + b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1 +
 b*x/a)) + 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) + b**3*x**
(11/2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(104) = 208.

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.47

$$\int x^{3/2}(a+bx)^{5/2}\,dx = -\frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{5}{2}}} - \frac{\frac{15\,\sqrt{bx+a}a^5b^4}{\sqrt{x}} - \frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}} + \frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}} + \frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}} - \frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^7 - \frac{5\,(bx+a)b^6}{x} + \frac{10\,(bx+a)^2b^5}{x^2} - \frac{10\,(bx+a)^3b^4}{x^3} + \frac{5\,(bx+a)^4b^3}{x^4} - \frac{(bx+a)^5b^2}{x^5}\right)}$$

input
```
integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)
/sqrt(x)))/b^(5/2) - 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a
)^(3/2)*a^5*b^3/x^(3/2) + 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x +
a)^(7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x + a
)*b^6/x + 10*(b*x + a)^2*b^5/x^2 - 10*(b*x + a)^3*b^4/x^3 + 5*(b*x + a)^4*
b^3/x^4 - (b*x + a)^5*b^2/x^5)
```

Giac [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{5/2}\,dx=\int x^{3/2}\,(a+b\,x)^{5/2}\,dx$$

input
```
int(x^(3/2)*(a + b*x)^(5/2),x)
```

output
```
int(x^(3/2)*(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int x^{3/2}(a+bx)^{5/2}\,dx=\frac{-15\sqrt{x}\,\sqrt{bx+a}\,a^4b+10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+248\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+336\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x}{640b^3}$$

input
```
int(x^(3/2)*(b*x+a)^(5/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(a + b*x)*a**4*b + 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x
 + 248*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 336*sqrt(x)*sqrt(a + b*x)*a*
b**4*x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*x**4 + 15*sqrt(b)*log((sqrt(a +
 b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(640*b**3)
```

3.447 $\int \sqrt{x}(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \sqrt{x}(a+bx)^{5/2}\,dx=\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{59}{96}a^2x^{3/2}\sqrt{a+bx}+\frac{17}{24}abx^{5/2}\sqrt{a+bx}+\frac{1}{4}b^2x^{7/2}\sqrt{a+bx}-\frac{5a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}$$

output
```
5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b+59/96*a^2*x^(3/2)*(b*x+a)^(1/2)+17/24*a*b
*x^(5/2)*(b*x+a)^(1/2)+1/4*b^2*x^(7/2)*(b*x+a)^(1/2)-5/64*a^4*arctanh(b^(1
/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(a+bx)^{5/2}\,dx=\frac{\sqrt{x}\sqrt{a+bx}(15a^3+118a^2bx+136ab^2x^2+48b^3x^3)}{192b}-\frac{5a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{32b^{3/2}}$$

input
```
Integrate[Sqrt[x]*(a + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*b^3*x^3)
)/(192*b) - (5*a^4*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/
(32*b^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}a\int \sqrt{x}(a+bx)^{3/2}dx+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}a\left(\frac{1}{2}a\int \sqrt{x}\sqrt{a+bx}dx+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

$$\Big\downarrow \; 65$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

input
```
Int[Sqrt[x]*(a + b*x)^(5/2),x]
```

output
```
(x^(3/2)*(a + b*x)^(5/2))/4 + (5*a*((x^(3/2)*(a + b*x)^(3/2))/3 + (a*((x^(
3/2)*Sqrt[a + b*x])/2 + (a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b
]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/4))/2))/8
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{(48b^3x^3+136a\,b^2x^2+118a^2bx+15a^3)\sqrt{x}\,\sqrt{bx+a}}{192b}-\frac{5a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{128b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	98
default	$\frac{\sqrt{x}\,(bx+a)^{\frac{7}{2}}}{4b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3}+\frac{5a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}$	116

input
```
int(x^(1/2)*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*b^3*x^3+136*a*b^2*x^2+118*a^2*b*x+15*a^3)*x^(1/2)*(b*x+a)^(1/2)/
b-5/128/b^(3/2)*a^4*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^
(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.32

$$\int \sqrt{x}(a+bx)^{5/2}\,dx = \left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b)\sqrt{}}{384\,b^2}\right.$$

input
```
integrate(x^(1/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a))
+ (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int \sqrt{x}(a+bx)^{5/2}\,dx = \frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1+\frac{bx}{a}}} + \frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1+\frac{bx}{a}}} + \frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1+\frac{bx}{a}}} + \frac{23\sqrt{a}b^2x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}} - \frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} + \frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(1/2)*(b*x+a)**(5/2),x)
```

output
```
5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 + b*x/a)) + 133*a**(5/2)*x**(3/2)/(192*sqr
t(1 + b*x/a)) + 127*a**(3/2)*b*x**(5/2)/(96*sqrt(1 + b*x/a)) + 23*sqrt(a)*
b**2*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))
/(64*b**(3/2)) + b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(86) = 172.

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.47

$$\int \sqrt{x}(a+bx)^{5/2}\,dx = \frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{3}{2}}} + \frac{\frac{15\sqrt{bx+a}a^4b^3}{\sqrt{x}} - \frac{55\,(bx+a)^{\frac{3}{2}}\,a^4b^2}{x^{\frac{3}{2}}} + \frac{73\,(bx+a)^{\frac{5}{2}}\,a^4b}{x^{\frac{5}{2}}} + \frac{15\,(bx+a)^{\frac{7}{2}}\,a^4}{x^{\frac{7}{2}}}}{192\left(b^5 - \frac{4\,(bx+a)b^4}{x} + \frac{6\,(bx+a)^2b^3}{x^2} - \frac{4\,(bx+a)^3b^2}{x^3} + \frac{(bx+a)^4b}{x^4}\right)}$$

input
```
integrate(x^(1/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(3/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 55*(b*x + a)
^(3/2)*a^4*b^2/x^(3/2) + 73*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(
7/2)*a^4/x^(7/2))/(b^5 - 4*(b*x + a)*b^4/x + 6*(b*x + a)^2*b^3/x^2 - 4*(b*
x + a)^3*b^2/x^3 + (b*x + a)^4*b/x^4)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{5/2}\,dx = \int \sqrt{x}\,{\left(a+b\,x\right)}^{5/2} \,d x$$

input
```
int(x^(1/2)*(a + b*x)^(5/2),x)
```

output
```
int(x^(1/2)*(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(a+bx)^{5/2}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b+118\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+136\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,b^4x^3-}{192b^2}$$

input
```
int(x^(1/2)*(b*x+a)^(5/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b + 118*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
 136*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x**
3 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(192*b
**2)
```

3.448 $\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx=\frac{11}{8}a^2\sqrt{x}\sqrt{a+bx}+\frac{13}{12}abx^{3/2}\sqrt{a+bx}+\frac{1}{3}b^2x^{5/2}\sqrt{a+bx}+\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

output
```
11/8*a^2*x^(1/2)*(b*x+a)^(1/2)+13/12*a*b*x^(3/2)*(b*x+a)^(1/2)+1/3*b^2*x^(
5/2)*(b*x+a)^(1/2)+5/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx=\frac{1}{24}\sqrt{x}\sqrt{a+bx}\left(33a^2+26abx+8b^2x^2\right)-\frac{5a^3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{8\sqrt{b}}$$

input
```
Integrate[(a + b*x)^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(33*a^2 + 26*a*b*x + 8*b^2*x^2))/24 - (5*a^3*Log[-(
Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{6}a\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

input
```
Int[(a + b*x)^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*(a + b*x)^(5/2))/3 + (5*a*((Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a*(Sq
rt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
))/4))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\left(8 b^{2} x^{2}+26 a b x +33 a^{2}\right) \sqrt{x}\, \sqrt{b x +a}}{24}+\frac{5 a^{3} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right) \sqrt{x \left(b x +a \right)}}{16 \sqrt{b}\, \sqrt{x}\, \sqrt{b x +a}}$	84
default	$\frac{\sqrt{x}\, \left(b x +a \right)^{\frac{5}{2}}}{3}+\frac{5 a \left(\frac{\sqrt{x}\, \left(b x +a \right)^{\frac{3}{2}}}{2}+\frac{3 a \left(\sqrt{x}\, \sqrt{b x +a}+\frac{a \sqrt{x \left(b x +a \right)}\, \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{2 \sqrt{b x +a}\, \sqrt{x}\, \sqrt{b}}\right)}{4}\right)}{6}$	94

input
```
int((b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^2*x^2+26*a*b*x+33*a^2)*x^(1/2)*(b*x+a)^(1/2)+5/16*a^3*ln((1/2*a+
b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^3x^2+26\,ab^2x+33\,a^2b)\sqrt{bx+a}\sqrt{x}}{48\,b}, -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(8\,b^3x^2+26\,ab^2x+33\,a^2b)\sqrt{bx+a}\sqrt{x}}{24\,b}\right]$$

input
```
integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a
^3*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (8*b^3*x^2 + 26*a*b^2
*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

Sympy [A] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{8}+\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{12}+\frac{\sqrt{a}b^2x^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}$$

input
```
integrate((b*x+a)**(5/2)/x**(1/2),x)
```

output
```
11*a**(5/2)*sqrt(x)*sqrt(1 + b*x/a)/8 + 13*a**(3/2)*b*x**(3/2)*sqrt(1 + b*
x/a)/12 + sqrt(a)*b**2*x**(5/2)*sqrt(1 + b*x/a)/3 + 5*a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(68) = 136$.

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = -\frac{5\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,\sqrt{b}} - \frac{\frac{15\,\sqrt{bx+a}a^3b^2}{\sqrt{x}} - \frac{40\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}} + \frac{33\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^3 - \frac{3\,(bx+a)b^2}{x} + \frac{3\,(bx+a)^2b}{x^2} - \frac{(bx+a)^3}{x^3}\right)}$$

input
```
integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/sqrt(b) - 1/24*(15*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^
(3/2)*a^3*b/x^(3/2) + 33*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x + a)*b
^2/x + 3*(b*x + a)^2*b/x^2 - (b*x + a)^3/x^3)
```

Giac [A] (verification not implemented)

Time = 75.00 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = -\frac{\left(\frac{15\,a^3\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b}+\frac{5\,a}{b}\right)+\frac{15\,a^2}{b}\right)\right)b}{24\,|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
-1/24*(15*a^3*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/s
qrt(b) - sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b
 + 5*a/b) + 15*a^2/b))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx=\int \frac{{\left(a+b\,x\right)}^{5/2}}{\sqrt{x}}\,dx$$

input
```
int((a + b*x)^(5/2)/x^(1/2),x)
```

output
```
int((a + b*x)^(5/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx=\frac{33\sqrt{x}\,\sqrt{bx+a}\,a^{2}b+26\sqrt{x}\,\sqrt{bx+a}\,a\,b^{2}x+8\sqrt{x}\,\sqrt{bx+a}\,b^{3}x^{2}+15\sqrt{b}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}}{\sqrt{a}}\right.}{24b}$$

input
```
int((b*x+a)^(5/2)/x^(1/2),x)
```

output
```
(33*sqrt(x)*sqrt(a + b*x)*a**2*b + 26*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b)
```

3.449 $\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{2a^2\sqrt{a+bx}}{\sqrt{x}} + \frac{9}{4}ab\sqrt{x}\sqrt{a+bx} + \frac{1}{2}b^2x^{3/2}\sqrt{a+bx} + \frac{15}{4}a^2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2*a^2*(b*x+a)^(1/2)/x^(1/2)+9/4*a*b*x^(1/2)*(b*x+a)^(1/2)+1/2*b^2*x^(3/2)
*(b*x+a)^(1/2)+15/4*a^2*b^(1/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = \frac{\sqrt{a+bx}(-8a^2+9abx+2b^2x^2)}{4\sqrt{x}} + \frac{15}{2}a^2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)$$

input
```
Integrate[(a + b*x)^(5/2)/x^(3/2),x]
```

output
```
(Sqrt[a + b*x]*(-8*a^2 + 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) + (15*a^2*Sqrt[
b]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {57, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$5b\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx-\frac{2(a+bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$5b\left(\frac{3}{4}a\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{65}$$

$$5b\left(\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{219}$$

$$5b\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}$$

input
```
Int[(a + b*x)^(5/2)/x^(3/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/Sqrt[x] + 5*b*((Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a*(Sq
rt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
))/4)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-2b^2x^2-9abx+8a^2)}{4\sqrt{x}}+\frac{15a^2\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{8\sqrt{x}\,\sqrt{bx+a}}$	84

input
```
int((b*x+a)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(-2*b^2*x^2-9*a*b*x+8*a^2)/x^(1/2)+15/8*a^2*b^(1/2)*ln(
(1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = \left[\frac{15\,a^2\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x^2+9\,abx-8\,a^2)\sqrt{bx+a}\sqrt{x}}{8\,x}, -\right.$$

input
```
integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqr
t(-b)*x*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (2*b^2*x^2 + 9*a*b*x - 8*
a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [A] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} + \frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1+\frac{bx}{a}}} + \frac{11\sqrt{a}b^2x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}} + \frac{15a^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4} + \frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(5/2)/x**(3/2),x)
```

output
```
-2*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) + a**(3/2)*b*sqrt(x)/(4*sqrt(1 + b*x
/a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sqrt(1 + b*x/a)) + 15*a**2*sqrt(b)*asin
h(sqrt(b)*sqrt(x)/sqrt(a))/4 + b**3*x**(5/2)/(2*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{15}{8}a^2\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right) - \frac{2\sqrt{bx+a}a^2}{\sqrt{x}} - \frac{\frac{7\sqrt{bx+a}a^2b^2}{\sqrt{x}} - \frac{9(bx+a)^{\frac{3}{2}}a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2(bx+a)b}{x}+\frac{(bx+a)^2}{x^2}\right)}$$

input
```
integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-15/8*a^2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b
*x + a)/sqrt(x))) - 2*sqrt(b*x + a)*a^2/sqrt(x) - 1/4*(7*sqrt(b*x + a)*a^2
*b^2/sqrt(x) - 9*(b*x + a)^(3/2)*a^2*b/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (
b*x + a)^2/x^2)
```

Giac [A] (verification not implemented)

Time = 75.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{\left(\frac{15\,a^2\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \frac{((2\,bx+7\,a)(bx+a)-15\,a^2)\sqrt{bx+a}}{\sqrt{(bx+a)b-ab}}\right)b^2}{4\,|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

output
```
-1/4*(15*a^2*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sq
rt(b) - ((2*b*x + 7*a)*(b*x + a) - 15*a^2)*sqrt(b*x + a)/sqrt((b*x + a)*b
- a*b))*b^2/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = \int \frac{(a+b\,x)^{5/2}}{x^{3/2}}\,dx$$

input
```
int((a + b*x)^(5/2)/x^(3/2),x)
```

output
```
int((a + b*x)^(5/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = \frac{-8\sqrt{x}\,\sqrt{bx+a}\,a^2+9\sqrt{x}\,\sqrt{bx+a}\,abx+2\sqrt{x}\,\sqrt{bx+a}\,b^2x^2+15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{4x}$$

input
```
int((b*x+a)^(5/2)/x^(3/2),x)
```

output
```
( - 8*sqrt(x)*sqrt(a + b*x)*a**2 + 9*sqrt(x)*sqrt(a + b*x)*a*b*x + 2*sqrt(
x)*sqrt(a + b*x)*b**2*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**2*x - 10*sqrt(b)*a**2*x)/(4*x)
```

3.450 $\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = -\frac{2a^2\sqrt{a+bx}}{3x^{3/2}} - \frac{14ab\sqrt{a+bx}}{3\sqrt{x}} + b^2\sqrt{x}\sqrt{a+bx} + 5ab^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2/3*a^2*(b*x+a)^(1/2)/x^(3/2)-14/3*a*b*(b*x+a)^(1/2)/x^(1/2)+b^2*x^(1/2)*
(b*x+a)^(1/2)+5*a*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = \frac{\sqrt{a+bx}(-2a^2-14abx+3b^2x^2)}{3x^{3/2}} + 10ab^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)$$

input
```
Integrate[(a + b*x)^(5/2)/x^(5/2),x]
```

output
```
(Sqrt[a + b*x]*(-2*a^2 - 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 10*a*b^(3/2)
*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {57, 57, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{5}{3}b\int \frac{(a+bx)^{3/2}}{x^{3/2}}dx-\frac{2(a+bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{5}{3}b\left(3b\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{65}$$

$$\frac{5}{3}b\left(3b\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{3}b\left(3b\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}$$

input
```
Int[(a + b*x)^(5/2)/x^(5/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/(3*x^(3/2)) + (5*b*((-2*(a + b*x)^(3/2))/Sqrt[x] + 3*
b*(Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sq
rt[b])))/3
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3b^2x^2+14abx+2a^2)}{3x^{\frac{3}{2}}}+\frac{5a\,b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$	82

input
```
int((b*x+a)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x+a)^(1/2)*(-3*b^2*x^2+14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*ln((1
/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = \left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(3\,b^2x^2-14\,abx-2\,a^2)\sqrt{bx+a}\sqrt{x}}{6\,x^2},\right.$$

input
```
integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sq
rt(-b)*b*x^2*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (3*b^2*x^2 - 14*a*b*
x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = -\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x} - \frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3} - \frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2} + 5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right) + b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}+1}$$

input
```
integrate((b*x+a)**(5/2)/x**(5/2),x)
```

output
```
-2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 14*a*b**(3/2)*sqrt(a/(b*x) + 1)/
3 - 5*a*b**(3/2)*log(a/(b*x))/2 + 5*a*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1)
+ b**(5/2)*x*sqrt(a/(b*x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = -\frac{5}{2}\,ab^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right) - \frac{4\sqrt{bx+a}ab}{\sqrt{x}} - \frac{\sqrt{bx+a}ab^2}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}} - \frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-5/2*a*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
+ a)/sqrt(x))) - 4*sqrt(b*x + a)*a*b/sqrt(x) - sqrt(b*x + a)*a*b^2/((b - (
b*x + a)/x)*sqrt(x)) - 2/3*(b*x + a)^(3/2)*a/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 75.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = -\frac{\left(\frac{15\,a\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}} - \frac{(15\,a^2b+(3\,(bx+a)b-20\,ab)(bx+a))\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{3}{2}}}\right)b^3}{3\,|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/3*(15*a*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt
(b) - (15*a^2*b + (3*(b*x + a)*b - 20*a*b)*(b*x + a))*sqrt(b*x + a)/((b*x
+ a)*b - a*b)^(3/2))*b^3/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = \int \frac{(a+b\,x)^{5/2}}{x^{5/2}}\,dx$$

input
```
int((a + b*x)^(5/2)/x^(5/2),x)
```

output
```
int((a + b*x)^(5/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = \frac{-4\sqrt{x}\,\sqrt{bx+a}\,a^2 - 28\sqrt{x}\,\sqrt{bx+a}\,abx + 6\sqrt{x}\,\sqrt{bx+a}\,b^2x^2 + 30\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right.}{6x^2}$$

input
```
int((b*x+a)^(5/2)/x^(5/2),x)
```

output
```
( - 4*sqrt(x)*sqrt(a + b*x)*a**2 - 28*sqrt(x)*sqrt(a + b*x)*a*b*x + 6*sqrt
(x)*sqrt(a + b*x)*b**2*x**2 + 30*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a*b*x**2 + 5*sqrt(b)*a*b*x**2)/(6*x**2)
```

3.451 $\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = -\frac{2a^2\sqrt{a+bx}}{5x^{5/2}} - \frac{22ab\sqrt{a+bx}}{15x^{3/2}} - \frac{46b^2\sqrt{a+bx}}{15\sqrt{x}} + 2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2/5*a^2*(b*x+a)^(1/2)/x^(5/2)-22/15*a*b*(b*x+a)^(1/2)/x^(3/2)-46/15*b^2*(
b*x+a)^(1/2)/x^(1/2)+2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = -\frac{2\sqrt{a+bx}(3a^2+11abx+23b^2x^2)}{15x^{5/2}} - 2b^{5/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)$$

input
```
Integrate[(a + b*x)^(5/2)/x^(7/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(3*a^2 + 11*a*b*x + 23*b^2*x^2))/(15*x^(5/2)) - 2*b^(5/2
)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {57, 57, 57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$b\int \frac{(a+bx)^{3/2}}{x^{5/2}}dx-\frac{2(a+bx)^{5/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 57}$$

$$b\left(b\int \frac{\sqrt{a+bx}}{x^{3/2}}dx-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 57}$$

$$b\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 65}$$

$$b\left(b\left(2b\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 219}$$

$$b\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}$$

input
```
Int[(a + b*x)^(5/2)/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(5/2))/(5*x^(5/2)) + b*((-2*(a + b*x)^(3/2))/(3*x^(3/2)) + b
*((-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
+ b*x]]))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{2\sqrt{bx+a}\,(23b^2x^2+11abx+3a^2)}{15x^{\frac{5}{2}}}+\frac{b^{\frac{5}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	80

input
```
int((b*x+a)^(5/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(1/2)*(23*b^2*x^2+11*a*b*x+3*a^2)/x^(5/2)+b^(5/2)*ln((1/2*a+
b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = \left[\frac{15\,b^{\frac{5}{2}}x^3\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(23\,b^2x^2+11\,abx+3\,a^2)\sqrt{bx+a}\sqrt{x}}{15\,x^3},\right.$$

input
```
integrate((b*x+a)^(5/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/15*(15*b^(5/2)*x^3*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2
*(23*b^2*x^2 + 11*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3, -2/15*(15*sqr
t(-b)*b^2*x^3*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (23*b^2*x^2 + 11*a*
b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3]
```

Sympy [A] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = -\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2}-\frac{22ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15x} -\frac{46b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15}-b^{\frac{5}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{5}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)$$

input
```
integrate((b*x+a)**(5/2)/x**(7/2),x)
```

output
```
-2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 22*a*b**(3/2)*sqrt(a/(b*x) +
1)/(15*x) - 46*b**(5/2)*sqrt(a/(b*x) + 1)/15 - b**(5/2)*log(a/(b*x)) + 2*b
**(5/2)*log(sqrt(a/(b*x) + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = -b^{\frac{5}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+a}b^2}{\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}b}{3\,x^{\frac{3}{2}}}-\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^(5/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-b^(5/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x))) - 2*sqrt(b*x + a)*b^2/sqrt(x) - 2/3*(b*x + a)^(3/2)*b/x^(3/2) - 2
/5*(b*x + a)^(5/2)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 76.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = -\frac{2\left(15\,b^{\frac{5}{2}}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\frac{\left(15\,a^2b^5+\left(23\,(bx+a)b^5-35\,ab^5\right)(bx+a)\right)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{5}{2}}}\right)b}{15\,|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(7/2),x, algorithm="giac")
```

output
```
-2/15*(15*b^(5/2)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)
)) + (15*a^2*b^5 + (23*(b*x + a)*b^5 - 35*a*b^5)*(b*x + a))*sqrt(b*x + a)/
((b*x + a)*b - a*b)^(5/2))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = \int \frac{(a+b\,x)^{5/2}}{x^{7/2}}\,dx$$

input
```
int((a + b*x)^(5/2)/x^(7/2),x)
```

output
```
int((a + b*x)^(5/2)/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{5/2}}{x^{7/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{22\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{46\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)b^2x^3+\frac{2\sqrt{b}\,b^2x^3}{3}}{x^3}$$

input
```
int((b*x+a)^(5/2)/x^(7/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 - 11*sqrt(x)*sqrt(a + b*x)*a*b*x - 23*
sqrt(x)*sqrt(a + b*x)*b**2*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*
sqrt(b))/sqrt(a))*b**2*x**3 + 5*sqrt(b)*b**2*x**3))/(15*x**3)
```

3.452 $\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = -\frac{2(a+bx)^{7/2}}{7ax^{7/2}}$$

output
```
-2/7*(b*x+a)^(7/2)/a/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = -\frac{2(a+bx)^{7/2}}{7ax^{7/2}}$$

input
```
Integrate[(a + b*x)^(5/2)/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(7/2))/(7*a*x^(7/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{2(a+bx)^{7/2}}{7ax^{7/2}}$$

input
```
Int[(a + b*x)^(5/2)/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(7/2))/(7*a*x^(7/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}}{7a\,x^{\frac{7}{2}}}$	16
orering	$-\frac{2(bx+a)^{\frac{7}{2}}}{7a\,x^{\frac{7}{2}}}$	16
risch	$-\frac{2\sqrt{bx+a}\,(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3})}{7x^{\frac{7}{2}}a}$	43
default	$-\frac{(bx+a)^{\frac{5}{2}}}{x^{\frac{7}{2}}}-\frac{5a\left(-\frac{(bx+a)^{\frac{3}{2}}}{2x^{\frac{7}{2}}}-\frac{3a\left(-\frac{\sqrt{bx+a}}{3x^{\frac{7}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}\right)}{4}\right)}{2}$	125

input
```
int((b*x+a)^(5/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x+a)^(7/2)/a/x^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = -\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}}{7\,ax^{\frac{7}{2}}}$$

input
```
integrate((b*x+a)^(5/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/(a*x^(7/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.

Time = 5.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.29

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = -\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{7x^3} - \frac{6ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{7x^2} - \frac{6b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{7x} - \frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{7a}$$

input
```
integrate((b*x+a)**(5/2)/x**(9/2),x)
```

output
```
-2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(7*x**3) - 6*a*b**(3/2)*sqrt(a/(b*x) + 1
)/(7*x**2) - 6*b**(5/2)*sqrt(a/(b*x) + 1)/(7*x) - 2*b**(7/2)*sqrt(a/(b*x)
+ 1)/(7*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = -\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,ax^{\frac{7}{2}}}$$

input
```
integrate((b*x+a)^(5/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/7*(b*x + a)^(7/2)/(a*x^(7/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = -\frac{2\,(bx+a)^{\frac{7}{2}}b^8}{7\,((bx+a)b-ab)^{\frac{7}{2}}a|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(9/2),x, algorithm="giac")
```

output
```
-2/7*(b*x + a)^(7/2)*b^8/(((b*x + a)*b - a*b)^(7/2)*a*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,a^2}{7}+\frac{6\,b^2\,x^2}{7}+\frac{6\,a\,b\,x}{7}+\frac{2\,b^3\,x^3}{7\,a}\right)}{x^{7/2}}$$

input
```
int((a + b*x)^(5/2)/x^(9/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a^2)/7 + (6*b^2*x^2)/7 + (6*a*b*x)/7 + (2*b^3*x^3)/(
7*a)))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.81

$$\int \frac{(a+bx)^{5/2}}{x^{9/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{7}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{7}-\frac{2\sqrt{b}\,b^3x^4}{7}}{a\,x^4}$$

input
```
int((b*x+a)^(5/2)/x^(9/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 3*s
qrt(x)*sqrt(a + b*x)*a*b**2*x**2 - sqrt(x)*sqrt(a + b*x)*b**3*x**3 - sqrt(
b)*b**3*x**4))/(7*a*x**4)
```

3.453 $\int \frac{(a+bx)^{5/2}}{x^{11/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}} dx = -\frac{2(a+bx)^{7/2}}{9ax^{9/2}} + \frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}$$

output
```
-2/9*(b*x+a)^(7/2)/a/x^(9/2)+4/63*b*(b*x+a)^(7/2)/a^2/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}} dx = -\frac{2(7a-2bx)(a+bx)^{7/2}}{63a^2x^{9/2}}$$

input
```
Integrate[(a + b*x)^(5/2)/x^(11/2),x]
```

output
```
(-2*(7*a - 2*b*x)*(a + b*x)^(7/2))/(63*a^2*x^(9/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}}\,dx$$

$$\Big\downarrow \ 55$$

$$-\frac{2b\int \frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}$$

$$\Big\downarrow \ 48$$

$$\frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}$$

input
```
Int[(a + b*x)^(5/2)/x^(11/2),x]
```

output
```
(-2*(a + b*x)^(7/2))/(9*a*x^(9/2)) + (4*b*(a + b*x)^(7/2))/(63*a^2*x^(7/2)
)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}(-2bx+7a)}{63x^{\frac{9}{2}}a^{2}}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}(-2bx+7a)}{63x^{\frac{9}{2}}a^{2}}$
risch	$-\frac{2\sqrt{bx+a}\,(-2b^{4}x^{4}+a\,x^{3}b^{3}+15a^{2}b^{2}x^{2}+19a^{3}bx+7a^{4})}{63x^{\frac{9}{2}}a^{2}}$
	$5a\left(-\frac{(bx+a)^{\frac{3}{2}}}{3x^{\frac{9}{2}}}-\frac{a\left(-\frac{\sqrt{bx+a}}{4x^{\frac{9}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{9a\,x^{\frac{9}{2}}}-\frac{8b\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{8}\right)}{2}\right.$

input
```
int((b*x+a)^(5/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*(b*x+a)^(7/2)*(-2*b*x+7*a)/x^(9/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}}\,dx = \frac{2\,(2\,b^4x^4 - ab^3x^3 - 15\,a^2b^2x^2 - 19\,a^3bx - 7\,a^4)\sqrt{bx+a}}{63\,a^2x^{\frac{9}{2}}}$$

input
```
integrate((b*x+a)^(5/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/63*(2*b^4*x^4 - a*b^3*x^3 - 15*a^2*b^2*x^2 - 19*a^3*b*x - 7*a^4)*sqrt(b*
x + a)/(a^2*x^(9/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(39) = 78.

Time = 16.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.59

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}}\,dx = -\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{9x^4} - \frac{38ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{63x^3} - \frac{10b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{21x^2} - \frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{63ax} + \frac{4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{63a^2}$$

input
```
integrate((b*x+a)**(5/2)/x**(11/2),x)
```

output
```
-2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(9*x**4) - 38*a*b**(3/2)*sqrt(a/(b*x) +
1)/(63*x**3) - 10*b**(5/2)*sqrt(a/(b*x) + 1)/(21*x**2) - 2*b**(7/2)*sqrt(a
/(b*x) + 1)/(63*a*x) + 4*b**(9/2)*sqrt(a/(b*x) + 1)/(63*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}}\,dx = \frac{2\left(\frac{9\,(bx+a)^{\frac{7}{2}}b}{x^{\frac{7}{2}}} - \frac{7\,(bx+a)^{\frac{9}{2}}}{x^{\frac{9}{2}}}\right)}{63\,a^2}$$

input
```
integrate((b*x+a)^(5/2)/x^(11/2),x, algorithm="maxima")
```

output
```
2/63*(9*(b*x + a)^(7/2)*b/x^(7/2) - 7*(b*x + a)^(9/2)/x^(9/2))/a^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}}\,dx = \frac{2\left(\frac{2\,(bx+a)b^9}{a^2} - \frac{9\,b^9}{a}\right)(bx+a)^{\frac{7}{2}}b}{63\,((bx+a)b-ab)^{\frac{9}{2}}|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(11/2),x, algorithm="giac")
```

output
```
2/63*(2*(b*x + a)*b^9/a^2 - 9*b^9/a)*(b*x + a)^(7/2)*b/(((b*x + a)*b - a*b
)^(9/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,a^2}{9} + \frac{10\,b^2\,x^2}{21} + \frac{38\,a\,b\,x}{63} + \frac{2\,b^3\,x^3}{63\,a} - \frac{4\,b^4\,x^4}{63\,a^2}\right)}{x^{9/2}}$$

input
```
int((a + b*x)^(5/2)/x^(11/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a^2)/9 + (10*b^2*x^2)/21 + (38*a*b*x)/63 + (2*b^3*x^
3)/(63*a) - (4*b^4*x^4)/(63*a^2)))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{(a+bx)^{5/2}}{x^{11/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9}-\frac{38\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63}-\frac{10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{21}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{63}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^4x^4}{63}-4}{a^2x^5}$$

input
```
int((b*x+a)^(5/2)/x^(11/2),x)
```

output
```
(2*( - 7*sqrt(x)*sqrt(a + b*x)*a**4 - 19*sqrt(x)*sqrt(a + b*x)*a**3*b*x -
15*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - sqrt(x)*sqrt(a + b*x)*a*b**3*x**
3 + 2*sqrt(x)*sqrt(a + b*x)*b**4*x**4 - 2*sqrt(b)*b**4*x**5))/(63*a**2*x**
5)
```

3.454 $\int \frac{(a+bx)^{5/2}}{x^{13/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}} dx = -\frac{2(a+bx)^{7/2}}{11ax^{11/2}} + \frac{8b(a+bx)^{7/2}}{99a^2x^{9/2}} - \frac{16b^2(a+bx)^{7/2}}{693a^3x^{7/2}}$$

output
```
-2/11*(b*x+a)^(7/2)/a/x^(11/2)+8/99*b*(b*x+a)^(7/2)/a^2/x^(9/2)-16/693*b^2
*(b*x+a)^(7/2)/a^3/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}} dx = -\frac{2(a+bx)^{7/2}(63a^2-28abx+8b^2x^2)}{693a^3x^{11/2}}$$

input
```
Integrate[(a + b*x)^(5/2)/x^(13/2),x]
```

output
```
(-2*(a + b*x)^(7/2)*(63*a^2 - 28*a*b*x + 8*b^2*x^2))/(693*a^3*x^(11/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\int \frac{(a+bx)^{5/2}}{x^{11/2}}dx}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{4b\left(\frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}$$

input
```
Int[(a + b*x)^(5/2)/x^(13/2),x]
```

output
```
(-2*(a + b*x)^(7/2))/(11*a*x^(11/2)) - (4*b*((-2*(a + b*x)^(7/2))/(9*a*x^(
9/2)) + (4*b*(a + b*x)^(7/2))/(63*a^2*x^(7/2))))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(8b^2x^2-28abx+63a^2\right)}{693x^{\frac{11}{2}}a^3}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(8b^2x^2-28abx+63a^2\right)}{693x^{\frac{11}{2}}a^3}$
risch	$-\frac{2\sqrt{bx+a}\left(8b^5x^5-4a\,b^4x^4+3a^2b^3x^3+113a^3b^2x^2+161a^4bx+63a^5\right)}{693x^{\frac{11}{2}}a^3}$
	$\left(3a\left(-\frac{\sqrt{bx+a}}{5x^{\frac{11}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{11a\,x^{\frac{11}{2}}}-\frac{10b\left(-\frac{2\sqrt{bx+a}}{9a\,x^{\frac{9}{2}}}-\frac{8b\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx\cdots}}{3a^2\sqrt{\cdots}}\cdots\right)}{5a}\cdots\right)}{7a}\cdots\right)}{9a}\cdots\right)}{11a}\cdots\right)}{10}\cdots\right)\cdots\right.$

input
```
int((b*x+a)^(5/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693*(b*x+a)^(7/2)*(8*b^2*x^2-28*a*b*x+63*a^2)/x^(11/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}}\,dx = -\frac{2\,(8\,b^5x^5-4\,ab^4x^4+3\,a^2b^3x^3+113\,a^3b^2x^2+161\,a^4bx+63\,a^5)\sqrt{bx+a}}{693\,a^3x^{\frac{11}{2}}}$$

input
```
integrate((b*x+a)^(5/2)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/693*(8*b^5*x^5 - 4*a*b^4*x^4 + 3*a^2*b^3*x^3 + 113*a^3*b^2*x^2 + 161*a^
4*b*x + 63*a^5)*sqrt(b*x + a)/(a^3*x^(11/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(63) = 126$.

Time = 44.57 (sec) , antiderivative size = 464, normalized size of antiderivative = 6.82

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}}\,dx = -\frac{126a^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{x\left(693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6\right)} - \frac{574a^6b^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}{693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6} - \frac{996a^5b^{\frac{13}{2}}x\sqrt{\frac{a}{bx}+1}}{693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6} - \frac{780a^4b^{\frac{15}{2}}x^2\sqrt{\frac{a}{bx}+1}}{693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6} - \frac{230a^3b^{\frac{17}{2}}x^3\sqrt{\frac{a}{bx}+1}}{693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6} - \frac{6a^2b^{\frac{19}{2}}x^4\sqrt{\frac{a}{bx}+1}}{693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6} - \frac{24ab^{\frac{21}{2}}x^5\sqrt{\frac{a}{bx}+1}}{693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6} - \frac{16b^{\frac{23}{2}}x^6\sqrt{\frac{a}{bx}+1}}{693a^5b^4x^4+1386a^4b^5x^5+693a^3b^6x^6}$$

input
```
integrate((b*x+a)**(5/2)/x**(13/2),x)
```

output
```
-126*a**7*b**(9/2)*sqrt(a/(b*x) + 1)/(x*(693*a**5*b**4*x**4 + 1386*a**4*b*
*5*x**5 + 693*a**3*b**6*x**6)) - 574*a**6*b**(11/2)*sqrt(a/(b*x) + 1)/(693
*a**5*b**4*x**4 + 1386*a**4*b**5*x**5 + 693*a**3*b**6*x**6) - 996*a**5*b**
(13/2)*x*sqrt(a/(b*x) + 1)/(693*a**5*b**4*x**4 + 1386*a**4*b**5*x**5 + 693
*a**3*b**6*x**6) - 780*a**4*b**(15/2)*x**2*sqrt(a/(b*x) + 1)/(693*a**5*b**
4*x**4 + 1386*a**4*b**5*x**5 + 693*a**3*b**6*x**6) - 230*a**3*b**(17/2)*x*
*3*sqrt(a/(b*x) + 1)/(693*a**5*b**4*x**4 + 1386*a**4*b**5*x**5 + 693*a**3*
b**6*x**6) - 6*a**2*b**(19/2)*x**4*sqrt(a/(b*x) + 1)/(693*a**5*b**4*x**4 +
 1386*a**4*b**5*x**5 + 693*a**3*b**6*x**6) - 24*a*b**(21/2)*x**5*sqrt(a/(b
*x) + 1)/(693*a**5*b**4*x**4 + 1386*a**4*b**5*x**5 + 693*a**3*b**6*x**6) -
 16*b**(23/2)*x**6*sqrt(a/(b*x) + 1)/(693*a**5*b**4*x**4 + 1386*a**4*b**5*
x**5 + 693*a**3*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}}\,dx = -\frac{2\left(\frac{99\,(bx+a)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}} - \frac{154\,(bx+a)^{\frac{9}{2}}b}{x^{\frac{9}{2}}} + \frac{63\,(bx+a)^{\frac{11}{2}}}{x^{\frac{11}{2}}}\right)}{693\,a^3}$$

input
```
integrate((b*x+a)^(5/2)/x^(13/2),x, algorithm="maxima")
```

output
```
-2/693*(99*(b*x + a)^(7/2)*b^2/x^(7/2) - 154*(b*x + a)^(9/2)*b/x^(9/2) + 6
3*(b*x + a)^(11/2)/x^(11/2))/a^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}}\,dx = -\frac{2\left(\frac{99\,b^5}{a} + 4\left(\frac{2\,(bx+a)b^5}{a^3} - \frac{11\,b^5}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b^7}{693\,((bx+a)b-ab)^{\frac{11}{2}}\,|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(13/2),x, algorithm="giac")
```

output
```
-2/693*(99*b^5/a + 4*(2*(b*x + a)*b^5/a^3 - 11*b^5/a^2)*(b*x + a))*(b*x +
a)^(7/2)*b^7/(((b*x + a)*b - a*b)^(11/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,a^2}{11} + \frac{226\,b^2\,x^2}{693} + \frac{46\,a\,b\,x}{99} + \frac{2\,b^3\,x^3}{231\,a} - \frac{8\,b^4\,x^4}{693\,a^2} + \frac{16\,b^5\,x^5}{693\,a^3}\right)}{x^{11/2}}$$

input
```
int((a + b*x)^(5/2)/x^(13/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a^2)/11 + (226*b^2*x^2)/693 + (46*a*b*x)/99 + (2*b^3
*x^3)/(231*a) - (8*b^4*x^4)/(693*a^2) + (16*b^5*x^5)/(693*a^3)))/x^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)^{5/2}}{x^{13/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5}{11} - \frac{46\sqrt{x}\,\sqrt{bx+a}\,a^4bx}{99} - \frac{226\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2}{693} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^3}{231} + \frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^4}{693} -}{a^3x^6}$$

input
```
int((b*x+a)^(5/2)/x^(13/2),x)
```

output
```
(2*( - 63*sqrt(x)*sqrt(a + b*x)*a**5 - 161*sqrt(x)*sqrt(a + b*x)*a**4*b*x
- 113*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*
b**3*x**3 + 4*sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 - 8*sqrt(x)*sqrt(a + b*x)*
b**5*x**5 + 8*sqrt(b)*b**5*x**6))/(693*a**3*x**6)
```

3.455 $\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = -\frac{2(a+bx)^{7/2}}{13ax^{13/2}}+\frac{12b(a+bx)^{7/2}}{143a^2x^{11/2}}-\frac{16b^2(a+bx)^{7/2}}{429a^3x^{9/2}}+\frac{32b^3(a+bx)^{7/2}}{3003a^4x^{7/2}}$$

output
```
-2/13*(b*x+a)^(7/2)/a/x^(13/2)+12/143*b*(b*x+a)^(7/2)/a^2/x^(11/2)-16/429*
b^2*(b*x+a)^(7/2)/a^3/x^(9/2)+32/3003*b^3*(b*x+a)^(7/2)/a^4/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = -\frac{2(a+bx)^{7/2}\left(231a^3-126a^2bx+56ab^2x^2-16b^3x^3\right)}{3003a^4x^{13/2}}$$

input
```
Integrate[(a + b*x)^(5/2)/x^(15/2),x]
```

output
```
(-2*(a + b*x)^(7/2)*(231*a^3 - 126*a^2*b*x + 56*a*b^2*x^2 - 16*b^3*x^3))/(
3003*a^4*x^(13/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}dx$$

$$\Big\downarrow \; 55$$

$$-\frac{6b\int \frac{(a+bx)^{5/2}}{x^{13/2}}dx}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{6b\left(-\frac{4b\int \frac{(a+bx)^{5/2}}{x^{11/2}}dx}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}$$

input
```
Int[(a + b*x)^(5/2)/x^(15/2),x]
```

output
```
(-2*(a + b*x)^(7/2))/(13*a*x^(13/2)) - (6*b*((-2*(a + b*x)^(7/2))/(11*a*x^
(11/2)) - (4*b*((-2*(a + b*x)^(7/2))/(9*a*x^(9/2)) + (4*b*(a + b*x)^(7/2))
/(63*a^2*x^(7/2))))/(11*a)))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-16b^{3}x^{3}+56a\,b^{2}x^{2}-126a^{2}bx+231a^{3}\right)}{3003x^{\frac{13}{2}}a^{4}}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-16b^{3}x^{3}+56a\,b^{2}x^{2}-126a^{2}bx+231a^{3}\right)}{3003x^{\frac{13}{2}}a^{4}}$
risch	$-\frac{2\sqrt{bx+a}\left(-16b^{6}x^{6}+8a\,x^{5}b^{5}-6a^{2}x^{4}b^{4}+5a^{3}x^{3}b^{3}+371a^{4}x^{2}b^{2}+567a^{5}xb+231a^{6}\right)}{3003x^{\frac{13}{2}}a^{4}}$
	$\Bigg(\Bigg(\Bigg(a\left(-\frac{2\sqrt{bx+a}}{13a\,x^{\frac{13}{2}}}-\frac{12b\left(-\frac{2\sqrt{bx+a}}{11a\,x^{\frac{11}{2}}}-\frac{10b\left(-\frac{2\sqrt{bx+a}}{9a\,x^{\frac{9}{2}}}-\frac{8b\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\cdots\right.}{\cdots}\right.}{\cdots}\right)}{9a}\right)}{11a}\right)}{13a}\right.$

input
```
int((b*x+a)^(5/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3003*(b*x+a)^(7/2)*(-16*b^3*x^3+56*a*b^2*x^2-126*a^2*b*x+231*a^3)/x^(13
/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = \frac{2\,(16\,b^6x^6-8\,ab^5x^5+6\,a^2b^4x^4-5\,a^3b^3x^3-371\,a^4b^2x^2-567\,a^5bx-231\,a^6)\sqrt{bx+a}}{3003\,a^4x^{\frac{13}{2}}}$$

input
```
integrate((b*x+a)^(5/2)/x^(15/2),x, algorithm="fricas")
```

output
```
2/3003*(16*b^6*x^6 - 8*a*b^5*x^5 + 6*a^2*b^4*x^4 - 5*a^3*b^3*x^3 - 371*a^4
*b^2*x^2 - 567*a^5*b*x - 231*a^6)*sqrt(b*x + a)/(a^4*x^(13/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. 2(87) = 174.

Time = 134.54 (sec) , antiderivative size = 702, normalized size of antiderivative = 7.63

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = -\frac{462a^9b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} - \frac{2520a^8b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} - \frac{5530a^7b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} - \frac{6100a^6b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} - \frac{3378a^5b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} - \frac{752a^4b^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} + \frac{10a^3b^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} + \frac{60a^2b^{\frac{33}{2}}x^7\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} + \frac{80ab^{\frac{35}{2}}x^8\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9} + \frac{32b^{\frac{37}{2}}x^9\sqrt{\frac{a}{bx}+1}}{3003a^7b^9x^6+9009a^6b^{10}x^7+9009a^5b^{11}x^8+3003a^4b^{12}x^9}$$

input
```
integrate((b*x+a)**(5/2)/x**(15/2),x)
```

output
```
-462*a**9*b**(19/2)*sqrt(a/(b*x) + 1)/(3003*a**7*b**9*x**6 + 9009*a**6*b**
10*x**7 + 9009*a**5*b**11*x**8 + 3003*a**4*b**12*x**9) - 2520*a**8*b**(21/
2)*x*sqrt(a/(b*x) + 1)/(3003*a**7*b**9*x**6 + 9009*a**6*b**10*x**7 + 9009*
a**5*b**11*x**8 + 3003*a**4*b**12*x**9) - 5530*a**7*b**(23/2)*x**2*sqrt(a/
(b*x) + 1)/(3003*a**7*b**9*x**6 + 9009*a**6*b**10*x**7 + 9009*a**5*b**11*x
**8 + 3003*a**4*b**12*x**9) - 6100*a**6*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(
3003*a**7*b**9*x**6 + 9009*a**6*b**10*x**7 + 9009*a**5*b**11*x**8 + 3003*a
**4*b**12*x**9) - 3378*a**5*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3003*a**7*b*
*9*x**6 + 9009*a**6*b**10*x**7 + 9009*a**5*b**11*x**8 + 3003*a**4*b**12*x*
*9) - 752*a**4*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(3003*a**7*b**9*x**6 + 900
9*a**6*b**10*x**7 + 9009*a**5*b**11*x**8 + 3003*a**4*b**12*x**9) + 10*a**3
*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(3003*a**7*b**9*x**6 + 9009*a**6*b**10*x
**7 + 9009*a**5*b**11*x**8 + 3003*a**4*b**12*x**9) + 60*a**2*b**(33/2)*x**
7*sqrt(a/(b*x) + 1)/(3003*a**7*b**9*x**6 + 9009*a**6*b**10*x**7 + 9009*a**
5*b**11*x**8 + 3003*a**4*b**12*x**9) + 80*a*b**(35/2)*x**8*sqrt(a/(b*x) +
1)/(3003*a**7*b**9*x**6 + 9009*a**6*b**10*x**7 + 9009*a**5*b**11*x**8 + 30
03*a**4*b**12*x**9) + 32*b**(37/2)*x**9*sqrt(a/(b*x) + 1)/(3003*a**7*b**9*
x**6 + 9009*a**6*b**10*x**7 + 9009*a**5*b**11*x**8 + 3003*a**4*b**12*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = \frac{2\left(\frac{429\,(bx+a)^{\frac{7}{2}}b^3}{x^{\frac{7}{2}}} - \frac{1001\,(bx+a)^{\frac{9}{2}}b^2}{x^{\frac{9}{2}}} + \frac{819\,(bx+a)^{\frac{11}{2}}b}{x^{\frac{11}{2}}} - \frac{231\,(bx+a)^{\frac{13}{2}}}{x^{\frac{13}{2}}}\right)}{3003\,a^4}$$

input
```
integrate((b*x+a)^(5/2)/x^(15/2),x, algorithm="maxima")
```

output
```
2/3003*(429*(b*x + a)^(7/2)*b^3/x^(7/2) - 1001*(b*x + a)^(9/2)*b^2/x^(9/2)
 + 819*(b*x + a)^(11/2)*b/x^(11/2) - 231*(b*x + a)^(13/2)/x^(13/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = -\frac{2\left(\frac{429\,b^{13}}{a}-2\left(\frac{143\,b^{13}}{a^2}+4\left(\frac{2\,(bx+a)b^{13}}{a^4}-\frac{13\,b^{13}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{3003\left((bx+a)b-ab\right)^{\frac{13}{2}}|b|}$$

input
```
integrate((b*x+a)^(5/2)/x^(15/2),x, algorithm="giac")
```

output
```
-2/3003*(429*b^13/a - 2*(143*b^13/a^2 + 4*(2*(b*x + a)*b^13/a^4 - 13*b^13/
a^3)*(b*x + a))*(b*x + a))*(b*x + a)^(7/2)*b/(((b*x + a)*b - a*b)^(13/2)*a
bs(b))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,a^2}{13}+\frac{106\,b^2\,x^2}{429}+\frac{54\,a\,b\,x}{143}+\frac{10\,b^3\,x^3}{3003\,a}-\frac{4\,b^4\,x^4}{1001\,a^2}+\frac{16\,b^5\,x^5}{3003\,a^3}-\frac{32\,b^6\,x^6}{3003\,a^4}\right)}{x^{13/2}}$$

input
```
int((a + b*x)^(5/2)/x^(15/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*a^2)/13 + (106*b^2*x^2)/429 + (54*a*b*x)/143 + (10*b
^3*x^3)/(3003*a) - (4*b^4*x^4)/(1001*a^2) + (16*b^5*x^5)/(3003*a^3) - (32*
b^6*x^6)/(3003*a^4)))/x^(13/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^{5/2}}{x^{15/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6}{13} - \frac{54\sqrt{x}\,\sqrt{bx+a}\,a^5bx}{143} - \frac{106\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x^2}{429} - \frac{10\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^3}{3003} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^4x^4}{1001}}{a^4x^7}$$

input
```
int((b*x+a)^(5/2)/x^(15/2),x)
```

output
```
(2*( - 231*sqrt(x)*sqrt(a + b*x)*a**6 - 567*sqrt(x)*sqrt(a + b*x)*a**5*b*x
 - 371*sqrt(x)*sqrt(a + b*x)*a**4*b**2*x**2 - 5*sqrt(x)*sqrt(a + b*x)*a**3
*b**3*x**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**4*x**4 - 8*sqrt(x)*sqrt(a + b
*x)*a*b**5*x**5 + 16*sqrt(x)*sqrt(a + b*x)*b**6*x**6 - 16*sqrt(b)*b**6*x**
7))/(3003*a**4*x**7)
```

3.456 $\int x^{5/2}\sqrt{2+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int x^{5/2}\sqrt{2+bx}\,dx=\frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

output
```
5/8*x^(1/2)*(b*x+2)^(1/2)/b^3-5/24*x^(3/2)*(b*x+2)^(1/2)/b^2+1/12*x^(5/2)*
(b*x+2)^(1/2)/b+1/4*x^(7/2)*(b*x+2)^(1/2)-5/4*arcsinh(1/2*b^(1/2)*x^(1/2)*
2^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int x^{5/2}\sqrt{2+bx}\,dx=\frac{\sqrt{x}\sqrt{2+bx}(15-5bx+2b^2x^2+6b^3x^3)}{24b^3}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{2b^{7/2}}$$

input
```
Integrate[x^(5/2)*Sqrt[2 + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5
*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt{bx+2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\int \frac{x^{5/2}}{\sqrt{bx+2}}dx+\frac{1}{4}x^{7/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\int \frac{x^{3/2}}{\sqrt{bx+2}}dx}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{63}$$

$$\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int\frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}$$

input
```
Int[x^(5/2)*Sqrt[2 + b*x],x]
```

output
```
(x^(7/2)*Sqrt[2 + b*x])/4 + ((x^(5/2)*Sqrt[2 + b*x])/(3*b) - (5*((x^(3/2)*
Sqrt[2 + b*x])/(2*b) - (3*((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]
*Sqrt[x])/Sqrt[2]])/b^(3/2)))/(2*b)))/(3*b))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66

method	result	size
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(42b^3x^3+14b^2x^2-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{1344}+\frac{5\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{32}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	71
risch	$\frac{\left(6b^3x^3+2b^2x^2-5bx+15\right)\sqrt{x}\,\sqrt{bx+2}}{24b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)\sqrt{x(bx+2)}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$	85
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{3}{2}}}{4b}-\frac{5\left(\frac{x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}\right)}{4b}$	121

input
```
int(x^(5/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-8/b^(7/2)/Pi^(1/2)*(-1/1344*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(42*b^3*x^3+
14*b^2*x^2-35*b*x+105)*(1/2*b*x+1)^(1/2)+5/32*Pi^(1/2)*arcsinh(1/2*b^(1/2)
*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int x^{5/2}\sqrt{2+bx}\,dx = \left[\frac{\left(6\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{24\,b^4},\right.$$

input
```
integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*((6*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15
*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/24*((6*b^4*x
^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arcta
n(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))/b^4]
```

Sympy [A] (verification not implemented)

Time = 17.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int x^{5/2}\sqrt{2+bx}\,dx = \frac{bx^{\frac{9}{2}}}{4\sqrt{bx+2}} + \frac{7x^{\frac{7}{2}}}{12\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{24b\sqrt{bx+2}} + \frac{5x^{\frac{3}{2}}}{24b^2\sqrt{bx+2}} + \frac{5\sqrt{x}}{4b^3\sqrt{bx+2}} - \frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

input
```
integrate(x**(5/2)*(b*x+2)**(1/2),x)
```

output
```
b*x**(9/2)/(4*sqrt(b*x + 2)) + 7*x**(7/2)/(12*sqrt(b*x + 2)) - x**(5/2)/(2
4*b*sqrt(b*x + 2)) + 5*x**(3/2)/(24*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(4*b**
3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(75) = 150.

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.51

$$\int x^{5/2}\sqrt{2+bx}\,dx = \frac{\frac{15\sqrt{bx+2}b^3}{\sqrt{x}} + \frac{73(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}} - \frac{55(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}} + \frac{15(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7 - \frac{4(bx+2)b^6}{x} + \frac{6(bx+2)^2b^5}{x^2} - \frac{4(bx+2)^3b^4}{x^3} + \frac{(bx+2)^4b^3}{x^4}\right)} + \frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) + 73*(b*x + 2)^(3/2)*b^2/x^(3/2) - 55*(
b*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x + 2)*
b^6/x + 6*(b*x + 2)^2*b^5/x^2 - 4*(b*x + 2)^3*b^4/x^3 + (b*x + 2)^4*b^3/x^
4) + 5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/s
qrt(x)))/b^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(75) = 150$.

Time = 11.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.70

$$\int x^{5/2}\sqrt{2+bx}\,dx = \frac{\frac{\left(\left((bx+2)\left(2\,(bx+2)\left(\frac{3\,(bx+2)}{b^3}-\frac{25}{b^3}\right)+\frac{163}{b^3}\right)-\frac{279}{b^3}\right)\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}-\frac{210\,\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)}{b^{\frac{5}{2}}}\right)|b|}{b}+}{24\,b}$$

input
```
integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/24*((((b*x + 2)*(2*(b*x + 2)*(3*(b*x + 2)/b^3 - 25/b^3) + 163/b^3) - 279
/b^3)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) - 210*log(abs(-sqrt(b*x + 2)*s
qrt(b) + sqrt((b*x + 2)*b - 2*b)))/b^(5/2))*abs(b)/b + 8*(((2*b*x - 9)*(b*
x + 2) + 33)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) + 30*sqrt(b)*log(abs(-s
qrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^4)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\sqrt{2+bx}\,dx = \int x^{5/2}\,\sqrt{b\,x+2}\,dx$$

input
```
int(x^(5/2)*(b*x + 2)^(1/2),x)
```

output
```
int(x^(5/2)*(b*x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int x^{5/2}\sqrt{2+bx}\,dx = \frac{6\sqrt{x}\,\sqrt{bx+2}\,b^4x^3+2\sqrt{x}\,\sqrt{bx+2}\,b^3x^2-5\sqrt{x}\,\sqrt{bx+2}\,b^2x+15\sqrt{x}\,\sqrt{bx+2}\,b-30\sqrt{}}{24b^4}$$

input
```
int(x^(5/2)*(b*x+2)^(1/2),x)
```

output
```
(6*sqrt(x)*sqrt(b*x + 2)*b**4*x**3 + 2*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 - 5
*sqrt(x)*sqrt(b*x + 2)*b**2*x + 15*sqrt(x)*sqrt(b*x + 2)*b - 30*sqrt(b)*lo
g((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/(24*b**4)
```

3.457 $\int x^{3/2}\sqrt{2+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int x^{3/2}\sqrt{2+bx}\,dx=-\frac{\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

output
```
-1/2*x^(1/2)*(b*x+2)^(1/2)/b^2+1/6*x^(3/2)*(b*x+2)^(1/2)/b+1/3*x^(5/2)*(b*
x+2)^(1/2)+arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int x^{3/2}\sqrt{2+bx}\,dx=\frac{\sqrt{x}\sqrt{2+bx}(-3+bx+2b^2x^2)}{6b^2}-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)*Sqrt[2 + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 2*b^2*x^2))/(6*b^2) - (2*ArcTanh[(Sqrt[
b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{bx+2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\int \frac{x^{3/2}}{\sqrt{bx+2}}dx+\frac{1}{3}x^{5/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{63}$$

$$\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}$$

input
```
Int[x^(3/2)*Sqrt[2 + b*x],x]
```

output
```
(x^(5/2)*Sqrt[2 + b*x])/3 + ((x^(3/2)*Sqrt[2 + b*x])/(2*b) - (3*((Sqrt[x]*
Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)))/(2*b))
/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
meijerg	$-\frac{4\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(-10b^{2}x^{2}-5bx+15\right)\sqrt{\frac{bx}{2}+1}}{120}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	63
risch	$\frac{\left(2b^{2}x^{2}+bx-3\right)\sqrt{x}\,\sqrt{bx+2}}{6b^{2}}+\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	76
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}$	100

input
```
int(x^(3/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/b^(5/2)/Pi^(1/2)*(1/120*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(-10*b^2*x^2-5
*b*x+15)*(1/2*b*x+1)^(1/2)-1/4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int x^{3/2}\sqrt{2+bx}\,dx = \left[\frac{(2\,b^3x^2+b^2x-3\,b)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{6\,b^3}, \frac{(2\,b^3x^2+b^2x\right.$$

input
```
integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*((2*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x
+ sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 + b^2*x - 3*b)*
sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))
/b^3]
```

Sympy [A] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int x^{3/2}\sqrt{2+bx}\,dx = \frac{bx^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{5x^{\frac{5}{2}}}{6\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{6b\sqrt{bx+2}}-\frac{\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x**(3/2)*(b*x+2)**(1/2),x)
```

output
```
b*x**(7/2)/(3*sqrt(b*x + 2)) + 5*x**(5/2)/(6*sqrt(b*x + 2)) - x**(3/2)/(6*
b*sqrt(b*x + 2)) - sqrt(x)/(b**2*sqrt(b*x + 2)) + asinh(sqrt(2)*sqrt(b)*sq
rt(x)/2)/b**(5/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(59) = 118$.

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.60

$$\int x^{3/2}\sqrt{2+bx}\,dx = -\frac{\frac{3\sqrt{bx+2}b^2}{\sqrt{x}} + \frac{8(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} - \frac{3(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5 - \frac{3(bx+2)b^4}{x} + \frac{3(bx+2)^2b^3}{x^2} - \frac{(bx+2)^3b^2}{x^3}\right)} - \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) + 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x +
 2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x + 2)*b^4/x + 3*(b*x + 2)^2*b^3/x^2 - (b*x
 + 2)^3*b^2/x^3) - 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + s
qrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(59) = 118$.

Time = 11.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.76

$$\int x^{3/2}\sqrt{2+bx}\,dx = \frac{\frac{\left(\left((2\,bx-9)(bx+2)+33\right)\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}+30\,\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)\right)|b|}{b^3} + \frac{6\left(\sqrt{(bx+2)b-2\,b}\cdots\right.}{\cdots}}{6\,b}$$

input
```
integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*((((2*b*x - 9)*(b*x + 2) + 33)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) +
 30*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*ab
s(b)/b^3 + 6*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3) - 6*sqrt(b)*
log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^3)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{2+bx}\,dx=\int x^{3/2}\,\sqrt{b\,x+2}\,dx$$

input
```
int(x^(3/2)*(b*x + 2)^(1/2),x)
```

output
```
int(x^(3/2)*(b*x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int x^{3/2}\sqrt{2+bx}\,dx=\frac{2\sqrt{x}\,\sqrt{bx+2}\,b^3x^2+\sqrt{x}\,\sqrt{bx+2}\,b^2x-3\sqrt{x}\,\sqrt{bx+2}\,b+6\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{6b^3}$$

input
```
int(x^(3/2)*(b*x+2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 + sqrt(x)*sqrt(b*x + 2)*b**2*x - 3*sqrt
(x)*sqrt(b*x + 2)*b + 6*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt
(2)))/(6*b**3)
```

3.458 $\int \sqrt{x}\sqrt{2+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \sqrt{x}\sqrt{2+bx}\,dx=\frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

output
```
1/2*x^(1/2)*(b*x+2)^(1/2)/b+1/2*x^(3/2)*(b*x+2)^(1/2)-arcsinh(1/2*b^(1/2)*
x^(1/2)*2^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \sqrt{x}\sqrt{2+bx}\,dx=\frac{\sqrt{x}(1+bx)\sqrt{2+bx}}{2b}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]*Sqrt[2 + b*x],x]
```

output
```
(Sqrt[x]*(1 + b*x)*Sqrt[2 + b*x])/(2*b) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sq
rt[2] - Sqrt[2 + b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{bx+2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{63}$$

$$\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

input
```
Int[Sqrt[x]*Sqrt[2 + b*x],x]
```

output
```
(x^(3/2)*Sqrt[2 + b*x])/2 + ((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[
b]*Sqrt[x])/Sqrt[2]])/b^(3/2))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

method	result	size
meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(3bx+3)\sqrt{\frac{bx}{2}+1}}{12}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	55
risch	$\frac{(bx+1)\sqrt{x}\,\sqrt{bx+2}}{2b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}$	68
default	$\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}$	79

input
```
int(x^(1/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^(3/2)/Pi^(1/2)*(-1/12*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(3*b*x+3)*(1/2
*b*x+1)^(1/2)+1/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int \sqrt{x}\sqrt{2+bx}\,dx$$
$$= \left[\frac{(b^2x+b)\sqrt{bx+2}\sqrt{x}+\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{2\,b^2}, \frac{(b^2x+b)\sqrt{bx+2}\sqrt{x}+2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}}{\sqrt{bx}}\right.}{2\,b^2}\right.$$

input
```
integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*
sqrt(b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + 2*sqrt
(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \frac{bx^{\frac{5}{2}}}{2\sqrt{bx+2}} + \frac{3x^{\frac{3}{2}}}{2\sqrt{bx+2}} + \frac{\sqrt{x}}{b\sqrt{bx+2}} - \frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x**(1/2)*(b*x+2)**(1/2),x)
```

output
```
b*x**(5/2)/(2*sqrt(b*x + 2)) + 3*x**(3/2)/(2*sqrt(b*x + 2)) + sqrt(x)/(b*s
qrt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(45) = 90$.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \frac{\frac{\sqrt{bx+2}b}{\sqrt{x}} + \frac{(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^3 - \frac{2\,(bx+2)b^2}{x} + \frac{(bx+2)^2 b}{x^2}} + \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
(sqrt(b*x + 2)*b/sqrt(x) + (b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x + 2)*b^2
/x + (b*x + 2)^2*b/x^2) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt
(b) + sqrt(b*x + 2)/sqrt(x)))/b^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(45) = 90$.

Time = 11.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.09

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \frac{\frac{\left(\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}(bx-3)-6\,\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)\right)|b|}{b^2} + \frac{4\left(2\,\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)+\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}\right)|b|}{b^2}}{2\,b}$$

input
```
integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/2*((sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3) - 6*sqrt(b)*log(abs(
-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^2 + 4*(2*sqrt
(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))) + sqrt((b*x
 + 2)*b - 2*b)*sqrt(b*x + 2))*abs(b)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \sqrt{x}\left(\frac{x}{2}+\frac{1}{2\,b}\right)\sqrt{b\,x+2}-\frac{\ln\left(b\,x+\sqrt{b}\,\sqrt{x}\,\sqrt{b\,x+2}+1\right)}{2\,b^{3/2}}$$

input
```
int(x^(1/2)*(b*x + 2)^(1/2),x)
```

output
```
x^(1/2)*(x/2 + 1/(2*b))*(b*x + 2)^(1/2) - log(b*x + b^(1/2)*x^(1/2)*(b*x +
 2)^(1/2) + 1)/(2*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \frac{\sqrt{x}\,\sqrt{bx+2}\,b^2x+\sqrt{x}\,\sqrt{bx+2}\,b-2\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{2b^2}$$

input
```
int(x^(1/2)*(b*x+2)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b**2*x + sqrt(x)*sqrt(b*x + 2)*b - 2*sqrt(b)*log((s
qrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/(2*b**2)
```

3.459 $\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx=\sqrt{x}\sqrt{2+bx}+\frac{2\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

output
```
x^(1/2)*(b*x+2)^(1/2)+2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx=\sqrt{x}\sqrt{2+bx}-\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

input
```
Integrate[Sqrt[2 + b*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[2 + b*x] - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b
]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{60}$$

$$\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx+\sqrt{x}\sqrt{bx+2}$$

$$\Big\downarrow \text{63}$$

$$2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}+\sqrt{x}\sqrt{bx+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{bx+2}$$

input
```
Int[Sqrt[2 + b*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

method	result	size
meijerg	$-\frac{-\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\sqrt{\frac{bx}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}\,\sqrt{\pi}}$	49
default	$\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	58
risch	$\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	58

input
```
int((b*x+2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^(1/2)/Pi^(1/2)*(-Pi^(1/2)*b^(1/2)*x^(1/2)*2^(1/2)*(1/2*b*x+1)^(1/2)-2
*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = \left[\frac{\sqrt{bx+2}b\sqrt{x}+\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{b}, \frac{\sqrt{bx+2}b\sqrt{x}-2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+2}}\right)}{b}\right]$$

input
```
integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x
) + 1))/b, (sqrt(b*x + 2)*b*sqrt(x) - 2*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/s
qrt(b*x + 2)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{bx+2} + \frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

input
```
integrate((b*x+2)**(1/2)/x**(1/2),x)
```

output
```
sqrt(x)*sqrt(b*x + 2) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = -\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}} - \frac{2\sqrt{bx+2}}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

input
```
integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))
/sqrt(b) - 2*sqrt(b*x + 2)/((b - (b*x + 2)/x)*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 5.80 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = -\frac{b\left(\frac{2\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2b}\right|\right)}{\sqrt{b}}-\frac{\sqrt{(bx+2)b-2b}\sqrt{bx+2}}{b}\right)}{|b|}$$

input
```
integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
-b*(2*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b) -
 sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b)/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{b\,x+2}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{\sqrt{b}}$$

input
```
int((b*x + 2)^(1/2)/x^(1/2),x)
```

output
```
x^(1/2)*(b*x + 2)^(1/2) - (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^
(1/2))))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+2}\,b+2\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{b}$$

input
```
int((b*x+2)^(1/2)/x^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b + 2*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))
/sqrt(2)))/b
```

3.460 $\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = -\frac{2\sqrt{2+bx}}{\sqrt{x}} + 2\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

output
```
-2*(b*x+2)^(1/2)/x^(1/2)+2*b^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = -\frac{2\sqrt{2+bx}}{\sqrt{x}} - 2\sqrt{b}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)$$

input
```
Integrate[Sqrt[2 + b*x]/x^(3/2),x]
```

output
```
(-2*Sqrt[2 + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b
*x]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {57, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$b\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

$$\Big\downarrow \text{63}$$

$$2b\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

$$\Big\downarrow \text{222}$$

$$2\sqrt{b}\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

input
```
Int[Sqrt[2 + b*x]/x^(3/2),x]
```

output
```
(-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result	size
meijerg	$-\frac{\sqrt{b}\left(\frac{4\sqrt{\pi}\,\sqrt{2}\,\sqrt{\frac{bx}{2}+1}}{\sqrt{x}\,\sqrt{b}}-4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	49
risch	$-\frac{2\sqrt{bx+2}}{\sqrt{x}}+\frac{\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	59

input
```
int((b*x+2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^(1/2)/Pi^(1/2)*(4*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(1/2*b*x+1)^(1/2
)-4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = \left[\frac{\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-2\sqrt{bx+2}\sqrt{x}}{x},\right.$$
$$\left.-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+2}}\right)+\sqrt{bx+2}\sqrt{x}\right)}{x}\right]$$

input
```
integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b*x + 2)
*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)) + sqrt(
b*x + 2)*sqrt(x))/x]
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = -2\sqrt{b}\sqrt{1+\frac{2}{bx}} - \sqrt{b}\log\left(\frac{1}{bx}\right) + 2\sqrt{b}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)$$

input
```
integrate((b*x+2)**(1/2)/x**(3/2),x)
```

output
```
-2*sqrt(b)*sqrt(1 + 2/(b*x)) - sqrt(b)*log(1/(b*x)) + 2*sqrt(b)*log(sqrt(1
 + 2/(b*x)) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = -\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right) - \frac{2\sqrt{bx+2}}{\sqrt{x}}$$

input
```
integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/s
qrt(x))) - 2*sqrt(b*x + 2)/sqrt(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 5.74 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = -\frac{2\,b^2\left(\frac{\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx+2}}{\sqrt{(bx+2)b-2\,b}}\right)}{|b|}$$

input
```
integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
-2*b^2*(log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b)
 + sqrt(b*x + 2)/sqrt((b*x + 2)*b - 2*b))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = \int \frac{\sqrt{b\,x+2}}{x^{3/2}}\,dx$$

input
```
int((b*x + 2)^(1/2)/x^(3/2),x)
```

output
```
int((b*x + 2)^(1/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+2}+2\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)x-2\sqrt{b}\,x}{x}$$

input
```
int((b*x+2)^(1/2)/x^(3/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(b*x + 2) + sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b
))/sqrt(2))*x - sqrt(b)*x))/x
```

3.461 $\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

output
```
-1/3*(b*x+2)^(3/2)/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

input
```
Integrate[Sqrt[2 + b*x]/x^(5/2),x]
```

output
```
-1/3*(2 + b*x)^(3/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+2}}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 48$$

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

input
```
Int[Sqrt[2 + b*x]/x^(5/2),x]
```

output
```
-1/3*(2 + b*x)^(3/2)/x^(3/2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	13
orering	$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	13
meijerg	$-\frac{2\sqrt{2}\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	17
risch	$-\frac{b^2x^2+4bx+4}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	26
default	$-\frac{2\sqrt{bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{bx+2}}{3\sqrt{x}}$	27

input
```
int((b*x+2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x+2)^(3/2)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx=-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3} - \frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

input
```
integrate((b*x+2)**(1/2)/x**(5/2),x)
```

output
```
-b**(3/2)*sqrt(1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{(bx+2)^{\frac{3}{2}}b^4}{3\left((bx+2)b-2\,b\right)^{\frac{3}{2}}|b|}$$

input
```
integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/3*(b*x + 2)^(3/2)*b^4/(((b*x + 2)*b - 2*b)^(3/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}+\frac{2}{3}\right)}{x^{3/2}}$$

input
```
int((b*x + 2)^(1/2)/x^(5/2),x)
```

output
```
-((b*x + 2)^(1/2)*((b*x)/3 + 2/3))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = \frac{-\sqrt{x}\,\sqrt{bx+2}\,bx-2\sqrt{x}\,\sqrt{bx+2}-\sqrt{b}\,b\,x^2}{3x^2}$$

input
```
int((b*x+2)^(1/2)/x^(5/2),x)
```

output
```
( - sqrt(x)*sqrt(b*x + 2)*b*x - 2*sqrt(x)*sqrt(b*x + 2) - sqrt(b)*b*x**2)/
(3*x**2)
```

3.462 $\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx=-\frac{(2+bx)^{3/2}}{5x^{5/2}}+\frac{b(2+bx)^{3/2}}{15x^{3/2}}$$

output
```
-1/5*(b*x+2)^(3/2)/x^(5/2)+1/15*b*(b*x+2)^(3/2)/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx=\frac{\sqrt{2+bx}(-6-bx+b^2x^2)}{15x^{5/2}}$$

input
```
Integrate[Sqrt[2 + b*x]/x^(7/2),x]
```

output
```
(Sqrt[2 + b*x]*(-6 - b*x + b^2*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+2}}{x^{7/2}} \, dx$$

$$\Big\downarrow \; 55$$

$$-\frac{1}{5} b \int \frac{\sqrt{bx+2}}{x^{5/2}} dx - \frac{(bx+2)^{3/2}}{5x^{5/2}}$$

$$\Big\downarrow \; 48$$

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}} - \frac{(bx+2)^{3/2}}{5x^{5/2}}$$

input
```
Int[Sqrt[2 + b*x]/x^(7/2),x]
```

output
```
-1/5*(2 + b*x)^(3/2)/x^(5/2) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.47

method	result	size
gosper	$\frac{(bx+2)^{\frac{3}{2}}(bx-3)}{15x^{\frac{5}{2}}}$	18
orering	$\frac{(bx+2)^{\frac{3}{2}}(bx-3)}{15x^{\frac{5}{2}}}$	18
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{1}{6}b^{2}x^{2}+\frac{1}{6}bx+1\right)\sqrt{\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
risch	$\frac{b^{3}x^{3}+b^{2}x^{2}-8bx-12}{15x^{\frac{5}{2}}\sqrt{bx+2}}$	33
default	$-\frac{2\sqrt{bx+2}}{5x^{\frac{5}{2}}}+\frac{b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}$	43

input
```
int((b*x+2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(b*x+2)^(3/2)*(b*x-3)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx = \frac{(b^2x^2-bx-6)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
1/15*(b^2*x^2 - b*x - 6)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx = \frac{b^{\frac{5}{2}}\sqrt{1+\frac{2}{bx}}}{15} - \frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{15x} - \frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{5x^2}$$

input
```
integrate((b*x+2)**(1/2)/x**(7/2),x)
```

output
```
b**(5/2)*sqrt(1 + 2/(b*x))/15 - b**(3/2)*sqrt(1 + 2/(b*x))/(15*x) - 2*sqrt
(b)*sqrt(1 + 2/(b*x))/(5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx = \frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}} - \frac{(bx+2)^{\frac{5}{2}}}{10\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(b*x + 2)^(5/2)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx = \frac{((bx+2)b^5-5\,b^5)(bx+2)^{\frac{3}{2}}b}{15\,((bx+2)b-2\,b)^{\frac{5}{2}}|b|}$$

input
```
integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
1/15*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2)^(3/2)*b/(((b*x + 2)*b - 2*b)^(5/2)*
abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx = -\frac{\sqrt{b\,x+2}\,\left(-\frac{b^2\,x^2}{15}+\frac{b\,x}{15}+\frac{2}{5}\right)}{x^{5/2}}$$

input
```
int((b*x + 2)^(1/2)/x^(7/2),x)
```

output
```
-((b*x + 2)^(1/2)*((b*x)/15 - (b^2*x^2)/15 + 2/5))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+2}\,b^2x^2-\sqrt{x}\,\sqrt{bx+2}\,bx-6\sqrt{x}\,\sqrt{bx+2}-\sqrt{b}\,b^2x^3}{15x^3}$$

input
```
int((b*x+2)^(1/2)/x^(7/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b**2*x**2 - sqrt(x)*sqrt(b*x + 2)*b*x - 6*sqrt(x)*s
qrt(b*x + 2) - sqrt(b)*b**2*x**3)/(15*x**3)
```

3.463 $\int \frac{\sqrt{2+bx}}{x^{9/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\sqrt{2+bx}}{x^{9/2}} \, dx = -\frac{(2+bx)^{3/2}}{7x^{7/2}} + \frac{2b(2+bx)^{3/2}}{35x^{5/2}} - \frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}$$

output
```
-1/7*(b*x+2)^(3/2)/x^(7/2)+2/35*b*(b*x+2)^(3/2)/x^(5/2)-2/105*b^2*(b*x+2)^
(3/2)/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2+bx}}{x^{9/2}} \, dx = \frac{\sqrt{2+bx}(-30-3bx+2b^2x^2-2b^3x^3)}{105x^{7/2}}$$

input
```
Integrate[Sqrt[2 + b*x]/x^(9/2),x]
```

output
```
(Sqrt[2 + b*x]*(-30 - 3*b*x + 2*b^2*x^2 - 2*b^3*x^3))/(105*x^(7/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+2}}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2}{7}b\int \frac{\sqrt{bx+2}}{x^{7/2}}dx-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2}{7}b\left(-\frac{1}{5}b\int \frac{\sqrt{bx+2}}{x^{5/2}}dx-\frac{(bx+2)^{3/2}}{5x^{5/2}}\right)-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(bx+2)^{3/2}}{7x^{7/2}}-\frac{2}{7}b\left(\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}\right)$$

input
```
Int[Sqrt[2 + b*x]/x^(9/2),x]
```

output
```
-1/7*(2 + b*x)^(3/2)/x^(7/2) - (2*b*(-1/5*(2 + b*x)^(3/2)/x^(5/2) + (b*(2
+ b*x)^(3/2))/(15*x^(3/2))))/7
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{(bx+2)^{\frac{3}{2}}\left(2b^{2}x^{2}-6bx+15\right)}{105x^{\frac{7}{2}}}$	27
orering	$-\frac{(bx+2)^{\frac{3}{2}}\left(2b^{2}x^{2}-6bx+15\right)}{105x^{\frac{7}{2}}}$	27
meijerg	$-\frac{2\sqrt{2}\left(\frac{1}{15}b^{3}x^{3}-\frac{1}{15}b^{2}x^{2}+\frac{1}{10}bx+1\right)\sqrt{\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39
risch	$-\frac{2b^{4}x^{4}+2b^{3}x^{3}-b^{2}x^{2}+36bx+60}{105x^{\frac{7}{2}}\sqrt{bx+2}}$	43
default	$-\frac{2\sqrt{bx+2}}{7x^{\frac{7}{2}}}+\frac{b\left(-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	59

input
```
int((b*x+2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(b*x+2)^(3/2)*(2*b^2*x^2-6*b*x+15)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx = -\frac{(2\,b^3x^3-2\,b^2x^2+3\,bx+30)\sqrt{bx+2}}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x + 30)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(53) = 106.

Time = 5.37 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.58

$$\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx = -\frac{2b^{\frac{19}{2}}x^5\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3} - \frac{6b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3} - \frac{3b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3} - \frac{34b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3} - \frac{132b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3} - \frac{120b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}$$

input
```
integrate((b*x+2)**(1/2)/x**(9/2),x)
```

output
```
-2*b**(19/2)*x**5*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b
**4*x**3) - 6*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x
**4 + 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(105*b**6*x**5 +
 420*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(105
*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 132*b**(11/2)*x*sqrt(1 + 2/(
b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9/2)*sqrt(
1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx = -\frac{(bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}} + \frac{(bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}} - \frac{(bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-1/12*(b*x + 2)^(3/2)*b^2/x^(3/2) + 1/10*(b*x + 2)^(5/2)*b/x^(5/2) - 1/28*
(b*x + 2)^(7/2)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx = -\frac{(35\,b^3 + 2\,((bx+2)b^3 - 7\,b^3)(bx+2))(bx+2)^{\frac{3}{2}}b^5}{105\,((bx+2)b - 2\,b)^{\frac{7}{2}}\,|b|}$$

input
```
integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
-1/105*(35*b^3 + 2*((b*x + 2)*b^3 - 7*b^3)*(b*x + 2))*(b*x + 2)^(3/2)*b^5/
(((b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx = -\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{105} - \frac{2\,b^2\,x^2}{105} + \frac{b\,x}{35} + \frac{2}{7}\right)}{x^{7/2}}$$

input
```
int((b*x + 2)^(1/2)/x^(9/2),x)
```

output
```
-((b*x + 2)^(1/2)*((b*x)/35 - (2*b^2*x^2)/105 + (2*b^3*x^3)/105 + 2/7))/x^
(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+2}\,b^3x^3+2\sqrt{x}\,\sqrt{bx+2}\,b^2x^2-3\sqrt{x}\,\sqrt{bx+2}\,bx-30\sqrt{x}\,\sqrt{bx+2}+2\sqrt{b}\,b^3x}{105x^4}$$

input
```
int((b*x+2)^(1/2)/x^(9/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(b*x + 2)*b**3*x**3 + 2*sqrt(x)*sqrt(b*x + 2)*b**2*x**2
- 3*sqrt(x)*sqrt(b*x + 2)*b*x - 30*sqrt(x)*sqrt(b*x + 2) + 2*sqrt(b)*b**3*
x**4)/(105*x**4)
```

3.464 $\int \frac{\sqrt{2+bx}}{x^{11/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\sqrt{2+bx}}{x^{11/2}} \, dx = -\frac{(2+bx)^{3/2}}{9x^{9/2}} + \frac{b(2+bx)^{3/2}}{21x^{7/2}} - \frac{2b^2(2+bx)^{3/2}}{105x^{5/2}} + \frac{2b^3(2+bx)^{3/2}}{315x^{3/2}}$$

output
```
-1/9*(b*x+2)^(3/2)/x^(9/2)+1/21*b*(b*x+2)^(3/2)/x^(7/2)-2/105*b^2*(b*x+2)^
(3/2)/x^(5/2)+2/315*b^3*(b*x+2)^(3/2)/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{2+bx}}{x^{11/2}} \, dx = \frac{\sqrt{2+bx}(-70-5bx+3b^2x^2-2b^3x^3+2b^4x^4)}{315x^{9/2}}$$

input
```
Integrate[Sqrt[2 + b*x]/x^(11/2),x]
```

output
```
(Sqrt[2 + b*x]*(-70 - 5*b*x + 3*b^2*x^2 - 2*b^3*x^3 + 2*b^4*x^4))/(315*x^(
9/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+2}}{x^{11/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{1}{3}b\int \frac{\sqrt{bx+2}}{x^{9/2}}dx-\frac{(bx+2)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{1}{3}b\left(-\frac{2}{7}b\int \frac{\sqrt{bx+2}}{x^{7/2}}dx-\frac{(bx+2)^{3/2}}{7x^{7/2}}\right)-\frac{(bx+2)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{1}{3}b\left(-\frac{2}{7}b\left(-\frac{1}{5}b\int \frac{\sqrt{bx+2}}{x^{5/2}}dx-\frac{(bx+2)^{3/2}}{5x^{5/2}}\right)-\frac{(bx+2)^{3/2}}{7x^{7/2}}\right)-\frac{(bx+2)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(bx+2)^{3/2}}{9x^{9/2}}-\frac{1}{3}b\left(-\frac{(bx+2)^{3/2}}{7x^{7/2}}-\frac{2}{7}b\left(\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}\right)\right)$$

input
```
Int[Sqrt[2 + b*x]/x^(11/2),x]
```

output
```
-1/9*(2 + b*x)^(3/2)/x^(9/2) - (b*(-1/7*(2 + b*x)^(3/2)/x^(7/2) - (2*b*(-1
/5*(2 + b*x)^(3/2)/x^(5/2) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))))/7))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
gosper	$\frac{(bx+2)^{\frac{3}{2}}\left(2b^{3}x^{3}-6b^{2}x^{2}+15bx-35\right)}{315x^{\frac{9}{2}}}$	35
orering	$\frac{(bx+2)^{\frac{3}{2}}\left(2b^{3}x^{3}-6b^{2}x^{2}+15bx-35\right)}{315x^{\frac{9}{2}}}$	35
meijerg	$-\frac{2\sqrt{2}\left(-\frac{1}{35}b^{4}x^{4}+\frac{1}{35}b^{3}x^{3}-\frac{3}{70}b^{2}x^{2}+\frac{1}{14}bx+1\right)\sqrt{\frac{bx}{2}+1}}{9x^{\frac{9}{2}}}$	47
risch	$\frac{2b^{5}x^{5}+2b^{4}x^{4}-b^{3}x^{3}+b^{2}x^{2}-80bx-140}{315x^{\frac{9}{2}}\sqrt{bx+2}}$	50
default	$-\frac{2\sqrt{bx+2}}{9x^{\frac{9}{2}}}+\frac{b\left(-\frac{\sqrt{bx+2}}{7x^{\frac{7}{2}}}-\frac{3b\left(-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}\right)}{9}$	75

input
```
int((b*x+2)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(b*x+2)^(3/2)*(2*b^3*x^3-6*b^2*x^2+15*b*x-35)/x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{2+bx}}{x^{11/2}}\,dx = \frac{(2\,b^4x^4 - 2\,b^3x^3 + 3\,b^2x^2 - 5\,bx - 70)\sqrt{bx+2}}{315\,x^{\frac{9}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
1/315*(2*b^4*x^4 - 2*b^3*x^3 + 3*b^2*x^2 - 5*b*x - 70)*sqrt(b*x + 2)/x^(9/
2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(71) = 142.

Time = 15.22 (sec) , antiderivative size = 428, normalized size of antiderivative = 5.35

$$\int \frac{\sqrt{2+bx}}{x^{11/2}}\,dx = \frac{2b^{\frac{33}{2}}x^7\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4} + \frac{10b^{\frac{31}{2}}x^6\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4} + \frac{15b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4} + \frac{5b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4} - \frac{80b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4} - \frac{456b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4} - \frac{880b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4} - \frac{560b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{315b^{12}x^7+1890b^{11}x^6+3780b^{10}x^5+2520b^9x^4}$$

input
```
integrate((b*x+2)**(1/2)/x**(11/2),x)
```

output
```
2*b**(33/2)*x**7*sqrt(1 + 2/(b*x))/(315*b**12*x**7 + 1890*b**11*x**6 + 378
0*b**10*x**5 + 2520*b**9*x**4) + 10*b**(31/2)*x**6*sqrt(1 + 2/(b*x))/(315*
b**12*x**7 + 1890*b**11*x**6 + 3780*b**10*x**5 + 2520*b**9*x**4) + 15*b**(
29/2)*x**5*sqrt(1 + 2/(b*x))/(315*b**12*x**7 + 1890*b**11*x**6 + 3780*b**1
0*x**5 + 2520*b**9*x**4) + 5*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(315*b**12*x
**7 + 1890*b**11*x**6 + 3780*b**10*x**5 + 2520*b**9*x**4) - 80*b**(25/2)*x
**3*sqrt(1 + 2/(b*x))/(315*b**12*x**7 + 1890*b**11*x**6 + 3780*b**10*x**5
+ 2520*b**9*x**4) - 456*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/(315*b**12*x**7 +
 1890*b**11*x**6 + 3780*b**10*x**5 + 2520*b**9*x**4) - 880*b**(21/2)*x*sqr
t(1 + 2/(b*x))/(315*b**12*x**7 + 1890*b**11*x**6 + 3780*b**10*x**5 + 2520*
b**9*x**4) - 560*b**(19/2)*sqrt(1 + 2/(b*x))/(315*b**12*x**7 + 1890*b**11*
x**6 + 3780*b**10*x**5 + 2520*b**9*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{2+bx}}{x^{11/2}}\,dx = \frac{(bx+2)^{\frac{3}{2}}b^3}{24\,x^{\frac{3}{2}}} - \frac{3\,(bx+2)^{\frac{5}{2}}b^2}{40\,x^{\frac{5}{2}}} + \frac{3\,(bx+2)^{\frac{7}{2}}b}{56\,x^{\frac{7}{2}}} - \frac{(bx+2)^{\frac{9}{2}}}{72\,x^{\frac{9}{2}}}$$

input
```
integrate((b*x+2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
1/24*(b*x + 2)^(3/2)*b^3/x^(3/2) - 3/40*(b*x + 2)^(5/2)*b^2/x^(5/2) + 3/56
*(b*x + 2)^(7/2)*b/x^(7/2) - 1/72*(b*x + 2)^(9/2)/x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{2+bx}}{x^{11/2}}\,dx = -\frac{(105\,b^9 - (63\,b^9 + 2\,((bx+2)b^9 - 9\,b^9)(bx+2))(bx+2))(bx+2)^{\frac{3}{2}}b}{315\,((bx+2)b - 2\,b)^{\frac{9}{2}}|b|}$$

input
```
integrate((b*x+2)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
-1/315*(105*b^9 - (63*b^9 + 2*((b*x + 2)*b^9 - 9*b^9)*(b*x + 2))*(b*x + 2)
)*(b*x + 2)^(3/2)*b/(((b*x + 2)*b - 2*b)^(9/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{2+bx}}{x^{11/2}}\,dx = -\frac{\sqrt{b\,x+2}\,\left(-\frac{2\,b^4\,x^4}{315} + \frac{2\,b^3\,x^3}{315} - \frac{b^2\,x^2}{105} + \frac{b\,x}{63} + \frac{2}{9}\right)}{x^{9/2}}$$

input
```
int((b*x + 2)^(1/2)/x^(11/2),x)
```

output
```
-((b*x + 2)^(1/2)*((b*x)/63 - (b^2*x^2)/105 + (2*b^3*x^3)/315 - (2*b^4*x^4
)/315 + 2/9))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{2+bx}}{x^{11/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{bx+2}\,b^4x^4 - 2\sqrt{x}\,\sqrt{bx+2}\,b^3x^3 + 3\sqrt{x}\,\sqrt{bx+2}\,b^2x^2 - 5\sqrt{x}\,\sqrt{bx+2}\,bx - 70\sqrt{x}}{315x^5}$$

input
```
int((b*x+2)^(1/2)/x^(11/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b**4*x**4 - 2*sqrt(x)*sqrt(b*x + 2)*b**3*x**3 + 3
*sqrt(x)*sqrt(b*x + 2)*b**2*x**2 - 5*sqrt(x)*sqrt(b*x + 2)*b*x - 70*sqrt(x
)*sqrt(b*x + 2) - 2*sqrt(b)*b**4*x**5)/(315*x**5)
```

3.465 $\int x^{5/2}(2+bx)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{11}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}bx^{9/2}\sqrt{2+bx}-\frac{3\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

output
```
3/8*x^(1/2)*(b*x+2)^(1/2)/b^3-1/8*x^(3/2)*(b*x+2)^(1/2)/b^2+1/20*x^(5/2)*(
b*x+2)^(1/2)/b+11/20*x^(7/2)*(b*x+2)^(1/2)+1/5*b*x^(9/2)*(b*x+2)^(1/2)-3/4
*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{\sqrt{x}\sqrt{2+bx}(15-5bx+2b^2x^2+22b^3x^3+8b^4x^4)}{40b^3}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{2b^{7/2}}$$

input
```
Integrate[x^(5/2)*(2 + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 22*b^3*x^3 + 8*b^4*x^4))/
(40*b^3) + (3*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/(2*b^(
7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {60, 60, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(bx+2)^{3/2}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{3}{5}\int x^{5/2}\sqrt{bx+2}dx+\frac{1}{5}x^{7/2}(bx+2)^{3/2}$$

$$\Big\downarrow \; 60$$

$$\frac{3}{5}\left(\frac{1}{4}\int \frac{x^{5/2}}{\sqrt{bx+2}}dx+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}$$

$$\Big\downarrow \; 60$$

$$\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\int \frac{x^{3/2}}{\sqrt{bx+2}}dx}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}$$

$$\Big\downarrow \; 60$$

$$\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}$$

$$\Big\downarrow \; 60$$

$$\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+$$
$$\frac{1}{5}x^{7/2}(bx+2)^{3/2}$$

$\downarrow$ 63

$$\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+$$
$$\frac{1}{5}x^{7/2}(bx+2)^{3/2}$$

$\downarrow$ 222

$$\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+$$
$$\frac{1}{5}x^{7/2}(bx+2)^{3/2}$$

input
```
Int[x^(5/2)*(2 + b*x)^(3/2),x]
```

output
```
(x^(7/2)*(2 + b*x)^(3/2))/5 + (3*((x^(7/2)*Sqrt[2 + b*x])/4 + ((x^(5/2)*Sq
rt[2 + b*x])/(3*b) - (5*((x^(3/2)*Sqrt[2 + b*x])/(2*b) - (3*((Sqrt[x]*Sqrt
[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)))/(2*b)))/(3
*b))/4))/5
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(56b^{4}x^{4}+154b^{3}x^{3}+14b^{2}x^{2}-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{280}-\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}}{b^{\frac{7}{2}}\sqrt{\pi}}$	79
risch	$\frac{\left(8b^{4}x^{4}+22b^{3}x^{3}+2b^{2}x^{2}-5bx+15\right)\sqrt{x}\,\sqrt{bx+2}}{40b^{3}}-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$	93
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{5}{2}}}{5b}-\frac{\frac{x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}}{4b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{3b}\right)}{4b}}{b}$	135

input
```
int(x^(5/2)*(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
24/b^(7/2)/Pi^(1/2)*(1/6720*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(56*b^4*x^4+1
54*b^3*x^3+14*b^2*x^2-35*b*x+105)*(1/2*b*x+1)^(1/2)-1/32*Pi^(1/2)*arcsinh(
1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \left[\frac{(8\,b^5x^4+22\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{40\,b^4}\right.$$

input
```
integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
[1/40*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)
*sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1
/40*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*s
qrt(x) + 30*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))/b^4]
```

Sympy [A] (verification not implemented)

Time = 50.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{b^2x^{\frac{11}{2}}}{5\sqrt{bx+2}} + \frac{19bx^{\frac{9}{2}}}{20\sqrt{bx+2}} + \frac{23x^{\frac{7}{2}}}{20\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{40b\sqrt{bx+2}} + \frac{x^{\frac{3}{2}}}{8b^2\sqrt{bx+2}} + \frac{3\sqrt{x}}{4b^3\sqrt{bx+2}} - \frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

input
```
integrate(x**(5/2)*(b*x+2)**(3/2),x)
```

output
```
b**2*x**(11/2)/(5*sqrt(b*x + 2)) + 19*b*x**(9/2)/(20*sqrt(b*x + 2)) + 23*x
**(7/2)/(20*sqrt(b*x + 2)) - x**(5/2)/(40*b*sqrt(b*x + 2)) + x**(3/2)/(8*b
**2*sqrt(b*x + 2)) + 3*sqrt(x)/(4*b**3*sqrt(b*x + 2)) - 3*asinh(sqrt(2)*sq
rt(b)*sqrt(x)/2)/(4*b**(7/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(88) = 176$.

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.53

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{\frac{15\sqrt{bx+2}b^4}{\sqrt{x}} - \frac{70(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}} - \frac{128(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}} + \frac{70(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}} - \frac{15(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8 - \frac{5(bx+2)b^7}{x} + \frac{10(bx+2)^2b^6}{x^2} - \frac{10(bx+2)^3b^5}{x^3} + \frac{5(bx+2)^4b^4}{x^4} - \frac{(bx+2)^5b^3}{x^5}\right)} + \frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) - 128*
(b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^
(9/2)/x^(9/2))/(b^8 - 5*(b*x + 2)*b^7/x + 10*(b*x + 2)^2*b^6/x^2 - 10*(b*x
 + 2)^3*b^5/x^3 + 5*(b*x + 2)^4*b^4/x^4 - (b*x + 2)^5*b^3/x^5) + 3/8*log(-
(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/
2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(88) = 176$.

Time = 16.74 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.19

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{\frac{20\left(\left((bx+2)\left(2\,(bx+2)\left(\frac{3\,(bx+2)}{b^3}-\frac{25}{b^3}\right)+\frac{163}{b^3}\right)-\frac{279}{b^3}\right)\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}-\frac{210\,\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)}{b^{\frac{5}{2}}}\right)|b|}{b}+\frac{3\left(((2\right.}{}}{}$$

input
```
integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
1/120*(20*(((b*x + 2)*(2*(b*x + 2)*(3*(b*x + 2)/b^3 - 25/b^3) + 163/b^3) -
 279/b^3)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) - 210*log(abs(-sqrt(b*x +
2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/b^(5/2))*abs(b)/b + 3*(((2*((4*b*x
- 33)*(b*x + 2) + 171)*(b*x + 2) - 745)*(b*x + 2) + 965)*sqrt((b*x + 2)*b
- 2*b)*sqrt(b*x + 2) + 630*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((
b*x + 2)*b - 2*b))))*abs(b)/b^4 + 80*(((2*b*x - 9)*(b*x + 2) + 33)*sqrt((b
*x + 2)*b - 2*b)*sqrt(b*x + 2) + 30*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b)
 + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^4)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \int x^{5/2}\,{\left(b\,x+2\right)}^{3/2}\,dx$$

input
```
int(x^(5/2)*(b*x + 2)^(3/2),x)
```

output
```
int(x^(5/2)*(b*x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{8\sqrt{x}\,\sqrt{bx+2}\,b^5x^4+22\sqrt{x}\,\sqrt{bx+2}\,b^4x^3+2\sqrt{x}\,\sqrt{bx+2}\,b^3x^2-5\sqrt{x}\,\sqrt{bx+2}\,b^2x+15\sqrt{x}\,\sqrt{b}}{40b^4}$$

input
```
int(x^(5/2)*(b*x+2)^(3/2),x)
```

output
```
(8*sqrt(x)*sqrt(b*x + 2)*b**5*x**4 + 22*sqrt(x)*sqrt(b*x + 2)*b**4*x**3 +
2*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 - 5*sqrt(x)*sqrt(b*x + 2)*b**2*x + 15*sq
rt(x)*sqrt(b*x + 2)*b - 30*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/s
qrt(2)))/(40*b**4)
```

3.466 $\int x^{3/2}(2+bx)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int x^{3/2}(2+bx)^{3/2}\,dx = -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b} +\frac{3}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}bx^{7/2}\sqrt{2+bx}+\frac{3\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

output

```
-3/8*x^(1/2)*(b*x+2)^(1/2)/b^2+1/8*x^(3/2)*(b*x+2)^(1/2)/b+3/4*x^(5/2)*(b*
x+2)^(1/2)+1/4*b*x^(7/2)*(b*x+2)^(1/2)+3/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(
1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int x^{3/2}(2+bx)^{3/2}\,dx = \frac{\sqrt{x}\sqrt{2+bx}(-3+bx+6b^2x^2+2b^3x^3)}{8b^2}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{2b^{5/2}}$$

input

```
Integrate[x^(3/2)*(2 + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 6*b^2*x^2 + 2*b^3*x^3))/(8*b^2) - (3*Ar
cTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(bx+2)^{3/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\int x^{3/2}\sqrt{bx+2}dx+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\left(\frac{1}{3}\int \frac{x^{3/2}}{\sqrt{bx+2}}dx+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{63}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \; 222$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

input
```
Int[x^(3/2)*(2 + b*x)^(3/2),x]
```

output
```
(x^(5/2)*(2 + b*x)^(3/2))/4 + (3*((x^(5/2)*Sqrt[2 + b*x])/3 + ((x^(3/2)*Sq
rt[2 + b*x])/(2*b) - (3*((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[2]])/b^(3/2)))/(2*b))/3))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.67

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(-10b^{3}x^{3}-30b^{2}x^{2}-5bx+15\right)\sqrt{\frac{bx}{2}+1}}{40}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}}{b^{\frac{5}{2}}\sqrt{\pi}}$	71
risch	$\frac{\left(2b^{3}x^{3}+6b^{2}x^{2}+bx-3\right)\sqrt{x}\,\sqrt{bx+2}}{8b^{2}}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	84
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}}{4b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{3b}\right)}{4b}$	114

input
```
int(x^(3/2)*(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
12/b^(5/2)/Pi^(1/2)*(-1/480*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(-10*b^3*x^3-
30*b^2*x^2-5*b*x+15)*(1/2*b*x+1)^(1/2)+1/16*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x
^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int x^{3/2}(2+bx)^{3/2}\,dx=\left[\frac{(2\,b^4x^3+6\,b^3x^2+b^2x-3\,b)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{8\,b^3},\frac{(2\,b^4x^3+6}{}\right.$$

input
```
integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*((2*b^4*x^3 + 6*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt
(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/8*((2*b^4*x^3 + 6
*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(-b)
*sqrt(x)/sqrt(b*x + 2)))/b^3]
```

Sympy [A] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

$$\int x^{3/2}(2+bx)^{3/2}\,dx = \frac{b^2x^{\frac{9}{2}}}{4\sqrt{bx+2}} + \frac{5bx^{\frac{7}{2}}}{4\sqrt{bx+2}} + \frac{13x^{\frac{5}{2}}}{8\sqrt{bx+2}} - \frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{bx+2}} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

input
```
integrate(x**(3/2)*(b*x+2)**(3/2),x)
```

output
```
b**2*x**(9/2)/(4*sqrt(b*x + 2)) + 5*b*x**(7/2)/(4*sqrt(b*x + 2)) + 13*x**(
5/2)/(8*sqrt(b*x + 2)) - x**(3/2)/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*
sqrt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(73) = 146.

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.54

$$\int x^{3/2}(2+bx)^{3/2}\,dx = -\frac{\frac{3\sqrt{bx+2}b^3}{\sqrt{x}} - \frac{11(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}} - \frac{11(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}} + \frac{3(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^6 - \frac{4(bx+2)b^5}{x} + \frac{6(bx+2)^2b^4}{x^2} - \frac{4(bx+2)^3b^3}{x^3} + \frac{(bx+2)^4b^2}{x^4}\right)} - \frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(3*sqrt(b*x + 2)*b^3/sqrt(x) - 11*(b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(b
*x + 2)^(5/2)*b/x^(5/2) + 3*(b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x + 2)*b^
5/x + 6*(b*x + 2)^2*b^4/x^2 - 4*(b*x + 2)^3*b^3/x^3 + (b*x + 2)^4*b^2/x^4)
 - 3/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqr
t(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(73) = 146$.

Time = 16.99 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.34

$$\int x^{3/2}(2+bx)^{3/2}\,dx = \frac{\left(\left((bx+2)\left(2\,(bx+2)\left(\frac{3\,(bx+2)}{b^3}-\frac{25}{b^3}\right)+\frac{163}{b^3}\right)-\frac{279}{b^3}\right)\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}-\frac{210\,\log\left(\left|-\sqrt{bx\cdots}\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
1/24*((((b*x + 2)*(2*(b*x + 2)*(3*(b*x + 2)/b^3 - 25/b^3) + 163/b^3) - 279
/b^3)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) - 210*log(abs(-sqrt(b*x + 2)*s
qrt(b) + sqrt((b*x + 2)*b - 2*b)))/b^(5/2))*abs(b) + 16*(((2*b*x - 9)*(b*x
 + 2) + 33)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) + 30*sqrt(b)*log(abs(-sq
rt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^3 + 48*(sqrt((b*
x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3) - 6*sqrt(b)*log(abs(-sqrt(b*x + 2)
*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^3)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(2+bx)^{3/2}\,dx = \int x^{3/2}\,{\left(b\,x+2\right)}^{3/2}\,dx$$

input
```
int(x^(3/2)*(b*x + 2)^(3/2),x)
```

output
```
int(x^(3/2)*(b*x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int x^{3/2}(2+bx)^{3/2}\,dx = \frac{2\sqrt{x}\,\sqrt{bx+2}\,b^4x^3+6\sqrt{x}\,\sqrt{bx+2}\,b^3x^2+\sqrt{x}\,\sqrt{bx+2}\,b^2x-3\sqrt{x}\,\sqrt{bx+2}\,b+6\sqrt{b}\log\left(\frac{\sqrt{bx+2}\cdots}{\cdots}\right.\cdots}{8b^3}$$

input
```
int(x^(3/2)*(b*x+2)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b**4*x**3 + 6*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 + s
qrt(x)*sqrt(b*x + 2)*b**2*x - 3*sqrt(x)*sqrt(b*x + 2)*b + 6*sqrt(b)*log((s
qrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/(8*b**3)
```

3.467 $\int \sqrt{x}(2+bx)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \sqrt{x}(2+bx)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{7}{6}x^{3/2}\sqrt{2+bx}+\frac{1}{3}bx^{5/2}\sqrt{2+bx}-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

output
```
1/2*x^(1/2)*(b*x+2)^(1/2)/b+7/6*x^(3/2)*(b*x+2)^(1/2)+1/3*b*x^(5/2)*(b*x+2
)^(1/2)-arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(2+bx)^{3/2}\,dx=\frac{\sqrt{x}\sqrt{2+bx}(3+7bx+2b^2x^2)}{6b}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]*(2 + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(3 + 7*b*x + 2*b^2*x^2))/(6*b) + (2*ArcTanh[(Sqrt[b
]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(bx+2)^{3/2}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\int \sqrt{x}\sqrt{bx+2}dx+\frac{1}{3}x^{3/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{ 63}$$

$$\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}$$

input
```
Int[Sqrt[x]*(2 + b*x)^(3/2),x]
```

output
```
(x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2 + b*x)^(3/2))/3 + ((Sqrt[x]*Sqrt[2
 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(2b^{2}x^{2}+7bx+3\right)\sqrt{\frac{bx}{2}+1}}{6}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	63
risch	$\frac{\left(2b^{2}x^{2}+7bx+3\right)\sqrt{x}\,\sqrt{bx+2}}{6b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}$	77
default	$\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{3b}$	93

input
```
int(x^(1/2)*(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
6/b^(3/2)/Pi^(1/2)*(1/36*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(2*b^2*x^2+7*b*x
+3)*(1/2*b*x+1)^(1/2)-1/6*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

$$\int \sqrt{x}(2+bx)^{3/2}\,dx = \left[\frac{(2\,b^3x^2+7\,b^2x+3\,b)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{6\,b^2}, \frac{(2\,b^3x^2+7\,b^2x+3\right.$$

input
```
integrate(x^(1/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*((2*b^3*x^2 + 7*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*
x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/6*((2*b^3*x^2 + 7*b^2*x + 3
*b)*sqrt(b*x + 2)*sqrt(x) + 6*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x +
2)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 3.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \sqrt{x}(2+bx)^{3/2}\,dx = \frac{b^2x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{11bx^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{17x^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x**(1/2)*(b*x+2)**(3/2),x)
```

output
```
b**2*x**(7/2)/(3*sqrt(b*x + 2)) + 11*b*x**(5/2)/(6*sqrt(b*x + 2)) + 17*x**
(3/2)/(6*sqrt(b*x + 2)) + sqrt(x)/(b*sqrt(b*x + 2)) - asinh(sqrt(2)*sqrt(b
)*sqrt(x)/2)/b**(3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.59

$$\int \sqrt{x}(2+bx)^{3/2}\,dx = \frac{\frac{3\sqrt{bx+2}b^2}{\sqrt{x}} - \frac{8(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} - \frac{3(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^4 - \frac{3(bx+2)b^3}{x} + \frac{3(bx+2)^2b^2}{x^2} - \frac{(bx+2)^3b}{x^3}\right)} + \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) - 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x +
2)^(5/2)/x^(5/2))/(b^4 - 3*(b*x + 2)*b^3/x + 3*(b*x + 2)^2*b^2/x^2 - (b*x
+ 2)^3*b/x^3) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt
(b*x + 2)/sqrt(x)))/b^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(58) = 116$.

Time = 16.62 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.53

$$\int \sqrt{x}(2+bx)^{3/2}\,dx = \frac{\frac{\left(((2bx-9)(bx+2)+33)\sqrt{(bx+2)b-2b}\sqrt{bx+2}+30\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2b}\right|\right)\right)|b|}{b^2} + \frac{12\left(\sqrt{(bx+2)b-2b}\sqrt{bx+2}(bx\right.}{}}{6}$$

input
```
integrate(x^(1/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
1/6*((((2*b*x - 9)*(b*x + 2) + 33)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) +
 30*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*ab
s(b)/b^2 + 12*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3) - 6*sqrt(b)
*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^2 +
24*(2*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))) +
 sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2))*abs(b)/b^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(2+bx)^{3/2}\,dx = \int \sqrt{x}\,(b\,x+2)^{3/2}\,dx$$

input
```
int(x^(1/2)*(b*x + 2)^(3/2),x)
```

output
```
int(x^(1/2)*(b*x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \sqrt{x}(2+bx)^{3/2}\,dx = \frac{2\sqrt{x}\,\sqrt{bx+2}\,b^3x^2+7\sqrt{x}\,\sqrt{bx+2}\,b^2x+3\sqrt{x}\,\sqrt{bx+2}\,b-6\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{6b^2}$$

input
```
int(x^(1/2)*(b*x+2)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 + 7*sqrt(x)*sqrt(b*x + 2)*b**2*x + 3*sq
rt(x)*sqrt(b*x + 2)*b - 6*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sq
rt(2)))/(6*b**2)
```

3.468 $\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx=\frac{5}{2}\sqrt{x}\sqrt{2+bx}+\frac{1}{2}bx^{3/2}\sqrt{2+bx}+\frac{3\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

output
```
5/2*x^(1/2)*(b*x+2)^(1/2)+1/2*b*x^(3/2)*(b*x+2)^(1/2)+3*arcsinh(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx=\frac{1}{2}\sqrt{x}\sqrt{2+bx}(5+bx)-\frac{3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

input
```
Integrate[(2 + b*x)^(3/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(5 + b*x))/2 - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 +
 b*x]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+2)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}\int \frac{\sqrt{bx+2}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}$$

$$\Big\downarrow \text{63}$$

$$\frac{3}{2}\left(2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{3}{2}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}$$

input
```
Int[(2 + b*x)^(3/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*(Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

method	result	size
meijerg	$\frac{4\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\left(\frac{bx}{8}+\frac{5}{8}\right)\sqrt{\frac{bx}{2}+1}+3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}\,\sqrt{\pi}}$	54
risch	$\frac{(bx+5)\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	65
default	$\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	72

input
```
int((b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/b^(1/2)/Pi^(1/2)*(4/3*Pi^(1/2)*b^(1/2)*x^(1/2)*2^(1/2)*(1/8*b*x+5/8)*(1/
2*b*x+1)^(1/2)+Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.65

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx = \left[\frac{(b^2x+5\,b)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{2\,b}, \frac{(b^2x+5\,b)\sqrt{bx+2}\sqrt{x}}{}\right.$$

input
```
integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
 2)*sqrt(b)*sqrt(x) + 1))/b, 1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) - 6*
sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))/b]
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx = \frac{b^2x^{\frac{5}{2}}}{2\sqrt{bx+2}} + \frac{7bx^{\frac{3}{2}}}{2\sqrt{bx+2}} + \frac{5\sqrt{x}}{\sqrt{bx+2}} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

input
```
integrate((b*x+2)**(3/2)/x**(1/2),x)
```

output
```
b**2*x**(5/2)/(2*sqrt(b*x + 2)) + 7*b*x**(3/2)/(2*sqrt(b*x + 2)) + 5*sqrt(
x)/sqrt(b*x + 2) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(43) = 86$.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx = -\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,\sqrt{b}} - \frac{\frac{3\,\sqrt{bx+2}b}{\sqrt{x}} - \frac{5\,(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^2 - \frac{2\,(bx+2)b}{x} + \frac{(bx+2)^2}{x^2}}$$

input
```
integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(
x)))/sqrt(b) - (3*sqrt(b*x + 2)*b/sqrt(x) - 5*(b*x + 2)^(3/2)/x^(3/2))/(b^
2 - 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2)
```

Giac [A] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx = \frac{\left(\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}\left(\frac{bx+2}{b}+\frac{3}{b}\right)-\frac{6\,\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)}{\sqrt{b}}\right)b}{2\,|b|}$$

input
```
integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)/b + 3/b) - 6*log(abs
(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx = \int \frac{{\left(b\,x+2\right)}^{3/2}}{\sqrt{x}}\,dx$$

input
```
int((b*x + 2)^(3/2)/x^(1/2),x)
```

output
```
int((b*x + 2)^(3/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+2}\,b^2x+5\sqrt{x}\,\sqrt{bx+2}\,b+6\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{2b}$$

input
```
int((b*x+2)^(3/2)/x^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b**2*x + 5*sqrt(x)*sqrt(b*x + 2)*b + 6*sqrt(b)*log(
(sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/(2*b)
```

3.469 $\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = -\frac{4\sqrt{2+bx}}{\sqrt{x}} + b\sqrt{x}\sqrt{2+bx} + 6\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

output
```
-4*(b*x+2)^(1/2)/x^(1/2)+b*x^(1/2)*(b*x+2)^(1/2)+6*b^(1/2)*arcsinh(1/2*b^(
1/2)*x^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = \frac{(-4+bx)\sqrt{2+bx}}{\sqrt{x}} - 12\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)$$

input
```
Integrate[(2 + b*x)^(3/2)/x^(3/2),x]
```

output
```
((-4 + b*x)*Sqrt[2 + b*x])/Sqrt[x] - 12*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/
(Sqrt[2] - Sqrt[2 + b*x])]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {57, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+2)^{3/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$3b\int \frac{\sqrt{bx+2}}{\sqrt{x}}dx-\frac{2(bx+2)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$3b\left(\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx+\sqrt{x}\sqrt{bx+2}\right)-\frac{2(bx+2)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{63}$$

$$3b\left(2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}+\sqrt{x}\sqrt{bx+2}\right)-\frac{2(bx+2)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{222}$$

$$3b\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{bx+2}\right)-\frac{2(bx+2)^{3/2}}{\sqrt{x}}$$

input
```
Int[(2 + b*x)^(3/2)/x^(3/2),x]
```

output
```
(-2*(2 + b*x)^(3/2))/Sqrt[x] + 3*b*(Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sq
rt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

method	result	size
meijerg	$\frac{3\sqrt{b}\left(-\frac{8\sqrt{\pi}\,\sqrt{2}\left(-\frac{bx}{4}+1\right)\sqrt{\frac{bx}{2}+1}}{3\sqrt{x}\,\sqrt{b}}+4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	55
risch	$\frac{b^{2}x^{2}-2bx-8}{\sqrt{x}\,\sqrt{bx+2}}+\frac{3\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	72

input
```
int((b*x+2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*b^(1/2)/Pi^(1/2)*(-8/3*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(-1/4*b*x+1)*(
1/2*b*x+1)^(1/2)+4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = \left[\frac{3\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\sqrt{bx+2}(bx-4)\sqrt{x}}{x}, -\frac{6\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{}}{\sqrt{bx+}}\right.}{}\right.$$

input
```
integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[(3*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + sqrt(b*x + 2)
*(b*x - 4)*sqrt(x))/x, -(6*sqrt(-b)*x*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2
)) - sqrt(b*x + 2)*(b*x - 4)*sqrt(x))/x]
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = 6\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{bx+2}}-\frac{2b\sqrt{x}}{\sqrt{bx+2}}-\frac{8}{\sqrt{x}\sqrt{bx+2}}$$

input
```
integrate((b*x+2)**(3/2)/x**(3/2),x)
```

output
```
6*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/sqrt(b*x + 2) -
 2*b*sqrt(x)/sqrt(b*x + 2) - 8/(sqrt(x)*sqrt(b*x + 2))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = -3\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right) - \frac{4\sqrt{bx+2}}{\sqrt{x}} - \frac{2\sqrt{bx+2}b}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

input
```
integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-3*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)
/sqrt(x))) - 4*sqrt(b*x + 2)/sqrt(x) - 2*sqrt(b*x + 2)*b/((b - (b*x + 2)/x
)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = \frac{\left(\frac{\sqrt{bx+2}(bx-4)}{\sqrt{(bx+2)b-2b}} - \frac{6\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2b}\right|\right)}{\sqrt{b}}\right)b^2}{|b|}$$

input
```
integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
(sqrt(b*x + 2)*(b*x - 4)/sqrt((b*x + 2)*b - 2*b) - 6*log(abs(-sqrt(b*x + 2
)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b))*b^2/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = \int \frac{(b\,x+2)^{3/2}}{x^{3/2}}\,dx$$

input
```
int((b*x + 2)^(3/2)/x^(3/2),x)
```

output
```
int((b*x + 2)^(3/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{bx+2}\,bx-8\sqrt{x}\,\sqrt{bx+2}+12\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)x-9\sqrt{b}\,x}{2x}$$

input
```
int((b*x+2)^(3/2)/x^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b*x - 8*sqrt(x)*sqrt(b*x + 2) + 12*sqrt(b)*log((s
qrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2))*x - 9*sqrt(b)*x)/(2*x)
```

3.470 $\int \frac{(2+bx)^{3/2}}{x^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}} dx = -\frac{4\sqrt{2+bx}}{3x^{3/2}} - \frac{8b\sqrt{2+bx}}{3\sqrt{x}} + 2b^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

output
```
-4/3*(b*x+2)^(1/2)/x^(3/2)-8/3*b*(b*x+2)^(1/2)/x^(1/2)+2*b^(3/2)*arcsinh(1
/2*b^(1/2)*x^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}} dx = -\frac{4\sqrt{2+bx}(1+2bx)}{3x^{3/2}} - 2b^{3/2} \log \left(-\sqrt{b}\sqrt{x} + \sqrt{2+bx}\right)$$

input
```
Integrate[(2 + b*x)^(3/2)/x^(5/2),x]
```

output
```
(-4*Sqrt[2 + b*x]*(1 + 2*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[2 + b*x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {57, 57, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+2)^{3/2}}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$b\int \frac{\sqrt{bx+2}}{x^{3/2}}dx-\frac{2(bx+2)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 57}$$

$$b\left(b\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx-\frac{2\sqrt{bx+2}}{\sqrt{x}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 63}$$

$$b\left(2b\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}-\frac{2\sqrt{bx+2}}{\sqrt{x}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 222}$$

$$b\left(2\sqrt{b}\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}$$

input
```
Int[(2 + b*x)^(3/2)/x^(5/2),x]
```

output
```
(-2*(2 + b*x)^(3/2))/(3*x^(3/2)) + b*((-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[
b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
meijerg	$\frac{3 b^{\frac{3}{2}} \left(-\frac{16 \sqrt{\pi}\, \sqrt{2}\, \left(2 b x +1\right) \sqrt{\frac{b x}{2}+1}}{9 x^{\frac{3}{2}} b^{\frac{3}{2}}}+\frac{8 \sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{\sqrt{b}\, \sqrt{x}\, \sqrt{2}}{2}\right)}{3}\right)}{4 \sqrt{\pi}}$	55
risch	$-\frac{4 \left(2 b^{2} x^{2}+5 b x +2\right)}{3 x^{\frac{3}{2}} \sqrt{b x +2}}+\frac{b^{\frac{3}{2}} \ln \left(\frac{b x +1}{\sqrt{b}}+\sqrt{b \,x^{2}+2 x}\right) \sqrt{x \left(b x +2\right)}}{\sqrt{x}\, \sqrt{b x +2}}$	73

input
```
int((b*x+2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*b^(3/2)/Pi^(1/2)*(-16/9*Pi^(1/2)/x^(3/2)*2^(1/2)/b^(3/2)*(2*b*x+1)*(1/
2*b*x+1)^(1/2)+8/3*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = \left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-4\,(2\,bx+1)\sqrt{bx+2}\sqrt{x}}{3\,x^2}, -\frac{2\left(3\,\sqrt{-b}bx^2\operatorname{arcta}\right.}{}\right.$$

input
```
integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 4*(2*b*
x + 1)*sqrt(b*x + 2)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(-b)*
sqrt(x)/sqrt(b*x + 2)) + 2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{8b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3} - b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) + 2b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right) - \frac{4\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

input
```
integrate((b*x+2)**(3/2)/x**(5/2),x)
```

output
```
-8*b**(3/2)*sqrt(1 + 2/(b*x))/3 - b**(3/2)*log(1/(b*x)) + 2*b**(3/2)*log(s
qrt(1 + 2/(b*x)) + 1) - 4*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = -b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right) - \frac{2\sqrt{bx+2}b}{\sqrt{x}} - \frac{2\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/s
qrt(x))) - 2*sqrt(b*x + 2)*b/sqrt(x) - 2/3*(b*x + 2)^(3/2)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2\,b^3\left(\frac{3\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)}{\sqrt{b}} + \frac{2\,(2\,(bx+2)b-3\,b)\sqrt{bx+2}}{((bx+2)b-2\,b)^{\frac{3}{2}}}\right)}{3\,|b|}$$

input
```
integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*b^3*(3*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqr
t(b) + 2*(2*(b*x + 2)*b - 3*b)*sqrt(b*x + 2)/((b*x + 2)*b - 2*b)^(3/2))/ab
s(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = \int \frac{(b\,x+2)^{3/2}}{x^{5/2}}\,dx$$

input
```
int((b*x + 2)^(3/2)/x^(5/2),x)
```

output
```
int((b*x + 2)^(3/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = \frac{-\frac{8\sqrt{x}\,\sqrt{bx+2}\,bx}{3}-\frac{4\sqrt{x}\,\sqrt{bx+2}}{3}+2\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)b\,x^2}{x^2}$$

input
```
int((b*x+2)^(3/2)/x^(5/2),x)
```

output
```
(2*( - 4*sqrt(x)*sqrt(b*x + 2)*b*x - 2*sqrt(x)*sqrt(b*x + 2) + 3*sqrt(b)*l
og((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2))*b*x**2))/(3*x**2)
```

3.471 $\int x^{5/2}(2+bx)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int x^{5/2}(2+bx)^{5/2}\,dx=\frac{5\sqrt{x}\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{9}{8}x^{7/2}\sqrt{2+bx}+\frac{5}{6}bx^{9/2}\sqrt{2+bx}+\frac{1}{6}b^2x^{11/2}\sqrt{2+bx}-\frac{5\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}$$

output
```
5/16*x^(1/2)*(b*x+2)^(1/2)/b^3-5/48*x^(3/2)*(b*x+2)^(1/2)/b^2+1/24*x^(5/2)
*(b*x+2)^(1/2)/b+9/8*x^(7/2)*(b*x+2)^(1/2)+5/6*b*x^(9/2)*(b*x+2)^(1/2)+1/6
*b^2*x^(11/2)*(b*x+2)^(1/2)-5/8*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68

$$\int x^{5/2}(2+bx)^{5/2}\,dx=\frac{\sqrt{x}\sqrt{2+bx}(15-5bx+2b^2x^2+54b^3x^3+40b^4x^4+8b^5x^5)}{48b^3}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{4b^{7/2}}$$

input
```
Integrate[x^(5/2)*(2 + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 54*b^3*x^3 + 40*b^4*x^4 +
 8*b^5*x^5))/(48*b^3) + (5*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b
*x])])/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {60, 60, 60, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(bx+2)^{5/2}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{5}{6}\int x^{5/2}(bx+2)^{3/2}dx+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{6}\left(\frac{3}{5}\int x^{5/2}\sqrt{bx+2}dx+\frac{1}{5}x^{7/2}(bx+2)^{3/2}\right)+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{6}\left(\frac{3}{5}\left(\frac{1}{4}\int \frac{x^{5/2}}{\sqrt{bx+2}}dx+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}\right)+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\int \frac{x^{3/2}}{\sqrt{bx+2}}dx}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}\right)+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int\frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}\right)+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

$\Big\downarrow$ 60

$$\frac{5}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int\frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}\right)+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

$\Big\downarrow$ 63

$$\frac{5}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int\frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}\right)+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

$\Big\downarrow$ 222

$$\frac{5}{6}\left(\frac{3}{5}\left(\frac{1}{4}\left(\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{7/2}(bx+2)^{3/2}\right)+\frac{1}{6}x^{7/2}(bx+2)^{5/2}$$

input
```
Int[x^(5/2)*(2 + b*x)^(5/2),x]
```

output
```
(x^(7/2)*(2 + b*x)^(5/2))/6 + (5*((x^(7/2)*(2 + b*x)^(3/2))/5 + (3*((x^(7/
2)*Sqrt[2 + b*x])/4 + ((x^(5/2)*Sqrt[2 + b*x])/(3*b) - (5*((x^(3/2)*Sqrt[2
 + b*x])/(2*b) - (3*((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[
x])/Sqrt[2]])/b^(3/2)))/(2*b)))/(3*b))/4))/5))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

method	result
meijerg	$-\frac{120\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(56b^5x^5+280b^4x^4+378b^3x^3+14b^2x^2-35bx+105)\sqrt{\frac{bx}{2}+1}}{40320}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{192}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$
risch	$\frac{(8b^5x^5+40b^4x^4+54b^3x^3+2b^2x^2-5bx+15)\sqrt{x}\,\sqrt{bx+2}}{48b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)\sqrt{x(bx+2)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{7}{2}}}{6b}-\frac{5\left(\frac{x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}}}{5b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{7}{2}}}{4b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3}+\frac{5\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{4b}\right)}{5b}\right)}{6b}$

input
```
int(x^(5/2)*(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-120/b^(7/2)/Pi^(1/2)*(-1/40320*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(56*b^5*x
^5+280*b^4*x^4+378*b^3*x^3+14*b^2*x^2-35*b*x+105)*(1/2*b*x+1)^(1/2)+1/192*
Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.14

$$\int x^{5/2}(2+bx)^{5/2}\,dx=\left[\frac{(8\,b^6x^5+40\,b^5x^4+54\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\right.}{48\,b^4}\right.$$

input
```
integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*
sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b^4, 1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2
*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqr
t(b*x + 2)))/b^4]
```

Sympy [A] (verification not implemented)

Time = 153.94 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int x^{5/2}(2+bx)^{5/2}\,dx = \frac{b^3x^{\frac{13}{2}}}{6\sqrt{bx+2}} + \frac{7b^2x^{\frac{11}{2}}}{6\sqrt{bx+2}} + \frac{67bx^{\frac{9}{2}}}{24\sqrt{bx+2}} + \frac{55x^{\frac{7}{2}}}{24\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{48b\sqrt{bx+2}} + \frac{5x^{\frac{3}{2}}}{48b^2\sqrt{bx+2}} + \frac{5\sqrt{x}}{8b^3\sqrt{bx+2}} - \frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}}$$

input
```
integrate(x**(5/2)*(b*x+2)**(5/2),x)
```

output
```
b**3*x**(13/2)/(6*sqrt(b*x + 2)) + 7*b**2*x**(11/2)/(6*sqrt(b*x + 2)) + 67
*b*x**(9/2)/(24*sqrt(b*x + 2)) + 55*x**(7/2)/(24*sqrt(b*x + 2)) - x**(5/2)
/(48*b*sqrt(b*x + 2)) + 5*x**(3/2)/(48*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(8*
b**3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(103) = 206.

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51

$$\int x^{5/2}(2+bx)^{5/2}\,dx = \frac{\frac{15\sqrt{bx+2}b^5}{\sqrt{x}} - \frac{85(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{198(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}} + \frac{198(bx+2)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}} - \frac{85(bx+2)^{\frac{9}{2}}b}{x^{\frac{9}{2}}} + \frac{15(bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^9 - \frac{6(bx+2)b^8}{x} + \frac{15(bx+2)^2b^7}{x^2} - \frac{20(bx+2)^3b^6}{x^3} + \frac{15(bx+2)^4b^5}{x^4} - \frac{6(bx+2)^5b^4}{x^5} + \frac{(bx+2)^6b^3}{x^6}\right)} + \frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
1/24*(15*sqrt(b*x + 2)*b^5/sqrt(x) - 85*(b*x + 2)^(3/2)*b^4/x^(3/2) + 198*
(b*x + 2)^(5/2)*b^3/x^(5/2) + 198*(b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(b*x +
2)^(9/2)*b/x^(9/2) + 15*(b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x + 2)*b^8/
x + 15*(b*x + 2)^2*b^7/x^2 - 20*(b*x + 2)^3*b^6/x^3 + 15*(b*x + 2)^4*b^5/x
^4 - 6*(b*x + 2)^5*b^4/x^5 + (b*x + 2)^6*b^3/x^6) + 5/16*log(-(sqrt(b) - s
qrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(103) = 206$.

Time = 22.49 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.72

$$\int x^{5/2}(2+bx)^{5/2}\,dx = \frac{\left(\left(\left(2\left((bx+2)\left(4\,(bx+2)\left(\frac{5\,(bx+2)}{b^5}-\frac{61}{b^5}\right)+\frac{1251}{b^5}\right)-\frac{3481}{b^5}\right)(bx+2)+\frac{11395}{b^5}\right)(bx+2)-\frac{11895}{b^5}\right)}{}$$

input
```
integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
1/240*((((2*((b*x + 2)*(4*(b*x + 2)*(5*(b*x + 2)/b^5 - 61/b^5) + 1251/b^5)
 - 3481/b^5)*(b*x + 2) + 11395/b^5)*(b*x + 2) - 11895/b^5)*sqrt((b*x + 2)*
b - 2*b)*sqrt(b*x + 2) - 6930*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x +
 2)*b - 2*b)))/b^(9/2))*b*abs(b) + 120*(((b*x + 2)*(2*(b*x + 2)*(3*(b*x +
2)/b^3 - 25/b^3) + 163/b^3) - 279/b^3)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x +
2) - 210*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/b^(5/2
))*abs(b)/b + 36*(((2*((4*b*x - 33)*(b*x + 2) + 171)*(b*x + 2) - 745)*(b*x
 + 2) + 965)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) + 630*sqrt(b)*log(abs(-
sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^4 + 320*(((2*b
*x - 9)*(b*x + 2) + 33)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) + 30*sqrt(b)
*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^4)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(2+bx)^{5/2}\,dx = \int x^{5/2}\,(b\,x+2)^{5/2}\,dx$$

input
```
int(x^(5/2)*(b*x + 2)^(5/2),x)
```

output
```
int(x^(5/2)*(b*x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int x^{5/2}(2+bx)^{5/2}\,dx = \frac{8\sqrt{x}\,\sqrt{bx+2}\,b^6x^5 + 40\sqrt{x}\,\sqrt{bx+2}\,b^5x^4 + 54\sqrt{x}\,\sqrt{bx+2}\,b^4x^3 + 2\sqrt{x}\,\sqrt{bx+2}\,b^3x^2 - 5\sqrt{x}\,\cdots}{48b^4}$$

input
```
int(x^(5/2)*(b*x+2)^(5/2),x)
```

output
```
(8*sqrt(x)*sqrt(b*x + 2)*b**6*x**5 + 40*sqrt(x)*sqrt(b*x + 2)*b**5*x**4 +
54*sqrt(x)*sqrt(b*x + 2)*b**4*x**3 + 2*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 - 5
*sqrt(x)*sqrt(b*x + 2)*b**2*x + 15*sqrt(x)*sqrt(b*x + 2)*b - 30*sqrt(b)*lo
g((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/(48*b**4)
```

3.472 $\int x^{3/2}(2+bx)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int x^{3/2}(2+bx)^{5/2}\,dx = -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{31}{20}x^{5/2}\sqrt{2+bx}+\frac{21}{20}bx^{7/2}\sqrt{2+bx}+\frac{1}{5}b^2x^{9/2}\sqrt{2+bx}+\frac{3\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

output
```
-3/8*x^(1/2)*(b*x+2)^(1/2)/b^2+1/8*x^(3/2)*(b*x+2)^(1/2)/b+31/20*x^(5/2)*(
b*x+2)^(1/2)+21/20*b*x^(7/2)*(b*x+2)^(1/2)+1/5*b^2*x^(9/2)*(b*x+2)^(1/2)+3
/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int x^{3/2}(2+bx)^{5/2}\,dx = \frac{\sqrt{x}\sqrt{2+bx}(-15+5bx+62b^2x^2+42b^3x^3+8b^4x^4)}{40b^2}-\frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{2b^{5/2}}$$

input
```
Integrate[x^(3/2)*(2 + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(-15 + 5*b*x + 62*b^2*x^2 + 42*b^3*x^3 + 8*b^4*x^4)
)/(40*b^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/(2*b
^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {60, 60, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(bx+2)^{5/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\int x^{3/2}(bx+2)^{3/2}dx+\frac{1}{5}x^{5/2}(bx+2)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\int x^{3/2}\sqrt{bx+2}dx+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\left(\frac{1}{3}\int \frac{x^{3/2}}{\sqrt{bx+2}}dx+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \; 63$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int\frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

$$\Big\downarrow \; 222$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{bx+2}\right)+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}$$

input

```
Int[x^(3/2)*(2 + b*x)^(5/2),x]
```

output

```
(x^(5/2)*(2 + b*x)^(3/2))/4 + (x^(5/2)*(2 + b*x)^(5/2))/5 + (3*((x^(5/2)*S
qrt[2 + b*x])/3 + ((x^(3/2)*Sqrt[2 + b*x])/(2*b) - (3*((Sqrt[x]*Sqrt[2 + b
*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)))/(2*b))/3))/4
```

Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

method	result	size
meijerg	$-\frac{60\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(-8b^{4}x^{4}-42b^{3}x^{3}-62b^{2}x^{2}-5bx+15\right)\sqrt{\frac{bx}{2}+1}}{2400}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{80}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	79
risch	$\frac{\left(8b^{4}x^{4}+42b^{3}x^{3}+62b^{2}x^{2}+5bx-15\right)\sqrt{x}\,\sqrt{bx+2}}{40b^{2}}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	93
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}}}{5b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{7}{2}}}{4b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3}+\frac{5\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{4b}\right)}{5b}$	126

input
```
int(x^(3/2)*(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-60/b^(5/2)/Pi^(1/2)*(1/2400*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(-8*b^4*x^4-
42*b^3*x^3-62*b^2*x^2-5*b*x+15)*(1/2*b*x+1)^(1/2)-1/80*Pi^(1/2)*arcsinh(1/
2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.20

$$\int x^{3/2}(2+bx)^{5/2}\,dx=\left[\frac{\left(8\,b^{5}x^{4}+42\,b^{4}x^{3}+62\,b^{3}x^{2}+5\,b^{2}x-15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right.}{40\,b^{3}}\right.$$

input
```
integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
[1/40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2
)*sqrt(x) + 15*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3,
1/40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)
*sqrt(x) - 30*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))/b^3]
```

Sympy [A] (verification not implemented)

Time = 27.76 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int x^{3/2}(2+bx)^{5/2}\,dx = \frac{b^3x^{\frac{11}{2}}}{5\sqrt{bx+2}} + \frac{29b^2x^{\frac{9}{2}}}{20\sqrt{bx+2}} + \frac{73bx^{\frac{7}{2}}}{20\sqrt{bx+2}} + \frac{129x^{\frac{5}{2}}}{40\sqrt{bx+2}} - \frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{bx+2}} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

input
```
integrate(x**(3/2)*(b*x+2)**(5/2),x)
```

output
```
b**3*x**(11/2)/(5*sqrt(b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(b*x + 2)) + 7
3*b*x**(7/2)/(20*sqrt(b*x + 2)) + 129*x**(5/2)/(40*sqrt(b*x + 2)) - x**(3/
2)/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(b*x + 2)) + 3*asinh(sqrt(2
)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(88) = 176.

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.53

$$\int x^{3/2}(2+bx)^{5/2}\,dx = -\frac{\frac{15\sqrt{bx+2}b^4}{\sqrt{x}} - \frac{70(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}} + \frac{128(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}} + \frac{70(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}} - \frac{15(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^7 - \frac{5(bx+2)b^6}{x} + \frac{10(bx+2)^2b^5}{x^2} - \frac{10(bx+2)^3b^4}{x^3} + \frac{5(bx+2)^4b^3}{x^4} - \frac{(bx+2)^5b^2}{x^5}\right)} - \frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) + 128
*(b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)
^(9/2)/x^(9/2))/(b^7 - 5*(b*x + 2)*b^6/x + 10*(b*x + 2)^2*b^5/x^2 - 10*(b*
x + 2)^3*b^4/x^3 + 5*(b*x + 2)^4*b^3/x^4 - (b*x + 2)^5*b^2/x^5) - 3/8*log(
-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5
/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(88) = 176$.

Time = 22.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.69

$$\int x^{3/2}(2+bx)^{5/2}\,dx = \frac{10\left(\left((bx+2)\left(2\,(bx+2)\left(\frac{3\,(bx+2)}{b^3}-\frac{25}{b^3}\right)+\frac{163}{b^3}\right)-\frac{279}{b^3}\right)\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}-\frac{210\log\left(\left|-\sqrt{\right.\right.}}{}}{}$$

input
```
integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
1/40*(10*(((b*x + 2)*(2*(b*x + 2)*(3*(b*x + 2)/b^3 - 25/b^3) + 163/b^3) -
279/b^3)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) - 210*log(abs(-sqrt(b*x + 2
)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/b^(5/2))*abs(b) + (((2*((4*b*x - 33)
*(b*x + 2) + 171)*(b*x + 2) - 745)*(b*x + 2) + 965)*sqrt((b*x + 2)*b - 2*b
)*sqrt(b*x + 2) + 630*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x +
 2)*b - 2*b))))*abs(b)/b^3 + 80*(((2*b*x - 9)*(b*x + 2) + 33)*sqrt((b*x +
2)*b - 2*b)*sqrt(b*x + 2) + 30*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sq
rt((b*x + 2)*b - 2*b))))*abs(b)/b^3 + 160*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*
x + 2)*(b*x - 3) - 6*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x +
2)*b - 2*b))))*abs(b)/b^3)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(2+bx)^{5/2}\,dx=\int x^{3/2}\,{\left(b\,x+2\right)}^{5/2}\,dx$$

input
```
int(x^(3/2)*(b*x + 2)^(5/2),x)
```

output
```
int(x^(3/2)*(b*x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x^{3/2}(2+bx)^{5/2}\,dx=\frac{8\sqrt{x}\,\sqrt{bx+2}\,b^5x^4+42\sqrt{x}\,\sqrt{bx+2}\,b^4x^3+62\sqrt{x}\,\sqrt{bx+2}\,b^3x^2+5\sqrt{x}\,\sqrt{bx+2}\,b^2x-15\sqrt{x}\,\sqrt{\cdots}}{40b^3}$$

input
```
int(x^(3/2)*(b*x+2)^(5/2),x)
```

output
```
(8*sqrt(x)*sqrt(b*x + 2)*b**5*x**4 + 42*sqrt(x)*sqrt(b*x + 2)*b**4*x**3 +
62*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 + 5*sqrt(x)*sqrt(b*x + 2)*b**2*x - 15*s
qrt(x)*sqrt(b*x + 2)*b + 30*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/
sqrt(2)))/(40*b**3)
```

3.473 $\int \sqrt{x}(2+bx)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \frac{5\sqrt{x}\sqrt{2+bx}}{8b}+\frac{59}{24}x^{3/2}\sqrt{2+bx}+\frac{17}{12}bx^{5/2}\sqrt{2+bx}+\frac{1}{4}b^2x^{7/2}\sqrt{2+bx}-\frac{5\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

output
```
5/8*x^(1/2)*(b*x+2)^(1/2)/b+59/24*x^(3/2)*(b*x+2)^(1/2)+17/12*b*x^(5/2)*(b
*x+2)^(1/2)+1/4*b^2*x^(7/2)*(b*x+2)^(1/2)-5/4*arcsinh(1/2*b^(1/2)*x^(1/2)*
2^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \frac{\sqrt{x}\sqrt{2+bx}(15+59bx+34b^2x^2+6b^3x^3)}{24b}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{2b^{3/2}}$$

input
```
Integrate[Sqrt[x]*(2 + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(15 + 59*b*x + 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5
*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(bx+2)^{5/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{4}\int \sqrt{x}(bx+2)^{3/2}dx+\frac{1}{4}x^{3/2}(bx+2)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{4}\left(\int \sqrt{x}\sqrt{bx+2}dx+\frac{1}{3}x^{3/2}(bx+2)^{3/2}\right)+\frac{1}{4}x^{3/2}(bx+2)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{3/2}(bx+2)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{3/2}(bx+2)^{5/2}$$

$$\Big\downarrow \text{63}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{3/2}(bx+2)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{5}{4}\left(\frac{1}{2}\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}\right)+\frac{1}{4}x^{3/2}(bx+2)^{5/2}$$

input
```
Int[Sqrt[x]*(2 + b*x)^(5/2),x]
```

output
```
(x^(3/2)*(2 + b*x)^(5/2))/4 + (5*((x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2
+ b*x)^(3/2))/3 + ((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x]
)/Sqrt[2]])/b^(3/2))/2))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.67

method	result	size
meijerg	$-\frac{30\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(6b^3x^3+34b^2x^2+59bx+15)\sqrt{\frac{bx}{2}+1}}{720}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{24}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	71
risch	$\frac{(6b^3x^3+34b^2x^2+59bx+15)\sqrt{x}\,\sqrt{bx+2}}{24b}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)\sqrt{x(bx+2)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}$	85
default	$\frac{\sqrt{x}\,(bx+2)^{\frac{7}{2}}}{4b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3}+\frac{5\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{4b}$	105

input
```
int(x^(1/2)*(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-30/b^(3/2)/Pi^(1/2)*(-1/720*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(6*b^3*x^3+3
4*b^2*x^2+59*b*x+15)*(1/2*b*x+1)^(1/2)+1/24*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x
^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.29

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \left[\frac{(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{24\,b^2}, \frac{(6\,b^4x\right.$$

input
```
integrate(x^(1/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*((6*b^4*x^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) +
15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/24*((6*b^4
*x^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*a
rctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \frac{b^3x^{\frac{9}{2}}}{4\sqrt{bx+2}} + \frac{23b^2x^{\frac{7}{2}}}{12\sqrt{bx+2}} + \frac{127bx^{\frac{5}{2}}}{24\sqrt{bx+2}} + \frac{133x^{\frac{3}{2}}}{24\sqrt{bx+2}} + \frac{5\sqrt{x}}{4b\sqrt{bx+2}} - \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}}$$

input
```
integrate(x**(1/2)*(b*x+2)**(5/2),x)
```

output
```
b**3*x**(9/2)/(4*sqrt(b*x + 2)) + 23*b**2*x**(7/2)/(12*sqrt(b*x + 2)) + 12
7*b*x**(5/2)/(24*sqrt(b*x + 2)) + 133*x**(3/2)/(24*sqrt(b*x + 2)) + 5*sqrt
(x)/(4*b*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(73) = 146$.

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.52

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \frac{\frac{15\sqrt{bx+2}b^3}{\sqrt{x}} - \frac{55(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}} + \frac{73(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}} + \frac{15(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^5 - \frac{4(bx+2)b^4}{x} + \frac{6(bx+2)^2b^3}{x^2} - \frac{4(bx+2)^3b^2}{x^3} + \frac{(bx+2)^4b}{x^4}\right)} + \frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) - 55*(b*x + 2)^(3/2)*b^2/x^(3/2) + 73*(
b*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x + 2)*
b^4/x + 6*(b*x + 2)^2*b^3/x^2 - 4*(b*x + 2)^3*b^2/x^3 + (b*x + 2)^4*b/x^4)
 + 5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqr
t(x)))/b^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(73) = 146$.

Time = 22.68 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.93

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \frac{\left(\left((bx+2)\left(2\,(bx+2)\left(\frac{3\,(bx+2)}{b^3}-\frac{25}{b^3}\right)+\frac{163}{b^3}\right)-\frac{279}{b^3}\right)\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}-\frac{210\,\log\left(\left|-\sqrt{bx\cdots}\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate(x^(1/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
1/24*((((b*x + 2)*(2*(b*x + 2)*(3*(b*x + 2)/b^3 - 25/b^3) + 163/b^3) - 279
/b^3)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) - 210*log(abs(-sqrt(b*x + 2)*s
qrt(b) + sqrt((b*x + 2)*b - 2*b)))/b^(5/2))*b*abs(b) + 24*(((2*b*x - 9)*(b
*x + 2) + 33)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) + 30*sqrt(b)*log(abs(-
sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^2 + 144*(sqrt(
(b*x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3) - 6*sqrt(b)*log(abs(-sqrt(b*x +
 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^2 + 192*(2*sqrt(b)*log(a
bs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))) + sqrt((b*x + 2)*b -
 2*b)*sqrt(b*x + 2))*abs(b)/b^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \int \sqrt{x}\,{\left(b\,x+2\right)}^{5/2}\,dx$$

input
```
int(x^(1/2)*(b*x + 2)^(5/2),x)
```

output
```
int(x^(1/2)*(b*x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int \sqrt{x}(2+bx)^{5/2}\,dx = \frac{6\sqrt{x}\,\sqrt{bx+2}\,b^4x^3+34\sqrt{x}\,\sqrt{bx+2}\,b^3x^2+59\sqrt{x}\,\sqrt{bx+2}\,b^2x+15\sqrt{x}\,\sqrt{bx+2}\,b-30\sqrt{b}\log\left(\right.}{24b^2}$$

input
```
int(x^(1/2)*(b*x+2)^(5/2),x)
```

output
```
(6*sqrt(x)*sqrt(b*x + 2)*b**4*x**3 + 34*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 +
59*sqrt(x)*sqrt(b*x + 2)*b**2*x + 15*sqrt(x)*sqrt(b*x + 2)*b - 30*sqrt(b)*
log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/(24*b**2)
```

3.474 $\int \frac{(2+bx)^{5/2}}{\sqrt{x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}} dx = \frac{11}{2}\sqrt{x}\sqrt{2+bx}+\frac{13}{6}bx^{3/2}\sqrt{2+bx}+\frac{1}{3}b^2x^{5/2}\sqrt{2+bx}+\frac{5\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

output
```
11/2*x^(1/2)*(b*x+2)^(1/2)+13/6*b*x^(3/2)*(b*x+2)^(1/2)+1/3*b^2*x^(5/2)*(b
*x+2)^(1/2)+5*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}} dx = \frac{1}{6}\sqrt{x}\sqrt{2+bx}\left(33+13bx+2b^2x^2\right) - \frac{5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

input
```
Integrate[(2 + b*x)^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(33 + 13*b*x + 2*b^2*x^2))/6 - (5*Log[-(Sqrt[b]*Sqr
t[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+2)^{5/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{3}\int \frac{(bx+2)^{3/2}}{\sqrt{x}}dx+\frac{1}{3}\sqrt{x}(bx+2)^{5/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{bx+2}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)+\frac{1}{3}\sqrt{x}(bx+2)^{5/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)+\frac{1}{3}\sqrt{x}(bx+2)^{5/2}$$

$$\Big\downarrow \text{ 63}$$

$$\frac{5}{3}\left(\frac{3}{2}\left(2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)+\frac{1}{3}\sqrt{x}(bx+2)^{5/2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{5}{3}\left(\frac{3}{2}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)+\frac{1}{3}\sqrt{x}(bx+2)^{5/2}$$

input
```
Int[(2 + b*x)^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*(2 + b*x)^(5/2))/3 + (5*((Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*(Sqrt[x
]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]))/2))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

method	result	size
meijerg	$-\frac{15\left(-\frac{8\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\left(\frac{1}{24}b^{2}x^{2}+\frac{13}{48}bx+\frac{11}{16}\right)\sqrt{\frac{bx}{2}+1}}{15}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3}\right)}{\sqrt{b}\,\sqrt{\pi}}$	63
risch	$\frac{\left(2b^{2}x^{2}+13bx+33\right)\sqrt{x}\,\sqrt{bx+2}}{6}+\frac{5\sqrt{x\left(bx+2\right)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	74
default	$\frac{\sqrt{x}\,\left(bx+2\right)^{\frac{5}{2}}}{3}+\frac{5\sqrt{x}\,\left(bx+2\right)^{\frac{3}{2}}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x\left(bx+2\right)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	84

input
```
int((b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-15/b^(1/2)/Pi^(1/2)*(-8/15*Pi^(1/2)*b^(1/2)*x^(1/2)*2^(1/2)*(1/24*b^2*x^2
+13/48*b*x+11/16)*(1/2*b*x+1)^(1/2)-1/3*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/
2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.45

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = \left[\frac{(2\,b^3x^2+13\,b^2x+33\,b)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{6\,b}, \frac{(2\,b^3x^2 -}{}\right.$$

input
```
integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/6*((2*b^3*x^2 + 13*b^2*x + 33*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log
(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 + 13*b^2*x +
 33*b)*sqrt(b*x + 2)*sqrt(x) - 30*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*
x + 2)))/b]
```

Sympy [A] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{b^3x^{\frac{7}{2}}}{3\sqrt{bx+2}} + \frac{17b^2x^{\frac{5}{2}}}{6\sqrt{bx+2}} + \frac{59bx^{\frac{3}{2}}}{6\sqrt{bx+2}} + \frac{11\sqrt{x}}{\sqrt{bx+2}} + \frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

input
```
integrate((b*x+2)**(5/2)/x**(1/2),x)
```

output
```
b**3*x**(7/2)/(3*sqrt(b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(b*x + 2)) + 59*
b*x**(3/2)/(6*sqrt(b*x + 2)) + 11*sqrt(x)/sqrt(b*x + 2) + 5*asinh(sqrt(2)*
sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(58) = 116$.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.55

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = -\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\sqrt{b}} - \frac{\frac{15\sqrt{bx+2}b^2}{\sqrt{x}} - \frac{40(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} + \frac{33(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^3 - \frac{3(bx+2)b^2}{x} + \frac{3(bx+2)^2b}{x^2} - \frac{(bx+2)^3}{x^3}\right)}$$

input
```
integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(
x)))/sqrt(b) - 1/3*(15*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^
(3/2) + 33*(b*x + 2)^(5/2)/x^(5/2))/(b^3 - 3*(b*x + 2)*b^2/x + 3*(b*x + 2)
^2*b/x^2 - (b*x + 2)^3/x^3)
```

Giac [A] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{\left(\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}\left((bx+2)\left(\frac{2\,(bx+2)}{b}+\frac{5}{b}\right)+\frac{15}{b}\right) - \frac{30\,\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-\right.\right.}{\sqrt{b}}}{6\,|b|}$$

input
```
integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)*(2*(b*x + 2)/b + 5/b
) + 15/b) - 30*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/
sqrt(b))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = \int \frac{(b\,x+2)^{5/2}}{\sqrt{x}}\,dx$$

input
```
int((b*x + 2)^(5/2)/x^(1/2),x)
```

output
```
int((b*x + 2)^(5/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,\sqrt{bx+2}\,b^3x^2+13\sqrt{x}\,\sqrt{bx+2}\,b^2x+33\sqrt{x}\,\sqrt{bx+2}\,b+30\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{6b}$$

input
```
int((b*x+2)^(5/2)/x^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 + 13*sqrt(x)*sqrt(b*x + 2)*b**2*x + 33*
sqrt(x)*sqrt(b*x + 2)*b + 30*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))
/sqrt(2)))/(6*b)
```

3.475 $\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{8\sqrt{2+bx}}{\sqrt{x}}+\frac{9}{2}b\sqrt{x}\sqrt{2+bx}+\frac{1}{2}b^2x^{3/2}\sqrt{2+bx}+15\sqrt{b}\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

output
```
-8*(b*x+2)^(1/2)/x^(1/2)+9/2*b*x^(1/2)*(b*x+2)^(1/2)+1/2*b^2*x^(3/2)*(b*x+
2)^(1/2)+15*b^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = \frac{\sqrt{2+bx}(-16+9bx+b^2x^2)}{2\sqrt{x}}-30\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)$$

input
```
Integrate[(2 + b*x)^(5/2)/x^(3/2),x]
```

output
```
(Sqrt[2 + b*x]*(-16 + 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 30*Sqrt[b]*ArcTanh[(
Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {57, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+2)^{5/2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{57}$$

$$5b\int \frac{(bx+2)^{3/2}}{\sqrt{x}}dx-\frac{2(bx+2)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$5b\left(\frac{3}{2}\int \frac{\sqrt{bx+2}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)-\frac{2(bx+2)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$5b\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)-\frac{2(bx+2)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{63}$$

$$5b\left(\frac{3}{2}\left(2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)-\frac{2(bx+2)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{222}$$

$$5b\left(\frac{3}{2}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{bx+2}\right)+\frac{1}{2}\sqrt{x}(bx+2)^{3/2}\right)-\frac{2(bx+2)^{5/2}}{\sqrt{x}}$$

input
```
Int[(2 + b*x)^(5/2)/x^(3/2),x]
```

output
```
(-2*(2 + b*x)^(5/2))/Sqrt[x] + 5*b*((Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*(Sqrt
[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]))/2)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

method	result	size
meijerg	$-\frac{15\sqrt{b}\left(\frac{16\sqrt{\pi}\,\sqrt{2}\left(-\frac{1}{16}b^{2}x^{2}-\frac{9}{16}bx+1\right)\sqrt{\frac{bx}{2}+1}}{15\sqrt{x}\,\sqrt{b}}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	63
risch	$\frac{b^{3}x^{3}+11b^{2}x^{2}+2bx-32}{2\sqrt{x}\,\sqrt{bx+2}}+\frac{15\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{2\sqrt{x}\,\sqrt{bx+2}}$	81

input
```
int((b*x+2)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15/2*b^(1/2)/Pi^(1/2)*(16/15*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(-1/16*b^2*
x^2-9/16*b*x+1)*(1/2*b*x+1)^(1/2)-2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = \left[\frac{15\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+(b^2x^2+9\,bx-16)\sqrt{bx+2}\sqrt{x}}{2\,x}, -\frac{30\,\sqrt{-b}x\,a\right.$$

input
```
integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(15*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x^2
 + 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))/x, -1/2*(30*sqrt(-b)*x*arctan(sqrt(-
b)*sqrt(x)/sqrt(b*x + 2)) - (b^2*x^2 + 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))/
x]
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = 15\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) + \frac{b^3x^{\frac{5}{2}}}{2\sqrt{bx+2}} + \frac{11b^2x^{\frac{3}{2}}}{2\sqrt{bx+2}} + \frac{b\sqrt{x}}{\sqrt{bx+2}} - \frac{16}{\sqrt{x}\sqrt{bx+2}}$$

input
```
integrate((b*x+2)**(5/2)/x**(3/2),x)
```

output
```
15*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**3*x**(5/2)/(2*sqrt(b*x +
2)) + 11*b**2*x**(3/2)/(2*sqrt(b*x + 2)) + b*sqrt(x)/sqrt(b*x + 2) - 16/(s
qrt(x)*sqrt(b*x + 2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{15}{2}\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right) - \frac{\frac{7\sqrt{bx+2}b^2}{\sqrt{x}} - \frac{9(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^2 - \frac{2(bx+2)b}{x} + \frac{(bx+2)^2}{x^2}} - \frac{8\sqrt{bx+2}}{\sqrt{x}}$$

input
```
integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-15/2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 2)/sqrt(x))) - (7*sqrt(b*x + 2)*b^2/sqrt(x) - 9*(b*x + 2)^(3/2)*b/x^(3/2)
)/(b^2 - 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2) - 8*sqrt(b*x + 2)/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = \frac{\left(\frac{((bx+7)(bx+2)-30)\sqrt{bx+2}}{\sqrt{(bx+2)b-2\,b}} - \frac{30\,\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)}{\sqrt{b}}\right)b^2}{2\,|b|}$$

input
```
integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

output
```
1/2*(((b*x + 7)*(b*x + 2) - 30)*sqrt(b*x + 2)/sqrt((b*x + 2)*b - 2*b) - 30
*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b))*b^2/a
bs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = \int \frac{(b\,x+2)^{5/2}}{x^{3/2}}\,dx$$

input
```
int((b*x + 2)^(5/2)/x^(3/2),x)
```

output
```
int((b*x + 2)^(5/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+2}\,b^2x^2+9\sqrt{x}\,\sqrt{bx+2}\,bx-16\sqrt{x}\,\sqrt{bx+2}+30\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)x-2}{2x}$$

input
```
int((b*x+2)^(5/2)/x^(3/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b**2*x**2 + 9*sqrt(x)*sqrt(b*x + 2)*b*x - 16*sqrt(x
)*sqrt(b*x + 2) + 30*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)
)*x - 20*sqrt(b)*x)/(2*x)
```

3.476 $\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx=-\frac{8\sqrt{2+bx}}{3x^{3/2}}-\frac{28b\sqrt{2+bx}}{3\sqrt{x}}+b^2\sqrt{x}\sqrt{2+bx}+10b^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

output
```
-8/3*(b*x+2)^(1/2)/x^(3/2)-28/3*b*(b*x+2)^(1/2)/x^(1/2)+b^2*x^(1/2)*(b*x+2
)^(1/2)+10*b^(3/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx=\frac{\sqrt{2+bx}(-8-28bx+3b^2x^2)}{3x^{3/2}}-10b^{3/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)$$

input
```
Integrate[(2 + b*x)^(5/2)/x^(5/2),x]
```

output
```
(Sqrt[2 + b*x]*(-8 - 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 10*b^(3/2)*Log[-(S
qrt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {57, 57, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+2)^{5/2}}{x^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{5}{3}b\int \frac{(bx+2)^{3/2}}{x^{3/2}}dx-\frac{2(bx+2)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{5}{3}b\left(3b\int \frac{\sqrt{bx+2}}{\sqrt{x}}dx-\frac{2(bx+2)^{3/2}}{\sqrt{x}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{3}b\left(3b\left(\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx+\sqrt{x}\sqrt{bx+2}\right)-\frac{2(bx+2)^{3/2}}{\sqrt{x}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{63}$$

$$\frac{5}{3}b\left(3b\left(2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}+\sqrt{x}\sqrt{bx+2}\right)-\frac{2(bx+2)^{3/2}}{\sqrt{x}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{5}{3}b\left(3b\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{bx+2}\right)-\frac{2(bx+2)^{3/2}}{\sqrt{x}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}$$

input
```
Int[(2 + b*x)^(5/2)/x^(5/2),x]
```

output
```
(-2*(2 + b*x)^(5/2))/(3*x^(3/2)) + (5*b*((-2*(2 + b*x)^(3/2))/Sqrt[x] + 3*
b*(Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b])
))/3
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

method	result	size
meijerg	$-\frac{15 b^{\frac{3}{2}} \left(\frac{32 \sqrt{\pi}\, \sqrt{2}\, \left(-\frac{3}{8} b^{2} x^{2}+\frac{7}{2} b x +1\right) \sqrt{\frac{b x}{2}+1}}{45 x^{\frac{3}{2}} b^{\frac{3}{2}}}-\frac{8 \sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{\sqrt{b}\, \sqrt{x}\, \sqrt{2}}{2}\right)}{3}\right)}{4 \sqrt{\pi}}$	63
risch	$\frac{3 b^{3} x^{3}-22 b^{2} x^{2}-64 b x -16}{3 x^{\frac{3}{2}} \sqrt{b x +2}}+\frac{5 b^{\frac{3}{2}} \ln\left(\frac{b x +1}{\sqrt{b}}+\sqrt{b \,x^{2}+2 x}\right) \sqrt{x \left(b x +2\right)}}{\sqrt{x}\, \sqrt{b x +2}}$	82

input
```
int((b*x+2)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-15/4*b^(3/2)/Pi^(1/2)*(32/45*Pi^(1/2)/x^(3/2)*2^(1/2)/b^(3/2)*(-3/8*b^2*x
^2+7/2*b*x+1)*(1/2*b*x+1)^(1/2)-8/3*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.50

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx = \left[\frac{15\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+(3\,b^2x^2-28\,bx-8)\sqrt{bx+2}\sqrt{x}}{3\,x^2}, -\frac{30\,\sqrt{-bb}}{}\right.$$

input
```
integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(15*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (3*b^2
*x^2 - 28*b*x - 8)*sqrt(b*x + 2)*sqrt(x))/x^2, -1/3*(30*sqrt(-b)*b*x^2*arc
tan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)) - (3*b^2*x^2 - 28*b*x - 8)*sqrt(b*x +
2)*sqrt(x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx = b^{\frac{5}{2}}x\sqrt{1+\frac{2}{bx}} - \frac{28b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3} - 5b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) + 10b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right) - \frac{8\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

input
```
integrate((b*x+2)**(5/2)/x**(5/2),x)
```

output
```
b**(5/2)*x*sqrt(1 + 2/(b*x)) - 28*b**(3/2)*sqrt(1 + 2/(b*x))/3 - 5*b**(3/2
)*log(1/(b*x)) + 10*b**(3/2)*log(sqrt(1 + 2/(b*x)) + 1) - 8*sqrt(b)*sqrt(1
 + 2/(b*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx = -5\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{8\,\sqrt{bx+2}b}{\sqrt{x}}-\frac{2\,\sqrt{bx+2}b^2}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}-\frac{4\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-5*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)
/sqrt(x))) - 8*sqrt(b*x + 2)*b/sqrt(x) - 2*sqrt(b*x + 2)*b^2/((b - (b*x +
2)/x)*sqrt(x)) - 4/3*(b*x + 2)^(3/2)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx = -\frac{b^3\left(\frac{30\,\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)}{\sqrt{b}}-\frac{((3\,(bx+2)b-40\,b)(bx+2)+60\,b)\sqrt{bx+2}}{((bx+2)b-2\,b)^{\frac{3}{2}}}\right)}{3\,|b|}$$

input
```
integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/3*b^3*(30*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sq
rt(b) - ((3*(b*x + 2)*b - 40*b)*(b*x + 2) + 60*b)*sqrt(b*x + 2)/((b*x + 2)
*b - 2*b)^(3/2))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx = \int \frac{(b\,x+2)^{5/2}}{x^{5/2}}\,dx$$

input
```
int((b*x + 2)^(5/2)/x^(5/2),x)
```

output
```
int((b*x + 2)^(5/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx = \frac{3\sqrt{x}\,\sqrt{bx+2}\,b^2x^2 - 28\sqrt{x}\,\sqrt{bx+2}\,bx - 8\sqrt{x}\,\sqrt{bx+2} + 30\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)b\,x^2}{3x^2}$$

input
```
int((b*x+2)^(5/2)/x^(5/2),x)
```

output
```
(3*sqrt(x)*sqrt(b*x + 2)*b**2*x**2 - 28*sqrt(x)*sqrt(b*x + 2)*b*x - 8*sqrt
(x)*sqrt(b*x + 2) + 30*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(
2))*b*x**2 + 5*sqrt(b)*b*x**2)/(3*x**2)
```

3.477 $\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx=\frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}$$

output
```
5/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b^3-5/12*a*x^(3/2)*(b*x+a)^(1/2)/b^2+1/3*x^(
5/2)*(b*x+a)^(1/2)/b-5/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2
)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx=\frac{\sqrt{x}\sqrt{a+bx}(15a^2-10abx+8b^2x^2)}{24b^3}+\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{4b^{7/2}}$$

input
```
Integrate[x^(5/2)/Sqrt[a + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2))/(24*b^3) + (5*a^3*
ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{65}$$

$$\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}$$

input
```
Int[x^(5/2)/Sqrt[a + b*x],x]
```

output
```
(x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a
*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])
/b^(3/2)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{(8b^2x^2-10abx+15a^2)\sqrt{x}\,\sqrt{bx+a}}{24b^3}-\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{x^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\frac{5a\left(\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4b}\right)}{6b}$	109

input
```
int(x^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^2*x^2-10*a*b*x+15*a^2)*x^(1/2)*(b*x+a)^(1/2)/b^3-5/16*a^3/b^(7/2
)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x
+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx+a}\sqrt{x}}{48\,b^4},\right.$$

input
```
integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*
a^3*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (8*b^3*x^2 - 10*a*b^
2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{1+\frac{bx}{a}}} + \frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1+\frac{bx}{a}}} - \frac{\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}} - \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)/(b*x+a)**(1/2),x)
```

output
```
5*a**(5/2)*sqrt(x)/(8*b**3*sqrt(1 + b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2
*sqrt(1 + b*x/a)) - sqrt(a)*x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*a**3*asinh
(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/
a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \frac{5\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}} - \frac{\frac{33\,\sqrt{bx+a}a^3b^2}{\sqrt{x}} - \frac{40\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}} + \frac{15\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^6 - \frac{3\,(bx+a)b^5}{x} + \frac{3\,(bx+a)^2b^4}{x^2} - \frac{(bx+a)^3b^3}{x^3}\right)}$$

input
```
integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) - 1/24*(33*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(
3/2)*a^3*b/x^(3/2) + 15*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x + a)*b^
5/x + 3*(b*x + a)^2*b^4/x^2 - (b*x + a)^3*b^3/x^3)
```

Giac [A] (verification not implemented)

Time = 75.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \frac{\left(15\,a^3\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\left(2\left(4\,bx-9\,a\right)(bx+a)+33\,a^2\right)\sqrt{(}}{24\,b^5}$$

input
```
integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*(15*a^3*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a
*b))) + (2*(4*b*x - 9*a)*(b*x + a) + 33*a^2)*sqrt((b*x + a)*b - a*b)*sqrt(
b*x + a))*abs(b)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \int \frac{x^{5/2}}{\sqrt{a+b\,x}}\,dx$$

input
```
int(x^(5/2)/(a + b*x)^(1/2),x)
```

output
```
int(x^(5/2)/(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^2b-10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{24b^4}$$

input
```
int(x^(5/2)/(b*x+a)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**2*b - 10*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**4)
```

3.478 $\int \frac{x^{3/2}}{\sqrt{a+bx}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{x^{3/2}}{\sqrt{a+bx}} \, dx = -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{3a^2 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}$$

output
```
-3/4*a*x^(1/2)*(b*x+a)^(1/2)/b^2+1/2*x^(3/2)*(b*x+a)^(1/2)/b+3/4*a^2*arcta
nh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}}{\sqrt{a+bx}} \, dx = \frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-3a+2bx) + 6a^2 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}}$$

input
```
Integrate[x^(3/2)/Sqrt[a + b*x],x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a + 2*b*x) + 6*a^2*ArcTanh[(Sqrt[b]*Sqr
t[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{65}$$

$$\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}$$

input
```
Int[x^(3/2)/Sqrt[a + b*x],x]
```

output
```
(x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{\left(-2 b x +3 a \right) \sqrt{x}\, \sqrt{b x +a}}{4 b^{2}}+\frac{3 a^{2} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right) \sqrt{x \left(b x +a \right)}}{8 b^{\frac{5}{2}} \sqrt{x}\, \sqrt{b x +a}}$	76
default	$\frac{x^{\frac{3}{2}} \sqrt{b x +a}}{2 b}-\frac{3 a \left(\frac{\sqrt{x}\, \sqrt{b x +a}}{b}-\frac{a \sqrt{x \left(b x +a \right)}\, \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right)}{2 b^{\frac{3}{2}} \sqrt{x}\, \sqrt{b x +a}}\right)}{4 b}$	87

input
```
int(x^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*x+3*a)*x^(1/2)*(b*x+a)^(1/2)/b^2+3/8*a^2/b^(5/2)*ln((1/2*a+b*x)
/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^2x-3\,ab)\sqrt{bx+a}\sqrt{x}}{8\,b^3},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(2\,b^2x-3\,ab)\sqrt{bx+a}\sqrt{x}}{4\,b^3}\right]$$

input
```
integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(s
qrt(-b)*sqrt(x)/sqrt(b*x + a)) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/
b^3]
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = -\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}x^{\frac{3}{2}}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)/(b*x+a)**(1/2),x)
```

output
```
-3*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 + b*x/a)) - sqrt(a)*x**(3/2)/(4*b*sqrt(
1 + b*x/a)) + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2
)/(2*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = -\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}} + \frac{\frac{5\,\sqrt{bx+a}a^2b}{\sqrt{x}} - \frac{3\,(bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^4 - \frac{2\,(bx+a)b^3}{x} + \frac{(bx+a)^2b^2}{x^2}\right)}$$

input
```
integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(5/2) + 1/4*(5*sqrt(b*x + a)*a^2*b/sqrt(x) - 3*(b*x + a)^(3/2)*
a^2/x^(3/2))/(b^4 - 2*(b*x + a)*b^3/x + (b*x + a)^2*b^2/x^2)
```

Giac [A] (verification not implemented)

Time = 75.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = -\frac{\left(3\,a^2\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right) - \sqrt{(bx+a)b-ab}(2\,bx-3\,a)\sqrt{bx+a}\right)|b|}{4\,b^4}$$

input
```
integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(3*a^2*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*
b))) - sqrt((b*x + a)*b - a*b)*(2*b*x - 3*a)*sqrt(b*x + a))*abs(b)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = \int \frac{x^{3/2}}{\sqrt{a+b\,x}}\,dx$$

input
```
int(x^(3/2)/(a + b*x)^(1/2),x)
```

output
```
int(x^(3/2)/(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^3}$$

input
```
int(x^(3/2)/(b*x+a)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(
b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**3)
```

3.479 $\int \frac{\sqrt{x}}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}} dx = \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

output
```
x^(1/2)*(b*x+a)^(1/2)/b-a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}} dx = \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[a + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x])/b - (2*a*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqr
t[a + b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

input
```
Int[Sqrt[x]/Sqrt[a + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b
^(3/2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	65
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	65

input
```
int(x^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1
/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \left[\frac{a\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}b\sqrt{x}}{2\,b^{2}}, \frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)+\sqrt{bx+a}b\sqrt{x}}{b^{2}}\right]$$

input
```
integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(
b*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)
) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{b} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x**(1/2)/(b*x+a)**(1/2),x)
```

output
```
sqrt(a)*sqrt(x)*sqrt(1 + b*x/a)/b - a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3
/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}} - \frac{\sqrt{bx+a}a}{\left(b^2-\frac{(bx+a)b}{x}\right)\sqrt{x}}$$

input
```
integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt
(x)))/b^(3/2) - sqrt(b*x + a)*a/((b^2 - (b*x + a)*b/x)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 75.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \frac{\left(a\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\sqrt{(bx+a)b-ab}\sqrt{bx+a}\right)|b|}{b^3}$$

input
```
integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
(a*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))) + sq
rt((b*x + a)*b - a*b)*sqrt(b*x + a))*abs(b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{a+b\,x}}{b} - \frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int(x^(1/2)/(a + b*x)^(1/2),x)
```

output
```
(x^(1/2)*(a + b*x)^(1/2))/b - (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2
) - a^(1/2))))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,b-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b^2}$$

input
```
int(x^(1/2)/(b*x+a)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*b - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b**2
```

3.480 $\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = -\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

output
```
(-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \; 65$$

$$2\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{x}\,\sqrt{bx+a}\,\sqrt{b}}$	48

input
```
int(1/x^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)
^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = \left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)}{b}\right]$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arc
tan(sqrt(-b)*sqrt(x)/sqrt(b*x + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = \frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**(1/2),x)
```

output
```
2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = -\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))
/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 76.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{|b|}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/abs(
b)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

input
```
int(1/(x^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
-(4*atan(((a + b*x)^(1/2) - a^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = \frac{2\sqrt{b}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{b}$$

input
```
int(1/x^(1/2)/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)))/b
```

3.481 $\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

output
```
-2*(b*x+a)^(1/2)/a/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \text{ 48}$$
$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16
default	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16
risch	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16
orering	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16

input
```
int(1/x^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x+a)^(1/2)/a/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x + a)/(a*sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**(1/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x) + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(b*x + a)/(a*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{bx+a}b^2}{\sqrt{(bx+a)b-ab}a|b|}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+b\,x}}{a\,\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(a + b*x)^(1/2)),x)
```

output
```
-(2*(a + b*x)^(1/2))/(a*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}-2\sqrt{b}\,x}{ax}$$

input
```
int(1/x^(3/2)/(b*x+a)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a + b*x) + sqrt(b)*x))/(a*x)
```

3.482 $\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{3ax^{3/2}} + \frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}$$

output
```
-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2(a-2bx)\sqrt{a+bx}}{3a^2x^{3/2}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*(a - 2*b*x)*Sqrt[a + b*x])/(3*a^2*x^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \ 55$$

$$-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

$$\Big\downarrow \ 48$$

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-2bx+a)}{3x^{\frac{3}{2}}a^{2}}$	22
risch	$-\frac{2\sqrt{bx+a}\,(-2bx+a)}{3x^{\frac{3}{2}}a^{2}}$	22
orering	$-\frac{2\sqrt{bx+a}\,(-2bx+a)}{3x^{\frac{3}{2}}a^{2}}$	22
default	$-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}$	33

input
```
int(1/x^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-2*b*x+a)/x^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = \frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3ax} + \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a^2}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**(1/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*a*x) + 4*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a**
2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = \frac{2\left(\frac{3\sqrt{bx+a}b}{\sqrt{x}} - \frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}b^3\left(\frac{2(bx+a)b}{a^2}-\frac{3b}{a}\right)}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}|b|}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x + a)*b^3*(2*(b*x + a)*b/a^2 - 3*b/a)/(((b*x + a)*b - a*b)^(3/
2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{\left(\frac{2}{3a}-\frac{4bx}{3a^2}\right)\sqrt{a+bx}}{x^{3/2}}$$

input
```
int(1/(x^(5/2)*(a + b*x)^(1/2)),x)
```

output
```
-((2/(3*a) - (4*b*x)/(3*a^2))*(a + b*x)^(1/2))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,bx}{3}-\frac{4\sqrt{b}\,b\,x^2}{3}}{a^2x^2}$$

input
```
int(1/x^(5/2)/(b*x+a)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a + 2*sqrt(x)*sqrt(a + b*x)*b*x - 2*sqrt(b)*b
*x**2))/(3*a**2*x**2)
```

3.483 $\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{5ax^{5/2}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}$$

output
```
-2/5*(b*x+a)^(1/2)/a/x^(5/2)+8/15*b*(b*x+a)^(1/2)/a^2/x^(3/2)-16/15*b^2*(b
*x+a)^(1/2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}(3a^2-4abx+8b^2x^2)}{15a^3x^{5/2}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}dx$$
$$\Big\downarrow \text{ 55}$$
$$-\frac{4b\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$
$$\Big\downarrow \text{ 55}$$
$$-\frac{4b\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$
$$\Big\downarrow \text{ 48}$$
$$-\frac{4b\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x])/(5*a*x^(5/2)) - (4*b*((-2*Sqrt[a + b*x])/(3*a*x^(3/2))
+ (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(8b^{2}x^{2}-4abx+3a^{2})}{15x^{\frac{5}{2}}a^{3}}$	35
risch	$-\frac{2\sqrt{bx+a}\,(8b^{2}x^{2}-4abx+3a^{2})}{15x^{\frac{5}{2}}a^{3}}$	35
orering	$-\frac{2\sqrt{bx+a}\,(8b^{2}x^{2}-4abx+3a^{2})}{15x^{\frac{5}{2}}a^{3}}$	35
default	$-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}$	55

input
```
int(1/x^(7/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(1/2)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(5/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(63) = 126.

Time = 2.42 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.22

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{4a^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{24ab^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{16b^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}$$

input
```
integrate(1/x**(7/2)/(b*x+a)**(1/2),x)
```

output
```
-6*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3
+ 15*a**3*b**6*x**4) - 4*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*a**5*b**4*
x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*a**2*b**(13/2)*x**2*sqrt
(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
- 24*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*
x**3 + 15*a**3*b**6*x**4) - 16*b**(17/2)*x**4*sqrt(a/(b*x) + 1)/(15*a**5*b
**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\left(\frac{15\sqrt{bx+a}b^2}{\sqrt{x}} - \frac{10(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} + \frac{3(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15a^3}$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(15*sqrt(b*x + a)*b^2/sqrt(x) - 10*(b*x + a)^(3/2)*b/x^(3/2) + 3*(b*
x + a)^(5/2)/x^(5/2))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\left(\frac{15b^5}{a} + 4\left(\frac{2(bx+a)b^5}{a^3} - \frac{5b^5}{a^2}\right)(bx+a)\right)\sqrt{bx+a}b}{15((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(15*b^5/a + 4*(2*(b*x + a)*b^5/a^3 - 5*b^5/a^2)*(b*x + a))*sqrt(b*x
+ a)*b/(((b*x + a)*b - a*b)^(5/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2}{5\,a} + \frac{16\,b^2\,x^2}{15\,a^3} - \frac{8\,b\,x}{15\,a^2}\right)}{x^{5/2}}$$

input
```
int(1/(x^(7/2)*(a + b*x)^(1/2)),x)
```

output
```
-((a + b*x)^(1/2)*(2/(5*a) + (16*b^2*x^2)/(15*a^3) - (8*b*x)/(15*a^2)))/x^
(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+\frac{16\sqrt{b}\,b^2x^3}{15}}{a^3x^3}$$

input
```
int(1/x^(7/2)/(b*x+a)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b*x - 8*sq
rt(x)*sqrt(a + b*x)*b**2*x**2 + 8*sqrt(b)*b**2*x**3))/(15*a**3*x**3)
```

3.484 $\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} + \frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}$$

output
```
-2/7*(b*x+a)^(1/2)/a/x^(7/2)+12/35*b*(b*x+a)^(1/2)/a^2/x^(5/2)-16/35*b^2*(
b*x+a)^(1/2)/a^3/x^(3/2)+32/35*b^3*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}(5a^3 - 6a^2bx + 8ab^2x^2 - 16b^3x^3)}{35a^4x^{7/2}}$$

input
```
Integrate[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(5*a^3 - 6*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/(35*a^4*
x^(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\int \frac{1}{x^{7/2}\sqrt{a+bx}}dx}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\left(-\frac{4b\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

input
```
Int[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x])/(7*a*x^(7/2)) - (6*b*((-2*Sqrt[a + b*x])/(5*a*x^(5/2))
- (4*b*((-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt
[x])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^{\frac{7}{2}}a^4}$	46
risch	$-\frac{2\sqrt{bx+a}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^{\frac{7}{2}}a^4}$	46
orering	$-\frac{2\sqrt{bx+a}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^{\frac{7}{2}}a^4}$	46
default	$-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}$	77

input
```
int(1/x^(9/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(1/2)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(7/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = \frac{2\,(16\,b^3x^3 - 8\,ab^2x^2 + 6\,a^2bx - 5\,a^3)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(
7/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(87) = 174.

Time = 6.80 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.30

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx =& -\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}\\
&-\frac{18a^5b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}\\
&-\frac{10a^4b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}\\
&+\frac{10a^3b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}\\
&+\frac{60a^2b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}\\
&+\frac{80ab^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}\\
&+\frac{32b^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}\end{aligned}$$

input
```
integrate(1/x**(9/2)/(b*x+a)**(1/2),x)
```

output
```
-10*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x
**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*a**5*b**(21/2)*x*sqrt
(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x*
*5 + 35*a**4*b**12*x**6) - 10*a**4*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(35*a*
*7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x
**6) + 10*a**3*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a
**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 60*a**2*b**(2
7/2)*x**4*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105
*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 80*a*b**(29/2)*x**5*sqrt(a/(b*x)
+ 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a
**4*b**12*x**6) + 32*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 +
 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = \frac{2\left(\frac{35\sqrt{bx+a}b^3}{\sqrt{x}} - \frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}} + \frac{21\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}} - \frac{5\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^4}$$

input
```
integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(35*sqrt(b*x + a)*b^3/sqrt(x) - 35*(b*x + a)^(3/2)*b^2/x^(3/2) + 21*(
b*x + a)^(5/2)*b/x^(5/2) - 5*(b*x + a)^(7/2)/x^(7/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = \frac{2\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{2\,(bx+a)b^3}{a^4} - \frac{7\,b^3}{a^3}\right) + \frac{35\,b^3}{a^2}\right) - \frac{35\,b^3}{a}\right)\sqrt{bx+a}b^5}{35\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

input
```
integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/35*(2*(b*x + a)*(4*(b*x + a)*(2*(b*x + a)*b^3/a^4 - 7*b^3/a^3) + 35*b^3/
a^2) - 35*b^3/a)*sqrt(b*x + a)*b^5/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2}{7\,a}+\frac{16\,b^2\,x^2}{35\,a^3}-\frac{32\,b^3\,x^3}{35\,a^4}-\frac{12\,b\,x}{35\,a^2}\right)}{x^{7/2}}$$

input
```
int(1/(x^(9/2)*(a + b*x)^(1/2)),x)
```

output
```
-((a + b*x)^(1/2)*(2/(7*a) + (16*b^2*x^2)/(35*a^3) - (32*b^3*x^3)/(35*a^4)
 - (12*b*x)/(35*a^2)))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{32\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{32\sqrt{b}\,b^3x^4}{35}}{a^4x^4}$$

input
```
int(1/x^(9/2)/(b*x+a)^(1/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 8
*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**3*x**3 -
16*sqrt(b)*b**3*x**4))/(35*a**4*x**4)
```

3.485 $\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx=-\frac{2a^2\sqrt{x}}{b^3\sqrt{a+bx}}-\frac{7a\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}$$

output
```
-2*a^2*x^(1/2)/b^3/(b*x+a)^(1/2)-7/4*a*x^(1/2)*(b*x+a)^(1/2)/b^3+1/2*x^(3/
2)*(b*x+a)^(1/2)/b^2+15/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx=\frac{\sqrt{x}(-15a^2-5abx+2b^2x^2)}{4b^3\sqrt{a+bx}}+\frac{15a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{2b^{7/2}}$$

input
```
Integrate[x^(5/2)/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(-15*a^2 - 5*a*b*x + 2*b^2*x^2))/(4*b^3*Sqrt[a + b*x]) + (15*a^2*
ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {57, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{5\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

input
```
Int[x^(5/2)/(a + b*x)^(3/2),x]
```

output
```
(-2*x^(5/2))/(b*Sqrt[a + b*x]) + (5*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*
((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/
b^(3/2)))/(4*b)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{(-2bx+7a)\sqrt{x}\,\sqrt{bx+a}}{4b^{3}}+\frac{\left(\frac{15a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8b^{\frac{7}{2}}}-\frac{2a^{2}\sqrt{b\left(x+\frac{a}{b}\right)^{2}-\left(x+\frac{a}{b}\right)a}}{b^{4}\left(x+\frac{a}{b}\right)}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	119

input
```
int(x^(5/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*x+7*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+(15/8*a^2/b^(7/2)*ln((1/2*a+b*
x)/b^(1/2)+(b*x^2+a*x)^(1/2))-2*a^2/b^4/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1
/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx = \left[\frac{15\,(a^2bx+a^3)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,b^3x^2-5\,ab^2x-15\,a^2b)\sqrt{b}}{8\,(b^5x+ab^4)}\right.$$

input
```
integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(
x) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5
*x + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqr
t(b*x + a)) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

Sympy [A] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx = -\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1+\frac{bx}{a}}} - \frac{5\sqrt{a}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx}{a}}} + \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)/(b*x+a)**(3/2),x)
```

output
```
-15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**2
*sqrt(1 + b*x/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) +
x**(5/2)/(2*sqrt(a)*b*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx = -\frac{8\,a^2b^2 - \frac{25\,(bx+a)a^2b}{x} + \frac{15\,(bx+a)^2a^2}{x^2}}{4\left(\frac{\sqrt{bx+a}b^5}{\sqrt{x}} - \frac{2\,(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)} - \frac{15\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(8*a^2*b^2 - 25*(b*x + a)*a^2*b/x + 15*(b*x + a)^2*a^2/x^2)/(sqrt(b*x
 + a)*b^5/sqrt(x) - 2*(b*x + a)^(3/2)*b^4/x^(3/2) + (b*x + a)^(5/2)*b^3/x^
(5/2)) - 15/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b
*x + a)/sqrt(x)))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx =$$

$$-\frac{\left(\frac{32\,a^3\sqrt{b}}{\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{15\,a^2\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}-2\,\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b}-\right.\right.}{8\,b^4}$$

input
```
integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(32*a^3*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2
+ a*b) + 15*a^2*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/s
qrt(b) - 2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)/b - 9*a/b))*
abs(b)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx = \int \frac{x^{5/2}}{(a+b\,x)^{3/2}}\,dx$$

input
```
int(x^(5/2)/(a + b*x)^(3/2),x)
```

output
```
int(x^(5/2)/(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2 - 10\sqrt{b}\,\sqrt{bx+a}\,a^2 - 15\sqrt{x}\,a^2b - 5\sqrt{x}\,a\,b^2x + 2\sqrt{}}{4\sqrt{bx+a}\,b^4}$$

input
```
int(x^(5/2)/(b*x+a)^(3/2),x)
```

output
```
(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**2 - 10*sqrt(b)*sqrt(a + b*x)*a**2 - 15*sqrt(x)*a**2*b - 5*sqrt(x)*a*b**2
*x + 2*sqrt(x)*b**3*x**2)/(4*sqrt(a + b*x)*b**4)
```

3.486 $\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx=\frac{2a\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

output
```
2*a*x^(1/2)/b^2/(b*x+a)^(1/2)+x^(1/2)*(b*x+a)^(1/2)/b^2-3*a*arctanh(b^(1/2
)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx=\frac{\sqrt{x}(3a+bx)}{b^2\sqrt{a+bx}}+\frac{6a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(3*a + b*x))/(b^2*Sqrt[a + b*x]) + (6*a*ArcTanh[(Sqrt[b]*Sqrt[x])
/(Sqrt[a] - Sqrt[a + b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {57, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \ 57$$

$$\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \ 60$$

$$\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \ 65$$

$$\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \ 219$$

$$\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

input
```
Int[x^(3/2)/(a + b*x)^(3/2),x]
```

output
```
(-2*x^(3/2))/(b*Sqrt[a + b*x]) + (3*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(52) = 104$.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.56

method	result	size
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b^{2}}+\frac{\left(-\frac{3a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{5}{2}}}+\frac{2a\sqrt{b\left(x+\frac{a}{b}\right)^{2}-\left(x+\frac{a}{b}\right)a}}{b^{3}\left(x+\frac{a}{b}\right)}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	106

input
```
int(x^(3/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*
x)^(1/2))+2*a/b^3/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2))*(x*(b*x+a))^(1/2)
/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.09

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx = \left[\frac{3\,(abx+a^2)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(b^2x+3\,ab)\sqrt{bx+a}\sqrt{x}}{2\,(b^4x+ab^3)}, \frac{3\,(ab$$

input
```
integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x)
+ a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x
 + a^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (b^2*x + 3*a*b)*
sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx = \frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1+\frac{bx}{a}}} - \frac{3a\,\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} + \frac{x^{\frac{3}{2}}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)/(b*x+a)**(3/2),x)
```

output
```
3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt(b)*sqrt(x)/sqrt(
a))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx = \frac{2\,ab - \frac{3\,(bx+a)a}{x}}{\frac{\sqrt{bx+a}b^3}{\sqrt{x}} - \frac{(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}} + \frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
(2*a*b - 3*(b*x + a)*a/x)/(sqrt(b*x + a)*b^3/sqrt(x) - (b*x + a)^(3/2)*b^2
/x^(3/2)) + 3/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b
*x + a)/sqrt(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(52) = 104.

Time = 15.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.69

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx = \frac{\left(\frac{8\,a^2\sqrt{b}}{\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab} + \frac{3\,a\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}} + \frac{2\,\sqrt{(bx+a)b-ab}\sqrt{bx+a}}{b}\right)|b|}{2\,b^3}$$

input
```
integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(8*a^2*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 +
a*b) + 3*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b
) + 2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b)*abs(b)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx = \int \frac{x^{3/2}}{(a+b\,x)^{3/2}}\,dx$$

input
```
int(x^(3/2)/(a + b*x)^(3/2),x)
```

output
```
int(x^(3/2)/(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx = \frac{-12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a+9\sqrt{b}\,\sqrt{bx+a}\,a+12\sqrt{x}\,ab+4\sqrt{x}\,b^2x}{4\sqrt{bx+a}\,b^3}$$

input
```
int(x^(3/2)/(b*x+a)^(3/2),x)
```

output
```
( - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a + 9*sqrt(b)*sqrt(a + b*x)*a + 12*sqrt(x)*a*b + 4*sqrt(x)*b**2*x)/(4*sq
rt(a + b*x)*b**3)
```

3.487 $\int \frac{\sqrt{x}}{(a+bx)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}} \, dx = -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

output
```
-2*x^(1/2)/b/(b*x+a)^(1/2)+2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2
)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}} \, dx = -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{4\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(a + b*x)^(3/2),x]
```

output
```
(-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (4*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
Sqrt[a + b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{b}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \text{65}$$

$$\frac{2\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

input
```
Int[Sqrt[x]/(a + b*x)^(3/2),x]
```

output
```
(-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x
]])/b^(3/2)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [F]

$$\int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{2}}}dx$$

input
```
int(x^(1/2)/(b*x+a)^(3/2),x)
```

output
```
int(x^(1/2)/(b*x+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = \left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}b\sqrt{x}}{b^3x+ab^2}, -\frac{2\left((bx+a)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)+\sqrt{bx+a}b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

input
```
integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*s
qrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqr
t(-b)*sqrt(x)/sqrt(b*x + a)) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = \frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{2\sqrt{x}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(1/2)/(b*x+a)**(3/2),x)
```

output
```
2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1 +
b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = -\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}} - \frac{2\sqrt{x}}{\sqrt{bx+a}b}$$

input
```
integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))
/b^(3/2) - 2*sqrt(x)/(sqrt(b*x + a)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(36) = 72$.

Time = 15.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = -\frac{\left(\frac{4\,a\sqrt{b}}{\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}\right)|b|}{b^2}$$

input
```
integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-(4*a*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)
+ log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b))*abs(b)
/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{x}}{{(a+b\,x)}^{3/2}}\,dx$$

input
```
int(x^(1/2)/(a + b*x)^(3/2),x)
```

output
```
int(x^(1/2)/(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)-2\sqrt{b}\,\sqrt{bx+a}-2\sqrt{x}\,b}{\sqrt{bx+a}\,b^2}$$

input
```
int(x^(1/2)/(b*x+a)^(3/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)) -
 sqrt(b)*sqrt(a + b*x) - sqrt(x)*b))/(sqrt(a + b*x)*b**2)
```

3.488 $\int \frac{1}{\sqrt{x}(a+bx)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}} \, dx = \frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

output
```
2*x^(1/2)/a/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}} \, dx = \frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
```

output
```
(2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{48}$$

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
```

output
```
(2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{2\sqrt{x}}{a\sqrt{bx+a}}$	16
default	$\frac{2\sqrt{x}}{a\sqrt{bx+a}}$	16
orering	$\frac{2\sqrt{x}}{a\sqrt{bx+a}}$	16

input
```
int(1/x^(1/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/a/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\sqrt{x}}{abx+a^2}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**(3/2),x)
```

output
```
2/(a*sqrt(b)*sqrt(a/(b*x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{\sqrt{bx+a}a}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)/(sqrt(b*x + a)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(15) = 30.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{4\,b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)|b|}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
4*b^(3/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*abs
(b))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{a+b\,x}}{a^2+b\,x\,a}$$

input
```
int(1/(x^(1/2)*(a + b*x)^(3/2)),x)
```

output
```
(2*x^(1/2)*(a + b*x)^(1/2))/(a^2 + a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}+2\sqrt{x}\,b}{\sqrt{bx+a}\,ab}$$

input

```
int(1/x^(1/2)/(b*x+a)^(3/2),x)
```

output

```
(2*(sqrt(b)*sqrt(a + b*x) + sqrt(x)*b))/(sqrt(a + b*x)*a*b)
```

3.489 $\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = \frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

output
```
2/a/x^(1/2)/(b*x+a)^(1/2)-4*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2(a+2bx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{2}{a\sqrt{x}\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(a + b*x)^(3/2)),x]
```

output
```
2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{2(2bx+a)}{\sqrt{x}\,\sqrt{bx+a}\,a^{2}}$	22
orering	$-\frac{2(2bx+a)}{\sqrt{x}\,\sqrt{bx+a}\,a^{2}}$	22
default	$-\frac{2}{a\sqrt{x}\,\sqrt{bx+a}}-\frac{4b\sqrt{x}}{a^{2}\sqrt{bx+a}}$	33
risch	$-\frac{2\sqrt{bx+a}}{a^{2}\sqrt{x}}-\frac{2b\sqrt{x}}{a^{2}\sqrt{bx+a}}$	33

input
```
int(1/x^(3/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*b*x+a)/x^(1/2)/(b*x+a)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2\,(2\,bx+a)\sqrt{bx+a}\sqrt{x}}{a^2bx^2+a^3x}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}+1}} - \frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**(3/2),x)
```

output
```
-2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2\,b\sqrt{x}}{\sqrt{bx+a}a^2} - \frac{2\,\sqrt{bx+a}}{a^2\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*b*sqrt(x)/(sqrt(b*x + a)*a^2) - 2*sqrt(b*x + a)/(a^2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(31) = 62$.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{4\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a|b|} - \frac{2\,\sqrt{bx+a}b^2}{\sqrt{(bx+a)b-ab}a^2|b|}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-4*b^(5/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a*
abs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^2*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2\,a\,\sqrt{a+b\,x}+4\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(a^3+b\,x\,a^2\right)}$$

input
```
int(1/(x^(3/2)*(a + b*x)^(3/2)),x)
```

output
```
-(2*a*(a + b*x)^(1/2) + 4*b*x*(a + b*x)^(1/2))/(x^(1/2)*(a^3 + a^2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = \frac{-4\sqrt{b}\,\sqrt{bx+a}\,x-2\sqrt{x}\,a-4\sqrt{x}\,bx}{\sqrt{bx+a}\,a^2x}$$

input
```
int(1/x^(3/2)/(b*x+a)^(3/2),x)
```

output
```
(2*( - 2*sqrt(b)*sqrt(a + b*x)*x - sqrt(x)*a - 2*sqrt(x)*b*x))/(sqrt(a + b
*x)*a**2*x)
```

3.490 $\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{3/2}\sqrt{a+bx}} - \frac{8\sqrt{a+bx}}{3a^2x^{3/2}} + \frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}$$

output
```
2/a/x^(3/2)/(b*x+a)^(1/2)-8/3*(b*x+a)^(1/2)/a^2/x^(3/2)+16/3*b*(b*x+a)^(1/
2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = -\frac{2(a^2-4abx-8b^2x^2)}{3a^3x^{3/2}\sqrt{a+bx}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*(a^2 - 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{55}$$

$$\frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{48}$$

$$\frac{4\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

input
```
Int[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

output
```
2/(a*x^(3/2)*Sqrt[a + b*x]) + (4*((-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*
Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2\left(-8b^{2}x^{2}-4abx+a^{2}\right)}{3x^{\frac{3}{2}}\sqrt{bx+a}\,a^{3}}$	33
orering	$-\frac{2\left(-8b^{2}x^{2}-4abx+a^{2}\right)}{3x^{\frac{3}{2}}\sqrt{bx+a}\,a^{3}}$	33
risch	$-\frac{2\sqrt{bx+a}\,\left(-5bx+a\right)}{3a^{3}x^{\frac{3}{2}}}+\frac{2b^{2}\sqrt{x}}{a^{3}\sqrt{bx+a}}$	41
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4b\left(-\frac{2}{a\sqrt{x}\,\sqrt{bx+a}}-\frac{4b\sqrt{x}}{a^{2}\sqrt{bx+a}}\right)}{3a}$	55

input
```
int(1/x^(5/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-8*b^2*x^2-4*a*b*x+a^2)/x^(3/2)/(b*x+a)^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{2\,(8\,b^2x^2+4\,abx-a^2)\sqrt{bx+a}\sqrt{x}}{3\,(a^3bx^3+a^4x^2)}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(58) = 116.

Time = 1.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.48

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = -\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3} + \frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3} + \frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3} + \frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**(3/2),x)
```

output
```
-2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a
**3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a
**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/
(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 16*b**(15/2)*x**3*
sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{2\,b^2\sqrt{x}}{\sqrt{bx+a}a^3} + \frac{2\left(\frac{6\,\sqrt{bx+a}b}{\sqrt{x}} - \frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) + 2/3*(6*sqrt(b*x + a)*b/sqrt(x) - (b*x
+ a)^(3/2)/x^(3/2))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(47) = 94.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{2\,\sqrt{bx+a}\left(\frac{5\,(bx+a)b^2|b|}{a^3} - \frac{6\,b^2|b|}{a^2}\right)}{3\,((bx+a)b-ab)^{\frac{3}{2}}} + \frac{4\,b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 + ab\right)a^2|b|}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x + a)*(5*(b*x + a)*b^2*abs(b)/a^3 - 6*b^2*abs(b)/a^2)/((b*x +
a)*b - a*b)^(3/2) + 4*b^(7/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b
- a*b))^2 + a*b)*a^2*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{\sqrt{a+b\,x}\,\left(\frac{8\,x}{3\,a^2}-\frac{2}{3\,a\,b}+\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}+\frac{a\,x^{3/2}}{b}}$$

input
```
int(1/(x^(5/2)*(a + b*x)^(3/2)),x)
```

output
```
((a + b*x)^(1/2)*((8*x)/(3*a^2) - 2/(3*a*b) + (16*b*x^2)/(3*a^3)))/(x^(5/2
) + (a*x^(3/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{-\frac{16\sqrt{b}\,\sqrt{bx+a}\,b\,x^2}{3}-\frac{2\sqrt{x}\,a^2}{3}+\frac{8\sqrt{x}\,abx}{3}+\frac{16\sqrt{x}\,b^2x^2}{3}}{\sqrt{bx+a}\,a^3x^2}$$

input
```
int(1/x^(5/2)/(b*x+a)^(3/2),x)
```

output
```
(2*( - 8*sqrt(b)*sqrt(a + b*x)*b*x**2 - sqrt(x)*a**2 + 4*sqrt(x)*a*b*x + 8
*sqrt(x)*b**2*x**2))/(3*sqrt(a + b*x)*a**3*x**2)
```

3.491 $\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} + \frac{16b\sqrt{a+bx}}{5a^3x^{3/2}} - \frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}$$

output
```
2/a/x^(5/2)/(b*x+a)^(1/2)-12/5*(b*x+a)^(1/2)/a^2/x^(5/2)+16/5*b*(b*x+a)^(1
/2)/a^3/x^(3/2)-32/5*b^2*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{2(a^3-2a^2bx+8ab^2x^2+16b^3x^3)}{5a^4x^{5/2}\sqrt{a+bx}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a +
b*x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}dx}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{55}$$

$$\frac{6\left(-\frac{4b\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{55}$$

$$\frac{6\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{48}$$

$$\frac{6\left(-\frac{4b\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

input
```
Int[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

output
```
2/(a*x^(5/2)*Sqrt[a + b*x]) + (6*((-2*Sqrt[a + b*x])/(5*a*x^(5/2)) - (4*b*
((-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/
(5*a)))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2\left(16 b^{3} x^{3}+8 a \,b^{2} x^{2}-2 a^{2} b x +a^{3}\right)}{5 x^{\frac{5}{2}} \sqrt{b x +a}\, a^{4}}$	44
orering	$-\frac{2\left(16 b^{3} x^{3}+8 a \,b^{2} x^{2}-2 a^{2} b x +a^{3}\right)}{5 x^{\frac{5}{2}} \sqrt{b x +a}\, a^{4}}$	44
risch	$-\frac{2 \sqrt{b x +a}\, \left(11 b^{2} x^{2}-3 a b x +a^{2}\right)}{5 a^{4} x^{\frac{5}{2}}}-\frac{2 b^{3} \sqrt{x}}{a^{4} \sqrt{b x +a}}$	52
default	$-\frac{2}{5 a \,x^{\frac{5}{2}} \sqrt{b x +a}}-\frac{6 b \left(-\frac{2}{3 a \,x^{\frac{3}{2}} \sqrt{b x +a}}-\frac{4 b \left(-\frac{2}{a \sqrt{x}\, \sqrt{b x +a}}-\frac{4 b \sqrt{x}}{a^{2} \sqrt{b x +a}}\right)}{3 a}\right)}{5 a}$	77

input
```
int(1/x^(7/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(16*b^3*x^3+8*a*b^2*x^2-2*a^2*b*x+a^3)/x^(5/2)/(b*x+a)^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{2\,(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3)\sqrt{bx+a}\sqrt{x}}{5\,(a^4bx^4+a^5x^3)}$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a
^4*b*x^4 + a^5*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(82) = 164.

