Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/16-1.1.1.2b

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**299**]. This is test number [16].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (299)	0.00 (0)
Mathematica	100.00 (299)	0.00 (0)
Fricas	72.91 (218)	27.09 (81)
Maple	69.23 (207)	30.77 (92)
Reduce	52.84 (158)	47.16 (141)
Giac	47.16 (141)	52.84 (158)
Maxima	47.16 (141)	52.84 (158)
Sympy	44.82 (134)	55.18 (165)
Mupad	41.14 (123)	58.86 (176)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

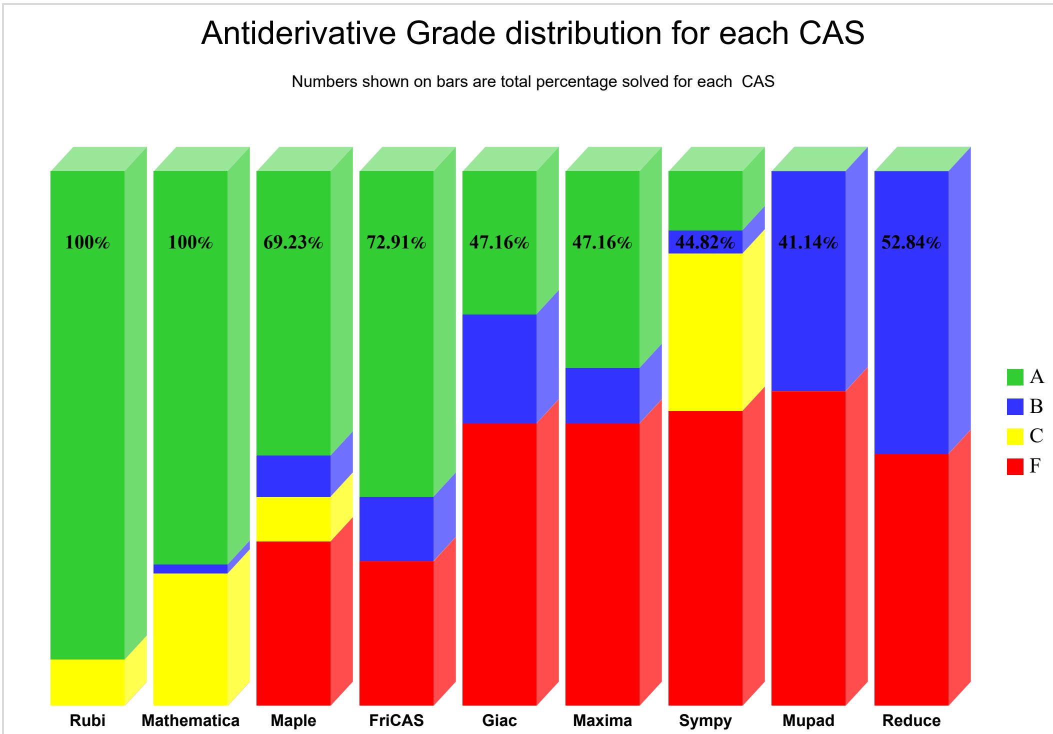
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