Time = 4.61 (sec) , antiderivative size = 348, normalized size of antiderivative = 4.00

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{2a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5} - \frac{10a^3b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5} - \frac{60a^2b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5} - \frac{80ab^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5} - \frac{32b^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}$$

input
```
integrate(1/x**(7/2)/(b*x+a)**(3/2),x)
```

output
```
-2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3
 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*b**(23/2)*x**2*sqrt(a
/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 +
5*a**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(5*a**7*b**9
*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 80*
a*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3
+ 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**5*sqrt(a/(b*x)
 + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4
*b**12*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{2\,b^3\sqrt{x}}{\sqrt{bx+a}a^4} - \frac{2\left(\frac{15\,\sqrt{bx+a}b^2}{\sqrt{x}} - \frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} + \frac{(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{5\,a^4}$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) - 2/5*(15*sqrt(b*x + a)*b^2/sqrt(x) - 5
*(b*x + a)^(3/2)*b/x^(3/2) + (b*x + a)^(5/2)/x^(5/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{4\,b^{\frac{9}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 + ab\right)a^3|b|} - \frac{2\left(\frac{15\,b^6}{a^2|b|} + (bx+a)\left(\frac{11\,(bx+a)b^6}{a^4|b|} - \frac{25\,b^6}{a^3|b|}\right)\right)\sqrt{bx+a}}{5\left((bx+a)b-ab\right)^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-4*b^(9/2)/((((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a^
3*abs(b)) - 2/5*(15*b^6/(a^2*abs(b)) + (b*x + a)*(11*(b*x + a)*b^6/(a^4*ab
s(b)) - 25*b^6/(a^3*abs(b))))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a\,b}-\frac{4\,x}{5\,a^2}+\frac{16\,b\,x^2}{5\,a^3}+\frac{32\,b^2\,x^3}{5\,a^4}\right)}{x^{7/2}+\frac{a\,x^{5/2}}{b}}$$

input
```
int(1/(x^(7/2)*(a + b*x)^(3/2)),x)
```

output
```
-((a + b*x)^(1/2)*(2/(5*a*b) - (4*x)/(5*a^2) + (16*b*x^2)/(5*a^3) + (32*b^
2*x^3)/(5*a^4)))/(x^(7/2) + (a*x^(5/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{\frac{32\sqrt{b}\,\sqrt{bx+a}\,b^2x^3}{5}-\frac{2\sqrt{x}\,a^3}{5}+\frac{4\sqrt{x}\,a^2bx}{5}-\frac{16\sqrt{x}\,a\,b^2x^2}{5}-\frac{32\sqrt{x}\,b^3x^3}{5}}{\sqrt{bx+a}\,a^4x^3}$$

input
```
int(1/x^(7/2)/(b*x+a)^(3/2),x)
```

output
```
(2*(16*sqrt(b)*sqrt(a + b*x)*b**2*x**3 - sqrt(x)*a**3 + 2*sqrt(x)*a**2*b*x
 - 8*sqrt(x)*a*b**2*x**2 - 16*sqrt(x)*b**3*x**3))/(5*sqrt(a + b*x)*a**4*x*
*3)
```

3.492 $\int \frac{x^{5/2}}{(a+bx)^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}} \, dx = -\frac{2a^2\sqrt{x}}{3b^3(a+bx)^{3/2}} + \frac{14a\sqrt{x}}{3b^3\sqrt{a+bx}} + \frac{\sqrt{x}\sqrt{a+bx}}{b^3} - \frac{5a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}$$

output
```
-2/3*a^2*x^(1/2)/b^3/(b*x+a)^(3/2)+14/3*a*x^(1/2)/b^3/(b*x+a)^(1/2)+x^(1/2
)*(b*x+a)^(1/2)/b^3-5*a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}} \, dx = \frac{\sqrt{x}(15a^2+20abx+3b^2x^2)}{3b^3(a+bx)^{3/2}} + \frac{10a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*(15*a^2 + 20*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2)) + (10*a*
ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {57, 57, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \ 57$$

$$\frac{5\int \frac{x^{3/2}}{(a+bx)^{3/2}}dx}{3b}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

$$\Big\downarrow \ 57$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3b}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

$$\Big\downarrow \ 60$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3b}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

$$\Big\downarrow \ 65$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3b}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3b}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

input
```
Int[x^(5/2)/(a + b*x)^(5/2),x]
```

output
```
(-2*x^(5/2))/(3*b*(a + b*x)^(3/2)) + (5*((-2*x^(3/2))/(b*Sqrt[a + b*x]) +
(3*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(3/2)))/b))/(3*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(70) = 140.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56

method	result	size
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b^{3}}+\frac{\left(-\frac{5a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{7}{2}}}+\frac{14a\sqrt{b\left(x+\frac{a}{b}\right)^{2}-\left(x+\frac{a}{b}\right)a}}{3b^{4}\left(x+\frac{a}{b}\right)}-\frac{2a^{2}\sqrt{b\left(x+\frac{a}{b}\right)^{2}-\left(x+\frac{a}{b}\right)a}}{3b^{5}\left(x+\frac{a}{b}\right)^{2}}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	147

input
```
int(x^(5/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(b*x+a)^(1/2)/b^3+(-5/2/b^(7/2)*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*
x)^(1/2))+14/3/b^4*a/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2)-2/3/b^5*a^2/(x+
a/b)^2*(b*(x+a/b)^2-(x+a/b)*a)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.24

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx = \left[\frac{15\,(ab^{2}x^{2}+2\,a^{2}bx+a^{3})\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(3\,b^{3}x^{2}+20\,ab^{2}x-}{6\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})}\right.$$

input
```
integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)
*sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a
)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(90) = 180$.

Time = 3.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.21

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx = -\frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}} - \frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}} + \frac{15a^{40}b^{\frac{45}{2}}x^{26}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}} + \frac{20a^{39}b^{\frac{47}{2}}x^{27}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}} + \frac{3a^{38}b^{\frac{49}{2}}x^{28}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)/(b*x+a)**(5/2),x)
```

output
```
-15*a**(81/2)*b**22*x**(51/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a
))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2
)*x**(53/2)*sqrt(1 + b*x/a)) - 15*a**(79/2)*b**23*x**(53/2)*sqrt(1 + b*x/a
)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 +
 b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 15*a**40*b**(
45/2)*x**26/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)
*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 20*a**39*b**(47/2)*x**27/(3*a**(79
/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*
sqrt(1 + b*x/a)) + 3*a**38*b**(49/2)*x**28/(3*a**(79/2)*b**(51/2)*x**(51/2
)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx = \frac{2\,ab^2 + \frac{10\,(bx+a)ab}{x} - \frac{15\,(bx+a)^2a}{x^2}}{3\left(\frac{(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} - \frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)} + \frac{5\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*a*b^2 + 10*(b*x + a)*a*b/x - 15*(b*x + a)^2*a/x^2)/((b*x + a)^(3/2)
*b^4/x^(3/2) - (b*x + a)^(5/2)*b^3/x^(5/2)) + 5/2*a*log(-(sqrt(b) - sqrt(b
*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(70) = 140.

Time = 15.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.07

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx = \frac{\left(\frac{15\,a\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}} + \frac{6\,\sqrt{(bx+a)b-ab}\sqrt{bx+a}}{b} + \frac{8\left(9\,a^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4 b+\cdots}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{\cdots}\right.\right.}\right.}{6\,b^4}$$

input
```
integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*(15*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b)
 + 6*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b + 8*(9*a^2*(sqrt(b*x + a)*sqr
t(b) - sqrt((b*x + a)*b - a*b))^4*b + 12*a^3*(sqrt(b*x + a)*sqrt(b) - sqrt
((b*x + a)*b - a*b))^2*b^2 + 7*a^4*b^3)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b
*x + a)*b - a*b))^2 + a*b)^3*sqrt(b)))*abs(b)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx = \int \frac{x^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

input
```
int(x^(5/2)/(a + b*x)^(5/2),x)
```

output
```
int(x^(5/2)/(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx = \frac{-30\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2 - 30\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abx - 5\sqrt{b}\,\sqrt{}}{6\sqrt{bx+a}\,b^4\,(bx+a}$$

input
```
int(x^(5/2)/(b*x+a)^(5/2),x)
```

output
```
( - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a**2 - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sq
rt(a))*a*b*x - 5*sqrt(b)*sqrt(a + b*x)*a**2 - 5*sqrt(b)*sqrt(a + b*x)*a*b*
x + 30*sqrt(x)*a**2*b + 40*sqrt(x)*a*b**2*x + 6*sqrt(x)*b**3*x**2)/(6*sqrt
(a + b*x)*b**4*(a + b*x))
```

3.493 $\int \frac{x^{3/2}}{(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}} \, dx = \frac{2a\sqrt{x}}{3b^2(a+bx)^{3/2}} - \frac{8\sqrt{x}}{3b^2\sqrt{a+bx}} + \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

output
```
2/3*a*x^(1/2)/b^2/(b*x+a)^(3/2)-8/3*x^(1/2)/b^2/(b*x+a)^(1/2)+2*arctanh(b^
(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}} \, dx = -\frac{2\sqrt{x}(3a+4bx)}{3b^2(a+bx)^{3/2}} + \frac{4\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(a + b*x)^(5/2),x]
```

output
```
(-2*Sqrt[x]*(3*a + 4*b*x))/(3*b^2*(a + b*x)^(3/2)) + (4*ArcTanh[(Sqrt[b]*S
qrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {57, 57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \ 57$$

$$\frac{\int \frac{\sqrt{x}}{(a+bx)^{3/2}}dx}{b}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

$$\Big\downarrow \ 57$$

$$\frac{\frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{b}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}}{b}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

$$\Big\downarrow \ 65$$

$$\frac{\frac{2\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}}{b}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}}{b}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

input
```
Int[x^(3/2)/(a + b*x)^(5/2),x]
```

output
```
(-2*x^(3/2))/(3*b*(a + b*x)^(3/2)) + ((-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}dx$$

input
```
int(x^(3/2)/(b*x+a)^(5/2),x)
```

output
```
int(x^(3/2)/(b*x+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.54

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = \left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\left(4\,b^2x+3\,ab\right)\sqrt{bx+a}}{3\left(b^5x^2+2\,ab^4x+a^2b^3\right)}\right.$$

input
```
integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt
(b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2
*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqr
t(-b)*sqrt(x)/sqrt(b*x + a)) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(68) = 136.

Time = 1.66 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.56

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = \frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}} + \frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}} - \frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}} - \frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)/(b*x+a)**(5/2),x)
```

output
```
6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))
/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*
x**(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*x/a)*a
sinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*
x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b**(23/2
)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**
(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b
**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(
1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = -\frac{2\left(b+\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}b^{2}} - \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(b + 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*b^2) - log(-(sqrt(b) - s
qrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(52) = 104.

Time = 15.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.29

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = -\frac{\left(\frac{3\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}\right)}{\sqrt{b}} + \frac{8\left(3\,a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+3\,a^{2}\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}b^{\frac{3}{2}}+2\,a^{3}b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}}\right)|b|}{3\,b^{3}}$$

input
```
integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b) +
 8*(3*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 3*a^
2*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 2*a^3*b^(5
/2))/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3)*abs(b)
/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = \int \frac{x^{3/2}}{(a+b\,x)^{5/2}}\,dx$$

input
```
int(x^(3/2)/(a + b*x)^(5/2),x)
```

output
```
int(x^(3/2)/(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a + 2\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bx - 2\sqrt{x}\,ab - \frac{8\sqrt{}}{}}{\sqrt{bx+a}\,b^{3}\,(bx+a)}$$

input
```
int(x^(3/2)/(b*x+a)^(5/2),x)
```

output
```
(2*(3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))
*a + 3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*b*x - 3*sqrt(x)*a*b - 4*sqrt(x)*b**2*x))/(3*sqrt(a + b*x)*b**3*(a + b*x)
)
```

3.494 $\int \frac{\sqrt{x}}{(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}} dx = \frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

output
```
2/3*x^(3/2)/a/(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}} dx = \frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

input
```
Integrate[Sqrt[x]/(a + b*x)^(5/2),x]
```

output
```
(2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 48}$$
$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

input
```
Int[Sqrt[x]/(a + b*x)^(5/2),x]
```

output
```
(2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3a(bx+a)^{\frac{3}{2}}}$	16
orering	$\frac{2x^{\frac{3}{2}}}{3a(bx+a)^{\frac{3}{2}}}$	16
default	$-\frac{\sqrt{x}}{b(bx+a)^{\frac{3}{2}}}+\frac{a\left(\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}\right)}{2b}$	54

input
```
int(x^(1/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/a/(b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{2\,\sqrt{bx+a}x^{\frac{3}{2}}}{3\,(ab^2x^2+2\,a^2bx+a^3)}$$

input
```
integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.59 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(1/2)/(b*x+a)**(5/2),x)
```

output
```
2*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

input
```
integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(15) = 30$.

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{4\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+a^{2}b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}b^{2}}$$

input
```
integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + a^2*b
^(5/2))*abs(b)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b
)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{2\,x^{3/2}\,\sqrt{a+b\,x}}{3\,\left(a^3+2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

input
```
int(x^(1/2)/(a + b*x)^(5/2),x)
```

output
```
(2*x^(3/2)*(a + b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 + 2*a^2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{\frac{2\sqrt{b}\,\sqrt{bx+a}\,a}{3}+\frac{2\sqrt{b}\,\sqrt{bx+a}\,bx}{3}+\frac{2\sqrt{x}\,b^2x}{3}}{\sqrt{bx+a}\,a\,b^2\,(bx+a)}$$

input
```
int(x^(1/2)/(b*x+a)^(5/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a + b*x)*a + sqrt(b)*sqrt(a + b*x)*b*x + sqrt(x)*b**2*x))
/(3*sqrt(a + b*x)*a*b**2*(a + b*x))
```

3.495 $\int \frac{1}{\sqrt{x}(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}} dx = \frac{2\sqrt{x}}{3a(a+bx)^{3/2}} + \frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}$$

output
```
2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}} dx = \frac{2\sqrt{x}(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

output
```
(2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}dx}{3a}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

output
```
(2*Sqrt[x])/(3*a*(a + b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2\sqrt{x}\,(2bx+3a)}{3(bx+a)^{\frac{3}{2}}\,a^{2}}$	24
orering	$\frac{2\sqrt{x}\,(2bx+3a)}{3(bx+a)^{\frac{3}{2}}\,a^{2}}$	24
default	$\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}$	32

input
```
int(1/x^(1/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(1/2)*(2*b*x+3*a)/(b*x+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\sqrt{x}}{3\,(a^2b^2x^2+2\,a^3bx+a^4)}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.

Time = 0.79 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{6a}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}} + \frac{4bx}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**(5/2),x)
```

output
```
6*a/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1
)) + 4*b*x/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b
*x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx = -\frac{2\left(b - \frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(b - 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(31) = 62.

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{8\left(3\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 + ab\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 + ab\right)^3 |b|}$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(5/2)/
(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{6\,a\,\sqrt{x}\,\sqrt{a+b\,x}+4\,b\,x^{3/2}\,\sqrt{a+b\,x}}{3\,a^4+6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

input
```
int(1/(x^(1/2)*(a + b*x)^(5/2)),x)
```

output
```
(6*a*x^(1/2)*(a + b*x)^(1/2) + 4*b*x^(3/2)*(a + b*x)^(1/2))/(3*a^4 + 3*a^2
*b^2*x^2 + 6*a^3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{-\frac{4\sqrt{b}\,\sqrt{bx+a}\,a}{3}-\frac{4\sqrt{b}\,\sqrt{bx+a}\,bx}{3}+2\sqrt{x}\,ab+\frac{4\sqrt{x}\,b^2x}{3}}{\sqrt{bx+a}\,a^2b\,(bx+a)}$$

input
```
int(1/x^(1/2)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 2*sqrt(b)*sqrt(a + b*x)*a - 2*sqrt(b)*sqrt(a + b*x)*b*x + 3*sqrt(x)
*a*b + 2*sqrt(x)*b**2*x))/(3*sqrt(a + b*x)*a**2*b*(a + b*x))
```

3.496 $\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx=\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}$$

output
```
2/3/a/x^(1/2)/(b*x+a)^(3/2)+8/3/a^2/x^(1/2)/(b*x+a)^(1/2)-16/3*(b*x+a)^(1/
2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx=-\frac{2(3a^2+12abx+8b^2x^2)}{3a^3\sqrt{x}(a+bx)^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\int \frac{1}{x^{3/2}(a+bx)^{3/2}}dx}{3a}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\left(\frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{x}\sqrt{a+bx}}\right)}{3a}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4\left(\frac{2}{a\sqrt{x}\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}\right)}{3a}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}$$

input
```
Int[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

output
```
2/(3*a*Sqrt[x]*(a + b*x)^(3/2)) + (4*(2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqr
t[a + b*x])/(a^2*Sqrt[x])))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2\left(8b^{2}x^{2}+12abx+3a^{2}\right)}{3\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}a^{3}}$	35
orering	$-\frac{2\left(8b^{2}x^{2}+12abx+3a^{2}\right)}{3\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}a^{3}}$	35
risch	$-\frac{2\sqrt{bx+a}}{a^{3}\sqrt{x}}-\frac{2b\left(5bx+6a\right)\sqrt{x}}{3\left(bx+a\right)^{\frac{3}{2}}a^{3}}$	41
default	$-\frac{2}{a\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{2\sqrt{x}}{3a\left(bx+a\right)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}\right)}{a}$	54

input
```
int(1/x^(3/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(8*b^2*x^2+12*a*b*x+3*a^2)/x^(1/2)/(b*x+a)^(3/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{2\,(8\,b^2x^2+12\,abx+3\,a^2)\sqrt{bx+a}\sqrt{x}}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2
*a^4*b*x^2 + a^5*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(58) = 116.

Time = 1.50 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.39

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2} - \frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2} - \frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**(5/2),x)
```

output
```
-6*a**2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**3*b
**6*x**2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*
x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x + 3*a**3*b**6*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = \frac{2\left(b^2-\frac{6\,(bx+a)b}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^3} - \frac{2\,\sqrt{bx+a}}{a^3\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(b^2 - 6*(b*x + a)*b/x)*x^(3/2)/((b*x + a)^(3/2)*a^3) - 2*sqrt(b*x + a
)/(a^3*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(46) = 92.

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{2\,\sqrt{bx+a}b^2}{\sqrt{(bx+a)b-ab}a^3|b|} - \frac{4\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4 b^{\frac{5}{2}} + 12\,a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2 b^{\frac{7}{2}} + 5\,a^2b^{\frac{9}{2}}\right)}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3 a^2|b|}$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(b
*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(5/2) + 12*a*(sqrt(b*x + a)
*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(7/2) + 5*a^2*b^(9/2))/(((sqrt(b*x
 + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*a^2*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{6\,a^2\,\sqrt{a+b\,x}+16\,b^2\,x^2\,\sqrt{a+b\,x}+24\,a\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b+3\,x\,a^3\,b^2\right)+3\,a^5\right)}$$

input
```
int(1/(x^(3/2)*(a + b*x)^(5/2)),x)
```

output
```
-(6*a^2*(a + b*x)^(1/2) + 16*b^2*x^2*(a + b*x)^(1/2) + 24*a*b*x*(a + b*x)^
(1/2))/(x^(1/2)*(x*(6*a^4*b + 3*a^3*b^2*x) + 3*a^5))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = \frac{\frac{16\sqrt{b}\,\sqrt{bx+a}\,ax}{3}+\frac{16\sqrt{b}\,\sqrt{bx+a}\,b\,x^2}{3}-2\sqrt{x}\,a^2-8\sqrt{x}\,abx-\frac{16\sqrt{x}\,b^2x^2}{3}}{\sqrt{bx+a}\,a^3x\,(bx+a)}$$

input
```
int(1/x^(3/2)/(b*x+a)^(5/2),x)
```

output
```
(2*(8*sqrt(b)*sqrt(a + b*x)*a*x + 8*sqrt(b)*sqrt(a + b*x)*b*x**2 - 3*sqrt(
x)*a**2 - 12*sqrt(x)*a*b*x - 8*sqrt(x)*b**2*x**2))/(3*sqrt(a + b*x)*a**3*x
*(a + b*x))
```

3.497 $\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx=\frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}$$

output
```
2/3/a/x^(3/2)/(b*x+a)^(3/2)+4/a^2/x^(3/2)/(b*x+a)^(1/2)-16/3*(b*x+a)^(1/2)
/a^3/x^(3/2)+32/3*b*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx=-\frac{2(a^3-6a^2bx-24ab^2x^2-16b^3x^3)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*(a^3 - 6*a^2*b*x - 24*a*b^2*x^2 - 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x
)^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}dx$$

$$\Big\downarrow \ 55$$

$$\frac{2\int \frac{1}{x^{5/2}(a+bx)^{3/2}}dx}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 55$$

$$\frac{2\left(\frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 55$$

$$\frac{2\left(\frac{4\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 48$$

$$\frac{2\left(\frac{4\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

input
```
Int[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

output
```
2/(3*a*x^(3/2)*(a + b*x)^(3/2)) + (2*(2/(a*x^(3/2)*Sqrt[a + b*x]) + (4*((-
2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/a))
/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2\left(-16b^{3}x^{3}-24a\,b^{2}x^{2}-6a^{2}bx+a^{3}\right)}{3x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}a^{4}}$	44
orering	$-\frac{2\left(-16b^{3}x^{3}-24a\,b^{2}x^{2}-6a^{2}bx+a^{3}\right)}{3x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}a^{4}}$	44
risch	$-\frac{2\sqrt{bx+a}\,\left(-8bx+a\right)}{3a^{4}x^{\frac{3}{2}}}+\frac{2b^{2}\left(8bx+9a\right)\sqrt{x}}{3\left(bx+a\right)^{\frac{3}{2}}a^{4}}$	49
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}}-\frac{2b\left(-\frac{2}{a\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{2\sqrt{x}}{3a\left(bx+a\right)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}\right)}{a}\right)}{a}$	76

input
```
int(1/x^(5/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-16*b^3*x^3-24*a*b^2*x^2-6*a^2*b*x+a^3)/x^(3/2)/(b*x+a)^(3/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{2\,(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3)\sqrt{bx+a}\sqrt{x}}{3\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a
^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(78) = 156.

Time = 2.75 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.01

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = -\frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**(5/2),x)
```

output
```
-2*a**4*b**(19/2)*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9
*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(a/(b*x) +
 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*
x**4) + 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b
**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2)*x**3*s
qrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 +
3*a**4*b**12*x**4) + 32*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x +
9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{2\left(\frac{9\sqrt{bx+a}b}{\sqrt{x}} - \frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4} - \frac{2\left(b^3 - \frac{9\,(bx+a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^4}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(9*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^4 - 2/3*(b^3 -
 9*(b*x + a)*b^2/x)*x^(3/2)/((b*x + a)^(3/2)*a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(62) = 124.

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.08

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{2\sqrt{bx+a}\left(\frac{8\,(bx+a)b^2|b|}{a^4} - \frac{9\,b^2|b|}{a^3}\right)}{3\,((bx+a)b-ab)^{\frac{3}{2}}} + \frac{8\left(3\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^4 b^{\frac{7}{2}} + 9\,a\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 b^{\frac{9}{2}} + 4\,a^2b^{\frac{11}{2}}\right)}{3\left(\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 + ab\right)^3 a^3|b|}$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x + a)*(8*(b*x + a)*b^2*abs(b)/a^4 - 9*b^2*abs(b)/a^3)/((b*x +
a)*b - a*b)^(3/2) + 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b
))^4*b^(7/2) + 9*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(
9/2) + 4*a^2*b^(11/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))
^2 + a*b)^3*a^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{32\,b^3\,x^3\,\sqrt{a+b\,x}-2\,a^3\,\sqrt{a+b\,x}+12\,a^2\,b\,x\,\sqrt{a+b\,x}+48\,a\,b^2\,x^2\,\sqrt{a+b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b+3\,x\,a^4\,b^2\right)+3\,a^6\right)}$$

input
```
int(1/(x^(5/2)*(a + b*x)^(5/2)),x)
```

output
```
(32*b^3*x^3*(a + b*x)^(1/2) - 2*a^3*(a + b*x)^(1/2) + 12*a^2*b*x*(a + b*x)
^(1/2) + 48*a*b^2*x^2*(a + b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b + 3*a^4*b^2*x)
 + 3*a^6))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{-\frac{32\sqrt{b}\,\sqrt{bx+a}\,ab\,x^2}{3}-\frac{32\sqrt{b}\,\sqrt{bx+a}\,b^2x^3}{3}-\frac{2\sqrt{x}\,a^3}{3}+4\sqrt{x}\,a^2bx+16\sqrt{x}\,a\,b^2x^2+\frac{32\sqrt{x}\,b^3x^3}{3}}{\sqrt{bx+a}\,a^4x^2\,(bx+a)}$$

input
```
int(1/x^(5/2)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 16*sqrt(b)*sqrt(a + b*x)*a*b*x**2 - 16*sqrt(b)*sqrt(a + b*x)*b**2*x
**3 - sqrt(x)*a**3 + 6*sqrt(x)*a**2*b*x + 24*sqrt(x)*a*b**2*x**2 + 16*sqrt
(x)*b**3*x**3))/(3*sqrt(a + b*x)*a**4*x**2*(a + b*x))
```

3.498 $\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx=\frac{5\sqrt{x}\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

output
```
5/2*x^(1/2)*(b*x+2)^(1/2)/b^3-5/6*x^(3/2)*(b*x+2)^(1/2)/b^2+1/3*x^(5/2)*(b
*x+2)^(1/2)/b-5*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx=\frac{\sqrt{x}\sqrt{2+bx}(15-5bx+2b^2x^2)}{6b^3}+\frac{10\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/Sqrt[2 + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2))/(6*b^3) + (10*ArcTanh[(Sq
rt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{bx+2}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\int \frac{x^{3/2}}{\sqrt{bx+2}}dx}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)}{3b}$$

$$\Big\downarrow \text{63}$$

$$\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)}{3b}$$

$$\Big\downarrow \text{222}$$

$$\frac{x^{5/2}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)}{3b}$$

input
```
Int[x^(5/2)/Sqrt[2 + b*x],x]
```

output
```
(x^(5/2)*Sqrt[2 + b*x])/(3*b) - (5*((x^(3/2)*Sqrt[2 + b*x])/(2*b) - (3*((S
qrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)))
/(2*b)))/(3*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\left(14b^2x^2-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{42}-5\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	63
risch	$\frac{\left(2b^2x^2-5bx+15\right)\sqrt{x}\,\sqrt{bx+2}}{6b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)\sqrt{x(bx+2)}}{2b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$	77
default	$\frac{x^{\frac{5}{2}}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)\sqrt{x(bx+2)}}{b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}\right)}{2b}\right)}{3b}$	104

input
```
int(x^(5/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/b^(7/2)/Pi^(1/2)*(1/336*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(14*b^2*x^2-35*
b*x+105)*(1/2*b*x+1)^(1/2)-5/8*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.38

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = \left[\frac{\left(2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{6\,b^4}, \frac{(2\,b^3x^2-5\,b}{}\right.$$

input
```
integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*((2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(
b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/6*((2*b^3*x^2 - 5*b^2*x +
 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*
x + 2)))/b^4]
```

Sympy [A] (verification not implemented)

Time = 7.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = \frac{x^{\frac{7}{2}}}{3\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{6b\sqrt{bx+2}} + \frac{5x^{\frac{3}{2}}}{6b^2\sqrt{bx+2}} + \frac{5\sqrt{x}}{b^3\sqrt{bx+2}} - \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

input
```
integrate(x**(5/2)/(b*x+2)**(1/2),x)
```

output
```
x**(7/2)/(3*sqrt(b*x + 2)) - x**(5/2)/(6*b*sqrt(b*x + 2)) + 5*x**(3/2)/(6*
b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(b**3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqr
t(b)*sqrt(x)/2)/b**(7/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(63) = 126.

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = -\frac{\frac{33\sqrt{bx+2}b^2}{\sqrt{x}} - \frac{40(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} + \frac{15(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^6 - \frac{3(bx+2)b^5}{x} + \frac{3(bx+2)^2b^4}{x^2} - \frac{(bx+2)^3b^3}{x^3}\right)} + \frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(33*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3/2) + 15*(b*
x + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x + 2)*b^5/x + 3*(b*x + 2)^2*b^4/x^2 - (
b*x + 2)^3*b^3/x^3) + 5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b)
+ sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = \frac{\left(((2\,bx-9)(bx+2)+33)\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}+30\,\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx}\right.\right.\right.}{6\,b^5}$$

input
```
integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(((2*b*x - 9)*(b*x + 2) + 33)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) +
30*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs
(b)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = \int \frac{x^{5/2}}{\sqrt{b\,x+2}}\,dx$$

input
```
int(x^(5/2)/(b*x + 2)^(1/2),x)
```

output
```
int(x^(5/2)/(b*x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = \frac{2\sqrt{x}\,\sqrt{bx+2}\,b^3x^2-5\sqrt{x}\,\sqrt{bx+2}\,b^2x+15\sqrt{x}\,\sqrt{bx+2}\,b-30\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{6b^4}$$

input
```
int(x^(5/2)/(b*x+2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b**3*x**2 - 5*sqrt(x)*sqrt(b*x + 2)*b**2*x + 15*s
qrt(x)*sqrt(b*x + 2)*b - 30*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/
sqrt(2)))/(6*b**4)
```

3.499 $\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx=-\frac{3\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{2b}+\frac{3\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

output
```
-3/2*x^(1/2)*(b*x+2)^(1/2)/b^2+1/2*x^(3/2)*(b*x+2)^(1/2)/b+3*arcsinh(1/2*b
^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx=\frac{\sqrt{x}(-3+bx)\sqrt{2+bx}}{2b^2}-\frac{6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/Sqrt[2 + b*x],x]
```

output
```
(Sqrt[x]*(-3 + b*x)*Sqrt[2 + b*x])/(2*b^2) - (6*ArcTanh[(Sqrt[b]*Sqrt[x])/
(Sqrt[2] - Sqrt[2 + b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{bx+2}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{63}$$

$$\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}$$

$$\Big\downarrow \text{222}$$

$$\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}$$

input
```
Int[x^(3/2)/Sqrt[2 + b*x],x]
```

output
```
(x^(3/2)*Sqrt[2 + b*x])/(2*b) - (3*((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(-5bx+15)\sqrt{\frac{bx}{2}+1}}{10}+3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55
risch	$\frac{(bx-3)\sqrt{x}\,\sqrt{bx+2}}{2b^{2}}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	68
default	$\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}\right)}{2b}$	83

input
```
int(x^(3/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/b^(5/2)/Pi^(1/2)*(-1/40*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(-5*b*x+15)*(1/
2*b*x+1)^(1/2)+3/4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \left[\frac{(b^2x-3\,b)\sqrt{bx+2}\sqrt{x}+3\,\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{2\,b^3}, \frac{(b^2x-3\,b)\sqrt{bx+2}\sqrt{x}-\ldots}{2\,b\ldots}\right.$$

input
```
integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) -
6*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)))/b^3]
```

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \frac{x^{\frac{5}{2}}}{2\sqrt{bx+2}} - \frac{x^{\frac{3}{2}}}{2b\sqrt{bx+2}} - \frac{3\sqrt{x}}{b^2\sqrt{bx+2}} + \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x**(3/2)/(b*x+2)**(1/2),x)
```

output
```
x**(5/2)/(2*sqrt(b*x + 2)) - x**(3/2)/(2*b*sqrt(b*x + 2)) - 3*sqrt(x)/(b**
2*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(48) = 96.

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \frac{\frac{5\sqrt{bx+2}b}{\sqrt{x}} - \frac{3\,(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^4 - \frac{2\,(bx+2)b^3}{x} + \frac{(bx+2)^2b^2}{x^2}} - \frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
(5*sqrt(b*x + 2)*b/sqrt(x) - 3*(b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x + 2)
*b^3/x + (b*x + 2)^2*b^2/x^2) - 3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))
/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \frac{\left(\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}(bx-3)-6\,\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2\,b}\right|\right)\right)|b|}{2\,b^4}$$

input
```
integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3) - 6*sqrt(b)*log(abs(-
sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \int \frac{x^{3/2}}{\sqrt{b\,x+2}}\,dx$$

input
```
int(x^(3/2)/(b*x + 2)^(1/2),x)
```

output
```
int(x^(3/2)/(b*x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+2}\,b^2x-3\sqrt{x}\,\sqrt{bx+2}\,b+6\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{2b^3}$$

input
```
int(x^(3/2)/(b*x+2)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b**2*x - 3*sqrt(x)*sqrt(b*x + 2)*b + 6*sqrt(b)*log(
(sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/(2*b**3)
```

3.500 $\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx=\frac{\sqrt{x}\sqrt{2+bx}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

output
```
x^(1/2)*(b*x+2)^(1/2)/b-2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx=\frac{\sqrt{x}\sqrt{2+bx}}{b}+\frac{4\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[2 + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x])/b + (4*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2
 + b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{bx+2}}\,dx$$
$$\Big\downarrow \; 60$$
$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}$$
$$\Big\downarrow \; 63$$
$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}$$
$$\Big\downarrow \; 222$$
$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

input
```
Int[Sqrt[x]/Sqrt[2 + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\sqrt{\frac{bx}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	49
default	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}$	62
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)\sqrt{x(bx+2)}}{b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}$	62

input
```
int(x^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^(3/2)/Pi^(1/2)*(1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(1/2*b*x+1)^(1/2)
-Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \left[\frac{\sqrt{bx+2}b\sqrt{x}+\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{b^{2}}, \frac{\sqrt{bx+2}b\sqrt{x}+2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+2}}\right)}{b^{2}}\right]$$

input
```
integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x
) + 1))/b^2, (sqrt(b*x + 2)*b*sqrt(x) + 2*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)
/sqrt(b*x + 2)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \frac{x^{\frac{3}{2}}}{\sqrt{bx+2}} + \frac{2\sqrt{x}}{b\sqrt{bx+2}} - \frac{2\,\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x**(1/2)/(b*x+2)**(1/2),x)
```

output
```
x**(3/2)/sqrt(b*x + 2) + 2*sqrt(x)/(b*sqrt(b*x + 2)) - 2*asinh(sqrt(2)*sqr
t(b)*sqrt(x)/2)/b**(3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(32) = 64.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}} - \frac{2\sqrt{bx+2}}{\left(b^2-\frac{(bx+2)b}{x}\right)\sqrt{x}}$$

input
```
integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
b^(3/2) - 2*sqrt(b*x + 2)/((b^2 - (b*x + 2)*b/x)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \frac{\left(2\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2b}\right|\right)+\sqrt{(bx+2)b-2b}\sqrt{bx+2}\right)|b|}{b^3}$$

input
```
integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
(2*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))) + sq
rt((b*x + 2)*b - 2*b)*sqrt(b*x + 2))*abs(b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{b^{3/2}}+\frac{\sqrt{x}\,\sqrt{b\,x+2}}{b}$$

input
```
int(x^(1/2)/(b*x + 2)^(1/2),x)
```

output
```
(4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(1/2))))/b^(3/2) + (x^(1/2
)*(b*x + 2)^(1/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+2}\,b-2\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{b^2}$$

input
```
int(x^(1/2)/(b*x+2)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b - 2*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))
/sqrt(2)))/b**2
```

3.501 $\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

output
```
2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = -\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

output
```
(-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx$$

$$\Big\downarrow \text{63}$$

$$2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

output
```
(2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

method	result	size
meijerg	$\frac{2\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}$	18
default	$\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	46

input
```
int(1/x^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = \left[\frac{\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+2}}\right)}{b}\right]$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
[log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1)/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(-b)*sqrt(x)/sqrt(b*x + 2))/b]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = \frac{2\,\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

input
```
integrate(1/x**(1/2)/(b*x+2)**(1/2),x)
```

output
```
2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = -\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))
/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 5.97 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = -\frac{2\sqrt{b}\log\left(\left|-\sqrt{bx+2}\sqrt{b}+\sqrt{(bx+2)b-2b}\right|\right)}{|b|}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/abs(
b)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = \frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{b\,x+2}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

input
```
int(1/(x^(1/2)*(b*x + 2)^(1/2)),x)
```

output
```
(4*atan((2^(1/2) - (b*x + 2)^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx = \frac{2\sqrt{b}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)}{b}$$

input
```
int(1/x^(1/2)/(b*x+2)^(1/2),x)
```

output
```
(2*sqrt(b)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)))/b
```

3.502 $\int \frac{1}{x^{3/2}\sqrt{2+bx}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}} \, dx = -\frac{\sqrt{2+bx}}{\sqrt{x}}$$

output
```
-(b*x+2)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}} \, dx = -\frac{\sqrt{2+bx}}{\sqrt{x}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

output
```
-(Sqrt[2 + b*x]/Sqrt[x])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

output
```
-(Sqrt[2 + b*x]/Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
default	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
risch	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
orering	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
meijerg	$-\frac{\sqrt{2}\,\sqrt{\frac{bx}{2}+1}}{\sqrt{x}}$	17

input
```
int(1/x^(3/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+2)^(1/2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{bx+2}}{\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*x + 2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\sqrt{b}\sqrt{1+\frac{2}{bx}}$$

input
```
integrate(1/x**(3/2)/(b*x+2)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(1 + 2/(b*x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{bx+2}}{\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x + 2)/sqrt(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{bx+2}b^2}{\sqrt{(bx+2)b-2\,b}|b|}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{b\,x+2}}{\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(b*x + 2)^(1/2)),x)
```

output
```
-(b*x + 2)^(1/2)/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = \frac{-\sqrt{x}\,\sqrt{bx+2}-\sqrt{b}\,x}{x}$$

input
```
int(1/x^(3/2)/(b*x+2)^(1/2),x)
```

output
```
( - (sqrt(x)*sqrt(b*x + 2) + sqrt(b)*x))/x
```

3.503 $\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{3x^{3/2}} + \frac{b\sqrt{2+bx}}{3\sqrt{x}}$$

output
```
-1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx = \frac{(-1+bx)\sqrt{2+bx}}{3x^{3/2}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

output
```
((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{bx+2}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

output
```
-1/3*Sqrt[2 + b*x]/x^(3/2) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.47

method	result	size
gosper	$\frac{\sqrt{bx+2}\,(bx-1)}{3x^{\frac{3}{2}}}$	18
orering	$\frac{\sqrt{bx+2}\,(bx-1)}{3x^{\frac{3}{2}}}$	18
meijerg	$-\frac{\sqrt{2}\,(-bx+1)\sqrt{\frac{bx}{2}+1}}{3x^{\frac{3}{2}}}$	23
risch	$\frac{b^{2}x^{2}+bx-2}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	25
default	$-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}$	27

input
```
int(1/x^(5/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+2)^(1/2)*(b*x-1)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx=\frac{\sqrt{bx+2}(bx-1)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*x + 2)*(b*x - 1)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx = \frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3} - \frac{\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

input
```
integrate(1/x**(5/2)/(b*x+2)**(1/2),x)
```

output
```
b**(3/2)*sqrt(1 + 2/(b*x))/3 - sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx = \frac{\sqrt{bx+2}b}{2\sqrt{x}} - \frac{(bx+2)^{\frac{3}{2}}}{6x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/6*(b*x + 2)^(3/2)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx = \frac{((bx+2)b-3b)\sqrt{bx+2}b^3}{3((bx+2)b-2b)^{\frac{3}{2}}|b|}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/3*((b*x + 2)*b - 3*b)*sqrt(b*x + 2)*b^3/(((b*x + 2)*b - 2*b)^(3/2)*abs(b
))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx=\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}-\frac{1}{3}\right)}{x^{3/2}}$$

input
```
int(1/(x^(5/2)*(b*x + 2)^(1/2)),x)
```

output
```
((b*x + 2)^(1/2)*((b*x)/3 - 1/3))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx=\frac{\sqrt{x}\,\sqrt{bx+2}\,bx-\sqrt{x}\,\sqrt{bx+2}-\sqrt{b}\,b\,x^2}{3x^2}$$

input
```
int(1/x^(5/2)/(b*x+2)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b*x + 2)*b*x - sqrt(x)*sqrt(b*x + 2) - sqrt(b)*b*x**2)/(3*x*
*2)
```

3.504 $\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}$$

output
```
-1/5*(b*x+2)^(1/2)/x^(5/2)+2/15*b*(b*x+2)^(1/2)/x^(3/2)-2/15*b^2*(b*x+2)^(
1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = \frac{\sqrt{2+bx}(-3+2bx-2b^2x^2)}{15x^{5/2}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

output
```
(Sqrt[2 + b*x]*(-3 + 2*b*x - 2*b^2*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{bx+2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2}{5}b\int \frac{1}{x^{5/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2}{5}b\left(-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{2}{5}b\left(\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

output
```
-1/5*Sqrt[2 + b*x]/x^(5/2) - (2*b*(-1/3*Sqrt[2 + b*x]/x^(3/2) + (b*Sqrt[2
+ b*x])/(3*Sqrt[x])))/5
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{\sqrt{bx+2}\,(2b^2x^2-2bx+3)}{15x^{\frac{5}{2}}}$	27
orering	$-\frac{\sqrt{bx+2}\,(2b^2x^2-2bx+3)}{15x^{\frac{5}{2}}}$	27
meijerg	$-\frac{\sqrt{2}\,\left(\frac{2}{3}b^2x^2-\frac{2}{3}bx+1\right)\sqrt{\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
risch	$-\frac{2b^3x^3+2b^2x^2-bx+6}{15x^{\frac{5}{2}}\sqrt{bx+2}}$	35
default	$-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}$	43

input
```
int(1/x^(7/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x+2)^(1/2)*(2*b^2*x^2-2*b*x+3)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = -\frac{(2\,b^2x^2-2\,bx+3)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(2*b^2*x^2 - 2*b*x + 3)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(53) = 106.

Time = 2.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.80

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = -\frac{2b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2} - \frac{6b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2} - \frac{3b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2} - \frac{4b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2} - \frac{12b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}$$

input
```
integrate(1/x**(7/2)/(b*x+2)**(1/2),x)
```

output
```
-2*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 60*b**4
*x**2) - 6*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 +
 60*b**4*x**2) - 3*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**
5*x**3 + 60*b**4*x**2) - 4*b**(11/2)*x*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 6
0*b**5*x**3 + 60*b**4*x**2) - 12*b**(9/2)*sqrt(1 + 2/(b*x))/(15*b**6*x**4
+ 60*b**5*x**3 + 60*b**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{bx+2}\,b^2}{4\sqrt{x}} + \frac{(bx+2)^{\frac{3}{2}}b}{6x^{\frac{3}{2}}} - \frac{(bx+2)^{\frac{5}{2}}}{20x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(b*x + 2)*b^2/sqrt(x) + 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/20*(b*x
 + 2)^(5/2)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = -\frac{\left(15\,b^5+2\left((bx+2)b^5-5\,b^5\right)(bx+2)\right)\sqrt{bx+2}b}{15\left((bx+2)b-2\,b\right)^{\frac{5}{2}}|b|}$$

input
```
integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/15*(15*b^5 + 2*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2))*sqrt(b*x + 2)*b/(((b*
x + 2)*b - 2*b)^(5/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{b\,x+2}\left(\frac{2\,b^2\,x^2}{15}-\frac{2\,b\,x}{15}+\frac{1}{5}\right)}{x^{5/2}}$$

input
```
int(1/(x^(7/2)*(b*x + 2)^(1/2)),x)
```

output
```
-((b*x + 2)^(1/2)*((2*b^2*x^2)/15 - (2*b*x)/15 + 1/5))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+2}\,b^2x^2+2\sqrt{x}\,\sqrt{bx+2}\,bx-3\sqrt{x}\,\sqrt{bx+2}+2\sqrt{b}\,b^2x^3}{15x^3}$$

input
```
int(1/x^(7/2)/(b*x+2)^(1/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(b*x + 2)*b**2*x**2 + 2*sqrt(x)*sqrt(b*x + 2)*b*x - 3*sq
rt(x)*sqrt(b*x + 2) + 2*sqrt(b)*b**2*x**3)/(15*x**3)
```

3.505 $\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}-\frac{2b^2\sqrt{2+bx}}{35x^{3/2}}+\frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}$$

output
```
-1/7*(b*x+2)^(1/2)/x^(7/2)+3/35*b*(b*x+2)^(1/2)/x^(5/2)-2/35*b^2*(b*x+2)^(
1/2)/x^(3/2)+2/35*b^3*(b*x+2)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = \frac{\sqrt{2+bx}(-5+3bx-2b^2x^2+2b^3x^3)}{35x^{7/2}}$$

input
```
Integrate[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

output
```
(Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{bx+2}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{3}{7}b\int \frac{1}{x^{7/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{3}{7}b\left(-\frac{2}{5}b\int \frac{1}{x^{5/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{5x^{5/2}}\right)-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{3}{7}b\left(-\frac{2}{5}b\left(-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)-\frac{\sqrt{bx+2}}{5x^{5/2}}\right)-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{3}{7}b\left(-\frac{2}{5}b\left(\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)-\frac{\sqrt{bx+2}}{5x^{5/2}}\right)-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

input
```
Int[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

output
```
-1/7*Sqrt[2 + b*x]/x^(7/2) - (3*b*(-1/5*Sqrt[2 + b*x]/x^(5/2) - (2*b*(-1/3
*Sqrt[2 + b*x]/x^(3/2) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])))/5))/7
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
gosper	$\frac{\sqrt{bx+2}\,(2b^3x^3-2b^2x^2+3bx-5)}{35x^{\frac{7}{2}}}$	35
orering	$\frac{\sqrt{bx+2}\,(2b^3x^3-2b^2x^2+3bx-5)}{35x^{\frac{7}{2}}}$	35
meijerg	$-\frac{\sqrt{2}\,(-\frac{2}{5}b^3x^3+\frac{2}{5}b^2x^2-\frac{3}{5}bx+1)\sqrt{\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39
risch	$\frac{2b^4x^4+2b^3x^3-b^2x^2+bx-10}{35x^{\frac{7}{2}}\sqrt{bx+2}}$	42
default	$-\frac{\sqrt{bx+2}}{7x^{\frac{7}{2}}}-\frac{3b\left(-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	59

input
```
int(1/x^(9/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x+2)^(1/2)*(2*b^3*x^3-2*b^2*x^2+3*b*x-5)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = \frac{(2\,b^3x^3 - 2\,b^2x^2 + 3\,bx - 5)\sqrt{bx+2}}{35\,x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/35*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x - 5)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(73) = 146$.

Time = 6.52 (sec) , antiderivative size = 374, normalized size of antiderivative = 4.68

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx &= \frac{2b^{\frac{31}{2}}x^6\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6 + 210b^{11}x^5 + 420b^{10}x^4 + 280b^9x^3} \\ &+ \frac{10b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6 + 210b^{11}x^5 + 420b^{10}x^4 + 280b^9x^3} \\ &+ \frac{15b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6 + 210b^{11}x^5 + 420b^{10}x^4 + 280b^9x^3} \\ &+ \frac{5b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6 + 210b^{11}x^5 + 420b^{10}x^4 + 280b^9x^3} \\ &- \frac{10b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6 + 210b^{11}x^5 + 420b^{10}x^4 + 280b^9x^3} \\ &- \frac{36b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6 + 210b^{11}x^5 + 420b^{10}x^4 + 280b^9x^3} \\ &- \frac{40b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6 + 210b^{11}x^5 + 420b^{10}x^4 + 280b^9x^3}\end{aligned}$$

input
```
integrate(1/x**(9/2)/(b*x+2)**(1/2),x)
```

output
```
2*b**(31/2)*x**6*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b
**10*x**4 + 280*b**9*x**3) + 10*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(35*b**12
*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) + 15*b**(27/2)*x*
*4*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 28
0*b**9*x**3) + 5*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**
11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) - 10*b**(23/2)*x**2*sqrt(1 + 2/(
b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) -
36*b**(21/2)*x*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**
10*x**4 + 280*b**9*x**3) - 40*b**(19/2)*sqrt(1 + 2/(b*x))/(35*b**12*x**6 +
 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = \frac{\sqrt{bx+2}b^3}{8\sqrt{x}} - \frac{(bx+2)^{\frac{3}{2}}b^2}{8\,x^{\frac{3}{2}}} + \frac{3\,(bx+2)^{\frac{5}{2}}b}{40\,x^{\frac{5}{2}}} - \frac{(bx+2)^{\frac{7}{2}}}{56\,x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(b*x + 2)*b^3/sqrt(x) - 1/8*(b*x + 2)^(3/2)*b^2/x^(3/2) + 3/40*(b*
x + 2)^(5/2)*b/x^(5/2) - 1/56*(b*x + 2)^(7/2)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = -\frac{(35\,b^3 - (35\,b^3 + 2\,((bx+2)b^3 - 7\,b^3)(bx+2))(bx+2))\sqrt{bx+2}b^5}{35\,((bx+2)b - 2\,b)^{\frac{7}{2}}|b|}$$

input
```
integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/35*(35*b^3 - (35*b^3 + 2*((b*x + 2)*b^3 - 7*b^3)*(b*x + 2))*(b*x + 2))*
sqrt(b*x + 2)*b^5/(((b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = \frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{35}-\frac{2\,b^2\,x^2}{35}+\frac{3\,b\,x}{35}-\frac{1}{7}\right)}{x^{7/2}}$$

input
```
int(1/(x^(9/2)*(b*x + 2)^(1/2)),x)
```

output
```
((b*x + 2)^(1/2)*((3*b*x)/35 - (2*b^2*x^2)/35 + (2*b^3*x^3)/35 - 1/7))/x^(
7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = \frac{2\sqrt{x}\,\sqrt{bx+2}\,b^3x^3-2\sqrt{x}\,\sqrt{bx+2}\,b^2x^2+3\sqrt{x}\,\sqrt{bx+2}\,bx-5\sqrt{x}\,\sqrt{bx+2}-2\sqrt{b}\,b^3x}{35x^4}$$

input
```
int(1/x^(9/2)/(b*x+2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(b*x + 2)*b**3*x**3 - 2*sqrt(x)*sqrt(b*x + 2)*b**2*x**2 + 3
*sqrt(x)*sqrt(b*x + 2)*b*x - 5*sqrt(x)*sqrt(b*x + 2) - 2*sqrt(b)*b**3*x**4
)/(35*x**4)
```

3.506 $\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx=-\frac{8\sqrt{x}}{b^3\sqrt{2+bx}}-\frac{7\sqrt{x}\sqrt{2+bx}}{2b^3}+\frac{x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

output
```
-8*x^(1/2)/b^3/(b*x+2)^(1/2)-7/2*x^(1/2)*(b*x+2)^(1/2)/b^3+1/2*x^(3/2)*(b*
x+2)^(1/2)/b^2+15*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx=\frac{\sqrt{x}(-30-5bx+b^2x^2)}{2b^3\sqrt{2+bx}}-\frac{30\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(2 + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(-30 - 5*b*x + b^2*x^2))/(2*b^3*Sqrt[2 + b*x]) - (30*ArcTanh[(Sqr
t[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {57, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(bx+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{5\int \frac{x^{3/2}}{\sqrt{bx+2}}dx}{b}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{2b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{2b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \text{63}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{2b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{5\left(\frac{x^{3/2}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{2b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

input
```
Int[x^(5/2)/(2 + b*x)^(3/2),x]
```

output
```
(-2*x^(5/2))/(b*Sqrt[2 + b*x]) + (5*((x^(3/2)*Sqrt[2 + b*x])/(2*b) - (3*((
Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2))
)/(2*b)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\left(-\frac{7}{2}b^{2}x^{2}+\frac{35}{2}bx+105\right)}{14\sqrt{\frac{bx}{2}+1}}+15\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	63
risch	$\frac{(bx-7)\sqrt{x}\,\sqrt{bx+2}}{2b^{3}}+\frac{\left(\frac{15\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2b^{\frac{7}{2}}}-\frac{8\sqrt{b\left(x+\frac{2}{b}\right)^{2}-2x-\frac{4}{b}}}{b^{4}\left(x+\frac{2}{b}\right)}\right)\sqrt{x(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	106

input
```
int(x^(5/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/b^(7/2)/Pi^(1/2)*(-1/112*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(-7/2*b^2*x^2+
35/2*b*x+105)/(1/2*b*x+1)^(1/2)+15/8*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*
2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.73

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx = \left[\frac{15\,(bx+2)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+(b^{3}x^{2}-5\,b^{2}x-30\,b)\sqrt{bx+2}\sqrt{x}}{2\,(b^{5}x+2\,b^{4})}, -\right.$$

input
```
integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(15*(b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) +
(b^3*x^2 - 5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4), -1/2*(3
0*(b*x + 2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)) - (b^3*x^2 - 5
*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx = \frac{x^{\frac{5}{2}}}{2b\sqrt{bx+2}} - \frac{5x^{\frac{3}{2}}}{2b^2\sqrt{bx+2}} - \frac{15\sqrt{x}}{b^3\sqrt{bx+2}} + \frac{15\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

input
```
integrate(x**(5/2)/(b*x+2)**(3/2),x)
```

output
```
x**(5/2)/(2*b*sqrt(b*x + 2)) - 5*x**(3/2)/(2*b**2*sqrt(b*x + 2)) - 15*sqrt
(x)/(b**3*sqrt(b*x + 2)) + 15*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx = -\frac{8\,b^2 - \frac{25\,(bx+2)b}{x} + \frac{15\,(bx+2)^2}{x^2}}{\frac{\sqrt{bx+2}b^5}{\sqrt{x}} - \frac{2\,(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}} - \frac{15\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-(8*b^2 - 25*(b*x + 2)*b/x + 15*(b*x + 2)^2/x^2)/(sqrt(b*x + 2)*b^5/sqrt(x
) - 2*(b*x + 2)^(3/2)*b^4/x^(3/2) + (b*x + 2)^(5/2)*b^3/x^(5/2)) - 15/2*lo
g(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^
(7/2)
```

Giac [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx = \frac{\left(\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}\left(\frac{bx+2}{b}-\frac{9}{b}\right)-\frac{15\,\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{\sqrt{b}}-\frac{6}{\left(\sqrt{bx+2}\sqrt{b}-\sqrt{\right.}}}{2\,b^4}$$

input
```
integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)/b - 9/b) - 15*log((s
qrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) - 64*sqrt(b)/((
sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx = \int \frac{x^{5/2}}{{(b\,x+2)}^{3/2}}\,dx$$

input
```
int(x^(5/2)/(b*x + 2)^(3/2),x)
```

output
```
int(x^(5/2)/(b*x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx = \frac{30\sqrt{b}\,\sqrt{bx+2}\,\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)-20\sqrt{b}\,\sqrt{bx+2}+\sqrt{x}\,b^3x^2-5\sqrt{x}\,b^2x-30\sqrt{x}\,b}{2\sqrt{bx+2}\,b^4}$$

input
```
int(x^(5/2)/(b*x+2)^(3/2),x)
```

output
```
(30*sqrt(b)*sqrt(b*x + 2)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)) -
 20*sqrt(b)*sqrt(b*x + 2) + sqrt(x)*b**3*x**2 - 5*sqrt(x)*b**2*x - 30*sqrt
(x)*b)/(2*sqrt(b*x + 2)*b**4)
```

3.507 $\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx=\frac{4\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{\sqrt{x}\sqrt{2+bx}}{b^2}-\frac{6\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

output
```
4*x^(1/2)/b^2/(b*x+2)^(1/2)+x^(1/2)*(b*x+2)^(1/2)/b^2-6*arcsinh(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx=\frac{\sqrt{x}(6+bx)}{b^2\sqrt{2+bx}}+\frac{12\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(2 + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(6 + b*x))/(b^2*Sqrt[2 + b*x]) + (12*ArcTanh[(Sqrt[b]*Sqrt[x])/(S
qrt[2] - Sqrt[2 + b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {57, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(bx+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{57}$$

$$\frac{3 \int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{3 \left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \text{63}$$

$$\frac{3 \left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{3 \left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

input
```
Int[x^(3/2)/(2 + b*x)^(3/2),x]
```

output
```
(-2*x^(3/2))/(b*Sqrt[2 + b*x]) + (3*((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSin
h[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(\frac{5bx}{2}+15\right)}{5\sqrt{\frac{bx}{2}+1}}-6\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b^{2}}+\frac{\left(-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{b\left(x+\frac{2}{b}\right)^{2}-2x-\frac{4}{b}}}{b^{3}\left(x+\frac{2}{b}\right)}\right)\sqrt{x(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	100

input
```
int(x^(3/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/b^(5/2)/Pi^(1/2)*(1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(5/2*b*x+15)/(1/
2*b*x+1)^(1/2)-3/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = \left[\frac{3\,(bx+2)\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+(b^2x+6\,b)\sqrt{bx+2}\sqrt{x}}{b^4x+2\,b^3}, \frac{6\,(bx+2)\sqrt{-\right.$$

input
```
integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
[(3*(b*x + 2)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*
x + 6*b)*sqrt(b*x + 2)*sqrt(x))/(b^4*x + 2*b^3), (6*(b*x + 2)*sqrt(-b)*arc
tan(sqrt(-b)*sqrt(x)/sqrt(b*x + 2)) + (b^2*x + 6*b)*sqrt(b*x + 2)*sqrt(x))
/(b^4*x + 2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = \frac{x^{\frac{3}{2}}}{b\sqrt{bx+2}} + \frac{6\sqrt{x}}{b^2\sqrt{bx+2}} - \frac{6\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x**(3/2)/(b*x+2)**(3/2),x)
```

output
```
x**(3/2)/(b*sqrt(b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(b*x + 2)) - 6*asinh(sqrt
(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = \frac{2\left(2\,b-\frac{3\,(bx+2)}{x}\right)}{\frac{\sqrt{bx+2}b^3}{\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
2*(2*b - 3*(b*x + 2)/x)/(sqrt(b*x + 2)*b^3/sqrt(x) - (b*x + 2)^(3/2)*b^2/x
^(3/2)) + 3*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2
)/sqrt(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(47) = 94.

Time = 1.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.71

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = \frac{\left(\frac{3\,\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{\sqrt{b}}+\frac{\sqrt{(bx+2)b-2\,b}\sqrt{bx+2}}{b}+\frac{16\,\sqrt{b}}{\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b}\right)|b|}{b^3}$$

input
```
integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
(3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + sqrt
((b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b + 16*sqrt(b)/((sqrt(b*x + 2)*sqrt(b) -
 sqrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = \int \frac{x^{3/2}}{(b\,x+2)^{3/2}}\,dx$$

input
```
int(x^(3/2)/(b*x + 2)^(3/2),x)
```

output
```
int(x^(3/2)/(b*x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = \frac{-12\sqrt{b}\,\sqrt{bx+2}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)+9\sqrt{b}\,\sqrt{bx+2}+2\sqrt{x}\,b^2x+12\sqrt{x}\,b}{2\sqrt{bx+2}\,b^3}$$

input
```
int(x^(3/2)/(b*x+2)^(3/2),x)
```

output
```
( - 12*sqrt(b)*sqrt(b*x + 2)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)
) + 9*sqrt(b)*sqrt(b*x + 2) + 2*sqrt(x)*b**2*x + 12*sqrt(x)*b)/(2*sqrt(b*x
 + 2)*b**3)
```

3.508 $\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx=-\frac{2\sqrt{x}}{b\sqrt{2+bx}}+\frac{2\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

output
```
-2*x^(1/2)/b/(b*x+2)^(1/2)+2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx=-\frac{2\sqrt{x}}{b\sqrt{2+bx}}-\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(2 + b*x)^(3/2),x]
```

output
```
(-2*Sqrt[x])/(b*Sqrt[2 + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]
])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {57, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(bx+2)^{3/2}}\,dx$$

$$\Big\downarrow \ 57$$

$$\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \ 63$$

$$\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

$$\Big\downarrow \ 222$$

$$\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

input
```
Int[Sqrt[x]/(2 + b*x)^(3/2),x]
```

output
```
(-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^
(3/2)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}}{\sqrt{\frac{bx}{2}+1}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	48

input
```
int(x^(1/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^(3/2)/Pi^(1/2)*(-1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)/(1/2*b*x+1)^(1/2
)+Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = \left[\frac{(bx+2)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-2\sqrt{bx+2}b\sqrt{x}}{b^3x+2\,b^2},\right.$$
$$\left.-\frac{2\left((bx+2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+2}}\right)+\sqrt{bx+2}b\sqrt{x}\right)}{b^3x+2\,b^2}\right]$$

input
```
integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
[((b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(
b*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2), -2*((b*x + 2)*sqrt(-b)*arctan(sqrt(-b
)*sqrt(x)/sqrt(b*x + 2)) + sqrt(b*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{bx+2}}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x**(1/2)/(b*x+2)**(3/2),x)
```

output
```
-2*sqrt(x)/(b*sqrt(b*x + 2)) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = -\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}} - \frac{2\sqrt{x}}{\sqrt{bx+2}b}$$

input
```
integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))
/b^(3/2) - 2*sqrt(x)/(sqrt(b*x + 2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(33) = 66.

Time = 1.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = -\frac{\left(\frac{\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{\sqrt{b}} + \frac{8\sqrt{b}}{\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b}\right)|b|}{b^2}$$

input
```
integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
-(log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + 8*sqr
t(b)/((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b
^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(b\,x+2)^{3/2}}\,dx$$

input
```
int(x^(1/2)/(b*x + 2)^(3/2),x)
```

output
```
int(x^(1/2)/(b*x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+2}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)-2\sqrt{b}\,\sqrt{bx+2}-2\sqrt{x}\,b}{\sqrt{bx+2}\,b^2}$$

input
```
int(x^(1/2)/(b*x+2)^(3/2),x)
```

output
```
(2*(sqrt(b)*sqrt(b*x + 2)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)) -
 sqrt(b)*sqrt(b*x + 2) - sqrt(x)*b))/(sqrt(b*x + 2)*b**2)
```

3.509 $\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx=\frac{\sqrt{x}}{\sqrt{2+bx}}$$

output
```
x^(1/2)/(b*x+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx=\frac{\sqrt{x}}{\sqrt{2+bx}}$$

input
```
Integrate[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

output
```
Sqrt[x]/Sqrt[2 + b*x]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(bx+2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 48}$$
$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

input
```
Int[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

output
```
Sqrt[x]/Sqrt[2 + b*x]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{\sqrt{x}}{\sqrt{bx+2}}$	12
default	$\frac{\sqrt{x}}{\sqrt{bx+2}}$	12
orering	$\frac{\sqrt{x}}{\sqrt{bx+2}}$	12
meijerg	$\frac{\sqrt{x}\,\sqrt{2}}{2\sqrt{\frac{bx}{2}+1}}$	17

input
```
int(1/x^(1/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)/(b*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{bx+2}}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(x)/sqrt(b*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{1}{\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

input
```
integrate(1/x**(1/2)/(b*x+2)**(3/2),x)
```

output
```
1/(sqrt(b)*sqrt(1 + 2/(b*x)))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{bx+2}}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(x)/sqrt(b*x + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(11) = 22.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{4\,b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
4*b^(3/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs
(b))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{b\,x+2}}$$

input
```
int(1/(x^(1/2)*(b*x + 2)^(3/2)),x)
```

output
```
x^(1/2)/(b*x + 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{bx+2}+\sqrt{x}\,b}{\sqrt{bx+2}\,b}$$

input
```
int(1/x^(1/2)/(b*x+2)^(3/2),x)
```

output
```
(sqrt(b)*sqrt(b*x + 2) + sqrt(x)*b)/(sqrt(b*x + 2)*b)
```

3.510 $\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = \frac{1}{\sqrt{x}\sqrt{2+bx}} - \frac{\sqrt{2+bx}}{\sqrt{x}}$$

output
```
1/x^(1/2)/(b*x+2)^(1/2)-(b*x+2)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = \frac{-1-bx}{\sqrt{x}\sqrt{2+bx}}$$

input
```
Integrate[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

output
```
(-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(bx+2)^{3/2}}dx$$

$$\Big\downarrow \ 55$$

$$\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx + \frac{1}{\sqrt{x}\sqrt{bx+2}}$$

$$\Big\downarrow \ 48$$

$$\frac{1}{\sqrt{x}\sqrt{bx+2}} - \frac{\sqrt{bx+2}}{\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

output
```
1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b*x]/Sqrt[x]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{bx+1}{\sqrt{x}\,\sqrt{bx+2}}$	18
orering	$-\frac{bx+1}{\sqrt{x}\,\sqrt{bx+2}}$	18
meijerg	$-\frac{\sqrt{2}\,(bx+1)}{2\sqrt{x}\,\sqrt{\frac{bx}{2}+1}}$	22
default	$-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}$	27
risch	$-\frac{\sqrt{bx+2}}{2\sqrt{x}}-\frac{b\sqrt{x}}{2\sqrt{bx+2}}$	27

input
```
int(1/x^(3/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+1)/x^(1/2)/(b*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = -\frac{\sqrt{bx+2}(bx+1)\sqrt{x}}{bx^2+2\,x}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(b*x + 2)*(b*x + 1)*sqrt(x)/(b*x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = -\frac{\sqrt{b}}{\sqrt{1+\frac{2}{bx}}} - \frac{1}{\sqrt{b}x\sqrt{1+\frac{2}{bx}}}$$

input
```
integrate(1/x**(3/2)/(b*x+2)**(3/2),x)
```

output
```
-sqrt(b)/sqrt(1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(1 + 2/(b*x)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = -\frac{b\sqrt{x}}{2\sqrt{bx+2}} - \frac{\sqrt{bx+2}}{2\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*b*sqrt(x)/sqrt(b*x + 2) - 1/2*sqrt(b*x + 2)/sqrt(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = -\frac{\sqrt{bx+2}b^2}{2\sqrt{(bx+2)b-2b}|b|} - \frac{2b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)|b|}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 2*b^(5/2)/(((sqr
t(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = -\frac{b\,x+1}{\sqrt{x}\,\sqrt{b\,x+2}}$$

input
```
int(1/(x^(3/2)*(b*x + 2)^(3/2)),x)
```

output
```
-(b*x + 1)/(x^(1/2)*(b*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx = \frac{-\sqrt{b}\,\sqrt{bx+2}\,x-\sqrt{x}\,bx-\sqrt{x}}{\sqrt{bx+2}\,x}$$

input
```
int(1/x^(3/2)/(b*x+2)^(3/2),x)
```

output
```
( - (sqrt(b)*sqrt(b*x + 2)*x + sqrt(x)*b*x + sqrt(x)))/(sqrt(b*x + 2)*x)
```

3.511 $\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} + \frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

output
```
1/x^(3/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(3/2)+2/3*b*(b*x+2)^(1/2)/x^(1
/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{-1+2bx+2b^2x^2}{3x^{3/2}\sqrt{2+bx}}$$

input
```
Integrate[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

output
```
(-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(bx+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{55}$$

$$2\int \frac{1}{x^{5/2}\sqrt{bx+2}}dx+\frac{1}{x^{3/2}\sqrt{bx+2}}$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)+\frac{1}{x^{3/2}\sqrt{bx+2}}$$

$$\Big\downarrow \text{48}$$

$$2\left(\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)+\frac{1}{x^{3/2}\sqrt{bx+2}}$$

input
```
Int[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

output
```
1/(x^(3/2)*Sqrt[2 + b*x]) + 2*(-1/3*Sqrt[2 + b*x]/x^(3/2) + (b*Sqrt[2 + b*
x])/(3*Sqrt[x]))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{2b^{2}x^{2}+2bx-1}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	27
orering	$\frac{2b^{2}x^{2}+2bx-1}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	27
meijerg	$-\frac{\sqrt{2}\left(-2b^{2}x^{2}-2bx+1\right)}{6x^{\frac{3}{2}}\sqrt{\frac{bx}{2}+1}}$	31
default	$-\frac{1}{3x^{\frac{3}{2}}\sqrt{bx+2}}-\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}\right)}{3}$	43
risch	$\frac{5b^{2}x^{2}+8bx-4}{12x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^{2}\sqrt{x}}{4\sqrt{bx+2}}$	43

input
```
int(1/x^(5/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*b^2*x^2+2*b*x-1)/x^(3/2)/(b*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx=\frac{(2\,b^{2}x^{2}+2\,bx-1)\sqrt{bx+2}\sqrt{x}}{3\,(bx^{3}+2\,x^{2})}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*b^2*x^2 + 2*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^3 + 2*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(49) = 98$.

Time = 1.45 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.21

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{2b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x} + \frac{6b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x} + \frac{3b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x} - \frac{2b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}$$

input
```
integrate(1/x**(5/2)/(b*x+2)**(3/2),x)
```

output
```
2*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x
) + 6*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b*
*4*x) + 3*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b
**4*x) - 2*b**(9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**
4*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{b^2\sqrt{x}}{4\sqrt{bx+2}} + \frac{\sqrt{bx+2}b}{2\sqrt{x}} - \frac{(bx+2)^{\frac{3}{2}}}{12x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*b^2*sqrt(x)/sqrt(b*x + 2) + 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/12*(b*x +
2)^(3/2)/x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|} + \frac{\left(5\,(bx+2)b^2|b|-12\,b^2|b|\right)\sqrt{bx+2}}{12\,\left((bx+2)b-2\,b\right)^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
b^(7/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b
)) + 1/12*(5*(b*x + 2)*b^2*abs(b) - 12*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2
)*b - 2*b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{\sqrt{b\,x+2}\,\left(\frac{2\,x}{3}+\frac{2\,b\,x^2}{3}-\frac{1}{3\,b}\right)}{x^{5/2}+\frac{2\,x^{3/2}}{b}}$$

input
```
int(1/(x^(5/2)*(b*x + 2)^(3/2)),x)
```

output
```
((b*x + 2)^(1/2)*((2*x)/3 + (2*b*x^2)/3 - 1/(3*b)))/(x^(5/2) + (2*x^(3/2))
/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{-2\sqrt{b}\,\sqrt{bx+2}\,b\,x^2+2\sqrt{x}\,b^2x^2+2\sqrt{x}\,bx-\sqrt{x}}{3\sqrt{bx+2}\,x^2}$$

input
```
int(1/x^(5/2)/(b*x+2)^(3/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(b*x + 2)*b*x**2 + 2*sqrt(x)*b**2*x**2 + 2*sqrt(x)*b*x -
 sqrt(x))/(3*sqrt(b*x + 2)*x**2)
```

3.512 $\int \frac{1}{x^{7/2}(2+bx)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}} \, dx = \frac{1}{x^{5/2}\sqrt{2+bx}} - \frac{3\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{5x^{3/2}} - \frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}$$

output
```
1/x^(5/2)/(b*x+2)^(1/2)-3/5*(b*x+2)^(1/2)/x^(5/2)+2/5*b*(b*x+2)^(1/2)/x^(3
/2)-2/5*b^2*(b*x+2)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}} \, dx = \frac{-1+bx-2b^2x^2-2b^3x^3}{5x^{5/2}\sqrt{2+bx}}$$

input
```
Integrate[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

output
```
(-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(bx+2)^{3/2}}dx$$

$$\Big\downarrow \text{55}$$

$$3\int \frac{1}{x^{7/2}\sqrt{bx+2}}dx+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{2}{5}b\int \frac{1}{x^{5/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{5x^{5/2}}\right)+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{2}{5}b\left(-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)-\frac{\sqrt{bx+2}}{5x^{5/2}}\right)+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

$$\Big\downarrow \text{48}$$

$$3\left(-\frac{2}{5}b\left(\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)-\frac{\sqrt{bx+2}}{5x^{5/2}}\right)+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

input
```
Int[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

output
```
1/(x^(5/2)*Sqrt[2 + b*x]) + 3*(-1/5*Sqrt[2 + b*x]/x^(5/2) - (2*b*(-1/3*Sqr
t[2 + b*x]/x^(3/2) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])))/5)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2b^3x^3+2b^2x^2-bx+1}{5x^{\frac{5}{2}}\sqrt{bx+2}}$	35
orering	$-\frac{2b^3x^3+2b^2x^2-bx+1}{5x^{\frac{5}{2}}\sqrt{bx+2}}$	35
meijerg	$-\frac{\sqrt{2}\,(2b^3x^3+2b^2x^2-bx+1)}{10x^{\frac{5}{2}}\sqrt{\frac{bx}{2}+1}}$	39
risch	$-\frac{11b^3x^3+16b^2x^2-8bx+8}{40x^{\frac{5}{2}}\sqrt{bx+2}}-\frac{b^3\sqrt{x}}{8\sqrt{bx+2}}$	51
default	$-\frac{1}{5x^{\frac{5}{2}}\sqrt{bx+2}}-\frac{3b\left(-\frac{1}{3x^{\frac{3}{2}}\sqrt{bx+2}}-\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}\right)}{3}\right)}{5}$	59

input
```
int(1/x^(7/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(2*b^3*x^3+2*b^2*x^2-b*x+1)/x^(5/2)/(b*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx = -\frac{(2\,b^3x^3+2\,b^2x^2-bx+1)\sqrt{bx+2}\sqrt{x}}{5\,(bx^4+2\,x^3)}$$

input
```
integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(2*b^3*x^3 + 2*b^2*x^2 - b*x + 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^4 + 2*x^
3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(70) = 140.

Time = 4.52 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.64

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx = -\frac{2b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2} - \frac{10b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2} - \frac{15b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2} - \frac{5b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2} - \frac{4b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}$$

input
```
integrate(1/x**(7/2)/(b*x+2)**(3/2),x)
```

output
```
-2*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**
10*x**3 + 40*b**9*x**2) - 10*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(5*b**12*x**
5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 15*b**(25/2)*x**3*sqrt
(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2
) - 5*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*
b**10*x**3 + 40*b**9*x**2) - 4*b**(19/2)*sqrt(1 + 2/(b*x))/(5*b**12*x**5 +
 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx = -\frac{b^3\sqrt{x}}{8\sqrt{bx+2}} - \frac{3\sqrt{bx+2}b^2}{8\sqrt{x}} + \frac{(bx+2)^{\frac{3}{2}}b}{8\,x^{\frac{3}{2}}} - \frac{(bx+2)^{\frac{5}{2}}}{40\,x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*b^3*sqrt(x)/sqrt(b*x + 2) - 3/8*sqrt(b*x + 2)*b^2/sqrt(x) + 1/8*(b*x
+ 2)^(3/2)*b/x^(3/2) - 1/40*(b*x + 2)^(5/2)/x^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(52) = 104.

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx = -\frac{b^{\frac{9}{2}}}{2\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|} - \frac{\left(\frac{60\,b^6}{|b|}+\left(\frac{11\,(bx+2)b^6}{|b|}-\frac{50\,b^6}{|b|}\right)(bx+2)\right)\sqrt{bx+2}}{40\,((bx+2)b-2\,b)^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*b^(9/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*
abs(b)) - 1/40*(60*b^6/abs(b) + (11*(b*x + 2)*b^6/abs(b) - 50*b^6/abs(b))*
(b*x + 2))*sqrt(b*x + 2)/((b*x + 2)*b - 2*b)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx = -\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b\,x^2}{5}-\frac{x}{5}+\frac{1}{5\,b}+\frac{2\,b^2\,x^3}{5}\right)}{x^{7/2}+\frac{2\,x^{5/2}}{b}}$$

input
```
int(1/(x^(7/2)*(b*x + 2)^(3/2)),x)
```

output
```
-((b*x + 2)^(1/2)*((2*b*x^2)/5 - x/5 + 1/(5*b) + (2*b^2*x^3)/5))/(x^(7/2)
+ (2*x^(5/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+2}\,b^2x^3-2\sqrt{x}\,b^3x^3-2\sqrt{x}\,b^2x^2+\sqrt{x}\,bx-\sqrt{x}}{5\sqrt{bx+2}\,x^3}$$

input
```
int(1/x^(7/2)/(b*x+2)^(3/2),x)
```

output
```
(2*sqrt(b)*sqrt(b*x + 2)*b**2*x**3 - 2*sqrt(x)*b**3*x**3 - 2*sqrt(x)*b**2*
x**2 + sqrt(x)*b*x - sqrt(x))/(5*sqrt(b*x + 2)*x**3)
```

3.513 $\int \frac{x^{5/2}}{(2+bx)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}} dx = -\frac{8\sqrt{x}}{3b^3(2+bx)^{3/2}} + \frac{28\sqrt{x}}{3b^3\sqrt{2+bx}} + \frac{\sqrt{x}\sqrt{2+bx}}{b^3} - \frac{10\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

output
```
-8/3*x^(1/2)/b^3/(b*x+2)^(3/2)+28/3*x^(1/2)/b^3/(b*x+2)^(1/2)+x^(1/2)*(b*x
+2)^(1/2)/b^3-10*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}} dx = \frac{\sqrt{x}(60+40bx+3b^2x^2)}{3b^3(2+bx)^{3/2}} + \frac{20\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2+bx}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(2 + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*(60 + 40*b*x + 3*b^2*x^2))/(3*b^3*(2 + b*x)^(3/2)) + (20*ArcTanh[
(Sqrt[b]*Sqrt[x])/(Sqrt[2] - Sqrt[2 + b*x])])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {57, 57, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(bx+2)^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{5\int \frac{x^{3/2}}{(bx+2)^{3/2}}dx}{3b}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{x}}{\sqrt{bx+2}}dx}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}\right)}{3b}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}\right)}{3b}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

$$\Big\downarrow \text{63}$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}\right)}{3b}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{bx+2}}\right)}{3b}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

input
```
Int[x^(5/2)/(2 + b*x)^(5/2),x]
```

output
```
(-2*x^(5/2))/(3*b*(2 + b*x)^(3/2)) + (5*((-2*x^(3/2))/(b*Sqrt[2 + b*x]) +
(3*((Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)))/b))/(3*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\left(\frac{21}{4}b^{2}x^{2}+70bx+105\right)}{21\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}-10\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	63
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b^{3}}+\frac{\left(-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{b^{\frac{7}{2}}}-\frac{8\sqrt{b\left(x+\frac{2}{b}\right)^{2}-2x-\frac{4}{b}}}{3b^{5}\left(x+\frac{2}{b}\right)^{2}}+\frac{28\sqrt{b\left(x+\frac{2}{b}\right)^{2}-2x-\frac{4}{b}}}{3b^{4}\left(x+\frac{2}{b}\right)}\right)\sqrt{x(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	136

input
```
int(x^(5/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/3/b^(7/2)/Pi^(1/2)*(1/56*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(21/4*b^2*x^2+
70*b*x+105)/(1/2*b*x+1)^(3/2)-15/4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.15

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx=\left[\frac{15\left(b^{2}x^{2}+4\,bx+4\right)\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(3\,b^{3}x^{2}+40\,b^{2}x+60\,b\right)\sqrt{bx}}{3\left(b^{6}x^{2}+4\,b^{5}x+4\,b^{4}\right)}\right.$$

input
```
integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(15*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqr
t(x) + 1) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt(x))/(b^6*x^2
+ 4*b^5*x + 4*b^4), 1/3*(30*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(-b)
*sqrt(x)/sqrt(b*x + 2)) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt
(x))/(b^6*x^2 + 4*b^5*x + 4*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.

Time = 3.06 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.62

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx = \frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}} + \frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}} + \frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}} - \frac{30b^{10}x^{\frac{27}{2}}\sqrt{bx+2}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}} - \frac{60b^{9}x^{\frac{25}{2}}\sqrt{bx+2}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}$$

input
```
integrate(x**(5/2)/(b*x+2)**(5/2),x)
```

output
```
3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(2
5/2)*sqrt(b*x + 2)) + 40*b**(21/2)*x**14/(3*b**(27/2)*x**(27/2)*sqrt(b*x +
 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) + 60*b**(19/2)*x**13/(3*b**(27/
2)*x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 30*b**
10*x**(27/2)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x
**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 60*b**9*x*
*(25/2)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27
/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx = \frac{2\left(2\,b^2 + \frac{10\,(bx+2)b}{x} - \frac{15\,(bx+2)^2}{x^2}\right)}{3\left(\frac{(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} - \frac{(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)} + \frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(2*b^2 + 10*(b*x + 2)*b/x - 15*(b*x + 2)^2/x^2)/((b*x + 2)^(3/2)*b^4/x
^(3/2) - (b*x + 2)^(5/2)*b^3/x^(5/2)) + 5*log(-(sqrt(b) - sqrt(b*x + 2)/sq
rt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(62) = 124$.

Time = 1.43 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.11

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx = \frac{\left(\frac{15\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2\right)}{\sqrt{b}} + \frac{3\sqrt{(bx+2)b-2b}\sqrt{bx+2}}{b} + \frac{16\left(9\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^4\sqrt{b}+\cdots\right.}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{\cdots}\right.\right.}\right.}{3\,b^4}$$

input
```
integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
1/3*(15*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) +
 3*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b + 16*(9*(sqrt(b*x + 2)*sqrt(b)
- sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 24*(sqrt(b*x + 2)*sqrt(b) - sqrt((b
*x + 2)*b - 2*b))^2*b^(3/2) + 28*b^(5/2))/((sqrt(b*x + 2)*sqrt(b) - sqrt((
b*x + 2)*b - 2*b))^2 + 2*b)^3)*abs(b)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx = \int \frac{x^{5/2}}{(b\,x+2)^{5/2}}\,dx$$

input
```
int(x^(5/2)/(b*x + 2)^(5/2),x)
```

output
```
int(x^(5/2)/(b*x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx = \frac{-30\sqrt{b}\,\sqrt{bx+2}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)bx-60\sqrt{b}\,\sqrt{bx+2}\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)-5\sqrt{b}\,\sqrt{bx+}}{3\sqrt{bx+2}\,b^4\,(bx+2)}$$

input
```
int(x^(5/2)/(b*x+2)^(5/2),x)
```

output
```
( - 30*sqrt(b)*sqrt(b*x + 2)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2)
)*b*x - 60*sqrt(b)*sqrt(b*x + 2)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqr
t(2)) - 5*sqrt(b)*sqrt(b*x + 2)*b*x - 10*sqrt(b)*sqrt(b*x + 2) + 3*sqrt(x)
*b**3*x**2 + 40*sqrt(x)*b**2*x + 60*sqrt(x)*b)/(3*sqrt(b*x + 2)*b**4*(b*x
+ 2))
```

3.514 $\int \frac{x^{3/2}}{(2+bx)^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}} dx = \frac{4\sqrt{x}}{3b^2(2+bx)^{3/2}} - \frac{8\sqrt{x}}{3b^2\sqrt{2+bx}} + \frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

output
```
4/3*x^(1/2)/b^2/(b*x+2)^(3/2)-8/3*x^(1/2)/b^2/(b*x+2)^(1/2)+2*arcsinh(1/2*
b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}} dx = -\frac{4\sqrt{x}(3+2bx)}{3b^2(2+bx)^{3/2}} - \frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(2 + b*x)^(5/2),x]
```

output
```
(-4*Sqrt[x]*(3 + 2*b*x))/(3*b^2*(2 + b*x)^(3/2)) - (2*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[2 + b*x]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {57, 57, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(bx+2)^{5/2}}\,dx$$

$$\Big\downarrow \ 57$$

$$\frac{\int \frac{\sqrt{x}}{(bx+2)^{3/2}}dx}{b}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

$$\Big\downarrow \ 57$$

$$\frac{\frac{\int \frac{1}{\sqrt{x}\sqrt{bx+2}}dx}{b}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}}{b}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

$$\Big\downarrow \ 63$$

$$\frac{\frac{2\int \frac{1}{\sqrt{bx+2}}d\sqrt{x}}{b}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}}{b}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

$$\Big\downarrow \ 222$$

$$\frac{\frac{2\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}}{b}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

input
```
Int[x^(3/2)/(2 + b*x)^(5/2),x]
```

output
```
(-2*x^(3/2))/(3*b*(2 + b*x)^(3/2)) + ((-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(10bx+15)}{15\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55

input
```
int(x^(3/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
4/3/b^(5/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(10*b*x+15)/(
1/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.51

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx = \left[\frac{3\,(b^2x^2+4\,bx+4)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-4\,(2\,b^2x+3\,b)\sqrt{bx+2}\sqrt{x}}{3\,(b^5x^2+4\,b^4x+4\,b^3)}, -\right.$$

input
```
integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt
(x) + 1) - 4*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x + 4
*b^3), -2/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt
(b*x + 2)) + 2*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x +
 4*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(65) = 130.

Time = 1.54 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.84

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx = -\frac{8b^{\frac{11}{2}}x^8}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}} - \frac{12b^{\frac{9}{2}}x^7}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}} + \frac{6b^5x^{\frac{15}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}} + \frac{12b^4x^{\frac{13}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}$$

input
```
integrate(x**(3/2)/(b*x+2)**(5/2),x)
```

output
```
-8*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x**(1
3/2)*sqrt(b*x + 2)) - 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2
) + 6*b**(13/2)*x**(13/2)*sqrt(b*x + 2)) + 6*b**5*x**(15/2)*sqrt(b*x + 2)*
asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*
b**(13/2)*x**(13/2)*sqrt(b*x + 2)) + 12*b**4*x**(13/2)*sqrt(b*x + 2)*asinh
(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(1
3/2)*x**(13/2)*sqrt(b*x + 2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx = -\frac{2\left(b+\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}b^2} - \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(b + 3*(b*x + 2)/x)*x^(3/2)/((b*x + 2)^(3/2)*b^2) - log(-(sqrt(b) - s
qrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(48) = 96.

Time = 1.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.30

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx = -\frac{\left(\frac{3\,\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{\sqrt{b}} + \frac{16\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^4\sqrt{b}+6\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2 b^{\frac{3}{2}}+8\,b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3}\right)|b|}{3\,b^3}$$

input
```
integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) +
 16*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 6*(sq
rt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(3/2) + 8*b^(5/2))/((sq
rt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3)*abs(b)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx=\int \frac{x^{3/2}}{{\left(b\,x+2\right)}^{5/2}} \,d x$$

input
```
int(x^(3/2)/(b*x + 2)^(5/2),x)
```

output
```
int(x^(3/2)/(b*x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx=\frac{2\sqrt{b}\,\sqrt{bx+2}\,\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)bx+4\sqrt{b}\,\sqrt{bx+2}\,\log\left(\frac{\sqrt{bx+2}+\sqrt{x}\,\sqrt{b}}{\sqrt{2}}\right)-\frac{8\sqrt{x}\,b^{2}x}{3}-4\sqrt{x}\,\cdots}{\sqrt{bx+2}\,b^{3}\left(bx+2\right)}$$

input
```
int(x^(3/2)/(b*x+2)^(5/2),x)
```

output
```
(2*(3*sqrt(b)*sqrt(b*x + 2)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(2))
*b*x + 6*sqrt(b)*sqrt(b*x + 2)*log((sqrt(b*x + 2) + sqrt(x)*sqrt(b))/sqrt(
2)) - 4*sqrt(x)*b**2*x - 6*sqrt(x)*b))/(3*sqrt(b*x + 2)*b**3*(b*x + 2))
```

3.515 $\int \frac{\sqrt{x}}{(2+bx)^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}} \, dx = \frac{x^{3/2}}{3(2+bx)^{3/2}}$$

output
```
1/3*x^(3/2)/(b*x+2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}} \, dx = \frac{x^{3/2}}{3(2+bx)^{3/2}}$$

input
```
Integrate[Sqrt[x]/(2 + b*x)^(5/2),x]
```

output
```
x^(3/2)/(3*(2 + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(bx+2)^{5/2}}\,dx$$

$$\Big\downarrow \; 48$$

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

input
```
Int[Sqrt[x]/(2 + b*x)^(5/2),x]
```

output
```
x^(3/2)/(3*(2 + b*x)^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x^{\frac{3}{2}}}{3(bx+2)^{\frac{3}{2}}}$	13
orering	$\frac{x^{\frac{3}{2}}}{3(bx+2)^{\frac{3}{2}}}$	13
meijerg	$\frac{x^{\frac{3}{2}}\sqrt{2}}{12\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	17
default	$-\frac{\sqrt{x}}{b(bx+2)^{\frac{3}{2}}}+\frac{\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}}{b}$	46

input
```
int(x^(1/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3/2)/(b*x+2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx=\frac{\sqrt{bx+2}\,x^{\frac{3}{2}}}{3\,(b^2x^2+4\,bx+4)}$$

input
```
integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*x + 2)*x^(3/2)/(b^2*x^2 + 4*b*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx = \frac{x^{\frac{3}{2}}}{3bx\sqrt{bx+2}+6\sqrt{bx+2}}$$

input
```
integrate(x**(1/2)/(b*x+2)**(5/2),x)
```

output
```
x**(3/2)/(3*b*x*sqrt(b*x + 2) + 6*sqrt(b*x + 2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx = \frac{x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x^(3/2)/(b*x + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(12) = 24.

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.56

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx = \frac{4\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{4}\sqrt{b}+4\,b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)^{3}b^{2}}$$

input
```
integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 4*b^(
5/2))*abs(b)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^
3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx = \frac{x^{3/2}}{3\,(b\,x+2)^{3/2}}$$

input
```
int(x^(1/2)/(b*x + 2)^(5/2),x)
```

output
```
x^(3/2)/(3*(b*x + 2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx = \frac{\sqrt{b}\,\sqrt{bx+2}\,bx+2\sqrt{b}\,\sqrt{bx+2}+\sqrt{x}\,b^2x}{3\sqrt{bx+2}\,b^2\,(bx+2)}$$

input
```
int(x^(1/2)/(b*x+2)^(5/2),x)
```

output
```
(sqrt(b)*sqrt(b*x + 2)*b*x + 2*sqrt(b)*sqrt(b*x + 2) + sqrt(x)*b**2*x)/(3*
sqrt(b*x + 2)*b**2*(b*x + 2))
```

3.516 $\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx = \frac{\sqrt{x}}{3(2+bx)^{3/2}} + \frac{\sqrt{x}}{3\sqrt{2+bx}}$$

output
```
1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx = \frac{\sqrt{x}(3+bx)}{3(2+bx)^{3/2}}$$

input
```
Integrate[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

output
```
(Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(bx+2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{3} \int \frac{1}{\sqrt{x}(bx+2)^{3/2}}dx + \frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{x}}{3\sqrt{bx+2}} + \frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

input
```
Int[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

output
```
Sqrt[x]/(3*(2 + b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 + b*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{\sqrt{x}\,(bx+3)}{3(bx+2)^{\frac{3}{2}}}$	18
orering	$\frac{\sqrt{x}\,(bx+3)}{3(bx+2)^{\frac{3}{2}}}$	18
meijerg	$\frac{\sqrt{x}\,\sqrt{2}\,(bx+3)}{12\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	22
default	$\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}$	26

input
```
int(1/x^(1/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(1/2)*(b*x+3)/(b*x+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx=\frac{(bx+3)\sqrt{bx+2}\sqrt{x}}{3\,(b^2x^2+4\,bx+4)}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^2 + 4*b*x + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(29) = 58$.

Time = 0.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx = \frac{bx}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}} + \frac{3}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

input
```
integrate(1/x**(1/2)/(b*x+2)**(5/2),x)
```

output
```
b*x/(3*b**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x))) + 3/(3*
b**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx = -\frac{\left(b-\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{6\,(bx+2)^{\frac{3}{2}}}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(b - 3*(b*x + 2)/x)*x^(3/2)/(b*x + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.14

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx = \frac{8\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3|b|}$$

input
```
integrate(1/x^(1/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
8/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(5/2)/
(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx = \frac{3\sqrt{x}\sqrt{b\,x+2}+b\,x^{3/2}\sqrt{b\,x+2}}{3\,b^2\,x^2+12\,b\,x+12}$$

input
```
int(1/(x^(1/2)*(b*x + 2)^(5/2)),x)
```

output
```
(3*x^(1/2)*(b*x + 2)^(1/2) + b*x^(3/2)*(b*x + 2)^(1/2))/(12*b*x + 3*b^2*x^
2 + 12)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx = \frac{-\sqrt{b}\,\sqrt{bx+2}\,bx-2\sqrt{b}\,\sqrt{bx+2}+\sqrt{x}\,b^2x+3\sqrt{x}\,b}{3\sqrt{bx+2}\,b\,(bx+2)}$$

input

```
int(1/x^(1/2)/(b*x+2)^(5/2),x)
```

output

```
( - sqrt(b)*sqrt(b*x + 2)*b*x - 2*sqrt(b)*sqrt(b*x + 2) + sqrt(x)*b**2*x +
 3*sqrt(x)*b)/(3*sqrt(b*x + 2)*b*(b*x + 2))
```

3.517 $\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = \frac{1}{3\sqrt{x}(2+bx)^{3/2}} + \frac{2}{3\sqrt{x}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3\sqrt{x}}$$

output
```
1/3/x^(1/2)/(b*x+2)^(3/2)+2/3/x^(1/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(1
/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = \frac{-3-6bx-2b^2x^2}{3\sqrt{x}(2+bx)^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

output
```
(-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(bx+2)^{5/2}}dx$$

$$\Big\downarrow \ 55$$

$$\frac{2}{3}\int \frac{1}{x^{3/2}(bx+2)^{3/2}}dx+\frac{1}{3\sqrt{x}(bx+2)^{3/2}}$$

$$\Big\downarrow \ 55$$

$$\frac{2}{3}\left(\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx+\frac{1}{\sqrt{x}\sqrt{bx+2}}\right)+\frac{1}{3\sqrt{x}(bx+2)^{3/2}}$$

$$\Big\downarrow \ 48$$

$$\frac{2}{3}\left(\frac{1}{\sqrt{x}\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{\sqrt{x}}\right)+\frac{1}{3\sqrt{x}(bx+2)^{3/2}}$$

input
```
Int[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

output
```
1/(3*Sqrt[x]*(2 + b*x)^(3/2)) + (2*(1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b
*x]/Sqrt[x]))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{2b^{2}x^{2}+6bx+3}{3\sqrt{x}\,(bx+2)^{\frac{3}{2}}}$	27
orering	$-\frac{2b^{2}x^{2}+6bx+3}{3\sqrt{x}\,(bx+2)^{\frac{3}{2}}}$	27
meijerg	$-\frac{\sqrt{2}\,(2b^{2}x^{2}+6bx+3)}{12\sqrt{x}\,\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	31
risch	$-\frac{\sqrt{bx+2}}{4\sqrt{x}}-\frac{b(5bx+12)\sqrt{x}}{12(bx+2)^{\frac{3}{2}}}$	33
default	$-\frac{1}{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}-2b\left(\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}\right)$	42

input
```
int(1/x^(3/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*b^2*x^2+6*b*x+3)/x^(1/2)/(b*x+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = -\frac{(2\,b^{2}x^{2}+6\,bx+3)\sqrt{bx+2}\sqrt{x}}{3\,(b^{2}x^{3}+4\,bx^{2}+4\,x)}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*b^2*x^2 + 6*b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^3 + 4*b*x^2 + 4*
x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.

Time = 1.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.13

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = -\frac{2b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4} - \frac{6b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4} - \frac{3b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}$$

input
```
integrate(1/x**(3/2)/(b*x+2)**(5/2),x)
```

output
```
-2*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) -
6*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 3*b*
*(9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = \frac{\left(b^2 - \frac{6\,(bx+2)b}{x}\right)x^{\frac{3}{2}}}{12\,(bx+2)^{\frac{3}{2}}} - \frac{\sqrt{bx+2}}{4\,\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(b^2 - 6*(b*x + 2)*b/x)*x^(3/2)/(b*x + 2)^(3/2) - 1/4*sqrt(b*x + 2)/s
qrt(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(37) = 74$.

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.64

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = -\frac{\sqrt{bx+2}b^2}{4\sqrt{(bx+2)b-2b}|b|} - \frac{3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^4 b^{\frac{5}{2}} + 24\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2 b^{\frac{7}{2}} + 20\,b^{\frac{9}{2}}}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2\,b\right)^3|b|}$$

input
```
integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 1/3*(3*(sqrt(b*x
 + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(5/2) + 24*(sqrt(b*x + 2)*sqr
t(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(7/2) + 20*b^(9/2))/(((sqrt(b*x + 2)*s
qrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = -\frac{3\sqrt{b\,x+2}+6\,b\,x\,\sqrt{b\,x+2}+2\,b^2\,x^2\,\sqrt{b\,x+2}}{\sqrt{x}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

input
```
int(1/(x^(3/2)*(b*x + 2)^(5/2)),x)
```

output
```
-(3*(b*x + 2)^(1/2) + 6*b*x*(b*x + 2)^(1/2) + 2*b^2*x^2*(b*x + 2)^(1/2))/(
x^(1/2)*(x*(12*b + 3*b^2*x) + 12))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+2}\,b\,x^2+4\sqrt{b}\,\sqrt{bx+2}\,x-2\sqrt{x}\,b^2x^2-6\sqrt{x}\,bx-3\sqrt{x}}{3\sqrt{bx+2}\,x\,(bx+2)}$$

input
```
int(1/x^(3/2)/(b*x+2)^(5/2),x)
```

output
```
(2*sqrt(b)*sqrt(b*x + 2)*b*x**2 + 4*sqrt(b)*sqrt(b*x + 2)*x - 2*sqrt(x)*b*
*2*x**2 - 6*sqrt(x)*b*x - 3*sqrt(x))/(3*sqrt(b*x + 2)*x*(b*x + 2))
```

3.518 $\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

output
```
1/3/x^(3/2)/(b*x+2)^(3/2)+1/x^(3/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(3/2
)+2/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{-1+3bx+6b^2x^2+2b^3x^3}{3x^{3/2}(2+bx)^{3/2}}$$

input
```
Integrate[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

output
```
(-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(bx+2)^{5/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\int \frac{1}{x^{5/2}(bx+2)^{3/2}}dx+\frac{1}{3x^{3/2}(bx+2)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$2\int \frac{1}{x^{5/2}\sqrt{bx+2}}dx+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{bx+2}}dx-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$2\left(\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}\right)+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}$$

input
```
Int[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

output
```
1/(3*x^(3/2)*(2 + b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 + b*x]) + 2*(-1/3*Sqrt[2
 + b*x]/x^(3/2) + (b*Sqrt[2 + b*x])/(3*Sqrt[x]))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2b^3x^3+6b^2x^2+3bx-1}{3x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}$	35
orering	$\frac{2b^3x^3+6b^2x^2+3bx-1}{3x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}$	35
meijerg	$-\frac{\sqrt{2}\left(-2b^3x^3-6b^2x^2-3bx+1\right)}{12x^{\frac{3}{2}}\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	39
risch	$\frac{4b^2x^2+7bx-2}{12x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^2(4bx+9)\sqrt{x}}{12(bx+2)^{\frac{3}{2}}}$	49
default	$-\frac{1}{3x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}-b\left(-\frac{1}{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}-2b\left(\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}\right)\right)$	58

input
```
int(1/x^(5/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*b^3*x^3+6*b^2*x^2+3*b*x-1)/x^(3/2)/(b*x+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{(2\,b^3x^3+6\,b^2x^2+3\,bx-1)\sqrt{bx+2}\sqrt{x}}{3\,(b^2x^4+4\,bx^3+4\,x^2)}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b^3*x^3 + 6*b^2*x^2 + 3*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^4 + 4
*b*x^3 + 4*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(66) = 132.

Time = 2.66 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.62

$$\begin{aligned}\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx &= \frac{2b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} \\ &+ \frac{10b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} + \frac{15b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} \\ &+ \frac{5b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} - \frac{2b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}\end{aligned}$$

input
```
integrate(1/x**(5/2)/(b*x+2)**(5/2),x)
```

output
```
2*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**1
0*x**2 + 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**12*x**4 +
18*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2*sqrt(1 + 2/
(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 5*b**(
21/2)*x*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 +
24*b**9*x) - 2*b**(19/2)*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 +
 36*b**10*x**2 + 24*b**9*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{3\,\sqrt{bx+2}b}{8\,\sqrt{x}} - \frac{\left(b^3 - \frac{9\,(bx+2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(bx+2)^{\frac{3}{2}}} - \frac{(bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

output
```
3/8*sqrt(b*x + 2)*b/sqrt(x) - 1/24*(b^3 - 9*(b*x + 2)*b^2/x)*x^(3/2)/(b*x
+ 2)^(3/2) - 1/24*(b*x + 2)^(3/2)/x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(49) = 98.

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{(4\,(bx+2)b^2|b| - 9\,b^2|b|)\sqrt{bx+2}}{12\,((bx+2)b-2\,b)^{\frac{3}{2}}} + \frac{3\left(\sqrt{bx+2}\sqrt{b} - \sqrt{(bx+2)b-2\,b}\right)^4 b^{\frac{7}{2}} + 18\left(\sqrt{bx+2}\sqrt{b} - \sqrt{(bx+2)b-2\,b}\right)^2 b^{\frac{9}{2}} + 16\,b^{\frac{11}{2}}}{3\left(\left(\sqrt{bx+2}\sqrt{b} - \sqrt{(bx+2)b-2\,b}\right)^2 + 2\,b\right)^3 |b|}$$

input
```
integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

output
```
1/12*(4*(b*x + 2)*b^2*abs(b) - 9*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*b -
2*b)^(3/2) + 1/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^
(7/2) + 18*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(9/2) + 1
6*b^(11/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3
*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{3\,b\,x\,\sqrt{b\,x+2}-\sqrt{b\,x+2}+6\,b^2\,x^2\,\sqrt{b\,x+2}+2\,b^3\,x^3\,\sqrt{b\,x+2}}{x^{3/2}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

input
```
int(1/(x^(5/2)*(b*x + 2)^(5/2)),x)
```

output
```
(3*b*x*(b*x + 2)^(1/2) - (b*x + 2)^(1/2) + 6*b^2*x^2*(b*x + 2)^(1/2) + 2*b
^3*x^3*(b*x + 2)^(1/2))/(x^(3/2)*(x*(12*b + 3*b^2*x) + 12))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{-2\sqrt{b}\,\sqrt{bx+2}\,b^2x^3-4\sqrt{b}\,\sqrt{bx+2}\,b\,x^2+2\sqrt{x}\,b^3x^3+6\sqrt{x}\,b^2x^2+3\sqrt{x}\,bx-\sqrt{x}}{3\sqrt{bx+2}\,x^2\,(bx+2)}$$

input
```
int(1/x^(5/2)/(b*x+2)^(5/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(b*x + 2)*b**2*x**3 - 4*sqrt(b)*sqrt(b*x + 2)*b*x**2 + 2
*sqrt(x)*b**3*x**3 + 6*sqrt(x)*b**2*x**2 + 3*sqrt(x)*b*x - sqrt(x))/(3*sqr
t(b*x + 2)*x**2*(b*x + 2))
```

3.519 $\int \frac{x^{3/2}}{\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{x^{3/2}}{\sqrt{1+x}} \, dx = -\frac{3}{4}\sqrt{x}\sqrt{1+x} + \frac{1}{2}x^{3/2}\sqrt{1+x} + \frac{3\operatorname{arcsinh}\left(\sqrt{x}\right)}{4}$$

output
```
-3/4*x^(1/2)*(1+x)^(1/2)+1/2*x^(3/2)*(1+x)^(1/2)+3/4*arcsinh(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}}{\sqrt{1+x}} \, dx = \frac{1}{4}\sqrt{x}\sqrt{1+x}(-3+2x) - \frac{3}{4}\log\left(-\sqrt{x} + \sqrt{1+x}\right)$$

input
```
Integrate[x^(3/2)/Sqrt[1 + x],x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(-3 + 2*x))/4 - (3*Log[-Sqrt[x] + Sqrt[1 + x]])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx\right)$$

$$\Big\downarrow \text{ 63}$$

$$\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}x^{3/2}\sqrt{x+1}-\frac{3}{4}\left(\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)\right)$$

input
```
Int[x^(3/2)/Sqrt[1 + x],x]
```

output
```
(x^(3/2)*Sqrt[1 + x])/2 - (3*(Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,(-10x+15)\sqrt{1+x}}{20}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{4}}{\sqrt{\pi}}$	33
risch	$\frac{(2x-3)\sqrt{x}\,\sqrt{1+x}}{4}+\frac{3\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{8\sqrt{x}\,\sqrt{1+x}}$	45
default	$\frac{x^{\frac{3}{2}}\sqrt{1+x}}{2}-\frac{3\sqrt{x}\,\sqrt{1+x}}{4}+\frac{3\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{8\sqrt{x}\,\sqrt{1+x}}$	50

input
```
int(x^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/2)*(-10*x+15)*(1+x)^(1/2)+3/4*Pi^(1/2)*arc
sinh(x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx = \frac{1}{4}\,(2\,x-3)\sqrt{x+1}\sqrt{x} - \frac{3}{8}\,\log\left(2\,\sqrt{x+1}\sqrt{x} - 2\,x - 1\right)$$

input
```
integrate(x^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*x - 3)*sqrt(x + 1)*sqrt(x) - 3/8*log(2*sqrt(x + 1)*sqrt(x) - 2*x -
1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.56

$$\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx = \begin{cases} \frac{\sqrt{x}(x+1)^{\frac{3}{2}}}{2} - \frac{5\sqrt{x}\sqrt{x+1}}{4} + \frac{3\,\operatorname{acosh}\left(\sqrt{x+1}\right)}{4} & \text{for}\ |x+1| > 1 \\ -\frac{3i\,\operatorname{asin}\left(\sqrt{x+1}\right)}{4} - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}} + \frac{7i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}} - \frac{5i\sqrt{x+1}}{4\sqrt{-x}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((sqrt(x)*(x + 1)**(3/2)/2 - 5*sqrt(x)*sqrt(x + 1)/4 + 3*acosh(sq
rt(x + 1))/4, Abs(x + 1) > 1), (-3*I*asin(sqrt(x + 1))/4 - I*(x + 1)**(5/2
)/(2*sqrt(-x)) + 7*I*(x + 1)**(3/2)/(4*sqrt(-x)) - 5*I*sqrt(x + 1)/(4*sqrt
(-x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx = -\frac{\frac{3\,(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}} - \frac{5\,\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^2}{x^2} - \frac{2\,(x+1)}{x} + 1\right)} + \frac{3}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) - \frac{3}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

input
```
integrate(x^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(3*(x + 1)^(3/2)/x^(3/2) - 5*sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*
(x + 1)/x + 1) + 3/8*log(sqrt(x + 1)/sqrt(x) + 1) - 3/8*log(sqrt(x + 1)/sq
rt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx = \frac{1}{4}\,(2\,x - 3)\sqrt{x+1}\sqrt{x} - \frac{3}{4}\,\log\left(\sqrt{x+1} - \sqrt{x}\right)$$

input
```
integrate(x^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*x - 3)*sqrt(x + 1)*sqrt(x) - 3/4*log(sqrt(x + 1) - sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx = \int \frac{x^{3/2}}{\sqrt{x+1}}\,dx$$

input
```
int(x^(3/2)/(x + 1)^(1/2),x)
```

output
```
int(x^(3/2)/(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx = \frac{\sqrt{x}\,\sqrt{x+1}\,x}{2} - \frac{3\sqrt{x}\,\sqrt{x+1}}{4} + \frac{3\log\left(\sqrt{x+1}+\sqrt{x}\right)}{4}$$

input
```
int(x^(3/2)/(1+x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(x + 1)*x - 3*sqrt(x)*sqrt(x + 1) + 3*log(sqrt(x + 1) + sqr
t(x)))/4
```

3.520 $\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx = \sqrt{x}\sqrt{1+x} - \text{arcsinh}\left(\sqrt{x}\right)$$

output
```
x^(1/2)*(1+x)^(1/2)-arcsinh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[x]/Sqrt[1 + x],x]
```

output
```
(Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]
]))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{63}$$

$$\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$

$$\Big\downarrow \text{222}$$

$$\sqrt{x}\sqrt{x+1}-\operatorname{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x]/Sqrt[1 + x],x]
```

output
```
Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{1+x}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{\sqrt{\pi}}$	27
default	$\sqrt{x}\,\sqrt{1+x}-\frac{\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{1+x}}$	39
risch	$\sqrt{x}\,\sqrt{1+x}-\frac{\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{1+x}}$	39

input
```
int(x^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/Pi^(1/2)*(Pi^(1/2)*x^(1/2)*(1+x)^(1/2)-Pi^(1/2)*arcsinh(x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x} + \frac{1}{2}\log\left(2\sqrt{x+1}\sqrt{x} - 2x - 1\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \begin{cases} \sqrt{x}\sqrt{x+1} - \operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ i \operatorname{asin}\left(\sqrt{x+1}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{-x}} + \frac{i\sqrt{x+1}}{\sqrt{-x}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((sqrt(x)*sqrt(x + 1) - acosh(sqrt(x + 1)), Abs(x + 1) > 1), (I*a
sin(sqrt(x + 1)) - I*(x + 1)**(3/2)/sqrt(-x) + I*sqrt(x + 1)/sqrt(-x), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(16) = 32$.

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)} - \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right) + \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
 1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - 2\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)$$

input
```
int(x^(1/2)/(x + 1)^(1/2),x)
```

output
```
x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - \log\left(\sqrt{x+1} + \sqrt{x}\right)$$

input
```
int(x^(1/2)/(1+x)^(1/2),x)
```

output
```
sqrt(x)*sqrt(x + 1) - log(sqrt(x + 1) + sqrt(x))
```

3.521 $\int \frac{1}{\sqrt{x}\sqrt{1+x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}} dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

output
```
2*arcsinh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}} dx = -2\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
-2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$
$$\Big\downarrow \text{ 63}$$
$$2\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$
$$\Big\downarrow \text{ 222}$$
$$2\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
2*ArcSinh[Sqrt[x]]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$2\ \mathrm{arcsinh}\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{x(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{\sqrt{x}\,\sqrt{1+x}}$	28

input
```
int(1/x^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -\log\left(2\sqrt{x+1}\sqrt{x} - 2x - 1\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \begin{cases} 2\,\mathrm{acosh}\left(\sqrt{x+1}\right) & \text{for}\ |x+1| > 1 \\ -2i\,\mathrm{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) - \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -2\,\log\left(\sqrt{x+1} - \sqrt{x}\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2*log(sqrt(x + 1) - sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 4\,\mathrm{atanh}\left(\frac{\sqrt{x+1}-1}{\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(x + 1)^(1/2)),x)
```

output
```
4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 2\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
int(1/x^(1/2)/(1+x)^(1/2),x)
```

output
```
2*log(sqrt(x + 1) + sqrt(x))
```

3.522 $\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{1+x}}{\sqrt{x}}$$

output
```
-2*(1+x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{1+x}}{\sqrt{x}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[1 + x]),x]
```

output
```
(-2*Sqrt[1 + x])/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{x+1}}\,dx$$

$$\Big\downarrow \ 48$$

$$-\frac{2\sqrt{x+1}}{\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*Sqrt[1 + x]),x]
```

output
```
(-2*Sqrt[1 + x])/Sqrt[x]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2\sqrt{1+x}}{\sqrt{x}}$	11
default	$-\frac{2\sqrt{1+x}}{\sqrt{x}}$	11
meijerg	$-\frac{2\sqrt{1+x}}{\sqrt{x}}$	11
risch	$-\frac{2\sqrt{1+x}}{\sqrt{x}}$	11
orering	$-\frac{2\sqrt{1+x}}{\sqrt{x}}$	11

input
```
int(1/x^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(1+x)^(1/2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = -\frac{2\left(\sqrt{x+1}\sqrt{x}+x\right)}{x}$$

input
```
integrate(1/x^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(x + 1)*sqrt(x) + x)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = \begin{cases} \frac{2i}{\sqrt{-1+\frac{1}{x+1}}} & \text{for } \frac{1}{|x+1|} > 1 \\ -\frac{2}{\sqrt{1-\frac{1}{x+1}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(3/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*I/sqrt(-1 + 1/(x + 1)), 1/Abs(x + 1) > 1), (-2/sqrt(1 - 1/(x
+ 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{x+1}}{\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(x + 1)/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{x+1}}{\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(x + 1)/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{x+1}}{\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(x + 1)^(1/2)),x)
```

output
```
-(2*(x + 1)^(1/2))/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx = \frac{-2\sqrt{x}\,\sqrt{x+1}-2x}{x}$$

input
```
int(1/x^(3/2)/(1+x)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(x + 1) + x))/x
```

3.523 $\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{1+x}}{3x^{3/2}} + \frac{4\sqrt{1+x}}{3\sqrt{x}}$$

output
```
-2/3*(1+x)^(1/2)/x^(3/2)+4/3*(1+x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx = \frac{2\sqrt{1+x}(-1+2x)}{3x^{3/2}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[1 + x]),x]
```

output
```
(2*Sqrt[1 + x]*(-1 + 2*x))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{x+1}}dx-\frac{2\sqrt{x+1}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4\sqrt{x+1}}{3\sqrt{x}}-\frac{2\sqrt{x+1}}{3x^{3/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[1 + x]),x]
```

output
```
(-2*Sqrt[1 + x])/(3*x^(3/2)) + (4*Sqrt[1 + x])/(3*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{2\sqrt{1+x}\,(-1+2x)}{3x^{\frac{3}{2}}}$	16
meijerg	$-\frac{2(1-2x)\sqrt{1+x}}{3x^{\frac{3}{2}}}$	16
orering	$\frac{2\sqrt{1+x}\,(-1+2x)}{3x^{\frac{3}{2}}}$	16
risch	$\frac{-\frac{2}{3}+\frac{2}{3}x+\frac{4}{3}x^{2}}{x^{\frac{3}{2}}\sqrt{1+x}}$	19
default	$-\frac{2\sqrt{1+x}}{3x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{3\sqrt{x}}$	22

input
```
int(1/x^(5/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x)^(1/2)*(-1+2*x)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx=\frac{2\left((2\,x-1)\sqrt{x+1}\sqrt{x}+2\,x^{2}\right)}{3\,x^{2}}$$

input
```
integrate(1/x^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*((2*x - 1)*sqrt(x + 1)*sqrt(x) + 2*x^2)/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.91

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx = \begin{cases} -\frac{4i(x+1)}{3\sqrt{-1+\frac{1}{x+1}}(x+1)-3\sqrt{-1+\frac{1}{x+1}}} + \frac{6i}{3\sqrt{-1+\frac{1}{x+1}}(x+1)-3\sqrt{-1+\frac{1}{x+1}}} & \text{for } \frac{1}{|x+1|} > 1 \\ \frac{4(x+1)}{3\sqrt{1-\frac{1}{x+1}}(x+1)-3\sqrt{1-\frac{1}{x+1}}} - \frac{6}{3\sqrt{1-\frac{1}{x+1}}(x+1)-3\sqrt{1-\frac{1}{x+1}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(5/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((-4*I*(x + 1)/(3*sqrt(-1 + 1/(x + 1))*(x + 1) - 3*sqrt(-1 + 1/(x
 + 1))) + 6*I/(3*sqrt(-1 + 1/(x + 1))*(x + 1) - 3*sqrt(-1 + 1/(x + 1))), 1
/Abs(x + 1) > 1), (4*(x + 1)/(3*sqrt(1 - 1/(x + 1))*(x + 1) - 3*sqrt(1 - 1
/(x + 1))) - 6/(3*sqrt(1 - 1/(x + 1))*(x + 1) - 3*sqrt(1 - 1/(x + 1))), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx = -\frac{2\,(x+1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}} + \frac{2\,\sqrt{x+1}}{\sqrt{x}}$$

input
```
integrate(1/x^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(x + 1)^(3/2)/x^(3/2) + 2*sqrt(x + 1)/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx = \frac{2\,(2\,x-1)\sqrt{x+1}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(2*x - 1)*sqrt(x + 1)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx = \frac{\left(\frac{4\,x}{3}-\frac{2}{3}\right)\,\sqrt{x+1}}{x^{3/2}}$$

input
```
int(1/(x^(5/2)*(x + 1)^(1/2)),x)
```

output
```
(((4*x)/3 - 2/3)*(x + 1)^(1/2))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx = \frac{\frac{4\sqrt{x}\,\sqrt{x+1}\,x}{3}-\frac{2\sqrt{x}\,\sqrt{x+1}}{3}-\frac{4x^2}{3}}{x^2}$$

input
```
int(1/x^(5/2)/(1+x)^(1/2),x)
```

output
```
(2*(2*sqrt(x)*sqrt(x + 1)*x - sqrt(x)*sqrt(x + 1) - 2*x**2))/(3*x**2)
```

3.524 $\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{1+x}}{5x^{5/2}} + \frac{8\sqrt{1+x}}{15x^{3/2}} - \frac{16\sqrt{1+x}}{15\sqrt{x}}$$

output
```
-2/5*(1+x)^(1/2)/x^(5/2)+8/15*(1+x)^(1/2)/x^(3/2)-16/15*(1+x)^(1/2)/x^(1/2
)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = -\frac{2\sqrt{1+x}(3-4x+8x^2)}{15x^{5/2}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[1 + x]),x]
```

output
```
(-2*Sqrt[1 + x]*(3 - 4*x + 8*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4}{5}\int \frac{1}{x^{5/2}\sqrt{x+1}}dx-\frac{2\sqrt{x+1}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4}{5}\left(-\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{x+1}}dx-\frac{2\sqrt{x+1}}{3x^{3/2}}\right)-\frac{2\sqrt{x+1}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{4}{5}\left(\frac{4\sqrt{x+1}}{3\sqrt{x}}-\frac{2\sqrt{x+1}}{3x^{3/2}}\right)-\frac{2\sqrt{x+1}}{5x^{5/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[1 + x]),x]
```

output
```
(-2*Sqrt[1 + x])/(5*x^(5/2)) - (4*((-2*Sqrt[1 + x])/(3*x^(3/2)) + (4*Sqrt[
1 + x])/(3*Sqrt[x])))/5
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2\sqrt{1+x}\,(8x^2-4x+3)}{15x^{\frac{5}{2}}}$	21
meijerg	$-\frac{2\left(\frac{8}{3}x^2-\frac{4}{3}x+1\right)\sqrt{1+x}}{5x^{\frac{5}{2}}}$	21
orering	$-\frac{2\sqrt{1+x}\,(8x^2-4x+3)}{15x^{\frac{5}{2}}}$	21
risch	$-\frac{2(8x^3+4x^2-x+3)}{15x^{\frac{5}{2}}\sqrt{1+x}}$	26
default	$-\frac{2\sqrt{1+x}}{5x^{\frac{5}{2}}}+\frac{8\sqrt{1+x}}{15x^{\frac{3}{2}}}-\frac{16\sqrt{1+x}}{15\sqrt{x}}$	32

input
```
int(1/x^(7/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(1+x)^(1/2)*(8*x^2-4*x+3)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = -\frac{2\left(8\,x^3+(8\,x^2-4\,x+3)\sqrt{x+1}\sqrt{x}\right)}{15\,x^3}$$

input
```
integrate(1/x^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(8*x^3 + (8*x^2 - 4*x + 3)*sqrt(x + 1)*sqrt(x))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 301, normalized size of antiderivative = 6.14

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = \begin{cases} \frac{16i(x+1)^2}{15\sqrt{-1+\frac{1}{x+1}}(x+1)^2-30\sqrt{-1+\frac{1}{x+1}}(x+1)+15\sqrt{-1+\frac{1}{x+1}}} - \frac{40i(x+1)}{15\sqrt{-1+\frac{1}{x+1}}(x+1)^2-30\sqrt{-1+\frac{1}{x+1}}(x+1)+15\sqrt{-} \\ -\frac{16(x+1)^2}{15\sqrt{1-\frac{1}{x+1}}(x+1)^2-30\sqrt{1-\frac{1}{x+1}}(x+1)+15\sqrt{1-\frac{1}{x+1}}} + \frac{40(x+1)}{15\sqrt{1-\frac{1}{x+1}}(x+1)^2-30\sqrt{1-\frac{1}{x+1}}(x+1)+15\sqrt{1-\frac{1}{x+1}}} \end{cases}$$

input
```
integrate(1/x**(7/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((16*I*(x + 1)**2/(15*sqrt(-1 + 1/(x + 1))*(x + 1)**2 - 30*sqrt(-
1 + 1/(x + 1))*(x + 1) + 15*sqrt(-1 + 1/(x + 1))) - 40*I*(x + 1)/(15*sqrt(
-1 + 1/(x + 1))*(x + 1)**2 - 30*sqrt(-1 + 1/(x + 1))*(x + 1) + 15*sqrt(-1
+ 1/(x + 1))) + 30*I/(15*sqrt(-1 + 1/(x + 1))*(x + 1)**2 - 30*sqrt(-1 + 1/
(x + 1))*(x + 1) + 15*sqrt(-1 + 1/(x + 1))), 1/Abs(x + 1) > 1), (-16*(x +
1)**2/(15*sqrt(1 - 1/(x + 1))*(x + 1)**2 - 30*sqrt(1 - 1/(x + 1))*(x + 1)
+ 15*sqrt(1 - 1/(x + 1))) + 40*(x + 1)/(15*sqrt(1 - 1/(x + 1))*(x + 1)**2
- 30*sqrt(1 - 1/(x + 1))*(x + 1) + 15*sqrt(1 - 1/(x + 1))) - 30/(15*sqrt(1
 - 1/(x + 1))*(x + 1)**2 - 30*sqrt(1 - 1/(x + 1))*(x + 1) + 15*sqrt(1 - 1/
(x + 1))), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = -\frac{2\,(x+1)^{\frac{5}{2}}}{5\,x^{\frac{5}{2}}} + \frac{4\,(x+1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}} - \frac{2\,\sqrt{x+1}}{\sqrt{x}}$$

input
```
integrate(1/x^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-2/5*(x + 1)^(5/2)/x^(5/2) + 4/3*(x + 1)^(3/2)/x^(3/2) - 2*sqrt(x + 1)/sqr
t(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = -\frac{2\,(4\,(2\,x-3)(x+1)+15)\sqrt{x+1}}{15\,x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(4*(2*x - 3)*(x + 1) + 15)*sqrt(x + 1)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = -\frac{\sqrt{x+1}\,\left(\frac{16\,x^2}{15}-\frac{8\,x}{15}+\frac{2}{5}\right)}{x^{5/2}}$$

input
```
int(1/(x^(7/2)*(x + 1)^(1/2)),x)
```

output
```
-((x + 1)^(1/2)*((16*x^2)/15 - (8*x)/15 + 2/5))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx = \frac{-\frac{16\sqrt{x}\,\sqrt{x+1}\,x^2}{15}+\frac{8\sqrt{x}\,\sqrt{x+1}\,x}{15}-\frac{2\sqrt{x}\,\sqrt{x+1}}{5}+\frac{16x^3}{15}}{x^3}$$

input
```
int(1/x^(7/2)/(1+x)^(1/2),x)
```

output
```
(2*( - 8*sqrt(x)*sqrt(x + 1)*x**2 + 4*sqrt(x)*sqrt(x + 1)*x - 3*sqrt(x)*sq
rt(x + 1) + 8*x**3))/(15*x**3)
```

3.525 $\int \frac{1}{\sqrt{x}\sqrt{3+2x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}} \, dx = \sqrt{2}\text{arcsinh}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

output
```
2^(1/2)*arcsinh(1/3*6^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}} \, dx = -\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{3+2x}\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[3 + 2*x]),x]
```

output
```
-(Sqrt[2]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[3 + 2*x]])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{2x+3}}dx$$

$$\Big\downarrow \text{ 63}$$

$$2\int \frac{1}{\sqrt{2x+3}}d\sqrt{x}$$

$$\Big\downarrow \text{ 222}$$

$$\sqrt{2}\text{arcsinh}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

input
```
Int[1/(Sqrt[x]*Sqrt[3 + 2*x]),x]
```

output
```
Sqrt[2]*ArcSinh[Sqrt[2/3]*Sqrt[x]]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
meijerg	$\sqrt{2}\, \operatorname{arcsinh}\left(\frac{\sqrt{x}\,\sqrt{3}\,\sqrt{2}}{3}\right)$	17
default	$\frac{\sqrt{x(2x+3)}\, \ln\left(\frac{\left(\frac{3}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^{2}+3x}\right)\sqrt{2}}{2\sqrt{x}\,\sqrt{2x+3}}$	48

input
```
int(1/x^(1/2)/(2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arcsinh(1/3*x^(1/2)*3^(1/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}}\, dx = \frac{1}{2}\,\sqrt{2}\log\left(2\,\sqrt{2}\sqrt{2\,x+3}\sqrt{x}+4\,x+3\right)$$

input
```
integrate(1/x^(1/2)/(3+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(2*sqrt(2)*sqrt(2*x + 3)*sqrt(x) + 4*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}}\, dx = \begin{cases} \sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{6}\sqrt{x+\frac{3}{2}}}{3}\right) & \text{for}\ \left|x+\frac{3}{2}\right| > \frac{3}{2} \\ -\sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{6}\sqrt{x+\frac{3}{2}}}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(3+2*x)**(1/2),x)
```

output
```
Piecewise((sqrt(2)*acosh(sqrt(6)*sqrt(x + 3/2)/3), Abs(x + 3/2) > 3/2), (-
sqrt(2)*I*asin(sqrt(6)*sqrt(x + 3/2)/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(13) = 26.

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}}\, dx = -\frac{1}{2}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2\,x+3}}{\sqrt{x}}}{\sqrt{2}+\frac{\sqrt{2\,x+3}}{\sqrt{x}}}\right)$$

input
```
integrate(1/x^(1/2)/(3+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2*x + 3)/sqrt(x))/(sqrt(2) + sqrt(2*x +
3)/sqrt(x)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}}\, dx = -\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{2\,x+3}\right)$$

input
```
integrate(1/x^(1/2)/(3+2*x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*log(-sqrt(2)*sqrt(x) + sqrt(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}}\,dx = -2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\left(\sqrt{3}-\sqrt{2\,x+3}\right)}{2\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(2*x + 3)^(1/2)),x)
```

output
```
-2*2^(1/2)*atanh((2^(1/2)*(3^(1/2) - (2*x + 3)^(1/2)))/(2*x^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{x}\sqrt{3+2x}}\,dx = \sqrt{2}\log\left(\frac{\sqrt{2x+3}+\sqrt{x}\,\sqrt{2}}{\sqrt{3}}\right)$$

input
```
int(1/x^(1/2)/(3+2*x)^(1/2),x)
```

output
```
sqrt(2)*log((sqrt(2*x + 3) + sqrt(x)*sqrt(2))/sqrt(3))
```

3.526 $\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx = \sqrt{2}\arcsin\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

output
```
2^(1/2)*arcsin(1/3*6^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx = -2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{3}-\sqrt{3-2x}}\right)$$

input
```
Integrate[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

output
```
-2*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[x])/(Sqrt[3] - Sqrt[3 - 2*x])]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {63, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}dx$$

$$\Big\downarrow \text{63}$$

$$2\int \frac{1}{\sqrt{3-2x}}d\sqrt{x}$$

$$\Big\downarrow \text{223}$$

$$\sqrt{2}\arcsin\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

input
```
Int[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

output
```
Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
meijerg	$\sqrt{2}\, \arcsin\left(\frac{\sqrt{x}\,\sqrt{3}\,\sqrt{2}}{3}\right)$	17
default	$\frac{\sqrt{(3-2x)x}\,\sqrt{2}\, \arcsin\left(\frac{4x}{3}-1\right)}{2\sqrt{3-2x}\,\sqrt{x}}$	31

input
```
int(1/(3-2*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arcsin(1/3*x^(1/2)*3^(1/2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\, dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}\sqrt{-2\,x+3}}{2\,x-3}\right)$$

input
```
integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(sqrt(2)*sqrt(x)*sqrt(-2*x + 3)/(2*x - 3))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\, dx = \begin{cases} -\sqrt{2}i\,\operatorname{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for}\ |x| > \frac{3}{2} \\ \sqrt{2}\,\operatorname{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3-2*x)**(1/2)/x**(1/2),x)
```

output
```
Piecewise((-sqrt(2)*I*acosh(sqrt(6)*sqrt(x)/3), Abs(x) > 3/2), (sqrt(2)*as
in(sqrt(6)*sqrt(x)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,x+3}}{2\,\sqrt{x}}\right)$$

input
```
integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx = \sqrt{2}\arcsin\left(\frac{1}{3}\,\sqrt{6}\sqrt{x}\right)$$

input
```
integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arcsin(1/3*sqrt(6)*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx = 2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(\sqrt{3}-\sqrt{3-2\,x}\right)}{2\,\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(3 - 2*x)^(1/2)),x)
```

output
```
2*2^(1/2)*atan((2^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*x^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx = -\sqrt{2}\log\left(\frac{\sqrt{-2x+3}+\sqrt{x}\,\sqrt{2}\,i}{\sqrt{3}}\right)i$$

input
```
int(1/(3-2*x)^(1/2)/x^(1/2),x)
```

output
```
- sqrt(2)*log((sqrt( - 2*x + 3) + sqrt(x)*sqrt(2)*i)/sqrt(3))*i
```

3.527 $\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = \sqrt{2}\text{arcsinh}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)$$

output
```
2^(1/2)*arcsinh(1/3*(-3+2*x)^(1/2)*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = -\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{-3+2x}\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

output
```
-(Sqrt[2]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[-3 + 2*x]])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{2x-3}}dx$$
$$\Big\downarrow \text{64}$$
$$\int \frac{1}{\sqrt{\frac{1}{2}(2x-3)+\frac{3}{2}}}d\sqrt{2x-3}$$
$$\Big\downarrow \text{222}$$
$$\sqrt{2}\text{arcsinh}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

input
```
Int[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

output
```
Sqrt[2]*ArcSinh[Sqrt[-3 + 2*x]/Sqrt[3]]
```

Defintions of rubi rules used

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(x-\frac{3}{2}\right)}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{3}\,\sqrt{2}}{3}\right)}{\sqrt{\operatorname{signum}\left(x-\frac{3}{2}\right)}}$	31
default	$\frac{\sqrt{(2x-3)x}\,\ln\left(\frac{\left(-\frac{3}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^{2}-3x}\right)\sqrt{2}}{2\sqrt{x}\,\sqrt{2x-3}}$	48

input
```
int(1/x^(1/2)/(2*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)/signum(x-3/2)^(1/2)*(-signum(x-3/2))^(1/2)*arcsin(1/3*x^(1/2)*3^(1
/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = \frac{1}{2}\,\sqrt{2}\log\left(-2\,\sqrt{2}\sqrt{2\,x-3}\sqrt{x}-4\,x+3\right)$$

input
```
integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(2*x - 3)*sqrt(x) - 4*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = \begin{cases} \sqrt{2}\,\text{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for } |x| > \frac{3}{2} \\ -\sqrt{2}i\,\text{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(-3+2*x)**(1/2),x)
```

output
```
Piecewise((sqrt(2)*acosh(sqrt(6)*sqrt(x)/3), Abs(x) > 3/2), (-sqrt(2)*I*as
in(sqrt(6)*sqrt(x)/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(17) = 34.

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = -\frac{1}{2}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2\,x-3}}{\sqrt{x}}}{\sqrt{2}+\frac{\sqrt{2\,x-3}}{\sqrt{x}}}\right)$$

input
```
integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2*x - 3)/sqrt(x))/(sqrt(2) + sqrt(2*x -
3)/sqrt(x)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = -\sqrt{2}\log\left(\sqrt{2}\sqrt{x}-\sqrt{2\,x-3}\right)$$

input
```
integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*log(sqrt(2)*sqrt(x) - sqrt(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = -2\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\left(-\sqrt{2\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\,\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(2*x - 3)^(1/2)),x)
```

output
```
-2*2^(1/2)*atanh((2^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*x^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx = \sqrt{2}\log\left(\frac{\sqrt{2x-3}+\sqrt{x}\,\sqrt{2}}{\sqrt{3}}\right)$$

input
```
int(1/x^(1/2)/(-3+2*x)^(1/2),x)
```

output
```
sqrt(2)*log((sqrt(2*x - 3) + sqrt(x)*sqrt(2))/sqrt(3))
```

3.528 $\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-3-2x}}\right)$$

output
```
2^(1/2)*arctan(2^(1/2)*x^(1/2)/(-3-2*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{-6-4x}\sqrt{x}}{3+2x}\right)$$

input
```
Integrate[1/(Sqrt[-3 - 2*x]*Sqrt[x]),x]
```

output
```
-(Sqrt[2]*ArcTan[(Sqrt[-6 - 4*x]*Sqrt[x])/(3 + 2*x)])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x-3}\sqrt{x}}dx$$

$$\Big\downarrow \text{ 65}$$

$$2\int \frac{1}{\frac{2x}{-2x-3}+1}d\frac{\sqrt{x}}{\sqrt{-2x-3}}$$

$$\Big\downarrow \text{ 216}$$

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-2x-3}}\right)$$

input
```
Int[1/(Sqrt[-3 - 2*x]*Sqrt[x]),x]
```

output
```
Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[x])/Sqrt[-3 - 2*x]]
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
meijerg	$-i\sqrt{2}\ \text{arcsinh}\left(\frac{\sqrt{x}\,\sqrt{3}\,\sqrt{2}}{3}\right)$	19
default	$\frac{\sqrt{(-3-2x)x}\,\sqrt{2}\ \arcsin\left(\frac{4x}{3}+1\right)}{2\sqrt{-3-2x}\,\sqrt{x}}$	31

input
```
int(1/(-3-2*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*arcsinh(1/3*x^(1/2)*3^(1/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}\sqrt{-2\,x-3}}{2\,x+3}\right)$$

input
```
integrate(1/(-3-2*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(sqrt(2)*sqrt(x)*sqrt(-2*x - 3)/(2*x + 3))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = \begin{cases} -\sqrt{2}i\,\text{acosh}\left(\frac{\sqrt{6}\sqrt{x+\frac{3}{2}}}{3}\right) & \text{for } \left|x+\frac{3}{2}\right| > \frac{3}{2} \\ -\sqrt{2}\,\text{asin}\left(\frac{\sqrt{6}\sqrt{x+\frac{3}{2}}}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-3-2*x)**(1/2)/x**(1/2),x)
```

output
```
Piecewise((-sqrt(2)*I*acosh(sqrt(6)*sqrt(x + 3/2)/3), Abs(x + 3/2) > 3/2),
 (-sqrt(2)*asin(sqrt(6)*sqrt(x + 3/2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,x-3}}{2\,\sqrt{x}}\right)$$

input
```
integrate(1/(-3-2*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x - 3)/sqrt(x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = i\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+\sqrt{2\,x+3}\right)$$

input
```
integrate(1/(-3-2*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(2)*log(-sqrt(2)*sqrt(x) + sqrt(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = 2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(-\sqrt{-2\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\,\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(- 2*x - 3)^(1/2)),x)
```

output
```
2*2^(1/2)*atan((2^(1/2)*(3^(1/2)*1i - (- 2*x - 3)^(1/2)))/(2*x^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-3-2x}\sqrt{x}}\,dx = -\sqrt{2}\log\left(\frac{\sqrt{-2x-3}+\sqrt{x}\,\sqrt{2}\,i}{\sqrt{3}}\right)i$$

input
```
int(1/(-3-2*x)^(1/2)/x^(1/2),x)
```

output
```
- sqrt(2)*log((sqrt( - 2*x - 3) + sqrt(x)*sqrt(2)*i)/sqrt(3))*i
```

3.529 $\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = 2\text{arcsinh}\left(\frac{\sqrt{x}}{2}\right)$$

output
```
2*arcsinh(1/2*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = -2\log\left(-\sqrt{x}+\sqrt{4+x}\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[4 + x]),x]
```

output
```
-2*Log[-Sqrt[x] + Sqrt[4 + x]]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x+4}}dx$$

$$\Big\downarrow \text{63}$$

$$2\int \frac{1}{\sqrt{x+4}}d\sqrt{x}$$

$$\Big\downarrow \text{222}$$

$$2\text{arcsinh}\left(\frac{\sqrt{x}}{2}\right)$$

input
```
Int[1/(Sqrt[x]*Sqrt[4 + x]),x]
```

output
```
2*ArcSinh[Sqrt[x]/2]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\frac{\sqrt{x}}{2}\right)$	9
default	$\frac{\sqrt{x(4+x)}\, \ln\left(x+2+\sqrt{x^2+4x}\right)}{\sqrt{x}\,\sqrt{4+x}}$	30

input
```
int(1/x^(1/2)/(4+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(1/2*x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = -\log\left(\sqrt{x+4}\sqrt{x}-x-2\right)$$

input
```
integrate(1/x^(1/2)/(4+x)^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(x + 4)*sqrt(x) - x - 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = \begin{cases} 2\operatorname{acosh}\left(\frac{\sqrt{x+4}}{2}\right) & \text{for } |x+4|>4 \\ -2i\operatorname{asin}\left(\frac{\sqrt{x+4}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(4+x)**(1/2),x)
```

output
```
Piecewise((2*acosh(sqrt(x + 4)/2), Abs(x + 4) > 4), (-2*I*asin(sqrt(x + 4)
/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = \log\left(\frac{\sqrt{x+4}}{\sqrt{x}}+1\right)-\log\left(\frac{\sqrt{x+4}}{\sqrt{x}}-1\right)$$

input
```
integrate(1/x^(1/2)/(4+x)^(1/2),x, algorithm="maxima")
```

output
```
log(sqrt(x + 4)/sqrt(x) + 1) - log(sqrt(x + 4)/sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = -2\log\left(\sqrt{x+4}-\sqrt{x}\right)$$

input
```
integrate(1/x^(1/2)/(4+x)^(1/2),x, algorithm="giac")
```

output
```
-2*log(sqrt(x + 4) - sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = 4\,\mathrm{atanh}\left(\frac{\sqrt{x+4}-2}{\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(x + 4)^(1/2)),x)
```

output
```
4*atanh(((x + 4)^(1/2) - 2)/x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}\sqrt{4+x}}\,dx = 2\log\left(\frac{\sqrt{x+4}}{2}+\frac{\sqrt{x}}{2}\right)$$

input
```
int(1/x^(1/2)/(4+x)^(1/2),x)
```

output
```
2*log((sqrt(x + 4) + sqrt(x))/2)
```

3.530 $\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = 2\arcsin\left(\frac{\sqrt{x}}{2}\right)$$

output
```
2*arcsin(1/2*x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = -\frac{2\sqrt{-4+x}\sqrt{x}\log\left(\sqrt{-4+x}-\sqrt{x}\right)}{\sqrt{-((-4+x)x)}}$$

input
```
Integrate[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

output
```
(-2*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] - Sqrt[x]])/Sqrt[-((-4 + x)*x)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}dx$$

$$\Big\downarrow \text{ 62}$$

$$\int \frac{1}{\sqrt{4x-x^2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{1-\frac{1}{16}(4-2x)^2}}d(4-2x)$$

$$\Big\downarrow \text{ 223}$$

$$-\arcsin\left(\frac{1}{4}(4-2x)\right)$$

input
```
Int[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

output
```
-ArcSin[(4 - 2*x)/4]
```

Defintions of rubi rules used

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
meijerg	$2 \arcsin\left(\frac{\sqrt{x}}{2}\right)$	9
default	$\frac{\sqrt{(4-x)x}\,\arcsin\left(-1+\frac{x}{2}\right)}{\sqrt{4-x}\,\sqrt{x}}$	27

input
```
int(1/(4-x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsin(1/2*x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = -2\arctan\left(\frac{\sqrt{x}\sqrt{-x+4}}{x-4}\right)$$

input
```
integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(x)*sqrt(-x + 4)/(x - 4))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{x}}{2}\right) & \text{for } |x| > 4 \\ 2\operatorname{asin}\left(\frac{\sqrt{x}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(4-x)**(1/2)/x**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(x)/2), Abs(x) > 4), (2*asin(sqrt(x)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = -2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

input
```
integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-2*arctan(sqrt(-x + 4)/sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = 2\arcsin\left(\frac{1}{2}\sqrt{x}\right)$$

input
```
integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
2*arcsin(1/2*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = -4\,\mathrm{atan}\left(\frac{\sqrt{4-x}-2}{\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(4 - x)^(1/2)),x)
```

output
```
-4*atan(((4 - x)^(1/2) - 2)/x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = -2\log\left(\frac{\sqrt{-x+4}}{2}+\frac{\sqrt{x}\,i}{2}\right)i$$

input
```
int(1/(4-x)^(1/2)/x^(1/2),x)
```

output
```
- 2*log((sqrt( - x + 4) + sqrt(x)*i)/2)*i
```

3.531 $\int x^{5/2}\sqrt{a-bx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int x^{5/2}\sqrt{a-bx}\,dx=-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{5a^4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}$$

output
```
-5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^3-5/96*a^2*x^(3/2)*(-b*x+a)^(1/2)/b^2-1
/24*a*x^(5/2)*(-b*x+a)^(1/2)/b+1/4*x^(7/2)*(-b*x+a)^(1/2)+5/64*a^4*arctan(
b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int x^{5/2}\sqrt{a-bx}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a-bx}(-15a^3-10a^2bx-8ab^2x^2+48b^3x^3)+30a^4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{192b^{7/2}}$$

input
```
Integrate[x^(5/2)*Sqrt[a - b*x],x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a - b*x]*(-15*a^3 - 10*a^2*b*x - 8*a*b^2*x^2 + 48*b^
3*x^3) + 30*a^4*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/(192
*b^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {60, 60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt{a-bx}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a-bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{8}a\left(\frac{5a\int \frac{x^{3/2}}{\sqrt{a-bx}}dx}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{8}a\left(\frac{5a\left(\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

$\big\downarrow$ 65

$$\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

$\big\downarrow$ 216

$$\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

input
```
Int[x^(5/2)*Sqrt[a - b*x],x]
```

output
```
(x^(7/2)*Sqrt[a - b*x])/4 + (a*(-1/3*(x^(5/2)*Sqrt[a - b*x])/b + (5*a*(-1/
2*(x^(3/2)*Sqrt[a - b*x])/b + (3*a*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)))/(4*b)))/(6*b)))/8
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{(-48b^3x^3+8ab^2x^2+10a^2bx+15a^3)\sqrt{x}\sqrt{-bx+a}}{192b^3}+\frac{5a^4\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)\sqrt{x(-bx+a)}}{128b^{\frac{7}{2}}\sqrt{x}\sqrt{-bx+a}}$	102
default	$-\frac{x^{\frac{5}{2}}(-bx+a)^{\frac{3}{2}}}{4b}+\frac{5a\left(-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}}{3b}+\frac{a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)}{2\sqrt{-bx+a}\sqrt{x}\sqrt{b}}\right)}{4b}\right)}{2b}\right)}{8b}$	135

input
```
int(x^(5/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*b^3*x^3+8*a*b^2*x^2+10*a^2*b*x+15*a^3)/b^3*x^(1/2)*(-b*x+a)^(1
/2)+5/128*a^4/b^(7/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-
b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.36

$$\int x^{5/2}\sqrt{a-bx}\,dx = \left[-\frac{15\,a^4\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2}{384\,b^4}\right.$$

$$\left.-\frac{15\,a^4\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)-(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\sqrt{x}}{192\,b^4}\right]$$

input
```
integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)
*sqrt(x))/b^4, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x
)/(b*x - a)) - (48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-
b*x + a)*sqrt(x))/b^4]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.81 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.54

$$\int x^{5/2}\sqrt{a-bx}\,dx = \begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} + \frac{ibx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}} + \frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} - \frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} \end{cases}$$

input
```
integrate(x**(5/2)*(-b*x+a)**(1/2),x)
```

output
```
Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(5/2)*
x**(3/2)/(192*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(5/2)/(96*b*sqrt(-1 +
 b*x/a)) - 7*I*sqrt(a)*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sqr
t(b)*sqrt(x)/sqrt(a))/(64*b**(7/2)) + I*b*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*
x/a)), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 - b*x/a)) + 5
*a**(5/2)*x**(3/2)/(192*b**2*sqrt(1 - b*x/a)) + a**(3/2)*x**(5/2)/(96*b*sq
rt(1 - b*x/a)) + 7*sqrt(a)*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin(sqr
t(b)*sqrt(x)/sqrt(a))/(64*b**(7/2)) - b*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.34

$$\int x^{5/2}\sqrt{a-bx}\,dx = -\frac{5\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{64\,b^{\frac{7}{2}}} + \frac{\frac{15\,\sqrt{-bx+a}a^4b^3}{\sqrt{x}} - \frac{73\,(-bx+a)^{\frac{3}{2}}\,a^4b^2}{x^{\frac{3}{2}}} - \frac{55\,(-bx+a)^{\frac{5}{2}}\,a^4b}{x^{\frac{5}{2}}} - \frac{15\,(-bx+a)^{\frac{7}{2}}\,a^4}{x^{\frac{7}{2}}}}{192\left(b^7 - \frac{4\,(bx-a)b^6}{x} + \frac{6\,(bx-a)^2b^5}{x^2} - \frac{4\,(bx-a)^3b^4}{x^3} + \frac{(bx-a)^4b^3}{x^4}\right)}$$

input
```
integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/192*(15*sqr
t(-b*x + a)*a^4*b^3/sqrt(x) - 73*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 55*(-b
*x + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^7 - 4*(b
*x - a)*b^6/x + 6*(b*x - a)^2*b^5/x^2 - 4*(b*x - a)^3*b^4/x^3 + (b*x - a)^
4*b^3/x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(93) = 186$.

Time = 149.93 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.02

$$\int x^{5/2}\sqrt{a-bx}\,dx = \frac{\frac{8\left(\frac{15\,a^3\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}b}-\sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(2\,(bx-a)\left(\frac{4\,(bx-a)}{b^2}+\frac{13\,a}{b^2}\right)+\frac{33\,a^2}{b^2}\right)\right)a|b|}{b^2} - \cdots}{\cdots}$$

input
```
integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*(8*(15*a^3*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b
)))/(sqrt(-b)*b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)*(4*
(b*x - a)/b^2 + 13*a/b^2) + 33*a^2/b^2))*a*abs(b)/b^2 - (105*a^4*log(abs(-
sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/(sqrt(-b)*b^2) - (2*(b
*x - a)*(4*(b*x - a)*(6*(b*x - a)/b^3 + 25*a/b^3) + 163*a^2/b^3) + 279*a^3
/b^3)*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a))*abs(b)/b)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\sqrt{a-bx}\,dx = \int x^{5/2}\,\sqrt{a-b\,x}\,dx$$

input
```
int(x^(5/2)*(a - b*x)^(1/2),x)
```

output
```
int(x^(5/2)*(a - b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int x^{5/2}\sqrt{a-bx}\,dx = \frac{-15\sqrt{x}\,\sqrt{-bx+a}\,a^3b-10\sqrt{x}\,\sqrt{-bx+a}\,a^2b^2x-8\sqrt{x}\,\sqrt{-bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{-}}{192b^4}$$

input
```
int(x^(5/2)*(-b*x+a)^(1/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(a - b*x)*a**3*b - 10*sqrt(x)*sqrt(a - b*x)*a**2*b**2*x
 - 8*sqrt(x)*sqrt(a - b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a - b*x)*b**4*x**
3 - 15*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**4*i)/(1
92*b**4)
```

3.532 $\int x^{3/2}\sqrt{a-bx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int x^{3/2}\sqrt{a-bx}\,dx=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

output
```
-1/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b^2-1/12*a*x^(3/2)*(-b*x+a)^(1/2)/b+1/3*x^
(5/2)*(-b*x+a)^(1/2)+1/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2
)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int x^{3/2}\sqrt{a-bx}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a-bx}(-3a^2-2abx+8b^2x^2)+6a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{24b^{5/2}}$$

input
```
Integrate[x^(3/2)*Sqrt[a - b*x],x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a - b*x]*(-3*a^2 - 2*a*b*x + 8*b^2*x^2) + 6*a^3*ArcT
an[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/(24*b^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{a-bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a-bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\left(\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{65}$$

$$\frac{1}{6}a\left(\frac{3a\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}a\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

input
```
Int[x^(3/2)*Sqrt[a - b*x],x]
```

output
```
(x^(5/2)*Sqrt[a - b*x])/3 + (a*(-1/2*(x^(3/2)*Sqrt[a - b*x])/b + (3*a*(-((
Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^
(3/2)))/(4*b)))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{\left(-8b^{2}x^{2}+2abx+3a^{2}\right)\sqrt{x}\,\sqrt{-bx+a}}{24b^{2}}+\frac{a^{3}\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x\left(-bx+a\right)}}{16b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	91
default	$-\frac{x^{\frac{3}{2}}\left(-bx+a\right)^{\frac{3}{2}}}{3b}+\frac{a\left(-\frac{\sqrt{x}\,\left(-bx+a\right)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}\right)}{2b}$	112

input
```
int(x^(3/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*b^2*x^2+2*a*b*x+3*a^2)/b^2*x^(1/2)*(-b*x+a)^(1/2)+1/16*a^3/b^(5/
2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/
2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.48

$$\int x^{3/2}\sqrt{a-bx}\,dx=\left[-\frac{3\,a^{3}\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(8\,b^{3}x^{2}-2\,ab^{2}x-3\,a^{2}b)\sqrt{-b}\right.$$

$$\left.-\frac{3\,a^{3}\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)-(8\,b^{3}x^{2}-2\,ab^{2}x-3\,a^{2}b)\sqrt{-bx+a}\sqrt{x}}{24\,b^{3}}\right]$$

(first fraction denominator: $48\,b^{3}$)

input
```
integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 - 2*(8*b^3*x^2 - 2*a*b^2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/24*
(3*a^3*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (8*b^3*x
^2 - 2*a*b^2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.55

$$\int x^{3/2}\sqrt{a-bx}\,dx=\begin{cases}\frac{ia^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{-1+\frac{bx}{a}}}-\frac{5i\sqrt{a}x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{ibx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for }\left|\frac{bx}{a}\right|>1\\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1-\frac{bx}{a}}}+\frac{5\sqrt{a}x^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(3/2)*(-b*x+a)**(1/2),x)
```

output
```
Piecewise((I*a**(5/2)*sqrt(x)/(8*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(3
/2)/(24*b*sqrt(-1 + b*x/a)) - 5*I*sqrt(a)*x**(5/2)/(12*sqrt(-1 + b*x/a)) -
 I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) + I*b*x**(7/2)/(3*sqrt
(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b**2*sqrt(1
- b*x/a)) + a**(3/2)*x**(3/2)/(24*b*sqrt(1 - b*x/a)) + 5*sqrt(a)*x**(5/2)/
(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) - b
*x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\int x^{3/2}\sqrt{a-bx}\,dx=-\frac{a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{3\sqrt{-bx+a}a^3b^2}{\sqrt{x}}-\frac{8(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^5-\frac{3(bx-a)b^4}{x}+\frac{3(bx-a)^2b^3}{x^2}-\frac{(bx-a)^3b^2}{x^3}\right)}$$

input
```
integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/24*(3*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) - 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)
^(5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x - a)*b^4/x + 3*(b*x - a)^2*b^3/x^2 - (b*
x - a)^3*b^2/x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(74) = 148$.

Time = 150.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.03

$$\int x^{3/2}\sqrt{a-bx}\,dx = -\frac{\frac{\left(\frac{15\,a^3\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}b}-\sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(2\,(bx-a)\left(\frac{4\,(bx-a)}{b^2}+\frac{13\,a}{b^2}\right)+\frac{33\,a^2}{b^2}\right)\right)|b|}{b}-\frac{6\left(\frac{3\,a^2b\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{\cdots}\right.\right.}{\sqrt{-b}}\cdots\right.}{\cdots}}{24\,b}$$

input
```
integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/24*((15*a^3*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b))
)/(sqrt(-b)*b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)*(4*(b
*x - a)/b^2 + 13*a/b^2) + 33*a^2/b^2))*abs(b)/b - 6*(3*a^2*b*log(abs(-sqrt
(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) - sqrt((b*x - a)*
b + a*b)*(2*b*x + 3*a)*sqrt(-b*x + a))*a*abs(b)/b^3)/b
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{a-bx}\,dx = \int x^{3/2}\,\sqrt{a-b\,x}\,dx$$

input
```
int(x^(3/2)*(a - b*x)^(1/2),x)
```

output
```
int(x^(3/2)*(a - b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int x^{3/2}\sqrt{a-bx}\,dx=\frac{-3\sqrt{x}\,\sqrt{-bx+a}\,a^2b-2\sqrt{x}\,\sqrt{-bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{-bx+a}\,b^3x^2-3\sqrt{b}\log\left(\frac{\sqrt{-bx+}}{}\right.}{24b^3}$$

input
```
int(x^(3/2)*(-b*x+a)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a - b*x)*a**2*b - 2*sqrt(x)*sqrt(a - b*x)*a*b**2*x + 8*
sqrt(x)*sqrt(a - b*x)*b**3*x**2 - 3*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*s
qrt(b)*i)/sqrt(a))*a**3*i)/(24*b**3)
```

3.533 $\int \sqrt{x}\sqrt{a-bx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \sqrt{x}\sqrt{a-bx}\,dx=-\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}$$

output
```
-1/4*a*x^(1/2)*(-b*x+a)^(1/2)/b+1/2*x^(3/2)*(-b*x+a)^(1/2)+1/4*a^2*arctan(
b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \sqrt{x}\sqrt{a-bx}\,dx=\frac{\sqrt{x}\sqrt{a-bx}(-a+2bx)}{4b}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{2b^{3/2}}$$

input
```
Integrate[Sqrt[x]*Sqrt[a - b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a - b*x]*(-a + 2*b*x))/(4*b) + (a^2*ArcTan[(Sqrt[b]*Sqrt[x])
/(-Sqrt[a] + Sqrt[a - b*x])])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{a-bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{65}$$

$$\frac{1}{4}a\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}$$

input
```
Int[Sqrt[x]*Sqrt[a - b*x],x]
```

output
```
(x^(3/2)*Sqrt[a - b*x])/2 + (a*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(
Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

method	result	size
risch	$-\frac{(-2bx+a)\sqrt{x}\,\sqrt{-bx+a}}{4b}+\frac{a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x(-bx+a)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	78
default	$-\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}$	89

input
```
int(x^(1/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*x+a)/b*x^(1/2)*(-b*x+a)^(1/2)+1/8/b^(3/2)*a^2*arctan(b^(1/2)*(x
-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \sqrt{x}\sqrt{a-bx}\,dx = \left[-\frac{a^2\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(2\,b^2x-ab)\sqrt{-bx+a}\sqrt{x}}{8\,b^2},\ -\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)-(2\,b^2x-ab)\sqrt{-bx+a}\sqrt{x}}{4\,b^2}\right]$$

input
```
integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) -
2*(2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/4*(a^2*sqrt(b)*arctan(sq
rt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (2*b^2*x - a*b)*sqrt(-b*x + a)*s
qrt(x))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.69

$$\int \sqrt{x}\sqrt{a-bx}\,dx = \begin{cases} \frac{ia^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{-1+\frac{bx}{a}}} - \frac{3i\sqrt{a}x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}} - \frac{ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + \frac{ibx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1-\frac{bx}{a}}} + \frac{3\sqrt{a}x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}} + \frac{a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} - \frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)*(-b*x+a)**(1/2),x)
```

output
```
Piecewise((I*a**(3/2)*sqrt(x)/(4*b*sqrt(-1 + b*x/a)) - 3*I*sqrt(a)*x**(3/2
)/(4*sqrt(-1 + b*x/a)) - I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)
) + I*b*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(3/2)
*sqrt(x)/(4*b*sqrt(1 - b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 - b*x/a)) +
a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) - b*x**(5/2)/(2*sqrt(a)*sq
rt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

$$\int \sqrt{x}\sqrt{a-bx}\,dx = -\frac{a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}} + \frac{\frac{\sqrt{-bx+a}a^2b}{\sqrt{x}} - \frac{(-bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^3 - \frac{2\,(bx-a)b^2}{x} + \frac{(bx-a)^2b}{x^2}\right)}$$

input
```
integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/4*(sqrt(-b*x
 + a)*a^2*b/sqrt(x) - (-b*x + a)^(3/2)*a^2/x^(3/2))/(b^3 - 2*(b*x - a)*b^2
/x + (b*x - a)^2*b/x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(55) = 110.

Time = 150.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.16

$$\int \sqrt{x}\sqrt{a-bx}\,dx$$

$$= \frac{\frac{4\left(\frac{ab\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} - \sqrt{(bx-a)b+ab}\sqrt{-bx+a}\right)a|b|}{b^2} - \frac{\left(\frac{3\,a^2b\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} - \sqrt{(bx-a)b+ab}(2\,bx+3}{b^2}}{4\,b}$$

input
```
integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(4*(a*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/s
qrt(-b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a))*a*abs(b)/b^2 - (3*a^2*b*
log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) - sq
rt((b*x - a)*b + a*b)*(2*b*x + 3*a)*sqrt(-b*x + a))*abs(b)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \sqrt{x}\sqrt{a-bx}\,dx = \sqrt{x}\left(\frac{x}{2}-\frac{a}{4\,b}\right)\sqrt{a-b\,x}-\frac{a^2\,\ln\left(a-2\,b\,x+2\,\sqrt{-b}\,\sqrt{x}\,\sqrt{a-b\,x}\right)}{8\,{\left(-b\right)}^{3/2}}$$

input
```
int(x^(1/2)*(a - b*x)^(1/2),x)
```

output
```
x^(1/2)*(x/2 - a/(4*b))*(a - b*x)^(1/2) - (a^2*log(a - 2*b*x + 2*(-b)^(1/2
)*x^(1/2)*(a - b*x)^(1/2)))/(8*(-b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\sqrt{a-bx}\,dx = \frac{-\sqrt{x}\,\sqrt{-bx+a}\,ab+2\sqrt{x}\,\sqrt{-bx+a}\,b^2x-\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)a^2i}{4b^2}$$

input
```
int(x^(1/2)*(-b*x+a)^(1/2),x)
```

output
```
( - sqrt(x)*sqrt(a - b*x)*a*b + 2*sqrt(x)*sqrt(a - b*x)*b**2*x - sqrt(b)*l
og((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**2*i)/(4*b**2)
```

3.534 $\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{a-bx} + \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

output
```
x^(1/2)*(-b*x+a)^(1/2)+a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{a-bx} + \frac{2a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{\sqrt{b}}$$

input
```
Integrate[Sqrt[a - b*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[a - b*x] + (2*a*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a -
 b*x])])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx+\sqrt{x}\sqrt{a-bx}$$

$$\Big\downarrow \text{65}$$

$$a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}+\sqrt{x}\sqrt{a-bx}$$

$$\Big\downarrow \text{216}$$

$$\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a-bx}$$

input
```
Int[Sqrt[a - b*x]/Sqrt[x],x]
```

output
```
Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b
]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

method	result	size
default	$\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}$	66
risch	$\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}$	66

input
```
int((-b*x+a)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(
1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx=\left[-\frac{a\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,\sqrt{-bx+a}b\sqrt{x}}{2\,b},\right.$$
$$\left.-\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)-\sqrt{-bx+a}b\sqrt{x}}{b}\right]$$

input
```
integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*
sqrt(-b*x + a)*b*sqrt(x))/b, -(a*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqr
t(x)/(b*x - a)) - sqrt(-b*x + a)*b*sqrt(x))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = \begin{cases} -\frac{i\sqrt{a}\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}} - \frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{ibx^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \sqrt{a}\sqrt{x}\sqrt{1-\frac{bx}{a}} + \frac{a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x+a)**(1/2)/x**(1/2),x)
```

output
```
Piecewise((-I*sqrt(a)*sqrt(x)/sqrt(-1 + b*x/a) - I*a*acosh(sqrt(b)*sqrt(x)
/sqrt(a))/sqrt(b) + I*b*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
1), (sqrt(a)*sqrt(x)*sqrt(1 - b*x/a) + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqr
t(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = -\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}} + \frac{\sqrt{-bx+a}\,a}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

input
```
integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + sqrt(-b*x + a)*a/((b
 - (b*x - a)/x)*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.

Time = 76.67 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = \frac{\left(\frac{a\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} + \frac{\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b}\right)b}{|b|}$$

input
```
integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
(a*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) +
 sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b)*b/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{a-b\,x} + \frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
int((a - b*x)^(1/2)/x^(1/2),x)
```

output
```
x^(1/2)*(a - b*x)^(1/2) + (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a
^(1/2))))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx = \frac{\sqrt{x}\,\sqrt{-bx+a}\,b-\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)ai}{b}$$

input
```
int((-b*x+a)^(1/2)/x^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a - b*x)*b - sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)
/sqrt(a))*a*i)/b
```

3.535 $\int \frac{\sqrt{a-bx}}{x^{3/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{\sqrt{a-bx}}{x^{3/2}} dx = -\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

output
```
-2*(-b*x+a)^(1/2)/x^(1/2)-2*b^(1/2)*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a-bx}}{x^{3/2}} dx = -\frac{2\sqrt{a-bx}}{\sqrt{x}} - 4\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)$$

input
```
Integrate[Sqrt[a - b*x]/x^(3/2),x]
```

output
```
(-2*Sqrt[a - b*x])/Sqrt[x] - 4*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a]
+ Sqrt[a - b*x])]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {57, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx}}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{57}$$

$$-b \int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx-\frac{2\sqrt{a-bx}}{\sqrt{x}}$$

$$\Big\downarrow \text{65}$$

$$-2b \int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}-\frac{2\sqrt{a-bx}}{\sqrt{x}}$$

$$\Big\downarrow \text{216}$$

$$-2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2\sqrt{a-bx}}{\sqrt{x}}$$

input
```
Int[Sqrt[a - b*x]/x^(3/2),x]
```

output
```
(-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b
*x]]
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

method	result	size
risch	$-\frac{2\sqrt{-bx+a}}{\sqrt{x}}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	66

input
```
int((-b*x+a)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-b*x+a)^(1/2)/x^(1/2)-b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^
(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx = \left[\frac{\sqrt{-b}x\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\sqrt{x}}{x}, \frac{2\left(\sqrt{b}x\arctan\left(\frac{\sqrt{-bx+}}{bx}\right.\right.}{}\right.$$

input
```
integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*sqrt(
-b*x + a)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(
b*x - a)) - sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx = \begin{cases} \frac{2i\sqrt{a}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}} + 2i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2ib\sqrt{x}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}} - 2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + \frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x+a)**(1/2)/x**(3/2),x)
```

output
```
Piecewise((2*I*sqrt(a)/(sqrt(x)*sqrt(-1 + b*x/a)) + 2*I*sqrt(b)*acosh(sqrt
(b)*sqrt(x)/sqrt(a)) - 2*I*b*sqrt(x)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a
) > 1), (-2*sqrt(a)/(sqrt(x)*sqrt(1 - b*x/a)) - 2*sqrt(b)*asin(sqrt(b)*sqr
t(x)/sqrt(a)) + 2*b*sqrt(x)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx = 2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right) - \frac{2\sqrt{-bx+a}}{\sqrt{x}}$$

input
```
integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)/sqrt
(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 75.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx = -\frac{2\,b^2\left(\frac{\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}}+\frac{\sqrt{-bx+a}}{\sqrt{(bx-a)b+ab}}\right)}{|b|}$$

input
```
integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
-2*b^2*(log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(
-b) + sqrt(-b*x + a)/sqrt((b*x - a)*b + a*b))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx = \int \frac{\sqrt{a-b\,x}}{x^{3/2}}\,dx$$

input
```
int((a - b*x)^(1/2)/x^(3/2),x)
```

output
```
int((a - b*x)^(1/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{-bx+a}+2\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)ix-2\sqrt{b}\,ix}{x}$$

input
```
int((-b*x+a)^(1/2)/x^(3/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a - b*x) + sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b
)*i)/sqrt(a))*i*x - sqrt(b)*i*x))/x
```

3.536 $\int \frac{\sqrt{a-bx}}{x^{5/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\sqrt{a-bx}}{x^{5/2}} \, dx = -\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

output
```
-2/3*(-b*x+a)^(3/2)/a/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx}}{x^{5/2}} \, dx = -\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

input
```
Integrate[Sqrt[a - b*x]/x^(5/2),x]
```

output
```
(-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

input
```
Int[Sqrt[a - b*x]/x^(5/2),x]
```

output
```
(-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	17
risch	$-\frac{2(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	17
orering	$-\frac{2(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	17
default	$-\frac{\sqrt{-bx+a}}{x^{\frac{3}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}\right)}{2}$	52

input
```
int((-b*x+a)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x+a)^(3/2)/a/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx=\frac{2\,(bx-a)\sqrt{-bx+a}}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(b*x - a)*sqrt(-b*x + a)/(a*x^(3/2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.00

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = \begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x} + \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x} + \frac{2ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3a} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x+a)**(1/2)/x**(5/2),x)
```

output
```
Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 2*b**(3/2)*sqrt(a/(b*x) -
1)/(3*a), Abs(a/(b*x)) > 1), (-2*I*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 2*I*
b**(3/2)*sqrt(-a/(b*x) + 1)/(3*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = -\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(-b*x + a)^(3/2)/(a*x^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = \frac{2\,(bx-a)\sqrt{-bx+a}b^4}{3\,((bx-a)b+ab)^{\frac{3}{2}}a|b|}$$

input
```
integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*(b*x - a)*sqrt(-b*x + a)*b^4/(((b*x - a)*b + a*b)^(3/2)*a*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = \frac{\left(\frac{2\,b\,x}{3\,a}-\frac{2}{3}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

input
```
int((a - b*x)^(1/2)/x^(5/2),x)
```

output
```
(((2*b*x)/(3*a) - 2/3)*(a - b*x)^(1/2))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a}{3}+\frac{2\sqrt{x}\,\sqrt{-bx+a}\,bx}{3}+\frac{2\sqrt{b}\,bi\,x^2}{3}}{a\,x^2}$$

input
```
int((-b*x+a)^(1/2)/x^(5/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a - b*x)*a + sqrt(x)*sqrt(a - b*x)*b*x + sqrt(b)*b*i*x
**2))/(3*a*x**2)
```

3.537 $\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = -\frac{2(a-bx)^{3/2}}{5ax^{5/2}} - \frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}$$

output
```
-2/5*(-b*x+a)^(3/2)/a/x^(5/2)-4/15*b*(-b*x+a)^(3/2)/a^2/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = -\frac{2\sqrt{a-bx}(3a^2-abx-2b^2x^2)}{15a^2x^{5/2}}$$

input
```
Integrate[Sqrt[a - b*x]/x^(7/2),x]
```

output
```
(-2*Sqrt[a - b*x]*(3*a^2 - a*b*x - 2*b^2*x^2))/(15*a^2*x^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{55, 48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 55$$

$$\frac{2b\int \frac{\sqrt{a-bx}}{x^{5/2}}dx}{5a}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

input
```
Int[Sqrt[a - b*x]/x^(7/2),x]
```

output
```
(-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2)
)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}(2bx+3a)}{15x^{\frac{5}{2}}a^{2}}$	25
orering	$-\frac{2(-bx+a)^{\frac{3}{2}}(2bx+3a)}{15x^{\frac{5}{2}}a^{2}}$	25
risch	$-\frac{2\sqrt{-bx+a}\,(-2b^{2}x^{2}-abx+3a^{2})}{15x^{\frac{5}{2}}a^{2}}$	36
default	$-\frac{\sqrt{-bx+a}}{2x^{\frac{5}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{4}$	75

input
```
int((-b*x+a)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-b*x+a)^(3/2)*(2*b*x+3*a)/x^(5/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = \frac{2\,(2\,b^2x^2+abx-3\,a^2)\sqrt{-bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

input
```
integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(2*b^2*x^2 + a*b*x - 3*a^2)*sqrt(-b*x + a)/(a^2*x^(5/2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 241, normalized size of antiderivative = 5.24

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = \begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{5x^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}{15a^2} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{6ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{x(-15a^3bx+15a^2b^2x^2)}-\frac{8ia^2b^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}-\frac{2iab^{\frac{7}{2}}x\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}+\frac{4ib^{\frac{9}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x+a)**(1/2)/x**(7/2),x)
```

output
```
Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(5*x**2) + 2*b**(3/2)*sqrt(a/(b*x)
 - 1)/(15*a*x) + 4*b**(5/2)*sqrt(a/(b*x) - 1)/(15*a**2), Abs(a/(b*x)) > 1)
, (6*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1)/(x*(-15*a**3*b*x + 15*a**2*b**2*x*
*2)) - 8*I*a**2*b**(5/2)*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x
**2) - 2*I*a*b**(7/2)*x*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x*
*2) + 4*I*b**(9/2)*x**2*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x*
*2), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = -\frac{2\left(\frac{5\,(-bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\,(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^2}$$

input
```
integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-2/15*(5*(-b*x + a)^(3/2)*b/x^(3/2) + 3*(-b*x + a)^(5/2)/x^(5/2))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = \frac{2\left(\frac{2\,(bx-a)b^5}{a^2}+\frac{5\,b^5}{a}\right)(bx-a)\sqrt{-bx+a}b}{15\,((bx-a)b+ab)^{\frac{5}{2}}|b|}$$

input
```
integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
2/15*(2*(b*x - a)*b^5/a^2 + 5*b^5/a)*(b*x - a)*sqrt(-b*x + a)*b/(((b*x - a
)*b + a*b)^(5/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = \frac{\sqrt{a-b\,x}\,\left(\frac{4\,b^2\,x^2}{15\,a^2}+\frac{2\,b\,x}{15\,a}-\frac{2}{5}\right)}{x^{5/2}}$$

input
```
int((a - b*x)^(1/2)/x^(7/2),x)
```

output
```
((a - b*x)^(1/2)*((4*b^2*x^2)/(15*a^2) + (2*b*x)/(15*a) - 2/5))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a^2}{5}+\frac{2\sqrt{x}\,\sqrt{-bx+a}\,abx}{15}+\frac{4\sqrt{x}\,\sqrt{-bx+a}\,b^2x^2}{15}-\frac{4\sqrt{b}\,b^2 i\,x^3}{15}}{a^2x^3}$$

input

```
int((-b*x+a)^(1/2)/x^(7/2),x)
```

output

```
(2*( - 3*sqrt(x)*sqrt(a - b*x)*a**2 + sqrt(x)*sqrt(a - b*x)*a*b*x + 2*sqrt
(x)*sqrt(a - b*x)*b**2*x**2 - 2*sqrt(b)*b**2*i*x**3))/(15*a**2*x**3)
```

3.538 $\int \frac{\sqrt{a-bx}}{x^{9/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{\sqrt{a-bx}}{x^{9/2}} \, dx = -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} - \frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}} - \frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}$$

output
```
-2/7*(-b*x+a)^(3/2)/a/x^(7/2)-8/35*b*(-b*x+a)^(3/2)/a^2/x^(5/2)-16/105*b^2
*(-b*x+a)^(3/2)/a^3/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a-bx}}{x^{9/2}} \, dx = -\frac{2\sqrt{a-bx}(15a^3-3a^2bx-4ab^2x^2-8b^3x^3)}{105a^3x^{7/2}}$$

input
```
Integrate[Sqrt[a - b*x]/x^(9/2),x]
```

output
```
(-2*Sqrt[a - b*x]*(15*a^3 - 3*a^2*b*x - 4*a*b^2*x^2 - 8*b^3*x^3))/(105*a^3
*x^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx}}{x^{9/2}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4b \int \frac{\sqrt{a-bx}}{x^{7/2}}dx}{7a} - \frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4b\left(\frac{2b\int \frac{\sqrt{a-bx}}{x^{5/2}}dx}{5a} - \frac{2(a-bx)^{3/2}}{5ax^{5/2}}\right)}{7a} - \frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4b\left(-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}} - \frac{2(a-bx)^{3/2}}{5ax^{5/2}}\right)}{7a} - \frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

input
```
Int[Sqrt[a - b*x]/x^(9/2),x]
```

output
```
(-2*(a - b*x)^(3/2))/(7*a*x^(7/2)) + (4*b*((-2*(a - b*x)^(3/2))/(5*a*x^(5/
2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2))))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}} \left(8b^{2}x^{2}+12abx+15a^{2}\right)}{105x^{\frac{7}{2}} a^{3}}$	36
orering	$-\frac{2(-bx+a)^{\frac{3}{2}} \left(8b^{2}x^{2}+12abx+15a^{2}\right)}{105x^{\frac{7}{2}} a^{3}}$	36
risch	$-\frac{2\sqrt{-bx+a}\, \left(-8b^{3}x^{3}-4a\, b^{2}x^{2}-3a^{2}bx+15a^{3}\right)}{105x^{\frac{7}{2}} a^{3}}$	47
default	$-\frac{\sqrt{-bx+a}}{3x^{\frac{7}{2}}}-\frac{a \left(-\frac{2\sqrt{-bx+a}}{7a\, x^{\frac{7}{2}}}+\frac{6b \left(-\frac{2\sqrt{-bx+a}}{5a\, x^{\frac{5}{2}}}+\frac{4b \left(-\frac{2\sqrt{-bx+a}}{3a\, x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}$	98

input
```
int((-b*x+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(-b*x+a)^(3/2)*(8*b^2*x^2+12*a*b*x+15*a^2)/x^(7/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = \frac{2\,(8\,b^3x^3+4\,ab^2x^2+3\,a^2bx-15\,a^3)\sqrt{-bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

input
```
integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
2/105*(8*b^3*x^3 + 4*a*b^2*x^2 + 3*a^2*b*x - 15*a^3)*sqrt(-b*x + a)/(a^3*x
^(7/2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.71 (sec) , antiderivative size = 707, normalized size of antiderivative = 9.96

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = \begin{cases} -\frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} + \frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} - \frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} \\ -\frac{30ia^5b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} + \frac{66ia^4b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} - \frac{34ia^3b^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} \end{cases}$$

input
```
integrate((-b*x+a)**(1/2)/x**(9/2),x)
```

output
```
Piecewise((-30*a**5*b**(9/2)*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a
**4*b**5*x**4 + 105*a**3*b**6*x**5) + 66*a**4*b**(11/2)*x*sqrt(a/(b*x) - 1
)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3
*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4
 + 105*a**3*b**6*x**5) + 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) - 1)/(105*a**5
*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**
4*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b*
*6*x**5) + 16*b**(19/2)*x**5*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a
**4*b**5*x**4 + 105*a**3*b**6*x**5), Abs(a/(b*x)) > 1), (-30*I*a**5*b**(9/
2)*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*
b**6*x**5) + 66*I*a**4*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3
- 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*I*a**3*b**(13/2)*x**2*sqrt
(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x*
*5) + 6*I*a**2*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210
*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*I*a*b**(17/2)*x**4*sqrt(-a/(b*x
) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 16
*I*b**(19/2)*x**5*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x
**4 + 105*a**3*b**6*x**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = -\frac{2\left(\frac{35\,(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{42\,(-bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(-bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

input
```
integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/105*(35*(-b*x + a)^(3/2)*b^2/x^(3/2) + 42*(-b*x + a)^(5/2)*b/x^(5/2) +
15*(-b*x + a)^(7/2)/x^(7/2))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = \frac{2\left(\frac{35\,b^7}{a}+4\left(\frac{2\,(bx-a)b^7}{a^3}+\frac{7\,b^7}{a^2}\right)(bx-a)\right)(bx-a)\sqrt{-bx+a}b}{105\left((bx-a)b+ab\right)^{\frac{7}{2}}|b|}$$

input
```
integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
2/105*(35*b^7/a + 4*(2*(b*x - a)*b^7/a^3 + 7*b^7/a^2)*(b*x - a))*(b*x - a)
*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = \frac{\sqrt{a-b\,x}\left(\frac{8\,b^2\,x^2}{105\,a^2}+\frac{16\,b^3\,x^3}{105\,a^3}+\frac{2\,b\,x}{35\,a}-\frac{2}{7}\right)}{x^{7/2}}$$

input
```
int((a - b*x)^(1/2)/x^(9/2),x)
```

output
```
((a - b*x)^(1/2)*((8*b^2*x^2)/(105*a^2) + (16*b^3*x^3)/(105*a^3) + (2*b*x)
/(35*a) - 2/7))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a^3}{7}+\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a^2bx}{35}+\frac{8\sqrt{x}\,\sqrt{-bx+a}\,a\,b^2x^2}{105}+\frac{16\sqrt{x}\,\sqrt{-bx+a}\,b^3x^3}{105}-\frac{16\sqrt{b}\,b^3i\,x^4}{105}}{a^3x^4}$$

input
```
int((-b*x+a)^(1/2)/x^(9/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a - b*x)*a**3 + 3*sqrt(x)*sqrt(a - b*x)*a**2*b*x +
4*sqrt(x)*sqrt(a - b*x)*a*b**2*x**2 + 8*sqrt(x)*sqrt(a - b*x)*b**3*x**3 -
8*sqrt(b)*b**3*i*x**4))/(105*a**3*x**4)
```

3.539 $\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx=-\frac{2(a-bx)^{3/2}}{9ax^{9/2}}-\frac{4b(a-bx)^{3/2}}{21a^2x^{7/2}}-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{5/2}}-\frac{32b^3(a-bx)^{3/2}}{315a^4x^{3/2}}$$

output
```
-2/9*(-b*x+a)^(3/2)/a/x^(9/2)-4/21*b*(-b*x+a)^(3/2)/a^2/x^(7/2)-16/105*b^2
*(-b*x+a)^(3/2)/a^3/x^(5/2)-32/315*b^3*(-b*x+a)^(3/2)/a^4/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx=-\frac{2\sqrt{a-bx}(35a^4-5a^3bx-6a^2b^2x^2-8ab^3x^3-16b^4x^4)}{315a^4x^{9/2}}$$

input
```
Integrate[Sqrt[a - b*x]/x^(11/2),x]
```

output
```
(-2*Sqrt[a - b*x]*(35*a^4 - 5*a^3*b*x - 6*a^2*b^2*x^2 - 8*a*b^3*x^3 - 16*b
^4*x^4))/(315*a^4*x^(9/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2b\int \frac{\sqrt{a-bx}}{x^{9/2}}dx}{3a}-\frac{2(a-bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{2b\left(\frac{4b\int \frac{\sqrt{a-bx}}{x^{7/2}}dx}{7a}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a-bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{2b\left(\frac{4b\left(\frac{2b\int \frac{\sqrt{a-bx}}{x^{5/2}}dx}{5a}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a-bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2b\left(\frac{4b\left(-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a-bx)^{3/2}}{9ax^{9/2}}$$

input
```
Int[Sqrt[a - b*x]/x^(11/2),x]
```

output
```
(-2*(a - b*x)^(3/2))/(9*a*x^(9/2)) + (2*b*((-2*(a - b*x)^(3/2))/(7*a*x^(7/
2)) + (4*b*((-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15
*a^2*x^(3/2))))/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}\left(16b^{3}x^{3}+24a\,b^{2}x^{2}+30a^{2}bx+35a^{3}\right)}{315x^{\frac{9}{2}}a^{4}}$	47
orering	$-\frac{2(-bx+a)^{\frac{3}{2}}\left(16b^{3}x^{3}+24a\,b^{2}x^{2}+30a^{2}bx+35a^{3}\right)}{315x^{\frac{9}{2}}a^{4}}$	47
risch	$-\frac{2\sqrt{-bx+a}\,\left(-16b^{4}x^{4}-8a\,x^{3}b^{3}-6a^{2}b^{2}x^{2}-5a^{3}bx+35a^{4}\right)}{315x^{\frac{9}{2}}a^{4}}$	58
default	$-\frac{\sqrt{-bx+a}}{4x^{\frac{9}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{9a\,x^{\frac{9}{2}}}+\frac{8b\left(-\frac{2\sqrt{-bx+a}}{7a\,x^{\frac{7}{2}}}+\frac{6b\left(-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{9a}\right)}{8}$	121

input
```
int((-b*x+a)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-b*x+a)^(3/2)*(16*b^3*x^3+24*a*b^2*x^2+30*a^2*b*x+35*a^3)/x^(9/2)/
a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx = \frac{2\left(16\,b^4x^4+8\,ab^3x^3+6\,a^2b^2x^2+5\,a^3bx-35\,a^4\right)\sqrt{-bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

input
```
integrate((-b*x+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/315*(16*b^4*x^4 + 8*a*b^3*x^3 + 6*a^2*b^2*x^2 + 5*a^3*b*x - 35*a^4)*sqrt
(-b*x + a)/(a^4*x^(9/2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 68.30 (sec) , antiderivative size = 1139, normalized size of antiderivative = 11.86

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x+a)**(1/2)/x**(11/2),x)
```

output
```
Piecewise((70*a**7*b**(19/2)*sqrt(a/(b*x) - 1)/(-315*a**7*b**9*x**4 + 945*
a**6*b**10*x**5 - 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 220*a**6*b*
*(21/2)*x*sqrt(a/(b*x) - 1)/(-315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 - 9
45*a**5*b**11*x**6 + 315*a**4*b**12*x**7) + 228*a**5*b**(23/2)*x**2*sqrt(a
/(b*x) - 1)/(-315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 - 945*a**5*b**11*x*
*6 + 315*a**4*b**12*x**7) - 80*a**4*b**(25/2)*x**3*sqrt(a/(b*x) - 1)/(-315
*a**7*b**9*x**4 + 945*a**6*b**10*x**5 - 945*a**5*b**11*x**6 + 315*a**4*b**
12*x**7) - 10*a**3*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-315*a**7*b**9*x**4 +
 945*a**6*b**10*x**5 - 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) + 60*a**
2*b**(29/2)*x**5*sqrt(a/(b*x) - 1)/(-315*a**7*b**9*x**4 + 945*a**6*b**10*x
**5 - 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 80*a*b**(31/2)*x**6*sqr
t(a/(b*x) - 1)/(-315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 - 945*a**5*b**11
*x**6 + 315*a**4*b**12*x**7) + 32*b**(33/2)*x**7*sqrt(a/(b*x) - 1)/(-315*a
**7*b**9*x**4 + 945*a**6*b**10*x**5 - 945*a**5*b**11*x**6 + 315*a**4*b**12
*x**7), Abs(a/(b*x)) > 1), (70*I*a**7*b**(19/2)*sqrt(-a/(b*x) + 1)/(-315*a
**7*b**9*x**4 + 945*a**6*b**10*x**5 - 945*a**5*b**11*x**6 + 315*a**4*b**12
*x**7) - 220*I*a**6*b**(21/2)*x*sqrt(-a/(b*x) + 1)/(-315*a**7*b**9*x**4 +
945*a**6*b**10*x**5 - 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) + 228*I*a
**5*b**(23/2)*x**2*sqrt(-a/(b*x) + 1)/(-315*a**7*b**9*x**4 + 945*a**6*b**1
0*x**5 - 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 80*I*a**4*b**(25/...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx = -\frac{2\left(\frac{105\,(-bx+a)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}+\frac{189\,(-bx+a)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{135\,(-bx+a)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}+\frac{35\,(-bx+a)^{\frac{9}{2}}}{x^{\frac{9}{2}}}\right)}{315\,a^4}$$

input
```
integrate((-b*x+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
-2/315*(105*(-b*x + a)^(3/2)*b^3/x^(3/2) + 189*(-b*x + a)^(5/2)*b^2/x^(5/2
) + 135*(-b*x + a)^(7/2)*b/x^(7/2) + 35*(-b*x + a)^(9/2)/x^(9/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx = \frac{2\left(\frac{105\,b^9}{a}+2\left(\frac{63\,b^9}{a^2}+4\left(\frac{2\,(bx-a)b^9}{a^4}+\frac{9\,b^9}{a^3}\right)(bx-a)\right)(bx-a)\right)(bx-a)\sqrt{-bx+a}b}{315\left((bx-a)b+ab\right)^{\frac{9}{2}}|b|}$$

input
```
integrate((-b*x+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
2/315*(105*b^9/a + 2*(63*b^9/a^2 + 4*(2*(b*x - a)*b^9/a^4 + 9*b^9/a^3)*(b*
x - a))*(b*x - a))*(b*x - a)*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(9/2)*a
bs(b))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx = \frac{\sqrt{a-b\,x}\left(\frac{4\,b^2\,x^2}{105\,a^2}+\frac{16\,b^3\,x^3}{315\,a^3}+\frac{32\,b^4\,x^4}{315\,a^4}+\frac{2\,b\,x}{63\,a}-\frac{2}{9}\right)}{x^{9/2}}$$

input
```
int((a - b*x)^(1/2)/x^(11/2),x)
```

output
```
((a - b*x)^(1/2)*((4*b^2*x^2)/(105*a^2) + (16*b^3*x^3)/(315*a^3) + (32*b^4
*x^4)/(315*a^4) + (2*b*x)/(63*a) - 2/9))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a-bx}}{x^{11/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a^4}{9}+\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a^3bx}{63}+\frac{4\sqrt{x}\,\sqrt{-bx+a}\,a^2b^2x^2}{105}+\frac{16\sqrt{x}\,\sqrt{-bx+a}\,a\,b^3x^3}{315}+\frac{32\sqrt{x}\,\sqrt{-bx+a}\,b^4x^4}{315}}{a^4x^5}$$

input
```
int((-b*x+a)^(1/2)/x^(11/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a - b*x)*a**4 + 5*sqrt(x)*sqrt(a - b*x)*a**3*b*x +
6*sqrt(x)*sqrt(a - b*x)*a**2*b**2*x**2 + 8*sqrt(x)*sqrt(a - b*x)*a*b**3*x*
*3 + 16*sqrt(x)*sqrt(a - b*x)*b**4*x**4 - 16*sqrt(b)*b**4*i*x**5))/(315*a*
*4*x**5)
```

3.540 $\int x^{5/2}(a-bx)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 150

$$\int x^{5/2}(a-bx)^{3/2}\,dx=-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{11}{40}ax^{7/2}\sqrt{a-bx}-\frac{1}{5}bx^{9/2}\sqrt{a-bx}+\frac{3a^5\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}$$

output
```
-3/128*a^4*x^(1/2)*(-b*x+a)^(1/2)/b^3-1/64*a^3*x^(3/2)*(-b*x+a)^(1/2)/b^2-
1/80*a^2*x^(5/2)*(-b*x+a)^(1/2)/b+11/40*a*x^(7/2)*(-b*x+a)^(1/2)-1/5*b*x^(
9/2)*(-b*x+a)^(1/2)+3/128*a^5*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int x^{5/2}(a-bx)^{3/2}\,dx=-\frac{\sqrt{x}\sqrt{a-bx}(15a^4+10a^3bx+8a^2b^2x^2-176ab^3x^3+128b^4x^4)}{640b^3}+\frac{3a^5\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{64b^{7/2}}$$

input
```
Integrate[x^(5/2)*(a - b*x)^(3/2),x]
```

output
```
-1/640*(Sqrt[x]*Sqrt[a - b*x]*(15*a^4 + 10*a^3*b*x + 8*a^2*b^2*x^2 - 176*a
*b^3*x^3 + 128*b^4*x^4))/b^3 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
 Sqrt[a - b*x])])/(64*b^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {60, 60, 60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a-bx)^{3/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\int x^{5/2}\sqrt{a-bx}dx+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a-bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\int \frac{x^{3/2}}{\sqrt{a-bx}}dx}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

$\Big\downarrow$ 65

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

$\Big\downarrow$ 216

$$\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

input
```
Int[x^(5/2)*(a - b*x)^(3/2),x]
```

output
```
(x^(7/2)*(a - b*x)^(3/2))/5 + (3*a*((x^(7/2)*Sqrt[a - b*x])/4 + (a*(-1/3*(
x^(5/2)*Sqrt[a - b*x])/b + (5*a*(-1/2*(x^(3/2)*Sqrt[a - b*x])/b + (3*a*(-(
(Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b
^(3/2)))/(4*b)))/(6*b)))/8))/10
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(128b^4x^4-176a\,x^3b^3+8a^2b^2x^2+10a^3bx+15a^4)\sqrt{x}\,\sqrt{-bx+a}}{640b^3}+\frac{3a^5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)\sqrt{x(-bx+a)}}{256b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$-\frac{x^{\frac{5}{2}}(-bx+a)^{\frac{5}{2}}}{5b}+\frac{a\left(-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}}{4b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{5}{2}}}{3b}+\frac{a\left(\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)}{2b}$

input
```
int(x^(5/2)*(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/640*(128*b^4*x^4-176*a*b^3*x^3+8*a^2*b^2*x^2+10*a^3*b*x+15*a^4)/b^3*x^(
1/2)*(-b*x+a)^(1/2)+3/256*a^5/b^(7/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a
*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \left[-\frac{15\,a^5\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(128\,b^5x^4-176\,ab^4x^3+8\,a^2b^3x^2+\right.}{1280\,b^4}$$

input
```
integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
 a) + 2*(128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a^3*b^2*x + 15*a
^4*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x
 + a)*sqrt(b)*sqrt(x)/(b*x - a)) + (128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^
3*x^2 + 10*a^3*b^2*x + 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 50.82 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.51