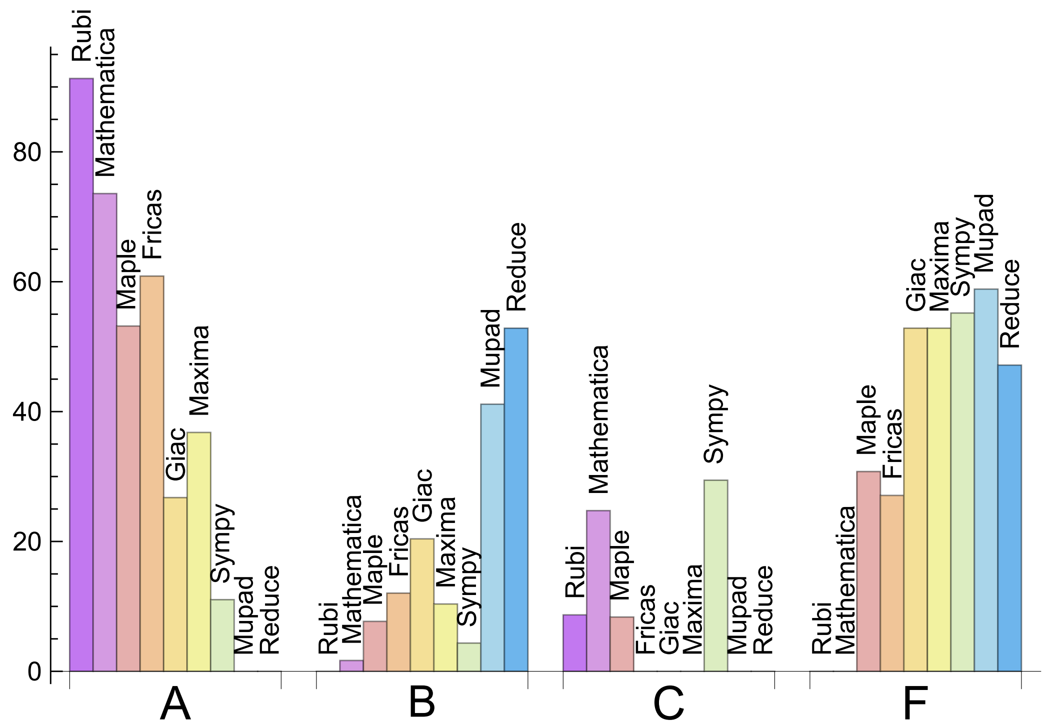
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.304	0.000	8.696	0.000
Mathematica	73.579	1.672	24.749	0.000
Fricas	60.870	12.040	0.000	27.090
Maple	53.177	7.692	8.361	30.769
Maxima	36.789	10.368	0.000	52.843
Giac	26.756	20.401	0.000	52.843
Sympy	11.037	4.348	29.431	55.184
Mupad	0.000	41.137	0.000	58.863
Reduce	0.000	52.843	0.000	47.157

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	81	100.00	0.00	0.00
Maple	92	100.00	0.00	0.00
Reduce	141	100.00	0.00	0.00
Giac	158	67.09	0.00	32.91
Maxima	158	93.67	0.00	6.33
Sympy	165	85.45	14.55	0.00
Mupad	176	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.09
Giac	0.15
Maple	0.15
Rubi	0.17
Reduce	0.17
Mupad	0.19
Mathematica	0.36
Sympy	13.07

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Reduce	54.22	1.12	51.00	1.00
Mupad	55.63	1.30	50.00	1.06
Mathematica	60.24	0.81	54.00	0.81
Maxima	64.75	1.29	45.00	0.98
Maple	66.84	1.11	43.00	0.89
Fricas	86.50	1.31	65.50	1.18
Rubi	89.38	0.97	67.00	1.00
Giac	95.40	1.79	55.00	1.35
Sympy	217.39	4.08	99.00	2.55

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

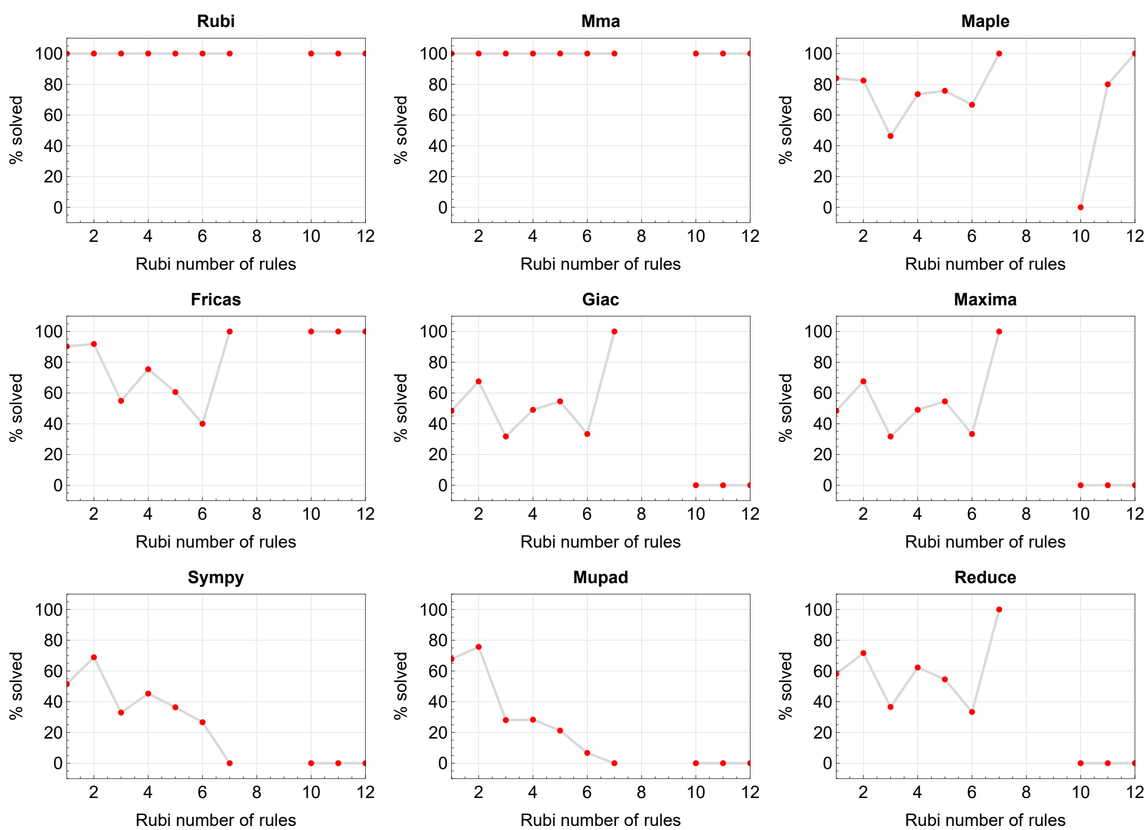


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

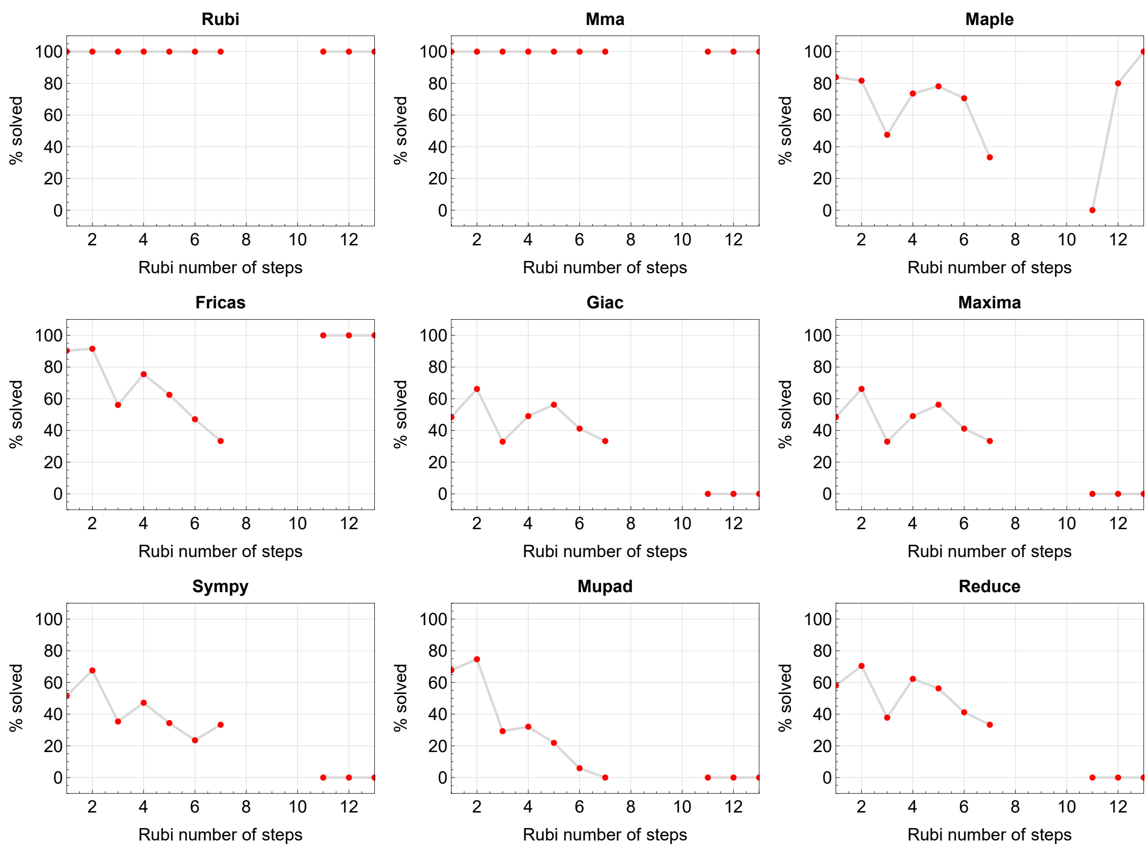


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