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{-1+\frac{bx}{a}}} - \frac{23ia^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}} + \frac{19i\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^5\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}} - \frac{ib^2}{5\sqrt{a}\sqrt{}} \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1-\frac{bx}{a}}} + \frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}} - \frac{19\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}} + \frac{3a^5\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}} + \frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}} \end{cases}$$

input
```
integrate(x**(5/2)*(-b*x+a)**(3/2),x)
```

output
```
Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**3*sqrt(-1 + b*x/a)) - I*a**(7/2)*x
**(3/2)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**(5/2)/(320*b*sqrt(-1 +
 b*x/a)) - 23*I*a**(3/2)*x**(7/2)/(80*sqrt(-1 + b*x/a)) + 19*I*sqrt(a)*b*x
**(9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(1
28*b**(7/2)) - I*b**2*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
 1), (-3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(
128*b**2*sqrt(1 - b*x/a)) + a**(5/2)*x**(5/2)/(320*b*sqrt(1 - b*x/a)) + 23
*a**(3/2)*x**(7/2)/(80*sqrt(1 - b*x/a)) - 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1
 - b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x*
*(11/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38

$$\int x^{5/2}(a-bx)^{3/2}\,dx = -\frac{3\,a^5\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{128\,b^{\frac{7}{2}}} + \frac{\frac{15\,\sqrt{-bx+a}a^5b^4}{\sqrt{x}} + \frac{70\,(-bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}} - \frac{128\,(-bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}} - \frac{70\,(-bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}} - \frac{15\,(-bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^8 - \frac{5\,(bx-a)b^7}{x} + \frac{10\,(bx-a)^2b^6}{x^2} - \frac{10\,(bx-a)^3b^5}{x^3} + \frac{5\,(bx-a)^4b^4}{x^4} - \frac{(bx-a)^5b^3}{x^5}\right)}$$

input
```
integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/640*(15*sq
rt(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) - 128*(
-b*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(
-b*x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x - a)*b^7/x + 10*(b*x - a)^2*b^6
/x^2 - 10*(b*x - a)^3*b^5/x^3 + 5*(b*x - a)^4*b^4/x^4 - (b*x - a)^5*b^3/x^
5)
```

Giac [F(-1)]

Timed out.

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \int x^{5/2}\,(a-b\,x)^{3/2}\,dx$$

input
```
int(x^(5/2)*(a - b*x)^(3/2),x)
```

output
```
int(x^(5/2)*(a - b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \frac{-15\sqrt{x}\,\sqrt{-bx+a}\,a^4b-10\sqrt{x}\,\sqrt{-bx+a}\,a^3b^2x-8\sqrt{x}\,\sqrt{-bx+a}\,a^2b^3x^2+176\sqrt{x}\,\sqrt{-bx+a}}{640b^4}$$

input
```
int(x^(5/2)*(-b*x+a)^(3/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(a - b*x)*a**4*b - 10*sqrt(x)*sqrt(a - b*x)*a**3*b**2*x
 - 8*sqrt(x)*sqrt(a - b*x)*a**2*b**3*x**2 + 176*sqrt(x)*sqrt(a - b*x)*a*b*
*4*x**3 - 128*sqrt(x)*sqrt(a - b*x)*b**5*x**4 - 15*sqrt(b)*log((sqrt(a - b
*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**5*i)/(640*b**4)
```

3.541 $\int x^{3/2}(a-bx)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int x^{3/2}(a-bx)^{3/2}\,dx=-\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{3}{8}ax^{5/2}\sqrt{a-bx}-\frac{1}{4}bx^{7/2}\sqrt{a-bx}+\frac{3a^4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}$$

output
```
-3/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^2-1/32*a^2*x^(3/2)*(-b*x+a)^(1/2)/b+3/8
*a*x^(5/2)*(-b*x+a)^(1/2)-1/4*b*x^(7/2)*(-b*x+a)^(1/2)+3/64*a^4*arctan(b^(
1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int x^{3/2}(a-bx)^{3/2}\,dx=\frac{-\sqrt{b}\sqrt{x}\sqrt{a-bx}(3a^3+2a^2bx-24ab^2x^2+16b^3x^3)+6a^4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{64b^{5/2}}$$

input
```
Integrate[x^(3/2)*(a - b*x)^(3/2),x]
```

output
```
(-(Sqrt[b]*Sqrt[x]*Sqrt[a - b*x]*(3*a^3 + 2*a^2*b*x - 24*a*b^2*x^2 + 16*b^
3*x^3)) + 6*a^4*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/(64*
b^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {60, 60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a-bx)^{3/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\int x^{3/2}\sqrt{a-bx}dx+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a-bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

$$\Big\downarrow \text{65}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\left(\frac{a\int\frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

$\big\downarrow$ 216

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

input
```
Int[x^(3/2)*(a - b*x)^(3/2),x]
```

output
```
(x^(5/2)*(a - b*x)^(3/2))/4 + (3*a*((x^(5/2)*Sqrt[a - b*x])/3 + (a*(-1/2*(
x^(3/2)*Sqrt[a - b*x])/b + (3*a*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)))/(4*b)))/6))/8
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))    Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{(16b^3x^3-24a\,b^2x^2+2a^2bx+3a^3)\sqrt{x}\,\sqrt{-bx+a}}{64b^2}+\frac{3a^4\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)\sqrt{x(-bx+a)}}{128b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	102
default	$-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}}{4b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{5}{2}}}{3b}+\frac{a\left(\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}$	129

input
```
int(x^(3/2)*(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(16*b^3*x^3-24*a*b^2*x^2+2*a^2*b*x+3*a^3)/b^2*x^(1/2)*(-b*x+a)^(1/2)
+3/128*a^4/b^(5/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x
+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int x^{3/2}(a-bx)^{3/2}\,dx = \left[-\frac{3\,a^4\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b}{128\,b^3}\right.$$

input
```
integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/128*(3*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) + 2*(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*s
qrt(x))/b^3, -1/64*(3*a^4*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b
*x - a)) + (16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x +
 a)*sqrt(x))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.58

$$\int x^{3/2}(a-bx)^{3/2}\,dx = \begin{cases} \frac{3ia^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{-1+\frac{bx}{a}}} - \frac{13ia^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{-1+\frac{bx}{a}}} + \frac{5i\sqrt{a}bx^{\frac{7}{2}}}{8\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}} - \frac{ib^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > \\ -\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1-\frac{bx}{a}}} + \frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1-\frac{bx}{a}}} - \frac{5\sqrt{a}bx^{\frac{7}{2}}}{8\sqrt{1-\frac{bx}{a}}} + \frac{3a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}} + \frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwis} \end{cases}$$

input
```
integrate(x**(3/2)*(-b*x+a)**(3/2),x)
```

output
```
Piecewise((3*I*a**(7/2)*sqrt(x)/(64*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x*
*(3/2)/(64*b*sqrt(-1 + b*x/a)) - 13*I*a**(3/2)*x**(5/2)/(32*sqrt(-1 + b*x/
a)) + 5*I*sqrt(a)*b*x**(7/2)/(8*sqrt(-1 + b*x/a)) - 3*I*a**4*acosh(sqrt(b)
*sqrt(x)/sqrt(a))/(64*b**(5/2)) - I*b**2*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x
/a)), Abs(b*x/a) > 1), (-3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 - b*x/a)) + a*
*(5/2)*x**(3/2)/(64*b*sqrt(1 - b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 -
 b*x/a)) - 5*sqrt(a)*b*x**(7/2)/(8*sqrt(1 - b*x/a)) + 3*a**4*asin(sqrt(b)*
sqrt(x)/sqrt(a))/(64*b**(5/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a))
, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36

$$\int x^{3/2}(a-bx)^{3/2}\,dx = -\frac{3\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{64\,b^{\frac{5}{2}}} + \frac{\frac{3\sqrt{-bx+a}a^4b^3}{\sqrt{x}} + \frac{11\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}} - \frac{11\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}} - \frac{3\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6 - \frac{4\,(bx-a)b^5}{x} + \frac{6\,(bx-a)^2b^4}{x^2} - \frac{4\,(bx-a)^3b^3}{x^3} + \frac{(bx-a)^4b^2}{x^4}\right)}$$

input
```
integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-3/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/64*(3*sqrt(
-b*x + a)*a^4*b^3/sqrt(x) + 11*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 11*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 3*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^6 - 4*(b*x
- a)*b^5/x + 6*(b*x - a)^2*b^4/x^2 - 4*(b*x - a)^3*b^3/x^3 + (b*x - a)^4*b
^2/x^4)
```

Giac [F(-1)]

Timed out.

$$\int x^{3/2}(a-bx)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a-bx)^{3/2}\,dx=\int x^{3/2}\,{\left(a-b\,x\right)}^{3/2}\,dx$$

input
```
int(x^(3/2)*(a - b*x)^(3/2),x)
```

output
```
int(x^(3/2)*(a - b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int x^{3/2}(a-bx)^{3/2}\,dx=\frac{-3\sqrt{x}\,\sqrt{-bx+a}\,a^3b-2\sqrt{x}\,\sqrt{-bx+a}\,a^2b^2x+24\sqrt{x}\,\sqrt{-bx+a}\,a\,b^3x^2-16\sqrt{x}\,\sqrt{-bx+a}\,b^4}{64b^3}$$

input
```
int(x^(3/2)*(-b*x+a)^(3/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a - b*x)*a**3*b - 2*sqrt(x)*sqrt(a - b*x)*a**2*b**2*x +
 24*sqrt(x)*sqrt(a - b*x)*a*b**3*x**2 - 16*sqrt(x)*sqrt(a - b*x)*b**4*x**3
 - 3*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**4*i)/(64*
b**3)
```

3.542 $\int \sqrt{x}(a-bx)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \sqrt{x}(a-bx)^{3/2}\,dx=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{7}{12}ax^{3/2}\sqrt{a-bx}-\frac{1}{3}bx^{5/2}\sqrt{a-bx}+\frac{a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}$$

output
```
-1/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b+7/12*a*x^(3/2)*(-b*x+a)^(1/2)-1/3*b*x^(5
/2)*(-b*x+a)^(1/2)+1/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a-bx)^{3/2}\,dx=-\frac{\sqrt{x}\sqrt{a-bx}(3a^2-14abx+8b^2x^2)}{24b}+\frac{a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{4b^{3/2}}$$

input
```
Integrate[Sqrt[x]*(a - b*x)^(3/2),x]
```

output
```
-1/24*(Sqrt[x]*Sqrt[a - b*x]*(3*a^2 - 14*a*b*x + 8*b^2*x^2))/b + (a^3*ArcT
an[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/(4*b^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a-bx)^{3/2}\,dx$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}a\int \sqrt{x}\sqrt{a-bx}dx+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

$$\Big\downarrow \ 65$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

input
```
Int[Sqrt[x]*(a - b*x)^(3/2),x]
```

output
```
(x^(3/2)*(a - b*x)^(3/2))/3 + (a*((x^(3/2)*Sqrt[a - b*x])/2 + (a*(-((Sqrt[
x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)
))/4))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\left(8 b^{2} x^{2}-14 a b x +3 a^{2}\right) \sqrt{x}\, \sqrt{-b x +a}}{24 b}+\frac{a^{3} \arctan \left(\frac{\sqrt{b}\, \left(x -\frac{a}{2 b}\right)}{\sqrt{-b \,x^{2}+a x}}\right) \sqrt{x \left(-b x +a \right)}}{16 b^{\frac{3}{2}} \sqrt{x}\, \sqrt{-b x +a}}$	91
default	$-\frac{\sqrt{x}\, \left(-b x +a \right)^{\frac{5}{2}}}{3 b}+\frac{a \left(\frac{\sqrt{x}\, \left(-b x +a \right)^{\frac{3}{2}}}{2}+\frac{3 a \left(\sqrt{x}\, \sqrt{-b x +a}+\frac{a \sqrt{x \left(-b x +a \right)}\, \arctan \left(\frac{\sqrt{b}\, \left(x -\frac{a}{2 b}\right)}{\sqrt{-b \,x^{2}+a x}}\right)}{2 \sqrt{-b x +a}\, \sqrt{x}\, \sqrt{b}}\right)}{4}\right)}{6 b}$	106

input
```
int(x^(1/2)*(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(8*b^2*x^2-14*a*b*x+3*a^2)/b*x^(1/2)*(-b*x+a)^(1/2)+1/16/b^(3/2)*a^3
*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)
/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.50

$$\int \sqrt{x}(a-bx)^{3/2}\,dx = \left[-\frac{3\,a^3\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(8\,b^3x^2-14\,ab^2x+3\,a^2b)\sqrt{-bx+a}}{48\,b^2}\right.$$
$$\left.-\frac{3\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)+(8\,b^3x^2-14\,ab^2x+3\,a^2b)\sqrt{-bx+a}\sqrt{x}}{24\,b^2}\right]$$

input
```
integrate(x^(1/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(8*b^3*x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/24
*(3*a^3*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) + (8*b^3*
x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.64

$$\int \sqrt{x}(a-bx)^{3/2}\,dx = \begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{-1+\frac{bx}{a}}} - \frac{17ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}} + \frac{11i\sqrt{a}bx^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} - \frac{ib^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1-\frac{bx}{a}}} + \frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1-\frac{bx}{a}}} - \frac{11\sqrt{a}bx^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)*(-b*x+a)**(3/2),x)
```

output
```
Piecewise((I*a**(5/2)*sqrt(x)/(8*b*sqrt(-1 + b*x/a)) - 17*I*a**(3/2)*x**(3
/2)/(24*sqrt(-1 + b*x/a)) + 11*I*sqrt(a)*b*x**(5/2)/(12*sqrt(-1 + b*x/a))
- I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) - I*b**2*x**(7/2)/(3*
sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b*sqrt(1
 - b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 - b*x/a)) - 11*sqrt(a)*b*x**(
5/2)/(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)
) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.33

$$\int \sqrt{x}(a-bx)^{3/2}\,dx = -\frac{a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{8\,b^{\frac{3}{2}}} + \frac{\frac{3\sqrt{-bx+a}a^3b^2}{\sqrt{x}} + \frac{8\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}} - \frac{3\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^4 - \frac{3\,(bx-a)b^3}{x} + \frac{3\,(bx-a)^2b^2}{x^2} - \frac{(bx-a)^3b}{x^3}\right)}$$

input
```
integrate(x^(1/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/24*(3*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)
^(5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x - a)*b^3/x + 3*(b*x - a)^2*b^2/x^2 - (b*
x - a)^3*b/x^3)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{x}(a-bx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a-bx)^{3/2}\,dx = \int \sqrt{x}\,(a-b\,x)^{3/2}\,dx$$

input
```
int(x^(1/2)*(a - b*x)^(3/2),x)
```

output
```
int(x^(1/2)*(a - b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(a-bx)^{3/2}\,dx = \frac{-3\sqrt{x}\,\sqrt{-bx+a}\,a^2b+14\sqrt{x}\,\sqrt{-bx+a}\,a\,b^2x-8\sqrt{x}\,\sqrt{-bx+a}\,b^3x^2-3\sqrt{b}\log\Bigl(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{}}{\sqrt{a}}}{24b^2}$$

input
```
int(x^(1/2)*(-b*x+a)^(3/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a - b*x)*a**2*b + 14*sqrt(x)*sqrt(a - b*x)*a*b**2*x - 8
*sqrt(x)*sqrt(a - b*x)*b**3*x**2 - 3*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*
sqrt(b)*i)/sqrt(a))*a**3*i)/(24*b**2)
```

3.543 $\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx=\frac{5}{4}a\sqrt{x}\sqrt{a-bx}-\frac{1}{2}bx^{3/2}\sqrt{a-bx}+\frac{3a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}$$

output
```
5/4*a*x^(1/2)*(-b*x+a)^(1/2)-1/2*b*x^(3/2)*(-b*x+a)^(1/2)+3/4*a^2*arctan(b
^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx=-\frac{1}{4}\sqrt{x}\sqrt{a-bx}(-5a+2bx)+\frac{3a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{2\sqrt{b}}$$

input
```
Integrate[(a - b*x)^(3/2)/Sqrt[x],x]
```

output
```
-1/4*(Sqrt[x]*Sqrt[a - b*x]*(-5*a + 2*b*x)) + (3*a^2*ArcTan[(Sqrt[b]*Sqrt[
x])/(-Sqrt[a] + Sqrt[a - b*x])])/(2*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}a\int \frac{\sqrt{a-bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

$$\Big\downarrow \text{65}$$

$$\frac{3}{4}a\left(a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

input
```
Int[(a - b*x)^(3/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a*(Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{(-2bx+5a)\sqrt{x}\,\sqrt{-bx+a}}{4}+\frac{3a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x(-bx+a)}}{8\sqrt{b}\,\sqrt{x}\,\sqrt{-bx+a}}$	77
default	$\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}$	83

input
```
int((-b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*b*x+5*a)*x^(1/2)*(-b*x+a)^(1/2)+3/8*a^2/b^(1/2)*arctan(b^(1/2)*(x-
1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx = \left[-\frac{3\,a^2\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(2\,b^2x-5\,ab)\sqrt{-bx+a}\sqrt{x}}{8\,b}, -\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)+(2\,b^2x-5\,ab)\sqrt{-bx+a}\sqrt{x}}{4\,b}\right]$$

input

```
integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output

```
[-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(b)*arcta
n(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) + (2*b^2*x - 5*a*b)*sqrt(-b*x
+ a)*sqrt(x))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.53

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx = \begin{cases} -\frac{5ia^{\frac{3}{2}}\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{7i\sqrt{a}bx^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}-\frac{ib^2x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{4}-\frac{\sqrt{a}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{2}+\frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} & \text{otherwise} \end{cases}$$

input

```
integrate((-b*x+a)**(3/2)/x**(1/2),x)
```

output

```
Piecewise((-5*I*a**(3/2)*sqrt(x)/(4*sqrt(-1 + b*x/a)) + 7*I*sqrt(a)*b*x**(
3/2)/(4*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqr
t(b)) - I*b**2*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (5*
a**(3/2)*sqrt(x)*sqrt(1 - b*x/a)/4 - sqrt(a)*b*x**(3/2)*sqrt(1 - b*x/a)/2
+ 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx = -\frac{3\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{4\,\sqrt{b}} + \frac{\frac{3\,\sqrt{-bx+a}a^2b}{\sqrt{x}} + \frac{5\,(-bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^2 - \frac{2\,(bx-a)b}{x} + \frac{(bx-a)^2}{x^2}\right)}$$

input
```
integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/4*(3*sqrt(-b
*x + a)*a^2*b/sqrt(x) + 5*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^2 - 2*(b*x - a)
*b/x + (b*x - a)^2/x^2)
```

Giac [A] (verification not implemented)

Time = 76.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx = \frac{\left(\frac{3\,a^2\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} - \sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(\frac{2\,(bx-a)}{b} - \frac{3\,a}{b}\right)\right)b}{4\,|b|}$$

input
```
integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*a^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sq
rt(-b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)/b - 3*a/b))*b
/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx=\int \frac{{\left(a-b\,x\right)}^{3/2}}{\sqrt{x}}\,dx$$

input
```
int((a - b*x)^(3/2)/x^(1/2),x)
```

output
```
int((a - b*x)^(3/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx=\frac{5\sqrt{x}\,\sqrt{-bx+a}\,ab-2\sqrt{x}\,\sqrt{-bx+a}\,b^2x-3\sqrt{b}\,\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)a^2i}{4b}$$

input
```
int((-b*x+a)^(3/2)/x^(1/2),x)
```

output
```
(5*sqrt(x)*sqrt(a - b*x)*a*b - 2*sqrt(x)*sqrt(a - b*x)*b**2*x - 3*sqrt(b)*
log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**2*i)/(4*b)
```

3.544 $\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx=-\frac{2a\sqrt{a-bx}}{\sqrt{x}}-b\sqrt{x}\sqrt{a-bx}-3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

output
```
-2*a*(-b*x+a)^(1/2)/x^(1/2)-b*x^(1/2)*(-b*x+a)^(1/2)-3*a*b^(1/2)*arctan(b^
(1/2)*x^(1/2)/(-b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx=\frac{(-2a-bx)\sqrt{a-bx}}{\sqrt{x}}-6a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)$$

input
```
Integrate[(a - b*x)^(3/2)/x^(3/2),x]
```

output
```
((-2*a - b*x)*Sqrt[a - b*x])/Sqrt[x] - 6*a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x]
)/(-Sqrt[a] + Sqrt[a - b*x])]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {57, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{57}$$

$$-3b\int \frac{\sqrt{a-bx}}{\sqrt{x}}dx-\frac{2(a-bx)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$-3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx+\sqrt{x}\sqrt{a-bx}\right)-\frac{2(a-bx)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{65}$$

$$-3b\left(a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}+\sqrt{x}\sqrt{a-bx}\right)-\frac{2(a-bx)^{3/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{216}$$

$$-3b\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a-bx}\right)-\frac{2(a-bx)^{3/2}}{\sqrt{x}}$$

input
```
Int[(a - b*x)^(3/2)/x^(3/2),x]
```

output
```
(-2*(a - b*x)^(3/2))/Sqrt[x] - 3*b*(Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(bx+2a)}{\sqrt{x}}-\frac{3a\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x(-bx+a)}}{2\sqrt{x}\,\sqrt{-bx+a}}$	74

input
```
int((-b*x+a)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-b*x+a)^(1/2)*(b*x+2*a)/x^(1/2)-3/2*a*b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)
/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx = \left[\frac{3\,a\sqrt{-b}x\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(bx+2\,a)\sqrt{-bx+a}\sqrt{x}}{2\,x}, \frac{3\,a\sqrt{b}x \cdots}{\cdots}\right.$$

input
```
integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*a*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) -
 2*(b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x, (3*a*sqrt(b)*x*arctan(sqrt(-b*x
+ a)*sqrt(b)*sqrt(x)/(b*x - a)) - (b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.94

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx = \begin{cases} \frac{2ia^{\frac{3}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}b\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}} + 3ia\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{ib^2x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}b\sqrt{x}}{\sqrt{1-\frac{bx}{a}}} - 3a\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + \frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x+a)**(3/2)/x**(3/2),x)
```

output
```
Piecewise((2*I*a**(3/2)/(sqrt(x)*sqrt(-1 + b*x/a)) - I*sqrt(a)*b*sqrt(x)/s
qrt(-1 + b*x/a) + 3*I*a*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a)) - I*b**2*x*
*(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(3/2)/(sqrt(x)*
sqrt(1 - b*x/a)) + sqrt(a)*b*sqrt(x)/sqrt(1 - b*x/a) - 3*a*sqrt(b)*asin(sq
rt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx = 3\,a\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right) - \frac{2\,\sqrt{-bx+a}a}{\sqrt{x}} - \frac{\sqrt{-bx+a}ab}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

input
```
integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
3*a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)*a/
sqrt(x) - sqrt(-b*x + a)*a*b/((b - (b*x - a)/x)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 76.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx = -\frac{\left(\frac{3\,a\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} + \frac{(bx+2\,a)\sqrt{-bx+a}}{\sqrt{(bx-a)b+ab}}\right)b^2}{|b|}$$

input
```
integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
-(3*a*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b
) + (b*x + 2*a)*sqrt(-b*x + a)/sqrt((b*x - a)*b + a*b))*b^2/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx = \int \frac{(a-b\,x)^{3/2}}{x^{3/2}}\,dx$$

input
```
int((a - b*x)^(3/2)/x^(3/2),x)
```

output
```
int((a - b*x)^(3/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx = \frac{-8\sqrt{x}\,\sqrt{-bx+a}\,a-4\sqrt{x}\,\sqrt{-bx+a}\,bx+12\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)aix-9\sqrt{b}\,aix}{4x}$$

input
```
int((-b*x+a)^(3/2)/x^(3/2),x)
```

output
```
( - 8*sqrt(x)*sqrt(a - b*x)*a - 4*sqrt(x)*sqrt(a - b*x)*b*x + 12*sqrt(b)*l
og((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a*i*x - 9*sqrt(b)*a*i*x)/(
4*x)
```

3.545 $\int \frac{(a-bx)^{3/2}}{x^{5/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}} dx = -\frac{2a\sqrt{a-bx}}{3x^{3/2}} + \frac{8b\sqrt{a-bx}}{3\sqrt{x}} + 2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

output
```
-2/3*a*(-b*x+a)^(1/2)/x^(3/2)+8/3*b*(-b*x+a)^(1/2)/x^(1/2)+2*b^(3/2)*arcta
n(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}} dx = \frac{2\sqrt{a-bx}(-a+4bx)}{3x^{3/2}} + 4b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)$$

input
```
Integrate[(a - b*x)^(3/2)/x^(5/2),x]
```

output
```
(2*Sqrt[a - b*x]*(-a + 4*b*x))/(3*x^(3/2)) + 4*b^(3/2)*ArcTan[(Sqrt[b]*Sqr
t[x])/(-Sqrt[a] + Sqrt[a - b*x])]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {57, 57, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}}dx$$

$$\Big\downarrow \ 57$$

$$-b\int \frac{\sqrt{a-bx}}{x^{3/2}}dx-\frac{2(a-bx)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 57$$

$$-b\left(-b\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx-\frac{2\sqrt{a-bx}}{\sqrt{x}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 65$$

$$-b\left(-2b\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}-\frac{2\sqrt{a-bx}}{\sqrt{x}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 216$$

$$-b\left(-2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2\sqrt{a-bx}}{\sqrt{x}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}$$

input
```
Int[(a - b*x)^(3/2)/x^(5/2),x]
```

output
```
(-2*(a - b*x)^(3/2))/(3*x^(3/2)) - b*((-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[
b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
risch	$-\frac{2\sqrt{-bx+a}\,(-4bx+a)}{3x^{\frac{3}{2}}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	71

input
```
int((-b*x+a)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x+a)^(1/2)*(-4*b*x+a)/x^(3/2)+b^(3/2)*arctan(b^(1/2)*(x-1/2*a/b)/
(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.77

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx = \left[\frac{3\sqrt{-b}bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(4\,bx-a)\sqrt{-bx+a}\sqrt{x}}{3\,x^2}, -\frac{2\,(3\right.$$

input
```
integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(s
qrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (4*b*x - a)*sqrt(-b*x + a)*sqrt
(x))/x^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.67

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx = \begin{cases} -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-2ib^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) & \text{for } \left|\frac{a}{b\right. \\ -\frac{2ia\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{8ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right) & \text{other} \end{cases}$$

input
```
integrate((-b*x+a)**(3/2)/x**(5/2),x)
```

output
```
Piecewise((-2*a*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 8*b**(3/2)*sqrt(a/(b*x)
- 1)/3 - 2*I*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + I*b**(3/2)*log(a/(b
*x)) + 2*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)), Abs(a/(b*x)) > 1), (-2*I*
a*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 8*I*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + I
*b**(3/2)*log(a/(b*x)) - 2*I*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx = -2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2\sqrt{-bx+a}b}{\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-2*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + a)*b/s
qrt(x) - 2/3*(-b*x + a)^(3/2)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 75.97 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx = \frac{2\left(\frac{3\,b^2\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}}+\frac{\left(4\,(bx-a)b^3+3\,ab^3\right)\sqrt{-bx+a}}{((bx-a)b+ab)^{\frac{3}{2}}}\right)b}{3\,|b|}$$

input
```
integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*b^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sq
rt(-b) + (4*(b*x - a)*b^3 + 3*a*b^3)*sqrt(-b*x + a)/((b*x - a)*b + a*b)^(3
/2))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx = \int \frac{(a-b\,x)^{3/2}}{x^{5/2}}\,dx$$

input
```
int((a - b*x)^(3/2)/x^(5/2),x)
```

output
```
int((a - b*x)^(3/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a}{3}+\frac{8\sqrt{x}\,\sqrt{-bx+a}\,bx}{3}-2\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)bi\,x^2}{x^2}$$

input
```
int((-b*x+a)^(3/2)/x^(5/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a - b*x)*a + 4*sqrt(x)*sqrt(a - b*x)*b*x - 3*sqrt(b)*l
og((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*b*i*x**2))/(3*x**2)
```

3.546 $\int x^{5/2}(a-bx)^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 175

$$\int x^{5/2}(a-bx)^{5/2}\,dx=-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{9}{32}a^2x^{7/2}\sqrt{a-bx}-\frac{5}{12}abx^{9/2}\sqrt{a-bx}+\frac{1}{6}b^2x^{11/2}\sqrt{a-bx}+\frac{5a^6\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}$$

output
```
-5/512*a^5*x^(1/2)*(-b*x+a)^(1/2)/b^3-5/768*a^4*x^(3/2)*(-b*x+a)^(1/2)/b^2
-1/192*a^3*x^(5/2)*(-b*x+a)^(1/2)/b+9/32*a^2*x^(7/2)*(-b*x+a)^(1/2)-5/12*a
*b*x^(9/2)*(-b*x+a)^(1/2)+1/6*b^2*x^(11/2)*(-b*x+a)^(1/2)+5/512*a^6*arctan
(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a-bx)^{5/2}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a-bx}(-15a^5-10a^4bx-8a^3b^2x^2+432a^2b^3x^3-640ab^4x^4+256b^5x^5)+30a^6\operatorname{arcta}}{1536b^{7/2}}$$

input
```
Integrate[x^(5/2)*(a - b*x)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a - b*x]*(-15*a^5 - 10*a^4*b*x - 8*a^3*b^2*x^2 + 432
*a^2*b^3*x^3 - 640*a*b^4*x^4 + 256*b^5*x^5) + 30*a^6*ArcTan[(Sqrt[b]*Sqrt[
x])/(-Sqrt[a] + Sqrt[a - b*x])])/(1536*b^(7/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {60, 60, 60, 60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a-bx)^{5/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\int x^{5/2}(a-bx)^{3/2}dx+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\int x^{5/2}\sqrt{a-bx}dx+\frac{1}{5}x^{7/2}(a-bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a-bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\int \frac{x^{3/2}}{\sqrt{a-bx}}dx}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\int\frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a-bx)^{3/2}\right)+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

$\downarrow$ 60

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(a\right.$$

$$\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

$\downarrow$ 65

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7}\right.$$

$$\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

$\downarrow$ 216

$$\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{5a\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}\right)+\frac{1}{4}x^{7/2}\sqrt{a-bx}\right)+\frac{1}{5}x^{7/2}(\right.$$

$$\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

input
```
Int[x^(5/2)*(a - b*x)^(5/2),x]
```

output
```
(x^(7/2)*(a - b*x)^(5/2))/6 + (5*a*((x^(7/2)*(a - b*x)^(3/2))/5 + (3*a*((x
^(7/2)*Sqrt[a - b*x])/4 + (a*(-1/3*(x^(5/2)*Sqrt[a - b*x])/b + (5*a*(-1/2*
(x^(3/2)*Sqrt[a - b*x])/b + (3*a*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)))/(4*b)))/(6*b)))/8))/10))/12
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\left(-256b^{5}x^{5}+640a\,b^{4}x^{4}-432a^{2}b^{3}x^{3}+8a^{3}b^{2}x^{2}+10a^{4}bx+15a^{5}\right)\sqrt{x}\,\sqrt{-bx+a}}{1536b^{3}}+\frac{5a^{6}\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x(-bx+a)}}{1024b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+a}}$ $5a\left(-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{7}{2}}}{5b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{7}{2}}}{4b}+\frac{a\left(\frac{\sqrt{x}\,(-bx+a)^{\frac{5}{2}}}{3}+\frac{5a\left(\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a}\cdots}{2\sqrt{\cdots}}\right.}{4}\cdots\right.}{6}\cdots\right.}{8b}\cdots\right.}{10b}\cdots\right.$

input
```
int(x^(5/2)*(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1536*(-256*b^5*x^5+640*a*b^4*x^4-432*a^2*b^3*x^3+8*a^3*b^2*x^2+10*a^4*b
*x+15*a^5)/b^3*x^(1/2)*(-b*x+a)^(1/2)+5/1024*a^6/b^(7/2)*arctan(b^(1/2)*(x
-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.24

$$\int x^{5/2}(a-bx)^{5/2}\,dx = \left[-\frac{15\,a^6\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3}{3072\,b^4}\right.$$

input
```
integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/3072*(15*a^6*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
 a) - 2*(256*b^6*x^5 - 640*a*b^5*x^4 + 432*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 1
0*a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/1536*(15*a^6*sqrt(
b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (256*b^6*x^5 - 640*a
*b^5*x^4 + 432*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*a^4*b^2*x - 15*a^5*b)*sqrt
(-b*x + a)*sqrt(x))/b^4]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 150.80 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.49

$$\int x^{5/2}(a-bx)^{5/2}\,dx = \begin{cases} \frac{5ia^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{-1+\frac{bx}{a}}} - \frac{55ia^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{67ia^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}b^2x^{\frac{11}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^6\operatorname{acosh}}{512b} \\ -\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1-\frac{bx}{a}}} + \frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}b^2x^{\frac{11}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{5a^6\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} \end{cases}$$

input
```
integrate(x**(5/2)*(-b*x+a)**(5/2),x)
```

output
```
Piecewise((5*I*a**(11/2)*sqrt(x)/(512*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(9/2
)*x**(3/2)/(1536*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x**(5/2)/(768*b*sqrt(
-1 + b*x/a)) - 55*I*a**(5/2)*x**(7/2)/(192*sqrt(-1 + b*x/a)) + 67*I*a**(3/
2)*b*x**(9/2)/(96*sqrt(-1 + b*x/a)) - 7*I*sqrt(a)*b**2*x**(11/2)/(12*sqrt(
-1 + b*x/a)) - 5*I*a**6*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + I*
b**3*x**(13/2)/(6*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(11/2
)*sqrt(x)/(512*b**3*sqrt(1 - b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*b**2*sqrt
(1 - b*x/a)) + a**(7/2)*x**(5/2)/(768*b*sqrt(1 - b*x/a)) + 55*a**(5/2)*x**
(7/2)/(192*sqrt(1 - b*x/a)) - 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 - b*x/a))
+ 7*sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 - b*x/a)) + 5*a**6*asin(sqrt(b)*sqrt
(x)/sqrt(a))/(512*b**(7/2)) - b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 - b*x/a)),
True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.38

$$\int x^{5/2}(a-bx)^{5/2}\,dx = -\frac{5\,a^6\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{512\,b^{\frac{7}{2}}} + \frac{\frac{15\sqrt{-bx+a}a^6b^5}{\sqrt{x}} + \frac{85\,(-bx+a)^{\frac{3}{2}}a^6b^4}{x^{\frac{3}{2}}} + \frac{198\,(-bx+a)^{\frac{5}{2}}a^6b^3}{x^{\frac{5}{2}}} - \frac{198\,(-bx+a)^{\frac{7}{2}}a^6b^2}{x^{\frac{7}{2}}} - \frac{85\,(-bx+a)^{\frac{9}{2}}a^6b}{x^{\frac{9}{2}}} - \frac{15\,(-bx+a)^{\frac{11}{2}}a^6}{x^{\frac{11}{2}}}}{1536\left(b^9 - \frac{6\,(bx-a)b^8}{x} + \frac{15\,(bx-a)^2b^7}{x^2} - \frac{20\,(bx-a)^3b^6}{x^3} + \frac{15\,(bx-a)^4b^5}{x^4} - \frac{6\,(bx-a)^5b^4}{x^5} + \frac{(bx-a)^6b^3}{x^6}\right)}$$

input
```
integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-5/512*a^6*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/1536*(15*s
qrt(-b*x + a)*a^6*b^5/sqrt(x) + 85*(-b*x + a)^(3/2)*a^6*b^4/x^(3/2) + 198*
(-b*x + a)^(5/2)*a^6*b^3/x^(5/2) - 198*(-b*x + a)^(7/2)*a^6*b^2/x^(7/2) -
85*(-b*x + a)^(9/2)*a^6*b/x^(9/2) - 15*(-b*x + a)^(11/2)*a^6/x^(11/2))/(b^
9 - 6*(b*x - a)*b^8/x + 15*(b*x - a)^2*b^7/x^2 - 20*(b*x - a)^3*b^6/x^3 +
15*(b*x - a)^4*b^5/x^4 - 6*(b*x - a)^5*b^4/x^5 + (b*x - a)^6*b^3/x^6)
```

Giac [F(-1)]

Timed out.

$$\int x^{5/2}(a-bx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(a-bx)^{5/2}\,dx=\int x^{5/2}\,{\left(a-b\,x\right)}^{5/2}\,dx$$

input
```
int(x^(5/2)*(a - b*x)^(5/2),x)
```

output
```
int(x^(5/2)*(a - b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a-bx)^{5/2}\,dx=\frac{-15\sqrt{x}\,\sqrt{-bx+a}\,a^5b-10\sqrt{x}\,\sqrt{-bx+a}\,a^4b^2x-8\sqrt{x}\,\sqrt{-bx+a}\,a^3b^3x^2+432\sqrt{x}\,\sqrt{-bx+a}$$
$$153$$

input
```
int(x^(5/2)*(-b*x+a)^(5/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(a - b*x)*a**5*b - 10*sqrt(x)*sqrt(a - b*x)*a**4*b**2*x
 - 8*sqrt(x)*sqrt(a - b*x)*a**3*b**3*x**2 + 432*sqrt(x)*sqrt(a - b*x)*a**2
*b**4*x**3 - 640*sqrt(x)*sqrt(a - b*x)*a*b**5*x**4 + 256*sqrt(x)*sqrt(a -
b*x)*b**6*x**5 - 15*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a
))*a**6*i)/(1536*b**4)
```

3.547 $\int x^{3/2}(a-bx)^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 150

$$\int x^{3/2}(a-bx)^{5/2}\,dx=-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{31}{80}a^2x^{5/2}\sqrt{a-bx}-\frac{21}{40}abx^{7/2}\sqrt{a-bx}+\frac{1}{5}b^2x^{9/2}\sqrt{a-bx}+\frac{3a^5\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}$$

output
```
-3/128*a^4*x^(1/2)*(-b*x+a)^(1/2)/b^2-1/64*a^3*x^(3/2)*(-b*x+a)^(1/2)/b+31
/80*a^2*x^(5/2)*(-b*x+a)^(1/2)-21/40*a*b*x^(7/2)*(-b*x+a)^(1/2)+1/5*b^2*x^
(9/2)*(-b*x+a)^(1/2)+3/128*a^5*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5
/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$\int x^{3/2}(a-bx)^{5/2}\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a-bx}(-15a^4-10a^3bx+248a^2b^2x^2-336ab^3x^3+128b^4x^4)+30a^5\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a}}\right)}{640b^{5/2}}$$

input
```
Integrate[x^(3/2)*(a - b*x)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a - b*x]*(-15*a^4 - 10*a^3*b*x + 248*a^2*b^2*x^2 - 3
36*a*b^3*x^3 + 128*b^4*x^4) + 30*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
Sqrt[a - b*x])])/(640*b^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {60, 60, 60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a-bx)^{5/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\int x^{3/2}(a-bx)^{3/2}dx+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\int x^{3/2}\sqrt{a-bx}dx+\frac{1}{4}x^{5/2}(a-bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a-bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\left(\frac{a\int\frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

$$\Big\downarrow \; 65$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\left(\frac{a\int\frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)+\frac{1}{3}x^{5/2}\sqrt{a-bx}\right)+\frac{1}{4}x^{5/2}(a-bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

input

```
Int[x^(3/2)*(a - b*x)^(5/2),x]
```

output

```
(x^(5/2)*(a - b*x)^(5/2))/5 + (a*((x^(5/2)*(a - b*x)^(3/2))/4 + (3*a*((x^(
5/2)*Sqrt[a - b*x])/3 + (a*(-1/2*(x^(3/2)*Sqrt[a - b*x])/b + (3*a*(-((Sqrt
[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2
)))/(4*b)))/6))/8))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(-128b^4x^4+336a\,x^3b^3-248a^2b^2x^2+10a^3bx+15a^4)\sqrt{x}\,\sqrt{-bx+a}}{640b^2}+\frac{3a^5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)\sqrt{x(-bx+a)}}{256b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{7}{2}}}{5b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{7}{2}}}{4b}+\frac{a\left(\frac{\sqrt{x}\,(-bx+a)^{\frac{5}{2}}}{3}+\frac{5a\left(\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)}{10b}$

input
```
int(x^(3/2)*(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/640*(-128*b^4*x^4+336*a*b^3*x^3-248*a^2*b^2*x^2+10*a^3*b*x+15*a^4)/b^2*
x^(1/2)*(-b*x+a)^(1/2)+3/256*a^5/b^(5/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^
2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

$$\int x^{3/2}(a-bx)^{5/2}\,dx=\left[-\frac{15\,a^5\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(128\,b^5x^4-336\,ab^4x^3+248\,a^2b^3x^2}{1280\,b^3}\right.$$

input
```
integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
 a) - 2*(128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 - 10*a^3*b^2*x - 15
*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b
*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^
2*b^3*x^2 - 10*a^3*b^2*x - 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.41 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.53

$$\int x^{3/2}(a-bx)^{5/2}\,dx=\begin{cases}\frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{-1+\frac{bx}{a}}}-\frac{129ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{-1+\frac{bx}{a}}}+\frac{73ia^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}-\frac{29i\sqrt{a}b^2x^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^5\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{ib^3x}{5\sqrt{a}\sqrt{-}} \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1-\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1-\frac{bx}{a}}}-\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}+\frac{29\sqrt{a}b^2x^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^5\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}}\end{cases}$$

input
```
integrate(x**(3/2)*(-b*x+a)**(5/2),x)
```

output
```
Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x
**(3/2)/(128*b*sqrt(-1 + b*x/a)) - 129*I*a**(5/2)*x**(5/2)/(320*sqrt(-1 +
b*x/a)) + 73*I*a**(3/2)*b*x**(7/2)/(80*sqrt(-1 + b*x/a)) - 29*I*sqrt(a)*b*
*2*x**(9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a)
)/(128*b**(5/2)) + I*b**3*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/
a) > 1), (-3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/
2)/(128*b*sqrt(1 - b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 - b*x/a)) -
 73*a**(3/2)*b*x**(7/2)/(80*sqrt(1 - b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(4
0*sqrt(1 - b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) -
 b**3*x**(11/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38

$$\int x^{3/2}(a-bx)^{5/2}\,dx = -\frac{3\,a^5\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{128\,b^{\frac{5}{2}}} + \frac{\frac{15\,\sqrt{-bx+a}a^5b^4}{\sqrt{x}} + \frac{70\,(-bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}} + \frac{128\,(-bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}} - \frac{70\,(-bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}} - \frac{15\,(-bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^7 - \frac{5\,(bx-a)b^6}{x} + \frac{10\,(bx-a)^2b^5}{x^2} - \frac{10\,(bx-a)^3b^4}{x^3} + \frac{5\,(bx-a)^4b^3}{x^4} - \frac{(bx-a)^5b^2}{x^5}\right)}$$

input
```
integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/640*(15*sq
rt(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) + 128*(
-b*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(
-b*x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x - a)*b^6/x + 10*(b*x - a)^2*b^5
/x^2 - 10*(b*x - a)^3*b^4/x^3 + 5*(b*x - a)^4*b^3/x^4 - (b*x - a)^5*b^2/x^
5)
```

Giac [F(-1)]

Timed out.

$$\int x^{3/2}(a-bx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a-bx)^{5/2}\,dx=\int x^{3/2}\,(a-b\,x)^{5/2}\,dx$$

input
```
int(x^(3/2)*(a - b*x)^(5/2),x)
```

output
```
int(x^(3/2)*(a - b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a-bx)^{5/2}\,dx=\frac{-15\sqrt{x}\,\sqrt{-bx+a}\,a^4b-10\sqrt{x}\,\sqrt{-bx+a}\,a^3b^2x+248\sqrt{x}\,\sqrt{-bx+a}\,a^2b^3x^2-336\sqrt{x}\,\sqrt{-bx}}{640b^3}$$

input
```
int(x^(3/2)*(-b*x+a)^(5/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(a - b*x)*a**4*b - 10*sqrt(x)*sqrt(a - b*x)*a**3*b**2*x
 + 248*sqrt(x)*sqrt(a - b*x)*a**2*b**3*x**2 - 336*sqrt(x)*sqrt(a - b*x)*a*
b**4*x**3 + 128*sqrt(x)*sqrt(a - b*x)*b**5*x**4 - 15*sqrt(b)*log((sqrt(a -
 b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**5*i)/(640*b**3)
```

3.548 $\int \sqrt{x}(a-bx)^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \sqrt{x}(a-bx)^{5/2}\,dx = -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{59}{96}a^2x^{3/2}\sqrt{a-bx}-\frac{17}{24}abx^{5/2}\sqrt{a-bx}+\frac{1}{4}b^2x^{7/2}\sqrt{a-bx}+\frac{5a^4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}$$

output

```
-5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b+59/96*a^2*x^(3/2)*(-b*x+a)^(1/2)-17/24*
a*b*x^(5/2)*(-b*x+a)^(1/2)+1/4*b^2*x^(7/2)*(-b*x+a)^(1/2)+5/64*a^4*arctan(
b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \sqrt{x}(a-bx)^{5/2}\,dx = \frac{\sqrt{x}\sqrt{a-bx}(-15a^3+118a^2bx-136ab^2x^2+48b^3x^3)}{192b}+\frac{5a^4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{32b^{3/2}}$$

input

```
Integrate[Sqrt[x]*(a - b*x)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[a - b*x]*(-15*a^3 + 118*a^2*b*x - 136*a*b^2*x^2 + 48*b^3*x^3
))/(192*b) + (5*a^4*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/
(32*b^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {60, 60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a-bx)^{5/2}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{8}a\int \sqrt{x}(a-bx)^{3/2}dx+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\int \sqrt{x}\sqrt{a-bx}dx+\frac{1}{3}x^{3/2}(a-bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{65}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a-bx}\right)+\frac{1}{3}x^{3/2}(a-bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

input
```
Int[Sqrt[x]*(a - b*x)^(5/2),x]
```

output
```
(x^(3/2)*(a - b*x)^(5/2))/4 + (5*a*((x^(3/2)*(a - b*x)^(3/2))/3 + (a*((x^(
3/2)*Sqrt[a - b*x])/2 + (a*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)))/4))/2))/8
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{\left(-48 b^{3} x^{3}+136 a \,b^{2} x^{2}-118 a^{2} b x +15 a^{3}\right) \sqrt{x}\, \sqrt{-b x +a}}{192 b}+\frac{5 a^{4} \arctan\left(\frac{\sqrt{b}\, \left(x -\frac{a}{2 b}\right)}{\sqrt{-b \,x^{2}+a x}}\right) \sqrt{x \left(-b x +a \right)}}{128 b^{\frac{3}{2}} \sqrt{x}\, \sqrt{-b x +a}}$	102
default	$-\frac{\sqrt{x}\, \left(-b x +a \right)^{\frac{7}{2}}}{4 b}+\frac{a \left(\frac{\sqrt{x}\, \left(-b x +a \right)^{\frac{5}{2}}}{3}+\frac{5 a \left(\frac{\sqrt{x}\, \left(-b x +a \right)^{\frac{3}{2}}}{2}+\frac{3 a \left(\sqrt{x}\, \sqrt{-b x +a}+\frac{a \sqrt{x \left(-b x +a \right)}\, \arctan\left(\frac{\sqrt{b}\, \left(x -\frac{a}{2 b}\right)}{\sqrt{-b \,x^{2}+a x}}\right)}{2 \sqrt{-b x +a}\, \sqrt{x}\, \sqrt{b}}\right)}{4}\right)}{6}\right)}{8 b}$	123

input
```
int(x^(1/2)*(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*b^3*x^3+136*a*b^2*x^2-118*a^2*b*x+15*a^3)/b*x^(1/2)*(-b*x+a)^(
1/2)+5/128/b^(3/2)*a^4*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(
-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\int \sqrt{x}(a-bx)^{5/2}\,dx = \left[-\frac{15\,a^4\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-\right.$$

$$\left.-\frac{15\,a^4\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)-(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\sqrt{x}}{192\,b^2}\right]$$

input
```
integrate(x^(1/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x +
 a)*sqrt(x))/b^2, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqr
t(x)/(b*x - a)) - (48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*
sqrt(-b*x + a)*sqrt(x))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.80 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.61

$$\int \sqrt{x}(a-bx)^{5/2}\,dx = \begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{-1+\frac{bx}{a}}} - \frac{133ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{127ia^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{23i\sqrt{a}b^2x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} + \frac{ib^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1-\frac{bx}{a}}} + \frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{23\sqrt{a}b^2x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}} + \frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} - \frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwis} \end{cases}$$

input
```
integrate(x**(1/2)*(-b*x+a)**(5/2),x)
```

output
```
Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b*sqrt(-1 + b*x/a)) - 133*I*a**(5/2)*x
**(3/2)/(192*sqrt(-1 + b*x/a)) + 127*I*a**(3/2)*b*x**(5/2)/(96*sqrt(-1 + b
*x/a)) - 23*I*sqrt(a)*b**2*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh
(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) + I*b**3*x**(9/2)/(4*sqrt(a)*sqrt(
-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 - b*x/a))
 + 133*a**(5/2)*x**(3/2)/(192*sqrt(1 - b*x/a)) - 127*a**(3/2)*b*x**(5/2)/(
96*sqrt(1 - b*x/a)) + 23*sqrt(a)*b**2*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a*
*4*asin(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) - b**3*x**(9/2)/(4*sqrt(a)*
sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \sqrt{x}(a-bx)^{5/2}\,dx = -\frac{5\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{64\,b^{\frac{3}{2}}} + \frac{\frac{15\,\sqrt{-bx+a}a^4b^3}{\sqrt{x}} + \frac{55\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}} + \frac{73\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}} - \frac{15\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^5 - \frac{4\,(bx-a)b^4}{x} + \frac{6\,(bx-a)^2b^3}{x^2} - \frac{4\,(bx-a)^3b^2}{x^3} + \frac{(bx-a)^4b}{x^4}\right)}$$

input
```
integrate(x^(1/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/192*(15*sqr
t(-b*x + a)*a^4*b^3/sqrt(x) + 55*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) + 73*(-b
*x + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^5 - 4*(b
*x - a)*b^4/x + 6*(b*x - a)^2*b^3/x^2 - 4*(b*x - a)^3*b^2/x^3 + (b*x - a)^
4*b/x^4)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{x}(a-bx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a-bx)^{5/2}\,dx=\int \sqrt{x}\,{\left(a-b\,x\right)}^{5/2}\,dx$$

input
```
int(x^(1/2)*(a - b*x)^(5/2),x)
```

output
```
int(x^(1/2)*(a - b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(a-bx)^{5/2}\,dx=\frac{-15\sqrt{x}\,\sqrt{-bx+a}\,a^3b+118\sqrt{x}\,\sqrt{-bx+a}\,a^2b^2x-136\sqrt{x}\,\sqrt{-bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{-bx+}}{192b^2}$$

input
```
int(x^(1/2)*(-b*x+a)^(5/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(a - b*x)*a**3*b + 118*sqrt(x)*sqrt(a - b*x)*a**2*b**2*
x - 136*sqrt(x)*sqrt(a - b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a - b*x)*b**4*
x**3 - 15*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**4*i)
/(192*b**2)
```

3.549 $\int \frac{(a-bx)^{5/2}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}} \, dx = \frac{11}{8}a^2\sqrt{x}\sqrt{a-bx} - \frac{13}{12}abx^{3/2}\sqrt{a-bx} + \frac{1}{3}b^2x^{5/2}\sqrt{a-bx} + \frac{5a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}$$

output
```
11/8*a^2*x^(1/2)*(-b*x+a)^(1/2)-13/12*a*b*x^(3/2)*(-b*x+a)^(1/2)+1/3*b^2*x
^(5/2)*(-b*x+a)^(1/2)+5/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/
2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}} \, dx = \frac{1}{24}\sqrt{x}\sqrt{a-bx}\left(33a^2-26abx+8b^2x^2\right) + \frac{5a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{4\sqrt{b}}$$

input
```
Integrate[(a - b*x)^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[a - b*x]*(33*a^2 - 26*a*b*x + 8*b^2*x^2))/24 + (5*a^3*ArcTan
[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/(4*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{6}a\int \frac{(a-bx)^{3/2}}{\sqrt{x}}dx+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\int \frac{\sqrt{a-bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

input
```
Int[(a - b*x)^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[x]*(a - b*x)^(5/2))/3 + (5*a*((Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a*(Sq
rt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b])
)/4))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\left(8 b^{2} x^{2}-26 a b x+33 a^{2}\right) \sqrt{x}\, \sqrt{-b x+a}}{24}+\frac{5 a^{3} \arctan\left(\frac{\sqrt{b}\, \left(x-\frac{a}{2 b}\right)}{\sqrt{-b\, x^{2}+a x}}\right) \sqrt{x \left(-b x+a\right)}}{16 \sqrt{b}\, \sqrt{x}\, \sqrt{-b x+a}}$	88
default	$\frac{\sqrt{x}\, \left(-b x+a\right)^{\frac{5}{2}}}{3}+\frac{5 a \left(\frac{\sqrt{x}\, \left(-b x+a\right)^{\frac{3}{2}}}{2}+\frac{3 a \left(\sqrt{x}\, \sqrt{-b x+a}+\frac{a \sqrt{x \left(-b x+a\right)}\, \arctan\left(\frac{\sqrt{b}\, \left(x-\frac{a}{2 b}\right)}{\sqrt{-b\, x^{2}+a x}}\right)}{2 \sqrt{-b x+a}\, \sqrt{x}\, \sqrt{b}}\right)}{4}\right)}{6}$	100

input
```
int((-b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^2*x^2-26*a*b*x+33*a^2)*x^(1/2)*(-b*x+a)^(1/2)+5/16*a^3/b^(1/2)*a
rctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(
-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.51

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = \left[-\frac{15\,a^3\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(8\,b^3x^2-26\,ab^2x+33\,a^2b)\sqrt{}}{48\,b}\right.$$
$$\left.-\frac{15\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)-(8\,b^3x^2-26\,ab^2x+33\,a^2b)\sqrt{-bx+a}\sqrt{x}}{24\,b}\right]$$

input
```
integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(8*b^3*x^2 - 26*a*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/24
*(15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (8*b^3
*x^2 - 26*a*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.46

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = \begin{cases} -\frac{11ia^{\frac{5}{2}}\sqrt{x}}{8\sqrt{-1+\frac{bx}{a}}}+\frac{59ia^{\frac{3}{2}}bx^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{17i\sqrt{a}b^2x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{ib^3x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{8}-\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{12}+\frac{\sqrt{a}b^2x^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{3}+\frac{5a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x+a)**(5/2)/x**(1/2),x)
```

output
```
Piecewise((-11*I*a**(5/2)*sqrt(x)/(8*sqrt(-1 + b*x/a)) + 59*I*a**(3/2)*b*x
**(3/2)/(24*sqrt(-1 + b*x/a)) - 17*I*sqrt(a)*b**2*x**(5/2)/(12*sqrt(-1 + b
*x/a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)) + I*b**3*x**(
7/2)/(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (11*a**(5/2)*sqrt(x)*s
qrt(1 - b*x/a)/8 - 13*a**(3/2)*b*x**(3/2)*sqrt(1 - b*x/a)/12 + sqrt(a)*b**
2*x**(5/2)*sqrt(1 - b*x/a)/3 + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqr
t(b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.30

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = -\frac{5\,a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{8\,\sqrt{b}} + \frac{\frac{15\,\sqrt{-bx+a}a^3b^2}{\sqrt{x}} + \frac{40\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}} + \frac{33\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^3 - \frac{3\,(bx-a)b^2}{x} + \frac{3\,(bx-a)^2b}{x^2} - \frac{(bx-a)^3}{x^3}\right)}$$

input
```
integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/24*(15*sqrt(
-b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 33*(-b*x +
 a)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x - a)*b^2/x + 3*(b*x - a)^2*b/x^2 - (b
*x - a)^3/x^3)
```

Giac [A] (verification not implemented)

Time = 75.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = \frac{\left(\frac{15\,a^3\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} + \sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(2\,(bx-a)\left(\frac{4\,(bx-a)}{b}\right.\right.\right.}{24\,|b|}$$

input
```
integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
1/24*(15*a^3*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/
sqrt(-b) + sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)*(4*(b*x - a
)/b - 5*a/b) + 15*a^2/b))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = \int \frac{(a-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

input
```
int((a - b*x)^(5/2)/x^(1/2),x)
```

output
```
int((a - b*x)^(5/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = \frac{33\sqrt{x}\,\sqrt{-bx+a}\,a^2b - 26\sqrt{x}\,\sqrt{-bx+a}\,a\,b^2x + 8\sqrt{x}\,\sqrt{-bx+a}\,b^3x^2 - 15\sqrt{b}\log\Big(\frac{\sqrt{-bx}}{}}{24b}$$

input
```
int((-b*x+a)^(5/2)/x^(1/2),x)
```

output
```
(33*sqrt(x)*sqrt(a - b*x)*a**2*b - 26*sqrt(x)*sqrt(a - b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a - b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*s
qrt(b)*i)/sqrt(a))*a**3*i)/(24*b)
```

3.550 $\int \frac{(a-bx)^{5/2}}{x^{3/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}} dx = -\frac{2a^2\sqrt{a-bx}}{\sqrt{x}} - \frac{9}{4}ab\sqrt{x}\sqrt{a-bx} + \frac{1}{2}b^2x^{3/2}\sqrt{a-bx} - \frac{15}{4}a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

output
```
-2*a^2*(-b*x+a)^(1/2)/x^(1/2)-9/4*a*b*x^(1/2)*(-b*x+a)^(1/2)+1/2*b^2*x^(3/
2)*(-b*x+a)^(1/2)-15/4*a^2*b^(1/2)*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}} dx = \frac{\sqrt{a-bx}(-8a^2-9abx+2b^2x^2)}{4\sqrt{x}} - \frac{15}{2}a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)$$

input
```
Integrate[(a - b*x)^(5/2)/x^(3/2),x]
```

output
```
(Sqrt[a - b*x]*(-8*a^2 - 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[
b]*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {57, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{57}$$

$$-5b\int \frac{(a-bx)^{3/2}}{\sqrt{x}}dx-\frac{2(a-bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$-5b\left(\frac{3}{4}a\int \frac{\sqrt{a-bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$-5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{65}$$

$$-5b\left(\frac{3}{4}a\left(a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}$$

$$\Big\downarrow \text{216}$$

$$-5b\left(\frac{3}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a-bx}\right)+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}$$

input
```
Int[(a - b*x)^(5/2)/x^(3/2),x]
```

output
```
(-2*(a - b*x)^(5/2))/Sqrt[x] - 5*b*((Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a*(Sq
rt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b])
)/4)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(-2b^2x^2+9abx+8a^2)}{4\sqrt{x}}-\frac{15a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)\sqrt{x(-bx+a)}}{8\sqrt{x}\,\sqrt{-bx+a}}$	88

input
```
int((-b*x+a)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b*x+a)^(1/2)*(-2*b^2*x^2+9*a*b*x+8*a^2)/x^(1/2)-15/8*a^2*b^(1/2)*ar
ctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-
b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx = \left[\frac{15\,a^2\sqrt{-b}x\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(2\,b^2x^2-9\,abx-8\,a^2)\sqrt{-bx+}}{8\,x}\right.$$

input
```
integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*a^2*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) + 2*(2*b^2*x^2 - 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x, 1/4*(15*a^2
*sqrt(b)*x*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) + (2*b^2*x^2 -
 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.72

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx = \begin{cases} \frac{2ia^{\frac{5}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}} + \frac{ia^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}} - \frac{11i\sqrt{a}b^2x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}} + \frac{15ia^2\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4} + \frac{ib^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}} - \frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1-\frac{bx}{a}}} + \frac{11\sqrt{a}b^2x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}} - \frac{15a^2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4} - \frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x+a)**(5/2)/x**(3/2),x)
```

output
```
Piecewise((2*I*a**(5/2)/(sqrt(x)*sqrt(-1 + b*x/a)) + I*a**(3/2)*b*sqrt(x)/
(4*sqrt(-1 + b*x/a)) - 11*I*sqrt(a)*b**2*x**(3/2)/(4*sqrt(-1 + b*x/a)) + 1
5*I*a**2*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/4 + I*b**3*x**(5/2)/(2*sqr
t(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(5/2)/(sqrt(x)*sqrt(1 - b*
x/a)) - a**(3/2)*b*sqrt(x)/(4*sqrt(1 - b*x/a)) + 11*sqrt(a)*b**2*x**(3/2)/
(4*sqrt(1 - b*x/a)) - 15*a**2*sqrt(b)*asin(sqrt(b)*sqrt(x)/sqrt(a))/4 - b*
*3*x**(5/2)/(2*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx = \frac{15}{4}\,a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right) - \frac{2\sqrt{-bx+a}a^2}{\sqrt{x}} - \frac{\frac{7\sqrt{-bx+a}a^2b^2}{\sqrt{x}} + \frac{9(-bx+a)^{\frac{3}{2}}a^2b}{x^{\frac{3}{2}}}}{4\left(b^2 - \frac{2(bx-a)b}{x} + \frac{(bx-a)^2}{x^2}\right)}$$

input
```
integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

output
```
15/4*a^2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x +
a)*a^2/sqrt(x) - 1/4*(7*sqrt(-b*x + a)*a^2*b^2/sqrt(x) + 9*(-b*x + a)^(3/2
)*a^2*b/x^(3/2))/(b^2 - 2*(b*x - a)*b/x + (b*x - a)^2/x^2)
```

Giac [A] (verification not implemented)

Time = 75.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx = -\frac{\left(\frac{15\,a^2\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} - \frac{((2\,bx-7\,a)(bx-a)-15\,a^2)\sqrt{-bx+a}}{\sqrt{(bx-a)b+ab}}\right)b^2}{4\,|b|}$$

input
```
integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

output
```
-1/4*(15*a^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/
sqrt(-b) - ((2*b*x - 7*a)*(b*x - a) - 15*a^2)*sqrt(-b*x + a)/sqrt((b*x - a
)*b + a*b))*b^2/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx = \int \frac{{\left(a-b\,x\right)}^{5/2}}{x^{3/2}}\,dx$$

input
```
int((a - b*x)^(5/2)/x^(3/2),x)
```

output
```
int((a - b*x)^(5/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx = \frac{-8\sqrt{x}\,\sqrt{-bx+a}\,a^2 - 9\sqrt{x}\,\sqrt{-bx+a}\,abx + 2\sqrt{x}\,\sqrt{-bx+a}\,b^2x^2 + 15\sqrt{b}\log\left(\frac{\sqrt{-bx+a+}}{\sqrt{a}}\right.}{4x}$$

input
```
int((-b*x+a)^(5/2)/x^(3/2),x)
```

output
```
( - 8*sqrt(x)*sqrt(a - b*x)*a**2 - 9*sqrt(x)*sqrt(a - b*x)*a*b*x + 2*sqrt(
x)*sqrt(a - b*x)*b**2*x**2 + 15*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(
b)*i)/sqrt(a))*a**2*i*x - 10*sqrt(b)*a**2*i*x)/(4*x)
```

3.551 $\int \frac{(a-bx)^{5/2}}{x^{5/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}} \, dx = -\frac{2a^2\sqrt{a-bx}}{3x^{3/2}} + \frac{14ab\sqrt{a-bx}}{3\sqrt{x}} + b^2\sqrt{x}\sqrt{a-bx} + 5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

output
```
-2/3*a^2*(-b*x+a)^(1/2)/x^(3/2)+14/3*a*b*(-b*x+a)^(1/2)/x^(1/2)+b^2*x^(1/2
)*(-b*x+a)^(1/2)+5*a*b^(3/2)*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}} \, dx = \frac{\sqrt{a-bx}(-2a^2+14abx+3b^2x^2)}{3x^{3/2}} + 10ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)$$

input
```
Integrate[(a - b*x)^(5/2)/x^(5/2),x]
```

output
```
(Sqrt[a - b*x]*(-2*a^2 + 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 10*a*b^(3/2)
*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {57, 57, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{57}$$

$$-\frac{5}{3}b\int \frac{(a-bx)^{3/2}}{x^{3/2}}dx-\frac{2(a-bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$-\frac{5}{3}b\left(-3b\int \frac{\sqrt{a-bx}}{\sqrt{x}}dx-\frac{2(a-bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{5}{3}b\left(-3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx+\sqrt{x}\sqrt{a-bx}\right)-\frac{2(a-bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{65}$$

$$-\frac{5}{3}b\left(-3b\left(a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}+\sqrt{x}\sqrt{a-bx}\right)-\frac{2(a-bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{5}{3}b\left(-3b\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a-bx}\right)-\frac{2(a-bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}$$

input
```
Int[(a - b*x)^(5/2)/x^(5/2),x]
```

output
```
(-2*(a - b*x)^(5/2))/(3*x^(3/2)) - (5*b*((-2*(a - b*x)^(3/2))/Sqrt[x] - 3*
b*(Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqr
t[b])))/3
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(-3b^2x^2-14abx+2a^2)}{3x^{\frac{3}{2}}}+\frac{5a\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)\sqrt{x(-bx+a)}}{2\sqrt{x}\,\sqrt{-bx+a}}$	86

input
```
int((-b*x+a)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x+a)^(1/2)*(-3*b^2*x^2-14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*arct
an(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*
x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.59

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx = \left[\frac{15\,a\sqrt{-b}bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(3\,b^2x^2+14\,abx-2\,a^2)\sqrt{-b}}{6\,x^2}\right.$$

input
```
integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*a*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
 a) + 2*(3*b^2*x^2 + 14*a*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2, -1/3*(
15*a*b^(3/2)*x^2*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (3*b^2
*x^2 + 14*a*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.63

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx = \begin{cases} -\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x} + \frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3} - 5iab^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2} + 5ab^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + \\ -\frac{2ia^2\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x} + \frac{14iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3} + \frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2} - 5iab^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right) + ib^{\frac{5}{2}}x\sqrt{} \end{cases}$$

input
```
integrate((-b*x+a)**(5/2)/x**(5/2),x)
```

output
```
Piecewise((-2*a**2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 14*a*b**(3/2)*sqrt(a/
(b*x) - 1)/3 - 5*I*a*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + 5*I*a*b**(3
/2)*log(a/(b*x))/2 + 5*a*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)) + b**(5/2)
*x*sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (-2*I*a**2*sqrt(b)*sqrt(-a/(b*x)
+ 1)/(3*x) + 14*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + 5*I*a*b**(3/2)*log(a/(
b*x))/2 - 5*I*a*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1) + I*b**(5/2)*x*sqrt(-
a/(b*x) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx = -5\,ab^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right) + \frac{4\sqrt{-bx+a}ab}{\sqrt{x}} + \frac{\sqrt{-bx+a}ab^2}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}} - \frac{2\,(-bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-5*a*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 4*sqrt(-b*x + a)*a
*b/sqrt(x) + sqrt(-b*x + a)*a*b^2/((b - (b*x - a)/x)*sqrt(x)) - 2/3*(-b*x
+ a)^(3/2)*a/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 75.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx = \frac{\left(\frac{15\,ab^2\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}}+\frac{\left(15\,a^2b^3+\left(3\,(bx-a)b^3+20\,ab^3\right)(bx-a)\right)\sqrt{-bx+a}}{((bx-a)b+ab)^{\frac{3}{2}}}\right)b}{3\,|b|}$$

input
```
integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
1/3*(15*a*b^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))
/sqrt(-b) + (15*a^2*b^3 + (3*(b*x - a)*b^3 + 20*a*b^3)*(b*x - a))*sqrt(-b*
x + a)/((b*x - a)*b + a*b)^(3/2))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx = \int \frac{(a-b\,x)^{5/2}}{x^{5/2}}\,dx$$

input
```
int((a - b*x)^(5/2)/x^(5/2),x)
```

output
```
int((a - b*x)^(5/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx = \frac{-4\sqrt{x}\,\sqrt{-bx+a}\,a^2+28\sqrt{x}\,\sqrt{-bx+a}\,abx+6\sqrt{x}\,\sqrt{-bx+a}\,b^2x^2-30\sqrt{b}\log\left(\frac{\sqrt{-bx+a}}{\sqrt{}}\right.}{6x^2}$$

input
```
int((-b*x+a)^(5/2)/x^(5/2),x)
```

output
```
( - 4*sqrt(x)*sqrt(a - b*x)*a**2 + 28*sqrt(x)*sqrt(a - b*x)*a*b*x + 6*sqrt
(x)*sqrt(a - b*x)*b**2*x**2 - 30*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt
(b)*i)/sqrt(a))*a*b*i*x**2 - 5*sqrt(b)*a*b*i*x**2)/(6*x**2)
```

3.552 $\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx=-\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{5a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}$$

output
```
-5/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b^3-5/12*a*x^(3/2)*(-b*x+a)^(1/2)/b^2-1/3*
x^(5/2)*(-b*x+a)^(1/2)/b+5/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^
(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx=-\frac{\sqrt{x}\sqrt{a-bx}(15a^2+10abx+8b^2x^2)}{24b^3}+\frac{5a^3\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{4b^{7/2}}$$

input
```
Integrate[x^(5/2)/Sqrt[a - b*x],x]
```

output
```
-1/24*(Sqrt[x]*Sqrt[a - b*x]*(15*a^2 + 10*a*b*x + 8*b^2*x^2))/b^3 + (5*a^3
*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {60, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5a\int \frac{x^{3/2}}{\sqrt{a-bx}}dx}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{5a\left(\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

$$\Big\downarrow \text{65}$$

$$\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

$$\Big\downarrow \text{216}$$

$$\frac{5a\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{6b}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

input
```
Int[x^(5/2)/Sqrt[a - b*x],x]
```

output
```
-1/3*(x^(5/2)*Sqrt[a - b*x])/b + (5*a*(-1/2*(x^(3/2)*Sqrt[a - b*x])/b + (3
*a*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]])/b^(3/2)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{(8b^2x^2+10abx+15a^2)\sqrt{x}\,\sqrt{-bx+a}}{24b^3}+\frac{5a^3\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)\sqrt{x(-bx+a)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	91
default	$-\frac{x^{\frac{5}{2}}\sqrt{-bx+a}}{3b}+\frac{5a\left(-\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2b}+\frac{3a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^2+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right)}{4b}\right)}{6b}$	116

input
```
int(x^(5/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(8*b^2*x^2+10*a*b*x+15*a^2)/b^3*x^(1/2)*(-b*x+a)^(1/2)+5/16*a^3/b^(7
/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1
/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.43

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx = \left[-\frac{15\,a^3\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(8\,b^3x^2+10\,ab^2x+15\,a^2b\right)\sqrt{-b}}{48\,b^4}\right.$$

$$\left.-\frac{15\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)+\left(8\,b^3x^2+10\,ab^2x+15\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{24\,b^4}\right]$$

input
```
integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) + 2*(8*b^3*x^2 + 10*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/
24*(15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) + (8*b
^3*x^2 + 10*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.57

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx = \begin{cases} \frac{5ia^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{1-\frac{bx}{a}}} + \frac{5a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(5/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((5*I*a**(5/2)*sqrt(x)/(8*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(3/2)*x
**(3/2)/(24*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(5/2)/(12*b*sqrt(-1 + b*
x/a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) - I*x**(7/2)/
(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(5/2)*sqrt(x)/(8*b**
3*sqrt(1 - b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*sqrt(1 - b*x/a)) + sqrt(
a)*x**(5/2)/(12*b*sqrt(1 - b*x/a)) + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/
(8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx = -\frac{5\,a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}} - \frac{\frac{33\sqrt{-bx+a}a^3b^2}{\sqrt{x}} + \frac{40(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}} + \frac{15(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^6 - \frac{3(bx-a)b^5}{x} + \frac{3(bx-a)^2b^4}{x^2} - \frac{(bx-a)^3b^3}{x^3}\right)}$$

input
```
integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) - 1/24*(33*sqrt(
-b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 15*(-b*x +
 a)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x - a)*b^5/x + 3*(b*x - a)^2*b^4/x^2 -
(b*x - a)^3*b^3/x^3)
```

Giac [A] (verification not implemented)

Time = 76.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx = \frac{\left(\frac{15\,a^3\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}b} - \sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(2\,(bx-a)\left(\frac{4\,(bx-a)}{b^2}+\cdots\right.\right.\right.}{24\,b^3}$$

input
```
integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*(15*a^3*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/
(sqrt(-b)*b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)*(4*(b*x
 - a)/b^2 + 13*a/b^2) + 33*a^2/b^2))*abs(b)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx = \int \frac{x^{5/2}}{\sqrt{a-b\,x}}\,dx$$

input
```
int(x^(5/2)/(a - b*x)^(1/2),x)
```

output
```
int(x^(5/2)/(a - b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx = \frac{-15\sqrt{x}\,\sqrt{-bx+a}\,a^2b-10\sqrt{x}\,\sqrt{-bx+a}\,a\,b^2x-8\sqrt{x}\,\sqrt{-bx+a}\,b^3x^2-15\sqrt{b}\log\Bigl(\frac{\sqrt{-bx+}}{}}{24b^4}$$

input
```
int(x^(5/2)/(-b*x+a)^(1/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(a - b*x)*a**2*b - 10*sqrt(x)*sqrt(a - b*x)*a*b**2*x -
8*sqrt(x)*sqrt(a - b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a - b*x) + sqrt(x
)*sqrt(b)*i)/sqrt(a))*a**3*i)/(24*b**4)
```

3.553 $\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx=-\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{3a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}$$

output
```
-3/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^2-1/2*x^(3/2)*(-b*x+a)^(1/2)/b+3/4*a^2*arc
tan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx=-\frac{\sqrt{x}\sqrt{a-bx}(3a+2bx)}{4b^2}+\frac{3a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{2b^{5/2}}$$

input
```
Integrate[x^(3/2)/Sqrt[a - b*x],x]
```

output
```
-1/4*(Sqrt[x]*Sqrt[a - b*x]*(3*a + 2*b*x))/b^2 + (3*a^2*ArcTan[(Sqrt[b]*Sq
rt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}dx$$
$$\Big\downarrow \text{60}$$
$$\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$
$$\Big\downarrow \text{60}$$
$$\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$
$$\Big\downarrow \text{65}$$
$$\frac{3a\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$
$$\Big\downarrow \text{216}$$
$$\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

input
```
Int[x^(3/2)/Sqrt[a - b*x],x]
```

output
```
-1/2*(x^(3/2)*Sqrt[a - b*x])/b + (3*a*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)))/(4*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{(2bx+3a)\sqrt{x}\,\sqrt{-bx+a}}{4b^{2}}+\frac{3a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)\sqrt{x(-bx+a)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	80
default	$-\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2b}+\frac{3a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right)}{4b}$	93

input
```
int(x^(3/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*b*x+3*a)/b^2*x^(1/2)*(-b*x+a)^(1/2)+3/8*a^2/b^(5/2)*arctan(b^(1/2)
*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx = \left[-\frac{3\,a^2\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,(2\,b^2x+3\,ab)\sqrt{-bx+a}\sqrt{x}}{8\,b^3}, -\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)+(2\,b^2x+3\,ab)\sqrt{-bx+a}\sqrt{x}}{4\,b^3}\right]$$

input
```
integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(b)*arc
tan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) + (2*b^2*x + 3*a*b)*sqrt(-b*
x + a)*sqrt(x))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.68

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx = \begin{cases} \frac{3ia^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}x^{\frac{3}{2}}}{4b\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}x^{\frac{3}{2}}}{4b\sqrt{1-\frac{bx}{a}}} + \frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((3*I*a**(3/2)*sqrt(x)/(4*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(
3/2)/(4*b*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b
**(5/2)) - I*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-3*a
**(3/2)*sqrt(x)/(4*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x**(3/2)/(4*b*sqrt(1 -
b*x/a)) + 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/(2*
sqrt(a)*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx = -\frac{3\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}} - \frac{\frac{5\,\sqrt{-bx+a}a^2b}{\sqrt{x}} + \frac{3\,(-bx+a)^{\frac{3}{2}}\,a^2}{x^{\frac{3}{2}}}}{4\left(b^4 - \frac{2\,(bx-a)b^3}{x} + \frac{(bx-a)^2b^2}{x^2}\right)}$$

input
```
integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) - 1/4*(5*sqrt(-b
*x + a)*a^2*b/sqrt(x) + 3*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^4 - 2*(b*x - a)
*b^3/x + (b*x - a)^2*b^2/x^2)
```

Giac [A] (verification not implemented)

Time = 75.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx = \frac{\left(\frac{3\,a^2b\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} - \sqrt{(bx-a)b+ab}(2\,bx+3\,a)\sqrt{-bx+a}\right)|b|}{4\,b^4}$$

input
```
integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*a^2*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/
sqrt(-b) - sqrt((b*x - a)*b + a*b)*(2*b*x + 3*a)*sqrt(-b*x + a))*abs(b)/b^
4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx=\int \frac{x^{3/2}}{\sqrt{a-b\,x}}\,dx$$

input
```
int(x^(3/2)/(a - b*x)^(1/2),x)
```

output
```
int(x^(3/2)/(a - b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx=\frac{-3\sqrt{x}\,\sqrt{-bx+a}\,ab-2\sqrt{x}\,\sqrt{-bx+a}\,b^2x-3\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)a^2i}{4b^3}$$

input
```
int(x^(3/2)/(-b*x+a)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a - b*x)*a*b - 2*sqrt(x)*sqrt(a - b*x)*b**2*x - 3*sqrt(
b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*a**2*i)/(4*b**3)
```

3.554 $\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

output
```
-x^(1/2)*(-b*x+a)^(1/2)/b+a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{2a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/Sqrt[a - b*x],x]
```

output
```
-((Sqrt[x]*Sqrt[a - b*x])/b) + (2*a*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + S
qrt[a - b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

input
```
Int[Sqrt[x]/Sqrt[a - b*x],x]
```

output
```
-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(3/2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	70
risch	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	70

input
```
int(x^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a
)^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx=\left[-\frac{a\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,\sqrt{-bx+a}b\sqrt{x}}{2\,b^{2}},\right.$$
$$\left.-\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)+\sqrt{-bx+a}b\sqrt{x}}{b^{2}}\right]$$

input
```
integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*
sqrt(-b*x + a)*b*sqrt(x))/b^2, -(a*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*s
qrt(x)/(b*x - a)) + sqrt(-b*x + a)*b*sqrt(x))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = \begin{cases} \frac{i\sqrt{a}\sqrt{x}}{b\sqrt{-1+\frac{bx}{a}}} - \frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{ix^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{\sqrt{a}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{b} + \frac{a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((I*sqrt(a)*sqrt(x)/(b*sqrt(-1 + b*x/a)) - I*a*acosh(sqrt(b)*sqrt
(x)/sqrt(a))/b**(3/2) - I*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a)
> 1), (-sqrt(a)*sqrt(x)*sqrt(1 - b*x/a)/b + a*asin(sqrt(b)*sqrt(x)/sqrt(a)
)/b**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = -\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}} - \frac{\sqrt{-bx+a}a}{\left(b^2 - \frac{(bx-a)b}{x}\right)\sqrt{x}}$$

input
```
integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) - sqrt(-b*x + a)*a/((b
^2 - (b*x - a)*b/x)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 75.63 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = \frac{\left(\frac{ab\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}} - \sqrt{(bx-a)b+ab}\sqrt{-bx+a}\right)|b|}{b^3}$$

input
```
integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
(a*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b)
 - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a))*abs(b)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = \frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{b^{3/2}} - \frac{\sqrt{x}\,\sqrt{a-b\,x}}{b}$$

input
```
int(x^(1/2)/(a - b*x)^(1/2),x)
```

output
```
(2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^(1/2))))/b^(3/2) - (x^(1/
2)*(a - b*x)^(1/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = \frac{-\sqrt{x}\,\sqrt{-bx+a}\,b-\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)ai}{b^{2}}$$

input
```
int(x^(1/2)/(-b*x+a)^(1/2),x)
```

output
```
( - (sqrt(x)*sqrt(a - b*x)*b + sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b
)*i)/sqrt(a))*a*i))/b**2
```

3.555 $\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

output
```
2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = \frac{4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

output
```
(4*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx$$

$$\Big\downarrow \; 65$$

$$2\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

output
```
(2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{\sqrt{x}\,\sqrt{-bx+a}\,\sqrt{b}}$	51

input
```
int(1/x^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/
b)/(-b*x^2+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)}{b}, -\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}\sqrt{b}\sqrt{x}}{bx-a}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)/b, -2*arcta
n(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = \begin{cases} -\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{2\,\mathrm{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), Abs(b*x/a) > 1), (
2*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 75.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = \frac{2\,b\log\left(\left|-\sqrt{-bx+a}\sqrt{-b}+\sqrt{(bx-a)b+ab}\right|\right)}{\sqrt{-b}|b|}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/(sqrt(-b)
*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a-b\,x}-\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

input
```
int(1/(x^(1/2)*(a - b*x)^(1/2)),x)
```

output
```
-(4*atan(((a - b*x)^(1/2) - a^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{b}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)i}{b}$$

input
```
int(1/x^(1/2)/(-b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(b)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))*i)/b
```

3.556 $\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

output
```
-2*(-b*x+a)^(1/2)/a/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx$$
$$\Big\downarrow \ 48$$
$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17
default	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17
risch	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17
orering	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17

input
```
int(1/x^(3/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-b*x+a)^(1/2)/a/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx=-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-b*x + a)/(a*sqrt(x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx=\begin{cases}-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a} & \text{for}\ \left|\frac{a}{bx}\right|>1\\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{a} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x**(3/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/a, Abs(a/(b*x)) > 1), (-2*I*sqrt(b
)*sqrt(-a/(b*x) + 1)/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-b*x + a)/(a*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{-bx+a}b^2}{\sqrt{(bx-a)b+ab}a|b|}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-b\,x}}{a\,\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(a - b*x)^(1/2)),x)
```

output
```
-(2*(a - b*x)^(1/2))/(a*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = \frac{-2\sqrt{x}\,\sqrt{-bx+a}-2\sqrt{b}\,ix}{ax}$$

input
```
int(1/x^(3/2)/(-b*x+a)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a - b*x) + sqrt(b)*i*x))/(a*x)
```

3.557 $\int \frac{1}{x^{5/2}\sqrt{a-bx}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}} \, dx = -\frac{2\sqrt{a-bx}}{3ax^{3/2}} - \frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}$$

output
```
-2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}} \, dx = -\frac{2\sqrt{a-bx}(a+2bx)}{3a^2x^{3/2}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2b\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{3a}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

input
```
Int[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3x^{\frac{3}{2}}a^{2}}$	23
risch	$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3x^{\frac{3}{2}}a^{2}}$	23
orering	$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3x^{\frac{3}{2}}a^{2}}$	23
default	$-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}$	35

input
```
int(1/x^(5/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x+a)^(1/2)*(2*b*x+a)/x^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx = -\frac{2\,(2\,bx+a)\sqrt{-bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*b*x + a)*sqrt(-b*x + a)/(a^2*x^(3/2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.85

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx = \begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3ax} - \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a^2} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{2ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} + \frac{2iab^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} - \frac{4ib^{\frac{7}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(5/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*a*x) - 4*b**(3/2)*sqrt(a/(b*x)
- 1)/(3*a**2), Abs(a/(b*x)) > 1), (2*I*a**2*b**(3/2)*sqrt(-a/(b*x) + 1)/(-
3*a**3*b*x + 3*a**2*b**2*x**2) + 2*I*a*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-3*a
**3*b*x + 3*a**2*b**2*x**2) - 4*I*b**(7/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a**
3*b*x + 3*a**2*b**2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx = -\frac{2\left(\frac{3\sqrt{-bx+a}b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(3*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx = -\frac{2\left(\frac{2\,(bx-a)b^3}{a^2}+\frac{3\,b^3}{a}\right)\sqrt{-bx+a}b}{3\,((bx-a)b+ab)^{\frac{3}{2}}|b|}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(2*(b*x - a)*b^3/a^2 + 3*b^3/a)*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)
^(3/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx = -\frac{\left(\frac{2}{3\,a}+\frac{4\,b\,x}{3\,a^2}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

input
```
int(1/(x^(5/2)*(a - b*x)^(1/2)),x)
```

output
```
-((2/(3*a) + (4*b*x)/(3*a^2))*(a - b*x)^(1/2))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a}{3}-\frac{4\sqrt{x}\,\sqrt{-bx+a}\,bx}{3}+\frac{4\sqrt{b}\,bi\,x^2}{3}}{a^2x^2}$$

input
```
int(1/x^(5/2)/(-b*x+a)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a - b*x)*a - 2*sqrt(x)*sqrt(a - b*x)*b*x + 2*sqrt(b)*b
*i*x**2))/(3*a**2*x**2)
```

3.558 $\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{5ax^{5/2}} - \frac{8b\sqrt{a-bx}}{15a^2x^{3/2}} - \frac{16b^2\sqrt{a-bx}}{15a^3\sqrt{x}}$$

output
```
-2/5*(-b*x+a)^(1/2)/a/x^(5/2)-8/15*b*(-b*x+a)^(1/2)/a^2/x^(3/2)-16/15*b^2*
(-b*x+a)^(1/2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}(3a^2+4abx+8b^2x^2)}{15a^3x^{5/2}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x]*(3*a^2 + 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{4b\int \frac{1}{x^{5/2}\sqrt{a-bx}}dx}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4b\left(\frac{2b\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{3a}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{4b\left(-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x])/(5*a*x^(5/2)) + (4*b*((-2*Sqrt[a - b*x])/(3*a*x^(3/2))
- (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}\,(8b^{2}x^{2}+4abx+3a^{2})}{15x^{\frac{5}{2}}a^{3}}$	36
risch	$-\frac{2\sqrt{-bx+a}\,(8b^{2}x^{2}+4abx+3a^{2})}{15x^{\frac{5}{2}}a^{3}}$	36
orering	$-\frac{2\sqrt{-bx+a}\,(8b^{2}x^{2}+4abx+3a^{2})}{15x^{\frac{5}{2}}a^{3}}$	36
default	$-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}$	58

input
```
int(1/x^(7/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-b*x+a)^(1/2)*(8*b^2*x^2+4*a*b*x+3*a^2)/x^(5/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = -\frac{2\,(8\,b^2x^2+4\,abx+3\,a^2)\sqrt{-bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(8*b^2*x^2 + 4*a*b*x + 3*a^2)*sqrt(-b*x + a)/(a^3*x^(5/2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 586, normalized size of antiderivative = 8.25

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = \begin{cases} -\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{15a^5b^4x^2-30a^4b^5x^3+15a^3b^6x^4} + \frac{4a^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{15a^5b^4x^2-30a^4b^5x^3+15a^3b^6x^4} - \frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{15a^5b^4x^2-30a^4b^5x^3+15a^3b^6x^4} + \\ -\frac{6ia^4b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{15a^5b^4x^2-30a^4b^5x^3+15a^3b^6x^4} + \frac{4ia^3b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{15a^5b^4x^2-30a^4b^5x^3+15a^3b^6x^4} - \frac{6ia^2b^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{15a^5b^4x^2-30a^4b^5x^3+15a^3b^6x^4} + \end{cases}$$

input
```
integrate(1/x**(7/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((-6*a**4*b**(9/2)*sqrt(a/(b*x) - 1)/(15*a**5*b**4*x**2 - 30*a**4
*b**5*x**3 + 15*a**3*b**6*x**4) + 4*a**3*b**(11/2)*x*sqrt(a/(b*x) - 1)/(15
*a**5*b**4*x**2 - 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*a**2*b**(13/2
)*x**2*sqrt(a/(b*x) - 1)/(15*a**5*b**4*x**2 - 30*a**4*b**5*x**3 + 15*a**3*
b**6*x**4) + 24*a*b**(15/2)*x**3*sqrt(a/(b*x) - 1)/(15*a**5*b**4*x**2 - 30
*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 16*b**(17/2)*x**4*sqrt(a/(b*x) - 1)
/(15*a**5*b**4*x**2 - 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4), Abs(a/(b*x))
 > 1), (-6*I*a**4*b**(9/2)*sqrt(-a/(b*x) + 1)/(15*a**5*b**4*x**2 - 30*a**4
*b**5*x**3 + 15*a**3*b**6*x**4) + 4*I*a**3*b**(11/2)*x*sqrt(-a/(b*x) + 1)/
(15*a**5*b**4*x**2 - 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*I*a**2*b**
(13/2)*x**2*sqrt(-a/(b*x) + 1)/(15*a**5*b**4*x**2 - 30*a**4*b**5*x**3 + 15
*a**3*b**6*x**4) + 24*I*a*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(15*a**5*b**4*
x**2 - 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 16*I*b**(17/2)*x**4*sqrt(-
a/(b*x) + 1)/(15*a**5*b**4*x**2 - 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = -\frac{2\left(\frac{15\sqrt{-bx+a}b^2}{\sqrt{x}} + \frac{10\,(-bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} + \frac{3\,(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^3}$$

input
```
integrate(1/x^(7/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(15*sqrt(-b*x + a)*b^2/sqrt(x) + 10*(-b*x + a)^(3/2)*b/x^(3/2) + 3*(
-b*x + a)^(5/2)/x^(5/2))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = -\frac{2\left(\frac{15\,b^5}{a}+4\left(\frac{2\,(bx-a)b^5}{a^3}+\frac{5\,b^5}{a^2}\right)(bx-a)\right)\sqrt{-bx+a}b}{15\left((bx-a)b+ab\right)^{\frac{5}{2}}|b|}$$

input
```
integrate(1/x^(7/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(15*b^5/a + 4*(2*(b*x - a)*b^5/a^3 + 5*b^5/a^2)*(b*x - a))*sqrt(-b*x
 + a)*b/(((b*x - a)*b + a*b)^(5/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = -\frac{\sqrt{a-b\,x}\left(\frac{2}{5\,a}+\frac{16\,b^2\,x^2}{15\,a^3}+\frac{8\,b\,x}{15\,a^2}\right)}{x^{5/2}}$$

input
```
int(1/(x^(7/2)*(a - b*x)^(1/2)),x)
```

output
```
-((a - b*x)^(1/2)*(2/(5*a) + (16*b^2*x^2)/(15*a^3) + (8*b*x)/(15*a^2)))/x^
(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{7/2}\sqrt{a-bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a^2}{5}-\frac{8\sqrt{x}\,\sqrt{-bx+a}\,abx}{15}-\frac{16\sqrt{x}\,\sqrt{-bx+a}\,b^2x^2}{15}+\frac{16\sqrt{b}\,b^2i\,x^3}{15}}{a^3x^3}$$

input
```
int(1/x^(7/2)/(-b*x+a)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a - b*x)*a**2 - 4*sqrt(x)*sqrt(a - b*x)*a*b*x - 8*sq
rt(x)*sqrt(a - b*x)*b**2*x**2 + 8*sqrt(b)*b**2*i*x**3))/(15*a**3*x**3)
```

3.559 $\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{7ax^{7/2}} - \frac{12b\sqrt{a-bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a-bx}}{35a^3x^{3/2}} - \frac{32b^3\sqrt{a-bx}}{35a^4\sqrt{x}}$$

output
```
-2/7*(-b*x+a)^(1/2)/a/x^(7/2)-12/35*b*(-b*x+a)^(1/2)/a^2/x^(5/2)-16/35*b^2
*(-b*x+a)^(1/2)/a^3/x^(3/2)-32/35*b^3*(-b*x+a)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}(5a^3+6a^2bx+8ab^2x^2+16b^3x^3)}{35a^4x^{7/2}}$$

input
```
Integrate[1/(x^(9/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x]*(5*a^3 + 6*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(35*a^4*
x^(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{6b\int \frac{1}{x^{7/2}\sqrt{a-bx}}dx}{7a}-\frac{2\sqrt{a-bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{6b\left(\frac{4b\int \frac{1}{x^{5/2}\sqrt{a-bx}}dx}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a-bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{6b\left(\frac{4b\left(\frac{2b\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{3a}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a-bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{6b\left(\frac{4b\left(-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a-bx}}{7ax^{7/2}}$$

input
```
Int[1/(x^(9/2)*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[a - b*x])/(7*a*x^(7/2)) + (6*b*((-2*Sqrt[a - b*x])/(5*a*x^(5/2))
+ (4*b*((-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt
[x])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}\,(16b^3x^3+8a\,b^2x^2+6a^2bx+5a^3)}{35x^{\frac{7}{2}}\,a^4}$	47
risch	$-\frac{2\sqrt{-bx+a}\,(16b^3x^3+8a\,b^2x^2+6a^2bx+5a^3)}{35x^{\frac{7}{2}}\,a^4}$	47
orering	$-\frac{2\sqrt{-bx+a}\,(16b^3x^3+8a\,b^2x^2+6a^2bx+5a^3)}{35x^{\frac{7}{2}}\,a^4}$	47
default	$-\frac{2\sqrt{-bx+a}}{7a\,x^{\frac{7}{2}}}+\frac{6b\left(-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}$	81

input
```
int(1/x^(9/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-b*x+a)^(1/2)*(16*b^3*x^3+8*a*b^2*x^2+6*a^2*b*x+5*a^3)/x^(7/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = -\frac{2\,(16\,b^3x^3+8\,ab^2x^2+6\,a^2bx+5\,a^3)\sqrt{-bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(9/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/35*(16*b^3*x^3 + 8*a*b^2*x^2 + 6*a^2*b*x + 5*a^3)*sqrt(-b*x + a)/(a^4*x
^(7/2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.34 (sec) , antiderivative size = 994, normalized size of antiderivative = 10.35

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(9/2)/(-b*x+a)**(1/2),x)
```

output
```
Piecewise((10*a**6*b**(19/2)*sqrt(a/(b*x) - 1)/(-35*a**7*b**9*x**3 + 105*a
**6*b**10*x**4 - 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*a**5*b**(2
1/2)*x*sqrt(a/(b*x) - 1)/(-35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 - 105*a
**5*b**11*x**5 + 35*a**4*b**12*x**6) + 10*a**4*b**(23/2)*x**2*sqrt(a/(b*x)
 - 1)/(-35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 - 105*a**5*b**11*x**5 + 35
*a**4*b**12*x**6) + 10*a**3*b**(25/2)*x**3*sqrt(a/(b*x) - 1)/(-35*a**7*b**
9*x**3 + 105*a**6*b**10*x**4 - 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) -
 60*a**2*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-35*a**7*b**9*x**3 + 105*a**6*b
**10*x**4 - 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 80*a*b**(29/2)*x**
5*sqrt(a/(b*x) - 1)/(-35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 - 105*a**5*b
**11*x**5 + 35*a**4*b**12*x**6) - 32*b**(31/2)*x**6*sqrt(a/(b*x) - 1)/(-35
*a**7*b**9*x**3 + 105*a**6*b**10*x**4 - 105*a**5*b**11*x**5 + 35*a**4*b**1
2*x**6), Abs(a/(b*x)) > 1), (10*I*a**6*b**(19/2)*sqrt(-a/(b*x) + 1)/(-35*a
**7*b**9*x**3 + 105*a**6*b**10*x**4 - 105*a**5*b**11*x**5 + 35*a**4*b**12*
x**6) - 18*I*a**5*b**(21/2)*x*sqrt(-a/(b*x) + 1)/(-35*a**7*b**9*x**3 + 105
*a**6*b**10*x**4 - 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 10*I*a**4*b
**(23/2)*x**2*sqrt(-a/(b*x) + 1)/(-35*a**7*b**9*x**3 + 105*a**6*b**10*x**4
 - 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 10*I*a**3*b**(25/2)*x**3*sq
rt(-a/(b*x) + 1)/(-35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 - 105*a**5*b**1
1*x**5 + 35*a**4*b**12*x**6) - 60*I*a**2*b**(27/2)*x**4*sqrt(-a/(b*x) +...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = -\frac{2\left(\frac{35\sqrt{-bx+a}b^3}{\sqrt{x}} + \frac{35(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}} + \frac{21(-bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}} + \frac{5(-bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^4}$$

input
```
integrate(1/x^(9/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/35*(35*sqrt(-b*x + a)*b^3/sqrt(x) + 35*(-b*x + a)^(3/2)*b^2/x^(3/2) + 2
1*(-b*x + a)^(5/2)*b/x^(5/2) + 5*(-b*x + a)^(7/2)/x^(7/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = -\frac{2\left(\frac{35\,b^7}{a}+2\left(\frac{35\,b^7}{a^2}+4\left(\frac{2\,(bx-a)b^7}{a^4}+\frac{7\,b^7}{a^3}\right)(bx-a)\right)(bx-a)\right)\sqrt{-bx+a}b}{35\left((bx-a)b+ab\right)^{\frac{7}{2}}|b|}$$

input
```
integrate(1/x^(9/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/35*(35*b^7/a + 2*(35*b^7/a^2 + 4*(2*(b*x - a)*b^7/a^4 + 7*b^7/a^3)*(b*x
 - a))*(b*x - a))*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = -\frac{\sqrt{a-b\,x}\left(\frac{2}{7\,a}+\frac{16\,b^2\,x^2}{35\,a^3}+\frac{32\,b^3\,x^3}{35\,a^4}+\frac{12\,b\,x}{35\,a^2}\right)}{x^{7/2}}$$

input
```
int(1/(x^(9/2)*(a - b*x)^(1/2)),x)
```

output
```
-((a - b*x)^(1/2)*(2/(7*a) + (16*b^2*x^2)/(35*a^3) + (32*b^3*x^3)/(35*a^4)
 + (12*b*x)/(35*a^2)))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{9/2}\sqrt{a-bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{-bx+a}\,a^3}{7}-\frac{12\sqrt{x}\,\sqrt{-bx+a}\,a^2bx}{35}-\frac{16\sqrt{x}\,\sqrt{-bx+a}\,a\,b^2x^2}{35}-\frac{32\sqrt{x}\,\sqrt{-bx+a}\,b^3x^3}{35}+\frac{32\sqrt{b}\,b^3i\,x^4}{35}}{a^4x^4}$$

input
```
int(1/x^(9/2)/(-b*x+a)^(1/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a - b*x)*a**3 - 6*sqrt(x)*sqrt(a - b*x)*a**2*b*x - 8
*sqrt(x)*sqrt(a - b*x)*a*b**2*x**2 - 16*sqrt(x)*sqrt(a - b*x)*b**3*x**3 +
16*sqrt(b)*b**3*i*x**4))/(35*a**4*x**4)
```

3.560 $\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx=\frac{2a^2\sqrt{x}}{b^3\sqrt{a-bx}}+\frac{7a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{15a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}$$

output
```
2*a^2*x^(1/2)/b^3/(-b*x+a)^(1/2)+7/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^3+1/2*x^(3
/2)*(-b*x+a)^(1/2)/b^2-15/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(
7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx=-\frac{\sqrt{x}(-15a^2+5abx+2b^2x^2)}{4b^3\sqrt{a-bx}}+\frac{15a^2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a-bx}}\right)}{2b^{7/2}}$$

input
```
Integrate[x^(5/2)/(a - b*x)^(3/2),x]
```

output
```
-1/4*(Sqrt[x]*(-15*a^2 + 5*a*b*x + 2*b^2*x^2))/(b^3*Sqrt[a - b*x]) + (15*a
^2*ArcTan[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a - b*x])])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {57, 60, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx$$

$$\Big\downarrow \ 57$$

$$\frac{2x^{5/2}}{b\sqrt{a-bx}}-\frac{5\int \frac{x^{3/2}}{\sqrt{a-bx}}dx}{b}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{5/2}}{b\sqrt{a-bx}}-\frac{5\left(\frac{3a\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{b}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{5/2}}{b\sqrt{a-bx}}-\frac{5\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{b}$$

$$\Big\downarrow \ 65$$

$$\frac{2x^{5/2}}{b\sqrt{a-bx}}-\frac{5\left(\frac{3a\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{b}$$

$$\Big\downarrow \ 216$$

$$\frac{2x^{5/2}}{b\sqrt{a-bx}} - \frac{5\left(\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{4b}-\frac{x^{3/2}\sqrt{a-bx}}{2b}\right)}{b}$$

input
```
Int[x^(5/2)/(a - b*x)^(3/2),x]
```

output
```
(2*x^(5/2))/(b*Sqrt[a - b*x]) - (5*(-1/2*(x^(3/2)*Sqrt[a - b*x])/b + (3*a*
(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
)/b^(3/2)))/(4*b)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

method	result	size
risch	$\frac{(2bx+7a)\sqrt{x}\,\sqrt{-bx+a}}{4b^{3}}+\frac{\left(-\frac{15a^{2}\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{8b^{\frac{7}{2}}}-\frac{2a^{2}\sqrt{-b\left(x-\frac{a}{b}\right)^{2}-\left(x-\frac{a}{b}\right)a}}{b^{4}\left(x-\frac{a}{b}\right)}\right)\sqrt{x(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	127

input
```
int(x^(5/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*x+7*a)/b^3*x^(1/2)*(-b*x+a)^(1/2)+(-15/8*a^2/b^(7/2)*arctan(b^(1/
2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))-2*a^2/b^4/(x-a/b)*(-b*(x-a/b)^2-(x-a/b)
*a)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.84

$$\int\frac{x^{5/2}}{(a-bx)^{3/2}}\,dx=\left[-\frac{15\,(a^{2}bx-a^{3})\sqrt{-b}\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(2\,b^{3}x^{2}+5\,ab^{2}x-1}{8\,(b^{5}x-ab^{4})}\right.$$

input
```
integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(15*(a^2*b*x - a^3)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*
sqrt(x) + a) - 2*(2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x)
)/(b^5*x - a*b^4), 1/4*(15*(a^2*b*x - a^3)*sqrt(b)*arctan(sqrt(-b*x + a)*s
qrt(b)*sqrt(x)/(b*x - a)) + (2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x +
 a)*sqrt(x))/(b^5*x - a*b^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.17

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx = \begin{cases} -\frac{15ia^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{-1+\frac{bx}{a}}} + \frac{5i\sqrt{a}x^{\frac{3}{2}}}{4b^2\sqrt{-1+\frac{bx}{a}}} + \frac{15ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} + \frac{ix^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1-\frac{bx}{a}}} - \frac{5\sqrt{a}x^{\frac{3}{2}}}{4b^2\sqrt{1-\frac{bx}{a}}} - \frac{15a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} - \frac{x^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(5/2)/(-b*x+a)**(3/2),x)
```

output
```
Piecewise((-15*I*a**(3/2)*sqrt(x)/(4*b**3*sqrt(-1 + b*x/a)) + 5*I*sqrt(a)*
x**(3/2)/(4*b**2*sqrt(-1 + b*x/a)) + 15*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(
a))/(4*b**(7/2)) + I*x**(5/2)/(2*sqrt(a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) >
 1), (15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 - b*x/a)) - 5*sqrt(a)*x**(3/2)/(4
*b**2*sqrt(1 - b*x/a)) - 15*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)
) - x**(5/2)/(2*sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx = \frac{8\,a^2b^2 - \frac{25\,(bx-a)a^2b}{x} + \frac{15\,(bx-a)^2a^2}{x^2}}{4\left(\frac{\sqrt{-bx+a}b^5}{\sqrt{x}} + \frac{2\,(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)} + \frac{15\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(8*a^2*b^2 - 25*(b*x - a)*a^2*b/x + 15*(b*x - a)^2*a^2/x^2)/(sqrt(-b*x
 + a)*b^5/sqrt(x) + 2*(-b*x + a)^(3/2)*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/
x^(5/2)) + 15/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx = \frac{\left(2\sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(\frac{2(bx-a)}{b^3}+\frac{9a}{b^3}\right)+\frac{32a^3}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}b}-\right.}{8b^2}$$

input
```
integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*(2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)/b^3 + 9*a/b^3)
+ 32*a^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sq
rt(-b)*b) - 15*a^2*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))
^2)/(sqrt(-b)*b^2))*abs(b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx = \int \frac{x^{5/2}}{(a-b\,x)^{3/2}}\,dx$$

input
```
int(x^(5/2)/(a - b*x)^(3/2),x)
```

output
```
int(x^(5/2)/(a - b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx = \frac{15\sqrt{b}\sqrt{-bx+a}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\sqrt{b}\,i}{\sqrt{a}}\right)a^2i-10\sqrt{b}\sqrt{-bx+a}\,a^2i+15\sqrt{x}\,a^2b-5\sqrt{x}\,a\,b^2}{4\sqrt{-bx+a}\,b^4}$$

input
```
int(x^(5/2)/(-b*x+a)^(3/2),x)
```

output
```
(15*sqrt(b)*sqrt(a - b*x)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))
*a**2*i - 10*sqrt(b)*sqrt(a - b*x)*a**2*i + 15*sqrt(x)*a**2*b - 5*sqrt(x)*
a*b**2*x - 2*sqrt(x)*b**3*x**2)/(4*sqrt(a - b*x)*b**4)
```

3.561 $\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx=\frac{2a\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{3a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

output
```
2*a*x^(1/2)/b^2/(-b*x+a)^(1/2)+x^(1/2)*(-b*x+a)^(1/2)/b^2-3*a*arctan(b^(1/
2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx=\frac{\sqrt{x}(3a-bx)}{b^2\sqrt{a-bx}}+\frac{6a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a-bx}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(a - b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(3*a - b*x))/(b^2*Sqrt[a - b*x]) + (6*a*ArcTan[(Sqrt[b]*Sqrt[x])/
(Sqrt[a] - Sqrt[a - b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {57, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{b}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{b}$$

$$\Big\downarrow \text{65}$$

$$\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{b}$$

input
```
Int[x^(3/2)/(a - b*x)^(3/2),x]
```

output
```
(2*x^(3/2))/(b*Sqrt[a - b*x]) - (3*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

method	result	size
risch	$\frac{\sqrt{x}\,\sqrt{-bx+a}}{b^{2}}+\frac{\left(-\frac{3a\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2b^{\frac{5}{2}}}-\frac{2a\sqrt{-b\left(x-\frac{a}{b}\right)^{2}-\left(x-\frac{a}{b}\right)a}}{b^{3}\left(x-\frac{a}{b}\right)}\right)\sqrt{x(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	114

input
```
int(x^(3/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)*(-b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*
x^2+a*x)^(1/2))-2*a/b^3/(x-a/b)*(-b*(x-a/b)^2-(x-a/b)*a)^(1/2))*(x*(-b*x+a
))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.27

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx = \left[-\frac{3\,(abx-a^2)\sqrt{-b}\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(b^2x-3\,ab)\sqrt{-bx+a}\right.$$

input
```
integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(3*(a*b*x - a^2)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqr
t(x) + a) - 2*(b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3), (3*
(a*b*x - a^2)*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) + (
b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.18

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx = \begin{cases} -\frac{3i\sqrt{a}\sqrt{x}}{b^2\sqrt{-1+\frac{bx}{a}}} + \frac{3ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} + \frac{ix^{\frac{3}{2}}}{\sqrt{a}b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1-\frac{bx}{a}}} - \frac{3a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} - \frac{x^{\frac{3}{2}}}{\sqrt{a}b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(-b*x+a)**(3/2),x)
```

output
```
Piecewise((-3*I*sqrt(a)*sqrt(x)/(b**2*sqrt(-1 + b*x/a)) + 3*I*a*acosh(sqrt
(b)*sqrt(x)/sqrt(a))/b**(5/2) + I*x**(3/2)/(sqrt(a)*b*sqrt(-1 + b*x/a)), A
bs(b*x/a) > 1), (3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 - b*x/a)) - 3*a*asin(sqrt(
b)*sqrt(x)/sqrt(a))/b**(5/2) - x**(3/2)/(sqrt(a)*b*sqrt(1 - b*x/a)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx = \frac{2\,ab - \frac{3\,(bx-a)a}{x}}{\frac{\sqrt{-bx+a}b^3}{\sqrt{x}} + \frac{(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}} + \frac{3\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
(2*a*b - 3*(b*x - a)*a/x)/(sqrt(-b*x + a)*b^3/sqrt(x) + (-b*x + a)^(3/2)*b
^2/x^(3/2)) + 3*a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(55) = 110.

Time = 15.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.83

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx =$$

$$-\frac{\left(\frac{8\,a^2\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab} + \frac{3\,a\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}} - \frac{2\,\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b}\right)|b|}{2\,b^3}$$

input
```
integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(8*a^2*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^
2 - a*b) + 3*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/
sqrt(-b) - 2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b)*abs(b)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx=\int \frac{x^{3/2}}{(a-b\,x)^{3/2}}\,dx$$

input
```
int(x^(3/2)/(a - b*x)^(3/2),x)
```

output
```
int(x^(3/2)/(a - b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx=\frac{12\sqrt{b}\,\sqrt{-bx+a}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)ai-9\sqrt{b}\,\sqrt{-bx+a}\,ai+12\sqrt{x}\,ab-4\sqrt{x}\,b^2x}{4\sqrt{-bx+a}\,b^3}$$

input
```
int(x^(3/2)/(-b*x+a)^(3/2),x)
```

output
```
(12*sqrt(b)*sqrt(a - b*x)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))
*a*i - 9*sqrt(b)*sqrt(a - b*x)*a*i + 12*sqrt(x)*a*b - 4*sqrt(x)*b**2*x)/(4
*sqrt(a - b*x)*b**3)
```

3.562 $\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

output
```
2*x^(1/2)/b/(-b*x+a)^(1/2)-2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2
)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{a-bx}} + \frac{4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a-bx}}\right)}{b^{3/2}}$$

input
```
Integrate[Sqrt[x]/(a - b*x)^(3/2),x]
```

output
```
(2*Sqrt[x])/(b*Sqrt[a - b*x]) + (4*ArcTan[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqr
t[a - b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {57, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

input
```
Int[Sqrt[x]/(a - b*x)^(3/2),x]
```

output
```
(2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
)/b^(3/2)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [F]

$$\int \frac{\sqrt{x}}{(-bx+a)^{\frac{3}{2}}}\,dx$$

input
```
int(x^(1/2)/(-b*x+a)^(3/2),x)
```

output
```
int(x^(1/2)/(-b*x+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \left[-\frac{(bx-a)\sqrt{-b}\log\left(-2\,bx-2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\,\sqrt{-bx+a}b\sqrt{x}}{b^3x-ab^2}, \frac{2\left((bx-\right.}{}\right.$$

input
```
integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-((b*x - a)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2), 2*((b*x - a)*sqrt(b)*arctan
(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - sqrt(-b*x + a)*b*sqrt(x))/(b^
3*x - a*b^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{2i\sqrt{x}}{\sqrt{a}b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{2\,\mathrm{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} + \frac{2\sqrt{x}}{\sqrt{a}b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(-b*x+a)**(3/2),x)
```

output
```
Piecewise((2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*I*sqrt(x)/(sqrt
(a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*asin(sqrt(b)*sqrt(x)/sqrt(a)
)/b**(3/2) + 2*sqrt(x)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}} + \frac{2\,\sqrt{x}}{\sqrt{-bx+a}b}$$

input
```
integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x
+ a)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(38) = 76$.

Time = 15.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = -\frac{\left(\frac{4\,a\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}+\frac{\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}}\right)|b|}{b^2}$$

input
```
integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-(4*a*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*
b) + log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b))*
abs(b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(a-b\,x)^{3/2}}\,dx$$

input
```
int(x^(1/2)/(a - b*x)^(3/2),x)
```

output
```
int(x^(1/2)/(a - b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{-bx+a}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)i-2\sqrt{b}\,\sqrt{-bx+a}\,i+2\sqrt{x}\,b}{\sqrt{-bx+a}\,b^2}$$

input
```
int(x^(1/2)/(-b*x+a)^(3/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a - b*x)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(a))
*i - sqrt(b)*sqrt(a - b*x)*i + sqrt(x)*b))/(sqrt(a - b*x)*b**2)
```

3.563 $\int \frac{1}{\sqrt{x}(a-bx)^{3/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}} dx = \frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

output
```
2*x^(1/2)/a/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}} dx = \frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

input
```
Integrate[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

output
```
(2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}} \, dx$$
$$\Big\downarrow \text{48}$$
$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

input
```
Int[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

output
```
(2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{2\sqrt{x}}{a\sqrt{-bx+a}}$	17
default	$\frac{2\sqrt{x}}{a\sqrt{-bx+a}}$	17
orering	$\frac{2\sqrt{x}}{a\sqrt{-bx+a}}$	17

input
```
int(1/x^(1/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/a/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = -\frac{2\sqrt{-bx+a}\sqrt{x}}{abx-a^2}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-b*x + a)*sqrt(x)/(a*b*x - a^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = \begin{cases} \frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ -\frac{2i}{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(-b*x+a)**(3/2),x)
```

output
```
Piecewise((2/(a*sqrt(b)*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-2*I/(a*sq
rt(b)*sqrt(-a/(b*x) + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{\sqrt{-bx+a}a}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)/(sqrt(-b*x + a)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = -\frac{4\sqrt{-b}b}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)|b|}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-4*sqrt(-b)*b/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*
b)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}\sqrt{a-b\,x}}{a^2-a\,b\,x}$$

input
```
int(1/(x^(1/2)*(a - b*x)^(3/2)),x)
```

output
```
(2*x^(1/2)*(a - b*x)^(1/2))/(a^2 - a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = \frac{-2\sqrt{b}\,\sqrt{-bx+a}\,i+2\sqrt{x}\,b}{\sqrt{-bx+a}\,ab}$$

input
```
int(1/x^(1/2)/(-b*x+a)^(3/2),x)
```

output
```
(2*( - sqrt(b)*sqrt(a - b*x)*i + sqrt(x)*b))/(sqrt(a - b*x)*a*b)
```

3.564 $\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx=\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

output
```
2/a/x^(1/2)/(-b*x+a)^(1/2)-4*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx=-\frac{2(a-2bx)}{a^2\sqrt{x}\sqrt{a-bx}}$$

input
```
Integrate[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

output
```
(-2*(a - 2*b*x))/(a^2*Sqrt[x]*Sqrt[a - b*x])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx$$
$$\Big\downarrow \text{55}$$
$$\frac{2\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{a}+\frac{2}{a\sqrt{x}\sqrt{a-bx}}$$
$$\Big\downarrow \text{48}$$
$$\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

output
```
2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqrt[a - b*x])/(a^2*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{2(-2bx+a)}{\sqrt{x}\,\sqrt{-bx+a}\,a^{2}}$	23
orering	$-\frac{2(-2bx+a)}{\sqrt{x}\,\sqrt{-bx+a}\,a^{2}}$	23
default	$-\frac{2}{a\sqrt{x}\,\sqrt{-bx+a}}+\frac{4b\sqrt{x}}{a^{2}\sqrt{-bx+a}}$	35
risch	$-\frac{2\sqrt{-bx+a}}{a^{2}\sqrt{x}}+\frac{2b\sqrt{x}}{a^{2}\sqrt{-bx+a}}$	35

input
```
int(1/x^(3/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*b*x+a)/x^(1/2)/(-b*x+a)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx = -\frac{2\,(2\,bx-a)\sqrt{-bx+a}\sqrt{x}}{a^2bx^2-a^3x}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*b*x - a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b*x^2 - a^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx = \begin{cases} -\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}-1}} + \frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{2iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-a^3b+a^2b^2x} - \frac{4ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-a^3b+a^2b^2x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(3/2)/(-b*x+a)**(3/2),x)
```

output
```
Piecewise((-2/(a*sqrt(b)*x*sqrt(a/(b*x) - 1)) + 4*sqrt(b)/(a**2*sqrt(a/(b*
x) - 1)), Abs(a/(b*x)) > 1), (2*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/(-a**3*b +
 a**2*b**2*x) - 4*I*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-a**3*b + a**2*b**2*x),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx = \frac{2\,b\sqrt{x}}{\sqrt{-bx+a}a^2} - \frac{2\,\sqrt{-bx+a}}{a^2\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*b*sqrt(x)/(sqrt(-b*x + a)*a^2) - 2*sqrt(-b*x + a)/(a^2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.29

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx = -\frac{4\,\sqrt{-b}b^2}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a|b|} - \frac{2\,\sqrt{-bx+a}b^2}{\sqrt{(bx-a)b+ab}a^2|b|}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-4*sqrt(-b)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 -
a*b)*a*abs(b)) - 2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^2*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx = -\frac{2\,a\,\sqrt{a-b\,x}-4\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(a^3-a^2\,b\,x\right)}$$

input
```
int(1/(x^(3/2)*(a - b*x)^(3/2)),x)
```

output
```
-(2*a*(a - b*x)^(1/2) - 4*b*x*(a - b*x)^(1/2))/(x^(1/2)*(a^3 - a^2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx = \frac{-4\sqrt{b}\,\sqrt{-bx+a}\,ix-2\sqrt{x}\,a+4\sqrt{x}\,bx}{\sqrt{-bx+a}\,a^2x}$$

input
```
int(1/x^(3/2)/(-b*x+a)^(3/2),x)
```

output
```
(2*( - 2*sqrt(b)*sqrt(a - b*x)*i*x - sqrt(x)*a + 2*sqrt(x)*b*x))/(sqrt(a -
 b*x)*a**2*x)
```

3.565 $\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = \frac{2}{ax^{3/2}\sqrt{a-bx}} - \frac{8\sqrt{a-bx}}{3a^2x^{3/2}} - \frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}$$

output
```
2/a/x^(3/2)/(-b*x+a)^(1/2)-8/3*(-b*x+a)^(1/2)/a^2/x^(3/2)-16/3*b*(-b*x+a)^
(1/2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = -\frac{2(a^2+4abx-8b^2x^2)}{3a^3x^{3/2}\sqrt{a-bx}}$$

input
```
Integrate[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

output
```
(-2*(a^2 + 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a - b*x])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\int \frac{1}{x^{5/2}\sqrt{a-bx}}dx}{a}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\left(\frac{2b\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{3a}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4\left(-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

input
```
Int[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

output
```
2/(a*x^(3/2)*Sqrt[a - b*x]) + (4*((-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*
Sqrt[a - b*x])/(3*a^2*Sqrt[x])))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2\left(-8b^{2}x^{2}+4abx+a^{2}\right)}{3x^{\frac{3}{2}}\sqrt{-bx+a}\,a^{3}}$	34
orering	$-\frac{2\left(-8b^{2}x^{2}+4abx+a^{2}\right)}{3x^{\frac{3}{2}}\sqrt{-bx+a}\,a^{3}}$	34
risch	$-\frac{2\sqrt{-bx+a}\,(5bx+a)}{3a^{3}x^{\frac{3}{2}}}+\frac{2b^{2}\sqrt{x}}{a^{3}\sqrt{-bx+a}}$	43
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\sqrt{-bx+a}}+\frac{4b\left(-\frac{2}{a\sqrt{x}\,\sqrt{-bx+a}}+\frac{4b\sqrt{x}}{a^{2}\sqrt{-bx+a}}\right)}{3a}$	58

input
```
int(1/x^(5/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-8*b^2*x^2+4*a*b*x+a^2)/x^(3/2)/(-b*x+a)^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = -\frac{2\,(8\,b^2x^2-4\,abx-a^2)\sqrt{-bx+a}\sqrt{x}}{3\,(a^3bx^3-a^4x^2)}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(8*b^2*x^2 - 4*a*b*x - a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b*x^3 - a^4*x
^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.85

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = \begin{cases} -\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} + \frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{16b^{\frac{15}{2}}}{3a^5b^4x-6a} \\ -\frac{2ia^3b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{6ia^2b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} + \frac{24iab^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} - \frac{16ib^{\frac{15}{2}}}{3a^5b^4x-6a} \end{cases}$$

input
```
integrate(1/x**(5/2)/(-b*x+a)**(3/2),x)
```

output
```
Piecewise((-2*a**3*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5
*x**2 + 3*a**3*b**6*x**3) - 6*a**2*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b
**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/
(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 16*b**(
15/2)*x**3*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b*
*6*x**3), Abs(a/(b*x)) > 1), (-2*I*a**3*b**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**
5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 6*I*a**2*b**(11/2)*x*sqr
t(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24
*I*a*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 +
 3*a**3*b**6*x**3) - 16*I*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x
 - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = \frac{2\,b^2\sqrt{x}}{\sqrt{-bx+a}a^3} - \frac{2\left(\frac{6\,\sqrt{-bx+a}b}{\sqrt{x}} + \frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*b^2*sqrt(x)/(sqrt(-b*x + a)*a^3) - 2/3*(6*sqrt(-b*x + a)*b/sqrt(x) + (-b
*x + a)^(3/2)/x^(3/2))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(50) = 100.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = -\frac{2\,\sqrt{-bx+a}\left(\frac{5\,(bx-a)b^2|b|}{a^3} + \frac{6\,b^2|b|}{a^2}\right)}{3\,((bx-a)b+ab)^{\frac{3}{2}}} - \frac{4\,\sqrt{-b}b^3}{\left(\left(\sqrt{-bx+a}\sqrt{-b} - \sqrt{(bx-a)b+ab}\right)^2 - ab\right)a^2|b|}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(-b*x + a)*(5*(b*x - a)*b^2*abs(b)/a^3 + 6*b^2*abs(b)/a^2)/((b*x
- a)*b + a*b)^(3/2) - 4*sqrt(-b)*b^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*
x - a)*b + a*b))^2 - a*b)*a^2*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = \frac{\sqrt{a-b\,x}\,\left(\frac{8\,x}{3\,a^2}+\frac{2}{3\,a\,b}-\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}-\frac{a\,x^{3/2}}{b}}$$

input
```
int(1/(x^(5/2)*(a - b*x)^(3/2)),x)
```

output
```
((a - b*x)^(1/2)*((8*x)/(3*a^2) + 2/(3*a*b) - (16*b*x^2)/(3*a^3)))/(x^(5/2
) - (a*x^(3/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = \frac{\frac{16\sqrt{b}\,\sqrt{-bx+a}\,bi\,x^2}{3}-\frac{2\sqrt{x}\,a^2}{3}-\frac{8\sqrt{x}\,abx}{3}+\frac{16\sqrt{x}\,b^2x^2}{3}}{\sqrt{-bx+a}\,a^3x^2}$$

input
```
int(1/x^(5/2)/(-b*x+a)^(3/2),x)
```

output
```
(2*(8*sqrt(b)*sqrt(a - b*x)*b*i*x**2 - sqrt(x)*a**2 - 4*sqrt(x)*a*b*x + 8*
sqrt(x)*b**2*x**2))/(3*sqrt(a - b*x)*a**3*x**2)
```

3.566 $\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 91

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = \frac{2}{ax^{5/2}\sqrt{a-bx}} - \frac{12\sqrt{a-bx}}{5a^2x^{5/2}} - \frac{16b\sqrt{a-bx}}{5a^3x^{3/2}} - \frac{32b^2\sqrt{a-bx}}{5a^4\sqrt{x}}$$

output
```
2/a/x^(5/2)/(-b*x+a)^(1/2)-12/5*(-b*x+a)^(1/2)/a^2/x^(5/2)-16/5*b*(-b*x+a)
^(1/2)/a^3/x^(3/2)-32/5*b^2*(-b*x+a)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = -\frac{2(a^3+2a^2bx+8ab^2x^2-16b^3x^3)}{5a^4x^{5/2}\sqrt{a-bx}}$$

input
```
Integrate[1/(x^(7/2)*(a - b*x)^(3/2)),x]
```

output
```
(-2*(a^3 + 2*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a -
b*x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}} \, dx$$

$$\Big\downarrow \ 55$$

$$\frac{6\int \frac{1}{x^{7/2}\sqrt{a-bx}}dx}{a}+\frac{2}{ax^{5/2}\sqrt{a-bx}}$$

$$\Big\downarrow \ 55$$

$$\frac{6\left(\frac{4b\int \frac{1}{x^{5/2}\sqrt{a-bx}}dx}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a-bx}}$$

$$\Big\downarrow \ 55$$

$$\frac{6\left(\frac{4b\left(\frac{2b\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{3a}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a-bx}}$$

$$\Big\downarrow \ 48$$

$$\frac{6\left(\frac{4b\left(-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a-bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a-bx}}$$

input
```
Int[1/(x^(7/2)*(a - b*x)^(3/2)),x]
```

output
```
2/(a*x^(5/2)*Sqrt[a - b*x]) + (6*((-2*Sqrt[a - b*x])/(5*a*x^(5/2)) + (4*b*
((-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])))/
(5*a)))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{2\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}+2a^{2}bx+a^{3}\right)}{5x^{\frac{5}{2}}\sqrt{-bx+a}\,a^{4}}$	45
orering	$-\frac{2\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}+2a^{2}bx+a^{3}\right)}{5x^{\frac{5}{2}}\sqrt{-bx+a}\,a^{4}}$	45
risch	$-\frac{2\sqrt{-bx+a}\,\left(11b^{2}x^{2}+3abx+a^{2}\right)}{5a^{4}x^{\frac{5}{2}}}+\frac{2b^{3}\sqrt{x}}{a^{4}\sqrt{-bx+a}}$	54
default	$-\frac{2}{5a\,x^{\frac{5}{2}}\sqrt{-bx+a}}+\frac{6b\left(-\frac{2}{3a\,x^{\frac{3}{2}}\sqrt{-bx+a}}+\frac{4b\left(-\frac{2}{a\sqrt{x}\,\sqrt{-bx+a}}+\frac{4b\sqrt{x}}{a^{2}\sqrt{-bx+a}}\right)}{3a}\right)}{5a}$	81

input
```
int(1/x^(7/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-16*b^3*x^3+8*a*b^2*x^2+2*a^2*b*x+a^3)/x^(5/2)/(-b*x+a)^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = -\frac{2\,(16\,b^3x^3-8\,ab^2x^2-2\,a^2bx-a^3)\sqrt{-bx+a}\sqrt{x}}{5\,(a^4bx^4-a^5x^3)}$$

input
```
integrate(1/x^(7/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(16*b^3*x^3 - 8*a*b^2*x^2 - 2*a^2*b*x - a^3)*sqrt(-b*x + a)*sqrt(x)/(
a^4*b*x^4 - a^5*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.30 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.79

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = \begin{cases} \frac{2a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx}-1}}{-5a^7b^9x^2+15a^6b^{10}x^3-15a^5b^{11}x^4+5a^4b^{12}x^5} + \frac{10a^3b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-5a^7b^9x^2+15a^6b^{10}x^3-15a^5b^{11}x^4+5a^4b^{12}x^5} - \frac{}{-5a^7b^9x^{\cdots}} \\ \frac{2ia^5b^{\frac{19}{2}}\sqrt{-\frac{a}{bx}+1}}{-5a^7b^9x^2+15a^6b^{10}x^3-15a^5b^{11}x^4+5a^4b^{12}x^5} + \frac{10ia^3b^{\frac{23}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-5a^7b^9x^2+15a^6b^{10}x^3-15a^5b^{11}x^4+5a^4b^{12}x^5} - \frac{}{-5a^7b^9x^{\cdots}} \end{cases}$$

input
```
integrate(1/x**(7/2)/(-b*x+a)**(3/2),x)
```

output
```
Piecewise((2*a**5*b**(19/2)*sqrt(a/(b*x) - 1)/(-5*a**7*b**9*x**2 + 15*a**6
*b**10*x**3 - 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) + 10*a**3*b**(23/2)*
x**2*sqrt(a/(b*x) - 1)/(-5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 - 15*a**5*b
**11*x**4 + 5*a**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) - 1)/
(-5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 - 15*a**5*b**11*x**4 + 5*a**4*b**1
2*x**5) + 80*a*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-5*a**7*b**9*x**2 + 15*a*
*6*b**10*x**3 - 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**
5*sqrt(a/(b*x) - 1)/(-5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 - 15*a**5*b**1
1*x**4 + 5*a**4*b**12*x**5), Abs(a/(b*x)) > 1), (2*I*a**5*b**(19/2)*sqrt(-
a/(b*x) + 1)/(-5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 - 15*a**5*b**11*x**4
+ 5*a**4*b**12*x**5) + 10*I*a**3*b**(23/2)*x**2*sqrt(-a/(b*x) + 1)/(-5*a**
7*b**9*x**2 + 15*a**6*b**10*x**3 - 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5)
 - 60*I*a**2*b**(25/2)*x**3*sqrt(-a/(b*x) + 1)/(-5*a**7*b**9*x**2 + 15*a**
6*b**10*x**3 - 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) + 80*I*a*b**(27/2)*
x**4*sqrt(-a/(b*x) + 1)/(-5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 - 15*a**5*
b**11*x**4 + 5*a**4*b**12*x**5) - 32*I*b**(29/2)*x**5*sqrt(-a/(b*x) + 1)/(
-5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 - 15*a**5*b**11*x**4 + 5*a**4*b**12
*x**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = \frac{2\,b^3\sqrt{x}}{\sqrt{-bx+a}a^4} - \frac{2\left(\frac{15\,\sqrt{-bx+a}b^2}{\sqrt{x}} + \frac{5\,(-bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} + \frac{(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{5\,a^4}$$

input
```
integrate(1/x^(7/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*b^3*sqrt(x)/(sqrt(-b*x + a)*a^4) - 2/5*(15*sqrt(-b*x + a)*b^2/sqrt(x) +
5*(-b*x + a)^(3/2)*b/x^(3/2) + (-b*x + a)^(5/2)/x^(5/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = -\frac{4\sqrt{-b}b^4}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a^3|b|} - \frac{2\left(\frac{15\,b^6}{a^2|b|}+(bx-a)\left(\frac{11\,(bx-a)b^6}{a^4|b|}+\frac{25\,b^6}{a^3|b|}\right)\right)\sqrt{-bx+a}}{5\left((bx-a)b+ab\right)^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-4*sqrt(-b)*b^4/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 -
a*b)*a^3*abs(b)) - 2/5*(15*b^6/(a^2*abs(b)) + (b*x - a)*(11*(b*x - a)*b^6/
(a^4*abs(b)) + 25*b^6/(a^3*abs(b))))*sqrt(-b*x + a)/((b*x - a)*b + a*b)^(5
/2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = \frac{\sqrt{a-b\,x}\,\left(\frac{4\,x}{5\,a^2}+\frac{2}{5\,a\,b}+\frac{16\,b\,x^2}{5\,a^3}-\frac{32\,b^2\,x^3}{5\,a^4}\right)}{x^{7/2}-\frac{a\,x^{5/2}}{b}}$$

input
```
int(1/(x^(7/2)*(a - b*x)^(3/2)),x)
```

output
```
((a - b*x)^(1/2)*((4*x)/(5*a^2) + 2/(5*a*b) + (16*b*x^2)/(5*a^3) - (32*b^2
*x^3)/(5*a^4)))/(x^(7/2) - (a*x^(5/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{7/2}(a-bx)^{3/2}}\,dx = \frac{\frac{32\sqrt{b}\,\sqrt{-bx+a}\,b^2 i\,x^3}{5} - \frac{2\sqrt{x}\,a^3}{5} - \frac{4\sqrt{x}\,a^2 bx}{5} - \frac{16\sqrt{x}\,a\,b^2x^2}{5} + \frac{32\sqrt{x}\,b^3x^3}{5}}{\sqrt{-bx+a}\,a^4x^3}$$

input

```
int(1/x^(7/2)/(-b*x+a)^(3/2),x)
```

output

```
(2*(16*sqrt(b)*sqrt(a - b*x)*b**2*i*x**3 - sqrt(x)*a**3 - 2*sqrt(x)*a**2*b
*x - 8*sqrt(x)*a*b**2*x**2 + 16*sqrt(x)*b**3*x**3))/(5*sqrt(a - b*x)*a**4*
x**3)
```

3.567 $\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx=\frac{2a^2\sqrt{x}}{3b^3(a-bx)^{3/2}}-\frac{14a\sqrt{x}}{3b^3\sqrt{a-bx}}-\frac{\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}$$

output
```
2/3*a^2*x^(1/2)/b^3/(-b*x+a)^(3/2)-14/3*a*x^(1/2)/b^3/(-b*x+a)^(1/2)-x^(1/
2)*(-b*x+a)^(1/2)/b^3+5*a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx=-\frac{\sqrt{x}(15a^2-20abx+3b^2x^2)}{3b^3(a-bx)^{3/2}}+\frac{10a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{b^{7/2}}$$

input
```
Integrate[x^(5/2)/(a - b*x)^(5/2),x]
```

output
```
-1/3*(Sqrt[x]*(15*a^2 - 20*a*b*x + 3*b^2*x^2))/(b^3*(a - b*x)^(3/2)) + (10
*a*ArcTan[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {57, 57, 60, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\int \frac{x^{3/2}}{(a-bx)^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\left(\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\int \frac{\sqrt{x}}{\sqrt{a-bx}}dx}{b}\right)}{3b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\left(\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\left(\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{2b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{b}\right)}{3b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\left(\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\left(\frac{a\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{b}\right)}{3b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\left(\frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\left(\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}\right)}{b}\right)}{3b}$$

input
```
Int[x^(5/2)/(a - b*x)^(5/2),x]
```

output
```
(2*x^(5/2))/(3*b*(a - b*x)^(3/2)) - (5*((2*x^(3/2))/(b*Sqrt[a - b*x]) - (3
*(-((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]
])/b^(3/2)))/b))/(3*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(75) = 150.

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.62

method	result	size
risch	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b^{3}}+\frac{\left(\frac{5a\arctan\left(\frac{\sqrt{b}\left(x-\frac{a}{2b}\right)}{\sqrt{-b\,x^{2}+ax}}\right)}{2b^{\frac{7}{2}}}+\frac{14a\sqrt{-b\left(x-\frac{a}{b}\right)^{2}-\left(x-\frac{a}{b}\right)a}}{3b^{4}\left(x-\frac{a}{b}\right)}+\frac{2a^{2}\sqrt{-b\left(x-\frac{a}{b}\right)^{2}-\left(x-\frac{a}{b}\right)a}}{3b^{5}\left(x-\frac{a}{b}\right)^{2}}\right)\sqrt{x(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	160

input
```
int(x^(5/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)*(-b*x+a)^(1/2)/b^3+(5/2/b^(7/2)*a*arctan(b^(1/2)*(x-1/2*a/b)/(-b*
x^2+a*x)^(1/2))+14/3/b^4*a/(x-a/b)*(-b*(x-a/b)^2-(x-a/b)*a)^(1/2)+2/3/b^5*
a^2/(x-a/b)^2*(-b*(x-a/b)^2-(x-a/b)*a)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(
-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.26

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx = \left[-\frac{15\left(ab^{2}x^{2}-2\,a^{2}bx+a^{3}\right)\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(3\,b^{3}x^{2}-\right.}{6\left(b^{6}x^{2}-2\,ab^{5}x+a^{2}b^{4}\right)}\right.$$

input
```
integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(15*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x
+ a)*sqrt(-b)*sqrt(x) + a) + 2*(3*b^3*x^2 - 20*a*b^2*x + 15*a^2*b)*sqrt(-b
*x + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), -1/3*(15*(a*b^2*x^2 - 2*
a^2*b*x + a^3)*sqrt(b)*arctan(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) +
(3*b^3*x^2 - 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/(b^6*x^2 - 2*a
*b^5*x + a^2*b^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 971, normalized size of antiderivative = 9.81

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)/(-b*x+a)**(5/2),x)
```

output
```
Piecewise((-30*I*a**(81/2)*b**22*x**(51/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*
sqrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**
(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 15*pi*a**(81/2)*b**22*x**(5
1/2)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) -
6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 30*I*a**(79/2)*b**23*x
**(53/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(6*a**(79/2)*b**(
51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1
 + b*x/a)) - 15*pi*a**(79/2)*b**23*x**(53/2)*sqrt(-1 + b*x/a)/(6*a**(79/2)
*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sq
rt(-1 + b*x/a)) + 30*I*a**40*b**(45/2)*x**26/(6*a**(79/2)*b**(51/2)*x**(51
/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) -
 40*I*a**39*b**(47/2)*x**27/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x
/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 6*I*a**38*b**(49
/2)*x**28/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*
b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (15*a**(81/2)*b**2
2*x**(51/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**
(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1
- b*x/a)) - 15*a**(79/2)*b**23*x**(53/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt
(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2
)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 15*a**40*b**(45/2)*x**26/(3*a*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx = \frac{2\,ab^2 + \frac{10\,(bx-a)ab}{x} - \frac{15\,(bx-a)^2 a}{x^2}}{3\left(\frac{(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)} - \frac{5\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*a*b^2 + 10*(b*x - a)*a*b/x - 15*(b*x - a)^2*a/x^2)/((-b*x + a)^(3/2
)*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^(5/2)) - 5*a*arctan(sqrt(-b*x + a)/
(sqrt(b)*sqrt(x)))/b^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(75) = 150$.

Time = 15.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.23

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx = \frac{\left(\frac{15\,a\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}b^2} - \frac{6\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b^3} - \frac{8\left(9\,a^2\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b\right.\right.}}{\left(\left(\sqrt{-bx+a}\right.\right.}\right.}{6\,b^2}$$

input
```
integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*(15*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/(sqrt
(-b)*b^2) - 6*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b^3 - 8*(9*a^2*(sqrt(
-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4 - 12*a^3*(sqrt(-b*x + a)*s
qrt(-b) - sqrt((b*x - a)*b + a*b))^2*b + 7*a^4*b^2)/(((sqrt(-b*x + a)*sqrt
(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx = \int \frac{x^{5/2}}{(a-b\,x)^{5/2}}\,dx$$

input
```
int(x^(5/2)/(a - b*x)^(5/2),x)
```

output
```
int(x^(5/2)/(a - b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx = \frac{-30\sqrt{b}\,\sqrt{-bx+a}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)a^2i+30\sqrt{b}\,\sqrt{-bx+a}\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)abix}{6\sqrt{-bx+a}\,b^4}$$

input
```
int(x^(5/2)/(-b*x+a)^(5/2),x)
```

output
```
( - 30*sqrt(b)*sqrt(a - b*x)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqrt(
a))*a**2*i + 30*sqrt(b)*sqrt(a - b*x)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)
*i)/sqrt(a))*a*b*i*x - 5*sqrt(b)*sqrt(a - b*x)*a**2*i + 5*sqrt(b)*sqrt(a -
 b*x)*a*b*i*x - 30*sqrt(x)*a**2*b + 40*sqrt(x)*a*b**2*x - 6*sqrt(x)*b**3*x
**2)/(6*sqrt(a - b*x)*b**4*(a - b*x))
```

3.568 $\int \frac{x^{3/2}}{(a-bx)^{5/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}} dx = \frac{2a\sqrt{x}}{3b^2(a-bx)^{3/2}} - \frac{8\sqrt{x}}{3b^2\sqrt{a-bx}} + \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

output
```
2/3*a*x^(1/2)/b^2/(-b*x+a)^(3/2)-8/3*x^(1/2)/b^2/(-b*x+a)^(1/2)+2*arctan(b
^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}} dx = \frac{2\sqrt{x}(-3a+4bx)}{3b^2(a-bx)^{3/2}} + \frac{4\arctan\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a-bx}}\right)}{b^{5/2}}$$

input
```
Integrate[x^(3/2)/(a - b*x)^(5/2),x]
```

output
```
(2*Sqrt[x]*(-3*a + 4*b*x))/(3*b^2*(a - b*x)^(3/2)) + (4*ArcTan[(Sqrt[b]*Sq
rt[x])/(-Sqrt[a] + Sqrt[a - b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {57, 57, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}dx$$

$$\Big\downarrow \ 57$$

$$\frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{\int \frac{\sqrt{x}}{(a-bx)^{3/2}}dx}{b}$$

$$\Big\downarrow \ 57$$

$$\frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{b}}{b}$$

$$\Big\downarrow \ 65$$

$$\frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\int \frac{1}{\frac{bx}{a-bx}+1}d\frac{\sqrt{x}}{\sqrt{a-bx}}}{b}}{b}$$

$$\Big\downarrow \ 216$$

$$\frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}}{b}$$

input
```
Int[x^(3/2)/(a - b*x)^(5/2),x]
```

output
```
(2*x^(3/2))/(3*b*(a - b*x)^(3/2)) - ((2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(3/2))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{(-bx+a)^{\frac{5}{2}}}dx$$

input
```
int(x^(3/2)/(-b*x+a)^(5/2),x)
```

output
```
int(x^(3/2)/(-b*x+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.63

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx = \left[-\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(4\,b^2x-3\,ab\right)}{3\left(b^5x^2-2\,ab^4x+a^2b^3\right)}\right.$$

input
```
integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*
sqrt(-b)*sqrt(x) + a) - 2*(4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^5*x
^2 - 2*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(b)*arcta
n(sqrt(-b*x + a)*sqrt(b)*sqrt(x)/(b*x - a)) - (4*b^2*x - 3*a*b)*sqrt(-b*x
+ a)*sqrt(x))/(b^5*x^2 - 2*a*b^4*x + a^2*b^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 833, normalized size of antiderivative = 11.11

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)/(-b*x+a)**(5/2),x)
```

output
```
Piecewise((-6*I*a**(39/2)*b**11*x**(27/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*s
qrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(
37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 3*pi*a**(39/2)*b**11*x**(27/
2)*sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*
a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 6*I*a**(37/2)*b**12*x**(
29/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/
2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 +
b*x/a)) - 3*pi*a**(37/2)*b**12*x**(29/2)*sqrt(-1 + b*x/a)/(3*a**(39/2)*b**
(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-
1 + b*x/a)) + 6*I*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*s
qrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) - 8*I*
a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) -
3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (6*a**
(39/2)*b**11*x**(27/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a*
*(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29
/2)*sqrt(1 - b*x/a)) - 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 - b*x/a)*asin(sq
rt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) -
3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*x/a)) - 6*a**19*b**(23/2)*x**14
/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*
x**(29/2)*sqrt(1 - b*x/a)) + 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(2...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx = \frac{2\left(b+\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}b^2} - \frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(b + 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*b^2) - 2*arctan(sqrt(-b*
x + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(55) = 110.

Time = 15.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.59

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx = \frac{\left(\frac{3\,\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}} + \frac{8\left(3\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}-3\,a^2\left(\sqrt{-bx+a}\sqrt{-b}-\cdots\right)\right.}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\cdots\right.}\right.}{3\,b^3}$$

input
```
integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(3*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b)
 + 8*(3*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) -
 3*a^2*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b +
2*a^3*sqrt(-b)*b^2)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2
 - a*b)^3)*abs(b)/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx = \int \frac{x^{3/2}}{\left(a-b\,x\right)^{5/2}}\,dx$$

input
```
int(x^(3/2)/(a - b*x)^(5/2),x)
```

output
```
int(x^(3/2)/(a - b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx = \frac{-2\sqrt{b}\,\sqrt{-bx+a}\,\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)ai+2\sqrt{b}\,\sqrt{-bx+a}\,\log\left(\frac{\sqrt{-bx+a}+\sqrt{x}\,\sqrt{b}\,i}{\sqrt{a}}\right)bix-2}{\sqrt{-bx+a}\,b^3\,(-bx+a)}$$

input
```
int(x^(3/2)/(-b*x+a)^(5/2),x)
```

output
```
(2*( - 3*sqrt(b)*sqrt(a - b*x)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i)/sqr
t(a))*a*i + 3*sqrt(b)*sqrt(a - b*x)*log((sqrt(a - b*x) + sqrt(x)*sqrt(b)*i
)/sqrt(a))*b*i*x - 3*sqrt(x)*a*b + 4*sqrt(x)*b**2*x))/(3*sqrt(a - b*x)*b**
3*(a - b*x))
```

3.569 $\int \frac{\sqrt{x}}{(a-bx)^{5/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}} \, dx = \frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

output
```
2/3*x^(3/2)/a/(-b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}} \, dx = \frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

input
```
Integrate[Sqrt[x]/(a - b*x)^(5/2),x]
```

output
```
(2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 48$$

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

input
```
Int[Sqrt[x]/(a - b*x)^(5/2),x]
```

output
```
(2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3a(-bx+a)^{\frac{3}{2}}}$	17
orering	$\frac{2x^{\frac{3}{2}}}{3a(-bx+a)^{\frac{3}{2}}}$	17
default	$\frac{\sqrt{x}}{b(-bx+a)^{\frac{3}{2}}}-\frac{a\left(\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{-bx+a}}\right)}{2b}$	56

input
```
int(x^(1/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/a/(-b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{2\sqrt{-bx+a}x^{\frac{3}{2}}}{3\left(ab^2x^2-2a^2bx+a^3\right)}$$

input
```
integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-b*x + a)*x^(3/2)/(a*b^2*x^2 - 2*a^2*b*x + a^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.32

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \begin{cases} \frac{2ix^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{2x^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(-b*x+a)**(5/2),x)
```

output
```
Piecewise((2*I*x**(3/2)/(-3*a**(5/2)*sqrt(-1 + b*x/a) + 3*a**(3/2)*b*x*sqr
t(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*x**(3/2)/(-3*a**(5/2)*sqrt(1 - b*x/a)
 + 3*a**(3/2)*b*x*sqrt(1 - b*x/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{2\,x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a}$$

input
```
integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)/((-b*x + a)^(3/2)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(16) = 32$.

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.64

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{4\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}+a^2\sqrt{-b}b^2\right)|b|}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3 b^2}$$

input
```
integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) + a^
2*sqrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b
))^2 - a*b)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{2\,x^{3/2}\,\sqrt{a-b\,x}}{3\,\left(a^3-2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

input
```
int(x^(1/2)/(a - b*x)^(5/2),x)
```

output
```
(2*x^(3/2)*(a - b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 - 2*a^2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{\frac{2\sqrt{b}\,\sqrt{-bx+a}\,ai}{3}-\frac{2\sqrt{b}\,\sqrt{-bx+a}\,bix}{3}+\frac{2\sqrt{x}\,b^2x}{3}}{\sqrt{-bx+a}\,a\,b^2\,(-bx+a)}$$

input
```
int(x^(1/2)/(-b*x+a)^(5/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a - b*x)*a*i - sqrt(b)*sqrt(a - b*x)*b*i*x + sqrt(x)*b**2
*x))/(3*sqrt(a - b*x)*a*b**2*(a - b*x))
```

3.570 $\int \frac{1}{\sqrt{x}(a-bx)^{5/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}} \, dx = \frac{2\sqrt{x}}{3a(a-bx)^{3/2}} + \frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}$$

output
```
2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}} \, dx = \frac{2\sqrt{x}(3a-2bx)}{3a^2(a-bx)^{3/2}}$$

input
```
Integrate[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

output
```
(2*Sqrt[x]*(3*a - 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}dx}{3a}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

input
```
Int[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

output
```
(2*Sqrt[x])/(3*a*(a - b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a - b*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{2\sqrt{x}\,(-2bx+3a)}{3(-bx+a)^{\frac{3}{2}}\,a^{2}}$	25
orering	$\frac{2\sqrt{x}\,(-2bx+3a)}{3(-bx+a)^{\frac{3}{2}}\,a^{2}}$	25
default	$\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{-bx+a}}$	34

input
```
int(1/x^(1/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(1/2)*(-2*b*x+3*a)/(-b*x+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx = -\frac{2\,(2\,bx-3\,a)\sqrt{-bx+a}\sqrt{x}}{3\,(a^2b^2x^2-2\,a^3bx+a^4)}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*b*x - 3*a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b^2*x^2 - 2*a^3*b*x + a^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.38

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx = \begin{cases} -\frac{6a}{-3a^3\sqrt{b}\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} + \frac{4bx}{-3a^3\sqrt{b}\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{6iab}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} - \frac{4ib^2x}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(-b*x+a)**(5/2),x)
```

output
```
Piecewise((-6*a/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/2)*x*sqr
t(a/(b*x) - 1)) + 4*b*x/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/
2)*x*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (6*I*a*b/(-3*a**3*b**(3/2)*sqr
t(-a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)) - 4*I*b**2*x/(-3*a
**3*b**(3/2)*sqrt(-a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx = \frac{2\left(b-\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(b - 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx = \frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}b^2}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3|b|}$$

input
```
integrate(1/x^(1/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt(-
b)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*ab
s(b))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx = \frac{6\,a\,\sqrt{x}\,\sqrt{a-b\,x}-4\,b\,x^{3/2}\,\sqrt{a-b\,x}}{3\,a^4-6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

input
```
int(1/(x^(1/2)*(a - b*x)^(5/2)),x)
```

output
```
(6*a*x^(1/2)*(a - b*x)^(1/2) - 4*b*x^(3/2)*(a - b*x)^(1/2))/(3*a^4 + 3*a^2
*b^2*x^2 - 6*a^3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx = \frac{\frac{4\sqrt{b}\,\sqrt{-bx+a}\,ai}{3}-\frac{4\sqrt{b}\,\sqrt{-bx+a}\,bix}{3}+2\sqrt{x}\,ab-\frac{4\sqrt{x}\,b^2x}{3}}{\sqrt{-bx+a}\,a^2b\,(-bx+a)}$$

input
```
int(1/x^(1/2)/(-b*x+a)^(5/2),x)
```

output
```
(2*(2*sqrt(b)*sqrt(a - b*x)*a*i - 2*sqrt(b)*sqrt(a - b*x)*b*i*x + 3*sqrt(x
)*a*b - 2*sqrt(x)*b**2*x))/(3*sqrt(a - b*x)*a**2*b*(a - b*x))
```

3.571 $\int \frac{1}{x^{3/2}(a-bx)^{5/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}} dx = \frac{2}{3a\sqrt{x}(a-bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}$$

output
```
2/3/a/x^(1/2)/(-b*x+a)^(3/2)+8/3/a^2/x^(1/2)/(-b*x+a)^(1/2)-16/3*(-b*x+a)^
(1/2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}} dx = -\frac{2(3a^2-12abx+8b^2x^2)}{3a^3\sqrt{x}(a-bx)^{3/2}}$$

input
```
Integrate[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

output
```
(-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4 \int \frac{1}{x^{3/2}(a-bx)^{3/2}}dx}{3a}+\frac{2}{3a\sqrt{x}(a-bx)^{3/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4 \left(\frac{2 \int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{a}+\frac{2}{a\sqrt{x}\sqrt{a-bx}}\right)}{3a}+\frac{2}{3a\sqrt{x}(a-bx)^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4 \left(\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}\right)}{3a}+\frac{2}{3a\sqrt{x}(a-bx)^{3/2}}$$

input
```
Int[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

output
```
2/(3*a*Sqrt[x]*(a - b*x)^(3/2)) + (4*(2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqr
t[a - b*x])/(a^2*Sqrt[x])))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{2\left(8b^{2}x^{2}-12abx+3a^{2}\right)}{3\sqrt{x}\,\left(-bx+a\right)^{\frac{3}{2}}a^{3}}$	36
orering	$-\frac{2\left(8b^{2}x^{2}-12abx+3a^{2}\right)}{3\sqrt{x}\,\left(-bx+a\right)^{\frac{3}{2}}a^{3}}$	36
risch	$-\frac{2\sqrt{-bx+a}}{a^{3}\sqrt{x}}+\frac{2b\left(-5bx+6a\right)\sqrt{x}}{3\left(-bx+a\right)^{\frac{3}{2}}a^{3}}$	43
default	$-\frac{2}{a\sqrt{x}\,\left(-bx+a\right)^{\frac{3}{2}}}+\frac{4b\left(\frac{2\sqrt{x}}{3a\left(-bx+a\right)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{-bx+a}}\right)}{a}$	57

input
```
int(1/x^(3/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(8*b^2*x^2-12*a*b*x+3*a^2)/x^(1/2)/(-b*x+a)^(3/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = -\frac{2\,(8\,b^2x^2-12\,abx+3\,a^2)\sqrt{-bx+a}\sqrt{x}}{3\,(a^3b^2x^3-2\,a^4bx^2+a^5x)}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(8*b^2*x^2 - 12*a*b*x + 3*a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b^2*x^3 -
2*a^4*b*x^2 + a^5*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.69

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = \begin{cases} -\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} + \frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} - \frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{6ia^2b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} + \frac{24iab^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} - \frac{16ib^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(3/2)/(-b*x+a)**(5/2),x)
```

output
```
Piecewise((-6*a**2*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a**4*b**5*x
 + 3*a**3*b**6*x**2) + 24*a*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6
*a**4*b**5*x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(3*
a**5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), Abs(a/(b*x)) > 1), (-6*I*a*
*2*b**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*
x**2) + 24*I*a*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x
 + 3*a**3*b**6*x**2) - 16*I*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4
 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = \frac{2\left(b^2 - \frac{6\,(bx-a)b}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^3} - \frac{2\,\sqrt{-bx+a}}{a^3\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(b^2 - 6*(b*x - a)*b/x)*x^(3/2)/((-b*x + a)^(3/2)*a^3) - 2*sqrt(-b*x +
 a)/(a^3*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(49) = 98.

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.82

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = -\frac{2\,\sqrt{-bx+a}b^2}{\sqrt{(bx-a)b+ab}a^3|b|} - \frac{4\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}b^2 - 12\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}b^3\right.}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^2|b|}$$

input
```
integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(
-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b)*b^2 - 12*a*(sqrt(
-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b^3 + 5*a^2*sqrt(
-b)*b^4)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*
a^2*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = \frac{6\,a^2\,\sqrt{a-b\,x}+16\,b^2\,x^2\,\sqrt{a-b\,x}-24\,a\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b-3\,a^3\,b^2\,x\right)-3\,a^5\right)}$$

input
```
int(1/(x^(3/2)*(a - b*x)^(5/2)),x)
```

output
```
(6*a^2*(a - b*x)^(1/2) + 16*b^2*x^2*(a - b*x)^(1/2) - 24*a*b*x*(a - b*x)^(
1/2))/(x^(1/2)*(x*(6*a^4*b - 3*a^3*b^2*x) - 3*a^5))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = \frac{\frac{16\sqrt{b}\,\sqrt{-bx+a}\,aix}{3}-\frac{16\sqrt{b}\,\sqrt{-bx+a}\,bi\,x^2}{3}-2\sqrt{x}\,a^2+8\sqrt{x}\,abx-\frac{16\sqrt{x}\,b^2x^2}{3}}{\sqrt{-bx+a}\,a^3x\,(-bx+a)}$$

input
```
int(1/x^(3/2)/(-b*x+a)^(5/2),x)
```

output
```
(2*(8*sqrt(b)*sqrt(a - b*x)*a*i*x - 8*sqrt(b)*sqrt(a - b*x)*b*i*x**2 - 3*s
qrt(x)*a**2 + 12*sqrt(x)*a*b*x - 8*sqrt(x)*b**2*x**2))/(3*sqrt(a - b*x)*a*
*3*x*(a - b*x))
```

3.572 $\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx=\frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}$$

output
```
2/3/a/x^(3/2)/(-b*x+a)^(3/2)+4/a^2/x^(3/2)/(-b*x+a)^(1/2)-16/3*(-b*x+a)^(1
/2)/a^3/x^(3/2)-32/3*b*(-b*x+a)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx=-\frac{2(a^3+6a^2bx-24ab^2x^2+16b^3x^3)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

input
```
Integrate[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

output
```
(-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x
)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}} \, dx$$

$$\Big\downarrow \; 55$$

$$\frac{2\int \frac{1}{x^{5/2}(a-bx)^{3/2}}dx}{a}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

$$\Big\downarrow \; 55$$

$$\frac{2\left(\frac{4\int \frac{1}{x^{5/2}\sqrt{a-bx}}dx}{a}+\frac{2}{ax^{3/2}\sqrt{a-bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

$$\Big\downarrow \; 55$$

$$\frac{2\left(\frac{4\left(\frac{2b\int \frac{1}{x^{3/2}\sqrt{a-bx}}dx}{3a}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a-bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

$$\Big\downarrow \; 48$$

$$\frac{2\left(\frac{4\left(-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a-bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

input
```
Int[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

output
```
2/(3*a*x^(3/2)*(a - b*x)^(3/2)) + (2*(2/(a*x^(3/2)*Sqrt[a - b*x]) + (4*((-
2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])))/a))
/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2\left(16 b^{3} x^{3}-24 a \,b^{2} x^{2}+6 a^{2} b x +a^{3}\right)}{3 x^{\frac{3}{2}} \left(-b x +a \right)^{\frac{3}{2}} a^{4}}$	45
orering	$-\frac{2\left(16 b^{3} x^{3}-24 a \,b^{2} x^{2}+6 a^{2} b x +a^{3}\right)}{3 x^{\frac{3}{2}} \left(-b x +a \right)^{\frac{3}{2}} a^{4}}$	45
risch	$-\frac{2 \sqrt{-b x +a}\, \left(8 b x +a \right)}{3 a^{4} x^{\frac{3}{2}}}+\frac{2 b^{2} \left(-8 b x +9 a \right) \sqrt{x}}{3 \left(-b x +a \right)^{\frac{3}{2}} a^{4}}$	51
default	$-\frac{2}{3 a \,x^{\frac{3}{2}} \left(-b x +a \right)^{\frac{3}{2}}}+\frac{2 b \left(-\frac{2}{a \sqrt{x}\, \left(-b x +a \right)^{\frac{3}{2}}}+\frac{4 b \left(\frac{2 \sqrt{x}}{3 a \left(-b x +a \right)^{\frac{3}{2}}}+\frac{4 \sqrt{x}}{3 a^{2} \sqrt{-b x +a}}\right)}{a}\right)}{a}$	80

input
```
int(1/x^(5/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(16*b^3*x^3-24*a*b^2*x^2+6*a^2*b*x+a^3)/x^(3/2)/(-b*x+a)^(3/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = -\frac{2\,(16\,b^3x^3-24\,ab^2x^2+6\,a^2bx+a^3)\sqrt{-bx+a}\sqrt{x}}{3\,(a^4b^2x^4-2\,a^5bx^3+a^6x^2)}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(16*b^3*x^3 - 24*a*b^2*x^2 + 6*a^2*b*x + a^3)*sqrt(-b*x + a)*sqrt(x)/
(a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 688, normalized size of antiderivative = 7.82

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = \begin{cases} \frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60a^2}{-3a^7b^9x+9a^6b^1} \\ \frac{2ia^4b^{\frac{19}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10ia^3b^{\frac{21}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60ia^2b}{-3a^7b^9x+9a^6b^1} \end{cases}$$

input
```
integrate(1/x**(5/2)/(-b*x+a)**(5/2),x)
```

output
```
Piecewise((2*a**4*b**(19/2)*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**
10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqr
t(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3
*a**4*b**12*x**4) - 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) - 1)/(-3*a**7*b**9
*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**
(25/2)*x**3*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5
*b**11*x**3 + 3*a**4*b**12*x**4) - 32*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-3
*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4),
 Abs(a/(b*x)) > 1), (2*I*a**4*b**(19/2)*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x
 + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*I*a**3*
b**(21/2)*x*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**
5*b**11*x**3 + 3*a**4*b**12*x**4) - 60*I*a**2*b**(23/2)*x**2*sqrt(-a/(b*x)
 + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**
12*x**4) + 80*I*a*b**(25/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a*
*6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) - 32*I*b**(27/2)*x*
*4*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x
**3 + 3*a**4*b**12*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = -\frac{2\left(\frac{9\sqrt{-bx+a}b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}+\frac{2\left(b^3-\frac{9\,(bx-a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^4}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(9*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^4 + 2/3*(b^
3 - 9*(b*x - a)*b^2/x)*x^(3/2)/((-b*x + a)^(3/2)*a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(66) = 132$.

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.35

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = -\frac{2\sqrt{-bx+a}\left(\frac{8\,(bx-a)b^2|b|}{a^4}+\frac{9\,b^2|b|}{a^3}\right)}{3\left((bx-a)b+ab\right)^{\frac{3}{2}}} - \frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}b^3 - 9\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}b^4 -\right.}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3 a^3|b|}$$

input
```
integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(-b*x + a)*(8*(b*x - a)*b^2*abs(b)/a^4 + 9*b^2*abs(b)/a^3)/((b*x
- a)*b + a*b)^(3/2) - 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b +
 a*b))^4*sqrt(-b)*b^3 - 9*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b +
a*b))^2*sqrt(-b)*b^4 + 4*a^2*sqrt(-b)*b^5)/(((sqrt(-b*x + a)*sqrt(-b) - sq
rt((b*x - a)*b + a*b))^2 - a*b)^3*a^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = \frac{2\,a^3\sqrt{a-b\,x}+32\,b^3\,x^3\sqrt{a-b\,x}+12\,a^2\,b\,x\sqrt{a-b\,x}-48\,a\,b^2\,x^2\sqrt{a-b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b-3\,a^4\,b^2\,x\right)-3\,a^6\right)}$$

input
```
int(1/(x^(5/2)*(a - b*x)^(5/2)),x)
```

output
```
(2*a^3*(a - b*x)^(1/2) + 32*b^3*x^3*(a - b*x)^(1/2) + 12*a^2*b*x*(a - b*x)
^(1/2) - 48*a*b^2*x^2*(a - b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b - 3*a^4*b^2*x)
 - 3*a^6))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = \frac{\frac{32\sqrt{b}\,\sqrt{-bx+a}\,abi\,x^2}{3} - \frac{32\sqrt{b}\,\sqrt{-bx+a}\,b^2i\,x^3}{3} - \frac{2\sqrt{x}\,a^3}{3} - 4\sqrt{x}\,a^2bx + 16\sqrt{x}\,a\,b^2x^2 - \frac{32\sqrt{x}\,b^3x^3}{3}}{\sqrt{-bx+a}\,a^4x^2\,(-bx+a)}$$

input
```
int(1/x^(5/2)/(-b*x+a)^(5/2),x)
```

output
```
(2*(16*sqrt(b)*sqrt(a - b*x)*a*b*i*x**2 - 16*sqrt(b)*sqrt(a - b*x)*b**2*i*
x**3 - sqrt(x)*a**3 - 6*sqrt(x)*a**2*b*x + 24*sqrt(x)*a*b**2*x**2 - 16*sqr
t(x)*b**3*x**3))/(3*sqrt(a - b*x)*a**4*x**2*(a - b*x))
```

3.573 $\int \frac{x^{3/2}}{\sqrt{1-x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{x^{3/2}}{\sqrt{1-x}} dx = -\frac{3}{4}\sqrt{1-x}\sqrt{x} - \frac{1}{2}\sqrt{1-x}x^{3/2} + \frac{3\arcsin\left(\sqrt{x}\right)}{4}$$

output
```
-3/4*(1-x)^(1/2)*x^(1/2)-1/2*(1-x)^(1/2)*x^(3/2)+3/4*arcsin(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}}{\sqrt{1-x}} dx = -\frac{1}{4}\sqrt{-((-1+x)x)}(3+2x) + \frac{3}{2}\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

input
```
Integrate[x^(3/2)/Sqrt[1 - x],x]
```

output
```
-1/4*(Sqrt[-((-1 + x)*x)]*(3 + 2*x)) + (3*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x]
)])/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx-\frac{1}{2}\sqrt{1-x}x^{3/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{x}}dx-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}$$

$$\Big\downarrow \text{62}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}dx-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{4}\left(-\frac{1}{2}\arcsin(1-2x)-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}$$

input
```
Int[x^(3/2)/Sqrt[1 - x],x]
```

output
```
-1/2*(Sqrt[1 - x]*x^(3/2)) + (3*(-(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/
2))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result	size
meijerg	$-\frac{i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,(10x+15)\sqrt{1-x}}{20}+\frac{3i\sqrt{\pi}\,\arcsin\left(\sqrt{x}\right)}{4}\right)}{\sqrt{\pi}}$	39
default	$-\frac{\sqrt{1-x}\,x^{\frac{3}{2}}}{2}-\frac{3\sqrt{1-x}\,\sqrt{x}}{4}+\frac{3\sqrt{x(1-x)}\,\arcsin(-1+2x)}{8\sqrt{x}\,\sqrt{1-x}}$	53
risch	$\frac{(2x+3)\sqrt{x}\,(-1+x)\sqrt{x(1-x)}}{4\sqrt{-x(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{x(1-x)}\,\arcsin(-1+2x)}{8\sqrt{x}\,\sqrt{1-x}}$	66

input
```
int(x^(3/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I/Pi^(1/2)*(-1/20*I*Pi^(1/2)*x^(1/2)*(10*x+15)*(1-x)^(1/2)+3/4*I*Pi^(1/2)
*arcsin(x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{4}\,(2\,x+3)\sqrt{x}\sqrt{-x+1} - \frac{3}{4}\,\arctan\left(\frac{\sqrt{x}\sqrt{-x+1}}{x-1}\right)$$

input
```
integrate(x^(3/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*x + 3)*sqrt(x)*sqrt(-x + 1) - 3/4*arctan(sqrt(x)*sqrt(-x + 1)/(x -
 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int \frac{x^{3/2}}{\sqrt{1-x}}\,dx = \begin{cases} -\frac{ix^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{ix^{\frac{3}{2}}}{4\sqrt{x-1}} + \frac{3i\sqrt{x}}{4\sqrt{x-1}} - \frac{3i\operatorname{acosh}\left(\sqrt{x}\right)}{4} & \text{for } |x| > 1 \\ \frac{x^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{x^{\frac{3}{2}}}{4\sqrt{1-x}} - \frac{3\sqrt{x}}{4\sqrt{1-x}} + \frac{3\operatorname{asin}\left(\sqrt{x}\right)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(1-x)**(1/2),x)
```

output
```
Piecewise((-I*x**(5/2)/(2*sqrt(x - 1)) - I*x**(3/2)/(4*sqrt(x - 1)) + 3*I*
sqrt(x)/(4*sqrt(x - 1)) - 3*I*acosh(sqrt(x))/4, Abs(x) > 1), (x**(5/2)/(2*
sqrt(1 - x)) + x**(3/2)/(4*sqrt(1 - x)) - 3*sqrt(x)/(4*sqrt(1 - x)) + 3*as
in(sqrt(x))/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{x^{3/2}}{\sqrt{1-x}}\,dx = -\frac{\frac{5\sqrt{-x+1}}{\sqrt{x}} + \frac{3(-x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{4\left(\frac{(x-1)^2}{x^2} - \frac{2(x-1)}{x} + 1\right)} - \frac{3}{4}\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

input
```
integrate(x^(3/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(5*sqrt(-x + 1)/sqrt(x) + 3*(-x + 1)^(3/2)/x^(3/2))/((x - 1)^2/x^2 -
2*(x - 1)/x + 1) - 3/4*arctan(sqrt(-x + 1)/sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{4}(2x+3)\sqrt{x}\sqrt{-x+1} + \frac{3}{4}\arcsin\left(\sqrt{x}\right)$$

input
```
integrate(x^(3/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*x + 3)*sqrt(x)*sqrt(-x + 1) + 3/4*arcsin(sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{1-x}}\,dx = \int \frac{x^{3/2}}{\sqrt{1-x}}\,dx$$

input
```
int(x^(3/2)/(1 - x)^(1/2),x)
```

output
```
int(x^(3/2)/(1 - x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{\sqrt{1-x}}\,dx = -\frac{\sqrt{x}\,\sqrt{1-x}\,x}{2} - \frac{3\sqrt{x}\,\sqrt{1-x}}{4} - \frac{3\log\left(\sqrt{1-x}+\sqrt{x}\,i\right)i}{4}$$

input
```
int(x^(3/2)/(1-x)^(1/2),x)
```

output
```
( - 2*sqrt(x)*sqrt( - x + 1)*x - 3*sqrt(x)*sqrt( - x + 1) - 3*log(sqrt( -
x + 1) + sqrt(x)*i)*i)/4
```

3.574 $\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx=-\sqrt{1-x}\sqrt{x}+\arcsin\left(\sqrt{x}\right)$$

output
```
-(1-x)^(1/2)*x^(1/2)+arcsin(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx=-\sqrt{-((-1+x)x)}+2\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

input
```
Integrate[Sqrt[x]/Sqrt[1 - x],x]
```

output
```
-Sqrt[-((-1 + x)*x)] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{x}}dx-\sqrt{1-x}\sqrt{x}$$

$$\Big\downarrow \text{62}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}dx-\sqrt{1-x}\sqrt{x}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)-\sqrt{1-x}\sqrt{x}$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2}\arcsin(1-2x)-\sqrt{1-x}\sqrt{x}$$

input
```
Int[Sqrt[x]/Sqrt[1 - x],x]
```

output
```
-(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

method	result	size
meijerg	$\frac{i\left(i\sqrt{\pi}\,\sqrt{x}\,\sqrt{1-x}-i\sqrt{\pi}\,\arcsin\left(\sqrt{x}\right)\right)}{\sqrt{\pi}}$	34
default	$-\sqrt{1-x}\,\sqrt{x}+\frac{\sqrt{x(1-x)}\,\arcsin(-1+2x)}{2\sqrt{x}\,\sqrt{1-x}}$	41
risch	$\frac{\sqrt{x}\,(-1+x)\sqrt{x(1-x)}}{\sqrt{-x(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{x(1-x)}\,\arcsin(-1+2x)}{2\sqrt{x}\,\sqrt{1-x}}$	60

input
```
int(x^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/Pi^(1/2)*(I*Pi^(1/2)*x^(1/2)*(1-x)^(1/2)-I*Pi^(1/2)*arcsin(x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = -\sqrt{x}\sqrt{-x+1} - \arctan\left(\frac{\sqrt{x}\sqrt{-x+1}}{x-1}\right)$$

input
```
integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(x)*sqrt(-x + 1) - arctan(sqrt(x)*sqrt(-x + 1)/(x - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = \begin{cases} -i\sqrt{x}\sqrt{x-1} - i\operatorname{acosh}\left(\sqrt{x}\right) & \text{for } |x| > 1 \\ \frac{x^{\frac{3}{2}}}{\sqrt{1-x}} - \frac{\sqrt{x}}{\sqrt{1-x}} + \operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(1-x)**(1/2),x)
```

output
```
Piecewise((-I*sqrt(x)*sqrt(x - 1) - I*acosh(sqrt(x)), Abs(x) > 1), (x**(3/
2)/sqrt(1 - x) - sqrt(x)/sqrt(1 - x) + asin(sqrt(x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = \frac{\sqrt{-x+1}}{\sqrt{x}\left(\frac{x-1}{x}-1\right)} - \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

input
```
integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-x + 1)/(sqrt(x)*((x - 1)/x - 1)) - arctan(sqrt(-x + 1)/sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = -\sqrt{x}\sqrt{-x+1} + \arcsin\left(\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(x)*sqrt(-x + 1) + arcsin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = 2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right) - \sqrt{x}\,\sqrt{1-x}$$

input
```
int(x^(1/2)/(1 - x)^(1/2),x)
```

output
```
2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - x^(1/2)*(1 - x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = -\sqrt{x}\,\sqrt{1-x} - \log\left(\sqrt{1-x} + \sqrt{x}\,i\right) i$$

input
```
int(x^(1/2)/(1-x)^(1/2),x)
```

output
```
- (sqrt(x)*sqrt( - x + 1) + log(sqrt( - x + 1) + sqrt(x)*i)*i)
```

3.575 $\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = 2\arcsin\left(\sqrt{x}\right)$$

output
```
2*arcsin(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

input
```
Integrate[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

output
```
(-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 62}$$

$$\int \frac{1}{\sqrt{x-x^2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)$$

$$\Big\downarrow \text{ 223}$$

$$-\arcsin(1-2x)$$

input
```
Int[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

output
```
-ArcSin[1 - 2*x]
```

Defintions of rubi rules used

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$2 \arcsin\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{x(1-x)}\, \arcsin(-1+2x)}{\sqrt{x}\, \sqrt{1-x}}$	27

input
```
int(1/(1-x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsin(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = -2 \arctan\left(\frac{\sqrt{x}\sqrt{-x+1}}{x-1}\right)$$

input
```
integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(x)*sqrt(-x + 1)/(x - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\sqrt{x}\right) & \text{for } |x| > 1 \\ 2\operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/x**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(x)), Abs(x) > 1), (2*asin(sqrt(x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = -2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

input
```
integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-2*arctan(sqrt(-x + 1)/sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = 2\arcsin\left(\sqrt{x}\right)$$

input
```
integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
2*arcsin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = -4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(1 - x)^(1/2)),x)
```

output
```
-4*atan(((1 - x)^(1/2) - 1)/x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx = -2\log\left(\sqrt{1-x}+\sqrt{x}\,i\right)i$$

input
```
int(1/(1-x)^(1/2)/x^(1/2),x)
```

output
```
 - 2*log(sqrt( - x + 1) + sqrt(x)*i)*i
```

3.576 $\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx=-\frac{2\sqrt{1-x}}{\sqrt{x}}$$

output
```
-2*(1-x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx=-\frac{2\sqrt{1-x}}{\sqrt{x}}$$

input
```
Integrate[1/(Sqrt[1 - x]*x^(3/2)),x]
```

output
```
(-2*Sqrt[1 - x])/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}x^{3/2}} \, dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{2\sqrt{1-x}}{\sqrt{x}}$$

input
```
Int[1/(Sqrt[1 - x]*x^(3/2)),x]
```

output
```
(-2*Sqrt[1 - x])/Sqrt[x]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2\sqrt{1-x}}{\sqrt{x}}$	13
default	$-\frac{2\sqrt{1-x}}{\sqrt{x}}$	13
meijerg	$-\frac{2\sqrt{1-x}}{\sqrt{x}}$	13
orering	$\frac{-2+2x}{\sqrt{1-x}\,\sqrt{x}}$	16
risch	$\frac{2\sqrt{x(1-x)}\,(-1+x)}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{-x(-1+x)}}$	33

input
```
int(1/(1-x)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(1-x)^(1/2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx = -\frac{2\sqrt{-x+1}}{\sqrt{x}}$$

input
```
integrate(1/(1-x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-x + 1)/sqrt(x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx = \begin{cases} -\frac{2i\sqrt{x-1}}{\sqrt{x}} & \text{for } |x|>1 \\ -\frac{2\sqrt{1-x}}{\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/x**(3/2),x)
```

output
```
Piecewise((-2*I*sqrt(x - 1)/sqrt(x), Abs(x) > 1), (-2*sqrt(1 - x)/sqrt(x),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx = -\frac{2\sqrt{-x+1}}{\sqrt{x}}$$

input
```
integrate(1/(1-x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x + 1)/sqrt(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx = -\frac{\sqrt{-x+1}-1}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{-x+1}-1}$$

input
```
integrate(1/(1-x)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
-(sqrt(-x + 1) - 1)/sqrt(x) + sqrt(x)/(sqrt(-x + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx = -\frac{2\sqrt{1-x}}{\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(1 - x)^(1/2)),x)
```

output
```
-(2*(1 - x)^(1/2))/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{1-x}x^{3/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{1-x}-2ix}{x}$$

input
```
int(1/(1-x)^(1/2)/x^(3/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt( - x + 1) + i*x))/x
```

3.577 $\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = -\frac{2\sqrt{1-x}}{3x^{3/2}} - \frac{4\sqrt{1-x}}{3\sqrt{x}}$$

output
```
-2/3*(1-x)^(1/2)/x^(3/2)-4/3*(1-x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = -\frac{2\sqrt{1-x}(1+2x)}{3x^{3/2}}$$

input
```
Integrate[1/(Sqrt[1 - x]*x^(5/2)),x]
```

output
```
(-2*Sqrt[1 - x]*(1 + 2*x))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}x^{5/2}} \, dx$$
$$\Big\downarrow \; 55$$
$$\frac{2}{3}\int \frac{1}{\sqrt{1-x}x^{3/2}}dx - \frac{2\sqrt{1-x}}{3x^{3/2}}$$
$$\Big\downarrow \; 48$$
$$-\frac{2\sqrt{1-x}}{3x^{3/2}} - \frac{4\sqrt{1-x}}{3\sqrt{x}}$$

input
```
Int[1/(Sqrt[1 - x]*x^(5/2)),x]
```

output
```
(-2*Sqrt[1 - x])/(3*x^(3/2)) - (4*Sqrt[1 - x])/(3*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{2(1+2x)\sqrt{1-x}}{3x^{\frac{3}{2}}}$	18
meijerg	$-\frac{2(1+2x)\sqrt{1-x}}{3x^{\frac{3}{2}}}$	18
orering	$\frac{2(-1+x)(1+2x)}{3x^{\frac{3}{2}}\sqrt{1-x}}$	21
default	$-\frac{2\sqrt{1-x}}{3x^{\frac{3}{2}}}-\frac{4\sqrt{1-x}}{3\sqrt{x}}$	26
risch	$\frac{2\sqrt{x(1-x)}\,(2x^2-x-1)}{3\sqrt{1-x}\,x^{\frac{3}{2}}\sqrt{-x(-1+x)}}$	40

input
```
int(1/(1-x)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(3/2)*(1+2*x)*(1-x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = -\frac{2\,(2\,x+1)\sqrt{-x+1}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(1/(1-x)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*x + 1)*sqrt(-x + 1)/x^(3/2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = \begin{cases} -\frac{4\sqrt{-1+\frac{1}{x}}}{3} - \frac{2\sqrt{-1+\frac{1}{x}}}{3x} & \text{for}\ \frac{1}{|x|} > 1 \\ -\frac{4i\sqrt{1-\frac{1}{x}}}{3} - \frac{2i\sqrt{1-\frac{1}{x}}}{3x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/x**(5/2),x)
```

output
```
Piecewise((-4*sqrt(-1 + 1/x)/3 - 2*sqrt(-1 + 1/x)/(3*x), 1/Abs(x) > 1), (-
4*I*sqrt(1 - 1/x)/3 - 2*I*sqrt(1 - 1/x)/(3*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = -\frac{2\sqrt{-x+1}}{\sqrt{x}} - \frac{2(-x+1)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

input
```
integrate(1/(1-x)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x + 1)/sqrt(x) - 2/3*(-x + 1)^(3/2)/x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = -\frac{\left(\sqrt{-x+1}-1\right)^3}{12\,x^{\frac{3}{2}}} - \frac{3\left(\sqrt{-x+1}-1\right)}{4\sqrt{x}} + \frac{x^{\frac{3}{2}}\left(\frac{9\left(\sqrt{-x+1}-1\right)^2}{x}+1\right)}{12\left(\sqrt{-x+1}-1\right)^3}$$

input
```
integrate(1/(1-x)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/12*(sqrt(-x + 1) - 1)^3/x^(3/2) - 3/4*(sqrt(-x + 1) - 1)/sqrt(x) + 1/12
*x^(3/2)*(9*(sqrt(-x + 1) - 1)^2/x + 1)/(sqrt(-x + 1) - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = -\frac{\left(\frac{4\,x}{3}+\frac{2}{3}\right)\sqrt{1-x}}{x^{3/2}}$$

input
```
int(1/(x^(5/2)*(1 - x)^(1/2)),x)
```

output
```
-(((4*x)/3 + 2/3)*(1 - x)^(1/2))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{1-x}x^{5/2}}\,dx = \frac{-\frac{4\sqrt{x}\,\sqrt{1-x}\,x}{3} - \frac{2\sqrt{x}\,\sqrt{1-x}}{3} + \frac{4i\,x^2}{3}}{x^2}$$

input
```
int(1/(1-x)^(1/2)/x^(5/2),x)
```

output
```
(2*( - 2*sqrt(x)*sqrt( - x + 1)*x - sqrt(x)*sqrt( - x + 1) + 2*i*x**2))/(3
*x**2)
```

3.578 $\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = -\frac{2\sqrt{1-x}}{5x^{5/2}} - \frac{8\sqrt{1-x}}{15x^{3/2}} - \frac{16\sqrt{1-x}}{15\sqrt{x}}$$

output
```
-2/5*(1-x)^(1/2)/x^(5/2)-8/15*(1-x)^(1/2)/x^(3/2)-16/15*(1-x)^(1/2)/x^(1/2
)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = -\frac{2\sqrt{1-x}(3+4x+8x^2)}{15x^{5/2}}$$

input
```
Integrate[1/(Sqrt[1 - x]*x^(7/2)),x]
```

output
```
(-2*Sqrt[1 - x]*(3 + 4*x + 8*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4}{5}\int \frac{1}{\sqrt{1-x}x^{5/2}}dx-\frac{2\sqrt{1-x}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-x}x^{3/2}}dx-\frac{2\sqrt{1-x}}{3x^{3/2}}\right)-\frac{2\sqrt{1-x}}{5x^{5/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4}{5}\left(-\frac{2\sqrt{1-x}}{3x^{3/2}}-\frac{4\sqrt{1-x}}{3\sqrt{x}}\right)-\frac{2\sqrt{1-x}}{5x^{5/2}}$$

input
```
Int[1/(Sqrt[1 - x]*x^(7/2)),x]
```

output
```
(4*((-2*Sqrt[1 - x])/(3*x^(3/2)) - (4*Sqrt[1 - x])/(3*Sqrt[x])))/5 - (2*Sq
rt[1 - x])/(5*x^(5/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.42

method	result	size
gosper	$-\frac{2\sqrt{1-x}\,(8x^{2}+4x+3)}{15x^{\frac{5}{2}}}$	23
meijerg	$-\frac{2\left(\frac{8}{3}x^{2}+\frac{4}{3}x+1\right)\sqrt{1-x}}{5x^{\frac{5}{2}}}$	23
orering	$\frac{2(-1+x)(8x^{2}+4x+3)}{15x^{\frac{5}{2}}\sqrt{1-x}}$	26
default	$-\frac{2\sqrt{1-x}}{5x^{\frac{5}{2}}}-\frac{8\sqrt{1-x}}{15x^{\frac{3}{2}}}-\frac{16\sqrt{1-x}}{15\sqrt{x}}$	38
risch	$\frac{2\sqrt{x(1-x)}\,(8x^{3}-4x^{2}-x-3)}{15\sqrt{1-x}\,x^{\frac{5}{2}}\sqrt{-x(-1+x)}}$	45

input
```
int(1/(1-x)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/x^(5/2)*(1-x)^(1/2)*(8*x^2+4*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = -\frac{2\,(8\,x^{2}+4\,x+3)\sqrt{-x+1}}{15\,x^{\frac{5}{2}}}$$

input
```
integrate(1/(1-x)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(8*x^2 + 4*x + 3)*sqrt(-x + 1)/x^(5/2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = \begin{cases} -\frac{16\sqrt{-1+\frac{1}{x}}}{15} - \frac{8\sqrt{-1+\frac{1}{x}}}{15x} - \frac{2\sqrt{-1+\frac{1}{x}}}{5x^2} & \text{for}\ \frac{1}{|x|} > 1 \\ -\frac{16i\sqrt{1-\frac{1}{x}}}{15} - \frac{8i\sqrt{1-\frac{1}{x}}}{15x} - \frac{2i\sqrt{1-\frac{1}{x}}}{5x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/x**(7/2),x)
```

output
```
Piecewise((-16*sqrt(-1 + 1/x)/15 - 8*sqrt(-1 + 1/x)/(15*x) - 2*sqrt(-1 + 1
/x)/(5*x**2), 1/Abs(x) > 1), (-16*I*sqrt(1 - 1/x)/15 - 8*I*sqrt(1 - 1/x)/(
15*x) - 2*I*sqrt(1 - 1/x)/(5*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = -\frac{2\sqrt{-x+1}}{\sqrt{x}} - \frac{4(-x+1)^{\frac{3}{2}}}{3x^{\frac{3}{2}}} - \frac{2(-x+1)^{\frac{5}{2}}}{5x^{\frac{5}{2}}}$$

input
```
integrate(1/(1-x)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x + 1)/sqrt(x) - 4/3*(-x + 1)^(3/2)/x^(3/2) - 2/5*(-x + 1)^(5/2)/
x^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(37) = 74$.

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = -\frac{\left(\sqrt{-x+1}-1\right)^5}{80\,x^{\frac{5}{2}}} - \frac{5\left(\sqrt{-x+1}-1\right)^3}{48\,x^{\frac{3}{2}}} - \frac{5\left(\sqrt{-x+1}-1\right)}{8\sqrt{x}} + \frac{\left(\frac{150\left(\sqrt{-x+1}-1\right)^4}{x^2} + \frac{25\left(\sqrt{-x+1}-1\right)^2}{x} + 3\right)x^{\frac{5}{2}}}{240\left(\sqrt{-x+1}-1\right)^5}$$

input
```
integrate(1/(1-x)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
-1/80*(sqrt(-x + 1) - 1)^5/x^(5/2) - 5/48*(sqrt(-x + 1) - 1)^3/x^(3/2) - 5
/8*(sqrt(-x + 1) - 1)/sqrt(x) + 1/240*(150*(sqrt(-x + 1) - 1)^4/x^2 + 25*(
sqrt(-x + 1) - 1)^2/x + 3)*x^(5/2)/(sqrt(-x + 1) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = -\frac{\sqrt{1-x}\left(\frac{16\,x^2}{15} + \frac{8\,x}{15} + \frac{2}{5}\right)}{x^{5/2}}$$

input
```
int(1/(x^(7/2)*(1 - x)^(1/2)),x)
```

output
```
-((1 - x)^(1/2)*((8*x)/15 + (16*x^2)/15 + 2/5))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{1-x}x^{7/2}}\,dx = \frac{-\frac{16\sqrt{x}\,\sqrt{1-x}\,x^2}{15} - \frac{8\sqrt{x}\,\sqrt{1-x}\,x}{15} - \frac{2\sqrt{x}\,\sqrt{1-x}}{5} + \frac{16i\,x^3}{15}}{x^3}$$

input
```
int(1/(1-x)^(1/2)/x^(7/2),x)
```

output
```
(2*( - 8*sqrt(x)*sqrt( - x + 1)*x**2 - 4*sqrt(x)*sqrt( - x + 1)*x - 3*sqrt
(x)*sqrt( - x + 1) + 8*i*x**3))/(15*x**3)
```

3.579 $\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = \frac{4\sqrt[4]{\frac{dx}{c}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{-c+dx}}{\sqrt{c}}\right)\middle|2\right)}{b\sqrt{c}\sqrt[4]{bx}}$$

output
```
4*(d*x/c)^(1/4)*EllipticE(sin(1/2*arctan((d*x-c)^(1/2)/c^(1/2))),2^(1/2))/
b/c^(1/2)/(b*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = -\frac{4x\sqrt{1-\frac{dx}{c}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{dx}{c}\right)}{(bx)^{5/4}\sqrt{-c+dx}}$$

input
```
Integrate[1/((b*x)^(5/4)*Sqrt[-c + d*x]),x]
```

output
```
(-4*x*Sqrt[1 - (d*x)/c]*Hypergeometric2F1[-1/4, 1/2, 3/4, (d*x)/c])/((b*x)
^(5/4)*Sqrt[-c + d*x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. $2(51) = 102$.

Time = 0.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{5/4}\sqrt{dx-c}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{d\int \frac{1}{\sqrt[4]{bx}\sqrt{dx-c}}dx}{bc}$$

$$\Big\downarrow \text{73}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\int \frac{\sqrt{bx}}{\sqrt{dx-c}}d\sqrt[4]{bx}}{b^2c}$$

$$\Big\downarrow \text{836}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\int \frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}\sqrt{dx-c}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\int \frac{1}{\sqrt{dx-c}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\left(\frac{\int \frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{dx-c}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\int \frac{1}{\sqrt{dx-c}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}$$

$$\Big\downarrow \text{765}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\left(\frac{\int \frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{dx-c}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\sqrt{1-\frac{dx}{c}}\int \frac{1}{\sqrt{1-\frac{dx}{c}}}d\sqrt[4]{bx}}{\sqrt{d}\sqrt{dx-c}}\right)}{b^2c}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\left(\frac{\int \frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{dx-c}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{dx-c}}\right)}{b^2c}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\left(\frac{\sqrt{1-\frac{dx}{c}}\int \frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{1-\frac{dx}{c}}}d\sqrt[4]{bx}}{\sqrt{d}\sqrt{dx-c}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{dx-c}}\right)}{b^2c}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\sqrt{1-\frac{dx}{c}}\int \frac{\sqrt{\frac{\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}}+1}}{\sqrt{1-\frac{\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}}}}d\sqrt[4]{bx}}{\sqrt{d}\sqrt{dx-c}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{dx-c}}\right)}{b^2c}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{4\sqrt{dx-c}}{bc\sqrt[4]{bx}}-\frac{4d\left(\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right)\middle|-1\right)}{d^{3/4}\sqrt{dx-c}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{dx-c}}\right)}{b^2c}$$

input
```
Int[1/((b*x)^(5/4)*Sqrt[-c + d*x]),x]
```

output
```
(4*Sqrt[-c + d*x])/(b*c*(b*x)^(1/4)) - (4*d*((b^(3/4)*c^(3/4)*Sqrt[1 - (d*
x)/c]*EllipticE[ArcSin[(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))], -1])/(d^(
3/4)*Sqrt[-c + d*x]) - (b^(3/4)*c^(3/4)*Sqrt[1 - (d*x)/c]*EllipticF[ArcSin
[(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))], -1])/(d^(3/4)*Sqrt[-c + d*x])))
/(b^2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{xd-c}}dx$$

input
```
int(1/(b*x)^(5/4)/(d*x-c)^(1/2),x)
```

output
```
int(1/(b*x)^(5/4)/(d*x-c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{dx-c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(d*x-c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x)^(3/4)*sqrt(d*x - c)/(b^2*d*x^3 - b^2*c*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = -\frac{i\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx}{c}\right)}{b^{\frac{5}{4}}\sqrt{c}\sqrt[4]{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(b*x)**(5/4)/(d*x-c)**(1/2),x)
```

output
```
-I*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x/c)/(b**(5/4)*sqrt(c)*x**(1/4
)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{dx-c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(d*x-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(d*x - c)), x)
```

Giac [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{dx-c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(d*x-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(d*x - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = \int \frac{1}{(b\,x)^{5/4}\,\sqrt{d\,x-c}}\,dx$$

input
```
int(1/((b*x)^(5/4)*(d*x - c)^(1/2)),x)
```

output
```
int(1/((b*x)^(5/4)*(d*x - c)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c+dx}}\,dx = -\frac{\int \frac{x^{\frac{3}{4}}\sqrt{dx-c}}{-d\,x^3+c\,x^2}dx}{b^{\frac{3}{4}}\sqrt{b}}$$

input
```
int(1/(b*x)^(5/4)/(d*x-c)^(1/2),x)
```

output
```
( - b**(1/4)*int((x**(3/4)*sqrt( - c + d*x))/(c*x**2 - d*x**3),x))/(sqrt(b
)*b)
```

3.580 $\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = \frac{4\sqrt[4]{-\frac{dx}{c}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{-c-dx}}{\sqrt{c}}\right)\middle|2\right)}{b\sqrt{c}\sqrt[4]{bx}}$$

output
```
4*(-d*x/c)^(1/4)*EllipticE(sin(1/2*arctan((-d*x-c)^(1/2)/c^(1/2))),2^(1/2)
)/b/c^(1/2)/(b*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = -\frac{4x\sqrt{1+\frac{dx}{c}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{dx}{c}\right)}{(bx)^{5/4}\sqrt{-c-dx}}$$

input
```
Integrate[1/((b*x)^(5/4)*Sqrt[-c - d*x]),x]
```

output
```
(-4*x*Sqrt[1 + (d*x)/c]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((d*x)/c)])/((b
*x)^(5/4)*Sqrt[-c - d*x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(53) = 106$.

Time = 0.35 (sec) , antiderivative size = 343, normalized size of antiderivative = 6.47, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {61, 73, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{d\int \frac{1}{\sqrt[4]{bx}\sqrt{-c-dx}}dx}{bc}+\frac{4\sqrt{-c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{4d\int \frac{\sqrt{bx}}{\sqrt{-c-dx}}d\sqrt[4]{bx}}{b^2c}+\frac{4\sqrt{-c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\int \frac{1}{\sqrt{-c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\int \frac{\sqrt{b}\sqrt{c}-\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}\sqrt{-c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}+\frac{4\sqrt{-c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\int \frac{1}{\sqrt{-c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\int \frac{\sqrt{b}\sqrt{c}-\sqrt{d}\sqrt{bx}}{\sqrt{-c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}+\frac{4\sqrt{-c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{4d\left(\frac{\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{bc+bdx}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{-c-dx}}-\frac{\int\frac{\sqrt{b}\sqrt{c}-\sqrt{d}\sqrt{bx}}{\sqrt{-c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}+\frac{4\sqrt{-c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{4d\left(\frac{\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{bc+bdx}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{-c-dx}}-\frac{\frac{\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{bc+bdx}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)E\left(2\right.}{\sqrt[4]{d}\sqrt{-c-dx}}}{\sqrt{d}}\right)}{b^2c}$$

$$\frac{4\sqrt{-c-dx}}{bc\sqrt[4]{bx}}$$

input
```
Int[1/((b*x)^(5/4)*Sqrt[-c - d*x]),x]
```

output
```
(4*Sqrt[-c - d*x])/(b*c*(b*x)^(1/4)) + (4*d*(-(((b*(b*x)^(1/4)*Sqrt[-c - d
*x])/(Sqrt[b]*Sqrt[c] + Sqrt[d]*Sqrt[b*x]) + (b^(1/4)*c^(1/4)*Sqrt[(b*c +
b*d*x)/(Sqrt[b]*Sqrt[c] + Sqrt[d]*Sqrt[b*x])^2]*(Sqrt[b]*Sqrt[c] + Sqrt[d]
*Sqrt[b*x])*EllipticE[2*ArcTan[(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))], 1
/2])/(d^(1/4)*Sqrt[-c - d*x]))/Sqrt[d]) + (b^(1/4)*c^(1/4)*Sqrt[(b*c + b*d
*x)/(Sqrt[b]*Sqrt[c] + Sqrt[d]*Sqrt[b*x])^2]*(Sqrt[b]*Sqrt[c] + Sqrt[d]*Sq
rt[b*x])*EllipticF[2*ArcTan[(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))], 1/2]
)/(2*d^(3/4)*Sqrt[-c - d*x])))/(b^2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-xd-c}}dx$$

input
```
int(1/(b*x)^(5/4)/(-d*x-c)^(1/2),x)
```

output
```
int(1/(b*x)^(5/4)/(-d*x-c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-dx-c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(-d*x-c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(b*x)^(3/4)*sqrt(-d*x - c)/(b^2*d*x^3 + b^2*c*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = -\frac{i\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dxe^{i\pi}}{c}\right)}{b^{\frac{5}{4}}\sqrt{c}\sqrt[4]{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(b*x)**(5/4)/(-d*x-c)**(1/2),x)
```

output
```
-I*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x*exp_polar(I*pi)/c)/(b**(5/4)
*sqrt(c)*x**(1/4)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-dx-c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(-d*x-c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(-d*x - c)), x)
```

Giac [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-dx-c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(-d*x-c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(-d*x - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = \int \frac{1}{(b\,x)^{5/4}\,\sqrt{-c-d\,x}}\,dx$$

input
```
int(1/((b*x)^(5/4)*(- c - d*x)^(1/2)),x)
```

output
```
int(1/((b*x)^(5/4)*(- c - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{-c-dx}}\,dx = -\frac{\left(\int \frac{x^{\frac{3}{4}}\sqrt{dx+c}}{d\,x^3+c\,x^2}dx\right)i}{b^{\frac{3}{4}}\sqrt{b}}$$

input
```
int(1/(b*x)^(5/4)/(-d*x-c)^(1/2),x)
```

output
```
( - b**(1/4)*int((x**(3/4)*sqrt(c + d*x))/(c*x**2 + d*x**3),x)*i)/(sqrt(b)
*b)
```

3.581 $\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 319

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = -\frac{4\sqrt{c+dx}}{bc\sqrt[4]{bx}}+\frac{4\sqrt{d}\sqrt[4]{bx}\sqrt{c+dx}}{bc\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)} - \frac{4\sqrt[4]{d}\sqrt{\frac{b(c+dx)}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b^{7/4}c^{3/4}\sqrt{c+dx}} + \frac{2\sqrt[4]{d}\sqrt{\frac{b(c+dx)}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),\frac{1}{2}\right)}{b^{7/4}c^{3/4}\sqrt{c+dx}}$$

output

```
-4*(d*x+c)^(1/2)/b/c/(b*x)^(1/4)+4*d^(1/2)*(b*x)^(1/4)*(d*x+c)^(1/2)/b/c/(
b^(1/2)*c^(1/2)+d^(1/2)*(b*x)^(1/2))-4*d^(1/4)*(b*(d*x+c)/(b^(1/2)*c^(1/2)
+d^(1/2)*(b*x)^(1/2))^2)^(1/2)*(b^(1/2)*c^(1/2)+d^(1/2)*(b*x)^(1/2))*Ellip
ticE(sin(2*arctan(d^(1/4)*(b*x)^(1/4)/b^(1/4)/c^(1/4))),1/2*2^(1/2))/b^(7/
4)/c^(3/4)/(d*x+c)^(1/2)+2*d^(1/4)*(b*(d*x+c)/(b^(1/2)*c^(1/2)+d^(1/2)*(b*
x)^(1/2))^2)^(1/2)*(b^(1/2)*c^(1/2)+d^(1/2)*(b*x)^(1/2))*InverseJacobiAM(2
*arctan(d^(1/4)*(b*x)^(1/4)/b^(1/4)/c^(1/4)),1/2*2^(1/2))/b^(7/4)/c^(3/4)/
(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.15

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = -\frac{4x\sqrt{1+\frac{dx}{c}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{dx}{c}\right)}{(bx)^{5/4}\sqrt{c+dx}}$$

input
```
Integrate[1/((b*x)^(5/4)*Sqrt[c + d*x]),x]
```

output
```
(-4*x*Sqrt[1 + (d*x)/c]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((d*x)/c)])/((b
*x)^(5/4)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {61, 73, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{d\int \frac{1}{\sqrt[4]{bx}\sqrt{c+dx}}dx}{bc}-\frac{4\sqrt{c+dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4d\int \frac{\sqrt{bx}}{\sqrt{c+dx}}d\sqrt[4]{bx}}{b^2c}-\frac{4\sqrt{c+dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{834}$$

$$\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\int \frac{1}{\sqrt{c+dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\int \frac{\sqrt{b}\sqrt{c}-\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}\sqrt{c+dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}-\frac{4\sqrt{c+dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\int \frac{1}{\sqrt{c+dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\int \frac{\sqrt{b}\sqrt{c}-\sqrt{d}\sqrt{bx}}{\sqrt{c+dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}-\frac{4\sqrt{c+dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{761}$$

$$\frac{4d\left(\frac{\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{bc+bdx}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx}}-\frac{\int \frac{\sqrt{b}\sqrt{c}-\sqrt{d}\sqrt{bx}}{\sqrt{c+dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}-\frac{4\sqrt{c+dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{4d\left(\frac{\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{bc+bdx}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx}}-\frac{\frac{\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{bc+bdx}{\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)^2}}\left(\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}\right)E\left(2\right.}{\sqrt[4]{d}\sqrt{c+dx}}}{\sqrt{d}}\right.}{b^2c}$$

$$\frac{4\sqrt{c+dx}}{bc\sqrt[4]{bx}}$$

input
```
Int[1/((b*x)^(5/4)*Sqrt[c + d*x]),x]
```

output
```
(-4*Sqrt[c + d*x])/(b*c*(b*x)^(1/4)) + (4*d*(-((-((b*(b*x)^(1/4)*Sqrt[c +
d*x])/(Sqrt[b]*Sqrt[c] + Sqrt[d]*Sqrt[b*x])) + (b^(1/4)*c^(1/4)*Sqrt[(b*c
+ b*d*x)/(Sqrt[b]*Sqrt[c] + Sqrt[d]*Sqrt[b*x])^2]*(Sqrt[b]*Sqrt[c] + Sqrt[
d]*Sqrt[b*x])*EllipticE[2*ArcTan[(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))],
 1/2])/(d^(1/4)*Sqrt[c + d*x]))/Sqrt[d]) + (b^(1/4)*c^(1/4)*Sqrt[(b*c + b*
d*x)/(Sqrt[b]*Sqrt[c] + Sqrt[d]*Sqrt[b*x])^2]*(Sqrt[b]*Sqrt[c] + Sqrt[d]*S
qrt[b*x])*EllipticF[2*ArcTan[(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))], 1/2
])/(2*d^(3/4)*Sqrt[c + d*x])))/(b^2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{xd+c}}dx$$

input
```
int(1/(b*x)^(5/4)/(d*x+c)^(1/2),x)
```

output
```
int(1/(b*x)^(5/4)/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x)^(3/4)*sqrt(d*x + c)/(b^2*d*x^3 + b^2*c*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.14

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dxe^{i\pi}}{c}\right)}{b^{\frac{5}{4}}\sqrt{c}\sqrt[4]{x}\,\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(b*x)**(5/4)/(d*x+c)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x*exp_polar(I*pi)/c)/(b**(5/4)*sq
rt(c)*x**(1/4)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = \int \frac{1}{(b\,x)^{5/4}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((b*x)^(5/4)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((b*x)^(5/4)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c+dx}}\,dx = \frac{\int \frac{x^{\frac{3}{4}}\sqrt{dx+c}}{d\,x^3+c\,x^2}dx}{b^{\frac{3}{4}}\sqrt{b}}$$

input
```
int(1/(b*x)^(5/4)/(d*x+c)^(1/2),x)
```

output
```
(b**(1/4)*int((x**(3/4)*sqrt(c + d*x))/(c*x**2 + d*x**3),x))/(sqrt(b)*b)
```

3.582 $\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 158

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = -\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}} - \frac{4\sqrt[4]{d}\sqrt{1-\frac{dx}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right)\middle|-1\right)}{b^{5/4}\sqrt[4]{c}\sqrt{c-dx}} + \frac{4\sqrt[4]{d}\sqrt{1-\frac{dx}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{b^{5/4}\sqrt[4]{c}\sqrt{c-dx}}$$

output
```
-4*(-d*x+c)^(1/2)/b/c/(b*x)^(1/4)-4*d^(1/4)*(1-d*x/c)^(1/2)*EllipticE(d^(1
/4)*(b*x)^(1/4)/b^(1/4)/c^(1/4),I)/b^(5/4)/c^(1/4)/(-d*x+c)^(1/2)+4*d^(1/4
)*(1-d*x/c)^(1/2)*EllipticF(d^(1/4)*(b*x)^(1/4)/b^(1/4)/c^(1/4),I)/b^(5/4)
/c^(1/4)/(-d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.31

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = -\frac{4x\sqrt{1-\frac{dx}{c}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{dx}{c}\right)}{(bx)^{5/4}\sqrt{c-dx}}$$

input
```
Integrate[1/((b*x)^(5/4)*Sqrt[c - d*x]),x]
```

output
```
(-4*x*Sqrt[1 - (d*x)/c]*Hypergeometric2F1[-1/4, 1/2, 3/4, (d*x)/c])/((b*x)
^(5/4)*Sqrt[c - d*x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{d\int \frac{1}{\sqrt[4]{bx}\sqrt{c-dx}}dx}{bc}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{4d\int \frac{\sqrt{bx}}{\sqrt{c-dx}}d\sqrt[4]{bx}}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{836}$$

$$-\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\int\frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}\sqrt{c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\int\frac{1}{\sqrt{c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d\left(\frac{\int\frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\int\frac{1}{\sqrt{c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}\right)}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{765}$$

$$-\frac{4d\left(\frac{\int\frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{\sqrt{b}\sqrt{c}\sqrt{1-\frac{dx}{c}}\int\frac{1}{\sqrt{1-\frac{dx}{c}}}d\sqrt[4]{bx}}{\sqrt{d}\sqrt{c-dx}}\right)}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{762}$$

$$-\frac{4d\left(\frac{\int\frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{c-dx}}d\sqrt[4]{bx}}{\sqrt{d}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{c-dx}}\right)}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{1390}$$

$$-\frac{4d\left(\frac{\sqrt{1-\frac{dx}{c}}\int\frac{\sqrt{b}\sqrt{c}+\sqrt{d}\sqrt{bx}}{\sqrt{1-\frac{dx}{c}}}d\sqrt[4]{bx}}{\sqrt{d}\sqrt{c-dx}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{c-dx}}\right)}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{1389}$$

$$-\frac{4d\left(\frac{\sqrt{b}\sqrt{c}\sqrt{1-\frac{dx}{c}}\int\frac{\sqrt{\frac{\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}}+1}}{\sqrt{1-\frac{\sqrt{d}\sqrt{bx}}{\sqrt{b}\sqrt{c}}}}d\sqrt[4]{bx}}{\sqrt{d}\sqrt{c-dx}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{c-dx}}\right)}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{4d\left(\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx}}-\frac{b^{3/4}c^{3/4}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt[4]{bx}}{\sqrt[4]{b}\sqrt[4]{c}}\right),-1\right)}{d^{3/4}\sqrt{c-dx}}\right)}{b^2c}-\frac{4\sqrt{c-dx}}{bc\sqrt[4]{bx}}$$

input
```
Int[1/((b*x)^(5/4)*Sqrt[c - d*x]),x]
```

output
```
(-4*Sqrt[c - d*x])/(b*c*(b*x)^(1/4)) - (4*d*((b^(3/4)*c^(3/4)*Sqrt[1 - (d*
x)/c]*EllipticE[ArcSin[(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))], -1])/(d^(
3/4)*Sqrt[c - d*x]) - (b^(3/4)*c^(3/4)*Sqrt[1 - (d*x)/c]*EllipticF[ArcSin[
(d^(1/4)*(b*x)^(1/4))/(b^(1/4)*c^(1/4))], -1])/(d^(3/4)*Sqrt[c - d*x])))/(
b^2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-xd+c}}dx$$

input
```
int(1/(b*x)^(5/4)/(-d*x+c)^(1/2),x)
```

output
```
int(1/(b*x)^(5/4)/(-d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-dx+c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(-d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(b*x)^(3/4)*sqrt(-d*x + c)/(b^2*d*x^3 - b^2*c*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.29

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dxe^{2i\pi}}{c}\right)}{b^{\frac{5}{4}}\sqrt{c}\sqrt[4]{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(b*x)**(5/4)/(-d*x+c)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x*exp_polar(2*I*pi)/c)/(b**(5/4)*
sqrt(c)*x**(1/4)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-dx+c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(-d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(-d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = \int \frac{1}{(bx)^{\frac{5}{4}}\sqrt{-dx+c}}\,dx$$

input
```
integrate(1/(b*x)^(5/4)/(-d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x)^(5/4)*sqrt(-d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = \int \frac{1}{{\left(b\,x\right)}^{5/4}\,\sqrt{c-d\,x}}\,dx$$

input
```
int(1/((b*x)^(5/4)*(c - d*x)^(1/2)),x)
```

output
```
int(1/((b*x)^(5/4)*(c - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(bx)^{5/4}\sqrt{c-dx}}\,dx = \frac{\int \frac{x^{\frac{3}{4}}\sqrt{-dx+c}}{-d\,x^3+c\,x^2}dx}{b^{\frac{3}{4}}\sqrt{b}}$$

input
```
int(1/(b*x)^(5/4)/(-d*x+c)^(1/2),x)
```

output
```
(b**(1/4)*int((x**(3/4)*sqrt(c - d*x))/(c*x**2 - d*x**3),x))/(sqrt(b)*b)
```

3.583 $\int x^3 \sqrt[3]{a+bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3 \sqrt[3]{a+bx}\,dx = -\frac{3a^3(a+bx)^{4/3}}{4b^4} + \frac{9a^2(a+bx)^{7/3}}{7b^4} - \frac{9a(a+bx)^{10/3}}{10b^4} + \frac{3(a+bx)^{13/3}}{13b^4}$$

output
```
-3/4*a^3*(b*x+a)^(4/3)/b^4+9/7*a^2*(b*x+a)^(7/3)/b^4-9/10*a*(b*x+a)^(10/3)
/b^4+3/13*(b*x+a)^(13/3)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^3 \sqrt[3]{a+bx}\,dx = \frac{3(a+bx)^{4/3}(-81a^3 + 108a^2bx - 126ab^2x^2 + 140b^3x^3)}{1820b^4}$$

input
```
Integrate[x^3*(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(4/3)*(-81*a^3 + 108*a^2*b*x - 126*a*b^2*x^2 + 140*b^3*x^3))/
(1820*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[3]{a+bx}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3 \sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

input
```
Int[x^3*(a + b*x)^(1/3),x]
```

output
```
(-3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a + b*x)^(7/3))/(7*b^4) - (9*a*
(a + b*x)^(10/3))/(10*b^4) + (3*(a + b*x)^(13/3))/(13*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{4}{3}}\left(-140b^{3}x^{3}+126a\,b^{2}x^{2}-108a^{2}bx+81a^{3}\right)}{1820b^{4}}$	43
pseudoelliptic	$-\frac{3(bx+a)^{\frac{4}{3}}\left(-140b^{3}x^{3}+126a\,b^{2}x^{2}-108a^{2}bx+81a^{3}\right)}{1820b^{4}}$	43
orering	$-\frac{3(bx+a)^{\frac{4}{3}}\left(-140b^{3}x^{3}+126a\,b^{2}x^{2}-108a^{2}bx+81a^{3}\right)}{1820b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{9a(bx+a)^{\frac{10}{3}}}{10}+\frac{9a^{2}(bx+a)^{\frac{7}{3}}}{7}-\frac{3a^{3}(bx+a)^{\frac{4}{3}}}{4}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{9a(bx+a)^{\frac{10}{3}}}{10}+\frac{9a^{2}(bx+a)^{\frac{7}{3}}}{7}-\frac{3a^{3}(bx+a)^{\frac{4}{3}}}{4}}{b^{4}}$	50
trager	$-\frac{3\left(-140b^{4}x^{4}-14a\,x^{3}b^{3}+18a^{2}b^{2}x^{2}-27a^{3}bx+81a^{4}\right)(bx+a)^{\frac{1}{3}}}{1820b^{4}}$	54
risch	$-\frac{3\left(-140b^{4}x^{4}-14a\,x^{3}b^{3}+18a^{2}b^{2}x^{2}-27a^{3}bx+81a^{4}\right)(bx+a)^{\frac{1}{3}}}{1820b^{4}}$	54

input
```
int(x^3*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/1820*(b*x+a)^(4/3)*(-140*b^3*x^3+126*a*b^2*x^2-108*a^2*b*x+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int x^3\sqrt[3]{a+bx}\,dx=\frac{3\,(140\,b^4x^4+14\,ab^3x^3-18\,a^2b^2x^2+27\,a^3bx-81\,a^4)(bx+a)^{\frac{1}{3}}}{1820\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/1820*(140*b^4*x^4 + 14*a*b^3*x^3 - 18*a^2*b^2*x^2 + 27*a^3*b*x - 81*a^4)
*(b*x + a)^(1/3)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(68) = 136$.

Time = 1.28 (sec) , antiderivative size = 1742, normalized size of antiderivative = 24.19

$$\int x^3 \sqrt[3]{a + bx}\, dx = \text{Too large to display}$$

input
```
integrate(x**3*(b*x+a)**(1/3),x)
```

output
```
-243*a**(73/3)*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x +
27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10
920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(73/3)/(1820*a**20*b
**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) -
1377*a**(70/3)*b*x*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*
x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4
+ 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1458*a**(70/3)*b*x/(182
0*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b*
*7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10
*x**6) - 3213*a**(67/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10
920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a
**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 3645*a**
(67/3)*b**2*x**2/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*
x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x*
*5 + 1820*a**14*b**10*x**6) - 3927*a**(64/3)*b**3*x**3*(1 + b*x/a)**(1/3)/
(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**1
7*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b
**10*x**6) + 4860*a**(64/3)*b**3*x**3/(1820*a**20*b**4 + 10920*a**19*b**5*
x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3\sqrt[3]{a+bx}\,dx=\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^4}-\frac{9\,(bx+a)^{\frac{10}{3}}a}{10\,b^4}+\frac{9\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^4}-\frac{3\,(bx+a)^{\frac{4}{3}}a^3}{4\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/13*(b*x + a)^(13/3)/b^4 - 9/10*(b*x + a)^(10/3)*a/b^4 + 9/7*(b*x + a)^(7
/3)*a^2/b^4 - 3/4*(b*x + a)^(4/3)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(56) = 112.

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.62

$$\int x^3\sqrt[3]{a+bx}\,dx$$
$$=\frac{3\left(\frac{13\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a}{b^3}+\frac{4\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}\cdots\right.}{b^3}\right.}{1820\,b}$$

input
```
integrate(x^3*(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/1820*(13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/
3)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^3 + 4*(35*(b*x + a)^(13/3) - 182*(b*
x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*
(b*x + a)^(1/3)*a^4)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3 \sqrt[3]{a+bx}\, dx = \frac{3\,{\left(a+b\,x\right)}^{13/3}}{13\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{4/3}}{4\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{7/3}}{7\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{10/3}}{10\,b^4}$$

input
```
int(x^3*(a + b*x)^(1/3),x)
```

output
```
(3*(a + b*x)^(13/3))/(13*b^4) - (3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(
a + b*x)^(7/3))/(7*b^4) - (9*a*(a + b*x)^(10/3))/(10*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int x^3 \sqrt[3]{a+bx}\, dx = \frac{3(bx+a)^{\frac{1}{3}} \left(140b^4x^4+14a\,b^3x^3-18a^2b^2x^2+27a^3bx-81a^4\right)}{1820b^4}$$

input
```
int(x^3*(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(1/3)*( - 81*a**4 + 27*a**3*b*x - 18*a**2*b**2*x**2 + 14*a*b
**3*x**3 + 140*b**4*x**4))/(1820*b**4)
```

3.584 $\int x^2\sqrt[3]{a+bx}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2\sqrt[3]{a+bx}\,dx=\frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}$$

output
```
3/4*a^2*(b*x+a)^(4/3)/b^3-6/7*a*(b*x+a)^(7/3)/b^3+3/10*(b*x+a)^(10/3)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2\sqrt[3]{a+bx}\,dx=\frac{3(a+bx)^{4/3}\left(9a^2-12abx+14b^2x^2\right)}{140b^3}$$

input
```
Integrate[x^2*(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(4/3)*(9*a^2 - 12*a*b*x + 14*b^2*x^2))/(140*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[3]{a+bx} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2 \sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

input
```
Int[x^2*(a + b*x)^(1/3),x]
```

output
```
(3*a^2*(a + b*x)^(4/3))/(4*b^3) - (6*a*(a + b*x)^(7/3))/(7*b^3) + (3*(a +
b*x)^(10/3))/(10*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{3(bx+a)^{\frac{4}{3}}(14b^2x^2-12abx+9a^2)}{140b^3}$	32
pseudoelliptic	$\frac{3(bx+a)^{\frac{4}{3}}(14b^2x^2-12abx+9a^2)}{140b^3}$	32
orering	$\frac{3(bx+a)^{\frac{4}{3}}(14b^2x^2-12abx+9a^2)}{140b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{6a(bx+a)^{\frac{7}{3}}}{7}+\frac{3a^2(bx+a)^{\frac{4}{3}}}{4}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{6a(bx+a)^{\frac{7}{3}}}{7}+\frac{3a^2(bx+a)^{\frac{4}{3}}}{4}}{b^3}$	38
trager	$\frac{3(14b^3x^3+2a\,b^2x^2-3a^2bx+9a^3)(bx+a)^{\frac{1}{3}}}{140b^3}$	43
risch	$\frac{3(14b^3x^3+2a\,b^2x^2-3a^2bx+9a^3)(bx+a)^{\frac{1}{3}}}{140b^3}$	43

input
```
int(x^2*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/140*(b*x+a)^(4/3)*(14*b^2*x^2-12*a*b*x+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x^2\sqrt[3]{a+bx}\,dx = \frac{3\,(14\,b^3x^3+2\,ab^2x^2-3\,a^2bx+9\,a^3)(bx+a)^{\frac{1}{3}}}{140\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/140*(14*b^3*x^3 + 2*a*b^2*x^2 - 3*a^2*b*x + 9*a^3)*(b*x + a)^(1/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.

Time = 0.85 (sec) , antiderivative size = 666, normalized size of antiderivative = 12.57

$$\int x^2\sqrt[3]{a+bx}\,dx = \frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{27a^{\frac{34}{3}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{72a^{\frac{31}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{81a^{\frac{31}{3}}bx}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{60a^{\frac{28}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{81a^{\frac{28}{3}}b^2x^2}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{60a^{\frac{25}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{27a^{\frac{25}{3}}b^3x^3}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{135a^{\frac{22}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{132a^{\frac{19}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{42a^{\frac{16}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}$$

input
```
integrate(x**2*(b*x+a)**(1/3),x)
```

output
```
27*a**(34/3)*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**
6*b**5*x**2 + 140*a**5*b**6*x**3) - 27*a**(34/3)/(140*a**8*b**3 + 420*a**7
*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 72*a**(31/3)*b*x*(1 +
 b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140
*a**5*b**6*x**3) - 81*a**(31/3)*b*x/(140*a**8*b**3 + 420*a**7*b**4*x + 420
*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 60*a**(28/3)*b**2*x**2*(1 + b*x/a)
**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b
**6*x**3) - 81*a**(28/3)*b**2*x**2/(140*a**8*b**3 + 420*a**7*b**4*x + 420*
a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 60*a**(25/3)*b**3*x**3*(1 + b*x/a)*
*(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b*
*6*x**3) - 27*a**(25/3)*b**3*x**3/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a
**6*b**5*x**2 + 140*a**5*b**6*x**3) + 135*a**(22/3)*b**4*x**4*(1 + b*x/a)*
*(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b*
*6*x**3) + 132*a**(19/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420
*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 42*a**(16/3)*b**
6*x**6*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5
*x**2 + 140*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2 \sqrt[3]{a+bx}\, dx = \frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^3} - \frac{6\,(bx+a)^{\frac{7}{3}}a}{7\,b^3} + \frac{3\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/10*(b*x + a)^(10/3)/b^3 - 6/7*(b*x + a)^(7/3)*a/b^3 + 3/4*(b*x + a)^(4/3
)*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int x^2 \sqrt[3]{a+bx}\,dx = \frac{3\left(\frac{10\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a}{b^2}+\frac{14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3}{b^2}\right)}{140\,b}$$

input
```
integrate(x^2*(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/140*(10*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^
2)*a/b^2 + (14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/
3)*a^2 - 140*(b*x + a)^(1/3)*a^3)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2 \sqrt[3]{a+bx}\,dx = \frac{42\,(a+b\,x)^{10/3}-120\,a\,(a+b\,x)^{7/3}+105\,a^2\,(a+b\,x)^{4/3}}{140\,b^3}$$

input
```
int(x^2*(a + b*x)^(1/3),x)
```

output
```
(42*(a + b*x)^(10/3) - 120*a*(a + b*x)^(7/3) + 105*a^2*(a + b*x)^(4/3))/(1
40*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x^2 \sqrt[3]{a+bx}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\left(14b^3x^3+2a\,b^2x^2-3a^2bx+9a^3\right)}{140b^3}$$

input
```
int(x^2*(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(1/3)*(9*a**3 - 3*a**2*b*x + 2*a*b**2*x**2 + 14*b**3*x**3))/
(140*b**3)
```

3.585 $\int x\sqrt[3]{a+bx}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\sqrt[3]{a+bx}\,dx=-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}$$

output
```
-3/4*a*(b*x+a)^(4/3)/b^2+3/7*(b*x+a)^(7/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\sqrt[3]{a+bx}\,dx=\frac{3\sqrt[3]{a+bx}(-3a^2+abx+4b^2x^2)}{28b^2}$$

input
```
Integrate[x*(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(1/3)*(-3*a^2 + a*b*x + 4*b^2*x^2))/(28*b^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[3]{a+bx}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{(a+bx)^{4/3}}{b}-\frac{a\sqrt[3]{a+bx}}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

input
```
Int[x*(a + b*x)^(1/3),x]
```

output
```
(-3*a*(a + b*x)^(4/3))/(4*b^2) + (3*(a + b*x)^(7/3))/(7*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{4}{3}}(-4bx+3a)}{28b^2}$	21
pseudoelliptic	$-\frac{3(bx+a)^{\frac{4}{3}}(-4bx+3a)}{28b^2}$	21
orering	$-\frac{3(bx+a)^{\frac{4}{3}}(-4bx+3a)}{28b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{4}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{4}}{b^2}$	26
trager	$-\frac{3(-4b^2x^2-abx+3a^2)(bx+a)^{\frac{1}{3}}}{28b^2}$	32
risch	$-\frac{3(-4b^2x^2-abx+3a^2)(bx+a)^{\frac{1}{3}}}{28b^2}$	32

input
```
int(x*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/28*(b*x+a)^(4/3)*(-4*b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\sqrt[3]{a+bx}\,dx=\frac{3\,(4\,b^2x^2+abx-3\,a^2)(bx+a)^{\frac{1}{3}}}{28\,b^2}$$

input
```
integrate(x*(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/28*(4*b^2*x^2 + a*b*x - 3*a^2)*(b*x + a)^(1/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.

Time = 0.56 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int x\sqrt[3]{a+bx}\,dx = -\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x} + \frac{9a^{\frac{13}{3}}}{28a^2b^2+28ab^3x} - \frac{6a^{\frac{10}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x} + \frac{9a^{\frac{10}{3}}bx}{28a^2b^2+28ab^3x} + \frac{15a^{\frac{7}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x} + \frac{12a^{\frac{4}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}$$

input
```
integrate(x*(b*x+a)**(1/3),x)
```

output
```
-9*a**(13/3)*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 9*a**(13/3)
/(28*a**2*b**2 + 28*a*b**3*x) - 6*a**(10/3)*b*x*(1 + b*x/a)**(1/3)/(28*a**
2*b**2 + 28*a*b**3*x) + 9*a**(10/3)*b*x/(28*a**2*b**2 + 28*a*b**3*x) + 15*
a**(7/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 12*a*
*(4/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x\sqrt[3]{a+bx}\,dx = \frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^2} - \frac{3\,(bx+a)^{\frac{4}{3}}a}{4\,b^2}$$

input
```
integrate(x*(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/7*(b*x + a)^(7/3)/b^2 - 3/4*(b*x + a)^(4/3)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int x\sqrt[3]{a+bx}\,dx = \frac{3\left(\frac{7\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a}{b}+\frac{2\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)}{b}\right)}{28\,b}$$

input
```
integrate(x*(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/28*(7*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a/b + 2*(2*(b*x + a)^(7/3)
 - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x\sqrt[3]{a+bx}\,dx = -\frac{21\,a\,{(a+b\,x)}^{4/3}-12\,{(a+b\,x)}^{7/3}}{28\,b^2}$$

input
```
int(x*(a + b*x)^(1/3),x)
```

output
```
-(21*a*(a + b*x)^(4/3) - 12*(a + b*x)^(7/3))/(28*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\sqrt[3]{a+bx}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\,(4b^2x^2+abx-3a^2)}{28b^2}$$

input
```
int(x*(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(1/3)*( - 3*a**2 + a*b*x + 4*b**2*x**2))/(28*b**2)
```

3.586 $\int \sqrt[3]{a+bx}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \sqrt[3]{a+bx}\,dx=\frac{3(a+bx)^{4/3}}{4b}$$

output
```
3/4*(b*x+a)^(4/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx}\,dx=\frac{3(a+bx)^{4/3}}{4b}$$

input
```
Integrate[(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(4/3))/(4*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx}\,dx$$
$$\Big\downarrow \; 17$$
$$\frac{3(a+bx)^{4/3}}{4b}$$

input
```
Int[(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(4/3))/(4*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
default	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
trager	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
risch	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
pseudoelliptic	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
orering	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13

input
```
int((b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*(b*x+a)^(4/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{a+bx}\,dx = \frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

input
```
integrate((b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/4*(b*x + a)^(4/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{a+bx}\,dx = \frac{3(a+bx)^{\frac{4}{3}}}{4b}$$

input
```
integrate((b*x+a)**(1/3),x)
```

output
```
3*(a + b*x)**(4/3)/(4*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{a+bx}\,dx = \frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

input
```
integrate((b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/4*(b*x + a)^(4/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{a+bx}\,dx = \frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

input
```
integrate((b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/4*(b*x + a)^(4/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{a+bx}\,dx=\frac{3\,{\left(a+b\,x\right)}^{4/3}}{4\,b}$$

input
```
int((a + b*x)^(1/3),x)
```

output
```
(3*(a + b*x)^(4/3))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{a+bx}\,dx=\frac{3(bx+a)^{\frac{4}{3}}}{4b}$$

input
```
int((b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(1/3)*(a + b*x))/(4*b)
```

3.587 $\int \frac{\sqrt[3]{a+bx}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{\sqrt[3]{a+bx}}{x} \, dx = 3\sqrt[3]{a+bx} - \sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

output
```
3*(b*x+a)^(1/3)-3^(1/2)*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/
2)/a^(1/3))-1/2*a^(1/3)*ln(x)+3/2*a^(1/3)*ln(a^(1/3)-(b*x+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt[3]{a+bx}}{x} \, dx = 3\sqrt[3]{a+bx} - \sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

input
```
Integrate[(a + b*x)^(1/3)/x,x]
```

output
```
3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3
))/Sqrt[3]] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[a^(2/3
) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$a\int \frac{1}{x(a+bx)^{2/3}}dx+3\sqrt[3]{a+bx}$$

$$\Big\downarrow \text{ 69}$$

$$a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}$$

$$\Big\downarrow \text{ 16}$$

$$a\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}$$

$$\Big\downarrow \text{ 1082}$$

$$a\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}$$

$$\Big\downarrow \text{ 217}$$

$$a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}$$

input

```
Int[(a + b*x)^(1/3)/x,x]
```

output

```
3*(a + b*x)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))
/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3
)])/(2*a^(2/3)))
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$3(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{a^{\frac{1}{3}}\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2}-a^{\frac{1}{3}}\sqrt{3}\,\arctan\Bigg($
derivativedivides	$3(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right.$
default	$3(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right.$

input
```
int((b*x+a)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3*(b*x+a)^(1/3)+a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a^(1/3)*ln((b*x+a)^(
2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(2/3*3^(1/2)/a^(
1/3)*(b*x+a)^(1/3)+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = -\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right) - \frac{1}{2}a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^(1/3)/x,x, algorithm="fricas")
```

output
```
-sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a
)/a) - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
) + a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = \frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{a}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+xe^{\frac{2i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{a}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+xe^{\frac{4i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x+a)**(1/3)/x,x)
```

output
```
4*a**(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(3*gamma
(7/3)) + 4*a**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_p
olar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(2*I*pi
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
4/3)/(3*gamma(7/3)) + 4*b**(1/3)*(a/b + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = -\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + 3\,(bx+a)^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^(1/3)/x,x, algorithm="maxima")
```

output
```
-sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
 - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) +
a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = -\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{1}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + 3\,(bx+a)^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^(1/3)/x,x, algorithm="giac")
```

output
```
-sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
 - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) +
a^(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3*(b*x + a)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = a^{1/3}\ln\left(9\,a\,(a+b\,x)^{1/3} -9\,a^{4/3}\right)+3\,(a+b\,x)^{1/3}+\frac{a^{1/3}\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{a^{1/3}\ln\Bigl(9\,a\,(a+\ldots}{}$$

input
```
int((a + b*x)^(1/3)/x,x)
```

output
```
a^(1/3)*log(9*a*(a + b*x)^(1/3) - 9*a^(4/3)) + 3*(a + b*x)^(1/3) + (a^(1/3
)*log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i -
1))/2 - (a^(1/3)*log(9*a*(a + b*x)^(1/3) + (9*a^(4/3)*(3^(1/2)*1i + 1))/2)
*(3^(1/2)*1i + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)a+6a^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}+2\log\left((bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}\right)a+2\,\mathrm{l}\ldots}{2\ldots}$$

input
```
int((b*x+a)^(1/3)/x,x)
```

output
```
(2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*a - 2*
sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*a + 6*a**
(2/3)*(a + b*x)**(1/3) + 2*log((a + b*x)**(1/6) + a**(1/6))*a + 2*log((a +
 b*x)**(1/6) - a**(1/6))*a - log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)*
*(1/3) + a**(1/3))*a - log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) +
a**(1/3))*a)/(2*a**(2/3))
```

3.588 $\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

output
```
-(b*x+a)^(1/3)/x-1/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3
))*3^(1/2)/a^(2/3)-1/6*b*ln(x)/a^(2/3)+1/2*b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(
2/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{6a^{2/3}\sqrt[3]{a+bx} + 2\sqrt{3}bx\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + bx\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+\right.}{6a^{2/3}x}$$

input
```
Integrate[(a + b*x)^(1/3)/x^2,x]
```

output
```
-1/6*(6*a^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(
1/3))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a
^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(2/3)*x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {51, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx}}{x^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}dx-\frac{\sqrt[3]{a+bx}}{x}$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}b\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}b\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}$$

input
```
Int[(a + b*x)^(1/3)/x^2,x]
```

output
```
-((a + b*x)^(1/3)/x) + (b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1
/3))/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^
(1/3)])/(2*a^(2/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

method	result
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}\right)$
pseudoelliptic	$\frac{-\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)bx+\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)bx-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)bx}{2}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}}{3a^{\frac{2}{3}}x}$

input
```
int((b*x+a)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
3*b*(-1/3*(b*x+a)^(1/3)/b/x+1/9/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/18/a^(
2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/9/a^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}\left(a^2\right)^{\frac{1}{6}}abx\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}}a+2\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{a^2}\right)+\left(a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(b\right.}{6\,a^2x}$$

input
```
integrate((b*x+a)^(1/3)/x^2,x, algorithm="fricas")
```

output
```
-1/6*(6*sqrt(1/3)*(a^2)^(1/6)*a*b*x*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)^(1
/3)*a + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*b*x*log((b*x + a
)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2/3)*b
*x*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x + a)^(1/3)*a^2)/(a^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 643, normalized size of antiderivative = 6.63

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(1/3)/x**2,x)
```

output
```
4*a**(7/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gam
ma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3
)*gamma(7/3)) + 4*a**(7/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*
(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) + 4*a**(7/3)*b*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3
*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) -
 4*a**(4/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x
)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(
a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*p
i/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3
)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2
*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) + 12*a**2*b**(4/3)*(a/b + x)**(1/3)
*exp(2*I*pi/3)*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}} - \frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}} + \frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{2}{3}}} - \frac{(bx+a)^{\frac{1}{3}}}{x}$$

input
```
integrate((b*x+a)^(1/3)/x^2,x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(2/3) - 1/6*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - (b*x + a)^(1/3)/x
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{1}{6}\,b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\,\log\left(\left|(bx+a)^{\frac{1}{3}}-\right.\right.}{a^{\frac{2}{3}}}\right.$$

input
```
integrate((b*x+a)^(1/3)/x^2,x, algorithm="giac")
```

output
```
-1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(2/3) + log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/
3) - 2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x + a)^(1/3)/(b*
x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = \frac{b\,\ln\left(3\,b\,(a+b\,x)^{1/3}-3\,a^{1/3}\,b\right)}{3\,a^{2/3}}-\frac{(a+b\,x)^{1/3}}{x}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+3\,b\,(a+b\,x)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{2/3}}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+3\,b\,(a+b\,x)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{2/3}}$$

input
```
int((a + b*x)^(1/3)/x^2,x)
```

output
```
(b*log(3*b*(a + b*x)^(1/3) - 3*a^(1/3)*b))/(3*a^(2/3)) - (a + b*x)^(1/3)/x
 - (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)^(1/3))*(b - 3^(1/
2)*b*1i))/(6*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)*b*1i))/2 + 3*b*(a + b
*x)^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-6a^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}+2\log\left((bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}\right)bx+$$

input
```
int((b*x+a)^(1/3)/x^2,x)
```

output
```
(2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b*x -
2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b*x - 6
*a**(2/3)*(a + b*x)**(1/3) + 2*log((a + b*x)**(1/6) + a**(1/6))*b*x + 2*lo
g((a + b*x)**(1/6) - a**(1/6))*b*x - log( - a**(1/6)*(a + b*x)**(1/6) + (a
 + b*x)**(1/3) + a**(1/3))*b*x - log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)
**(1/3) + a**(1/3))*b*x)/(6*a**(2/3)*x)
```

3.589 $\int \frac{\sqrt[3]{a+bx}}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 127

$$\int \frac{\sqrt[3]{a+bx}}{x^3} \, dx = -\frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax} + \frac{b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}} + \frac{b^2 \log(x)}{18a^{5/3}} - \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{6a^{5/3}}$$

output
```
-1/2*(b*x+a)^(1/3)/x^2-1/6*b*(b*x+a)^(1/3)/a/x+1/9*b^2*arctan(1/3*(a^(1/3)
+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)+1/18*b^2*ln(x)/a^(5/3)-
1/6*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt[3]{a+bx}}{x^3} \, dx = \frac{-\frac{3a^{2/3}\sqrt[3]{a+bx}(3a+bx)}{x^2} + 2\sqrt{3}b^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right) + b^2 \log\left(a^{2/3} + \sqrt[3]{a}\right.}{18a^{5/3}}$$

input
```
Integrate[(a + b*x)^(1/3)/x^3,x]
```

output
```
((-3*a^(2/3)*(a + b*x)^(1/3)*(3*a + b*x))/x^2 + 2*Sqrt[3]*b^2*ArcTan[(1 +
(2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b^2*Log[a^(1/3) - (a + b*x)^(1/3
)] + b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(
5/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx}}{x^3} dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{6} b \int \frac{1}{x^2 (a+bx)^{2/3}} dx - \frac{\sqrt[3]{a+bx}}{2x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6} b \left(-\frac{2b \int \frac{1}{x(a+bx)^{2/3}} dx}{3a} - \frac{\sqrt[3]{a+bx}}{ax} \right) - \frac{\sqrt[3]{a+bx}}{2x^2}$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{6} b \left(-\frac{2b \left(-\frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}} d\sqrt[3]{a+bx}}{2a^{2/3}} - \frac{3 \int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}} d\sqrt[3]{a+bx}}{2\sqrt[3]{a}} - \frac{\log(x)}{2a^{2/3}} \right)}{3a} - \frac{\sqrt[3]{a+bx}}{ax} \right) - \frac{\sqrt[3]{a+bx}}{2x^2}$$

$$\Big\downarrow \ 16$$

$$\frac{1}{6}b\left(-\frac{2b\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)-\frac{\sqrt[3]{a+bx}}{2x^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{6}b\left(-\frac{2b\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)-\frac{\sqrt[3]{a+bx}}{2x^2}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{6}b\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)-\frac{\sqrt[3]{a+bx}}{2x^2}$$

input
```
Int[(a + b*x)^(1/3)/x^3,x]
```

output
```
-1/2*(a + b*x)^(1/3)/x^2 + (b*(-((a + b*x)^(1/3)/(a*x)) - (2*b*(-((Sqrt[3]
*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a
^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))))/(3*a)))/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

method	result
derivativedivides	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}}{9a}\right.$
default	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}}{9a}\right.$
pseudoelliptic	$\frac{2b^2\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^2-2b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^2+b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2-3bx(bx+}{18a^{\frac{5}{3}}x^2}$

input
```
int((b*x+a)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3*b^2*(-(1/18/a*(b*x+a)^(4/3)+1/9*(b*x+a)^(1/3))/b^2/x^2-1/9/a*(1/3/a^(2/3
)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(
1/3)+a^(2/3))-1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1
/3)+1))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-(-a^2)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-a^2)^{\frac{1}{3}}a-2(-a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-(-a^2)^{\frac{1}{3}}}}{a^2}\right)+(-a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a\right.}{18}$$

input
```
integrate((b*x+a)^(1/3)/x^3,x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a^2)^(1/3))*arctan(-sqrt(1/3)*((-a^2)^
(1/3)*a - 2*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^2
)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)*(b*x
 + a)^(1/3)) - 2*(-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (-a^2)^(2/3)
) - 3*(a^2*b*x + 3*a^3)*(b*x + a)^(1/3))/(a^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 2266, normalized size of antiderivative = 17.84

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(1/3)/x**3,x)
```

output
```
-4*a**(16/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3)
)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) -
 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(
27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma
(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*
(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*exp(-2*I*pi/3)*l
og(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/
(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamm
a(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(
2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) +
81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x
)**3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*log(1 - b**(1
/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp
(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**
3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(-2*I*pi/3...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}} + \frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}} - \frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{5}{3}}} - \frac{(bx+a)^{\frac{4}{3}}b^2+2\,(bx+a)^{\frac{1}{3}}ab^2}{6\,\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

input
```
integrate((b*x+a)^(1/3)/x^3,x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(5/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(5/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(5/3) - 1/6*((b*x + a
)^(4/3)*b^2 + 2*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 +
a^3)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx$$
$$= \frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left((bx+a)^{\frac{4}{3}}b^3+2\,(bx+a)^{\frac{1}{3}}a\right.}{ab^2x^2}}{18\,b}$$

input
```
integrate((b*x+a)^(1/3)/x^3,x, algorithm="giac")
```

output
```
1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(5/3) + b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(5/3) - 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x + a
)^(4/3)*b^3 + 2*(b*x + a)^(1/3)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{b^2\ln\left(\frac{b^2}{(-a)^{2/3}}-\frac{b^2(a+bx)^{1/3}}{a}\right)}{9(-a)^{5/3}} - \frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1\mathrm{i}}{2(-a)^{2/3}}+\frac{b^2(a+bx)^{1/3}}{a}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{18(-a)^{5/3}} - \frac{\frac{b^2(a+bx)^{1/3}}{3}+\frac{b^2(a+bx)^{4/3}}{6a}}{(a+bx)^2-2a(a+bx)+a^2} + \frac{b^2\ln\left(\frac{b^2(a+bx)^{1/3}}{a}-\frac{b^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{2/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9(-a)^{5/3}}$$

input
```
int((a + b*x)^(1/3)/x^3,x)
```

output
```
(b^2*log(b^2/(-a)^(2/3) - (b^2*(a + b*x)^(1/3))/a))/(9*(-a)^(5/3)) - (log(
(3^(1/2)*b^2*1i + b^2)/(2*(-a)^(2/3)) + (b^2*(a + b*x)^(1/3))/a)*(3^(1/2)*
b^2*1i + b^2))/(18*(-a)^(5/3)) - ((b^2*(a + b*x)^(1/3))/3 + (b^2*(a + b*x)
^(4/3))/(6*a))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + (b^2*log((b^2*(a + b*
x)^(1/3))/a - (b^2*((3^(1/2)*1i)/2 - 1/2))/(-a)^(2/3))*((3^(1/2)*1i)/2 - 1
/2))/(9*(-a)^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2+2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2-9a^{\frac{5}{3}}(bx+a)^{\frac{1}{3}}-3a^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}bx-2\mathrm{l}}{}$$

input
```
int((b*x+a)^(1/3)/x^3,x)
```

output
```
( - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b**
2*x**2 + 2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3))
)*b**2*x**2 - 9*a**(2/3)*(a + b*x)**(1/3)*a - 3*a**(2/3)*(a + b*x)**(1/3)*
b*x - 2*log((a + b*x)**(1/6) + a**(1/6))*b**2*x**2 - 2*log((a + b*x)**(1/6
) - a**(1/6))*b**2*x**2 + log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1
/3) + a**(1/3))*b**2*x**2 + log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/
3) + a**(1/3))*b**2*x**2)/(18*a**(2/3)*a*x**2)
```

3.590 $\int x^3(a+bx)^{2/3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3(a+bx)^{2/3}\,dx=-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}$$

output
```
-3/5*a^3*(b*x+a)^(5/3)/b^4+9/8*a^2*(b*x+a)^(8/3)/b^4-9/11*a*(b*x+a)^(11/3)
/b^4+3/14*(b*x+a)^(14/3)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^3(a+bx)^{2/3}\,dx=\frac{3(a+bx)^{5/3}\left(-81a^3+135a^2bx-180ab^2x^2+220b^3x^3\right)}{3080b^4}$$

input
```
Integrate[x^3*(a + b*x)^(2/3),x]
```

output
```
(3*(a + b*x)^(5/3)*(-81*a^3 + 135*a^2*b*x - 180*a*b^2*x^2 + 220*b^3*x^3))/
(3080*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{2/3}\,dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

input
```
Int[x^3*(a + b*x)^(2/3),x]
```

output
```
(-3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a + b*x)^(8/3))/(8*b^4) - (9*a*
(a + b*x)^(11/3))/(11*b^4) + (3*(a + b*x)^(14/3))/(14*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{5}{3}}\left(-220b^{3}x^{3}+180a\,b^{2}x^{2}-135a^{2}bx+81a^{3}\right)}{3080b^{4}}$	43
pseudoelliptic	$-\frac{3(bx+a)^{\frac{5}{3}}\left(-220b^{3}x^{3}+180a\,b^{2}x^{2}-135a^{2}bx+81a^{3}\right)}{3080b^{4}}$	43
orering	$-\frac{3(bx+a)^{\frac{5}{3}}\left(-220b^{3}x^{3}+180a\,b^{2}x^{2}-135a^{2}bx+81a^{3}\right)}{3080b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{14}{3}}}{14}-\frac{9a(bx+a)^{\frac{11}{3}}}{11}+\frac{9a^{2}(bx+a)^{\frac{8}{3}}}{8}-\frac{3a^{3}(bx+a)^{\frac{5}{3}}}{5}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{14}{3}}}{14}-\frac{9a(bx+a)^{\frac{11}{3}}}{11}+\frac{9a^{2}(bx+a)^{\frac{8}{3}}}{8}-\frac{3a^{3}(bx+a)^{\frac{5}{3}}}{5}}{b^{4}}$	50
trager	$-\frac{3\left(-220b^{4}x^{4}-40a\,x^{3}b^{3}+45a^{2}b^{2}x^{2}-54a^{3}bx+81a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080b^{4}}$	54
risch	$-\frac{3\left(-220b^{4}x^{4}-40a\,x^{3}b^{3}+45a^{2}b^{2}x^{2}-54a^{3}bx+81a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080b^{4}}$	54

input
```
int(x^3*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/3080*(b*x+a)^(5/3)*(-220*b^3*x^3+180*a*b^2*x^2-135*a^2*b*x+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int x^3(a+bx)^{2/3}\,dx=\frac{3\,(220\,b^4x^4+40\,ab^3x^3-45\,a^2b^2x^2+54\,a^3bx-81\,a^4)(bx+a)^{\frac{2}{3}}}{3080\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3/3080*(220*b^4*x^4 + 40*a*b^3*x^3 - 45*a^2*b^2*x^2 + 54*a^3*b*x - 81*a^4)
*(b*x + a)^(2/3)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(68) = 136$.

Time = 1.32 (sec) , antiderivative size = 1742, normalized size of antiderivative = 24.19

$$\int x^3(a+bx)^{2/3}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(b*x+a)**(2/3),x)
```

output
```
-243*a**(74/3)*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x +
46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18
480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(74/3)/(3080*a**20*b
**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 +
 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) -
1296*a**(71/3)*b*x*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*
x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4
+ 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 1458*a**(71/3)*b*x/(308
0*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b*
*7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10
*x**6) - 2808*a**(68/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18
480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a
**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 3645*a**
(68/3)*b**2*x**2/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*
x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x*
*5 + 3080*a**14*b**10*x**6) - 3120*a**(65/3)*b**3*x**3*(1 + b*x/a)**(2/3)/
(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**1
7*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b
**10*x**6) + 4860*a**(65/3)*b**3*x**3/(3080*a**20*b**4 + 18480*a**19*b**5*
x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{2/3}\,dx = \frac{3\,(bx+a)^{\frac{14}{3}}}{14\,b^4} - \frac{9\,(bx+a)^{\frac{11}{3}}\,a}{11\,b^4} + \frac{9\,(bx+a)^{\frac{8}{3}}\,a^2}{8\,b^4} - \frac{3\,(bx+a)^{\frac{5}{3}}\,a^3}{5\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3/14*(b*x + a)^(14/3)/b^4 - 9/11*(b*x + a)^(11/3)*a/b^4 + 9/8*(b*x + a)^(8
/3)*a^2/b^4 - 3/5*(b*x + a)^(5/3)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(56) = 112.

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.62

$$\int x^3(a+bx)^{2/3}\,dx = \frac{3\left(\frac{7\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^2-220\,(bx+a)^{\frac{2}{3}}a^3\right)a}{b^3}+\frac{2\left(110\,(bx+a)^{\frac{14}{3}}-560\,(bx+a)^{\frac{11}{3}}a+1155\,(bx+a)^{\frac{8}{3}}\right.}{b^3}\right.}{3080\,b}$$

input
```
integrate(x^3*(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3/3080*(7*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/
3)*a^2 - 220*(b*x + a)^(2/3)*a^3)*a/b^3 + 2*(110*(b*x + a)^(14/3) - 560*(b
*x + a)^(11/3)*a + 1155*(b*x + a)^(8/3)*a^2 - 1232*(b*x + a)^(5/3)*a^3 + 7
70*(b*x + a)^(2/3)*a^4)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{2/3}\,dx = \frac{3\,{\left(a+b\,x\right)}^{14/3}}{14\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4}$$

input
```
int(x^3*(a + b*x)^(2/3),x)
```

output
```
(3*(a + b*x)^(14/3))/(14*b^4) - (3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(
a + b*x)^(8/3))/(8*b^4) - (9*a*(a + b*x)^(11/3))/(11*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int x^3(a+bx)^{2/3}\,dx = \frac{3(bx+a)^{\frac{2}{3}}\left(220b^4x^4+40a\,b^3x^3-45a^2b^2x^2+54a^3bx-81a^4\right)}{3080b^4}$$

input
```
int(x^3*(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(2/3)*( - 81*a**4 + 54*a**3*b*x - 45*a**2*b**2*x**2 + 40*a*b
**3*x**3 + 220*b**4*x**4))/(3080*b**4)
```

3.591 $\int x^2(a+bx)^{2/3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2(a+bx)^{2/3}\,dx=\frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}$$

output
```
3/5*a^2*(b*x+a)^(5/3)/b^3-3/4*a*(b*x+a)^(8/3)/b^3+3/11*(b*x+a)^(11/3)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^{2/3}\,dx=\frac{3(a+bx)^{5/3}\left(9a^2-15abx+20b^2x^2\right)}{220b^3}$$

input
```
Integrate[x^2*(a + b*x)^(2/3),x]
```

output
```
(3*(a + b*x)^(5/3)*(9*a^2 - 15*a*b*x + 20*b^2*x^2))/(220*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{2/3}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

input
```
Int[x^2*(a + b*x)^(2/3),x]
```

output
```
(3*a^2*(a + b*x)^(5/3))/(5*b^3) - (3*a*(a + b*x)^(8/3))/(4*b^3) + (3*(a +
b*x)^(11/3))/(11*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{3(bx+a)^{\frac{5}{3}}(20b^2x^2-15abx+9a^2)}{220b^3}$	32
pseudoelliptic	$\frac{3(bx+a)^{\frac{5}{3}}(20b^2x^2-15abx+9a^2)}{220b^3}$	32
orering	$\frac{3(bx+a)^{\frac{5}{3}}(20b^2x^2-15abx+9a^2)}{220b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{3a(bx+a)^{\frac{8}{3}}}{4}+\frac{3a^2(bx+a)^{\frac{5}{3}}}{5}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{3a(bx+a)^{\frac{8}{3}}}{4}+\frac{3a^2(bx+a)^{\frac{5}{3}}}{5}}{b^3}$	38
trager	$\frac{3(20b^3x^3+5a\,b^2x^2-6a^2bx+9a^3)(bx+a)^{\frac{2}{3}}}{220b^3}$	43
risch	$\frac{3(20b^3x^3+5a\,b^2x^2-6a^2bx+9a^3)(bx+a)^{\frac{2}{3}}}{220b^3}$	43

input
```
int(x^2*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/220*(b*x+a)^(5/3)*(20*b^2*x^2-15*a*b*x+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x^2(a+bx)^{2/3}\,dx = \frac{3\,(20\,b^3x^3+5\,ab^2x^2-6\,a^2bx+9\,a^3)(bx+a)^{\frac{2}{3}}}{220\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3/220*(20*b^3*x^3 + 5*a*b^2*x^2 - 6*a^2*b*x + 9*a^3)*(b*x + a)^(2/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.

Time = 0.88 (sec) , antiderivative size = 666, normalized size of antiderivative = 12.57

$$\int x^2(a+bx)^{2/3}\,dx = \frac{27a^{\frac{35}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} - \frac{27a^{\frac{35}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} + \frac{63a^{\frac{32}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} - \frac{81a^{\frac{32}{3}}bx}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} + \frac{42a^{\frac{29}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} - \frac{81a^{\frac{29}{3}}b^2x^2}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} + \frac{78a^{\frac{26}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} - \frac{27a^{\frac{26}{3}}b^3x^3}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} + \frac{207a^{\frac{23}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} + \frac{195a^{\frac{20}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3} + \frac{60a^{\frac{17}{3}}b^6x^6\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}$$

input
```
integrate(x**2*(b*x+a)**(2/3),x)
```

output
```
27*a**(35/3)*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**
6*b**5*x**2 + 220*a**5*b**6*x**3) - 27*a**(35/3)/(220*a**8*b**3 + 660*a**7
*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 63*a**(32/3)*b*x*(1 +
 b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220
*a**5*b**6*x**3) - 81*a**(32/3)*b*x/(220*a**8*b**3 + 660*a**7*b**4*x + 660
*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 42*a**(29/3)*b**2*x**2*(1 + b*x/a)
**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b
**6*x**3) - 81*a**(29/3)*b**2*x**2/(220*a**8*b**3 + 660*a**7*b**4*x + 660*
a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 78*a**(26/3)*b**3*x**3*(1 + b*x/a)*
*(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b*
*6*x**3) - 27*a**(26/3)*b**3*x**3/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a
**6*b**5*x**2 + 220*a**5*b**6*x**3) + 207*a**(23/3)*b**4*x**4*(1 + b*x/a)*
*(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b*
*6*x**3) + 195*a**(20/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660
*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 60*a**(17/3)*b**
6*x**6*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5
*x**2 + 220*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2(a+bx)^{2/3}\,dx = \frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^3} - \frac{3\,(bx+a)^{\frac{8}{3}}a}{4\,b^3} + \frac{3\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3/11*(b*x + a)^(11/3)/b^3 - 3/4*(b*x + a)^(8/3)*a/b^3 + 3/5*(b*x + a)^(5/3
)*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int x^2(a+bx)^{2/3}\,dx = \frac{3\left(\frac{11\left(5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}a+20\,(bx+a)^{\frac{2}{3}}a^2\right)a}{b^2}+\frac{40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^2-220\,(bx+a)^{\frac{2}{3}}a^3}{b^2}\right)}{440\,b}$$

input
```
integrate(x^2*(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3/440*(11*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a
^2)*a/b^2 + (40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(
5/3)*a^2 - 220*(b*x + a)^(2/3)*a^3)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2(a+bx)^{2/3}\,dx = \frac{60\,(a+b\,x)^{11/3}-165\,a\,(a+b\,x)^{8/3}+132\,a^2\,(a+b\,x)^{5/3}}{220\,b^3}$$

input
```
int(x^2*(a + b*x)^(2/3),x)
```

output
```
(60*(a + b*x)^(11/3) - 165*a*(a + b*x)^(8/3) + 132*a^2*(a + b*x)^(5/3))/(2
20*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x^2(a+bx)^{2/3}\,dx = \frac{3(bx+a)^{\frac{2}{3}}\left(20b^3x^3+5a\,b^2x^2-6a^2bx+9a^3\right)}{220b^3}$$

input
```
int(x^2*(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(2/3)*(9*a**3 - 6*a**2*b*x + 5*a*b**2*x**2 + 20*b**3*x**3))/
(220*b**3)
```

3.592 $\int x(a+bx)^{2/3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x(a+bx)^{2/3}\,dx=-\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}$$

output
```
-3/5*a*(b*x+a)^(5/3)/b^2+3/8*(b*x+a)^(8/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x(a+bx)^{2/3}\,dx=\frac{3(a+bx)^{2/3}\left(-3a^2+2abx+5b^2x^2\right)}{40b^2}$$

input
```
Integrate[x*(a + b*x)^(2/3),x]
```

output
```
(3*(a + b*x)^(2/3)*(-3*a^2 + 2*a*b*x + 5*b^2*x^2))/(40*b^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{2/3}\,dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(\frac{(a+bx)^{5/3}}{b}-\frac{a(a+bx)^{2/3}}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

input
```
Int[x*(a + b*x)^(2/3),x]
```

output
```
(-3*a*(a + b*x)^(5/3))/(5*b^2) + (3*(a + b*x)^(8/3))/(8*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{5}{3}}(-5bx+3a)}{40b^2}$	21
pseudoelliptic	$-\frac{3(bx+a)^{\frac{5}{3}}(-5bx+3a)}{40b^2}$	21
orering	$-\frac{3(bx+a)^{\frac{5}{3}}(-5bx+3a)}{40b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{3a(bx+a)^{\frac{5}{3}}}{5}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{3a(bx+a)^{\frac{5}{3}}}{5}}{b^2}$	26
trager	$-\frac{3(-5b^2x^2-2abx+3a^2)(bx+a)^{\frac{2}{3}}}{40b^2}$	32
risch	$-\frac{3(-5b^2x^2-2abx+3a^2)(bx+a)^{\frac{2}{3}}}{40b^2}$	32

input
```
int(x*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/40*(b*x+a)^(5/3)*(-5*b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int x(a+bx)^{2/3}\,dx = \frac{3\,(5\,b^2x^2+2\,abx-3\,a^2)(bx+a)^{\frac{2}{3}}}{40\,b^2}$$

input
```
integrate(x*(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3/40*(5*b^2*x^2 + 2*a*b*x - 3*a^2)*(b*x + a)^(2/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.

Time = 0.58 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int x(a+bx)^{2/3}\,dx = -\frac{9a^{\frac{14}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x} + \frac{9a^{\frac{14}{3}}}{40a^2b^2+40ab^3x} - \frac{3a^{\frac{11}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x} + \frac{9a^{\frac{11}{3}}bx}{40a^2b^2+40ab^3x} + \frac{21a^{\frac{8}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x} + \frac{15a^{\frac{5}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}$$

input
```
integrate(x*(b*x+a)**(2/3),x)
```

output
```
-9*a**(14/3)*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 9*a**(14/3)
/(40*a**2*b**2 + 40*a*b**3*x) - 3*a**(11/3)*b*x*(1 + b*x/a)**(2/3)/(40*a**
2*b**2 + 40*a*b**3*x) + 9*a**(11/3)*b*x/(40*a**2*b**2 + 40*a*b**3*x) + 21*
a**(8/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 15*a*
*(5/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(a+bx)^{2/3}\,dx = \frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^2} - \frac{3\,(bx+a)^{\frac{5}{3}}a}{5\,b^2}$$

input
```
integrate(x*(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3/8*(b*x + a)^(8/3)/b^2 - 3/5*(b*x + a)^(5/3)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int x(a+bx)^{2/3}\,dx = \frac{3\left(\frac{4\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}a\right)a}{b}+\frac{5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}a+20\,(bx+a)^{\frac{2}{3}}a^2}{b}\right)}{40\,b}$$

input
```
integrate(x*(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3/40*(4*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)*a/b + (5*(b*x + a)^(8/3)
 - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x(a+bx)^{2/3}\,dx = -\frac{24\,a\,(a+b\,x)^{5/3}-15\,(a+b\,x)^{8/3}}{40\,b^2}$$

input
```
int(x*(a + b*x)^(2/3),x)
```

output
```
-(24*a*(a + b*x)^(5/3) - 15*(a + b*x)^(8/3))/(40*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int x(a+bx)^{2/3}\,dx = \frac{3(bx+a)^{\frac{2}{3}}\,(5b^2x^2+2abx-3a^2)}{40b^2}$$

input
```
int(x*(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(2/3)*( - 3*a**2 + 2*a*b*x + 5*b**2*x**2))/(40*b**2)
```

3.593 $\int (a+bx)^{2/3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (a+bx)^{2/3}\,dx = \frac{3(a+bx)^{5/3}}{5b}$$

output
```
3/5*(b*x+a)^(5/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+bx)^{2/3}\,dx = \frac{3(a+bx)^{5/3}}{5b}$$

input
```
Integrate[(a + b*x)^(2/3),x]
```

output
```
(3*(a + b*x)^(5/3))/(5*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{2/3} \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{3(a+bx)^{5/3}}{5b}$$

input
```
Int[(a + b*x)^(2/3),x]
```

output
```
(3*(a + b*x)^(5/3))/(5*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
default	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
trager	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
risch	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
pseudoelliptic	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
orering	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13

input
```
int((b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/5*(b*x+a)^(5/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{2/3}\,dx = \frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate((b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3/5*(b*x + a)^(5/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{2/3}\,dx=\frac{3(a+bx)^{\frac{5}{3}}}{5b}$$

input
```
integrate((b*x+a)**(2/3),x)
```

output
```
3*(a + b*x)**(5/3)/(5*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{2/3}\,dx=\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate((b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3/5*(b*x + a)^(5/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{2/3}\,dx=\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate((b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3/5*(b*x + a)^(5/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{2/3}\,dx=\frac{3\,{\left(a+b\,x\right)}^{5/3}}{5\,b}$$

input
```
int((a + b*x)^(2/3),x)
```

output
```
(3*(a + b*x)^(5/3))/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{2/3}\,dx=\frac{3(bx+a)^{\frac{5}{3}}}{5b}$$

input
```
int((b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(2/3)*(a + b*x))/(5*b)
```

3.594 $\int \frac{(a+bx)^{2/3}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{(a+bx)^{2/3}}{x} dx = \frac{3}{2}(a+bx)^{2/3} + \sqrt{3}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

output
```
3/2*(b*x+a)^(2/3)+3^(1/2)*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(
1/2)/a^(1/3))-1/2*a^(2/3)*ln(x)+3/2*a^(2/3)*ln(a^(1/3)-(b*x+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)^{2/3}}{x} dx = \frac{3}{2}(a+bx)^{2/3} + \sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \frac{1}{2}a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

input
```
Integrate[(a + b*x)^(2/3)/x,x]
```

output
```
(3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^
(1/3))/Sqrt[3]] + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(2/3)*Log[a^
(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{2/3}}{x}dx$$

$$\Big\downarrow \text{ 60}$$

$$a\int \frac{1}{x\sqrt[3]{a+bx}}dx+\frac{3}{2}(a+bx)^{2/3}$$

$$\Big\downarrow \text{ 67}$$

$$a\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx)^{2/3}$$

$$\Big\downarrow \text{ 16}$$

$$a\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx)^{2/3}$$

$$\Big\downarrow \text{ 1082}$$

$$a\left(-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx)^{2/3}$$

$$\Big\downarrow \ 217$$

$$a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx)^{2/3}$$

input
```
Int[(a + b*x)^(2/3)/x,x]
```

output
```
(3*(a + b*x)^(2/3))/2 + a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3
))/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/
3)])/(2*a^(1/3)))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{3(bx+a)^{\frac{2}{3}}}{2}+a^{\frac{2}{3}}\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{a^{\frac{2}{3}}\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2}+a^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\right.$
derivativedivides	$\frac{3(bx+a)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}\right)$
default	$\frac{3(bx+a)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}\right)$

input
```
int((b*x+a)^(2/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3/2*(b*x+a)^(2/3)+a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a^(2/3)*ln((b*x+a)
^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+a^(2/3)*3^(1/2)*arctan(2/3*3^(1/2)/a
^(1/3)*(b*x+a)^(1/3)+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(a^2\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}}{3\,a}\right) - \frac{1}{2}\left(a^2\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right) + \left(a^2\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^(2/3)/x,x, algorithm="fricas")
```

output
```
sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(a^2)^(1/3)*(b*x + a
)^(1/3))/a) - 1/2*(a^2)^(1/3)*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2
)^(2/3)*(b*x + a)^(1/3)) + (a^2)^(1/3)*log((b*x + a)^(1/3)*a - (a^2)^(2/3)
) + 3/2*(b*x + a)^(2/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \frac{5a^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{5a^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{5a^{\frac{2}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{5b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*x+a)**(2/3)/x,x)
```

output
```
5*a**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(3*gamma
(8/3)) + 5*a**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(3*gamma(8/3)) + 5*a**(2/3)*exp(-2*I*pi
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
5/3)/(3*gamma(8/3)) + 5*b**(2/3)*(a/b + x)**(2/3)*gamma(5/3)/(2*gamma(8/3)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^(2/3)/x,x, algorithm="maxima")
```

output
```
sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a
^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/2*(b*x + a)^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{2}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

input
```
integrate((b*x+a)^(2/3)/x,x, algorithm="giac")
```

output
```
sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a
^(2/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/2*(b*x + a)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \frac{3\,(a+b\,x)^{2/3}}{2}$$

$$+a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-9\,a^{7/3}\right)+\frac{a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-\frac{9\,a^{7/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{a^{2/3}\,\ln}{}$$

input

```
int((a + b*x)^(2/3)/x,x)
```

output

```
(3*(a + b*x)^(2/3))/2 + a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - 9*a^(7/3)) + (
a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(
1/2)*1i - 1))/2 - (a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)
*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)a+2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)a+3a^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}+2\log\left((bx\right.}{}$$

input

```
int((b*x+a)^(2/3)/x,x)
```

output

```
( - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*a +
 2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*a + 3*
a**(1/3)*(a + b*x)**(2/3) + 2*log((a + b*x)**(1/6) + a**(1/6))*a + 2*log((
a + b*x)**(1/6) - a**(1/6))*a - log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*
x)**(1/3) + a**(1/3))*a - log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3)
 + a**(1/3))*a)/(2*a**(1/3))
```

3.595 $\int \frac{(a+bx)^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = -\frac{(a+bx)^{2/3}}{x}+\frac{2b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

output
```
-(b*x+a)^(2/3)/x+2/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3
))*3^(1/2)/a^(1/3)-1/3*b*ln(x)/a^(1/3)+b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = \frac{-3\sqrt[3]{a}(a+bx)^{2/3}+2\sqrt{3}bx\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-bx\,l}{3\sqrt[3]{a}x}$$

input
```
Integrate[(a + b*x)^(2/3)/x^2,x]
```

output
```
(-3*a^(1/3)*(a + b*x)^(2/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3)
)/a^(1/3))/Sqrt[3]] + 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] - b*x*Log[a^(2/
3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(1/3)*x)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{2/3}}{x^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{2}{3}b\int \frac{1}{x\sqrt[3]{a+bx}}dx-\frac{(a+bx)^{2/3}}{x}$$

$$\Big\downarrow \text{67}$$

$$\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)-\frac{(a+bx)^{2/3}}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)-\frac{(a+bx)^{2/3}}{x}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}b\left(-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)-\frac{(a+bx)^{2/3}}{x}$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{3}b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)-\frac{(a+bx)^{2/3}}{x}$$

input
```
Int[(a + b*x)^(2/3)/x^2,x]
```

output
```
-((a + b*x)^(2/3)/x) + (2*b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1
/3))/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(
1/3)])/(2*a^(1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{x}+\frac{2b\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}\right)}{3}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3bx}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{1}{3}}}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3bx}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{1}{3}}}\right)$
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)bx+2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)bx-\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)bx-3(bx+a)^{\frac{2}{3}}a^{\frac{1}{3}}}{3x\,a^{\frac{1}{3}}}$

input
```
int((b*x+a)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)^(2/3)/x+2/3*b*(1/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(1/3)*ln
((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3
^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.68

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+3\,\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}b\cdots}{3\cdots}\right.$$

input
```
integrate((b*x+a)^(2/3)/x^2,x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(1/3)*a*b*x*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(b*x
+ a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x + a)^(1/3)*a^(2/3) + 3*a)/x) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b*x +
a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1/3))
 - 3*(b*x + a)^(2/3)*a)/(a*x), 1/3*(6*sqrt(1/3)*a^(2/3)*b*x*arctan(sqrt(1/
3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*b*x*log((b*x + a)^(2/3
) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3)
 - a^(1/3)) - 3*(b*x + a)^(2/3)*a)/(a*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 643, normalized size of antiderivative = 6.84

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(2/3)/x**2,x)
```

output
```
10*a**(8/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*ga
mma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/
3)*gamma(8/3)) + 10*a**(8/3)*b*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**
(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma
(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a
**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)
) - 10*a**(5/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(5/3)*b**2*(a/b + x)*exp(-2*I*pi/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/
3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gam
ma(8/3)) - 10*a**(5/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) -
9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) + 15*a**2*b**(5/3)*(a/b + x)*
*(2/3)*exp(2*I*pi/3)*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*
b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = \frac{2\,\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}} - \frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{1}{3}}} + \frac{2\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}} - \frac{(bx+a)^{\frac{2}{3}}}{x}$$

input
```
integrate((b*x+a)^(2/3)/x^2,x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(1/3) - 1/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 2/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(1/3) - (b*x + a)^(2/3)/x
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = \frac{1}{3}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}} + \frac{2\log\left(\right.}{}\right.$$

input
```
integrate((b*x+a)^(2/3)/x^2,x, algorithm="giac")
```

output
```
1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(1/3) - log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3)
+ 2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x + a)^(2/3)/(b*x))
*b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = \frac{2\,b\,\ln\left(4\,a^{1/3}\,b^2-4\,b^2\,(a+b\,x)^{1/3}\right)}{3\,a^{1/3}} - \frac{(a+b\,x)^{2/3}}{x} - \frac{\ln\left(a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2-4\,b^2\,(a+b\,x)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{1/3}} - \frac{\ln\left(a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2-4\,b^2\,(a+b\,x)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{1/3}}$$

input
```
int((a + b*x)^(2/3)/x^2,x)
```

output
```
(2*b*log(4*a^(1/3)*b^2 - 4*b^2*(a + b*x)^(1/3)))/(3*a^(1/3)) - (a + b*x)^(
2/3)/x - (log(a^(1/3)*(b - 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*x)^(1/3))*(b - 3
^(1/2)*b*1i))/(3*a^(1/3)) - (log(a^(1/3)*(b + 3^(1/2)*b*1i)^2 - 4*b^2*(a +
 b*x)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx+2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-3a^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}+2\log\left((b\right.}{}$$

input
```
int((b*x+a)^(2/3)/x^2,x)
```

output
```
( - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b*x
 + 2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b*x
- 3*a**(1/3)*(a + b*x)**(2/3) + 2*log((a + b*x)**(1/6) + a**(1/6))*b*x + 2
*log((a + b*x)**(1/6) - a**(1/6))*b*x - log( - a**(1/6)*(a + b*x)**(1/6) +
 (a + b*x)**(1/3) + a**(1/3))*b*x - log(a**(1/6)*(a + b*x)**(1/6) + (a + b
*x)**(1/3) + a**(1/3))*b*x)/(3*a**(1/3)*x)
```

3.596 $\int \frac{(a+bx)^{2/3}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 127

$$\int \frac{(a+bx)^{2/3}}{x^3} dx = -\frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax} - \frac{b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}} + \frac{b^2\log(x)}{18a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}$$

output
```
-1/2*(b*x+a)^(2/3)/x^2-1/3*b*(b*x+a)^(2/3)/a/x-1/9*b^2*arctan(1/3*(a^(1/3)
+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)+1/18*b^2*ln(x)/a^(4/3)-
1/6*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{2/3}}{x^3} dx = -\frac{(a+bx)^{2/3}(a+2(a+bx))}{6ax^2} - \frac{b^2 \arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{4/3}} + \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{4/3}}$$

input
```
Integrate[(a + b*x)^(2/3)/x^3,x]
```

output
```
-1/6*((a + b*x)^(2/3)*(a + 2*(a + b*x)))/(a*x^2) - (b^2*ArcTan[1/Sqrt[3] +
 (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) - (b^2*Log[a^
(1/3) - (a + b*x)^(1/3)])/(9*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*
x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(4/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{2/3}}{x^3} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3} b \int \frac{1}{x^2 \sqrt[3]{a+bx}} dx - \frac{(a+bx)^{2/3}}{2x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3} b \left(-\frac{b \int \frac{1}{x \sqrt[3]{a+bx}} dx}{3a} - \frac{(a+bx)^{2/3}}{ax}\right) - \frac{(a+bx)^{2/3}}{2x^2}$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3} b \left(-\frac{b \left(\frac{3}{2} \int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}} d\sqrt[3]{a+bx} - \frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}} d\sqrt[3]{a+bx}}{2\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a} - \frac{(a+bx)^{2/3}}{ax}\right) - \frac{(a+bx)^{2/3}}{2x^2}$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}b\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)-\frac{(a+bx)^{2/3}}{2x^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{3}b\left(-\frac{b\left(-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)-\frac{(a+bx)^{2/3}}{2x^2}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}b\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)-\frac{(a+bx)^{2/3}}{2x^2}$$

input
```
Int[(a + b*x)^(2/3)/x^3,x]
```

output
```
-1/2*(a + b*x)^(2/3)/x^2 + (b*(-((a + b*x)^(2/3)/(a*x)) - (b*((Sqrt[3]*Arc
Tan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3
)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))))/(3*a)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(2bx+3a)}{6x^2a}-\frac{b^2\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}\right)}{9a}$
derivativedivides	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{5}{3}}}{9a}+\frac{(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{9a}\right.$
default	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{5}{3}}}{9a}+\frac{(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{9a}\right.$
pseudoelliptic	$\frac{-2b^2\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^2-2b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^2+b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2-6bx\,a^{\frac{1}{3}}}{18a^{\frac{4}{3}}x^2}$

input
```
int((b*x+a)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+a)^(2/3)*(2*b*x+3*a)/x^2/a-1/9*b^2/a*(1/a^(1/3)*ln((b*x+a)^(1/3)
-a^(1/3))-1/2/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+3^(1
/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx-3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)+a^{\frac{2}{3}}\cdots}{\cdots}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}a^{\frac{2}{3}}b^2x^2\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}b^2x^2\log\left(\cdots\right.}{18\,a^2x^2}$$

input
```
integrate((b*x+a)^(2/3)/x^3,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x - 3*sqrt(1/3)*(2*
(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) + a^(2/3)*b^2*x^2*log((b*x + a)^(2/3)
+ (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(1/
3) - a^(1/3)) - 3*(2*a*b*x + 3*a^2)*(b*x + a)^(2/3))/(a^2*x^2), -1/18*(6*s
qrt(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^
(1/3)) - a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a
^(2/3)) + 2*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(2*a*b*x +
3*a^2)*(b*x + a)^(2/3))/(a^2*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 2266, normalized size of antiderivative = 17.84

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(2/3)/x**3,x)
```

output
```
-10*a**(17/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3
))*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp
(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3
) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**
2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**
(1/3))*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)
*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma
(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)
*b**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamm
a(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi
/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*
a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b
+ x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/
(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gam
ma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b*
*3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3
))*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp
(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3
) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx = -\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}} + \frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}} - \frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{4}{3}}} - \frac{2\,(bx+a)^{\frac{5}{3}}b^2+(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

input
```
integrate((b*x+a)^(2/3)/x^3,x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(4/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(4/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(4/3) - 1/6*(2*(b*x
+ a)^(5/3)*b^2 + (b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 +
 a^3)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx =$$

$$-\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2\,(bx+a)^{\frac{5}{3}}b^3+(bx+a)^{\frac{2}{3}}ab\right.}{ab^2x^2}}{18\,b}$$

input
```
integrate((b*x+a)^(2/3)/x^3,x, algorithm="giac")
```

output
```
-1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(4/3) - b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(4/3) + 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*(b*x
+ a)^(5/3)*b^3 + (b*x + a)^(2/3)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx = \frac{(-1)^{1/3}\,b^2\,\ln\left(\left(a+b\,x\right)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{9\,a^{4/3}} - \frac{\frac{b^2\,(a+b\,x)^{2/3}}{6}+\frac{b^2\,(a+b\,x)^{5/3}}{3\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2} + \frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{4/3}} - \frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{4/3}}$$

input
```
int((a + b*x)^(2/3)/x^3,x)
```

output
```
((-1)^(1/3)*b^2*log((a + b*x)^(1/3) - (-1)^(2/3)*a^(1/3)))/(9*a^(4/3)) - (
(b^2*(a + b*x)^(2/3))/6 + (b^2*(a + b*x)^(5/3))/(3*a))/((a + b*x)^2 - 2*a*
(a + b*x) + a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-
1)^(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2)
)/(9*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^
(2/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(
9*a^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2-9a^{\frac{4}{3}}(bx+a)^{\frac{2}{3}}-6a^{\frac{1}{3}}(b}{}$$

input
```
int((b*x+a)^(2/3)/x^3,x)
```

output

```
(2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b**2*x
**2 - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b
**2*x**2 - 9*a**(1/3)*(a + b*x)**(2/3)*a - 6*a**(1/3)*(a + b*x)**(2/3)*b*x
 - 2*log((a + b*x)**(1/6) + a**(1/6))*b**2*x**2 - 2*log((a + b*x)**(1/6) -
 a**(1/6))*b**2*x**2 + log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3)
 + a**(1/3))*b**2*x**2 + log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3)
+ a**(1/3))*b**2*x**2)/(18*a**(1/3)*a*x**2)
```

3.597 $\int x^3(a+bx)^{4/3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^3(a+bx)^{4/3}\,dx=-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}$$

output
```
-3/7*a^3*(b*x+a)^(7/3)/b^4+9/10*a^2*(b*x+a)^(10/3)/b^4-9/13*a*(b*x+a)^(13/
3)/b^4+3/16*(b*x+a)^(16/3)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int x^3(a+bx)^{4/3}\,dx=\frac{3(a+bx)^{7/3}\left(-81a^3+189a^2bx-315ab^2x^2+455b^3x^3\right)}{7280b^4}$$

input
```
Integrate[x^3*(a + b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(7/3)*(-81*a^3 + 189*a^2*b*x - 315*a*b^2*x^2 + 455*b^3*x^3))/
(7280*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{4/3}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

input
```
Int[x^3*(a + b*x)^(4/3),x]
```

output
```
(-3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a + b*x)^(10/3))/(10*b^4) - (9*
a*(a + b*x)^(13/3))/(13*b^4) + (3*(a + b*x)^(16/3))/(16*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{7}{3}}\left(-455b^{3}x^{3}+315a\,b^{2}x^{2}-189a^{2}bx+81a^{3}\right)}{7280b^{4}}$	43
pseudoelliptic	$-\frac{3(bx+a)^{\frac{7}{3}}\left(-455b^{3}x^{3}+315a\,b^{2}x^{2}-189a^{2}bx+81a^{3}\right)}{7280b^{4}}$	43
orering	$-\frac{3(bx+a)^{\frac{7}{3}}\left(-455b^{3}x^{3}+315a\,b^{2}x^{2}-189a^{2}bx+81a^{3}\right)}{7280b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{16}{3}}}{16}-\frac{9a(bx+a)^{\frac{13}{3}}}{13}+\frac{9a^{2}(bx+a)^{\frac{10}{3}}}{10}-\frac{3a^{3}(bx+a)^{\frac{7}{3}}}{7}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{16}{3}}}{16}-\frac{9a(bx+a)^{\frac{13}{3}}}{13}+\frac{9a^{2}(bx+a)^{\frac{10}{3}}}{10}-\frac{3a^{3}(bx+a)^{\frac{7}{3}}}{7}}{b^{4}}$	50
trager	$-\frac{3\left(-455b^{5}x^{5}-595a\,b^{4}x^{4}-14a^{2}b^{3}x^{3}+18a^{3}b^{2}x^{2}-27a^{4}bx+81a^{5}\right)(bx+a)^{\frac{1}{3}}}{7280b^{4}}$	65
risch	$-\frac{3\left(-455b^{5}x^{5}-595a\,b^{4}x^{4}-14a^{2}b^{3}x^{3}+18a^{3}b^{2}x^{2}-27a^{4}bx+81a^{5}\right)(bx+a)^{\frac{1}{3}}}{7280b^{4}}$	65

input
```
int(x^3*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/7280*(b*x+a)^(7/3)*(-455*b^3*x^3+315*a*b^2*x^2-189*a^2*b*x+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int x^3(a+bx)^{4/3}\,dx = \frac{3\,(455\,b^5x^5+595\,ab^4x^4+14\,a^2b^3x^3-18\,a^3b^2x^2+27\,a^4bx-81\,a^5)(bx+a)^{\frac{1}{3}}}{7280\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
3/7280*(455*b^5*x^5 + 595*a*b^4*x^4 + 14*a^2*b^3*x^3 - 18*a^3*b^2*x^2 + 27
*a^4*b*x - 81*a^5)*(b*x + a)^(1/3)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1844 vs. $2(68) = 136$.

Time = 1.40 (sec) , antiderivative size = 1844, normalized size of antiderivative = 25.61

$$\int x^3(a+bx)^{4/3}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(b*x+a)**(4/3),x)
```

output
```
-243*a**(76/3)*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x +
109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 +
 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 243*a**(76/3)/(7280*a**2
0*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x
**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x*
*6) - 1377*a**(73/3)*b*x*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19
*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b
**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(73/3)
*b*x/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 1456
00*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280
*a**14*b**10*x**6) - 3213*a**(70/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(7280*a**
20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*
x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x
**6) + 3645*a**(70/3)*b**2*x**2/(7280*a**20*b**4 + 43680*a**19*b**5*x + 10
9200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 4
3680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) - 3927*a**(67/3)*b**3*x**3*(
1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**
6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**
9*x**5 + 7280*a**14*b**10*x**6) + 4860*a**(67/3)*b**3*x**3/(7280*a**20*b**
4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{4/3}\,dx = \frac{3\,(bx+a)^{\frac{16}{3}}}{16\,b^4} - \frac{9\,(bx+a)^{\frac{13}{3}}a}{13\,b^4} + \frac{9\,(bx+a)^{\frac{10}{3}}a^2}{10\,b^4} - \frac{3\,(bx+a)^{\frac{7}{3}}a^3}{7\,b^4}$$

input
```
integrate(x^3*(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
3/16*(b*x + a)^(16/3)/b^4 - 9/13*(b*x + a)^(13/3)*a/b^4 + 9/10*(b*x + a)^(
10/3)*a^2/b^4 - 3/7*(b*x + a)^(7/3)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.68

$$\int x^3(a+bx)^{4/3}\,dx = \frac{3\left(\frac{52\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a^2}{b^3}+\frac{32\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}\cdots\right.}{b^3}\cdots\right.}{\cdots}$$

input
```
integrate(x^3*(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
3/7280*(52*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/
3)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a^2/b^3 + 32*(35*(b*x + a)^(13/3) - 182*
(b*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 4
55*(b*x + a)^(1/3)*a^4)*a/b^3 + 5*(91*(b*x + a)^(16/3) - 560*(b*x + a)^(13
/3)*a + 1456*(b*x + a)^(10/3)*a^2 - 2080*(b*x + a)^(7/3)*a^3 + 1820*(b*x +
 a)^(4/3)*a^4 - 1456*(b*x + a)^(1/3)*a^5)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^3(a+bx)^{4/3}\,dx=\frac{3\,{\left(a+b\,x\right)}^{16/3}}{16\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{7/3}}{7\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{10/3}}{10\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{13/3}}{13\,b^4}$$

input
```
int(x^3*(a + b*x)^(4/3),x)
```

output
```
(3*(a + b*x)^(16/3))/(16*b^4) - (3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(
a + b*x)^(10/3))/(10*b^4) - (9*a*(a + b*x)^(13/3))/(13*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int x^3(a+bx)^{4/3}\,dx=\frac{3(bx+a)^{\frac{1}{3}}\left(455b^5x^5+595a\,b^4x^4+14a^2b^3x^3-18a^3b^2x^2+27a^4bx-81a^5\right)}{7280b^4}$$

input
```
int(x^3*(b*x+a)^(4/3),x)
```

output
```
(3*(a + b*x)**(1/3)*( - 81*a**5 + 27*a**4*b*x - 18*a**3*b**2*x**2 + 14*a**
2*b**3*x**3 + 595*a*b**4*x**4 + 455*b**5*x**5))/(7280*b**4)
```

3.598 $\int x^2(a+bx)^{4/3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2(a+bx)^{4/3}\,dx=\frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}$$

output
```
3/7*a^2*(b*x+a)^(7/3)/b^3-3/5*a*(b*x+a)^(10/3)/b^3+3/13*(b*x+a)^(13/3)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^{4/3}\,dx=\frac{3(a+bx)^{7/3}\left(9a^2-21abx+35b^2x^2\right)}{455b^3}$$

input
```
Integrate[x^2*(a + b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(7/3)*(9*a^2 - 21*a*b*x + 35*b^2*x^2))/(455*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{4/3}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

input
```
Int[x^2*(a + b*x)^(4/3),x]
```

output
```
(3*a^2*(a + b*x)^(7/3))/(7*b^3) - (3*a*(a + b*x)^(10/3))/(5*b^3) + (3*(a +
 b*x)^(13/3))/(13*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{3(bx+a)^{\frac{7}{3}}(35b^2x^2-21abx+9a^2)}{455b^3}$	32
pseudoelliptic	$\frac{3(bx+a)^{\frac{7}{3}}(35b^2x^2-21abx+9a^2)}{455b^3}$	32
orering	$\frac{3(bx+a)^{\frac{7}{3}}(35b^2x^2-21abx+9a^2)}{455b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{3a(bx+a)^{\frac{10}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{7}{3}}}{7}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{3a(bx+a)^{\frac{10}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{7}{3}}}{7}}{b^3}$	38
trager	$\frac{3(35b^4x^4+49a\,x^3b^3+2a^2b^2x^2-3a^3bx+9a^4)(bx+a)^{\frac{1}{3}}}{455b^3}$	54
risch	$\frac{3(35b^4x^4+49a\,x^3b^3+2a^2b^2x^2-3a^3bx+9a^4)(bx+a)^{\frac{1}{3}}}{455b^3}$	54

input
```
int(x^2*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/455*(b*x+a)^(7/3)*(35*b^2*x^2-21*a*b*x+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)^{4/3}\,dx = \frac{3\,(35\,b^4x^4+49\,ab^3x^3+2\,a^2b^2x^2-3\,a^3bx+9\,a^4)(bx+a)^{\frac{1}{3}}}{455\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
3/455*(35*b^4*x^4 + 49*a*b^3*x^3 + 2*a^2*b^2*x^2 - 3*a^3*b*x + 9*a^4)*(b*x
 + a)^(1/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(49) = 98$.

Time = 0.96 (sec) , antiderivative size = 733, normalized size of antiderivative = 13.83

$$\int x^2(a+bx)^{4/3}\,dx = \frac{27a^{\frac{37}{3}}\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{27a^{\frac{37}{3}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{72a^{\frac{34}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{81a^{\frac{34}{3}}bx}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{60a^{\frac{31}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{81a^{\frac{31}{3}}b^2x^2}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{165a^{\frac{28}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{27a^{\frac{28}{3}}b^3x^3}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{555a^{\frac{25}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{762a^{\frac{22}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{462a^{\frac{19}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{105a^{\frac{16}{3}}b^7x^7\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}$$

input
```
integrate(x**2*(b*x+a)**(4/3),x)
```

output
```
27*a**(37/3)*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a
**6*b**5*x**2 + 455*a**5*b**6*x**3) - 27*a**(37/3)/(455*a**8*b**3 + 1365*a
**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 72*a**(34/3)*b*x*
(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2
 + 455*a**5*b**6*x**3) - 81*a**(34/3)*b*x/(455*a**8*b**3 + 1365*a**7*b**4*
x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 60*a**(31/3)*b**2*x**2*(1
+ b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 +
455*a**5*b**6*x**3) - 81*a**(31/3)*b**2*x**2/(455*a**8*b**3 + 1365*a**7*b*
*4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 165*a**(28/3)*b**3*x**3
*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**
2 + 455*a**5*b**6*x**3) - 27*a**(28/3)*b**3*x**3/(455*a**8*b**3 + 1365*a**
7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 555*a**(25/3)*b**4*
x**4*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5
*x**2 + 455*a**5*b**6*x**3) + 762*a**(22/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(
455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**
3) + 462*a**(19/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7
*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 105*a**(16/3)*b**7*x
**7*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*
x**2 + 455*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2(a+bx)^{4/3}\,dx = \frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^3} - \frac{3\,(bx+a)^{\frac{10}{3}}a}{5\,b^3} + \frac{3\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^3}$$

input
```
integrate(x^2*(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
3/13*(b*x + a)^(13/3)/b^3 - 3/5*(b*x + a)^(10/3)*a/b^3 + 3/7*(b*x + a)^(7/
3)*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.96

$$\int x^2(a+bx)^{4/3}\,dx = \frac{3\left(\frac{65\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a^2}{b^2}+\frac{13\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a}{b^2}\right.}{910\,b}$$

input
```
integrate(x^2*(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
3/910*(65*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^
2)*a^2/b^2 + 13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a
)^(4/3)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^2 + 2*(35*(b*x + a)^(13/3) - 18
2*(b*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 +
 455*(b*x + a)^(1/3)*a^4)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2(a+bx)^{4/3}\,dx = \frac{105\,{\left(a+b\,x\right)}^{13/3}-273\,a\,{\left(a+b\,x\right)}^{10/3}+195\,a^2\,{\left(a+b\,x\right)}^{7/3}}{455\,b^3}$$

input
```
int(x^2*(a + b*x)^(4/3),x)
```

output
```
(105*(a + b*x)^(13/3) - 273*a*(a + b*x)^(10/3) + 195*a^2*(a + b*x)^(7/3))/
(455*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)^{4/3}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\,(35b^4x^4+49a\,b^3x^3+2a^2b^2x^2-3a^3bx+9a^4)}{455b^3}$$

input
```
int(x^2*(b*x+a)^(4/3),x)
```

output
```
(3*(a + b*x)**(1/3)*(9*a**4 - 3*a**3*b*x + 2*a**2*b**2*x**2 + 49*a*b**3*x*
*3 + 35*b**4*x**4))/(455*b**3)
```

3.599 $\int x(a+bx)^{4/3} dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x(a+bx)^{4/3} dx = -\frac{3a(a+bx)^{7/3}}{7b^2} + \frac{3(a+bx)^{10/3}}{10b^2}$$

output
```
-3/7*a*(b*x+a)^(7/3)/b^2+3/10*(b*x+a)^(10/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int x(a+bx)^{4/3} dx = \frac{3(a+bx)^{7/3}(-3a+7bx)}{70b^2}$$

input
```
Integrate[x*(a + b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(7/3)*(-3*a + 7*b*x))/(70*b^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{4/3}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{(a+bx)^{7/3}}{b}-\frac{a(a+bx)^{4/3}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

input
```
Int[x*(a + b*x)^(4/3),x]
```

output
```
(-3*a*(a + b*x)^(7/3))/(7*b^2) + (3*(a + b*x)^(10/3))/(10*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{7}{3}}(-7bx+3a)}{70b^2}$	21
pseudoelliptic	$-\frac{3(bx+a)^{\frac{7}{3}}(-7bx+3a)}{70b^2}$	21
orering	$-\frac{3(bx+a)^{\frac{7}{3}}(-7bx+3a)}{70b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{3a(bx+a)^{\frac{7}{3}}}{7}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{3a(bx+a)^{\frac{7}{3}}}{7}}{b^2}$	26
trager	$-\frac{3(-7b^3x^3-11a\,b^2x^2-a^2bx+3a^3)(bx+a)^{\frac{1}{3}}}{70b^2}$	43
risch	$-\frac{3(-7b^3x^3-11a\,b^2x^2-a^2bx+3a^3)(bx+a)^{\frac{1}{3}}}{70b^2}$	43

input
```
int(x*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/70*(b*x+a)^(7/3)*(-7*b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x(a+bx)^{4/3}\,dx = \frac{3\,(7\,b^3x^3+11\,ab^2x^2+a^2bx-3\,a^3)(bx+a)^{\frac{1}{3}}}{70\,b^2}$$

input
```
integrate(x*(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
3/70*(7*b^3*x^3 + 11*a*b^2*x^2 + a^2*b*x - 3*a^3)*(b*x + a)^(1/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int x(a+bx)^{4/3}\,dx = \begin{cases} -\frac{9a^3\sqrt[3]{a+bx}}{70b^2} + \frac{3a^2x\sqrt[3]{a+bx}}{70b} + \frac{33ax^2\sqrt[3]{a+bx}}{70} + \frac{3bx^3\sqrt[3]{a+bx}}{10} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**(4/3),x)
```

output
```
Piecewise((-9*a**3*(a + b*x)**(1/3)/(70*b**2) + 3*a**2*x*(a + b*x)**(1/3)/
(70*b) + 33*a*x**2*(a + b*x)**(1/3)/70 + 3*b*x**3*(a + b*x)**(1/3)/10, Ne(
b, 0)), (a**(4/3)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x(a+bx)^{4/3}\,dx = \frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^2} - \frac{3\,(bx+a)^{\frac{7}{3}}a}{7\,b^2}$$

input
```
integrate(x*(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
3/10*(b*x + a)^(10/3)/b^2 - 3/7*(b*x + a)^(7/3)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.47

$$\int x(a+bx)^{4/3}\,dx = \frac{3\left(\frac{35\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a^2}{b}+\frac{20\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a}{b}+\frac{14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(b\cdots}{b}\right.}{140\,b}$$

input
```
integrate(x*(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
3/140*(35*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a^2/b + 20*(2*(b*x + a)^
(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)*a/b + (14*(b*x + a)^
(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2 - 140*(b*x + a)^(1
/3)*a^3)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x(a+bx)^{4/3}\,dx = -\frac{30\,a\,(a+b\,x)^{7/3}-21\,(a+b\,x)^{10/3}}{70\,b^2}$$

input
```
int(x*(a + b*x)^(4/3),x)
```

output
```
-(30*a*(a + b*x)^(7/3) - 21*(a + b*x)^(10/3))/(70*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int x(a+bx)^{4/3}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\left(7b^3x^3+11a\,b^2x^2+a^2bx-3a^3\right)}{70b^2}$$

input
```
int(x*(b*x+a)^(4/3),x)
```

output
```
(3*(a + b*x)**(1/3)*( - 3*a**3 + a**2*b*x + 11*a*b**2*x**2 + 7*b**3*x**3))
/(70*b**2)
```

3.600 $\int (a+bx)^{4/3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (a+bx)^{4/3}\,dx=\frac{3(a+bx)^{7/3}}{7b}$$

output
```
3/7*(b*x+a)^(7/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+bx)^{4/3}\,dx=\frac{3(a+bx)^{7/3}}{7b}$$

input
```
Integrate[(a + b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(7/3))/(7*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{4/3}\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{3(a+bx)^{7/3}}{7b}$$

input
```
Int[(a + b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(7/3))/(7*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
default	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
pseudoelliptic	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
orering	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
trager	$\frac{3(b^2x^2+2abx+a^2)(bx+a)^{\frac{1}{3}}}{7b}$	29
risch	$\frac{3(b^2x^2+2abx+a^2)(bx+a)^{\frac{1}{3}}}{7b}$	29

input
```
int((b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/7*(b*x+a)^(7/3)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (a+bx)^{4/3}\,dx = \frac{3\,(b^2x^2+2\,abx+a^2)(bx+a)^{\frac{1}{3}}}{7\,b}$$

input
```
integrate((b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
3/7*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{4/3}\,dx=\frac{3(a+bx)^{\frac{7}{3}}}{7b}$$

input
```
integrate((b*x+a)**(4/3),x)
```

output
```
3*(a + b*x)**(7/3)/(7*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{4/3}\,dx=\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b}$$

input
```
integrate((b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
3/7*(b*x + a)^(7/3)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int (a+bx)^{4/3}\,dx=\frac{3\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+28\,(bx+a)^{\frac{1}{3}}a^2+7\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a\right)}{14\,b}$$

input
```
integrate((b*x+a)^(4/3),x, algorithm="giac")
```

output
```
3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 28*(b*x + a)^(1/3)*a^2 + 7
*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (a+bx)^{4/3}\,dx = \frac{3\,{\left(a+b\,x\right)}^{7/3}}{7\,b}$$

input
```
int((a + b*x)^(4/3),x)
```

output
```
(3*(a + b*x)^(7/3))/(7*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (a+bx)^{4/3}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\,(b^2x^2+2abx+a^2)}{7b}$$

input
```
int((b*x+a)^(4/3),x)
```

output
```
(3*(a + b*x)**(1/3)*(a**2 + 2*a*b*x + b**2*x**2))/(7*b)
```

3.601 $\int \frac{(a+bx)^{4/3}}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3} - \sqrt{3}a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

output
```
3*a*(b*x+a)^(1/3)+3/4*(b*x+a)^(4/3)-3^(1/2)*a^(4/3)*arctan(1/3*(a^(1/3)+2*
(b*x+a)^(1/3))*3^(1/2)/a^(1/3))-1/2*a^(4/3)*ln(x)+3/2*a^(4/3)*ln(a^(1/3)-(
b*x+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = \frac{3}{4}\sqrt[3]{a+bx}(5a+bx)-\sqrt{3}a^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

input
```
Integrate[(a + b*x)^(4/3)/x,x]
```

output
```
(3*(a + b*x)^(1/3)*(5*a + b*x))/4 - Sqrt[3]*a^(4/3)*ArcTan[(1 + (2*(a + b*
x)^(1/3))/a^(1/3))/Sqrt[3]] + a^(4/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^
(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {60, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{4/3}}{x}dx$$

$$\Big\downarrow \text{60}$$

$$a\int \frac{\sqrt[3]{a+bx}}{x}dx+\frac{3}{4}(a+bx)^{4/3}$$

$$\Big\downarrow \text{60}$$

$$a\left(a\int \frac{1}{x(a+bx)^{2/3}}dx+3\sqrt[3]{a+bx}\right)+\frac{3}{4}(a+bx)^{4/3}$$

$$\Big\downarrow \text{69}$$

$$a\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)+\frac{3}{4}(a+bx)^{4/3}$$

$$\Big\downarrow \text{16}$$

$$a\left(a\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)+\frac{3}{4}(a+bx)^{4/3}$$

$$\Big\downarrow \textbf{1082}$$

$$a\left(a\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)+\frac{3}{4}(a+bx)^{4/3}$$

$$\Big\downarrow \textbf{217}$$

$$a\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)+\frac{3}{4}(a+bx)^{4/3}$$

input
```
Int[(a + b*x)^(4/3)/x,x]
```

output
```
(3*(a + b*x)^(4/3))/4 + a*(3*(a + b*x)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 + (
2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*L
og[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{3(bx+a)^{\frac{1}{3}}(bx+5a)}{4}-\frac{a^{\frac{4}{3}}\left(2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)+\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left((bx+a)^{\frac{1}{3}}-\right.\right.}{2}$
derivativedivides	$\frac{3(bx+a)^{\frac{4}{3}}}{4}+3a(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{}\right.}{3}\right.$
default	$\frac{3(bx+a)^{\frac{4}{3}}}{4}+3a(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{}\right.}{3}\right.$

input
```
int((b*x+a)^(4/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3/4*(b*x+a)^(1/3)*(b*x+5*a)-1/2*a^(4/3)*(2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(
b*x+a)^(1/3))*3^(1/2)/a^(1/3))+ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2
/3))-2*ln((b*x+a)^(1/3)-a^(1/3)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = -\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right) - \frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+5\,a)(bx+a)^{\frac{1}{3}}$$

input
```
integrate((b*x+a)^(4/3)/x,x, algorithm="fricas")
```

output
```
-sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a
)/a) - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
) + a^(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + 5*a)*(b*x + a)^(1/
3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = \frac{7a^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{7a^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{7a^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{7a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\Gamma\left(\frac{7}{3}\right)}{\Gamma\left(\frac{10}{3}\right)} + \frac{7b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}{4\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x+a)**(4/3)/x,x)
```

output
```
7*a**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(3*gamma
(10/3)) + 7*a**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(3*gamma(10/3)) + 7*a**(4/3)*exp(2*I*
pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamm
a(7/3)/(3*gamma(10/3)) + 7*a*b**(1/3)*(a/b + x)**(1/3)*gamma(7/3)/gamma(10
/3) + 7*b**(4/3)*(a/b + x)**(4/3)*gamma(7/3)/(4*gamma(10/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = -\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + \frac{3}{4}\,(bx+a)^{\frac{4}{3}} + 3\,(bx+a)^{\frac{1}{3}}a$$

input
```
integrate((b*x+a)^(4/3)/x,x, algorithm="maxima")
```

output
```
-sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
 - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) +
a^(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + a)^(4/3) + 3*(b*x + a)
^(1/3)*a
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = -\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{4}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + \frac{3}{4}\,(bx+a)^{\frac{4}{3}} + 3\,(bx+a)^{\frac{1}{3}}a$$

input
```
integrate((b*x+a)^(4/3)/x,x, algorithm="giac")
```

output
```
-sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
 - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) +
a^(4/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/4*(b*x + a)^(4/3) + 3*(b*x
 + a)^(1/3)*a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = 3\,a\,(a+b\,x)^{1/3} + \frac{3\,(a+b\,x)^{4/3}}{4} + a^{4/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3} - 9\,a^{7/3}\right) + \frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(-1+\sqrt{3}\,1i\right)}{2} - 9\,a^2\,(a+b\,x)^{1/3}\right)\,\left(-1+\sqrt{3}\,1i\right)}{2} - \frac{a^{4/3}\,\ln\left(\right.}{}$$

input
```
int((a + b*x)^(4/3)/x,x)
```

output
```
3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 + a^(4/3)*log(9*a^2*(a + b*x)^
(1/3) - 9*a^(7/3)) + (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i - 1))/2 - 9*a^2*(
a + b*x)^(1/3))*(3^(1/2)*1i - 1))/2 - (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i
+ 1))/2 + 9*a^2*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = \frac{4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)a^2 - 4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)a^2 + 15a^{\frac{5}{3}}(bx+a)^{\frac{1}{3}} + 3a^{\frac{2}{3}}(bx+}{}$$

input
```
int((b*x+a)^(4/3)/x,x)
```

output
```
(4*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*a**2 -
 4*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*a**2 +
 15*a**(2/3)*(a + b*x)**(1/3)*a + 3*a**(2/3)*(a + b*x)**(1/3)*b*x + 4*log(
(a + b*x)**(1/6) + a**(1/6))*a**2 + 4*log((a + b*x)**(1/6) - a**(1/6))*a**
2 - 2*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3))*a**2
 - 2*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3))*a**2)/(4
*a**(2/3))
```

3.602 $\int \frac{(a+bx)^{4/3}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{(a+bx)^{4/3}}{x^2} dx = 3b\sqrt[3]{a+bx} - \frac{a\sqrt[3]{a+bx}}{x} - \frac{4\sqrt[3]{a}b \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{2}{3}\sqrt[3]{a}b\log(x) + 2\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

output
```
3*b*(b*x+a)^(1/3)-a*(b*x+a)^(1/3)/x-4/3*a^(1/3)*b*arctan(1/3*(a^(1/3)+2*(b
*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)-2/3*a^(1/3)*b*ln(x)+2*a^(1/3)*b*ln(a
^(1/3)-(b*x+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{4/3}}{x^2} dx = \frac{1}{3}\left(-\frac{3(a-3bx)\sqrt[3]{a+bx}}{x} - 4\sqrt{3}\sqrt[3]{a}b \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - 2\sqrt[3]{a}b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)\right)$$

input
```
Integrate[(a + b*x)^(4/3)/x^2,x]
```

output
```
((-3*(a - 3*b*x)*(a + b*x)^(1/3))/x - 4*Sqrt[3]*a^(1/3)*b*ArcTan[(1 + (2*(
a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1
/3)] - 2*a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)
])/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{4/3}}{x^2} \, dx$$

$$\Big\downarrow \text{51}$$

$$\frac{4}{3}b\int \frac{\sqrt[3]{a+bx}}{x}dx-\frac{(a+bx)^{4/3}}{x}$$

$$\Big\downarrow \text{60}$$

$$\frac{4}{3}b\left(a\int \frac{1}{x(a+bx)^{2/3}}dx+3\sqrt[3]{a+bx}\right)-\frac{(a+bx)^{4/3}}{x}$$

$$\Big\downarrow \text{69}$$

$$\frac{4}{3}b\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)-\frac{(a+bx)^{4/3}}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{4}{3}b\left(a\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)-\frac{(a+bx)^{4/3}}{x}$$

$$\Big\downarrow \ 1082$$

$$\frac{4}{3}b\left(a\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)-\frac{(a+bx)^{4/3}}{x}$$

$$\Big\downarrow \ 217$$

$$\frac{4}{3}b\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx}\right)-\frac{(a+bx)^{4/3}}{x}$$

input
```
Int[(a + b*x)^(4/3)/x^2,x]
```

output
```
-((a + b*x)^(4/3)/x) + (4*b*(3*(a + b*x)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 +
 (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3
*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3)))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{(9bx-3a)(bx+a)^{\frac{1}{3}}-2a^{\frac{1}{3}}bx\left(2\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right.\right.}{3x}$
derivativedivides	$3b\left((bx+a)^{\frac{1}{3}}-a\left(\frac{(bx+a)^{\frac{1}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{4\sqrt{3}\arctan}{}\right.\right.$
default	$3b\left((bx+a)^{\frac{1}{3}}-a\left(\frac{(bx+a)^{\frac{1}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{4\sqrt{3}\arctan}{}\right.\right.$

input
```
int((b*x+a)^(4/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((9*b*x-3*a)*(b*x+a)^(1/3)-2*a^(1/3)*b*x*(2*arctan(2/3*3^(1/2)/a^(1/3)
*(b*x+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)
+a^(2/3))-2*ln((b*x+a)^(1/3)-a^(1/3))))/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int\frac{(a+bx)^{4/3}}{x^2}\,dx=$$
$$-\frac{4\sqrt{3}a^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4a^{\frac{1}{3}}bx\log\left((bx+\right.}{3x}$$

input
```
integrate((b*x+a)^(4/3)/x^2,x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(3)*a^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3)
+ sqrt(3)*a)/a) + 2*a^(1/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1
/3) + a^(2/3)) - 4*a^(1/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3*(3*b*x -
 a)*(b*x + a)^(1/3))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 719, normalized size of antiderivative = 6.66

$$\int \frac{(a+bx)^{4/3}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(4/3)/x**2,x)
```

output
```
28*a**(10/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*p
i/3)*gamma(10/3)) + 28*a**(10/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_p
olar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*
a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(10/3)*b*exp(-2*I*pi/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/
3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*ga
mma(10/3)) - 28*a**(7/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/
b + x)**(1/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a
**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(7/3)*b**2*(a/b + x)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(
9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(
10/3)) - 28*a**(7/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*
gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 84*a**3*b**(
4/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma
(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 63*a**2*b**(7/3)*
(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3
) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{4/3}}{x^2}\,dx = -\frac{4}{3}\sqrt{3}a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{2}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{4}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + 3\,(bx+a)^{\frac{1}{3}}b - \frac{(bx+a)^{\frac{1}{3}}a}{x}$$

input
```
integrate((b*x+a)^(4/3)/x^2,x, algorithm="maxima")
```

output
```
-4/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^
(1/3)) - 2/3*a^(1/3)*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(
2/3)) + 4/3*a^(1/3)*b*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)*b
 - (b*x + a)^(1/3)*a/x
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{4/3}}{x^2}\,dx = -\frac{1}{3}\left(4\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) + 2\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) - 4\,a^{\frac{1}{3}}\log\left(\left|(\right.\right.$$

input
```
integrate((b*x+a)^(4/3)/x^2,x, algorithm="giac")
```

output
```
-1/3*(4*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a
^(1/3)) + 2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
)) - 4*a^(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) - 9*(b*x + a)^(1/3) + 3
*(b*x + a)^(1/3)*a/(b*x))*b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{4/3}}{x^2}\,dx = 3\,b\,(a+b\,x)^{1/3} + \frac{4\,a^{1/3}\,b\,\ln\left(12\,a^{4/3}\,b-12\,a\,b\,(a+b\,x)^{1/3}\right)}{3} - \frac{a\,(a+b\,x)^{1/3}}{x} + \frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}-6\,a^{4/3}\,b\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3} - \frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}+6\,a^{4/3}\,b\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

input
```
int((a + b*x)^(4/3)/x^2,x)
```

output
```
3*b*(a + b*x)^(1/3) + (4*a^(1/3)*b*log(12*a^(4/3)*b - 12*a*b*(a + b*x)^(1/
3)))/3 - (a*(a + b*x)^(1/3))/x + (2*a^(1/3)*b*log(12*a*b*(a + b*x)^(1/3) -
 6*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (2*a^(1/3)*b*log(12*a
*b*(a + b*x)^(1/3) + 6*a^(4/3)*b*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)^{4/3}}{x^2}\,dx = \frac{4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)abx - 4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)abx - 3a^{\frac{5}{3}}(bx+a)^{\frac{1}{3}} + 9a^{\frac{2}{3}}(bx}{}$$

input
```
int((b*x+a)^(4/3)/x^2,x)
```

output
```
(4*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*a*b*x
- 4*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*a*b*x
 - 3*a**(2/3)*(a + b*x)**(1/3)*a + 9*a**(2/3)*(a + b*x)**(1/3)*b*x + 4*log
((a + b*x)**(1/6) + a**(1/6))*a*b*x + 4*log((a + b*x)**(1/6) - a**(1/6))*a
*b*x - 2*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3))*a
*b*x - 2*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3))*a*b*
x)/(3*a**(2/3)*x)
```

3.603 $\int \frac{(a+bx)^{4/3}}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 125

$$\int \frac{(a+bx)^{4/3}}{x^3} \, dx = -\frac{a\sqrt[3]{a+bx}}{2x^2} - \frac{7b\sqrt[3]{a+bx}}{6x} - \frac{2b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}$$

output
```
-1/2*a*(b*x+a)^(1/3)/x^2-7/6*b*(b*x+a)^(1/3)/x-2/9*b^2*arctan(1/3*(a^(1/3)
+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)-1/9*b^2*ln(x)/a^(2/3)+1
/3*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx = \frac{1}{18}\left(-\frac{3\sqrt[3]{a+bx}(3a+7bx)}{x^2} - \frac{4\sqrt{3}b^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{4b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{2/3}} - \frac{2b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{a^{2/3}}\right)$$

input
```
Integrate[(a + b*x)^(4/3)/x^3,x]
```

output
```
((-3*(a + b*x)^(1/3)*(3*a + 7*b*x))/x^2 - (4*Sqrt[3]*b^2*ArcTan[(1 + (2*(a
 + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*b^2*Log[a^(1/3) - (a + b*x)
^(1/3)])/a^(2/3) - (2*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x
)^(2/3)])/a^(2/3))/18
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 51, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx$$

$$\Big\downarrow \text{51}$$

$$\frac{2}{3}b\int \frac{\sqrt[3]{a+bx}}{x^2}dx-\frac{(a+bx)^{4/3}}{2x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{2}{3}b\left(\frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}dx-\frac{\sqrt[3]{a+bx}}{x}\right)-\frac{(a+bx)^{4/3}}{2x^2}$$

$$\Big\downarrow \text{69}$$

$$\frac{2}{3}b\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}\right)-\frac{(a+bx)^{4/3}}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{2}{3}b\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}\right)-\frac{(a+bx)^{4/3}}{2x^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}b\left(\frac{1}{3}b\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}\right)-\frac{(a+bx)^{4/3}}{2x^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{3}b\left(\frac{1}{3}b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx}}{x}\right)-\frac{(a+bx)^{4/3}}{2x^2}$$

input
```
Int[(a + b*x)^(4/3)/x^3,x]
```

output
```
-1/2*(a + b*x)^(4/3)/x^2 + (2*b*(-((a + b*x)^(1/3)/x) + (b*(-((Sqrt[3]*Arc
Tan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/
3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))))/3))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

method	result
derivativedivides	$3b^2\left(-\frac{\frac{7(bx+a)^{\frac{4}{3}}}{18}-\frac{2a(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}}{\ldots}\right.}{27\ldots}\right.$
default	$3b^2\left(-\frac{\frac{7(bx+a)^{\frac{4}{3}}}{18}-\frac{2a(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}}{\ldots}\right.}{27\ldots}\right.$
pseudoelliptic	$\frac{-4b^2\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^2+4b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^2-2b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2-21bx(\ldots}{18x^2a^{\frac{2}{3}}}$

input
```
int((b*x+a)^(4/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3*b^2*(-(7/18*(b*x+a)^(4/3)-2/9*a*(b*x+a)^(1/3))/b^2/x^2+2/27/a^(2/3)*ln((
b*x+a)^(1/3)-a^(1/3))-1/27/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+
a^(2/3))-2/27/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+
1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx =$$

$$-\frac{12\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{a^2}\right)+2\,(a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2\right.}{18\,a^2x^2}$$

input
```
integrate((b*x+a)^(4/3)/x^3,x, algorithm="fricas")
```

output
```
-1/18*(12*sqrt(1/3)*(a^2)^(1/6)*a*b^2*x^2*arctan(sqrt(1/3)*(a^2)^(1/6)*((a
^2)^(1/3)*a + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 2*(a^2)^(2/3)*b^2*x^2*
log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 4*(
a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x + 3
*a^3)*(b*x + a)^(1/3))/(a^2*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 2266, normalized size of antiderivative = 18.13

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(4/3)/x**3,x)
```

output
```
28*a**(19/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3)
)*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp
(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10
/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*
b**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma
(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi
/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 5
4*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3
))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*ex
p(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(1
0/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)
*b**3*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*
gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3
) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b*
*3*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3
))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*ex
p(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(1
0/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16...
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx = -\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}} - \frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{2}{3}}} + \frac{2\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{2}{3}}} - \frac{7\,(bx+a)^{\frac{4}{3}}b^2-4\,(bx+a)^{\frac{1}{3}}ab^2}{6\,\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

input
```
integrate((b*x+a)^(4/3)/x^3,x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(2/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - 1/6*(7*(b*x +
 a)^(4/3)*b^2 - 4*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^
2)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx =$$

$$-\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2\,b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7\,(bx+a)^{\frac{4}{3}}b^3-4\,(bx+a)^{\frac{1}{3}}\right)}{b^2x^2}}{18\,b}$$

input
```
integrate((b*x+a)^(4/3)/x^3,x, algorithm="giac")
```

output
```
-1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(2/3) + 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(2/3) - 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(7*(b*
x + a)^(4/3)*b^3 - 4*(b*x + a)^(1/3)*a*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx = \frac{2\,b^2\,\ln\left(2\,b^2\,(a+b\,x)^{1/3}-2\,a^{1/3}\,b^2\right)}{9\,a^{2/3}} - \frac{\frac{7\,b^2\,(a+b\,x)^{4/3}}{6}-\frac{2\,a\,b^2\,(a+b\,x)^{1/3}}{3}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2} - \frac{\ln\left(2\,b^2\,(a+b\,x)^{1/3}+a^{1/3}\,\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)\right)\,\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{9\,a^{2/3}} + \frac{b^2\,\ln\left(2\,b^2\,(a+b\,x)^{1/3}-9\,a^{1/3}\,b^2\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{a^{2/3}}$$

input
```
int((a + b*x)^(4/3)/x^3,x)
```

output
```
(2*b^2*log(2*b^2*(a + b*x)^(1/3) - 2*a^(1/3)*b^2))/(9*a^(2/3)) - ((7*b^2*(
a + b*x)^(4/3))/6 - (2*a*b^2*(a + b*x)^(1/3))/3)/((a + b*x)^2 - 2*a*(a + b
*x) + a^2) - (log(2*b^2*(a + b*x)^(1/3) + a^(1/3)*(3^(1/2)*b^2*1i + b^2))*
(3^(1/2)*b^2*1i + b^2))/(9*a^(2/3)) + (b^2*log(2*b^2*(a + b*x)^(1/3) - 9*a
^(1/3)*b^2*((3^(1/2)*1i)/9 - 1/9))*((3^(1/2)*1i)/9 - 1/9))/a^(2/3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx = \frac{4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2 - 4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2 - 9a^{\frac{5}{3}}(bx+a)^{\frac{1}{3}} - 21a^{\frac{2}{3}}(}{}$$

input
```
int((b*x+a)^(4/3)/x^3,x)
```

output
```
(4*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b**2*x
**2 - 4*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b
**2*x**2 - 9*a**(2/3)*(a + b*x)**(1/3)*a - 21*a**(2/3)*(a + b*x)**(1/3)*b*
x + 4*log((a + b*x)**(1/6) + a**(1/6))*b**2*x**2 + 4*log((a + b*x)**(1/6)
- a**(1/6))*b**2*x**2 - 2*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1
/3) + a**(1/3))*b**2*x**2 - 2*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(
1/3) + a**(1/3))*b**2*x**2)/(18*a**(2/3)*x**2)
```

3.604 $\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx = -\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}$$

output
```
-3/2*a^3*(b*x+a)^(2/3)/b^4+9/5*a^2*(b*x+a)^(5/3)/b^4-9/8*a*(b*x+a)^(8/3)/b
^4+3/11*(b*x+a)^(11/3)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}\left(-81a^3+54a^2bx-45ab^2x^2+40b^3x^3\right)}{440b^4}$$

input
```
Integrate[x^3/(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(2/3)*(-81*a^3 + 54*a^2*b*x - 45*a*b^2*x^2 + 40*b^3*x^3))/(44
0*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

input
```
Int[x^3/(a + b*x)^(1/3),x]
```

output
```
(-3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a + b*x)^(5/3))/(5*b^4) - (9*a*
(a + b*x)^(8/3))/(8*b^4) + (3*(a + b*x)^(11/3))/(11*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^{3}x^{3}+45a\,b^{2}x^{2}-54a^{2}bx+81a^{3}\right)}{440b^{4}}$	43
trager	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^{3}x^{3}+45a\,b^{2}x^{2}-54a^{2}bx+81a^{3}\right)}{440b^{4}}$	43
risch	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^{3}x^{3}+45a\,b^{2}x^{2}-54a^{2}bx+81a^{3}\right)}{440b^{4}}$	43
pseudoelliptic	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^{3}x^{3}+45a\,b^{2}x^{2}-54a^{2}bx+81a^{3}\right)}{440b^{4}}$	43
orering	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^{3}x^{3}+45a\,b^{2}x^{2}-54a^{2}bx+81a^{3}\right)}{440b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{9a(bx+a)^{\frac{8}{3}}}{8}+\frac{9a^{2}(bx+a)^{\frac{5}{3}}}{5}-\frac{3a^{3}(bx+a)^{\frac{2}{3}}}{2}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{9a(bx+a)^{\frac{8}{3}}}{8}+\frac{9a^{2}(bx+a)^{\frac{5}{3}}}{5}-\frac{3a^{3}(bx+a)^{\frac{2}{3}}}{2}}{b^{4}}$	50

input
```
int(x^3/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/440*(b*x+a)^(2/3)*(-40*b^3*x^3+45*a*b^2*x^2-54*a^2*b*x+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx=\frac{3\,(40\,b^3x^3-45\,ab^2x^2+54\,a^2bx-81\,a^3)(bx+a)^{\frac{2}{3}}}{440\,b^4}$$

input
```
integrate(x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/440*(40*b^3*x^3 - 45*a*b^2*x^2 + 54*a^2*b*x - 81*a^3)*(b*x + a)^(2/3)/b^
4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. 2(68) = 136.

Time = 1.25 (sec) , antiderivative size = 1640, normalized size of antiderivative = 22.78

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(b*x+a)**(1/3),x)
```

output
```
-243*a**(71/3)*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 66
00*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a*
*15*b**9*x**5 + 440*a**14*b**10*x**6) + 243*a**(71/3)/(440*a**20*b**4 + 26
40*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16
*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1296*a**(68/3)
*b*x*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b
**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x
**5 + 440*a**14*b**10*x**6) + 1458*a**(68/3)*b*x/(440*a**20*b**4 + 2640*a*
*19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8
*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 2808*a**(65/3)*b**2
*x**2*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*
b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*
x**5 + 440*a**14*b**10*x**6) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b**4 +
2640*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**
16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 3120*a**(62/
3)*b**3*x**3*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600
*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**1
5*b**9*x**5 + 440*a**14*b**10*x**6) + 4860*a**(62/3)*b**3*x**3/(440*a**20*
b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6
600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 17...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^4} - \frac{9\,(bx+a)^{\frac{8}{3}}a}{8\,b^4} + \frac{9\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^4} - \frac{3\,(bx+a)^{\frac{2}{3}}a^3}{2\,b^4}$$

input
```
integrate(x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/11*(b*x + a)^(11/3)/b^4 - 9/8*(b*x + a)^(8/3)*a/b^4 + 9/5*(b*x + a)^(5/3
)*a^2/b^4 - 3/2*(b*x + a)^(2/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx = \frac{3\left(40\,(bx+a)^{\frac{11}{3}} - 165\,(bx+a)^{\frac{8}{3}}a + 264\,(bx+a)^{\frac{5}{3}}a^2 - 220\,(bx+a)^{\frac{2}{3}}a^3\right)}{440\,b^4}$$

input
```
integrate(x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/440*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3)*a
^2 - 220*(b*x + a)^(2/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx = \frac{3\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4} - \frac{3\,a^3\,{\left(a+b\,x\right)}^{2/3}}{2\,b^4} + \frac{9\,a^2\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4} - \frac{9\,a\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}$$

input
```
int(x^3/(a + b*x)^(1/3),x)
```

output
```
(3*(a + b*x)^(11/3))/(11*b^4) - (3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(
a + b*x)^(5/3))/(5*b^4) - (9*a*(a + b*x)^(8/3))/(8*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx = \frac{3(bx+a)^{\frac{2}{3}}\,(40b^3x^3-45a\,b^2x^2+54a^2bx-81a^3)}{440b^4}$$

input
```
int(x^3/(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(2/3)*( - 81*a**3 + 54*a**2*b*x - 45*a*b**2*x**2 + 40*b**3*x
**3))/(440*b**4)
```

3.605 $\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{3a^2(a+bx)^{2/3}}{2b^3} - \frac{6a(a+bx)^{5/3}}{5b^3} + \frac{3(a+bx)^{8/3}}{8b^3}$$

output
```
3/2*a^2*(b*x+a)^(2/3)/b^3-6/5*a*(b*x+a)^(5/3)/b^3+3/8*(b*x+a)^(8/3)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}\left(9a^2-6abx+5b^2x^2\right)}{40b^3}$$

input
```
Integrate[x^2/(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(2/3)*(9*a^2 - 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{a+bx}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

input
```
Int[x^2/(a + b*x)^(1/3),x]
```

output
```
(3*a^2*(a + b*x)^(2/3))/(2*b^3) - (6*a*(a + b*x)^(5/3))/(5*b^3) + (3*(a +
b*x)^(8/3))/(8*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{3(bx+a)^{\frac{2}{3}}\left(5b^{2}x^{2}-6abx+9a^{2}\right)}{40b^{3}}$	32
trager	$\frac{3(bx+a)^{\frac{2}{3}}\left(5b^{2}x^{2}-6abx+9a^{2}\right)}{40b^{3}}$	32
risch	$\frac{3(bx+a)^{\frac{2}{3}}\left(5b^{2}x^{2}-6abx+9a^{2}\right)}{40b^{3}}$	32
pseudoelliptic	$\frac{3(bx+a)^{\frac{2}{3}}\left(5b^{2}x^{2}-6abx+9a^{2}\right)}{40b^{3}}$	32
orering	$\frac{3(bx+a)^{\frac{2}{3}}\left(5b^{2}x^{2}-6abx+9a^{2}\right)}{40b^{3}}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{6a(bx+a)^{\frac{5}{3}}}{5}+\frac{3a^{2}(bx+a)^{\frac{2}{3}}}{2}}{b^{3}}$	38
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{6a(bx+a)^{\frac{5}{3}}}{5}+\frac{3a^{2}(bx+a)^{\frac{2}{3}}}{2}}{b^{3}}$	38

input
```
int(x^2/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/40*(b*x+a)^(2/3)*(5*b^2*x^2-6*a*b*x+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(5\,b^2x^2-6\,abx+9\,a^2)(bx+a)^{\frac{2}{3}}}{40\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/40*(5*b^2*x^2 - 6*a*b*x + 9*a^2)*(b*x + a)^(2/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(49) = 98$.

Time = 0.81 (sec) , antiderivative size = 600, normalized size of antiderivative = 11.32

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{27a^{\frac{32}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} - \frac{27a^{\frac{32}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} + \frac{63a^{\frac{29}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} - \frac{81a^{\frac{29}{3}}bx}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} + \frac{42a^{\frac{26}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} - \frac{81a^{\frac{26}{3}}b^2x^2}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} + \frac{18a^{\frac{23}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} - \frac{27a^{\frac{23}{3}}b^3x^3}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} + \frac{27a^{\frac{20}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3} + \frac{15a^{\frac{17}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}$$

input
```
integrate(x**2/(b*x+a)**(1/3),x)
```

output
```
27*a**(32/3)*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6
*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(32/3)/(40*a**8*b**3 + 120*a**7*b*
*4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 + b*x
/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*
b**6*x**3) - 81*a**(29/3)*b*x/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b
**5*x**2 + 40*a**5*b**6*x**3) + 42*a**(26/3)*b**2*x**2*(1 + b*x/a)**(2/3)/
(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3)
- 81*a**(26/3)*b**2*x**2/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x
**2 + 40*a**5*b**6*x**3) + 18*a**(23/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a
**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*
a**(23/3)*b**3*x**3/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3) + 27*a**(20/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**8*b
**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(1
7/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a*
*6*b**5*x**2 + 40*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^3} - \frac{6\,(bx+a)^{\frac{5}{3}}a}{5\,b^3} + \frac{3\,(bx+a)^{\frac{2}{3}}a^2}{2\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/8*(b*x + a)^(8/3)/b^3 - 6/5*(b*x + a)^(5/3)*a/b^3 + 3/2*(b*x + a)^(2/3)*
a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{3\left(5\,(bx+a)^{\frac{8}{3}} - 16\,(bx+a)^{\frac{5}{3}}a + 20\,(bx+a)^{\frac{2}{3}}a^2\right)}{40\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/40*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b
^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{15\,(a+b\,x)^{8/3}-48\,a\,(a+b\,x)^{5/3}+60\,a^2\,(a+b\,x)^{2/3}}{40\,b^3}$$

input
```
int(x^2/(a + b*x)^(1/3),x)
```

output
```
(15*(a + b*x)^(8/3) - 48*a*(a + b*x)^(5/3) + 60*a^2*(a + b*x)^(2/3))/(40*b
^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx = \frac{3(bx+a)^{\frac{2}{3}}\,(5b^2x^2-6abx+9a^2)}{40b^3}$$

input
```
int(x^2/(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(2/3)*(9*a**2 - 6*a*b*x + 5*b**2*x**2))/(40*b**3)
```

3.606 $\int \frac{x}{\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = -\frac{3a(a+bx)^{2/3}}{2b^2}+\frac{3(a+bx)^{5/3}}{5b^2}$$

output
```
-3/2*a*(b*x+a)^(2/3)/b^2+3/5*(b*x+a)^(5/3)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}(-3a+2bx)}{10b^2}$$

input
```
Integrate[x/(a + b*x)^(1/3),x]
```

output
```
(3*(a + b*x)^(2/3)*(-3*a + 2*b*x))/(10*b^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{a+bx}} \, dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{(a+bx)^{2/3}}{b} - \frac{a}{b\sqrt[3]{a+bx}}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{3(a+bx)^{5/3}}{5b^2} - \frac{3a(a+bx)^{2/3}}{2b^2}$$

input
```
Int[x/(a + b*x)^(1/3),x]
```

output
```
(-3*a*(a + b*x)^(2/3))/(2*b^2) + (3*(a + b*x)^(5/3))/(5*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
trager	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
risch	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
pseudoelliptic	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
orering	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-\frac{3a(bx+a)^{\frac{2}{3}}}{2}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-\frac{3a(bx+a)^{\frac{2}{3}}}{2}}{b^2}$	26

input
```
int(x/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/10*(b*x+a)^(2/3)*(-2*b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(2\,bx-3\,a)(bx+a)^{\frac{2}{3}}}{10\,b^2}$$

input
```
integrate(x/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/10*(2*b*x - 3*a)*(b*x + a)^(2/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(31) = 62$.

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.76

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = -\frac{9a^{\frac{11}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x} + \frac{9a^{\frac{11}{3}}}{10a^2b^2+10ab^3x} - \frac{3a^{\frac{8}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x} + \frac{9a^{\frac{8}{3}}bx}{10a^2b^2+10ab^3x} + \frac{6a^{\frac{5}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}$$

input
```
integrate(x/(b*x+a)**(1/3),x)
```

output
```
-9*a**(11/3)*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x) + 9*a**(11/3)
/(10*a**2*b**2 + 10*a*b**3*x) - 3*a**(8/3)*b*x*(1 + b*x/a)**(2/3)/(10*a**2
*b**2 + 10*a*b**3*x) + 9*a**(8/3)*b*x/(10*a**2*b**2 + 10*a*b**3*x) + 6*a**
(5/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^2} - \frac{3\,(bx+a)^{\frac{2}{3}}a}{2\,b^2}$$

input
```
integrate(x/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/5*(b*x + a)^(5/3)/b^2 - 3/2*(b*x + a)^(2/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = \frac{3\left(2\,(bx+a)^{\frac{5}{3}} - 5\,(bx+a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

input
```
integrate(x/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/10*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = -\frac{15\,a\,(a+b\,x)^{2/3} - 6\,(a+b\,x)^{5/3}}{10\,b^2}$$

input
```
int(x/(a + b*x)^(1/3),x)
```

output
```
-(15*a*(a + b*x)^(2/3) - 6*(a + b*x)^(5/3))/(10*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{x}{\sqrt[3]{a+bx}}\,dx = \frac{3(bx+a)^{\frac{2}{3}}\,(2bx-3a)}{10b^2}$$

input
```
int(x/(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(2/3)*( - 3*a + 2*b*x))/(10*b**2)
```

3.607 $\int \frac{1}{\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2b}$$

output
```
3/2*(b*x+a)^(2/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2b}$$

input
```
Integrate[(a + b*x)^(-1/3),x]
```

output
```
(3*(a + b*x)^(2/3))/(2*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx}} \, dx$$
$$\Big\downarrow \; 17$$
$$\frac{3(a+bx)^{2/3}}{2b}$$

input
```
Int[(a + b*x)^(-1/3),x]
```

output
```
(3*(a + b*x)^(2/3))/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
default	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
trager	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
risch	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
pseudoelliptic	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
orering	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13

input
```
int(1/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(b*x+a)^(2/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(1/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
3/2*(b*x + a)^(2/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{\frac{2}{3}}}{2b}$$

input
```
integrate(1/(b*x+a)**(1/3),x)
```

output
```
3*(a + b*x)**(2/3)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(1/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
3/2*(b*x + a)^(2/3)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(1/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
3/2*(b*x + a)^(2/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx=\frac{3\,{\left(a+b\,x\right)}^{2/3}}{2\,b}$$

input
```
int(1/(a + b*x)^(1/3),x)
```

output
```
(3*(a + b*x)^(2/3))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx=\frac{3(bx+a)^{\frac{2}{3}}}{2b}$$

input
```
int(1/(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(2/3))/(2*b)
```

3.608 $\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

output
```
3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))/a^(1/3)-1/2*
ln(x)/a^(1/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2\sqrt[3]{a}}$$

input
```
Integrate[1/(x*(a + b*x)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/
3) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^
(2/3)])/(2*a^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{a+bx}}dx$$

$$\Big\downarrow \text{ 67}$$

$$\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

input
```
Int[1/(x*(a + b*x)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[
x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75
default	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)+2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}$	75

input

```
int(1/x/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output

```
1/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(
b*x+a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a
)^(1/3)+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.70

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$$
$$= \left[\frac{\sqrt{3}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a\right.}{2\,a} \right.$$

input

```
integrate(1/x/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x + a)^(2/3)*a
^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)
*a^(2/3) + 3*a)/x) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a, 1/2*(2*sqrt(3)*
a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3
)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log
((b*x + a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.96

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(b*x+a)**(1/3),x)
```

output
```
2*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma
(5/3)) + 2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*p
i/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(-2*I*pi/3)*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(3*a
**(1/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}} + \frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

input
```
integrate(1/x/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + l
og((b*x + a)^(1/3) - a^(1/3))/a^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}} + \frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}$$

input
```
integrate(1/x/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + l
og(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{1/3}} + \frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}} - \frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}$$

input
```
int(1/(x*(a + b*x)^(1/3)),x)
```

output
```
log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(1/3) + (log(9*(a + b*x)^(1/3) - (9*a
^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a^(1/3)) - (log(9*(a +
b*x)^(1/3) - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a^(1/3
))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.78

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2\log\left((bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}\right)+2\log\left((bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}\right)}{2a^{\frac{1}{3}}}$$

input
```
int(1/x/(b*x+a)^(1/3),x)
```

output
```
( - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3))) + 2
*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3))) + 2*log(
(a + b*x)**(1/6) + a**(1/6)) + 2*log((a + b*x)**(1/6) - a**(1/6)) - log( -
 a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)) - log(a**(1/6)*(
a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)))/(2*a**(1/3))
```

3.609 $\int \frac{1}{x^2 \sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{1}{x^2 \sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{ax} - \frac{b \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

output
```
-(b*x+a)^(2/3)/a/x-1/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1
/3))*3^(1/2)/a^(4/3)+1/6*b*ln(x)/a^(4/3)-1/2*b*ln(a^(1/3)-(b*x+a)^(1/3))/a
^(4/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \sqrt[3]{a+bx}}\,dx = -\frac{6\sqrt[3]{a}(a+bx)^{2/3} + 2\sqrt{3}bx\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - bx\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a}\right.}{6a^{4/3}x}$$

input
```
Integrate[1/(x^2*(a + b*x)^(1/3)),x]
```

output
```
-1/6*(6*a^(1/3)*(a + b*x)^(2/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(
1/3))/a^(1/3))/Sqrt[3]] + 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] - b*x*Log[a
^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(4/3)*x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[3]{a+bx}} \, dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{b \int \frac{1}{x \sqrt[3]{a+bx}}dx}{3a}-\frac{(a+bx)^{2/3}}{ax}$$

$$\Big\downarrow \text{67}$$

$$-\frac{b \left(\frac{3}{2} \int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log (x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}$$

$$\Big\downarrow \text{16}$$

$$-\frac{b \left(\frac{3}{2} \int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log (x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{b\left(-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}$$

$\Big\downarrow$ 217

$$-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}$$

input
```
Int[1/(x^2*(a + b*x)^(1/3)),x]
```

output
```
-((a + b*x)^(2/3)/(a*x)) - (b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^
(1/3))/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)
^(1/3)])/(2*a^(1/3))))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{ax}-\frac{b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{4}{3}}}$
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)bx-2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)bx+\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)bx-6(bx+a)^{\frac{2}{3}}a^{\frac{1}{3}}}{6a^{\frac{4}{3}}x}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$

input
```
int(1/x^2/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)^(2/3)/a/x-1/3*b/a^(4/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/6*b/a^(4/3)*l
n((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3*b/a^(4/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.06

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx-3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x}\right)+(-a)^{\frac{2}{3}}\,bx\,\mathrm{lo}}{6\,a^2x}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}\,bx\log\left((bx+a)^{\frac{2}{3}}-(bx+\right.}{6\,a^2x}$$

input
```
integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2*(b*
x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3
)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b*x*log((b*x +
a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log
((b*x + a)^(1/3) + (-a)^(1/3)) - 6*(b*x + a)^(2/3)*a)/(a^2*x), -1/6*(6*sqr
t(1/3)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) - (-a
)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x*log((b*x + a)^(2/3) - (b*x
+ a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b*x*log((b*x + a)^(1/3)
 + (-a)^(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 831, normalized size of antiderivative = 8.31

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(b*x+a)**(1/3),x)
```

output
```
-2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/
3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**
(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9
*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)
*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b +
x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(
2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b
**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*
(a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3)) + 6*a*b**3*(a/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(9*a...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}} - \frac{(bx+a)^{\frac{2}{3}}b}{(bx+a)a-a^2} + \frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}} - \frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

input
```
integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(4/3) - (b*x + a)^(2/3)*b/((b*x + a)*a - a^2) + 1/6*b*log((b*x + a)^(2/3)
 + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x + a)^(1/3)
- a^(1/3))/a^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{1}{6}\,b\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} + \frac{2\,\log\left(\left|(bx+a)^{\frac{1}{3}}-a\right.\right.}{a^{\frac{4}{3}}}\right.$$

input
```
integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
-1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(4/3) - log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/
3) + 2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x + a)^(2/3)/(a*
b*x))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{(a+b\,x)^{2/3}}{a\,x}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}} + \frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}} - \frac{b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{4/3}}$$

input
```
int(1/(x^2*(a + b*x)^(1/3)),x)
```

output
```
(log((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b - 3^
(1/2)*b*1i))/(6*a^(4/3)) - (a + b*x)^(2/3)/(a*x) + (log((b + 3^(1/2)*b*1i)
^2/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3)
) - (b*log((a + b*x)^(1/3) - a^(1/3)))/(3*a^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.66

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-6a^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}-2\log\left((bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}\right)bx-}{}$$

input
```
int(1/x^2/(b*x+a)^(1/3),x)
```

output
```
(2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b*x -
2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b*x - 6
*a**(1/3)*(a + b*x)**(2/3) - 2*log((a + b*x)**(1/6) + a**(1/6))*b*x - 2*lo
g((a + b*x)**(1/6) - a**(1/6))*b*x + log( - a**(1/6)*(a + b*x)**(1/6) + (a
 + b*x)**(1/3) + a**(1/3))*b*x + log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)
**(1/3) + a**(1/3))*b*x)/(6*a**(1/3)*a*x)
```

3.610 $\int \frac{1}{x^3 \sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{1}{x^3 \sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{2b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}$$

output
```
-1/2*(b*x+a)^(2/3)/a/x^2+2/3*b*(b*x+a)^(2/3)/a^2/x+2/9*b^2*arctan(1/3*(a^(
1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)-1/9*b^2*ln(x)/a^(7/
3)+1/3*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx=-\frac{(a+bx)^{2/3}(7a-4(a+bx))}{6a^2x^2}+\frac{2b^2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}+\frac{2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{7/3}}-\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{7/3}}$$

input
```
Integrate[1/(x^3*(a + b*x)^(1/3)),x]
```

output
```
-1/6*((a + b*x)^(2/3)*(7*a - 4*(a + b*x)))/(a^2*x^2) + (2*b^2*ArcTan[1/Sqr
t[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (2*b^
2*Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(7/3)) - (b^2*Log[a^(2/3) + a^(1/3)
*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(7/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {52, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{2b\int \frac{1}{x^2\sqrt[3]{a+bx}}dx}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}$$

$$\Big\downarrow \text{52}$$

$$-\frac{2b\left(-\frac{b\int \frac{1}{x\sqrt[3]{a+bx}}dx}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 67

$$-\frac{2b\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 16

$$-\frac{2b\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 1082

$$-\frac{2b\left(-\frac{b\left(-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 217

$$-\frac{2b\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx)^{2/3}}{ax}\right)}{3a}-\frac{(a+bx)^{2/3}}{2ax^2}$$

input
```
Int[1/(x^3*(a + b*x)^(1/3)),x]
```

output
```
-1/2*(a + b*x)^(2/3)/(a*x^2) - (2*b*(-((a + b*x)^(2/3)/(a*x)) - (b*((Sqrt[
3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x]/(2*
a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))))/(3*a)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(-4bx+3a)}{6a^2x^2}+\frac{2b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}-\frac{b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\right.}{}\right.}{9a^{\frac{7}{3}}}$
pseudoelliptic	$\frac{4b^2\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^2+4b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^2-2b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2+12bx\,a^{\frac{1}{}}}{18a^{\frac{7}{3}}x^2}$
derivativedivides	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}}{3a}\right.}{3a}\right.$
default	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}}{3a}\right.}{3a}\right.$

input
```
int(1/x^3/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+a)^(2/3)*(-4*b*x+3*a)/a^2/x^2+2/9*b^2/a^(7/3)*ln((b*x+a)^(1/3)-a
^(1/3))-1/9*b^2/a^(7/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+2/
9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+3\,\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-2\,a^{\frac{2}{3}}b^2x^2\log\left((bx+\right.}{18\,a^3x^2}\right.$$

input
```
integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*
(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3
) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(
1/3) - a^(1/3)) + 3*(4*a*b*x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2), 1/18*(12
*sqrt(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/
a^(1/3)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(4*a*b*
x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 2730, normalized size of antiderivative = 21.00

$$\int \frac{1}{x^3 \sqrt[3]{a+bx}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x+a)**(1/3),x)
```

output
```
4*a**(14/3)*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamm
a(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*
b**(10/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3
)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4
/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(
5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a
**(14/3)*b**(10/3)*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*g
amma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) -
 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11
/3)*b**(13/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)
*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) +
 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b*
*(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}} - \frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}} + \frac{2\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}} + \frac{4\,(bx+a)^{\frac{5}{3}}b^2-7\,(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

input
```
integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(7/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3) + 1/6*(4*(b*x +
a)^(5/3)*b^2 - 7*(b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3
 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}} - \frac{2\,b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}} + \frac{4\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}} + \frac{3\left(4\,(bx+a)^{\frac{5}{3}}b^3-7\,(bx+a)\right.}{a^2b^2x^2}}{18\,b}$$

input
```
integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(7/3) - 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(7/3) + 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x
 + a)^(5/3)*b^3 - 7*(b*x + a)^(2/3)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \frac{2\,b^2\,\ln\left({\left(a+b\,x\right)}^{1/3}-a^{1/3}\right)}{9\,a^{7/3}}-\frac{\frac{7\,b^2\,{\left(a+b\,x\right)}^{2/3}}{6\,a}-\frac{2\,b^2\,{\left(a+b\,x\right)}^{5/3}}{3\,a^2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}-\frac{\ln\left(\frac{4\,b^4\,{\left(a+b\,x\right)}^{1/3}}{9\,a^4}-\frac{{\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}^2}{9\,a^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{9\,a^{7/3}}+\frac{b^2\,\ln\left(\frac{4\,b^4\,{\left(a+b\,x\right)}^{1/3}}{9\,a^4}-\frac{9\,b^4\,{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2}{a^{11/3}}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}{a^{7/3}}$$

input
```
int(1/(x^3*(a + b*x)^(1/3)),x)
```

output
```
(2*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(7/3)) - ((7*b^2*(a + b*x)^(2/
3))/(6*a) - (2*b^2*(a + b*x)^(5/3))/(3*a^2))/((a + b*x)^2 - 2*a*(a + b*x)
+ a^2) - (log((4*b^4*(a + b*x)^(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(
9*a^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*a^(7/3)) + (b^2*log((4*b^4*(a + b*
x)^(1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1i)/9 - 1/9)^2)/a^(11/3))*((3^(1/2)*1
i)/9 - 1/9))/a^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = \frac{-4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2+4\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2-9a^{\frac{4}{3}}(bx+a)^{\frac{2}{3}}+12a^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}bx+4}{}$$

input
```
int(1/x^3/(b*x+a)^(1/3),x)
```

output
```
( - 4*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b**
2*x**2 + 4*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3))
)*b**2*x**2 - 9*a**(1/3)*(a + b*x)**(2/3)*a + 12*a**(1/3)*(a + b*x)**(2/3)
*b*x + 4*log((a + b*x)**(1/6) + a**(1/6))*b**2*x**2 + 4*log((a + b*x)**(1/
6) - a**(1/6))*b**2*x**2 - 2*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)*
*(1/3) + a**(1/3))*b**2*x**2 - 2*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)
**(1/3) + a**(1/3))*b**2*x**2)/(18*a**(1/3)*a**2*x**2)
```

3.611 $\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx=\frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}$$

output
```
3/2*a^3*(b*x-a)^(2/3)/b^4+9/5*a^2*(b*x-a)^(5/3)/b^4+9/8*a*(b*x-a)^(8/3)/b^
4+3/11*(b*x-a)^(11/3)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx=\frac{3(-a+bx)^{2/3}\left(81a^3+54a^2bx+45ab^2x^2+40b^3x^3\right)}{440b^4}$$

input
```
Integrate[x^3/(-a + b*x)^(1/3),x]
```

output
```
(3*(-a + b*x)^(2/3)*(81*a^3 + 54*a^2*b*x + 45*a*b^2*x^2 + 40*b^3*x^3))/(44
0*b^4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{bx-a}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^3}{b^3\sqrt[3]{bx-a}}+\frac{3a^2(bx-a)^{2/3}}{b^3}+\frac{3a(bx-a)^{5/3}}{b^3}+\frac{(bx-a)^{8/3}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

input
```
Int[x^3/(-a + b*x)^(1/3),x]
```

output
```
(3*a^3*(-a + b*x)^(2/3))/(2*b^4) + (9*a^2*(-a + b*x)^(5/3))/(5*b^4) + (9*a
*(-a + b*x)^(8/3))/(8*b^4) + (3*(-a + b*x)^(11/3))/(11*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{3(bx-a)^{\frac{2}{3}}\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)}{440b^{4}}$	45
trager	$\frac{3(bx-a)^{\frac{2}{3}}\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)}{440b^{4}}$	45
pseudoelliptic	$\frac{3(bx-a)^{\frac{2}{3}}\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)}{440b^{4}}$	45
risch	$-\frac{3(-bx+a)\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)}{440b^{4}(bx-a)^{\frac{1}{3}}}$	51
orering	$-\frac{3(-bx+a)\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)}{440b^{4}(bx-a)^{\frac{1}{3}}}$	51
derivativedivides	$\frac{\frac{3(bx-a)^{\frac{11}{3}}}{11}+\frac{9a(bx-a)^{\frac{8}{3}}}{8}+\frac{9a^{2}(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^{3}(bx-a)^{\frac{2}{3}}}{2}}{b^{4}}$	58
default	$\frac{\frac{3(bx-a)^{\frac{11}{3}}}{11}+\frac{9a(bx-a)^{\frac{8}{3}}}{8}+\frac{9a^{2}(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^{3}(bx-a)^{\frac{2}{3}}}{2}}{b^{4}}$	58

input
```
int(x^3/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/440/b^4*(b*x-a)^(2/3)*(40*b^3*x^3+45*a*b^2*x^2+54*a^2*b*x+81*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(40\,b^3x^3+45\,ab^2x^2+54\,a^2bx+81\,a^3)(bx-a)^{\frac{2}{3}}}{440\,b^4}$$

input
```
integrate(x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

output
```
3/440*(40*b^3*x^3 + 45*a*b^2*x^2 + 54*a^2*b*x + 81*a^3)*(b*x - a)^(2/3)/b^
4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 4974, normalized size of antiderivative = 62.18

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx = \text{Too large to display}$$

input

```
integrate(x**3/(b*x-a)**(1/3),x)
```

output

```
Piecewise((243*a**(71/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*e
xp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi
/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3)
- 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 2
43*a**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 660
0*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**
14*b**10*x**6*exp(I*pi/3)) - 1296*a**(68/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*
pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a
**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16
*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**1
0*x**6*exp(I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3) - 264
0*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17
*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**
9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 2808*a**(65/3)*b*
*2*x**2*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640
*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*
b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9
*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**
2*x**2/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 66...
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(bx-a)^{\frac{11}{3}}}{11\,b^4} + \frac{9\,(bx-a)^{\frac{8}{3}}a}{8\,b^4} + \frac{9\,(bx-a)^{\frac{5}{3}}a^2}{5\,b^4} + \frac{3\,(bx-a)^{\frac{2}{3}}a^3}{2\,b^4}$$

input
```
integrate(x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

output
```
3/11*(b*x - a)^(11/3)/b^4 + 9/8*(b*x - a)^(8/3)*a/b^4 + 9/5*(b*x - a)^(5/3
)*a^2/b^4 + 3/2*(b*x - a)^(2/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx = \frac{3\left(40\,(bx-a)^{\frac{11}{3}} + 165\,(bx-a)^{\frac{8}{3}}a + 264\,(bx-a)^{\frac{5}{3}}a^2 + 220\,(bx-a)^{\frac{2}{3}}a^3\right)}{440\,b^4}$$

input
```
integrate(x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

output
```
3/440*(40*(b*x - a)^(11/3) + 165*(b*x - a)^(8/3)*a + 264*(b*x - a)^(5/3)*a
^2 + 220*(b*x - a)^(2/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,{\left(b\,x-a\right)}^{11/3}}{11\,b^4} + \frac{9\,a\,{\left(b\,x-a\right)}^{8/3}}{8\,b^4} + \frac{3\,a^3\,{\left(b\,x-a\right)}^{2/3}}{2\,b^4} + \frac{9\,a^2\,{\left(b\,x-a\right)}^{5/3}}{5\,b^4}$$

input
```
int(x^3/(b*x - a)^(1/3),x)
```

output
```
(3*(b*x - a)^(11/3))/(11*b^4) + (9*a*(b*x - a)^(8/3))/(8*b^4) + (3*a^3*(b*
x - a)^(2/3))/(2*b^4) + (9*a^2*(b*x - a)^(5/3))/(5*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx = \frac{3(bx-a)^{\frac{2}{3}}\left(40b^3x^3+45a\,b^2x^2+54a^2bx+81a^3\right)}{440b^4}$$

input
```
int(x^3/(b*x-a)^(1/3),x)
```

output
```
(3*( - a + b*x)**(2/3)*(81*a**3 + 54*a**2*b*x + 45*a*b**2*x**2 + 40*b**3*x
**3))/(440*b**4)
```

3.612 $\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx=\frac{3a^2(-a+bx)^{2/3}}{2b^3}+\frac{6a(-a+bx)^{5/3}}{5b^3}+\frac{3(-a+bx)^{8/3}}{8b^3}$$

output
```
3/2*a^2*(b*x-a)^(2/3)/b^3+6/5*a*(b*x-a)^(5/3)/b^3+3/8*(b*x-a)^(8/3)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx=\frac{3(-a+bx)^{2/3}\,(9a^2+6abx+5b^2x^2)}{40b^3}$$

input
```
Integrate[x^2/(-a + b*x)^(1/3),x]
```

output
```
(3*(-a + b*x)^(2/3)*(9*a^2 + 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{bx-a}}\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{b^2\sqrt[3]{bx-a}}+\frac{2a(bx-a)^{2/3}}{b^2}+\frac{(bx-a)^{5/3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2(bx-a)^{2/3}}{2b^3}+\frac{3(bx-a)^{8/3}}{8b^3}+\frac{6a(bx-a)^{5/3}}{5b^3}$$

input
```
Int[x^2/(-a + b*x)^(1/3),x]
```

output
```
(3*a^2*(-a + b*x)^(2/3))/(2*b^3) + (6*a*(-a + b*x)^(5/3))/(5*b^3) + (3*(-a
 + b*x)^(8/3))/(8*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{3(bx-a)^{\frac{2}{3}}(5b^2x^2+6abx+9a^2)}{40b^3}$	34
trager	$\frac{3(bx-a)^{\frac{2}{3}}(5b^2x^2+6abx+9a^2)}{40b^3}$	34
pseudoelliptic	$\frac{3(bx-a)^{\frac{2}{3}}(5b^2x^2+6abx+9a^2)}{40b^3}$	34
risch	$-\frac{3(-bx+a)(5b^2x^2+6abx+9a^2)}{40b^3(bx-a)^{\frac{1}{3}}}$	40
orering	$-\frac{3(-bx+a)(5b^2x^2+6abx+9a^2)}{40b^3(bx-a)^{\frac{1}{3}}}$	40
derivativedivides	$\frac{\frac{3(bx-a)^{\frac{8}{3}}}{8}+\frac{6a(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx-a)^{\frac{2}{3}}}{2}}{b^3}$	44
default	$\frac{\frac{3(bx-a)^{\frac{8}{3}}}{8}+\frac{6a(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx-a)^{\frac{2}{3}}}{2}}{b^3}$	44

input
```
int(x^2/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/40/b^3*(b*x-a)^(2/3)*(5*b^2*x^2+6*a*b*x+9*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(5\,b^2x^2+6\,abx+9\,a^2)(bx-a)^{\frac{2}{3}}}{40\,b^3}$$

input
```
integrate(x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

output
```
3/40*(5*b^2*x^2 + 6*a*b*x + 9*a^2)*(b*x - a)^(2/3)/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 1326, normalized size of antiderivative = 22.47

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate(x**2/(b*x-a)**(1/3),x)
```

output
```
Piecewise((-27*a**(32/3)*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**
4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)
/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3
) + 63*a**(29/3)*b*x*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x
- 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)
/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3
) - 42*a**(26/3)*b**2*x**2*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b
**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*e
xp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a*
*5*b**6*x**3) + 18*a**(23/3)*b**3*x**3*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3
+ 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)
*b**3*x**3*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*
x**2 + 40*a**5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(-1 + b*x/a)**(2/3)/(-4
0*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) +
15*a**(17/3)*b**5*x**5*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*
x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3), Abs(b*x/a) > 1), (-27*a**(32/
3)*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120
*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**
8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a*
*(29/3)*b*x*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(bx-a)^{\frac{8}{3}}}{8\,b^3} + \frac{6\,(bx-a)^{\frac{5}{3}}a}{5\,b^3} + \frac{3\,(bx-a)^{\frac{2}{3}}a^2}{2\,b^3}$$

input
```
integrate(x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

output
```
3/8*(b*x - a)^(8/3)/b^3 + 6/5*(b*x - a)^(5/3)*a/b^3 + 3/2*(b*x - a)^(2/3)*
a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx = \frac{3\left(5\,(bx-a)^{\frac{8}{3}} + 16\,(bx-a)^{\frac{5}{3}}a + 20\,(bx-a)^{\frac{2}{3}}a^2\right)}{40\,b^3}$$

input
```
integrate(x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

output
```
3/40*(5*(b*x - a)^(8/3) + 16*(b*x - a)^(5/3)*a + 20*(b*x - a)^(2/3)*a^2)/b
^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx = \frac{48\,a\,(b\,x-a)^{5/3} + 15\,(b\,x-a)^{8/3} + 60\,a^2\,(b\,x-a)^{2/3}}{40\,b^3}$$

input
```
int(x^2/(b*x - a)^(1/3),x)
```

output
```
(48*a*(b*x - a)^(5/3) + 15*(b*x - a)^(8/3) + 60*a^2*(b*x - a)^(2/3))/(40*b
^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx = \frac{3(bx-a)^{\frac{2}{3}}\,(5b^2x^2+6abx+9a^2)}{40b^3}$$

input
```
int(x^2/(b*x-a)^(1/3),x)
```

output
```
(3*( - a + b*x)**(2/3)*(9*a**2 + 6*a*b*x + 5*b**2*x**2))/(40*b**3)
```

3.613 $\int \frac{x}{\sqrt[3]{-a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \frac{3a(-a+bx)^{2/3}}{2b^2} + \frac{3(-a+bx)^{5/3}}{5b^2}$$

output
```
3/2*a*(b*x-a)^(2/3)/b^2+3/5*(b*x-a)^(5/3)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \frac{3(-a+bx)^{2/3}(3a+2bx)}{10b^2}$$

input
```
Integrate[x/(-a + b*x)^(1/3),x]
```

output
```
(3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{bx-a}}dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(\frac{a}{b\sqrt[3]{bx-a}}+\frac{(bx-a)^{2/3}}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

input
```
Int[x/(-a + b*x)^(1/3),x]
```

output
```
(3*a*(-a + b*x)^(2/3))/(2*b^2) + (3*(-a + b*x)^(5/3))/(5*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{3(2bx+3a)(bx-a)^{\frac{2}{3}}}{10b^{2}}$	23
trager	$\frac{3(2bx+3a)(bx-a)^{\frac{2}{3}}}{10b^{2}}$	23
pseudoelliptic	$\frac{3(2bx+3a)(bx-a)^{\frac{2}{3}}}{10b^{2}}$	23
risch	$-\frac{3(-bx+a)(2bx+3a)}{10b^{2}(bx-a)^{\frac{1}{3}}}$	29
orering	$-\frac{3(-bx+a)(2bx+3a)}{10b^{2}(bx-a)^{\frac{1}{3}}}$	29
derivativedivides	$\frac{\frac{3(bx-a)^{\frac{5}{3}}}{5}+\frac{3a(bx-a)^{\frac{2}{3}}}{2}}{b^{2}}$	30
default	$\frac{\frac{3(bx-a)^{\frac{5}{3}}}{5}+\frac{3a(bx-a)^{\frac{2}{3}}}{2}}{b^{2}}$	30

input
```
int(x/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/10/b^2*(2*b*x+3*a)*(b*x-a)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(2\,bx+3\,a)(bx-a)^{\frac{2}{3}}}{10\,b^{2}}$$

input
```
integrate(x/(b*x-a)^(1/3),x, algorithm="fricas")
```

output
```
3/10*(2*b*x + 3*a)*(b*x - a)^(2/3)/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 486, normalized size of antiderivative = 12.79

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \begin{cases} -\frac{9a^{\frac{11}{3}}\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{3a^{\frac{8}{3}}bx\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{6a^{\frac{5}{3}}b^2x^2\left(-1+\cdots\right.}{-10a^2b^2e^{\frac{i\pi}{3}}+10\cdots} \\ \frac{9a^{\frac{11}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{3a^{\frac{8}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{6a^{\frac{5}{3}}b^2x^2\left(1-\frac{bx}{a}\cdots\right.}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab\cdots} \end{cases}$$

input
```
integrate(x/(b*x-a)**(1/3),x)
```

output
```
Piecewise((-9*a**(11/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp
(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*pi/
3) + 10*a*b**3*x*exp(I*pi/3)) + 3*a**(8/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*p
i/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*
x/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 6*a**(5/3)*b**2*
x**2*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**
3*x*exp(I*pi/3)), Abs(b*x/a) > 1), (9*a**(11/3)*(1 - b*x/a)**(2/3)/(-10*a*
*2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**
2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 3*a**(8/3)*b*x*(1 - b*x/a)**(2/
3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/
(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 6*a**(5/3)*b**2*x*
*2*(1 - b*x/a)**(2/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(bx-a)^{\frac{5}{3}}}{5\,b^2} + \frac{3\,(bx-a)^{\frac{2}{3}}a}{2\,b^2}$$

input
```
integrate(x/(b*x-a)^(1/3),x, algorithm="maxima")
```

output
```
3/5*(b*x - a)^(5/3)/b^2 + 3/2*(b*x - a)^(2/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \frac{3\left(2\,(bx-a)^{\frac{5}{3}}+5\,(bx-a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

input
```
integrate(x/(b*x-a)^(1/3),x, algorithm="giac")
```

output
```
3/10*(2*(b*x - a)^(5/3) + 5*(b*x - a)^(2/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \frac{15\,a\,(b\,x-a)^{2/3}+6\,(b\,x-a)^{5/3}}{10\,b^2}$$

input
```
int(x/(b*x - a)^(1/3),x)
```

output
```
(15*a*(b*x - a)^(2/3) + 6*(b*x - a)^(5/3))/(10*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt[3]{-a+bx}}\,dx = \frac{3(bx-a)^{\frac{2}{3}}\,(2bx+3a)}{10b^2}$$

input
```
int(x/(b*x-a)^(1/3),x)
```

output
```
(3*( - a + b*x)**(2/3)*(3*a + 2*b*x))/(10*b**2)
```

3.614 $\int \frac{1}{\sqrt[3]{-a+bx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx=\frac{3(-a+bx)^{2/3}}{2b}$$

output
```
3/2*(b*x-a)^(2/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx=\frac{3(-a+bx)^{2/3}}{2b}$$

input
```
Integrate[(-a + b*x)^(-1/3),x]
```

output
```
(3*(-a + b*x)^(2/3))/(2*b)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{bx-a}}\,dx$$
$$\Big\downarrow \; 17$$
$$\frac{3(bx-a)^{2/3}}{2b}$$

input
```
Int[(-a + b*x)^(-1/3),x]
```

output
```
(3*(-a + b*x)^(2/3))/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
derivativedivides	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
default	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
trager	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
pseudoelliptic	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
risch	$-\frac{3(-bx+a)}{2b(bx-a)^{\frac{1}{3}}}$	21
orering	$-\frac{3(-bx+a)}{2b(bx-a)^{\frac{1}{3}}}$	21

input
```
int(1/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(b*x-a)^(2/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(1/(b*x-a)^(1/3),x, algorithm="fricas")
```

output
```
3/2*(b*x - a)^(2/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx = \frac{3(-a+bx)^{\frac{2}{3}}}{2b}$$

input
```
integrate(1/(b*x-a)**(1/3),x)
```

output
```
3*(-a + b*x)**(2/3)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(1/(b*x-a)^(1/3),x, algorithm="maxima")
```

output
```
3/2*(b*x - a)^(2/3)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(1/(b*x-a)^(1/3),x, algorithm="giac")
```

output
```
3/2*(b*x - a)^(2/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx = \frac{3\,(b\,x-a)^{2/3}}{2\,b}$$

input
```
int(1/(b*x - a)^(1/3),x)
```

output
```
(3*(b*x - a)^(2/3))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx = \frac{3(bx-a)^{\frac{2}{3}}}{2b}$$

input
```
int(1/(b*x-a)^(1/3),x)
```

output
```
(3*( - a + b*x)**(2/3))/(2*b)
```

3.615 $\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

output
```
-3^(1/2)*arctan(1/3*(a^(1/3)-2*(b*x-a)^(1/3))*3^(1/2)/a^(1/3))/a^(1/3)+1/2
*ln(x)/a^(1/3)-3/2*ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{2\sqrt[3]{a}}$$

input
```
Integrate[1/(x*(-a + b*x)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - (2*(-a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(
1/3) + (-a + b*x)^(1/3)] + Log[a^(2/3) - a^(1/3)*(-a + b*x)^(1/3) + (-a +
b*x)^(2/3)])/(2*a^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{bx-a}}dx$$

$$\Big\downarrow \text{68}$$

$$\frac{3}{2}\int \frac{1}{a^{2/3}-\sqrt[3]{bx-a}\sqrt[3]{a}+(bx-a)^{2/3}}d\sqrt[3]{bx-a}-\frac{3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx-a}}d\sqrt[3]{bx-a}}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{2}\int \frac{1}{a^{2/3}-\sqrt[3]{bx-a}\sqrt[3]{a}+(bx-a)^{2/3}}d\sqrt[3]{bx-a}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-(bx-a)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{bx-a}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{bx-a}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

input
```
Int[1/(x*(-a + b*x)^(1/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 - (2*(-a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3)) +
Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}-2(bx-a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)+\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)-\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2}}{a^{\frac{1}{3}}}$	79
derivativedivides	$-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}$	83
default	$-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}$	83

input
```
int(1/x/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-(3^(1/2)*arctan(1/3*(a^(1/3)-2*(b*x-a)^(1/3))*3^(1/2)/a^(1/3))+ln(a^(1/3)
+(b*x-a)^(1/3))-1/2*ln((b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a^(2/3)))/a^(1/
3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.48

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \left[\frac{\sqrt{3}a\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+(-a)^{\frac{2}{3}}\log\left((bx-\right.}{2\,a}\right.$$

input
```
integrate(1/x/(b*x-a)^(1/3),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*a*sqrt((-a)^(1/3)/a)*log((2*b*x + sqrt(3)*(2*(b*x - a)^(2/3)
*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*
x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*log((b*x - a)^(2/3) + (b*x
- a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)^(1/3) - (
-a)^(1/3)))/a, 1/2*(2*sqrt(3)*a*sqrt(-(-a)^(1/3)/a)*arctan(1/3*sqrt(3)*(2*
(b*x - a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*log((b*x -
 a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b
*x - a)^(1/3) - (-a)^(1/3)))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = -\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+xe^{\frac{i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} - \frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+xe^{i\pi}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} - \frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+xe^{\frac{5i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(b*x-a)**(1/3),x)
```

output
```
-2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**
(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*log(1 - b**(1/3)*(-a/b + x)*
*(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*ex
p(2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3
))*gamma(2/3)/(3*a**(1/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} + \frac{\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}} - \frac{\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

input
```
integrate(1/x/(b*x-a)^(1/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(1/3)
+ 1/2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - l
og((b*x - a)^(1/3) + a^(1/3))/a^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = -\frac{\sqrt{3}(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{a} + \frac{(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{2\,a} - \frac{(-a)^{\frac{2}{3}}\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a}$$

input
```
integrate(1/x/(b*x-a)^(1/3),x, algorithm="giac")
```

output
```
-sqrt(3)*(-a)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-
a)^(1/3))/a + 1/2*(-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1
/3) + (-a)^(2/3))/a - (-a)^(2/3)*log(abs((b*x - a)^(1/3) - (-a)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{\ln\left(9\,(b\,x-a)^{1/3}-9\,(-a)^{1/3}\right)}{(-a)^{1/3}} + \frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}} - \frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}}$$

input
```
int(1/(x*(b*x - a)^(1/3)),x)
```

output
```
log(9*(b*x - a)^(1/3) - 9*(-a)^(1/3))/(-a)^(1/3) + (log(9*(b*x - a)^(1/3)
- (9*(-a)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-a)^(1/3)) -
(log(9*(b*x - a)^(1/3) - (9*(-a)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
+ 1))/(2*(-a)^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.66

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx-a)^{\frac{1}{6}}-a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)-2\sqrt{3}\,atan\left(\frac{2(bx-a)^{\frac{1}{6}}+a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)-2\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)+\log\left(-a^{\frac{1}{6}}(bx-a)^{\frac{1}{6}}\right.}{2a^{\frac{1}{3}}}$$

input
```
int(1/x/(b*x-a)^(1/3),x)
```

output
```
(2*sqrt(3)*atan((2*( - a + b*x)**(1/6) - a**(1/6)*sqrt(3))/a**(1/6)) - 2*s
qrt(3)*atan((2*( - a + b*x)**(1/6) + a**(1/6)*sqrt(3))/a**(1/6)) - 2*log((
 - a + b*x)**(1/3) + a**(1/3)) + log( - a**(1/6)*( - a + b*x)**(1/6)*sqrt(
3) + ( - a + b*x)**(1/3) + a**(1/3)) + log(a**(1/6)*( - a + b*x)**(1/6)*sq
rt(3) + ( - a + b*x)**(1/3) + a**(1/3)))/(2*a**(1/3))
```

3.616 $\int \frac{1}{x^2 \sqrt[3]{-a+bx}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{1}{x^2 \sqrt[3]{-a+bx}} \, dx = \frac{(-a+bx)^{2/3}}{ax} - \frac{b \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}$$

output
```
(b*x-a)^(2/3)/a/x-1/3*b*arctan(1/3*(a^(1/3)-2*(b*x-a)^(1/3))*3^(1/2)/a^(1/
3))*3^(1/2)/a^(4/3)+1/6*b*ln(x)/a^(4/3)-1/2*b*ln(a^(1/3)+(b*x-a)^(1/3))/a^
(4/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2 \sqrt[3]{-a+bx}} \, dx = \frac{6\sqrt[3]{a}(-a+bx)^{2/3} - 2\sqrt{3}bx \arctan\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2bx\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right) + bx\log\left(a^{2/3} - \sqrt[3]{a}\right.}{6a^{4/3}x}$$

input
```
Integrate[1/(x^2*(-a + b*x)^(1/3)),x]
```

output
```
(6*a^(1/3)*(-a + b*x)^(2/3) - 2*Sqrt[3]*b*x*ArcTan[(1 - (2*(-a + b*x)^(1/3
))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) + (-a + b*x)^(1/3)] + b*x*Log[a^(
2/3) - a^(1/3)*(-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(6*a^(4/3)*x)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {52, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[3]{bx-a}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{b\int \frac{1}{x\sqrt[3]{bx-a}}dx}{3a}+\frac{(bx-a)^{2/3}}{ax}$$

$$\Big\downarrow \text{68}$$

$$\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}-\sqrt[3]{bx-a}\sqrt[3]{a}+(bx-a)^{2/3}}d\sqrt[3]{bx-a}-\frac{3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx-a}}d\sqrt[3]{bx-a}}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}$$

$$\Big\downarrow \text{16}$$

$$\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}-\sqrt[3]{bx-a}\sqrt[3]{a}+(bx-a)^{2/3}}d\sqrt[3]{bx-a}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}$$

$$\Big\downarrow \text{1082}$$

$$\frac{b\left(\frac{3\int\frac{1}{-(bx-a)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{bx-a}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}$$

$$\Big\downarrow \text{217}$$

$$\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{bx-a}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}$$

input
```
Int[1/(x^2*(-a + b*x)^(1/3)),x]
```

output
```
(-a + b*x)^(2/3)/(a*x) + (b*(-((Sqrt[3]*ArcTan[(1 - (2*(-a + b*x)^(1/3))/a
^(1/3))/Sqrt[3]])/a^(1/3)) + Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b
*x)^(1/3)])/(2*a^(1/3))))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{b\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}-2(bx-a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x+b\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)x-\frac{b\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x}{2}-3(bx-a)^{\frac{2}{3}}a^{\frac{1}{3}}}{3a^{\frac{4}{3}}x}$
risch	$-\frac{-bx+a}{ax(bx-a)^{\frac{1}{3}}}-\frac{b\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{4}{3}}}$
derivativedivides	$3b\left(\frac{(bx-a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$
default	$3b\left(\frac{(bx-a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$

input
```
int(1/x^2/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*3^(1/2)*arctan(1/3*(a^(1/3)-2*(b*x-a)^(1/3))*3^(1/2)/a^(1/3))*x+b*
ln(a^(1/3)+(b*x-a)^(1/3))*x-1/2*b*ln((b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a
^(2/3))*x-3*(b*x-a)^(2/3)*a^(1/3))/a^(4/3)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.18

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx$$
$$= \left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+(-a)^{\frac{2}{3}}\,bx\log}{6\,a^2x} \right.$$

input
```
integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2*(b*
x - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3
)/a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*b*x*log((b*x -
a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log
((b*x - a)^(1/3) - (-a)^(1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x), 1/6*(6*sqrt
(1/3)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x - a)^(1/3) + (-a)
^(1/3))*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*b*x*log((b*x - a)^(2/3) + (b*x -
 a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*x - a)^(1/3)
- (-a)^(1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 838, normalized size of antiderivative = 8.14

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx = \text{Too large to display}$$

input

```
integrate(1/x**2/(b*x-a)**(1/3),x)
```

output

```
-2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*
exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*
exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)
*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 -
 b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b
**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(
4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)
/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gam
ma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*
a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*ex
p_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*g
amma(5/3)) - 2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 -
b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b*
*(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b
+ x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b**(10/3)*(-a/b + x)**(
7/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)
/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gam
ma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) +...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}} + \frac{(bx-a)^{\frac{2}{3}}b}{(bx-a)a+a^2} + \frac{b\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}} - \frac{b\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

input
```
integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^
(4/3) + (b*x - a)^(2/3)*b/((b*x - a)*a + a^2) + 1/6*b*log((b*x - a)^(2/3)
- (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x - a)^(1/3) +
 a^(1/3))/a^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx = -\frac{1}{6}\,b\left(\frac{2\sqrt{3}(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{a^2} - \frac{(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{a^2} \cdot\right.$$

input
```
integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

output
```
-1/6*b*(2*sqrt(3)*(-a)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^
(1/3))/(-a)^(1/3))/a^2 - (-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*
(-a)^(1/3) + (-a)^(2/3))/a^2 + 2*(-a)^(2/3)*log(abs((b*x - a)^(1/3) - (-a)
^(1/3)))/a^2 - 6*(b*x - a)^(2/3)/(a*b*x))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx = \frac{(b\,x-a)^{2/3}}{a\,x} - \frac{b\,\ln\left((b\,x-a)^{1/3}+a^{1/3}\right)}{3\,a^{4/3}} + \frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}} + \frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}$$

input
```
int(1/(x^2*(b*x - a)^(1/3)),x)
```

output
```
(b*x - a)^(2/3)/(a*x) - (b*log((b*x - a)^(1/3) + a^(1/3)))/(3*a^(4/3)) + (
log((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x - a)^(1/3))/a^2)*(b - 3^(
1/2)*b*1i))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*
x - a)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx-a)^{\frac{1}{6}}-a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)bx-2\sqrt{3}\,atan\left(\frac{2(bx-a)^{\frac{1}{6}}+a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)bx+6a^{\frac{1}{3}}(bx-a)^{\frac{2}{3}}-2\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{6a^{\frac{4}{3}}x}$$

input
```
int(1/x^2/(b*x-a)^(1/3),x)
```

output
```
(2*sqrt(3)*atan((2*( - a + b*x)**(1/6) - a**(1/6)*sqrt(3))/a**(1/6))*b*x -
 2*sqrt(3)*atan((2*( - a + b*x)**(1/6) + a**(1/6)*sqrt(3))/a**(1/6))*b*x +
 6*a**(1/3)*( - a + b*x)**(2/3) - 2*log(( - a + b*x)**(1/3) + a**(1/3))*b*
x + log( - a**(1/6)*( - a + b*x)**(1/6)*sqrt(3) + ( - a + b*x)**(1/3) + a*
*(1/3))*b*x + log(a**(1/6)*( - a + b*x)**(1/6)*sqrt(3) + ( - a + b*x)**(1/
3) + a**(1/3))*b*x)/(6*a**(1/3)*a*x)
```

3.617 $\int \frac{1}{x^3 \sqrt[3]{-a+bx}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{1}{x^3 \sqrt[3]{-a+bx}} \, dx = \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} - \frac{2b^2 \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} + \frac{b^2 \log(x)}{9a^{7/3}} - \frac{b^2 \log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}}$$

output
```
1/2*(b*x-a)^(2/3)/a/x^2+2/3*b*(b*x-a)^(2/3)/a^2/x-2/9*b^2*arctan(1/3*(a^(1
/3)-2*(b*x-a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)+1/9*b^2*ln(x)/a^(7/3
)-1/3*b^2*ln(a^(1/3)+(b*x-a)^(1/3))/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx = \frac{(-a+bx)^{2/3}(7a+4(-a+bx))}{6a^2x^2} - \frac{2b^2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} - \frac{2b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{9a^{7/3}} + \frac{b^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{9a^{7/3}}$$

input
```
Integrate[1/(x^3*(-a + b*x)^(1/3)),x]
```

output
```
((-a + b*x)^(2/3)*(7*a + 4*(-a + b*x)))/(6*a^2*x^2) - (2*b^2*ArcTan[1/Sqrt
[3] - (2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (2*b^
2*Log[a^(1/3) + (-a + b*x)^(1/3)])/(9*a^(7/3)) + (b^2*Log[a^(2/3) - a^(1/3
)*(-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(9*a^(7/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {52, 52, 68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt[3]{bx-a}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{2b\int \frac{1}{x^2\sqrt[3]{bx-a}}dx}{3a} + \frac{(bx-a)^{2/3}}{2ax^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{2b\left(\frac{b\int \frac{1}{x\sqrt[3]{bx-a}}dx}{3a}+\frac{(bx-a)^{2/3}}{ax}\right)}{3a}+\frac{(bx-a)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 68

$$\frac{2b\left(\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}-\sqrt[3]{bx-a}\sqrt[3]{a}+(bx-a)^{2/3}}d\sqrt[3]{bx-a}-\frac{3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{bx-a}}d\sqrt[3]{bx-a}}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}\right)}{3a}+\frac{(bx-a)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 16

$$\frac{2b\left(\frac{b\left(\frac{3}{2}\int \frac{1}{a^{2/3}-\sqrt[3]{bx-a}\sqrt[3]{a}+(bx-a)^{2/3}}d\sqrt[3]{bx-a}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}\right)}{3a}+\frac{(bx-a)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 1082

$$\frac{2b\left(\frac{b\left(\frac{3\int \frac{1}{-(bx-a)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{bx-a}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}\right)}{3a}+\frac{(bx-a)^{2/3}}{2ax^2}$$

$\Big\downarrow$ 217

$$\frac{2b\left(\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{bx-a}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}\right)}{3a}+\frac{(bx-a)^{2/3}}{ax}\right)}{3a}+\frac{(bx-a)^{2/3}}{2ax^2}$$

input
```
Int[1/(x^3*(-a + b*x)^(1/3)),x]
```

output
```
(-a + b*x)^(2/3)/(2*a*x^2) + (2*b*((-a + b*x)^(2/3)/(a*x) + (b*(-((Sqrt[3]
*ArcTan[(1 - (2*(-a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3)) + Log[x]/(2*
a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))))/(3*a)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{(-bx+a)(4bx+3a)}{6a^2x^2(bx-a)^{\frac{1}{3}}}-\frac{2b^2\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{b^2\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\cdots}{}\right.}{}\right.}{9a^{\frac{7}{3}}}$
pseudoelliptic	$\frac{-4b^2\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}-2(bx-a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^2-4b^2\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)x^2+2b^2\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2+12b(b\cdots}{18a^{\frac{7}{3}}x^2}$
derivativedivides	$3b^2\left(\frac{(bx-a)^{\frac{2}{3}}}{6ab^2x^2}+\frac{\frac{2(bx-a)^{\frac{2}{3}}}{9abx}+\frac{2\left(-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-\cdots\right)}{3}\right)}{3a^{\frac{1}{3}}}\cdots\right)}{9a}}{a}\cdots\right.$
default	$3b^2\left(\frac{(bx-a)^{\frac{2}{3}}}{6ab^2x^2}+\frac{\frac{2(bx-a)^{\frac{2}{3}}}{9abx}+\frac{2\left(-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-\cdots\right)}{3}\right)}{3a^{\frac{1}{3}}}\cdots\right)}{9a}}{a}\cdots\right.$

input
```
int(1/x^3/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b*x+a)*(4*b*x+3*a)/a^2/x^2/(b*x-a)^(1/3)-2/9*b^2*ln(a^(1/3)+(b*x-a)
^(1/3))/a^(7/3)+1/9*b^2/a^(7/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a^(
2/3))+2/9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(1/3)-
1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx$$
$$= \left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+3\,\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+2\,(-a)^{\frac{2}{3}}\,b}{}\right.$$

input
```
integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(
2*(b*x - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)
^(1/3)/a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + 2*(-a)^(2/3)*b^2*x^2*
log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 4*(-a)^(2
/3)*b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2)*(b*x -
 a)^(2/3))/(a^3*x^2), 1/18*(12*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 2*(-
a)^(2/3)*b^2*x^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(
2/3)) - 4*(-a)^(2/3)*b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*
x + 3*a^2)*(b*x - a)^(2/3))/(a^3*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 2744, normalized size of antiderivative = 20.18

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x-a)**(1/3),x)
```

output
```
-4*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3
)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma
(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4
*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/
b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b
 + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)
*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*p
i/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 4*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7
*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-
a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(
10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(
2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*log(1 - b
**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a
/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(1
0/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*ex...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3 \sqrt[3]{-a+bx}}\,dx = \frac{2\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{7}{3}}} + \frac{b^2 \log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}} - \frac{2b^2 \log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}} + \frac{4(bx-a)^{\frac{5}{3}}b^2+7(bx-a)^{\frac{2}{3}}ab^2}{6\left((bx-a)^2a^2+2(bx-a)a^3+a^4\right)}$$

input
```
integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/
a^(7/3) + 1/9*b^2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) - 2/9*b^2*log((b*x - a)^(1/3) + a^(1/3))/a^(7/3) + 1/6*(4*(b*x -
a)^(5/3)*b^2 + 7*(b*x - a)^(2/3)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3
 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 \sqrt[3]{-a+bx}}\,dx = \frac{\frac{4\sqrt{3}b^3 \arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{(-a)^{\frac{1}{3}}a^2} - \frac{2b^3 \log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{(-a)^{\frac{1}{3}}a^2} - \frac{4(-a)^{\frac{2}{3}}b^3 \log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^3} + 3\left(4(\cdots\right.}{18b}$$

input
```
integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

output
```
1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-
a)^(1/3))/((-a)^(1/3)*a^2) - 2*b^3*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(
-a)^(1/3) + (-a)^(2/3))/((-a)^(1/3)*a^2) - 4*(-a)^(2/3)*b^3*log(abs((b*x -
 a)^(1/3) - (-a)^(1/3)))/a^3 + 3*(4*(b*x - a)^(5/3)*b^3 + 7*(b*x - a)^(2/3
)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx = \frac{\frac{7\,b^2\,(b\,x-a)^{2/3}}{6\,a}+\frac{2\,b^2\,(b\,x-a)^{5/3}}{3\,a^2}}{(a-b\,x)^2-2\,a\,(a-b\,x)+a^2} - \frac{\ln\left(\frac{4\,b^4\,(b\,x-a)^{1/3}}{9\,a^4}-\frac{\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)^2}{9\,(-a)^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{9\,(-a)^{7/3}} + \frac{2\,b^2\,\ln\left(\frac{4\,b^4\,(b\,x-a)^{1/3}}{9\,a^4}-\frac{4\,b^4}{9\,(-a)^{11/3}}\right)}{9\,(-a)^{7/3}} + \frac{b^2\,\ln\left(\frac{4\,b^4\,(b\,x-a)^{1/3}}{9\,a^4}-\frac{9\,b^4\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2}{(-a)^{11/3}}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{(-a)^{7/3}}$$

input
```
int(1/(x^3*(b*x - a)^(1/3)),x)
```

output
```
((7*b^2*(b*x - a)^(2/3))/(6*a) + (2*b^2*(b*x - a)^(5/3))/(3*a^2))/((a - b*
x)^2 - 2*a*(a - b*x) + a^2) - (log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (3^(1
/2)*b^2*1i + b^2)^2/(9*(-a)^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*(-a)^(7/3)
) + (2*b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (4*b^4)/(9*(-a)^(11/3))))
/(9*(-a)^(7/3)) + (b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (9*b^4*((3^(1
/2)*1i)/9 - 1/9)^2)/(-a)^(11/3))*((3^(1/2)*1i)/9 - 1/9))/(-a)^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx$$
$$= \frac{4\sqrt{3}\,atan\left(\frac{2(bx-a)^{\frac{1}{6}}-a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)b^2x^2 - 4\sqrt{3}\,atan\left(\frac{2(bx-a)^{\frac{1}{6}}+a^{\frac{1}{6}}\sqrt{3}}{a^{\frac{1}{6}}}\right)b^2x^2 + 9a^{\frac{4}{3}}(bx-a)^{\frac{2}{3}} + 12a^{\frac{1}{3}}(bx-a)^{\frac{2}{3}}\,bx -}{}$$

input
```
int(1/x^3/(b*x-a)^(1/3),x)
```

output
```
(4*sqrt(3)*atan((2*( - a + b*x)**(1/6) - a**(1/6)*sqrt(3))/a**(1/6))*b**2*
x**2 - 4*sqrt(3)*atan((2*( - a + b*x)**(1/6) + a**(1/6)*sqrt(3))/a**(1/6))
*b**2*x**2 + 9*a**(1/3)*( - a + b*x)**(2/3)*a + 12*a**(1/3)*( - a + b*x)**
(2/3)*b*x - 4*log(( - a + b*x)**(1/3) + a**(1/3))*b**2*x**2 + 2*log( - a**
(1/6)*( - a + b*x)**(1/6)*sqrt(3) + ( - a + b*x)**(1/3) + a**(1/3))*b**2*x
**2 + 2*log(a**(1/6)*( - a + b*x)**(1/6)*sqrt(3) + ( - a + b*x)**(1/3) + a
**(1/3))*b**2*x**2)/(18*a**(1/3)*a**2*x**2)
```

3.618 $\int \frac{x^3}{(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = -\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}$$

output
```
-3*a^3*(b*x+a)^(1/3)/b^4+9/4*a^2*(b*x+a)^(4/3)/b^4-9/7*a*(b*x+a)^(7/3)/b^4
+3/10*(b*x+a)^(10/3)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}(-81a^3+27a^2bx-18ab^2x^2+14b^3x^3)}{140b^4}$$

input
```
Integrate[x^3/(a + b*x)^(2/3),x]
```

output
```
(3*(a + b*x)^(1/3)*(-81*a^3 + 27*a^2*b*x - 18*a*b^2*x^2 + 14*b^3*x^3))/(14
0*b^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^{2/3}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

input
```
Int[x^3/(a + b*x)^(2/3),x]
```

output
```
(-3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b*x)^(4/3))/(4*b^4) - (9*a*(a +
 b*x)^(7/3))/(7*b^4) + (3*(a + b*x)^(10/3))/(10*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
trager	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
risch	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
pseudoelliptic	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
orering	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{9a(bx+a)^{\frac{7}{3}}}{7}+\frac{9a^{2}(bx+a)^{\frac{4}{3}}}{4}-3a^{3}(bx+a)^{\frac{1}{3}}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{9a(bx+a)^{\frac{7}{3}}}{7}+\frac{9a^{2}(bx+a)^{\frac{4}{3}}}{4}-3a^{3}(bx+a)^{\frac{1}{3}}}{b^{4}}$	50

input
```
int(x^3/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/140*(b*x+a)^(1/3)*(-14*b^3*x^3+18*a*b^2*x^2-27*a^2*b*x+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = \frac{3\,(14\,b^3x^3 - 18\,ab^2x^2 + 27\,a^2bx - 81\,a^3)(bx+a)^{\frac{1}{3}}}{140\,b^4}$$

input
```
integrate(x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3/140*(14*b^3*x^3 - 18*a*b^2*x^2 + 27*a^2*b*x - 81*a^3)*(b*x + a)^(1/3)/b^
4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(66) = 132$.

Time = 1.25 (sec) , antiderivative size = 1640, normalized size of antiderivative = 23.43

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(b*x+a)**(2/3),x)
```

output
```
-243*a**(70/3)*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 210
0*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**1
5*b**9*x**5 + 140*a**14*b**10*x**6) + 243*a**(70/3)/(140*a**20*b**4 + 840*
a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b*
*8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 1377*a**(67/3)*b*x
*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x
**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 +
140*a**14*b**10*x**6) + 1458*a**(67/3)*b*x/(140*a**20*b**4 + 840*a**19*b**
5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 +
 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 3213*a**(64/3)*b**2*x**2*(1
 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2
 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140
*a**14*b**10*x**6) + 3645*a**(64/3)*b**2*x**2/(140*a**20*b**4 + 840*a**19*
b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**
4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 3927*a**(61/3)*b**3*x**3
*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x
**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 +
140*a**14*b**10*x**6) + 4860*a**(61/3)*b**3*x**3/(140*a**20*b**4 + 840*a**
19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*
x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 2583*a**(58/3)*b**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = \frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^4} - \frac{9\,(bx+a)^{\frac{7}{3}}a}{7\,b^4} + \frac{9\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^4} - \frac{3\,(bx+a)^{\frac{1}{3}}a^3}{b^4}$$

input
```
integrate(x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3/10*(b*x + a)^(10/3)/b^4 - 9/7*(b*x + a)^(7/3)*a/b^4 + 9/4*(b*x + a)^(4/3
)*a^2/b^4 - 3*(b*x + a)^(1/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = \frac{3\left(14\,(bx+a)^{\frac{10}{3}} - 60\,(bx+a)^{\frac{7}{3}}a + 105\,(bx+a)^{\frac{4}{3}}a^2 - 140\,(bx+a)^{\frac{1}{3}}a^3\right)}{140\,b^4}$$

input
```
integrate(x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3/140*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^
2 - 140*(b*x + a)^(1/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = \frac{3\,(a+b\,x)^{10/3}}{10\,b^4} - \frac{3\,a^3\,(a+b\,x)^{1/3}}{b^4} + \frac{9\,a^2\,(a+b\,x)^{4/3}}{4\,b^4} - \frac{9\,a\,(a+b\,x)^{7/3}}{7\,b^4}$$

input
```
int(x^3/(a + b*x)^(2/3),x)
```

output
```
(3*(a + b*x)^(10/3))/(10*b^4) - (3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a +
b*x)^(4/3))/(4*b^4) - (9*a*(a + b*x)^(7/3))/(7*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{x^3}{(a+bx)^{2/3}}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\left(14b^3x^3-18a\,b^2x^2+27a^2bx-81a^3\right)}{140b^4}$$

input
```
int(x^3/(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(1/3)*( - 81*a**3 + 27*a**2*b*x - 18*a*b**2*x**2 + 14*b**3*x
**3))/(140*b**4)
```

3.619 $\int \frac{x^2}{(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx=\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}$$

output
```
3*a^2*(b*x+a)^(1/3)/b^3-3/2*a*(b*x+a)^(4/3)/b^3+3/7*(b*x+a)^(7/3)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx=\frac{3\sqrt[3]{a+bx}(9a^2-3abx+2b^2x^2)}{14b^3}$$

input
```
Integrate[x^2/(a + b*x)^(2/3),x]
```

output
```
(3*(a + b*x)^(1/3)*(9*a^2 - 3*a*b*x + 2*b^2*x^2))/(14*b^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^{2/3}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{b^2 (a+bx)^{2/3}}-\frac{2 a \sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3 a^2 \sqrt[3]{a+bx}}{b^3}+\frac{3 (a+bx)^{7/3}}{7 b^3}-\frac{3 a (a+bx)^{4/3}}{2 b^3}$$

input
```
Int[x^2/(a + b*x)^(2/3),x]
```

output
```
(3*a^2*(a + b*x)^(1/3))/b^3 - (3*a*(a + b*x)^(4/3))/(2*b^3) + (3*(a + b*x)
^(7/3))/(7*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{3(bx+a)^{\frac{1}{3}}(2b^2x^2-3abx+9a^2)}{14b^3}$	32
trager	$\frac{3(bx+a)^{\frac{1}{3}}(2b^2x^2-3abx+9a^2)}{14b^3}$	32
risch	$\frac{3(bx+a)^{\frac{1}{3}}(2b^2x^2-3abx+9a^2)}{14b^3}$	32
pseudoelliptic	$\frac{3(bx+a)^{\frac{1}{3}}(2b^2x^2-3abx+9a^2)}{14b^3}$	32
orering	$\frac{3(bx+a)^{\frac{1}{3}}(2b^2x^2-3abx+9a^2)}{14b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{2}+3a^2(bx+a)^{\frac{1}{3}}}{b^3}$	37
default	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{2}+3a^2(bx+a)^{\frac{1}{3}}}{b^3}$	37

input
```
int(x^2/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/14*(b*x+a)^(1/3)*(2*b^2*x^2-3*a*b*x+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx = \frac{3\,(2\,b^2x^2-3\,abx+9\,a^2)(bx+a)^{\frac{1}{3}}}{14\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3/14*(2*b^2*x^2 - 3*a*b*x + 9*a^2)*(b*x + a)^(1/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(48) = 96$.