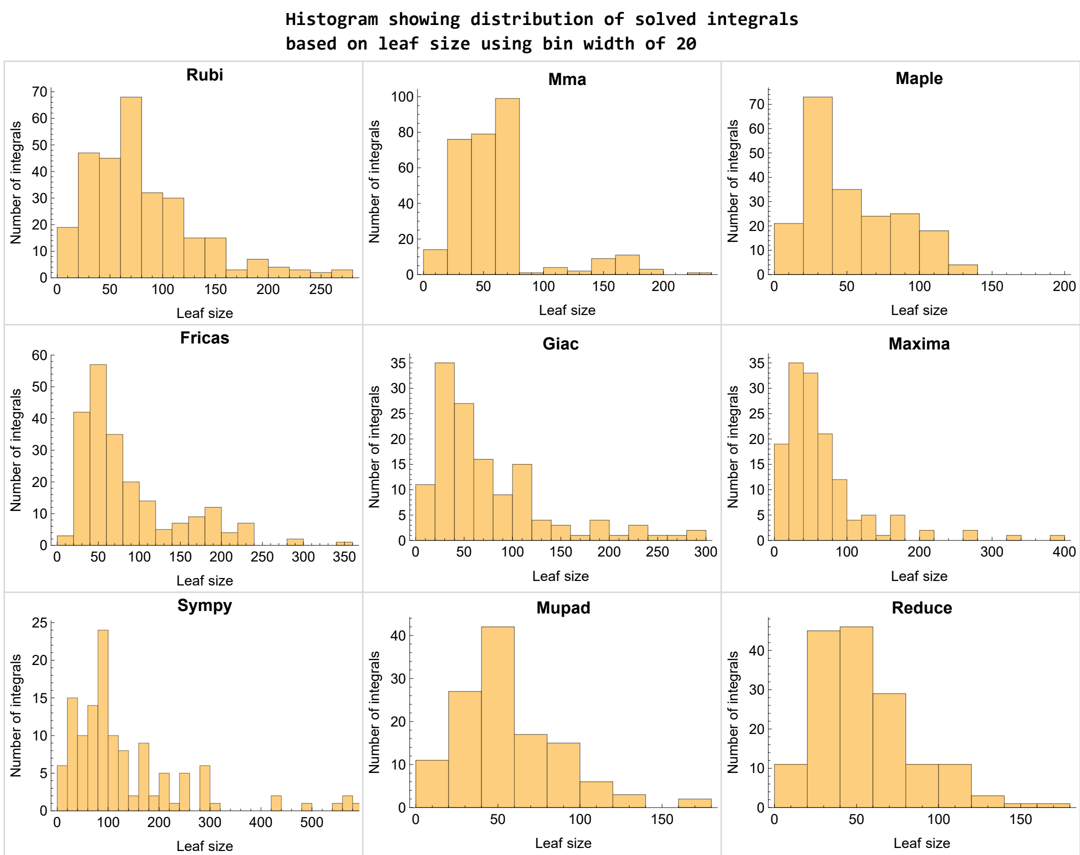


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

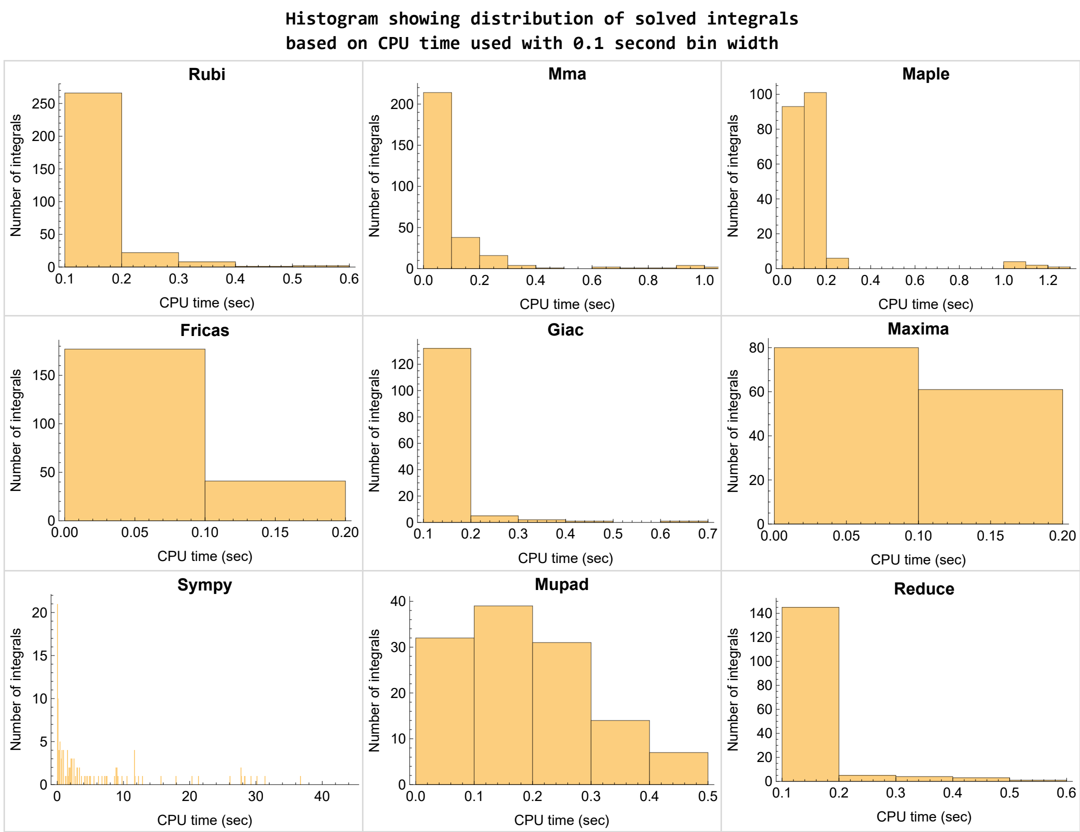


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