Time = 0.82 (sec) , antiderivative size = 600, normalized size of antiderivative = 11.76

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx = \frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} - \frac{27a^{\frac{31}{3}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} + \frac{72a^{\frac{28}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} - \frac{81a^{\frac{28}{3}}bx}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} + \frac{60a^{\frac{25}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} - \frac{81a^{\frac{25}{3}}b^2x^2}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} + \frac{18a^{\frac{22}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} - \frac{27a^{\frac{22}{3}}b^3x^3}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} + \frac{9a^{\frac{19}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3} + \frac{6a^{\frac{16}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}$$

input
```
integrate(x**2/(b*x+a)**(2/3),x)
```

output
```
27*a**(31/3)*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b
**5*x**2 + 14*a**5*b**6*x**3) - 27*a**(31/3)/(14*a**8*b**3 + 42*a**7*b**4*
x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 72*a**(28/3)*b*x*(1 + b*x/a)*
*(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x
**3) - 81*a**(28/3)*b*x/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2
 + 14*a**5*b**6*x**3) + 60*a**(25/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(14*a**8
*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 81*a**(2
5/3)*b**2*x**2/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**
5*b**6*x**3) + 18*a**(22/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 4
2*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 27*a**(22/3)*b**3
*x**3/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x*
*3) + 9*a**(19/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**
4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 6*a**(16/3)*b**5*x**5*(1 +
b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5
*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx = \frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^3} - \frac{3\,(bx+a)^{\frac{4}{3}}a}{2\,b^3} + \frac{3\,(bx+a)^{\frac{1}{3}}a^2}{b^3}$$

input
```
integrate(x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3/7*(b*x + a)^(7/3)/b^3 - 3/2*(b*x + a)^(4/3)*a/b^3 + 3*(b*x + a)^(1/3)*a^
2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx = \frac{3\left(2\,(bx+a)^{\frac{7}{3}} - 7\,(bx+a)^{\frac{4}{3}}a + 14\,(bx+a)^{\frac{1}{3}}a^2\right)}{14\,b^3}$$

input
```
integrate(x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b^
3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx = \frac{6\,(a+b\,x)^{7/3}-21\,a\,(a+b\,x)^{4/3}+42\,a^2\,(a+b\,x)^{1/3}}{14\,b^3}$$

input
```
int(x^2/(a + b*x)^(2/3),x)
```

output
```
(6*(a + b*x)^(7/3) - 21*a*(a + b*x)^(4/3) + 42*a^2*(a + b*x)^(1/3))/(14*b^
3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(a+bx)^{2/3}}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\,(2b^2x^2-3abx+9a^2)}{14b^3}$$

input
```
int(x^2/(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(1/3)*(9*a**2 - 3*a*b*x + 2*b**2*x**2))/(14*b**3)
```

3.620 $\int \frac{x}{(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x}{(a+bx)^{2/3}}\,dx = -\frac{3a\sqrt[3]{a+bx}}{b^2} + \frac{3(a+bx)^{4/3}}{4b^2}$$

output
```
-3*a*(b*x+a)^(1/3)/b^2+3/4*(b*x+a)^(4/3)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+bx)^{2/3}}\,dx = \frac{3(-3a+bx)\sqrt[3]{a+bx}}{4b^2}$$

input
```
Integrate[x/(a + b*x)^(2/3),x]
```

output
```
(3*(-3*a + b*x)*(a + b*x)^(1/3))/(4*b^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{\sqrt[3]{a+bx}}{b}-\frac{a}{b(a+bx)^{2/3}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

input
```
Int[x/(a + b*x)^(2/3),x]
```

output
```
(-3*a*(a + b*x)^(1/3))/b^2 + (3*(a + b*x)^(4/3))/(4*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^{2}}$	21
trager	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^{2}}$	21
risch	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^{2}}$	21
pseudoelliptic	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^{2}}$	21
orering	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^{2}}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{4}{3}}}{4}-3a(bx+a)^{\frac{1}{3}}}{b^{2}}$	26
default	$\frac{\frac{3(bx+a)^{\frac{4}{3}}}{4}-3a(bx+a)^{\frac{1}{3}}}{b^{2}}$	26

input
```
int(x/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(b*x+a)^(1/3)*(-b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x}{(a+bx)^{2/3}}\,dx=\frac{3\,(bx+a)^{\frac{1}{3}}(bx-3\,a)}{4\,b^{2}}$$

input
```
integrate(x/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3/4*(b*x + a)^(1/3)*(b*x - 3*a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.06

$$\int \frac{x}{(a+bx)^{2/3}}\,dx = -\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x} + \frac{9a^{\frac{10}{3}}}{4a^2b^2+4ab^3x} - \frac{6a^{\frac{7}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x} + \frac{9a^{\frac{7}{3}}bx}{4a^2b^2+4ab^3x} + \frac{3a^{\frac{4}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}$$

input
```
integrate(x/(b*x+a)**(2/3),x)
```

output
```
-9*a**(10/3)*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x) + 9*a**(10/3)/(
4*a**2*b**2 + 4*a*b**3*x) - 6*a**(7/3)*b*x*(1 + b*x/a)**(1/3)/(4*a**2*b**2
 + 4*a*b**3*x) + 9*a**(7/3)*b*x/(4*a**2*b**2 + 4*a*b**3*x) + 3*a**(4/3)*b*
*2*x**2*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+bx)^{2/3}}\,dx = \frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b^2} - \frac{3\,(bx+a)^{\frac{1}{3}}a}{b^2}$$

input
```
integrate(x/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3/4*(b*x + a)^(4/3)/b^2 - 3*(b*x + a)^(1/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+bx)^{2/3}}\,dx = \frac{3\left((bx+a)^{\frac{4}{3}} - 4\,(bx+a)^{\frac{1}{3}}a\right)}{4\,b^2}$$

input
```
integrate(x/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3/4*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx)^{2/3}}\,dx = -\frac{12\,a\,(a+b\,x)^{1/3} - 3\,(a+b\,x)^{4/3}}{4\,b^2}$$

input
```
int(x/(a + b*x)^(2/3),x)
```

output
```
-(12*a*(a + b*x)^(1/3) - 3*(a + b*x)^(4/3))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x}{(a+bx)^{2/3}}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\,(bx-3a)}{4b^2}$$

input
```
int(x/(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(1/3)*( - 3*a + b*x))/(4*b**2)
```

3.621 $\int \frac{1}{(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{b}$$

output
```
3*(b*x+a)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{b}$$

input
```
Integrate[(a + b*x)^(-2/3),x]
```

output
```
(3*(a + b*x)^(1/3))/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{2/3}} \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{3\sqrt[3]{a+bx}}{b}$$

input
```
Int[(a + b*x)^(-2/3),x]
```

output
```
(3*(a + b*x)^(1/3))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
default	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
trager	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
risch	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
pseudoelliptic	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
orering	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13

input
```
int(1/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*(b*x+a)^(1/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

input
```
integrate(1/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
3*(b*x + a)^(1/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{b}$$

input
```
integrate(1/(b*x+a)**(2/3),x)
```

output
```
3*(a + b*x)**(1/3)/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

input
```
integrate(1/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
3*(b*x + a)^(1/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

input
```
integrate(1/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
3*(b*x + a)^(1/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\,(a+b\,x)^{1/3}}{b}$$

input
```
int(1/(a + b*x)^(2/3),x)
```

output
```
(3*(a + b*x)^(1/3))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3(bx+a)^{\frac{1}{3}}}{b}$$

input
```
int(1/(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(1/3))/b
```

3.622 $\int \frac{1}{x(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

output
```
-3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))/a^(2/3)-1/2
*ln(x)/a^(2/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{2/3}}$$

input
```
Integrate[1/(x*(a + b*x)^(2/3)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[
a^(1/3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a +
b*x)^(2/3)])/a^(2/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^{2/3}}dx$$

$$\Big\downarrow \text{69}$$

$$-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

input
```
Int[1/(x*(a + b*x)^(2/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - L
og[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2a^{\frac{2}{3}}}$	75
derivativedivides	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}$	76
default	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}$	76

input
```
int(1/x/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))-ln((
b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+2*ln((b*x+a)^(1/3)-a^(1/3)))/a
^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(57) = 114.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx =$$

$$-\frac{2\sqrt{3}(a^2)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3\,a^2}\right)+(a^2)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right.}{2\,a^2}$$

input
```
integrate(1/x/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*
a + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*log((b*x + a)^(2/3)*
a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2/3)*log((b*x
+ a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.88

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = \frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(b*x+a)**(2/3),x)
```

output
```
log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4
/3)) + exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3
)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b**
(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3
)*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}} + \frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
 - 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) +
log((b*x + a)^(1/3) - a^(1/3))/a^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}} + \frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
 - 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) +
log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = \frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{2/3}} + \frac{\ln\left(\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-9\,(a+b\,x)^{1/3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}} - \frac{\ln\left(\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+9\,(a+b\,x)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}$$

input
```
int(1/(x*(a + b*x)^(2/3)),x)
```

output
```
log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(2/3) + (log((9*a^(1/3)*(3^(1/2)*1i -
 1))/2 - 9*(a + b*x)^(1/3))*(3^(1/2)*1i - 1))/(2*a^(2/3)) - (log((9*a^(1/3
)*(3^(1/2)*1i + 1))/2 + 9*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/(2*a^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2\log\left((bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}\right)+2\log}{}$$

input
```
int(1/x/(b*x+a)^(2/3),x)
```

output
```
(2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3))) - 2*sq
rt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3))) + 2*log((a
+ b*x)**(1/6) + a**(1/6)) + 2*log((a + b*x)**(1/6) - a**(1/6)) - log( - a*
*(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)) - log(a**(1/6)*(a +
 b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)))/(2*a**(2/3))
```

3.623 $\int \frac{1}{x^2(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx}}{ax}+\frac{2b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} + \frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}$$

output
```
-(b*x+a)^(1/3)/a/x+2/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1
/3))*3^(1/2)/a^(5/3)+1/3*b*ln(x)/a^(5/3)-b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(5/
3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = \frac{-3a^{2/3}\sqrt[3]{a+bx}+2\sqrt{3}bx\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+bx\,\mathrm{l}}{3a^{5/3}x}$$

input
```
Integrate[1/(x^2*(a + b*x)^(2/3)),x]
```

output
```
(-3*a^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3)
)/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a^(2/
3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(5/3)*x)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^{2/3}}dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{2b\int \frac{1}{x(a+bx)^{2/3}}dx}{3a}-\frac{\sqrt[3]{a+bx}}{ax}$$

$$\Big\downarrow \text{69}$$

$$-\frac{2b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2b\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2b\left(\frac{3\int\frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}$$

$\big\downarrow$ 217

$$-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}$$

input
```
Int[1/(x^2*(a + b*x)^(2/3)),x]
```

output
```
-((a + b*x)^(1/3)/(a*x)) - (2*b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3)
)/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a +
 b*x)^(1/3)])/(2*a^(2/3))))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)bx-2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)bx+\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)bx-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}}{3a^{\frac{5}{3}}x}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}}{a}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}}{a}\right)$

input
```
int(1/x^2/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*b*x-2
*ln((b*x+a)^(1/3)-a^(1/3))*b*x+ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2
/3))*b*x-3*(b*x+a)^(1/3)*a^(2/3))/a^(5/3)/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(75) = 150.

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.66

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = \frac{6\sqrt{\frac{1}{3}}abx\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-a^2\right)^{\frac{1}{3}}a-2\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx$$

input
```
integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
1/3*(6*sqrt(1/3)*a*b*x*sqrt(-(-a^2)^(1/3))*arctan(-sqrt(1/3)*((-a^2)^(1/3)
*a - 2*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^2)^(2/
3)*b*x*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)*(b*x + a)^(1/
3)) - 2*(-a^2)^(2/3)*b*x*log((b*x + a)^(1/3)*a - (-a^2)^(2/3)) - 3*(b*x +
a)^(1/3)*a^2)/(a^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 830, normalized size of antiderivative = 8.47

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(b*x+a)**(2/3),x)
```

output
```
-2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4
/3)) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1
/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(
2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*p
i/3)*gamma(4/3)) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(9
*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)
*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x
)**(5/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1
/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b*
*(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a
/b + x)**(5/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1
/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3)
 - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)
*b**(8/3)*(a/b + x)**(5/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3
)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/
3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/
3)*gamma(4/3)) + 3*a*b**2*(a/b + x)*exp(2*I*pi/3)*gamma(1/3)/(9*a**3*b*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = \frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{5}{3}}} - \frac{(bx+a)^{\frac{1}{3}}b}{(bx+a)a-a^2} + \frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{5}{3}}} - \frac{2\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{5}{3}}}$$

input
```
integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(5/3) - (b*x + a)^(1/3)*b/((b*x + a)*a - a^2) + 1/3*b*log((b*x + a)^(2/3)
+ (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2/3*b*log((b*x + a)^(1/3) -
 a^(1/3))/a^(5/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = \frac{1}{3}\,b\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}} + \frac{\log\left((bx+a)^{\frac{2}{3}} + (bx+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}} - \frac{2\log\right.$$

input
```
integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
1/3*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(5/3) + log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3
) - 2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x + a)^(1/3)/(a*b
*x))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = -\frac{(a+b\,x)^{1/3}}{a\,x} + \frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}} + \frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}} + \frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}} + \frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}} - \frac{2\,b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{5/3}}$$

input
```
int(1/(x^2*(a + b*x)^(2/3)),x)
```

output
```
(log((3*(b - 3^(1/2)*b*1i))/a^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b - 3^(1/2
)*b*1i))/(3*a^(5/3)) - (a + b*x)^(1/3)/(a*x) + (log((3*(b + 3^(1/2)*b*1i))
/a^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(3*a^(5/3)) - (2*b
*log((a + b*x)^(1/3) - a^(1/3)))/(3*a^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx+2\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-3a^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}-2\log(}{}$$

input
```
int(1/x^2/(b*x+a)^(2/3),x)
```

output
```
( - 2*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b*x
 + 2*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b*x
- 3*a**(2/3)*(a + b*x)**(1/3) - 2*log((a + b*x)**(1/6) + a**(1/6))*b*x - 2
*log((a + b*x)**(1/6) - a**(1/6))*b*x + log( - a**(1/6)*(a + b*x)**(1/6) +
 (a + b*x)**(1/3) + a**(1/3))*b*x + log(a**(1/6)*(a + b*x)**(1/6) + (a + b
*x)**(1/3) + a**(1/3))*b*x)/(3*a**(2/3)*a*x)
```

3.624 $\int \frac{1}{x^3(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}$$

output
```
-1/2*(b*x+a)^(1/3)/a/x^2+5/6*b*(b*x+a)^(1/3)/a^2/x-5/9*b^2*arctan(1/3*(a^(
1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)-5/18*b^2*ln(x)/a^(8
/3)+5/6*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx}(8a-5(a+bx))}{6a^2x^2}-\frac{5b^2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{8/3}}-\frac{5b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{8/3}}$$

input
```
Integrate[1/(x^3*(a + b*x)^(2/3)),x]
```

output
```
-1/6*((a + b*x)^(1/3)*(8*a - 5*(a + b*x)))/(a^2*x^2) - (5*b^2*ArcTan[1/Sqr
t[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (5*b^
2*Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(8/3)) - (5*b^2*Log[a^(2/3) + a^(1/
3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(8/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\int \frac{1}{x^2(a+bx)^{2/3}}dx}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

$$\Big\downarrow \text{52}$$

$$-\frac{5b\left(-\frac{2b\int \frac{1}{x(a+bx)^{2/3}}dx}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

$$\Big\downarrow \text{69}$$

$$-\frac{5b\left(-\frac{2b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2a^{2/3}}-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

$$\Big\downarrow \; 16$$

$$-\frac{5b\left(-\frac{2b\left(-\frac{3\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

$$\Big\downarrow \; 1082$$

$$-\frac{5b\left(-\frac{2b\left(\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{5b\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx}}{ax}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

input

```
Int[1/(x^3*(a + b*x)^(2/3)),x]
```

output
```
-1/2*(a + b*x)^(1/3)/(a*x^2) - (5*b*(-((a + b*x)^(1/3)/(a*x)) - (2*b*(-((S
qrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x
]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))))/(3*a)))/(
6*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-10b^2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^2+10b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^2-5b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)x^2+15b\cdots}{18a^{\frac{8}{3}}x^2}$
derivativedivides	$3b^2\left(-\frac{(bx+a)^{\frac{1}{3}}}{6a\,b^2x^2}-\frac{5\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+\cdots}{a^{\cdots}}\cdots\right)}{3}\right)}{9a^{\frac{2}{3}}}\cdots}{a}\cdots\right)}{6a}\cdots\right)$
default	$3b^2\left(-\frac{(bx+a)^{\frac{1}{3}}}{6a\,b^2x^2}-\frac{5\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+\cdots}{a^{\cdots}}\cdots\right)}{3}\right)}{9a^{\frac{2}{3}}}\cdots}{a}\cdots\right)}{6a}\cdots\right)$

input
```
int(1/x^3/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-10*b^2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3)
)*x^2+10*b^2*ln((b*x+a)^(1/3)-a^(1/3))*x^2-5*b^2*ln((b*x+a)^(2/3)+a^(1/3)*
(b*x+a)^(1/3)+a^(2/3))*x^2+15*b*x*(b*x+a)^(1/3)*a^(2/3)-9*(b*x+a)^(1/3)*a^
(5/3))/a^(8/3)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx = -\frac{30\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{a^2}\right)+5\,(a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2\right.}{18\,a^4x^2}$$

input
```
integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/18*(30*sqrt(1/3)*(a^2)^(1/6)*a*b^2*x^2*arctan(sqrt(1/3)*(a^2)^(1/6)*((a
^2)^(1/3)*a + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 5*(a^2)^(2/3)*b^2*x^2*
log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 10*
(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) - 3*(5*a^2*b*x -
3*a^3)*(b*x + a)^(1/3))/(a^4*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 2728, normalized size of antiderivative = 20.98

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x+a)**(2/3),x)
```

output
```
10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gam
ma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54
*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)
*b**(8/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I
*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/
3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3
) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*
b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(8
/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*ex
p(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*
gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) -
 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10
/3)*b**(11/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)
*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)
+ 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b*
*(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx = -\frac{5\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{8}{3}}} - \frac{5\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{8}{3}}} + \frac{5\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{8}{3}}} + \frac{5\,(bx+a)^{\frac{4}{3}}b^2-8\,(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

input
```
integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
-5/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(8/3) - 5/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(8/3) + 5/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(8/3) + 1/6*(5*(b*x
+ a)^(4/3)*b^2 - 8*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a
^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx =$$

$$-\frac{\frac{10\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5\,b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5\,(bx+a)^{\frac{4}{3}}b^3-8\,(bx+\cdots\right.}{a^2b^2x^2}}{18\,b}$$

input
```
integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
-1/18*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(8/3) + 5*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2
/3))/a^(8/3) - 10*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(5*(
b*x + a)^(4/3)*b^3 - 8*(b*x + a)^(1/3)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx = \frac{5\,b^2\,\ln\left({\left(a+b\,x\right)}^{1/3}-a^{1/3}\right)}{9\,a^{8/3}} - \frac{\frac{4\,b^2\,{\left(a+b\,x\right)}^{1/3}}{3\,a}-\frac{5\,b^2\,{\left(a+b\,x\right)}^{4/3}}{6\,a^2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2} + \frac{5\,b^2\,\ln\left(\frac{5\,b^2\,{\left(a+b\,x\right)}^{1/3}}{a^2}-\frac{5\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}} - \frac{5\,b^2\,\ln\left(\frac{5\,b^2\,{\left(a+b\,x\right)}^{1/3}}{a^2}+\frac{5\,b^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}}$$

input
```
int(1/(x^3*(a + b*x)^(2/3)),x)
```

output
```
(5*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(8/3)) - ((4*b^2*(a + b*x)^(1/
3))/(3*a) - (5*b^2*(a + b*x)^(4/3))/(6*a^2))/((a + b*x)^2 - 2*a*(a + b*x)
+ a^2) + (5*b^2*log((5*b^2*(a + b*x)^(1/3))/a^2 - (5*b^2*((3^(1/2)*1i)/2 -
 1/2))/a^(5/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)) - (5*b^2*log((5*b^2*(a
 + b*x)^(1/3))/a^2 + (5*b^2*((3^(1/2)*1i)/2 + 1/2))/a^(5/3))*((3^(1/2)*1i)
/2 + 1/2))/(9*a^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3(a+bx)^{2/3}}\,dx = \frac{10\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2-10\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2-9a^{\frac{5}{3}}(bx+a)^{\frac{1}{3}}+1$$

input
```
int(1/x^3/(b*x+a)^(2/3),x)
```

output
```
(10*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)*sqrt(3)))*b**2*
x**2 - 10*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))
*b**2*x**2 - 9*a**(2/3)*(a + b*x)**(1/3)*a + 15*a**(2/3)*(a + b*x)**(1/3)*
b*x + 10*log((a + b*x)**(1/6) + a**(1/6))*b**2*x**2 + 10*log((a + b*x)**(1
/6) - a**(1/6))*b**2*x**2 - 5*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)
**(1/3) + a**(1/3))*b**2*x**2 - 5*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x
)**(1/3) + a**(1/3))*b**2*x**2)/(18*a**(2/3)*a**2*x**2)
```

3.625 $\int \frac{x^3}{(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \frac{3a^3}{b^4\sqrt[3]{a+bx}} + \frac{9a^2(a+bx)^{2/3}}{2b^4} - \frac{9a(a+bx)^{5/3}}{5b^4} + \frac{3(a+bx)^{8/3}}{8b^4}$$

output
```
3*a^3/b^4/(b*x+a)^(1/3)+9/2*a^2*(b*x+a)^(2/3)/b^4-9/5*a*(b*x+a)^(5/3)/b^4+
3/8*(b*x+a)^(8/3)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \frac{3(81a^3+27a^2bx-9ab^2x^2+5b^3x^3)}{40b^4\sqrt[3]{a+bx}}$$

input
```
Integrate[x^3/(a + b*x)^(4/3),x]
```

output
```
(3*(81*a^3 + 27*a^2*b*x - 9*a*b^2*x^2 + 5*b^3*x^3))/(40*b^4*(a + b*x)^(1/3
))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^{4/3}} \, dx$$

$$\Big\downarrow \; 53$$

$$\int \left(-\frac{a^3}{b^3 (a+bx)^{4/3}}+\frac{3 a^2}{b^3 \sqrt[3]{a+bx}}-\frac{3 a (a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{3 a^3}{b^4 \sqrt[3]{a+bx}}+\frac{9 a^2 (a+bx)^{2/3}}{2 b^4}-\frac{9 a (a+bx)^{5/3}}{5 b^4}+\frac{3 (a+bx)^{8/3}}{8 b^4}$$

input
```
Int[x^3/(a + b*x)^(4/3),x]
```

output
```
(3*a^3)/(b^4*(a + b*x)^(1/3)) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) - (9*a*(a
+ b*x)^(5/3))/(5*b^4) + (3*(a + b*x)^(8/3))/(8*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
trager	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
pseudoelliptic	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
orering	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
risch	$\frac{3(5b^2x^2-14abx+41a^2)(bx+a)^{\frac{2}{3}}}{40b^4}+\frac{3a^3}{b^4(bx+a)^{\frac{1}{3}}}$	48
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{9a(bx+a)^{\frac{5}{3}}}{5}+\frac{9a^2(bx+a)^{\frac{2}{3}}}{2}+\frac{3a^3}{(bx+a)^{\frac{1}{3}}}}{b^4}$	49
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{9a(bx+a)^{\frac{5}{3}}}{5}+\frac{9a^2(bx+a)^{\frac{2}{3}}}{2}+\frac{3a^3}{(bx+a)^{\frac{1}{3}}}}{b^4}$	49

input
```
int(x^3/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/40/(b*x+a)^(1/3)*(5*b^3*x^3-9*a*b^2*x^2+27*a^2*b*x+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \frac{3\,(5\,b^3x^3-9\,ab^2x^2+27\,a^2bx+81\,a^3)(bx+a)^{\frac{2}{3}}}{40\,(b^5x+ab^4)}$$

input
```
integrate(x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
3/40*(5*b^3*x^3 - 9*a*b^2*x^2 + 27*a^2*b*x + 81*a^3)*(b*x + a)^(2/3)/(b^5*
x + a*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(66) = 132$.

Time = 1.29 (sec) , antiderivative size = 1538, normalized size of antiderivative = 21.97

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(b*x+a)**(4/3),x)
```

output
```
243*a**(68/3)*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a
**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**
9*x**5 + 40*a**14*b**10*x**6) - 243*a**(68/3)/(40*a**20*b**4 + 240*a**19*b
**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 +
240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1296*a**(65/3)*b*x*(1 + b*x/a
)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**
17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*
x**6) - 1458*a**(65/3)*b*x/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b
**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5
 + 40*a**14*b**10*x**6) + 2808*a**(62/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*
a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3
+ 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 3645*
a**(62/3)*b**2*x**2/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**
2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a
**14*b**10*x**6) + 3120*a**(59/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**20*b
**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a
**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 4860*a**(59/
3)*b**3*x**3/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800
*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b*
*10*x**6) + 1830*a**(56/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**20*b**4 ...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^4} - \frac{9\,(bx+a)^{\frac{5}{3}}a}{5\,b^4} + \frac{9\,(bx+a)^{\frac{2}{3}}a^2}{2\,b^4} + \frac{3\,a^3}{(bx+a)^{\frac{1}{3}}b^4}$$

input
```
integrate(x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
3/8*(b*x + a)^(8/3)/b^4 - 9/5*(b*x + a)^(5/3)*a/b^4 + 9/2*(b*x + a)^(2/3)*
a^2/b^4 + 3*a^3/((b*x + a)^(1/3)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \frac{3\,a^3}{(bx+a)^{\frac{1}{3}}b^4} + \frac{3\left(5\,(bx+a)^{\frac{8}{3}}b^{28} - 24\,(bx+a)^{\frac{5}{3}}ab^{28} + 60\,(bx+a)^{\frac{2}{3}}a^2b^{28}\right)}{40\,b^{32}}$$

input
```
integrate(x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
3*a^3/((b*x + a)^(1/3)*b^4) + 3/40*(5*(b*x + a)^(8/3)*b^28 - 24*(b*x + a)^
(5/3)*a*b^28 + 60*(b*x + a)^(2/3)*a^2*b^28)/b^32
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \frac{3\,(a+b\,x)^{8/3}}{8\,b^4} + \frac{9\,a^2\,(a+b\,x)^{2/3}}{2\,b^4} + \frac{3\,a^3}{b^4\,(a+b\,x)^{1/3}} - \frac{9\,a\,(a+b\,x)^{5/3}}{5\,b^4}$$

input
```
int(x^3/(a + b*x)^(4/3),x)
```

output
```
(3*(a + b*x)^(8/3))/(8*b^4) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) + (3*a^3)/(b
^4*(a + b*x)^(1/3)) - (9*a*(a + b*x)^(5/3))/(5*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{x^3}{(a+bx)^{4/3}}\,dx = \frac{\frac{3}{8}b^3x^3 - \frac{27}{40}a\,b^2x^2 + \frac{81}{40}a^2bx + \frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}\,b^4}$$

input
```
int(x^3/(b*x+a)^(4/3),x)
```

output
```
(3*(81*a**3 + 27*a**2*b*x - 9*a*b**2*x**2 + 5*b**3*x**3))/(40*(a + b*x)**(
1/3)*b**4)
```

3.626 $\int \frac{x^2}{(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = -\frac{3a^2}{b^3\sqrt[3]{a+bx}} - \frac{3a(a+bx)^{2/3}}{b^3} + \frac{3(a+bx)^{5/3}}{5b^3}$$

output
```
-3*a^2/b^3/(b*x+a)^(1/3)-3*a*(b*x+a)^(2/3)/b^3+3/5*(b*x+a)^(5/3)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = \frac{3(-9a^2-3abx+b^2x^2)}{5b^3\sqrt[3]{a+bx}}$$

input
```
Integrate[x^2/(a + b*x)^(4/3),x]
```

output
```
(3*(-9*a^2 - 3*a*b*x + b^2*x^2))/(5*b^3*(a + b*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^{4/3}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

input
```
Int[x^2/(a + b*x)^(4/3),x]
```

output
```
(-3*a^2)/(b^3*(a + b*x)^(1/3)) - (3*a*(a + b*x)^(2/3))/b^3 + (3*(a + b*x)^
(5/3))/(5*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{\frac{3}{5}b^{2}x^{2}-\frac{9}{5}abx-\frac{27}{5}a^{2}}{(bx+a)^{\frac{1}{3}}b^{3}}$	31
gosper	$-\frac{3(-b^{2}x^{2}+3abx+9a^{2})}{5(bx+a)^{\frac{1}{3}}b^{3}}$	32
trager	$-\frac{3(-b^{2}x^{2}+3abx+9a^{2})}{5(bx+a)^{\frac{1}{3}}b^{3}}$	32
orering	$-\frac{3(-b^{2}x^{2}+3abx+9a^{2})}{5(bx+a)^{\frac{1}{3}}b^{3}}$	32
risch	$-\frac{3(-bx+4a)(bx+a)^{\frac{2}{3}}}{5b^{3}}-\frac{3a^{2}}{b^{3}(bx+a)^{\frac{1}{3}}}$	37
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-3a(bx+a)^{\frac{2}{3}}-\frac{3a^{2}}{(bx+a)^{\frac{1}{3}}}}{b^{3}}$	38
default	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-3a(bx+a)^{\frac{2}{3}}-\frac{3a^{2}}{(bx+a)^{\frac{1}{3}}}}{b^{3}}$	38

input
```
int(x^2/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/5*(b^2*x^2-3*a*b*x-9*a^2)/(b*x+a)^(1/3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = \frac{3\,(b^2x^2-3\,abx-9\,a^2)(bx+a)^{\frac{2}{3}}}{5\,(b^4x+ab^3)}$$

input
```
integrate(x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
3/5*(b^2*x^2 - 3*a*b*x - 9*a^2)*(b*x + a)^(2/3)/(b^4*x + a*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(46) = 92$.

Time = 0.83 (sec) , antiderivative size = 534, normalized size of antiderivative = 10.90

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = -\frac{27a^{\frac{29}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} + \frac{27a^{\frac{29}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} - \frac{63a^{\frac{26}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} + \frac{81a^{\frac{26}{3}}bx}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} - \frac{42a^{\frac{23}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} + \frac{81a^{\frac{23}{3}}b^2x^2}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} - \frac{3a^{\frac{20}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} + \frac{27a^{\frac{20}{3}}b^3x^3}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3} + \frac{3a^{\frac{17}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}$$

input
```
integrate(x**2/(b*x+a)**(4/3),x)
```

output
```
-27*a**(29/3)*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b
**5*x**2 + 5*a**5*b**6*x**3) + 27*a**(29/3)/(5*a**8*b**3 + 15*a**7*b**4*x
+ 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 63*a**(26/3)*b*x*(1 + b*x/a)**(2
/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3)
+ 81*a**(26/3)*b*x/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a
**5*b**6*x**3) - 42*a**(23/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(5*a**8*b**3 +
15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81*a**(23/3)*b**2
*x**2/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3
) - 3*a**(20/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x
 + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 27*a**(20/3)*b**3*x**3/(5*a**8*
b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 3*a**(17/3
)*b**4*x**4*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**
5*x**2 + 5*a**5*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = \frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^3} - \frac{3\,(bx+a)^{\frac{2}{3}}a}{b^3} - \frac{3\,a^2}{(bx+a)^{\frac{1}{3}}b^3}$$

input
```
integrate(x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
3/5*(b*x + a)^(5/3)/b^3 - 3*(b*x + a)^(2/3)*a/b^3 - 3*a^2/((b*x + a)^(1/3)
*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = -\frac{3\left(\frac{5\,a^2}{(bx+a)^{\frac{1}{3}}b} - \frac{(bx+a)^{\frac{5}{3}}b^4 - 5\,(bx+a)^{\frac{2}{3}}ab^4}{b^5}\right)}{5\,b^2}$$

input
```
integrate(x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
-3/5*(5*a^2/((b*x + a)^(1/3)*b) - ((b*x + a)^(5/3)*b^4 - 5*(b*x + a)^(2/3)
*a*b^4)/b^5)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = -\frac{15\,a\,(a+b\,x)-3\,(a+b\,x)^2+15\,a^2}{5\,b^3\,(a+b\,x)^{1/3}}$$

input
```
int(x^2/(a + b*x)^(4/3),x)
```

output
```
-(15*a*(a + b*x) - 3*(a + b*x)^2 + 15*a^2)/(5*b^3*(a + b*x)^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(a+bx)^{4/3}}\,dx = \frac{\frac{3}{5}b^2x^2-\frac{9}{5}abx-\frac{27}{5}a^2}{(bx+a)^{\frac{1}{3}}\,b^3}$$

input
```
int(x^2/(b*x+a)^(4/3),x)
```

output
```
(3*( - 9*a**2 - 3*a*b*x + b**2*x**2))/(5*(a + b*x)**(1/3)*b**3)
```

3.627 $\int \frac{x}{(a+bx)^{4/3}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x}{(a+bx)^{4/3}} \, dx = \frac{3a}{b^2 \sqrt[3]{a+bx}} + \frac{3(a+bx)^{2/3}}{2b^2}$$

output
```
3*a/b^2/(b*x+a)^(1/3)+3/2*(b*x+a)^(2/3)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+bx)^{4/3}} \, dx = \frac{3(3a+bx)}{2b^2 \sqrt[3]{a+bx}}$$

input
```
Integrate[x/(a + b*x)^(4/3),x]
```

output
```
(3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^{4/3}}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{1}{b\sqrt[3]{a+bx}}-\frac{a}{b(a+bx)^{4/3}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

input
```
Int[x/(a + b*x)^(4/3),x]
```

output
```
(3*a)/(b^2*(a + b*x)^(1/3)) + (3*(a + b*x)^(2/3))/(2*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}} b^{2}}$	20
trager	$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}} b^{2}}$	20
orering	$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}} b^{2}}$	20
pseudoelliptic	$\frac{3bx+9a}{2(bx+a)^{\frac{1}{3}} b^{2}}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{2}{3}}}{2}+\frac{3a}{(bx+a)^{\frac{1}{3}}}}{b^{2}}$	25
default	$\frac{\frac{3(bx+a)^{\frac{2}{3}}}{2}+\frac{3a}{(bx+a)^{\frac{1}{3}}}}{b^{2}}$	25
risch	$\frac{3a}{b^{2}(bx+a)^{\frac{1}{3}}}+\frac{3(bx+a)^{\frac{2}{3}}}{2b^{2}}$	27

input
```
int(x/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/2/(b*x+a)^(1/3)*(b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a+bx)^{4/3}}\,dx = \frac{3\,(bx+3\,a)(bx+a)^{\frac{2}{3}}}{2\,(b^{3}x+ab^{2})}$$

input
```
integrate(x/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
3/2*(b*x + 3*a)*(b*x + a)^(2/3)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{x}{(a+bx)^{4/3}}\,dx = \begin{cases} \frac{9a}{2b^2\sqrt[3]{a+bx}} + \frac{3x}{2b\sqrt[3]{a+bx}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x+a)**(4/3),x)
```

output
```
Piecewise((9*a/(2*b**2*(a + b*x)**(1/3)) + 3*x/(2*b*(a + b*x)**(1/3)), Ne(
b, 0)), (x**2/(2*a**(4/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+bx)^{4/3}}\,dx = \frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b^2} + \frac{3\,a}{(bx+a)^{\frac{1}{3}}b^2}$$

input
```
integrate(x/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
3/2*(b*x + a)^(2/3)/b^2 + 3*a/((b*x + a)^(1/3)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx)^{4/3}}\,dx = \frac{3\left(\frac{(bx+a)^{\frac{2}{3}}}{b} + \frac{2\,a}{(bx+a)^{\frac{1}{3}}b}\right)}{2\,b}$$

input
```
integrate(x/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
3/2*((b*x + a)^(2/3)/b + 2*a/((b*x + a)^(1/3)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{x}{(a+bx)^{4/3}}\,dx = \frac{9\,a+3\,b\,x}{2\,b^2\,(a+b\,x)^{1/3}}$$

input
```
int(x/(a + b*x)^(4/3),x)
```

output
```
(9*a + 3*b*x)/(2*b^2*(a + b*x)^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x}{(a+bx)^{4/3}}\,dx = \frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}}\,b^2}$$

input
```
int(x/(b*x+a)^(4/3),x)
```

output
```
(3*(3*a + b*x))/(2*(a + b*x)**(1/3)*b**2)
```

3.628 $\int \frac{1}{(a+bx)^{4/3}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{(a+bx)^{4/3}} \, dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

output
```
-3/b/(b*x+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^{4/3}} \, dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

input
```
Integrate[(a + b*x)^(-4/3),x]
```

output
```
-3/(b*(a + b*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{4/3}} \, dx$$

$$\Big\downarrow \; 17$$

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

input
```
Int[(a + b*x)^(-4/3),x]
```

output
```
-3/(b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
derivativedivides	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
default	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
trager	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
pseudoelliptic	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
orering	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13

input
```
int(1/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/b/(b*x+a)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx)^{4/3}}\,dx = -\frac{3\,(bx+a)^{\frac{2}{3}}}{b^2x+ab}$$

input
```
integrate(1/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
-3*(b*x + a)^(2/3)/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{4/3}}\,dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

input
```
integrate(1/(b*x+a)**(4/3),x)
```

output
```
-3/(b*(a + b*x)**(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{4/3}}\,dx = -\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

input
```
integrate(1/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
-3/((b*x + a)^(1/3)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{4/3}}\,dx = -\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

input
```
integrate(1/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
-3/((b*x + a)^(1/3)*b)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{4/3}}\,dx = -\frac{3}{b\,(a+b\,x)^{1/3}}$$

input
```
int(1/(a + b*x)^(4/3),x)
```

output
```
-3/(b*(a + b*x)^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^{4/3}}\,dx = -\frac{3}{(bx+a)^{\frac{1}{3}}\,b}$$

input
```
int(1/(b*x+a)^(4/3),x)
```

output
```
( - 3)/((a + b*x)**(1/3)*b)
```

3.629 $\int \frac{1}{x(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx=\frac{3}{a\sqrt[3]{a+bx}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

output
```
3/a/(b*x+a)^(1/3)+3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(
1/3))/a^(4/3)-1/2*ln(x)/a^(4/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx=\frac{\frac{6\sqrt[3]{a}}{\sqrt[3]{a+bx}}+2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\right.}{2a^{4/3}}$$

input
```
Integrate[1/(x*(a + b*x)^(4/3)),x]
```

output
```
((6*a^(1/3))/(a + b*x)^(1/3) + 2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a
^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3
)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(2*a^(4/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx)^{4/3}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{\int \frac{1}{x\sqrt[3]{a+bx}}dx}{a}+\frac{3}{a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{67}$$

$$\frac{\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}$$

input
```
Int[1/(x*(a + b*x)^(4/3)),x]
```

output
```
3/(a*(a + b*x)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))
/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)
])/(2*a^(1/3)))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

method	result	size
pseudoelliptic	$\frac{\left(\sqrt{3}\, \arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)+\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2}\right)(bx+a)^{\frac{1}{3}}+3a^{\frac{1}{3}}}{a^{\frac{4}{3}}(bx+a)^{\frac{1}{3}}}$	92
derivativedivides	$\frac{3}{a(bx+a)^{\frac{1}{3}}}+\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}}{a}$	95
default	$\frac{3}{a(bx+a)^{\frac{1}{3}}}+\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}}{a}$	95

input
```
int(1/x/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
((3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))+ln((b*x+a)
^(1/3)-a^(1/3))-1/2*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3)))*(b*x+
a)^(1/3)+3*a^(1/3))/a^(4/3)/(b*x+a)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.06

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx = \left[\frac{\sqrt{3}(abx+a^2)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-}{}\right.$$

$$-\frac{(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{2\sqrt{3}(abx+a^2)\arctan}{}}{2\,(a^2bx+a^3)}$$

input
```
integrate(1/x/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*(a*b*x + a^2)*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x
+ a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x + a)^(1/3)*a^(2/3) + 3*a)/x) - (b*x + a)*a^(2/3)*log((b*x + a)^(2/3) + (
b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x + a)*a^(2/3)*log((b*x + a)^(1/3
) - a^(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*b*x + a^3), -1/2*((b*x + a)*a^(2/
3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(b*x + a)*
a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(a*b*x + a^2)*arctan(1/
3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 6*(b*x + a)^(2/
3)*a)/(a^2*b*x + a^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.98

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx = -\frac{\Gamma\left(-\frac{1}{3}\right)}{a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{2}{3}\right)} - \frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)} - \frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)} - \frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x/(b*x+a)**(4/3),x)
```

output
```
-gamma(-1/3)/(a*b**(1/3)*(a/b + x)**(1/3)*gamma(2/3)) - log(1 - b**(1/3)*(
a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3)) - exp(2*I*pi
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(
-1/3)/(3*a**(4/3)*gamma(2/3)) - exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)*
*(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}} + \frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}} + \frac{3}{(bx+a)^{\frac{1}{3}}a}$$

input
```
integrate(1/x/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + l
og((b*x + a)^(1/3) - a^(1/3))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}} + \frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}} + \frac{3}{(bx+a)^{\frac{1}{3}}a}$$

input
```
integrate(1/x/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + l
og(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.23

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx = \frac{\ln\left(9\,a\,(a+b\,x)^{1/3}-9\,a^{4/3}\right)}{a^{4/3}} + \frac{3}{a\,(a+b\,x)^{1/3}} + \frac{\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{4/3}} - \frac{\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{4/3}}$$

input
```
int(1/(x*(a + b*x)^(4/3)),x)
```

output
```
log(9*a*(a + b*x)^(1/3) - 9*a^(4/3))/a^(4/3) + 3/(a*(a + b*x)^(1/3)) + (lo
g(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1)
)/(2*a^(4/3)) - (log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i + 1)^2)/
4)*(3^(1/2)*1i + 1))/(2*a^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx = \frac{-2(bx+a)^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2(bx+a)^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)+2(bx+a}{}$$

input
```
int(1/x/(b*x+a)^(4/3),x)
```

output
```
( - 2*(a + b*x)**(1/3)*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1
/6)*sqrt(3))) + 2*(a + b*x)**(1/3)*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**(
1/6))/(a**(1/6)*sqrt(3))) + 2*(a + b*x)**(1/3)*log((a + b*x)**(1/6) + a**(
1/6)) + 2*(a + b*x)**(1/3)*log((a + b*x)**(1/6) - a**(1/6)) - (a + b*x)**(
1/3)*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3)) - (a
+ b*x)**(1/3)*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3))
 + 6*a**(1/3))/(2*a**(1/3)*(a + b*x)**(1/3)*a)
```

3.630 $\int \frac{1}{x^2(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = -\frac{4b}{a^2\sqrt[3]{a+bx}} - \frac{1}{ax\sqrt[3]{a+bx}} - \frac{4b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}$$

output
```
-4*b/a^2/(b*x+a)^(1/3)-1/a/x/(b*x+a)^(1/3)-4/3*b*arctan(1/3*(a^(1/3)+2*(b*
x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)+2/3*b*ln(x)/a^(7/3)-2*b*ln(a^
(1/3)-(b*x+a)^(1/3))/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = \frac{-\frac{3\sqrt[3]{a}(a+4bx)}{x\sqrt[3]{a+bx}} - 4\sqrt{3}b\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + 2b\log\left(a\right.}{3a^{7/3}}$$

input
```
Integrate[1/(x^2*(a + b*x)^(4/3)),x]
```

output
```
((-3*a^(1/3)*(a + 4*b*x))/(x*(a + b*x)^(1/3)) - 4*Sqrt[3]*b*ArcTan[(1 + (2
*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 4*b*Log[a^(1/3) - (a + b*x)^(1/3)] +
 2*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(7/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {52, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx$$

$$\Big\downarrow \ 52$$

$$-\frac{4b\int \frac{1}{x(a+bx)^{4/3}}dx}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}$$

$$\Big\downarrow \ 61$$

$$-\frac{4b\left(\frac{\int \frac{1}{x\sqrt[3]{a+bx}}dx}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}$$

$$\Big\downarrow \ 67$$

$$-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}$$

$$\Big\downarrow \ 16$$

$$-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 1082

$$-\frac{4b\left(\frac{-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 217

$$-\frac{4b\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}$$

input
```
Int[1/(x^2*(a + b*x)^(4/3)),x]
```

output
```
-(1/(a*x*(a + b*x)^(1/3))) - (4*b*(3/(a*(a + b*x)^(1/3)) + ((Sqrt[3]*ArcTa
n[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3))
 + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3)))/a))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{a^2x}-\frac{b\left(\frac{9}{(bx+a)^{\frac{1}{3}}}+\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right.}{a^{\frac{1}{3}}}}{3a^2}$
pseudoelliptic	$-\frac{(bx+a)^{\frac{2}{3}}}{a^2x}-\frac{4b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{7}{3}}}+\frac{2b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{4b\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)}{3a^{\frac{7}{3}}}$
derivativedivides	$3b\left(-\frac{1}{a^2(bx+a)^{\frac{1}{3}}}+\frac{-\frac{(bx+a)^{\frac{2}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)}{a^{\frac{1}{3}}}\right.}{3}\right.}{9a^{\frac{1}{3}}}}{a^2}\right.$
default	$3b\left(-\frac{1}{a^2(bx+a)^{\frac{1}{3}}}+\frac{-\frac{(bx+a)^{\frac{2}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)}{a^{\frac{1}{3}}}\right.}{3}\right.}{9a^{\frac{1}{3}}}}{a^2}\right.$

input
```
int(1/x^2/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(b*x+a)^(2/3)/x-1/3*b/a^2*(9/(b*x+a)^(1/3)+4/a^(1/3)*ln((b*x+a)^(1/
3)-a^(1/3))-2/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+4*3^
(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.60

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}(ab^2x^2+a^2bx)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx-3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3}{x}\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}(ab^2x^2+a^2bx)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2\,(b^2x^2+abx)(-a)^{\frac{2}{3}}\log}{3}$$

input
```
integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/3*(6*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*
sqrt(1/3)*(2*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a
)*sqrt((-a)^(1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + 2*(b^2*x^2
 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-
a)^(2/3)) - 4*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3
)) - 3*(4*a*b*x + a^2)*(b*x + a)^(2/3))/(a^3*b*x^2 + a^4*x), -1/3*(12*sqrt
(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x +
 a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(b^2*x^2 + a*b*x)*(-a)^(2
/3)*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 4*(b^
2*x^2 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(4*a*b*x +
 a^2)*(b*x + a)^(2/3))/(a^3*b*x^2 + a^4*x)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 857, normalized size of antiderivative = 7.58

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = \text{Too large to display}$$

input

```
integrate(1/x**2/(b*x+a)**(4/3),x)
```

output

```
-9*a**(4/3)*b**(2/3)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1
/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)
*gamma(2/3)) + 12*a**(1/3)*b**(5/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-
9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b
+ x)**(4/3)*exp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(
a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*e
xp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b
**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**
(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)*
*(4/3)*exp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*
(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*
exp(2*I*pi/3)*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/
b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b ...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = -\frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}} - \frac{4\,(bx+a)b-3\,ab}{(bx+a)^{\frac{4}{3}}a^2-(bx+a)^{\frac{1}{3}}a^3} + \frac{2\,b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}} - \frac{4\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{7}{3}}}$$

input
```
integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
-4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(7/3) - (4*(b*x + a)*b - 3*a*b)/((b*x + a)^(4/3)*a^2 - (b*x + a)^(1/3)*a^
3) + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7/3
) - 4/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = -\frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}} + \frac{2\,b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}} - \frac{4\,b\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}} - \frac{4\,(bx+a)b-3\,ab}{\left((bx+a)^{\frac{4}{3}}-(bx+a)^{\frac{1}{3}}a\right)a^2}$$

input
```
integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
-4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(7/3) + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(7/3) - 4/3*b*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) - (4*(b*x + a)*b
 - 3*a*b)/(((b*x + a)^(4/3) - (b*x + a)^(1/3)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.53

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = -\frac{\frac{3\,b}{a}-\frac{4\,b\,(a+b\,x)}{a^2}}{a\,(a+b\,x)^{1/3}-(a+b\,x)^{4/3}} + \frac{\ln\left(a^{7/3}\,\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\,\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}} + \frac{\ln\left(a^{7/3}\,\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\,\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}} - \frac{4\,b\,\ln\left(16\,a^{7/3}\,b^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)}{3\,a^{7/3}}$$

input
```
int(1/(x^2*(a + b*x)^(4/3)),x)
```

output
```
(log(a^(7/3)*(2*b - 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a + b*x)^(1/3))*(2*b - 3
^(1/2)*b*2i))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x))/a^2)/(a*(a + b*x)^(
1/3) - (a + b*x)^(4/3)) + (log(a^(7/3)*(2*b + 3^(1/2)*b*2i)^2 - 16*a^2*b^2
*(a + b*x)^(1/3))*(2*b + 3^(1/2)*b*2i))/(3*a^(7/3)) - (4*b*log(16*a^(7/3)*
b^2 - 16*a^2*b^2*(a + b*x)^(1/3)))/(3*a^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = \frac{4(bx+a)^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-4(bx+a)^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)bx-4(b}{}$$

input
```
int(1/x^2/(b*x+a)^(4/3),x)
```

output
```
(4*(a + b*x)**(1/3)*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(1/6)
*sqrt(3)))*b*x - 4*(a + b*x)**(1/3)*sqrt(3)*atan((2*(a + b*x)**(1/6) - a**
(1/6))/(a**(1/6)*sqrt(3)))*b*x - 4*(a + b*x)**(1/3)*log((a + b*x)**(1/6) +
 a**(1/6))*b*x - 4*(a + b*x)**(1/3)*log((a + b*x)**(1/6) - a**(1/6))*b*x +
 2*(a + b*x)**(1/3)*log( - a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) +
a**(1/3))*b*x + 2*(a + b*x)**(1/3)*log(a**(1/6)*(a + b*x)**(1/6) + (a + b*
x)**(1/3) + a**(1/3))*b*x - 3*a**(1/3)*a - 12*a**(1/3)*b*x)/(3*a**(1/3)*(a
 + b*x)**(1/3)*a**2*x)
```

3.631 $\int \frac{1}{x^3(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{14b^2}{3a^3\sqrt[3]{a+bx}} - \frac{1}{2ax^2\sqrt[3]{a+bx}} + \frac{7b}{6a^2x\sqrt[3]{a+bx}} + \frac{14b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}$$

output
```
14/3*b^2/a^3/(b*x+a)^(1/3)-1/2/a/x^2/(b*x+a)^(1/3)+7/6*b/a^2/x/(b*x+a)^(1/
3)+14/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/
a^(10/3)-7/9*b^2*ln(x)/a^(10/3)+7/3*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(10/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{\frac{3\sqrt[3]{a}(-3a^2+7abx+28b^2x^2)}{x^2\sqrt[3]{a+bx}} + 28\sqrt{3}b^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}{18a^{10/3}}$$

input
```
Integrate[1/(x^3*(a + b*x)^(4/3)),x]
```

output
```
((3*a^(1/3)*(-3*a^2 + 7*a*b*x + 28*b^2*x^2))/(x^2*(a + b*x)^(1/3)) + 28*Sq
rt[3]*b^2*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^2*Log[a
^(1/3) - (a + b*x)^(1/3)] - 14*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) +
 (a + b*x)^(2/3)])/(18*a^(10/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {52, 52, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx$$

$$\Big\downarrow \text{52}$$

$$-\frac{7b\int \frac{1}{x^2(a+bx)^{4/3}}dx}{6a}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{7b\left(-\frac{4b\int \frac{1}{x(a+bx)^{4/3}}dx}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}\right)}{6a}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{7b\left(-\frac{4b\left(\frac{\int \frac{1}{x\sqrt[3]{a+bx}}dx}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}\right)}{6a}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{67}$$

$$-\frac{7b\left(-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{a+bx}}d\sqrt[3]{a+bx}}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}\right)}{6a}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 16

$$-\frac{7b\left(-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{a+bx}\sqrt[3]{a}+(a+bx)^{2/3}}d\sqrt[3]{a+bx}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}\right)}{6a}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 1082

$$-\frac{7b\left(-\frac{4b\left(\frac{-\frac{3\int \frac{1}{-(a+bx)^{2/3}-3}d\left(\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}\right)}{6a}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{7b\left(-\frac{4b\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx}}\right)}{3a}-\frac{1}{ax\sqrt[3]{a+bx}}\right)}{6a}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

input
```
Int[1/(x^3*(a + b*x)^(4/3)),x]
```

output
```
-1/2*1/(a*x^2*(a + b*x)^(1/3)) - (7*b*(-(1/(a*x*(a + b*x)^(1/3))) - (4*b*(
3/(a*(a + b*x)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))
/Sqrt[3]])/a^(1/3) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)
])/(2*a^(1/3)))/a))/(3*a)))/(6*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(-10bx+3a)}{6a^{3}x^{2}}+\frac{b^{2}\left(\frac{27}{(bx+a)^{\frac{1}{3}}}+\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{14\sqrt{3}\arctan\left(\frac{\sqrt{3}}{\ldots}\right)}{a\ldots}\right.}{9a^{3}}$
derivativedivides	$3b^{2}\left(-\frac{\frac{-\frac{5(bx+a)^{\frac{5}{3}}}{9}+\frac{13a(bx+a)^{\frac{2}{3}}}{18}}{b^{2}x^{2}}-\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{1}{3}}}+\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{1}{3}}}-\frac{14\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+\ldots}{a^{\ldots}}\right.}{3}\right.}{27a^{\frac{1}{3}}}}{a^{3}}\right.$
default	$3b^{2}\left(-\frac{\frac{-\frac{5(bx+a)^{\frac{5}{3}}}{9}+\frac{13a(bx+a)^{\frac{2}{3}}}{18}}{b^{2}x^{2}}-\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{1}{3}}}+\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{1}{3}}}-\frac{14\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+\ldots}{a^{\ldots}}\right.}{3}\right.}{27a^{\frac{1}{3}}}}{a^{3}}\right.$
pseudoelliptic	$\frac{\frac{14\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b^{2}x^{2}(bx+a)^{\frac{1}{3}}}{9}+\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)b^{2}x^{2}(bx+a)^{\frac{1}{3}}}{9}-\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)b\ldots}{9}}{a^{\frac{10}{3}}x^{2}(bx+a)^{\frac{1}{3}}}$

input
```
int(1/x^3/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x+a)^(2/3)*(-10*b*x+3*a)/a^3/x^2+1/9*b^2/a^3*(27/(b*x+a)^(1/3)+14/
a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-7/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+
a)^(1/3)+a^(2/3))+14*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)
^(1/3)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \left[\frac{42\sqrt{\frac{1}{3}}(ab^3x^3+a^2b^2x^2)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)\right.}{x}\right.}{}\right.$$

$$-\frac{14\,(b^3x^3+ab^2x^2)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-28\,(b^3x^3+ab^2x^2)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-}{18\,(a^4bx^3+a^5x^2)}$$

input
```
integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/18*(42*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-1/a^(2/3))*log((2*b*x
+ 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sq
rt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 14*(b^3*x^3 + a*b^2
*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2
8*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(28*a*b
^2*x^2 + 7*a^2*b*x - 3*a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2), -1/18*
(14*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^
(1/3) + a^(2/3)) - 28*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) -
a^(1/3)) - 84*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*arctan(sqrt(1/3)*(2*(b*x
 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^2 + 7*a^2*b*x - 3*
a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 2793, normalized size of antiderivative = 18.74

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x+a)**(4/3),x)
```

output
```
54*a**(13/3)*b**(5/3)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**
(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*
pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma
(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 20
1*a**(10/3)*b**(8/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a
/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)
*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi
/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) + 231*a**(7/3)*b**(11/3)*(a/b + x)**2*exp(2*I*pi/3)*gamma(-1/3)/(-54*
a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b
 + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3
)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*p
i/3)*gamma(2/3)) - 84*a**(4/3)*b**(14/3)*(a/b + x)**3*exp(2*I*pi/3)*gamma(
-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(1
9/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3
)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a/b + x)**(1/3)*exp(2*I*pi/3)*
log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/
b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*
exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}} + \frac{28\,(bx+a)^2b^2-49\,(bx+a)ab^2+18\,a^2b^2}{6\left((bx+a)^{\frac{7}{3}}a^3-2\,(bx+a)^{\frac{4}{3}}a^4+(bx+a)^{\frac{1}{3}}a^5\right)} - \frac{7\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{10}{3}}} + \frac{14\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{10}{3}}}$$

input
```
integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) + 1/6*(28*(b*x + a)^2*b^2 - 49*(b*x + a)*a*b^2 + 18*a^2*b^2)/((b
*x + a)^(7/3)*a^3 - 2*(b*x + a)^(4/3)*a^4 + (b*x + a)^(1/3)*a^5) - 7/9*b^2
*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(10/3) + 14/9*
b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(10/3)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}} - \frac{7\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{10}{3}}} + \frac{14\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{10}{3}}} + \frac{3\,b^2}{(bx+a)^{\frac{1}{3}}a^3} + \frac{10\,(bx+a)^{\frac{5}{3}}b^2-13\,(bx+a)^{\frac{2}{3}}ab^2}{6\,a^3b^2x^2}$$

input
```
integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) - 7/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(10/3) + 14/9*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(10/3) + 3*b^
2/((b*x + a)^(1/3)*a^3) + 1/6*(10*(b*x + a)^(5/3)*b^2 - 13*(b*x + a)^(2/3)
*a*b^2)/(a^3*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{\frac{3\,b^2}{a}+\frac{14\,b^2\,(a+b\,x)^2}{3\,a^3}-\frac{49\,b^2\,(a+b\,x)}{6\,a^2}}{(a+b\,x)^{7/3}-2\,a\,(a+b\,x)^{4/3}+a^2\,(a+b\,x)^{1/3}} + \frac{\ln\left(588\,a^3\,b^4\,(a+b\,x)^{1/3}-3\,a^{10/3}\,\left(-7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)^2\right)\,\left(-7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)}{9\,a^{10/3}} - \frac{\ln\left(588\,a^3\,b^4\,(a+b\,x)^{1/3}-3\,a^{10/3}\,\left(7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)^2\right)\,\left(7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)}{9\,a^{10/3}} + \frac{14\,b^2\,\ln\left(588\,a^3\,b^4\,(a+b\,x)^{1/3}-588\,a^{10/3}\,b^4\right)}{9\,a^{10/3}}$$

input
```
int(1/(x^3*(a + b*x)^(4/3)),x)
```

output
```
((3*b^2)/a + (14*b^2*(a + b*x)^2)/(3*a^3) - (49*b^2*(a + b*x))/(6*a^2))/((
a + b*x)^(7/3) - 2*a*(a + b*x)^(4/3) + a^2*(a + b*x)^(1/3)) + (log(588*a^3
*b^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^(1/2)*b^2*7i - 7*b^2)^2)*(3^(1/2)*b^2
*7i - 7*b^2))/(9*a^(10/3)) - (log(588*a^3*b^4*(a + b*x)^(1/3) - 3*a^(10/3)
*(3^(1/2)*b^2*7i + 7*b^2)^2)*(3^(1/2)*b^2*7i + 7*b^2))/(9*a^(10/3)) + (14*
b^2*log(588*a^3*b^4*(a + b*x)^(1/3) - 588*a^(10/3)*b^4))/(9*a^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{-28(bx+a)^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}+a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x^2+28(bx+a)^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2(bx+a)^{\frac{1}{6}}-a^{\frac{1}{6}}}{a^{\frac{1}{6}}\sqrt{3}}\right)b^2x}{}$$

input
```
int(1/x^3/(b*x+a)^(4/3),x)
```

output
```
( - 28*(a + b*x)**(1/3)*sqrt(3)*atan((2*(a + b*x)**(1/6) + a**(1/6))/(a**(
1/6)*sqrt(3)))*b**2*x**2 + 28*(a + b*x)**(1/3)*sqrt(3)*atan((2*(a + b*x)**
(1/6) - a**(1/6))/(a**(1/6)*sqrt(3)))*b**2*x**2 + 28*(a + b*x)**(1/3)*log(
(a + b*x)**(1/6) + a**(1/6))*b**2*x**2 + 28*(a + b*x)**(1/3)*log((a + b*x)
**(1/6) - a**(1/6))*b**2*x**2 - 14*(a + b*x)**(1/3)*log( - a**(1/6)*(a + b
*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3))*b**2*x**2 - 14*(a + b*x)**(1/3)*
log(a**(1/6)*(a + b*x)**(1/6) + (a + b*x)**(1/3) + a**(1/3))*b**2*x**2 - 9
*a**(1/3)*a**2 + 21*a**(1/3)*a*b*x + 84*a**(1/3)*b**2*x**2)/(18*a**(1/3)*(
a + b*x)**(1/3)*a**3*x**2)
```

3.632 $\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}$$

output
```
3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))*3^(1/2)/a)/a-1/2*ln(x)/a+3/2*ln
(a-(b^3*x+a^3)^(1/3))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx =\frac{2\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)+2\log\left(a-\sqrt[3]{a^3+b^3x}\right)-\log\left(a^2+a\sqrt[3]{a^3+b^3x}+(a^3+b^3x)^{2/3}\right)}{2a}$$

input
```
Integrate[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] + 2*Log[a - (a^
3 + b^3*x)^(1/3)] - Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/3)]
)/(2*a)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}dx$$

$$\Big\downarrow \ 67$$

$$-\frac{3\int \frac{1}{a-\sqrt[3]{a^3+b^3x}}d\sqrt[3]{a^3+b^3x}}{2a}+\frac{3}{2}\int \frac{1}{a^2+\sqrt[3]{a^3+b^3x}a+\left(a^3+b^3x\right)^{2/3}}d\sqrt[3]{a^3+b^3x}-\frac{\log(x)}{2a}$$

$$\Big\downarrow \ 16$$

$$\frac{3}{2}\int \frac{1}{a^2+\sqrt[3]{a^3+b^3x}a+\left(a^3+b^3x\right)^{2/3}}d\sqrt[3]{a^3+b^3x}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

$$\Big\downarrow \ 1082$$

$$-\frac{3\int \frac{1}{-\left(a^3+b^3x\right)^{2/3}-3}d\left(\frac{2\sqrt[3]{a^3+b^3x}}{a}+1\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

$$\Big\downarrow \ 217$$

$$\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3+b^3x}}{a}+1}{\sqrt{3}}\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

input
```
Int[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*x)^(1/3))/a)/Sqrt[3]])/a - Log[x]/(2*a)
 + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

method	result	size
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)+2\ln\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)-\ln\left(a^{2}+a\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a}$	85
derivativedivides	$\frac{\ln\left(a-\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^{2}+a\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	86
default	$\frac{\ln\left(a-\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^{2}+a\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	86

input
```
int(1/x/(b^3*x+a^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))*3^(1/2)/a)+2*ln(-a+(b^3*
x+a^3)^(1/3))-ln(a^2+a*(b^3*x+a^3)^(1/3)+(b^3*x+a^3)^(2/3)))/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx$$
$$=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a}$$

input
```
integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) -
 log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) + 2*log(-a + (b^3*
x + a^3)^(1/3)))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = \frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} + \frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} - \frac{\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x/(b**3*x+a**3)**(1/3),x)
```

output
```
exp(I*pi/3)*log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gam
ma(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a*
*3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(
2*I*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\,(b^3x+a^3)^{\frac{1}{3}}\right)}{3\,a}\right)}{a} - \frac{\log\left(a^2+(b^3x+a^3)^{\frac{1}{3}}a+(b^3x+a^3)^{\frac{2}{3}}\right)}{2\,a} + \frac{\log\left(-a+(b^3x+a^3)^{\frac{1}{3}}\right)}{a}$$

input
```
integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(-a + (b^3*x + a^3)^
(1/3))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\,(b^3x+a^3)^{\frac{1}{3}}\right)}{3\,a}\right)}{a} - \frac{\log\left(a^2+(b^3x+a^3)^{\frac{1}{3}}a+(b^3x+a^3)^{\frac{2}{3}}\right)}{2\,a} + \frac{\log\left(\left|-a+(b^3x+a^3)^{\frac{1}{3}}\right|\right)}{a}$$

input
```
integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(abs(-a + (b^3*x + a
^3)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = \frac{\ln\left(9\,{\left(a^3+x\,b^3\right)}^{1/3}-9\,a\right)}{a} + \frac{\ln\left(9\,{\left(a^3+x\,b^3\right)}^{1/3}-\frac{9\,a\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a} - \frac{\ln\left(9\,{\left(a^3+x\,b^3\right)}^{1/3}-\frac{9\,a\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}$$

input
```
int(1/(x*(b^3*x + a^3)^(1/3)),x)
```

output
```
log(9*(b^3*x + a^3)^(1/3) - 9*a)/a + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3^
(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(b^3*x + a^3)^(1/3) -
 (9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.32

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(b^3x+a^3)^{\frac{1}{6}}-\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)-2\sqrt{3}\,atan\left(\frac{2(b^3x+a^3)^{\frac{1}{6}}+\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)+2\log\left((b^3x+a^3)^{\frac{1}{6}}-\sqrt{a}\right)+2\log\left((b^3x+a^3)^{\frac{1}{6}}\right.}{2a}$$

input
```
int(1/x/(b^3*x+a^3)^(1/3),x)
```

output
```
(2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) - sqrt(a))/(sqrt(a)*sqrt(3))) -
2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) + sqrt(a))/(sqrt(a)*sqrt(3))) + 2
*log((a**3 + b**3*x)**(1/6) - sqrt(a)) + 2*log((a**3 + b**3*x)**(1/6) + sq
rt(a)) - log( - sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3) +
a) - log(sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3) + a))/(2*
a)
```

3.633 $\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}$$

output
```
3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))*3^(1/2)/a)/a-1/2*ln(x)/a+3/2*l
n(a-(-b^3*x+a^3)^(1/3))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx =\frac{2\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)+2\log\left(a-\sqrt[3]{a^3-b^3x}\right)-\log\left(a^2+a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}\right)}{2a}$$

input
```
Integrate[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] + 2*Log[a - (a^
3 - b^3*x)^(1/3)] - Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/3)]
)/(2*a)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}dx$$

$$\Big\downarrow \text{67}$$

$$-\frac{3\int \frac{1}{a-\sqrt[3]{a^3-b^3x}}d\sqrt[3]{a^3-b^3x}}{2a}+\frac{3}{2}\int \frac{1}{a^2+\sqrt[3]{a^3-b^3x}a+\left(a^3-b^3x\right)^{2/3}}d\sqrt[3]{a^3-b^3x}-\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{2}\int \frac{1}{a^2+\sqrt[3]{a^3-b^3x}a+\left(a^3-b^3x\right)^{2/3}}d\sqrt[3]{a^3-b^3x}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{3\int \frac{1}{-\left(a^3-b^3x\right)^{2/3}-3}d\left(\frac{2\sqrt[3]{a^3-b^3x}}{a}+1\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3-b^3x}}{a}+1}{\sqrt{3}}\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

input
```
Int[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + (2*(a^3 - b^3*x)^(1/3))/a)/Sqrt[3]])/a - Log[x]/(2*a)
 + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

method	result	size
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)+2\ln\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)-\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a}$	89
derivativedivides	$\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}+\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}$	90
default	$\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}+\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}$	90

input
```
int(1/x/(-b^3*x+a^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))*3^(1/2)/a)+2*ln(-a+(-b^
3*x+a^3)^(1/3))-ln(a^2+a*(-b^3*x+a^3)^(1/3)+(-b^3*x+a^3)^(2/3)))/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(-b^{3}x+\right.\right.}{2a}$$

input
```
integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a)
- log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) + 2*log(-a + (-
b^3*x + a^3)^(1/3)))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.86

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = -\frac{e^{-\frac{2i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} + \frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} - \frac{\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x/(-b**3*x+a**3)**(1/3),x)
```

output
```
-exp(-2*I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*
gamma(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(I*pi)/(b*(-a*
*3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(
5*I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} - \frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a} + \frac{\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}$$

input
```
integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(-a + (-b^3*x + a
^3)^(1/3))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} - \frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a} + \frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

input
```
integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(abs(-a + (-b^3*x
 + a^3)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = \frac{\ln\left(9\,{\left(a^3-b^3\,x\right)}^{1/3}-9\,a\right)}{a} + \frac{\ln\left(9\,{\left(a^3-b^3\,x\right)}^{1/3}-\frac{9\,a\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a} - \frac{\ln\left(9\,{\left(a^3-b^3\,x\right)}^{1/3}-\frac{9\,a\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}$$

input
```
int(1/(x*(a^3 - b^3*x)^(1/3)),x)
```

output
```
log(9*(a^3 - b^3*x)^(1/3) - 9*a)/a + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3^
(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(a^3 - b^3*x)^(1/3) -
 (9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.37

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2(-b^3x+a^3)^{\frac{1}{6}}-\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)-2\sqrt{3}\,atan\left(\frac{2(-b^3x+a^3)^{\frac{1}{6}}+\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)+2\log\left((-b^3x+a^3)^{\frac{1}{6}}-\sqrt{a}\right)+2\log\left((-b^3x\right.}{}$$

input
```
int(1/x/(-b^3*x+a^3)^(1/3),x)
```

output
```
(2*sqrt(3)*atan((2*(a**3 - b**3*x)**(1/6) - sqrt(a))/(sqrt(a)*sqrt(3))) -
2*sqrt(3)*atan((2*(a**3 - b**3*x)**(1/6) + sqrt(a))/(sqrt(a)*sqrt(3))) + 2
*log((a**3 - b**3*x)**(1/6) - sqrt(a)) + 2*log((a**3 - b**3*x)**(1/6) + sq
rt(a)) - log( - sqrt(a)*(a**3 - b**3*x)**(1/6) + (a**3 - b**3*x)**(1/3) +
a) - log(sqrt(a)*(a**3 - b**3*x)**(1/6) + (a**3 - b**3*x)**(1/3) + a))/(2*
a)
```

3.634 $\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

output
```
-3^(1/2)*arctan(1/3*(a-2*(b^3*x-a^3)^(1/3))*3^(1/2)/a)/a+1/2*ln(x)/a-3/2*l
n(a+(b^3*x-a^3)^(1/3))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)-2\log\left(a+\sqrt[3]{-a^3+b^3x}\right)+\log\left(a^2-a\sqrt[3]{-a^3+b^3x}+(-a^3+b^3x)^{2/}\right.}{2a}$$

input
```
Integrate[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a + (
-a^3 + b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x)^(
2/3)])/(2*a)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{b^3x-a^3}}dx$$

$$\Big\downarrow \text{68}$$

$$-\frac{3\int \frac{1}{a+\sqrt[3]{b^3x-a^3}}d\sqrt[3]{b^3x-a^3}}{2a}+\frac{3}{2}\int \frac{1}{a^2-\sqrt[3]{b^3x-a^3}a+\left(b^3x-a^3\right)^{2/3}}d\sqrt[3]{b^3x-a^3}+\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{2}\int \frac{1}{a^2-\sqrt[3]{b^3x-a^3}a+\left(b^3x-a^3\right)^{2/3}}d\sqrt[3]{b^3x-a^3}-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}+\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-\left(b^3x-a^3\right)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b^3x-a^3}}{a}\right)}{a}-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}+\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b^3x-a^3}}{a}}{\sqrt{3}}\right)}{a}-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}+\frac{\log(x)}{2a}$$

input
```
Int[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 - (2*(-a^3 + b^3*x)^(1/3))/a)/Sqrt[3]])/a) + Log[x]/(
2*a) - (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a-2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)-2\ln\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)+\ln\left(a^{2}-a\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a}$	90
derivativedivides	$-\frac{\ln\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{\frac{\ln\left(a^{2}-a\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	96
default	$-\frac{\ln\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{\frac{\ln\left(a^{2}-a\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	96

input
```
int(1/x/(b^3*x-a^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*3^(1/2)*arctan(1/3*(a-2*(b^3*x-a^3)^(1/3))*3^(1/2)/a)-2*ln(a+(b^3*
x-a^3)^(1/3))+ln(a^2-a*(b^3*x-a^3)^(1/3)+(b^3*x-a^3)^(2/3)))/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

input
```
integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a)
+ log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) - 2*log(a + (b^3*
x - a^3)^(1/3)))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = -\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} + \frac{\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} - \frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x/(b**3*x-a**3)**(1/3),x)
```

output
```
-exp(-I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*ga
mma(-1/3)/(3*a*gamma(2/3)) + log(-a*exp_polar(I*pi)/(b*(-a**3/b**3 + x)**(
1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(5*I
*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} + \frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a} - \frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$$

input
```
integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(a + (b^3*x - a^3)^
(1/3))/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} + \frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a} - \frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

input
```
integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(abs(a + (b^3*x - a
^3)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = -\frac{\ln\left(9\,a+9\,{\left(b^3\,x-a^3\right)}^{1/3}\right)}{a} - \frac{\ln\left(\frac{9\,a\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}+9\,{\left(b^3\,x-a^3\right)}^{1/3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a} + \frac{\ln\left(\frac{9\,a\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}+9\,{\left(b^3\,x-a^3\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}$$

input
```
int(1/(x*(b^3*x - a^3)^(1/3)),x)
```

output
```
(log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1))
/(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)
*1i - 1))/(2*a) - log(9*a + 9*(b^3*x - a^3)^(1/3))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2\left(b^3x-a^3\right)^{\frac{1}{6}}-\sqrt{a}\,\sqrt{3}}{\sqrt{a}}\right)-2\sqrt{3}\,atan\left(\frac{2\left(b^3x-a^3\right)^{\frac{1}{6}}+\sqrt{a}\,\sqrt{3}}{\sqrt{a}}\right)-2\log\left(\left(b^3x-a^3\right)^{\frac{1}{3}}+a\right)+\log\left(-\sqrt{a}\,(b^3x\right.}{2a}$$

input
```
int(1/x/(b^3*x-a^3)^(1/3),x)
```

output
```
(2*sqrt(3)*atan((2*( - a**3 + b**3*x)**(1/6) - sqrt(a)*sqrt(3))/sqrt(a)) -
 2*sqrt(3)*atan((2*( - a**3 + b**3*x)**(1/6) + sqrt(a)*sqrt(3))/sqrt(a)) -
 2*log(( - a**3 + b**3*x)**(1/3) + a) + log( - sqrt(a)*( - a**3 + b**3*x)*
*(1/6)*sqrt(3) + ( - a**3 + b**3*x)**(1/3) + a) + log(sqrt(a)*( - a**3 + b
**3*x)**(1/6)*sqrt(3) + ( - a**3 + b**3*x)**(1/3) + a))/(2*a)
```

3.635 $\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a} + \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

output
```
-3^(1/2)*arctan(1/3*(a-2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a)/a+1/2*ln(x)/a-3/2*
ln(a+(-b^3*x-a^3)^(1/3))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right) - 2\log\left(a+\sqrt[3]{-a^3-b^3x}\right) + \log\left(a^2 - a\sqrt[3]{-a^3-b^3x} + (-a^3-b^3x)^{2/}\right.}{2a}$$

input
```
Integrate[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a + (
-a^3 - b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x)^(
2/3)])/(2*a)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {68, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}dx$$

$$\Big\downarrow \text{68}$$

$$-\frac{3\int \frac{1}{a+\sqrt[3]{-a^3-b^3x}}d\sqrt[3]{-a^3-b^3x}}{2a}+\frac{3}{2}\int \frac{1}{a^2-\sqrt[3]{-a^3-b^3x}a+(-a^3-b^3x)^{2/3}}d\sqrt[3]{-a^3-b^3x}+\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{2}\int \frac{1}{a^2-\sqrt[3]{-a^3-b^3x}a+(-a^3-b^3x)^{2/3}}d\sqrt[3]{-a^3-b^3x}-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}+\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-(-a^3-b^3x)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a}-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}+\frac{\log(x)}{2a}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}+\frac{\log(x)}{2a}$$

input
```
Int[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 - (2*(-a^3 - b^3*x)^(1/3))/a)/Sqrt[3]])/a) + Log[x]/(
2*a) - (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a-2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)-2\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)+\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a}$	94
derivativedivides	$\frac{\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$	100
default	$\frac{\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$	100

input
```
int(1/x/(-b^3*x-a^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*3^(1/2)*arctan(1/3*(a-2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a)-2*ln(a+(-b^
3*x-a^3)^(1/3))+ln(a^2-a*(-b^3*x-a^3)^(1/3)+(-b^3*x-a^3)^(2/3)))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(-b^3x-\right.\right.}{2a}$$

input
```
integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
 + log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) - 2*log(a + (-
b^3*x - a^3)^(1/3)))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.83

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} - \frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)} + \frac{e^{\frac{2i\pi}{3}}\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x/(-b**3*x-a**3)**(1/3),x)
```

output
```
log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*
a*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3/b**3 + x)*
*(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) + exp(2*I*pi/3)*log(-a*exp_polar
(2*I*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a} + \frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2\,a} - \frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$$

input
```
integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^
2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(a + (-b^3*x - a
^3)^(1/3))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a} + \frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2\,a} - \frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

input
```
integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^
2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(abs(a + (-b^3*x
 - a^3)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = -\frac{\ln\left(9\,a+9\,{\left(-a^3-x\,b^3\right)}^{1/3}\right)}{a} - \frac{\ln\left(\frac{9\,a\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}+9\,{\left(-a^3-x\,b^3\right)}^{1/3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a} + \frac{\ln\left(\frac{9\,a\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}+9\,{\left(-a^3-x\,b^3\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}$$

input
```
int(1/(x*(- b^3*x - a^3)^(1/3)),x)
```

output
```
(log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1
))/(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(
1/2)*1i - 1))/(2*a) - log(9*a + 9*(- b^3*x - a^3)^(1/3))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(b^3x+a^3)^{\frac{1}{6}}-\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{2(b^3x+a^3)^{\frac{1}{6}}+\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)-2\log\left((b^3x+a^3)^{\frac{1}{6}}-\sqrt{a}\right)-2\log\left((b^3x+a^3\right.}{2a}$$

input
```
int(1/x/(-b^3*x-a^3)^(1/3),x)
```

output
```
( - 2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) - sqrt(a))/(sqrt(a)*sqrt(3)))
 + 2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) + sqrt(a))/(sqrt(a)*sqrt(3)))
- 2*log((a**3 + b**3*x)**(1/6) - sqrt(a)) - 2*log((a**3 + b**3*x)**(1/6) +
 sqrt(a)) + log( - sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3)
 + a) + log(sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3) + a))/
(2*a)
```

3.636 $\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

output
```
-3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))*3^(1/2)/a)/a^2-1/2*ln(x)/a^2+3
/2*ln(a-(b^3*x+a^3)^(1/3))/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)-2\log\left(a-\sqrt[3]{a^3+b^3x}\right)+\log\left(a^2+a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}\right)}{2a^2}$$

input
```
Integrate[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
- (a^3 + b^3*x)^(1/3)] + Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(
2/3)])/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}dx$$

$$\Big\downarrow \ 69$$

$$-\frac{3\int \frac{1}{a-\sqrt[3]{a^3+b^3x}}d\sqrt[3]{a^3+b^3x}}{2a^2}-\frac{3\int \frac{1}{a^2+\sqrt[3]{a^3+b^3x}a+\left(a^3+b^3x\right)^{2/3}}d\sqrt[3]{a^3+b^3x}}{2a}-\frac{\log(x)}{2a^2}$$

$$\Big\downarrow \ 16$$

$$-\frac{3\int \frac{1}{a^2+\sqrt[3]{a^3+b^3x}a+\left(a^3+b^3x\right)^{2/3}}d\sqrt[3]{a^3+b^3x}}{2a}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{3\int \frac{1}{-\left(a^3+b^3x\right)^{2/3}-3}d\left(\frac{2\sqrt[3]{a^3+b^3x}}{a}+1\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

$$\Big\downarrow \ 217$$

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3+b^3x}}{a}+1}{\sqrt{3}}\right)}{a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

input
```
Int[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*x)^(1/3))/a)/Sqrt[3]])/a^2) - Log[x]/
(2*a^2) + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)+2\ln\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)-\ln\left(a^{2}+a\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$	85
derivativedivides	$\frac{-\frac{\ln\left(a^{2}+a\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a-\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	87
default	$\frac{-\frac{\ln\left(a^{2}+a\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a-\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	87

input
```
int(1/x/(b^3*x+a^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))*3^(1/2)/a)+2*ln(-a+(b^3
*x+a^3)^(1/3))-ln(a^2+a*(b^3*x+a^3)^(1/3)+(b^3*x+a^3)^(2/3)))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \frac{1}{x\left(a^{3}+b^{3}x\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)+\log\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)-2\,\log\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{2\,a^{2}}$$

input
```
integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a)
+ log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) - 2*log(-a + (b^3
*x + a^3)^(1/3)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.86

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = \frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} + \frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} + \frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(b**3*x+a**3)**(2/3),x)
```

output
```
log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(-
2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/a)*gamma(1/
3)/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*ex
p_polar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2} - \frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2\,a^2} + \frac{\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

input
```
integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(-a + (b^3*x +
a^3)^(1/3))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

input
```
integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="giac")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (b^3*
x + a^3)^(1/3)))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = \frac{\ln\left(9a-9\left(a^3+x\,b^3\right)^{1/3}\right)}{a^2} + \frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2} - \frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}+\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}$$

input
```
int(1/(x*(b^3*x + a^3)^(2/3)),x)
```

output
```
log(9*a - 9*(b^3*x + a^3)^(1/3))/a^2 + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(
3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x + a^3)^(1/3)
 + (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2\left(b^3x+a^3\right)^{\frac{1}{6}}-\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{2\left(b^3x+a^3\right)^{\frac{1}{6}}+\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)+2\log\left(\left(b^3x+a^3\right)^{\frac{1}{6}}-\sqrt{}\right.}{}$$

input
```
int(1/x/(b^3*x+a^3)^(2/3),x)
```

output
```
( - 2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) - sqrt(a))/(sqrt(a)*sqrt(3)))
 + 2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) + sqrt(a))/(sqrt(a)*sqrt(3)))
+ 2*log((a**3 + b**3*x)**(1/6) - sqrt(a)) + 2*log((a**3 + b**3*x)**(1/6) +
 sqrt(a)) - log( - sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3)
 + a) - log(sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3) + a))/
(2*a**2)
```

3.637 $\int \frac{1}{x(a^3-b^3x)^{2/3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

output
```
-3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))*3^(1/2)/a)/a^2-1/2*ln(x)/a^2+
3/2*ln(a-(-b^3*x+a^3)^(1/3))/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)-2\log\left(a-\sqrt[3]{a^3-b^3x}\right)+\log\left(a^2+a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}\right)}{2a^2}$$

input
```
Integrate[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
- (a^3 - b^3*x)^(1/3)] + Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(
2/3)])/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}dx$$

$$\Big\downarrow \text{69}$$

$$-\frac{3\int \frac{1}{a-\sqrt[3]{a^3-b^3x}}d\sqrt[3]{a^3-b^3x}}{2a^2}-\frac{3\int \frac{1}{a^2+\sqrt[3]{a^3-b^3x}a+\left(a^3-b^3x\right)^{2/3}}d\sqrt[3]{a^3-b^3x}}{2a}-\frac{\log(x)}{2a^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{3\int \frac{1}{a^2+\sqrt[3]{a^3-b^3x}a+\left(a^3-b^3x\right)^{2/3}}d\sqrt[3]{a^3-b^3x}}{2a}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-\left(a^3-b^3x\right)^{2/3}-3}d\left(\frac{2\sqrt[3]{a^3-b^3x}}{a}+1\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3-b^3x}}{a}+1}{\sqrt{3}}\right)}{a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

input
```
Int[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 + (2*(a^3 - b^3*x)^(1/3))/a)/Sqrt[3]])/a^2) - Log[x]/
(2*a^2) + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)+2\ln\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)-\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$	89
derivativedivides	$\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	91
default	$\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	91

input
```
int(1/x/(-b^3*x+a^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))*3^(1/2)/a)+2*ln(-a+(-b
^3*x+a^3)^(1/3))-ln(a^2+a*(-b^3*x+a^3)^(1/3)+(-b^3*x+a^3)^(2/3)))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a^{3}-b^{3}x\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)-2\log\left(-a+\left(-b^{3}x+\right.\right.}{2a^{2}}$$

input
```
integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a)
 + log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) - 2*log(-a + (
-b^3*x + a^3)^(1/3)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = \frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} - \frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} + \frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(-b**3*x+a**3)**(2/3),x)
```

output
```
log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(1/3)/(3*a**2*
gamma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi
)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(-a**3/b**3
+ x)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

input
```
integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(
a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(-a + (-b^3*
x + a^3)^(1/3))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2\,a^2}+\frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

input
```
integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="giac")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(
a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (-
b^3*x + a^3)^(1/3)))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx=\frac{\ln\left(9\,a-9\left(a^3-b^3\,x\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\left(a^3-b^3\,x\right)^{1/3}-\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(9\left(a^3-b^3\,x\right)^{1/3}+\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}$$

input
```
int(1/(x*(a^3 - b^3*x)^(2/3)),x)
```

output
```
log(9*a - 9*(a^3 - b^3*x)^(1/3))/a^2 + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(
3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(a^3 - b^3*x)^(1/3)
 + (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.34

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2(-b^3x+a^3)^{\frac{1}{6}}-\sqrt{a}}{\sqrt{a}\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{2(-b^3x+a^3)^{\frac{1}{6}}+\sqrt{a}}{\sqrt{a}\sqrt{3}}\right)+2\log\left((-b^3x+a^3)^{\frac{1}{6}}\right.}{}$$

input
```
int(1/x/(-b^3*x+a^3)^(2/3),x)
```

output
```
( - 2*sqrt(3)*atan((2*(a**3 - b**3*x)**(1/6) - sqrt(a))/(sqrt(a)*sqrt(3)))
 + 2*sqrt(3)*atan((2*(a**3 - b**3*x)**(1/6) + sqrt(a))/(sqrt(a)*sqrt(3)))
+ 2*log((a**3 - b**3*x)**(1/6) - sqrt(a)) + 2*log((a**3 - b**3*x)**(1/6) +
 sqrt(a)) - log( - sqrt(a)*(a**3 - b**3*x)**(1/6) + (a**3 - b**3*x)**(1/3)
 + a) - log(sqrt(a)*(a**3 - b**3*x)**(1/6) + (a**3 - b**3*x)**(1/3) + a))/
(2*a**2)
```

3.638 $\int \frac{1}{x(-a^3+b^3x)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{1}{x(-a^3+b^3x)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

output
```
-3^(1/2)*arctan(1/3*(a-2*(b^3*x-a^3)^(1/3))*3^(1/2)/a)/a^2-1/2*ln(x)/a^2+3
/2*ln(a+(b^3*x-a^3)^(1/3))/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{1}{x(-a^3+b^3x)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right) - 2\log\left(a+\sqrt[3]{-a^3+b^3x}\right) + \log\left(a^2-a\sqrt[3]{-a^3+b^3x}+(-a^3+b^3x)^{2/3}\right)}{2a^2}$$

input
```
Integrate[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
 + (-a^3 + b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*
x)^(2/3)])/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(b^3x-a^3\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 70}$$

$$\frac{3\int \frac{1}{a+\sqrt[3]{b^3x-a^3}}d\sqrt[3]{b^3x-a^3}}{2a^2}+\frac{3\int \frac{1}{a^2-\sqrt[3]{b^3x-a^3}a+\left(b^3x-a^3\right)^{2/3}}d\sqrt[3]{b^3x-a^3}}{2a}-\frac{\log(x)}{2a^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3\int \frac{1}{a^2-\sqrt[3]{b^3x-a^3}a+\left(b^3x-a^3\right)^{2/3}}d\sqrt[3]{b^3x-a^3}}{2a}-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{3\int \frac{1}{-\left(b^3x-a^3\right)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{b^3x-a^3}}{a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b^3x-a^3}}{a}}{\sqrt{3}}\right)}{a^2}+\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}$$

input
```
Int[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 - (2*(-a^3 + b^3*x)^(1/3))/a)/Sqrt[3]])/a^2) - Log[x]
/(2*a^2) + (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a-2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)+2\ln\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)-\ln\left(a^{2}-a\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$	92
derivativedivides	$\frac{\ln\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}+\frac{-\frac{\ln\left(a^{2}-a\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}$	95
default	$\frac{\ln\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}+\frac{-\frac{\ln\left(a^{2}-a\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}$	95

input
```
int(1/x/(b^3*x-a^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*3^(1/2)*arctan(1/3*(a-2*(b^3*x-a^3)^(1/3))*3^(1/2)/a)+2*ln(a+(b^3*
x-a^3)^(1/3))-ln(a^2-a*(b^3*x-a^3)^(1/3)+(b^3*x-a^3)^(2/3)))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(-a^{3}+b^{3}x\right)^{2/3}}\,dx=\frac{2\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\,\sqrt{3}\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)-\log\left(a^{2}-\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)+2}{2\,a^{2}}$$

input
```
integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a)
- log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) + 2*log(a + (b^3*
x - a^3)^(1/3)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = -\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} + \frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} - \frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(b**3*x-a**3)**(2/3),x)
```

output
```
-exp(-I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma
(1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*
pi)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3
+ x)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2} - \frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2\,a^2} + \frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

input
```
integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(a + (b^3*x - a
^3)^(1/3))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

input
```
integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (b^3*x
 - a^3)^(1/3)))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = \frac{\ln\left(9\,a+9\left(b^3\,x-a^3\right)^{1/3}\right)}{a^2} + \frac{\ln\left(9\left(b^3\,x-a^3\right)^{1/3}+\frac{9\,a\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2} - \frac{\ln\left(9\left(b^3\,x-a^3\right)^{1/3}-\frac{9\,a\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2}$$

input
```
int(1/(x*(b^3*x - a^3)^(2/3)),x)
```

output
```
log(9*a + 9*(b^3*x - a^3)^(1/3))/a^2 + (log(9*(b^3*x - a^3)^(1/3) + (9*a*(
3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x - a^3)^(1/3)
 - (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.16

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2\left(b^3x-a^3\right)^{\frac{1}{6}}-\sqrt{a}\,\sqrt{3}}{\sqrt{a}}\right)-2\sqrt{3}\,atan\left(\frac{2\left(b^3x-a^3\right)^{\frac{1}{6}}+\sqrt{a}\,\sqrt{3}}{\sqrt{a}}\right)+2\log\left(\left(b^3x-a^3\right)\right.}{}$$

input
```
int(1/x/(b^3*x-a^3)^(2/3),x)
```

output
```
(2*sqrt(3)*atan((2*( - a**3 + b**3*x)**(1/6) - sqrt(a)*sqrt(3))/sqrt(a)) -
 2*sqrt(3)*atan((2*( - a**3 + b**3*x)**(1/6) + sqrt(a)*sqrt(3))/sqrt(a)) +
 2*log(( - a**3 + b**3*x)**(1/3) + a) - log( - sqrt(a)*( - a**3 + b**3*x)*
*(1/6)*sqrt(3) + ( - a**3 + b**3*x)**(1/3) + a) - log(sqrt(a)*( - a**3 + b
**3*x)**(1/6)*sqrt(3) + ( - a**3 + b**3*x)**(1/3) + a))/(2*a**2)
```

3.639 $\int \frac{1}{x(-a^3-b^3x)^{2/3}} dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{1}{x(-a^3-b^3x)^{2/3}} dx = -\frac{\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

output
```
-3^(1/2)*arctan(1/3*(a-2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a)/a^2-1/2*ln(x)/a^2+
3/2*ln(a+(-b^3*x-a^3)^(1/3))/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.39

$$\int \frac{1}{x(-a^3-b^3x)^{2/3}} dx = -\frac{2\sqrt{3}\arctan\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right) - 2\log\left(a+\sqrt[3]{-a^3-b^3x}\right) + \log\left(a^2-a\sqrt[3]{-a^3-b^3x}+(-a^3-b^3x)^{2/3}\right)}{2a^2}$$

input
```
Integrate[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
 + (-a^3 - b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*
x)^(2/3)])/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (-a^3-b^3x)^{2/3}} \, dx$$

$$\Big\downarrow \text{70}$$

$$\frac{3\int \frac{1}{a+\sqrt[3]{-a^3-b^3x}}d\sqrt[3]{-a^3-b^3x}}{2a^2}+\frac{3\int \frac{1}{a^2-\sqrt[3]{-a^3-b^3x}a+(-a^3-b^3x)^{2/3}}d\sqrt[3]{-a^3-b^3x}}{2a}-\frac{\log(x)}{2a^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\int \frac{1}{a^2-\sqrt[3]{-a^3-b^3x}a+(-a^3-b^3x)^{2/3}}d\sqrt[3]{-a^3-b^3x}}{2a}-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\int \frac{1}{-(-a^3-b^3x)^{2/3}-3}d\left(1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}+\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}$$

input
```
Int[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 - (2*(-a^3 - b^3*x)^(1/3))/a)/Sqrt[3]])/a^2) - Log[x]
/(2*a^2) + (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a-2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)+2\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)-\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$	96
derivativedivides	$\frac{-\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	99
default	$\frac{-\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	99

input
```
int(1/x/(-b^3*x-a^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*3^(1/2)*arctan(1/3*(a-2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a)+2*ln(a+(-b^
3*x-a^3)^(1/3))-ln(a^2-a*(-b^3*x-a^3)^(1/3)+(-b^3*x-a^3)^(2/3)))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\left(-a^{3}-b^{3}x\right)^{2/3}}\,dx=\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$$

input
```
integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
 - log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) + 2*log(a + (-
b^3*x - a^3)^(1/3)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.75

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = \frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} - \frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)} + \frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(-b**3*x-a**3)**(2/3),x)
```

output
```
exp(-2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamm
a(4/3)) - exp(-I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3
)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(a**3/b**3 + x)**(1/3)*exp
_polar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2} - \frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2\,a^2} + \frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

input
```
integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(
a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(a + (-b^3*x
 - a^3)^(1/3))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

input
```
integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(
a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (-b
^3*x - a^3)^(1/3)))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = \frac{\ln\left(9\,a+9\left(-a^3-x\,b^3\right)^{1/3}\right)}{a^2} + \frac{\ln\left(9\left(-a^3-x\,b^3\right)^{1/3}+\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2} - \frac{\ln\left(9\left(-a^3-x\,b^3\right)^{1/3}-\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}$$

input
```
int(1/(x*(- b^3*x - a^3)^(2/3)),x)
```

output
```
log(9*a + 9*(- b^3*x - a^3)^(1/3))/a^2 + (log(9*(- b^3*x - a^3)^(1/3) + (9
*a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(- b^3*x - a^3)
^(1/3) - (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2\left(b^3x+a^3\right)^{\frac{1}{6}}-\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{2\left(b^3x+a^3\right)^{\frac{1}{6}}+\sqrt{a}}{\sqrt{a}\,\sqrt{3}}\right)+2\log\left(\left(b^3x+a^3\right)^{\frac{1}{6}}-\right.}{}$$

input
```
int(1/x/(-b^3*x-a^3)^(2/3),x)
```

output
```
( - 2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) - sqrt(a))/(sqrt(a)*sqrt(3)))
 + 2*sqrt(3)*atan((2*(a**3 + b**3*x)**(1/6) + sqrt(a))/(sqrt(a)*sqrt(3)))
+ 2*log((a**3 + b**3*x)**(1/6) - sqrt(a)) + 2*log((a**3 + b**3*x)**(1/6) +
 sqrt(a)) - log( - sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3)
 + a) - log(sqrt(a)*(a**3 + b**3*x)**(1/6) + (a**3 + b**3*x)**(1/3) + a))/
(2*a**2)
```

3.640 $\int x^{5/2}\sqrt[4]{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\frac{16a^3\sqrt{x}\sqrt[4]{a+bx}}{231b^3}-\frac{8a^2x^{3/2}\sqrt[4]{a+bx}}{231b^2}+\frac{4ax^{5/2}\sqrt[4]{a+bx}}{165b}+\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}-\frac{32a^{9/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{231b^{7/2}(a+bx)^{3/4}}$$

output
```
16/231*a^3*x^(1/2)*(b*x+a)^(1/4)/b^3-8/231*a^2*x^(3/2)*(b*x+a)^(1/4)/b^2+4
/165*a*x^(5/2)*(b*x+a)^(1/4)/b+4/15*x^(7/2)*(b*x+a)^(1/4)-32/231*a^(9/2)*(
1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2)
)/b^(7/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.32

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\frac{2x^{7/2}\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[x^(5/2)*(a + b*x)^(1/4),x]
```

output
```
(2*x^(7/2)*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 7/2, 9/2, -((b*x)/a)])/
(7*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {60, 60, 60, 60, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{15}a\int \frac{x^{5/2}}{(a+bx)^{3/4}}dx+\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{15}a\left(\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\int \frac{x^{3/2}}{(a+bx)^{3/4}}dx}{11b}\right)+\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{15}a\left(\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}dx}{7b}\right)}{11b}\right)+\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{15}a\left(\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{2a\int\frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b}\right)}{7b}\right)}{11b}\right)+$$

$$\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{15}a\left(\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2}\right)}{7b}\right)}{11b}\right)+$$

$$\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{765}$$

$$\frac{1}{15}a\left(\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)}{11b}\right)+$$

$$\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{15}a\left(\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a^{5/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)}{11b}\right)+$$

$$\frac{4}{15}x^{7/2}\sqrt[4]{a+bx}$$

input
```
Int[x^(5/2)*(a + b*x)^(1/4),x]
```

output
```
(4*x^(7/2)*(a + b*x)^(1/4))/15 + (a*((4*x^(5/2)*(a + b*x)^(1/4))/(11*b) -
(10*a*((4*x^(3/2)*(a + b*x)^(1/4))/(7*b) - (6*a*((4*Sqrt[x]*(a + b*x)^(1/4
))/(3*b) - (8*a^(5/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/
4)/a^(1/4)], -1])/(3*b^2*Sqrt[-(a/b) + (a + b*x)/b])))/(7*b)))/(11*b)))/15
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int x^{\frac{5}{2}}(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(5/2)*(b*x+a)^(1/4),x)
```

output
```
int(x^(5/2)*(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(5/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.21

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\frac{2\sqrt[4]{a}x^{\frac{7}{2}}{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{7}$$

input
```
integrate(x**(5/2)*(b*x+a)**(1/4),x)
```

output
```
2*a**(1/4)*x**(7/2)*hyper((-1/4, 7/2), (9/2,), b*x*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)*x^(5/2), x)
```

Giac [F]

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\int x^{5/2}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(5/2)*(a + b*x)^(1/4),x)
```

output
```
int(x^(5/2)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int x^{5/2}\sqrt[4]{a+bx}\,dx=\frac{\frac{16\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^3}{231}-\frac{8\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^2bx}{231}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}a\,b^2x^2}{165}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}b^3x^3}{15}-\frac{8\left(\int\frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^2+ax}dx\right)a}{231}}{b^3}$$

input
```
int(x^(5/2)*(b*x+a)^(1/4),x)
```

output
```
(4*(20*sqrt(x)*(a + b*x)**(1/4)*a**3 - 10*sqrt(x)*(a + b*x)**(1/4)*a**2*b*
x + 7*sqrt(x)*(a + b*x)**(1/4)*a*b**2*x**2 + 77*sqrt(x)*(a + b*x)**(1/4)*b
**3*x**3 - 10*int((sqrt(x)*(a + b*x)**(1/4))/(a*x + b*x**2),x)*a**4))/(115
5*b**3)
```

3.641 $\int x^{3/2}\sqrt[4]{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx=-\frac{8a^2\sqrt{x}\sqrt[4]{a+bx}}{77b^2}+\frac{4ax^{3/2}\sqrt[4]{a+bx}}{77b}+\frac{4}{11}x^{5/2}\sqrt[4]{a+bx}+\frac{16a^{7/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{77b^{5/2}(a+bx)^{3/4}}$$

output
```
-8/77*a^2*x^(1/2)*(b*x+a)^(1/4)/b^2+4/77*a*x^(3/2)*(b*x+a)^(1/4)/b+4/11*x^
(5/2)*(b*x+a)^(1/4)+16/77*a^(7/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arct
an(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(5/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.38

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx=\frac{2x^{5/2}\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[x^(3/2)*(a + b*x)^(1/4),x]
```

output
```
(2*x^(5/2)*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 5/2, 7/2, -((b*x)/a)])/
(5*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{11}a\int \frac{x^{3/2}}{(a+bx)^{3/4}}dx+\frac{4}{11}x^{5/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{11}a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}dx}{7b}\right)+\frac{4}{11}x^{5/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{11}a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{2a\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b}\right)}{7b}\right)+\frac{4}{11}x^{5/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{11}a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2}\right)}{7b}\right)+\frac{4}{11}x^{5/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{765}$$

$$\frac{1}{11}a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)+\frac{4}{11}x^{5/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \; 762$$

$$\frac{1}{11}a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a^{5/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)+\frac{4}{11}x^{5/2}\sqrt[4]{a+bx}$$

input
```
Int[x^(3/2)*(a + b*x)^(1/4),x]
```

output
```
(4*x^(5/2)*(a + b*x)^(1/4))/11 + (a*((4*x^(3/2)*(a + b*x)^(1/4))/(7*b) - (
6*a*((4*Sqrt[x]*(a + b*x)^(1/4))/(3*b) - (8*a^(5/4)*Sqrt[1 - (a + b*x)/a]*
EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(3*b^2*Sqrt[-(a/b) + (a +
b*x)/b])))/(7*b)))/11
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int x^{\frac{3}{2}}(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(3/2)*(b*x+a)^(1/4),x)
```

output
```
int(x^(3/2)*(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx = \frac{2\sqrt[4]{a}x^{\frac{5}{2}}{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**(3/2)*(b*x+a)**(1/4),x)
```

output
```
2*a**(1/4)*x**(5/2)*hyper((-1/4, 5/2), (7/2,), b*x*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx=\int x^{3/2}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(3/2)*(a + b*x)^(1/4),x)
```

output
```
int(x^(3/2)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int x^{3/2}\sqrt[4]{a+bx}\,dx=\frac{-\frac{8\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^2}{77}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}abx}{77}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}b^2x^2}{11}+\frac{4\left(\int\frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^2+ax}dx\right)a^3}{77}}{b^2}$$

input
```
int(x^(3/2)*(b*x+a)^(1/4),x)
```

output
```
(4*( - 2*sqrt(x)*(a + b*x)**(1/4)*a**2 + sqrt(x)*(a + b*x)**(1/4)*a*b*x +
7*sqrt(x)*(a + b*x)**(1/4)*b**2*x**2 + int((sqrt(x)*(a + b*x)**(1/4))/(a*x
 + b*x**2),x)*a**3))/(77*b**2)
```

3.642 $\int \sqrt{x}\sqrt[4]{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx = \frac{4a\sqrt{x}\sqrt[4]{a+bx}}{21b}+\frac{4}{7}x^{3/2}\sqrt[4]{a+bx} - \frac{8a^{5/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{21b^{3/2}(a+bx)^{3/4}}$$

output
```
4/21*a*x^(1/2)*(b*x+a)^(1/4)/b+4/7*x^(3/2)*(b*x+a)^(1/4)-8/21*a^(5/2)*(1+b
*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b
^(3/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx = \frac{2x^{3/2}\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[Sqrt[x]*(a + b*x)^(1/4),x]
```

output
```
(2*x^(3/2)*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 3/2, 5/2, -((b*x)/a)])/
(3*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {60, 60, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{7}a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}dx+\frac{4}{7}x^{3/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{7}a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{2a\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b}\right)+\frac{4}{7}x^{3/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{7}a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2}\right)+\frac{4}{7}x^{3/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{765}$$

$$\frac{1}{7}a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)+\frac{4}{7}x^{3/2}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{7}a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a^{5/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)+\frac{4}{7}x^{3/2}\sqrt[4]{a+bx}$$

input
```
Int[Sqrt[x]*(a + b*x)^(1/4),x]
```

output
```
(4*x^(3/2)*(a + b*x)^(1/4))/7 + (a*((4*Sqrt[x]*(a + b*x)^(1/4))/(3*b) - (8
*a^(5/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)],
-1])/(3*b^2*Sqrt[-(a/b) + (a + b*x)/b])))/7
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \sqrt{x}\,(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(1/2)*(b*x+a)^(1/4),x)
```

output
```
int(x^(1/2)*(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx=\frac{2\sqrt[4]{a}x^{\frac{3}{2}}{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**(1/2)*(b*x+a)**(1/4),x)
```

output
```
2*a**(1/4)*x**(3/2)*hyper((-1/4, 3/2), (5/2,), b*x*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx = \int \sqrt{x}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(1/2)*(a + b*x)^(1/4),x)
```

output
```
int(x^(1/2)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \sqrt{x}\sqrt[4]{a+bx}\,dx = \frac{\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}a}{21}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}bx}{7}-\frac{2\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^{2}+ax}dx\right)a^{2}}{21}}{b}$$

input
```
int(x^(1/2)*(b*x+a)^(1/4),x)
```

output
```
(2*(2*sqrt(x)*(a + b*x)**(1/4)*a + 6*sqrt(x)*(a + b*x)**(1/4)*b*x - int((s
qrt(x)*(a + b*x)**(1/4))/(a*x + b*x**2),x)*a**2))/(21*b)
```

3.643 $\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = \frac{4}{3}\sqrt{x}\sqrt[4]{a+bx} + \frac{4a^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a+bx)^{3/4}}$$

output
```
4/3*x^(1/2)*(b*x+a)^(1/4)+4/3*a^(3/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*
arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(1/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx}{a}\right)}{\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x)/a)])/
(1 + (b*x)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {60, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}dx$$

$$\Big\downarrow \ 60$$

$$\frac{1}{3}a\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx+\frac{4}{3}\sqrt{x}\sqrt[4]{a+bx}$$

$$\Big\downarrow \ 73$$

$$\frac{4a\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b}+\frac{4}{3}\sqrt{x}\sqrt[4]{a+bx}$$

$$\Big\downarrow \ 765$$

$$\frac{4a\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}+\frac{4}{3}\sqrt{x}\sqrt[4]{a+bx}$$

$$\Big\downarrow \ 762$$

$$\frac{4a^{5/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}+\frac{4}{3}\sqrt{x}\sqrt[4]{a+bx}$$

input
```
Int[(a + b*x)^(1/4)/Sqrt[x],x]
```

output
```
(4*Sqrt[x]*(a + b*x)^(1/4))/3 + (4*a^(5/4)*Sqrt[1 - (a + b*x)/a]*EllipticF
[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(3*b*Sqrt[-(a/b) + (a + b*x)/b])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{\sqrt{x}}dx$$

input
```
int((b*x+a)^(1/4)/x^(1/2),x)
```

output
```
int((b*x+a)^(1/4)/x^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = 2\sqrt[4]{a}\sqrt{x}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)$$

input
```
integrate((b*x+a)**(1/4)/x**(1/2),x)
```

output
```
2*a**(1/4)*sqrt(x)*hyper((-1/4, 1/2), (3/2,), b*x*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/sqrt(x), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = \int \frac{(a+b\,x)^{1/4}}{\sqrt{x}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(1/2),x)
```

output
```
int((a + b*x)^(1/4)/x^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt{x}}\,dx = \frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{3} + \frac{\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^2+ax}dx\right)a}{3}$$

input
```
int((b*x+a)^(1/4)/x^(1/2),x)
```

output
```
(4*sqrt(x)*(a + b*x)**(1/4) + int((sqrt(x)*(a + b*x)**(1/4))/(a*x + b*x**2
),x)*a)/3
```

3.644 $\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx = -\frac{2\sqrt[4]{a+bx}}{\sqrt{x}} + \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{(a+bx)^{3/4}}$$

output
```
-2*(b*x+a)^(1/4)/x^(1/2)+2*a^(1/2)*b^(1/2)*(1+b*x/a)^(3/4)*InverseJacobiAM
(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx = -\frac{2\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt{x}\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(3/2),x]
```

output
```
(-2*(a + b*x)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, -((b*x)/a)])/(Sqrt[
x]*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {57, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{1}{2}b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx-\frac{2\sqrt[4]{a+bx}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 73}$$

$$2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{2\sqrt[4]{a+bx}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2\sqrt[4]{a}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{\sqrt{x}}$$

input
```
Int[(a + b*x)^(1/4)/x^(3/2),x]
```

output
```
(-2*(a + b*x)^(1/4))/Sqrt[x] + (2*a^(1/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[
ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{2}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(3/2),x)
```

output
```
int((b*x+a)^(1/4)/x^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx=\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx=-\frac{2\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{x}}$$

input
```
integrate((b*x+a)**(1/4)/x**(3/2),x)
```

output
```
-2*a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x*exp_polar(I*pi)/a)/sqrt(x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/x^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx = \int \frac{(a+b\,x)^{1/4}}{x^{3/2}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(3/2),x)
```

output
```
int((a + b*x)^(1/4)/x^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/2}}\,dx = \frac{-4(bx+a)^{\frac{1}{4}} - \sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^3+a\,x^2}dx\right)a}{\sqrt{x}}$$

input

```
int((b*x+a)^(1/4)/x^(3/2),x)
```

output

```
( - 4*(a + b*x)**(1/4) - sqrt(x)*int((sqrt(x)*(a + b*x)**(1/4))/(a*x**2 +
b*x**3),x)*a)/sqrt(x)
```

3.645 $\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx = -\frac{2\sqrt[4]{a+bx}}{3x^{3/2}}-\frac{b\sqrt[4]{a+bx}}{3a\sqrt{x}} - \frac{b^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{3\sqrt{a}(a+bx)^{3/4}}$$

output
```
-2/3*(b*x+a)^(1/4)/x^(3/2)-1/3*b*(b*x+a)^(1/4)/a/x^(1/2)-1/3*b^(3/2)*(1+b*
x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^
(1/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx = -\frac{2\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3x^{3/2}\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(5/2),x]
```

output
```
(-2*(a + b*x)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, -((b*x)/a)])/(3*x^
(3/2)*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {57, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{1}{6}b\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx-\frac{2\sqrt[4]{a+bx}}{3x^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{6}b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)-\frac{2\sqrt[4]{a+bx}}{3x^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}b\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)-\frac{2\sqrt[4]{a+bx}}{3x^{3/2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{1}{6}b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)-\frac{2\sqrt[4]{a+bx}}{3x^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{6}b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)-\frac{2\sqrt[4]{a+bx}}{3x^{3/2}}$$

input
```
Int[(a + b*x)^(1/4)/x^(5/2),x]
```

output
```
(-2*(a + b*x)^(1/4))/(3*x^(3/2)) + (b*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]) -
(2*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(
a^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/6
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{5}{2}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(5/2),x)
```

output
```
int((b*x+a)^(1/4)/x^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\, dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{5}{2}}}\, dx$$

input
```
integrate((b*x+a)^(1/4)/x^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/x^(5/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx = -\frac{2\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{3x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(1/4)/x**(5/2),x)
```

output
```
-2*a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x*exp_polar(I*pi)/a)/(3*x**(3/2
))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/x^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx = \int \frac{(a+b\,x)^{1/4}}{x^{5/2}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(5/2),x)
```

output
```
int((a + b*x)^(1/4)/x^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/2}}\,dx = \frac{-4(bx+a)^{\frac{1}{4}} - \sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^4+a\,x^3}dx\right)ax}{5\sqrt{x}\,x}$$

input
```
int((b*x+a)^(1/4)/x^(5/2),x)
```

output
```
( - 4*(a + b*x)**(1/4) - sqrt(x)*int((sqrt(x)*(a + b*x)**(1/4))/(a*x**3 +
b*x**4),x)*a*x)/(5*sqrt(x)*x)
```

3.646 $\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx = -\frac{2\sqrt[4]{a+bx}}{5x^{5/2}} - \frac{b\sqrt[4]{a+bx}}{15ax^{3/2}} + \frac{b^2\sqrt[4]{a+bx}}{6a^2\sqrt{x}} + \frac{b^{5/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{6a^{3/2}(a+bx)^{3/4}}$$

output
```
-2/5*(b*x+a)^(1/4)/x^(5/2)-1/15*b*(b*x+a)^(1/4)/a/x^(3/2)+1/6*b^2*(b*x+a)^
(1/4)/a^2/x^(1/2)+1/6*b^(5/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b
^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^(3/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx = -\frac{2\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5x^{5/2}\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, -((b*x)/a)])/(5*x^
(5/2)*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {57, 61, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 57$$

$$\frac{1}{10}b\int \frac{1}{x^{5/2}(a+bx)^{3/4}}dx-\frac{2\sqrt[4]{a+bx}}{5x^{5/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{1}{10}b\left(-\frac{5b\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)-\frac{2\sqrt[4]{a+bx}}{5x^{5/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{1}{10}b\left(-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)-\frac{2\sqrt[4]{a+bx}}{5x^{5/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{10}b\left(-\frac{5b\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)-\frac{2\sqrt[4]{a+bx}}{5x^{5/2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{1}{10}b\left(-\frac{5b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)-\frac{2\sqrt[4]{a+bx}}{5x^{5/2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{10}b\left(-\frac{5b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)-\frac{2\sqrt[4]{a+bx}}{5x^{5/2}}$$

input
```
Int[(a + b*x)^(1/4)/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(1/4))/(5*x^(5/2)) + (b*((-2*(a + b*x)^(1/4))/(3*a*x^(3/2))
- (5*b*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (a + b*x)/a]*Ellipt
icF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[-(a/b) + (a + b*x)
/b])))/(6*a)))/10
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{2}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(7/2),x)
```

output
```
int((b*x+a)^(1/4)/x^(7/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(7/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/x^(7/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx = -\frac{2\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{5x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**(1/4)/x**(7/2),x)
```

output
```
-2*a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x*exp_polar(I*pi)/a)/(5*x**(5/2
))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/x^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx=\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx=\int \frac{(a+b\,x)^{1/4}}{x^{7/2}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(7/2),x)
```

output
```
int((a + b*x)^(1/4)/x^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/2}}\,dx=\frac{-4(bx+a)^{\frac{1}{4}}-\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^5+a\,x^4}dx\right)a\,x^2}{9\sqrt{x}\,x^2}$$

input
```
int((b*x+a)^(1/4)/x^(7/2),x)
```

output
```
( - 4*(a + b*x)**(1/4) - sqrt(x)*int((sqrt(x)*(a + b*x)**(1/4))/(a*x**4 +
b*x**5),x)*a*x**2)/(9*sqrt(x)*x**2)
```

3.647 $\int x^{5/2}(a+bx)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 169

$$\int x^{5/2}(a+bx)^{3/4}\,dx=-\frac{16a^4\sqrt{x}}{221b^3\sqrt[4]{a+bx}}+\frac{8a^3x^{3/2}}{663b^2\sqrt[4]{a+bx}}-\frac{4a^2x^{5/2}}{663b\sqrt[4]{a+bx}}+\frac{12ax^{7/2}}{221\sqrt[4]{a+bx}}+\frac{4}{17}x^{7/2}(a+bx)^{3/4}+\frac{32a^{9/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{221b^{7/2}\sqrt[4]{a+bx}}$$

output

```
-16/221*a^4*x^(1/2)/b^3/(b*x+a)^(1/4)+8/663*a^3*x^(3/2)/b^2/(b*x+a)^(1/4)-
4/663*a^2*x^(5/2)/b/(b*x+a)^(1/4)+12/221*a*x^(7/2)/(b*x+a)^(1/4)+4/17*x^(7
/2)*(b*x+a)^(3/4)+32/221*a^(9/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arcta
n(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.28

$$\int x^{5/2}(a+bx)^{3/4}\,dx=\frac{2x^{7/2}(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[x^(5/2)*(a + b*x)^(3/4),x]
```

output
```
(2*x^(7/2)*(a + b*x)^(3/4)*Hypergeometric2F1[-3/4, 7/2, 9/2, -((b*x)/a)])/
(7*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {60, 60, 60, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^{3/4}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{17}a\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}dx+\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}dx}{13b}\right)+\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx}{3b}\right)}{13b}\right)+\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)}{3b}\right)}{13b}\right)+$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$$\Big\downarrow \; 73$$

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)}{3b}\right)}{13b}\right)+$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$$\Big\downarrow \; 836$$

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+\cdots}\right.}{5b^2}\right.}{3b}\right.}{13b}\right.$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$\downarrow$ 27

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right.}{3b}\right.}{13b}\right.$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$\downarrow$ 765

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a}\cdots}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}\cdots}\right.}{5b^2}\right.}{3b}\right.}{13b}\right.$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$\downarrow$ 762

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{13b}\right)$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$$\downarrow \text{1390}$$

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\mathrm{Ellipt}}{}\right.}{5b^2}\right.}{3b}\right.}{13b}\right.$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$\downarrow$ 1389

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\mathrm{E}}{}\right)}{5b^2}\right)}{3b}\right)}{13b}\right)$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

$\downarrow$ 327

$$\frac{3}{17}a\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+\cdots}{a}}\cdots}{\cdots}\right.}{5b^2}\right.}{3b}\right.}{13b}\right.$$

$$\frac{4}{17}x^{7/2}(a+bx)^{3/4}$$

input
```
Int[x^(5/2)*(a + b*x)^(3/4),x]
```

output
```
(4*x^(7/2)*(a + b*x)^(3/4))/17 + (3*a*((4*x^(5/2)*(a + b*x)^(3/4))/(13*b)
- (10*a*((4*x^(3/2)*(a + b*x)^(3/4))/(9*b) - (2*a*((4*Sqrt[x]*(a + b*x)^(3
/4))/(5*b) - (8*a*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*
x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a
+ b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a
 + b*x)/b]))/(5*b^2)))/(3*b)))/(13*b)))/17
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int x^{\frac{5}{2}}(bx+a)^{\frac{3}{4}}\,dx$$

input
```
int(x^(5/2)*(b*x+a)^(3/4),x)
```

output
```
int(x^(5/2)*(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int x^{5/2}(a+bx)^{3/4}\,dx=\int (bx+a)^{\frac{3}{4}}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(5/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.18

$$\int x^{5/2}(a+bx)^{3/4}\,dx = \frac{2a^{\frac{3}{4}}x^{\frac{7}{2}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{7}$$

input
```
integrate(x**(5/2)*(b*x+a)**(3/4),x)
```

output
```
2*a**(3/4)*x**(7/2)*hyper((-3/4, 7/2), (9/2,), b*x*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^{5/2}(a+bx)^{3/4}\,dx = \int (bx+a)^{\frac{3}{4}}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)*x^(5/2), x)
```

Giac [F]

$$\int x^{5/2}(a+bx)^{3/4}\,dx = \int (bx+a)^{\frac{3}{4}}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{3/4}\,dx=\int x^{5/2}\,(a+b\,x)^{3/4}\,dx$$

input
```
int(x^(5/2)*(a + b*x)^(3/4),x)
```

output
```
int(x^(5/2)*(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int x^{5/2}(a+bx)^{3/4}\,dx=\frac{\frac{16\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^3}{221}-\frac{40\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^2bx}{663}+\frac{12\sqrt{x}\,(bx+a)^{\frac{3}{4}}a\,b^2x^2}{221}+\frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}b^3x^3}{17}-\frac{8\left(\int\frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^2+ax}dx\right)a^4}{221}}{b^3}$$

input
```
int(x^(5/2)*(b*x+a)^(3/4),x)
```

output
```
(4*(12*sqrt(x)*(a + b*x)**(3/4)*a**3 - 10*sqrt(x)*(a + b*x)**(3/4)*a**2*b*
x + 9*sqrt(x)*(a + b*x)**(3/4)*a*b**2*x**2 + 39*sqrt(x)*(a + b*x)**(3/4)*b
**3*x**3 - 6*int((sqrt(x)*(a + b*x)**(3/4))/(a*x + b*x**2),x)*a**4))/(663*
b**3)
```

3.648 $\int x^{3/2}(a+bx)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 145

$$\int x^{3/2}(a+bx)^{3/4}\,dx=\frac{8a^3\sqrt{x}}{65b^2\sqrt[4]{a+bx}}-\frac{4a^2x^{3/2}}{195b\sqrt[4]{a+bx}}+\frac{4ax^{5/2}}{39\sqrt[4]{a+bx}}+\frac{4}{13}x^{5/2}(a+bx)^{3/4}-\frac{16a^{7/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx}}$$

output
```
8/65*a^3*x^(1/2)/b^2/(b*x+a)^(1/4)-4/195*a^2*x^(3/2)/b/(b*x+a)^(1/4)+4/39*
a*x^(5/2)/(b*x+a)^(1/4)+4/13*x^(5/2)*(b*x+a)^(3/4)-16/65*a^(7/2)*((b*x+a)/
a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(5/
2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.32

$$\int x^{3/2}(a+bx)^{3/4}\,dx=\frac{2x^{5/2}(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[x^(3/2)*(a + b*x)^(3/4),x]
```

output
```
(2*x^(5/2)*(a + b*x)^(3/4)*Hypergeometric2F1[-3/4, 5/2, 7/2, -((b*x)/a)])/
(5*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {60, 60, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^{3/4}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{13}a\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}dx+\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx}{3b}\right)+\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)}{3b}\right)+\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{73}$$

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)}{3b}\right)+\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$\downarrow$ 836

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)+\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$\downarrow$ 27

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)+\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$\downarrow$ 765

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)+$$

$$\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$$\downarrow 762$$

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right.$$

$$\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$$\downarrow 1390$$

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2}\right.}{3b}\right.$$

$$\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{1389}$$

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+}}{\sqrt[4]{a}}\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2}\right.}{3b}\right.$$

$$\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{13}a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\cdots}\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2}\right.}{3b}\right.$$

$$\frac{4}{13}x^{5/2}(a+bx)^{3/4}$$

input
```
Int[x^(3/2)*(a + b*x)^(3/4),x]
```

output
```
(4*x^(5/2)*(a + b*x)^(3/4))/13 + (3*a*((4*x^(3/2)*(a + b*x)^(3/4))/(9*b) -
 (2*a*((4*Sqrt[x]*(a + b*x)^(3/4))/(5*b) - (8*a*((a^(3/4)*Sqrt[1 - (a + b*
x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b
*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a
^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2)))/(3*b)))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int x^{\frac{3}{2}} (bx+a)^{\frac{3}{4}}\, dx$$

input
```
int(x^(3/2)*(b*x+a)^(3/4),x)
```

output
```
int(x^(3/2)*(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int x^{3/2}(a+bx)^{3/4}\, dx = \int (bx+a)^{\frac{3}{4}} x^{\frac{3}{2}}\, dx$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.21

$$\int x^{3/2}(a+bx)^{3/4}\, dx = \frac{2a^{\frac{3}{4}} x^{\frac{5}{2}} {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**(3/2)*(b*x+a)**(3/4),x)
```

output
```
2*a**(3/4)*x**(5/2)*hyper((-3/4, 5/2), (7/2,), b*x*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^{3/2}(a+bx)^{3/4}\,dx=\int (bx+a)^{\frac{3}{4}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}(a+bx)^{3/4}\,dx=\int (bx+a)^{\frac{3}{4}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{3/4}\,dx=\int x^{3/2}\,(a+b\,x)^{3/4}\,dx$$

input
```
int(x^(3/2)*(a + b*x)^(3/4),x)
```

output
```
int(x^(3/2)*(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int x^{3/2}(a+bx)^{3/4}\,dx = \frac{-\frac{8\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^2}{65} + \frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}abx}{39} + \frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}b^2x^2}{13} + \frac{4\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^2+ax}dx\right)a^3}{65}}{b^2}$$

input
```
int(x^(3/2)*(b*x+a)^(3/4),x)
```

output
```
(4*( - 6*sqrt(x)*(a + b*x)**(3/4)*a**2 + 5*sqrt(x)*(a + b*x)**(3/4)*a*b*x
+ 15*sqrt(x)*(a + b*x)**(3/4)*b**2*x**2 + 3*int((sqrt(x)*(a + b*x)**(3/4))
/(a*x + b*x**2),x)*a**3))/(195*b**2)
```

3.649 $\int \sqrt{x}(a+bx)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \sqrt{x}(a+bx)^{3/4}\,dx = -\frac{4a^2\sqrt{x}}{15b\sqrt[4]{a+bx}}+\frac{4ax^{3/2}}{15\sqrt[4]{a+bx}}+\frac{4}{9}x^{3/2}(a+bx)^{3/4}+\frac{8a^{5/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx}}$$

output
```
-4/15*a^2*x^(1/2)/b/(b*x+a)^(1/4)+4/15*a*x^(3/2)/(b*x+a)^(1/4)+4/9*x^(3/2)
*(b*x+a)^(3/4)+8/15*a^(5/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(
1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.39

$$\int \sqrt{x}(a+bx)^{3/4}\,dx = \frac{2x^{3/2}(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[Sqrt[x]*(a + b*x)^(3/4),x]
```

output
```
(2*x^(3/2)*(a + b*x)^(3/4)*Hypergeometric2F1[-3/4, 3/2, 5/2, -((b*x)/a)])/
(3*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {60, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^{3/4}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx+\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)+\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)+\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{836}$$

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)+$$
$$\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)+\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$\downarrow$ 765

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)+\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$\downarrow$ 762

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)+\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$\downarrow$ 1390

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)+\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$\downarrow$ 1389

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)+$$

$$\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{3}a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)$$

$$\frac{4}{9}x^{3/2}(a+bx)^{3/4}$$

input
```
Int[Sqrt[x]*(a + b*x)^(3/4),x]
```

output
```
(4*x^(3/2)*(a + b*x)^(3/4))/9 + (a*((4*Sqrt[x]*(a + b*x)^(3/4))/(5*b) - (8
*a*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4
)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*Ellip
ticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5
*b^2)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \sqrt{x}\,(bx+a)^{\frac{3}{4}}\,dx$$

input
```
int(x^(1/2)*(b*x+a)^(3/4),x)
```

output
```
int(x^(1/2)*(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \sqrt{x}(a+bx)^{3/4}\,dx=\int (bx+a)^{\frac{3}{4}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.26

$$\int \sqrt{x}(a+bx)^{3/4}\,dx = \frac{2a^{\frac{3}{4}}x^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**(1/2)*(b*x+a)**(3/4),x)
```

output
```
2*a**(3/4)*x**(3/2)*hyper((-3/4, 3/2), (5/2,), b*x*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int \sqrt{x}(a+bx)^{3/4}\,dx = \int (bx+a)^{\frac{3}{4}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}(a+bx)^{3/4}\,dx = \int (bx+a)^{\frac{3}{4}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{3/4}\,dx=\int \sqrt{x}\,{\left(a+b\,x\right)}^{3/4}\,dx$$

input
```
int(x^(1/2)*(a + b*x)^(3/4),x)
```

output
```
int(x^(1/2)*(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \sqrt{x}(a+bx)^{3/4}\,dx=\frac{\frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}a}{15}+\frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}bx}{9}-\frac{2\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^{2}+ax}dx\right)a^{2}}{15}}{b}$$

input
```
int(x^(1/2)*(b*x+a)^(3/4),x)
```

output
```
(2*(6*sqrt(x)*(a + b*x)**(3/4)*a + 10*sqrt(x)*(a + b*x)**(3/4)*b*x - 3*int
((sqrt(x)*(a + b*x)**(3/4))/(a*x + b*x**2),x)*a**2))/(45*b)
```

3.650 $\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx=\frac{12a\sqrt{x}}{5\sqrt[4]{a+bx}}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}-\frac{12a^{3/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx}}$$

output
```
12/5*a*x^(1/2)/(b*x+a)^(1/4)+4/5*x^(1/2)*(b*x+a)^(3/4)-12/5*a^(3/2)*((b*x+
a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^
(1/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx=\frac{2\sqrt{x}(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx}{a}\right)}{\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x)^(3/4)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(a + b*x)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x)/a)])/
(1 + (b*x)/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.54, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{5}a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{73}$$

$$\frac{12a\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{836}$$

$$\frac{12a\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{27}$$

$$\frac{12a\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{765}$$

$$\frac{12a\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{762}$$

$$\frac{12a\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{1390}$$

$$\frac{12a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{1389}$$

$$\frac{12a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

$$\Big\downarrow \text{327}$$

$$\frac{12a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b}+\frac{4}{5}\sqrt{x}(a+bx)^{3/4}$$

input
```
Int[(a + b*x)^(3/4)/Sqrt[x],x]
```

output
```
(4*Sqrt[x]*(a + b*x)^(3/4))/5 + (12*a*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*Elli
pticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (
a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -
1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{3}{4}}}{\sqrt{x}}dx$$

input
```
int((b*x+a)^(3/4)/x^(1/2),x)
```

output
```
int((b*x+a)^(3/4)/x^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)/sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx = 2a^{\frac{3}{4}}\sqrt{x}\,{}_2F_1\left(\begin{matrix} -\frac{3}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)$$

input
```
integrate((b*x+a)**(3/4)/x**(1/2),x)
```

output
```
2*a**(3/4)*sqrt(x)*hyper((-3/4, 1/2), (3/2,), b*x*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)/sqrt(x), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx = \int \frac{(a+b\,x)^{3/4}}{\sqrt{x}}\,dx$$

input
```
int((a + b*x)^(3/4)/x^(1/2),x)
```

output
```
int((a + b*x)^(3/4)/x^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/4}}{\sqrt{x}}\,dx = \frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{5} + \frac{3\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^2+ax}dx\right)a}{5}$$

input
```
int((b*x+a)^(3/4)/x^(1/2),x)
```

output
```
(4*sqrt(x)*(a + b*x)**(3/4) + 3*int((sqrt(x)*(a + b*x)**(3/4))/(a*x + b*x*
*2),x)*a)/5
```

3.651 $\int \frac{(a+bx)^{3/4}}{x^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx=\frac{6b\sqrt{x}}{\sqrt[4]{a+bx}}-\frac{2(a+bx)^{3/4}}{\sqrt{x}}-\frac{6\sqrt{a}\sqrt{b}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx}}$$

output
```
6*b*x^(1/2)/(b*x+a)^(1/4)-2*(b*x+a)^(3/4)/x^(1/2)-6*a^(1/2)*b^(1/2)*((b*x+
a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/(b
*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx=-\frac{2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt{x}\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x)^(3/4)/x^(3/2),x]
```

output
```
(-2*(a + b*x)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x)/a)])/(Sqrt[
x]*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {57, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{3}{2}b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

$$\Big\downarrow \text{73}$$

$$6\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

$$\Big\downarrow \text{836}$$

$$6\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

$$\Big\downarrow \text{27}$$

$$6\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

$$\Big\downarrow \text{765}$$

$$6\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

$$\Big\downarrow \ 762$$

$$\frac{6\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)-\frac{2(a+bx)^{3/4}}{\sqrt{x}}}{}$$

$$\Big\downarrow \ 1390$$

$$6\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

$$\Big\downarrow \ 1389$$

$$6\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

$$\Big\downarrow \ 327$$

$$6\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)-\frac{2(a+bx)^{3/4}}{\sqrt{x}}$$

input
```
Int[(a + b*x)^(3/4)/x^(3/2),x]
```

output
```
(-2*(a + b*x)^(3/4))/Sqrt[x] + 6*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE
[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/
4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/S
qrt[-(a/b) + (a + b*x)/b])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{3}{2}}}dx$$

input
```
int((b*x+a)^(3/4)/x^(3/2),x)
```

output
```
int((b*x+a)^(3/4)/x^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx=\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)/x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx=-\frac{2a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{x}}$$

input
```
integrate((b*x+a)**(3/4)/x**(3/2),x)
```

output
```
-2*a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x*exp_polar(I*pi)/a)/sqrt(x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx=\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx = \int \frac{(a+b\,x)^{3/4}}{x^{3/2}}\,dx$$

input
```
int((a + b*x)^(3/4)/x^(3/2),x)
```

output
```
int((a + b*x)^(3/4)/x^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/4}}{x^{3/2}}\,dx = \frac{4(bx+a)^{\frac{3}{4}} + 3\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^3+a\,x^2}dx\right)a}{\sqrt{x}}$$

input
```
int((b*x+a)^(3/4)/x^(3/2),x)
```

output
```
(4*(a + b*x)**(3/4) + 3*sqrt(x)*int((sqrt(x)*(a + b*x)**(3/4))/(a*x**2 + b
*x**3),x)*a)/sqrt(x)
```

3.652 $\int \frac{(a+bx)^{3/4}}{x^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}} dx = -\frac{b}{\sqrt{x}\sqrt[4]{a+bx}} - \frac{2(a+bx)^{3/4}}{3x^{3/2}} - \frac{b^{3/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx}}$$

output
```
-b/x^(1/2)/(b*x+a)^(1/4)-2/3*(b*x+a)^(3/4)/x^(3/2)-b^(3/2)*((b*x+a)/a)^(1/
4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(1/2)/(b*
x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}} dx = -\frac{2(a+bx)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x)^(3/4)/x^(5/2),x]
```

output
```
(-2*(a + b*x)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x)/a)])/(3*x^
(3/2)*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {57, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 57$$

$$\frac{1}{2}b\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2}b\left(\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}b\left(\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$$\Big\downarrow \ 836$$

$$\frac{1}{2}b\left(\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$\downarrow$ 765

$$\frac{1}{2}b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$\downarrow$ 762

$$\frac{1}{2}b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$\downarrow$ 1390

$$\frac{1}{2}b\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$\downarrow$ 1389

$$\frac{1}{2}b\left(\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

$\big\downarrow$ 327

$$\frac{1}{2}b\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)-\frac{2(a+bx)^{3/4}}{3x^{3/2}}$$

input

```
Int[(a + b*x)^(3/4)/x^(5/2),x]
```

output

```
(-2*(a + b*x)^(3/4))/(3*x^(3/2)) + (b*((-2*(a + b*x)^(3/4))/(a*Sqrt[x]) +
(2*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4
)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*Ellip
ticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/a)
)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{5}{2}}}dx$$

input
```
int((b*x+a)^(3/4)/x^(5/2),x)
```

output
```
int((b*x+a)^(3/4)/x^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)/x^(5/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}}\,dx = -\frac{2a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{3x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(3/4)/x**(5/2),x)
```

output
```
-2*a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x*exp_polar(I*pi)/a)/(3*x**(3/2
))
```

Maxima [F]

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)/x^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}}\,dx = \int \frac{(a+b\,x)^{3/4}}{x^{5/2}}\,dx$$

input
```
int((a + b*x)^(3/4)/x^(5/2),x)
```

output
```
int((a + b*x)^(3/4)/x^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/4}}{x^{5/2}}\,dx = \frac{-4(bx+a)^{\frac{3}{4}} - 3\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^4+a\,x^3}dx\right)ax}{3\sqrt{x}\,x}$$

input
```
int((b*x+a)^(3/4)/x^(5/2),x)
```

output
```
( - 4*(a + b*x)**(3/4) - 3*sqrt(x)*int((sqrt(x)*(a + b*x)**(3/4))/(a*x**3
+ b*x**4),x)*a*x)/(3*sqrt(x)*x)
```

3.653 $\int \frac{(a+bx)^{3/4}}{x^{7/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}} dx = -\frac{b}{5x^{3/2}\sqrt[4]{a+bx}}+\frac{b^2}{10a\sqrt{x}\sqrt[4]{a+bx}} -\frac{2(a+bx)^{3/4}}{5x^{5/2}}+\frac{3b^{5/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}\sqrt[4]{a+bx}}$$

output
```
-1/5*b/x^(3/2)/(b*x+a)^(1/4)+1/10*b^2/a/x^(1/2)/(b*x+a)^(1/4)-2/5*(b*x+a)^
(3/4)/x^(5/2)+3/10*b^(5/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1
/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.39

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}} dx = -\frac{2(a+bx)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5x^{5/2}\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x)^(3/4)/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, -((b*x)/a)])/(5*x^
(5/2)*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {57, 61, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{3}{10}b\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}dx-\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3}{10}b\left(-\frac{b\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)-\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3}{10}b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)-\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)-\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$$\Big\downarrow \text{836}$$

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)-\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$\downarrow$ 27

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)-\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$\downarrow$ 765

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)-\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$\downarrow$ 762

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)-$$

$$\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$\downarrow$ 1390

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3}}{3ax^{3/2}}\right.$$

$$\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$\downarrow$ 1389

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+b}{3ax^3}\right.$$

$$\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

$\downarrow$ 327

$$\frac{3}{10}b\left(-\frac{b\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a}{3}\right.$$

$$\frac{2(a+bx)^{3/4}}{5x^{5/2}}$$

input
```
Int[(a + b*x)^(3/4)/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(3/4))/(5*x^(5/2)) + (3*b*((-2*(a + b*x)^(3/4))/(3*a*x^(3/2)
) - (b*((-2*(a + b*x)^(3/4))/(a*Sqrt[x]) + (2*((a^(3/4)*Sqrt[1 - (a + b*x)
/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x
)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(
1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/a))/(2*a)))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{7}{2}}}dx$$

input
```
int((b*x+a)^(3/4)/x^(7/2),x)
```

output
```
int((b*x+a)^(3/4)/x^(7/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}}\,dx=\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(7/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)/x^(7/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}}\,dx=-\frac{2a^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{5}{2},-\frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{5x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**(3/4)/x**(7/2),x)
```

output
```
-2*a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x*exp_polar(I*pi)/a)/(5*x**(5/2
))
```

Maxima [F]

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}}\,dx=\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)/x^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}}\,dx = \int \frac{(a+b\,x)^{3/4}}{x^{7/2}}\,dx$$

input
```
int((a + b*x)^(3/4)/x^(7/2),x)
```

output
```
int((a + b*x)^(3/4)/x^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/4}}{x^{7/2}}\,dx = \frac{-4(bx+a)^{\frac{3}{4}} - 3\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^5+a\,x^4}dx\right)a\,x^2}{7\sqrt{x}\,x^2}$$

input
```
int((b*x+a)^(3/4)/x^(7/2),x)
```

output
```
( - 4*(a + b*x)**(3/4) - 3*sqrt(x)*int((sqrt(x)*(a + b*x)**(3/4))/(a*x**4
+ b*x**5),x)*a*x**2)/(7*sqrt(x)*x**2)
```

3.654 $\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 145

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx = -\frac{3b}{35x^{5/2}\sqrt[4]{a+bx}}+\frac{b^2}{70ax^{3/2}\sqrt[4]{a+bx}}-\frac{b^3}{20a^2\sqrt{x}\sqrt[4]{a+bx}} -\frac{2(a+bx)^{3/4}}{7x^{7/2}}-\frac{3b^{7/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx}}$$

output
```
-3/35*b/x^(5/2)/(b*x+a)^(1/4)+1/70*b^2/a/x^(3/2)/(b*x+a)^(1/4)-1/20*b^3/a^
2/x^(1/2)/(b*x+a)^(1/4)-2/7*(b*x+a)^(3/4)/x^(7/2)-3/20*b^(7/2)*((b*x+a)/a)
^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)
/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx = -\frac{2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{4},-\frac{5}{2},-\frac{bx}{a}\right)}{7x^{7/2}\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x)^(3/4)/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(3/4)*Hypergeometric2F1[-7/2, -3/4, -5/2, -((b*x)/a)])/(7*x^
(7/2)*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.59, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {57, 61, 61, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{3}{14}b\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}dx-\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3}{14}b\left(-\frac{7b\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}dx}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)-\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)-\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)-\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)-\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \; 836$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)-\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)-\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \ 765$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right.$$

$$\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\Big\downarrow \ 762$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-2(\right.$$

$$\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$\downarrow$ 1390

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/}}{3ax^{3/2}}}{10a}\right.$$

$$\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$$\downarrow 1389$$

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx}{3ax^3}\right)}{10a}\right.$$

$$\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

$\downarrow$ 327

$$\frac{3}{14}b\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a\ldots}{3\ldots}\right)}{10a}\ldots\right.$$

$$\frac{2(a+bx)^{3/4}}{7x^{7/2}}$$

input
```
Int[(a + b*x)^(3/4)/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(3/4))/(7*x^(7/2)) + (3*b*((-2*(a + b*x)^(3/4))/(5*a*x^(5/2)
) - (7*b*((-2*(a + b*x)^(3/4))/(3*a*x^(3/2)) - (b*((-2*(a + b*x)^(3/4))/(a
*Sqrt[x]) + (2*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^
(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b
*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a +
b*x)/b]))/a))/(2*a)))/(10*a)))/14
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{9}{2}}}dx$$

input
```
int((b*x+a)^(3/4)/x^(9/2),x)
```

output
```
int((b*x+a)^(3/4)/x^(9/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(9/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)/x^(9/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.25

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx = -\frac{2a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{7}{2}, -\frac{3}{4} \\ -\frac{5}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{7x^{\frac{7}{2}}}$$

input
```
integrate((b*x+a)**(3/4)/x**(9/2),x)
```

output
```
-2*a**(3/4)*hyper((-7/2, -3/4), (-5/2,), b*x*exp_polar(I*pi)/a)/(7*x**(7/2
))
```

Maxima [F]

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/4)/x^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx=\int \frac{(bx+a)^{\frac{3}{4}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x+a)^(3/4)/x^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/4)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx=\int \frac{(a+b\,x)^{3/4}}{x^{9/2}}\,dx$$

input
```
int((a + b*x)^(3/4)/x^(9/2),x)
```

output
```
int((a + b*x)^(3/4)/x^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/4}}{x^{9/2}}\,dx=\frac{-4(bx+a)^{\frac{3}{4}}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^6+a\,x^5}dx\right)a\,x^3}{11\sqrt{x}\,x^3}$$

input
```
int((b*x+a)^(3/4)/x^(9/2),x)
```

output
```
( - 4*(a + b*x)**(3/4) - 3*sqrt(x)*int((sqrt(x)*(a + b*x)**(3/4))/(a*x**5
+ b*x**6),x)*a*x**3)/(11*sqrt(x)*x**3)
```

3.655 $\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\,dx = -\frac{16a^3\sqrt{x}}{39b^3\sqrt[4]{a+bx}} + \frac{8a^2x^{3/2}}{117b^2\sqrt[4]{a+bx}} - \frac{4ax^{5/2}}{117b\sqrt[4]{a+bx}} + \frac{4x^{7/2}}{13\sqrt[4]{a+bx}} + \frac{32a^{7/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx}}$$

output
```
-16/39*a^3*x^(1/2)/b^3/(b*x+a)^(1/4)+8/117*a^2*x^(3/2)/b^2/(b*x+a)^(1/4)-4
/117*a*x^(5/2)/b/(b*x+a)^(1/4)+4/13*x^(7/2)/(b*x+a)^(1/4)+32/39*a^(7/2)*((
b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2)
)/b^(7/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.32

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\,dx = \frac{2x^{7/2}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(5/2)/(a + b*x)^(1/4),x]
```

output
```
(2*x^(7/2)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 7/2, 9/2, -((b*x)/a)
])/(7*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.43, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {60, 60, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}dx}{13b}$$

$$\Big\downarrow \text{60}$$

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx}{3b}\right)}{13b}$$

$$\Big\downarrow \text{60}$$

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)}{3b}\right)}{13b}$$

$\downarrow$ 836

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)}{13b}$$

$\downarrow$ 27

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)}{13b}$$

$\downarrow$ 765

$$\frac{\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{13b}}{}$$

$\downarrow$ 762

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{13b}$$

$\downarrow$ 1390

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{1389}$$

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{13b}$$

$$\Big\downarrow \text{327}$$

$$\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2}\right.}{3b}\right.}{13b}$$

input
```
Int[x^(5/2)/(a + b*x)^(1/4),x]
```

output
```
(4*x^(5/2)*(a + b*x)^(3/4))/(13*b) - (10*a*((4*x^(3/2)*(a + b*x)^(3/4))/(9
*b) - (2*a*((4*Sqrt[x]*(a + b*x)^(3/4))/(5*b) - (8*a*((a^(3/4)*Sqrt[1 - (a
 + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (
a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1
/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2)))/(3*b)))/(13*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(5/2)/(b*x+a)^(1/4),x)
```

output
```
int(x^(5/2)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\, dx = \int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{1}{4}}}\, dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^(5/2)/(b*x + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.20

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\, dx = \frac{2x^{\frac{7}{2}}{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

input
```
integrate(x**(5/2)/(b*x+a)**(1/4),x)
```

output
```
2*x**(7/2)*hyper((1/4, 7/2), (9/2,), b*x*exp_polar(I*pi)/a)/(7*a**(1/4))
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(b*x + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{5/2}}{(a+b\,x)^{1/4}}\,dx$$

input
```
int(x^(5/2)/(a + b*x)^(1/4),x)
```

output
```
int(x^(5/2)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{\sqrt{x}\,x^2}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(5/2)/(b*x+a)^(1/4),x)
```

output
```
int((sqrt(x)*x**2)/(a + b*x)**(1/4),x)
```

3.656 $\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \frac{8a^2\sqrt{x}}{15b^2\sqrt[4]{a+bx}} - \frac{4ax^{3/2}}{45b\sqrt[4]{a+bx}} + \frac{4x^{5/2}}{9\sqrt[4]{a+bx}} - \frac{16a^{5/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx}}$$

output
```
8/15*a^2*x^(1/2)/b^2/(b*x+a)^(1/4)-4/45*a*x^(3/2)/b/(b*x+a)^(1/4)+4/9*x^(5
/2)/(b*x+a)^(1/4)-16/15*a^(5/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan
(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.38

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \frac{2x^{5/2}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(3/2)/(a + b*x)^(1/4),x]
```

output
```
(2*x^(5/2)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 5/2, 7/2, -((b*x)/a)
])/(5*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.47, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {60, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)}{3b}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)}{3b}$$

$$\Big\downarrow \text{836}$$

$$\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}$$

$\downarrow$ 27

$$\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}$$

$\downarrow$ 765

$$\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}$$

$\downarrow$ 762

$$\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}$$

$\downarrow$ 1390

$$\frac{2a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}$$

$\big\downarrow$ 1389

$$\frac{2a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}$$

$\big\downarrow$ 327

$$\frac{2a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}$$

input
```
Int[x^(3/2)/(a + b*x)^(1/4),x]
```

output
```
(4*x^(3/2)*(a + b*x)^(3/4))/(9*b) - (2*a*((4*Sqrt[x]*(a + b*x)^(3/4))/(5*b
) - (8*a*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/
a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]
*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b
]))/(5*b^2)))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(3/2)/(b*x+a)^(1/4),x)
```

output
```
int(x^(3/2)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^(3/2)/(b*x + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \frac{2x^{\frac{5}{2}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

input
```
integrate(x**(3/2)/(b*x+a)**(1/4),x)
```

output
```
2*x**(5/2)*hyper((1/4, 5/2), (7/2,), b*x*exp_polar(I*pi)/a)/(5*a**(1/4))
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(b*x + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{3/2}}{(a+b\,x)^{1/4}}\,dx$$

input
```
int(x^(3/2)/(a + b*x)^(1/4),x)
```

output
```
int(x^(3/2)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}\,dx = \int \frac{\sqrt{x}\,x}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(3/2)/(b*x+a)^(1/4),x)
```

output
```
int((sqrt(x)*x)/(a + b*x)**(1/4),x)
```

3.657 $\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}} \, dx = -\frac{4a\sqrt{x}}{5b\sqrt[4]{a+bx}} + \frac{4x^{3/2}}{5\sqrt[4]{a+bx}} + \frac{8a^{3/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx}}$$

output
```
-4/5*a*x^(1/2)/b/(b*x+a)^(1/4)+4/5*x^(3/2)/(b*x+a)^(1/4)+8/5*a^(3/2)*((b*x
+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b
^(3/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}} \, dx = \frac{2x^{3/2}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3\sqrt[4]{a+bx}}$$

input
```
Integrate[Sqrt[x]/(a + b*x)^(1/4),x]
```

output
```
(2*x^(3/2)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 3/2, 5/2, -((b*x)/a)
])/(3*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.52, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}$$

$$\Big\downarrow \text{ 836}$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}$$

$$\Big\downarrow \ 762$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}$$

$$\Big\downarrow \ 1390$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}$$

$$\Big\downarrow \ 1389$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}$$

$$\Big\downarrow \ 327$$

$$\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}$$

input
```
Int[Sqrt[x]/(a + b*x)^(1/4),x]
```

output
```
(4*Sqrt[x]*(a + b*x)^(3/4))/(5*b) - (8*a*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*E
llipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]
- (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)]
, -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{\sqrt{x}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(1/2)/(b*x+a)^(1/4),x)
```

output
```
int(x^(1/2)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b*x + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}\,dx = \frac{2x^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

input
```
integrate(x**(1/2)/(b*x+a)**(1/4),x)
```

output
```
2*x**(3/2)*hyper((1/4, 3/2), (5/2,), b*x*exp_polar(I*pi)/a)/(3*a**(1/4))
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(b*x + a)^(1/4), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}\,dx=\int \frac{\sqrt{x}}{{\left(a+b\,x\right)}^{1/4}} \,d x$$

input
```
int(x^(1/2)/(a + b*x)^(1/4),x)
```

output
```
int(x^(1/2)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}\,dx=\int \frac{\sqrt{x}}{\left(b x +a \right)^{\frac{1}{4}}}d x$$

input
```
int(x^(1/2)/(b*x+a)^(1/4),x)
```

output
```
int(sqrt(x)/(a + b*x)**(1/4),x)
```

3.658 $\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx=\frac{4\sqrt{x}}{\sqrt[4]{a+bx}}-\frac{4\sqrt{a}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx}}$$

output
```
4*x^(1/2)/(b*x+a)^(1/4)-4*a^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arct
an(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(1/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx=\frac{2\sqrt{x}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx}{a}\right)}{\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)^(1/4)),x]
```

output
```
(2*Sqrt[x]*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x)/a)
])/(a + b*x)^(1/4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{b}$$

$$\Big\downarrow \text{836}$$

$$\frac{4\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b}$$

$$\Big\downarrow \text{765}$$

$$\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b}$$

$$\Big\downarrow \text{762}$$

$$\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b}$$

$$\Big\downarrow \ 1390$$

$$\frac{4\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b}$$

$$\Big\downarrow \ 1389$$

$$\frac{4\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b}$$

$$\Big\downarrow \ 327$$

$$\frac{4\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)^(1/4)),x]
```

output
```
(4*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4
)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*Ellip
ticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(1/2)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(1/2)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx=\int \frac{1}{(bx+a)^{\frac{1}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b*x^2 + a*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx=\frac{2\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**(1/4),x)
```

output
```
2*sqrt(x)*hyper((1/4, 1/2), (3/2,), b*x*exp_polar(I*pi)/a)/a**(1/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{\sqrt{x}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(1/2)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(1/2)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}\,dx = \frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}+\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a}{3\sqrt{bx+a}}$$

input
```
int(1/x^(1/2)/(b*x+a)^(1/4),x)
```

output
```
(4*sqrt(x)*(a + b*x)**(1/4) + sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/4))
/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a)/(3*sqrt(a + b*x))
```

3.659 $\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx=-\frac{2}{\sqrt{x}\sqrt[4]{a+bx}}-\frac{2\sqrt{b}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx}}$$

output
```
-2/x^(1/2)/(b*x+a)^(1/4)-2*b^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arc
tan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(1/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx=-\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt{x}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)^(1/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x)/a)])/(Sq
rt[x]*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{836}$$

$$\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{765}$$

$$\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{762}$$

$$\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(a + b*x)^(1/4)),x]
```

output
```
(-2*(a + b*x)^(3/4))/(a*Sqrt[x]) + (2*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*Elli
pticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (
a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -
1])/Sqrt[-(a/b) + (a + b*x)/b]))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{x^{\frac{3}{2}} (bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(3/2)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(3/2)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b*x^3 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\sqrt{x}}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**(1/4),x)
```

output
```
-2*hyper((-1/2, 1/4), (1/2,), b*x*exp_polar(I*pi)/a)/(a**(1/4)*sqrt(x))
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{3/2}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(3/2)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(3/2)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}\,dx = \frac{-4\sqrt{x}\,(bx+a)^{\frac{1}{4}} - \sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^4+2ab\,x^3+a^2x^2}dx\right)ax}{\sqrt{bx+a}\,x}$$

input
```
int(1/x^(3/2)/(b*x+a)^(1/4),x)
```

output
```
( - 4*sqrt(x)*(a + b*x)**(1/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/
4))/(a**2*x**2 + 2*a*b*x**3 + b**2*x**4),x)*a*x)/(sqrt(a + b*x)*x)
```

3.660 $\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx = -\frac{2}{3x^{3/2}\sqrt[4]{a+bx}}+\frac{b}{3a\sqrt{x}\sqrt[4]{a+bx}} + \frac{b^{3/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx}}$$

output
```
-2/3/x^(3/2)/(b*x+a)^(1/4)+1/3*b/a/x^(1/2)/(b*x+a)^(1/4)+b^(3/2)*((b*x+a)/
a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(3/
2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3x^{3/2}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)^(1/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x)/a)])/(3
*x^(3/2)*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {61, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{b\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{b\left(\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$$\Big\downarrow \text{836}$$

$$-\frac{b\left(\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$\Big\downarrow$ 765

$$-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$\Big\downarrow$ 762

$$-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$\Big\downarrow$ 1390

$$-\frac{b\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$\Big\downarrow$ 1389

$$-\frac{b\left(\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

$$\Big\downarrow \textbf{327}$$

$$-\frac{b\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}$$

input
```
Int[1/(x^(5/2)*(a + b*x)^(1/4)),x]
```

output
```
(-2*(a + b*x)^(3/4))/(3*a*x^(3/2)) - (b*((-2*(a + b*x)^(3/4))/(a*Sqrt[x])
+ (2*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1
/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*Ell
ipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/
a))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{x^{\frac{5}{2}} (bx+a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(5/2)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(5/2)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{5/2} \sqrt[4]{a+bx}} \, dx = \int \frac{1}{(bx+a)^{\frac{1}{4}} x^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b*x^4 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx = -\frac{2{}_2F_1\left(\begin{matrix} -\frac{3}{2},\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{3\sqrt[4]{a}x^{\frac{3}{2}}}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**(1/4),x)
```

output
```
-2*hyper((-3/2, 1/4), (-1/2,), b*x*exp_polar(I*pi)/a)/(3*a**(1/4)*x**(3/2)
)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx=\int \frac{1}{x^{5/2}\,{\left(a+b\,x\right)}^{1/4}}\,dx$$

input
```
int(1/(x^(5/2)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(5/2)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}\,dx=\frac{-4\sqrt{x}\,(bx+a)^{\frac{1}{4}}-\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^5+2ab\,x^4+a^2x^3}dx\right)a\,x^2}{5\sqrt{bx+a}\,x^2}$$

input
```
int(1/x^(5/2)/(b*x+a)^(1/4),x)
```

output
```
( - 4*sqrt(x)*(a + b*x)**(1/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/
4))/(a**2*x**3 + 2*a*b*x**4 + b**2*x**5),x)*a*x**2)/(5*sqrt(a + b*x)*x**2)
```

3.661 $\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = -\frac{2}{5x^{5/2}\sqrt[4]{a+bx}}+\frac{b}{15ax^{3/2}\sqrt[4]{a+bx}} -\frac{7b^2}{30a^2\sqrt{x}\sqrt[4]{a+bx}}-\frac{7b^{5/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{10a^{5/2}\sqrt[4]{a+bx}}$$

output
```
-2/5/x^(5/2)/(b*x+a)^(1/4)+1/15*b/a/x^(3/2)/(b*x+a)^(1/4)-7/30*b^2/a^2/x^(
1/2)/(b*x+a)^(1/4)-7/10*b^(5/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan
(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5x^{5/2}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x)^(1/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x)/a)])/(5
*x^(5/2)*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.67, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {61, 61, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{7b\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}dx}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{7b\left(-\frac{b\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{7b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{7b\left(-\frac{b\left(\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$\Big\downarrow$ 836

$$-\frac{7b\left(-\frac{b\left(\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$\Big\downarrow$ 27

$$-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$\Big\downarrow$ 765

$$-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$\Big\downarrow$ 762

$$-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$\Big\downarrow$ 1390

$$-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$\Big\downarrow$ 1389

$$-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

$\Big\downarrow$ 327

$$-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/}}{3ax^{3/2}}\right.}{10a}$$

$$\frac{2(a+bx)^{3/4}}{5ax^{5/2}}$$

input
```
Int[1/(x^(7/2)*(a + b*x)^(1/4)),x]
```

output
```
(-2*(a + b*x)^(3/4))/(5*a*x^(5/2)) - (7*b*((-2*(a + b*x)^(3/4))/(3*a*x^(3/
2)) - (b*((-2*(a + b*x)^(3/4))/(a*Sqrt[x]) + (2*((a^(3/4)*Sqrt[1 - (a + b*
x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b
*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a
^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/a))/(2*a)))/(10*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{x^{\frac{7}{2}} (bx+a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(7/2)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(7/2)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}} x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b*x^5 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = -\frac{2 {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{5\sqrt[4]{a}x^{\frac{5}{2}}}$$

input
```
integrate(1/x**(7/2)/(b*x+a)**(1/4),x)
```

output
```
-2*hyper((-5/2, 1/4), (-3/2,), b*x*exp_polar(I*pi)/a)/(5*a**(1/4)*x**(5/2)
)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{7/2}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(7/2)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(7/2)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}\,dx = \frac{-4\sqrt{x}\,(bx+a)^{\frac{1}{4}}-\sqrt{bx+a}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^6+2ab\,x^5+a^2x^4}dx\right)a\,x^3}{9\sqrt{bx+a}\,x^3}$$

input
```
int(1/x^(7/2)/(b*x+a)^(1/4),x)
```

output
```
( - 4*sqrt(x)*(a + b*x)**(1/4) - sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/
4))/(a**2*x**4 + 2*a*b*x**5 + b**2*x**6),x)*a*x**3)/(9*sqrt(a + b*x)*x**3)
```

3.662 $\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \frac{80a^2\sqrt{x}\sqrt[4]{a+bx}}{77b^3} - \frac{40ax^{3/2}\sqrt[4]{a+bx}}{77b^2} + \frac{4x^{5/2}\sqrt[4]{a+bx}}{11b} - \frac{160a^{7/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{77b^{7/2}(a+bx)^{3/4}}$$

output
```
80/77*a^2*x^(1/2)*(b*x+a)^(1/4)/b^3-40/77*a*x^(3/2)*(b*x+a)^(1/4)/b^2+4/11
*x^(5/2)*(b*x+a)^(1/4)/b-160/77*a^(7/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/
2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(7/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \frac{2x^{7/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7(a+bx)^{3/4}}$$

input
```
Integrate[x^(5/2)/(a + b*x)^(3/4),x]
```

output
```
(2*x^(7/2)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 7/2, 9/2, -((b*x)/a)
])/(7*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 60, 60, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\int \frac{x^{3/2}}{(a+bx)^{3/4}}dx}{11b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}dx}{7b}\right)}{11b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{2a\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{765}$$

$$\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{762}$$

$$\frac{4x^{5/2}\sqrt[4]{a+bx}}{11b}-\frac{10a\left(\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a^{5/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}\right)}{11b}$$

input
```
Int[x^(5/2)/(a + b*x)^(3/4),x]
```

output
```
(4*x^(5/2)*(a + b*x)^(1/4))/(11*b) - (10*a*((4*x^(3/2)*(a + b*x)^(1/4))/(7
*b) - (6*a*((4*Sqrt[x]*(a + b*x)^(1/4))/(3*b) - (8*a^(5/4)*Sqrt[1 - (a + b
*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(3*b^2*Sqrt[-(a/b)
+ (a + b*x)/b])))/(7*b)))/(11*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(5/2)/(b*x+a)^(3/4),x)
```

output
```
int(x^(5/2)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^(5/2)/(b*x + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \frac{2x^{\frac{7}{2}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

input
```
integrate(x**(5/2)/(b*x+a)**(3/4),x)
```

output
```
2*x**(7/2)*hyper((3/4, 7/2), (9/2,), b*x*exp_polar(I*pi)/a)/(7*a**(3/4))
```

Maxima [F]

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(b*x + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{5/2}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(5/2)/(a + b*x)^(3/4),x)
```

output
```
int(x^(5/2)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{5/2}}{(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}\,x^2}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(5/2)/(b*x+a)^(3/4),x)
```

output
```
int((sqrt(x)*x**2)/(a + b*x)**(3/4),x)
```

3.663 $\int \frac{x^{3/2}}{(a+bx)^{3/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = -\frac{8a\sqrt{x}\sqrt[4]{a+bx}}{7b^2}+\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b} + \frac{16a^{5/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{7b^{5/2}(a+bx)^{3/4}}$$

output
```
-8/7*a*x^(1/2)*(b*x+a)^(1/4)/b^2+4/7*x^(3/2)*(b*x+a)^(1/4)/b+16/7*a^(5/2)*
(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2
))/b^(5/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = \frac{2x^{5/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5(a+bx)^{3/4}}$$

input
```
Integrate[x^(3/2)/(a + b*x)^(3/4),x]
```

output
```
(2*x^(5/2)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 5/2, 7/2, -((b*x)/a)
])/(5*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {60, 60, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}dx}{7b}$$

$$\Big\downarrow \; 60$$

$$\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{2a\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \; 73$$

$$\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2}\right)}{7b}$$

$$\Big\downarrow \; 765$$

$$\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}$$

$$\Big\downarrow \; 762$$

$$\frac{4x^{3/2}\sqrt[4]{a+bx}}{7b}-\frac{6a\left(\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a^{5/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{7b}$$

input
```
Int[x^(3/2)/(a + b*x)^(3/4),x]
```

output
```
(4*x^(3/2)*(a + b*x)^(1/4))/(7*b) - (6*a*((4*Sqrt[x]*(a + b*x)^(1/4))/(3*b
) - (8*a^(5/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1
/4)], -1])/(3*b^2*Sqrt[-(a/b) + (a + b*x)/b])))/(7*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(3/2)/(b*x+a)^(3/4),x)
```

output
```
int(x^(3/2)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^(3/2)/(b*x + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = \frac{2x^{\frac{5}{2}}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

input
```
integrate(x**(3/2)/(b*x+a)**(3/4),x)
```

output
```
2*x**(5/2)*hyper((3/4, 5/2), (7/2,), b*x*exp_polar(I*pi)/a)/(5*a**(3/4))
```

Maxima [F]

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(b*x + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{3/2}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(3/2)/(a + b*x)^(3/4),x)
```

output
```
int(x^(3/2)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{3/2}}{(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}\,x}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(3/2)/(b*x+a)^(3/4),x)
```

output
```
int((sqrt(x)*x)/(a + b*x)**(3/4),x)
```

3.664 $\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx=\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx)^{3/4}}$$

output
```
4/3*x^(1/2)*(b*x+a)^(1/4)/b-8/3*a^(3/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/
2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(3/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx=\frac{2x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3(a+bx)^{3/4}}$$

input
```
Integrate[Sqrt[x]/(a + b*x)^(3/4),x]
```

output
```
(2*x^(3/2)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 3/2, 5/2, -((b*x)/a)
])/(3*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {60, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}} \, dx$$

$$\Big\downarrow \ 60$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{2a\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b}$$

$$\Big\downarrow \ 73$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2}$$

$$\Big\downarrow \ 765$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$

$$\Big\downarrow \ 762$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}}{3b}-\frac{8a^{5/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$

input
```
Int[Sqrt[x]/(a + b*x)^(3/4),x]
```

output
```
(4*Sqrt[x]*(a + b*x)^(1/4))/(3*b) - (8*a^(5/4)*Sqrt[1 - (a + b*x)/a]*Ellip
ticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(3*b^2*Sqrt[-(a/b) + (a + b*x)/
b])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(1/2)/(b*x+a)^(3/4),x)
```

output
```
int(x^(1/2)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(sqrt(x)/(b*x + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx = \frac{2x^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bxe^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

input
```
integrate(x**(1/2)/(b*x+a)**(3/4),x)
```

output
```
2*x**(3/2)*hyper((3/4, 3/2), (5/2,), b*x*exp_polar(I*pi)/a)/(3*a**(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(b*x + a)^(3/4), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(1/2)/(a + b*x)^(3/4),x)
```

output
```
int(x^(1/2)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(1/2)/(b*x+a)^(3/4),x)
```

output
```
int(sqrt(x)/(a + b*x)**(3/4),x)
```

3.665 $\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx=\frac{4\sqrt{a}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}$$

output
```
4*a^(1/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/
2)),2^(1/2))/b^(1/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx=\frac{2\sqrt{x}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx}{a}\right)}{(a+bx)^{3/4}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)^(3/4)),x]
```

output
```
(2*Sqrt[x]*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x)/a)
])/(a + b*x)^(3/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx$$

$$\Big\downarrow \; 73$$

$$\frac{4\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{b}$$

$$\Big\downarrow \; 765$$

$$\frac{4\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{b\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$

$$\Big\downarrow \; 762$$

$$\frac{4\sqrt[4]{a}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{b\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)^(3/4)),x]
```

output
```
(4*a^(1/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)]
, -1])/(b*Sqrt[-(a/b) + (a + b*x)/b])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(1/2)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(1/2)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*sqrt(x)/(b*x^2 + a*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx = \frac{2\sqrt{x}\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**(3/4),x)
```

output
```
2*sqrt(x)*hyper((1/2, 3/4), (3/2,), b*x*exp_polar(I*pi)/a)/a**(3/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx = \int \frac{1}{\sqrt{x}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/(x^(1/2)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(1/2)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,ax+\sqrt{bx+a}\,b\,x^2}dx$$

input
```
int(1/x^(1/2)/(b*x+a)^(3/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x + sqrt(a + b*x)*b*x**2),
x)
```

3.666 $\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}} - \frac{2\sqrt{b}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a+bx)^{3/4}}$$

output
```
-2*(b*x+a)^(1/4)/a/x^(1/2)-2*b^(1/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*a
rctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^(1/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = -\frac{2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt{x}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)^(3/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x)/a)])/(Sq
rt[x]*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}$$

$$\Big\downarrow \text{765}$$

$$-\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}$$

$$\Big\downarrow \text{762}$$

$$-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(a + b*x)^(3/4)),x]
```

output
```
(-2*(a + b*x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcS
in[(a + b*x)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{x^{\frac{3}{2}} (bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(3/2)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(3/2)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*sqrt(x)/(b*x^3 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\sqrt{x}}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**(3/4),x)
```

output
```
-2*hyper((-1/2, 3/4), (1/2,), b*x*exp_polar(I*pi)/a)/(a**(3/4)*sqrt(x))
```

Maxima [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{3/2}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/(x^(3/2)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(3/2)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a\,x^2+\sqrt{bx+a}\,b\,x^3}dx$$

input
```
int(1/x^(3/2)/(b*x+a)^(3/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x**2 + sqrt(a + b*x)*b*x**
3),x)
```

3.667 $\int \frac{1}{x^{5/2}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}+\frac{5b\sqrt[4]{a+bx}}{3a^2\sqrt{x}} + \frac{5b^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}$$

output
```
-2/3*(b*x+a)^(1/4)/a/x^(3/2)+5/3*b*(b*x+a)^(1/4)/a^2/x^(1/2)+5/3*b^(3/2)*(
1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2)
)/a^(3/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}}\,dx = -\frac{2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3x^{3/2}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)^(3/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x)/a)])/(3
*x^(3/2)*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {61, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{5b\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{5b\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}$$

$$\Big\downarrow \text{765}$$

$$-\frac{5b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$-\frac{5b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}$$

input
```
Int[1/(x^(5/2)*(a + b*x)^(3/4)),x]
```

output
```
(-2*(a + b*x)^(1/4))/(3*a*x^(3/2)) - (5*b*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]
) - (2*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1
])/(a^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/(6*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{x^{\frac{5}{2}} (bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(5/2)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(5/2)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}} dx = \int \frac{1}{(bx+a)^{\frac{3}{4}} x^{\frac{5}{2}}} dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*sqrt(x)/(b*x^4 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}} dx = -\frac{2 {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{3a^{\frac{3}{4}} x^{\frac{3}{2}}}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**(3/4),x)
```

output
```
-2*hyper((-3/2, 3/4), (-1/2,), b*x*exp_polar(I*pi)/a)/(3*a**(3/4)*x**(3/2)
)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{5/2}\,{\left(a+b\,x\right)}^{3/4}}\,dx$$

input
```
int(1/(x^(5/2)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(5/2)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{3/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a\,x^3+\sqrt{bx+a}\,b\,x^4}dx$$

input
```
int(1/x^(5/2)/(b*x+a)^(3/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x**3 + sqrt(a + b*x)*b*x**
4),x)
```

3.668 $\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx}}{5ax^{5/2}}+\frac{3b\sqrt[4]{a+bx}}{5a^2x^{3/2}}-\frac{3b^2\sqrt[4]{a+bx}}{2a^3\sqrt{x}} - \frac{3b^{5/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{2a^{5/2}(a+bx)^{3/4}}$$

output
```
-2/5*(b*x+a)^(1/4)/a/x^(5/2)+3/5*b*(b*x+a)^(1/4)/a^2/x^(3/2)-3/2*b^2*(b*x+
a)^(1/4)/a^3/x^(1/2)-3/2*b^(5/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arcta
n(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^(5/2)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx = -\frac{2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5x^{5/2}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x)^(3/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x)/a)])/(5
*x^(5/2)*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {61, 61, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{9b\int \frac{1}{x^{5/2}(a+bx)^{3/4}}dx}{10a}-\frac{2\sqrt[4]{a+bx}}{5ax^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{9b\left(-\frac{5b\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)}{10a}-\frac{2\sqrt[4]{a+bx}}{5ax^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{9b\left(-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)}{10a}-\frac{2\sqrt[4]{a+bx}}{5ax^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{9b\left(-\frac{5b\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)}{10a}-\frac{2\sqrt[4]{a+bx}}{5ax^{5/2}}$$

$$\Big\downarrow \text{765}$$

$$-\frac{9b\left(-\frac{5b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)}{10a}-\frac{2\sqrt[4]{a+bx}}{5ax^{5/2}}$$

$$\Big\downarrow 762$$

$$-\frac{9b\left(-\frac{5b\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a}-\frac{2\sqrt[4]{a+bx}}{3ax^{3/2}}\right)}{10a}-\frac{2\sqrt[4]{a+bx}}{5ax^{5/2}}$$

input
```
Int[1/(x^(7/2)*(a + b*x)^(3/4)),x]
```

output
```
(-2*(a + b*x)^(1/4))/(5*a*x^(5/2)) - (9*b*((-2*(a + b*x)^(1/4))/(3*a*x^(3/
2)) - (5*b*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (a + b*x)/a]*El
lipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[-(a/b) + (a +
b*x)/b])))/(6*a)))/(10*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{x^{\frac{7}{2}} \left(bx+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(7/2)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(7/2)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx=\int \frac{1}{\left(bx+a\right)^{\frac{3}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*sqrt(x)/(b*x^5 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle|\, \frac{bxe^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^{\frac{5}{2}}}$$

input
```
integrate(1/x**(7/2)/(b*x+a)**(3/4),x)
```

output
```
-2*hyper((-5/2, 3/4), (-3/2,), b*x*exp_polar(I*pi)/a)/(5*a**(3/4)*x**(5/2)
)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx=\int \frac{1}{x^{7/2}\,{\left(a+b\,x\right)}^{3/4}} \,dx$$

input
```
int(1/(x^(7/2)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(7/2)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{3/4}}\,dx=\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a\,x^{4}+\sqrt{bx+a}\,b\,x^{5}}dx$$

input
```
int(1/x^(7/2)/(b*x+a)^(3/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x**4 + sqrt(a + b*x)*b*x**
5),x)
```

3.669 $\int \frac{x^{7/2}}{(a+bx)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 151

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = -\frac{224a^3\sqrt{x}}{39b^4\sqrt[4]{a+bx}} + \frac{112a^2x^{3/2}}{117b^3\sqrt[4]{a+bx}} - \frac{56ax^{5/2}}{117b^2\sqrt[4]{a+bx}} + \frac{4x^{7/2}}{13b\sqrt[4]{a+bx}} + \frac{448a^{7/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{39b^{9/2}\sqrt[4]{a+bx}}$$

output
```
-224/39*a^3*x^(1/2)/b^4/(b*x+a)^(1/4)+112/117*a^2*x^(3/2)/b^3/(b*x+a)^(1/4
)-56/117*a*x^(5/2)/b^2/(b*x+a)^(1/4)+4/13*x^(7/2)/b/(b*x+a)^(1/4)+448/39*a
^(7/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))
),2^(1/2))/b^(9/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.33

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = \frac{2x^{9/2}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{9}{2},\frac{11}{2},-\frac{bx}{a}\right)}{9a\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(7/2)/(a + b*x)^(5/4),x]
```

output
```
(2*x^(9/2)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 9/2, 11/2, -((b*x)/a
)])/(9*a*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.57, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {57, 60, 60, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{14\int \frac{x^{5/2}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}dx}{13b}\right)}{b}-\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx}{3b}\right)}{13b}\right)}{b}-\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)}{3b}\right)}{13b}\right)}{b}-\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 73

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)}{3b}\right)}{13b}\right)}{b}-\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 836

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)}{13b}\right)}{b}$$

$$\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 27

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)}{13b}\right)}{b}-$$

$$\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 765

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{13b}\right)}{b}$$

$$\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 762$$

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\,d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\cdots\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\cdots\right.}{5b^2}\cdots\right.}{3b}\cdots\right.}{13b}\cdots\right.}{b}\cdots$$

$$\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$$\downarrow \; 1390$$

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\cdots\right.}{5b^2}\cdots\right.}{3b}\cdots\right.}{13b}\cdots\right.}{b}$$

$$\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt{\frac{a+bx}{b}-}}\right.}{5b^2}\right.}{3b}\right.}{13b}\right.}{b}$$

$$\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 327

$$\frac{14\left(\frac{4x^{5/2}(a+bx)^{3/4}}{13b}-\frac{10a\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\cdots\right.}{\sqrt{\frac{a+\cdots}{b}\cdots}}\right.}{5b^2}\cdots\right.}{3b}\cdots\right.}{13b}\cdots\right.}{b}\cdots$$

$$\frac{4x^{7/2}}{b\sqrt[4]{a+bx}}$$

input

```
Int[x^(7/2)/(a + b*x)^(5/4),x]
```

output

```
(-4*x^(7/2))/(b*(a + b*x)^(1/4)) + (14*((4*x^(5/2)*(a + b*x)^(3/4))/(13*b)
 - (10*a*((4*x^(3/2)*(a + b*x)^(3/4))/(9*b) - (2*a*((4*Sqrt[x]*(a + b*x)^(
3/4))/(5*b) - (8*a*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b
*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a
 + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (
a + b*x)/b]))/(5*b^2)))/(3*b)))/(13*b)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{x^{\frac{7}{2}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(7/2)/(b*x+a)^(5/4),x)
```

output
```
int(x^(7/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{7}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(7/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(7/2)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.19

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = \frac{2x^{\frac{9}{2}}\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{9a^{\frac{5}{4}}}$$

input
```
integrate(x**(7/2)/(b*x+a)**(5/4),x)
```

output
```
2*x**(9/2)*hyper((5/4, 9/2), (11/2,), b*x*exp_polar(I*pi)/a)/(9*a**(5/4))
```

Maxima [F]

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{7}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(7/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/(b*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{7}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(7/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^(7/2)/(b*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{7/2}}{(a+b\,x)^{5/4}}\,dx$$

input
```
int(x^(7/2)/(a + b*x)^(5/4),x)
```

output
```
int(x^(7/2)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{7/2}}{(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}\,x^3}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(7/2)/(b*x+a)^(5/4),x)
```

output
```
int((sqrt(x)*x**3)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.670 $\int \frac{x^{5/2}}{(a+bx)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}} dx = \frac{16a^2\sqrt{x}}{3b^3\sqrt[4]{a+bx}} - \frac{8ax^{3/2}}{9b^2\sqrt[4]{a+bx}} + \frac{4x^{5/2}}{9b\sqrt[4]{a+bx}} - \frac{32a^{5/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx}}$$

output
```
16/3*a^2*x^(1/2)/b^3/(b*x+a)^(1/4)-8/9*a*x^(3/2)/b^2/(b*x+a)^(1/4)+4/9*x^(
5/2)/b/(b*x+a)^(1/4)-32/3*a^(5/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arct
an(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.39

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}} dx = \frac{2x^{7/2}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7a\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(5/2)/(a + b*x)^(5/4),x]
```

output
```
(2*x^(7/2)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 7/2, 9/2, -((b*x)/a)
])/(7*a*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.63, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {57, 60, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \; 57$$

$$\frac{10\int \frac{x^{3/2}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 60$$

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx}{3b}\right)}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 60$$

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)}{3b}\right)}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 73$$

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)}{3b}\right)}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 836

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 27

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{3b}\right)}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 765

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 762

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{b}-\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 1390

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{b}$$

$$\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 1389

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{3b}\right)}{b}$$

$$\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 327

$$\frac{10\left(\frac{4x^{3/2}(a+bx)^{3/4}}{9b}-\frac{2a\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2}\right.}{3b}\right.}{b}$$

$$\frac{4x^{5/2}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(5/2)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(5/2))/(b*(a + b*x)^(1/4)) + (10*((4*x^(3/2)*(a + b*x)^(3/4))/(9*b)
- (2*a*((4*Sqrt[x]*(a + b*x)^(3/4))/(5*b) - (8*a*((a^(3/4)*Sqrt[1 - (a + b
*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a +
b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/
a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2)))/(3*b)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(5/2)/(b*x+a)^(5/4),x)
```

output
```
int(x^(5/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(5/2)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}}\,dx = \frac{2x^{\frac{7}{2}}{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

input
```
integrate(x**(5/2)/(b*x+a)**(5/4),x)
```

output
```
2*x**(7/2)*hyper((5/4, 7/2), (9/2,), b*x*exp_polar(I*pi)/a)/(7*a**(5/4))
```

Maxima [F]

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}}\,dx=\int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(b*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}}\,dx=\int \frac{x^{\frac{5}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(5/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/(b*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{5/2}}{(a+b\,x)^{5/4}}\,dx$$

input
```
int(x^(5/2)/(a + b*x)^(5/4),x)
```

output
```
int(x^(5/2)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{5/2}}{(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}\,x^2}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(5/2)/(b*x+a)^(5/4),x)
```

output
```
int((sqrt(x)*x**2)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.671 $\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx=-\frac{24a\sqrt{x}}{5b^2\sqrt[4]{a+bx}}+\frac{4x^{3/2}}{5b\sqrt[4]{a+bx}}+\frac{48a^{3/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx}}$$

output
```
-24/5*a*x^(1/2)/b^2/(b*x+a)^(1/4)+4/5*x^(3/2)/b/(b*x+a)^(1/4)+48/5*a^(3/2)
*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1
/2))/b^(5/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx=\frac{2x^{5/2}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5a\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(3/2)/(a + b*x)^(5/4),x]
```

output
```
(2*x^(5/2)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 5/2, 7/2, -((b*x)/a)
])/(5*a*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.72, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {57, 60, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx$$

$$\Big\downarrow \ 57$$

$$\frac{6\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \ 60$$

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \ 73$$

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \ 836$$

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \ 27$$

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 765

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 762

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 1390

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 1389

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \ 327$$

$$\frac{6\left(\frac{4\sqrt{x}(a+bx)^{3/4}}{5b}-\frac{8a\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{5b^2}\right)}{b}-\frac{4x^{3/2}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(3/2)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(3/2))/(b*(a + b*x)^(1/4)) + (6*((4*Sqrt[x]*(a + b*x)^(3/4))/(5*b) -
 (8*a*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(
1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*El
lipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))
/(5*b^2)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(3/2)/(b*x+a)^(5/4),x)
```

output
```
int(x^(3/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/2)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx = \frac{2x^{\frac{5}{2}}\,{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

input
```
integrate(x**(3/2)/(b*x+a)**(5/4),x)
```

output
```
2*x**(5/2)*hyper((5/4, 5/2), (7/2,), b*x*exp_polar(I*pi)/a)/(5*a**(5/4))
```

Maxima [F]

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(b*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(3/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(b*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{3/2}}{(a+b\,x)^{5/4}}\,dx$$

input
```
int(x^(3/2)/(a + b*x)^(5/4),x)
```

output
```
int(x^(3/2)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{3/2}}{(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}\,x}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(3/2)/(b*x+a)^(5/4),x)
```

output
```
int((sqrt(x)*x)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.672 $\int \frac{\sqrt{x}}{(a+bx)^{5/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}} \, dx = \frac{4\sqrt{x}}{b\sqrt[4]{a+bx}} - \frac{8\sqrt{a}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx}}$$

output
```
4*x^(1/2)/b/(b*x+a)^(1/4)-8*a^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*ar
ctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}} \, dx = \frac{2x^{3/2}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3a\sqrt[4]{a+bx}}$$

input
```
Integrate[Sqrt[x]/(a + b*x)^(5/4),x]
```

output
```
(2*x^(3/2)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 3/2, 5/2, -((b*x)/a)
])/(3*a*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {57, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \; 57$$

$$\frac{2\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{b}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 73$$

$$\frac{8\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 836$$

$$\frac{8\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{8\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 765$$

$$\frac{8\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 762$$

$$\frac{8\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 1390$$

$$\frac{8\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 1389$$

$$\frac{8\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 327$$

$$\frac{8\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b^2}-\frac{4\sqrt{x}}{b\sqrt[4]{a+bx}}$$

input
```
Int[Sqrt[x]/(a + b*x)^(5/4),x]
```

output
```
(-4*Sqrt[x])/(b*(a + b*x)^(1/4)) + (8*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*Elli
pticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (
a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -
1])/Sqrt[-(a/b) + (a + b*x)/b]))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{\sqrt{x}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(1/2)/(b*x+a)^(5/4),x)
```

output
```
int(x^(1/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}}\, dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{5}{4}}}\, dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}}\,dx = \frac{2x^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

input
```
integrate(x**(1/2)/(b*x+a)**(5/4),x)
```

output
```
2*x**(3/2)*hyper((5/4, 3/2), (5/2,), b*x*exp_polar(I*pi)/a)/(3*a**(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(b*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(1/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(b*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}}{(a+b\,x)^{5/4}}\,dx$$

input
```
int(x^(1/2)/(a + b*x)^(5/4),x)
```

output
```
int(x^(1/2)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(1/2)/(b*x+a)^(5/4),x)
```

output
```
int(sqrt(x)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.673 $\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \frac{4\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx}}$$

output
```
4*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(
1/2))/a^(1/2)/b^(1/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \frac{2\sqrt{x}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{bx}{a}\right)}{a\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x)^(5/4)),x]
```

output
```
(2*Sqrt[x]*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/2, 5/4, 3/2, -((b*x)/a)
])/(a*(a + b*x)^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(57) = 114.

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{ab}$$

$$\Big\downarrow \text{836}$$

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{ab}$$

$$\Big\downarrow \text{765}$$

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab}$$

$\Big\downarrow$ 762

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab}$$

$\Big\downarrow$ 1390

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab}$$

$\Big\downarrow$ 1389

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab}$$

$\Big\downarrow$ 327

$$\frac{4\sqrt{x}}{a\sqrt[4]{a+bx}}-\frac{4\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{ab}$$

input
```
Int[1/(Sqrt[x]*(a + b*x)^(5/4)),x]
```

output
```
(4*Sqrt[x])/(a*(a + b*x)^(1/4)) - (4*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*Ellip
ticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a
^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1
])/Sqrt[-(a/b) + (a + b*x)/b]))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{x}\,(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(1/x^(1/2)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(1/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b^2*x^3 + 2*a*b*x^2 + a^2*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \frac{2\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

input
```
integrate(1/x**(1/2)/(b*x+a)**(5/4),x)
```

output
```
2*sqrt(x)*hyper((1/2, 5/4), (3/2,), b*x*exp_polar(I*pi)/a)/a**(5/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{\sqrt{x}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/(x^(1/2)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(1/2)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,ax+\sqrt{bx+a}\,b\,x^2}dx$$

input
```
int(1/x^(1/2)/(b*x+a)^(5/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x + sqrt(a + b*x)*b*x**2),
x)
```

3.674 $\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx = -\frac{2}{a\sqrt{x}\sqrt[4]{a+bx}} - \frac{6\sqrt{b}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx}}$$

output
```
-2/a/x^(1/2)/(b*x+a)^(1/4)-6*b^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*a
rctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{4},\frac{1}{2},-\frac{bx}{a}\right)}{a\sqrt{x}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x)^(5/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x)/a)])/(a*
Sqrt[x]*(a + b*x)^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {61, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{3\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{3\left(\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{3\left(\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 836}$$

$$\frac{3\left(\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \textbf{765}$$

$$\frac{3\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \textbf{762}$$

$$\frac{3\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \textbf{1390}$$

$$\frac{3\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \textbf{1389}$$

$$\frac{3\left(\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 327$$

$$\frac{3\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{a}+\frac{4}{a\sqrt{x}\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(3/2)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*Sqrt[x]*(a + b*x)^(1/4)) + (3*((-2*(a + b*x)^(3/4))/(a*Sqrt[x]) + (2*
((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)],
 -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*Elliptic
F[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/a))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{x^{\frac{3}{2}} (bx+a)^{\frac{5}{4}}} dx$$

input
```
int(1/x^(3/2)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(3/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}} \, dx = \int \frac{1}{(bx+a)^{\frac{5}{4}} x^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\sqrt{x}}$$

input
```
integrate(1/x**(3/2)/(b*x+a)**(5/4),x)
```

output
```
-2*hyper((-1/2, 5/4), (1/2,), b*x*exp_polar(I*pi)/a)/(a**(5/4)*sqrt(x))
```

Maxima [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{3/2}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/(x^(3/2)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(3/2)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/2}(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a\,x^2+\sqrt{bx+a}\,b\,x^3}dx$$

input
```
int(1/x^(3/2)/(b*x+a)^(5/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x**2 + sqrt(a + b*x)*b*x**
3),x)
```

3.675 $\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx = -\frac{2}{3ax^{3/2}\sqrt[4]{a+bx}}+\frac{7b}{3a^2\sqrt{x}\sqrt[4]{a+bx}} + \frac{7b^{3/2}\sqrt[4]{\frac{a+bx}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{a^{5/2}\sqrt[4]{a+bx}}$$

output
```
-2/3/a/x^(3/2)/(b*x+a)^(1/4)+7/3*b/a^2/x^(1/2)/(b*x+a)^(1/4)+7*b^(3/2)*((b
*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))
/a^(5/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3ax^{3/2}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x)^(5/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x)/a)])/(3
*a*x^(3/2)*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {61, 61, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx$$

$$\Big\downarrow \text{61}$$

$$\frac{7\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(-\frac{b\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{7\left(-\frac{b\left(\frac{2\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 836$$

$$\frac{7\left(-\frac{b\left(\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 765$$

$$\frac{7\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{\frac{a}{4}}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 762

$$\frac{7\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{\frac{a}{4}}+\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 1390

$$\frac{7\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}+$$

$$\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 1389

$$\frac{7\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}+$$

$$\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 327

$$\frac{7\left(-\frac{b\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{a}$$

$$\frac{4}{ax^{3/2}\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(5/2)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*x^(3/2)*(a + b*x)^(1/4)) + (7*((-2*(a + b*x)^(3/4))/(3*a*x^(3/2)) - (
b*((-2*(a + b*x)^(3/4))/(a*Sqrt[x]) + (2*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*E
llipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]
- (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)]
, -1])/Sqrt[-(a/b) + (a + b*x)/b]))/a))/(2*a)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{x^{\frac{5}{2}} (bx+a)^{\frac{5}{4}}} dx$$

input
```
int(1/x^(5/2)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(5/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}} dx = \int \frac{1}{(bx+a)^{\frac{5}{4}} x^{\frac{5}{2}}} dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b^2*x^5 + 2*a*b*x^4 + a^2*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}} dx = -\frac{2 {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix} \middle| \frac{bxe^{i\pi}}{a}\right)}{3a^{\frac{5}{4}} x^{\frac{3}{2}}}$$

input
```
integrate(1/x**(5/2)/(b*x+a)**(5/4),x)
```

output
```
-2*hyper((-3/2, 5/4), (-1/2,), b*x*exp_polar(I*pi)/a)/(3*a**(5/4)*x**(3/2)
)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{5/2}\,{\left(a+b\,x\right)}^{5/4}}\,dx$$

input
```
int(1/(x^(5/2)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(5/2)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/2}(a+bx)^{5/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a\,x^3+\sqrt{bx+a}\,b\,x^4}dx$$

input
```
int(1/x^(5/2)/(b*x+a)^(5/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x**3 + sqrt(a + b*x)*b*x**
4),x)
```

3.676 $\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx = -\frac{2}{5ax^{5/2}\sqrt[4]{a+bx}}+\frac{11b}{15a^2x^{3/2}\sqrt[4]{a+bx}} -\frac{77b^2}{30a^3\sqrt{x}\sqrt[4]{a+bx}}-\frac{77b^{5/2}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{10a^{7/2}\sqrt[4]{a+bx}}$$

output
```
-2/5/a/x^(5/2)/(b*x+a)^(1/4)+11/15*b/a^2/x^(3/2)/(b*x+a)^(1/4)-77/30*b^2/a
^3/x^(1/2)/(b*x+a)^(1/4)-77/10*b^(5/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2
*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5ax^{5/2}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x)^(5/4)),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*x)/a)])/(5
*a*x^(5/2)*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.83, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {61, 61, 61, 61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{11\int \frac{1}{x^{7/2}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{11\left(-\frac{7b\int \frac{1}{x^{5/2}\sqrt[4]{a+bx}}dx}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)}{a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\int \frac{1}{x^{3/2}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)}{a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{b\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)}{a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 73

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)}{a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 836

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)}{4a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 27

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)}{4a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 765

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/4}}{5ax^{5/2}}\right)}{a}+\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$\downarrow$ 762

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\frac{2(a+bx)^{3/}}{5ax^{5/2}}}{a}$$

$$\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$\downarrow$ 1390

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}-\right.}{a}$$

$$\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$\downarrow$ 1389

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(2\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/4}}{3ax^{3/2}}\right)}{10a}}{a}$$

$$\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

$$\downarrow \text{327}$$

$$\frac{11\left(-\frac{7b\left(-\frac{b\left(\frac{2\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a}-\frac{2(a+bx)^{3/4}}{a\sqrt{x}}\right)}{2a}-\frac{2(a+bx)^{3/}}{3ax^{3/2}}\right)}{10a}\right)}{a}$$

$$\frac{4}{ax^{5/2}\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(7/2)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*x^(5/2)*(a + b*x)^(1/4)) + (11*((-2*(a + b*x)^(3/4))/(5*a*x^(5/2)) -
(7*b*((-2*(a + b*x)^(3/4))/(3*a*x^(3/2)) - (b*((-2*(a + b*x)^(3/4))/(a*Sqr
t[x]) + (2*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4
)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/
a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)
/b]))/a))/(2*a)))/(10*a)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{x^{\frac{7}{2}} (bx+a)^{\frac{5}{4}}}dx$$

input
```
int(1/x^(7/2)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(7/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx=\int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(x)/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}x^{\frac{5}{2}}}$$

input
```
integrate(1/x**(7/2)/(b*x+a)**(5/4),x)
```

output
```
-2*hyper((-5/2, 5/4), (-3/2,), b*x*exp_polar(I*pi)/a)/(5*a**(5/4)*x**(5/2)
)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx=\int \frac{1}{x^{7/2}\,{\left(a+b\,x\right)}^{5/4}} \,d x$$

input
```
int(1/(x^(7/2)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(7/2)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{7/2}(a+bx)^{5/4}}\,dx=\int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a\,x^{4}+\sqrt{bx+a}\,b\,x^{5}}dx$$

input
```
int(1/x^(7/2)/(b*x+a)^(5/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x**4 + sqrt(a + b*x)*b*x**
5),x)
```

3.677 $\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \frac{4\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt{d}\sqrt[4]{a+bx}}$$

output
```
4*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*(d*x)^(1/2)/a^(1/2)/d
^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/d^(1/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \frac{2x\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{bx}{a}\right)}{a\sqrt{dx}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*x)^(5/4)),x]
```

output
```
(2*x*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/2, 5/4, 3/2, -((b*x)/a)])/(a*
Sqrt[d*x]*(a + b*x)^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 159 vs. 2(69) = 138.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{\int \frac{1}{\sqrt{dx}\sqrt[4]{a+bx}}dx}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\int \frac{\sqrt{a+bx}}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}d\sqrt[4]{a+bx}}{ab}$$

$$\Big\downarrow \text{ 836}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}d\sqrt[4]{a+bx}\right)}{ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}d\sqrt[4]{a+bx}\right)}{ab}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}\right)}{ab}$$

$$\Big\downarrow \text{762}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}\right)}{ab}$$

$$\Big\downarrow \text{1390}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}\right)}{ab}$$

$$\Big\downarrow \text{1389}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}\right)}{ab}$$

$$\Big\downarrow \text{327}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a+bx}}-\frac{4\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{d(a+bx)}{b}-\frac{ad}{b}}}\right)}{ab}$$

input
```
Int[1/(Sqrt[d*x]*(a + b*x)^(5/4)),x]
```

output
```
(4*Sqrt[d*x])/(a*d*(a + b*x)^(1/4)) - (4*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*E
llipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-((a*d)/b) + (d*(a + b
*x))/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/
a^(1/4)], -1])/Sqrt[-((a*d)/b) + (d*(a + b*x))/b]))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{xd}\,(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(1/(x*d)^(1/2)/(b*x+a)^(5/4),x)
```

output
```
int(1/(x*d)^(1/2)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*sqrt(d*x)/(b^2*d*x^3 + 2*a*b*d*x^2 + a^2*d*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \frac{2\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\sqrt{d}}$$

input
```
integrate(1/(d*x)**(1/2)/(b*x+a)**(5/4),x)
```

output
```
2*sqrt(x)*hyper((1/2, 5/4), (3/2,), b*x*exp_polar(I*pi)/a)/(a**(5/4)*sqrt(
d))
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \int \frac{1}{\sqrt{d\,x}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a + b*x)^(5/4)),x)
```

output
```
int(1/((d*x)^(1/2)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}(a+bx)^{5/4}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(b*x+a)^(5/4),x)
```

output
```
(sqrt(d)*int((sqrt(x)*(a + b*x)**(3/4))/(a**2*x + 2*a*b*x**2 + b**2*x**3),
x))/d
```

3.678 $\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = -\frac{4\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt{d}\sqrt[4]{-a-bx}}$$

output
```
-4*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*(d*x)^(1/2)/a^(1/2)/
d^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/d^(1/2)/(-b*x-a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = -\frac{2x\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{bx}{a}\right)}{a\sqrt{dx}\sqrt[4]{-a-bx}}$$

input
```
Integrate[1/(Sqrt[d*x]*(-a - b*x)^(5/4)),x]
```

output
```
(-2*x*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/2, 5/4, 3/2, -((b*x)/a)])/(a
*Sqrt[d*x]*(-a - b*x)^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 311 vs. 2(72) = 144.

Time = 0.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {61, 73, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}dx$$

$$\Big\downarrow \ 61$$

$$\frac{\int \frac{1}{\sqrt{dx}\sqrt[4]{-a-bx}}dx}{a}-\frac{4\sqrt{dx}}{ad\sqrt[4]{-a-bx}}$$

$$\Big\downarrow \ 73$$

$$-\frac{4\int \frac{\sqrt{-a-bx}}{\sqrt{-\frac{(-a-bx)d}{b}-\frac{ad}{b}}}d\sqrt[4]{-a-bx}}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{-a-bx}}$$

$$\Big\downarrow \ 834$$

$$-\frac{4\left(\sqrt{a}\int \frac{1}{\sqrt{-\frac{(-a-bx)d}{b}-\frac{ad}{b}}}d\sqrt[4]{-a-bx}-\sqrt{a}\int \frac{\sqrt{a}-\sqrt{-a-bx}}{\sqrt{a}\sqrt{-\frac{(-a-bx)d}{b}-\frac{ad}{b}}}d\sqrt[4]{-a-bx}\right)}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{-a-bx}}$$

$$\Big\downarrow \ 27$$

$$-\frac{4\left(\sqrt{a}\int \frac{1}{\sqrt{-\frac{(-a-bx)d}{b}-\frac{ad}{b}}}d\sqrt[4]{-a-bx}-\int \frac{\sqrt{a}-\sqrt{-a-bx}}{\sqrt{-\frac{(-a-bx)d}{b}-\frac{ad}{b}}}d\sqrt[4]{-a-bx}\right)}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{-a-bx}}$$

$$\Big\downarrow \ 761$$

$$-\frac{4\left(\frac{\sqrt[4]{a}\sqrt{-\frac{bx}{(\sqrt{-a-bx}+\sqrt{a})^2}}(\sqrt{-a-bx}+\sqrt{a})\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-a-bx}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{-\frac{d(-a-bx)}{b}-\frac{ad}{b}}}-\int\frac{\sqrt{a}-\sqrt{-a-bx}}{\sqrt{-\frac{(-a-bx)d}{b}-\frac{ad}{b}}}d\sqrt[4]{-a-bx}\right)}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{-a-bx}}$$

$\downarrow$ 1510

$$-\frac{4\left(\frac{\sqrt[4]{a}\sqrt{-\frac{bx}{(\sqrt{-a-bx}+\sqrt{a})^2}}(\sqrt{-a-bx}+\sqrt{a})\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-a-bx}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{-\frac{d(-a-bx)}{b}-\frac{ad}{b}}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx}{(\sqrt{-a-bx}+\sqrt{a})^2}}(\sqrt{-a-bx}+\sqrt{a})E\left(2\arctan\right.}{\sqrt{-\frac{d(-a-bx)}{b}-\frac{ad}{b}}}\right.}{ab}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{-a-bx}}$$

input
```
Int[1/(Sqrt[d*x]*(-a - b*x)^(5/4)),x]
```

output
```
(-4*Sqrt[d*x])/(a*d*(-a - b*x)^(1/4)) - (4*(-((b*(-a - b*x)^(1/4)*Sqrt[-((
a*d)/b) - (d*(-a - b*x))/b])/(d*(Sqrt[a] + Sqrt[-a - b*x]))) - (a^(1/4)*Sq
rt[-((b*x)/(Sqrt[a] + Sqrt[-a - b*x])^2)]*(Sqrt[a] + Sqrt[-a - b*x])*Ellip
ticE[2*ArcTan[(-a - b*x)^(1/4)/a^(1/4)], 1/2])/Sqrt[-((a*d)/b) - (d*(-a -
b*x))/b] + (a^(1/4)*Sqrt[-((b*x)/(Sqrt[a] + Sqrt[-a - b*x])^2)]*(Sqrt[a] +
 Sqrt[-a - b*x])*EllipticF[2*ArcTan[(-a - b*x)^(1/4)/a^(1/4)], 1/2])/(2*Sq
rt[-((a*d)/b) - (d*(-a - b*x))/b])))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{\sqrt{xd}\,(-bx-a)^{\frac{5}{4}}}dx$$

input
```
int(1/(x*d)^(1/2)/(-b*x-a)^(5/4),x)
```

output
```
int(1/(x*d)^(1/2)/(-b*x-a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = \int \frac{1}{(-bx-a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(-b*x-a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x - a)^(3/4)*sqrt(d*x)/(b^2*d*x^3 + 2*a*b*d*x^2 + a^2*d*x), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = \frac{2\sqrt{x}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\sqrt{d}}$$

input
```
integrate(1/(d*x)**(1/2)/(-b*x-a)**(5/4),x)
```

output
```
2*sqrt(x)*exp(3*I*pi/4)*hyper((1/2, 5/4), (3/2,), b*x*exp_polar(I*pi)/a)/(
a**(5/4)*sqrt(d))
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = \int \frac{1}{(-bx-a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(-b*x-a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x - a)^(5/4)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = \int \frac{1}{(-bx-a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(-b*x-a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x - a)^(5/4)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = \int \frac{1}{\sqrt{d\,x}\,{\left(-a-b\,x\right)}^{5/4}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(- a - b*x)^(5/4)),x)
```

output
```
int(1/((d*x)^(1/2)*(- a - b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}(-a-bx)^{5/4}}\,dx = \frac{\sqrt{d}\,(-1)^{\frac{1}{4}}\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)i}{d}$$

input
```
int(1/(d*x)^(1/2)/(-b*x-a)^(5/4),x)
```

output
```
(sqrt(d)*( - 1)**(1/4)*int((sqrt(x)*(a + b*x)**(3/4))/(a**2*x + 2*a*b*x**2
 + b**2*x**3),x)*i)/d
```

3.679 $\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx=\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}-\frac{4\sqrt[4]{1-\frac{bx}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}\sqrt{dx}}{\sqrt{a}\sqrt{d}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt{d}\sqrt[4]{a-bx}}$$

output
```
4*(d*x)^(1/2)/a/d/(-b*x+a)^(1/4)-4*(1-b*x/a)^(1/4)*EllipticE(sin(1/2*arcsi
n(b^(1/2)*(d*x)^(1/2)/a^(1/2)/d^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/d^(1/2)/(
-b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx=\frac{2x\sqrt[4]{1-\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},\frac{bx}{a}\right)}{a\sqrt{dx}\sqrt[4]{a-bx}}$$

input
```
Integrate[1/(Sqrt[d*x]*(a - b*x)^(5/4)),x]
```

output
```
(2*x*(1 - (b*x)/a)^(1/4)*Hypergeometric2F1[1/2, 5/4, 3/2, (b*x)/a])/(a*Sqr
t[d*x]*(a - b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec), antiderivative size = 166, normalized size of antiderivative = 1.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {61, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}-\frac{\int \frac{1}{\sqrt{dx}\sqrt[4]{a-bx}}dx}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{4\int \frac{\sqrt{a-bx}}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}d\sqrt[4]{a-bx}}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

$$\Big\downarrow \text{ 836}$$

$$\frac{4\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a-bx}}{\sqrt{a}\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}d\sqrt[4]{a-bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}d\sqrt[4]{a-bx}\right)}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a-bx}}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}d\sqrt[4]{a-bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}d\sqrt[4]{a-bx}\right)}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a-bx}}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}d\sqrt[4]{a-bx}-\frac{\sqrt{a}\sqrt{1-\frac{a-bx}{a}}\int \frac{1}{\sqrt{1-\frac{a-bx}{a}}}d\sqrt[4]{a-bx}}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}\right)}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

$$\Big\downarrow \text{762}$$

$$\frac{4\left(\int \frac{\sqrt{a}+\sqrt{a-bx}}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}d\sqrt[4]{a-bx}-\frac{a^{3/4}\sqrt{1-\frac{a-bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a-bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}\right)}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{4\left(\frac{\sqrt{1-\frac{a-bx}{a}}\int \frac{\sqrt{a}+\sqrt{a-bx}}{\sqrt{1-\frac{a-bx}{a}}}d\sqrt[4]{a-bx}}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a-bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a-bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}\right)}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{4\left(\frac{\sqrt{a}\sqrt{1-\frac{a-bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a-bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a-bx}}{\sqrt{a}}}}d\sqrt[4]{a-bx}}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a-bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a-bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}\right)}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

$$\Big\downarrow \text{327}$$

$$\frac{4\left(\frac{a^{3/4}\sqrt{1-\frac{a-bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a-bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a-bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a-bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{ad}{b}-\frac{d(a-bx)}{b}}}\right)}{ab}+\frac{4\sqrt{dx}}{ad\sqrt[4]{a-bx}}$$

input
```
Int[1/(Sqrt[d*x]*(a - b*x)^(5/4)),x]
```

output
```
(4*Sqrt[d*x])/(a*d*(a - b*x)^(1/4)) + (4*((a^(3/4)*Sqrt[1 - (a - b*x)/a]*E
llipticE[ArcSin[(a - b*x)^(1/4)/a^(1/4)], -1])/Sqrt[(a*d)/b - (d*(a - b*x)
)/b] - (a^(3/4)*Sqrt[1 - (a - b*x)/a]*EllipticF[ArcSin[(a - b*x)^(1/4)/a^(
1/4)], -1])/Sqrt[(a*d)/b - (d*(a - b*x))/b]))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{xd}\,(-bx+a)^{\frac{5}{4}}}dx$$

input
```
int(1/(x*d)^(1/2)/(-b*x+a)^(5/4),x)
```

output
```
int(1/(x*d)^(1/2)/(-b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(-b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x + a)^(3/4)*sqrt(d*x)/(b^2*d*x^3 - 2*a*b*d*x^2 + a^2*d*x), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx = \frac{2\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bxe^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}\sqrt{d}}$$

input
```
integrate(1/(d*x)**(1/2)/(-b*x+a)**(5/4),x)
```

output
```
2*sqrt(x)*hyper((1/2, 5/4), (3/2,), b*x*exp_polar(2*I*pi)/a)/(a**(5/4)*sqr
t(d))
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(-b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x + a)^(5/4)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(-b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x + a)^(5/4)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx = \int \frac{1}{\sqrt{d\,x}\,(a-b\,x)^{5/4}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a - b*x)^(5/4)),x)
```

output
```
int(1/((d*x)^(1/2)*(a - b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}(a-bx)^{5/4}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{x}\,(-bx+a)^{\frac{3}{4}}}{b^2x^3-2ab\,x^2+a^2x}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(-b*x+a)^(5/4),x)
```

output
```
(sqrt(d)*int((sqrt(x)*(a - b*x)**(3/4))/(a**2*x - 2*a*b*x**2 + b**2*x**3),
x))/d
```

3.680 $\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 240

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = -\frac{4\sqrt{dx}}{ad\sqrt[4]{-a+bx}}+\frac{4\sqrt{dx}\sqrt[4]{-a+bx}}{ad\left(\sqrt{a}+\sqrt{-a+bx}\right)} - \frac{4\sqrt{\frac{bx}{\left(\sqrt{a}+\sqrt{-a+bx}\right)^2}}\left(\sqrt{a}+\sqrt{-a+bx}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b\sqrt{dx}} + \frac{2\sqrt{\frac{bx}{\left(\sqrt{a}+\sqrt{-a+bx}\right)^2}}\left(\sqrt{a}+\sqrt{-a+bx}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}b\sqrt{dx}}$$

output
```
-4*(d*x)^(1/2)/a/d/(b*x-a)^(1/4)+4*(d*x)^(1/2)*(b*x-a)^(1/4)/a/d/(a^(1/2)+
(b*x-a)^(1/2))-4*(b*x/(a^(1/2)+(b*x-a)^(1/2))^2)^(1/2)*(a^(1/2)+(b*x-a)^(1
/2))*EllipticE(sin(2*arctan((b*x-a)^(1/4)/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b
/(d*x)^(1/2)+2*(b*x/(a^(1/2)+(b*x-a)^(1/2))^2)^(1/2)*(a^(1/2)+(b*x-a)^(1/2
))*InverseJacobiAM(2*arctan((b*x-a)^(1/4)/a^(1/4)),1/2*2^(1/2))/a^(3/4)/b/
(d*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = -\frac{2x\sqrt[4]{1-\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},\frac{bx}{a}\right)}{a\sqrt{dx}\sqrt[4]{-a+bx}}$$

input
```
Integrate[1/(Sqrt[d*x]*(-a + b*x)^(5/4)),x]
```

output
```
(-2*x*(1 - (b*x)/a)^(1/4)*Hypergeometric2F1[1/2, 5/4, 3/2, (b*x)/a])/(a*Sq
rt[d*x]*(-a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {61, 73, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(bx-a)^{5/4}}\,dx$$

$$\Big\downarrow \text{61}$$

$$\frac{\int \frac{1}{\sqrt{dx}\sqrt[4]{bx-a}}dx}{a}-\frac{4\sqrt{dx}}{ad\sqrt[4]{bx-a}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4\int \frac{\sqrt{bx-a}}{\sqrt{\frac{(bx-a)d}{b}+\frac{ad}{b}}}d\sqrt[4]{bx-a}}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{bx-a}}$$

$$\Big\downarrow \text{834}$$

$$\frac{4\left(\sqrt{a}\int \frac{1}{\sqrt{\frac{(bx-a)d}{b}+\frac{ad}{b}}}d\sqrt[4]{bx-a}-\sqrt{a}\int \frac{\sqrt{a}-\sqrt{bx-a}}{\sqrt{a}\sqrt{\frac{(bx-a)d}{b}+\frac{ad}{b}}}d\sqrt[4]{bx-a}\right)}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{bx-a}}$$

$$\Big\downarrow \; 27$$

$$\frac{4\left(\sqrt{a}\int \frac{1}{\sqrt{\frac{(bx-a)d}{b}+\frac{ad}{b}}}d\sqrt[4]{bx-a}-\int \frac{\sqrt{a}-\sqrt{bx-a}}{\sqrt{\frac{(bx-a)d}{b}+\frac{ad}{b}}}d\sqrt[4]{bx-a}\right)}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{bx-a}}$$

$$\Big\downarrow \; 761$$

$$\frac{4\left(\frac{\sqrt[4]{a}\sqrt{\frac{bx}{(\sqrt{bx-a}+\sqrt{a})^2}}(\sqrt{bx-a}+\sqrt{a})\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{bx-a}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{\frac{d(bx-a)}{b}+\frac{ad}{b}}}-\int \frac{\sqrt{a}-\sqrt{bx-a}}{\sqrt{\frac{(bx-a)d}{b}+\frac{ad}{b}}}d\sqrt[4]{bx-a}\right)}{ab}-\frac{4\sqrt{dx}}{ad\sqrt[4]{bx-a}}$$

$$\Big\downarrow \; 1510$$

$$\frac{4\left(\frac{\sqrt[4]{a}\sqrt{\frac{bx}{(\sqrt{bx-a}+\sqrt{a})^2}}(\sqrt{bx-a}+\sqrt{a})\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{bx-a}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{\frac{d(bx-a)}{b}+\frac{ad}{b}}}-\frac{\sqrt[4]{a}\sqrt{\frac{bx}{(\sqrt{bx-a}+\sqrt{a})^2}}(\sqrt{bx-a}+\sqrt{a})E\left(2\arctan\left(\frac{\sqrt[4]{bx-a}}{\sqrt[4]{a}}\right.\right.}{\sqrt{\frac{d(bx-a)}{b}+\frac{ad}{b}}}\right.}{ab}$$

$$\frac{4\sqrt{dx}}{ad\sqrt[4]{bx-a}}$$

input
```
Int[1/(Sqrt[d*x]*(-a + b*x)^(5/4)),x]
```

output
```
(-4*Sqrt[d*x])/(a*d*(-a + b*x)^(1/4)) + (4*((b*(-a + b*x)^(1/4)*Sqrt[(a*d)
/b + (d*(-a + b*x))/b])/(d*(Sqrt[a] + Sqrt[-a + b*x])) - (a^(1/4)*Sqrt[(b*
x)/(Sqrt[a] + Sqrt[-a + b*x])^2]*(Sqrt[a] + Sqrt[-a + b*x])*EllipticE[2*Ar
cTan[(-a + b*x)^(1/4)/a^(1/4)], 1/2])/Sqrt[(a*d)/b + (d*(-a + b*x))/b] + (
a^(1/4)*Sqrt[(b*x)/(Sqrt[a] + Sqrt[-a + b*x])^2]*(Sqrt[a] + Sqrt[-a + b*x]
)*EllipticF[2*ArcTan[(-a + b*x)^(1/4)/a^(1/4)], 1/2])/(2*Sqrt[(a*d)/b + (d
*(-a + b*x))/b])))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [F]

$$\int \frac{1}{\sqrt{xd}\,(bx-a)^{\frac{5}{4}}}dx$$

input
```
int(1/(x*d)^(1/2)/(b*x-a)^(5/4),x)
```

output
```
int(1/(x*d)^(1/2)/(b*x-a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = \int \frac{1}{(bx-a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b*x-a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x - a)^(3/4)*sqrt(d*x)/(b^2*d*x^3 - 2*a*b*d*x^2 + a^2*d*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.15

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = \frac{2\sqrt{x}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx}{a}\right)}{a^{\frac{5}{4}}\sqrt{d}}$$

input
```
integrate(1/(d*x)**(1/2)/(b*x-a)**(5/4),x)
```

output
```
2*sqrt(x)*exp(3*I*pi/4)*hyper((1/2, 5/4), (3/2,), b*x/a)/(a**(5/4)*sqrt(d)
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = \int \frac{1}{(bx-a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b*x-a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x - a)^(5/4)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = \int \frac{1}{(bx-a)^{\frac{5}{4}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b*x-a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x - a)^(5/4)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = \int \frac{1}{\sqrt{d\,x}\,(b\,x-a)^{5/4}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(b*x - a)^(5/4)),x)
```

output
```
int(1/((d*x)^(1/2)*(b*x - a)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}(-a+bx)^{5/4}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{x}\,(bx-a)^{\frac{3}{4}}}{b^2x^3-2ab\,x^2+a^2x}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(b*x-a)^(5/4),x)
```

output
```
(sqrt(d)*int((sqrt(x)*( - a + b*x)**(3/4))/(a**2*x - 2*a*b*x**2 + b**2*x**
3),x))/d
```

3.681 $\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \frac{2\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),2\right)}{\sqrt{3}}$$

output
```
2/3*2^(3/4)*InverseJacobiAM(1/2*arctan(1/2*6^(1/2)*x^(1/2)),2^(1/2))*3^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \sqrt[4]{2}\sqrt{x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x}{2}\right)$$

input
```
Integrate[1/(Sqrt[x]*(2 + 3*x)^(3/4)),x]
```

output
```
2^(1/4)*Sqrt[x]*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x)/2]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 142 vs. $2(32) = 64$.

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 4.44, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {73, 764}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(3x+2)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4}{3}\int \frac{1}{\sqrt{\frac{1}{3}(3x+2)-\frac{2}{3}}}d\sqrt[4]{3x+2}$$

$$\Big\downarrow \text{764}$$

$$\frac{2^{3/4}\sqrt{\sqrt{2}\sqrt{3x+2}-2}\sqrt{\frac{\sqrt{2}\sqrt{3x+2}+2}{2-\sqrt{2}\sqrt{3x+2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{2^{3/4}\sqrt[4]{3x+2}}{\sqrt{\sqrt{2}\sqrt{3x+2}-2}}\right),\frac{1}{2}\right)}{3\sqrt{x}\sqrt{\frac{1}{2-\sqrt{2}\sqrt{3x+2}}}}$$

input
```
Int[1/(Sqrt[x]*(2 + 3*x)^(3/4)),x]
```

output
```
(2^(3/4)*Sqrt[-2 + Sqrt[2]*Sqrt[2 + 3*x]]*Sqrt[(2 + Sqrt[2]*Sqrt[2 + 3*x])
/(2 - Sqrt[2]*Sqrt[2 + 3*x])]*EllipticF[ArcSin[(2^(3/4)*(2 + 3*x)^(1/4))/S
qrt[-2 + Sqrt[2]*Sqrt[2 + 3*x]]], 1/2])/(3*Sqrt[x]*Sqrt[(2 - Sqrt[2]*Sqrt[
2 + 3*x])^(-1)])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 764
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[(a - q*x^2)/(a + q*x^2)]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[a + b*x^
4]*Sqrt[a/(a + q*x^2)]))*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2],
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

method	result	size
meijerg	$2^{\frac{1}{4}}\sqrt{x}\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x}{2}\right)$	17

input
```
int(1/x^(1/2)/(2+3*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
2^(1/4)*x^(1/2)*hypergeom([1/2,3/4],[3/2],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \int \frac{1}{(3\,x+2)^{\frac{3}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(2+3*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x + 2)^(1/4)*sqrt(x)/(3*x^2 + 2*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \sqrt[4]{2}\sqrt{x}\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{3xe^{i\pi}}{2}\right)$$

input
```
integrate(1/x**(1/2)/(2+3*x)**(3/4),x)
```

output
```
2**(1/4)*sqrt(x)*hyper((1/2, 3/4), (3/2,), 3*x*exp_polar(I*pi)/2)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \int \frac{1}{(3\,x+2)^{\frac{3}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(2+3*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x + 2)^(3/4)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \int \frac{1}{(3\,x+2)^{\frac{3}{4}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(2+3*x)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x + 2)^(3/4)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \int \frac{1}{\sqrt{x}\,(3\,x+2)^{3/4}}\,dx$$

input
```
int(1/(x^(1/2)*(3*x + 2)^(3/4)),x)
```

output
```
int(1/(x^(1/2)*(3*x + 2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}(2+3x)^{3/4}}\,dx = \int \frac{\sqrt{x}\,(3x+2)^{\frac{3}{4}}}{3\sqrt{3x+2}\,x^2+2\sqrt{3x+2}\,x}dx$$

input
```
int(1/x^(1/2)/(2+3*x)^(3/4),x)
```

output
```
int((sqrt(x)*(3*x + 2)**(3/4))/(3*sqrt(3*x + 2)*x**2 + 2*sqrt(3*x + 2)*x),
x)
```

3.682 $\int x^{11/4} \sqrt[4]{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 155

$$\int x^{11/4} \sqrt[4]{a+bx}\,dx = \frac{77a^3x^{3/4}\sqrt[4]{a+bx}}{1536b^3} - \frac{11a^2x^{7/4}\sqrt[4]{a+bx}}{384b^2} + \frac{ax^{11/4}\sqrt[4]{a+bx}}{48b} + \frac{1}{4}x^{15/4}\sqrt[4]{a+bx} + \frac{77a^4\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{1024b^{15/4}} - \frac{77a^4\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{1024b^{15/4}}$$

output
```
77/1536*a^3*x^(3/4)*(b*x+a)^(1/4)/b^3-11/384*a^2*x^(7/4)*(b*x+a)^(1/4)/b^2
+1/48*a*x^(11/4)*(b*x+a)^(1/4)/b+1/4*x^(15/4)*(b*x+a)^(1/4)+77/1024*a^4*ar
ctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(15/4)-77/1024*a^4*arctanh(b^(1/4)*x
^(1/4)/(b*x+a)^(1/4))/b^(15/4)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int x^{11/4}\sqrt[4]{a+bx}\,dx = \frac{2b^{3/4}x^{3/4}\sqrt[4]{a+bx}(77a^3-44a^2bx+32ab^2x^2+384b^3x^3)+231a^4\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{3072b^{15/4}}$$

input
```
Integrate[x^(11/4)*(a + b*x)^(1/4),x]
```

output
```
(2*b^(3/4)*x^(3/4)*(a + b*x)^(1/4)*(77*a^3 - 44*a^2*b*x + 32*a*b^2*x^2 + 3
84*b^3*x^3) + 231*a^4*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] - 231*a^4*
ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])/(3072*b^(15/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {60, 60, 60, 60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11/4}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{16}a\int \frac{x^{11/4}}{(a+bx)^{3/4}}dx+\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\int \frac{x^{7/4}}{(a+bx)^{3/4}}dx}{12b}\right)+\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\int \frac{x^{3/4}}{(a+bx)^{3/4}}dx}{8b}\right)}{12b}\right)+\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int\frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{4b}\right)}{8b}\right)}{12b}\right)+$$

$$\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 73

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int\frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{8b}\right)}{12b}\right)+$$

$$\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 854

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int\frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}\right)}{12b}\right)+$$

$$\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}\right)}{8b}\right)}{12b}\right)$$

$$\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)}{12b}\right)+$$

$$\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{16}a\left(\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)}{12b}\right)$$

$$\frac{1}{4}x^{15/4}\sqrt[4]{a+bx}$$

input
```
Int[x^(11/4)*(a + b*x)^(1/4),x]
```

output
```
(x^(15/4)*(a + b*x)^(1/4))/4 + (a*((x^(11/4)*(a + b*x)^(1/4))/(3*b) - (11*
a*((x^(7/4)*(a + b*x)^(1/4))/(2*b) - (7*a*((x^(3/4)*(a + b*x)^(1/4))/b - (
3*a*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(
1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4))))/b))/(8*b)))/(12*b)))/16
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int x^{\frac{11}{4}} (bx + a)^{\frac{1}{4}} \, dx$$

input
```
int(x^(11/4)*(b*x+a)^(1/4),x)
```

output
```
int(x^(11/4)*(b*x+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.61

$$\int x^{11/4} \sqrt[4]{a + bx} \, dx = -\frac{231 \left(\frac{a^{16}}{b^{15}}\right)^{\frac{1}{4}} b^3 \log\left(\frac{77\left(\left(\frac{a^{16}}{b^{15}}\right)^{\frac{1}{4}} b^4 x + (bx+a)^{\frac{1}{4}} a^4 x^{\frac{3}{4}}\right)}{x}\right) - 231 \left(\frac{a^{16}}{b^{15}}\right)^{\frac{1}{4}} b^3 \log\left(-\frac{77\left(\left(\frac{a^{16}}{b^{15}}\right)^{\frac{1}{4}} b^4 x - (bx+a)^{\frac{1}{4}} a^4 x^{\frac{3}{4}}\right)}{x}\right) - 2}{}$$

input
```
integrate(x^(11/4)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/6144*(231*(a^16/b^15)^(1/4)*b^3*log(77*((a^16/b^15)^(1/4)*b^4*x + (b*x
+ a)^(1/4)*a^4*x^(3/4))/x) - 231*(a^16/b^15)^(1/4)*b^3*log(-77*((a^16/b^15
)^(1/4)*b^4*x - (b*x + a)^(1/4)*a^4*x^(3/4))/x) - 231*I*(a^16/b^15)^(1/4)*
b^3*log(-77*(I*(a^16/b^15)^(1/4)*b^4*x - (b*x + a)^(1/4)*a^4*x^(3/4))/x) +
 231*I*(a^16/b^15)^(1/4)*b^3*log(-77*(-I*(a^16/b^15)^(1/4)*b^4*x - (b*x +
a)^(1/4)*a^4*x^(3/4))/x) - 4*(384*b^3*x^3 + 32*a*b^2*x^2 - 44*a^2*b*x + 77
*a^3)*(b*x + a)^(1/4)*x^(3/4))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 71.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.24

$$\int x^{11/4}\sqrt[4]{a+bx}\,dx = \frac{\sqrt[4]{a}x^{\frac{15}{4}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**(11/4)*(b*x+a)**(1/4),x)
```

output
```
a**(1/4)*x**(15/4)*gamma(15/4)*hyper((-1/4, 15/4), (19/4,), b*x*exp_polar(
I*pi)/a)/gamma(19/4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34

$$\int x^{11/4}\sqrt[4]{a+bx}\,dx = \frac{\frac{231(bx+a)^{\frac{1}{4}}a^4b^3}{x^{\frac{1}{4}}} + \frac{351(bx+a)^{\frac{5}{4}}a^4b^2}{x^{\frac{5}{4}}} - \frac{275(bx+a)^{\frac{9}{4}}a^4b}{x^{\frac{9}{4}}} + \frac{77(bx+a)^{\frac{13}{4}}a^4}{x^{\frac{13}{4}}}}{1536\left(b^7 - \frac{4(bx+a)b^6}{x} + \frac{6(bx+a)^2b^5}{x^2} - \frac{4(bx+a)^3b^4}{x^3} + \frac{(bx+a)^4b^3}{x^4}\right)} - \frac{77\left(\frac{2a^4\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} - \frac{a^4\log\left(-\frac{b^{\frac{1}{4}} - \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}} + \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{2048\,b^3}$$

input
```
integrate(x^(11/4)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
1/1536*(231*(b*x + a)^(1/4)*a^4*b^3/x^(1/4) + 351*(b*x + a)^(5/4)*a^4*b^2/
x^(5/4) - 275*(b*x + a)^(9/4)*a^4*b/x^(9/4) + 77*(b*x + a)^(13/4)*a^4/x^(1
3/4))/(b^7 - 4*(b*x + a)*b^6/x + 6*(b*x + a)^2*b^5/x^2 - 4*(b*x + a)^3*b^4
/x^3 + (b*x + a)^4*b^3/x^4) - 77/2048*(2*a^4*arctan((b*x + a)^(1/4)/(b^(1/
4)*x^(1/4)))/b^(3/4) - a^4*log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/
4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4))/b^3
```

Giac [F]

$$\int x^{11/4}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{11}{4}}\,dx$$

input
```
integrate(x^(11/4)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{11/4}\sqrt[4]{a+bx}\,dx=\int x^{11/4}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(11/4)*(a + b*x)^(1/4),x)
```

output
```
int(x^(11/4)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int x^{11/4}\sqrt[4]{a+bx}\,dx = \frac{308x^{\frac{3}{4}}(bx+a)^{\frac{1}{4}}\,a^3 - 176x^{\frac{7}{4}}(bx+a)^{\frac{1}{4}}\,a^2b + 128x^{\frac{11}{4}}(bx+a)^{\frac{1}{4}}\,a\,b^2 + 1536x^{\frac{15}{4}}(bx+a)}{6144b^3}$$

input
```
int(x^(11/4)*(b*x+a)^(1/4),x)
```

output
```
(308*x**(3/4)*(a + b*x)**(1/4)*a**3 - 176*x**(3/4)*(a + b*x)**(1/4)*a**2*b
*x + 128*x**(3/4)*(a + b*x)**(1/4)*a*b**2*x**2 + 1536*x**(3/4)*(a + b*x)**
(1/4)*b**3*x**3 - 231*int((a + b*x)**(1/4)/(x**(1/4)*a + x**(1/4)*b*x),x)*
a**4)/(6144*b**3)
```

3.683 $\int x^{7/4} \sqrt[4]{a+bx}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int x^{7/4} \sqrt[4]{a+bx}\, dx = -\frac{7a^2x^{3/4}\sqrt[4]{a+bx}}{96b^2} + \frac{ax^{7/4}\sqrt[4]{a+bx}}{24b} + \frac{1}{3}x^{11/4}\sqrt[4]{a+bx} - \frac{7a^3\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{64b^{11/4}} + \frac{7a^3\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{64b^{11/4}}$$

output
```
-7/96*a^2*x^(3/4)*(b*x+a)^(1/4)/b^2+1/24*a*x^(7/4)*(b*x+a)^(1/4)/b+1/3*x^(
11/4)*(b*x+a)^(1/4)-7/64*a^3*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(11/4
)+7/64*a^3*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.78

$$\int x^{7/4} \sqrt[4]{a+bx}\, dx = \frac{2b^{3/4}x^{3/4}\sqrt[4]{a+bx}(-7a^2+4abx+32b^2x^2) - 21a^3\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right) + 21a^3\text{arctanh}}{192b^{11/4}}$$

input
```
Integrate[x^(7/4)*(a + b*x)^(1/4),x]
```

output
```
(2*b^(3/4)*x^(3/4)*(a + b*x)^(1/4)*(-7*a^2 + 4*a*b*x + 32*b^2*x^2) - 21*a^
3*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] + 21*a^3*ArcTanh[(b^(1/4)*x^(1
/4))/(a + b*x)^(1/4)])/(192*b^(11/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {60, 60, 60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/4} \sqrt[4]{a+bx} \, dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{12} a \int \frac{x^{7/4}}{(a+bx)^{3/4}} dx + \frac{1}{3} x^{11/4} \sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{12} a \left(\frac{x^{7/4} \sqrt[4]{a+bx}}{2b} - \frac{7a \int \frac{x^{3/4}}{(a+bx)^{3/4}} dx}{8b} \right) + \frac{1}{3} x^{11/4} \sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{12} a \left(\frac{x^{7/4} \sqrt[4]{a+bx}}{2b} - \frac{7a \left(\frac{x^{3/4} \sqrt[4]{a+bx}}{b} - \frac{3a \int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}} dx}{4b} \right)}{8b} \right) + \frac{1}{3} x^{11/4} \sqrt[4]{a+bx}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{12} a \left(\frac{x^{7/4} \sqrt[4]{a+bx}}{2b} - \frac{7a \left(\frac{x^{3/4} \sqrt[4]{a+bx}}{b} - \frac{3a \int \frac{\sqrt{x}}{(a+bx)^{3/4}} d\sqrt[4]{x}}{b} \right)}{8b} \right) + \frac{1}{3} x^{11/4} \sqrt[4]{a+bx}$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{12}a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int\frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}\right)+\frac{1}{3}x^{11/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{12}a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int\frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}\right)}{8b}\right)+\frac{1}{3}x^{11/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{12}a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)+$$

$$\frac{1}{3}x^{11/4}\sqrt[4]{a+bx}$$

$\downarrow$ 219

$$\frac{1}{12}a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)+$$

$$\frac{1}{3}x^{11/4}\sqrt[4]{a+bx}$$

input
```
Int[x^(7/4)*(a + b*x)^(1/4),x]
```

output
```
(x^(11/4)*(a + b*x)^(1/4))/3 + (a*((x^(7/4)*(a + b*x)^(1/4))/(2*b) - (7*a*
((x^(3/4)*(a + b*x)^(1/4))/b - (3*a*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*
x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4))
))/b))/(8*b)))/12
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int x^{\frac{7}{4}}(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(7/4)*(b*x+a)^(1/4),x)
```

output
```
int(x^(7/4)*(b*x+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.82

$$\int x^{7/4}\sqrt[4]{a+bx}\,dx = \frac{21\left(\frac{a^{12}}{b^{11}}\right)^{\frac{1}{4}} b^2\log\left(\frac{7\left(\left(\frac{a^{12}}{b^{11}}\right)^{\frac{1}{4}} b^3x+(bx+a)^{\frac{1}{4}}a^3x^{\frac{3}{4}}\right)}{x}\right) - 21\left(\frac{a^{12}}{b^{11}}\right)^{\frac{1}{4}} b^2\log\left(-\frac{7\left(\left(\frac{a^{12}}{b^{11}}\right)^{\frac{1}{4}} b^3x-(bx+\cdots}{x}\cdots\right.}{\cdots}$$

input
```
integrate(x^(7/4)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
1/384*(21*(a^12/b^11)^(1/4)*b^2*log(7*((a^12/b^11)^(1/4)*b^3*x + (b*x + a)
^(1/4)*a^3*x^(3/4))/x) - 21*(a^12/b^11)^(1/4)*b^2*log(-7*((a^12/b^11)^(1/4
)*b^3*x - (b*x + a)^(1/4)*a^3*x^(3/4))/x) - 21*I*(a^12/b^11)^(1/4)*b^2*log
(-7*(I*(a^12/b^11)^(1/4)*b^3*x - (b*x + a)^(1/4)*a^3*x^(3/4))/x) + 21*I*(a
^12/b^11)^(1/4)*b^2*log(-7*(-I*(a^12/b^11)^(1/4)*b^3*x - (b*x + a)^(1/4)*a
^3*x^(3/4))/x) + 4*(32*b^2*x^2 + 4*a*b*x - 7*a^2)*(b*x + a)^(1/4)*x^(3/4))
/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.77 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.28

$$\int x^{7/4}\sqrt[4]{a+bx}\,dx = \frac{\sqrt[4]{a}x^{\frac{11}{4}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**(7/4)*(b*x+a)**(1/4),x)
```

output
```
a**(1/4)*x**(11/4)*gamma(11/4)*hyper((-1/4, 11/4), (15/4,), b*x*exp_polar(
I*pi)/a)/gamma(15/4)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.34

$$\int x^{7/4}\sqrt[4]{a+bx}\,dx = -\frac{\frac{21\,(bx+a)^{\frac{1}{4}}a^3b^2}{x^{\frac{1}{4}}}+\frac{18\,(bx+a)^{\frac{5}{4}}a^3b}{x^{\frac{5}{4}}}-\frac{7\,(bx+a)^{\frac{9}{4}}a^3}{x^{\frac{9}{4}}}}{96\left(b^5-\frac{3\,(bx+a)b^4}{x}+\frac{3\,(bx+a)^2b^3}{x^2}-\frac{(bx+a)^3b^2}{x^3}\right)} + \frac{7\left(\frac{2\,a^3\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a^3\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^2}$$

input
```
integrate(x^(7/4)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/96*(21*(b*x + a)^(1/4)*a^3*b^2/x^(1/4) + 18*(b*x + a)^(5/4)*a^3*b/x^(5/
4) - 7*(b*x + a)^(9/4)*a^3/x^(9/4))/(b^5 - 3*(b*x + a)*b^4/x + 3*(b*x + a)
^2*b^3/x^2 - (b*x + a)^3*b^2/x^3) + 7/128*(2*a^3*arctan((b*x + a)^(1/4)/(b
^(1/4)*x^(1/4)))/b^(3/4) - a^3*log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4))/(b
^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4))/b^2
```

Giac [F]

$$\int x^{7/4}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{7}{4}}\,dx$$

input
```
integrate(x^(7/4)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/4}\sqrt[4]{a+bx}\,dx=\int x^{7/4}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(7/4)*(a + b*x)^(1/4),x)
```

output
```
int(x^(7/4)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int x^{7/4}\sqrt[4]{a+bx}\,dx=\frac{-28x^{\frac{3}{4}}(bx+a)^{\frac{1}{4}}a^2+16x^{\frac{7}{4}}(bx+a)^{\frac{1}{4}}ab+128x^{\frac{11}{4}}(bx+a)^{\frac{1}{4}}b^2+21\left(\int\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}a+x^{\frac{5}{4}}b}dx\right)a^3}{384b^2}$$

input
```
int(x^(7/4)*(b*x+a)^(1/4),x)
```

output
```
( - 28*x**(3/4)*(a + b*x)**(1/4)*a**2 + 16*x**(3/4)*(a + b*x)**(1/4)*a*b*x
 + 128*x**(3/4)*(a + b*x)**(1/4)*b**2*x**2 + 21*int((a + b*x)**(1/4)/(x**(
1/4)*a + x**(1/4)*b*x),x)*a**3)/(384*b**2)
```

3.684 $\int x^{3/4}\sqrt[4]{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx=\frac{ax^{3/4}\sqrt[4]{a+bx}}{8b}+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}+\frac{3a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{7/4}}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{7/4}}$$

output
```
1/8*a*x^(3/4)*(b*x+a)^(1/4)/b+1/2*x^(7/4)*(b*x+a)^(1/4)+3/16*a^2*arctan(b^
(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(7/4)-3/16*a^2*arctanh(b^(1/4)*x^(1/4)/(b*x
+a)^(1/4))/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx=\frac{2b^{3/4}x^{3/4}\sqrt[4]{a+bx}(a+4bx)+3a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)-3a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{7/4}}$$

input
```
Integrate[x^(3/4)*(a + b*x)^(1/4),x]
```

output
```
(2*b^(3/4)*x^(3/4)*(a + b*x)^(1/4)*(a + 4*b*x) + 3*a^2*ArcTan[(b^(1/4)*x^(
1/4))/(a + b*x)^(1/4)] - 3*a^2*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])
/(16*b^(7/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {60, 60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{8}a\int \frac{x^{3/4}}{(a+bx)^{3/4}}dx+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{8}a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{4b}\right)+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{8}a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{8}a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}\right)+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 216

$$\frac{1}{8}a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 219

$$\frac{1}{8}a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)+\frac{1}{2}x^{7/4}\sqrt[4]{a+bx}$$

input
```
Int[x^(3/4)*(a + b*x)^(1/4),x]
```

output
```
(x^(7/4)*(a + b*x)^(1/4))/2 + (a*((x^(3/4)*(a + b*x)^(1/4))/b - (3*a*(-1/2
*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1
/4))/(a + b*x)^(1/4)]/(2*b^(3/4))))/b))/8
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int x^{\frac{3}{4}}(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(3/4)*(b*x+a)^(1/4),x)
```

output
```
int(x^(3/4)*(b*x+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.03

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx = -\frac{3\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b\log\left(\frac{3\left(\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x+(bx+a)^{\frac{1}{4}}a^2x^{\frac{3}{4}}\right)}{x}\right)-3\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b\log\left(-\frac{3\left(\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x-(bx+a)^{\frac{1}{4}}a^2x^{\frac{3}{4}}\right)}{x}\right)-3i\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b\log}{}$$

input
```
integrate(x^(3/4)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/32*(3*(a^8/b^7)^(1/4)*b*log(3*((a^8/b^7)^(1/4)*b^2*x + (b*x + a)^(1/4)*
a^2*x^(3/4))/x) - 3*(a^8/b^7)^(1/4)*b*log(-3*((a^8/b^7)^(1/4)*b^2*x - (b*x
 + a)^(1/4)*a^2*x^(3/4))/x) - 3*I*(a^8/b^7)^(1/4)*b*log(-3*(I*(a^8/b^7)^(1
/4)*b^2*x - (b*x + a)^(1/4)*a^2*x^(3/4))/x) + 3*I*(a^8/b^7)^(1/4)*b*log(-3
*(-I*(a^8/b^7)^(1/4)*b^2*x - (b*x + a)^(1/4)*a^2*x^(3/4))/x) - 4*(4*b*x +
a)*(b*x + a)^(1/4)*x^(3/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx=\frac{\sqrt[4]{a}x^{\frac{7}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**(3/4)*(b*x+a)**(1/4),x)
```

output
```
a**(1/4)*x**(7/4)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x*exp_polar(I*p
i)/a)/gamma(11/4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx=\frac{\frac{3(bx+a)^{\frac{1}{4}}a^2b}{x^{\frac{1}{4}}}+\frac{(bx+a)^{\frac{5}{4}}a^2}{x^{\frac{5}{4}}}}{8\left(b^3-\frac{2(bx+a)b^2}{x}+\frac{(bx+a)^2b}{x^2}\right)}-\frac{3\left(\frac{2a^2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{32\,b}$$

input
```
integrate(x^(3/4)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
1/8*(3*(b*x + a)^(1/4)*a^2*b/x^(1/4) + (b*x + a)^(5/4)*a^2/x^(5/4))/(b^3 -
 2*(b*x + a)*b^2/x + (b*x + a)^2*b/x^2) - 3/32*(2*a^2*arctan((b*x + a)^(1/
4)/(b^(1/4)*x^(1/4)))/b^(3/4) - a^2*log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4
))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4))/b
```

Giac [F]

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}\,dx$$

input
```
integrate(x^(3/4)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx = \int x^{3/4}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(3/4)*(a + b*x)^(1/4),x)
```

output
```
int(x^(3/4)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int x^{3/4}\sqrt[4]{a+bx}\,dx = \frac{4x^{\frac{3}{4}}(bx+a)^{\frac{1}{4}}\,a+16x^{\frac{7}{4}}(bx+a)^{\frac{1}{4}}\,b-3\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}a+x^{\frac{5}{4}}b}dx\right)a^2}{32b}$$

input
```
int(x^(3/4)*(b*x+a)^(1/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4)*a + 16*x**(3/4)*(a + b*x)**(1/4)*b*x - 3*int(
(a + b*x)**(1/4)/(x**(1/4)*a + x**(1/4)*b*x),x)*a**2)/(32*b)
```

3.685 $\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}} \, dx = x^{3/4}\sqrt[4]{a+bx} - \frac{a \arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}} + \frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}$$

output
```
x^(3/4)*(b*x+a)^(1/4)-1/2*a*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(3/4)+
1/2*a*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}} \, dx = x^{3/4}\sqrt[4]{a+bx} - \frac{a \arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}} + \frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(1/4),x]
```

output
```
x^(3/4)*(a + b*x)^(1/4) - (a*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])/(2
*b^(3/4)) + (a*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])/(2*b^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}a\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx+x^{3/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{73}$$

$$a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}+x^{3/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{854}$$

$$a\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+x^{3/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{827}$$

$$a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)+x^{3/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{216}$$

$$a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)+x^{3/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)+x^{3/4}\sqrt[4]{a+bx}$$

input
```
Int[(a + b*x)^(1/4)/x^(1/4),x]
```

output
```
x^(3/4)*(a + b*x)^(1/4) + a*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)
]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4)))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(1/4),x)
```

output
```
int((b*x+a)^(1/4)/x^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}}\,dx = \frac{1}{4}\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+(bx+a)^{\frac{1}{4}}ax^{\frac{3}{4}}}{x}\right) - \frac{1}{4}\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx-(bx+a)^{\frac{1}{4}}ax^{\frac{3}{4}}}{x}\right) + \frac{1}{4}i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+(bx+a)^{\frac{1}{4}}ax^{\frac{3}{4}}}{x}\right) - \frac{1}{4}i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+(bx+a)^{\frac{1}{4}}ax^{\frac{3}{4}}}{x}\right) + (bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}$$

input
```
integrate((b*x+a)^(1/4)/x^(1/4),x, algorithm="fricas")
```

output
```
1/4*(a^4/b^3)^(1/4)*log(((a^4/b^3)^(1/4)*b*x + (b*x + a)^(1/4)*a*x^(3/4))/
x) - 1/4*(a^4/b^3)^(1/4)*log(-((a^4/b^3)^(1/4)*b*x - (b*x + a)^(1/4)*a*x^(
3/4))/x) + 1/4*I*(a^4/b^3)^(1/4)*log((I*(a^4/b^3)^(1/4)*b*x + (b*x + a)^(1
/4)*a*x^(3/4))/x) - 1/4*I*(a^4/b^3)^(1/4)*log((-I*(a^4/b^3)^(1/4)*b*x + (b
*x + a)^(1/4)*a*x^(3/4))/x) + (b*x + a)^(1/4)*x^(3/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}}\,dx = \frac{\sqrt[4]{a}x^{\frac{3}{4}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x+a)**(1/4)/x**(1/4),x)
```

output
```
a**(1/4)*x**(3/4)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x*exp_polar(I*pi
)/a)/gamma(7/4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}}\,dx = \frac{a\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{2\,b^{\frac{3}{4}}} - \frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{4\,b^{\frac{3}{4}}} - \frac{(bx+a)^{\frac{1}{4}}a}{\left(b-\frac{bx+a}{x}\right)x^{\frac{1}{4}}}$$

input
```
integrate((b*x+a)^(1/4)/x^(1/4),x, algorithm="maxima")
```

output
```
1/2*a*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(3/4) - 1/4*a*log(-(b^(1
/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4
) - (b*x + a)^(1/4)*a/((b - (b*x + a)/x)*x^(1/4))
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}}\,dx = \int \frac{(a+b\,x)^{1/4}}{x^{1/4}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(1/4),x)
```

output
```
int((a + b*x)^(1/4)/x^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{x}}\,dx = x^{\frac{3}{4}}(bx+a)^{\frac{1}{4}} + \frac{\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}a+x^{\frac{5}{4}}b}dx\right)a}{4}$$

input
```
int((b*x+a)^(1/4)/x^(1/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4) + int((a + b*x)**(1/4)/(x**(1/4)*a + x**(1/4)
*b*x),x)*a)/4
```

3.686 $\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx=-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}-2\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)+2\sqrt[4]{b}\,\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)$$

output
```
-4*(b*x+a)^(1/4)/x^(1/4)-2*b^(1/4)*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))+2
*b^(1/4)*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx=-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}-2\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)+2\sqrt[4]{b}\,\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)$$

input
```
Integrate[(a + b*x)^(1/4)/x^(5/4),x]
```

output
```
(-4*(a + b*x)^(1/4))/x^(1/4) - 2*b^(1/4)*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x
)^(1/4)] + 2*b^(1/4)*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {57, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx$$

$$\Big\downarrow \ 57$$

$$b\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}$$

$$\Big\downarrow \ 73$$

$$4b\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}$$

$$\Big\downarrow \ 854$$

$$4b\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}$$

$$\Big\downarrow \ 827$$

$$4b\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}$$

$$\Big\downarrow \ 216$$

$$4b\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}$$

$$\Big\downarrow \ 219$$

$$4b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{x}}$$

input
```
Int[(a + b*x)^(1/4)/x^(5/4),x]
```

output
```
(-4*(a + b*x)^(1/4))/x^(1/4) + 4*b*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x
)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4)))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{5}{4}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(5/4),x)
```

output
```
int((b*x+a)^(1/4)/x^(5/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx = \frac{b^{\frac{1}{4}}x\log\left(\frac{b^{\frac{1}{4}}x+(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) - b^{\frac{1}{4}}x\log\left(-\frac{b^{\frac{1}{4}}x-(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) + i\,b^{\frac{1}{4}}x\log\left(\frac{i\,b^{\frac{1}{4}}x+(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) - i}{x}$$

input
```
integrate((b*x+a)^(1/4)/x^(5/4),x, algorithm="fricas")
```

output
```
(b^(1/4)*x*log((b^(1/4)*x + (b*x + a)^(1/4)*x^(3/4))/x) - b^(1/4)*x*log(-(
b^(1/4)*x - (b*x + a)^(1/4)*x^(3/4))/x) + I*b^(1/4)*x*log((I*b^(1/4)*x + (
b*x + a)^(1/4)*x^(3/4))/x) - I*b^(1/4)*x*log((-I*b^(1/4)*x + (b*x + a)^(1/
4)*x^(3/4))/x) - 4*(b*x + a)^(1/4)*x^(3/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx = \frac{\sqrt[4]{a}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x+a)**(1/4)/x**(5/4),x)
```

output
```
a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x*exp_polar(I*pi)/a)/(x
**(1/4)*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx = 2\,b^{\frac{1}{4}}\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right) - b^{\frac{1}{4}}\log\left(-\frac{b^{\frac{1}{4}} - \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}} + \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right) - \frac{4\,(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}$$

input
```
integrate((b*x+a)^(1/4)/x^(5/4),x, algorithm="maxima")
```

output
```
2*b^(1/4)*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4))) - b^(1/4)*log(-(b^(1/4
) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4))) - 4*(b*x
 + a)^(1/4)/x^(1/4)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx = \int \frac{(a+b\,x)^{1/4}}{x^{5/4}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(5/4),x)
```

output
```
int((a + b*x)^(1/4)/x^(5/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{5/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{5}{4}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(5/4),x)
```

output
```
int((a + b*x)**(1/4)/(x**(1/4)*x),x)
```

3.687 $\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = -\frac{4(a+bx)^{5/4}}{5ax^{5/4}}$$

output
```
-4/5*(b*x+a)^(5/4)/a/x^(5/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = -\frac{4(a+bx)^{5/4}}{5ax^{5/4}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(9/4),x]
```

output
```
(-4*(a + b*x)^(5/4))/(5*a*x^(5/4))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}} \, dx$$
$$\Big\downarrow \ 48$$
$$-\frac{4(a+bx)^{5/4}}{5ax^{5/4}}$$

input
```
Int[(a + b*x)^(1/4)/x^(9/4),x]
```

output
```
(-4*(a + b*x)^(5/4))/(5*a*x^(5/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{5}{4}}}{5a\,x^{\frac{5}{4}}}$	16
risch	$-\frac{4(bx+a)^{\frac{5}{4}}}{5a\,x^{\frac{5}{4}}}$	16
orering	$-\frac{4(bx+a)^{\frac{5}{4}}}{5a\,x^{\frac{5}{4}}}$	16

input
```
int((b*x+a)^(1/4)/x^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-4/5*(b*x+a)^(5/4)/a/x^(5/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = -\frac{4\,(bx+a)^{\frac{5}{4}}}{5\,ax^{\frac{5}{4}}}$$

input
```
integrate((b*x+a)^(1/4)/x^(9/4),x, algorithm="fricas")
```

output
```
-4/5*(b*x + a)^(5/4)/(a*x^(5/4))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(19) = 38$.

Time = 2.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{x\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{a\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x+a)**(1/4)/x**(9/4),x)
```

output
```
b**(1/4)*(a/(b*x) + 1)**(1/4)*gamma(-5/4)/(x*gamma(-1/4)) + b**(5/4)*(a/(b
*x) + 1)**(1/4)*gamma(-5/4)/(a*gamma(-1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = -\frac{4\,(bx+a)^{\frac{5}{4}}}{5\,ax^{\frac{5}{4}}}$$

input
```
integrate((b*x+a)^(1/4)/x^(9/4),x, algorithm="maxima")
```

output
```
-4/5*(b*x + a)^(5/4)/(a*x^(5/4))
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{9}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(9/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(9/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = -\frac{\left(\frac{4\,b\,x}{5\,a}+\frac{4}{5}\right)\,{\left(a+b\,x\right)}^{1/4}}{x^{5/4}}$$

input
```
int((a + b*x)^(1/4)/x^(9/4),x)
```

output
```
-(((4*b*x)/(5*a) + 4/5)*(a + b*x)^(1/4))/x^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}\,dx = -\frac{4(bx+a)^{\frac{5}{4}}}{5x^{\frac{5}{4}}a}$$

input
```
int((b*x+a)^(1/4)/x^(9/4),x)
```

output
```
( - 4*(a + b*x)**(1/4)*(a + b*x))/(5*x**(1/4)*a*x)
```

3.688 $\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = -\frac{4(a+bx)^{5/4}}{9ax^{9/4}} + \frac{16b(a+bx)^{5/4}}{45a^2x^{5/4}}$$

output
```
-4/9*(b*x+a)^(5/4)/a/x^(9/4)+16/45*b*(b*x+a)^(5/4)/a^2/x^(5/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = -\frac{4\sqrt[4]{a+bx}(5a^2+abx-4b^2x^2)}{45a^2x^{9/4}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(13/4),x]
```

output
```
(-4*(a + b*x)^(1/4)*(5*a^2 + a*b*x - 4*b^2*x^2))/(45*a^2*x^(9/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}dx}{9a}-\frac{4(a+bx)^{5/4}}{9ax^{9/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{16b(a+bx)^{5/4}}{45a^2x^{5/4}}-\frac{4(a+bx)^{5/4}}{9ax^{9/4}}$$

input
```
Int[(a + b*x)^(1/4)/x^(13/4),x]
```

output
```
(-4*(a + b*x)^(5/4))/(9*a*x^(9/4)) + (16*b*(a + b*x)^(5/4))/(45*a^2*x^(5/4
))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{5}{4}}(-4bx+5a)}{45x^{\frac{9}{4}}a^{2}}$	24
orering	$-\frac{4(bx+a)^{\frac{5}{4}}(-4bx+5a)}{45x^{\frac{9}{4}}a^{2}}$	24
risch	$-\frac{4(bx+a)^{\frac{1}{4}}(-4b^{2}x^{2}+abx+5a^{2})}{45x^{\frac{9}{4}}a^{2}}$	34

input
```
int((b*x+a)^(1/4)/x^(13/4),x,method=_RETURNVERBOSE)
```

output
```
-4/45*(b*x+a)^(5/4)*(-4*b*x+5*a)/x^(9/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = \frac{4\,(4\,b^2x^2 - abx - 5\,a^2)(bx+a)^{\frac{1}{4}}}{45\,a^2x^{\frac{9}{4}}}$$

input
```
integrate((b*x+a)^(1/4)/x^(13/4),x, algorithm="fricas")
```

output
```
4/45*(4*b^2*x^2 - a*b*x - 5*a^2)*(b*x + a)^(1/4)/(a^2*x^(9/4))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(39) = 78.

Time = 23.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{9}{4}\right)}{4x^2\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{9}{4}\right)}{4ax\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{9}{4}\right)}{a^2\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x+a)**(1/4)/x**(13/4),x)
```

output
```
-5*b**(1/4)*(a/(b*x) + 1)**(1/4)*gamma(-9/4)/(4*x**2*gamma(-1/4)) - b**(5/
4)*(a/(b*x) + 1)**(1/4)*gamma(-9/4)/(4*a*x*gamma(-1/4)) + b**(9/4)*(a/(b*x
) + 1)**(1/4)*gamma(-9/4)/(a**2*gamma(-1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = \frac{4\left(\frac{9\,(bx+a)^{\frac{5}{4}}b}{x^{\frac{5}{4}}} - \frac{5\,(bx+a)^{\frac{9}{4}}}{x^{\frac{9}{4}}}\right)}{45\,a^2}$$

input
```
integrate((b*x+a)^(1/4)/x^(13/4),x, algorithm="maxima")
```

output
```
4/45*(9*(b*x + a)^(5/4)*b/x^(5/4) - 5*(b*x + a)^(9/4)/x^(9/4))/a^2
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{13}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(13/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(13/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = -\frac{(a+b\,x)^{1/4}\left(\frac{4\,b\,x}{45\,a}-\frac{16\,b^2\,x^2}{45\,a^2}+\frac{4}{9}\right)}{x^{9/4}}$$

input
```
int((a + b*x)^(1/4)/x^(13/4),x)
```

output
```
-((a + b*x)^(1/4)*((4*b*x)/(45*a) - (16*b^2*x^2)/(45*a^2) + 4/9))/x^(9/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\left(4b^2x^2-abx-5a^2\right)}{45x^{\frac{9}{4}}a^2}$$

input
```
int((b*x+a)^(1/4)/x^(13/4),x)
```

output

```
(4*(a + b*x)**(1/4)*( - 5*a**2 - a*b*x + 4*b**2*x**2))/(45*x**(1/4)*a**2*x
**2)
```

3.689 $\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = -\frac{4(a+bx)^{5/4}}{13ax^{13/4}} + \frac{32b(a+bx)^{5/4}}{117a^2x^{9/4}} - \frac{128b^2(a+bx)^{5/4}}{585a^3x^{5/4}}$$

output
```
-4/13*(b*x+a)^(5/4)/a/x^(13/4)+32/117*b*(b*x+a)^(5/4)/a^2/x^(9/4)-128/585*
b^2*(b*x+a)^(5/4)/a^3/x^(5/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = -\frac{4(a+bx)^{5/4}\,(45a^2-40abx+32b^2x^2)}{585a^3x^{13/4}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(17/4),x]
```

output
```
(-4*(a + b*x)^(5/4)*(45*a^2 - 40*a*b*x + 32*b^2*x^2))/(585*a^3*x^(13/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{8b\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}dx}{13a}-\frac{4(a+bx)^{5/4}}{13ax^{13/4}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{8b\left(-\frac{4b\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}dx}{9a}-\frac{4(a+bx)^{5/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{5/4}}{13ax^{13/4}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{8b\left(\frac{16b(a+bx)^{5/4}}{45a^2x^{5/4}}-\frac{4(a+bx)^{5/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{5/4}}{13ax^{13/4}}$$

input
```
Int[(a + b*x)^(1/4)/x^(17/4),x]
```

output
```
(-4*(a + b*x)^(5/4))/(13*a*x^(13/4)) - (8*b*((-4*(a + b*x)^(5/4))/(9*a*x^(
9/4)) + (16*b*(a + b*x)^(5/4))/(45*a^2*x^(5/4))))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{5}{4}} \left(32b^{2}x^{2}-40abx+45a^{2}\right)}{585x^{\frac{13}{4}} a^{3}}$	35
orering	$-\frac{4(bx+a)^{\frac{5}{4}} \left(32b^{2}x^{2}-40abx+45a^{2}\right)}{585x^{\frac{13}{4}} a^{3}}$	35
risch	$-\frac{4(bx+a)^{\frac{1}{4}} \left(32b^{3}x^{3}-8a\,b^{2}x^{2}+5a^{2}bx+45a^{3}\right)}{585x^{\frac{13}{4}} a^{3}}$	46

input
```
int((b*x+a)^(1/4)/x^(17/4),x,method=_RETURNVERBOSE)
```

output
```
-4/585*(b*x+a)^(5/4)*(32*b^2*x^2-40*a*b*x+45*a^2)/x^(13/4)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = -\frac{4\,(32\,b^3x^3 - 8\,ab^2x^2 + 5\,a^2bx + 45\,a^3)(bx+a)^{\frac{1}{4}}}{585\,a^3x^{\frac{13}{4}}}$$

input
```
integrate((b*x+a)^(1/4)/x^(17/4),x, algorithm="fricas")
```

output
```
-4/585*(32*b^3*x^3 - 8*a*b^2*x^2 + 5*a^2*b*x + 45*a^3)*(b*x + a)^(1/4)/(a^
3*x^(13/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(1/4)/x**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = -\frac{4\left(\frac{117\,(bx+a)^{\frac{5}{4}}b^2}{x^{\frac{5}{4}}} - \frac{130\,(bx+a)^{\frac{9}{4}}b}{x^{\frac{9}{4}}} + \frac{45\,(bx+a)^{\frac{13}{4}}}{x^{\frac{13}{4}}}\right)}{585\,a^3}$$

input
```
integrate((b*x+a)^(1/4)/x^(17/4),x, algorithm="maxima")
```

output
```
-4/585*(117*(b*x + a)^(5/4)*b^2/x^(5/4) - 130*(b*x + a)^(9/4)*b/x^(9/4) +
45*(b*x + a)^(13/4)/x^(13/4))/a^3
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{17}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(17/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(17/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = -\frac{(a+b\,x)^{1/4}\left(\frac{128\,b^3\,x^3}{585\,a^3}-\frac{32\,b^2\,x^2}{585\,a^2}+\frac{4\,b\,x}{117\,a}+\frac{4}{13}\right)}{x^{13/4}}$$

input
```
int((a + b*x)^(1/4)/x^(17/4),x)
```

output
```
-((a + b*x)^(1/4)*((128*b^3*x^3)/(585*a^3) - (32*b^2*x^2)/(585*a^2) + (4*b
*x)/(117*a) + 4/13))/x^(13/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\left(-32b^3x^3+8a\,b^2x^2-5a^2bx-45a^3\right)}{585x^{\frac{13}{4}}a^3}$$

input
```
int((b*x+a)^(1/4)/x^(17/4),x)
```

output
```
(4*(a + b*x)**(1/4)*( - 45*a**3 - 5*a**2*b*x + 8*a*b**2*x**2 - 32*b**3*x**
3))/(585*x**(1/4)*a**3*x**3)
```

3.690 $\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx=-\frac{4(a+bx)^{5/4}}{17ax^{17/4}}+\frac{48b(a+bx)^{5/4}}{221a^2x^{13/4}}-\frac{128b^2(a+bx)^{5/4}}{663a^3x^{9/4}}+\frac{512b^3(a+bx)^{5/4}}{3315a^4x^{5/4}}$$

output
```
-4/17*(b*x+a)^(5/4)/a/x^(17/4)+48/221*b*(b*x+a)^(5/4)/a^2/x^(13/4)-128/663
*b^2*(b*x+a)^(5/4)/a^3/x^(9/4)+512/3315*b^3*(b*x+a)^(5/4)/a^4/x^(5/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx=-\frac{4(a+bx)^{5/4}\left(195a^3-180a^2bx+160ab^2x^2-128b^3x^3\right)}{3315a^4x^{17/4}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(21/4),x]
```

output
```
(-4*(a + b*x)^(5/4)*(195*a^3 - 180*a^2*b*x + 160*a*b^2*x^2 - 128*b^3*x^3))
/(3315*a^4*x^(17/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{12b\int \frac{\sqrt[4]{a+bx}}{x^{17/4}}dx}{17a}-\frac{4(a+bx)^{5/4}}{17ax^{17/4}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{12b\left(-\frac{8b\int \frac{\sqrt[4]{a+bx}}{x^{13/4}}dx}{13a}-\frac{4(a+bx)^{5/4}}{13ax^{13/4}}\right)}{17a}-\frac{4(a+bx)^{5/4}}{17ax^{17/4}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{12b\left(-\frac{8b\left(-\frac{4b\int \frac{\sqrt[4]{a+bx}}{x^{9/4}}dx}{9a}-\frac{4(a+bx)^{5/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{5/4}}{13ax^{13/4}}\right)}{17a}-\frac{4(a+bx)^{5/4}}{17ax^{17/4}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{12b\left(-\frac{8b\left(\frac{16b(a+bx)^{5/4}}{45a^2x^{5/4}}-\frac{4(a+bx)^{5/4}}{9ax^{9/4}}\right)}{13a}-\frac{4(a+bx)^{5/4}}{13ax^{13/4}}\right)}{17a}-\frac{4(a+bx)^{5/4}}{17ax^{17/4}}$$

input
```
Int[(a + b*x)^(1/4)/x^(21/4),x]
```

output
```
(-4*(a + b*x)^(5/4))/(17*a*x^(17/4)) - (12*b*((-4*(a + b*x)^(5/4))/(13*a*x
^(13/4)) - (8*b*((-4*(a + b*x)^(5/4))/(9*a*x^(9/4)) + (16*b*(a + b*x)^(5/4
))/(45*a^2*x^(5/4))))/(13*a)))/(17*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{5}{4}}\left(-128b^{3}x^{3}+160a\,b^{2}x^{2}-180a^{2}bx+195a^{3}\right)}{3315x^{\frac{17}{4}}a^{4}}$	46
orering	$-\frac{4(bx+a)^{\frac{5}{4}}\left(-128b^{3}x^{3}+160a\,b^{2}x^{2}-180a^{2}bx+195a^{3}\right)}{3315x^{\frac{17}{4}}a^{4}}$	46
risch	$-\frac{4(bx+a)^{\frac{1}{4}}\left(-128b^{4}x^{4}+32a\,x^{3}b^{3}-20a^{2}b^{2}x^{2}+15a^{3}bx+195a^{4}\right)}{3315x^{\frac{17}{4}}a^{4}}$	57

input
```
int((b*x+a)^(1/4)/x^(21/4),x,method=_RETURNVERBOSE)
```

output
```
-4/3315*(b*x+a)^(5/4)*(-128*b^3*x^3+160*a*b^2*x^2-180*a^2*b*x+195*a^3)/x^(
17/4)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx = \frac{4\,(128\,b^4x^4 - 32\,ab^3x^3 + 20\,a^2b^2x^2 - 15\,a^3bx - 195\,a^4)(bx+a)^{\frac{1}{4}}}{3315\,a^4x^{\frac{17}{4}}}$$

input
```
integrate((b*x+a)^(1/4)/x^(21/4),x, algorithm="fricas")
```

output
```
4/3315*(128*b^4*x^4 - 32*a*b^3*x^3 + 20*a^2*b^2*x^2 - 15*a^3*b*x - 195*a^4
)*(b*x + a)^(1/4)/(a^4*x^(17/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(1/4)/x**(21/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx = \frac{4\left(\frac{663\,(bx+a)^{\frac{5}{4}}b^3}{x^{\frac{5}{4}}} - \frac{1105\,(bx+a)^{\frac{9}{4}}b^2}{x^{\frac{9}{4}}} + \frac{765\,(bx+a)^{\frac{13}{4}}b}{x^{\frac{13}{4}}} - \frac{195\,(bx+a)^{\frac{17}{4}}}{x^{\frac{17}{4}}}\right)}{3315\,a^4}$$

input
```
integrate((b*x+a)^(1/4)/x^(21/4),x, algorithm="maxima")
```

output
```
4/3315*(663*(b*x + a)^(5/4)*b^3/x^(5/4) - 1105*(b*x + a)^(9/4)*b^2/x^(9/4)
 + 765*(b*x + a)^(13/4)*b/x^(13/4) - 195*(b*x + a)^(17/4)/x^(17/4))/a^4
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{21}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(21/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(21/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx = -\frac{(a+b\,x)^{1/4}\left(\frac{128\,b^3\,x^3}{3315\,a^3}-\frac{16\,b^2\,x^2}{663\,a^2}-\frac{512\,b^4\,x^4}{3315\,a^4}+\frac{4\,b\,x}{221\,a}+\frac{4}{17}\right)}{x^{17/4}}$$

input
```
int((a + b*x)^(1/4)/x^(21/4),x)
```

output
```
-((a + b*x)^(1/4)*((128*b^3*x^3)/(3315*a^3) - (16*b^2*x^2)/(663*a^2) - (51
2*b^4*x^4)/(3315*a^4) + (4*b*x)/(221*a) + 4/17))/x^(17/4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt[4]{a+bx}}{x^{21/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\left(128b^4x^4-32a\,b^3x^3+20a^2b^2x^2-15a^3bx-195a^4\right)}{3315x^{\frac{17}{4}}a^4}$$

input
```
int((b*x+a)^(1/4)/x^(21/4),x)
```

output
```
(4*(a + b*x)**(1/4)*( - 195*a**4 - 15*a**3*b*x + 20*a**2*b**2*x**2 - 32*a*
b**3*x**3 + 128*b**4*x**4))/(3315*x**(1/4)*a**4*x**4)
```

3.691 $\int x^{9/4}\sqrt[4]{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 151

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx=\frac{3a^3\sqrt[4]{x}\sqrt[4]{a+bx}}{28b^3}-\frac{3a^2x^{5/4}\sqrt[4]{a+bx}}{70b^2}+\frac{ax^{9/4}\sqrt[4]{a+bx}}{35b}+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}+\frac{3a^{7/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{28b^4x^{3/4}}$$

output
```
3/28*a^3*x^(1/4)*(b*x+a)^(1/4)/b^3-3/70*a^2*x^(5/4)*(b*x+a)^(1/4)/b^2+1/35
*a*x^(9/4)*(b*x+a)^(1/4)/b+2/7*x^(13/4)*(b*x+a)^(1/4)+3/28*a^(7/2)*(b*x/(b
*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2
)),2^(1/2))/b^4/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.31

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx=\frac{4x^{13/4}\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{13}{4},\frac{17}{4},-\frac{bx}{a}\right)}{13\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[x^(9/4)*(a + b*x)^(1/4),x]
```

output
```
(4*x^(13/4)*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 13/4, 17/4, -((b*x)/a)
])/(13*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {60, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{14}a\int \frac{x^{9/4}}{(a+bx)^{3/4}}dx+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int\frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$\downarrow$ 73

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$\downarrow$ 768

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)}{10b}\right)+\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$\downarrow$ 858

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)+$$

$$\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 807

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)+$$

$$\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 229

$$\frac{1}{14}a\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)+$$

$$\frac{2}{7}x^{13/4}\sqrt[4]{a+bx}$$

input
```
Int[x^(9/4)*(a + b*x)^(1/4),x]
```

output
```
(2*x^(13/4)*(a + b*x)^(1/4))/7 + (a*((2*x^(9/4)*(a + b*x)^(1/4))/(5*b) - (
9*a*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b) - (5*a*((2*x^(1/4)*(a + b*x)^(1/4))
/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt
[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(6*b)))/(10*b)))/14
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(9/4)*(b*x+a)^(1/4),x)
```

output
```
int(x^(9/4)*(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{9}{4}}\,dx$$

input
```
integrate(x^(9/4)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(9/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.92 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.25

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx = \frac{\sqrt[4]{a}x^{\frac{13}{4}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**(9/4)*(b*x+a)**(1/4),x)
```

output
```
a**(1/4)*x**(13/4)*gamma(13/4)*hyper((-1/4, 13/4), (17/4,), b*x*exp_polar(
I*pi)/a)/gamma(17/4)
```

Maxima [F]

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}x^{\frac{9}{4}}\,dx$$

input
```
integrate(x^(9/4)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)*x^(9/4), x)
```

Giac [F]

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{9}{4}}\,dx$$

input
```
integrate(x^(9/4)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx=\int x^{9/4}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(9/4)*(a + b*x)^(1/4),x)
```

output
```
int(x^(9/4)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int x^{9/4}\sqrt[4]{a+bx}\,dx=\frac{60x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}\,a^3-24x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}\,a^2b+16x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}\,a\,b^2+160x^{\frac{13}{4}}(bx+a)^{\frac{1}{4}}\,b^3-}{560b^3}$$

input
```
int(x^(9/4)*(b*x+a)^(1/4),x)
```

output
```
(60*x**(1/4)*(a + b*x)**(1/4)*a**3 - 24*x**(1/4)*(a + b*x)**(1/4)*a**2*b*x
 + 16*x**(1/4)*(a + b*x)**(1/4)*a*b**2*x**2 + 160*x**(1/4)*(a + b*x)**(1/4
)*b**3*x**3 - 15*int((a + b*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**4)
/(560*b**3)
```

3.692 $\int x^{5/4} \sqrt[4]{a+bx}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int x^{5/4} \sqrt[4]{a+bx}\, dx = -\frac{a^2 \sqrt[4]{x} \sqrt[4]{a+bx}}{6b^2} + \frac{a x^{5/4} \sqrt[4]{a+bx}}{15b} + \frac{2}{5} x^{9/4} \sqrt[4]{a+bx} - \frac{a^{5/2} \left(\frac{bx}{a+bx}\right)^{3/4} (a+bx)^{3/4} \text{EllipticF}\left(\frac{1}{2} \arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right), 2\right)}{6b^3 x^{3/4}}$$

output
```
-1/6*a^2*x^(1/4)*(b*x+a)^(1/4)/b^2+1/15*a*x^(5/4)*(b*x+a)^(1/4)/b+2/5*x^(9
/4)*(b*x+a)^(1/4)-1/6*a^(5/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJac
obiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^3/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int x^{5/4} \sqrt[4]{a+bx}\, dx = \frac{4x^{9/4} \sqrt[4]{a+bx} \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{9}{4}, \frac{13}{4}, -\frac{bx}{a}\right)}{9 \sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[x^(5/4)*(a + b*x)^(1/4),x]
```

output
```
(4*x^(9/4)*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 9/4, 13/4, -((b*x)/a)])
/(9*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/4}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{10}a\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{10}a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{10}a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{10}a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{1}{10}a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 858

$$\frac{1}{10}a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 807

$$\frac{1}{10}a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

$\Big\downarrow$ 229

$$\frac{1}{10}a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)+\frac{2}{5}x^{9/4}\sqrt[4]{a+bx}$$

input
```
Int[x^(5/4)*(a + b*x)^(1/4),x]
```

output
```
(2*x^(9/4)*(a + b*x)^(1/4))/5 + (a*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b) - (5
*a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)
*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/
4))))/(6*b)))/10
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(5/4)*(b*x+a)^(1/4),x)
```

output
```
int(x^(5/4)*(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int x^{5/4}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}\,dx$$

input
```
integrate(x^(5/4)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(5/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.29

$$\int x^{5/4}\sqrt[4]{a+bx}\,dx=\frac{\sqrt[4]{a}x^{\frac{9}{4}}\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**(5/4)*(b*x+a)**(1/4),x)
```

output
```
a**(1/4)*x**(9/4)*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x*exp_polar(I*p
i)/a)/gamma(13/4)
```

Maxima [F]

$$\int x^{5/4}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}\,dx$$

input
```
integrate(x^(5/4)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)*x^(5/4), x)
```

Giac [F]

$$\int x^{5/4}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}\,dx$$

input
```
integrate(x^(5/4)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/4}\sqrt[4]{a+bx}\,dx=\int x^{5/4}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(5/4)*(a + b*x)^(1/4),x)
```

output
```
int(x^(5/4)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int x^{5/4}\sqrt[4]{a+bx}\,dx=\frac{-20x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}a^2+8x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}ab+48x^{\frac{9}{4}}(bx+a)^{\frac{1}{4}}b^2+5\left(\int\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a^3}{120b^2}$$

input
```
int(x^(5/4)*(b*x+a)^(1/4),x)
```

output
```
( - 20*x**(1/4)*(a + b*x)**(1/4)*a**2 + 8*x**(1/4)*(a + b*x)**(1/4)*a*b*x
+ 48*x**(1/4)*(a + b*x)**(1/4)*b**2*x**2 + 5*int((a + b*x)**(1/4)/(x**(3/4
)*a + x**(3/4)*b*x),x)*a**3)/(120*b**2)
```

3.693 $\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx = \frac{a\sqrt[4]{x}\sqrt[4]{a+bx}}{3b}+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx} + \frac{a^{3/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3b^2x^{3/4}}$$

output
```
1/3*a*x^(1/4)*(b*x+a)^(1/4)/b+2/3*x^(5/4)*(b*x+a)^(1/4)+1/3*a^(3/2)*(b*x/(
b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/
2)),2^(1/2))/b^2/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx = \frac{4x^{5/4}\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{9}{4},-\frac{bx}{a}\right)}{5\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[x^(1/4)*(a + b*x)^(1/4),x]
```

output
```
(4*x^(5/4)*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 9/4, -((b*x)/a)])/
(5*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{6}a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{6}a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{6}a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{6}a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)+\frac{2}{3}x^{5/4}\sqrt[4]{a+bx}$$

input
```
Int[x^(1/4)*(a + b*x)^(1/4),x]
```

output
```
(2*x^(5/4)*(a + b*x)^(1/4))/3 + (a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqr
t[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b
]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}\,dx$$

input
```
int(x^(1/4)*(b*x+a)^(1/4),x)
```

output
```
int(x^(1/4)*(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx=\int (bx+a)^{\frac{1}{4}}x^{\frac{1}{4}}\,dx$$

input
```
integrate(x^(1/4)*(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.36

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx = \frac{\sqrt[4]{a}x^{\frac{5}{4}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(1/4)*(b*x+a)**(1/4),x)
```

output
```
a**(1/4)*x**(5/4)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x*exp_polar(I*pi
)/a)/gamma(9/4)
```

Maxima [F]

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}x^{\frac{1}{4}}\,dx$$

input
```
integrate(x^(1/4)*(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)*x^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx = \int (bx+a)^{\frac{1}{4}}x^{\frac{1}{4}}\,dx$$

input
```
integrate(x^(1/4)*(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)*x^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx=\int x^{1/4}\,(a+b\,x)^{1/4}\,dx$$

input
```
int(x^(1/4)*(a + b*x)^(1/4),x)
```

output
```
int(x^(1/4)*(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \sqrt[4]{x}\sqrt[4]{a+bx}\,dx=\frac{4x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}\,a+8x^{\frac{5}{4}}(bx+a)^{\frac{1}{4}}\,b-\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a^{2}}{12b}$$

input
```
int(x^(1/4)*(b*x+a)^(1/4),x)
```

output
```
(4*x**(1/4)*(a + b*x)**(1/4)*a + 8*x**(1/4)*(a + b*x)**(1/4)*b*x - int((a
+ b*x)**(1/4)/(x**(3/4)*a + x**(3/4)*b*x),x)*a**2)/(12*b)
```

3.694 $\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx = 2\sqrt[4]{x}\sqrt[4]{a+bx} - \frac{2\sqrt{a}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{bx^{3/4}}$$

output
```
2*x^(1/4)*(b*x+a)^(1/4)-2*a^(1/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*Invers
eJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx = \frac{4\sqrt[4]{x}\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx}{a}\right)}{\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(3/4),x]
```

output
```
(4*x^(1/4)*(a + b*x)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x)/a)])/
(1 + (b*x)/a)^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx+2\sqrt[4]{x}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 73}$$

$$2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}+2\sqrt[4]{x}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{(a+bx)^{3/4}}+2\sqrt[4]{x}\sqrt[4]{a+bx}$$

$$\Big\downarrow \text{ 858}$$

$$2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 807}$$

$$2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 229}$$

$$2\sqrt[4]{x}\sqrt[4]{a+bx}-\frac{2\sqrt{a}\sqrt{b}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{(a+bx)^{3/4}}$$

input
```
Int[(a + b*x)^(1/4)/x^(3/4),x]
```

output
```
2*x^(1/4)*(a + b*x)^(1/4) - (2*Sqrt[a]*Sqrt[b]*(1 + a/(b*x))^(3/4)*x^(3/4)
*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(a + b*x)^(3/4)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(3/4),x)
```

output
```
int((b*x+a)^(1/4)/x^(3/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/x^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx = \frac{\sqrt[4]{a}\sqrt[4]{x}\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x+a)**(1/4)/x**(3/4),x)
```

output
```
a**(1/4)*x**(1/4)*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x*exp_polar(I*pi
)/a)/gamma(5/4)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/x^(3/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx=\int \frac{{\left(a+b\,x\right)}^{1/4}}{x^{3/4}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(3/4),x)
```

output
```
int((a + b*x)^(1/4)/x^(3/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{3/4}}\,dx=2x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}+\frac{\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{3}{4}}a+x^{\frac{7}{4}}b}dx\right)a}{2}$$

input
```
int((b*x+a)^(1/4)/x^(3/4),x)
```

output
```
(4*x**(1/4)*(a + b*x)**(1/4) + int((a + b*x)**(1/4)/(x**(3/4)*a + x**(3/4)
*b*x),x)*a)/2
```

3.695 $\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{3x^{3/4}} - \frac{4\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3\sqrt{a}x^{3/4}}$$

output
```
-4/3*(b*x+a)^(1/4)/x^(3/4)-4/3*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJa
cobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(1/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx = -\frac{4\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{1}{4},-\frac{bx}{a}\right)}{3x^{3/4}\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(7/4),x]
```

output
```
(-4*(a + b*x)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x)/a)])/(3*x^(
3/4)*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {57, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{1}{3}b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4}{3}b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{4b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3\sqrt{a}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3x^{3/4}}$$

input
```
Int[(a + b*x)^(1/4)/x^(7/4),x]
```

output
```
(-4*(a + b*x)^(1/4))/(3*x^(3/4)) - (4*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*
EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*Sqrt[a]*(a + b*x)^(3
/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{4}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(7/4),x)
```

output
```
int((b*x+a)^(1/4)/x^(7/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/x^(7/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx = -\frac{2\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx}\right)}{\sqrt{x}}$$

input
```
integrate((b*x+a)**(1/4)/x**(7/4),x)
```

output
```
-2*b**(1/4)*hyper((-1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x))/sqrt(x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(7/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/x^(7/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(7/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx=\int \frac{{\left(a+b\,x\right)}^{1/4}}{x^{7/4}} \,d x$$

input
```
int((a + b*x)^(1/4)/x^(7/4),x)
```

output
```
int((a + b*x)^(1/4)/x^(7/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{7/4}}\,dx=\frac{-4(bx+a)^{\frac{1}{4}}-x^{\frac{3}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{7}{4}} a+x^{\frac{11}{4}} b}d x\right) a}{2x^{\frac{3}{4}}}$$

input
```
int((b*x+a)^(1/4)/x^(7/4),x)
```

output
```
( - 4*(a + b*x)**(1/4) - x**(3/4)*int((a + b*x)**(1/4)/(x**(3/4)*a*x + x**
(3/4)*b*x**2),x)*a)/(2*x**(3/4))
```

3.696 $\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{7x^{7/4}} - \frac{4b\sqrt[4]{a+bx}}{21ax^{3/4}} + \frac{8b\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{21a^{3/2}x^{3/4}}$$

output
```
-4/7*(b*x+a)^(1/4)/x^(7/4)-4/21*b*(b*x+a)^(1/4)/a/x^(3/4)+8/21*b*(b*x/(b*x
+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))
,2^(1/2))/a^(3/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = -\frac{4\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{4},-\frac{3}{4},-\frac{bx}{a}\right)}{7x^{7/4}\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(11/4),x]
```

output
```
(-4*(a + b*x)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x)/a)])/(7*x^
(7/4)*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {57, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{1}{7}b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{7}b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{7}b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{1}{7}b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{1}{7}b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}$$

$\Big\downarrow$ 807

$$\frac{1}{7}b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}$$

$\Big\downarrow$ 229

$$\frac{1}{7}b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)-\frac{4\sqrt[4]{a+bx}}{7x^{7/4}}$$

input
```
Int[(a + b*x)^(1/4)/x^(11/4),x]
```

output
```
(-4*(a + b*x)^(1/4))/(7*x^(7/4)) + (b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4))
+ (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x]
)/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/7
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{11}{4}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(11/4),x)
```

output
```
int((b*x+a)^(1/4)/x^(11/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{11}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(11/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/x^(11/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = -\frac{2\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx}\right)}{3x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(1/4)/x**(11/4),x)
```

output
```
-2*b**(1/4)*hyper((-1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x))/(3*x**(3/2
))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{11}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(11/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/x^(11/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{11}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(11/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = \int \frac{(a+b\,x)^{1/4}}{x^{11/4}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(11/4),x)
```

output
```
int((a + b*x)^(1/4)/x^(11/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{11/4}}\,dx = \frac{-4(bx+a)^{\frac{1}{4}} - x^{\frac{7}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{11}{4}}a+x^{\frac{15}{4}}b}dx\right)a}{6x^{\frac{7}{4}}}$$

input
```
int((b*x+a)^(1/4)/x^(11/4),x)
```

output
```
( - 4*(a + b*x)**(1/4) - x**(3/4)*int((a + b*x)**(1/4)/(x**(3/4)*a*x**2 +
x**(3/4)*b*x**3),x)*a*x)/(6*x**(3/4)*x)
```

3.697 $\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{11x^{11/4}} - \frac{4b\sqrt[4]{a+bx}}{77ax^{7/4}} + \frac{8b^2\sqrt[4]{a+bx}}{77a^2x^{3/4}} - \frac{16b^2\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{77a^{5/2}x^{3/4}}$$

output
```
-4/11*(b*x+a)^(1/4)/x^(11/4)-4/77*b*(b*x+a)^(1/4)/a/x^(7/4)+8/77*b^2*(b*x+
a)^(1/4)/a^2/x^(3/4)-16/77*b^2*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJa
cobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(5/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = -\frac{4\sqrt[4]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},-\frac{7}{4},-\frac{bx}{a}\right)}{11x^{11/4}\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/4)/x^(15/4),x]
```

output
```
(-4*(a + b*x)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x)/a)])/(11*
x^(11/4)*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {57, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}dx$$

$$\Big\downarrow \ 57$$

$$\frac{1}{11}b\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

$$\Big\downarrow \ 61$$

$$\frac{1}{11}b\left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

$$\Big\downarrow \ 61$$

$$\frac{1}{11}b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{11}b\left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

$$\Big\downarrow \; 768$$

$$\frac{1}{11}b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

$$\Big\downarrow \; 858$$

$$\frac{1}{11}b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{1}{11}b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{1}{11}b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)-\frac{4\sqrt[4]{a+bx}}{11x^{11/4}}$$

input

```
Int[(a + b*x)^(1/4)/x^(15/4),x]
```

output
```
(-4*(a + b*x)^(1/4))/(11*x^(11/4)) + (b*((-4*(a + b*x)^(1/4))/(7*a*x^(7/4)
) - (6*b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3
/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*
(a + b*x)^(3/4))))/(7*a)))/11
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{15}{4}}}dx$$

input
```
int((b*x+a)^(1/4)/x^(15/4),x)
```

output
```
int((b*x+a)^(1/4)/x^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{15}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(15/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)/x^(15/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 66.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = \frac{\sqrt[4]{a}\Gamma\left(-\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix} -\frac{11}{4}, -\frac{1}{4} \\ -\frac{7}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{x^{\frac{11}{4}}\Gamma\left(-\frac{7}{4}\right)}$$

input
```
integrate((b*x+a)**(1/4)/x**(15/4),x)
```

output
```
a**(1/4)*gamma(-11/4)*hyper((-11/4, -1/4), (-7/4,), b*x*exp_polar(I*pi)/a)
/(x**(11/4)*gamma(-7/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{15}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(15/4),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/4)/x^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = \int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{15}{4}}}\,dx$$

input
```
integrate((b*x+a)^(1/4)/x^(15/4),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/4)/x^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = \int \frac{(a+b\,x)^{1/4}}{x^{15/4}}\,dx$$

input
```
int((a + b*x)^(1/4)/x^(15/4),x)
```

output
```
int((a + b*x)^(1/4)/x^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx}}{x^{15/4}}\,dx = \frac{-4(bx+a)^{\frac{1}{4}} - x^{\frac{11}{4}}\left(\int \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{15}{4}}a+x^{\frac{19}{4}}b}dx\right)a}{10x^{\frac{11}{4}}}$$

input
```
int((b*x+a)^(1/4)/x^(15/4),x)
```

output
```
( - 4*(a + b*x)**(1/4) - x**(3/4)*int((a + b*x)**(1/4)/(x**(3/4)*a*x**3 +
x**(3/4)*b*x**4),x)*a*x**2)/(10*x**(3/4)*x**2)
```

3.698 $\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = \frac{15a^2\sqrt[4]{x}(a+bx)^{3/4}}{32b^3} - \frac{3ax^{5/4}(a+bx)^{3/4}}{8b^2} + \frac{x^{9/4}(a+bx)^{3/4}}{3b} - \frac{15a^3\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{64b^{13/4}} - \frac{15a^3\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{64b^{13/4}}$$

output
```
15/32*a^2*x^(1/4)*(b*x+a)^(3/4)/b^3-3/8*a*x^(5/4)*(b*x+a)^(3/4)/b^2+1/3*x^
(9/4)*(b*x+a)^(3/4)/b-15/64*a^3*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(1
3/4)-15/64*a^3*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = \frac{2\sqrt[4]{b}\sqrt[4]{x}(a+bx)^{3/4}\left(45a^2-36abx+32b^2x^2\right) - 45a^3\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right) - 45a^3\text{arctanh}\left(\right.}{192b^{13/4}}$$

input
```
Integrate[x^(9/4)/(a + b*x)^(1/4),x]
```

output
```
(2*b^(1/4)*x^(1/4)*(a + b*x)^(3/4)*(45*a^2 - 36*a*b*x + 32*b^2*x^2) - 45*a
^3*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] - 45*a^3*ArcTanh[(b^(1/4)*x^(
1/4))/(a + b*x)^(1/4)])/(192*b^(13/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {60, 60, 60, 73, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}dx}{4b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}dx}{8b}\right)}{4b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}dx}{4b}\right)}{8b}\right)}{4b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{8b}\right)}{4b}$$

$$\Big\downarrow \; 770$$

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}\right)}{4b}$$

$$\Big\downarrow \; 756$$

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{1}{2}\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b}\right)}{8b}\right)}{4b}$$

$$\Big\downarrow \; 216$$

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{8b}\right)}{4b}$$

$\Big\downarrow$ 219

$$\frac{x^{9/4}(a+bx)^{3/4}}{3b}-\frac{3a\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{8b}\right)}{4b}$$

input
```
Int[x^(9/4)/(a + b*x)^(1/4),x]
```

output
```
(x^(9/4)*(a + b*x)^(3/4))/(3*b) - (3*a*((x^(5/4)*(a + b*x)^(3/4))/(2*b) -
(5*a*((x^(1/4)*(a + b*x)^(3/4))/b - (a*(ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)
^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4
))))/b))/(8*b)))/(4*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(9/4)/(b*x+a)^(1/4),x)
```

output
```
int(x^(9/4)/(b*x+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.12

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = -\frac{45\,b^3\left(\frac{a^{12}}{b^{13}}\right)^{\frac{1}{4}}\log\left(\frac{15\left((bx+a)^{\frac{3}{4}}a^3x^{\frac{1}{4}}+(b^4x+ab^3)\left(\frac{a^{12}}{b^{13}}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-45\,b^3\left(\frac{a^{12}}{b^{13}}\right)^{\frac{1}{4}}\log\left(\frac{15\left((bx+a)^{\frac{3}{4}}a^3x^{\frac{1}{4}}-(b^4x+ab^3)\left(\frac{a^{12}}{b^{13}}\right)^{\frac{1}{4}}\right)}{bx+a}\right.}{}$$

input
```
integrate(x^(9/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/384*(45*b^3*(a^12/b^13)^(1/4)*log(15*((b*x + a)^(3/4)*a^3*x^(1/4) + (b^
4*x + a*b^3)*(a^12/b^13)^(1/4))/(b*x + a)) - 45*b^3*(a^12/b^13)^(1/4)*log(
15*((b*x + a)^(3/4)*a^3*x^(1/4) - (b^4*x + a*b^3)*(a^12/b^13)^(1/4))/(b*x
+ a)) - 45*I*b^3*(a^12/b^13)^(1/4)*log(15*((b*x + a)^(3/4)*a^3*x^(1/4) - (
I*b^4*x + I*a*b^3)*(a^12/b^13)^(1/4))/(b*x + a)) + 45*I*b^3*(a^12/b^13)^(1
/4)*log(15*((b*x + a)^(3/4)*a^3*x^(1/4) - (-I*b^4*x - I*a*b^3)*(a^12/b^13)
^(1/4))/(b*x + a)) - 4*(32*b^2*x^2 - 36*a*b*x + 45*a^2)*(b*x + a)^(3/4)*x^
(1/4))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = \frac{x^{\frac{13}{4}}\Gamma\left(\frac{13}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**(9/4)/(b*x+a)**(1/4),x)
```

output
```
x**(13/4)*gamma(13/4)*hyper((1/4, 13/4), (17/4,), b*x*exp_polar(I*pi)/a)/(
a**(1/4)*gamma(17/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = \frac{15\,a^3\left(\frac{2\,\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{1}{4}}}\right)}{128\,b^3} - \frac{\frac{113\,(bx+a)^{\frac{3}{4}}a^3b^2}{x^{\frac{3}{4}}}-\frac{126\,(bx+a)^{\frac{7}{4}}a^3b}{x^{\frac{7}{4}}}+\frac{45\,(bx+a)^{\frac{11}{4}}a^3}{x^{\frac{11}{4}}}}{96\left(b^6-\frac{3\,(bx+a)b^5}{x}+\frac{3\,(bx+a)^2b^4}{x^2}-\frac{(bx+a)^3b^3}{x^3}\right)}$$

input
```
integrate(x^(9/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
15/128*a^3*(2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(1/4) + log(-(b^
(1/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(1
/4))/b^3 - 1/96*(113*(b*x + a)^(3/4)*a^3*b^2/x^(3/4) - 126*(b*x + a)^(7/4)
*a^3*b/x^(7/4) + 45*(b*x + a)^(11/4)*a^3/x^(11/4))/(b^6 - 3*(b*x + a)*b^5/
x + 3*(b*x + a)^2*b^4/x^2 - (b*x + a)^3*b^3/x^3)
```

Giac [F]

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(9/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^(9/4)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{9/4}}{(a+b\,x)^{1/4}}\,dx$$

input
```
int(x^(9/4)/(a + b*x)^(1/4),x)
```

output
```
int(x^(9/4)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{9/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(9/4)/(b*x+a)^(1/4),x)
```

output
```
int((x**(1/4)*x**2)/(a + b*x)**(1/4),x)
```

3.699 $\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx=-\frac{5a\sqrt[4]{x}(a+bx)^{3/4}}{8b^2}+\frac{x^{5/4}(a+bx)^{3/4}}{2b}+\frac{5a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{9/4}}+\frac{5a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{9/4}}$$

output
```
-5/8*a*x^(1/4)*(b*x+a)^(3/4)/b^2+1/2*x^(5/4)*(b*x+a)^(3/4)/b+5/16*a^2*arct
an(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(9/4)+5/16*a^2*arctanh(b^(1/4)*x^(1/4)
/(b*x+a)^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx=\frac{2\sqrt[4]{b}\sqrt[4]{x}(a+bx)^{3/4}(-5a+4bx)+5a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)+5a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{9/4}}$$

input
```
Integrate[x^(5/4)/(a + b*x)^(1/4),x]
```

output
```
(2*b^(1/4)*x^(1/4)*(a + b*x)^(3/4)*(-5*a + 4*b*x) + 5*a^2*ArcTan[(b^(1/4)*
x^(1/4))/(a + b*x)^(1/4)] + 5*a^2*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4
)])/(16*b^(9/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {60, 60, 73, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}dx}{8b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}dx}{4b}\right)}{8b}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{8b}$$

$$\Big\downarrow \text{770}$$

$$\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}$$

$$\Big\downarrow \text{756}$$

$$\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{1}{2}\int\frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b}\right)}{8b}$$

$\downarrow$ 216

$$\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{8b}$$

$\downarrow$ 219

$$\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{8b}$$

input
```
Int[x^(5/4)/(a + b*x)^(1/4),x]
```

output
```
(x^(5/4)*(a + b*x)^(3/4))/(2*b) - (5*a*((x^(1/4)*(a + b*x)^(3/4))/b - (a*(
ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x
^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4))))/b))/(8*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(5/4)/(b*x+a)^(1/4),x)
```

output
```
int(x^(5/4)/(b*x+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.48

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx = \frac{5\,b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{5\left((bx+a)^{\frac{3}{4}}a^2x^{\frac{1}{4}}+(b^3x+ab^2)\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-5\,b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{5\left((bx+a)^{\frac{3}{4}}a^2x^{\frac{1}{4}}-(b^3x+ab^2\right.}{bx+a}\right.}{}$$

input
```
integrate(x^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
1/32*(5*b^2*(a^8/b^9)^(1/4)*log(5*((b*x + a)^(3/4)*a^2*x^(1/4) + (b^3*x +
a*b^2)*(a^8/b^9)^(1/4))/(b*x + a)) - 5*b^2*(a^8/b^9)^(1/4)*log(5*((b*x + a
)^(3/4)*a^2*x^(1/4) - (b^3*x + a*b^2)*(a^8/b^9)^(1/4))/(b*x + a)) - 5*I*b^
2*(a^8/b^9)^(1/4)*log(5*((b*x + a)^(3/4)*a^2*x^(1/4) - (I*b^3*x + I*a*b^2)
*(a^8/b^9)^(1/4))/(b*x + a)) + 5*I*b^2*(a^8/b^9)^(1/4)*log(5*((b*x + a)^(3
/4)*a^2*x^(1/4) - (-I*b^3*x - I*a*b^2)*(a^8/b^9)^(1/4))/(b*x + a)) + 4*(4*
b*x - 5*a)*(b*x + a)^(3/4)*x^(1/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.83 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx = \frac{x^{\frac{9}{4}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**(5/4)/(b*x+a)**(1/4),x)
```

output
```
x**(9/4)*gamma(9/4)*hyper((1/4, 9/4), (13/4,), b*x*exp_polar(I*pi)/a)/(a**
(1/4)*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx = -\frac{5\,a^2\left(\frac{2\,\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{1}{4}}}\right)}{32\,b^2} + \frac{\frac{9\,(bx+a)^{\frac{3}{4}}a^2b}{x^{\frac{3}{4}}} - \frac{5\,(bx+a)^{\frac{7}{4}}a^2}{x^{\frac{7}{4}}}}{8\left(b^4 - \frac{2\,(bx+a)b^3}{x} + \frac{(bx+a)^2b^2}{x^2}\right)}$$

input
```
integrate(x^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
-5/32*a^2*(2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(1/4) + log(-(b^(
1/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(1/
4))/b^2 + 1/8*(9*(b*x + a)^(3/4)*a^2*b/x^(3/4) - 5*(b*x + a)^(7/4)*a^2/x^(
7/4))/(b^4 - 2*(b*x + a)*b^3/x + (b*x + a)^2*b^2/x^2)
```

Giac [F]

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(5/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^(5/4)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{5/4}}{(a+b\,x)^{1/4}}\,dx$$

input
```
int(x^(5/4)/(a + b*x)^(1/4),x)
```

output
```
int(x^(5/4)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
int(x^(5/4)/(b*x+a)^(1/4),x)
```

output
```
int((x**(1/4)*x)/(a + b*x)**(1/4),x)
```

3.700 $\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx=\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{5/4}}-\frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{5/4}}$$

output
```
x^(1/4)*(b*x+a)^(3/4)/b-1/2*a*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(5/4
)-1/2*a*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx=\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{5/4}}-\frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{5/4}}$$

input
```
Integrate[x^(1/4)/(a + b*x)^(1/4),x]
```

output
```
(x^(1/4)*(a + b*x)^(3/4))/b - (a*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]
)/(2*b^(5/4)) - (a*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])/(2*b^(5/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {60, 73, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}dx}{4b}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}$$

$$\Big\downarrow \text{770}$$

$$\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}$$

$$\Big\downarrow \text{756}$$

$$\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{1}{2}\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}$$

$$\downarrow 219$$

$$\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}$$

input
```
Int[x^(1/4)/(a + b*x)^(1/4),x]
```

output
```
(x^(1/4)*(a + b*x)^(3/4))/b - (a*(ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)
]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4))))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(1/4)/(b*x+a)^(1/4),x)
```

output
```
int(x^(1/4)/(b*x+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx =$$

$$-\frac{b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}ax^{\frac{1}{4}}+(b^2x+ab)\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}}{bx+a}\right)-b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}ax^{\frac{1}{4}}-(b^2x+ab)\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}}{bx+a}\right)-ib\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{(bx\cdots}{\cdots}\right.}{4b}$$

input
```
integrate(x^(1/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/4*(b*(a^4/b^5)^(1/4)*log(((b*x + a)^(3/4)*a*x^(1/4) + (b^2*x + a*b)*(a^
4/b^5)^(1/4))/(b*x + a)) - b*(a^4/b^5)^(1/4)*log(((b*x + a)^(3/4)*a*x^(1/4
) - (b^2*x + a*b)*(a^4/b^5)^(1/4))/(b*x + a)) - I*b*(a^4/b^5)^(1/4)*log(((
b*x + a)^(3/4)*a*x^(1/4) - (I*b^2*x + I*a*b)*(a^4/b^5)^(1/4))/(b*x + a)) +
 I*b*(a^4/b^5)^(1/4)*log(((b*x + a)^(3/4)*a*x^(1/4) - (-I*b^2*x - I*a*b)*(
a^4/b^5)^(1/4))/(b*x + a)) - 4*(b*x + a)^(3/4)*x^(1/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx = \frac{x^{\frac{5}{4}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(1/4)/(b*x+a)**(1/4),x)
```

output
```
x**(5/4)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x*exp_polar(I*pi)/a)/(a**(
1/4)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx = \frac{a\left(\frac{2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{1}{4}}}\right)}{4\,b} - \frac{(bx+a)^{\frac{3}{4}}a}{\left(b^2-\frac{(bx+a)b}{x}\right)x^{\frac{3}{4}}}$$

input
```
integrate(x^(1/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
1/4*a*(2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(1/4) + log(-(b^(1/4)
 - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(1/4))/
b - (b*x + a)^(3/4)*a/((b^2 - (b*x + a)*b/x)*x^(3/4))
```

Giac [F]

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(1/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^(1/4)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{1/4}}{(a+b\,x)^{1/4}}\,dx$$

input
```
int(x^(1/4)/(a + b*x)^(1/4),x)
```

output
```
int(x^(1/4)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(1/4)/(b*x+a)^(1/4),x)
```

output
```
int(x**(1/4)/(a + b*x)**(1/4),x)
```

3.701 $\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx = \frac{2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}} + \frac{2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}}$$

output
```
2*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(1/4)+2*arctanh(b^(1/4)*x^(1/4)/
(b*x+a)^(1/4))/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx = \frac{2\left(\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right) + \text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)\right)}{\sqrt[4]{b}}$$

input
```
Integrate[1/(x^(3/4)*(a + b*x)^(1/4)),x]
```

output
```
(2*(ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] + ArcTanh[(b^(1/4)*x^(1/4))/
(a + b*x)^(1/4)]))/b^(1/4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {73, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{73}$$

$$4\int \frac{1}{\sqrt[4]{a+bx}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{770}$$

$$4\int \frac{1}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{756}$$

$$4\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{1}{2}\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)$$

input

```
Int[1/(x^(3/4)*(a + b*x)^(1/4)),x]
```

output
```
4*(ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4
)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4)))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{1}{x^{\frac{3}{4}} (bx+a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(3/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(3/4)/(b*x+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^{3/4} \sqrt[4]{a+bx}} dx = \frac{\log\left(\frac{(bx+a)^{\frac{3}{4}} x^{\frac{1}{4}} + \frac{bx+a}{b^{\frac{1}{4}}}}{bx+a}\right)}{b^{\frac{1}{4}}} - \frac{\log\left(\frac{(bx+a)^{\frac{3}{4}} x^{\frac{1}{4}} - \frac{bx+a}{b^{\frac{1}{4}}}}{bx+a}\right)}{b^{\frac{1}{4}}} + \frac{i \log\left(\frac{(bx+a)^{\frac{3}{4}} x^{\frac{1}{4}} + \frac{i bx + i a}{b^{\frac{1}{4}}}}{bx+a}\right)}{b^{\frac{1}{4}}} - \frac{i \log\left(\frac{(bx+a)^{\frac{3}{4}} x^{\frac{1}{4}} + \frac{-i bx - i a}{b^{\frac{1}{4}}}}{bx+a}\right)}{b^{\frac{1}{4}}}$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
log(((b*x + a)^(3/4)*x^(1/4) + (b*x + a)/b^(1/4))/(b*x + a))/b^(1/4) - log
(((b*x + a)^(3/4)*x^(1/4) - (b*x + a)/b^(1/4))/(b*x + a))/b^(1/4) + I*log(
((b*x + a)^(3/4)*x^(1/4) + (I*b*x + I*a)/b^(1/4))/(b*x + a))/b^(1/4) - I*l
og(((b*x + a)^(3/4)*x^(1/4) + (-I*b*x - I*a)/b^(1/4))/(b*x + a))/b^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx = \frac{\sqrt[4]{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(3/4)/(b*x+a)**(1/4),x)
```

output
```
x**(1/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x*exp_polar(I*pi)/a)/(a**(
1/4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx = -\frac{2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{1}{4}}}$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
-2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(1/4) - log(-(b^(1/4) - (b*
x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(1/4)
```

Giac [F]

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{3/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(3/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(3/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(3/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)),x)
```

3.702 $\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx=-\frac{4(a+bx)^{3/4}}{3ax^{3/4}}$$

output
```
-4/3*(b*x+a)^(3/4)/a/x^(3/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx=-\frac{4(a+bx)^{3/4}}{3ax^{3/4}}$$

input
```
Integrate[1/(x^(7/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(3*a*x^(3/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}dx$$
$$\Big\downarrow \text{ 48}$$
$$-\frac{4(a+bx)^{3/4}}{3ax^{3/4}}$$

input
```
Int[1/(x^(7/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(3*a*x^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{3}{4}}}{3a\,x^{\frac{3}{4}}}$	16
risch	$-\frac{4(bx+a)^{\frac{3}{4}}}{3a\,x^{\frac{3}{4}}}$	16
orering	$-\frac{4(bx+a)^{\frac{3}{4}}}{3a\,x^{\frac{3}{4}}}$	16

input
```
int(1/x^(7/4)/(b*x+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(b*x+a)^(3/4)/a/x^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx = -\frac{4\,(bx+a)^{\frac{3}{4}}}{3\,ax^{\frac{3}{4}}}$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
-4/3*(b*x + a)^(3/4)/(a*x^(3/4))
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx = \frac{b^{\frac{3}{4}}\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{a\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**(7/4)/(b*x+a)**(1/4),x)
```

output
```
b**(3/4)*(a/(b*x) + 1)**(3/4)*gamma(-3/4)/(a*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx = -\frac{4\,(bx+a)^{\frac{3}{4}}}{3\,ax^{\frac{3}{4}}}$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
-4/3*(b*x + a)^(3/4)/(a*x^(3/4))
```

Giac [F]

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(7/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{7/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(7/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(7/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{\frac{7}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(7/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*x),x)
```

3.703 $\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = -\frac{4(a+bx)^{3/4}}{7ax^{7/4}}+\frac{16b(a+bx)^{3/4}}{21a^2x^{3/4}}$$

output
```
-4/7*(b*x+a)^(3/4)/a/x^(7/4)+16/21*b*(b*x+a)^(3/4)/a^2/x^(3/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = -\frac{4(3a-4bx)(a+bx)^{3/4}}{21a^2x^{7/4}}$$

input
```
Integrate[1/(x^(11/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(3*a - 4*b*x)*(a + b*x)^(3/4))/(21*a^2*x^(7/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}dx}{7a}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{16b(a+bx)^{3/4}}{21a^2x^{3/4}}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}$$

input
```
Int[1/(x^(11/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(7*a*x^(7/4)) + (16*b*(a + b*x)^(3/4))/(21*a^2*x^(3/4
))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{3}{4}}(-4bx+3a)}{21x^{\frac{7}{4}}a^{2}}$	24
risch	$-\frac{4(bx+a)^{\frac{3}{4}}(-4bx+3a)}{21x^{\frac{7}{4}}a^{2}}$	24
orering	$-\frac{4(bx+a)^{\frac{3}{4}}(-4bx+3a)}{21x^{\frac{7}{4}}a^{2}}$	24

input
```
int(1/x^(11/4)/(b*x+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-4/21*(b*x+a)^(3/4)*(-4*b*x+3*a)/x^(7/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = \frac{4\,(4\,bx-3\,a)(bx+a)^{\frac{3}{4}}}{21\,a^2x^{\frac{7}{4}}}$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
4/21*(4*b*x - 3*a)*(b*x + a)^(3/4)/(a^2*x^(7/4))
```

Sympy [A] (verification not implemented)

Time = 11.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4ax\Gamma\left(\frac{1}{4}\right)} + \frac{b^{\frac{7}{4}}\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{a^2\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**(11/4)/(b*x+a)**(1/4),x)
```

output
```
-3*b**(3/4)*(a/(b*x) + 1)**(3/4)*gamma(-7/4)/(4*a*x*gamma(1/4)) + b**(7/4)
*(a/(b*x) + 1)**(3/4)*gamma(-7/4)/(a**2*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = \frac{4\left(\frac{7\,(bx+a)^{\frac{3}{4}}b}{x^{\frac{3}{4}}} - \frac{3\,(bx+a)^{\frac{7}{4}}}{x^{\frac{7}{4}}}\right)}{21\,a^2}$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
4/21*(7*(b*x + a)^(3/4)*b/x^(3/4) - 3*(b*x + a)^(7/4)/x^(7/4))/a^2
```

Giac [F]

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(11/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{11/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(11/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(11/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{\frac{11}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(11/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*x**2),x)
```

3.704 $\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = -\frac{4(a+bx)^{3/4}}{11ax^{11/4}}+\frac{32b(a+bx)^{3/4}}{77a^2x^{7/4}}-\frac{128b^2(a+bx)^{3/4}}{231a^3x^{3/4}}$$

output
```
-4/11*(b*x+a)^(3/4)/a/x^(11/4)+32/77*b*(b*x+a)^(3/4)/a^2/x^(7/4)-128/231*b
^2*(b*x+a)^(3/4)/a^3/x^(3/4)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = -\frac{4(a+bx)^{3/4}(21a^2-24abx+32b^2x^2)}{231a^3x^{11/4}}$$

input
```
Integrate[1/(x^(15/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4)*(21*a^2 - 24*a*b*x + 32*b^2*x^2))/(231*a^3*x^(11/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \ 55$$

$$-\frac{8b\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}dx}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}$$

$$\Big\downarrow \ 55$$

$$-\frac{8b\left(-\frac{4b\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}dx}{7a}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}$$

$$\Big\downarrow \ 48$$

$$-\frac{8b\left(\frac{16b(a+bx)^{3/4}}{21a^2x^{3/4}}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}$$

input
```
Int[1/(x^(15/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(11*a*x^(11/4)) - (8*b*((-4*(a + b*x)^(3/4))/(7*a*x^(
7/4)) + (16*b*(a + b*x)^(3/4))/(21*a^2*x^(3/4))))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{3}{4}}\left(32b^{2}x^{2}-24abx+21a^{2}\right)}{231x^{\frac{11}{4}}a^{3}}$	35
risch	$-\frac{4(bx+a)^{\frac{3}{4}}\left(32b^{2}x^{2}-24abx+21a^{2}\right)}{231x^{\frac{11}{4}}a^{3}}$	35
orering	$-\frac{4(bx+a)^{\frac{3}{4}}\left(32b^{2}x^{2}-24abx+21a^{2}\right)}{231x^{\frac{11}{4}}a^{3}}$	35

input
```
int(1/x^(15/4)/(b*x+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-4/231*(b*x+a)^(3/4)*(32*b^2*x^2-24*a*b*x+21*a^2)/x^(11/4)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = -\frac{4\,(32\,b^2x^2-24\,abx+21\,a^2)(bx+a)^{\frac{3}{4}}}{231\,a^3x^{\frac{11}{4}}}$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
-4/231*(32*b^2*x^2 - 24*a*b*x + 21*a^2)*(b*x + a)^(3/4)/(a^3*x^(11/4))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(63) = 126.

Time = 92.33 (sec) , antiderivative size = 396, normalized size of antiderivative = 5.82

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = \frac{21a^4b^{\frac{19}{4}}\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{1}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{1}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{1}{4}\right)} + \frac{18a^3b^{\frac{23}{4}}x\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{1}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{1}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{1}{4}\right)} + \frac{5a^2b^{\frac{27}{4}}x^2\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{1}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{1}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{1}{4}\right)} + \frac{40ab^{\frac{31}{4}}x^3\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{1}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{1}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{1}{4}\right)} + \frac{32b^{\frac{35}{4}}x^4\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{1}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{1}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**(15/4)/(b*x+a)**(1/4),x)
```

output
```
21*a**4*b**(19/4)*(a/(b*x) + 1)**(3/4)*gamma(-11/4)/(16*a**5*b**4*x**2*gam
ma(1/4) + 32*a**4*b**5*x**3*gamma(1/4) + 16*a**3*b**6*x**4*gamma(1/4)) + 1
8*a**3*b**(23/4)*x*(a/(b*x) + 1)**(3/4)*gamma(-11/4)/(16*a**5*b**4*x**2*ga
mma(1/4) + 32*a**4*b**5*x**3*gamma(1/4) + 16*a**3*b**6*x**4*gamma(1/4)) +
5*a**2*b**(27/4)*x**2*(a/(b*x) + 1)**(3/4)*gamma(-11/4)/(16*a**5*b**4*x**2
*gamma(1/4) + 32*a**4*b**5*x**3*gamma(1/4) + 16*a**3*b**6*x**4*gamma(1/4))
 + 40*a*b**(31/4)*x**3*(a/(b*x) + 1)**(3/4)*gamma(-11/4)/(16*a**5*b**4*x**
2*gamma(1/4) + 32*a**4*b**5*x**3*gamma(1/4) + 16*a**3*b**6*x**4*gamma(1/4)
) + 32*b**(35/4)*x**4*(a/(b*x) + 1)**(3/4)*gamma(-11/4)/(16*a**5*b**4*x**2
*gamma(1/4) + 32*a**4*b**5*x**3*gamma(1/4) + 16*a**3*b**6*x**4*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = -\frac{4\left(\frac{77\,(bx+a)^{\frac{3}{4}}b^2}{x^{\frac{3}{4}}} - \frac{66\,(bx+a)^{\frac{7}{4}}b}{x^{\frac{7}{4}}} + \frac{21\,(bx+a)^{\frac{11}{4}}}{x^{\frac{11}{4}}}\right)}{231\,a^3}$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
-4/231*(77*(b*x + a)^(3/4)*b^2/x^(3/4) - 66*(b*x + a)^(7/4)*b/x^(7/4) + 21
*(b*x + a)^(11/4)/x^(11/4))/a^3
```

Giac [F]

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{15}{4}}}\,dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(15/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{15/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(15/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(15/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{\frac{15}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(15/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*x**3),x)
```

3.705 $\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx = -\frac{4(a+bx)^{3/4}}{15ax^{15/4}} + \frac{16b(a+bx)^{3/4}}{55a^2x^{11/4}} - \frac{128b^2(a+bx)^{3/4}}{385a^3x^{7/4}} + \frac{512b^3(a+bx)^{3/4}}{1155a^4x^{3/4}}$$

output
```
-4/15*(b*x+a)^(3/4)/a/x^(15/4)+16/55*b*(b*x+a)^(3/4)/a^2/x^(11/4)-128/385*
b^2*(b*x+a)^(3/4)/a^3/x^(7/4)+512/1155*b^3*(b*x+a)^(3/4)/a^4/x^(3/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx = -\frac{4(a+bx)^{3/4}\left(77a^3-84a^2bx+96ab^2x^2-128b^3x^3\right)}{1155a^4x^{15/4}}$$

input
```
Integrate[1/(x^(19/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4)*(77*a^3 - 84*a^2*b*x + 96*a*b^2*x^2 - 128*b^3*x^3))/(1
155*a^4*x^(15/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{15ax^{15/4}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\left(-\frac{8b\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}dx}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{15ax^{15/4}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}dx}{7a}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{15ax^{15/4}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{4b\left(-\frac{8b\left(\frac{16b(a+bx)^{3/4}}{21a^2x^{3/4}}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{15ax^{15/4}}$$

input
```
Int[1/(x^(19/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(15*a*x^(15/4)) - (4*b*((-4*(a + b*x)^(3/4))/(11*a*x^
(11/4)) - (8*b*((-4*(a + b*x)^(3/4))/(7*a*x^(7/4)) + (16*b*(a + b*x)^(3/4)
)/(21*a^2*x^(3/4))))/(11*a)))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{3}{4}}\left(-128b^{3}x^{3}+96a\,b^{2}x^{2}-84a^{2}bx+77a^{3}\right)}{1155x^{\frac{15}{4}}a^{4}}$	46
risch	$-\frac{4(bx+a)^{\frac{3}{4}}\left(-128b^{3}x^{3}+96a\,b^{2}x^{2}-84a^{2}bx+77a^{3}\right)}{1155x^{\frac{15}{4}}a^{4}}$	46
orering	$-\frac{4(bx+a)^{\frac{3}{4}}\left(-128b^{3}x^{3}+96a\,b^{2}x^{2}-84a^{2}bx+77a^{3}\right)}{1155x^{\frac{15}{4}}a^{4}}$	46

input
```
int(1/x^(19/4)/(b*x+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-4/1155*(b*x+a)^(3/4)*(-128*b^3*x^3+96*a*b^2*x^2-84*a^2*b*x+77*a^3)/x^(15/
4)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx = \frac{4\,(128\,b^3x^3 - 96\,ab^2x^2 + 84\,a^2bx - 77\,a^3)(bx+a)^{\frac{3}{4}}}{1155\,a^4x^{\frac{15}{4}}}$$

input
```
integrate(1/x^(19/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
4/1155*(128*b^3*x^3 - 96*a*b^2*x^2 + 84*a^2*b*x - 77*a^3)*(b*x + a)^(3/4)/
(a^4*x^(15/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(19/4)/(b*x+a)**(1/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx = \frac{4\left(\frac{385\,(bx+a)^{\frac{3}{4}}b^3}{x^{\frac{3}{4}}} - \frac{495\,(bx+a)^{\frac{7}{4}}b^2}{x^{\frac{7}{4}}} + \frac{315\,(bx+a)^{\frac{11}{4}}b}{x^{\frac{11}{4}}} - \frac{77\,(bx+a)^{\frac{15}{4}}}{x^{\frac{15}{4}}}\right)}{1155\,a^4}$$

input
```
integrate(1/x^(19/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
4/1155*(385*(b*x + a)^(3/4)*b^3/x^(3/4) - 495*(b*x + a)^(7/4)*b^2/x^(7/4)
+ 315*(b*x + a)^(11/4)*b/x^(11/4) - 77*(b*x + a)^(15/4)/x^(15/4))/a^4
```

Giac [F]

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx=\int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{19}{4}}}\,dx$$

input
```
integrate(1/x^(19/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(19/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx=\int \frac{1}{x^{19/4}\,{(a+b\,x)}^{1/4}}\,dx$$

input
```
int(1/(x^(19/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(19/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{19/4}\sqrt[4]{a+bx}}\,dx=\int \frac{1}{x^{\frac{19}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(19/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*x**4),x)
```

3.706 $\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx = -\frac{7ax^{3/4}(a+bx)^{3/4}}{15b^2}+\frac{2x^{7/4}(a+bx)^{3/4}}{5b} + \frac{7a^3\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{10\sqrt{2}b^3\sqrt[4]{x}\sqrt[4]{a+bx}}$$

output
```
-7/15*a*x^(3/4)*(b*x+a)^(3/4)/b^2+2/5*x^(7/4)*(b*x+a)^(3/4)/b+7/20*a^3*(-b
*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1
/2)/b^3/x^(1/4)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx = \frac{4x^{11/4}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{4},\frac{15}{4},-\frac{bx}{a}\right)}{11\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(7/4)/(a + b*x)^(1/4),x]
```

output
```
(4*x^(11/4)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 11/4, 15/4, -((b*x)
/a)])/(11*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}$$

$$\Big\downarrow \text{839}$$

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{10b}$$

$\downarrow$ 813

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}$$

$\downarrow$ 858

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}$$

$\downarrow$ 807

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}$$

$\downarrow$ 212

$$\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}$$

input
```
Int[x^(7/4)/(a + b*x)^(1/4),x]
```

output
```
(2*x^(7/4)*(a + b*x)^(3/4))/(5*b) - (7*a*((2*x^(3/4)*(a + b*x)^(3/4))/(3*b
) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/
4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)
^(1/4))))/b))/(10*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(7/4)/(b*x+a)^(1/4),x)
```

output
```
int(x^(7/4)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^(7/4)/(b*x + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.32

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx = \frac{x^{\frac{11}{4}}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\,\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**(7/4)/(b*x+a)**(1/4),x)
```

output
```
x**(11/4)*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x*exp_polar(I*pi)/a)/(
a**(1/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^(7/4)/(b*x + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx=\int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^(7/4)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx=\int \frac{x^{7/4}}{(a+b\,x)^{1/4}}\,dx$$

input
```
int(x^(7/4)/(a + b*x)^(1/4),x)
```

output
```
int(x^(7/4)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}\,dx=\int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(7/4)/(b*x+a)^(1/4),x)
```

output
```
int((x**(3/4)*x)/(a + b*x)**(1/4),x)
```

3.707 $\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \frac{2x^{3/4}(a+bx)^{3/4}}{3b} - \frac{a^2\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|\,2\right)}{\sqrt{2}b^2\sqrt[4]{x}\sqrt[4]{a+bx}}$$

output
```
2/3*x^(3/4)*(b*x+a)^(3/4)/b-1/2*a^2*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(s
in(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^2/x^(1/4)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \frac{4x^{7/4}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{11}{4},-\frac{bx}{a}\right)}{7\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(3/4)/(a + b*x)^(1/4),x]
```

output
```
(4*x^(7/4)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 7/4, 11/4, -((b*x)/a
)])/(7*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}$$

$$\Big\downarrow \text{839}$$

$$\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}$$

$$\Big\downarrow \text{813}$$

$$\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}$$

$$\Big\downarrow \text{858}$$

$$\frac{2x^{3/4}(a+bx)^{3/4}}{3b} - \frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}} + \frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}$$

$\downarrow$ 807

$$\frac{2x^{3/4}(a+bx)^{3/4}}{3b} - \frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}} + \frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}$$

$\downarrow$ 212

$$\frac{2x^{3/4}(a+bx)^{3/4}}{3b} - \frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}} + \frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}$$

input
```
Int[x^(3/4)/(a + b*x)^(1/4),x]
```

output
```
(2*x^(3/4)*(a + b*x)^(3/4))/(3*b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (S
qrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt
[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(3/4)/(b*x+a)^(1/4),x)
```

output
```
int(x^(3/4)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^(3/4)/(b*x + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \frac{x^{\frac{7}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**(3/4)/(b*x+a)**(1/4),x)
```

output
```
x**(7/4)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x*exp_polar(I*pi)/a)/(a**
(1/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^(3/4)/(b*x + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^(3/4)/(b*x + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{3/4}}{(a+b\,x)^{1/4}}\,dx$$

input
```
int(x^(3/4)/(a + b*x)^(1/4),x)
```

output
```
int(x^(3/4)/(a + b*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(x^(3/4)/(b*x+a)^(1/4),x)
```

output
```
int(x**(3/4)/(a + b*x)**(1/4),x)
```

3.708 $\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx=\frac{\sqrt{2}a\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{b\sqrt[4]{x}\sqrt[4]{a+bx}}$$

output
```
2^(1/2)*a*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),
2^(1/2))/b/x^(1/4)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx=\frac{4x^{3/4}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx}{a}\right)}{3\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(1/4)*(a + b*x)^(1/4)),x]
```

output
```
(4*x^(3/4)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x)/a)
])/(3*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{73}$$

$$4\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{839}$$

$$4\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)$$

$$\Big\downarrow \text{813}$$

$$4\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)$$

$$\Big\downarrow \text{858}$$

$$4\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)$$

$$\Big\downarrow \text{807}$$

$$4\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)$$

$$\Big\downarrow \text{212}$$

$$4\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)$$

input
```
Int[1/(x^(1/4)*(a + b*x)^(1/4)),x]
```

output
```
4*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*Elli
pticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))
)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{1}{4}} (bx+a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(1/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(1/4)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}} dx = \int \frac{1}{(bx+a)^{\frac{1}{4}} x^{\frac{1}{4}}} dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b*x^2 + a*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x**(1/4)/(b*x+a)**(1/4),x)
```

output
```
x**(3/4)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x*exp_polar(I*pi)/a)/(a**(
1/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{1/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(1/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(1/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(1/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(1/4)),x)
```

3.709 $\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx=-\frac{4}{\sqrt[4]{x}\sqrt[4]{a+bx}}+\frac{4\sqrt[4]{\frac{bx}{a+bx}}\sqrt[4]{a+bx}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{x}}$$

output
```
-4/x^(1/4)/(b*x+a)^(1/4)+4*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/4)*EllipticE(sin
(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(1/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx=-\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx}{a}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(5/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x)/a)])/(x^
(1/4)*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 839}$$

$$\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 813}$$

$$\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{807}$$

$$\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{212}$$

$$\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

input
```
Int[1/(x^(5/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sq
rt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[
b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{5}{4}} (bx+a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(5/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(5/4)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b*x^3 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx}\right)}{\sqrt[4]{b}\sqrt{x}}$$

input
```
integrate(1/x**(5/4)/(b*x+a)**(1/4),x)
```

output
```
-2*hyper((1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x))/(b**(1/4)*sqrt(x))
```

Maxima [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{5/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(5/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(5/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\left(bx+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(5/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(1/4)*x),x)
```

3.710 $\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx = \frac{8b}{5a\sqrt[4]{x}\sqrt[4]{a+bx}} - \frac{4(a+bx)^{3/4}}{5ax^{5/4}} - \frac{8b\sqrt[4]{\frac{bx}{a+bx}}\sqrt[4]{a+bx}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{x}}$$

output
```
8/5*b/a/x^(1/4)/(b*x+a)^(1/4)-4/5*(b*x+a)^(3/4)/a/x^(5/4)-8/5*b*(b*x/(b*x+
a))^(1/4)*(b*x+a)^(1/4)*EllipticE(sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2
^(1/2))/a^(3/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},-\frac{1}{4},-\frac{bx}{a}\right)}{5x^{5/4}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(9/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x)/a)])/(5
*x^(5/4)*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{2b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2b\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

$$\Big\downarrow \text{839}$$

$$-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

$$\Big\downarrow \text{813}$$

$$-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}$$

input
```
Int[1/(x^(9/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(5*a*x^(5/4)) - (2*b*((-4*(a + b*x)^(3/4))/(a*x^(1/4)
) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/
4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)
^(1/4))))/a))/(5*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{9}{4}} (bx + a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(9/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(9/4)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{9/4} \sqrt[4]{a + bx}} dx = \int \frac{1}{(bx + a)^{\frac{1}{4}} x^{\frac{9}{4}}} dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b*x^4 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx}\right)}{3\sqrt[4]{b}x^{\frac{3}{2}}}$$

input
```
integrate(1/x**(9/4)/(b*x+a)**(1/4),x)
```

output
```
-2*hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x))/(3*b**(1/4)*x**(3/2)
)
```

Maxima [F]

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(9/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(9/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{9/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(9/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(9/4)*(a + b*x)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}\,dx = -\frac{4(bx+a)^{\frac{3}{4}}}{3x^{\frac{3}{4}}\sqrt{x}\,a}$$

input
```
int(1/x^(9/4)/(b*x+a)^(1/4),x)
```

output
```
( - 4*x**(1/4)*(a + b*x)**(3/4))/(3*sqrt(x)*a*x)
```

3.711 $\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 130

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx = -\frac{16b^2}{15a^2\sqrt[4]{x}\sqrt[4]{a+bx}}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}+\frac{8b(a+bx)^{3/4}}{15a^2x^{5/4}}+\frac{16b^2\sqrt[4]{\frac{bx}{a+bx}}\sqrt[4]{a+bx}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{x}}$$

output
```
-16/15*b^2/a^2/x^(1/4)/(b*x+a)^(1/4)-4/9*(b*x+a)^(3/4)/a/x^(9/4)+8/15*b*(b
*x+a)^(3/4)/a^2/x^(5/4)+16/15*b^2*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/4)*Ellipt
icE(sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(5/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},-\frac{5}{4},-\frac{bx}{a}\right)}{9x^{9/4}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(13/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x)/a)])/(9
*x^(9/4)*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{2b\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{2b\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{8b\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$$\Big\downarrow \text{839}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$$\Big\downarrow \text{813}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$\downarrow$ 807

$$-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

$\downarrow$ 212

$$-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}$$

input
```
Int[1/(x^(13/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(9*a*x^(9/4)) - (2*b*((-4*(a + b*x)^(3/4))/(5*a*x^(5/
4)) - (2*b*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^
(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sq
rt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/(5*a)))/(3*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{13}{4}} (bx + a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(13/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(13/4)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx=\int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b*x^5 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx=\frac{\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},\frac{1}{4}\\-\frac{5}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt[4]{a}x^{\frac{9}{4}}\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate(1/x**(13/4)/(b*x+a)**(1/4),x)
```

output
```
gamma(-9/4)*hyper((-9/4, 1/4), (-5/4,), b*x*exp_polar(I*pi)/a)/(a**(1/4)*x
**(9/4)*gamma(-5/4))
```

Maxima [F]

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(13/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(13/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{13/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(13/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(13/4)*(a + b*x)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}\,dx = \frac{4(bx+a)^{\frac{3}{4}}\,(4bx-3a)}{21x^{\frac{7}{4}}\sqrt{x}\,a^2}$$

input
```
int(1/x^(13/4)/(b*x+a)^(1/4),x)
```

output
```
(4*x**(1/4)*(a + b*x)**(3/4)*( - 3*a + 4*b*x))/(21*sqrt(x)*a**2*x**2)
```

3.712 $\int \frac{x^{11/4}}{(a+bx)^{3/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}} \, dx = \frac{77a^2x^{3/4}\sqrt[4]{a+bx}}{96b^3} - \frac{11ax^{7/4}\sqrt[4]{a+bx}}{24b^2} + \frac{x^{11/4}\sqrt[4]{a+bx}}{3b} + \frac{77a^3\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{64b^{15/4}} - \frac{77a^3\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{64b^{15/4}}$$

output
```
77/96*a^2*x^(3/4)*(b*x+a)^(1/4)/b^3-11/24*a*x^(7/4)*(b*x+a)^(1/4)/b^2+1/3*
x^(11/4)*(b*x+a)^(1/4)/b+77/64*a^3*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b
^(15/4)-77/64*a^3*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(15/4)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}} \, dx = \frac{2b^{3/4}x^{3/4}\sqrt[4]{a+bx}(77a^2-44abx+32b^2x^2)+231a^3\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)-231a^3\text{arctan}}{192b^{15/4}}$$

input
```
Integrate[x^(11/4)/(a + b*x)^(3/4),x]
```

output
```
(2*b^(3/4)*x^(3/4)*(a + b*x)^(1/4)*(77*a^2 - 44*a*b*x + 32*b^2*x^2) + 231*
a^3*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] - 231*a^3*ArcTanh[(b^(1/4)*x
^(1/4))/(a + b*x)^(1/4)])/(192*b^(15/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {60, 60, 60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}}dx$$

$$\Big\downarrow \; 60$$

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\int \frac{x^{7/4}}{(a+bx)^{3/4}}dx}{12b}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\int \frac{x^{3/4}}{(a+bx)^{3/4}}dx}{8b}\right)}{12b}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{4b}\right)}{8b}\right)}{12b}$$

$$\Big\downarrow \; 73$$

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{8b}\right)}{12b}$$

$\big\downarrow$ 854

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int\frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}\right)}{12b}$$

$\big\downarrow$ 827

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int\frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}\right)}{8b}\right)}{12b}$$

$\big\downarrow$ 216

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)}{12b}$$

$\downarrow$ 219

$$\frac{x^{11/4}\sqrt[4]{a+bx}}{3b}-\frac{11a\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)}{12b}$$

input
```
Int[x^(11/4)/(a + b*x)^(3/4),x]
```

output
```
(x^(11/4)*(a + b*x)^(1/4))/(3*b) - (11*a*((x^(7/4)*(a + b*x)^(1/4))/(2*b)
- (7*a*((x^(3/4)*(a + b*x)^(1/4))/b - (3*a*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/
(a + b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b
^(3/4))))/b))/(8*b)))/(12*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(11/4)/(b*x+a)^(3/4),x)
```

output
```
int(x^(11/4)/(b*x+a)^(3/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.78

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}}\,dx = -\frac{231\,b^3\left(\frac{a^{12}}{b^{15}}\right)^{\frac{1}{4}}\log\left(\frac{77\left(b^4x\left(\frac{a^{12}}{b^{15}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}a^3x^{\frac{3}{4}}\right)}{x}\right)-231\,b^3\left(\frac{a^{12}}{b^{15}}\right)^{\frac{1}{4}}\log\left(-\frac{77\left(b^4x\left(\frac{a^{12}}{b^{15}}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}a^3x^{\frac{3}{4}}\right)}{x}\right)-23}{}$$

input
```
integrate(x^(11/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/384*(231*b^3*(a^12/b^15)^(1/4)*log(77*(b^4*x*(a^12/b^15)^(1/4) + (b*x +
 a)^(1/4)*a^3*x^(3/4))/x) - 231*b^3*(a^12/b^15)^(1/4)*log(-77*(b^4*x*(a^12
/b^15)^(1/4) - (b*x + a)^(1/4)*a^3*x^(3/4))/x) - 231*I*b^3*(a^12/b^15)^(1/
4)*log(-77*(I*b^4*x*(a^12/b^15)^(1/4) - (b*x + a)^(1/4)*a^3*x^(3/4))/x) +
231*I*b^3*(a^12/b^15)^(1/4)*log(-77*(-I*b^4*x*(a^12/b^15)^(1/4) - (b*x + a
)^(1/4)*a^3*x^(3/4))/x) - 4*(32*b^2*x^2 - 44*a*b*x + 77*a^2)*(b*x + a)^(1/
4)*x^(3/4))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 66.75 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}}\,dx = \frac{x^{\frac{15}{4}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\right|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**(11/4)/(b*x+a)**(3/4),x)
```

output
```
x**(15/4)*gamma(15/4)*hyper((3/4, 15/4), (19/4,), b*x*exp_polar(I*pi)/a)/(
a**(3/4)*gamma(19/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.31

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}}\,dx = -\frac{\frac{153\,(bx+a)^{\frac{1}{4}}a^3b^2}{x^{\frac{1}{4}}} - \frac{198\,(bx+a)^{\frac{5}{4}}a^3b}{x^{\frac{5}{4}}} + \frac{77\,(bx+a)^{\frac{9}{4}}a^3}{x^{\frac{9}{4}}}}{96\left(b^6 - \frac{3\,(bx+a)b^5}{x} + \frac{3\,(bx+a)^2b^4}{x^2} - \frac{(bx+a)^3b^3}{x^3}\right)} - \frac{77\left(\frac{2\,a^3\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} - \frac{a^3\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^3}$$

input
```
integrate(x^(11/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/96*(153*(b*x + a)^(1/4)*a^3*b^2/x^(1/4) - 198*(b*x + a)^(5/4)*a^3*b/x^(
5/4) + 77*(b*x + a)^(9/4)*a^3/x^(9/4))/(b^6 - 3*(b*x + a)*b^5/x + 3*(b*x +
 a)^2*b^4/x^2 - (b*x + a)^3*b^3/x^3) - 77/128*(2*a^3*arctan((b*x + a)^(1/4
)/(b^(1/4)*x^(1/4)))/b^(3/4) - a^3*log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4)
)/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4))/b^3
```

Giac [F]

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(11/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(11/4)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{11/4}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(11/4)/(a + b*x)^(3/4),x)
```

output
```
int(x^(11/4)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{11/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(11/4)/(b*x+a)^(3/4),x)
```

output
```
int((x**(3/4)*x**2)/(a + b*x)**(3/4),x)
```

3.713 $\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = -\frac{7ax^{3/4}\sqrt[4]{a+bx}}{8b^2}+\frac{x^{7/4}\sqrt[4]{a+bx}}{2b} - \frac{21a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{11/4}}+\frac{21a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{11/4}}$$

output
```
-7/8*a*x^(3/4)*(b*x+a)^(1/4)/b^2+1/2*x^(7/4)*(b*x+a)^(1/4)/b-21/16*a^2*arc
tan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(11/4)+21/16*a^2*arctanh(b^(1/4)*x^(1
/4)/(b*x+a)^(1/4))/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = \frac{2b^{3/4}x^{3/4}\sqrt[4]{a+bx}(-7a+4bx)-21a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)+21a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{11/4}}$$

input
```
Integrate[x^(7/4)/(a + b*x)^(3/4),x]
```

output
```
(2*b^(3/4)*x^(3/4)*(a + b*x)^(1/4)*(-7*a + 4*b*x) - 21*a^2*ArcTan[(b^(1/4)
*x^(1/4))/(a + b*x)^(1/4)] + 21*a^2*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1
/4)])/(16*b^(11/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {60, 60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\int \frac{x^{3/4}}{(a+bx)^{3/4}}dx}{8b}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{4b}\right)}{8b}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{8b}$$

$$\Big\downarrow \text{854}$$

$$\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}$$

$$\Big\downarrow \text{827}$$

$$\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}\right)}{8b}$$

$\downarrow$ 216

$$\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}$$

$\downarrow$ 219

$$\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}$$

input
```
Int[x^(7/4)/(a + b*x)^(3/4),x]
```

output
```
(x^(7/4)*(a + b*x)^(1/4))/(2*b) - (7*a*((x^(3/4)*(a + b*x)^(1/4))/b - (3*a
*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4
)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4))))/b))/(8*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(7/4)/(b*x+a)^(3/4),x)
```

output
```
int(x^(7/4)/(b*x+a)^(3/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.06

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = \frac{21\,b^2\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(\frac{21\left(b^3x\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}a^2x^{\frac{3}{4}}\right)}{x}\right)-21\,b^2\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(-\frac{21\left(b^3x\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}-(bx+a\right.}{x}\right.}{}$$

input
```
integrate(x^(7/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
1/32*(21*b^2*(a^8/b^11)^(1/4)*log(21*(b^3*x*(a^8/b^11)^(1/4) + (b*x + a)^(
1/4)*a^2*x^(3/4))/x) - 21*b^2*(a^8/b^11)^(1/4)*log(-21*(b^3*x*(a^8/b^11)^(
1/4) - (b*x + a)^(1/4)*a^2*x^(3/4))/x) - 21*I*b^2*(a^8/b^11)^(1/4)*log(-21
*(I*b^3*x*(a^8/b^11)^(1/4) - (b*x + a)^(1/4)*a^2*x^(3/4))/x) + 21*I*b^2*(a
^8/b^11)^(1/4)*log(-21*(-I*b^3*x*(a^8/b^11)^(1/4) - (b*x + a)^(1/4)*a^2*x^
(3/4))/x) + 4*(4*b*x - 7*a)*(b*x + a)^(1/4)*x^(3/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = \frac{x^{\frac{11}{4}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**(7/4)/(b*x+a)**(3/4),x)
```

output
```
x**(11/4)*gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*x*exp_polar(I*pi)/a)/(
a**(3/4)*gamma(15/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = \frac{\frac{11\,(bx+a)^{\frac{1}{4}}a^2b}{x^{\frac{1}{4}}} - \frac{7\,(bx+a)^{\frac{5}{4}}a^2}{x^{\frac{5}{4}}}}{8\left(b^4 - \frac{2\,(bx+a)b^3}{x} + \frac{(bx+a)^2b^2}{x^2}\right)} + \frac{21\left(\frac{2\,a^2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} - \frac{a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{32\,b^2}$$

input
```
integrate(x^(7/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
1/8*(11*(b*x + a)^(1/4)*a^2*b/x^(1/4) - 7*(b*x + a)^(5/4)*a^2/x^(5/4))/(b^
4 - 2*(b*x + a)*b^3/x + (b*x + a)^2*b^2/x^2) + 21/32*(2*a^2*arctan((b*x +
a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(3/4) - a^2*log(-(b^(1/4) - (b*x + a)^(1/4)/
x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4))/b^2
```

Giac [F]

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(7/4)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{7/4}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(7/4)/(a + b*x)^(3/4),x)
```

output
```
int(x^(7/4)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{7/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(7/4)/(b*x+a)^(3/4),x)
```

output
```
int((x**(3/4)*x)/(a + b*x)**(3/4),x)
```

3.714 $\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = \frac{x^{3/4}\sqrt[4]{a+bx}}{b} + \frac{3a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{7/4}} - \frac{3a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{7/4}}$$

output
```
x^(3/4)*(b*x+a)^(1/4)/b+3/2*a*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(7/4
)-3/2*a*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = \frac{x^{3/4}\sqrt[4]{a+bx}}{b} + \frac{3a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{7/4}} - \frac{3a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{7/4}}$$

input
```
Integrate[x^(3/4)/(a + b*x)^(3/4),x]
```

output
```
(x^(3/4)*(a + b*x)^(1/4))/b + (3*a*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4
)])/(2*b^(7/4)) - (3*a*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])/(2*b^(7
/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{4b}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}$$

$$\Big\downarrow \text{854}$$

$$\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}$$

$$\Big\downarrow \text{827}$$

$$\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}$$

$\Big\downarrow$ 219

$$\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}$$

input
```
Int[x^(3/4)/(a + b*x)^(3/4),x]
```

output
```
(x^(3/4)*(a + b*x)^(1/4))/b - (3*a*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x
)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4)))
)/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(x^(3/4)/(b*x+a)^(3/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.51

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = -\frac{3\,b\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(b^2x\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}ax^{\frac{3}{4}}\right)}{x}\right)-3\,b\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(b^2x\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}ax^{\frac{3}{4}}\right)}{x}\right)-3i\,b\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(\cdots\right.}{4\,b}$$

input
```
integrate(x^(3/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/4*(3*b*(a^4/b^7)^(1/4)*log(3*(b^2*x*(a^4/b^7)^(1/4) + (b*x + a)^(1/4)*a
*x^(3/4))/x) - 3*b*(a^4/b^7)^(1/4)*log(-3*(b^2*x*(a^4/b^7)^(1/4) - (b*x +
a)^(1/4)*a*x^(3/4))/x) - 3*I*b*(a^4/b^7)^(1/4)*log(-3*(I*b^2*x*(a^4/b^7)^(
1/4) - (b*x + a)^(1/4)*a*x^(3/4))/x) + 3*I*b*(a^4/b^7)^(1/4)*log(-3*(-I*b^
2*x*(a^4/b^7)^(1/4) - (b*x + a)^(1/4)*a*x^(3/4))/x) - 4*(b*x + a)^(1/4)*x^
(3/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.44

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = \frac{x^{\frac{7}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**(3/4)/(b*x+a)**(3/4),x)
```

output
```
x**(7/4)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x*exp_polar(I*pi)/a)/(a**
(3/4)*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = -\frac{3\left(\frac{2\,a\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} - \frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{4\,b} - \frac{(bx+a)^{\frac{1}{4}}a}{\left(b^2-\frac{(bx+a)b}{x}\right)x^{\frac{1}{4}}}$$

input
```
integrate(x^(3/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
-3/4*(2*a*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(3/4) - a*log(-(b^(1
/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4
))/b - (b*x + a)^(1/4)*a/((b^2 - (b*x + a)*b/x)*x^(1/4))
```

Giac [F]

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(3/4)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{3/4}}{{\left(a+b\,x\right)}^{3/4}} \,dx$$

input
```
int(x^(3/4)/(a + b*x)^(3/4),x)
```

output
```
int(x^(3/4)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{3/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(x**(3/4)/(a + b*x)**(3/4),x)
```

3.715 $\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = -\frac{2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{3/4}} + \frac{2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{3/4}}$$

output
```
-2*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(3/4)+2*arctanh(b^(1/4)*x^(1/4)
/(b*x+a)^(1/4))/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = -\frac{2\left(\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right) - \text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)\right)}{b^{3/4}}$$

input
```
Integrate[1/(x^(1/4)*(a + b*x)^(3/4)),x]
```

output
```
(-2*(ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] - ArcTanh[(b^(1/4)*x^(1/4))
/(a + b*x)^(1/4)]))/b^(3/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{ 73}$$

$$4\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{ 854}$$

$$4\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 827}$$

$$4\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{ 216}$$

$$4\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$4\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)$$

input
```
Int[1/(x^(1/4)*(a + b*x)^(3/4)),x]
```

output
```
4*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/
4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4)))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{1}{x^{\frac{1}{4}} (bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(1/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(1/4)/(b*x+a)^(3/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.32

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = \frac{1}{b^3}^{\frac{1}{4}} \log\left(\frac{b\frac{1}{b^3}^{\frac{1}{4}}x + (bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) - \frac{1}{b^3}^{\frac{1}{4}} \log\left(-\frac{b\frac{1}{b^3}^{\frac{1}{4}}x - (bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) + i\,\frac{1}{b^3}^{\frac{1}{4}} \log\left(\frac{i\,b\frac{1}{b^3}^{\frac{1}{4}}x + (bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) - i\,\frac{1}{b^3}^{\frac{1}{4}} \log\left(\frac{-i\,b\frac{1}{b^3}^{\frac{1}{4}}x + (bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right)$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
(b^(-3))^(1/4)*log((b*(b^(-3))^(1/4)*x + (b*x + a)^(1/4)*x^(3/4))/x) - (b^
(-3))^(1/4)*log(-(b*(b^(-3))^(1/4)*x - (b*x + a)^(1/4)*x^(3/4))/x) + I*(b^
(-3))^(1/4)*log((I*b*(b^(-3))^(1/4)*x + (b*x + a)^(1/4)*x^(3/4))/x) - I*(b
^(-3))^(1/4)*log((-I*b*(b^(-3))^(1/4)*x + (b*x + a)^(1/4)*x^(3/4))/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x**(1/4)/(b*x+a)**(3/4),x)
```

output
```
x**(3/4)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x*exp_polar(I*pi)/a)/(a**(
3/4)*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = \frac{2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(3/4) - log(-(b^(1/4) - (b*x
 + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{1/4}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/(x^(1/4)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(1/4)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(1/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)),x)
```

3.716 $\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{a\sqrt[4]{x}}$$

output
```
-4*(b*x+a)^(1/4)/a/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{a\sqrt[4]{x}}$$

input
```
Integrate[1/(x^(5/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4))/(a*x^(1/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx$$
$$\Big\downarrow \text{48}$$
$$-\frac{4\sqrt[4]{a+bx}}{a\sqrt[4]{x}}$$

input
```
Int[1/(x^(5/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4))/(a*x^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{1}{4}}}{a\,x^{\frac{1}{4}}}$	16
risch	$-\frac{4(bx+a)^{\frac{1}{4}}}{a\,x^{\frac{1}{4}}}$	16
orering	$-\frac{4(bx+a)^{\frac{1}{4}}}{a\,x^{\frac{1}{4}}}$	16

input
```
int(1/x^(5/4)/(b*x+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-4*(b*x+a)^(1/4)/a/x^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = -\frac{4\,(bx+a)^{\frac{1}{4}}}{ax^{\frac{1}{4}}}$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
-4*(b*x + a)^(1/4)/(a*x^(1/4))
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{1}{4}\right)}{a\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**(5/4)/(b*x+a)**(3/4),x)
```

output
```
b**(1/4)*(a/(b*x) + 1)**(1/4)*gamma(-1/4)/(a*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = -\frac{4\,(bx+a)^{\frac{1}{4}}}{ax^{\frac{1}{4}}}$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
-4*(b*x + a)^(1/4)/(a*x^(1/4))
```

Giac [F]

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(5/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = -\frac{4\,{(a+b\,x)}^{1/4}}{a\,x^{1/4}}$$

input
```
int(1/(x^(5/4)*(a + b*x)^(3/4)),x)
```

output
```
-(4*(a + b*x)^(1/4))/(a*x^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^{5/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(5/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*x),x)
```

3.717 $\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}+\frac{16b\sqrt[4]{a+bx}}{5a^2\sqrt[4]{x}}$$

output
```
-4/5*(b*x+a)^(1/4)/a/x^(5/4)+16/5*b*(b*x+a)^(1/4)/a^2/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = -\frac{4(a-4bx)\sqrt[4]{a+bx}}{5a^2x^{5/4}}$$

input
```
Integrate[1/(x^(9/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a - 4*b*x)*(a + b*x)^(1/4))/(5*a^2*x^(5/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{4b\int \frac{1}{x^{5/4}(a+bx)^{3/4}}dx}{5a}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{16b\sqrt[4]{a+bx}}{5a^2\sqrt[4]{x}}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}$$

input
```
Int[1/(x^(9/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4))/(5*a*x^(5/4)) + (16*b*(a + b*x)^(1/4))/(5*a^2*x^(1/4)
)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{1}{4}}(-4bx+a)}{5x^{\frac{5}{4}}a^{2}}$	22
risch	$-\frac{4(bx+a)^{\frac{1}{4}}(-4bx+a)}{5x^{\frac{5}{4}}a^{2}}$	22
orering	$-\frac{4(bx+a)^{\frac{1}{4}}(-4bx+a)}{5x^{\frac{5}{4}}a^{2}}$	22

input
```
int(1/x^(9/4)/(b*x+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-4/5*(b*x+a)^(1/4)*(-4*b*x+a)/x^(5/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = \frac{4\,(4\,bx-a)(bx+a)^{\frac{1}{4}}}{5\,a^2x^{\frac{5}{4}}}$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
4/5*(4*b*x - a)*(b*x + a)^(1/4)/(a^2*x^(5/4))
```

Sympy [A] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = -\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{4ax\Gamma\left(\frac{3}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{a^2\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**(9/4)/(b*x+a)**(3/4),x)
```

output
```
-b**(1/4)*(a/(b*x) + 1)**(1/4)*gamma(-5/4)/(4*a*x*gamma(3/4)) + b**(5/4)*(
a/(b*x) + 1)**(1/4)*gamma(-5/4)/(a**2*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = \frac{4\left(\frac{5\,(bx+a)^{\frac{1}{4}}b}{x^{\frac{1}{4}}} - \frac{(bx+a)^{\frac{5}{4}}}{x^{\frac{5}{4}}}\right)}{5\,a^2}$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
4/5*(5*(b*x + a)^(1/4)*b/x^(1/4) - (b*x + a)^(5/4)/x^(5/4))/a^2
```

Giac [F]

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(9/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = -\frac{\left(\frac{4}{5\,a}-\frac{16\,b\,x}{5\,a^2}\right)\,{\left(a+b\,x\right)}^{1/4}}{x^{5/4}}$$

input
```
int(1/(x^(9/4)*(a + b*x)^(3/4)),x)
```

output
```
-((4/(5*a) - (16*b*x)/(5*a^2))*(a + b*x)^(1/4))/x^(5/4)
```

Reduce [F]

$$\int \frac{1}{x^{9/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{\frac{9}{4}}\,(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(9/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*x**2),x)
```

3.718 $\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}} + \frac{32b\sqrt[4]{a+bx}}{45a^2x^{5/4}} - \frac{128b^2\sqrt[4]{a+bx}}{45a^3\sqrt[4]{x}}$$

output
```
-4/9*(b*x+a)^(1/4)/a/x^(9/4)+32/45*b*(b*x+a)^(1/4)/a^2/x^(5/4)-128/45*b^2*
(b*x+a)^(1/4)/a^3/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}(5a^2-8abx+32b^2x^2)}{45a^3x^{9/4}}$$

input
```
Integrate[1/(x^(13/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4)*(5*a^2 - 8*a*b*x + 32*b^2*x^2))/(45*a^3*x^(9/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}} \, dx$$

$$\Big\downarrow \ 55$$

$$-\frac{8b\int \frac{1}{x^{9/4}(a+bx)^{3/4}}dx}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}$$

$$\Big\downarrow \ 55$$

$$-\frac{8b\left(-\frac{4b\int \frac{1}{x^{5/4}(a+bx)^{3/4}}dx}{5a}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}$$

$$\Big\downarrow \ 48$$

$$-\frac{8b\left(\frac{16b\sqrt[4]{a+bx}}{5a^2\sqrt[4]{x}}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}$$

input
```
Int[1/(x^(13/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4))/(9*a*x^(9/4)) - (8*b*((-4*(a + b*x)^(1/4))/(5*a*x^(5/
4)) + (16*b*(a + b*x)^(1/4))/(5*a^2*x^(1/4))))/(9*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{1}{4}}\left(32b^{2}x^{2}-8abx+5a^{2}\right)}{45x^{\frac{9}{4}}a^{3}}$	35
risch	$-\frac{4(bx+a)^{\frac{1}{4}}\left(32b^{2}x^{2}-8abx+5a^{2}\right)}{45x^{\frac{9}{4}}a^{3}}$	35
orering	$-\frac{4(bx+a)^{\frac{1}{4}}\left(32b^{2}x^{2}-8abx+5a^{2}\right)}{45x^{\frac{9}{4}}a^{3}}$	35

input
```
int(1/x^(13/4)/(b*x+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-4/45*(b*x+a)^(1/4)*(32*b^2*x^2-8*a*b*x+5*a^2)/x^(9/4)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx = -\frac{4\,(32\,b^2x^2 - 8\,abx + 5\,a^2)(bx+a)^{\frac{1}{4}}}{45\,a^3x^{\frac{9}{4}}}$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
-4/45*(32*b^2*x^2 - 8*a*b*x + 5*a^2)*(b*x + a)^(1/4)/(a^3*x^(9/4))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(63) = 126.

Time = 63.97 (sec) , antiderivative size = 396, normalized size of antiderivative = 5.82

$$\begin{aligned}\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx &= \frac{5a^4b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx}+1}\,\Gamma\left(-\frac{9}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{3}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{3}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{2a^3b^{\frac{21}{4}}x\sqrt[4]{\frac{a}{bx}+1}\,\Gamma\left(-\frac{9}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{3}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{3}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{21a^2b^{\frac{25}{4}}x^2\sqrt[4]{\frac{a}{bx}+1}\,\Gamma\left(-\frac{9}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{3}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{3}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{56ab^{\frac{29}{4}}x^3\sqrt[4]{\frac{a}{bx}+1}\,\Gamma\left(-\frac{9}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{3}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{3}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{32b^{\frac{33}{4}}x^4\sqrt[4]{\frac{a}{bx}+1}\,\Gamma\left(-\frac{9}{4}\right)}{16a^5b^4x^2\Gamma\left(\frac{3}{4}\right)+32a^4b^5x^3\Gamma\left(\frac{3}{4}\right)+16a^3b^6x^4\Gamma\left(\frac{3}{4}\right)}\end{aligned}$$

input
```
integrate(1/x**(13/4)/(b*x+a)**(3/4),x)
```

output
```
5*a**4*b**(17/4)*(a/(b*x) + 1)**(1/4)*gamma(-9/4)/(16*a**5*b**4*x**2*gamma
(3/4) + 32*a**4*b**5*x**3*gamma(3/4) + 16*a**3*b**6*x**4*gamma(3/4)) + 2*a
**3*b**(21/4)*x*(a/(b*x) + 1)**(1/4)*gamma(-9/4)/(16*a**5*b**4*x**2*gamma(
3/4) + 32*a**4*b**5*x**3*gamma(3/4) + 16*a**3*b**6*x**4*gamma(3/4)) + 21*a
**2*b**(25/4)*x**2*(a/(b*x) + 1)**(1/4)*gamma(-9/4)/(16*a**5*b**4*x**2*gam
ma(3/4) + 32*a**4*b**5*x**3*gamma(3/4) + 16*a**3*b**6*x**4*gamma(3/4)) + 5
6*a*b**(29/4)*x**3*(a/(b*x) + 1)**(1/4)*gamma(-9/4)/(16*a**5*b**4*x**2*gam
ma(3/4) + 32*a**4*b**5*x**3*gamma(3/4) + 16*a**3*b**6*x**4*gamma(3/4)) + 3
2*b**(33/4)*x**4*(a/(b*x) + 1)**(1/4)*gamma(-9/4)/(16*a**5*b**4*x**2*gamma
(3/4) + 32*a**4*b**5*x**3*gamma(3/4) + 16*a**3*b**6*x**4*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx = -\frac{4\left(\frac{45\,(bx+a)^{\frac{1}{4}}b^2}{x^{\frac{1}{4}}} - \frac{18\,(bx+a)^{\frac{5}{4}}b}{x^{\frac{5}{4}}} + \frac{5\,(bx+a)^{\frac{9}{4}}}{x^{\frac{9}{4}}}\right)}{45\,a^3}$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
-4/45*(45*(b*x + a)^(1/4)*b^2/x^(1/4) - 18*(b*x + a)^(5/4)*b/x^(5/4) + 5*(
b*x + a)^(9/4)/x^(9/4))/a^3
```

Giac [F]

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(13/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx = -\frac{(a+b\,x)^{1/4}\left(\frac{4}{9\,a}+\frac{128\,b^2\,x^2}{45\,a^3}-\frac{32\,b\,x}{45\,a^2}\right)}{x^{9/4}}$$

input
```
int(1/(x^(13/4)*(a + b*x)^(3/4)),x)
```

output
```
-((a + b*x)^(1/4)*(4/(9*a) + (128*b^2*x^2)/(45*a^3) - (32*b*x)/(45*a^2)))/
x^(9/4)
```

Reduce [F]

$$\int \frac{1}{x^{13/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{\frac{13}{4}}\,(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(13/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*x**3),x)
```

3.719 $\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx=-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}+\frac{16b\sqrt[4]{a+bx}}{39a^2x^{9/4}}-\frac{128b^2\sqrt[4]{a+bx}}{195a^3x^{5/4}}+\frac{512b^3\sqrt[4]{a+bx}}{195a^4\sqrt[4]{x}}$$

output
```
-4/13*(b*x+a)^(1/4)/a/x^(13/4)+16/39*b*(b*x+a)^(1/4)/a^2/x^(9/4)-128/195*b
^2*(b*x+a)^(1/4)/a^3/x^(5/4)+512/195*b^3*(b*x+a)^(1/4)/a^4/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx=\frac{4\sqrt[4]{a+bx}(-15a^3+20a^2bx-32ab^2x^2+128b^3x^3)}{195a^4x^{13/4}}$$

input
```
Integrate[1/(x^(17/4)*(a + b*x)^(3/4)),x]
```

output
```
(4*(a + b*x)^(1/4)*(-15*a^3 + 20*a^2*b*x - 32*a*b^2*x^2 + 128*b^3*x^3))/(1
95*a^4*x^(13/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{12b\int \frac{1}{x^{13/4}(a+bx)^{3/4}}dx}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{12b\left(-\frac{8b\int \frac{1}{x^{9/4}(a+bx)^{3/4}}dx}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{12b\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^{5/4}(a+bx)^{3/4}}dx}{5a}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{12b\left(-\frac{8b\left(\frac{16b\sqrt[4]{a+bx}}{5a^2\sqrt[4]{x}}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}$$

input
```
Int[1/(x^(17/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4))/(13*a*x^(13/4)) - (12*b*((-4*(a + b*x)^(1/4))/(9*a*x^
(9/4)) - (8*b*((-4*(a + b*x)^(1/4))/(5*a*x^(5/4)) + (16*b*(a + b*x)^(1/4))
/(5*a^2*x^(1/4))))/(9*a)))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{1}{4}} \left(-128b^{3}x^{3}+32a\,b^{2}x^{2}-20a^{2}bx+15a^{3}\right)}{195x^{\frac{13}{4}} a^{4}}$	46
risch	$-\frac{4(bx+a)^{\frac{1}{4}} \left(-128b^{3}x^{3}+32a\,b^{2}x^{2}-20a^{2}bx+15a^{3}\right)}{195x^{\frac{13}{4}} a^{4}}$	46
orering	$-\frac{4(bx+a)^{\frac{1}{4}} \left(-128b^{3}x^{3}+32a\,b^{2}x^{2}-20a^{2}bx+15a^{3}\right)}{195x^{\frac{13}{4}} a^{4}}$	46

input
```
int(1/x^(17/4)/(b*x+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-4/195*(b*x+a)^(1/4)*(-128*b^3*x^3+32*a*b^2*x^2-20*a^2*b*x+15*a^3)/x^(13/4
)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx = \frac{4\,(128\,b^3x^3 - 32\,ab^2x^2 + 20\,a^2bx - 15\,a^3)(bx+a)^{\frac{1}{4}}}{195\,a^4x^{\frac{13}{4}}}$$

input
```
integrate(1/x^(17/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
4/195*(128*b^3*x^3 - 32*a*b^2*x^2 + 20*a^2*b*x - 15*a^3)*(b*x + a)^(1/4)/(
a^4*x^(13/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(17/4)/(b*x+a)**(3/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx = \frac{4\left(\frac{195\,(bx+a)^{\frac{1}{4}}b^3}{x^{\frac{1}{4}}} - \frac{117\,(bx+a)^{\frac{5}{4}}b^2}{x^{\frac{5}{4}}} + \frac{65\,(bx+a)^{\frac{9}{4}}b}{x^{\frac{9}{4}}} - \frac{15\,(bx+a)^{\frac{13}{4}}}{x^{\frac{13}{4}}}\right)}{195\,a^4}$$

input
```
integrate(1/x^(17/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
4/195*(195*(b*x + a)^(1/4)*b^3/x^(1/4) - 117*(b*x + a)^(5/4)*b^2/x^(5/4) +
 65*(b*x + a)^(9/4)*b/x^(9/4) - 15*(b*x + a)^(13/4)/x^(13/4))/a^4
```

Giac [F]

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{17}{4}}}\,dx$$

input
```
integrate(1/x^(17/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(17/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx = -\frac{(a+b\,x)^{1/4}\left(\frac{4}{13\,a}+\frac{128\,b^2\,x^2}{195\,a^3}-\frac{512\,b^3\,x^3}{195\,a^4}-\frac{16\,b\,x}{39\,a^2}\right)}{x^{13/4}}$$

input
```
int(1/(x^(17/4)*(a + b*x)^(3/4)),x)
```

output
```
-((a + b*x)^(1/4)*(4/(13*a) + (128*b^2*x^2)/(195*a^3) - (512*b^3*x^3)/(195
*a^4) - (16*b*x)/(39*a^2)))/x^(13/4)
```

Reduce [F]

$$\int \frac{1}{x^{17/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{\frac{17}{4}}\,(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(17/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*x**4),x)
```

3.720 $\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 130

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \frac{3a^2\sqrt[4]{x}\sqrt[4]{a+bx}}{2b^3} - \frac{3ax^{5/4}\sqrt[4]{a+bx}}{5b^2} + \frac{2x^{9/4}\sqrt[4]{a+bx}}{5b} + \frac{3a^{5/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{2b^4x^{3/4}}$$

output
```
3/2*a^2*x^(1/4)*(b*x+a)^(1/4)/b^3-3/5*a*x^(5/4)*(b*x+a)^(1/4)/b^2+2/5*x^(9
/4)*(b*x+a)^(1/4)/b+3/2*a^(5/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJ
acobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^4/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.36

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \frac{4x^{13/4}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{13}{4},\frac{17}{4},-\frac{bx}{a}\right)}{13(a+bx)^{3/4}}$$

input
```
Integrate[x^(9/4)/(a + b*x)^(3/4),x]
```

output
```
(4*x^(13/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 13/4, 17/4, -((b*x)
/a)])/(13*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \ 73$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \; 768$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \; 858$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \; 807$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \; 229$$

$$\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}$$

input

```
Int[x^(9/4)/(a + b*x)^(3/4),x]
```

output
```
(2*x^(9/4)*(a + b*x)^(1/4))/(5*b) - (9*a*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b
) - (5*a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x
^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*
x)^(3/4))))/(6*b)))/(10*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(9/4)/(b*x+a)^(3/4),x)
```

output
```
int(x^(9/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(9/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^(9/4)/(b*x + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \frac{x^{\frac{13}{4}}\Gamma\left(\frac{13}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{13}{4}\\\frac{17}{4}\end{matrix}\right|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**(9/4)/(b*x+a)**(3/4),x)
```

output
```
x**(13/4)*gamma(13/4)*hyper((3/4, 13/4), (17/4,), b*x*exp_polar(I*pi)/a)/(
a**(3/4)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(9/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^(9/4)/(b*x + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(9/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(9/4)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{9/4}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(9/4)/(a + b*x)^(3/4),x)
```

output
```
int(x^(9/4)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{9/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(9/4)/(b*x+a)^(3/4),x)
```

output
```
int((x**(1/4)*x**2)/(a + b*x)**(3/4),x)
```

3.721 $\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = -\frac{5a\sqrt[4]{x}\sqrt[4]{a+bx}}{3b^2}+\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b} - \frac{5a^{3/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3b^3x^{3/4}}$$

output
```
-5/3*a*x^(1/4)*(b*x+a)^(1/4)/b^2+2/3*x^(5/4)*(b*x+a)^(1/4)/b-5/3*a^(3/2)*(
b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a
)^(1/2)),2^(1/2))/b^3/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = \frac{4x^{9/4}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{13}{4},-\frac{bx}{a}\right)}{9(a+bx)^{3/4}}$$

input
```
Integrate[x^(5/4)/(a + b*x)^(3/4),x]
```

output
```
(4*x^(9/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 9/4, 13/4, -((b*x)/a
)])/(9*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}$$

$$\Big\downarrow \ 73$$

$$\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}$$

$$\Big\downarrow \ 768$$

$$\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}$$

$$\Big\downarrow \ 858$$

$$\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}$$

$$\Big\downarrow \text{807}$$

$$\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}$$

$$\Big\downarrow \text{229}$$

$$\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}$$

input
```
Int[x^(5/4)/(a + b*x)^(3/4),x]
```

output
```
(2*x^(5/4)*(a + b*x)^(1/4))/(3*b) - (5*a*((2*x^(1/4)*(a + b*x)^(1/4))/b +
(2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/
Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(6*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(5/4)/(b*x+a)^(3/4),x)
```

output
```
int(x^(5/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(5/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^(5/4)/(b*x + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.34

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = \frac{x^{\frac{9}{4}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**(5/4)/(b*x+a)**(3/4),x)
```

output
```
x**(9/4)*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x*exp_polar(I*pi)/a)/(a**
(3/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(5/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^(5/4)/(b*x + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(5/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(5/4)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{5/4}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(5/4)/(a + b*x)^(3/4),x)
```

output
```
int(x^(5/4)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{5/4}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(5/4)/(b*x+a)^(3/4),x)
```

output
```
int((x**(1/4)*x)/(a + b*x)**(3/4),x)
```

3.722 $\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx=\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}+\frac{2\sqrt{a}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{b^2x^{3/4}}$$

output
```
2*x^(1/4)*(b*x+a)^(1/4)/b+2*a^(1/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*Inve
rseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^2/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx=\frac{4x^{5/4}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{9}{4},-\frac{bx}{a}\right)}{5(a+bx)^{3/4}}$$

input
```
Integrate[x^(1/4)/(a + b*x)^(3/4),x]
```

output
```
(4*x^(5/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[3/4, 5/4, 9/4, -((b*x)/a)
])/(5*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}$$

input
```
Int[x^(1/4)/(a + b*x)^(3/4),x]
```

output
```
(2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*Ell
ipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(1/4)/(b*x+a)^(3/4),x)
```

output
```
int(x^(1/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(1/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^(1/4)/(b*x + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx = \frac{x^{\frac{5}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(1/4)/(b*x+a)**(3/4),x)
```

output
```
x**(5/4)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x*exp_polar(I*pi)/a)/(a**(
3/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(1/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^(1/4)/(b*x + a)^(3/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^(1/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^(1/4)/(b*x + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{1/4}}{(a+b\,x)^{3/4}}\,dx$$

input
```
int(x^(1/4)/(a + b*x)^(3/4),x)
```

output
```
int(x^(1/4)/(a + b*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(x^(1/4)/(b*x+a)^(3/4),x)
```

output
```
int(x**(1/4)/(a + b*x)**(3/4),x)
```

3.723 $\int \frac{1}{x^{3/4}(a+bx)^{3/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}} dx = -\frac{4\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{\sqrt{a}bx^{3/4}}$$

output
```
-4*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b
*x+a)^(1/2)),2^(1/2))/a^(1/2)/b/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}} dx = \frac{4\sqrt[4]{x}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx}{a}\right)}{(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(3/4)*(a + b*x)^(3/4)),x]
```

output
```
(4*x^(1/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x)/a)
])/(a + b*x)^(3/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$4\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{768}$$

$$\frac{4x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{4x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{(a+bx)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{4\sqrt{b}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{a}(a+bx)^{3/4}}$$

input
```
Int[1/(x^(3/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*Sqrt[b]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])
/Sqrt[b]]/2, 2])/(Sqrt[a]*(a + b*x)^(3/4))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{3}{4}} (bx + a)^{\frac{3}{4}}} dx$$

input
```
int(1/x^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(3/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b*x^2 + a*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}}\,dx = \frac{\sqrt[4]{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(3/4)/(b*x+a)**(3/4),x)
```

output
```
x**(1/4)*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x*exp_polar(I*pi)/a)/(a**(
3/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{3/4}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/(x^(3/4)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(3/4)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(3/4)),x)
```

3.724 $\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}} + \frac{8\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3a^{3/2}x^{3/4}}$$

output
```
-4/3*(b*x+a)^(1/4)/a/x^(3/4)+8/3*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*Inverse
JacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(3/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},-\frac{bx}{a}\right)}{3x^{3/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(7/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x)/a)])/(3*
x^(3/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}$$

input
```
Int[1/(x^(7/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4
)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^
(3/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{7}{4}} (bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(7/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(7/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}} x^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b*x^3 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{1}{4} \end{matrix} \middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}x^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**(7/4)/(b*x+a)**(3/4),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x*exp_polar(I*pi)/a)/(a**(3/4)*x*
*(3/4)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(7/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(7/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{7/4}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/(x^(7/4)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(7/4)*(a + b*x)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^{7/4}(a+bx)^{3/4}}\,dx = -\frac{4(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}\sqrt{x}\,a}$$

input
```
int(1/x^(7/4)/(b*x+a)^(3/4),x)
```

output
```
( - 4*x**(3/4)*(a + b*x)**(1/4))/(sqrt(x)*a*x)
```

3.725 $\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}+\frac{8b\sqrt[4]{a+bx}}{7a^2x^{3/4}} - \frac{16b\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{7a^{5/2}x^{3/4}}$$

output
```
-4/7*(b*x+a)^(1/4)/a/x^(7/4)+8/7*b*(b*x+a)^(1/4)/a^2/x^(3/4)-16/7*b*(b*x/(
b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/
2)),2^(1/2))/a^(5/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},-\frac{bx}{a}\right)}{7x^{7/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(11/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x)/a)])/(7
*x^(7/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}$$

$$\Big\downarrow \, 807$$

$$-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}$$

$$\Big\downarrow \, 229$$

$$-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}$$

input
```
Int[1/(x^(11/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(a + b*x)^(1/4))/(7*a*x^(7/4)) - (6*b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/
4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqr
t[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/(7*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{11}{4}} (bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^(11/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(11/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b*x^4 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{3}{4} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**(11/4)/(b*x+a)**(3/4),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 3/4), (-3/4,), b*x*exp_polar(I*pi)/a)/(a**(3/4)*x
**(7/4)*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(11/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(11/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{11/4}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/(x^(11/4)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(11/4)*(a + b*x)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^{11/4}(a+bx)^{3/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\,(4bx-a)}{5x^{\frac{5}{4}}\sqrt{x}\,a^2}$$

input
```
int(1/x^(11/4)/(b*x+a)^(3/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4)*( - a + 4*b*x))/(5*sqrt(x)*a**2*x**2)
```

3.726 $\int \frac{1}{x^{15/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 130

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}+\frac{40b\sqrt[4]{a+bx}}{77a^2x^{7/4}}-\frac{80b^2\sqrt[4]{a+bx}}{77a^3x^{3/4}}+\frac{160b^2\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{77a^{7/2}x^{3/4}}$$

output
```
-4/11*(b*x+a)^(1/4)/a/x^(11/4)+40/77*b*(b*x+a)^(1/4)/a^2/x^(7/4)-80/77*b^2
*(b*x+a)^(1/4)/a^3/x^(3/4)+160/77*b^2*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*In
verseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(7/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{3}{4},-\frac{7}{4},-\frac{bx}{a}\right)}{11x^{11/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(15/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x)/a)])/(
11*x^(11/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {61, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{10b\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{10b\left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

$\Big\downarrow$ 858

$$-\frac{10b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

$\Big\downarrow$ 807

$$-\frac{10b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

$\Big\downarrow$ 229

$$-\frac{10b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}$$

input

```
Int[1/(x^(15/4)*(a + b*x)^(3/4)),x]
```

output

```
(-4*(a + b*x)^(1/4))/(11*a*x^(11/4)) - (10*b*((-4*(a + b*x)^(1/4))/(7*a*x^
(7/4)) - (6*b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x
))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(
3/2)*(a + b*x)^(3/4))))/(7*a)))/(11*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{15}{4}} (bx+a)^{\frac{3}{4}}} dx$$

input
```
int(1/x^(15/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/x^(15/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}} dx = \int \frac{1}{(bx+a)^{\frac{3}{4}} x^{\frac{15}{4}}} dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b*x^5 + a*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}} dx = \text{Timed out}$$

input
```
integrate(1/x**(15/4)/(b*x+a)**(3/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{15}{4}}}\,dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(15/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}x^{\frac{15}{4}}}\,dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*x^(15/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{x^{15/4}\,{\left(a+b\,x\right)}^{3/4}}\,dx$$

input
```
int(1/(x^(15/4)*(a + b*x)^(3/4)),x)
```

output
```
int(1/(x^(15/4)*(a + b*x)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^{15/4}(a+bx)^{3/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\left(-32b^2x^2+8abx-5a^2\right)}{45x^{\frac{9}{4}}\sqrt{x}\,a^3}$$

input
```
int(1/x^(15/4)/(b*x+a)^(3/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4)*( - 5*a**2 + 8*a*b*x - 32*b**2*x**2))/(45*sqr
t(x)*a**3*x**3)
```

3.727 $\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 132

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx=-\frac{4a^2\sqrt[4]{x}}{b^3\sqrt[4]{a+bx}}-\frac{13a\sqrt[4]{x}(a+bx)^{3/4}}{8b^3}+\frac{x^{5/4}(a+bx)^{3/4}}{2b^2}+\frac{45a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{13/4}}+\frac{45a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{13/4}}$$

output
```
-4*a^2*x^(1/4)/b^3/(b*x+a)^(1/4)-13/8*a*x^(1/4)*(b*x+a)^(3/4)/b^3+1/2*x^(5
/4)*(b*x+a)^(3/4)/b^2+45/16*a^2*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(1
3/4)+45/16*a^2*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx=\frac{\frac{2\sqrt[4]{b}\sqrt[4]{x}(-45a^2-9abx+4b^2x^2)}{\sqrt[4]{a+bx}}+45a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)+45a^2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{13/4}}$$

input
```
Integrate[x^(9/4)/(a + b*x)^(5/4),x]
```

output
```
((2*b^(1/4)*x^(1/4)*(-45*a^2 - 9*a*b*x + 4*b^2*x^2))/(a + b*x)^(1/4) + 45*
a^2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] + 45*a^2*ArcTanh[(b^(1/4)*x^
(1/4))/(a + b*x)^(1/4)])/(16*b^(13/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {57, 60, 60, 73, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{9\int \frac{x^{5/4}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{9\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}dx}{8b}\right)}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{9\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}dx}{4b}\right)}{8b}\right)}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{9\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int\frac{1}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{8b}\right)}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{770}$$

$$\frac{9\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int\frac{1}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}\right)}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{756}$$

$$\frac{9\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{1}{2}\int\frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b}\right)}{8b}\right)}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{216}$$

$$\frac{9\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{8b}\right)}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

$\downarrow$ 219

$$\frac{9\left(\frac{x^{5/4}(a+bx)^{3/4}}{2b}-\frac{5a\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{8b}\right)}{b}-\frac{4x^{9/4}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(9/4)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(9/4))/(b*(a + b*x)^(1/4)) + (9*((x^(5/4)*(a + b*x)^(3/4))/(2*b) - (
5*a*((x^(1/4)*(a + b*x)^(3/4))/b - (a*(ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^
(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4)
)))/b))/(8*b)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(9/4)/(b*x+a)^(5/4),x)
```

output
```
int(x^(9/4)/(b*x+a)^(5/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.50

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx = \frac{45\,(b^4x+ab^3)\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}\log\left(\frac{45\left((bx+a)^{\frac{3}{4}}a^2x^{\frac{1}{4}}+(b^4x+ab^3)\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-45\,(b^4x+ab^3)\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}\mathrm{lo}}{}$$

input
```
integrate(x^(9/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
1/32*(45*(b^4*x + a*b^3)*(a^8/b^13)^(1/4)*log(45*((b*x + a)^(3/4)*a^2*x^(1
/4) + (b^4*x + a*b^3)*(a^8/b^13)^(1/4))/(b*x + a)) - 45*(b^4*x + a*b^3)*(a
^8/b^13)^(1/4)*log(45*((b*x + a)^(3/4)*a^2*x^(1/4) - (b^4*x + a*b^3)*(a^8/
b^13)^(1/4))/(b*x + a)) - 45*(I*b^4*x + I*a*b^3)*(a^8/b^13)^(1/4)*log(45*(
(b*x + a)^(3/4)*a^2*x^(1/4) - (I*b^4*x + I*a*b^3)*(a^8/b^13)^(1/4))/(b*x +
 a)) - 45*(-I*b^4*x - I*a*b^3)*(a^8/b^13)^(1/4)*log(45*((b*x + a)^(3/4)*a^
2*x^(1/4) - (-I*b^4*x - I*a*b^3)*(a^8/b^13)^(1/4))/(b*x + a)) + 4*(4*b^2*x
^2 - 9*a*b*x - 45*a^2)*(b*x + a)^(3/4)*x^(1/4))/(b^4*x + a*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx = \frac{x^{\frac{13}{4}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**(9/4)/(b*x+a)**(5/4),x)
```

output
```
x**(13/4)*gamma(13/4)*hyper((5/4, 13/4), (17/4,), b*x*exp_polar(I*pi)/a)/(
a**(5/4)*gamma(17/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx = -\frac{32\,a^2b^2 - \frac{81\,(bx+a)a^2b}{x} + \frac{45\,(bx+a)^2a^2}{x^2}}{8\left(\frac{(bx+a)^{\frac{1}{4}}b^5}{x^{\frac{1}{4}}} - \frac{2\,(bx+a)^{\frac{5}{4}}b^4}{x^{\frac{5}{4}}} + \frac{(bx+a)^{\frac{9}{4}}b^3}{x^{\frac{9}{4}}}\right)} - \frac{45\,a^2\left(\frac{2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}} - \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}} + \frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{1}{4}}}\right)}{32\,b^3}$$

input
```
integrate(x^(9/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/8*(32*a^2*b^2 - 81*(b*x + a)*a^2*b/x + 45*(b*x + a)^2*a^2/x^2)/((b*x +
a)^(1/4)*b^5/x^(1/4) - 2*(b*x + a)^(5/4)*b^4/x^(5/4) + (b*x + a)^(9/4)*b^3
/x^(9/4)) - 45/32*a^2*(2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(1/4)
 + log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(
1/4)))/b^(1/4))/b^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(9/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument
Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{9/4}}{{\left(a+b\,x\right)}^{5/4}}\,dx$$

input
```
int(x^(9/4)/(a + b*x)^(5/4),x)
```

output
```
int(x^(9/4)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{9/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(9/4)/(b*x+a)^(5/4),x)
```

output
```
int((x**(1/4)*x**2)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.728 $\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = \frac{4a\sqrt[4]{x}}{b^2\sqrt[4]{a+bx}}+\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b^2} - \frac{5a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{9/4}} - \frac{5a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{9/4}}$$

output
```
4*a*x^(1/4)/b^2/(b*x+a)^(1/4)+x^(1/4)*(b*x+a)^(3/4)/b^2-5/2*a*arctan(b^(1/
4)*x^(1/4)/(b*x+a)^(1/4))/b^(9/4)-5/2*a*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1
/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = \frac{\frac{2\sqrt[4]{b}\sqrt[4]{x}(5a+bx)}{\sqrt[4]{a+bx}} - 5a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right) - 5a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{9/4}}$$

input
```
Integrate[x^(5/4)/(a + b*x)^(5/4),x]
```

output
```
((2*b^(1/4)*x^(1/4)*(5*a + b*x))/(a + b*x)^(1/4) - 5*a*ArcTan[(b^(1/4)*x^(
1/4))/(a + b*x)^(1/4)] - 5*a*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])/(
2*b^(9/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {57, 60, 73, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{5\int \frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{5/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{5\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}dx}{4b}\right)}{b}-\frac{4x^{5/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{b}-\frac{4x^{5/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{770}$$

$$\frac{5\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\int \frac{1}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{b}-\frac{4x^{5/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{756}$$

$$\frac{5\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{1}{2}\int\frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{5/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{216}$$

$$\frac{5\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{b}-\frac{4x^{5/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{\sqrt[4]{x}(a+bx)^{3/4}}{b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}\right)}{b}-\frac{4x^{5/4}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(5/4)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(5/4))/(b*(a + b*x)^(1/4)) + (5*((x^(1/4)*(a + b*x)^(3/4))/b - (a*(A
rcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x^
(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4))))/b))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(5/4)/(b*x+a)^(5/4),x)
```

output
```
int(x^(5/4)/(b*x+a)^(5/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.07

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = -\frac{5\left(b^3x+ab^2\right)\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{5\left((bx+a)^{\frac{3}{4}}ax^{\frac{1}{4}}+(b^3x+ab^2)\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-5\left(b^3x+ab^2\right)\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{5\left((bx+a)^{\frac{3}{4}}ax^{\frac{1}{4}}-(b^3x+ab^2\right.}{bx+a}\right.}{}$$

input
```
integrate(x^(5/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/4*(5*(b^3*x + a*b^2)*(a^4/b^9)^(1/4)*log(5*((b*x + a)^(3/4)*a*x^(1/4) +
 (b^3*x + a*b^2)*(a^4/b^9)^(1/4))/(b*x + a)) - 5*(b^3*x + a*b^2)*(a^4/b^9)
^(1/4)*log(5*((b*x + a)^(3/4)*a*x^(1/4) - (b^3*x + a*b^2)*(a^4/b^9)^(1/4))
/(b*x + a)) - 5*(I*b^3*x + I*a*b^2)*(a^4/b^9)^(1/4)*log(5*((b*x + a)^(3/4)
*a*x^(1/4) - (I*b^3*x + I*a*b^2)*(a^4/b^9)^(1/4))/(b*x + a)) - 5*(-I*b^3*x
 - I*a*b^2)*(a^4/b^9)^(1/4)*log(5*((b*x + a)^(3/4)*a*x^(1/4) - (-I*b^3*x -
 I*a*b^2)*(a^4/b^9)^(1/4))/(b*x + a)) - 4*(b*x + 5*a)*(b*x + a)^(3/4)*x^(1
/4))/(b^3*x + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.36

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = \frac{x^{\frac{9}{4}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**(5/4)/(b*x+a)**(5/4),x)
```

output
```
x**(9/4)*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x*exp_polar(I*pi)/a)/(a**
(5/4)*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = \frac{4\,ab-\frac{5\,(bx+a)a}{x}}{\frac{(bx+a)^{\frac{1}{4}}b^3}{x^{\frac{1}{4}}}-\frac{(bx+a)^{\frac{5}{4}}b^2}{x^{\frac{5}{4}}}} + \frac{5\,a\left(\frac{2\,\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{1}{4}}}\right)}{4\,b^2}$$

input
```
integrate(x^(5/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
(4*a*b - 5*(b*x + a)*a/x)/((b*x + a)^(1/4)*b^3/x^(1/4) - (b*x + a)^(5/4)*b
^2/x^(5/4)) + 5/4*a*(2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(1/4) +
 log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/
4)))/b^(1/4))/b^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(5/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument
 Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{5/4}}{{(a+b\,x)}^{5/4}} \,dx$$

input
```
int(x^(5/4)/(a + b*x)^(5/4),x)
```

output
```
int(x^(5/4)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{5/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}\,a + (bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(5/4)/(b*x+a)^(5/4),x)
```

output
```
int((x**(1/4)*x)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.729 $\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = -\frac{4\sqrt[4]{x}}{b\sqrt[4]{a+bx}}+\frac{2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{5/4}}+\frac{2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{5/4}}$$

output
```
-4*x^(1/4)/b/(b*x+a)^(1/4)+2*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(5/4)
+2*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = \frac{2\left(-\frac{2\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)+\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)\right)}{b^{5/4}}$$

input
```
Integrate[x^(1/4)/(a + b*x)^(5/4),x]
```

output
```
(2*((-2*b^(1/4)*x^(1/4))/(a + b*x)^(1/4) + ArcTan[(b^(1/4)*x^(1/4))/(a + b
*x)^(1/4)] + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]))/b^(5/4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {57, 73, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{\int \frac{1}{x^{3/4}\sqrt[4]{a+bx}}dx}{b}-\frac{4\sqrt[4]{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4\int \frac{1}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}-\frac{4\sqrt[4]{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{770}$$

$$\frac{4\int \frac{1}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}-\frac{4\sqrt[4]{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{756}$$

$$\frac{4\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{1}{2}\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b}-\frac{4\sqrt[4]{x}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{216}$$

$$\frac{4\left(\frac{1}{2}\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{4\sqrt[4]{x}}{b\sqrt[4]{a+bx}}$$

$$\downarrow 219$$

$$\frac{4\left(\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{4\sqrt[4]{x}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(1/4)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(1/4))/(b*(a + b*x)^(1/4)) + (4*(ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^
(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(1/4)
)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(1/4)/(b*x+a)^(5/4),x)
```

output
```
int(x^(1/4)/(b*x+a)^(5/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = \frac{(b^2x+ab)\frac{1}{b^5}^{\frac{1}{4}}\log\left(\frac{(b^2x+ab)\frac{1}{b^5}^{\frac{1}{4}}+(bx+a)^{\frac{3}{4}}x^{\frac{1}{4}}}{bx+a}\right)-(b^2x+ab)\frac{1}{b^5}^{\frac{1}{4}}\log\left(-\frac{(b^2x+ab)\frac{1}{b^5}^{\frac{1}{4}}-(bx+a)^{\frac{3}{4}}x}{bx+a}\right.}{}$$

input
```
integrate(x^(1/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
((b^2*x + a*b)*(b^(-5))^(1/4)*log(((b^2*x + a*b)*(b^(-5))^(1/4) + (b*x + a
)^(3/4)*x^(1/4))/(b*x + a)) - (b^2*x + a*b)*(b^(-5))^(1/4)*log(-((b^2*x +
a*b)*(b^(-5))^(1/4) - (b*x + a)^(3/4)*x^(1/4))/(b*x + a)) + (I*b^2*x + I*a
*b)*(b^(-5))^(1/4)*log(((I*b^2*x + I*a*b)*(b^(-5))^(1/4) + (b*x + a)^(3/4)
*x^(1/4))/(b*x + a)) + (-I*b^2*x - I*a*b)*(b^(-5))^(1/4)*log(((-I*b^2*x -
I*a*b)*(b^(-5))^(1/4) + (b*x + a)^(3/4)*x^(1/4))/(b*x + a)) - 4*(b*x + a)^
(3/4)*x^(1/4))/(b^2*x + a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = \frac{x^{\frac{5}{4}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(1/4)/(b*x+a)**(5/4),x)
```

output
```
x**(5/4)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x*exp_polar(I*pi)/a)/(a**(
5/4)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = -\frac{\frac{2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{1}{4}}}}{b}-\frac{4\,x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}b}$$

input
```
integrate(x^(1/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
-(2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(1/4) + log(-(b^(1/4) - (b
*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(1/4))/b - 4
*x^(1/4)/((b*x + a)^(1/4)*b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(1/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{1/4}}{{(a+b\,x)}^{5/4}}\,dx$$

input
```
int(x^(1/4)/(a + b*x)^(5/4),x)
```

output
```
int(x^(1/4)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(1/4)/(b*x+a)^(5/4),x)
```

output
```
int(x**(1/4)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.730 $\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx=\frac{4\sqrt[4]{x}}{a\sqrt[4]{a+bx}}$$

output
```
4*x^(1/4)/a/(b*x+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx=\frac{4\sqrt[4]{x}}{a\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(3/4)*(a + b*x)^(5/4)),x]
```

output
```
(4*x^(1/4))/(a*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx$$

$$\Big\downarrow \; 48$$

$$\frac{4\sqrt[4]{x}}{a\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(3/4)*(a + b*x)^(5/4)),x]
```

output
```
(4*x^(1/4))/(a*(a + b*x)^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{4x^{\frac{1}{4}}}{a(bx+a)^{\frac{1}{4}}}$	16
orering	$\frac{4x^{\frac{1}{4}}}{a(bx+a)^{\frac{1}{4}}}$	16

input
```
int(1/x^(3/4)/(b*x+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
4*x^(1/4)/a/(b*x+a)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx = \frac{4\,(bx+a)^{\frac{3}{4}}x^{\frac{1}{4}}}{abx+a^2}$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
4*(b*x + a)^(3/4)*x^(1/4)/(a*b*x + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx = \frac{\Gamma\left(\frac{1}{4}\right)}{a\sqrt[4]{b}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(3/4)/(b*x+a)**(5/4),x)
```

output
```
gamma(1/4)/(a*b**(1/4)*(a/(b*x) + 1)**(1/4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx = \frac{4\,x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}a}$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
4*x^(1/4)/((b*x + a)^(1/4)*a)
```

Giac [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{3/4}\,{(a+b\,x)}^{5/4}}\,dx$$

input
```
int(1/(x^(3/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(3/4)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(bx+a)^{\frac{1}{4}}\,a + x^{\frac{7}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

input
```
int(1/x^(3/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*a + x**(3/4)*(a + b*x)**(1/4)*b*x),x)
```

3.731 $\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx=\frac{4}{ax^{3/4}\sqrt[4]{a+bx}}-\frac{16(a+bx)^{3/4}}{3a^2x^{3/4}}$$

output
```
4/a/x^(3/4)/(b*x+a)^(1/4)-16/3*(b*x+a)^(3/4)/a^2/x^(3/4)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx=-\frac{4(a+4bx)}{3a^2x^{3/4}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(7/4)*(a + b*x)^(5/4)),x]
```

output
```
(-4*(a + 4*b*x))/(3*a^2*x^(3/4)*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{3/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4}{ax^{3/4}\sqrt[4]{a+bx}}-\frac{16(a+bx)^{3/4}}{3a^2x^{3/4}}$$

input
```
Int[1/(x^(7/4)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*x^(3/4)*(a + b*x)^(1/4)) - (16*(a + b*x)^(3/4))/(3*a^2*x^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{4(4bx+a)}{3x^{\frac{3}{4}}(bx+a)^{\frac{1}{4}}a^{2}}$	22
orering	$-\frac{4(4bx+a)}{3x^{\frac{3}{4}}(bx+a)^{\frac{1}{4}}a^{2}}$	22
risch	$-\frac{4(bx+a)^{\frac{3}{4}}}{3a^{2}x^{\frac{3}{4}}}-\frac{4b\,x^{\frac{1}{4}}}{a^{2}(bx+a)^{\frac{1}{4}}}$	33

input
```
int(1/x^(7/4)/(b*x+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(4*b*x+a)/x^(3/4)/(b*x+a)^(1/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx = -\frac{4\,(4\,bx+a)(bx+a)^{\frac{3}{4}}x^{\frac{1}{4}}}{3\,(a^2bx^2+a^3x)}$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
-4/3*(4*b*x + a)*(b*x + a)^(3/4)*x^(1/4)/(a^2*b*x^2 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right)}{4a\sqrt[4]{b}x\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(\frac{5}{4}\right)} + \frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{a^2\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(7/4)/(b*x+a)**(5/4),x)
```

output
```
gamma(-3/4)/(4*a*b**(1/4)*x*(a/(b*x) + 1)**(1/4)*gamma(5/4)) + b**(3/4)*ga
mma(-3/4)/(a**2*(a/(b*x) + 1)**(1/4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx = -\frac{4\,bx^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}a^2} - \frac{4\,(bx+a)^{\frac{3}{4}}}{3\,a^2x^{\frac{3}{4}}}$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
-4*b*x^(1/4)/((b*x + a)^(1/4)*a^2) - 4/3*(b*x + a)^(3/4)/(a^2*x^(3/4))
```

Giac [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(7/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{7/4}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/(x^(7/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(7/4)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{\frac{7}{4}}\,(bx+a)^{\frac{1}{4}}\,a+x^{\frac{11}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

input
```
int(1/x^(7/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*a*x + x**(3/4)*(a + b*x)**(1/4)*b*x**2),x
)
```

3.732 $\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = \frac{4}{ax^{7/4}\sqrt[4]{a+bx}} - \frac{32(a+bx)^{3/4}}{7a^2x^{7/4}} + \frac{128b(a+bx)^{3/4}}{21a^3x^{3/4}}$$

output
```
4/a/x^(7/4)/(b*x+a)^(1/4)-32/7*(b*x+a)^(3/4)/a^2/x^(7/4)+128/21*b*(b*x+a)^
(3/4)/a^3/x^(3/4)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = -\frac{4(3a^2-8abx-32b^2x^2)}{21a^3x^{7/4}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(11/4)*(a + b*x)^(5/4)),x]
```

output
```
(-4*(3*a^2 - 8*a*b*x - 32*b^2*x^2))/(21*a^3*x^(7/4)*(a + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{8\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{7/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{55}$$

$$\frac{8\left(-\frac{4b\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}dx}{7a}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{a}+\frac{4}{ax^{7/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{48}$$

$$\frac{8\left(\frac{16b(a+bx)^{3/4}}{21a^2x^{3/4}}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{a}+\frac{4}{ax^{7/4}\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(11/4)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*x^(7/4)*(a + b*x)^(1/4)) + (8*((-4*(a + b*x)^(3/4))/(7*a*x^(7/4)) + (
16*b*(a + b*x)^(3/4))/(21*a^2*x^(3/4))))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{4(-32b^2x^2-8abx+3a^2)}{21x^{\frac{7}{4}}(bx+a)^{\frac{1}{4}}a^3}$	35
orering	$-\frac{4(-32b^2x^2-8abx+3a^2)}{21x^{\frac{7}{4}}(bx+a)^{\frac{1}{4}}a^3}$	35
risch	$-\frac{4(bx+a)^{\frac{3}{4}}(-11bx+3a)}{21a^3x^{\frac{7}{4}}}+\frac{4b^2x^{\frac{1}{4}}}{a^3(bx+a)^{\frac{1}{4}}}$	43

input
```
int(1/x^(11/4)/(b*x+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-4/21*(-32*b^2*x^2-8*a*b*x+3*a^2)/x^(7/4)/(b*x+a)^(1/4)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = \frac{4\,(32\,b^2x^2+8\,abx-3\,a^2)(bx+a)^{\frac{3}{4}}x^{\frac{1}{4}}}{21\,(a^3bx^3+a^4x^2)}$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
4/21*(32*b^2*x^2 + 8*a*b*x - 3*a^2)*(b*x + a)^(3/4)*x^(1/4)/(a^3*b*x^3 + a
^4*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(58) = 116.

Time = 33.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.89

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = -\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16a^5b^4x\Gamma\left(\frac{5}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{5}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{5}{4}\right)} + \frac{5a^2b^{\frac{23}{4}}x\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16a^5b^4x\Gamma\left(\frac{5}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{5}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{5}{4}\right)} + \frac{40ab^{\frac{27}{4}}x^2\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16a^5b^4x\Gamma\left(\frac{5}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{5}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{5}{4}\right)} + \frac{32b^{\frac{31}{4}}x^3\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16a^5b^4x\Gamma\left(\frac{5}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{5}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(11/4)/(b*x+a)**(5/4),x)
```

output
```
-3*a**3*b**(19/4)*(a/(b*x) + 1)**(3/4)*gamma(-7/4)/(16*a**5*b**4*x*gamma(5
/4) + 32*a**4*b**5*x**2*gamma(5/4) + 16*a**3*b**6*x**3*gamma(5/4)) + 5*a**
2*b**(23/4)*x*(a/(b*x) + 1)**(3/4)*gamma(-7/4)/(16*a**5*b**4*x*gamma(5/4)
+ 32*a**4*b**5*x**2*gamma(5/4) + 16*a**3*b**6*x**3*gamma(5/4)) + 40*a*b**(
27/4)*x**2*(a/(b*x) + 1)**(3/4)*gamma(-7/4)/(16*a**5*b**4*x*gamma(5/4) + 3
2*a**4*b**5*x**2*gamma(5/4) + 16*a**3*b**6*x**3*gamma(5/4)) + 32*b**(31/4)
*x**3*(a/(b*x) + 1)**(3/4)*gamma(-7/4)/(16*a**5*b**4*x*gamma(5/4) + 32*a**
4*b**5*x**2*gamma(5/4) + 16*a**3*b**6*x**3*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = \frac{4\,b^2x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}a^3} + \frac{4\left(\frac{14\,(bx+a)^{\frac{3}{4}}b}{x^{\frac{3}{4}}} - \frac{3\,(bx+a)^{\frac{7}{4}}}{x^{\frac{7}{4}}}\right)}{21\,a^3}$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
4*b^2*x^(1/4)/((b*x + a)^(1/4)*a^3) + 4/21*(14*(b*x + a)^(3/4)*b/x^(3/4) -
 3*(b*x + a)^(7/4)/x^(7/4))/a^3
```