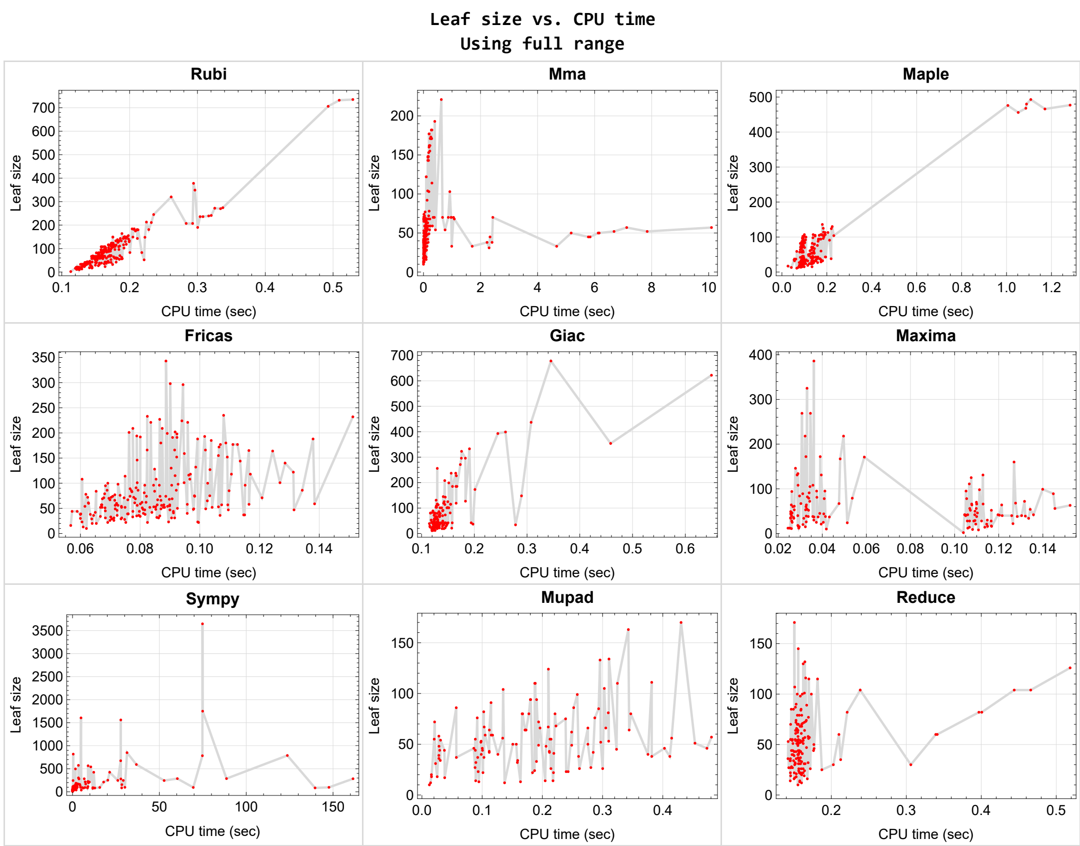


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {149, 163, 178, 194, 207, 220, 229, 230, 235, 236, 251, 252, 256, 257, 275, 280}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