Giac [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(11/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{11/4}\,{\left(a+b\,x\right)}^{5/4}}\,dx$$

input
```
int(1/(x^(11/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(11/4)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{\frac{11}{4}}\,(bx+a)^{\frac{1}{4}}\,a+x^{\frac{15}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

input
```
int(1/x^(11/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*a*x**2 + x**(3/4)*(a + b*x)**(1/4)*b*x**3
),x)
```

3.733 $\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = \frac{4}{ax^{11/4}\sqrt[4]{a+bx}} - \frac{48(a+bx)^{3/4}}{11a^2x^{11/4}} + \frac{384b(a+bx)^{3/4}}{77a^3x^{7/4}} - \frac{512b^2(a+bx)^{3/4}}{77a^4x^{3/4}}$$

output
```
4/a/x^(11/4)/(b*x+a)^(1/4)-48/11*(b*x+a)^(3/4)/a^2/x^(11/4)+384/77*b*(b*x+
a)^(3/4)/a^3/x^(7/4)-512/77*b^2*(b*x+a)^(3/4)/a^4/x^(3/4)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = -\frac{4(7a^3-12a^2bx+32ab^2x^2+128b^3x^3)}{77a^4x^{11/4}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(15/4)*(a + b*x)^(5/4)),x]
```

output
```
(-4*(7*a^3 - 12*a^2*b*x + 32*a*b^2*x^2 + 128*b^3*x^3))/(77*a^4*x^(11/4)*(a
 + b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{12\int \frac{1}{x^{15/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{11/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{12\left(-\frac{8b\int \frac{1}{x^{11/4}\sqrt[4]{a+bx}}dx}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}\right)}{a}+\frac{4}{ax^{11/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{12\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^{7/4}\sqrt[4]{a+bx}}dx}{7a}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}\right)}{a}+\frac{4}{ax^{11/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{12\left(-\frac{8b\left(\frac{16b(a+bx)^{3/4}}{21a^2x^{3/4}}-\frac{4(a+bx)^{3/4}}{7ax^{7/4}}\right)}{11a}-\frac{4(a+bx)^{3/4}}{11ax^{11/4}}\right)}{a}+\frac{4}{ax^{11/4}\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(15/4)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*x^(11/4)*(a + b*x)^(1/4)) + (12*((-4*(a + b*x)^(3/4))/(11*a*x^(11/4))
 - (8*b*((-4*(a + b*x)^(3/4))/(7*a*x^(7/4)) + (16*b*(a + b*x)^(3/4))/(21*a
^2*x^(3/4))))/(11*a)))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{4\left(128 b^{3} x^{3}+32 a\,b^{2} x^{2}-12 a^{2} b x +7 a^{3}\right)}{77 x^{\frac{11}{4}} \left(b x +a \right)^{\frac{1}{4}} a^{4}}$	46
orering	$-\frac{4\left(128 b^{3} x^{3}+32 a\,b^{2} x^{2}-12 a^{2} b x +7 a^{3}\right)}{77 x^{\frac{11}{4}} \left(b x +a \right)^{\frac{1}{4}} a^{4}}$	46
risch	$-\frac{4 \left(b x +a \right)^{\frac{3}{4}} \left(51 b^{2} x^{2}-19 a b x +7 a^{2}\right)}{77 a^{4} x^{\frac{11}{4}}}-\frac{4 b^{3} x^{\frac{1}{4}}}{a^{4} \left(b x +a \right)^{\frac{1}{4}}}$	54

input
```
int(1/x^(15/4)/(b*x+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-4/77*(128*b^3*x^3+32*a*b^2*x^2-12*a^2*b*x+7*a^3)/x^(11/4)/(b*x+a)^(1/4)/a
^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = -\frac{4\,(128\,b^3x^3+32\,ab^2x^2-12\,a^2bx+7\,a^3)(bx+a)^{\frac{3}{4}}x^{\frac{1}{4}}}{77\,(a^4bx^4+a^5x^3)}$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
-4/77*(128*b^3*x^3 + 32*a*b^2*x^2 - 12*a^2*b*x + 7*a^3)*(b*x + a)^(3/4)*x^
(1/4)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(15/4)/(b*x+a)**(5/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = -\frac{4\,b^3x^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}a^4} - \frac{4\left(\frac{77\,(bx+a)^{\frac{3}{4}}b^2}{x^{\frac{3}{4}}} - \frac{33\,(bx+a)^{\frac{7}{4}}b}{x^{\frac{7}{4}}} + \frac{7\,(bx+a)^{\frac{11}{4}}}{x^{\frac{11}{4}}}\right)}{77\,a^4}$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
-4*b^3*x^(1/4)/((b*x + a)^(1/4)*a^4) - 4/77*(77*(b*x + a)^(3/4)*b^2/x^(3/4
) - 33*(b*x + a)^(7/4)*b/x^(7/4) + 7*(b*x + a)^(11/4)/x^(11/4))/a^4
```

Giac [F]

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{15}{4}}}\,dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(15/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{15/4}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/(x^(15/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(15/4)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{15/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{\frac{15}{4}}\,(bx+a)^{\frac{1}{4}}\,a+x^{\frac{19}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

input
```
int(1/x^(15/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(1/4)*a*x**3 + x**(3/4)*(a + b*x)**(1/4)*b*x**4
),x)
```

3.734 $\int \frac{x^{11/4}}{(a+bx)^{5/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}} \, dx = -\frac{4a^2x^{3/4}}{b^3\sqrt[4]{a+bx}} - \frac{17ax^{3/4}(a+bx)^{3/4}}{15b^3} + \frac{2x^{7/4}(a+bx)^{3/4}}{5b^2} + \frac{77a^3\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{10\sqrt{2}b^4\sqrt[4]{x}\sqrt[4]{a+bx}}$$

output
```
-4*a^2*x^(3/4)/b^3/(b*x+a)^(1/4)-17/15*a*x^(3/4)*(b*x+a)^(3/4)/b^3+2/5*x^(
7/4)*(b*x+a)^(3/4)/b^2+77/20*a^3*(-b*x/a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(
1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)/b^4/x^(1/4)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.37

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}} \, dx = \frac{4x^{15/4}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{15}{4},\frac{19}{4},-\frac{bx}{a}\right)}{15a\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(11/4)/(a + b*x)^(5/4),x]
```

output
```
(4*x^(15/4)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 15/4, 19/4, -((b*x)
/a)])/(15*a*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {57, 60, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{11\int \frac{x^{7/4}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 839

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 813

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$\Big\downarrow$ 858

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \textbf{807}$$

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{212}$$

$$\frac{11\left(\frac{2x^{7/4}(a+bx)^{3/4}}{5b}-\frac{7a\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{10b}\right)}{b}-\frac{4x^{11/4}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(11/4)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(11/4))/(b*(a + b*x)^(1/4)) + (11*((2*x^(7/4)*(a + b*x)^(3/4))/(5*b)
 - (7*a*((2*x^(3/4)*(a + b*x)^(3/4))/(3*b) - (2*a*(x^(3/4)/(2*(a + b*x)^(1
/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt
[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b))/(10*b)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(11/4)/(b*x+a)^(5/4),x)
```

output
```
int(x^(11/4)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(11/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(11/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}}\,dx = \frac{x^{\frac{15}{4}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**(11/4)/(b*x+a)**(5/4),x)
```

output
```
x**(15/4)*gamma(15/4)*hyper((5/4, 15/4), (19/4,), b*x*exp_polar(I*pi)/a)/(
a**(5/4)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}}\,dx=\int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(11/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^(11/4)/(b*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}}\,dx=\int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(11/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^(11/4)/(b*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}}\,dx=\int \frac{x^{11/4}}{(a+b\,x)^{5/4}}\,dx$$

input
```
int(x^(11/4)/(a + b*x)^(5/4),x)
```

output
```
int(x^(11/4)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{11/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(11/4)/(b*x+a)^(5/4),x)
```

output
```
int((x**(3/4)*x**2)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.735 $\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx=\frac{4ax^{3/4}}{b^2\sqrt[4]{a+bx}}+\frac{2x^{3/4}(a+bx)^{3/4}}{3b^2}-\frac{7a^2\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{\sqrt{2}b^3\sqrt[4]{x}\sqrt[4]{a+bx}}$$

output
```
4*a*x^(3/4)/b^2/(b*x+a)^(1/4)+2/3*x^(3/4)*(b*x+a)^(3/4)/b^2-7/2*a^2*(-b*x/
a-b^2*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))*2^(1/2)
/b^3/x^(1/4)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx=\frac{4x^{11/4}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{11}{4},\frac{15}{4},-\frac{bx}{a}\right)}{11a\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(7/4)/(a + b*x)^(5/4),x]
```

output
```
(4*x^(11/4)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 11/4, 15/4, -((b*x)
/a)])/(11*a*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {57, 60, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{7\int \frac{x^{3/4}}{\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{a\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{2b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{839}$$

$$\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{813}$$

$$\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{212}$$

$$\frac{7\left(\frac{2x^{3/4}(a+bx)^{3/4}}{3b}-\frac{2a\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}\right)}{b}-\frac{4x^{7/4}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(7/4)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(7/4))/(b*(a + b*x)^(1/4)) + (7*((2*x^(3/4)*(a + b*x)^(3/4))/(3*b) -
 (2*a*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*
EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1
/4))))/b))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(7/4)/(b*x+a)^(5/4),x)
```

output
```
int(x^(7/4)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(7/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx = \frac{x^{\frac{11}{4}}\Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**(7/4)/(b*x+a)**(5/4),x)
```

output
```
x**(11/4)*gamma(11/4)*hyper((5/4, 11/4), (15/4,), b*x*exp_polar(I*pi)/a)/(
a**(5/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^(7/4)/(b*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^(7/4)/(b*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{7/4}}{(a+b\,x)^{5/4}}\,dx$$

input
```
int(x^(7/4)/(a + b*x)^(5/4),x)
```

output
```
int(x^(7/4)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{7/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(7/4)/(b*x+a)^(5/4),x)
```

output
```
int((x**(3/4)*x)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.736 $\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = -\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}+\frac{3\sqrt{2}a\sqrt[4]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right)\middle|2\right)}{b^2\sqrt[4]{x}\sqrt[4]{a+bx}}$$

output
```
-4*x^(3/4)/b/(b*x+a)^(1/4)+3*2^(1/2)*a*(-b*x/a-b^2*x^2/a^2)^(1/4)*Elliptic
E(sin(1/2*arcsin(1+2*b*x/a)),2^(1/2))/b^2/x^(1/4)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = \frac{4x^{7/4}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{11}{4},-\frac{bx}{a}\right)}{7a\sqrt[4]{a+bx}}$$

input
```
Integrate[x^(3/4)/(a + b*x)^(5/4),x]
```

output
```
(4*x^(7/4)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[5/4, 7/4, 11/4, -((b*x)/a
)])/(7*a*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {57, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{3\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{b}-\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{12\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{b}-\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{839}$$

$$\frac{12\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{b}-\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{813}$$

$$\frac{12\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{b}-\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{12\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}-\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{12\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}-\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{212}$$

$$\frac{12\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{b}-\frac{4x^{3/4}}{b\sqrt[4]{a+bx}}$$

input
```
Int[x^(3/4)/(a + b*x)^(5/4),x]
```

output
```
(-4*x^(3/4))/(b*(a + b*x)^(1/4)) + (12*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqr
t[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b
]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{5}{4}}}dx$$

input
```
int(x^(3/4)/(b*x+a)^(5/4),x)
```

output
```
int(x^(3/4)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.43

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = \frac{x^{\frac{7}{4}}\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**(3/4)/(b*x+a)**(5/4),x)
```

output
```
x**(7/4)*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x*exp_polar(I*pi)/a)/(a**
(5/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^(3/4)/(b*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^(3/4)/(b*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{3/4}}{(a+b\,x)^{5/4}}\,dx$$

input
```
int(x^(3/4)/(a + b*x)^(5/4),x)
```

output
```
int(x^(3/4)/(a + b*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{3/4}}{(a+bx)^{5/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{1}{4}}\,a+(bx+a)^{\frac{1}{4}}\,bx}dx$$

input
```
int(x^(3/4)/(b*x+a)^(5/4),x)
```

output
```
int(x**(3/4)/((a + b*x)**(1/4)*a + (a + b*x)**(1/4)*b*x),x)
```

3.737 $\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}} dx = -\frac{4\sqrt[4]{x}\sqrt[4]{\frac{a+bx}{bx}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx}}$$

output
```
-4*x^(1/4)*((b*x+a)/b/x)^(1/4)*EllipticE(sin(1/2*arctan(1/b^(1/2)/x^(1/2)*
a^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/(b*x+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}} dx = \frac{4x^{3/4}\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx}{a}\right)}{3a\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(1/4)*(a + b*x)^(5/4)),x]
```

output
```
(4*x^(3/4)*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x)/a)
])/(3*a*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{4x^{3/4}}{a\sqrt[4]{a+bx}}-\frac{2\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{4x^{3/4}}{a\sqrt[4]{a+bx}}-\frac{8\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}$$

$$\Big\downarrow \text{839}$$

$$\frac{4x^{3/4}}{a\sqrt[4]{a+bx}}-\frac{8\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}$$

$$\Big\downarrow \text{813}$$

$$\frac{4x^{3/4}}{a\sqrt[4]{a+bx}}-\frac{8\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}$$

$$\Big\downarrow \text{858}$$

$$\frac{4x^{3/4}}{a\sqrt[4]{a+bx}}-\frac{8\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}$$

$$\Big\downarrow \text{807}$$

$$\frac{4x^{3/4}}{a\sqrt[4]{a+bx}}-\frac{8\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}$$

$$\Big\downarrow \text{212}$$

$$\frac{4x^{3/4}}{a\sqrt[4]{a+bx}}-\frac{8\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}$$

input
```
Int[1/(x^(1/4)*(a + b*x)^(5/4)),x]
```

output
```
(4*x^(3/4))/(a*(a + b*x)^(1/4)) - (8*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[
a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]
/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{1}{4}} (bx+a)^{\frac{5}{4}}}dx$$

input
```
int(1/x^(1/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(1/4)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b^2*x^3 + 2*a*b*x^2 + a^2*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x**(1/4)/(b*x+a)**(5/4),x)
```

output
```
x**(3/4)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x*exp_polar(I*pi)/a)/(a**(
5/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{1/4}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/(x^(1/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(1/4)*(a + b*x)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(bx+a)^{\frac{1}{4}}\,a + x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}\,b}dx$$

input
```
int(1/x^(1/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(1/4)*a + x**(1/4)*(a + b*x)**(1/4)*b*x),x)
```

3.738 $\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = -\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}+\frac{8\sqrt[4]{\frac{bx}{a+bx}}\sqrt[4]{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{x}}$$

output
```
-4/a/x^(1/4)/(b*x+a)^(1/4)+8*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/4)*EllipticE(s
in(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(3/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{bx}{a}\right)}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(5/4)*(a + b*x)^(5/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x)/a)])/(a*
x^(1/4)*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{2\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{2\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 839}$$

$$\frac{2\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 813}$$

$$\frac{2\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{2\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{212}$$

$$\frac{2\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{a}+\frac{4}{a\sqrt[4]{x}\sqrt[4]{a+bx}}$$

input

```
Int[1/(x^(5/4)*(a + b*x)^(5/4)),x]
```

output

```
4/(a*x^(1/4)*(a + b*x)^(1/4)) + (2*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*
b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*Elli
pticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))
))/a))/a
```

Defintions of rubi rules used

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{5}{4}} (bx+a)^{\frac{5}{4}}}dx$$

input
```
int(1/x^(5/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(5/4)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}\sqrt[4]{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**(5/4)/(b*x+a)**(5/4),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x*exp_polar(I*pi)/a)/(a**(5/4)*x*
*(1/4)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{5/4}\,{\left(a+b\,x\right)}^{5/4}}\,dx$$

input
```
int(1/(x^(5/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(5/4)*(a + b*x)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^{5/4}(a+bx)^{5/4}}\,dx = \frac{4x^{\frac{1}{4}}(bx+a)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{bx+a}\,a}$$

input
```
int(1/x^(5/4)/(b*x+a)^(5/4),x)
```

output
```
(4*x**(1/4)*(a + b*x)**(1/4))/(sqrt(x)*sqrt(a + b*x)*a)
```

3.739 $\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx = \frac{4}{ax^{5/4}\sqrt[4]{a+bx}}+\frac{48b}{5a^2\sqrt[4]{x}\sqrt[4]{a+bx}} -\frac{24(a+bx)^{3/4}}{5a^2x^{5/4}}-\frac{48b\sqrt[4]{\frac{bx}{a+bx}}\sqrt[4]{a+bx}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{5a^{5/2}\sqrt[4]{x}}$$

output
```
4/a/x^(5/4)/(b*x+a)^(1/4)+48/5*b/a^2/x^(1/4)/(b*x+a)^(1/4)-24/5*(b*x+a)^(3
/4)/a^2/x^(5/4)-48/5*b*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/4)*EllipticE(sin(1/2
*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/2))/a^(5/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},-\frac{1}{4},-\frac{bx}{a}\right)}{5ax^{5/4}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(9/4)*(a + b*x)^(5/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*x)/a)])/(5
*a*x^(5/4)*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}dx$$

$$\Big\downarrow \; 61$$

$$\frac{6\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 61$$

$$\frac{6\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 61$$

$$\frac{6\left(-\frac{2b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \; 73$$

$$\frac{6\left(-\frac{2b\left(\frac{8b\int\frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 839

$$\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int\frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 813

$$\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 858

$$\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 807

$$\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 212

$$\frac{6\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{a}+\frac{4}{ax^{5/4}\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(9/4)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*x^(5/4)*(a + b*x)^(1/4)) + (6*((-4*(a + b*x)^(3/4))/(5*a*x^(5/4)) - (
2*b*((-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4))
+ (Sqrt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/
Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a))/(5*a)))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{9}{4}} (bx+a)^{\frac{5}{4}}} dx$$

input
```
int(1/x^(9/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(9/4)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx=\int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b^2*x^5 + 2*a*b*x^4 + a^2*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx=\frac{\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{5}{4}\\-\frac{1}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}x^{\frac{5}{4}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**(9/4)/(b*x+a)**(5/4),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 5/4), (-1/4,), b*x*exp_polar(I*pi)/a)/(a**(5/4)*x
**(5/4)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(9/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(9/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{9/4}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/(x^(9/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(9/4)*(a + b*x)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^{9/4}(a+bx)^{5/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\,(-4bx-a)}{3x^{\frac{3}{4}}\sqrt{x}\,\sqrt{bx+a}\,a^2}$$

input
```
int(1/x^(9/4)/(b*x+a)^(5/4),x)
```

output
```
(4*x**(1/4)*(a + b*x)**(1/4)*( - a - 4*b*x))/(3*sqrt(x)*sqrt(a + b*x)*a**2
*x)
```

3.740 $\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = \frac{4}{ax^{9/4}\sqrt[4]{a+bx}} - \frac{32b^2}{3a^3\sqrt[4]{x}\sqrt[4]{a+bx}} - \frac{40(a+bx)^{3/4}}{9a^2x^{9/4}} + \frac{16b(a+bx)^{3/4}}{3a^3x^{5/4}} + \frac{32b^2\sqrt[4]{\frac{bx}{a+bx}}\sqrt[4]{a+bx}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{3a^{7/2}\sqrt[4]{x}}$$

output

```
4/a/x^(9/4)/(b*x+a)^(1/4)-32/3*b^2/a^3/x^(1/4)/(b*x+a)^(1/4)-40/9*(b*x+a)^
(3/4)/a^2/x^(9/4)+16/3*b*(b*x+a)^(3/4)/a^3/x^(5/4)+32/3*b^2*(b*x/(b*x+a))^
(1/4)*(b*x+a)^(1/4)*EllipticE(sin(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2))),2^(1/
2))/a^(7/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{5}{4},-\frac{5}{4},-\frac{bx}{a}\right)}{9ax^{9/4}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(13/4)*(a + b*x)^(5/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x)/a)])/(9
*a*x^(9/4)*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {61, 61, 61, 61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{10\int \frac{1}{x^{13/4}\sqrt[4]{a+bx}}dx}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{10\left(-\frac{2b\int \frac{1}{x^{9/4}\sqrt[4]{a+bx}}dx}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{10\left(-\frac{2b\left(-\frac{2b\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 73

$$\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 839

$$\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 813

$$\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+$$

$$\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 858

$$\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

$\downarrow$ 807

$$\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{\frac{4^{a}}{ax^{9/4}\sqrt[4]{a+bx}}}+$$

$$\Big\downarrow \; 212$$

$$\frac{10\left(-\frac{2b\left(-\frac{2b\left(\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}\right)}{5a}-\frac{4(a+bx)^{3/4}}{5ax^{5/4}}\right)}{3a}-\frac{4(a+bx)^{3/4}}{9ax^{9/4}}\right)}{a}+\frac{4}{ax^{9/4}\sqrt[4]{a+bx}}$$

input
```
Int[1/(x^(13/4)*(a + b*x)^(5/4)),x]
```

output
```
4/(a*x^(9/4)*(a + b*x)^(1/4)) + (10*((-4*(a + b*x)^(3/4))/(9*a*x^(9/4)) -
(2*b*((-4*(a + b*x)^(3/4))/(5*a*x^(5/4)) - (2*b*((-4*(a + b*x)^(3/4))/(a*x
^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sqrt[a]*(1 + a/(b*x))^(1/4)
*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a
+ b*x)^(1/4))))/a))/(5*a)))/(3*a)))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{13}{4}} (bx+a)^{\frac{5}{4}}}dx$$

input
```
int(1/x^(13/4)/(b*x+a)^(5/4),x)
```

output
```
int(1/x^(13/4)/(b*x+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 110.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = \frac{\Gamma\left(-\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{9}{4}, \frac{5}{4} \\ -\frac{5}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate(1/x**(13/4)/(b*x+a)**(5/4),x)
```

output
```
gamma(-9/4)*hyper((-9/4, 5/4), (-5/4,), b*x*exp_polar(I*pi)/a)/(a**(5/4)*x
**(9/4)*gamma(-5/4))
```

Maxima [F]

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(13/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/4)*x^(13/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = \int \frac{1}{x^{13/4}\,(a+b\,x)^{5/4}}\,dx$$

input
```
int(1/(x^(13/4)*(a + b*x)^(5/4)),x)
```

output
```
int(1/(x^(13/4)*(a + b*x)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^{13/4}(a+bx)^{5/4}}\,dx = \frac{4(bx+a)^{\frac{1}{4}}\,(32b^2x^2+8abx-3a^2)}{21x^{\frac{7}{4}}\sqrt{x}\,\sqrt{bx+a}\,a^3}$$

input
```
int(1/x^(13/4)/(b*x+a)^(5/4),x)
```

output
```
(4*x**(1/4)*(a + b*x)**(1/4)*( - 3*a**2 + 8*a*b*x + 32*b**2*x**2))/(21*sqr
t(x)*sqrt(a + b*x)*a**3*x**2)
```

3.741 $\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = -\frac{4a^2x^{3/4}}{3b^3(a+bx)^{3/4}} - \frac{15ax^{3/4}\sqrt[4]{a+bx}}{8b^3} + \frac{x^{7/4}\sqrt[4]{a+bx}}{2b^2} - \frac{77a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{15/4}} + \frac{77a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{16b^{15/4}}$$

output
```
-4/3*a^2*x^(3/4)/b^3/(b*x+a)^(3/4)-15/8*a*x^(3/4)*(b*x+a)^(1/4)/b^3+1/2*x^
(7/4)*(b*x+a)^(1/4)/b^2-77/16*a^2*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^
(15/4)+77/16*a^2*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(15/4)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = \frac{\frac{2b^{3/4}x^{3/4}(-77a^2-33abx+12b^2x^2)}{(a+bx)^{3/4}} - 231a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right) + 231a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{48b^{15/4}}$$

input
```
Integrate[x^(11/4)/(a + b*x)^(7/4),x]
```

output
```
((2*b^(3/4)*x^(3/4)*(-77*a^2 - 33*a*b*x + 12*b^2*x^2))/(a + b*x)^(3/4) - 2
31*a^2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)] + 231*a^2*ArcTanh[(b^(1/4
)*x^(1/4))/(a + b*x)^(1/4)])/(48*b^(15/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {57, 60, 60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{11\int \frac{x^{7/4}}{(a+bx)^{3/4}}dx}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{11\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\int \frac{x^{3/4}}{(a+bx)^{3/4}}dx}{8b}\right)}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{11\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{4b}\right)}{8b}\right)}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{11\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int\frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{8b}\right)}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{854}$$

$$\frac{11\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int\frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{8b}\right)}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{827}$$

$$\frac{11\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int\frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int\frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}\right)}{8b}\right)}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{11\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

$\Big\downarrow$ 219

$$\frac{11\left(\frac{x^{7/4}\sqrt[4]{a+bx}}{2b}-\frac{7a\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{8b}\right)}{3b}-\frac{4x^{11/4}}{3b(a+bx)^{3/4}}$$

input
```
Int[x^(11/4)/(a + b*x)^(7/4),x]
```

output
```
(-4*x^(11/4))/(3*b*(a + b*x)^(3/4)) + (11*((x^(7/4)*(a + b*x)^(1/4))/(2*b)
 - (7*a*((x^(3/4)*(a + b*x)^(1/4))/b - (3*a*(-1/2*ArcTan[(b^(1/4)*x^(1/4))
/(a + b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*
b^(3/4))))/b))/(8*b)))/(3*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{7}{4}}}dx$$

input
```
int(x^(11/4)/(b*x+a)^(7/4),x)
```

output
```
int(x^(11/4)/(b*x+a)^(7/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.12

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = \frac{231\,(b^4x+ab^3)\left(\frac{a^8}{b^{15}}\right)^{\frac{1}{4}}\log\left(\frac{77\left(b^4x\left(\frac{a^8}{b^{15}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}a^2x^{\frac{3}{4}}\right)}{x}\right)-231\,(b^4x+ab^3)\left(\frac{a^8}{b^{15}}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
integrate(x^(11/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
1/96*(231*(b^4*x + a*b^3)*(a^8/b^15)^(1/4)*log(77*(b^4*x*(a^8/b^15)^(1/4)
+ (b*x + a)^(1/4)*a^2*x^(3/4))/x) - 231*(b^4*x + a*b^3)*(a^8/b^15)^(1/4)*l
og(-77*(b^4*x*(a^8/b^15)^(1/4) - (b*x + a)^(1/4)*a^2*x^(3/4))/x) - 231*(I*
b^4*x + I*a*b^3)*(a^8/b^15)^(1/4)*log(-77*(I*b^4*x*(a^8/b^15)^(1/4) - (b*x
 + a)^(1/4)*a^2*x^(3/4))/x) - 231*(-I*b^4*x - I*a*b^3)*(a^8/b^15)^(1/4)*lo
g(-77*(-I*b^4*x*(a^8/b^15)^(1/4) - (b*x + a)^(1/4)*a^2*x^(3/4))/x) + 4*(12
*b^2*x^2 - 33*a*b*x - 77*a^2)*(b*x + a)^(1/4)*x^(3/4))/(b^4*x + a*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 61.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = \frac{x^{\frac{15}{4}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**(11/4)/(b*x+a)**(7/4),x)
```

output
```
x**(15/4)*gamma(15/4)*hyper((7/4, 15/4), (19/4,), b*x*exp_polar(I*pi)/a)/(
a**(7/4)*gamma(19/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = -\frac{32\,a^2b^2-\frac{121\,(bx+a)a^2b}{x}+\frac{77\,(bx+a)^2a^2}{x^2}}{24\left(\frac{(bx+a)^{\frac{3}{4}}b^5}{x^{\frac{3}{4}}}-\frac{2\,(bx+a)^{\frac{7}{4}}b^4}{x^{\frac{7}{4}}}+\frac{(bx+a)^{\frac{11}{4}}b^3}{x^{\frac{11}{4}}}\right)} + \frac{77\left(\frac{2\,a^2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{32\,b^3}$$

input
```
integrate(x^(11/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
-1/24*(32*a^2*b^2 - 121*(b*x + a)*a^2*b/x + 77*(b*x + a)^2*a^2/x^2)/((b*x
+ a)^(3/4)*b^5/x^(3/4) - 2*(b*x + a)^(7/4)*b^4/x^(7/4) + (b*x + a)^(11/4)*
b^3/x^(11/4)) + 77/32*(2*a^2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(
3/4) - a^2*log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(
1/4)/x^(1/4)))/b^(3/4))/b^3
```

Giac [F]

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(11/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^(11/4)/(b*x + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{11/4}}{(a+b\,x)^{7/4}}\,dx$$

input
```
int(x^(11/4)/(a + b*x)^(7/4),x)
```

output
```
int(x^(11/4)/(a + b*x)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^{11/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{11}{4}}}{(bx+a)^{\frac{3}{4}}\,a+(bx+a)^{\frac{3}{4}}\,bx}dx$$

input

```
int(x^(11/4)/(b*x+a)^(7/4),x)
```

output

```
int((x**(3/4)*x**2)/((a + b*x)**(3/4)*a + (a + b*x)**(3/4)*b*x),x)
```

3.742 $\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx = \frac{4ax^{3/4}}{3b^2(a+bx)^{3/4}} + \frac{x^{3/4}\sqrt[4]{a+bx}}{b^2} + \frac{7a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{11/4}} - \frac{7a\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{11/4}}$$

output
```
4/3*a*x^(3/4)/b^2/(b*x+a)^(3/4)+x^(3/4)*(b*x+a)^(1/4)/b^2+7/2*a*arctan(b^(
1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(11/4)-7/2*a*arctanh(b^(1/4)*x^(1/4)/(b*x+a)
^(1/4))/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx = \frac{\frac{2b^{3/4}x^{3/4}(7a+3bx)}{(a+bx)^{3/4}} + 21a\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right) - 21a\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{6b^{11/4}}$$

input
```
Integrate[x^(7/4)/(a + b*x)^(7/4),x]
```

output
```
((2*b^(3/4)*x^(3/4)*(7*a + 3*b*x))/(a + b*x)^(3/4) + 21*a*ArcTan[(b^(1/4)*
x^(1/4))/(a + b*x)^(1/4)] - 21*a*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)
])/(6*b^(11/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {57, 60, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{7\int \frac{x^{3/4}}{(a+bx)^{3/4}}dx}{3b}-\frac{4x^{7/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{4b}\right)}{3b}-\frac{4x^{7/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{7\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{3b}-\frac{4x^{7/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{854}$$

$$\frac{7\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}\right)}{3b}-\frac{4x^{7/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{827}$$

$$\frac{7\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}\right)}{3b}-\frac{4x^{7/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{7\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{3b}-\frac{4x^{7/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{7\left(\frac{x^{3/4}\sqrt[4]{a+bx}}{b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}\right)}{3b}-\frac{4x^{7/4}}{3b(a+bx)^{3/4}}$$

input

```
Int[x^(7/4)/(a + b*x)^(7/4),x]
```

output

```
(-4*x^(7/4))/(3*b*(a + b*x)^(3/4)) + (7*((x^(3/4)*(a + b*x)^(1/4))/b - (3*
a*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/
4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3/4))))/b))/(3*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{7}{4}}}dx$$

input
```
int(x^(7/4)/(b*x+a)^(7/4),x)
```

output
```
int(x^(7/4)/(b*x+a)^(7/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.57

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx =$$

$$-\frac{21\left(b^3x+ab^2\right)\left(\frac{a^4}{b^{11}}\right)^{\frac{1}{4}}\log\left(\frac{7\left(b^3x\left(\frac{a^4}{b^{11}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}ax^{\frac{3}{4}}\right)}{x}\right)-21\left(b^3x+ab^2\right)\left(\frac{a^4}{b^{11}}\right)^{\frac{1}{4}}\log\left(-\frac{7\left(b^3x\left(\frac{a^4}{b^{11}}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}\cdots\right)}{x}\cdots\right.}{\cdots}$$

input
```
integrate(x^(7/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
-1/12*(21*(b^3*x + a*b^2)*(a^4/b^11)^(1/4)*log(7*(b^3*x*(a^4/b^11)^(1/4) +
 (b*x + a)^(1/4)*a*x^(3/4))/x) - 21*(b^3*x + a*b^2)*(a^4/b^11)^(1/4)*log(-
7*(b^3*x*(a^4/b^11)^(1/4) - (b*x + a)^(1/4)*a*x^(3/4))/x) + 21*(-I*b^3*x -
 I*a*b^2)*(a^4/b^11)^(1/4)*log(-7*(I*b^3*x*(a^4/b^11)^(1/4) - (b*x + a)^(1
/4)*a*x^(3/4))/x) + 21*(I*b^3*x + I*a*b^2)*(a^4/b^11)^(1/4)*log(-7*(-I*b^3
*x*(a^4/b^11)^(1/4) - (b*x + a)^(1/4)*a*x^(3/4))/x) - 4*(3*b*x + 7*a)*(b*x
 + a)^(1/4)*x^(3/4))/(b^3*x + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.94 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.35

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx = \frac{x^{\frac{11}{4}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**(7/4)/(b*x+a)**(7/4),x)
```

output
```
x**(11/4)*gamma(11/4)*hyper((7/4, 11/4), (15/4,), b*x*exp_polar(I*pi)/a)/(
a**(7/4)*gamma(15/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx = \frac{4\,ab-\frac{7\,(bx+a)a}{x}}{3\left(\frac{(bx+a)^{\frac{3}{4}}b^3}{x^{\frac{3}{4}}}-\frac{(bx+a)^{\frac{7}{4}}b^2}{x^{\frac{7}{4}}}\right)} - \frac{7\left(\frac{2\,a\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}\right)}{4\,b^2}$$

input
```
integrate(x^(7/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
1/3*(4*a*b - 7*(b*x + a)*a/x)/((b*x + a)^(3/4)*b^3/x^(3/4) - (b*x + a)^(7/
4)*b^2/x^(7/4)) - 7/4*(2*a*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(3/
4) - a*log(-(b^(1/4) - (b*x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)
/x^(1/4)))/b^(3/4))/b^2
```

Giac [F]

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(7/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^(7/4)/(b*x + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{7/4}}{(a+b\,x)^{7/4}}\,dx$$

input
```
int(x^(7/4)/(a + b*x)^(7/4),x)
```

output
```
int(x^(7/4)/(a + b*x)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^{7/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{7}{4}}}{(bx+a)^{\frac{3}{4}}a+(bx+a)^{\frac{3}{4}}bx}dx$$

input
```
int(x^(7/4)/(b*x+a)^(7/4),x)
```

output
```
int((x**(3/4)*x)/((a + b*x)**(3/4)*a + (a + b*x)**(3/4)*b*x),x)
```

3.743 $\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = -\frac{4x^{3/4}}{3b(a+bx)^{3/4}} - \frac{2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{7/4}} + \frac{2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{7/4}}$$

output
```
-4/3*x^(3/4)/b/(b*x+a)^(3/4)-2*arctan(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(7/
4)+2*arctanh(b^(1/4)*x^(1/4)/(b*x+a)^(1/4))/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = -\frac{4x^{3/4}}{3b(a+bx)^{3/4}} - \frac{2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{7/4}} + \frac{2\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{b^{7/4}}$$

input
```
Integrate[x^(3/4)/(a + b*x)^(7/4),x]
```

output
```
(-4*x^(3/4))/(3*b*(a + b*x)^(3/4)) - (2*ArcTan[(b^(1/4)*x^(1/4))/(a + b*x)
^(1/4)])/b^(7/4) + (2*ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)])/b^(7/4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {57, 73, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{\int \frac{1}{\sqrt[4]{x}(a+bx)^{3/4}}dx}{b}-\frac{4x^{3/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4\int \frac{\sqrt{x}}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}-\frac{4x^{3/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{854}$$

$$\frac{4\int \frac{\sqrt{x}}{1-bx}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{b}-\frac{4x^{3/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{827}$$

$$\frac{4\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}\sqrt{x}+1}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}\right)}{b}-\frac{4x^{3/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{4\left(\frac{\int \frac{1}{1-\sqrt{b}\sqrt{x}}d\frac{\sqrt[4]{x}}{\sqrt[4]{a+bx}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}-\frac{4x^{3/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{x}}{\sqrt[4]{a+bx}}\right)}{2b^{3/4}}\right)}{b}-\frac{4x^{3/4}}{3b(a+bx)^{3/4}}$$

input
```
Int[x^(3/4)/(a + b*x)^(7/4),x]
```

output
```
(-4*x^(3/4))/(3*b*(a + b*x)^(3/4)) + (4*(-1/2*ArcTan[(b^(1/4)*x^(1/4))/(a
+ b*x)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x^(1/4))/(a + b*x)^(1/4)]/(2*b^(3
/4))))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{7}{4}}}dx$$

input
```
int(x^(3/4)/(b*x+a)^(7/4),x)
```

output
```
int(x^(3/4)/(b*x+a)^(7/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.64

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = \frac{3\,(b^2x+ab)\frac{1}{b^7}^{\frac{1}{4}}\log\left(\frac{b^2\frac{1}{b^7}^{\frac{1}{4}}x+(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) - 3\,(b^2x+ab)\frac{1}{b^7}^{\frac{1}{4}}\log\left(-\frac{b^2\frac{1}{b^7}^{\frac{1}{4}}x-(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{x}\right) -}{}$$

input
```
integrate(x^(3/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
1/3*(3*(b^2*x + a*b)*(b^(-7))^(1/4)*log((b^2*(b^(-7))^(1/4)*x + (b*x + a)^
(1/4)*x^(3/4))/x) - 3*(b^2*x + a*b)*(b^(-7))^(1/4)*log(-(b^2*(b^(-7))^(1/4
)*x - (b*x + a)^(1/4)*x^(3/4))/x) - 3*(-I*b^2*x - I*a*b)*(b^(-7))^(1/4)*lo
g((I*b^2*(b^(-7))^(1/4)*x + (b*x + a)^(1/4)*x^(3/4))/x) - 3*(I*b^2*x + I*a
*b)*(b^(-7))^(1/4)*log((-I*b^2*(b^(-7))^(1/4)*x + (b*x + a)^(1/4)*x^(3/4))
/x) - 4*(b*x + a)^(1/4)*x^(3/4))/(b^2*x + a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = \frac{x^{\frac{7}{4}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**(3/4)/(b*x+a)**(7/4),x)
```

output
```
x**(7/4)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x*exp_polar(I*pi)/a)/(a**
(7/4)*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = \frac{\frac{2\arctan\left(\frac{(bx+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}{b^{\frac{1}{4}}+\frac{(bx+a)^{\frac{1}{4}}}{x^{\frac{1}{4}}}}\right)}{b^{\frac{3}{4}}}}{b} - \frac{4\,x^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}b}$$

input
```
integrate(x^(3/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
(2*arctan((b*x + a)^(1/4)/(b^(1/4)*x^(1/4)))/b^(3/4) - log(-(b^(1/4) - (b*
x + a)^(1/4)/x^(1/4))/(b^(1/4) + (b*x + a)^(1/4)/x^(1/4)))/b^(3/4))/b - 4/
3*x^(3/4)/((b*x + a)^(3/4)*b)
```

Giac [F]

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(3/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^(3/4)/(b*x + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{3/4}}{(a+b\,x)^{7/4}}\,dx$$

input
```
int(x^(3/4)/(a + b*x)^(7/4),x)
```

output
```
int(x^(3/4)/(a + b*x)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^{3/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{3}{4}}}{(bx+a)^{\frac{3}{4}}a+(bx+a)^{\frac{3}{4}}bx}dx$$

input
```
int(x^(3/4)/(b*x+a)^(7/4),x)
```

output
```
int(x**(3/4)/((a + b*x)**(3/4)*a + (a + b*x)**(3/4)*b*x),x)
```

3.744 $\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \frac{4x^{3/4}}{3a(a+bx)^{3/4}}$$

output
```
4/3*x^(3/4)/a/(b*x+a)^(3/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \frac{4x^{3/4}}{3a(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(1/4)*(a + b*x)^(7/4)),x]
```

output
```
(4*x^(3/4))/(3*a*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}} \, dx$$
$$\Big\downarrow \text{48}$$
$$\frac{4x^{3/4}}{3a(a+bx)^{3/4}}$$

input
```
Int[1/(x^(1/4)*(a + b*x)^(7/4)),x]
```

output
```
(4*x^(3/4))/(3*a*(a + b*x)^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{4x^{\frac{3}{4}}}{3a(bx+a)^{\frac{3}{4}}}$	16
orering	$\frac{4x^{\frac{3}{4}}}{3a(bx+a)^{\frac{3}{4}}}$	16

input
```
int(1/x^(1/4)/(b*x+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
4/3*x^(3/4)/a/(b*x+a)^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \frac{4\,(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{3\,(abx+a^2)}$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
4/3*(b*x + a)^(1/4)*x^(3/4)/(a*b*x + a^2)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \frac{\Gamma\left(\frac{3}{4}\right)}{ab^{\frac{3}{4}}\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x**(1/4)/(b*x+a)**(7/4),x)
```

output
```
gamma(3/4)/(a*b**(3/4)*(a/(b*x) + 1)**(3/4)*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \frac{4\,x^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}a}$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
4/3*x^(3/4)/((b*x + a)^(3/4)*a)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(1/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \frac{4\,x^{3/4}\,{(a+b\,x)}^{1/4}}{3\,a^2+3\,b\,x\,a}$$

input
```
int(1/(x^(1/4)*(a + b*x)^(7/4)),x)
```

output
```
(4*x^(3/4)*(a + b*x)^(1/4))/(3*a^2 + 3*a*b*x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{x}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(bx+a)^{\frac{3}{4}}\,a + x^{\frac{5}{4}}\,(bx+a)^{\frac{3}{4}}\,b}dx$$

input
```
int(1/x^(1/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*a + x**(1/4)*(a + b*x)**(3/4)*b*x),x)
```

3.745 $\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = \frac{4}{3a\sqrt[4]{x}(a+bx)^{3/4}} - \frac{16\sqrt[4]{a+bx}}{3a^2\sqrt[4]{x}}$$

output
```
4/3/a/x^(1/4)/(b*x+a)^(3/4)-16/3*(b*x+a)^(1/4)/a^2/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = -\frac{4(3a+4bx)}{3a^2\sqrt[4]{x}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(5/4)*(a + b*x)^(7/4)),x]
```

output
```
(-4*(3*a + 4*b*x))/(3*a^2*x^(1/4)*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx$$

$$\Big\downarrow \; 55$$

$$\frac{4\int \frac{1}{x^{5/4}(a+bx)^{3/4}}dx}{3a}+\frac{4}{3a\sqrt[4]{x}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 48$$

$$\frac{4}{3a\sqrt[4]{x}(a+bx)^{3/4}}-\frac{16\sqrt[4]{a+bx}}{3a^2\sqrt[4]{x}}$$

input
```
Int[1/(x^(5/4)*(a + b*x)^(7/4)),x]
```

output
```
4/(3*a*x^(1/4)*(a + b*x)^(3/4)) - (16*(a + b*x)^(1/4))/(3*a^2*x^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{4(4bx+3a)}{3x^{\frac{1}{4}}(bx+a)^{\frac{3}{4}}a^{2}}$	24
orering	$-\frac{4(4bx+3a)}{3x^{\frac{1}{4}}(bx+a)^{\frac{3}{4}}a^{2}}$	24
risch	$-\frac{4(bx+a)^{\frac{1}{4}}}{a^{2}x^{\frac{1}{4}}}-\frac{4b\,x^{\frac{3}{4}}}{3a^{2}(bx+a)^{\frac{3}{4}}}$	33

input
```
int(1/x^(5/4)/(b*x+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(4*b*x+3*a)/x^(1/4)/(b*x+a)^(3/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = -\frac{4\,(4\,bx+3\,a)(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{3\,(a^2bx^2+a^3x)}$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
-4/3*(4*b*x + 3*a)*(b*x + a)^(1/4)*x^(3/4)/(a^2*b*x^2 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = \frac{3\Gamma\left(-\frac{1}{4}\right)}{4ab^{\frac{3}{4}}x\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt[4]{b}\Gamma\left(-\frac{1}{4}\right)}{a^2\left(\frac{a}{bx}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x**(5/4)/(b*x+a)**(7/4),x)
```

output
```
3*gamma(-1/4)/(4*a*b**(3/4)*x*(a/(b*x) + 1)**(3/4)*gamma(7/4)) + b**(1/4)*
gamma(-1/4)/(a**2*(a/(b*x) + 1)**(3/4)*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = -\frac{4\,bx^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}a^2} - \frac{4\,(bx+a)^{\frac{1}{4}}}{a^2x^{\frac{1}{4}}}$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
-4/3*b*x^(3/4)/((b*x + a)^(3/4)*a^2) - 4*(b*x + a)^(1/4)/(a^2*x^(1/4))
```

Giac [F]

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(5/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = -\frac{12\,a\,(a+b\,x)^{1/4}+16\,b\,x\,(a+b\,x)^{1/4}}{x^{1/4}\,\left(3\,a^3+3\,b\,x\,a^2\right)}$$

input
```
int(1/(x^(5/4)*(a + b*x)^(7/4)),x)
```

output
```
-(12*a*(a + b*x)^(1/4) + 16*b*x*(a + b*x)^(1/4))/(x^(1/4)*(3*a^3 + 3*a^2*b
*x))
```

Reduce [F]

$$\int \frac{1}{x^{5/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(bx+a)^{\frac{3}{4}}\,a+x^{\frac{9}{4}}\,(bx+a)^{\frac{3}{4}}\,b}dx$$

input
```
int(1/x^(5/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*a*x + x**(1/4)*(a + b*x)**(3/4)*b*x**2),x
)
```

3.746 $\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = \frac{4}{3ax^{5/4}(a+bx)^{3/4}} - \frac{32\sqrt[4]{a+bx}}{15a^2x^{5/4}} + \frac{128b\sqrt[4]{a+bx}}{15a^3\sqrt[4]{x}}$$

output
```
4/3/a/x^(5/4)/(b*x+a)^(3/4)-32/15*(b*x+a)^(1/4)/a^2/x^(5/4)+128/15*b*(b*x+
a)^(1/4)/a^3/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = -\frac{4(3a^2-24abx-32b^2x^2)}{15a^3x^{5/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(9/4)*(a + b*x)^(7/4)),x]
```

output
```
(-4*(3*a^2 - 24*a*b*x - 32*b^2*x^2))/(15*a^3*x^(5/4)*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{8\int \frac{1}{x^{9/4}(a+bx)^{3/4}}dx}{3a}+\frac{4}{3ax^{5/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{8\left(-\frac{4b\int \frac{1}{x^{5/4}(a+bx)^{3/4}}dx}{5a}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{3a}+\frac{4}{3ax^{5/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{8\left(\frac{16b\sqrt[4]{a+bx}}{5a^2\sqrt[4]{x}}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{3a}+\frac{4}{3ax^{5/4}(a+bx)^{3/4}}$$

input
```
Int[1/(x^(9/4)*(a + b*x)^(7/4)),x]
```

output
```
4/(3*a*x^(5/4)*(a + b*x)^(3/4)) + (8*((-4*(a + b*x)^(1/4))/(5*a*x^(5/4)) +
 (16*b*(a + b*x)^(1/4))/(5*a^2*x^(1/4))))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{4\left(-32b^{2}x^{2}-24abx+3a^{2}\right)}{15x^{\frac{5}{4}}\left(bx+a\right)^{\frac{3}{4}}a^{3}}$	35
orering	$-\frac{4\left(-32b^{2}x^{2}-24abx+3a^{2}\right)}{15x^{\frac{5}{4}}\left(bx+a\right)^{\frac{3}{4}}a^{3}}$	35
risch	$-\frac{4\left(bx+a\right)^{\frac{1}{4}}\left(-9bx+a\right)}{5a^{3}x^{\frac{5}{4}}}+\frac{4b^{2}x^{\frac{3}{4}}}{3a^{3}\left(bx+a\right)^{\frac{3}{4}}}$	41

input
```
int(1/x^(9/4)/(b*x+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-4/15*(-32*b^2*x^2-24*a*b*x+3*a^2)/x^(5/4)/(b*x+a)^(3/4)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = \frac{4\,(32\,b^2x^2+24\,abx-3\,a^2)(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{15\,(a^3bx^3+a^4x^2)}$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
4/15*(32*b^2*x^2 + 24*a*b*x - 3*a^2)*(b*x + a)^(1/4)*x^(3/4)/(a^3*b*x^3 +
a^4*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(60) = 120.

Time = 21.35 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = -\frac{3a^3b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{16a^5b^4x\Gamma\left(\frac{7}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{7}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{7}{4}\right)} + \frac{21a^2b^{\frac{21}{4}}x\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{16a^5b^4x\Gamma\left(\frac{7}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{7}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{7}{4}\right)} + \frac{56ab^{\frac{25}{4}}x^2\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{16a^5b^4x\Gamma\left(\frac{7}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{7}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{7}{4}\right)} + \frac{32b^{\frac{29}{4}}x^3\sqrt[4]{\frac{a}{bx}+1}\Gamma\left(-\frac{5}{4}\right)}{16a^5b^4x\Gamma\left(\frac{7}{4}\right)+32a^4b^5x^2\Gamma\left(\frac{7}{4}\right)+16a^3b^6x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x**(9/4)/(b*x+a)**(7/4),x)
```

output
```
-3*a**3*b**(17/4)*(a/(b*x) + 1)**(1/4)*gamma(-5/4)/(16*a**5*b**4*x*gamma(7
/4) + 32*a**4*b**5*x**2*gamma(7/4) + 16*a**3*b**6*x**3*gamma(7/4)) + 21*a*
*2*b**(21/4)*x*(a/(b*x) + 1)**(1/4)*gamma(-5/4)/(16*a**5*b**4*x*gamma(7/4)
 + 32*a**4*b**5*x**2*gamma(7/4) + 16*a**3*b**6*x**3*gamma(7/4)) + 56*a*b**
(25/4)*x**2*(a/(b*x) + 1)**(1/4)*gamma(-5/4)/(16*a**5*b**4*x*gamma(7/4) +
32*a**4*b**5*x**2*gamma(7/4) + 16*a**3*b**6*x**3*gamma(7/4)) + 32*b**(29/4
)*x**3*(a/(b*x) + 1)**(1/4)*gamma(-5/4)/(16*a**5*b**4*x*gamma(7/4) + 32*a*
*4*b**5*x**2*gamma(7/4) + 16*a**3*b**6*x**3*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = \frac{4\,b^2x^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}a^3} + \frac{4\left(\frac{10\,(bx+a)^{\frac{1}{4}}b}{x^{\frac{1}{4}}} - \frac{(bx+a)^{\frac{5}{4}}}{x^{\frac{5}{4}}}\right)}{5\,a^3}$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
4/3*b^2*x^(3/4)/((b*x + a)^(3/4)*a^3) + 4/5*(10*(b*x + a)^(1/4)*b/x^(1/4)
- (b*x + a)^(5/4)/x^(5/4))/a^3
```

Giac [F]

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/x^(9/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(9/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = \frac{(a+b\,x)^{1/4}\left(\frac{32\,x}{5\,a^2}-\frac{4}{5\,a\,b}+\frac{128\,b\,x^2}{15\,a^3}\right)}{x^{9/4}+\frac{a\,x^{5/4}}{b}}$$

input
```
int(1/(x^(9/4)*(a + b*x)^(7/4)),x)
```

output
```
((a + b*x)^(1/4)*((32*x)/(5*a^2) - 4/(5*a*b) + (128*b*x^2)/(15*a^3)))/(x^(
9/4) + (a*x^(5/4))/b)
```

Reduce [F]

$$\int \frac{1}{x^{9/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{\frac{9}{4}}\,(bx+a)^{\frac{3}{4}}\,a+x^{\frac{13}{4}}\,(bx+a)^{\frac{3}{4}}\,b}dx$$

input
```
int(1/x^(9/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*a*x**2 + x**(1/4)*(a + b*x)**(3/4)*b*x**3
),x)
```

3.747 $\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx=\frac{4}{3ax^{9/4}(a+bx)^{3/4}}-\frac{16\sqrt[4]{a+bx}}{9a^2x^{9/4}}+\frac{128b\sqrt[4]{a+bx}}{45a^3x^{5/4}}-\frac{512b^2\sqrt[4]{a+bx}}{45a^4\sqrt[4]{x}}$$

output
```
4/3/a/x^(9/4)/(b*x+a)^(3/4)-16/9*(b*x+a)^(1/4)/a^2/x^(9/4)+128/45*b*(b*x+a
)^(1/4)/a^3/x^(5/4)-512/45*b^2*(b*x+a)^(1/4)/a^4/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx=-\frac{4(5a^3-12a^2bx+96ab^2x^2+128b^3x^3)}{45a^4x^{9/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(13/4)*(a + b*x)^(7/4)),x]
```

output
```
(-4*(5*a^3 - 12*a^2*b*x + 96*a*b^2*x^2 + 128*b^3*x^3))/(45*a^4*x^(9/4)*(a
+ b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\int \frac{1}{x^{13/4}(a+bx)^{3/4}}dx}{a}+\frac{4}{3ax^{9/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\left(-\frac{8b\int \frac{1}{x^{9/4}(a+bx)^{3/4}}dx}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{a}+\frac{4}{3ax^{9/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^{5/4}(a+bx)^{3/4}}dx}{5a}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{a}+\frac{4}{3ax^{9/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4\left(-\frac{8b\left(\frac{16b\sqrt[4]{a+bx}}{5a^2\sqrt[4]{x}}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{a}+\frac{4}{3ax^{9/4}(a+bx)^{3/4}}$$

input
```
Int[1/(x^(13/4)*(a + b*x)^(7/4)),x]
```

output
```
4/(3*a*x^(9/4)*(a + b*x)^(3/4)) + (4*((-4*(a + b*x)^(1/4))/(9*a*x^(9/4)) -
 (8*b*((-4*(a + b*x)^(1/4))/(5*a*x^(5/4)) + (16*b*(a + b*x)^(1/4))/(5*a^2*
x^(1/4))))/(9*a)))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{4\left(128 b^{3} x^{3}+96 a\,b^{2} x^{2}-12 a^{2} b x +5 a^{3}\right)}{45 x^{\frac{9}{4}} \left(b x +a \right)^{\frac{3}{4}} a^{4}}$	46
orering	$-\frac{4\left(128 b^{3} x^{3}+96 a\,b^{2} x^{2}-12 a^{2} b x +5 a^{3}\right)}{45 x^{\frac{9}{4}} \left(b x +a \right)^{\frac{3}{4}} a^{4}}$	46
risch	$-\frac{4 \left(b x +a \right)^{\frac{1}{4}} \left(113 b^{2} x^{2}-17 a b x +5 a^{2}\right)}{45 a^{4} x^{\frac{9}{4}}}-\frac{4 b^{3} x^{\frac{3}{4}}}{3 a^{4} \left(b x +a \right)^{\frac{3}{4}}}$	54

input
```
int(1/x^(13/4)/(b*x+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-4/45*(128*b^3*x^3+96*a*b^2*x^2-12*a^2*b*x+5*a^3)/x^(9/4)/(b*x+a)^(3/4)/a^
4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx = -\frac{4\,(128\,b^3x^3+96\,ab^2x^2-12\,a^2bx+5\,a^3)(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{45\,(a^4bx^4+a^5x^3)}$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
-4/45*(128*b^3*x^3 + 96*a*b^2*x^2 - 12*a^2*b*x + 5*a^3)*(b*x + a)^(1/4)*x^
(3/4)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(13/4)/(b*x+a)**(7/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx = -\frac{4\,b^3x^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}a^4} - \frac{4\left(\frac{135\,(bx+a)^{\frac{1}{4}}b^2}{x^{\frac{1}{4}}} - \frac{27\,(bx+a)^{\frac{5}{4}}b}{x^{\frac{5}{4}}} + \frac{5\,(bx+a)^{\frac{9}{4}}}{x^{\frac{9}{4}}}\right)}{45\,a^4}$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
-4/3*b^3*x^(3/4)/((b*x + a)^(3/4)*a^4) - 4/45*(135*(b*x + a)^(1/4)*b^2/x^(
1/4) - 27*(b*x + a)^(5/4)*b/x^(5/4) + 5*(b*x + a)^(9/4)/x^(9/4))/a^4
```

Giac [F]

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/x^(13/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(13/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx = -\frac{(a+b\,x)^{1/4}\left(\frac{4}{9\,a\,b}-\frac{16\,x}{15\,a^2}+\frac{128\,b\,x^2}{15\,a^3}+\frac{512\,b^2\,x^3}{45\,a^4}\right)}{x^{13/4}+\frac{a\,x^{9/4}}{b}}$$

input
```
int(1/(x^(13/4)*(a + b*x)^(7/4)),x)
```

output
```
-((a + b*x)^(1/4)*(4/(9*a*b) - (16*x)/(15*a^2) + (128*b*x^2)/(15*a^3) + (5
12*b^2*x^3)/(45*a^4)))/(x^(13/4) + (a*x^(9/4))/b)
```

Reduce [F]

$$\int \frac{1}{x^{13/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{\frac{13}{4}}\,(bx+a)^{\frac{3}{4}}\,a+x^{\frac{17}{4}}\,(bx+a)^{\frac{3}{4}}\,b}dx$$

input
```
int(1/x^(13/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*a*x**3 + x**(1/4)*(a + b*x)**(3/4)*b*x**4
),x)
```

3.748 $\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 113

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = \frac{4}{3ax^{13/4}(a+bx)^{3/4}} - \frac{64\sqrt[4]{a+bx}}{39a^2x^{13/4}} + \frac{256b\sqrt[4]{a+bx}}{117a^3x^{9/4}} - \frac{2048b^2\sqrt[4]{a+bx}}{585a^4x^{5/4}} + \frac{8192b^3\sqrt[4]{a+bx}}{585a^5\sqrt[4]{x}}$$

output
```
4/3/a/x^(13/4)/(b*x+a)^(3/4)-64/39*(b*x+a)^(1/4)/a^2/x^(13/4)+256/117*b*(b
*x+a)^(1/4)/a^3/x^(9/4)-2048/585*b^2*(b*x+a)^(1/4)/a^4/x^(5/4)+8192/585*b^
3*(b*x+a)^(1/4)/a^5/x^(1/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = -\frac{4(45a^4-80a^3bx+192a^2b^2x^2-1536ab^3x^3-2048b^4x^4)}{585a^5x^{13/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(17/4)*(a + b*x)^(7/4)),x]
```

output
```
(-4*(45*a^4 - 80*a^3*b*x + 192*a^2*b^2*x^2 - 1536*a*b^3*x^3 - 2048*b^4*x^4
))/(585*a^5*x^(13/4)*(a + b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx$$

$$\Big\downarrow \; 55$$

$$\frac{16\int \frac{1}{x^{17/4}(a+bx)^{3/4}}dx}{3a}+\frac{4}{3ax^{13/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 55$$

$$\frac{16\left(-\frac{12b\int \frac{1}{x^{13/4}(a+bx)^{3/4}}dx}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}\right)}{3a}+\frac{4}{3ax^{13/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 55$$

$$\frac{16\left(-\frac{12b\left(-\frac{8b\int \frac{1}{x^{9/4}(a+bx)^{3/4}}dx}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}\right)}{3a}+\frac{4}{3ax^{13/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 55$$

$$\frac{16\left(-\frac{12b\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^{5/4}(a+bx)^{3/4}}dx}{5a}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}\right)}{3a}+\frac{4}{3ax^{13/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \; 48$$

$$\frac{16\left(-\frac{12b\left(-\frac{8b\left(\frac{16b\sqrt[4]{a+bx}}{5a^2\sqrt[4]{x}}-\frac{4\sqrt[4]{a+bx}}{5ax^{5/4}}\right)}{9a}-\frac{4\sqrt[4]{a+bx}}{9ax^{9/4}}\right)}{13a}-\frac{4\sqrt[4]{a+bx}}{13ax^{13/4}}\right)}{3a}+\frac{4}{3ax^{13/4}(a+bx)^{3/4}}$$

input

```
Int[1/(x^(17/4)*(a + b*x)^(7/4)),x]
```

output

```
4/(3*a*x^(13/4)*(a + b*x)^(3/4)) + (16*((-4*(a + b*x)^(1/4))/(13*a*x^(13/4
)) - (12*b*((-4*(a + b*x)^(1/4))/(9*a*x^(9/4)) - (8*b*((-4*(a + b*x)^(1/4)
)/(5*a*x^(5/4)) + (16*b*(a + b*x)^(1/4))/(5*a^2*x^(1/4))))/(9*a)))/(13*a))
)/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{4\left(-2048 b^{4} x^{4}-1536 a \,x^{3} b^{3}+192 a^{2} b^{2} x^{2}-80 a^{3} b x +45 a^{4}\right)}{585 x^{\frac{13}{4}} \left(b x +a \right)^{\frac{3}{4}} a^{5}}$	57
orering	$-\frac{4\left(-2048 b^{4} x^{4}-1536 a \,x^{3} b^{3}+192 a^{2} b^{2} x^{2}-80 a^{3} b x +45 a^{4}\right)}{585 x^{\frac{13}{4}} \left(b x +a \right)^{\frac{3}{4}} a^{5}}$	57
risch	$-\frac{4 \left(b x +a \right)^{\frac{1}{4}} \left(-1853 b^{3} x^{3}+317 a \,b^{2} x^{2}-125 a^{2} b x +45 a^{3}\right)}{585 a^{5} x^{\frac{13}{4}}}+\frac{4 b^{4} x^{\frac{3}{4}}}{3 a^{5} \left(b x +a \right)^{\frac{3}{4}}}$	65

input
```
int(1/x^(17/4)/(b*x+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-4/585*(-2048*b^4*x^4-1536*a*b^3*x^3+192*a^2*b^2*x^2-80*a^3*b*x+45*a^4)/x^
(13/4)/(b*x+a)^(3/4)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = \frac{4\,(2048\,b^4x^4+1536\,ab^3x^3-192\,a^2b^2x^2+80\,a^3bx-45\,a^4)(bx+a)^{\frac{1}{4}}x^{\frac{3}{4}}}{585\,(a^5bx^5+a^6x^4)}$$

input
```
integrate(1/x^(17/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
4/585*(2048*b^4*x^4 + 1536*a*b^3*x^3 - 192*a^2*b^2*x^2 + 80*a^3*b*x - 45*a
^4)*(b*x + a)^(1/4)*x^(3/4)/(a^5*b*x^5 + a^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(17/4)/(b*x+a)**(7/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = \frac{4\,b^4x^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}a^5} + \frac{4\left(\frac{2340\,(bx+a)^{\frac{1}{4}}b^3}{x^{\frac{1}{4}}} - \frac{702\,(bx+a)^{\frac{5}{4}}b^2}{x^{\frac{5}{4}}} + \frac{260\,(bx+a)^{\frac{9}{4}}b}{x^{\frac{9}{4}}} - \frac{45\,(bx+a)^{\frac{13}{4}}}{x^{\frac{13}{4}}}\right)}{585\,a^5}$$

input
```
integrate(1/x^(17/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
4/3*b^4*x^(3/4)/((b*x + a)^(3/4)*a^5) + 4/585*(2340*(b*x + a)^(1/4)*b^3/x^
(1/4) - 702*(b*x + a)^(5/4)*b^2/x^(5/4) + 260*(b*x + a)^(9/4)*b/x^(9/4) -
45*(b*x + a)^(13/4)/x^(13/4))/a^5
```

Giac [F]

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{17}{4}}}\,dx$$

input
```
integrate(1/x^(17/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(17/4)), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = \frac{(a+b\,x)^{1/4}\left(\frac{64\,x}{117\,a^2}-\frac{4}{13\,a\,b}-\frac{256\,b\,x^2}{195\,a^3}+\frac{2048\,b^2\,x^3}{195\,a^4}+\frac{8192\,b^3\,x^4}{585\,a^5}\right)}{x^{17/4}+\frac{a\,x^{13/4}}{b}}$$

input
```
int(1/(x^(17/4)*(a + b*x)^(7/4)),x)
```

output
```
((a + b*x)^(1/4)*((64*x)/(117*a^2) - 4/(13*a*b) - (256*b*x^2)/(195*a^3) +
(2048*b^2*x^3)/(195*a^4) + (8192*b^3*x^4)/(585*a^5)))/(x^(17/4) + (a*x^(13
/4))/b)
```

Reduce [F]

$$\int \frac{1}{x^{17/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{\frac{17}{4}}\,(bx+a)^{\frac{3}{4}}\,a + x^{\frac{21}{4}}\,(bx+a)^{\frac{3}{4}}\,b}dx$$

input
```
int(1/x^(17/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(3/4)*a*x**4 + x**(1/4)*(a + b*x)**(3/4)*b*x**5
),x)
```

3.749 $\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 154

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = -\frac{4a^2x^{5/4}}{3b^3(a+bx)^{3/4}} + \frac{13a^2\sqrt[4]{x}\sqrt[4]{a+bx}}{2b^4} - \frac{19ax^{5/4}\sqrt[4]{a+bx}}{15b^3} + \frac{2x^{9/4}\sqrt[4]{a+bx}}{5b^2} + \frac{13a^{5/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\mathrm{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{2b^5x^{3/4}}$$

output
```
-4/3*a^2*x^(5/4)/b^3/(b*x+a)^(3/4)+13/2*a^2*x^(1/4)*(b*x+a)^(1/4)/b^4-19/1
5*a*x^(5/4)*(b*x+a)^(1/4)/b^3+2/5*x^(9/4)*(b*x+a)^(1/4)/b^2+13/2*a^(5/2)*(
b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a
)^(1/2)),2^(1/2))/b^5/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.32

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = \frac{4x^{17/4}\left(1+\frac{bx}{a}\right)^{3/4}\mathrm{Hypergeometric2F1}\left(\frac{7}{4},\frac{17}{4},\frac{21}{4},-\frac{bx}{a}\right)}{17a(a+bx)^{3/4}}$$

input
```
Integrate[x^(13/4)/(a + b*x)^(7/4),x]
```

output
```
(4*x^(17/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[7/4, 17/4, 21/4, -((b*x)
/a)])/(17*a*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {57, 60, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{13\int \frac{x^{9/4}}{(a+bx)^{3/4}}dx}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int\frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ \textbf{73}$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int\frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ \textbf{768}$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ 858$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ 807$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ 229$$

$$\frac{13\left(\frac{2x^{9/4}\sqrt[4]{a+bx}}{5b}-\frac{9a\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{10b}\right)}{3b}-\frac{4x^{13/4}}{3b(a+bx)^{3/4}}$$

input
```
Int[x^(13/4)/(a + b*x)^(7/4),x]
```

output
```
(-4*x^(13/4))/(3*b*(a + b*x)^(3/4)) + (13*((2*x^(9/4)*(a + b*x)^(1/4))/(5*
b) - (9*a*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b) - (5*a*((2*x^(1/4)*(a + b*x)^
(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a
]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(6*b)))/(10*b)))/(3
*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{13}{4}}}{(bx+a)^{\frac{7}{4}}}dx$$

input
```
int(x^(13/4)/(b*x+a)^(7/4),x)
```

output
```
int(x^(13/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{13}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(13/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(13/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 159.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = \frac{x^{\frac{17}{4}}\Gamma\left(\frac{17}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{17}{4}\\ \frac{21}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{21}{4}\right)}$$

input
```
integrate(x**(13/4)/(b*x+a)**(7/4),x)
```

output
```
x**(17/4)*gamma(17/4)*hyper((7/4, 17/4), (21/4,), b*x*exp_polar(I*pi)/a)/(
a**(7/4)*gamma(21/4))
```

Maxima [F]

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{13}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(13/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^(13/4)/(b*x + a)^(7/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(13/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,3,2,0]%%%} / %%%{1,[0,0,0,0,4]%%%} Error: Bad Argu
ment Valu
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{13/4}}{(a+b\,x)^{7/4}}\,dx$$

input
```
int(x^(13/4)/(a + b*x)^(7/4),x)
```

output
```
int(x^(13/4)/(a + b*x)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^{13/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{13}{4}}}{(bx+a)^{\frac{3}{4}}\,a+(bx+a)^{\frac{3}{4}}\,bx}dx$$

input
```
int(x^(13/4)/(b*x+a)^(7/4),x)
```

output
```
int((x**(1/4)*x**3)/((a + b*x)**(3/4)*a + (a + b*x)**(3/4)*b*x),x)
```

3.750 $\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = -\frac{4a^2\sqrt[4]{x}}{3b^3(a+bx)^{3/4}} - \frac{11a\sqrt[4]{x}\sqrt[4]{a+bx}}{3b^3} + \frac{2x^{5/4}\sqrt[4]{a+bx}}{3b^2} - \frac{5a^{3/2}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{b^4x^{3/4}}$$

output
```
-4/3*a^2*x^(1/4)/b^3/(b*x+a)^(3/4)-11/3*a*x^(1/4)*(b*x+a)^(1/4)/b^3+2/3*x^
(5/4)*(b*x+a)^(1/4)/b^2-5*a^(3/2)*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*Invers
eJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/b^4/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.39

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = \frac{4x^{13/4}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{13}{4},\frac{17}{4},-\frac{bx}{a}\right)}{13a(a+bx)^{3/4}}$$

input
```
Integrate[x^(9/4)/(a + b*x)^(7/4),x]
```

output
```
(4*x^(13/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[7/4, 13/4, 17/4, -((b*x)
/a)])/(13*a*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {57, 60, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{3\int \frac{x^{5/4}}{(a+bx)^{3/4}}dx}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{6b}\right)}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{6b}\right)}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{3\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{6b}\right)}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \; 768$$

$$\frac{3\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{6b}\right)}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \; 858$$

$$\frac{3\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{3\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int\frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{3\left(\frac{2x^{5/4}\sqrt[4]{a+bx}}{3b}-\frac{5a\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{6b}\right)}{b}-\frac{4x^{9/4}}{3b(a+bx)^{3/4}}$$

input
```
Int[x^(9/4)/(a + b*x)^(7/4),x]
```

output
```
(-4*x^(9/4))/(3*b*(a + b*x)^(3/4)) + (3*((2*x^(5/4)*(a + b*x)^(1/4))/(3*b)
 - (5*a*((2*x^(1/4)*(a + b*x)^(1/4))/b + (2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^
(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x
)^(3/4))))/(6*b)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{7}{4}}}dx$$

input
```
int(x^(9/4)/(b*x+a)^(7/4),x)
```

output
```
int(x^(9/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(9/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(9/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = \frac{x^{\frac{13}{4}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix} \middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**(9/4)/(b*x+a)**(7/4),x)
```

output
```
x**(13/4)*gamma(13/4)*hyper((7/4, 13/4), (17/4,), b*x*exp_polar(I*pi)/a)/(
a**(7/4)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(9/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^(9/4)/(b*x + a)^(7/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(9/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,2,0]%%%} / %%%{1,[0,0,0,0,1]%%%} Error: Bad Argum
ent Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{9/4}}{(a+b\,x)^{7/4}}\,dx$$

input
```
int(x^(9/4)/(a + b*x)^(7/4),x)
```

output
```
int(x^(9/4)/(a + b*x)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^{9/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{9}{4}}}{(bx+a)^{\frac{3}{4}}\,a+(bx+a)^{\frac{3}{4}}\,bx}dx$$

input
```
int(x^(9/4)/(b*x+a)^(7/4),x)
```

output
```
int((x**(1/4)*x**2)/((a + b*x)**(3/4)*a + (a + b*x)**(3/4)*b*x),x)
```

3.751 $\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx=\frac{4a\sqrt[4]{x}}{3b^2(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b^2}+\frac{10\sqrt{a}\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3b^3x^{3/4}}$$

output
```
4/3*a*x^(1/4)/b^2/(b*x+a)^(3/4)+2*x^(1/4)*(b*x+a)^(1/4)/b^2+10/3*a^(1/2)*(
b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a
)^(1/2)),2^(1/2))/b^3/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx=\frac{4x^{9/4}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{9}{4},\frac{13}{4},-\frac{bx}{a}\right)}{9a(a+bx)^{3/4}}$$

input
```
Integrate[x^(5/4)/(a + b*x)^(7/4),x]
```

output
```
(4*x^(9/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[7/4, 9/4, 13/4, -((b*x)/a
)])/(9*a*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {57, 60, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}dx$$

$$\Big\downarrow \ 57$$

$$\frac{5\int \frac{\sqrt[4]{x}}{(a+bx)^{3/4}}dx}{3b}-\frac{4x^{5/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ 60$$

$$\frac{5\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{a\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{2b}\right)}{3b}-\frac{4x^{5/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ 73$$

$$\frac{5\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2a\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{b}\right)}{3b}-\frac{4x^{5/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ 768$$

$$\frac{5\left(\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}-\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{b(a+bx)^{3/4}}\right)}{3b}-\frac{4x^{5/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \ 858$$

$$\frac{5\left(\frac{2ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{3b}-\frac{4x^{5/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{5\left(\frac{ax^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{b(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{3b}-\frac{4x^{5/4}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{5\left(\frac{2\sqrt{a}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx)^{3/4}}+\frac{2\sqrt[4]{x}\sqrt[4]{a+bx}}{b}\right)}{3b}-\frac{4x^{5/4}}{3b(a+bx)^{3/4}}$$

input
```
Int[x^(5/4)/(a + b*x)^(7/4),x]
```

output
```
(-4*x^(5/4))/(3*b*(a + b*x)^(3/4)) + (5*((2*x^(1/4)*(a + b*x)^(1/4))/b + (
2*Sqrt[a]*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/S
qrt[b]]/2, 2])/(Sqrt[b]*(a + b*x)^(3/4))))/(3*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{7}{4}}}dx$$

input
```
int(x^(5/4)/(b*x+a)^(7/4),x)
```

output
```
int(x^(5/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx=\int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(5/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(5/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.35

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx=\frac{x^{\frac{9}{4}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**(5/4)/(b*x+a)**(7/4),x)
```

output
```
x**(9/4)*gamma(9/4)*hyper((7/4, 9/4), (13/4,), b*x*exp_polar(I*pi)/a)/(a**
(7/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(5/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^(5/4)/(b*x + a)^(7/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(5/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,1,2,0]%%%} / %%%{1,[0,0,0,0,1]%%%} Error: Bad Argu
ment Valu
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{5/4}}{{(a+b\,x)}^{7/4}}\,dx$$

input
```
int(x^(5/4)/(a + b*x)^(7/4),x)
```

output
```
int(x^(5/4)/(a + b*x)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^{5/4}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}\,a+(bx+a)^{\frac{3}{4}}\,bx}dx$$

input
```
int(x^(5/4)/(b*x+a)^(7/4),x)
```

output
```
int((x**(1/4)*x)/((a + b*x)**(3/4)*a + (a + b*x)**(3/4)*b*x),x)
```

3.752 $\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx = -\frac{4\sqrt[4]{x}}{3b(a+bx)^{3/4}} - \frac{4\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3\sqrt{a}b^2x^{3/4}}$$

output
```
-4/3*x^(1/4)/b/(b*x+a)^(3/4)-4/3*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*Inverse
JacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(1/2)/b^2/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx = \frac{4x^{5/4}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{9}{4},-\frac{bx}{a}\right)}{5a(a+bx)^{3/4}}$$

input
```
Integrate[x^(1/4)/(a + b*x)^(7/4),x]
```

output
```
(4*x^(5/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[5/4, 7/4, 9/4, -((b*x)/a)
])/(5*a*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {57, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3b}-\frac{4\sqrt[4]{x}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{4\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3b}-\frac{4\sqrt[4]{x}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{4x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3b(a+bx)^{3/4}}-\frac{4\sqrt[4]{x}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{4x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3b(a+bx)^{3/4}}-\frac{4\sqrt[4]{x}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{2x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3b(a+bx)^{3/4}}-\frac{4\sqrt[4]{x}}{3b(a+bx)^{3/4}}$$

$$\Big\downarrow \text{ 229}$$

$$-\frac{4x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx)^{3/4}}-\frac{4\sqrt[4]{x}}{3b(a+bx)^{3/4}}$$

input
```
Int[x^(1/4)/(a + b*x)^(7/4),x]
```

output
```
(-4*x^(1/4))/(3*b*(a + b*x)^(3/4)) - (4*(1 + a/(b*x))^(3/4)*x^(3/4)*Ellipt
icF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x)^
(3/4))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{7}{4}}}dx$$

input
```
int(x^(1/4)/(b*x+a)^(7/4),x)
```

output
```
int(x^(1/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(1/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx = \frac{x^{\frac{5}{4}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(1/4)/(b*x+a)**(7/4),x)
```

output
```
x**(5/4)*gamma(5/4)*hyper((5/4, 7/4), (9/4,), b*x*exp_polar(I*pi)/a)/(a**(
7/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(1/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^(1/4)/(b*x + a)^(7/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx = \int \frac{x^{\frac{1}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^(1/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^(1/4)/(b*x + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx=\int \frac{x^{1/4}}{{\left(a+b\,x\right)}^{7/4}} \,d x$$

input
```
int(x^(1/4)/(a + b*x)^(7/4),x)
```

output
```
int(x^(1/4)/(a + b*x)^(7/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{x}}{(a+bx)^{7/4}}\,dx=\int \frac{x^{\frac{1}{4}}}{\left(b x +a \right)^{\frac{3}{4}} a +\left(b x +a \right)^{\frac{3}{4}} b x}d x$$

input
```
int(x^(1/4)/(b*x+a)^(7/4),x)
```

output
```
int(x**(1/4)/((a + b*x)**(3/4)*a + (a + b*x)**(3/4)*b*x),x)
```

3.753 $\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx = \frac{4\sqrt[4]{x}}{3a(a+bx)^{3/4}} - \frac{8\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3a^{3/2}bx^{3/4}}$$

output
```
4/3*x^(1/4)/a/(b*x+a)^(3/4)-8/3*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJ
acobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(3/2)/b/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx = \frac{4\sqrt[4]{x}\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{5}{4},-\frac{bx}{a}\right)}{a(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(3/4)*(a + b*x)^(7/4)),x]
```

output
```
(4*x^(1/4)*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, -((b*x)/a)
])/(a*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{2\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}+\frac{4\sqrt[4]{x}}{3a(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{8\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}+\frac{4\sqrt[4]{x}}{3a(a+bx)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{8x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}+\frac{4\sqrt[4]{x}}{3a(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{4\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{8x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{4\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{4\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{8\sqrt{b}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}$$

input
```
Int[1/(x^(3/4)*(a + b*x)^(7/4)),x]
```

output
```
(4*x^(1/4))/(3*a*(a + b*x)^(3/4)) - (8*Sqrt[b]*(1 + a/(b*x))^(3/4)*x^(3/4)
*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(
3/4))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{3}{4}} \left(bx+a\right)^{\frac{7}{4}}}dx$$

input
```
int(1/x^(3/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/x^(3/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{\left(bx+a\right)^{\frac{7}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b^2*x^3 + 2*a*b*x^2 + a^2*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx=\frac{\sqrt[4]{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(3/4)/(b*x+a)**(7/4),x)
```

output
```
x**(1/4)*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x*exp_polar(I*pi)/a)/(a**(
7/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx=\int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx=\int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x^(3/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{3/4}\,{\left(a+b\,x\right)}^{7/4}}\,dx$$

input
```
int(1/(x^(3/4)*(a + b*x)^(7/4)),x)
```

output
```
int(1/(x^(3/4)*(a + b*x)^(7/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^{3/4}(a+bx)^{7/4}}\,dx = \frac{4x^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}\,\sqrt{x}\,a}$$

input
```
int(1/x^(3/4)/(b*x+a)^(7/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4))/(3*sqrt(x)*a*(a + b*x))
```

3.754 $\int \frac{1}{x^{7/4}(a+bx)^{7/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx = \frac{4}{3ax^{3/4}(a+bx)^{3/4}} - \frac{8\sqrt[4]{a+bx}}{3a^2x^{3/4}} + \frac{16\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{3a^{5/2}x^{3/4}}$$

output
```
4/3/a/x^(3/4)/(b*x+a)^(3/4)-8/3*(b*x+a)^(1/4)/a^2/x^(3/4)+16/3*(b*x/(b*x+a
))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2
^(1/2))/a^(5/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{1}{4},-\frac{bx}{a}\right)}{3ax^{3/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(7/4)*(a + b*x)^(7/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-3/4, 7/4, 1/4, -((b*x)/a)])/(3*
a*x^(3/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx$$

$$\Big\downarrow \text{61}$$

$$\frac{2\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{a}+\frac{4}{3ax^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{a}+\frac{4}{3ax^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{a}+\frac{4}{3ax^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{2\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{a}+\frac{4}{3ax^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{2\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{a}+\frac{4}{3ax^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{807}$$

$$\frac{2\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{a}+\frac{4}{3ax^{3/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{229}$$

$$\frac{2\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{a}+\frac{4}{3ax^{3/4}(a+bx)^{3/4}}$$

input
```
Int[1/(x^(7/4)*(a + b*x)^(7/4)),x]
```

output
```
4/(3*a*x^(3/4)*(a + b*x)^(3/4)) + (2*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) +
 (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])
/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{7}{4}} (bx+a)^{\frac{7}{4}}} dx$$

input
```
int(1/x^(7/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/x^(7/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}} dx = \int \frac{1}{(bx+a)^{\frac{7}{4}} x^{\frac{7}{4}}} dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{7}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}} x^{\frac{3}{4}} \Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**(7/4)/(b*x+a)**(7/4),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 7/4), (1/4,), b*x*exp_polar(I*pi)/a)/(a**(7/4)*x*
*(3/4)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}} x^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(7/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/x^(7/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(7/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{7/4}\,(a+b\,x)^{7/4}}\,dx$$

input
```
int(1/(x^(7/4)*(a + b*x)^(7/4)),x)
```

output
```
int(1/(x^(7/4)*(a + b*x)^(7/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^{7/4}(a+bx)^{7/4}}\,dx = \frac{-\frac{16bx}{3}-4a}{x^{\frac{1}{4}}\,(bx+a)^{\frac{3}{4}}\sqrt{x}\,a^2}$$

input
```
int(1/x^(7/4)/(b*x+a)^(7/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4)*( - 3*a - 4*b*x))/(3*sqrt(x)*a**2*x*(a + b*x)
)
```

3.755 $\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx = \frac{4}{3ax^{7/4}(a+bx)^{3/4}} - \frac{40\sqrt[4]{a+bx}}{21a^2x^{7/4}} + \frac{80b\sqrt[4]{a+bx}}{21a^3x^{3/4}} - \frac{160b\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{21a^{7/2}x^{3/4}}$$

output
```
4/3/a/x^(7/4)/(b*x+a)^(3/4)-40/21*(b*x+a)^(1/4)/a^2/x^(7/4)+80/21*b*(b*x+a
)^(1/4)/a^3/x^(3/4)-160/21*b*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJaco
biAM(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(7/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{7}{4},-\frac{3}{4},-\frac{bx}{a}\right)}{7ax^{7/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(11/4)*(a + b*x)^(7/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-7/4, 7/4, -3/4, -((b*x)/a)])/(7
*a*x^(7/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {61, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx$$

$$\Big\downarrow \text{61}$$

$$\frac{10\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{10\left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{10\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{10\left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{10\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{10\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{10\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{10\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{3a}+\frac{4}{3ax^{7/4}(a+bx)^{3/4}}$$

input
```
Int[1/(x^(11/4)*(a + b*x)^(7/4)),x]
```

output
```
4/(3*a*x^(7/4)*(a + b*x)^(3/4)) + (10*((-4*(a + b*x)^(1/4))/(7*a*x^(7/4))
- (6*b*((-4*(a + b*x)^(1/4))/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4
)*x^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a
 + b*x)^(3/4))))/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{11}{4}} (bx+a)^{\frac{7}{4}}} dx$$

input
```
int(1/x^(11/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/x^(11/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\, dx = \int \frac{1}{(bx+a)^{\frac{7}{4}} x^{\frac{11}{4}}}\, dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b^2*x^5 + 2*a*b*x^4 + a^2*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 66.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\, dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{7}{4} \\ -\frac{3}{4} \end{matrix} \middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{7}{4}} x^{\frac{7}{4}} \Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**(11/4)/(b*x+a)**(7/4),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 7/4), (-3/4,), b*x*exp_polar(I*pi)/a)/(a**(7/4)*x
**(7/4)*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(11/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/x^(11/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(11/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{11/4}\,(a+b\,x)^{7/4}}\,dx$$

input
```
int(1/(x^(11/4)*(a + b*x)^(7/4)),x)
```

output
```
int(1/(x^(11/4)*(a + b*x)^(7/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^{11/4}(a+bx)^{7/4}}\,dx = \frac{\frac{128}{15}b^2x^2+\frac{32}{5}abx-\frac{4}{5}a^2}{x^{\frac{5}{4}}\,(bx+a)^{\frac{3}{4}}\,\sqrt{x}\,a^3}$$

input
```
int(1/x^(11/4)/(b*x+a)^(7/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4)*( - 3*a**2 + 24*a*b*x + 32*b**2*x**2))/(15*sq
rt(x)*a**3*x**2*(a + b*x))
```

3.756 $\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 151

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx = \frac{4}{3ax^{11/4}(a+bx)^{3/4}} - \frac{56\sqrt[4]{a+bx}}{33a^2x^{11/4}} + \frac{80b\sqrt[4]{a+bx}}{33a^3x^{7/4}} - \frac{160b^2\sqrt[4]{a+bx}}{33a^4x^{3/4}} + \frac{320b^2\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{33a^{9/2}x^{3/4}}$$

output
```
4/3/a/x^(11/4)/(b*x+a)^(3/4)-56/33*(b*x+a)^(1/4)/a^2/x^(11/4)+80/33*b*(b*x
+a)^(1/4)/a^3/x^(7/4)-160/33*b^2*(b*x+a)^(1/4)/a^4/x^(3/4)+320/33*b^2*(b*x
/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b*x+a)^(
1/2)),2^(1/2))/a^(9/2)/x^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx = -\frac{4\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{7}{4},-\frac{7}{4},-\frac{bx}{a}\right)}{11ax^{11/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/(x^(15/4)*(a + b*x)^(7/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-11/4, 7/4, -7/4, -((b*x)/a)])/(
11*a*x^(11/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {61, 61, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{14\int \frac{1}{x^{15/4}(a+bx)^{3/4}}dx}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{14\left(-\frac{10b\int \frac{1}{x^{11/4}(a+bx)^{3/4}}dx}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{14\left(-\frac{10b\left(-\frac{6b\int \frac{1}{x^{7/4}(a+bx)^{3/4}}dx}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{14\left(-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{x^{3/4}(a+bx)^{3/4}}dx}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \ 73$$

$$\frac{14\left(-\frac{10b\left(-\frac{6b\left(-\frac{8b\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{x}}{3a}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \ 768$$

$$\frac{14\left(-\frac{10b\left(-\frac{6b\left(-\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}x^{3/4}}d\sqrt[4]{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \ 858$$

$$\frac{14\left(-\frac{10b\left(-\frac{6b\left(\frac{8bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{x}}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{807}$$

$$\frac{14\left(-\frac{10b\left(-\frac{6b\left(\frac{4bx^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{3/4}}d\sqrt{x}}{3a(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

$$\Big\downarrow \textbf{229}$$

$$\frac{14\left(-\frac{10b\left(-\frac{6b\left(\frac{8b^{3/2}x^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right),2\right)}{3a^{3/2}(a+bx)^{3/4}}-\frac{4\sqrt[4]{a+bx}}{3ax^{3/4}}\right)}{7a}-\frac{4\sqrt[4]{a+bx}}{7ax^{7/4}}\right)}{11a}-\frac{4\sqrt[4]{a+bx}}{11ax^{11/4}}\right)}{3a}+\frac{4}{3ax^{11/4}(a+bx)^{3/4}}$$

input

```
Int[1/(x^(15/4)*(a + b*x)^(7/4)),x]
```

output

```
4/(3*a*x^(11/4)*(a + b*x)^(3/4)) + (14*((-4*(a + b*x)^(1/4))/(11*a*x^(11/4
)) - (10*b*((-4*(a + b*x)^(1/4))/(7*a*x^(7/4)) - (6*b*((-4*(a + b*x)^(1/4)
)/(3*a*x^(3/4)) + (8*b^(3/2)*(1 + a/(b*x))^(3/4)*x^(3/4)*EllipticF[ArcTan[
(Sqrt[a]*Sqrt[x])/Sqrt[b]]/2, 2])/(3*a^(3/2)*(a + b*x)^(3/4))))/(7*a)))/(1
1*a)))/(3*a)
```

Defintions of rubi rules used

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{15}{4}} (bx + a)^{\frac{7}{4}}} dx$$

input
```
int(1/x^(15/4)/(b*x+a)^(7/4),x)
```

output
```
int(1/x^(15/4)/(b*x+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{15/4}(a + bx)^{7/4}} dx = \int \frac{1}{(bx + a)^{\frac{7}{4}} x^{\frac{15}{4}}} dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*x^(1/4)/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(15/4)/(b*x+a)**(7/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{15}{4}}}\,dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(15/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{4}}x^{\frac{15}{4}}}\,dx$$

input
```
integrate(1/x^(15/4)/(b*x+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/4)*x^(15/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx = \int \frac{1}{x^{15/4}\,{(a+b\,x)}^{7/4}}\,dx$$

input
```
int(1/(x^(15/4)*(a + b*x)^(7/4)),x)
```

output
```
int(1/(x^(15/4)*(a + b*x)^(7/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^{15/4}(a+bx)^{7/4}}\,dx = \frac{-\frac{512}{45}b^3x^3 - \frac{128}{15}a\,b^2x^2 + \frac{16}{15}a^2bx - \frac{4}{9}a^3}{x^{\frac{9}{4}}\,(bx+a)^{\frac{3}{4}}\,\sqrt{x}\,a^4}$$

input
```
int(1/x^(15/4)/(b*x+a)^(7/4),x)
```

output
```
(4*x**(3/4)*(a + b*x)**(1/4)*( - 5*a**3 + 12*a**2*b*x - 96*a*b**2*x**2 - 1
28*b**3*x**3))/(45*sqrt(x)*a**4*x**3*(a + b*x))
```

3.757 $\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx=-\frac{4}{\sqrt[4]{x}\sqrt[4]{a+bx}}+\frac{4\sqrt[4]{\frac{bx}{a+bx}}\sqrt[4]{a+bx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{x}}$$

output
```
-4/x^(1/4)/(b*x+a)^(1/4)+4*(b*x/(b*x+a))^(1/4)*(b*x+a)^(1/4)*EllipticE(sin
(1/2*arcsin(a^(1/2)/(b*x+a)^(1/2)),2^(1/2))/a^(1/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx=-\frac{4\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx}{a}\right)}{\sqrt[4]{x}\sqrt[4]{a+bx}}$$

input
```
Integrate[1/(x^(5/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x)/a)])/(x^
(1/4)*(a + b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a+bx}}dx}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a+bx}}d\sqrt[4]{x}}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 839}$$

$$\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{1}{2}a\int \frac{\sqrt{x}}{(a+bx)^{5/4}}d\sqrt[4]{x}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 813}$$

$$\frac{8b\left(\frac{x^{3/4}}{2\sqrt[4]{a+bx}}-\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{5/4}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\sqrt[4]{x}\left(\frac{ax}{b}+1\right)^{5/4}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{807}$$

$$\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}\int \frac{1}{\left(\frac{\sqrt{x}a}{b}+1\right)^{5/4}}d\sqrt{x}}{4b\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{212}$$

$$\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{\frac{a}{bx}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|2\right)}{2\sqrt{b}\sqrt[4]{a+bx}}+\frac{x^{3/4}}{2\sqrt[4]{a+bx}}\right)}{a}-\frac{4(a+bx)^{3/4}}{a\sqrt[4]{x}}$$

input
```
Int[1/(x^(5/4)*(a + b*x)^(1/4)),x]
```

output
```
(-4*(a + b*x)^(3/4))/(a*x^(1/4)) + (8*b*(x^(3/4)/(2*(a + b*x)^(1/4)) + (Sq
rt[a]*(1 + a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[
b]]/2, 2])/(2*Sqrt[b]*(a + b*x)^(1/4))))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{5}{4}} (bx + a)^{\frac{1}{4}}} dx$$

input
```
int(1/x^(5/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/x^(5/4)/(b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/4)*x^(3/4)/(b*x^3 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = -\frac{2\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx}\right)}{\sqrt[4]{b}\sqrt{x}}$$

input
```
integrate(1/x**(5/4)/(b*x+a)**(1/4),x)
```

output
```
-2*hyper((1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x))/(b**(1/4)*sqrt(x))
```

Maxima [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/4)*x^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{5/4}\,(a+b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(5/4)*(a + b*x)^(1/4)),x)
```

output
```
int(1/(x^(5/4)*(a + b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a+bx}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(bx+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(5/4)/(b*x+a)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x)**(1/4)*x),x)
```

3.758 $\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = -\frac{4}{\sqrt[4]{x}\sqrt[4]{a-bx}}+\frac{4\sqrt[4]{a-bx}\sqrt[4]{1-\frac{a}{a-bx}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a-bx}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{x}}$$

output
```
-4/x^(1/4)/(-b*x+a)^(1/4)+4*(-b*x+a)^(1/4)*(1-a/(-b*x+a))^(1/4)*EllipticE(
sin(1/2*arcsin(a^(1/2)/(-b*x+a)^(1/2))),2^(1/2))/a^(1/2)/x^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = -\frac{4\sqrt[4]{1-\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{bx}{a}\right)}{\sqrt[4]{x}\sqrt[4]{a-bx}}$$

input
```
Integrate[1/(x^(5/4)*(a - b*x)^(1/4)),x]
```

output
```
(-4*(1 - (b*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x)/a])/(x^(1/
4)*(a - b*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {61, 73, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{2b\int \frac{1}{\sqrt[4]{x}\sqrt[4]{a-bx}}dx}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{8b\int \frac{\sqrt{x}}{\sqrt[4]{a-bx}}d\sqrt[4]{x}}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{840}$$

$$-\frac{8b\left(-\frac{a\int \frac{1}{\sqrt{x}\sqrt[4]{a-bx}}d\sqrt[4]{x}}{2b}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{842}$$

$$-\frac{8b\left(-\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx}}x^{3/4}}d\sqrt[4]{x}}{2b\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int \frac{1}{\sqrt[4]{x}\sqrt[4]{1-\frac{ax}{b}}}d\frac{1}{\sqrt[4]{x}}}{2b\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \ 807$$

$$-\frac{8b\left(\frac{a\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}\int \frac{1}{\sqrt[4]{1-\frac{a\sqrt{x}}{b}}}d\sqrt{x}}{4b\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}$$

$$\Big\downarrow \ 226$$

$$-\frac{8b\left(\frac{\sqrt{a}\sqrt[4]{x}\sqrt[4]{1-\frac{a}{bx}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{2b\sqrt[4]{x}}\right)}{a}-\frac{4(a-bx)^{3/4}}{a\sqrt[4]{x}}$$

input
```
Int[1/(x^(5/4)*(a - b*x)^(1/4)),x]
```

output
```
(-4*(a - b*x)^(3/4))/(a*x^(1/4)) - (8*b*(-1/2*(a - b*x)^(3/4)/(b*x^(1/4))
+ (Sqrt[a]*(1 - a/(b*x))^(1/4)*x^(1/4)*EllipticE[ArcSin[(Sqrt[a]*Sqrt[x])/
Sqrt[b]]/2, 2])/(2*Sqrt[b]*(a - b*x)^(1/4))))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 840
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{\frac{5}{4}} \left(-bx+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(5/4)/(-b*x+a)^(1/4),x)
```

output
```
int(1/x^(5/4)/(-b*x+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = \int \frac{1}{(-bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(-b*x+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x + a)^(3/4)*x^(3/4)/(b*x^3 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = \frac{2ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx}\right)}{\sqrt[4]{b}\sqrt{x}}$$

input
```
integrate(1/x**(5/4)/(-b*x+a)**(1/4),x)
```

output
```
2*I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(b*x))/(b**(1/4)*sqrt(x))
```

Maxima [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = \int \frac{1}{(-bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(-b*x+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x + a)^(1/4)*x^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = \int \frac{1}{(-bx+a)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/x^(5/4)/(-b*x+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x + a)^(1/4)*x^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = \int \frac{1}{x^{5/4}\,(a-b\,x)^{1/4}}\,dx$$

input
```
int(1/(x^(5/4)*(a - b*x)^(1/4)),x)
```

output
```
int(1/(x^(5/4)*(a - b*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/4}\sqrt[4]{a-bx}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\left(-bx+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^(5/4)/(-b*x+a)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a - b*x)**(1/4)*x),x)
```

3.759 $\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = -\frac{4\left(\frac{bx}{a+bx}\right)^{3/4}(a+bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx}}\right),2\right)}{\sqrt{a}b(cx)^{3/4}}$$

output
```
-4*(b*x/(b*x+a))^(3/4)*(b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(b
*x+a)^(1/2)),2^(1/2))/a^(1/2)/b/(c*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = \frac{4x\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx}{a}\right)}{(cx)^{3/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/((c*x)^(3/4)*(a + b*x)^(3/4)),x]
```

output
```
(4*x*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x)/a)])/((c
*x)^(3/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{cx}}{c}$$

$$\Big\downarrow \text{768}$$

$$\frac{4(cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}(cx)^{3/4}}d\sqrt[4]{cx}}{c(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{4(cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{cx}\left(\frac{axc^2}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{cx}}}{c(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2(cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a\sqrt{cx}c}{b}+1\right)^{3/4}}d\sqrt{cx}}{c(a+bx)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{4\sqrt{b}(cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{cx}}{\sqrt{b}}\right),2\right)}{\sqrt{a}c^{3/2}(a+bx)^{3/4}}$$

input
```
Int[1/((c*x)^(3/4)*(a + b*x)^(3/4)),x]
```

output
```
(-4*Sqrt[b]*(1 + a/(b*x))^(3/4)*(c*x)^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sqrt
[c]*Sqrt[c*x])/Sqrt[b]]/2, 2])/(Sqrt[a]*c^(3/2)*(a + b*x)^(3/4))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{4}} (bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(c*x)^(3/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*(c*x)^(1/4)/(b*c*x^2 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = \frac{\sqrt[4]{x}\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/4)/(b*x+a)**(3/4),x)
```

output
```
x**(1/4)*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x*exp_polar(I*pi)/a)/(a**(
3/4)*c**(3/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*(c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*(c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(c\,x)^{3/4}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(3/4)*(a + b*x)^(3/4)),x)
```

output
```
int(1/((c*x)^(3/4)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx)^{3/4}}\,dx = \frac{\int \frac{1}{x^{\frac{3}{4}}(bx+a)^{\frac{3}{4}}}dx}{c^{\frac{3}{4}}}$$

input
```
int(1/(c*x)^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(3/4)),x)/c**(3/4)
```

3.760 $\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = \frac{4\left(-\frac{bx}{a-bx}\right)^{3/4}(a-bx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a-bx}}\right),2\right)}{\sqrt{a}b(-cx)^{3/4}}$$

output
```
4*(-b*x/(-b*x+a))^(3/4)*(-b*x+a)^(3/4)*InverseJacobiAM(1/2*arcsin(a^(1/2)/
(-b*x+a)^(1/2)),2^(1/2))/a^(1/2)/b/(-c*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = \frac{4x\left(1-\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{bx}{a}\right)}{(-cx)^{3/4}(a-bx)^{3/4}}$$

input
```
Integrate[1/((-(c*x))^(3/4)*(a - b*x)^(3/4)),x]
```

output
```
(4*x*(1 - (b*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x)/a])/((-(c*
x))^(3/4)*(a - b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$-\frac{4\int \frac{1}{(a-bx)^{3/4}}d\sqrt[4]{-cx}}{c}$$

$$\Big\downarrow \text{768}$$

$$-\frac{4(-cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx}\right)^{3/4}(-cx)^{3/4}}d\sqrt[4]{-cx}}{c(a-bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{4(-cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\int \frac{1}{\sqrt[4]{-cx}\left(1-\frac{ac^2x}{b}\right)^{3/4}}d\frac{1}{\sqrt[4]{-cx}}}{c(a-bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2(-cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\int \frac{1}{\left(\frac{a\sqrt{-cx}c}{b}+1\right)^{3/4}}d\sqrt{-cx}}{c(a-bx)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{4\sqrt{b}(-cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}\sqrt{c}\sqrt{-cx}}{\sqrt{b}}\right),2\right)}{\sqrt{a}c^{3/2}(a-bx)^{3/4}}$$

input
```
Int[1/((-(c*x))^(3/4)*(a - b*x)^(3/4)),x]
```

output
```
(4*Sqrt[b]*(1 - a/(b*x))^(3/4)*(-(c*x))^(3/4)*EllipticF[ArcTan[(Sqrt[a]*Sq
rt[c]*Sqrt[-(c*x)])/Sqrt[b]]/2, 2])/(Sqrt[a]*c^(3/2)*(a - b*x)^(3/4))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(-cx\right)^{\frac{3}{4}} \left(-bx+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(-c*x)^(3/4)/(-b*x+a)^(3/4),x)
```

output
```
int(1/(-c*x)^(3/4)/(-b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{3}{4}}\,(-cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-c*x)^(3/4)/(-b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((-b*x + a)^(1/4)*(-c*x)^(1/4)/(b*c*x^2 - a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = -\frac{2i\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{x}}$$

input
```
integrate(1/(-c*x)**(3/4)/(-b*x+a)**(3/4),x)
```

output
```
-2*I*hyper((1/2, 3/4), (3/2,), a/(b*x))/(b**(3/4)*c**(3/4)*sqrt(x))
```

Maxima [F]

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{3}{4}}\,(-cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-c*x)^(3/4)/(-b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x + a)^(3/4)*(-c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{3}{4}}\,(-cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-c*x)^(3/4)/(-b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x + a)^(3/4)*(-c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{{(-c\,x)}^{3/4}\,{(a-b\,x)}^{3/4}}\,dx$$

input
```
int(1/((-c*x)^(3/4)*(a - b*x)^(3/4)),x)
```

output
```
int(1/((-c*x)^(3/4)*(a - b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(-cx)^{3/4}(a-bx)^{3/4}}\,dx = -\frac{\left(\int \frac{1}{x^{\frac{3}{4}}(-bx+a)^{\frac{3}{4}}}dx\right)(-1)^{\frac{1}{4}}}{c^{\frac{3}{4}}}$$

input
```
int(1/(-c*x)^(3/4)/(-b*x+a)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(a - b*x)**(3/4)),x)/(c**(3/4)*( - 1)**(3/4))
```

3.761 $\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = -\frac{2\sqrt{2}a\left(\frac{bx}{a}-\frac{b^2x^2}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(1-\frac{2bx}{a}\right),2\right)}{b(cx)^{3/4}(a-bx)^{3/4}}$$

output
```
-2*2^(1/2)*a*(b*x/a-b^2*x^2/a^2)^(3/4)*InverseJacobiAM(1/2*arcsin(1-2*b*x/
a),2^(1/2))/b/(c*x)^(3/4)/(-b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = \frac{4x\left(1-\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{bx}{a}\right)}{(cx)^{3/4}(a-bx)^{3/4}}$$

input
```
Integrate[1/((c*x)^(3/4)*(a - b*x)^(3/4)),x]
```

output
```
(4*x*(1 - (b*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x)/a])/((c*x)
^(3/4)*(a - b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {73, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4\int \frac{1}{(a-bx)^{3/4}}d\sqrt[4]{cx}}{c}$$

$$\Big\downarrow \text{768}$$

$$\frac{4(cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx}\right)^{3/4}(cx)^{3/4}}d\sqrt[4]{cx}}{c(a-bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{4(cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\int \frac{1}{\sqrt[4]{cx}\left(1-\frac{ac^2x}{b}\right)^{3/4}}d\frac{1}{\sqrt[4]{cx}}}{c(a-bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2(cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac\sqrt{cx}}{b}\right)^{3/4}}d\sqrt{cx}}{c(a-bx)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{4\sqrt{b}(cx)^{3/4}\left(1-\frac{a}{bx}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}\sqrt{c}\sqrt{cx}}{\sqrt{b}}\right),2\right)}{\sqrt{a}c^{3/2}(a-bx)^{3/4}}$$

input
```
Int[1/((c*x)^(3/4)*(a - b*x)^(3/4)),x]
```

output
```
(-4*Sqrt[b]*(1 - a/(b*x))^(3/4)*(c*x)^(3/4)*EllipticF[ArcSin[(Sqrt[a]*Sqrt
[c]*Sqrt[c*x])/Sqrt[b]]/2, 2])/(Sqrt[a]*c^(3/2)*(a - b*x)^(3/4))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{4}} (-bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(3/4)/(-b*x+a)^(3/4),x)
```

output
```
int(1/(c*x)^(3/4)/(-b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{3}{4}} (cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(-b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x + a)^(1/4)*(c*x)^(1/4)/(b*c*x^2 - a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = \frac{2ie^{-\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\right|\frac{a}{bx}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{x}}$$

input
```
integrate(1/(c*x)**(3/4)/(-b*x+a)**(3/4),x)
```

output
```
2*I*exp(-I*pi/4)*hyper((1/2, 3/4), (3/2,), a/(b*x))/(b**(3/4)*c**(3/4)*sqr
t(x))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{3}{4}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(-b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x + a)^(3/4)*(c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{(-bx+a)^{\frac{3}{4}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(-b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x + a)^(3/4)*(c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = \int \frac{1}{(c\,x)^{3/4}\,(a-b\,x)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(3/4)*(a - b*x)^(3/4)),x)
```

output
```
int(1/((c*x)^(3/4)*(a - b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/4}(a-bx)^{3/4}}\,dx = \frac{\int \frac{1}{x^{\frac{3}{4}}(-bx+a)^{\frac{3}{4}}}dx}{c^{\frac{3}{4}}}$$

input
```
int(1/(c*x)^(3/4)/(-b*x+a)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(a - b*x)**(3/4)),x)/c**(3/4)
```

3.762 $\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = \frac{2\sqrt{2}a\left(-\frac{bx}{a}-\frac{b^2x^2}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(1+\frac{2bx}{a}\right),2\right)}{b(-cx)^{3/4}(a+bx)^{3/4}}$$

output
```
2*2^(1/2)*a*(-b*x/a-b^2*x^2/a^2)^(3/4)*InverseJacobiAM(1/2*arcsin(1+2*b*x/
a),2^(1/2))/b/(-c*x)^(3/4)/(b*x+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = \frac{4x\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx}{a}\right)}{(-cx)^{3/4}(a+bx)^{3/4}}$$

input
```
Integrate[1/((-(c*x))^(3/4)*(a + b*x)^(3/4)),x]
```

output
```
(4*x*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x)/a)])/((-
(c*x))^(3/4)*(a + b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {73, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$-\frac{4\int \frac{1}{(a+bx)^{3/4}}d\sqrt[4]{-cx}}{c}$$

$$\Big\downarrow \text{768}$$

$$-\frac{4(-cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx}+1\right)^{3/4}(-cx)^{3/4}}d\sqrt[4]{-cx}}{c(a+bx)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{4(-cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{-cx}\left(\frac{axc^2}{b}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{-cx}}}{c(a+bx)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2(-cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\int \frac{1}{\left(1-\frac{ac\sqrt{-cx}}{b}\right)^{3/4}}d\sqrt{-cx}}{c(a+bx)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{4\sqrt{b}(-cx)^{3/4}\left(\frac{a}{bx}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}\sqrt{c}\sqrt{-cx}}{\sqrt{b}}\right),2\right)}{\sqrt{a}c^{3/2}(a+bx)^{3/4}}$$

input
```
Int[1/((-(c*x))^(3/4)*(a + b*x)^(3/4)),x]
```

output
```
(4*Sqrt[b]*(1 + a/(b*x))^(3/4)*(-(c*x))^(3/4)*EllipticF[ArcSin[(Sqrt[a]*Sq
rt[c]*Sqrt[-(c*x)])/Sqrt[b]]/2, 2])/(Sqrt[a]*c^(3/2)*(a + b*x)^(3/4))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(-cx)^{\frac{3}{4}}(bx+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(-c*x)^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(-c*x)^(3/4)/(b*x+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}(-cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-c*x)^(3/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(1/4)*(-c*x)^(1/4)/(b*c*x^2 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = \frac{\sqrt[4]{x}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-c*x)**(3/4)/(b*x+a)**(3/4),x)
```

output
```
x**(1/4)*exp(-3*I*pi/4)*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x*exp_polar
(I*pi)/a)/(a**(3/4)*c**(3/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\,(-cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-c*x)^(3/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/4)*(-c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{4}}\,(-cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-c*x)^(3/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/4)*(-c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = \int \frac{1}{(-c\,x)^{3/4}\,(a+b\,x)^{3/4}}\,dx$$

input
```
int(1/((-c*x)^(3/4)*(a + b*x)^(3/4)),x)
```

output
```
int(1/((-c*x)^(3/4)*(a + b*x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(-cx)^{3/4}(a+bx)^{3/4}}\,dx = -\frac{\left(\int \frac{1}{x^{\frac{3}{4}}(bx+a)^{\frac{3}{4}}}dx\right)(-1)^{\frac{1}{4}}}{c^{\frac{3}{4}}}$$

input
```
int(1/(-c*x)^(3/4)/(b*x+a)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x)**(3/4)),x)/(c**(3/4)*( - 1)**(3/4))
```

3.763 $\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = -4\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{1}{\sqrt{x}}\right),2\right)$$

output
```
-4*InverseJacobiAM(1/2*arcsin(1/x^(1/2)),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = 4\sqrt[4]{-1+x}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},1-x\right)$$

input
```
Integrate[1/((-1 + x)^(3/4)*x^(3/4)),x]
```

output
```
4*(-1 + x)^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, 1 - x]
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)^{3/4}x^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$4\int \frac{1}{x^{3/4}}d\sqrt[4]{x-1}$$

$$\Big\downarrow \text{768}$$

$$\frac{4\left(\frac{1}{x-1}+1\right)^{3/4}(x-1)^{3/4}\int \frac{1}{\left(1+\frac{1}{x-1}\right)^{3/4}(x-1)^{3/4}}d\sqrt[4]{x-1}}{x^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{4\left(\frac{1}{x-1}+1\right)^{3/4}(x-1)^{3/4}\int \frac{1}{\sqrt[4]{x-1}x^{3/4}}d\frac{1}{\sqrt[4]{x-1}}}{x^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2\left(\frac{1}{x-1}+1\right)^{3/4}(x-1)^{3/4}\int \frac{1}{\left(\sqrt{x-1}+1\right)^{3/4}}d\sqrt{x-1}}{x^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{4\left(\frac{1}{x-1}+1\right)^{3/4}(x-1)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{x-1}\right),2\right)}{x^{3/4}}$$

input
```
Int[1/((-1 + x)^(3/4)*x^(3/4)),x]
```

output
```
(-4*(1 + (-1 + x)^(-1))^(3/4)*(-1 + x)^(3/4)*EllipticF[ArcTan[Sqrt[-1 + x]
]/2, 2])/x^(3/4)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

method	result	size
meijerg	$\frac{4(-\operatorname{signum}(-1+x))^{\frac{3}{4}} x^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{5}{4}\right],x\right)}{\operatorname{signum}(-1+x)^{\frac{3}{4}}}$	27

input
```
int(1/(-1+x)^(3/4)/x^(3/4),x,method=_RETURNVERBOSE)
```

output
```
4/signum(-1+x)^(3/4)*(-signum(-1+x))^(3/4)*x^(1/4)*hypergeom([1/4,3/4],[5/
4],x)
```

Fricas [F]

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{(x-1)^{\frac{3}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-1+x)^(3/4)/x^(3/4),x, algorithm="fricas")
```

output
```
integral((x - 1)^(1/4)*x^(1/4)/(x^2 - x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = -\frac{i\sqrt[4]{x}e^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\right|x\right)}{\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-1+x)**(3/4)/x**(3/4),x)
```

output
```
-I*x**(1/4)*exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 3/4), (5/4,), x)/gamma(5/4
)
```

Maxima [F]

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{(x-1)^{\frac{3}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-1+x)^(3/4)/x^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((x - 1)^(3/4)*x^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{(x-1)^{\frac{3}{4}}x^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-1+x)^(3/4)/x^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((x - 1)^(3/4)*x^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{x^{3/4}\,(x-1)^{3/4}}\,dx$$

input
```
int(1/(x^(3/4)*(x - 1)^(3/4)),x)
```

output
```
int(1/(x^(3/4)*(x - 1)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(-1+x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(x-1)^{\frac{3}{4}}}dx$$

input
```
int(1/(-1+x)^(3/4)/x^(3/4),x)
```

output
```
int(1/(x**(3/4)*(x - 1)**(3/4)),x)
```

3.764 $\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = -4 \operatorname{EllipticF}\left(\frac{1}{2} \arcsin\left(\frac{1}{\sqrt{x}}\right), 2\right)$$

output
```
-4*InverseJacobiAM(1/2*arcsin(1/x^(1/2)),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = 4 \sqrt[4]{-1+x} \, \text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, 1-x\right)$$

input
```
Integrate[1/(((-1 + x)/x)^(3/4)*x^(3/2)),x]
```

output
```
4*(-1 + x)^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, 1 - x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2035, 2073, 858, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{x-1}{x}\right)^{3/4} x^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2035}$$

$$2 \int \frac{1}{\left(-\frac{1-x}{x}\right)^{3/4} x} d\sqrt{x}$$

$$\Big\downarrow \text{ 2073}$$

$$2 \int \frac{1}{\left(1-\frac{1}{x}\right)^{3/4} x} d\sqrt{x}$$

$$\Big\downarrow \text{ 858}$$

$$-2 \int \frac{1}{(1-x)^{3/4}} d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 230}$$

$$-4 \operatorname{EllipticF}\left(\frac{1}{2} \arcsin\left(\frac{1}{\sqrt{x}}\right), 2\right)$$

input
```
Int[1/(((-1 + x)/x)^(3/4)*x^(3/2)),x]
```

output
```
-4*EllipticF[ArcSin[1/Sqrt[x]]/2, 2]
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Maple [F]

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{\frac{3}{4}} x^{\frac{3}{2}}}dx$$

input
```
int(1/((-1+x)/x)^(3/4)/x^(3/2),x)
```

output
```
int(1/((-1+x)/x)^(3/4)/x^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = \int \frac{1}{x^{\frac{3}{2}} \left(\frac{x-1}{x}\right)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/((-1+x)/x)^(3/4)/x^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x)*((x - 1)/x)^(1/4)/(x^2 - x), x)
```

Sympy [F]

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = \int \frac{1}{x^{\frac{3}{2}} \left(1 - \frac{1}{x}\right)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/((-1+x)/x)**(3/4)/x**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*(1 - 1/x)**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = \int \frac{1}{x^{\frac{3}{2}} \left(\frac{x-1}{x}\right)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/((-1+x)/x)^(3/4)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^(3/2)*((x - 1)/x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = \int \frac{1}{x^{\frac{3}{2}} \left(\frac{x-1}{x}\right)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/((-1+x)/x)^(3/4)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x^(3/2)*((x - 1)/x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = \int \frac{1}{x^{3/2} \left(\frac{x-1}{x}\right)^{3/4}} \, dx$$

input
```
int(1/(x^(3/2)*((x - 1)/x)^(3/4)),x)
```

output
```
int(1/(x^(3/2)*((x - 1)/x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{\left(\frac{-1+x}{x}\right)^{3/4} x^{3/2}} \, dx = \int \frac{x^{\frac{5}{4}} (x-1)^{\frac{3}{4}}}{\sqrt{x-1}\, x^3 - \sqrt{x-1}\, x^2} dx$$

input
```
int(1/((-1+x)/x)^(3/4)/x^(3/2),x)
```

output
```
int((x**(5/4)*(x - 1)**(3/4))/(sqrt(x - 1)*x**3 - sqrt(x - 1)*x**2),x)
```

3.765 $\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = -4E\left(\frac{1}{2}\arcsin\left(\frac{1}{\sqrt{x}}\right)\middle|2\right)$$

output
```
-4*EllipticE(sin(1/2*arcsin(1/x^(1/2))),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = \frac{4}{3}(-1+x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},1-x\right)$$

input
```
Integrate[1/((-1 + x)^(1/4)*x^(5/4)),x]
```

output
```
(4*(-1 + x)^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, 1 - x])/3
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 5.36, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {61, 73, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x-1}x^{5/4}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-2\int \frac{1}{\sqrt[4]{x-1}\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\int \frac{\sqrt{x-1}}{\sqrt[4]{x}}d\sqrt[4]{x-1}$$

$$\Big\downarrow \text{839}$$

$$\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}-\frac{1}{2}\int \frac{\sqrt{x-1}}{x^{5/4}}d\sqrt[4]{x-1}\right)$$

$$\Big\downarrow \text{813}$$

$$\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}-\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}\int \frac{1}{\left(1+\frac{1}{x-1}\right)^{5/4}(x-1)^{3/4}}d\sqrt[4]{x-1}}{2\sqrt[4]{x}}\right)$$

$$\Big\downarrow \text{858}$$

$$\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}\int \frac{1}{\sqrt[4]{x-1}x^{5/4}}d\frac{1}{\sqrt[4]{x-1}}}{2\sqrt[4]{x}}+\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}\int \frac{1}{\left(\sqrt{x-1}+1\right)^{5/4}}d\sqrt{x-1}}{4\sqrt[4]{x}}+\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}\right)$$

$$\Big\downarrow \textbf{212}$$

$$\frac{4(x-1)^{3/4}}{\sqrt[4]{x}}-8\left(\frac{\sqrt[4]{\frac{1}{x-1}+1}\sqrt[4]{x-1}E\left(\frac{1}{2}\arctan\left(\sqrt{x-1}\right)\middle|2\right)}{2\sqrt[4]{x}}+\frac{(x-1)^{3/4}}{2\sqrt[4]{x}}\right)$$

input
```
Int[1/((-1 + x)^(1/4)*x^(5/4)),x]
```

output
```
(4*(-1 + x)^(3/4))/x^(1/4) - 8*((-1 + x)^(3/4)/(2*x^(1/4)) + ((1 + (-1 + x
)^(-1))^(1/4)*(-1 + x)^(1/4)*EllipticE[ArcTan[Sqrt[-1 + x]]/2, 2])/(2*x^(1
/4)))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

method	result	size
meijerg	$-\frac{4(-\operatorname{signum}(-1+x))^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],x\right)}{\operatorname{signum}(-1+x)^{\frac{1}{4}} x^{\frac{1}{4}}}$	27
risch	$\frac{4(-1+x)^{\frac{3}{4}}}{x^{\frac{1}{4}}}-\frac{8(-\operatorname{signum}(-1+x))^{\frac{1}{4}} \sqrt{x}\, \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x\right) (x(-1+x))^{\frac{1}{4}}}{3 \operatorname{signum}(-1+x)^{\frac{1}{4}} (-1+x)^{\frac{1}{4}}}$	50

input
```
int(1/(-1+x)^(1/4)/x^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-4/signum(-1+x)^(1/4)*(-signum(-1+x))^(1/4)/x^(1/4)*hypergeom([-1/4,1/4],[
3/4],x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = \int \frac{1}{(x-1)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-1+x)^(1/4)/x^(5/4),x, algorithm="fricas")
```

output
```
integral((x - 1)^(3/4)*x^(3/4)/(x^3 - x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = -\frac{ie^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|x\right)}{\sqrt[4]{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(-1+x)**(1/4)/x**(5/4),x)
```

output
```
-I*exp(I*pi/4)*gamma(-1/4)*hyper((-1/4, 1/4), (3/4,), x)/(x**(1/4)*gamma(3
/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = \int \frac{1}{(x-1)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-1+x)^(1/4)/x^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((x - 1)^(1/4)*x^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = \int \frac{1}{(x-1)^{\frac{1}{4}}x^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-1+x)^(1/4)/x^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((x - 1)^(1/4)*x^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = \int \frac{1}{x^{5/4}\,(x-1)^{1/4}}\,dx$$

input
```
int(1/(x^(5/4)*(x - 1)^(1/4)),x)
```

output
```
int(1/(x^(5/4)*(x - 1)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{-1+x}x^{5/4}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(x-1)^{\frac{1}{4}}}dx$$

input
```
int(1/(-1+x)^(1/4)/x^(5/4),x)
```

output
```
int(1/(x**(1/4)*(x - 1)**(1/4)*x),x)
```

3.766 $\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = -4E\left(\frac{1}{2}\arcsin\left(\frac{1}{\sqrt{x}}\right)\middle|2\right)$$

output
```
-4*EllipticE(sin(1/2*arcsin(1/x^(1/2))),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = \frac{4}{3}(-1+x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},1-x\right)$$

input
```
Integrate[1/((((-1 + x)/x)^(1/4)*x^(3/2)),x]
```

output
```
(4*(-1 + x)^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, 1 - x])/3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2035, 2073, 858, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{\frac{x-1}{x}}x^{3/2}}dx$$

$$\Big\downarrow \text{ 2035}$$

$$2\int \frac{1}{\sqrt[4]{-\frac{1-x}{x}}x}d\sqrt{x}$$

$$\Big\downarrow \text{ 2073}$$

$$2\int \frac{1}{\sqrt[4]{1-\frac{1}{x}}x}d\sqrt{x}$$

$$\Big\downarrow \text{ 858}$$

$$-2\int \frac{1}{\sqrt[4]{1-x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 226}$$

$$-4E\left(\frac{1}{2}\arcsin\left(\frac{1}{\sqrt{x}}\right)\middle|2\right)$$

input
```
Int[1/(((-1 + x)/x)^(1/4)*x^(3/2)),x]
```

output
```
-4*EllipticE[ArcSin[1/Sqrt[x]]/2, 2]
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

method	result	size
risch	$\frac{-4+4x}{\sqrt{x}\,\left(\frac{-1+x}{x}\right)^{\frac{1}{4}}}-\frac{8\left(-\operatorname{signum}\left(-1+x\right)\right)^{\frac{1}{4}} x^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x\right) \left(x\left(-1+x\right)\right)^{\frac{1}{4}}}{3\operatorname{signum}\left(-1+x\right)^{\frac{1}{4}} \left(\frac{-1+x}{x}\right)^{\frac{1}{4}}}$	61

input
```
int(1/((-1+x)/x)^(1/4)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(-1+x)/x^(1/2)/((-1+x)/x)^(1/4)-8/3/signum(-1+x)^(1/4)*(-signum(-1+x))^(
1/4)*x^(1/4)*hypergeom([1/4,3/4],[7/4],x)/((-1+x)/x)^(1/4)*(x*(-1+x))^(1/4
)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(\frac{x-1}{x}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((-1+x)/x)^(1/4)/x^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x)*((x - 1)/x)^(3/4)/(x^2 - x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt[4]{1-\frac{1}{x}}}\,dx$$

input
```
integrate(1/((-1+x)/x)**(1/4)/x**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*(1 - 1/x)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(\frac{x-1}{x}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((-1+x)/x)^(1/4)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x^(3/2)*((x - 1)/x)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(\frac{x-1}{x}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((-1+x)/x)^(1/4)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x^(3/2)*((x - 1)/x)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = \int \frac{1}{x^{3/2}\left(\frac{x-1}{x}\right)^{1/4}}\,dx$$

input
```
int(1/(x^(3/2)*((x - 1)/x)^(1/4)),x)
```

output
```
int(1/(x^(3/2)*((x - 1)/x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{\frac{-1+x}{x}}x^{3/2}}\,dx = \int \frac{(x-1)^{\frac{1}{4}}}{x^{\frac{5}{4}}\sqrt{x-1}}dx$$

input
```
int(1/((-1+x)/x)^(1/4)/x^(3/2),x)
```

output
```
int((x**(3/4)*(x - 1)**(1/4))/(sqrt(x - 1)*x**2),x)
```

3.767 $\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = -2\sqrt{2}\,\mathrm{EllipticF}\left(\frac{1}{2}\arcsin(1-2x),2\right)$$

output
```
2*2^(1/2)*InverseJacobiAM(1/2*arcsin(-1+2*x),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = -\frac{4\sqrt[4]{-((-1+x)x)}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},1-x\right)}{\sqrt[4]{x}}$$

input
```
Integrate[1/((1 - x)^(3/4)*x^(3/4)),x]
```

output
```
(-4*(-((-1 + x)*x))^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, 1 - x])/x^(1/4)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {73, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx$$

$$\Big\downarrow \text{73}$$

$$-4\int \frac{1}{x^{3/4}}d\sqrt[4]{1-x}$$

$$\Big\downarrow \text{768}$$

$$-\frac{4\left(1-\frac{1}{1-x}\right)^{3/4}(1-x)^{3/4}\int \frac{1}{\left(1-\frac{1}{1-x}\right)^{3/4}(1-x)^{3/4}}d\sqrt[4]{1-x}}{x^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{4\left(1-\frac{1}{1-x}\right)^{3/4}(1-x)^{3/4}\int \frac{1}{\sqrt[4]{1-x}x^{3/4}}d\frac{1}{\sqrt[4]{1-x}}}{x^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2\left(1-\frac{1}{1-x}\right)^{3/4}(1-x)^{3/4}\int \frac{1}{\left(1-\sqrt{1-x}\right)^{3/4}}d\sqrt{1-x}}{x^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{4\left(1-\frac{1}{1-x}\right)^{3/4}(1-x)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{1-x}\right),2\right)}{x^{3/4}}$$

input
```
Int[1/((1 - x)^(3/4)*x^(3/4)),x]
```

output
```
(4*(1 - (1 - x)^(-1))^(3/4)*(1 - x)^(3/4)*EllipticF[ArcSin[Sqrt[1 - x]]/2,
 2])/x^(3/4)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
meijerg	$4x^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{5}{4}\right], x\right)$	13

input
```
int(1/(1-x)^(3/4)/x^(3/4),x,method=_RETURNVERBOSE)
```

output
```
4*x^(1/4)*hypergeom([1/4,3/4],[5/4],x)
```

Fricas [F]

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}(-x+1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1-x)^(3/4)/x^(3/4),x, algorithm="fricas")
```

output
```
integral(-x^(1/4)*(-x + 1)^(1/4)/(x^2 - x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = \frac{2ie^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{1}{x}\right)}{\sqrt{x}}$$

input
```
integrate(1/(1-x)**(3/4)/x**(3/4),x)
```

output
```
2*I*exp(-I*pi/4)*hyper((1/2, 3/4), (3/2,), 1/x)/sqrt(x)
```

Maxima [F]

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}(-x+1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1-x)^(3/4)/x^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/(x^(3/4)*(-x + 1)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}(-x+1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1-x)^(3/4)/x^(3/4),x, algorithm="giac")
```

output
```
integrate(1/(x^(3/4)*(-x + 1)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{x^{3/4}\,(1-x)^{3/4}}\,dx$$

input
```
int(1/(x^(3/4)*(1 - x)^(3/4)),x)
```

output
```
int(1/(x^(3/4)*(1 - x)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(1-x)^{3/4}x^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(1-x)^{\frac{3}{4}}}dx$$

input
```
int(1/(1-x)^(3/4)/x^(3/4),x)
```

output
```
int(1/(x**(3/4)*( - x + 1)**(3/4)),x)
```

3.768 $\int \frac{1}{(x-x^2)^{3/4}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{(x-x^2)^{3/4}} \, dx = -2\sqrt{2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin(1-2x),2\right)$$

output
```
2*2^(1/2)*InverseJacobiAM(1/2*arcsin(-1+2*x),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{(x-x^2)^{3/4}} \, dx = -\frac{4\sqrt[4]{-((-1+x)x)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},1-x\right)}{\sqrt[4]{x}}$$

input
```
Integrate[(x - x^2)^(-3/4),x]
```

output
```
(-4*(-((-1 + x)*x))^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, 1 - x])/x^(1/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-x^2)^{3/4}} \, dx$$

$$\Big\downarrow \; 1090$$

$$-\sqrt{2} \int \frac{1}{\left(1-(1-2x)^2\right)^{3/4}} d(1-2x)$$

$$\Big\downarrow \; 230$$

$$-2\sqrt{2} \operatorname{EllipticF}\left(\frac{1}{2}\arcsin(1-2x),2\right)$$

input
```
Int[(x - x^2)^(-3/4),x]
```

output
```
-2*Sqrt[2]*EllipticF[ArcSin[1 - 2*x]/2, 2]
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
meijerg	$4x^{\frac{1}{4}}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{5}{4}\right],x\right)$	13

input
```
int(1/(-x^2+x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
4*x^(1/4)*hypergeom([1/4,3/4],[5/4],x)
```

Fricas [F]

$$\int \frac{1}{(x-x^2)^{3/4}}\,dx = \int \frac{1}{(-x^2+x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-x^2+x)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-x^2 + x)^(1/4)/(x^2 - x), x)
```

Sympy [F]

$$\int \frac{1}{(x-x^2)^{3/4}}\,dx = \int \frac{1}{(-x^2+x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-x**2+x)**(3/4),x)
```

output
```
Integral((-x**2 + x)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(x-x^2)^{3/4}}\,dx = \int \frac{1}{(-x^2+x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-x^2+x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-x^2 + x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(x-x^2)^{3/4}}\,dx = \int \frac{1}{(-x^2+x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-x^2+x)^(3/4),x, algorithm="giac")
```

output
```
integrate((-x^2 + x)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{1}{(x-x^2)^{3/4}}\,dx = \frac{4\,x\,(1-x)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ x\right)}{(x-x^2)^{3/4}}$$

input
```
int(1/(x - x^2)^(3/4),x)
```

output
```
(4*x*(1 - x)^(3/4)*hypergeom([1/4, 3/4], 5/4, x))/(x - x^2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(x-x^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}(1-x)^{\frac{3}{4}}}dx$$

input
```
int(1/(-x^2+x)^(3/4),x)
```

output
```
int(1/(x**(3/4)*( - x + 1)**(3/4)),x)
```

3.769 $\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = -\frac{4(1-x)^{3/4}}{\sqrt[4]{x}} + 2\sqrt{2}E\left(\frac{1}{2}\arcsin(1-2x)\middle|2\right)$$

output
```
-4*(1-x)^(3/4)/x^(1/4)-2*2^(1/2)*EllipticE(sin(1/2*arcsin(-1+2*x)),2^(1/2)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = -\frac{4\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},x\right)}{\sqrt[4]{x}}$$

input
```
Integrate[1/((1 - x)^(1/4)*x^(5/4)),x]
```

output
```
(-4*Hypergeometric2F1[-1/4, 1/4, 3/4, x])/x^(1/4)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(36) = 72$.

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.42, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {61, 73, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}dx$$

$$\Big\downarrow \text{61}$$

$$-2\int \frac{1}{\sqrt[4]{1-x}\sqrt[4]{x}}dx-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}$$

$$\Big\downarrow \text{73}$$

$$8\int \frac{\sqrt{1-x}}{\sqrt[4]{x}}d\sqrt[4]{1-x}-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}$$

$$\Big\downarrow \text{840}$$

$$8\left(-\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt[4]{x}}d\sqrt[4]{1-x}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}$$

$$\Big\downarrow \text{842}$$

$$8\left(-\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}\int \frac{1}{\sqrt[4]{1-\frac{1}{1-x}}(1-x)^{3/4}}d\sqrt[4]{1-x}}{2\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}$$

$$\Big\downarrow \text{858}$$

$$8\left(\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}\int \frac{1}{\sqrt[4]{1-x}\sqrt[4]{x}}d\frac{1}{\sqrt[4]{1-x}}}{2\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}$$

$$\Big\downarrow \text{807}$$

$$8\left(\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}\int \frac{1}{\sqrt[4]{1-\sqrt{1-x}}}d\sqrt{1-x}}{4\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}$$

$$\Big\downarrow \text{226}$$

$$8\left(\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}E\left(\frac{1}{2}\arcsin\left(\sqrt{1-x}\right)\middle|2\right)}{2\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}$$

input
```
Int[1/((1 - x)^(1/4)*x^(5/4)),x]
```

output
```
(-4*(1 - x)^(3/4))/x^(1/4) + 8*(-1/2*x^(3/4)/(1 - x)^(1/4) + ((1 - (1 - x)
^(-1))^(1/4)*(1 - x)^(1/4)*EllipticE[ArcSin[Sqrt[1 - x]]/2, 2])/(2*x^(1/4)
))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 840
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

method	result	size
meijerg	$-\frac{4 \operatorname{hypergeom}\left(\left[-\frac{1}{4}, \frac{1}{4}\right], \left[\frac{3}{4}\right], x\right)}{x^{\frac{1}{4}}}$	13
risch	$\frac{4(-1+x)(x(1-x))^{\frac{1}{4}}}{(-x(-1+x))^{\frac{1}{4}} x^{\frac{1}{4}} (1-x)^{\frac{1}{4}}} - \frac{8\sqrt{x}\ \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], x\right)(x(1-x))^{\frac{1}{4}}}{3(1-x)^{\frac{1}{4}}}$	62

input
```
int(1/(1-x)^(1/4)/x^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-4/x^(1/4)*hypergeom([-1/4,1/4],[3/4],x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = \int \frac{1}{x^{\frac{5}{4}}(-x+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(1-x)^(1/4)/x^(5/4),x, algorithm="fricas")
```

output
```
integral(-x^(3/4)*(-x + 1)^(3/4)/(x^3 - x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = \frac{2ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{1}{x}\right)}{\sqrt{x}}$$

input
```
integrate(1/(1-x)**(1/4)/x**(5/4),x)
```

output
```
2*I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), 1/x)/sqrt(x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = \int \frac{1}{x^{\frac{5}{4}}(-x+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(1-x)^(1/4)/x^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/(x^(5/4)*(-x + 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = \int \frac{1}{x^{\frac{5}{4}}(-x+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(1-x)^(1/4)/x^(5/4),x, algorithm="giac")
```

output
```
integrate(1/(x^(5/4)*(-x + 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = \int \frac{1}{x^{5/4}\,(1-x)^{1/4}}\,dx$$

input
```
int(1/(x^(5/4)*(1 - x)^(1/4)),x)
```

output
```
int(1/(x^(5/4)*(1 - x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}\,dx = \int \frac{1}{x^{\frac{5}{4}}(1-x)^{\frac{1}{4}}}dx$$

input
```
int(1/(1-x)^(1/4)/x^(5/4),x)
```

output
```
int(1/(x**(1/4)*( - x + 1)**(1/4)*x),x)
```

3.770 $\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx = -\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}+2\sqrt{2}E\left(\frac{1}{2}\arcsin(1-2x)\middle|2\right)$$

output
```
-4*(1-x)^(3/4)/x^(1/4)-2*2^(1/2)*EllipticE(sin(1/2*arcsin(-1+2*x)),2^(1/2)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx = -\frac{4(-((-1+x)x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},1-x\right)}{3x^{3/4}}$$

input
```
Integrate[1/(x*(x - x^2)^(1/4)),x]
```

output
```
(-4*(-((-1 + x)*x))^(3/4)*Hypergeometric2F1[3/4, 5/4, 7/4, 1 - x])/(3*x^(3
/4))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1137, 61, 73, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{x-x^2}}dx$$

$$\Big\downarrow \text{1137}$$

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\int \frac{1}{\sqrt[4]{1-x}x^{5/4}}dx}{\sqrt[4]{x-x^2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\left(-2\int \frac{1}{\sqrt[4]{1-x}\sqrt[4]{x}}dx-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\left(8\int \frac{\sqrt{1-x}}{\sqrt[4]{x}}d\sqrt[4]{1-x}-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-x^2}}$$

$$\Big\downarrow \text{840}$$

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\left(8\left(-\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt[4]{x}}d\sqrt[4]{1-x}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-x^2}}$$

$$\Big\downarrow \text{842}$$

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\left(8\left(-\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}\int \frac{1}{\sqrt[4]{1-\frac{1}{1-x}}(1-x)^{3/4}}d\sqrt[4]{1-x}}{2\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-x^2}}$$

$\Big\downarrow$ 858

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\left(8\left(\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}\int \frac{1}{\sqrt[4]{1-x}\sqrt[4]{x}}d\frac{1}{\sqrt[4]{1-x}}}{2\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-x^2}}$$

$\Big\downarrow$ 807

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\left(8\left(\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}\int \frac{1}{\sqrt[4]{1-\sqrt{1-x}}}d\sqrt{1-x}}{4\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-x^2}}$$

$\Big\downarrow$ 226

$$\frac{\sqrt[4]{1-x}\sqrt[4]{x}\left(8\left(\frac{\sqrt[4]{1-\frac{1}{1-x}}\sqrt[4]{1-x}E\left(\frac{1}{2}\arcsin\left(\sqrt{1-x}\right)|2\right)}{2\sqrt[4]{x}}-\frac{x^{3/4}}{2\sqrt[4]{1-x}}\right)-\frac{4(1-x)^{3/4}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-x^2}}$$

input

```
Int[1/(x*(x - x^2)^(1/4)),x]
```

output

```
((1 - x)^(1/4)*x^(1/4)*((-4*(1 - x)^(3/4))/x^(1/4) + 8*(-1/2*x^(3/4)/(1 -
x)^(1/4) + ((1 - (1 - x)^(-1))^(1/4)*(1 - x)^(1/4)*EllipticE[ArcSin[Sqrt[1
 - x]]/2, 2])/(2*x^(1/4)))))/(x - x^2)^(1/4)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 840
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1137
```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

method	result	size
meijerg	$-\frac{4\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],x\right)}{x^{\frac{1}{4}}}$	13
risch	$\frac{-4+4x}{\left(-x(-1+x)\right)^{\frac{1}{4}}}-\frac{8x^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x\right)}{3}$	27

input
```
int(1/x/(-x^2+x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-4/x^(1/4)*hypergeom([-1/4,1/4],[3/4],x)
```

Fricas [F]

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx=\int \frac{1}{\left(-x^2+x\right)^{\frac{1}{4}}x}\,dx$$

input
```
integrate(1/x/(-x^2+x)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-x^2 + x)^(3/4)/(x^3 - x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx = \int \frac{1}{x\sqrt[4]{-x\,(x-1)}}\,dx$$

input
```
integrate(1/x/(-x**2+x)**(1/4),x)
```

output
```
Integral(1/(x*(-x*(x - 1))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx = \int \frac{1}{(-x^2+x)^{\frac{1}{4}}x}\,dx$$

input
```
integrate(1/x/(-x^2+x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-x^2 + x)^(1/4)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx = \int \frac{1}{(-x^2+x)^{\frac{1}{4}}x}\,dx$$

input
```
integrate(1/x/(-x^2+x)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-x^2 + x)^(1/4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx = \int \frac{1}{x\,(x-x^2)^{1/4}}\,dx$$

input
```
int(1/(x*(x - x^2)^(1/4)),x)
```

output
```
int(1/(x*(x - x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{x-x^2}}\,dx = \int \frac{1}{x^{\frac{5}{4}}\,(1-x)^{\frac{1}{4}}}dx$$

input
```
int(1/x/(-x^2+x)^(1/4),x)
```

output
```
int(1/(x**(1/4)*( - x + 1)**(1/4)*x),x)
```

3.771 $\int (cx)^m(a+bx)^4\,dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int (cx)^m(a+bx)^4\,dx=\frac{a^4(cx)^{1+m}}{c(1+m)}+\frac{4a^3b(cx)^{2+m}}{c^2(2+m)}+\frac{6a^2b^2(cx)^{3+m}}{c^3(3+m)}+\frac{4ab^3(cx)^{4+m}}{c^4(4+m)}+\frac{b^4(cx)^{5+m}}{c^5(5+m)}$$

output
```
a^4*(c*x)^(1+m)/c/(1+m)+4*a^3*b*(c*x)^(2+m)/c^2/(2+m)+6*a^2*b^2*(c*x)^(3+m
)/c^3/(3+m)+4*a*b^3*(c*x)^(4+m)/c^4/(4+m)+b^4*(c*x)^(5+m)/c^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int (cx)^m(a+bx)^4\,dx=x(cx)^m\left(\frac{a^4}{1+m}+\frac{4a^3bx}{2+m}+\frac{6a^2b^2x^2}{3+m}+\frac{4ab^3x^3}{4+m}+\frac{b^4x^4}{5+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x)^4,x]
```

output
```
x*(c*x)^m*(a^4/(1 + m) + (4*a^3*b*x)/(2 + m) + (6*a^2*b^2*x^2)/(3 + m) + (
4*a*b^3*x^3)/(4 + m) + (b^4*x^4)/(5 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4(cx)^m\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^4(cx)^m+\frac{4a^3b(cx)^{m+1}}{c}+\frac{6a^2b^2(cx)^{m+2}}{c^2}+\frac{4ab^3(cx)^{m+3}}{c^3}+\frac{b^4(cx)^{m+4}}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4(cx)^{m+1}}{c(m+1)}+\frac{4a^3b(cx)^{m+2}}{c^2(m+2)}+\frac{6a^2b^2(cx)^{m+3}}{c^3(m+3)}+\frac{4ab^3(cx)^{m+4}}{c^4(m+4)}+\frac{b^4(cx)^{m+5}}{c^5(m+5)}$$

input
```
Int[(c*x)^m*(a + b*x)^4,x]
```

output
```
(a^4*(c*x)^(1 + m))/(c*(1 + m)) + (4*a^3*b*(c*x)^(2 + m))/(c^2*(2 + m)) +
(6*a^2*b^2*(c*x)^(3 + m))/(c^3*(3 + m)) + (4*a*b^3*(c*x)^(4 + m))/(c^4*(4
+ m)) + (b^4*(c*x)^(5 + m))/(c^5*(5 + m))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^4 x\,e^{m \ln(cx)}}{1+m}+\frac{b^4 x^5 e^{m \ln(cx)}}{5+m}+\frac{4a\,b^3 x^4 e^{m \ln(cx)}}{4+m}+\frac{6a^2 b^2 x^3 e^{m \ln(cx)}}{3+m}+\frac{4a^3 b\,x^2 e^{m \ln(cx)}}{2+m}$
gosper	$\frac{x(b^4 m^4 x^4+4a\,b^3 m^4 x^3+10b^4 m^3 x^4+6a^2 b^2 m^4 x^2+44a\,b^3 m^3 x^3+35b^4 m^2 x^4+4a^3 b\,m^4 x+72a^2 b^2 m^3 x^2+164a\,b^3 m^2 x^3+50m\,x^4}{}$
risch	$\frac{x(b^4 m^4 x^4+4a\,b^3 m^4 x^3+10b^4 m^3 x^4+6a^2 b^2 m^4 x^2+44a\,b^3 m^3 x^3+35b^4 m^2 x^4+4a^3 b\,m^4 x+72a^2 b^2 m^3 x^2+164a\,b^3 m^2 x^3+50m\,x^4}{}$
orering	$\frac{x(b^4 m^4 x^4+4a\,b^3 m^4 x^3+10b^4 m^3 x^4+6a^2 b^2 m^4 x^2+44a\,b^3 m^3 x^3+35b^4 m^2 x^4+4a^3 b\,m^4 x+72a^2 b^2 m^3 x^2+164a\,b^3 m^2 x^3+50m\,x^4}{}$
parallelrisch	$\frac{x^5 (cx)^m b^4 m^4+10x^5 (cx)^m b^4 m^3+35x^5 (cx)^m b^4 m^2+50x^5 (cx)^m b^4 m+x (cx)^m a^4 m^4+120x^4 (cx)^m a\,b^3+14x (cx)^m a^4 m^3+240x}{}$

input
```
int((c*x)^m*(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/(1+m)*x*exp(m*ln(c*x))+b^4/(5+m)*x^5*exp(m*ln(c*x))+4*a*b^3/(4+m)*x^4*
exp(m*ln(c*x))+6*a^2*b^2/(3+m)*x^3*exp(m*ln(c*x))+4*a^3*b/(2+m)*x^2*exp(m*
ln(c*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(104) = 208$.

Time = 0.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.43

$$\int (cx)^m (a+bx)^4\,dx$$
$$= \frac{((b^4 m^4 + 10\,b^4 m^3 + 35\,b^4 m^2 + 50\,b^4 m + 24\,b^4)x^5 + 4\,(ab^3 m^4 + 11\,ab^3 m^3 + 41\,ab^3 m^2 + 61\,ab^3 m + 30\,ab^3}{}$$

input
```
integrate((c*x)^m*(b*x+a)^4,x, algorithm="fricas")
```

output
```
((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m^
4 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2*
m^4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 + 4
*(a^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 +
(a^4*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*(c*x)^m/(m^5
+ 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1261 vs. $2(94) = 188$.

Time = 0.34 (sec) , antiderivative size = 1261, normalized size of antiderivative = 12.12

$$\int (cx)^m (a+bx)^4 \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x+a)**4,x)
```

output
```
Piecewise(((-a**4/(4*x**4) - 4*a**3*b/(3*x**3) - 3*a**2*b**2/x**2 - 4*a*b*
*3/x + b**4*log(x))/c**5, Eq(m, -5)), ((-a**4/(3*x**3) - 2*a**3*b/x**2 - 6
*a**2*b**2/x + 4*a*b**3*log(x) + b**4*x)/c**4, Eq(m, -4)), ((-a**4/(2*x**2
) - 4*a**3*b/x + 6*a**2*b**2*log(x) + 4*a*b**3*x + b**4*x**2/2)/c**3, Eq(m
, -3)), ((-a**4/x + 4*a**3*b*log(x) + 6*a**2*b**2*x + 2*a*b**3*x**2 + b**4
*x**3/3)/c**2, Eq(m, -2)), ((a**4*log(x) + 4*a**3*b*x + 3*a**2*b**2*x**2 +
 4*a*b**3*x**3/3 + b**4*x**4/4)/c, Eq(m, -1)), (a**4*m**4*x*(c*x)**m/(m**5
 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 14*a**4*m**3*x*(c*x)**m/(
m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 71*a**4*m**2*x*(c*x)*
*m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 154*a**4*m*x*(c*x
)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 120*a**4*x*(c*x
)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 4*a**3*b*m**4*x
**2*(c*x)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 52*a**3
*b*m**3*x**2*(c*x)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120)
+ 236*a**3*b*m**2*x**2*(c*x)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274
*m + 120) + 428*a**3*b*m*x**2*(c*x)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**
2 + 274*m + 120) + 240*a**3*b*x**2*(c*x)**m/(m**5 + 15*m**4 + 85*m**3 + 22
5*m**2 + 274*m + 120) + 6*a**2*b**2*m**4*x**3*(c*x)**m/(m**5 + 15*m**4 + 8
5*m**3 + 225*m**2 + 274*m + 120) + 72*a**2*b**2*m**3*x**3*(c*x)**m/(m**5 +
 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 294*a**2*b**2*m**2*x**3*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int (cx)^m (a+bx)^4 \, dx = \frac{b^4 c^m x^5 x^m}{m+5} + \frac{4\, a b^3 c^m x^4 x^m}{m+4} + \frac{6\, a^2 b^2 c^m x^3 x^m}{m+3} + \frac{4\, a^3 b c^m x^2 x^m}{m+2} + \frac{(cx)^{m+1}\, a^4}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x+a)^4,x, algorithm="maxima")
```

output
```
b^4*c^m*x^5*x^m/(m + 5) + 4*a*b^3*c^m*x^4*x^m/(m + 4) + 6*a^2*b^2*c^m*x^3*
x^m/(m + 3) + 4*a^3*b*c^m*x^2*x^m/(m + 2) + (c*x)^(m + 1)*a^4/(c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(104) = 208.

Time = 0.12 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.99

$$\int (cx)^m (a+bx)^4 \, dx = \frac{(cx)^m\, b^4 m^4 x^5 + 4\,(cx)^m\, a b^3 m^4 x^4 + 10\,(cx)^m\, b^4 m^3 x^5 + 6\,(cx)^m\, a^2 b^2 m^4 x^3 + 44\,(cx)^m\, a b^3 m^3 x^4 + 35\,(cx)^m}{}$$

input
```
integrate((c*x)^m*(b*x+a)^4,x, algorithm="giac")
```

output
```
((c*x)^m*b^4*m^4*x^5 + 4*(c*x)^m*a*b^3*m^4*x^4 + 10*(c*x)^m*b^4*m^3*x^5 +
6*(c*x)^m*a^2*b^2*m^4*x^3 + 44*(c*x)^m*a*b^3*m^3*x^4 + 35*(c*x)^m*b^4*m^2*
x^5 + 4*(c*x)^m*a^3*b*m^4*x^2 + 72*(c*x)^m*a^2*b^2*m^3*x^3 + 164*(c*x)^m*a
*b^3*m^2*x^4 + 50*(c*x)^m*b^4*m*x^5 + (c*x)^m*a^4*m^4*x + 52*(c*x)^m*a^3*b
*m^3*x^2 + 294*(c*x)^m*a^2*b^2*m^2*x^3 + 244*(c*x)^m*a*b^3*m*x^4 + 24*(c*x
)^m*b^4*x^5 + 14*(c*x)^m*a^4*m^3*x + 236*(c*x)^m*a^3*b*m^2*x^2 + 468*(c*x)
^m*a^2*b^2*m*x^3 + 120*(c*x)^m*a*b^3*x^4 + 71*(c*x)^m*a^4*m^2*x + 428*(c*x
)^m*a^3*b*m*x^2 + 240*(c*x)^m*a^2*b^2*x^3 + 154*(c*x)^m*a^4*m*x + 240*(c*x
)^m*a^3*b*x^2 + 120*(c*x)^m*a^4*x)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*
m + 120)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.53

$$\int (cx)^m (a+bx)^4 \, dx = (c\,x)^m \left(\frac{b^4\,x^5\,(m^4+10\,m^3+35\,m^2+50\,m+24)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{a^4\,x\,(m^4+14\,m^3+71\,m^2+154\,m+120)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{4\,a\,b^3\,x^4\,(m^4+11\,m^3+41\,m^2+61\,m+30)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{4\,a^3\,b\,x^2\,(m^4+13\,m^3+59\,m^2+107\,m+60)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{6\,a^2\,b^2\,x^3\,(m^4+12\,m^3+49\,m^2+78\,m+40)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} \right)$$

input
```
int((c*x)^m*(a + b*x)^4,x)
```

output
```
(c*x)^m*((b^4*x^5*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^2 +
85*m^3 + 15*m^4 + m^5 + 120) + (a^4*x*(154*m + 71*m^2 + 14*m^3 + m^4 + 120
))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (4*a*b^3*x^4*(61*m +
41*m^2 + 11*m^3 + m^4 + 30))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 12
0) + (4*a^3*b*x^2*(107*m + 59*m^2 + 13*m^3 + m^4 + 60))/(274*m + 225*m^2 +
 85*m^3 + 15*m^4 + m^5 + 120) + (6*a^2*b^2*x^3*(78*m + 49*m^2 + 12*m^3 + m
^4 + 40))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.71

$$\int (cx)^m (a+bx)^4 \, dx = \frac{x^m c^m x(b^4m^4x^4+4a\,b^3m^4x^3+10b^4m^3x^4+6a^2b^2m^4x^2+44a\,b^3m^3x^3+35b^4m^2x^4+4a^3b\,m^4x+72a^2b^2n}{}$$

input
```
int((c*x)^m*(b*x+a)^4,x)
```

output
```
(x**m*c**m*x*(a**4*m**4 + 14*a**4*m**3 + 71*a**4*m**2 + 154*a**4*m + 120*a
**4 + 4*a**3*b*m**4*x + 52*a**3*b*m**3*x + 236*a**3*b*m**2*x + 428*a**3*b*
m*x + 240*a**3*b*x + 6*a**2*b**2*m**4*x**2 + 72*a**2*b**2*m**3*x**2 + 294*
a**2*b**2*m**2*x**2 + 468*a**2*b**2*m*x**2 + 240*a**2*b**2*x**2 + 4*a*b**3
*m**4*x**3 + 44*a*b**3*m**3*x**3 + 164*a*b**3*m**2*x**3 + 244*a*b**3*m*x**
3 + 120*a*b**3*x**3 + b**4*m**4*x**4 + 10*b**4*m**3*x**4 + 35*b**4*m**2*x*
*4 + 50*b**4*m*x**4 + 24*b**4*x**4))/(m**5 + 15*m**4 + 85*m**3 + 225*m**2
+ 274*m + 120)
```

3.772 $\int (cx)^m(a+bx)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int (cx)^m(a+bx)^3\,dx=\frac{a^3(cx)^{1+m}}{c(1+m)}+\frac{3a^2b(cx)^{2+m}}{c^2(2+m)}+\frac{3ab^2(cx)^{3+m}}{c^3(3+m)}+\frac{b^3(cx)^{4+m}}{c^4(4+m)}$$

output
```
a^3*(c*x)^(1+m)/c/(1+m)+3*a^2*b*(c*x)^(2+m)/c^2/(2+m)+3*a*b^2*(c*x)^(3+m)/
c^3/(3+m)+b^3*(c*x)^(4+m)/c^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int (cx)^m(a+bx)^3\,dx=x(cx)^m\left(\frac{a^3}{1+m}+\frac{3a^2bx}{2+m}+\frac{3ab^2x^2}{3+m}+\frac{b^3x^3}{4+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x)^3,x]
```

output
```
x*(c*x)^m*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^
3*x^3)/(4 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(cx)^m\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(a^3(cx)^m+\frac{3a^2b(cx)^{m+1}}{c}+\frac{3ab^2(cx)^{m+2}}{c^2}+\frac{b^3(cx)^{m+3}}{c^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3(cx)^{m+1}}{c(m+1)}+\frac{3a^2b(cx)^{m+2}}{c^2(m+2)}+\frac{3ab^2(cx)^{m+3}}{c^3(m+3)}+\frac{b^3(cx)^{m+4}}{c^4(m+4)}$$

input
```
Int[(c*x)^m*(a + b*x)^3,x]
```

output
```
(a^3*(c*x)^(1 + m))/(c*(1 + m)) + (3*a^2*b*(c*x)^(2 + m))/(c^2*(2 + m)) +
(3*a*b^2*(c*x)^(3 + m))/(c^3*(3 + m)) + (b^3*(c*x)^(4 + m))/(c^4*(4 + m))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^3 x\, e^{m \ln(cx)}}{1+m}+\frac{b^3 x^4 e^{m \ln(cx)}}{4+m}+\frac{3a\, b^2 x^3 e^{m \ln(cx)}}{3+m}+\frac{3a^2 b\, x^2 e^{m \ln(cx)}}{2+m}$
gosper	$\frac{x\left(b^3 m^3 x^3+3a\, b^2 m^3 x^2+6b^3 m^2 x^3+3a^2 b\, m^3 x+21a\, b^2 m^2 x^2+11m\, x^3 b^3+a^3 m^3+24a^2 b\, m^2 x+42m\, x^2 a\, b^2+6b^3 x^3+9a^3 m^2+57}{(4+m)(3+m)(2+m)(1+m)}$
risch	$\frac{x\left(b^3 m^3 x^3+3a\, b^2 m^3 x^2+6b^3 m^2 x^3+3a^2 b\, m^3 x+21a\, b^2 m^2 x^2+11m\, x^3 b^3+a^3 m^3+24a^2 b\, m^2 x+42m\, x^2 a\, b^2+6b^3 x^3+9a^3 m^2+57}{(4+m)(3+m)(2+m)(1+m)}$
orering	$\frac{x\left(b^3 m^3 x^3+3a\, b^2 m^3 x^2+6b^3 m^2 x^3+3a^2 b\, m^3 x+21a\, b^2 m^2 x^2+11m\, x^3 b^3+a^3 m^3+24a^2 b\, m^2 x+42m\, x^2 a\, b^2+6b^3 x^3+9a^3 m^2+57}{(4+m)(3+m)(2+m)(1+m)}$
parallelrisch	$\frac{x^4 (cx)^m b^3 m^3+6x^4 (cx)^m b^3 m^2+3x^3 (cx)^m a\, b^2 m^3+11x^4 (cx)^m b^3 m+21x^3 (cx)^m a\, b^2 m^2+3x^2 (cx)^m a^2 b\, m^3+6x^4 (cx)^m b^3+42}{(4}$

input
```
int((c*x)^m*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/(1+m)*x*exp(m*ln(c*x))+b^3/(4+m)*x^4*exp(m*ln(c*x))+3*a*b^2/(3+m)*x^3*
exp(m*ln(c*x))+3*a^2*b/(2+m)*x^2*exp(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.96

$$\int (cx)^m (a+bx)^3\, dx = \frac{((b^3 m^3+6\, b^3 m^2+11\, b^3 m+6\, b^3)x^4+3\,(ab^2 m^3+7\, ab^2 m^2+14\, ab^2 m+8\, ab^2)x^3+3\,(a^2 bm^3+8\, a^2 bm^2+}{m^4+10\, m^3+35\, m^2+50\, m+24}$$

input
```
integrate((c*x)^m*(b*x+a)^3,x, algorithm="fricas")
```

output
```
((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 1
2*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*(c*x)^m/(m^4 +
 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(71) = 142$.

Time = 0.30 (sec) , antiderivative size = 702, normalized size of antiderivative = 8.67

$$\int (cx)^m (a+bx)^3\,dx$$
$$= \begin{cases} \frac{-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)}{c^4} \\ \frac{-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x}{c^3} \\ \frac{-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}}{c^2} \\ \frac{a^3\log(x)+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3}}{c} \\ \frac{a^3m^3x(cx)^m}{m^4+10m^3+35m^2+50m+24}+\frac{9a^3m^2x(cx)^m}{m^4+10m^3+35m^2+50m+24}+\frac{26a^3mx(cx)^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^3x(cx)^m}{m^4+10m^3+35m^2+50m+24}+\frac{3}{m^4+10} \end{cases}$$

input
```
integrate((c*x)**m*(b*x+a)**3,x)
```

output
```
Piecewise(((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x))
/c**4, Eq(m, -4)), ((-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*
x)/c**3, Eq(m, -3)), ((-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/
2)/c**2, Eq(m, -2)), ((a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x
**3/3)/c, Eq(m, -1)), (a**3*m**3*x*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50
*m + 24) + 9*a**3*m**2*x*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 26*a**3*m*x*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**3*x*(
c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 3*a**2*b*m**3*x**2*(c*x)*
*m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b*m**2*x**2*(c*x)**m/(
m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*(c*x)**m/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*(c*x)**m/(m**4 + 10*m**3 +
 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*(c*x)**m/(m**4 + 10*m**3 + 35*m
**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*(c*x)**m/(m**4 + 10*m**3 + 35*m**2
+ 50*m + 24) + 42*a*b**2*m*x**3*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m
+ 24) + 24*a*b**2*x**3*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b
**3*m**3*x**4*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**
2*x**4*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*(c
*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*x**4*(c*x)**m/(m**4
 + 10*m**3 + 35*m**2 + 50*m + 24), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int (cx)^m (a+bx)^3\,dx = \frac{b^3c^mx^4x^m}{m+4} + \frac{3\,ab^2c^mx^3x^m}{m+3} + \frac{3\,a^2bc^mx^2x^m}{m+2} + \frac{(cx)^{m+1}\,a^3}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x+a)^3,x, algorithm="maxima")
```

output
```
b^3*c^m*x^4*x^m/(m + 4) + 3*a*b^2*c^m*x^3*x^m/(m + 3) + 3*a^2*b*c^m*x^2*x^
m/(m + 2) + (c*x)^(m + 1)*a^3/(c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(81) = 162.

Time = 0.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.16

$$\int (cx)^m (a+bx)^3\,dx$$
$$= \frac{(cx)^m\,b^3m^3x^4 + 3\,(cx)^m\,ab^2m^3x^3 + 6\,(cx)^m\,b^3m^2x^4 + 3\,(cx)^m\,a^2bm^3x^2 + 21\,(cx)^m\,ab^2m^2x^3 + 11\,(cx)^m\,b^3}{}$$

input
```
integrate((c*x)^m*(b*x+a)^3,x, algorithm="giac")
```

output
```
((c*x)^m*b^3*m^3*x^4 + 3*(c*x)^m*a*b^2*m^3*x^3 + 6*(c*x)^m*b^3*m^2*x^4 + 3
*(c*x)^m*a^2*b*m^3*x^2 + 21*(c*x)^m*a*b^2*m^2*x^3 + 11*(c*x)^m*b^3*m*x^4 +
 (c*x)^m*a^3*m^3*x + 24*(c*x)^m*a^2*b*m^2*x^2 + 42*(c*x)^m*a*b^2*m*x^3 + 6
*(c*x)^m*b^3*x^4 + 9*(c*x)^m*a^3*m^2*x + 57*(c*x)^m*a^2*b*m*x^2 + 24*(c*x)
^m*a*b^2*x^3 + 26*(c*x)^m*a^3*m*x + 36*(c*x)^m*a^2*b*x^2 + 24*(c*x)^m*a^3*
x)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.09

$$\int (cx)^m(a+bx)^3\,dx=(c\,x)^m\left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

input
```
int((c*x)^m*(a + b*x)^3,x)
```

output
```
(c*x)^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 +
 24) + (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 +
24) + (3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4
 + 24) + (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 +
 m^4 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int (cx)^m(a+bx)^3\,dx = \frac{x^m c^m x(b^3m^3x^3+3a\,b^2m^3x^2+6b^3m^2x^3+3a^2b\,m^3x+21a\,b^2m^2x^2+11b^3m\,x^3+a^3m^3+24a^2b\,m^2x+42}{m^4+10m^3+35m^2+50m+}$$

input
```
int((c*x)^m*(b*x+a)^3,x)
```

output
```
(x**m*c**m*x*(a**3*m**3 + 9*a**3*m**2 + 26*a**3*m + 24*a**3 + 3*a**2*b*m**
3*x + 24*a**2*b*m**2*x + 57*a**2*b*m*x + 36*a**2*b*x + 3*a*b**2*m**3*x**2
+ 21*a*b**2*m**2*x**2 + 42*a*b**2*m*x**2 + 24*a*b**2*x**2 + b**3*m**3*x**3
 + 6*b**3*m**2*x**3 + 11*b**3*m*x**3 + 6*b**3*x**3))/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24)
```

3.773 $\int (cx)^m (a+bx)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int (cx)^m (a+bx)^2 \, dx = \frac{a^2 (cx)^{1+m}}{c(1+m)} + \frac{2ab(cx)^{2+m}}{c^2(2+m)} + \frac{b^2 (cx)^{3+m}}{c^3(3+m)}$$

output
```
a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(2+m)/c^2/(2+m)+b^2*(c*x)^(3+m)/c^3/(3
+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int (cx)^m (a+bx)^2 \, dx = x(cx)^m \left(\frac{a^2}{1+m} + \frac{2abx}{2+m} + \frac{b^2 x^2}{3+m} \right)$$

input
```
Integrate[(c*x)^m*(a + b*x)^2,x]
```

output
```
x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(cx)^m\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(a^2(cx)^m+\frac{2ab(cx)^{m+1}}{c}+\frac{b^2(cx)^{m+2}}{c^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2ab(cx)^{m+2}}{c^2(m+2)}+\frac{b^2(cx)^{m+3}}{c^3(m+3)}$$

input
```
Int[(c*x)^m*(a + b*x)^2,x]
```

output
```
(a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(2 + m))/(c^2*(2 + m)) + (b
^2*(c*x)^(3 + m))/(c^3*(3 + m))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
norman	$\frac{a^2 x\, \mathrm{e}^{m\ln(cx)}}{1+m}+\frac{b^2 x^3 \mathrm{e}^{m\ln(cx)}}{3+m}+\frac{2ab\, x^2 \mathrm{e}^{m\ln(cx)}}{2+m}$
gosper	$\frac{x(b^2m^2x^2+2ab\,m^2x+3m\,x^2b^2+a^2m^2+8mxab+2b^2x^2+5m\,a^2+6abx+6a^2)(cx)^m}{(3+m)(2+m)(1+m)}$
risch	$\frac{x(b^2m^2x^2+2ab\,m^2x+3m\,x^2b^2+a^2m^2+8mxab+2b^2x^2+5m\,a^2+6abx+6a^2)(cx)^m}{(3+m)(2+m)(1+m)}$
orering	$\frac{x(b^2m^2x^2+2ab\,m^2x+3m\,x^2b^2+a^2m^2+8mxab+2b^2x^2+5m\,a^2+6abx+6a^2)(cx)^m}{(3+m)(2+m)(1+m)}$
parallelrisch	$\frac{x^3(cx)^m b^2m^2+3x^3(cx)^m b^2m+2x^2(cx)^m ab\,m^2+2x^3(cx)^m b^2+8x^2(cx)^m abm+x(cx)^m a^2m^2+6x^2(cx)^m ab+5x(cx)^m a^2m+6}{(3+m)(2+m)(1+m)}$

input
```
int((c*x)^m*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*exp(m*ln(c*x))+b^2/(3+m)*x^3*exp(m*ln(c*x))+2*a*b/(2+m)*x^2*ex
p(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int (cx)^m(a+bx)^2\,dx = \frac{((b^2m^2+3\,b^2m+2\,b^2)x^3+2\,(abm^2+4\,abm+3\,ab)x^2+(a^2m^2+5\,a^2m+6\,a^2)x)(cx)^m}{m^3+6\,m^2+11\,m+6}$$

input
```
integrate((c*x)^m*(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^
2*m^2 + 5*a^2*m + 6*a^2)*x)*(c*x)^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(49) = 98$.

Time = 0.23 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.57

$$\int (cx)^m (a+bx)^2 \, dx = \begin{cases} \frac{-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)}{c^3} \\ \frac{-\frac{a^2}{x} + 2ab \log(x) + b^2 x}{c^2} \\ \frac{a^2 \log(x) + 2abx + \frac{b^2x^2}{2}}{c} \\ \frac{a^2m^2x(cx)^m}{m^3+6m^2+11m+6} + \frac{5a^2mx(cx)^m}{m^3+6m^2+11m+6} + \frac{6a^2x(cx)^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2(cx)^m}{m^3+6m^2+11m+6} + \frac{8abmx^2(cx)^m}{m^3+6m^2+11m+6} + \frac{6abx^2(cx)^m}{m^3+6m^2+11m+6} + \end{cases}$$

input
```
integrate((c*x)**m*(b*x+a)**2,x)
```

output
```
Piecewise(((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x))/c**3, Eq(m, -3)), ((-a
**2/x + 2*a*b*log(x) + b**2*x)/c**2, Eq(m, -2)), ((a**2*log(x) + 2*a*b*x +
 b**2*x**2/2)/c, Eq(m, -1)), (a**2*m**2*x*(c*x)**m/(m**3 + 6*m**2 + 11*m +
 6) + 5*a**2*m*x*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a**2*x*(c*x)**m/(
m**3 + 6*m**2 + 11*m + 6) + 2*a*b*m**2*x**2*(c*x)**m/(m**3 + 6*m**2 + 11*m
 + 6) + 8*a*b*m*x**2*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a*b*x**2*(c*x
)**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3*(c*x)**m/(m**3 + 6*m**2 +
 11*m + 6) + 3*b**2*m*x**3*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 2*b**2*x*
*3*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (cx)^m (a+bx)^2 \, dx = \frac{b^2c^mx^3x^m}{m+3} + \frac{2\,abc^mx^2x^m}{m+2} + \frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x+a)^2,x, algorithm="maxima")
```

output
```
b^2*c^m*x^3*x^m/(m + 3) + 2*a*b*c^m*x^2*x^m/(m + 2) + (c*x)^(m + 1)*a^2/(c
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

$$\int (cx)^m (a+bx)^2 \, dx = \frac{(cx)^m b^2 m^2 x^3 + 2 (cx)^m abm^2 x^2 + 3 (cx)^m b^2 m x^3 + (cx)^m a^2 m^2 x + 8 (cx)^m abmx^2 + 2 (cx)^m b^2 x^3 + 5 (c}{m^3 + 6 m^2 + 11 m + 6}$$

input
```
integrate((c*x)^m*(b*x+a)^2,x, algorithm="giac")
```

output
```
((c*x)^m*b^2*m^2*x^3 + 2*(c*x)^m*a*b*m^2*x^2 + 3*(c*x)^m*b^2*m*x^3 + (c*x)
^m*a^2*m^2*x + 8*(c*x)^m*a*b*m*x^2 + 2*(c*x)^m*b^2*x^3 + 5*(c*x)^m*a^2*m*x
 + 6*(c*x)^m*a*b*x^2 + 6*(c*x)^m*a^2*x)/(m^3 + 6*m^2 + 11*m + 6)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (cx)^m (a+bx)^2 \, dx = (c\,x)^m \left(\frac{a^2\,x\,(m^2+5\,m+6)}{m^3+6\,m^2+11\,m+6} + \frac{b^2\,x^3\,(m^2+3\,m+2)}{m^3+6\,m^2+11\,m+6} + \frac{2\,a\,b\,x^2\,(m^2+4\,m+3)}{m^3+6\,m^2+11\,m+6}\right)$$

input
```
int((c*x)^m*(a + b*x)^2,x)
```

output
```
(c*x)^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m
+ m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m +
 6*m^2 + m^3 + 6))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int (cx)^m (a+bx)^2 \, dx = \frac{x^m c^m x (b^2 m^2 x^2 + 2ab\, m^2 x + 3b^2 m\, x^2 + a^2 m^2 + 8abmx + 2b^2 x^2 + 5a^2 m + 6abx + 6a^2)}{m^3 + 6m^2 + 11m + 6}$$

input
```
int((c*x)^m*(b*x+a)^2,x)
```

output
```
(x**m*c**m*x*(a**2*m**2 + 5*a**2*m + 6*a**2 + 2*a*b*m**2*x + 8*a*b*m*x + 6
*a*b*x + b**2*m**2*x**2 + 3*b**2*m*x**2 + 2*b**2*x**2))/(m**3 + 6*m**2 + 1
1*m + 6)
```

3.774 $\int (cx)^m(a+bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int (cx)^m(a+bx)\,dx=\frac{a(cx)^{1+m}}{c(1+m)}+\frac{b(cx)^{2+m}}{c^2(2+m)}$$

output
```
a*(c*x)^(1+m)/c/(1+m)+b*(c*x)^(2+m)/c^2/(2+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int (cx)^m(a+bx)\,dx=x(cx)^m\left(\frac{a}{1+m}+\frac{bx}{2+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x),x]
```

output
```
x*(c*x)^m*(a/(1 + m) + (b*x)/(2 + m))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(cx)^m\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a(cx)^m+\frac{b(cx)^{m+1}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(cx)^{m+1}}{c(m+1)}+\frac{b(cx)^{m+2}}{c^2(m+2)}$$

input
```
Int[(c*x)^m*(a + b*x),x]
```

output
```
(a*(c*x)^(1 + m))/(c*(1 + m)) + (b*(c*x)^(2 + m))/(c^2*(2 + m))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{x(bmx+am+bx+2a)(cx)^m}{(2+m)(1+m)}$	32
risch	$\frac{x(bmx+am+bx+2a)(cx)^m}{(2+m)(1+m)}$	32
orering	$\frac{x(bmx+am+bx+2a)(cx)^m}{(2+m)(1+m)}$	32
norman	$\frac{ax\,e^{m\ln(cx)}}{1+m}+\frac{b\,x^2 e^{m\ln(cx)}}{2+m}$	34
parallelrisch	$\frac{x^2(cx)^m bm+x^2(cx)^m b+x(cx)^m am+2x(cx)^m a}{(2+m)(1+m)}$	52

input
```
int((c*x)^m*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x+a*m+b*x+2*a)*(c*x)^m/(2+m)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (cx)^m(a+bx)\,dx = \frac{((bm+b)x^2+(am+2\,a)x)(cx)^m}{m^2+3\,m+2}$$

input
```
integrate((c*x)^m*(b*x+a),x, algorithm="fricas")
```

output
```
((b*m + b)*x^2 + (a*m + 2*a)*x)*(c*x)^m/(m^2 + 3*m + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(27) = 54$.

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.83

$$\int (cx)^m(a+bx)\,dx = \begin{cases} \frac{-\frac{a}{x}+b\log{(x)}}{c^2} & \text{for}\ m=-2 \\ \frac{a\log{(x)}+bx}{c} & \text{for}\ m=-1 \\ \frac{amx(cx)^m}{m^2+3m+2}+\frac{2ax(cx)^m}{m^2+3m+2}+\frac{bmx^2(cx)^m}{m^2+3m+2}+\frac{bx^2(cx)^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(b*x+a),x)
```

output
```
Piecewise(((-a/x + b*log(x))/c**2, Eq(m, -2)), ((a*log(x) + b*x)/c, Eq(m,
-1)), (a*m*x*(c*x)**m/(m**2 + 3*m + 2) + 2*a*x*(c*x)**m/(m**2 + 3*m + 2) +
 b*m*x**2*(c*x)**m/(m**2 + 3*m + 2) + b*x**2*(c*x)**m/(m**2 + 3*m + 2), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (cx)^m(a+bx)\,dx = \frac{bc^m x^2 x^m}{m+2} + \frac{(cx)^{m+1}a}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x+a),x, algorithm="maxima")
```

output
```
b*c^m*x^2*x^m/(m + 2) + (c*x)^(m + 1)*a/(c*(m + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int (cx)^m(a+bx)\,dx = \frac{(cx)^m\,bmx^2+(cx)^m\,amx+(cx)^m\,bx^2+2\,(cx)^m\,ax}{m^2+3\,m+2}$$

input
```
integrate((c*x)^m*(b*x+a),x, algorithm="giac")
```

output
```
((c*x)^m*b*m*x^2 + (c*x)^m*a*m*x + (c*x)^m*b*x^2 + 2*(c*x)^m*a*x)/(m^2 + 3
*m + 2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (cx)^m(a+bx)\,dx = \frac{x\,(c\,x)^m\,(2\,a+a\,m+b\,x+b\,m\,x)}{m^2+3\,m+2}$$

input
```
int((c*x)^m*(a + b*x),x)
```

output
```
(x*(c*x)^m*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (cx)^m(a+bx)\,dx = \frac{x^m c^m x(bmx+am+bx+2a)}{m^2+3m+2}$$

input
```
int((c*x)^m*(b*x+a),x)
```

output
```
(x**m*c**m*x*(a*m + 2*a + b*m*x + b*x))/(m**2 + 3*m + 2)
```

3.775 $\int \frac{(cx)^m}{a+bx} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(cx)^m}{a+bx} dx = \frac{(cx)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)}{ac(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([1, 1+m],[2+m],-b*x/a)/a/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^m}{a+bx} dx = \frac{x(cx)^m \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)}{a(1+m)}$$

input
```
Integrate[(c*x)^m/(a + b*x),x]
```

output
```
(x*(c*x)^m*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{a+bx} \, dx$$
$$\Big\downarrow \; 74$$
$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{ac(m+1)}$$

input
```
Int[(c*x)^m/(a + b*x),x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*c*(1 + m
))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{bx+a} dx$$

input
```
int((c*x)^m/(b*x+a),x)
```

output
```
int((c*x)^m/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{a+bx}\,dx = \int \frac{(cx)^m}{bx+a}\,dx$$

input
```
integrate((c*x)^m/(b*x+a),x, algorithm="fricas")
```

output
```
integral((c*x)^m/(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{(cx)^m}{a+bx}\,dx = \frac{c^m m x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)} + \frac{c^m x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)}$$

input
```
integrate((c*x)**m/(b*x+a),x)
```

output
```
c**m*m*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(
a*gamma(m + 2)) + c**m*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1
)*gamma(m + 1)/(a*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(cx)^m}{a+bx}\,dx = \int \frac{(cx)^m}{bx+a}\,dx$$

input
```
integrate((c*x)^m/(b*x+a),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x + a), x)
```

Giac [F]

$$\int \frac{(cx)^m}{a+bx}\,dx = \int \frac{(cx)^m}{bx+a}\,dx$$

input
```
integrate((c*x)^m/(b*x+a),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{a+bx}\,dx = \int \frac{(c\,x)^m}{a+b\,x}\,dx$$

input
```
int((c*x)^m/(a + b*x),x)
```

output
```
int((c*x)^m/(a + b*x), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{a+bx}\,dx = \frac{c^m\left(x^m - \left(\int \frac{x^m}{b\,x^2+ax}dx\right)am\right)}{bm}$$

input
```
int((c*x)^m/(b*x+a),x)
```

output
```
(c**m*(x**m - int(x**m/(a*x + b*x**2),x)*a*m))/(b*m)
```

3.776 $\int \frac{(cx)^m}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(cx)^m}{(a+bx)^2} \, dx = \frac{(cx)^{1+m} \operatorname{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{a}\right)}{a^2c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([2, 1+m],[2+m],-b*x/a)/a^2/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^m}{(a+bx)^2} \, dx = \frac{x(cx)^m \operatorname{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[(c*x)^m/(a + b*x)^2,x]
```

output
```
(x*(c*x)^m*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(a+bx)^2} \, dx$$
$$\Big\downarrow \text{ 74}$$
$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{bx}{a}\right)}{a^2 c(m+1)}$$

input
```
Int[(c*x)^m/(a + b*x)^2,x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*c*(1 +
 m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{(bx+a)^2}dx$$

input
```
int((c*x)^m/(b*x+a)^2,x)
```

output
```
int((c*x)^m/(b*x+a)^2,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(a+bx)^2}\,dx = \int \frac{(cx)^m}{(bx+a)^2}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((c*x)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 282, normalized size of antiderivative = 8.29

$$\int \frac{(cx)^m}{(a+bx)^2}\,dx = -\frac{ac^m m^2 x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} - \frac{ac^m m x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} + \frac{ac^m m x^{m+1}\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} + \frac{ac^m x^{m+1}\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} - \frac{bc^m m^2 x x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} - \frac{bc^m m x x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)}$$

input
```
integrate((c*x)**m/(b*x+a)**2,x)
```

output
```
-a*c**m*m**2*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m
+ 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - a*c**m*m*x**(m + 1)*ler
chphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a
**2*b*x*gamma(m + 2)) + a*c**m*m*x**(m + 1)*gamma(m + 1)/(a**3*gamma(m + 2
) + a**2*b*x*gamma(m + 2)) + a*c**m*x**(m + 1)*gamma(m + 1)/(a**3*gamma(m
+ 2) + a**2*b*x*gamma(m + 2)) - b*c**m*m**2*x*x**(m + 1)*lerchphi(b*x*exp_
polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(
m + 2)) - b*c**m*m*x*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*
gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(cx)^m}{(a+bx)^2} \, dx = \int \frac{(cx)^m}{(bx+a)^2} \, dx$$

input
```
integrate((c*x)^m/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x + a)^2, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(a+bx)^2} \, dx = \int \frac{(cx)^m}{(bx+a)^2} \, dx$$

input
```
integrate((c*x)^m/(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(a+bx)^2}\,dx = \int \frac{(c\,x)^m}{(a+b\,x)^2}\,dx$$

input
```
int((c*x)^m/(a + b*x)^2,x)
```

output
```
int((c*x)^m/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(a+bx)^2}\,dx = \frac{c^m\left(x^m - \left(\int \frac{x^m}{b^2m\,x^3+2abm\,x^2-b^2x^3+a^2mx-2ab\,x^2-a^2x}dx\right)a^2m^2 + \left(\int \frac{x^m}{b^2m\,x^3+2abm\,x^2-b^2x^3+a^2mx-2ab\,x^2-a^2x}dx\right)a^2m\right.}{b\,(bmx+am}$$

input
```
int((c*x)^m/(b*x+a)^2,x)
```

output
```
(c**m*(x**m - int(x**m/(a**2*m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + b*
*2*m*x**3 - b**2*x**3),x)*a**2*m**2 + int(x**m/(a**2*m*x - a**2*x + 2*a*b*
m*x**2 - 2*a*b*x**2 + b**2*m*x**3 - b**2*x**3),x)*a**2*m - int(x**m/(a**2*
m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**3 - b**2*x**3),x)*a*b
*m**2*x + int(x**m/(a**2*m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m
*x**3 - b**2*x**3),x)*a*b*m*x))/(b*(a*m - a + b*m*x - b*x))
```

3.777 $\int \frac{(cx)^m}{(a+bx)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(cx)^m}{(a+bx)^3} dx = \frac{(cx)^{1+m} \text{Hypergeometric2F1}\left(3, 1+m, 2+m, -\frac{bx}{a}\right)}{a^3c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([3, 1+m],[2+m],-b*x/a)/a^3/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^m}{(a+bx)^3} dx = \frac{x(cx)^m \text{Hypergeometric2F1}\left(3, 1+m, 2+m, -\frac{bx}{a}\right)}{a^3(1+m)}$$

input
```
Integrate[(c*x)^m/(a + b*x)^3,x]
```

output
```
(x*(c*x)^m*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(a+bx)^3} \, dx$$
$$\Big\downarrow \; 74$$
$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(3, m+1, m+2, -\frac{bx}{a}\right)}{a^3 c(m+1)}$$

input
```
Int[(c*x)^m/(a + b*x)^3,x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*c*(1 +
 m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [F]

$$\int \frac{(cx)^m}{(bx+a)^3}dx$$

input
```
int((c*x)^m/(b*x+a)^3,x)
```

output
```
int((c*x)^m/(b*x+a)^3,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(a+bx)^3}\,dx = \int \frac{(cx)^m}{(bx+a)^3}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((c*x)^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 758, normalized size of antiderivative = 22.29

$$\int \frac{(cx)^m}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**m/(b*x+a)**3,x)
```

output
```
a**2*c**m*m**3*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(
m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*g
amma(m + 2)) - a**2*c**m*m**2*x**(m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2)
 + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a**2*c**m*m*
x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*
gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) +
a**2*c**m*m*x**(m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamm
a(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + 2*a**2*c**m*x**(m + 1)*gamma(m
 + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*ga
mma(m + 2)) + 2*a*b*c**m*m**3*x*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a,
 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2
*a**3*b**2*x**2*gamma(m + 2)) - a*b*c**m*m**2*x*x**(m + 1)*gamma(m + 1)/(2
*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m +
2)) - 2*a*b*c**m*m*x*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*
gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*
x**2*gamma(m + 2)) + a*b*c**m*x*x**(m + 1)*gamma(m + 1)/(2*a**5*gamma(m +
2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + b**2*c**m*
m**3*x**2*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1
)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(
m + 2)) - b**2*c**m*m*x**2*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1...
```

Maxima [F]

$$\int \frac{(cx)^m}{(a+bx)^3}\,dx = \int \frac{(cx)^m}{(bx+a)^3}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x + a)^3, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(a+bx)^3}\,dx = \int \frac{(cx)^m}{(bx+a)^3}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(a+bx)^3}\,dx = \int \frac{(c\,x)^m}{(a+b\,x)^3}\,dx$$

input
```
int((c*x)^m/(a + b*x)^3,x)
```

output
```
int((c*x)^m/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(a+bx)^3}\,dx = \frac{c^m \left(x^m - \left(\int \frac{x^m}{b^3m\,x^4+3a\,b^2m\,x^3-2b^3x^4+3a^2bm\,x^2-6a\,b^2x^3+a^3mx-6a^2b\,x^2-2a^3x}dx\right) a^3m^2 + 2\left(\int \frac{}{b^3m\,x^4+3a\,b^2m\,x^3-2b^3x^4+3a}\right.\right.}{}$$

input
```
int((c*x)^m/(b*x+a)^3,x)
```

output
```
(c**m*(x**m - int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x
**2 + 3*a*b**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**3
*m**2 + 2*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2
+ 3*a*b**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**3*m -
 2*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b
**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**2*b*m**2*x +
 4*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b
**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**2*b*m*x - in
t(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b**2*m
*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a*b**2*m**2*x**2 + 2
*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b**
2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a*b**2*m*x**2))/(
b*(a**2*m - 2*a**2 + 2*a*b*m*x - 4*a*b*x + b**2*m*x**2 - 2*b**2*x**2))
```

3.778 $\int (cx)^m(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int (cx)^m(a+bx)^{3/2}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m(a+bx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-m,\frac{7}{2},1+\frac{bx}{a}\right)}{5b}$$

output
```
2/5*(c*x)^m*(b*x+a)^(5/2)*hypergeom([5/2, -m],[7/2],1+b*x/a)/b/((-b*x/a)^m
)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (cx)^m(a+bx)^{3/2}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m(a+bx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-m,\frac{7}{2},1+\frac{bx}{a}\right)}{5b}$$

input
```
Integrate[(c*x)^m*(a + b*x)^(3/2),x]
```

output
```
(2*(c*x)^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(
5*b*(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}(cx)^m\,dx$$

$$\Big\downarrow \text{77}$$

$$(cx)^m\left(-\frac{bx}{a}\right)^{-m}\int \left(-\frac{bx}{a}\right)^m(a+bx)^{3/2}dx$$

$$\Big\downarrow \text{75}$$

$$\frac{2(a+bx)^{5/2}(cx)^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{5}{2},-m,\frac{7}{2},\frac{bx}{a}+1\right)}{5b}$$

input
```
Int[(c*x)^m*(a + b*x)^(3/2),x]
```

output
```
(2*(c*x)^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(
5*b*(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int (cx)^m (bx+a)^{\frac{3}{2}} dx$$

input
```
int((c*x)^m*(b*x+a)^(3/2),x)
```

output
```
int((c*x)^m*(b*x+a)^(3/2),x)
```

Fricas [F]

$$\int (cx)^m (a+bx)^{3/2} dx = \int (bx+a)^{\frac{3}{2}} (cx)^m dx$$

input
```
integrate((c*x)^m*(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/2)*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int (cx)^m (a+bx)^{3/2} dx = \frac{a^{\frac{3}{2}} c^m x^{m+1} \Gamma(m+1) \, {}_2F_1\left(\begin{matrix} -\frac{3}{2}, m+1 \\ m+2 \end{matrix} \middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

input
```
integrate((c*x)**m*(b*x+a)**(3/2),x)
```

output
```
a**(3/2)*c**m*x**(m + 1)*gamma(m + 1)*hyper((-3/2, m + 1), (m + 2,), b*x*e
xp_polar(I*pi)/a)/gamma(m + 2)
```

Maxima [F]

$$\int (cx)^m (a+bx)^{3/2} \, dx = \int (bx+a)^{\frac{3}{2}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m (a+bx)^{3/2} \, dx = \int (bx+a)^{\frac{3}{2}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m (a+bx)^{3/2} \, dx = \int (c\,x)^m\,(a+b\,x)^{3/2} \, dx$$

input
```
int((c*x)^m*(a + b*x)^(3/2),x)
```

output
```
int((c*x)^m*(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int (cx)^m(a+bx)^{3/2}\,dx = \frac{2c^m\left(3x^m\sqrt{bx+a}\,a^2+4x^m\sqrt{bx+a}\,ab\,m^2x+14x^m\sqrt{bx+a}\,abmx+6x^m\sqrt{bx+a}\,abx+4x^m\right.}{}$$

input
```
int((c*x)^m*(b*x+a)^(3/2),x)
```

output
```
(2*c**m*(3*x**m*sqrt(a + b*x)*a**2 + 4*x**m*sqrt(a + b*x)*a*b*m**2*x + 14*
x**m*sqrt(a + b*x)*a*b*m*x + 6*x**m*sqrt(a + b*x)*a*b*x + 4*x**m*sqrt(a +
b*x)*b**2*m**2*x**2 + 8*x**m*sqrt(a + b*x)*b**2*m*x**2 + 3*x**m*sqrt(a + b
*x)*b**2*x**2 - 24*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46
*a*m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2
),x)*a**3*m**4 - 108*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x +
46*a*m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x*
*2),x)*a**3*m**3 - 138*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x
+ 46*a*m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*
x**2),x)*a**3*m**2 - 45*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x
 + 46*a*m*x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b
*x**2),x)*a**3*m))/(b*(8*m**3 + 36*m**2 + 46*m + 15))
```

3.779 $\int (cx)^m \sqrt{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int (cx)^m \sqrt{a+bx}\,dx = \frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m(a+bx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-m,\frac{5}{2},1+\frac{bx}{a}\right)}{3b}$$

output
```
2/3*(c*x)^m*(b*x+a)^(3/2)*hypergeom([3/2, -m],[5/2],1+b*x/a)/b/((-b*x/a)^m
)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (cx)^m \sqrt{a+bx}\,dx = \frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m(a+bx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-m,\frac{5}{2},1+\frac{bx}{a}\right)}{3b}$$

input
```
Integrate[(c*x)^m*Sqrt[a + b*x],x]
```

output
```
(2*(c*x)^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(
3*b*(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(cx)^m\,dx$$

$$\Big\downarrow \text{77}$$

$$(cx)^m\left(-\frac{bx}{a}\right)^{-m}\int \left(-\frac{bx}{a}\right)^m\sqrt{a+bx}dx$$

$$\Big\downarrow \text{75}$$

$$\frac{2(a+bx)^{3/2}(cx)^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{3}{2},-m,\frac{5}{2},\frac{bx}{a}+1\right)}{3b}$$

input
```
Int[(c*x)^m*Sqrt[a + b*x],x]
```

output
```
(2*(c*x)^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(
3*b*(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int (cx)^m \sqrt{bx+a}dx$$

input
```
int((c*x)^m*(b*x+a)^(1/2),x)
```

output
```
int((c*x)^m*(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int (cx)^m\sqrt{a+bx}\,dx = \int \sqrt{bx+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x + a)*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int (cx)^m \sqrt{a+bx}\,dx = \frac{\sqrt{a}c^m x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

input
```
integrate((c*x)**m*(b*x+a)**(1/2),x)
```

output
```
sqrt(a)*c**m*x**(m + 1)*gamma(m + 1)*hyper((-1/2, m + 1), (m + 2,), b*x*ex
p_polar(I*pi)/a)/gamma(m + 2)
```

Maxima [F]

$$\int (cx)^m \sqrt{a+bx}\,dx = \int \sqrt{bx+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x + a)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \sqrt{a+bx}\,dx = \int \sqrt{bx+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x + a)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m\sqrt{a+bx}\,dx=\int (c\,x)^m\,\sqrt{a+b\,x}\,dx$$

input
```
int((c*x)^m*(a + b*x)^(1/2),x)
```

output
```
int((c*x)^m*(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int (cx)^m\sqrt{a+bx}\,dx$$
$$=\frac{2c^m\left(x^m\sqrt{bx+a}\,a+2x^m\sqrt{bx+a}\,bmx+x^m\sqrt{bx+a}\,bx-4\left(\int\frac{x^m\sqrt{bx+a}}{4b\,m^2x^2+4a\,m^2x+8bm\,x^2+8amx+3b\,x^2+3ax}dx\right)a^{\cdots}\right.}{b\,(4m^2\cdots}$$

input
```
int((c*x)^m*(b*x+a)^(1/2),x)
```

output
```
(2*c**m*(x**m*sqrt(a + b*x)*a + 2*x**m*sqrt(a + b*x)*b*m*x + x**m*sqrt(a +
 b*x)*b*x - 4*int((x**m*sqrt(a + b*x))/(4*a*m**2*x + 8*a*m*x + 3*a*x + 4*b
*m**2*x**2 + 8*b*m*x**2 + 3*b*x**2),x)*a**2*m**3 - 8*int((x**m*sqrt(a + b*
x))/(4*a*m**2*x + 8*a*m*x + 3*a*x + 4*b*m**2*x**2 + 8*b*m*x**2 + 3*b*x**2)
,x)*a**2*m**2 - 3*int((x**m*sqrt(a + b*x))/(4*a*m**2*x + 8*a*m*x + 3*a*x +
 4*b*m**2*x**2 + 8*b*m*x**2 + 3*b*x**2),x)*a**2*m))/(b*(4*m**2 + 8*m + 3))
```

3.780 $\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx = \frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*(c*x)^m*(b*x+a)^(1/2)*hypergeom([1/2, -m],[3/2],1+b*x/a)/b/((-b*x/a)^m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx = \frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

input
```
Integrate[(c*x)^m/Sqrt[a + b*x],x]
```

output
```
(2*(c*x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*
(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{a+bx}} \, dx$$
$$\Big\downarrow \ 77$$
$$(cx)^m \left(-\frac{bx}{a}\right)^{-m} \int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}dx$$
$$\Big\downarrow \ 75$$
$$\frac{2\sqrt{a+bx}(cx)^m \left(-\frac{bx}{a}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{bx}{a}+1\right)}{b}$$

input
```
Int[(c*x)^m/Sqrt[a + b*x],x]
```

output
```
(2*(c*x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*
(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{bx+a}}dx$$

input
```
int((c*x)^m/(b*x+a)^(1/2),x)
```

output
```
int((c*x)^m/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx = \int \frac{(cx)^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^m/sqrt(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx = \frac{c^m x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma(m+2)}$$

input
```
integrate((c*x)**m/(b*x+a)**(1/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/(sqrt(a)*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx = \int \frac{(cx)^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx = \int \frac{(cx)^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx = \int \frac{(c\,x)^m}{\sqrt{a+b\,x}}\,dx$$

input
```
int((c*x)^m/(a + b*x)^(1/2),x)
```

output
```
int((c*x)^m/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx}}\,dx$$
$$= \frac{2c^m\left(x^m\sqrt{bx+a}-2\left(\int \frac{x^m\sqrt{bx+a}}{2bm\,x^2+2amx+b\,x^2+ax}dx\right)a\,m^2-\left(\int \frac{x^m\sqrt{bx+a}}{2bm\,x^2+2amx+b\,x^2+ax}dx\right)am\right)}{b\,(2m+1)}$$

input
```
int((c*x)^m/(b*x+a)^(1/2),x)
```

output
```
(2*c**m*(x**m*sqrt(a + b*x) - 2*int((x**m*sqrt(a + b*x))/(2*a*m*x + a*x +
2*b*m*x**2 + b*x**2),x)*a*m**2 - int((x**m*sqrt(a + b*x))/(2*a*m*x + a*x +
 2*b*m*x**2 + b*x**2),x)*a*m))/(b*(2*m + 1))
```

3.781 $\int \frac{(cx)^m}{(a+bx)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{(cx)^m}{(a+bx)^{3/2}} \, dx = -\frac{2\left(-\frac{bx}{a}\right)^{-m} (cx)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},-m,\frac{1}{2},1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}$$

output
```
-2*(c*x)^m*hypergeom([-1/2, -m],[1/2],1+b*x/a)/b/((-b*x/a)^m)/(b*x+a)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{(a+bx)^{3/2}} \, dx = -\frac{2\left(-\frac{bx}{a}\right)^{-m} (cx)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},-m,\frac{1}{2},1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}$$

input
```
Integrate[(c*x)^m/(a + b*x)^(3/2),x]
```

output
```
(-2*(c*x)^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))
^m*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \; 77$$

$$(cx)^m \left(-\frac{bx}{a}\right)^{-m} \int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \; 75$$

$$-\frac{2(cx)^m \left(-\frac{bx}{a}\right)^{-m} \text{Hypergeometric2F1}\left(-\frac{1}{2},-m,\frac{1}{2},\frac{bx}{a}+1\right)}{b\sqrt{a+bx}}$$

input
```
Int[(c*x)^m/(a + b*x)^(3/2),x]
```

output
```
(-2*(c*x)^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))
^m*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{(cx)^m}{(bx+a)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^m/(b*x+a)^(3/2),x)
```

output
```
int((c*x)^m/(b*x+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(a+bx)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x + a)*(c*x)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{(cx)^m}{(a+bx)^{3/2}}\,dx = \frac{c^m x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{2}}\Gamma(m+2)}$$

input
```
integrate((c*x)**m/(b*x+a)**(3/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/(a**(3/2)*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(cx)^m}{(a+bx)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(a+bx)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(a+bx)^{3/2}}\,dx = \int \frac{(c\,x)^m}{(a+b\,x)^{3/2}}\,dx$$

input
```
int((c*x)^m/(a + b*x)^(3/2),x)
```

output
```
int((c*x)^m/(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(a+bx)^{3/2}}\,dx = \frac{2c^m\left(x^m\sqrt{bx+a}-2\left(\int \frac{x^m\sqrt{bx+a}}{2b^2m\,x^3+4abm\,x^2-b^2x^3+2a^2mx-2ab\,x^2-a^2x}dx\right)a^2m^2+\left(\int \frac{}{2b^2m\,x^3+4abm}\right.\right.}{}$$

input
```
int((c*x)^m/(b*x+a)^(3/2),x)
```

output
```
(2*c**m*(x**m*sqrt(a + b*x) - 2*int((x**m*sqrt(a + b*x))/(2*a**2*m*x - a**
2*x + 4*a*b*m*x**2 - 2*a*b*x**2 + 2*b**2*m*x**3 - b**2*x**3),x)*a**2*m**2
+ int((x**m*sqrt(a + b*x))/(2*a**2*m*x - a**2*x + 4*a*b*m*x**2 - 2*a*b*x**
2 + 2*b**2*m*x**3 - b**2*x**3),x)*a**2*m - 2*int((x**m*sqrt(a + b*x))/(2*a
**2*m*x - a**2*x + 4*a*b*m*x**2 - 2*a*b*x**2 + 2*b**2*m*x**3 - b**2*x**3),
x)*a*b*m**2*x + int((x**m*sqrt(a + b*x))/(2*a**2*m*x - a**2*x + 4*a*b*m*x*
*2 - 2*a*b*x**2 + 2*b**2*m*x**3 - b**2*x**3),x)*a*b*m*x))/(b*(2*a*m - a +
2*b*m*x - b*x))
```

3.782 $\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = -\frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m \,\text{Hypergeometric2F1}\left(-\frac{3}{2},-m,-\frac{1}{2},1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}$$

output
```
-2/3*(c*x)^m*hypergeom([-3/2, -m],[-1/2],1+b*x/a)/b/((-b*x/a)^m)/(b*x+a)^(
3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = -\frac{2\left(-\frac{bx}{a}\right)^{-m}(cx)^m \,\text{Hypergeometric2F1}\left(-\frac{3}{2},-m,-\frac{1}{2},1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}$$

input
```
Integrate[(c*x)^m/(a + b*x)^(5/2),x]
```

output
```
(-2*(c*x)^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/
a))^m*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(a+bx)^{5/2}} \, dx$$

$$\Big\downarrow \; 77$$

$$(cx)^m \left(-\frac{bx}{a}\right)^{-m} \int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \; 75$$

$$-\frac{2(cx)^m \left(-\frac{bx}{a}\right)^{-m} \text{Hypergeometric2F1}\left(-\frac{3}{2},-m,-\frac{1}{2},\frac{bx}{a}+1\right)}{3b(a+bx)^{3/2}}$$

input
```
Int[(c*x)^m/(a + b*x)^(5/2),x]
```

output
```
(-2*(c*x)^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/
a))^m*(a + b*x)^(3/2))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{(cx)^m}{(bx+a)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^m/(b*x+a)^(5/2),x)
```

output
```
int((c*x)^m/(b*x+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x + a)*(c*x)^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = \frac{c^m x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix} \frac{5}{2}, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{2}}\Gamma(m+2)}$$

input
```
integrate((c*x)**m/(b*x+a)**(5/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m + 1)*hyper((5/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/(a**(5/2)*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = \int \frac{(c\,x)^m}{(a+b\,x)^{5/2}}\,dx$$

input
```
int((c*x)^m/(a + b*x)^(5/2),x)
```

output
```
int((c*x)^m/(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(a+bx)^{5/2}}\,dx = \frac{2c^m\left(x^m\sqrt{bx+a}-2\left(\int \frac{x^m\sqrt{bx+a}}{2b^3m\,x^4+6a\,b^2m\,x^3-3b^3x^4+6a^2bm\,x^2-9a\,b^2x^3+2a^3mx-9a^2b\,x^2-3a^3x}dx\right)a^3m$$

input
```
int((c*x)^m/(b*x+a)^(5/2),x)
```

output
```
(2*c**m*(x**m*sqrt(a + b*x) - 2*int((x**m*sqrt(a + b*x))/(2*a**3*m*x - 3*a
**3*x + 6*a**2*b*m*x**2 - 9*a**2*b*x**2 + 6*a*b**2*m*x**3 - 9*a*b**2*x**3
+ 2*b**3*m*x**4 - 3*b**3*x**4),x)*a**3*m**2 + 3*int((x**m*sqrt(a + b*x))/(
2*a**3*m*x - 3*a**3*x + 6*a**2*b*m*x**2 - 9*a**2*b*x**2 + 6*a*b**2*m*x**3
- 9*a*b**2*x**3 + 2*b**3*m*x**4 - 3*b**3*x**4),x)*a**3*m - 4*int((x**m*sqr
t(a + b*x))/(2*a**3*m*x - 3*a**3*x + 6*a**2*b*m*x**2 - 9*a**2*b*x**2 + 6*a
*b**2*m*x**3 - 9*a*b**2*x**3 + 2*b**3*m*x**4 - 3*b**3*x**4),x)*a**2*b*m**2
*x + 6*int((x**m*sqrt(a + b*x))/(2*a**3*m*x - 3*a**3*x + 6*a**2*b*m*x**2 -
 9*a**2*b*x**2 + 6*a*b**2*m*x**3 - 9*a*b**2*x**3 + 2*b**3*m*x**4 - 3*b**3*
x**4),x)*a**2*b*m*x - 2*int((x**m*sqrt(a + b*x))/(2*a**3*m*x - 3*a**3*x +
6*a**2*b*m*x**2 - 9*a**2*b*x**2 + 6*a*b**2*m*x**3 - 9*a*b**2*x**3 + 2*b**3
*m*x**4 - 3*b**3*x**4),x)*a*b**2*m**2*x**2 + 3*int((x**m*sqrt(a + b*x))/(2
*a**3*m*x - 3*a**3*x + 6*a**2*b*m*x**2 - 9*a**2*b*x**2 + 6*a*b**2*m*x**3 -
 9*a*b**2*x**3 + 2*b**3*m*x**4 - 3*b**3*x**4),x)*a*b**2*m*x**2))/(b*(2*a**
2*m - 3*a**2 + 4*a*b*m*x - 6*a*b*x + 2*b**2*m*x**2 - 3*b**2*x**2))
```

3.783 $\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx = \frac{2x^{2+m}\left(-\frac{bx}{a}\right)^{-2-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-2-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*x^(2+m)*(-b*x/a)^(-2-m)*(b*x+a)^(1/2)*hypergeom([1/2, -2-m],[3/2],1+b*x/
a)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx = \frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-2-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b^3}$$

input
```
Integrate[x^(2 + m)/Sqrt[a + b*x],x]
```

output
```
(2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])
/(b^3*(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m+2}}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{ 77}$$

$$\frac{a^2x^m\left(-\frac{bx}{a}\right)^{-m}\int \frac{\left(-\frac{bx}{a}\right)^{m+2}}{\sqrt{a+bx}}dx}{b^2}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{2a^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m-2,\frac{3}{2},\frac{bx}{a}+1\right)}{b^3}$$

input
```
Int[x^(2 + m)/Sqrt[a + b*x],x]
```

output
```
(2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])
/(b^3*(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{x^{2+m}}{\sqrt{bx+a}}dx$$

input
```
int(x^(2+m)/(b*x+a)^(1/2),x)
```

output
```
int(x^(2+m)/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+2}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m + 2)/sqrt(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx = \frac{x^{m+3}\Gamma(m+3)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+3\\ m+4\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(m+4\right)}$$

input
```
integrate(x**(2+m)/(b*x+a)**(1/2),x)
```

output
```
x**(m + 3)*gamma(m + 3)*hyper((1/2, m + 3), (m + 4,), b*x*exp_polar(I*pi)/
a)/(sqrt(a)*gamma(m + 4))
```

Maxima [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+2}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m + 2)/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+2}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m + 2)/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+2}}{\sqrt{a+b\,x}}\,dx$$

input
```
int(x^(m + 2)/(a + b*x)^(1/2),x)
```

output
```
int(x^(m + 2)/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx$$
$$= \frac{8x^m\sqrt{bx+a}\,a^2m^2 + 24x^m\sqrt{bx+a}\,a^2m + 16x^m\sqrt{bx+a}\,a^2 - 8x^m\sqrt{bx+a}\,ab\,m^2x - 20x^m\sqrt{bx+a}\,abm}{}$$

input
```
int(x^(2+m)/(b*x+a)^(1/2),x)
```

output
```
(2*(4*x**m*sqrt(a + b*x)*a**2*m**2 + 12*x**m*sqrt(a + b*x)*a**2*m + 8*x**m
*sqrt(a + b*x)*a**2 - 4*x**m*sqrt(a + b*x)*a*b*m**2*x - 10*x**m*sqrt(a + b
*x)*a*b*m*x - 4*x**m*sqrt(a + b*x)*a*b*x + 4*x**m*sqrt(a + b*x)*b**2*m**2*
x**2 + 8*x**m*sqrt(a + b*x)*b**2*m*x**2 + 3*x**m*sqrt(a + b*x)*b**2*x**2 -
 32*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*m*x + 15*a*x
 + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x)*a**3*m**6
- 240*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*m*x + 15*a
*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x)*a**3*m**
5 - 680*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*m*x + 15
*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x)*a**3*m
**4 - 900*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*m*x +
15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x)*a**3
*m**3 - 548*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*m*x
+ 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x)*a*
*3*m**2 - 120*int((x**m*sqrt(a + b*x))/(8*a*m**3*x + 36*a*m**2*x + 46*a*m*
x + 15*a*x + 8*b*m**3*x**2 + 36*b*m**2*x**2 + 46*b*m*x**2 + 15*b*x**2),x)*
a**3*m))/(b**3*(8*m**3 + 36*m**2 + 46*m + 15))
```

3.784 $\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx = \frac{2x^{1+m}\left(-\frac{bx}{a}\right)^{-1-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-1-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*x^(1+m)*(-b*x/a)^(-1-m)*(b*x+a)^(1/2)*hypergeom([1/2, -1-m],[3/2],1+b*x/
a)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx = -\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-1-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b^2}$$

input
```
Integrate[x^(1 + m)/Sqrt[a + b*x],x]
```

output
```
(-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/
(b^2*(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m+1}}{\sqrt{a+bx}} \, dx$$

$$\Big\downarrow \; 77$$

$$-\frac{ax^m \left(-\frac{bx}{a}\right)^{-m} \int \frac{\left(-\frac{bx}{a}\right)^{m+1}}{\sqrt{a+bx}}dx}{b}$$

$$\Big\downarrow \; 75$$

$$-\frac{2ax^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{2},-m-1,\frac{3}{2},\frac{bx}{a}+1\right)}{b^2}$$

input
```
Int[x^(1 + m)/Sqrt[a + b*x],x]
```

output
```
(-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/
(b^2*(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{x^{1+m}}{\sqrt{bx+a}}dx$$

input
```
int(x^(1+m)/(b*x+a)^(1/2),x)
```

output
```
int(x^(1+m)/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+1}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m + 1)/sqrt(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx = \frac{x^{m+2}\Gamma(m+2)\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, m+2 \\ m+3 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(m+3\right)}$$

input
```
integrate(x**(1+m)/(b*x+a)**(1/2),x)
```

output
```
x**(m + 2)*gamma(m + 2)*hyper((1/2, m + 2), (m + 3,), b*x*exp_polar(I*pi)/
a)/(sqrt(a)*gamma(m + 3))
```

Maxima [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+1}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m + 1)/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+1}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m + 1)/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m+1}}{\sqrt{a+b\,x}}\,dx$$

input
```
int(x^(m + 1)/(a + b*x)^(1/2),x)
```

output
```
int(x^(m + 1)/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx$$
$$= \frac{-4x^m\sqrt{bx+a}\,am - 4x^m\sqrt{bx+a}\,a + 4x^m\sqrt{bx+a}\,bmx + 2x^m\sqrt{bx+a}\,bx + 16\left(\int \frac{x^m\sqrt{bx+a}}{4b\,m^2x^2+4a\,m^2x+8bm\,x^2+}\right.}{}$$

input
```
int(x^(1+m)/(b*x+a)^(1/2),x)
```

output
```
(2*( - 2*x**m*sqrt(a + b*x)*a*m - 2*x**m*sqrt(a + b*x)*a + 2*x**m*sqrt(a +
 b*x)*b*m*x + x**m*sqrt(a + b*x)*b*x + 8*int((x**m*sqrt(a + b*x))/(4*a*m**
2*x + 8*a*m*x + 3*a*x + 4*b*m**2*x**2 + 8*b*m*x**2 + 3*b*x**2),x)*a**2*m**
4 + 24*int((x**m*sqrt(a + b*x))/(4*a*m**2*x + 8*a*m*x + 3*a*x + 4*b*m**2*x
**2 + 8*b*m*x**2 + 3*b*x**2),x)*a**2*m**3 + 22*int((x**m*sqrt(a + b*x))/(4
*a*m**2*x + 8*a*m*x + 3*a*x + 4*b*m**2*x**2 + 8*b*m*x**2 + 3*b*x**2),x)*a*
*2*m**2 + 6*int((x**m*sqrt(a + b*x))/(4*a*m**2*x + 8*a*m*x + 3*a*x + 4*b*m
**2*x**2 + 8*b*m*x**2 + 3*b*x**2),x)*a**2*m))/(b**2*(4*m**2 + 8*m + 3))
```

3.785 $\int \frac{x^m}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx = \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*x^m*(b*x+a)^(1/2)*hypergeom([1/2, -m],[3/2],1+b*x/a)/b/((-b*x/a)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx = \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

input
```
Integrate[x^m/Sqrt[a + b*x],x]
```

output
```
(2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((
b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 77}$$

$$x^m\left(-\frac{bx}{a}\right)^{-m}\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 75}$$

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{bx}{a}+1\right)}{b}$$

input
```
Int[x^m/Sqrt[a + b*x],x]
```

output
```
(2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((
b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{x^m}{\sqrt{bx+a}}dx$$

input
```
int(x^m/(b*x+a)^(1/2),x)
```

output
```
int(x^m/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx = \int \frac{x^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/sqrt(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx = \frac{x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(m+2\right)}$$

input
```
integrate(x**m/(b*x+a)**(1/2),x)
```

output
```
x**(m + 1)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/
a)/(sqrt(a)*gamma(m + 2))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx = \int \frac{x^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx = \int \frac{x^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^m/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx = \int \frac{x^m}{\sqrt{a+b\,x}}\,dx$$

input
```
int(x^m/(a + b*x)^(1/2),x)
```

output
```
int(x^m/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx}}\,dx$$
$$= \frac{2x^m\sqrt{bx+a} - 4\left(\int \frac{x^m\sqrt{bx+a}}{2bm\,x^2+2amx+b\,x^2+ax}dx\right)a\,m^2 - 2\left(\int \frac{x^m\sqrt{bx+a}}{2bm\,x^2+2amx+b\,x^2+ax}dx\right)am}{b\,(2m+1)}$$

input
```
int(x^m/(b*x+a)^(1/2),x)
```

output
```
(2*(x**m*sqrt(a + b*x) - 2*int((x**m*sqrt(a + b*x))/(2*a*m*x + a*x + 2*b*m
*x**2 + b*x**2),x)*a*m**2 - int((x**m*sqrt(a + b*x))/(2*a*m*x + a*x + 2*b*
m*x**2 + b*x**2),x)*a*m))/(b*(2*m + 1))
```

3.786 $\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx=\frac{2x^{-1+m}\left(-\frac{bx}{a}\right)^{1-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*x^(-1+m)*(-b*x/a)^(1-m)*(b*x+a)^(1/2)*hypergeom([1/2, 1-m],[3/2],1+b*x/a
)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx=-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-m,\frac{3}{2},1+\frac{bx}{a}\right)}{a}$$

input
```
Integrate[x^(-1 + m)/Sqrt[a + b*x],x]
```

output
```
(-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*
(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-1}}{\sqrt{a+bx}}dx$$
$$\Big\downarrow \text{ 77}$$
$$-\frac{bx^m\left(-\frac{bx}{a}\right)^{-m}\int \frac{\left(-\frac{bx}{a}\right)^{m-1}}{\sqrt{a+bx}}dx}{a}$$
$$\Big\downarrow \text{ 75}$$
$$-\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},1-m,\frac{3}{2},\frac{bx}{a}+1\right)}{a}$$

input
```
Int[x^(-1 + m)/Sqrt[a + b*x],x]
```

output
```
(-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*
(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{x^{m-1}}{\sqrt{bx+a}}dx$$

input
```
int(x^(m-1)/(b*x+a)^(1/2),x)
```

output
```
int(x^(m-1)/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m-1}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m - 1)/sqrt(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx = \frac{a^m a^{-m-\frac{1}{2}} x^m \Gamma(m)\, {}_2F_1\left(\begin{matrix}\frac{1}{2}, m\\ m+1\end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+1)}$$

input
```
integrate(x**(-1+m)/(b*x+a)**(1/2),x)
```

output
```
a**m*a**(-m - 1/2)*x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I
*pi)/a)/gamma(m + 1)
```

Maxima [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx=\int \frac{x^{m-1}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m - 1)/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx=\int \frac{x^{m-1}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx=\int \frac{x^{m-1}}{\sqrt{a+b\,x}}\,dx$$

input
```
int(x^(m - 1)/(a + b*x)^(1/2),x)
```

output
```
int(x^(m - 1)/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^m\sqrt{bx+a}}{b\,x^2+ax}dx$$

input
```
int(x^(-1+m)/(b*x+a)^(1/2),x)
```

output
```
int((x**m*sqrt(a + b*x))/(a*x + b*x**2),x)
```

3.787 $\int \frac{x^{-2+m}}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}} dx = \frac{2x^{-2+m}\left(-\frac{bx}{a}\right)^{2-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},2-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*x^(-2+m)*(-b*x/a)^(2-m)*(b*x+a)^(1/2)*hypergeom([1/2, 2-m],[3/2],1+b*x/a
)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}} dx = \frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},2-m,\frac{3}{2},1+\frac{bx}{a}\right)}{a^2}$$

input
```
Integrate[x^(-2 + m)/Sqrt[a + b*x],x]
```

output
```
(2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a
^2*(-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-2}}{\sqrt{a+bx}} \, dx$$

$$\Big\downarrow \text{77}$$

$$\frac{b^2 x^m \left(-\frac{bx}{a}\right)^{-m} \int \frac{\left(-\frac{bx}{a}\right)^{m-2}}{\sqrt{a+bx}}dx}{a^2}$$

$$\Big\downarrow \text{75}$$

$$\frac{2bx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},2-m,\frac{3}{2},\frac{bx}{a}+1\right)}{a^2}$$

input
```
Int[x^(-2 + m)/Sqrt[a + b*x],x]
```

output
```
(2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a
^2*(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{x^{-2+m}}{\sqrt{bx+a}}dx$$

input
```
int(x^(-2+m)/(b*x+a)^(1/2),x)
```

output
```
int(x^(-2+m)/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m-2}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m - 2)/sqrt(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx = \frac{x^{m-1}\Gamma(m-1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m-1\\m\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma(m)}$$

input
```
integrate(x**(-2+m)/(b*x+a)**(1/2),x)
```

output
```
x**(m - 1)*gamma(m - 1)*hyper((1/2, m - 1), (m,), b*x*exp_polar(I*pi)/a)/(
sqrt(a)*gamma(m))
```

Maxima [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m-2}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m - 2)/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m-2}}{\sqrt{bx+a}}\,dx$$

input
```
integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m - 2)/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^{m-2}}{\sqrt{a+b\,x}}\,dx$$

input
```
int(x^(m - 2)/(a + b*x)^(1/2),x)
```

output
```
int(x^(m - 2)/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx = \int \frac{x^m\sqrt{bx+a}}{b\,x^3+a\,x^2}dx$$

input
```
int(x^(-2+m)/(b*x+a)^(1/2),x)
```

output
```
int((x**m*sqrt(a + b*x))/(a*x**2 + b*x**3),x)
```

3.788 $\int \frac{x^m}{\sqrt{2+3x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{x^m}{\sqrt{2+3x}} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, 1+m, 2+m, -\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

output
```
1/2*x^(1+m)*hypergeom([1/2, 1+m],[2+m],-3/2*x)*2^(1/2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{2+3x}} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, 1+m, 2+m, -\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

input
```
Integrate[x^m/Sqrt[2 + 3*x],x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m
))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{3x+2}}\,dx$$
$$\Big\downarrow \text{ 74}$$
$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},m+1,m+2,-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

input
```
Int[x^m/Sqrt[2 + 3*x],x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m
))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
meijerg	$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],\left[2+m\right],-\frac{3x}{2}\right)\sqrt{2}}{2+2m}$	29

input
```
int(x^m/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)*2^(1/2)/(1+m)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x+2}}\,dx$$

input
```
integrate(x^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/sqrt(3*x + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx = \frac{2\cdot 2^m\sqrt{3}\cdot 3^{-m}\sqrt{x+\frac{2}{3}}e^{i\pi m}{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3x}{2}+1\right)}{3}$$

input
```
integrate(x**m/(2+3*x)**(1/2),x)
```

output
```
2*2**m*sqrt(3)*sqrt(x + 2/3)*exp(I*pi*m)*hyper((1/2, -m), (3/2,), 3*x/2 +
1)/(3*3**m)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x+2}}\,dx$$

input
```
integrate(x^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(3*x + 2), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x+2}}\,dx$$

input
```
integrate(x^m/(2+3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x+2}}\,dx$$

input
```
int(x^m/(3*x + 2)^(1/2),x)
```

output
```
int(x^m/(3*x + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx$$
$$= \frac{2x^m\sqrt{3x+2} - 8\left(\int \frac{x^m\sqrt{3x+2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m^2 - 4\left(\int \frac{x^m\sqrt{3x+2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m}{6m+3}$$

input
```
int(x^m/(2+3*x)^(1/2),x)
```

output
```
(2*(x**m*sqrt(3*x + 2) - 4*int((x**m*sqrt(3*x + 2))/(6*m*x**2 + 4*m*x + 3*
x**2 + 2*x),x)*m**2 - 2*int((x**m*sqrt(3*x + 2))/(6*m*x**2 + 4*m*x + 3*x**
2 + 2*x),x)*m))/(3*(2*m + 1))
```

3.789 $\int \frac{x^m}{\sqrt{2-3x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{x^m}{\sqrt{2-3x}} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, 1+m, 2+m, \frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

output
```
1/2*x^(1+m)*hypergeom([1/2, 1+m],[2+m],3/2*x)*2^(1/2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{2-3x}} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, 1+m, 2+m, \frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

input
```
Integrate[x^m/Sqrt[2 - 3*x],x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx$$

$$\Big\downarrow \; 74$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},m+1,m+2,\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

input

```
Int[x^m/Sqrt[2 - 3*x],x]
```

output

```
(x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Defintions of rubi rules used

rule 74

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
meijerg	$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)\sqrt{2}}{2+2m}$	29

input
```
int(x^m/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],3/2*x)*2^(1/2)/(1+m)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = \int \frac{x^m}{\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(x^m/(2-3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-x^m*sqrt(-3*x + 2)/(3*x - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = -\frac{2\cdot 2^m\sqrt{3}\cdot 3^{-m}i\sqrt{x-\frac{2}{3}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\,\frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{3}$$

input
```
integrate(x**m/(2-3*x)**(1/2),x)
```

output
```
-2*2**m*sqrt(3)*I*sqrt(x - 2/3)*hyper((1/2, -m), (3/2,), 3*(x - 2/3)*exp_p
olar(I*pi)/2)/(3*3**m)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = \int \frac{x^m}{\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(x^m/(2-3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = \int \frac{x^m}{\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(x^m/(2-3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = \int \frac{x^m}{\sqrt{2-3\,x}}\,dx$$

input
```
int(x^m/(2 - 3*x)^(1/2),x)
```

output
```
int(x^m/(2 - 3*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx$$
$$= \frac{-2x^m\sqrt{-3x+2} - 8\left(\int \frac{x^m\sqrt{-3x+2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m^2 - 4\left(\int \frac{x^m\sqrt{-3x+2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m}{6m+3}$$

input
```
int(x^m/(2-3*x)^(1/2),x)
```

output
```
(2*( - x**m*sqrt( - 3*x + 2) - 4*int((x**m*sqrt( - 3*x + 2))/(6*m*x**2 - 4
*m*x + 3*x**2 - 2*x),x)*m**2 - 2*int((x**m*sqrt( - 3*x + 2))/(6*m*x**2 - 4
*m*x + 3*x**2 - 2*x),x)*m))/(3*(2*m + 1))
```

3.790 $\int \frac{x^m}{\sqrt{-2+3x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{x^m}{\sqrt{-2+3x}} dx = \left(\frac{3}{2}\right)^{-1-m} \sqrt{-2+3x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-\frac{3x}{2}\right)$$

output
```
(3/2)^(-1-m)*(-2+3*x)^(1/2)*hypergeom([1/2, -m],[3/2],1-3/2*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{-2+3x}} dx = \left(\frac{3}{2}\right)^{-1-m} \sqrt{-2+3x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-\frac{3x}{2}\right)$$

input
```
Integrate[x^m/Sqrt[-2 + 3*x],x]
```

output
```
(3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{75\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{3x-2}}\,dx$$
$$\Big\downarrow \text{ 75}$$
$$\left(\frac{3}{2}\right)^{-m-1}\sqrt{3x-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-\frac{3x}{2}\right)$$

input
```
Int[x^m/Sqrt[-2 + 3*x],x]
```

output
```
(3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-\frac{2}{3}+x\right)}\,x^{1+m}\,\operatorname{hypergeom}\left(\left[\frac{1}{2},1+m\right],\left[2+m\right],\frac{3x}{2}\right)}{2\sqrt{\operatorname{signum}\left(-\frac{2}{3}+x\right)}\,\left(1+m\right)}$	43

input
```
int(x^m/(-2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)/signum(-2/3+x)^(1/2)*(-signum(-2/3+x))^(1/2)/(1+m)*x^(1+m)*hyp
ergeom([1/2,1+m],[2+m],3/2*x)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x-2}}\,dx$$

input
```
integrate(x^m/(-2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/sqrt(3*x - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = -\frac{\sqrt{2}ix^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{3x}{2}\right)}{2\Gamma\left(m+2\right)}$$

input
```
integrate(x**m/(-2+3*x)**(1/2),x)
```

output
```
-sqrt(2)*I*x**(m + 1)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x/2)/(2
*gamma(m + 2))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x-2}}\,dx$$

input
```
integrate(x^m/(-2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(3*x - 2), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x-2}}\,dx$$

input
```
integrate(x^m/(-2+3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(3*x - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = \int \frac{x^m}{\sqrt{3\,x-2}}\,dx$$

input
```
int(x^m/(3*x - 2)^(1/2),x)
```

output
```
int(x^m/(3*x - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = \frac{2x^m\sqrt{3x-2}+8\left(\int \frac{x^m\sqrt{3x-2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m^2+4\left(\int \frac{x^m\sqrt{3x-2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m}{6m+3}$$

input
```
int(x^m/(-2+3*x)^(1/2),x)
```

output
```
(2*(x**m*sqrt(3*x - 2) + 4*int((x**m*sqrt(3*x - 2))/(6*m*x**2 - 4*m*x + 3*
x**2 - 2*x),x)*m**2 + 2*int((x**m*sqrt(3*x - 2))/(6*m*x**2 - 4*m*x + 3*x**
2 - 2*x),x)*m))/(3*(2*m + 1))
```

3.791 $\int \frac{x^m}{\sqrt{-2-3x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx=-2^{1+m}3^{-1-m}\sqrt{-2-3x}(-x)^{-m}x^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{3x}{2}\right)$$

output
```
-2^(1+m)*3^(-1-m)*(-2-3*x)^(1/2)*x^m*hypergeom([1/2, -m],[3/2],1+3/2*x)/((
-x)^m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx=-\frac{2}{3}\left(1+\frac{1}{2}(-2-3x)\right)^{-m}\sqrt{-2-3x}x^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{3x}{2}\right)$$

input
```
Integrate[x^m/Sqrt[-2 - 3*x],x]
```

output
```
(-2*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/(3*(1
 + (-2 - 3*x)/2)^m)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {77, 27, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{-3x-2}}\,dx$$

$$\Big\downarrow \text{77}$$

$$\left(\frac{2}{3}\right)^m (-x)^{-m} x^m \int \frac{\left(\frac{3}{2}\right)^m (-x)^m}{\sqrt{-3x-2}}dx$$

$$\Big\downarrow \text{27}$$

$$(-x)^{-m} x^m \int \frac{(-x)^m}{\sqrt{-3x-2}}dx$$

$$\Big\downarrow \text{75}$$

$$-2^{m+1} 3^{-m-1} \sqrt{-3x-2} (-x)^{-m} x^m \text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{3x}{2}+1\right)$$

input
```
Int[x^m/Sqrt[-2 - 3*x],x]
```

output
```
-((2^(1 + m)*3^(-1 - m)*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2,
 1 + (3*x)/2])/(-x)^m)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

method	result	size
meijerg	$-\frac{i x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2},1+m\right],\left[2+m\right],-\frac{3x}{2}\right) \sqrt{2}}{2(1+m)}$	30

input
```
int(x^m/(-2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)*2^(1/2)/(1+m)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = \int \frac{x^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
integrate(x^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-x^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = -\frac{2\cdot 2^m\sqrt{3}\cdot 3^{-m}i\sqrt{x+\frac{2}{3}}e^{i\pi m}{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3x}{2}+1\right)}{3}$$

input
```
integrate(x**m/(-2-3*x)**(1/2),x)
```

output
```
-2*2**m*sqrt(3)*I*sqrt(x + 2/3)*exp(I*pi*m)*hyper((1/2, -m), (3/2,), 3*x/2
 + 1)/(3*3**m)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = \int \frac{x^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
integrate(x^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(-3*x - 2), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = \int \frac{x^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
integrate(x^m/(-2-3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(-3*x - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = \int \frac{x^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
int(x^m/(- 3*x - 2)^(1/2),x)
```

output
```
int(x^m/(- 3*x - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = \frac{-2x^m\sqrt{-3x-2}+8\left(\int \frac{x^m\sqrt{-3x-2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m^2+4\left(\int \frac{x^m\sqrt{-3x-2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m}{6m+3}$$

input
```
int(x^m/(-2-3*x)^(1/2),x)
```

output
```
(2*( - x**m*sqrt( - 3*x - 2) + 4*int((x**m*sqrt( - 3*x - 2))/(6*m*x**2 + 4
*m*x + 3*x**2 + 2*x),x)*m**2 + 2*int((x**m*sqrt( - 3*x - 2))/(6*m*x**2 + 4
*m*x + 3*x**2 + 2*x),x)*m))/(3*(2*m + 1))
```

3.792 $\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx = \frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*(-x)^m*(b*x+a)^(1/2)*hypergeom([1/2, -m],[3/2],1+b*x/a)/b/((-b*x/a)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx = \frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

input
```
Integrate[(-x)^m/Sqrt[a + b*x],x]
```

output
```
(2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(
-((b*x)/a))^m)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-x)^m}{\sqrt{a+bx}} \, dx$$
$$\Big\downarrow \text{77}$$
$$(-x)^m \left(-\frac{bx}{a}\right)^{-m} \int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}dx$$
$$\Big\downarrow \text{75}$$
$$\frac{2(-x)^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{bx}{a}+1\right)}{b}$$

input
```
Int[(-x)^m/Sqrt[a + b*x],x]
```

output
```
(2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(
-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{(-x)^m}{\sqrt{bx+a}}dx$$

input
```
int((-x)^m/(b*x+a)^(1/2),x)
```

output
```
int((-x)^m/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx = \int \frac{(-x)^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((-x)^m/sqrt(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx = \frac{x^{m+1}e^{i\pi m}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(m+2\right)}$$

input
```
integrate((-x)**m/(b*x+a)**(1/2),x)
```

output
```
x**(m + 1)*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_
polar(I*pi)/a)/(sqrt(a)*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx = \int \frac{(-x)^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-x)^m/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx = \int \frac{(-x)^m}{\sqrt{bx+a}}\,dx$$

input
```
integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((-x)^m/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx = \int \frac{(-x)^m}{\sqrt{a+b\,x}}\,dx$$

input
```
int((-x)^m/(a + b*x)^(1/2),x)
```

output
```
int((-x)^m/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx$$
$$= \frac{2(-1)^m \left(x^m\sqrt{bx+a} - 2\left(\int \frac{x^m\sqrt{bx+a}}{2bm\,x^2+2amx+b\,x^2+ax}dx\right) a\,m^2 - \left(\int \frac{x^m\sqrt{bx+a}}{2bm\,x^2+2amx+b\,x^2+ax}dx\right) am\right)}{b\,(2m+1)}$$

input
```
int((-x)^m/(b*x+a)^(1/2),x)
```

output
```
(2*( - 1)**m*(x**m*sqrt(a + b*x) - 2*int((x**m*sqrt(a + b*x))/(2*a*m*x + a
*x + 2*b*m*x**2 + b*x**2),x)*a*m**2 - int((x**m*sqrt(a + b*x))/(2*a*m*x +
a*x + 2*b*m*x**2 + b*x**2),x)*a*m))/(b*(2*m + 1))
```

3.793 $\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = -\frac{(-x)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+m,2+m,-\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

output
```
-1/2*(-x)^(1+m)*hypergeom([1/2, 1+m],[2+m],-3/2*x)*2^(1/2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = \frac{(-x)^m x\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+m,2+m,-\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

input
```
Integrate[(-x)^m/Sqrt[2 + 3*x],x]
```

output
```
((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-x)^m}{\sqrt{3x+2}}dx$$
$$\Big\downarrow \text{74}$$
$$-\frac{(-x)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},m+1,m+2,-\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

input
```
Int[(-x)^m/Sqrt[2 + 3*x],x]
```

output
```
-(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(
1 + m)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^{m} x \,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],\left[2+m\right],-\frac{3 x}{2}\right)}{2+2 m}$	30

input
```
int((-x)^m/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Fricas [F]

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x+2}}\,dx$$

input
```
integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((-x)^m/sqrt(3*x + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = \frac{2\cdot 2^m\sqrt{3}\cdot 3^{-m}\sqrt{x+\frac{2}{3}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x+\frac{2}{3}\right)e^{2i\pi}}{2}\right)}{3}$$

input
```
integrate((-x)**m/(2+3*x)**(1/2),x)
```

output
```
2*2**m*sqrt(3)*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3)*exp_pola
r(2*I*pi)/2)/(3*3**m)
```

Maxima [F]

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x+2}}\,dx$$

input
```
integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-x)^m/sqrt(3*x + 2), x)
```

Giac [F]

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x+2}}\,dx$$

input
```
integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((-x)^m/sqrt(3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x+2}}\,dx$$

input
```
int((-x)^m/(3*x + 2)^(1/2),x)
```

output
```
int((-x)^m/(3*x + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx$$
$$= \frac{2(-1)^m \left(x^m\sqrt{3x+2} - 4\left(\int \frac{x^m\sqrt{3x+2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m^2 - 2\left(\int \frac{x^m\sqrt{3x+2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m\right)}{6m+3}$$

input
```
int((-x)^m/(2+3*x)^(1/2),x)
```

output
```
(2*( - 1)**m*(x**m*sqrt(3*x + 2) - 4*int((x**m*sqrt(3*x + 2))/(6*m*x**2 +
4*m*x + 3*x**2 + 2*x),x)*m**2 - 2*int((x**m*sqrt(3*x + 2))/(6*m*x**2 + 4*m
*x + 3*x**2 + 2*x),x)*m))/(3*(2*m + 1))
```

3.794 $\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx=-\frac{(-x)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+m,2+m,\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

output
```
-1/2*(-x)^(1+m)*hypergeom([1/2, 1+m],[2+m],3/2*x)*2^(1/2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx=\frac{(-x)^m x\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+m,2+m,\frac{3x}{2}\right)}{\sqrt{2}(1+m)}$$

input
```
Integrate[(-x)^m/Sqrt[2 - 3*x],x]
```

output
```
((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-x)^m}{\sqrt{2-3x}} \, dx$$

$$\Big\downarrow \; 74$$

$$-\frac{(-x)^{m+1} \text{Hypergeometric2F1}\left(\frac{1}{2},m+1,m+2,\frac{3x}{2}\right)}{\sqrt{2}(m+1)}$$

input
```
Int[(-x)^m/Sqrt[2 - 3*x],x]
```

output
```
-(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1
 + m)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^{m} x \,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],\left[2+m\right],\frac{3x}{2}\right)}{2+2m}$	30

input
```
int((-x)^m/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],3/2*x)
```

Fricas [F]

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(-x)^m*sqrt(-3*x + 2)/(3*x - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx = -\frac{2\cdot 2^m\sqrt{3}\cdot 3^{-m}i\sqrt{x-\frac{2}{3}}e^{i\pi m}{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\right|\frac{3(x-\frac{2}{3})e^{i\pi}}{2}\right)}{3}$$

input
```
integrate((-x)**m/(2-3*x)**(1/2),x)
```

output
```
-2*2**m*sqrt(3)*I*sqrt(x - 2/3)*exp(I*pi*m)*hyper((1/2, -m), (3/2,), 3*(x
- 2/3)*exp_polar(I*pi)/2)/(3*3**m)
```

Maxima [F]

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-x)^m/sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((-x)^m/sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{2-3\,x}}\,dx$$

input
```
int((-x)^m/(2 - 3*x)^(1/2),x)
```

output
```
int((-x)^m/(2 - 3*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx$$
$$= \frac{2(-1)^m \left(-x^m\sqrt{-3x+2} - 4\left(\int \frac{x^m\sqrt{-3x+2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m^2 - 2\left(\int \frac{x^m\sqrt{-3x+2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m\right)}{6m+3}$$

input
```
int((-x)^m/(2-3*x)^(1/2),x)
```

output
```
(2*( - 1)**m*( - x**m*sqrt( - 3*x + 2) - 4*int((x**m*sqrt( - 3*x + 2))/(6*
m*x**2 - 4*m*x + 3*x**2 - 2*x),x)*m**2 - 2*int((x**m*sqrt( - 3*x + 2))/(6*
m*x**2 - 4*m*x + 3*x**2 - 2*x),x)*m))/(3*(2*m + 1))
```

3.795 $\int \frac{(-x)^m}{\sqrt{-2+3x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(-x)^m}{\sqrt{-2+3x}} dx = 2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-\frac{3x}{2}\right)$$

output
```
2^(1+m)*3^(-1-m)*(-x)^m*(-2+3*x)^(1/2)*hypergeom([1/2, -m],[3/2],1-3/2*x)/
(x^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(-x)^m}{\sqrt{-2+3x}} dx = 2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-\frac{3x}{2}\right)$$

input
```
Integrate[(-x)^m/Sqrt[-2 + 3*x],x]
```

output
```
(2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2
, 1 - (3*x)/2])/x^m
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {77, 27, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-x)^m}{\sqrt{3x-2}}dx$$

$$\Big\downarrow \text{ 77}$$

$$\left(\frac{2}{3}\right)^m (-x)^m x^{-m} \int \frac{\left(\frac{3}{2}\right)^m x^m}{\sqrt{3x-2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$(-x)^m x^{-m} \int \frac{x^m}{\sqrt{3x-2}}dx$$

$$\Big\downarrow \text{ 75}$$

$$2^{m+1}3^{-m-1}\sqrt{3x-2}(-x)^m x^{-m} \text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-\frac{3x}{2}\right)$$

input
```
Int[(-x)^m/Sqrt[-2 + 3*x],x]
```

output
```
(2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2
, 1 - (3*x)/2])/x^m
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^{m}\sqrt{-\operatorname{signum}\left(-\frac{2}{3}+x\right)}\,x\,\operatorname{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)}{2\sqrt{\operatorname{signum}\left(-\frac{2}{3}+x\right)}\,(1+m)}$	44

input
```
int((-x)^m/(-2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-x)^m/signum(-2/3+x)^(1/2)*(-signum(-2/3+x))^(1/2)/(1+m)*x*hy
pergeom([1/2,1+m],[2+m],3/2*x)
```

Fricas [F]

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x-2}}\,dx$$

input
```
integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((-x)^m/sqrt(3*x - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = -\frac{\sqrt{2}ix^{m+1}e^{i\pi m}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\ m+2\end{matrix}\middle|\frac{3x}{2}\right)}{2\Gamma\left(m+2\right)}$$

input
```
integrate((-x)**m/(-2+3*x)**(1/2),x)
```

output
```
-sqrt(2)*I*x**(m + 1)*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,
), 3*x/2)/(2*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x-2}}\,dx$$

input
```
integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-x)^m/sqrt(3*x - 2), x)
```

Giac [F]

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x-2}}\,dx$$

input
```
integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((-x)^m/sqrt(3*x - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = \int \frac{(-x)^m}{\sqrt{3\,x-2}}\,dx$$

input
```
int((-x)^m/(3*x - 2)^(1/2),x)
```

output
```
int((-x)^m/(3*x - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = \frac{2(-1)^m \left(x^m\sqrt{3x-2}+4\left(\int \frac{x^m\sqrt{3x-2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m^2+2\left(\int \frac{x^m\sqrt{3x-2}}{6m\,x^2-4mx+3x^2-2x}dx\right)m\right)}{6m+3}$$

input
```
int((-x)^m/(-2+3*x)^(1/2),x)
```

output
```
(2*( - 1)**m*(x**m*sqrt(3*x - 2) + 4*int((x**m*sqrt(3*x - 2))/(6*m*x**2 -
4*m*x + 3*x**2 - 2*x),x)*m**2 + 2*int((x**m*sqrt(3*x - 2))/(6*m*x**2 - 4*m
*x + 3*x**2 - 2*x),x)*m))/(3*(2*m + 1))
```

3.796 $\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = -\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2-3x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{3x}{2}\right)$$

output
```
-(3/2)^(-1-m)*(-2-3*x)^(1/2)*hypergeom([1/2, -m],[3/2],1+3/2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = -\frac{2}{3}\left(1+\frac{1}{2}(-2-3x)\right)^{-m}\sqrt{-2-3x}x^{-m}\left(-x^2\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{3x}{2}\right)$$

input
```
Integrate[(-x)^m/Sqrt[-2 - 3*x],x]
```

output
```
(-2*Sqrt[-2 - 3*x]*(-x^2)^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/
(3*(1 + (-2 - 3*x)/2)^m*x^m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-x)^m}{\sqrt{-3x-2}}\,dx$$

$$\Big\downarrow \ 75$$

$$-\left(\frac{3}{2}\right)^{-m-1}\sqrt{-3x-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{3x}{2}+1\right)$$

input
```
Int[(-x)^m/Sqrt[-2 - 3*x],x]
```

output
```
-((3/2)^(-1 - m)*Sqrt[-2 - 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/
2])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
meijerg	$-\frac{i\sqrt{2}\,(-x)^{m} x \,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)}{2(1+m)}$	31

input
```
int((-x)^m/(-2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Fricas [F]

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(-x)^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = -\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m} i\sqrt{x+\frac{2}{3}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x+\frac{2}{3}\right)e^{2i\pi}}{2}\right)}{3}$$

input
```
integrate((-x)**m/(-2-3*x)**(1/2),x)
```

output
```
-2*2**m*sqrt(3)*I*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3)*exp_p
olar(2*I*pi)/2)/(3*3**m)
```

Maxima [F]

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-x)^m/sqrt(-3*x - 2), x)
```

Giac [F]

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((-x)^m/sqrt(-3*x - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = \int \frac{(-x)^m}{\sqrt{-3\,x-2}}\,dx$$

input
```
int((-x)^m/(- 3*x - 2)^(1/2),x)
```

output
```
int((-x)^m/(- 3*x - 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx$$
$$= \frac{2(-1)^m \left(-x^m\sqrt{-3x-2}+4\left(\int \frac{x^m\sqrt{-3x-2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m^2+2\left(\int \frac{x^m\sqrt{-3x-2}}{6m\,x^2+4mx+3x^2+2x}dx\right)m\right)}{6m+3}$$

input
```
int((-x)^m/(-2-3*x)^(1/2),x)
```

output
```
(2*( - 1)**m*( - x**m*sqrt( - 3*x - 2) + 4*int((x**m*sqrt( - 3*x - 2))/(6*
m*x**2 + 4*m*x + 3*x**2 + 2*x),x)*m**2 + 2*int((x**m*sqrt( - 3*x - 2))/(6*
m*x**2 + 4*m*x + 3*x**2 + 2*x),x)*m))/(3*(2*m + 1))
```

3.797 $\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 13

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \frac{x^m}{\sqrt{a+bx}}$$

output
```
x^m/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \frac{x^m}{\sqrt{a+bx}}$$

input
```
Integrate[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

output
```
x^m/Sqrt[a + b*x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-1}(2am+b(2m-1)x)}{2(a+bx)^{3/2}} \, dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{x^{m-1}(2am-b(1-2m)x)}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{83}$$

$$\frac{x^m}{\sqrt{a+bx}}$$

input
```
Int[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

output
```
x^m/Sqrt[a + b*x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x^{m}}{\sqrt{bx+a}}$	12
orering	$\frac{x\,x^{m-1}(2am+b(-1+2m)x)}{\sqrt{bx+a}\,(2bmx+2am-bx)}$	44

input
```
int(1/2*x^(m-1)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
x^m/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \frac{xx^{m-1}}{\sqrt{bx+a}}$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="fr
icas")
```

output
```
x*x^(m - 1)/sqrt(b*x + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.73 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.69

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \frac{aa^m a^{-m-\frac{3}{2}} m x^m \Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, m\\ m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+1)} + \frac{bx^{m+1}\cdot(2m-1)\,\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

input
```
integrate(1/2*x**(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)**(3/2),x)
```

output
```
a*a**m*a**(-m - 3/2)*m*x**m*gamma(m)*hyper((3/2, m), (m + 1,), b*x*exp_pol
ar(I*pi)/a)/gamma(m + 1) + b*x**(m + 1)*(2*m - 1)*gamma(m + 1)*hyper((3/2,
 m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \frac{x^m}{\sqrt{bx+a}}$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="ma
xima")
```

output
```
x^m/sqrt(b*x + a)
```

Giac [F]

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \int \frac{(b(2m-1)x+2\,am)x^{m-1}}{2\,(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(1/2*(b*(2*m - 1)*x + 2*a*m)*x^(m - 1)/(b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \frac{x^m}{\sqrt{a+b\,x}}$$

input
```
int((x^(m - 1)*(2*a*m + b*x*(2*m - 1)))/(2*(a + b*x)^(3/2)),x)
```

output
```
x^m/(a + b*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx = \frac{x^m}{\sqrt{bx+a}}$$

input
```
int(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x)
```

output
```
x**m/sqrt(a + b*x)
```

3.798 $\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx$

Optimal result

Integrand size = 34, antiderivative size = 13

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx = \frac{x^m}{\sqrt{a+bx}}$$

output
```
x^m/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx = \frac{x^m}{\sqrt{a+bx}}$$

input
```
Integrate[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

output
```
x^m/Sqrt[a + b*x]
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 7.08, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{mx^{m-1}}{\sqrt{a+bx}}-\frac{bx^m}{2(a+bx)^{3/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^m\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(-\frac{1}{2},-m,\frac{1}{2},\frac{bx}{a}+1\right)}{\sqrt{a+bx}}-\frac{2mx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},1-m,\frac{3}{2},\frac{bx}{a}+1\right)}{a}$$

input
```
Int[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

output
```
(x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/((-((b*x)/a))^m*Sqrt[a
 + b*x]) - (2*m*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (
b*x)/a])/(a*(-((b*x)/a))^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(11) = 22.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

method	result	size
orering	$\frac{2x(bx+a)\left(-\frac{b\,x^{m}}{2(bx+a)^{\frac{3}{2}}}+\frac{m\,x^{m-1}}{\sqrt{bx+a}}\right)}{2bmx+2am-bx}$	53

input
```
int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(m-1)/(b*x+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2*x*(b*x+a)/(2*b*m*x+2*a*m-b*x)*(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(m-1)/(b*x+a
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx=\frac{x^m}{\sqrt{bx+a}}$$

input
```
integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
x^m/sqrt(b*x + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 6.15

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx=\frac{a^m a^{-m-\frac{1}{2}} m x^m \Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m\\ m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+1)}-\frac{bx^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

input
```
integrate(-1/2*b*x**m/(b*x+a)**(3/2)+m*x**(-1+m)/(b*x+a)**(1/2),x)
```

output
```
a**m*a**(-m - 1/2)*m*x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar
(I*pi)/a)/gamma(m + 1) - b*x**(m + 1)*gamma(m + 1)*hyper((3/2, m + 1), (m
+ 2,), b*x*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx=\frac{x^m}{\sqrt{bx+a}}$$

input
```
integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
x^m/sqrt(b*x + a)
```

Giac [F]

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx=\int \frac{mx^{m-1}}{\sqrt{bx+a}}-\frac{bx^m}{2\,(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate(m*x^(m - 1)/sqrt(b*x + a) - 1/2*b*x^m/(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx=\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x}}-\frac{b\,x^m}{2\,{(a+b\,x)}^{3/2}}\,dx$$

input
```
int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)),x)
```

output
```
int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx=\frac{x^m}{\sqrt{bx+a}}$$

input
```
int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

output
```
x**m/sqrt(a + b*x)
```

3.799 $\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 23

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {7, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{1-n}{2}+\frac{n-3}{2}}}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 7}$$

$$\int \frac{1}{x\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 7
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
pseudoelliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

input
```
int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)}{a}\right]$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(s
qrt(-a)/sqrt(b*x + a))/a]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x)^(1/2)),x)
```

output
```
-(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a}\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int(1/x/(b*x+a)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.800 $\int \frac{x^m}{\sqrt{1-x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{x^m}{\sqrt{1-x}} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, 1+m, 2+m, x\right)}{1+m}$$

output
```
x^(1+m)*hypergeom([1/2, 1+m],[2+m],x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^m}{\sqrt{1-x}} dx = -2\sqrt{1-x} \text{Hypergeometric2F1}\left(\frac{1}{2}, -m, \frac{3}{2}, 1-x\right)$$

input
```
Integrate[x^m/Sqrt[1 - x],x]
```

output
```
-2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-x}} \, dx$$
$$\Big\downarrow \; 75$$
$$-2\sqrt{1-x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)$$

input
```
Int[x^m/Sqrt[1 - x],x]
```

output
```
-2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - x]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
meijerg	$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],\left[2+m\right],x\right)}{1+m}$	23

input
```
int(x^m/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*hypergeom([1/2,1+m],[2+m],x)/(1+m)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{1-x}}\,dx = \int \frac{x^m}{\sqrt{-x+1}}\,dx$$

input
```
integrate(x^m/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-x^m*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^m}{\sqrt{1-x}}\,dx = -2i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)$$

input
```
integrate(x**m/(1-x)**(1/2),x)
```

output
```
-2*I*sqrt(x - 1)*hyper((1/2, -m), (3/2,), (x - 1)*exp_polar(I*pi))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{1-x}}\,dx = \int \frac{x^m}{\sqrt{-x+1}}\,dx$$

input
```
integrate(x^m/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(-x + 1), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{1-x}}\,dx = \int \frac{x^m}{\sqrt{-x+1}}\,dx$$

input
```
integrate(x^m/(1-x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(-x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{1-x}}\,dx = \int \frac{x^m}{\sqrt{1-x}}\,dx$$

input
```
int(x^m/(1 - x)^(1/2),x)
```

output
```
int(x^m/(1 - x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{1-x}}\,dx = \frac{-2x^m\sqrt{1-x} - 4\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m^2 - 2\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m}{2m+1}$$

input

```
int(x^m/(1-x)^(1/2),x)
```

output

```
(2*( - x**m*sqrt( - x + 1) - 2*int((x**m*sqrt( - x + 1))/(2*m*x**2 - 2*m*x
 + x**2 - x),x)*m**2 - int((x**m*sqrt( - x + 1))/(2*m*x**2 - 2*m*x + x**2
- x),x)*m))/(2*m + 1)
```

3.801 $\int \frac{(cx)^m}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx=\frac{(cx)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+m,2+m,x\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([1/2, 1+m],[2+m],x)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx=-2\sqrt{1-x}x^{-m}(cx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)$$

input
```
Integrate[(c*x)^m/Sqrt[1 - x],x]
```

output
```
(-2*Sqrt[1 - x]*(c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, 1 - x])/x^m
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx$$
$$\Big\downarrow \; 74$$
$$\frac{(cx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},m+1,m+2,x\right)}{c(m+1)}$$

input
```
Int[(c*x)^m/Sqrt[1 - x],x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, x])/(c*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
meijerg	$\frac{(cx)^m x\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],x\right)}{1+m}$	24

input
```
int((c*x)^m/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx = \int \frac{(cx)^m}{\sqrt{-x+1}}\,dx$$

input
```
integrate((c*x)^m/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(c*x)^m*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx = \frac{c^m x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|xe^{2i\pi}\right)}{\Gamma(m+2)}$$

input
```
integrate((c*x)**m/(1-x)**(1/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), x*exp_polar(2*I
*pi))/gamma(m + 2)
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx = \int \frac{(cx)^m}{\sqrt{-x+1}}\,dx$$

input
```
integrate((c*x)^m/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt(-x + 1), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx = \int \frac{(cx)^m}{\sqrt{-x+1}}\,dx$$

input
```
integrate((c*x)^m/(1-x)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(-x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx = \int \frac{(c\,x)^m}{\sqrt{1-x}}\,dx$$

input
```
int((c*x)^m/(1 - x)^(1/2),x)
```

output
```
int((c*x)^m/(1 - x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{1-x}}\,dx$$
$$= \frac{2c^m\left(-x^m\sqrt{1-x}-2\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m^2-\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m\right)}{2m+1}$$

input
```
int((c*x)^m/(1-x)^(1/2),x)
```

output
```
(2*c**m*( - x**m*sqrt( - x + 1) - 2*int((x**m*sqrt( - x + 1))/(2*m*x**2 -
2*m*x + x**2 - x),x)*m**2 - int((x**m*sqrt( - x + 1))/(2*m*x**2 - 2*m*x +
x**2 - x),x)*m))/(2*m + 1)
```

3.802 $\int \frac{x^m}{\sqrt{a-ax}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{x^m}{\sqrt{a-ax}} \, dx = -\frac{2\sqrt{a-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)}{a}$$

output
```
-2*(-a*x+a)^(1/2)*hypergeom([1/2, -m],[3/2],1-x)/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{a-ax}} \, dx = -\frac{2\sqrt{a-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)}{a}$$

input
```
Integrate[x^m/Sqrt[a - a*x],x]
```

output
```
(-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - x])/a
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a-ax}}dx$$
$$\Big\downarrow \text{ 75}$$
$$-\frac{2\sqrt{a-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)}{a}$$

input
```
Int[x^m/Sqrt[a - a*x],x]
```

output
```
(-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - x])/a
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{-ax+a}}dx$$

input
```
int(x^m/(-a*x+a)^(1/2),x)
```

output
```
int(x^m/(-a*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{a-ax}}\,dx = \int \frac{x^m}{\sqrt{-ax+a}}\,dx$$

input
```
integrate(x^m/(-a*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*x + a)*x^m/(a*x - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{x^m}{\sqrt{a-ax}}\,dx = -\frac{2i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)}{\sqrt{a}}$$

input
```
integrate(x**m/(-a*x+a)**(1/2),x)
```

output
```
-2*I*sqrt(x - 1)*hyper((1/2, -m), (3/2,), (x - 1)*exp_polar(I*pi))/sqrt(a)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a-ax}}\,dx = \int \frac{x^m}{\sqrt{-ax+a}}\,dx$$

input
```
integrate(x^m/(-a*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(-a*x + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a-ax}}\,dx = \int \frac{x^m}{\sqrt{-ax+a}}\,dx$$

input
```
integrate(x^m/(-a*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(-a*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a-ax}}\,dx = \int \frac{x^m}{\sqrt{a-a\,x}}\,dx$$

input
```
int(x^m/(a - a*x)^(1/2),x)
```

output
```
int(x^m/(a - a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a-ax}}\,dx = \frac{2\sqrt{a}\left(-x^m\sqrt{1-x}-2\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m^2-\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m\right)}{a\,(2m+1)}$$

input
```
int(x^m/(-a*x+a)^(1/2),x)
```

output
```
(2*sqrt(a)*( - x**m*sqrt( - x + 1) - 2*int((x**m*sqrt( - x + 1))/(2*m*x**2
 - 2*m*x + x**2 - x),x)*m**2 - int((x**m*sqrt( - x + 1))/(2*m*x**2 - 2*m*x
 + x**2 - x),x)*m))/(a*(2*m + 1))
```

3.803 $\int \frac{(cx)^m}{\sqrt{a-ax}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{(cx)^m}{\sqrt{a-ax}}\,dx = -\frac{2x^{-m}(cx)^m\sqrt{a-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)}{a}$$

output
```
-2*(c*x)^m*(-a*x+a)^(1/2)*hypergeom([1/2, -m],[3/2],1-x)/a/(x^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^m}{\sqrt{a-ax}}\,dx = -\frac{2x^{-m}(cx)^m\sqrt{a-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)}{a}$$

input
```
Integrate[(c*x)^m/Sqrt[a - a*x],x]
```

output
```
(-2*(c*x)^m*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - x])/(a*x^m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{a-ax}}dx$$

$$\Big\downarrow \text{77}$$

$$x^{-m}(cx)^m \int \frac{x^m}{\sqrt{a-ax}}dx$$

$$\Big\downarrow \text{75}$$

$$-\frac{2\sqrt{a-ax}x^{-m}(cx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-x\right)}{a}$$

input
```
Int[(c*x)^m/Sqrt[a - a*x],x]
```

output
```
(-2*(c*x)^m*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - x])/(a*x^m)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{-ax + a}} dx$$

input
```
int((c*x)^m/(-a*x+a)^(1/2),x)
```

output
```
int((c*x)^m/(-a*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\sqrt{a - ax}} \, dx = \int \frac{(cx)^m}{\sqrt{-ax + a}} \, dx$$

input
```
integrate((c*x)^m/(-a*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*x + a)*(c*x)^m/(a*x - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^m}{\sqrt{a - ax}} \, dx = \frac{c^m x^{m+1} \Gamma(m+1) \, {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, m+1 \\ m+2 \end{matrix}\right| x e^{2i\pi}\right)}{\sqrt{a} \Gamma(m+2)}$$

input
```
integrate((c*x)**m/(-a*x+a)**(1/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), x*exp_polar(2*I
*pi))/(sqrt(a)*gamma(m + 2))
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{a-ax}}\,dx = \int \frac{(cx)^m}{\sqrt{-ax+a}}\,dx$$

input
```
integrate((c*x)^m/(-a*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt(-a*x + a), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{a-ax}}\,dx = \int \frac{(cx)^m}{\sqrt{-ax+a}}\,dx$$

input
```
integrate((c*x)^m/(-a*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(-a*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{a-ax}}\,dx = \int \frac{(c\,x)^m}{\sqrt{a-a\,x}}\,dx$$

input
```
int((c*x)^m/(a - a*x)^(1/2),x)
```

output
```
int((c*x)^m/(a - a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{a-ax}}\,dx$$
$$= \frac{2c^m\sqrt{a}\left(-x^m\sqrt{1-x}-2\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m^2-\left(\int \frac{x^m\sqrt{1-x}}{2m\,x^2-2mx+x^2-x}dx\right)m\right)}{a\,(2m+1)}$$

input
```
int((c*x)^m/(-a*x+a)^(1/2),x)
```

output
```
(2*c**m*sqrt(a)*( - x**m*sqrt( - x + 1) - 2*int((x**m*sqrt( - x + 1))/(2*m
*x**2 - 2*m*x + x**2 - x),x)*m**2 - int((x**m*sqrt( - x + 1))/(2*m*x**2 -
2*m*x + x**2 - x),x)*m))/(a*(2*m + 1))
```

3.804 $\int x^3(a+bx)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 83

$$\int x^3(a+bx)^p\,dx=-\frac{a^3(a+bx)^{1+p}}{b^4(1+p)}+\frac{3a^2(a+bx)^{2+p}}{b^4(2+p)}-\frac{3a(a+bx)^{3+p}}{b^4(3+p)}+\frac{(a+bx)^{4+p}}{b^4(4+p)}$$

output
```
-a^3*(b*x+a)^(p+1)/b^4/(p+1)+3*a^2*(b*x+a)^(2+p)/b^4/(2+p)-3*a*(b*x+a)^(3+
p)/b^4/(3+p)+(b*x+a)^(4+p)/b^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int x^3(a+bx)^p\,dx=\frac{(a+bx)^{1+p}\left(-\frac{a^3}{1+p}+\frac{3a^2(a+bx)}{2+p}-\frac{3a(a+bx)^2}{3+p}+\frac{(a+bx)^3}{4+p}\right)}{b^4}$$

input
```
Integrate[x^3*(a + b*x)^p,x]
```

output
```
((a + b*x)^(1 + p)*(-(a^3/(1 + p)) + (3*a^2*(a + b*x))/(2 + p) - (3*a*(a +
 b*x)^2)/(3 + p) + (a + b*x)^3/(4 + p)))/b^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^p\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{p+1}}{b^3}-\frac{3a(a+bx)^{p+2}}{b^3}+\frac{(a+bx)^{p+3}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3(a+bx)^{p+1}}{b^4(p+1)}+\frac{3a^2(a+bx)^{p+2}}{b^4(p+2)}-\frac{3a(a+bx)^{p+3}}{b^4(p+3)}+\frac{(a+bx)^{p+4}}{b^4(p+4)}$$

input
```
Int[x^3*(a + b*x)^p,x]
```

output
```
-((a^3*(a + b*x)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x)^(2 + p))/(b^4*
(2 + p)) - (3*a*(a + b*x)^(3 + p))/(b^4*(3 + p)) + (a + b*x)^(4 + p)/(b^4*
(4 + p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

method	result
gosper	$-\frac{(bx+a)^{p+1}\left(-b^{3}p^{3}x^{3}-6b^{3}p^{2}x^{3}+3a\,b^{2}p^{2}x^{2}-11b^{3}p\,x^{3}+9a\,b^{2}p\,x^{2}-6b^{3}x^{3}-6a^{2}bpx+6a\,b^{2}x^{2}-6a^{2}bx+6a^{3}\right)}{b^{4}\left(p^{4}+10p^{3}+35p^{2}+50p+24\right)}$
orering	$-\frac{(bx+a)^{p}\left(-b^{3}p^{3}x^{3}-6b^{3}p^{2}x^{3}+3a\,b^{2}p^{2}x^{2}-11b^{3}p\,x^{3}+9a\,b^{2}p\,x^{2}-6b^{3}x^{3}-6a^{2}bpx+6a\,b^{2}x^{2}-6a^{2}bx+6a^{3}\right)(bx+a)}{b^{4}\left(p^{4}+10p^{3}+35p^{2}+50p+24\right)}$
risch	$-\frac{\left(-b^{4}p^{3}x^{4}-a\,b^{3}p^{3}x^{3}-6b^{4}p^{2}x^{4}-3a\,b^{3}p^{2}x^{3}-11b^{4}p\,x^{4}+3a^{2}b^{2}p^{2}x^{2}-2x^{3}ap\,b^{3}-6b^{4}x^{4}+3a^{2}p\,x^{2}b^{2}-6xp\,a^{3}b+6a^{4}\right)(bx+a)^{p}}{(3+p)(4+p)(2+p)(p+1)b^{4}}$
norman	$\frac{x^{4}e^{p\ln(bx+a)}}{4+p}+\frac{ap\,x^{3}e^{p\ln(bx+a)}}{b(p^{2}+7p+12)}-\frac{6a^{4}e^{p\ln(bx+a)}}{b^{4}(p^{4}+10p^{3}+35p^{2}+50p+24)}-\frac{3a^{2}p\,x^{2}e^{p\ln(bx+a)}}{b^{2}(p^{3}+9p^{2}+26p+24)}+\frac{6p\,a^{3}x\,e^{p\ln(bx+a)}}{b^{3}(p^{4}+10p^{3}+35p^{2}+50p+}$
parallelrisch	$\frac{x^{4}(bx+a)^{p}b^{4}p^{3}+6x^{4}(bx+a)^{p}b^{4}p^{2}+x^{3}(bx+a)^{p}a\,b^{3}p^{3}+11x^{4}(bx+a)^{p}b^{4}p+3x^{3}(bx+a)^{p}a\,b^{3}p^{2}+6x^{4}(bx+a)^{p}b^{4}+2x^{3}(bx+a)^{p}a\,b}{b^{4}(p^{4}+10p^{3}+35p^{2}+50p+24)}$

input
```
int(x^3*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(b*x+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p^3*x^3-6*b^3*p^2*x
^3+3*a*b^2*p^2*x^2-11*b^3*p*x^3+9*a*b^2*p*x^2-6*b^3*x^3-6*a^2*b*p*x+6*a*b^
2*x^2-6*a^2*b*x+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.72

$$\int x^3(a+bx)^p\,dx = \frac{(6\,a^3bpx+(b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4)x^4-6\,a^4+(ab^3p^3+3\,ab^3p^2+2\,ab^3p)x^3-3\,(a^2b^2p^2+a^2b^2p)x^2}{b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4}$$

input
```
integrate(x^3*(b*x+a)^p,x, algorithm="fricas")
```

output
```
(6*a^3*b*p*x + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^3 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^2)*(
b*x + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. $2(71) = 142$.

Time = 0.55 (sec) , antiderivative size = 1318, normalized size of antiderivative = 15.88

$$\int x^3(a+bx)^p\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*
a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a*
*3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**
2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3) + 18*a*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**
7*x**3) + 6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3), Eq(p, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4
*a*b**5*x + 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2)
 - 12*a**2*b*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*
a**2*b*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b
+ x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 +
 4*a*b**5*x + 2*b**6*x**2), Eq(p, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2
*b**5*x) + 6*a**3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**
4 + 2*b**5*x) - 3*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4
+ 2*b**5*x), Eq(p, -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(
2*b**2) + x**3/(3*b), Eq(p, -1)), (-6*a**4*(a + b*x)**p/(b**4*p**4 + 10*b*
*4*p**3 + 35*b**4*p**2 + 50*b**4*p + 24*b**4) + 6*a**3*b*p*x*(a + b*x)**p/
(b**4*p**4 + 10*b**4*p**3 + 35*b**4*p**2 + 50*b**4*p + 24*b**4) - 3*a**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

$$\int x^3(a+bx)^p\,dx = \frac{((p^3+6\,p^2+11\,p+6)b^4x^4+(p^3+3\,p^2+2\,p)ab^3x^3-3\,(p^2+p)a^2b^2x^2+6\,a^3bpx-6\,a^4)(bx+a)^p}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

input
```
integrate(x^3*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*x^4 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^3 - 3*(p^2
 + p)*a^2*b^2*x^2 + 6*a^3*b*p*x - 6*a^4)*(b*x + a)^p/((p^4 + 10*p^3 + 35*p
^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(83) = 166.

Time = 0.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.72

$$\int x^3(a+bx)^p\,dx = \frac{(bx+a)^pb^4p^3x^4+(bx+a)^pab^3p^3x^3+6\,(bx+a)^pb^4p^2x^4+3\,(bx+a)^pab^3p^2x^3+11\,(bx+a)^pb^4px^4-3\,(b}{b^4p^4+10\,b^4p^3+}$$

input
```
integrate(x^3*(b*x+a)^p,x, algorithm="giac")
```

output
```
((b*x + a)^p*b^4*p^3*x^4 + (b*x + a)^p*a*b^3*p^3*x^3 + 6*(b*x + a)^p*b^4*p
^2*x^4 + 3*(b*x + a)^p*a*b^3*p^2*x^3 + 11*(b*x + a)^p*b^4*p*x^4 - 3*(b*x +
 a)^p*a^2*b^2*p^2*x^2 + 2*(b*x + a)^p*a*b^3*p*x^3 + 6*(b*x + a)^p*b^4*x^4
- 3*(b*x + a)^p*a^2*b^2*p*x^2 + 6*(b*x + a)^p*a^3*b*p*x - 6*(b*x + a)^p*a^
4)/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.12

$$\int x^3(a+bx)^p\,dx=(a+b\,x)^p\left(\frac{x^4\,\left(p^3+6\,p^2+11\,p+6\right)}{p^4+10\,p^3+35\,p^2+50\,p+24}-\frac{6\,a^4}{b^4\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{6\,a^3\,p\,x}{b^3\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{a\,p\,x^3\,\left(p^2+3\,p+2\right)}{b\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{3\,a^2\,p\,x^2\,\left(p+1\right)}{b^2\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}\right)$$

input
```
int(x^3*(a + b*x)^p,x)
```

output
```
(a + b*x)^p*((x^4*(11*p + 6*p^2 + p^3 + 6))/(50*p + 35*p^2 + 10*p^3 + p^4
+ 24) - (6*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (6*a^3*p*x)/(b
^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^3*(3*p + p^2 + 2))/(b*(50
*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^2*(p + 1))/(b^2*(50*p + 35*
p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.71

$$\int x^3(a+bx)^p\,dx$$
$$=\frac{(bx+a)^p\left(b^4p^3x^4+a\,b^3p^3x^3+6b^4p^2x^4+3a\,b^3p^2x^3+11b^4p\,x^4-3a^2b^2p^2x^2+2a\,b^3p\,x^3+6b^4x^4-3a^2b^2p\right.}{b^4\left(p^4+10p^3+35p^2+50p+24\right)}$$

input
```
int(x^3*(b*x+a)^p,x)
```

output
```
((a + b*x)**p*( - 6*a**4 + 6*a**3*b*p*x - 3*a**2*b**2*p**2*x**2 - 3*a**2*b
**2*p*x**2 + a*b**3*p**3*x**3 + 3*a*b**3*p**2*x**3 + 2*a*b**3*p*x**3 + b**
4*p**3*x**4 + 6*b**4*p**2*x**4 + 11*b**4*p*x**4 + 6*b**4*x**4))/(b**4*(p**
4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.805 $\int x^2(a+bx)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int x^2(a+bx)^p\,dx=\frac{a^2(a+bx)^{1+p}}{b^3(1+p)}-\frac{2a(a+bx)^{2+p}}{b^3(2+p)}+\frac{(a+bx)^{3+p}}{b^3(3+p)}$$

output
```
a^2*(b*x+a)^(p+1)/b^3/(p+1)-2*a*(b*x+a)^(2+p)/b^3/(2+p)+(b*x+a)^(3+p)/b^3/
(3+p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x^2(a+bx)^p\,dx=\frac{(a+bx)^{1+p}\left(2a^2-2ab(1+p)x+b^2(2+3p+p^2)\,x^2\right)}{b^3(1+p)(2+p)(3+p)}$$

input
```
Integrate[x^2*(a + b*x)^p,x]
```

output
```
((a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/(b
^3*(1 + p)*(2 + p)*(3 + p))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^p\,dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{p+1}}{b^2}+\frac{(a+bx)^{p+2}}{b^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

input
```
Int[x^2*(a + b*x)^p,x]
```

output
```
(a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(2 +
p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result
gosper	$\frac{(bx+a)^{p+1}(b^2p^2x^2+3b^2p\,x^2-2abpx+2b^2x^2-2abx+2a^2)}{b^3(p^3+6p^2+11p+6)}$
orering	$\frac{(bx+a)(b^2p^2x^2+3b^2p\,x^2-2abpx+2b^2x^2-2abx+2a^2)(bx+a)^p}{b^3(p^3+6p^2+11p+6)}$
risch	$\frac{(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)(bx+a)^p}{(2+p)(3+p)(p+1)b^3}$
norman	$\frac{x^3\mathrm{e}^{p\ln(bx+a)}}{3+p}+\frac{ap\,x^2\mathrm{e}^{p\ln(bx+a)}}{b(p^2+5p+6)}+\frac{2a^3\mathrm{e}^{p\ln(bx+a)}}{b^3(p^3+6p^2+11p+6)}-\frac{2p\,a^2x\,\mathrm{e}^{p\ln(bx+a)}}{b^2(p^3+6p^2+11p+6)}$
parallelrisch	$\frac{x^3(bx+a)^pa\,b^3p^2+3x^3(bx+a)^pa\,b^3p+x^2(bx+a)^pa^2b^2p^2+2x^3(bx+a)^pa\,b^3+x^2(bx+a)^pa^2b^2p-2x(bx+a)^pa^3bp+2(bx+a)^pa^4}{(3+p)(2+p)(p+1)b^3a}$

input
```
int(x^2*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(b*x+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+
2*b^2*x^2-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

$$\int x^2(a+bx)^p\,dx = -\frac{(2\,a^2bpx-(b^3p^2+3\,b^3p+2\,b^3)x^3-2\,a^3-(ab^2p^2+ab^2p)x^2)(bx+a)^p}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

input
```
integrate(x^2*(b*x+a)^p,x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b
^2*p)*x^2)*(b*x + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(51) = 102$.

Time = 0.38 (sec) , antiderivative size = 597, normalized size of antiderivative = 9.95

$$\int x^2(a+bx)^p\,dx$$
$$= \begin{cases} \frac{a^px^3}{3} \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} - \frac{2a^2bpx(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2p^2x^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2px^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{b^3p^2x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a
*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) +
 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b*
*3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b
**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3
- a*x/b**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p/(b**3*p**3 + 6*
b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**2*(a + b*x)**p/(b**3*p**3
 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p*x**2*(a + b*x)**p/(b**3*p*
*3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3
*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b*
*3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b*
*3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x^2(a+bx)^p\,dx=\frac{((p^2+3\,p+2)b^3x^3+(p^2+p)ab^2x^2-2\,a^2bpx+2\,a^3)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3}$$

input
```
integrate(x^2*(b*x+a)^p,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*a^3)*(b*x
 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(60) = 120.

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.33

$$\int x^2(a+bx)^p\,dx$$
$$=\frac{(bx+a)^pb^3p^2x^3+(bx+a)^pab^2p^2x^2+3\,(bx+a)^pb^3px^3+(bx+a)^pab^2px^2+2\,(bx+a)^pb^3x^3-2\,(bx+a}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

input
```
integrate(x^2*(b*x+a)^p,x, algorithm="giac")
```

output
```
((b*x + a)^p*b^3*p^2*x^3 + (b*x + a)^p*a*b^2*p^2*x^2 + 3*(b*x + a)^p*b^3*p
*x^3 + (b*x + a)^p*a*b^2*p*x^2 + 2*(b*x + a)^p*b^3*x^3 - 2*(b*x + a)^p*a^2
*b*p*x + 2*(b*x + a)^p*a^3)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.20

$$\int x^2(a+bx)^p\,dx = \begin{cases} \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3} & \text{if } p=-1 \\ \frac{x}{b^2}-\frac{a^2}{b^3\,(a+b\,x)}-\frac{2\,a\,\ln(a+b\,x)}{b^3} & \text{if } p=-2 \\ \frac{\ln(a+b\,x)+\frac{2\,a}{a+b\,x}-\frac{a^2}{2\,(a+b\,x)^2}}{b^3} & \text{if } p=-3 \\ \frac{2\,(a+b\,x)^{p+1}\,\left(8\,a^2-8\,a\,b\,p\,x-8\,a\,b\,x+4\,b^2\,p^2\,x^2+12\,b^2\,p\,x^2+8\,b^2\,x^2\right)}{b^3\,(8\,p^3+48\,p^2+88\,p+48)} & \text{if } p\neq -1\wedge p\neq -2\wedge p\neq -3 \end{cases}$$

input
```
int(x^2*(a + b*x)^p,x)
```

output
```
piecewise(p == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), p ==
-2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, p == -3, (log(a
+ b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, p ~= -1 & p ~= -2 & p
 ~= -3, (2*(a + b*x)^(p + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*p*x^2 - 8*a*b*x +
 4*b^2*p^2*x^2 - 8*a*b*p*x))/(b^3*(88*p + 48*p^2 + 8*p^3 + 48)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int x^2(a+bx)^p\,dx = \frac{(bx+a)^p\,(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+a\,b^2p\,x^2+2b^3x^3-2a^2bpx+2a^3)}{b^3\,(p^3+6p^2+11p+6)}$$

input
```
int(x^2*(b*x+a)^p,x)
```

output
```
((a + b*x)**p*(2*a**3 - 2*a**2*b*p*x + a*b**2*p**2*x**2 + a*b**2*p*x**2 +
b**3*p**2*x**3 + 3*b**3*p*x**3 + 2*b**3*x**3))/(b**3*(p**3 + 6*p**2 + 11*p
 + 6))
```

3.806 $\int x(a+bx)^p\,dx$

Optimal result

Integrand size = 9, antiderivative size = 39

$$\int x(a+bx)^p\,dx = -\frac{a(a+bx)^{1+p}}{b^2(1+p)}+\frac{(a+bx)^{2+p}}{b^2(2+p)}$$

output
```
-a*(b*x+a)^(p+1)/b^2/(p+1)+(b*x+a)^(2+p)/b^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int x(a+bx)^p\,dx = \frac{(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)}$$

input
```
Integrate[x*(a + b*x)^p,x]
```

output
```
((a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^p\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{(a+bx)^{p+1}}{b}-\frac{a(a+bx)^p}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

input
```
Int[x*(a + b*x)^p,x]
```

output
```
-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{(bx+a)^{p+1}(-xpb-bx+a)}{b^2(p^2+3p+2)}$	36
orering	$-\frac{(bx+a)^{p}(-xpb-bx+a)(bx+a)}{b^2(p^2+3p+2)}$	39
risch	$-\frac{(-b^2p\,x^2-abpx-b^2x^2+a^2)(bx+a)^p}{b^2(2+p)(p+1)}$	50
parallelrisch	$\frac{x^2(bx+a)^p b^2p+x^2(bx+a)^p b^2+x(bx+a)^p abp-(bx+a)^p a^2}{b^2(p^2+3p+2)}$	69
norman	$\frac{x^2 e^{p\ln(bx+a)}}{2+p}+\frac{pax\,e^{p\ln(bx+a)}}{b(p^2+3p+2)}-\frac{a^2 e^{p\ln(bx+a)}}{b^2(p^2+3p+2)}$	73

input
```
int(x*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(b*x+a)^(p+1)/(p^2+3*p+2)*(-b*p*x-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int x(a+bx)^p\,dx=\frac{(abpx+(b^2p+b^2)x^2-a^2)(bx+a)^p}{b^2p^2+3\,b^2p+2\,b^2}$$

input
```
integrate(x*(b*x+a)^p,x, algorithm="fricas")
```

output
```
(a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 5.15

$$\int x(a+bx)^p\,dx = \begin{cases} \frac{a^px^2}{2} & \text{for}\ b=0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\ p=-2 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for}\ p=-1 \\ -\frac{a^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{abpx(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2px^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2x^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**p,x)
```

output
```
Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(
a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -2)), (-a*log
(a/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*
p + 2*b**2) + a*b*p*x*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*
p*x**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)*
*p/(b**2*p**2 + 3*b**2*p + 2*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int x(a+bx)^p\,dx = \frac{(b^2(p+1)x^2 + abpx - a^2)(bx+a)^p}{(p^2+3\,p+2)b^2}$$

input
```
integrate(x*(b*x+a)^p,x, algorithm="maxima")
```

output
```
(b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

$$\int x(a+bx)^p\,dx = \frac{(bx+a)^p b^2 p x^2 + (bx+a)^p abpx + (bx+a)^p b^2 x^2 - (bx+a)^p a^2}{b^2 p^2 + 3\,b^2 p + 2\,b^2}$$

input
```
integrate(x*(b*x+a)^p,x, algorithm="giac")
```

output
```
((b*x + a)^p*b^2*p*x^2 + (b*x + a)^p*a*b*p*x + (b*x + a)^p*b^2*x^2 - (b*x
+ a)^p*a^2)/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.41

$$\int x(a+bx)^p\,dx = \begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if } p=-1 \\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if } p=-2 \\ \frac{2\left(\frac{(a+b\,x)^{p+2}}{2\,p+4}-\frac{a\,(a+b\,x)^{p+1}}{2\,p+2}\right)}{b^2} & \text{if } p\neq -1 \land p\neq -2 \end{cases}$$

input
```
int(x*(a + b*x)^p,x)
```

output
```
piecewise(p == -1, -(a*log(a + b*x) - b*x)/b^2, p == -2, (log(a + b*x) + a
/(a + b*x))/b^2, p ~= -1 & p ~= -2, (2*((a + b*x)^(p + 2)/(2*p + 4) - (a*(
a + b*x)^(p + 1))/(2*p + 2)))/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int x(a+bx)^p\,dx=\frac{(bx+a)^p\,(b^2p\,x^2+abpx+b^2x^2-a^2)}{b^2\,(p^2+3p+2)}$$

input
```
int(x*(b*x+a)^p,x)
```

output
```
((a + b*x)**p*( - a**2 + a*b*p*x + b**2*p*x**2 + b**2*x**2))/(b**2*(p**2 +
 3*p + 2))
```

3.807 $\int (a+bx)^p \, dx$

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (a+bx)^p \, dx = \frac{(a+bx)^{1+p}}{b(1+p)}$$

output
```
(b*x+a)^(p+1)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p \, dx = \frac{(a+bx)^{1+p}}{b(1+p)}$$

input
```
Integrate[(a + b*x)^p,x]
```

output
```
(a + b*x)^(1 + p)/(b*(1 + p))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^p\,dx$$
$$\Big\downarrow \; 17$$
$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

input
```
Int[(a + b*x)^p,x]
```

output
```
(a + b*x)^(1 + p)/(b*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(bx+a)^{p+1}}{b(p+1)}$	19
default	$\frac{(bx+a)^{p+1}}{b(p+1)}$	19
risch	$\frac{(bx+a)(bx+a)^{p}}{b(p+1)}$	22
orering	$\frac{(bx+a)(bx+a)^{p}}{b(p+1)}$	22
parallelrisch	$\frac{x(bx+a)^{p}ab+(bx+a)^{p}a^{2}}{(p+1)ab}$	36
norman	$\frac{x\,\mathrm{e}^{p\ln(bx+a)}}{p+1}+\frac{a\,\mathrm{e}^{p\ln(bx+a)}}{b(p+1)}$	37

input
```
int((b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)^(p+1)/b/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^p\,dx=\frac{(bx+a)(bx+a)^p}{bp+b}$$

input
```
integrate((b*x+a)^p,x, algorithm="fricas")
```

output
```
(b*x + a)*(b*x + a)^p/(b*p + b)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^p \, dx = \frac{\begin{cases} \frac{(a+bx)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

input
```
integrate((b*x+a)**p,x)
```

output
```
Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x), True))/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p \, dx = \frac{(bx+a)^{p+1}}{b(p+1)}$$

input
```
integrate((b*x+a)^p,x, algorithm="maxima")
```

output
```
(b*x + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p \, dx = \frac{(bx+a)^{p+1}}{b(p+1)}$$

input
```
integrate((b*x+a)^p,x, algorithm="giac")
```

output
```
(b*x + a)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^p\,dx=\frac{(a+b\,x)^{p+1}}{b\,(p+1)}$$

input
```
int((a + b*x)^p,x)
```

output
```
(a + b*x)^(p + 1)/(b*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int (a+bx)^p\,dx=\frac{(bx+a)^p\,(bx+a)}{b\,(p+1)}$$

input
```
int((b*x+a)^p,x)
```

output
```
((a + b*x)**p*(a + b*x))/(b*(p + 1))
```

3.808 $\int \frac{(a+bx)^p}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{(a+bx)^p}{x} dx = -\frac{(a+bx)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)}$$

output
```
-(b*x+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^p}{x} dx = -\frac{(a+bx)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx}{a}\right)}{a(1+p)}$$

input
```
Integrate[(a + b*x)^p/x,x]
```

output
```
-(((a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/(a*(
1 + p)))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x} \, dx$$

$$\Big\downarrow \; 75$$

$$-\frac{(a+bx)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx}{a}+1\right)}{a(p+1)}$$

input
```
Int[(a + b*x)^p/x,x]
```

output
```
-(((a + b*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x)/a])/(a*(
1 + p)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^p}{x} dx$$

input
```
int((b*x+a)^p/x,x)
```

output
```
int((b*x+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x}\,dx=\int \frac{(bx+a)^p}{x}\,dx$$

input
```
integrate((b*x+a)^p/x,x, algorithm="fricas")
```

output
```
integral((b*x + a)^p/x, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26)=52$.

Time = 0.72 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int \frac{(a+bx)^p}{x}\,dx=-\frac{b^{p+1}p\left(\frac{a}{b}+x\right)^{p+1}\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,p+1\right)\Gamma(p+1)}{a\Gamma\left(p+2\right)}-\frac{b^{p+1}\left(\frac{a}{b}+x\right)^{p+1}\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,p+1\right)\Gamma(p+1)}{a\Gamma\left(p+2\right)}$$

input
```
integrate((b*x+a)**p/x,x)
```

output
```
-b**(p + 1)*p*(a/b + x)**(p + 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)*gamma(p
 + 1)/(a*gamma(p + 2)) - b**(p + 1)*(a/b + x)**(p + 1)*lerchphi(b*(a/b + x
)/a, 1, p + 1)*gamma(p + 1)/(a*gamma(p + 2))
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x}\,dx = \int \frac{(bx+a)^p}{x}\,dx$$

input
```
integrate((b*x+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x}\,dx = \int \frac{(bx+a)^p}{x}\,dx$$

input
```
integrate((b*x+a)^p/x,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x}\,dx = \int \frac{(a+b\,x)^p}{x}\,dx$$

input
```
int((a + b*x)^p/x,x)
```

output
```
int((a + b*x)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x}\,dx = \frac{(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)ap}{p}$$

input
```
int((b*x+a)^p/x,x)
```

output
```
((a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*a*p)/p
```

3.809 $\int \frac{(a+bx)^p}{x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{(a+bx)^p}{x^2} \, dx = \frac{b(a+bx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)}$$

output
```
b*(b*x+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x/a)/a^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^p}{x^2} \, dx = \frac{b(a+bx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx}{a}\right)}{a^2(1+p)}$$

input
```
Integrate[(a + b*x)^p/x^2,x]
```

output
```
(b*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a])/(a^2
*(1 + p))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{75\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x^2} \, dx$$

$$\Big\downarrow \ 75$$

$$\frac{b(a+bx)^{p+1} \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx}{a}+1\right)}{a^2(p+1)}$$

input
```
Int[(a + b*x)^p/x^2,x]
```

output
```
(b*(a + b*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x)/a])/(a^2
*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^p}{x^2} dx$$

input
```
int((b*x+a)^p/x^2,x)
```

output
```
int((b*x+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x^2}\,dx = \int \frac{(bx+a)^p}{x^2}\,dx$$

input
```
integrate((b*x+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*x + a)^p/x^2, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(27) = 54.

Time = 0.97 (sec) , antiderivative size = 333, normalized size of antiderivative = 9.51

$$\int \frac{(a+bx)^p}{x^2}\,dx = \frac{ab^{p+2}p^2\left(\frac{a}{b}+x\right)^{p+1}\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,p+1\right)\Gamma(p+1)}{-a^3\Gamma(p+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(p+2)} + \frac{ab^{p+2}p\left(\frac{a}{b}+x\right)^{p+1}\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,p+1\right)\Gamma(p+1)}{-a^3\Gamma(p+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(p+2)} - \frac{ab^{p+2}p\left(\frac{a}{b}+x\right)^{p+1}\Gamma(p+1)}{-a^3\Gamma(p+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(p+2)} - \frac{ab^{p+2}\left(\frac{a}{b}+x\right)^{p+1}\Gamma(p+1)}{-a^3\Gamma(p+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(p+2)} - \frac{bb^{p+2}p^2\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^{p+1}\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,p+1\right)\Gamma(p+1)}{-a^3\Gamma(p+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(p+2)} - \frac{bb^{p+2}p\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^{p+1}\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,p+1\right)\Gamma(p+1)}{-a^3\Gamma(p+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(p+2)}$$

input
```
integrate((b*x+a)**p/x**2,x)
```

output
```
a*b**(p + 2)*p**2*(a/b + x)**(p + 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)*gam
ma(p + 1)/(-a**3*gamma(p + 2) + a**2*b*(a/b + x)*gamma(p + 2)) + a*b**(p +
 2)*p*(a/b + x)**(p + 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)*gamma(p + 1)/(-
a**3*gamma(p + 2) + a**2*b*(a/b + x)*gamma(p + 2)) - a*b**(p + 2)*p*(a/b +
 x)**(p + 1)*gamma(p + 1)/(-a**3*gamma(p + 2) + a**2*b*(a/b + x)*gamma(p +
 2)) - a*b**(p + 2)*(a/b + x)**(p + 1)*gamma(p + 1)/(-a**3*gamma(p + 2) +
a**2*b*(a/b + x)*gamma(p + 2)) - b*b**(p + 2)*p**2*(a/b + x)*(a/b + x)**(p
 + 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)*gamma(p + 1)/(-a**3*gamma(p + 2) +
 a**2*b*(a/b + x)*gamma(p + 2)) - b*b**(p + 2)*p*(a/b + x)*(a/b + x)**(p +
 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)*gamma(p + 1)/(-a**3*gamma(p + 2) + a
**2*b*(a/b + x)*gamma(p + 2))
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x^2}\,dx = \int \frac{(bx+a)^p}{x^2}\,dx$$

input
```
integrate((b*x+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x^2}\,dx = \int \frac{(bx+a)^p}{x^2}\,dx$$

input
```
integrate((b*x+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x^2}\,dx = \int \frac{(a+b\,x)^p}{x^2}\,dx$$

input
```
int((a + b*x)^p/x^2,x)
```

output
```
int((a + b*x)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x^2}\,dx = \frac{-(bx+a)^p + \left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right) bpx}{x}$$

input
```
int((b*x+a)^p/x^2,x)
```

output
```
( - (a + b*x)**p + int((a + b*x)**p/(a*x + b*x**2),x)*b*p*x)/x
```

3.810 $\int \frac{(a+bx)^p}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{(a+bx)^p}{x^3} dx = -\frac{b^2(a+bx)^{1+p} \text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)}$$

output
```
-b^2*(b*x+a)^(p+1)*hypergeom([3, p+1],[2+p],1+b*x/a)/a^3/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^p}{x^3} dx = -\frac{b^2(a+bx)^{1+p} \text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{bx}{a}\right)}{a^3(1+p)}$$

input
```
Integrate[(a + b*x)^p/x^3,x]
```

output
```
-((b^2*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x)/a])/
(a^3*(1 + p)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x^3} \, dx$$

$$\Big\downarrow \text{75}$$

$$-\frac{b^2(a+bx)^{p+1} \text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{bx}{a}+1\right)}{a^3(p+1)}$$

input
```
Int[(a + b*x)^p/x^3,x]
```

output
```
-((b^2*(a + b*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*x)/a])/
(a^3*(1 + p)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int \frac{(bx+a)^p}{x^3}dx$$

input
```
int((b*x+a)^p/x^3,x)
```

output
```
int((b*x+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x^3}\,dx=\int \frac{(bx+a)^p}{x^3}\,dx$$

input
```
integrate((b*x+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((b*x + a)^p/x^3, x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(31) = 62$.

Time = 1.95 (sec) , antiderivative size = 899, normalized size of antiderivative = 23.66

$$\int \frac{(a+bx)^p}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)**p/x**3,x)
```

output
```
-a**2*b**(p + 3)*p**3*(a/b + x)**(p + 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)
*gamma(p + 1)/(2*a**5*gamma(p + 2) - 4*a**4*b*(a/b + x)*gamma(p + 2) + 2*a
**3*b**2*(a/b + x)**2*gamma(p + 2)) + a**2*b**(p + 3)*p**2*(a/b + x)**(p +
 1)*gamma(p + 1)/(2*a**5*gamma(p + 2) - 4*a**4*b*(a/b + x)*gamma(p + 2) +
2*a**3*b**2*(a/b + x)**2*gamma(p + 2)) + a**2*b**(p + 3)*p*(a/b + x)**(p +
 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)*gamma(p + 1)/(2*a**5*gamma(p + 2) -
4*a**4*b*(a/b + x)*gamma(p + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(p + 2)) -
 a**2*b**(p + 3)*p*(a/b + x)**(p + 1)*gamma(p + 1)/(2*a**5*gamma(p + 2) -
4*a**4*b*(a/b + x)*gamma(p + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(p + 2)) -
 2*a**2*b**(p + 3)*(a/b + x)**(p + 1)*gamma(p + 1)/(2*a**5*gamma(p + 2) -
4*a**4*b*(a/b + x)*gamma(p + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(p + 2)) +
 2*a*b*b**(p + 3)*p**3*(a/b + x)*(a/b + x)**(p + 1)*lerchphi(b*(a/b + x)/a
, 1, p + 1)*gamma(p + 1)/(2*a**5*gamma(p + 2) - 4*a**4*b*(a/b + x)*gamma(p
 + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(p + 2)) - a*b*b**(p + 3)*p**2*(a/b
+ x)*(a/b + x)**(p + 1)*gamma(p + 1)/(2*a**5*gamma(p + 2) - 4*a**4*b*(a/b
+ x)*gamma(p + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(p + 2)) - 2*a*b*b**(p +
 3)*p*(a/b + x)*(a/b + x)**(p + 1)*lerchphi(b*(a/b + x)/a, 1, p + 1)*gamma
(p + 1)/(2*a**5*gamma(p + 2) - 4*a**4*b*(a/b + x)*gamma(p + 2) + 2*a**3*b*
*2*(a/b + x)**2*gamma(p + 2)) + a*b*b**(p + 3)*(a/b + x)*(a/b + x)**(p + 1
)*gamma(p + 1)/(2*a**5*gamma(p + 2) - 4*a**4*b*(a/b + x)*gamma(p + 2) +...
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x^3} \, dx = \int \frac{(bx+a)^p}{x^3} \, dx$$

input
```
integrate((b*x+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x^3}\,dx = \int \frac{(bx+a)^p}{x^3}\,dx$$

input
```
integrate((b*x+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x^3}\,dx = \int \frac{(a+b\,x)^p}{x^3}\,dx$$

input
```
int((a + b*x)^p/x^3,x)
```

output
```
int((a + b*x)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x^3}\,dx = \frac{-(bx+a)^p\,a-(bx+a)^p\,bpx+\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p^2x^2-\left(\int \frac{(bx+a)^p}{b\,x^2+ax}dx\right)b^2p\,x^2}{2a\,x^2}$$

input
```
int((b*x+a)^p/x^3,x)
```

output
```
( - (a + b*x)**p*a - (a + b*x)**p*b*p*x + int((a + b*x)**p/(a*x + b*x**2),
x)*b**2*p**2*x**2 - int((a + b*x)**p/(a*x + b*x**2),x)*b**2*p*x**2)/(2*a*x
**2)
```

3.811 $\int x^{3/2}(a+bx)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int x^{3/2}(a+bx)^p\,dx=\frac{2}{5}x^{5/2}(a+bx)^p\left(\frac{a+bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx}{a}\right)$$

output
```
2/5*x^(5/2)*(b*x+a)^p*hypergeom([5/2, -p],[7/2],-b*x/a)/(((b*x+a)/a)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x^{3/2}(a+bx)^p\,dx=\frac{2}{5}x^{5/2}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx}{a}\right)$$

input
```
Integrate[x^(3/2)*(a + b*x)^p,x]
```

output
```
(2*x^(5/2)*(a + b*x)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x)/a)])/(5*(1
+ (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^p\,dx$$

$$\Big\downarrow 76$$

$$(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int x^{3/2}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow 74$$

$$\frac{2}{5}x^{5/2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx}{a}\right)$$

input
```
Int[x^(3/2)*(a + b*x)^p,x]
```

output
```
(2*x^(5/2)*(a + b*x)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x)/a)])/(5*(1
+ (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int x^{\frac{3}{2}} (bx + a)^p \, dx$$

input
```
int(x^(3/2)*(b*x+a)^p,x)
```

output
```
int(x^(3/2)*(b*x+a)^p,x)
```

Fricas [F]

$$\int x^{3/2} (a + bx)^p \, dx = \int (bx + a)^p x^{\frac{3}{2}} \, dx$$

input
```
integrate(x^(3/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x + a)^p*x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 68.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^{3/2}(a+bx)^p\,dx=\frac{2a^px^{\frac{5}{2}}{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**(3/2)*(b*x+a)**p,x)
```

output
```
2*a**p*x**(5/2)*hyper((5/2, -p), (7/2,), b*x*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^{3/2}(a+bx)^p\,dx=\int (bx+a)^px^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}(a+bx)^p\,dx=\int (bx+a)^px^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^p\,dx=\int x^{3/2}\,(a+b\,x)^p\,dx$$

input
```
int(x^(3/2)*(a + b*x)^p,x)
```

output
```
int(x^(3/2)*(a + b*x)^p, x)
```

Reduce [F]

$$\int x^{3/2}(a+bx)^p\,dx=\frac{-12\sqrt{x}\,(bx+a)^p\,a^2p+8\sqrt{x}\,(bx+a)^p\,ab\,p^2x+4\sqrt{x}\,(bx+a)^p\,abpx+8\sqrt{x}\,(bx+a)^p\,b^2p^2x^2+1}{}$$

input
```
int(x^(3/2)*(b*x+a)^p,x)
```

output
```
(2*( - 6*sqrt(x)*(a + b*x)**p*a**2*p + 4*sqrt(x)*(a + b*x)**p*a*b*p**2*x +
 2*sqrt(x)*(a + b*x)**p*a*b*p*x + 4*sqrt(x)*(a + b*x)**p*b**2*p**2*x**2 +
8*sqrt(x)*(a + b*x)**p*b**2*p*x**2 + 3*sqrt(x)*(a + b*x)**p*b**2*x**2 + 24
*int((sqrt(x)*(a + b*x)**p)/(8*a*p**3*x + 36*a*p**2*x + 46*a*p*x + 15*a*x
+ 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**3*p**4 +
 108*int((sqrt(x)*(a + b*x)**p)/(8*a*p**3*x + 36*a*p**2*x + 46*a*p*x + 15*
a*x + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**3*p*
*3 + 138*int((sqrt(x)*(a + b*x)**p)/(8*a*p**3*x + 36*a*p**2*x + 46*a*p*x +
 15*a*x + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**
3*p**2 + 45*int((sqrt(x)*(a + b*x)**p)/(8*a*p**3*x + 36*a*p**2*x + 46*a*p*
x + 15*a*x + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*
a**3*p))/(b**2*(8*p**3 + 36*p**2 + 46*p + 15))
```

3.812 $\int \sqrt{x}(a+bx)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \sqrt{x}(a+bx)^p\,dx = \frac{2}{3}x^{3/2}(a+bx)^p\left(\frac{a+bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx}{a}\right)$$

output
```
2/3*x^(3/2)*(b*x+a)^p*hypergeom([3/2, -p],[5/2],-b*x/a)/(((b*x+a)/a)^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \sqrt{x}(a+bx)^p\,dx = \frac{2}{3}x^{3/2}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx}{a}\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x)^p,x]
```

output
```
(2*x^(3/2)*(a + b*x)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x)/a)])/(3*(1
+ (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^p\,dx$$

$$\Big\downarrow \text{76}$$

$$(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int \sqrt{x}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{2}{3}x^{3/2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx}{a}\right)$$

input
```
Int[Sqrt[x]*(a + b*x)^p,x]
```

output
```
(2*x^(3/2)*(a + b*x)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x)/a)])/(3*(1
+ (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Maple [F]

$$\int \sqrt{x}\,(bx+a)^{p}\,dx$$

input
```
int(x^(1/2)*(b*x+a)^p,x)
```

output
```
int(x^(1/2)*(b*x+a)^p,x)
```

Fricas [F]

$$\int \sqrt{x}(a+bx)^{p}\,dx=\int (bx+a)^{p}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x + a)^p*sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \sqrt{x}(a+bx)^p\,dx = \frac{2a^p x^{\frac{3}{2}}\, {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**(1/2)*(b*x+a)**p,x)
```

output
```
2*a**p*x**(3/2)*hyper((3/2, -p), (5/2,), b*x*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int \sqrt{x}(a+bx)^p\,dx = \int (bx+a)^p\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}(a+bx)^p\,dx = \int (bx+a)^p\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^p\,dx = \int \sqrt{x}\,(a+b\,x)^p\,dx$$

input
```
int(x^(1/2)*(a + b*x)^p,x)
```

output
```
int(x^(1/2)*(a + b*x)^p, x)
```

Reduce [F]

$$\int \sqrt{x}(a+bx)^p\,dx$$
$$= \frac{4\sqrt{x}\,(bx+a)^p\,ap + 4\sqrt{x}\,(bx+a)^p\,bpx + 2\sqrt{x}\,(bx+a)^p\,bx - 8\left(\int \frac{\sqrt{x}\,(bx+a)^p}{4b\,p^2x^2+4a\,p^2x+8bp\,x^2+8apx+3b\,x^2+3ax}dx\right) a^2}{b\,(4p^2+8p}$$

input
```
int(x^(1/2)*(b*x+a)^p,x)
```

output
```
(2*(2*sqrt(x)*(a + b*x)**p*a*p + 2*sqrt(x)*(a + b*x)**p*b*p*x + sqrt(x)*(a
 + b*x)**p*b*x - 4*int((sqrt(x)*(a + b*x)**p)/(4*a*p**2*x + 8*a*p*x + 3*a*
x + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*p**3 - 8*int((sqrt(x)*(
a + b*x)**p)/(4*a*p**2*x + 8*a*p*x + 3*a*x + 4*b*p**2*x**2 + 8*b*p*x**2 +
3*b*x**2),x)*a**2*p**2 - 3*int((sqrt(x)*(a + b*x)**p)/(4*a*p**2*x + 8*a*p*
x + 3*a*x + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*p))/(b*(4*p**2
+ 8*p + 3))
```

3.813 $\int \frac{(a+bx)^p}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx = 2\sqrt{x}(a+bx)^p\left(\frac{a+bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx}{a}\right)$$

output
```
2*x^(1/2)*(b*x+a)^p*hypergeom([1/2, -p],[3/2],-b*x/a)/(((b*x+a)/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx = 2\sqrt{x}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx}{a}\right)$$

input
```
Integrate[(a + b*x)^p/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(a + b*x)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x)/a)])/(1 + (
b*x)/a)^p
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx$$

$$\Big\downarrow \; 76$$

$$(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int \frac{\left(\frac{bx}{a}+1\right)^p}{\sqrt{x}}dx$$

$$\Big\downarrow \; 74$$

$$2\sqrt{x}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx}{a}\right)$$

input
```
Int[(a + b*x)^p/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(a + b*x)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x)/a)])/(1 + (
b*x)/a)^p
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int \frac{(bx+a)^p}{\sqrt{x}}dx$$

input
```
int((b*x+a)^p/x^(1/2),x)
```

output
```
int((b*x+a)^p/x^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx=\int \frac{(bx+a)^p}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^p/x^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^p/sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx=2a^p\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)$$

input
```
integrate((b*x+a)**p/x**(1/2),x)
```

output
```
2*a**p*sqrt(x)*hyper((1/2, -p), (3/2,), b*x*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx = \int \frac{(bx+a)^p}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^p/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/sqrt(x), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx = \int \frac{(bx+a)^p}{\sqrt{x}}\,dx$$

input
```
integrate((b*x+a)^p/x^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx = \int \frac{(a+b\,x)^p}{\sqrt{x}}\,dx$$

input
```
int((a + b*x)^p/x^(1/2),x)
```

output
```
int((a + b*x)^p/x^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(bx+a)^p+4\left(\int \frac{\sqrt{x}\,(bx+a)^p}{2bp\,x^2+2apx+b\,x^2+ax}dx\right)a\,p^2+2\left(\int \frac{\sqrt{x}\,(bx+a)^p}{2bp\,x^2+2apx+b\,x^2+ax}dx\right)ap}{2p+1}$$

input
```
int((b*x+a)^p/x^(1/2),x)
```

output
```
(2*(sqrt(x)*(a + b*x)**p + 2*int((sqrt(x)*(a + b*x)**p)/(2*a*p*x + a*x + 2
*b*p*x**2 + b*x**2),x)*a*p**2 + int((sqrt(x)*(a + b*x)**p)/(2*a*p*x + a*x
+ 2*b*p*x**2 + b*x**2),x)*a*p))/(2*p + 1)
```

3.814 $\int \frac{(a+bx)^p}{x^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = -\frac{2(a+bx)^p\left(\frac{a+bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt{x}}$$

output
```
-2*(b*x+a)^p*hypergeom([-1/2, -p],[1/2],-b*x/a)/x^(1/2)/(((b*x+a)/a)^p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = -\frac{2(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt{x}}$$

input
```
Integrate[(a + b*x)^p/x^(3/2),x]
```

output
```
(-2*(a + b*x)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x)/a)])/(Sqrt[x]*(1
+ (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 76$$

$$(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int \frac{\left(\frac{bx}{a}+1\right)^p}{x^{3/2}}dx$$

$$\Big\downarrow \; 74$$

$$-\frac{2(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx}{a}\right)}{\sqrt{x}}$$

input
```
Int[(a + b*x)^p/x^(3/2),x]
```

output
```
(-2*(a + b*x)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x)/a)])/(Sqrt[x]*(1
+ (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int \frac{(bx+a)^p}{x^{\frac{3}{2}}}dx$$

input
```
int((b*x+a)^p/x^(3/2),x)
```

output
```
int((b*x+a)^p/x^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = \int \frac{(bx+a)^p}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/x^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^p/x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = -\frac{2a^p\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{x}}$$

input
```
integrate((b*x+a)**p/x**(3/2),x)
```

output
```
-2*a**p*hyper((-1/2, -p), (1/2,), b*x*exp_polar(I*pi)/a)/sqrt(x)
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = \int \frac{(bx+a)^p}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/x^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/x^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = \int \frac{(bx+a)^p}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/x^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = \int \frac{(a+b\,x)^p}{x^{3/2}}\,dx$$

input
```
int((a + b*x)^p/x^(3/2),x)
```

output
```
int((a + b*x)^p/x^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x^{3/2}}\,dx = \frac{2(bx+a)^p + 4\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^p}{2bp\,x^3+2ap\,x^2-b\,x^3-a\,x^2}dx\right)a\,p^2 - 2\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^p}{2bp\,x^3+2ap\,x^2-b\,x^3-a\,x^2}dx\right)ap}{\sqrt{x}\,(2p-1)}$$

input
```
int((b*x+a)^p/x^(3/2),x)
```

output
```
(2*((a + b*x)**p + 2*sqrt(x)*int((sqrt(x)*(a + b*x)**p)/(2*a*p*x**2 - a*x*
*2 + 2*b*p*x**3 - b*x**3),x)*a*p**2 - sqrt(x)*int((sqrt(x)*(a + b*x)**p)/(
2*a*p*x**2 - a*x**2 + 2*b*p*x**3 - b*x**3),x)*a*p))/(sqrt(x)*(2*p - 1))
```

3.815 $\int \frac{(a+bx)^p}{x^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{(a+bx)^p}{x^{5/2}} dx = -\frac{2(a+bx)^p \left(\frac{a+bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},-\frac{bx}{a}\right)}{3x^{3/2}}$$

output
```
-2/3*(b*x+a)^p*hypergeom([-3/2, -p],[-1/2],-b*x/a)/x^(3/2)/(((b*x+a)/a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^p}{x^{5/2}} dx = -\frac{2(a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},-\frac{bx}{a}\right)}{3x^{3/2}}$$

input
```
Integrate[(a + b*x)^p/x^(5/2),x]
```

output
```
(-2*(a + b*x)^p*Hypergeometric2F1[-3/2, -p, -1/2, -((b*x)/a)])/(3*x^(3/2)*
(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^p}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 76}$$

$$(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \int \frac{\left(\frac{bx}{a}+1\right)^p}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{2(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{3}{2},-p,-\frac{1}{2},-\frac{bx}{a}\right)}{3x^{3/2}}$$

input
```
Int[(a + b*x)^p/x^(5/2),x]
```

output
```
(-2*(a + b*x)^p*Hypergeometric2F1[-3/2, -p, -1/2, -((b*x)/a)])/(3*x^(3/2)*
(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int \frac{(bx+a)^p}{x^{\frac{5}{2}}}dx$$

input
```
int((b*x+a)^p/x^(5/2),x)
```

output
```
int((b*x+a)^p/x^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^p}{x^{5/2}}\,dx = \int \frac{(bx+a)^p}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/x^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x + a)^p/x^(5/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 161.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^p}{x^{5/2}}\,dx = -\frac{2a^p {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -p \\ -\frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{3x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**p/x**(5/2),x)
```

output
```
-2*a**p*hyper((-3/2, -p), (-1/2,), b*x*exp_polar(I*pi)/a)/(3*x**(3/2))
```

Maxima [F]

$$\int \frac{(a+bx)^p}{x^{5/2}}\,dx = \int \frac{(bx+a)^p}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/x^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p/x^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx)^p}{x^{5/2}}\,dx = \int \frac{(bx+a)^p}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)^p/x^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x + a)^p/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^p}{x^{5/2}}\,dx = \int \frac{(a+b\,x)^p}{x^{5/2}}\,dx$$

input
```
int((a + b*x)^p/x^(5/2),x)
```

output
```
int((a + b*x)^p/x^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^p}{x^{5/2}}\,dx = \frac{2(bx+a)^p + 4\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^p}{2bp\,x^4+2ap\,x^3-3b\,x^4-3a\,x^3}dx\right)a\,p^2x - 6\sqrt{x}\left(\int \frac{\sqrt{x}\,(bx+a)^p}{2bp\,x^4+2ap\,x^3-3b\,x^4-3a\,x^3}dx}{\sqrt{x}\,x\,(2p-3)}$$

input
```
int((b*x+a)^p/x^(5/2),x)
```

output
```
(2*((a + b*x)**p + 2*sqrt(x)*int((sqrt(x)*(a + b*x)**p)/(2*a*p*x**3 - 3*a*
x**3 + 2*b*p*x**4 - 3*b*x**4),x)*a*p**2*x - 3*sqrt(x)*int((sqrt(x)*(a + b*
x)**p)/(2*a*p*x**3 - 3*a*x**3 + 2*b*p*x**4 - 3*b*x**4),x)*a*p*x))/(sqrt(x)
*x*(2*p - 3))
```

3.816 $\int x^m(a+bx)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int x^m(a+bx)^p\,dx=\frac{x^{1+m}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,2+m+p,2+m,-\frac{bx}{a}\right)}{a(1+m)}$$

output

```
x^(1+m)*(b*x+a)^(p+1)*hypergeom([1, 2+m+p],[2+m],-b*x/a)/a/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int x^m(a+bx)^p\,dx=\frac{x^{1+m}(a+bx)^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1+m,-p,2+m,-\frac{bx}{a}\right)}{1+m}$$

input

```
Integrate[x^m*(a + b*x)^p,x]
```

output

```
(x^(1 + m)*(a + b*x)^p*Hypergeometric2F1[1 + m, -p, 2 + m, -((b*x)/a)])/((
1 + m)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(a+bx)^p\,dx$$

$$\Big\downarrow \text{76}$$

$$(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int x^m\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{m+1}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(m+1,-p,m+2,-\frac{bx}{a}\right)}{m+1}$$

input
```
Int[x^m*(a + b*x)^p,x]
```

output
```
(x^(1 + m)*(a + b*x)^p*Hypergeometric2F1[1 + m, -p, 2 + m, -((b*x)/a)])/((
1 + m)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int x^m(bx+a)^p\,dx$$

input
```
int(x^m*(b*x+a)^p,x)
```

output
```
int(x^m*(b*x+a)^p,x)
```

Fricas [F]

$$\int x^m(a+bx)^p\,dx=\int (bx+a)^p x^m\,dx$$

input
```
integrate(x^m*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x + a)^p*x^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int x^m(a+bx)^p\,dx = \frac{a^p x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-p,m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

input
```
integrate(x**m*(b*x+a)**p,x)
```

output
```
a**p*x**(m + 1)*gamma(m + 1)*hyper((-p, m + 1), (m + 2,), b*x*exp_polar(I*
pi)/a)/gamma(m + 2)
```

Maxima [F]

$$\int x^m(a+bx)^p\,dx = \int (bx+a)^p x^m\,dx$$

input
```
integrate(x^m*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^m, x)
```

Giac [F]

$$\int x^m(a+bx)^p\,dx = \int (bx+a)^p x^m\,dx$$

input
```
integrate(x^m*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m(a+bx)^p\,dx=\int x^m\,(a+b\,x)^p\,dx$$

input
```
int(x^m*(a + b*x)^p,x)
```

output
```
int(x^m*(a + b*x)^p, x)
```

Reduce [F]

$$\int x^m(a+bx)^p\,dx$$
$$=\frac{x^m(bx+a)^p\,ap+x^m(bx+a)^p\,bmx+x^m(bx+a)^p\,bpx-\left(\int\frac{x^m(bx+a)^p}{b\,m^2x^2+2bmp\,x^2+b\,p^2x^2+a\,m^2x+2ampx+a\,p^2x+bm\,x^2+}\right.}{}$$

input
```
int(x^m*(b*x+a)^p,x)
```

output
```
(x**m*(a + b*x)**p*a*p + x**m*(a + b*x)**p*b*m*x + x**m*(a + b*x)**p*b*p*x
 - int((x**m*(a + b*x)**p)/(a*m**2*x + 2*a*m*p*x + a*m*x + a*p**2*x + a*p*
x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**
2*m**3*p - 2*int((x**m*(a + b*x)**p)/(a*m**2*x + 2*a*m*p*x + a*m*x + a*p**
2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*p**2*x**2 + b*p*x*
*2),x)*a**2*m**2*p**2 - int((x**m*(a + b*x)**p)/(a*m**2*x + 2*a*m*p*x + a*
m*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*p**2*x*
*2 + b*p*x**2),x)*a**2*m**2*p - int((x**m*(a + b*x)**p)/(a*m**2*x + 2*a*m*
p*x + a*m*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b
*p**2*x**2 + b*p*x**2),x)*a**2*m*p**3 - int((x**m*(a + b*x)**p)/(a*m**2*x
+ 2*a*m*p*x + a*m*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*
x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m*p**2)/(b*(m**2 + 2*m*p + m + p**2
 + p))
```

3.817 $\int (cx)^m (a+bx)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int (cx)^m (a+bx)^p \, dx = \frac{(cx)^{1+m}(a+bx)^{1+p} \text{Hypergeometric2F1}\left(1,2+m+p,2+m,-\frac{bx}{a}\right)}{ac(1+m)}$$

output
```
(c*x)^(1+m)*(b*x+a)^(p+1)*hypergeom([1, 2+m+p],[2+m],-b*x/a)/a/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int (cx)^m (a+bx)^p \, dx = \frac{x(cx)^m(a+bx)^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1+m,-p,2+m,-\frac{bx}{a}\right)}{1+m}$$

input
```
Integrate[(c*x)^m*(a + b*x)^p,x]
```

output
```
(x*(c*x)^m*(a + b*x)^p*Hypergeometric2F1[1 + m, -p, 2 + m, -((b*x)/a)])/((
1 + m)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (a+bx)^p \, dx$$

$$\Big\downarrow \; 76$$

$$(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \int (cx)^m \left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \; 74$$

$$\frac{(cx)^{m+1}(a+bx)^p \left(\frac{bx}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(m+1,-p,m+2,-\frac{bx}{a}\right)}{c(m+1)}$$

input
```
Int[(c*x)^m*(a + b*x)^p,x]
```

output
```
((c*x)^(1 + m)*(a + b*x)^p*Hypergeometric2F1[1 + m, -p, 2 + m, -((b*x)/a)]
)/(c*(1 + m)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int (cx)^m (bx + a)^p \, dx$$

input
```
int((c*x)^m*(b*x+a)^p,x)
```

output
```
int((c*x)^m*(b*x+a)^p,x)
```

Fricas [F]

$$\int (cx)^m (a + bx)^p \, dx = \int (bx + a)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x + a)^p*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (cx)^m(a+bx)^p\,dx = \frac{a^p c^m x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix} -p, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

input
```
integrate((c*x)**m*(b*x+a)**p,x)
```

output
```
a**p*c**m*x**(m + 1)*gamma(m + 1)*hyper((-p, m + 1), (m + 2,), b*x*exp_pol
ar(I*pi)/a)/gamma(m + 2)
```

Maxima [F]

$$\int (cx)^m(a+bx)^p\,dx = \int (bx+a)^p(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m(a+bx)^p\,dx = \int (bx+a)^p(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m(a+bx)^p\,dx=\int (c\,x)^m\,(a+b\,x)^p\,dx$$

input
```
int((c*x)^m*(a + b*x)^p,x)
```

output
```
int((c*x)^m*(a + b*x)^p, x)
```

Reduce [F]

$$\int (cx)^m(a+bx)^p\,dx$$
$$=\frac{c^m\left(x^m(bx+a)^p\,ap+x^m(bx+a)^p\,bmx+x^m(bx+a)^p\,bpx-\left(\int\frac{x^m(bx+a)^p}{b\,m^2x^2+2bmp\,x^2+b\,p^2x^2+a\,m^2x+2ampx+a\,p^2x+bm\right.}{}$$

input
```
int((c*x)^m*(b*x+a)^p,x)
```

output
```
(c**m*(x**m*(a + b*x)**p*a*p + x**m*(a + b*x)**p*b*m*x + x**m*(a + b*x)**p
*b*p*x - int((x**m*(a + b*x)**p)/(a*m**2*x + 2*a*m*p*x + a*m*x + a*p**2*x
+ a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*p**2*x**2 + b*p*x**2),
x)*a**2*m**3*p - 2*int((x**m*(a + b*x)**p)/(a*m**2*x + 2*a*m*p*x + a*m*x +
 a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*p**2*x**2 +
b*p*x**2),x)*a**2*m**2*p**2 - int((x**m*(a + b*x)**p)/(a*m**2*x + 2*a*m*p*
x + a*m*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x**2 + b*p
**2*x**2 + b*p*x**2),x)*a**2*m**2*p - int((x**m*(a + b*x)**p)/(a*m**2*x +
2*a*m*p*x + a*m*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2 + b*m*x*
*2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m*p**3 - int((x**m*(a + b*x)**p)/(a*m
**2*x + 2*a*m*p*x + a*m*x + a*p**2*x + a*p*x + b*m**2*x**2 + 2*b*m*p*x**2
+ b*m*x**2 + b*p**2*x**2 + b*p*x**2),x)*a**2*m*p**2))/(b*(m**2 + 2*m*p + m
 + p**2 + p))
```

3.818 $\int x^{-4+p}(a+bx)^{-p}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int x^{-4+p}(a+bx)^{-p}\,dx=-\frac{x^{-3+p}(a+bx)^{1-p}}{a(3-p)}+\frac{2bx^{-2+p}(a+bx)^{1-p}}{a^2(2-p)(3-p)}-\frac{2b^2x^{-1+p}(a+bx)^{1-p}}{a^3(1-p)(2-p)(3-p)}$$

output
```
-x^(-3+p)*(b*x+a)^(1-p)/a/(3-p)+2*b*x^(-2+p)*(b*x+a)^(1-p)/a^2/(2-p)/(3-p)
-2*b^2*x^(-1+p)*(b*x+a)^(1-p)/a^3/(1-p)/(2-p)/(3-p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int x^{-4+p}(a+bx)^{-p}\,dx=\frac{x^{-3+p}(a+bx)^{1-p}\left(a^2(2-3p+p^2)+2ab(-1+p)x+2b^2x^2\right)}{a^3(-3+p)(-2+p)(-1+p)}$$

input
```
Integrate[x^(-4 + p)/(a + b*x)^p,x]
```

output
```
(x^(-3 + p)*(a + b*x)^(1 - p)*(a^2*(2 - 3*p + p^2) + 2*a*b*(-1 + p)*x + 2*
b^2*x^2))/(a^3*(-3 + p)*(-2 + p)*(-1 + p))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p-4}(a+bx)^{-p}\,dx$$

$$\Big\downarrow \; 55$$

$$-\frac{2b\int x^{p-3}(a+bx)^{-p}dx}{a(3-p)}-\frac{x^{p-3}(a+bx)^{1-p}}{a(3-p)}$$

$$\Big\downarrow \; 55$$

$$-\frac{2b\left(-\frac{b\int x^{p-2}(a+bx)^{-p}dx}{a(2-p)}-\frac{x^{p-2}(a+bx)^{1-p}}{a(2-p)}\right)}{a(3-p)}-\frac{x^{p-3}(a+bx)^{1-p}}{a(3-p)}$$

$$\Big\downarrow \; 48$$

$$-\frac{2b\left(\frac{bx^{p-1}(a+bx)^{1-p}}{a^2(1-p)(2-p)}-\frac{x^{p-2}(a+bx)^{1-p}}{a(2-p)}\right)}{a(3-p)}-\frac{x^{p-3}(a+bx)^{1-p}}{a(3-p)}$$

input
```
Int[x^(-4 + p)/(a + b*x)^p,x]
```

output
```
-((x^(-3 + p)*(a + b*x)^(1 - p))/(a*(3 - p))) - (2*b*(-((x^(-2 + p)*(a + b
*x)^(1 - p))/(a*(2 - p))) + (b*x^(-1 + p)*(a + b*x)^(1 - p))/(a^2*(1 - p)*
(2 - p))))/(a*(3 - p))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{-3+p}(bx+a)(bx+a)^{-p}\left(a^2p^2+2abpx+2b^2x^2-3a^2p-2abx+2a^2\right)}{a^3(-3+p)(-2+p)(-1+p)}$	77
orering	$\frac{(bx+a)x\left(a^2p^2+2abpx+2b^2x^2-3a^2p-2abx+2a^2\right)x^{-4+p}(bx+a)^{-p}}{(-3+p)(-2+p)(-1+p)a^3}$	78

input
```
int(x^(-4+p)/((b*x+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x^(-3+p)/a^3/(-3+p)/(-2+p)/(-1+p)*(b*x+a)/((b*x+a)^p)*(a^2*p^2+2*a*b*p*x+2
*b^2*x^2-3*a^2*p-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int x^{-4+p}(a+bx)^{-p}\,dx = \frac{(2\,ab^2px^3+2\,b^3x^4+(a^2bp^2-a^2bp)x^2+(a^3p^2-3\,a^3p+2\,a^3)x)x^{p-4}}{(a^3p^3-6\,a^3p^2+11\,a^3p-6\,a^3)(bx+a)^p}$$

input
```
integrate(x^(-4+p)/((b*x+a)^p),x, algorithm="fricas")
```

output
```
(2*a*b^2*p*x^3 + 2*b^3*x^4 + (a^2*b*p^2 - a^2*b*p)*x^2 + (a^3*p^2 - 3*a^3*
p + 2*a^3)*x)*x^(p - 4)/((a^3*p^3 - 6*a^3*p^2 + 11*a^3*p - 6*a^3)*(b*x + a
)^p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(76) = 152.

Time = 10.74 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.53

$$\begin{aligned}\int x^{-4+p}(a+bx)^{-p}\,dx &= \frac{a^2p^2x^{p-3}\left(1+\frac{bx}{a}\right)^{3-p}\Gamma(p-3)}{a^2a^p\Gamma(p)+2aa^pbx\Gamma(p)+a^pb^2x^2\Gamma(p)} \\ &\quad - \frac{3a^2px^{p-3}\left(1+\frac{bx}{a}\right)^{3-p}\Gamma(p-3)}{a^2a^p\Gamma(p)+2aa^pbx\Gamma(p)+a^pb^2x^2\Gamma(p)} \\ &\quad + \frac{2a^2x^{p-3}\left(1+\frac{bx}{a}\right)^{3-p}\Gamma(p-3)}{a^2a^p\Gamma(p)+2aa^pbx\Gamma(p)+a^pb^2x^2\Gamma(p)} \\ &\quad + \frac{2abpxx^{p-3}\left(1+\frac{bx}{a}\right)^{3-p}\Gamma(p-3)}{a^2a^p\Gamma(p)+2aa^pbx\Gamma(p)+a^pb^2x^2\Gamma(p)} \\ &\quad - \frac{2abxx^{p-3}\left(1+\frac{bx}{a}\right)^{3-p}\Gamma(p-3)}{a^2a^p\Gamma(p)+2aa^pbx\Gamma(p)+a^pb^2x^2\Gamma(p)} \\ &\quad + \frac{2b^2x^2x^{p-3}\left(1+\frac{bx}{a}\right)^{3-p}\Gamma(p-3)}{a^2a^p\Gamma(p)+2aa^pbx\Gamma(p)+a^pb^2x^2\Gamma(p)}\end{aligned}$$

input
```
integrate(x**(-4+p)/((b*x+a)**p),x)
```

output
```
a**2*p**2*x**(p - 3)*(1 + b*x/a)**(3 - p)*gamma(p - 3)/(a**2*a**p*gamma(p)
 + 2*a*a**p*b*x*gamma(p) + a**p*b**2*x**2*gamma(p)) - 3*a**2*p*x**(p - 3)*
(1 + b*x/a)**(3 - p)*gamma(p - 3)/(a**2*a**p*gamma(p) + 2*a*a**p*b*x*gamma
(p) + a**p*b**2*x**2*gamma(p)) + 2*a**2*x**(p - 3)*(1 + b*x/a)**(3 - p)*ga
mma(p - 3)/(a**2*a**p*gamma(p) + 2*a*a**p*b*x*gamma(p) + a**p*b**2*x**2*ga
mma(p)) + 2*a*b*p*x*x**(p - 3)*(1 + b*x/a)**(3 - p)*gamma(p - 3)/(a**2*a**
p*gamma(p) + 2*a*a**p*b*x*gamma(p) + a**p*b**2*x**2*gamma(p)) - 2*a*b*x*x*
*(p - 3)*(1 + b*x/a)**(3 - p)*gamma(p - 3)/(a**2*a**p*gamma(p) + 2*a*a**p*
b*x*gamma(p) + a**p*b**2*x**2*gamma(p)) + 2*b**2*x**2*x**(p - 3)*(1 + b*x/
a)**(3 - p)*gamma(p - 3)/(a**2*a**p*gamma(p) + 2*a*a**p*b*x*gamma(p) + a**
p*b**2*x**2*gamma(p))
```

Maxima [F]

$$\int x^{-4+p}(a+bx)^{-p}\,dx=\int \frac{x^{p-4}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-4+p)/((b*x+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^(p - 4)/(b*x + a)^p, x)
```

Giac [F]

$$\int x^{-4+p}(a+bx)^{-p}\,dx=\int \frac{x^{p-4}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-4+p)/((b*x+a)^p),x, algorithm="giac")
```

output
```
integrate(x^(p - 4)/(b*x + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int x^{-4+p}(a+bx)^{-p}\,dx = \frac{\frac{x\,x^{p-4}\,(p^2-3\,p+2)}{p^3-6\,p^2+11\,p-6}+\frac{2\,b^3\,x^{p-4}\,x^4}{a^3\,(p^3-6\,p^2+11\,p-6)}+\frac{2\,b^2\,p\,x^{p-4}\,x^3}{a^2\,(p^3-6\,p^2+11\,p-6)}+\frac{b\,p\,x^{p-4}\,x^2\,(p-1)}{a\,(p^3-6\,p^2+11\,p-6)}}{(a+b\,x)^p}$$

input
```
int(x^(p - 4)/(a + b*x)^p,x)
```

output
```
((x*x^(p - 4)*(p^2 - 3*p + 2))/(11*p - 6*p^2 + p^3 - 6) + (2*b^3*x^(p - 4)
*x^4)/(a^3*(11*p - 6*p^2 + p^3 - 6)) + (2*b^2*p*x^(p - 4)*x^3)/(a^2*(11*p
- 6*p^2 + p^3 - 6)) + (b*p*x^(p - 4)*x^2*(p - 1))/(a*(11*p - 6*p^2 + p^3 -
 6)))/(a + b*x)^p
```

Reduce [F]

$$\int x^{-4+p}(a+bx)^{-p}\,dx = \int \frac{x^p}{(bx+a)^p\,x^4}dx$$

input
```
int(x^(-4+p)/((b*x+a)^p),x)
```

output
```
int(x**p/((a + b*x)**p*x**4),x)
```

3.819 $\int x^{-3+p}(a+bx)^{-p}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int x^{-3+p}(a+bx)^{-p}\,dx=-\frac{x^{-2+p}(a+bx)^{1-p}}{a(2-p)}+\frac{bx^{-1+p}(a+bx)^{1-p}}{a^2(1-p)(2-p)}$$

output
```
-x^(-2+p)*(b*x+a)^(1-p)/a/(2-p)+b*x^(-1+p)*(b*x+a)^(1-p)/a^2/(1-p)/(2-p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int x^{-3+p}(a+bx)^{-p}\,dx=\frac{x^{-2+p}(a+bx)^{1-p}(a(-1+p)+bx)}{a^2(-2+p)(-1+p)}$$

input
```
Integrate[x^(-3 + p)/(a + b*x)^p,x]
```

output
```
(x^(-2 + p)*(a + b*x)^(1 - p)*(a*(-1 + p) + b*x))/(a^2*(-2 + p)*(-1 + p))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p-3}(a+bx)^{-p}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{b\int x^{p-2}(a+bx)^{-p}dx}{a(2-p)}-\frac{x^{p-2}(a+bx)^{1-p}}{a(2-p)}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{bx^{p-1}(a+bx)^{1-p}}{a^2(1-p)(2-p)}-\frac{x^{p-2}(a+bx)^{1-p}}{a(2-p)}$$

input
```
Int[x^(-3 + p)/(a + b*x)^p,x]
```

output
```
-((x^(-2 + p)*(a + b*x)^(1 - p))/(a*(2 - p))) + (b*x^(-1 + p)*(a + b*x)^(1
 - p))/(a^2*(1 - p)*(2 - p))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{x^{-2+p}(bx+a)(bx+a)^{-p}(ap+bx-a)}{a^{2}(-2+p)(-1+p)}$	44
orering	$\frac{x^{-3+p}(ap+bx-a)x(bx+a)(bx+a)^{-p}}{(-2+p)(-1+p)a^{2}}$	45

input
```
int(x^(-3+p)/((b*x+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x^(-2+p)/a^2/(-2+p)/(-1+p)*(b*x+a)/((b*x+a)^p)*(a*p+b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int x^{-3+p}(a+bx)^{-p}\,dx=\frac{(abpx^{2}+b^{2}x^{3}+(a^{2}p-a^{2})x)x^{p-3}}{(a^{2}p^{2}-3\,a^{2}p+2\,a^{2})(bx+a)^{p}}$$

input
```
integrate(x^(-3+p)/((b*x+a)^p),x, algorithm="fricas")
```

output
```
(a*b*p*x^2 + b^2*x^3 + (a^2*p - a^2)*x)*x^(p - 3)/((a^2*p^2 - 3*a^2*p + 2*
a^2)*(b*x + a)^p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(42) = 84$.

Time = 10.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.94

$$\int x^{-3+p}(a+bx)^{-p}\,dx = \frac{apx^{p-2}\left(1+\frac{bx}{a}\right)^{2-p}\Gamma(p-2)}{aa^p\Gamma(p)+a^pbx\Gamma(p)} - \frac{ax^{p-2}\left(1+\frac{bx}{a}\right)^{2-p}\Gamma(p-2)}{aa^p\Gamma(p)+a^pbx\Gamma(p)} + \frac{bx x^{p-2}\left(1+\frac{bx}{a}\right)^{2-p}\Gamma(p-2)}{aa^p\Gamma(p)+a^pbx\Gamma(p)}$$

input
```
integrate(x**(-3+p)/((b*x+a)**p),x)
```

output
```
a*p*x**(p - 2)*(1 + b*x/a)**(2 - p)*gamma(p - 2)/(a*a**p*gamma(p) + a**p*b
*x*gamma(p)) - a*x**(p - 2)*(1 + b*x/a)**(2 - p)*gamma(p - 2)/(a*a**p*gamm
a(p) + a**p*b*x*gamma(p)) + b*x*x**(p - 2)*(1 + b*x/a)**(2 - p)*gamma(p -
2)/(a*a**p*gamma(p) + a**p*b*x*gamma(p))
```

Maxima [F]

$$\int x^{-3+p}(a+bx)^{-p}\,dx = \int \frac{x^{p-3}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-3+p)/((b*x+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^(p - 3)/(b*x + a)^p, x)
```

Giac [F]

$$\int x^{-3+p}(a+bx)^{-p}\,dx=\int \frac{x^{p-3}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-3+p)/((b*x+a)^p),x, algorithm="giac")
```

output
```
integrate(x^(p - 3)/(b*x + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

$$\int x^{-3+p}(a+bx)^{-p}\,dx=\frac{\frac{x\,x^{p-3}\,(p-1)}{p^2-3\,p+2}+\frac{b^2\,x^{p-3}\,x^3}{a^2\,(p^2-3\,p+2)}+\frac{b\,p\,x^{p-3}\,x^2}{a\,(p^2-3\,p+2)}}{(a+b\,x)^p}$$

input
```
int(x^(p - 3)/(a + b*x)^p,x)
```

output
```
((x*x^(p - 3)*(p - 1))/(p^2 - 3*p + 2) + (b^2*x^(p - 3)*x^3)/(a^2*(p^2 - 3
*p + 2)) + (b*p*x^(p - 3)*x^2)/(a*(p^2 - 3*p + 2)))/(a + b*x)^p
```

Reduce [F]

$$\int x^{-3+p}(a+bx)^{-p}\,dx=\int \frac{x^p}{(bx+a)^p\,x^3}dx$$

input
```
int(x^(-3+p)/((b*x+a)^p),x)
```

output
```
int(x**p/((a + b*x)**p*x**3),x)
```

3.820 $\int x^{-2+p}(a+bx)^{-p}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int x^{-2+p}(a+bx)^{-p}\,dx=-\frac{x^{-1+p}(a+bx)^{1-p}}{a(1-p)}$$

output
```
-x^(-1+p)*(b*x+a)^(1-p)/a/(1-p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int x^{-2+p}(a+bx)^{-p}\,dx=\frac{x^{-1+p}(a+bx)^{1-p}}{a(-1+p)}$$

input
```
Integrate[x^(-2 + p)/(a + b*x)^p,x]
```

output
```
(x^(-1 + p)*(a + b*x)^(1 - p))/(a*(-1 + p))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p-2}(a+bx)^{-p}\,dx$$

$$\Big\downarrow \, 48$$

$$-\frac{x^{p-1}(a+bx)^{1-p}}{a(1-p)}$$

input
```
Int[x^(-2 + p)/(a + b*x)^p,x]
```

output
```
-((x^(-1 + p)*(a + b*x)^(1 - p))/(a*(1 - p)))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{x^{-1+p}(bx+a)(bx+a)^{-p}}{a(-1+p)}$	29
orering	$\frac{x(bx+a)x^{-2+p}(bx+a)^{-p}}{a(-1+p)}$	30
parallelrisch	$\frac{(x^2x^{-2+p}b+x\,x^{-2+p}a)(bx+a)^{-p}}{a(-1+p)}$	38

input
```
int(x^(-2+p)/((b*x+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x^(-1+p)/a/(-1+p)*(b*x+a)/((b*x+a)^p)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int x^{-2+p}(a+bx)^{-p}\,dx = \frac{(bx^2+ax)x^{p-2}}{(ap-a)(bx+a)^p}$$

input
```
integrate(x^(-2+p)/((b*x+a)^p),x, algorithm="fricas")
```

output
```
(b*x^2 + a*x)*x^(p - 2)/((a*p - a)*(b*x + a)^p)
```

Sympy [A] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int x^{-2+p}(a+bx)^{-p}\,dx = \frac{a^{-p}x^{p-1}\left(1+\frac{bx}{a}\right)^{1-p}\Gamma(p-1)}{\Gamma(p)}$$

input
```
integrate(x**(-2+p)/((b*x+a)**p),x)
```

output
```
x**(p - 1)*(1 + b*x/a)**(1 - p)*gamma(p - 1)/(a**p*gamma(p))
```

Maxima [F]

$$\int x^{-2+p}(a+bx)^{-p}\,dx=\int \frac{x^{p-2}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-2+p)/((b*x+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^(p - 2)/(b*x + a)^p, x)
```

Giac [F]

$$\int x^{-2+p}(a+bx)^{-p}\,dx=\int \frac{x^{p-2}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-2+p)/((b*x+a)^p),x, algorithm="giac")
```

output
```
integrate(x^(p - 2)/(b*x + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int x^{-2+p}(a+bx)^{-p}\,dx=\frac{x^p\,(a+b\,x)}{a\,x\,(p-1)\,(a+b\,x)^p}$$

input
```
int(x^(p - 2)/(a + b*x)^p,x)
```

output
```
(x^p*(a + b*x))/(a*x*(p - 1)*(a + b*x)^p)
```

Reduce [F]

$$\int x^{-2+p}(a+bx)^{-p}\,dx=\int \frac{x^p}{(bx+a)^p\,x^2}dx$$

input
```
int(x^(-2+p)/((b*x+a)^p),x)
```

output
```
int(x**p/((a + b*x)**p*x**2),x)
```

3.821 $\int x^{-1+p}(a+bx)^{-p}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\frac{x^p(a+bx)^{1-p}\,\text{Hypergeometric2F1}\left(1,1,1+p,-\frac{bx}{a}\right)}{ap}$$

output
```
x^p*(b*x+a)^(1-p)*hypergeom([1, 1],[p+1],-b*x/a)/a/p
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\frac{x^p(a+bx)^{-p}\left(1+\frac{bx}{a}\right)^p\text{Hypergeometric2F1}\left(p,p,1+p,-\frac{bx}{a}\right)}{p}$$

input
```
Integrate[x^(-1 + p)/(a + b*x)^p,x]
```

output
```
(x^p*(1 + (b*x)/a)^p*Hypergeometric2F1[p, p, 1 + p, -((b*x)/a)])/(p*(a + b
*x)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p-1}(a+bx)^{-p}\,dx$$

$$\Big\downarrow \text{ 76}$$

$$(a+bx)^{-p}\left(\frac{bx}{a}+1\right)^{p}\int x^{p-1}\left(\frac{bx}{a}+1\right)^{-p}dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{x^p(a+bx)^{-p}\left(\frac{bx}{a}+1\right)^p \text{Hypergeometric2F1}\left(p,p,p+1,-\frac{bx}{a}\right)}{p}$$

input
```
Int[x^(-1 + p)/(a + b*x)^p,x]
```

output
```
(x^p*(1 + (b*x)/a)^p*Hypergeometric2F1[p, p, 1 + p, -((b*x)/a)])/(p*(a + b
*x)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Maple [F]

$$\int x^{-1+p}(bx+a)^{-p}\,dx$$

input
```
int(x^(-1+p)/((b*x+a)^p),x)
```

output
```
int(x^(-1+p)/((b*x+a)^p),x)
```

Fricas [F]

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\int\frac{x^{p-1}}{(bx+a)^{p}}\,dx$$

input
```
integrate(x^(-1+p)/((b*x+a)^p),x, algorithm="fricas")
```

output
```
integral(x^(p - 1)/(b*x + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\frac{a^{-p}x^p\Gamma(p)\,{}_2F_1\left(\begin{matrix} p,p \\ p+1 \end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(p+1)}$$

input
```
integrate(x**(-1+p)/((b*x+a)**p),x)
```

output
```
x**p*gamma(p)*hyper((p, p), (p + 1,), b*x*exp_polar(I*pi)/a)/(a**p*gamma(p
 + 1))
```

Maxima [F]

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\int\frac{x^{p-1}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-1+p)/((b*x+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^(p - 1)/(b*x + a)^p, x)
```

Giac [F]

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\int\frac{x^{p-1}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(-1+p)/((b*x+a)^p),x, algorithm="giac")
```

output
```
integrate(x^(p - 1)/(b*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\int \frac{x^{p-1}}{(a+b\,x)^p}\,dx$$

input
```
int(x^(p - 1)/(a + b*x)^p,x)
```

output
```
int(x^(p - 1)/(a + b*x)^p, x)
```

Reduce [F]

$$\int x^{-1+p}(a+bx)^{-p}\,dx=\int \frac{x^p}{(bx+a)^p\,x}dx$$

input
```
int(x^(-1+p)/((b*x+a)^p),x)
```

output
```
int(x**p/((a + b*x)**p*x),x)
```

3.822 $\int x^p(a+bx)^{-p}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int x^p(a+bx)^{-p}\,dx=\frac{x^{1+p}(a+bx)^{1-p}\,\text{Hypergeometric2F1}\left(1,2,2+p,-\frac{bx}{a}\right)}{a(1+p)}$$

output
```
x^(p+1)*(b*x+a)^(1-p)*hypergeom([1, 2],[2+p],-b*x/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^p(a+bx)^{-p}\,dx=\frac{x^{1+p}(a+bx)^{-p}\left(1+\frac{bx}{a}\right)^p\text{Hypergeometric2F1}\left(p,1+p,2+p,-\frac{bx}{a}\right)}{1+p}$$

input
```
Integrate[x^p/(a + b*x)^p,x]
```

output
```
(x^(1 + p)*(1 + (b*x)/a)^p*Hypergeometric2F1[p, 1 + p, 2 + p, -((b*x)/a)])
/((1 + p)*(a + b*x)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{76, 74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p(a+bx)^{-p}\,dx$$

$$\Big\downarrow \ 76$$

$$(a+bx)^{-p}\left(\frac{bx}{a}+1\right)^p\int x^p\left(\frac{bx}{a}+1\right)^{-p}dx$$

$$\Big\downarrow \ 74$$

$$\frac{x^{p+1}(a+bx)^{-p}\left(\frac{bx}{a}+1\right)^p \text{Hypergeometric2F1}\left(p,p+1,p+2,-\frac{bx}{a}\right)}{p+1}$$

input
```
Int[x^p/(a + b*x)^p,x]
```

output
```
(x^(1 + p)*(1 + (b*x)/a)^p*Hypergeometric2F1[p, 1 + p, 2 + p, -((b*x)/a)])
/((1 + p)*(a + b*x)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int x^p(bx+a)^{-p}\,dx$$

input
```
int(x^p/((b*x+a)^p),x)
```

output
```
int(x^p/((b*x+a)^p),x)
```

Fricas [F]

$$\int x^p(a+bx)^{-p}\,dx=\int \frac{x^p}{(bx+a)^p}\,dx$$

input
```
integrate(x^p/((b*x+a)^p),x, algorithm="fricas")
```

output
```
integral(x^p/(b*x + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^p(a+bx)^{-p}\,dx=\frac{a^{-p}x^{p+1}\Gamma(p+1)\,{}_2F_1\left(\begin{matrix}p,p+1\\p+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(p+2)}$$

input
```
integrate(x**p/((b*x+a)**p),x)
```

output
```
x**(p + 1)*gamma(p + 1)*hyper((p, p + 1), (p + 2,), b*x*exp_polar(I*pi)/a)
/(a**p*gamma(p + 2))
```

Maxima [F]

$$\int x^p(a+bx)^{-p}\,dx=\int\frac{x^p}{(bx+a)^p}\,dx$$

input
```
integrate(x^p/((b*x+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^p/(b*x + a)^p, x)
```

Giac [F]

$$\int x^p(a+bx)^{-p}\,dx=\int\frac{x^p}{(bx+a)^p}\,dx$$

input
```
integrate(x^p/((b*x+a)^p),x, algorithm="giac")
```

output
```
integrate(x^p/(b*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^p(a+bx)^{-p}\,dx = \int \frac{x^p}{(a+b\,x)^p}\,dx$$

input
```
int(x^p/(a + b*x)^p,x)
```

output
```
int(x^p/(a + b*x)^p, x)
```

Reduce [F]

$$\int x^p(a+bx)^{-p}\,dx = \int \frac{x^p}{(bx+a)^p}dx$$

input
```
int(x^p/((b*x+a)^p),x)
```

output
```
int(x**p/(a + b*x)**p,x)
```

3.823 $\int x^{1+p}(a+bx)^{-p}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^{1+p}(a+bx)^{-p}\,dx=\frac{x^{2+p}(a+bx)^{1-p}\,\text{Hypergeometric2F1}\left(1,3,3+p,-\frac{bx}{a}\right)}{a(2+p)}$$

output
```
x^(2+p)*(b*x+a)^(1-p)*hypergeom([1, 3],[3+p],-b*x/a)/a/(2+p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^{1+p}(a+bx)^{-p}\,dx =\frac{x^{2+p}(a+bx)^{-p}\left(1+\frac{bx}{a}\right)^p\text{Hypergeometric2F1}\left(p,2+p,3+p,-\frac{bx}{a}\right)}{2+p}$$

input
```
Integrate[x^(1 + p)/(a + b*x)^p,x]
```

output
```
(x^(2 + p)*(1 + (b*x)/a)^p*Hypergeometric2F1[p, 2 + p, 3 + p, -((b*x)/a)])
/((2 + p)*(a + b*x)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p+1}(a+bx)^{-p}\,dx$$

$$\Big\downarrow \text{ 76}$$

$$(a+bx)^{-p}\left(\frac{bx}{a}+1\right)^p\int x^{p+1}\left(\frac{bx}{a}+1\right)^{-p}dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{x^{p+2}(a+bx)^{-p}\left(\frac{bx}{a}+1\right)^p \text{Hypergeometric2F1}\left(p,p+2,p+3,-\frac{bx}{a}\right)}{p+2}$$

input
```
Int[x^(1 + p)/(a + b*x)^p,x]
```

output
```
(x^(2 + p)*(1 + (b*x)/a)^p*Hypergeometric2F1[p, 2 + p, 3 + p, -((b*x)/a)])
/((2 + p)*(a + b*x)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int x^{p+1}(bx+a)^{-p}\,dx$$

input
```
int(x^(p+1)/((b*x+a)^p),x)
```

output
```
int(x^(p+1)/((b*x+a)^p),x)
```

Fricas [F]

$$\int x^{1+p}(a+bx)^{-p}\,dx=\int\frac{x^{p+1}}{(bx+a)^{p}}\,dx$$

input
```
integrate(x^(p+1)/((b*x+a)^p),x, algorithm="fricas")
```

output
```
integral(x^(p + 1)/(b*x + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^{1+p}(a+bx)^{-p}\,dx = \frac{a^{-p}x^{p+2}\Gamma(p+2)\,{}_2F_1\left(\begin{matrix} p, p+2 \\ p+3 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(p+3)}$$

input
```
integrate(x**(p+1)/((b*x+a)**p),x)
```

output
```
x**(p + 2)*gamma(p + 2)*hyper((p, p + 2), (p + 3,), b*x*exp_polar(I*pi)/a)
/(a**p*gamma(p + 3))
```

Maxima [F]

$$\int x^{1+p}(a+bx)^{-p}\,dx = \int \frac{x^{p+1}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(p+1)/((b*x+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^(p + 1)/(b*x + a)^p, x)
```

Giac [F]

$$\int x^{1+p}(a+bx)^{-p}\,dx = \int \frac{x^{p+1}}{(bx+a)^p}\,dx$$

input
```
integrate(x^(p+1)/((b*x+a)^p),x, algorithm="giac")
```

output
```
integrate(x^(p + 1)/(b*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+p}(a+bx)^{-p}\,dx=\int \frac{x^{p+1}}{(a+b\,x)^p}\,dx$$

input
```
int(x^(p + 1)/(a + b*x)^p,x)
```

output
```
int(x^(p + 1)/(a + b*x)^p, x)
```

Reduce [F]

$$\int x^{1+p}(a+bx)^{-p}\,dx=\int \frac{x^p x}{(bx+a)^p}dx$$

input
```
int(x^(p+1)/((b*x+a)^p),x)
```

output
```
int((x**p*x)/(a + b*x)**p,x)
```

3.824 $\int (bx)^m (2+dx)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int (bx)^m (2+dx)^p \, dx = \frac{2^p (bx)^{1+m} \text{Hypergeometric2F1}\left(1+m,-p,2+m,-\frac{dx}{2}\right)}{b(1+m)}$$

output
```
2^p*(b*x)^(1+m)*hypergeom([-p, 1+m],[2+m],-1/2*d*x)/b/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (bx)^m (2+dx)^p \, dx = \frac{2^p x (bx)^m \text{Hypergeometric2F1}\left(1+m,-p,2+m,-\frac{dx}{2}\right)}{1+m}$$

input
```
Integrate[(b*x)^m*(2 + d*x)^p,x]
```

output
```
(2^p*x*(b*x)^m*Hypergeometric2F1[1 + m, -p, 2 + m, -1/2*(d*x)])/(1 + m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^m (dx+2)^p\,dx$$

$$\Big\downarrow \text{74}$$

$$\frac{2^p (bx)^{m+1} \text{Hypergeometric2F1}\left(m+1,-p,m+2,-\frac{dx}{2}\right)}{b(m+1)}$$

input
```
Int[(b*x)^m*(2 + d*x)^p,x]
```

output
```
(2^p*(b*x)^(1 + m)*Hypergeometric2F1[1 + m, -p, 2 + m, -1/2*(d*x)])/(b*(1
+ m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
meijerg	$\frac{2^{p} (bx)^{m} x \,\text{hypergeom}\left(\left[-p,1+m\right],\left[2+m\right],-\frac{xd}{2}\right)}{1+m}$	32

input
```
int((b*x)^m*(d*x+2)^p,x,method=_RETURNVERBOSE)
```

output
```
2^p*(b*x)^m/(1+m)*x*hypergeom([-p,1+m],[2+m],-1/2*x*d)
```

Fricas [F]

$$\int (bx)^m(2+dx)^p\,dx=\int (bx)^m\,(dx+2)^p\,dx$$

input
```
integrate((b*x)^m*(d*x+2)^p,x, algorithm="fricas")
```

output
```
integral((b*x)^m*(d*x + 2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (bx)^m(2+dx)^p\,dx=\frac{2^p b^m x^{m+1}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-p,m+1\\ m+2\end{matrix}\middle|\frac{dxe^{i\pi}}{2}\right)}{\Gamma\left(m+2\right)}$$

input
```
integrate((b*x)**m*(d*x+2)**p,x)
```

output
```
2**p*b**m*x**(m + 1)*gamma(m + 1)*hyper((-p, m + 1), (m + 2,), d*x*exp_pol
ar(I*pi)/2)/gamma(m + 2)
```

Maxima [F]

$$\int (bx)^m(2+dx)^p\,dx = \int (bx)^m\,(dx+2)^p\,dx$$

input
```
integrate((b*x)^m*(d*x+2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x)^m*(d*x + 2)^p, x)
```

Giac [F]

$$\int (bx)^m(2+dx)^p\,dx = \int (bx)^m\,(dx+2)^p\,dx$$

input
```
integrate((b*x)^m*(d*x+2)^p,x, algorithm="giac")
```

output
```
integrate((b*x)^m*(d*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (bx)^m(2+dx)^p\,dx = \int (b\,x)^m\,(d\,x+2)^p\,dx$$

input
```
int((b*x)^m*(d*x + 2)^p,x)
```

output
```
int((b*x)^m*(d*x + 2)^p, x)
```

Reduce [F]

$$\int (bx)^m(2+dx)^p\,dx = \frac{b^m\left(x^m(dx+2)^p\,dmx + x^m(dx+2)^p\,dpx + 2x^m(dx+2)^p\,p - 4\left(\int \frac{x^m(dx+2)^p}{d\,m^2x^2+2dmp\,x^2+d\,p^2x^2+dm\,x^2+dp\,x^2+2m^2x+\right.\right.}{}$$

input
```
int((b*x)^m*(d*x+2)^p,x)
```

output
```
(b**m*(x**m*(d*x + 2)**p*d*m*x + x**m*(d*x + 2)**p*d*p*x + 2*x**m*(d*x + 2
)**p*p - 4*int((x**m*(d*x + 2)**p)/(d*m**2*x**2 + 2*d*m*p*x**2 + d*m*x**2
+ d*p**2*x**2 + d*p*x**2 + 2*m**2*x + 4*m*p*x + 2*m*x + 2*p**2*x + 2*p*x),
x)*m**3*p - 8*int((x**m*(d*x + 2)**p)/(d*m**2*x**2 + 2*d*m*p*x**2 + d*m*x*
*2 + d*p**2*x**2 + d*p*x**2 + 2*m**2*x + 4*m*p*x + 2*m*x + 2*p**2*x + 2*p*
x),x)*m**2*p**2 - 4*int((x**m*(d*x + 2)**p)/(d*m**2*x**2 + 2*d*m*p*x**2 +
d*m*x**2 + d*p**2*x**2 + d*p*x**2 + 2*m**2*x + 4*m*p*x + 2*m*x + 2*p**2*x
+ 2*p*x),x)*m**2*p - 4*int((x**m*(d*x + 2)**p)/(d*m**2*x**2 + 2*d*m*p*x**2
 + d*m*x**2 + d*p**2*x**2 + d*p*x**2 + 2*m**2*x + 4*m*p*x + 2*m*x + 2*p**2
*x + 2*p*x),x)*m*p**3 - 4*int((x**m*(d*x + 2)**p)/(d*m**2*x**2 + 2*d*m*p*x
**2 + d*m*x**2 + d*p**2*x**2 + d*p*x**2 + 2*m**2*x + 4*m*p*x + 2*m*x + 2*p
**2*x + 2*p*x),x)*m*p**2))/(d*(m**2 + 2*m*p + m + p**2 + p))
```

3.825 $\int (bx)^m (c-bcx)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int (bx)^m (c-bcx)^p \, dx = -\frac{(c-bcx)^{1+p} \text{Hypergeometric2F1}(-m,1+p,2+p,1-bx)}{bc(1+p)}$$

output
```
-(-b*c*x+c)^(p+1)*hypergeom([-m, p+1],[2+p],-b*x+1)/b/c/(p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int (bx)^m (c-bcx)^p \, dx = \frac{x(bx)^m (1-bx)^{-p} (c-bcx)^p \text{Hypergeometric2F1}(1+m,-p,2+m,bx)}{1+m}$$

input
```
Integrate[(b*x)^m*(c - b*c*x)^p,x]
```

output
```
(x*(b*x)^m*(c - b*c*x)^p*Hypergeometric2F1[1 + m, -p, 2 + m, b*x])/((1 + m
)*(1 - b*x)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{75\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^m(c-bcx)^p\,dx$$

$$\Big\downarrow \text{75}$$

$$-\frac{(c-bcx)^{p+1}\text{Hypergeometric2F1}(-m,p+1,p+2,1-bx)}{bc(p+1)}$$

input
```
Int[(b*x)^m*(c - b*c*x)^p,x]
```

output
```
-(((c - b*c*x)^(1 + p)*Hypergeometric2F1[-m, 1 + p, 2 + p, 1 - b*x])/(b*c*
(1 + p)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Maple [F]

$$\int (bx)^m\,(-bcx+c)^p\,dx$$

input
```
int((b*x)^m*(-b*c*x+c)^p,x)
```

output
```
int((b*x)^m*(-b*c*x+c)^p,x)
```

Fricas [F]

$$\int (bx)^m (c-bcx)^p \, dx = \int (-bcx+c)^p (bx)^m \, dx$$

input
```
integrate((b*x)^m*(-b*c*x+c)^p,x, algorithm="fricas")
```

output
```
integral((-b*c*x + c)^p*(b*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int (bx)^m (c-bcx)^p \, dx = \frac{b^m c^p x^{m+1} \Gamma(m+1) \, {}_2F_1\left(\begin{matrix} -p, m+1 \\ m+2 \end{matrix} \middle| bxe^{2i\pi}\right)}{\Gamma(m+2)}$$

input
```
integrate((b*x)**m*(-b*c*x+c)**p,x)
```

output
```
b**m*c**p*x**(m + 1)*gamma(m + 1)*hyper((-p, m + 1), (m + 2,), b*x*exp_pol
ar(2*I*pi))/gamma(m + 2)
```

Maxima [F]

$$\int (bx)^m (c-bcx)^p \, dx = \int (-bcx+c)^p (bx)^m \, dx$$

input
```
integrate((b*x)^m*(-b*c*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((-b*c*x + c)^p*(b*x)^m, x)
```

Giac [F]

$$\int (bx)^m (c-bcx)^p \, dx = \int (-bcx+c)^p (bx)^m \, dx$$

input
```
integrate((b*x)^m*(-b*c*x+c)^p,x, algorithm="giac")
```

output
```
integrate((-b*c*x + c)^p*(b*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (bx)^m (c-bcx)^p \, dx = \int (b\,x)^m\,(c-b\,c\,x)^p \, dx$$

input
```
int((b*x)^m*(c - b*c*x)^p,x)
```

output
```
int((b*x)^m*(c - b*c*x)^p, x)
```

Reduce [F]

$$\int (bx)^m (c - bcx)^p \, dx$$
$$= \frac{b^m \left(x^m (-bcx + c)^p bmx + x^m (-bcx + c)^p bpx - x^m (-bcx + c)^p p - \left(\int \frac{x^m (-bcx+c)^p}{b m^2 x^2 + 2bmp x^2 + b p^2 x^2 + bm x^2 + bp x^2 - m}\right.\right.}{}$$

input

```
int((b*x)^m*(-b*c*x+c)^p,x)
```

output

```
(b**m*(x**m*( - b*c*x + c)**p*b*m*x + x**m*( - b*c*x + c)**p*b*p*x - x**m*
( - b*c*x + c)**p*p - int((x**m*( - b*c*x + c)**p)/(b*m**2*x**2 + 2*b*m*p*
x**2 + b*m*x**2 + b*p**2*x**2 + b*p*x**2 - m**2*x - 2*m*p*x - m*x - p**2*x
 - p*x),x)*m**3*p - 2*int((x**m*( - b*c*x + c)**p)/(b*m**2*x**2 + 2*b*m*p*
x**2 + b*m*x**2 + b*p**2*x**2 + b*p*x**2 - m**2*x - 2*m*p*x - m*x - p**2*x
 - p*x),x)*m**2*p**2 - int((x**m*( - b*c*x + c)**p)/(b*m**2*x**2 + 2*b*m*p
*x**2 + b*m*x**2 + b*p**2*x**2 + b*p*x**2 - m**2*x - 2*m*p*x - m*x - p**2*
x - p*x),x)*m**2*p - int((x**m*( - b*c*x + c)**p)/(b*m**2*x**2 + 2*b*m*p*x
**2 + b*m*x**2 + b*p**2*x**2 + b*p*x**2 - m**2*x - 2*m*p*x - m*x - p**2*x
- p*x),x)*m*p**3 - int((x**m*( - b*c*x + c)**p)/(b*m**2*x**2 + 2*b*m*p*x**
2 + b*m*x**2 + b*p**2*x**2 + b*p*x**2 - m**2*x - 2*m*p*x - m*x - p**2*x -
p*x),x)*m*p**2))/(b*(m**2 + 2*m*p + m + p**2 + p))
```

3.826 $\int (bx)^m(c+dx)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int (bx)^m(c+dx)^p\,dx = \frac{(bx)^{1+m}(c+dx)^{1+p}\,\text{Hypergeometric2F1}\left(1,2+m+p,2+m,-\frac{dx}{c}\right)}{bc(1+m)}$$

output
```
(b*x)^(1+m)*(d*x+c)^(p+1)*hypergeom([1, 2+m+p],[2+m],-d*x/c)/b/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int (bx)^m(c+dx)^p\,dx = \frac{x(bx)^m(c+dx)^p\left(1+\frac{dx}{c}\right)^{-p}\text{Hypergeometric2F1}\left(1+m,-p,2+m,-\frac{dx}{c}\right)}{1+m}$$

input
```
Integrate[(b*x)^m*(c + d*x)^p,x]
```

output
```
(x*(b*x)^m*(c + d*x)^p*Hypergeometric2F1[1 + m, -p, 2 + m, -((d*x)/c)])/((
1 + m)*(1 + (d*x)/c)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^m (c+dx)^p \, dx$$

$$\Big\downarrow \text{ 76}$$

$$(c+dx)^p \left(\frac{dx}{c}+1\right)^{-p} \int (bx)^m \left(\frac{dx}{c}+1\right)^p dx$$

$$\Big\downarrow \text{ 74}$$

$$\frac{(bx)^{m+1}(c+dx)^p \left(\frac{dx}{c}+1\right)^{-p} \text{Hypergeometric2F1}\left(m+1,-p,m+2,-\frac{dx}{c}\right)}{b(m+1)}$$

input
```
Int[(b*x)^m*(c + d*x)^p,x]
```

output
```
((b*x)^(1 + m)*(c + d*x)^p*Hypergeometric2F1[1 + m, -p, 2 + m, -((d*x)/c)]
)/(b*(1 + m)*(1 + (d*x)/c)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Maple [F]

$$\int (bx)^m (xd+c)^p\,dx$$

input
```
int((b*x)^m*(d*x+c)^p,x)
```

output
```
int((b*x)^m*(d*x+c)^p,x)
```

Fricas [F]

$$\int (bx)^m(c+dx)^p\,dx=\int (bx)^m\,(dx+c)^p\,dx$$

input
```
integrate((b*x)^m*(d*x+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x)^m*(d*x + c)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (bx)^m (c+dx)^p\,dx = \frac{b^m c^p x^{m+1} \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -p, m+1 \\ m+2 \end{matrix}\middle| \frac{dxe^{i\pi}}{c}\right)}{\Gamma(m+2)}$$

input
```
integrate((b*x)**m*(d*x+c)**p,x)
```

output
```
b**m*c**p*x**(m + 1)*gamma(m + 1)*hyper((-p, m + 1), (m + 2,), d*x*exp_pol
ar(I*pi)/c)/gamma(m + 2)
```

Maxima [F]

$$\int (bx)^m (c+dx)^p\,dx = \int (bx)^m (dx+c)^p\,dx$$

input
```
integrate((b*x)^m*(d*x+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x)^m*(d*x + c)^p, x)
```

Giac [F]

$$\int (bx)^m (c+dx)^p\,dx = \int (bx)^m (dx+c)^p\,dx$$

input
```
integrate((b*x)^m*(d*x+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x)^m*(d*x + c)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (bx)^m (c+dx)^p \, dx = \int (b\,x)^m \, (c+d\,x)^p \, dx$$

input
```
int((b*x)^m*(c + d*x)^p,x)
```

output
```
int((b*x)^m*(c + d*x)^p, x)
```

Reduce [F]

$$\int (bx)^m (c+dx)^p \, dx$$
$$= \frac{b^m \left(x^m (dx+c)^p cp + x^m (dx+c)^p dmx + x^m (dx+c)^p dpx - \left(\int \frac{x^m (dx+c)^p}{d\,m^2 x^2 + 2dmp\,x^2 + d\,p^2 x^2 + c\,m^2 x + 2cmpx + c\,p^2 x + dm\right.}{}$$

input
```
int((b*x)^m*(d*x+c)^p,x)
```

output
```
(b**m*(x**m*(c + d*x)**p*c*p + x**m*(c + d*x)**p*d*m*x + x**m*(c + d*x)**p
*d*p*x - int((x**m*(c + d*x)**p)/(c*m**2*x + 2*c*m*p*x + c*m*x + c*p**2*x
+ c*p*x + d*m**2*x**2 + 2*d*m*p*x**2 + d*m*x**2 + d*p**2*x**2 + d*p*x**2),
x)*c**2*m**3*p - 2*int((x**m*(c + d*x)**p)/(c*m**2*x + 2*c*m*p*x + c*m*x +
 c*p**2*x + c*p*x + d*m**2*x**2 + 2*d*m*p*x**2 + d*m*x**2 + d*p**2*x**2 +
d*p*x**2),x)*c**2*m**2*p**2 - int((x**m*(c + d*x)**p)/(c*m**2*x + 2*c*m*p*
x + c*m*x + c*p**2*x + c*p*x + d*m**2*x**2 + 2*d*m*p*x**2 + d*m*x**2 + d*p
**2*x**2 + d*p*x**2),x)*c**2*m**2*p - int((x**m*(c + d*x)**p)/(c*m**2*x +
2*c*m*p*x + c*m*x + c*p**2*x + c*p*x + d*m**2*x**2 + 2*d*m*p*x**2 + d*m*x*
*2 + d*p**2*x**2 + d*p*x**2),x)*c**2*m*p**3 - int((x**m*(c + d*x)**p)/(c*m
**2*x + 2*c*m*p*x + c*m*x + c*p**2*x + c*p*x + d*m**2*x**2 + 2*d*m*p*x**2
+ d*m*x**2 + d*p**2*x**2 + d*p*x**2),x)*c**2*m*p**2))/(d*(m**2 + 2*m*p + m
 + p**2 + p))
```

3.827 $\int x^{-1+p}(a+bx)^{-1-p}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx=\frac{x^p(a+bx)^{-p}}{ap}$$

output
```
x^p/a/p/((b*x+a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx=\frac{x^p(a+bx)^{-p}}{ap}$$

input
```
Integrate[x^(-1 + p)*(a + b*x)^(-1 - p),x]
```

output
```
x^p/(a*p*(a + b*x)^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p-1}(a+bx)^{-p-1}\,dx$$

$$\Big\downarrow \; 48$$

$$\frac{x^p(a+bx)^{-p}}{ap}$$

input
```
Int[x^(-1 + p)*(a + b*x)^(-1 - p),x]
```

output
```
x^p/(a*p*(a + b*x)^p)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{p}(bx+a)^{-p}}{ap}$	20
orering	$\frac{x(bx+a)x^{-1+p}(bx+a)^{-1-p}}{ap}$	30
parallelrisch	$\frac{x^{2}x^{-1+p}(bx+a)^{-1-p}b+x\,x^{-1+p}(bx+a)^{-1-p}a}{ap}$	49

input
```
int(x^(-1+p)*(b*x+a)^(-1-p),x,method=_RETURNVERBOSE)
```

output
```
x^p/a/p*(b*x+a)^(-p)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx=\frac{(bx^2+ax)(bx+a)^{-p-1}x^{p-1}}{ap}$$

input
```
integrate(x^(-1+p)*(b*x+a)^(-1-p),x, algorithm="fricas")
```

output
```
(b*x^2 + a*x)*(b*x + a)^(-p - 1)*x^(p - 1)/(a*p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.88 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx=\frac{a^pa^{-2p-1}x^p\left(1+\frac{bx}{a}\right)^{-p}\Gamma(p)}{\Gamma\left(p+1\right)}$$

input
```
integrate(x**(-1+p)*(b*x+a)**(-1-p),x)
```

output
```
a**p*a**(-2*p - 1)*x***p*gamma(p)/((1 + b*x/a)**p*gamma(p + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx=\frac{e^{(-p\log(bx+a)+p\log(x))}}{ap}$$

input
```
integrate(x^(-1+p)*(b*x+a)^(-1-p),x, algorithm="maxima")
```

output
```
e^(-p*log(b*x + a) + p*log(x))/(a*p)
```

Giac [F]

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx=\int (bx+a)^{-p-1}x^{p-1}\,dx$$

input
```
integrate(x^(-1+p)*(b*x+a)^(-1-p),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(-p - 1)*x^(p - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx=\frac{x^p}{a\,p\,(a+b\,x)^p}$$

input
```
int(x^(p - 1)/(a + b*x)^(p + 1),x)
```

output
```
x^p/(a*p*(a + b*x)^p)
```

Reduce [F]

$$\int x^{-1+p}(a+bx)^{-1-p}\,dx = \int \frac{x^p}{(bx+a)^p\,ax+(bx+a)^p\,b\,x^2}dx$$

input
```
int(x^(-1+p)*(b*x+a)^(-1-p),x)
```

output
```
int(x**p/((a + b*x)**p*a*x + (a + b*x)**p*b*x**2),x)
```

3.828 $\int x^{-3-p}(a+bx)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int x^{-3-p}(a+bx)^p\,dx=-\frac{x^{-2-p}(a+bx)^{1+p}}{a(2+p)}+\frac{bx^{-1-p}(a+bx)^{1+p}}{a^2(1+p)(2+p)}$$

output
```
-x^(-2-p)*(b*x+a)^(p+1)/a/(2+p)+b*x^(-1-p)*(b*x+a)^(p+1)/a^2/(p+1)/(2+p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int x^{-3-p}(a+bx)^p\,dx=-\frac{x^{-2-p}(a+ap-bx)(a+bx)^{1+p}}{a^2(1+p)(2+p)}$$

input
```
Integrate[x^(-3 - p)*(a + b*x)^p,x]
```

output
```
-((x^(-2 - p)*(a + a*p - b*x)*(a + b*x)^(1 + p))/(a^2*(1 + p)*(2 + p)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-p-3}(a+bx)^p\,dx$$

$$\Big\downarrow \ 55$$

$$-\frac{b\int x^{-p-2}(a+bx)^pdx}{a(p+2)}-\frac{x^{-p-2}(a+bx)^{p+1}}{a(p+2)}$$

$$\Big\downarrow \ 48$$

$$\frac{bx^{-p-1}(a+bx)^{p+1}}{a^2(p+1)(p+2)}-\frac{x^{-p-2}(a+bx)^{p+1}}{a(p+2)}$$

input
```
Int[x^(-3 - p)*(a + b*x)^p,x]
```

output
```
-((x^(-2 - p)*(a + b*x)^(1 + p))/(a*(2 + p))) + (b*x^(-1 - p)*(a + b*x)^(1
 + p))/(a^2*(1 + p)*(2 + p))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{x^{-2-p}(bx+a)^{p+1}(ap-bx+a)}{a^2(2+p)(p+1)}$	41
orering	$-\frac{(bx+a)^p x^{-3-p}(ap-bx+a)x(bx+a)}{(2+p)(p+1)a^2}$	45

input
```
int(x^(-3-p)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-x^(-2-p)/a^2/(2+p)/(p+1)*(b*x+a)^(p+1)*(a*p-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int x^{-3-p}(a+bx)^p\,dx = -\frac{(abpx^2 - b^2x^3 + (a^2p + a^2)x)(bx+a)^p x^{-p-3}}{a^2p^2 + 3\,a^2p + 2\,a^2}$$

input
```
integrate(x^(-3-p)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
-(a*b*p*x^2 - b^2*x^3 + (a^2*p + a^2)*x)*(b*x + a)^p*x^(-p - 3)/(a^2*p^2 +
 3*a^2*p + 2*a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(46) = 92$.

Time = 0.93 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int x^{-3-p}(a+bx)^p\,dx = -\frac{aa^p p x^{-p-2}\left(1+\frac{bx}{a}\right)^{p+2}\Gamma(-p-2)}{a\Gamma(-p)+bx\Gamma(-p)} - \frac{aa^p x^{-p-2}\left(1+\frac{bx}{a}\right)^{p+2}\Gamma(-p-2)}{a\Gamma(-p)+bx\Gamma(-p)} + \frac{a^p b x x^{-p-2}\left(1+\frac{bx}{a}\right)^{p+2}\Gamma(-p-2)}{a\Gamma(-p)+bx\Gamma(-p)}$$

input
```
integrate(x**(-3-p)*(b*x+a)**p,x)
```

output
```
-a*a**p*p*x**(-p - 2)*(1 + b*x/a)**(p + 2)*gamma(-p - 2)/(a*gamma(-p) + b*
x*gamma(-p)) - a*a**p*x**(-p - 2)*(1 + b*x/a)**(p + 2)*gamma(-p - 2)/(a*ga
mma(-p) + b*x*gamma(-p)) + a**p*b*x*x**(-p - 2)*(1 + b*x/a)**(p + 2)*gamma
(-p - 2)/(a*gamma(-p) + b*x*gamma(-p))
```

Maxima [F]

$$\int x^{-3-p}(a+bx)^p\,dx = \int (bx+a)^p x^{-p-3}\,dx$$

input
```
integrate(x^(-3-p)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^(-p - 3), x)
```

Giac [F]

$$\int x^{-3-p}(a+bx)^p\,dx=\int (bx+a)^p x^{-p-3}\,dx$$

input
```
integrate(x^(-3-p)*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^(-p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int x^{-3-p}(a+bx)^p\,dx=-(a+b\,x)^p\left(\frac{x\,(p+1)}{x^{p+3}\,(p^2+3\,p+2)}-\frac{b^2\,x^3}{a^2\,x^{p+3}\,(p^2+3\,p+2)}+\frac{b\,p\,x^2}{a\,x^{p+3}\,(p^2+3\,p+2)}\right)$$

input
```
int((a + b*x)^p/x^(p + 3),x)
```

output
```
-(a + b*x)^p*((x*(p + 1))/(x^(p + 3)*(3*p + p^2 + 2)) - (b^2*x^3)/(a^2*x^(
p + 3)*(3*p + p^2 + 2)) + (b*p*x^2)/(a*x^(p + 3)*(3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int x^{-3-p}(a+bx)^p\,dx=\frac{(bx+a)^p\,(-abpx+b^2x^2-a^2p-a^2)}{x^pa^2x^2\,(p^2+3p+2)}$$

input
```
int(x^(-3-p)*(b*x+a)^p,x)
```

output
```
((a + b*x)**p*( - a**2*p - a**2 - a*b*p*x + b**2*x**2))/(x**p*a**2*x**2*(p
**2 + 3*p + 2))
```

3.829 $\int x^{2p-3(1+p)}(a+bx)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx=-\frac{x^{-2-p}(a+bx)^{1+p}}{a(2+p)}+\frac{bx^{-1-p}(a+bx)^{1+p}}{a^2(1+p)(2+p)}$$

output
```
-x^(-2-p)*(b*x+a)^(p+1)/a/(2+p)+b*x^(-1-p)*(b*x+a)^(p+1)/a^2/(p+1)/(2+p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx=-\frac{x^{-2-p}(a+ap-bx)(a+bx)^{1+p}}{a^2(1+p)(2+p)}$$

input
```
Integrate[x^(2*p - 3*(1 + p))*(a + b*x)^p,x]
```

output
```
-((x^(-2 - p)*(a + a*p - b*x)*(a + b*x)^(1 + p))/(a^2*(1 + p)*(2 + p)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2p-3(p+1)}(a+bx)^p\,dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{b\int x^{-p-2}(a+bx)^pdx}{a(p+2)}-\frac{x^{-p-2}(a+bx)^{p+1}}{a(p+2)}$$

$$\Big\downarrow \text{48}$$

$$\frac{bx^{-p-1}(a+bx)^{p+1}}{a^2(p+1)(p+2)}-\frac{x^{-p-2}(a+bx)^{p+1}}{a(p+2)}$$

input
```
Int[x^(2*p - 3*(1 + p))*(a + b*x)^p,x]
```

output
```
-((x^(-2 - p)*(a + b*x)^(1 + p))/(a*(2 + p))) + (b*x^(-1 - p)*(a + b*x)^(1
 + p))/(a^2*(1 + p)*(2 + p))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{x^{-2-p} (b x +a)^{p +1} (a p -b x +a)}{a^{2} (2+p) (p +1)}$	41
orering	$-\frac{(b x +a)^{p} x^{-3-p} (a p -b x +a) x (b x +a)}{(2+p) (p +1) a^{2}}$	45

input
```
int(x^(-3-p)*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-x^(-2-p)/a^2/(2+p)/(p+1)*(b*x+a)^(p+1)*(a*p-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx = -\frac{(abpx^2 - b^2x^3 + (a^2p + a^2)x)(bx + a)^p x^{-p-3}}{a^2p^2 + 3\,a^2p + 2\,a^2}$$

input
```
integrate(x^(-3-p)*(b*x+a)^p,x, algorithm="fricas")
```

output
```
-(a*b*p*x^2 - b^2*x^3 + (a^2*p + a^2)*x)*(b*x + a)^p*x^(-p - 3)/(a^2*p^2 +
 3*a^2*p + 2*a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(46) = 92$.

Time = 0.93 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx = -\frac{aa^p p x^{-p-2}\left(1+\frac{bx}{a}\right)^{p+2}\Gamma(-p-2)}{a\Gamma(-p)+bx\Gamma(-p)} - \frac{aa^p x^{-p-2}\left(1+\frac{bx}{a}\right)^{p+2}\Gamma(-p-2)}{a\Gamma(-p)+bx\Gamma(-p)} + \frac{a^p bx x^{-p-2}\left(1+\frac{bx}{a}\right)^{p+2}\Gamma(-p-2)}{a\Gamma(-p)+bx\Gamma(-p)}$$

input
```
integrate(x**(-3-p)*(b*x+a)**p,x)
```

output
```
-a*a**p*p*x**(-p - 2)*(1 + b*x/a)**(p + 2)*gamma(-p - 2)/(a*gamma(-p) + b*
x*gamma(-p)) - a*a**p*x**(-p - 2)*(1 + b*x/a)**(p + 2)*gamma(-p - 2)/(a*ga
mma(-p) + b*x*gamma(-p)) + a**p*b*x*x**(-p - 2)*(1 + b*x/a)**(p + 2)*gamma
(-p - 2)/(a*gamma(-p) + b*x*gamma(-p))
```

Maxima [F]

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx = \int (bx+a)^p x^{-p-3}\,dx$$

input
```
integrate(x^(-3-p)*(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^p*x^(-p - 3), x)
```

Giac [F]

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx=\int (bx+a)^p x^{-p-3}\,dx$$

input
```
integrate(x^(-3-p)*(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x + a)^p*x^(-p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx=-(a+b\,x)^p\left(\frac{x\,(p+1)}{x^{p+3}\,(p^2+3\,p+2)}-\frac{b^2\,x^3}{a^2\,x^{p+3}\,(p^2+3\,p+2)}+\frac{b\,p\,x^2}{a\,x^{p+3}\,(p^2+3\,p+2)}\right)$$

input
```
int((a + b*x)^p/x^(p + 3),x)
```

output
```
-(a + b*x)^p*((x*(p + 1))/(x^(p + 3)*(3*p + p^2 + 2)) - (b^2*x^3)/(a^2*x^(
p + 3)*(3*p + p^2 + 2)) + (b*p*x^2)/(a*x^(p + 3)*(3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int x^{2p-3(1+p)}(a+bx)^p\,dx=\frac{(bx+a)^p\,(-abpx+b^2x^2-a^2p-a^2)}{x^pa^2x^2\,(p^2+3p+2)}$$

input
```
int(x^(-3-p)*(b*x+a)^p,x)
```

output
```
((a + b*x)**p*( - a**2*p - a**2 - a*b*p*x + b**2*x**2))/(x**p*a**2*x**2*(p
**2 + 3*p + 2))
```

3.830 $\int (2-3x)^p x^m \, dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int (2-3x)^p x^m \, dx = \frac{2^p x^{1+m} \operatorname{Hypergeometric2F1}\left(1+m,-p,2+m,\frac{3x}{2}\right)}{1+m}$$

output
```
2^p*x^(1+m)*hypergeom([-p, 1+m],[2+m],3/2*x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2-3x)^p x^m \, dx = \frac{2^p x^{1+m} \operatorname{Hypergeometric2F1}\left(1+m,-p,2+m,\frac{3x}{2}\right)}{1+m}$$

input
```
Integrate[(2 - 3*x)^p*x^m,x]
```

output
```
(2^p*x^(1 + m)*Hypergeometric2F1[1 + m, -p, 2 + m, (3*x)/2])/(1 + m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(2-3x)^p\,dx$$

$$\Big\downarrow \text{74}$$

$$\frac{2^p x^{m+1} \text{Hypergeometric2F1}\left(m+1,-p,m+2,\frac{3x}{2}\right)}{m+1}$$

input
```
Int[(2 - 3*x)^p*x^m,x]
```

output
```
(2^p*x^(1 + m)*Hypergeometric2F1[1 + m, -p, 2 + m, (3*x)/2])/(1 + m)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{1+m} \operatorname{hypergeom}\left(\left[-p , 1+m \right], \left[2+m \right], \frac{3 x}{2}\right)}{1+m}$	30

input
```
int((2-3*x)^p*x^m,x,method=_RETURNVERBOSE)
```

output
```
2^p*x^(1+m)*hypergeom([-p,1+m],[2+m],3/2*x)/(1+m)
```

Fricas [F]

$$\int (2-3x)^p x^m \, dx = \int x^m (-3\,x+2)^p \, dx$$

input
```
integrate((2-3*x)^p*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*(-3*x + 2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int (2-3x)^p x^m \, dx = \frac{2^p x^{m+1} \Gamma(m+1) \, {}_2F_1\left(\begin{matrix} -p, m+1 \\ m+2 \end{matrix} \middle| \frac{3xe^{2i\pi}}{2}\right)}{\Gamma(m+2)}$$

input
```
integrate((2-3*x)**p*x**m,x)
```

output
```
2**p*x**(m + 1)*gamma(m + 1)*hyper((-p, m + 1), (m + 2,), 3*x*exp_polar(2*
I*pi)/2)/gamma(m + 2)
```

Maxima [F]

$$\int (2-3x)^p x^m\,dx = \int x^m(-3\,x+2)^p\,dx$$

input
```
integrate((2-3*x)^p*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*(-3*x + 2)^p, x)
```

Giac [F]

$$\int (2-3x)^p x^m\,dx = \int x^m(-3\,x+2)^p\,dx$$

input
```
integrate((2-3*x)^p*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*(-3*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-3x)^p x^m\,dx = \int x^m\,(2-3\,x)^p\,dx$$

input
```
int(x^m*(2 - 3*x)^p,x)
```

output
```
int(x^m*(2 - 3*x)^p, x)
```

Reduce [F]

$$\int (2-3x)^p x^m \, dx$$
$$= \frac{3x^m(-3x+2)^p mx + 3x^m(-3x+2)^p px - 2x^m(-3x+2)^p p - 4\left(\int \frac{x^m(-3x+2)^p}{3m^2x^2+6mp\,x^2+3p^2x^2-2m^2x-4mpx+3m\,x^2-}\right.}{}$$

input
```
int((2-3*x)^p*x^m,x)
```

output
```
(3*x**m*( - 3*x + 2)**p*m*x + 3*x**m*( - 3*x + 2)**p*p*x - 2*x**m*( - 3*x
+ 2)**p*p - 4*int((x**m*( - 3*x + 2)**p)/(3*m**2*x**2 - 2*m**2*x + 6*m*p*x
**2 - 4*m*p*x + 3*m*x**2 - 2*m*x + 3*p**2*x**2 - 2*p**2*x + 3*p*x**2 - 2*p
*x),x)*m**3*p - 8*int((x**m*( - 3*x + 2)**p)/(3*m**2*x**2 - 2*m**2*x + 6*m
*p*x**2 - 4*m*p*x + 3*m*x**2 - 2*m*x + 3*p**2*x**2 - 2*p**2*x + 3*p*x**2 -
 2*p*x),x)*m**2*p**2 - 4*int((x**m*( - 3*x + 2)**p)/(3*m**2*x**2 - 2*m**2*
x + 6*m*p*x**2 - 4*m*p*x + 3*m*x**2 - 2*m*x + 3*p**2*x**2 - 2*p**2*x + 3*p
*x**2 - 2*p*x),x)*m**2*p - 4*int((x**m*( - 3*x + 2)**p)/(3*m**2*x**2 - 2*m
**2*x + 6*m*p*x**2 - 4*m*p*x + 3*m*x**2 - 2*m*x + 3*p**2*x**2 - 2*p**2*x +
 3*p*x**2 - 2*p*x),x)*m*p**3 - 4*int((x**m*( - 3*x + 2)**p)/(3*m**2*x**2 -
 2*m**2*x + 6*m*p*x**2 - 4*m*p*x + 3*m*x**2 - 2*m*x + 3*p**2*x**2 - 2*p**2
*x + 3*p*x**2 - 2*p*x),x)*m*p**2)/(3*(m**2 + 2*m*p + m + p**2 + p))
```

3.831 $\int (2-3x)^p x^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int (2-3x)^p x^{5/2}\,dx = \frac{1}{7}2^{1+p}x^{7/2}\,\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{3x}{2}\right)$$

output
```
1/7*2^(p+1)*x^(7/2)*hypergeom([7/2, -p],[9/2],3/2*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2-3x)^p x^{5/2}\,dx = \frac{1}{7}2^{1+p}x^{7/2}\,\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{3x}{2}\right)$$

input
```
Integrate[(2 - 3*x)^p*x^(5/2),x]
```

output
```
(2^(1 + p)*x^(7/2)*Hypergeometric2F1[7/2, -p, 9/2, (3*x)/2])/7
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(2-3x)^p\,dx$$

$$\Big\downarrow \text{74}$$

$$\frac{1}{7}2^{p+1}x^{7/2}\,\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},\frac{3x}{2}\right)$$

input
```
Int[(2 - 3*x)^p*x^(5/2),x]
```

output
```
(2^(1 + p)*x^(7/2)*Hypergeometric2F1[7/2, -p, 9/2, (3*x)/2])/7
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
meijerg	$\frac{2^{p+1} x^{\frac{7}{2}} \operatorname{hypergeom}\left(\left[\frac{7}{2},-p\right],\left[\frac{9}{2}\right],\frac{3x}{2}\right)}{7}$	22

input
```
int((2-3*x)^p*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*2^(p+1)*x^(7/2)*hypergeom([7/2,-p],[9/2],3/2*x)
```

Fricas [F]

$$\int (2-3x)^p x^{5/2}\,dx = \int x^{\frac{5}{2}}(-3\,x+2)^p\,dx$$

input
```
integrate((2-3*x)^p*x^(5/2),x, algorithm="fricas")
```

output
```
integral(x^(5/2)*(-3*x + 2)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (2-3x)^p x^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((2-3*x)**p*x**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (2-3x)^p x^{5/2}\,dx = \int x^{\frac{5}{2}}(-3\,x+2)^p\,dx$$

input
```
integrate((2-3*x)^p*x^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)*(-3*x + 2)^p, x)
```

Giac [F]

$$\int (2-3x)^p x^{5/2}\,dx = \int x^{\frac{5}{2}}(-3\,x+2)^p\,dx$$

input
```
integrate((2-3*x)^p*x^(5/2),x, algorithm="giac")
```

output
```
integrate(x^(5/2)*(-3*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-3x)^p x^{5/2}\,dx = \int x^{5/2}\,(2-3\,x)^p\,dx$$

input
```
int(x^(5/2)*(2 - 3*x)^p,x)
```

output
```
int(x^(5/2)*(2 - 3*x)^p, x)
```

Reduce [F]

$$\int (2-3x)^p x^{5/2}\,dx = \frac{144\sqrt{x}\,(-3x+2)^p\,p^3x^3 - 96\sqrt{x}\,(-3x+2)^p\,p^3x^2 + 648\sqrt{x}\,(-3x+2)^p\,p^2x^3 - 192\sqrt{x}\,(-3x}{}$$

input
```
int((2-3*x)^p*x^(5/2),x)
```

output
```
(2*(72*sqrt(x)*( - 3*x + 2)**p*p**3*x**3 - 48*sqrt(x)*( - 3*x + 2)**p*p**3
*x**2 + 324*sqrt(x)*( - 3*x + 2)**p*p**2*x**3 - 96*sqrt(x)*( - 3*x + 2)**p
*p**2*x**2 - 80*sqrt(x)*( - 3*x + 2)**p*p**2*x + 414*sqrt(x)*( - 3*x + 2)*
*p*p*x**3 - 36*sqrt(x)*( - 3*x + 2)**p*p*x**2 - 40*sqrt(x)*( - 3*x + 2)**p
*p*x - 80*sqrt(x)*( - 3*x + 2)**p*p + 135*sqrt(x)*( - 3*x + 2)**p*x**3 - 1
280*int((sqrt(x)*( - 3*x + 2)**p)/(48*p**4*x**2 - 32*p**4*x + 384*p**3*x**
2 - 256*p**3*x + 1032*p**2*x**2 - 688*p**2*x + 1056*p*x**2 - 704*p*x + 315
*x**2 - 210*x),x)*p**5 - 10240*int((sqrt(x)*( - 3*x + 2)**p)/(48*p**4*x**2
 - 32*p**4*x + 384*p**3*x**2 - 256*p**3*x + 1032*p**2*x**2 - 688*p**2*x +
1056*p*x**2 - 704*p*x + 315*x**2 - 210*x),x)*p**4 - 27520*int((sqrt(x)*( -
 3*x + 2)**p)/(48*p**4*x**2 - 32*p**4*x + 384*p**3*x**2 - 256*p**3*x + 103
2*p**2*x**2 - 688*p**2*x + 1056*p*x**2 - 704*p*x + 315*x**2 - 210*x),x)*p*
*3 - 28160*int((sqrt(x)*( - 3*x + 2)**p)/(48*p**4*x**2 - 32*p**4*x + 384*p
**3*x**2 - 256*p**3*x + 1032*p**2*x**2 - 688*p**2*x + 1056*p*x**2 - 704*p*
x + 315*x**2 - 210*x),x)*p**2 - 8400*int((sqrt(x)*( - 3*x + 2)**p)/(48*p**
4*x**2 - 32*p**4*x + 384*p**3*x**2 - 256*p**3*x + 1032*p**2*x**2 - 688*p**
2*x + 1056*p*x**2 - 704*p*x + 315*x**2 - 210*x),x)*p))/(9*(16*p**4 + 128*p
**3 + 344*p**2 + 352*p + 105))
```

3.832 $\int (2-3x)^{5/2} x^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int (2-3x)^{5/2} x^m \, dx = \frac{4\sqrt{2}x^{1+m} \text{Hypergeometric2F1}\left(-\frac{5}{2},1+m,2+m,\frac{3x}{2}\right)}{1+m}$$

output
```
4*2^(1/2)*x^(1+m)*hypergeom([-5/2, 1+m],[2+m],3/2*x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (2-3x)^{5/2} x^m \, dx = \frac{4\sqrt{2}x^{1+m} \text{Hypergeometric2F1}\left(-\frac{5}{2},1+m,2+m,\frac{3x}{2}\right)}{1+m}$$

input
```
Integrate[(2 - 3*x)^(5/2)*x^m,x]
```

output
```
(4*Sqrt[2]*x^(1 + m)*Hypergeometric2F1[-5/2, 1 + m, 2 + m, (3*x)/2])/(1 +
m)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-3x)^{5/2}x^m\,dx$$

$$\Big\downarrow \ 74$$

$$\frac{4\sqrt{2}x^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},m+1,m+2,\frac{3x}{2}\right)}{m+1}$$

input
```
Int[(2 - 3*x)^(5/2)*x^m,x]
```

output
```
(4*Sqrt[2]*x^(1 + m)*Hypergeometric2F1[-5/2, 1 + m, 2 + m, (3*x)/2])/(1 +
m)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
meijerg	$\frac{4\sqrt{2}\,x^{1+m}\,\text{hypergeom}\left(\left[-\frac{5}{2},1+m\right],\left[2+m\right],\frac{3x}{2}\right)}{1+m}$	29

input
```
int((2-3*x)^(5/2)*x^m,x,method=_RETURNVERBOSE)
```

output
```
4*2^(1/2)*x^(1+m)*hypergeom([-5/2,1+m],[2+m],3/2*x)/(1+m)
```

Fricas [F]

$$\int (2-3x)^{5/2}x^m\,dx = \int x^m(-3\,x+2)^{\frac{5}{2}}\,dx$$

input
```
integrate((2-3*x)^(5/2)*x^m,x, algorithm="fricas")
```

output
```
integral((9*x^2 - 12*x + 4)*x^m*sqrt(-3*x + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int (2-3x)^{5/2}x^m\,dx = \frac{18\cdot 2^m\sqrt{3}\cdot 3^{-m}i\left(x-\frac{2}{3}\right)^{\frac{7}{2}}\,{}_2F_1\left(\begin{matrix}\frac{7}{2},-m\\ \frac{9}{2}\end{matrix}\middle|\frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{7}$$

input
```
integrate((2-3*x)**(5/2)*x**m,x)
```

output
```
18*2**m*sqrt(3)*I*(x - 2/3)**(7/2)*hyper((7/2, -m), (9/2,), 3*(x - 2/3)*ex
p_polar(I*pi)/2)/(7*3**m)
```

Maxima [F]

$$\int (2-3x)^{5/2}x^m\,dx = \int x^m(-3\,x+2)^{\frac{5}{2}}\,dx$$

input
```
integrate((2-3*x)^(5/2)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*(-3*x + 2)^(5/2), x)
```

Giac [F]

$$\int (2-3x)^{5/2}x^m\,dx = \int x^m(-3\,x+2)^{\frac{5}{2}}\,dx$$

input
```
integrate((2-3*x)^(5/2)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*(-3*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-3x)^{5/2}x^m\,dx = \int x^m\,(2-3\,x)^{5/2}\,dx$$

input
```
int(x^m*(2 - 3*x)^(5/2),x)
```

output
```
int(x^m*(2 - 3*x)^(5/2), x)
```

Reduce [F]

$$\int (2-3x)^{5/2}x^m\,dx = \frac{144x^m\sqrt{-3x+2}\,m^3x^3 - 192x^m\sqrt{-3x+2}\,m^3x^2 + 64x^m\sqrt{-3x+2}\,m^3x + 648x^m\sqrt{-3x+}}{}$$

input
```
int((2-3*x)^(5/2)*x^m,x)
```

output
```
(2*(72*x**m*sqrt( - 3*x + 2)*m**3*x**3 - 96*x**m*sqrt( - 3*x + 2)*m**3*x**
2 + 32*x**m*sqrt( - 3*x + 2)*m**3*x + 324*x**m*sqrt( - 3*x + 2)*m**2*x**3
- 552*x**m*sqrt( - 3*x + 2)*m**2*x**2 + 224*x**m*sqrt( - 3*x + 2)*m**2*x +
 414*x**m*sqrt( - 3*x + 2)*m*x**3 - 792*x**m*sqrt( - 3*x + 2)*m*x**2 + 464
*x**m*sqrt( - 3*x + 2)*m*x + 135*x**m*sqrt( - 3*x + 2)*x**3 - 270*x**m*sqr
t( - 3*x + 2)*x**2 + 180*x**m*sqrt( - 3*x + 2)*x - 40*x**m*sqrt( - 3*x + 2
) - 1280*int((x**m*sqrt( - 3*x + 2))/(48*m**4*x**2 - 32*m**4*x + 384*m**3*
x**2 - 256*m**3*x + 1032*m**2*x**2 - 688*m**2*x + 1056*m*x**2 - 704*m*x +
315*x**2 - 210*x),x)*m**5 - 10240*int((x**m*sqrt( - 3*x + 2))/(48*m**4*x**
2 - 32*m**4*x + 384*m**3*x**2 - 256*m**3*x + 1032*m**2*x**2 - 688*m**2*x +
 1056*m*x**2 - 704*m*x + 315*x**2 - 210*x),x)*m**4 - 27520*int((x**m*sqrt(
 - 3*x + 2))/(48*m**4*x**2 - 32*m**4*x + 384*m**3*x**2 - 256*m**3*x + 1032
*m**2*x**2 - 688*m**2*x + 1056*m*x**2 - 704*m*x + 315*x**2 - 210*x),x)*m**
3 - 28160*int((x**m*sqrt( - 3*x + 2))/(48*m**4*x**2 - 32*m**4*x + 384*m**3
*x**2 - 256*m**3*x + 1032*m**2*x**2 - 688*m**2*x + 1056*m*x**2 - 704*m*x +
 315*x**2 - 210*x),x)*m**2 - 8400*int((x**m*sqrt( - 3*x + 2))/(48*m**4*x**
2 - 32*m**4*x + 384*m**3*x**2 - 256*m**3*x + 1032*m**2*x**2 - 688*m**2*x +
 1056*m*x**2 - 704*m*x + 315*x**2 - 210*x),x)*m))/(16*m**4 + 128*m**3 + 34
4*m**2 + 352*m + 105)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file