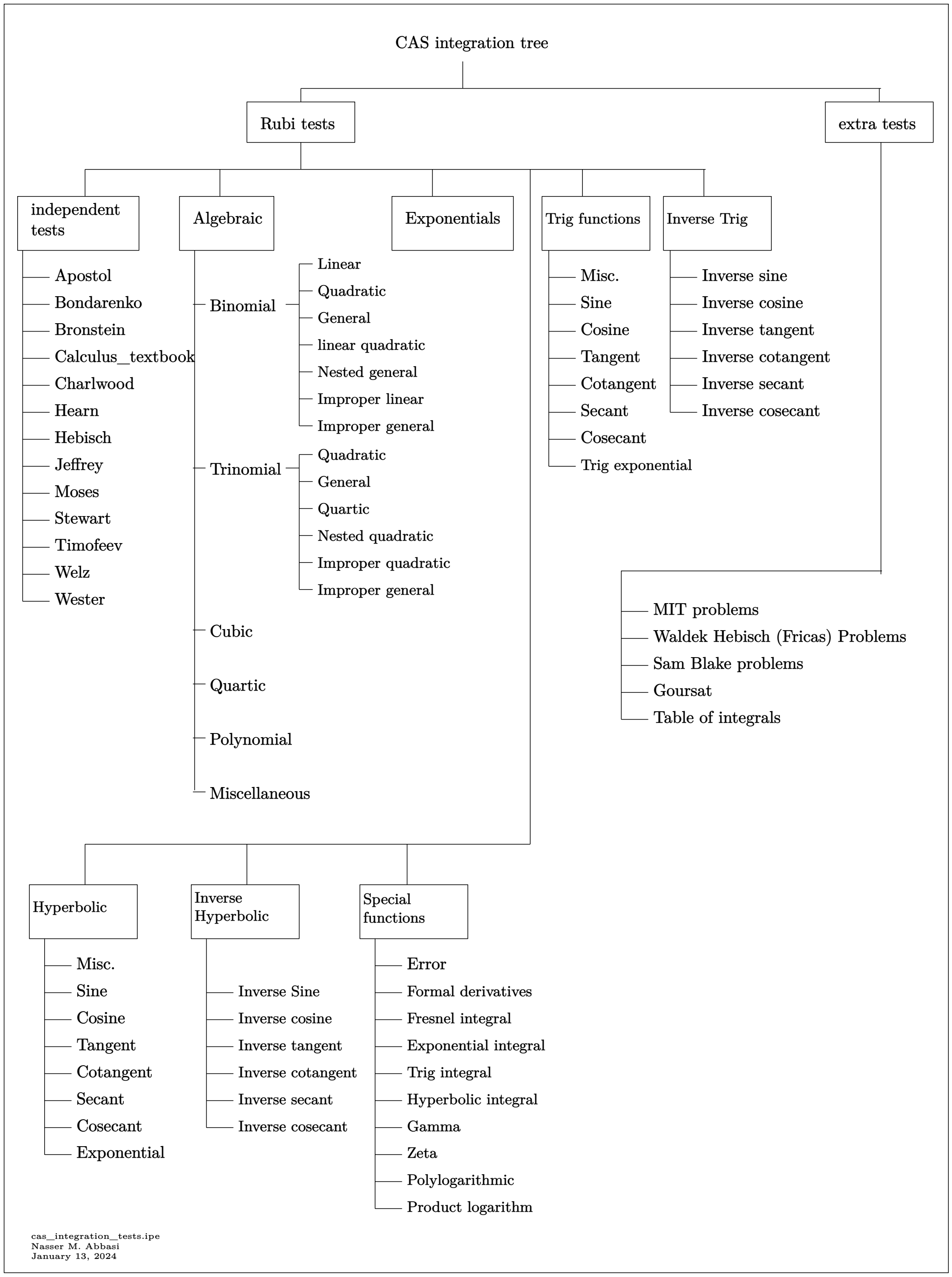
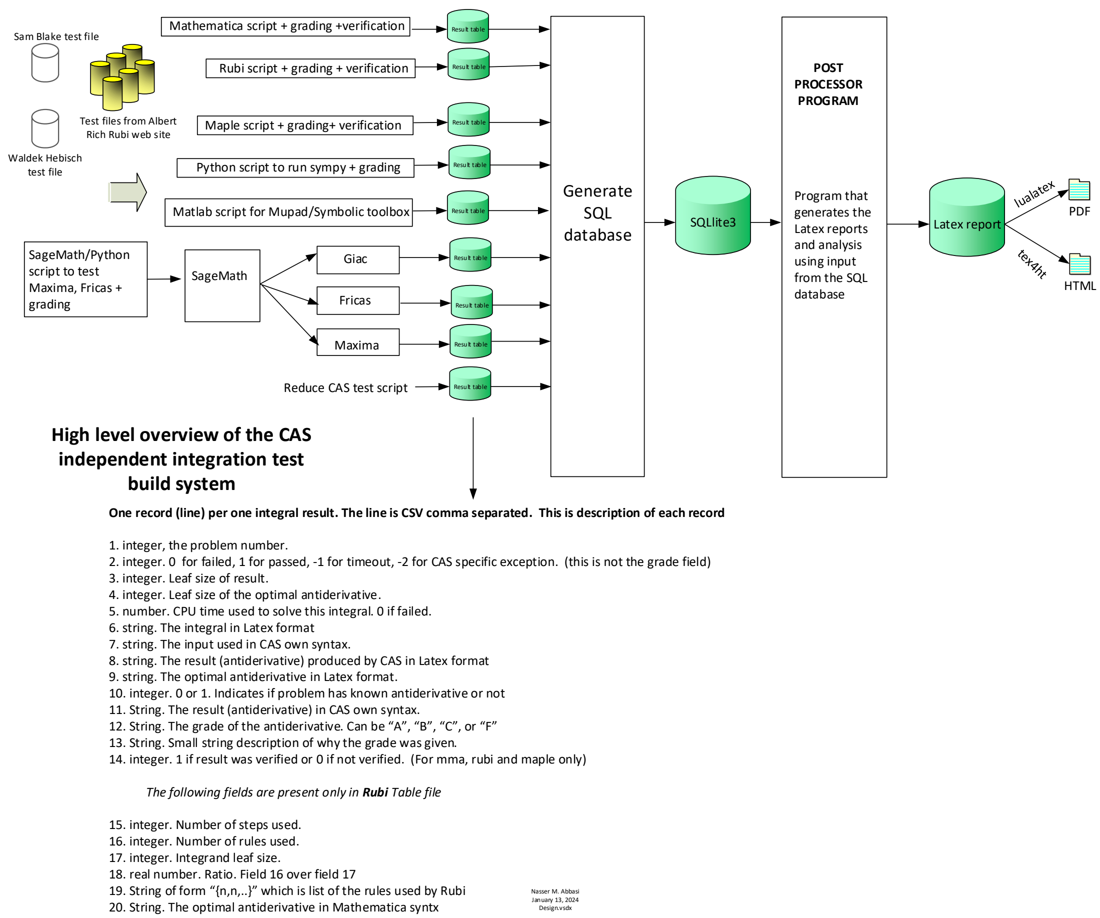


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 184, 185, 186, 187, 188, 189, 190, 191, 194, 197, 198, 199, 200, 201, 202, 203, 204, 207, 210, 211, 212, 213, 214, 215, 216, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { }

C grade { 148, 150, 151, 152, 162, 164, 165, 166, 177, 179, 180, 181, 182, 183, 192, 193, 195, 196, 205, 206, 208, 209, 217, 218, 219, 221 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 157, 158, 159, 160, 161, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 184, 185, 186, 187, 188, 189, 190, 191, 197, 198, 199, 200, 201, 202, 203, 204, 210, 212, 213, 214, 215, 216, 231, 232, 233, 234, 237, 238, 239, 253, 254, 255, 258, 259, 260, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 54, 64, 65, 66, 114 }

C grade { 148, 149, 150, 151, 152, 162, 163, 164, 165, 166, 177, 178, 179, 180, 181, 182, 183, 192, 193, 194, 195, 196, 205, 206, 207, 208, 209, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 144, 145, 146, 147, 158, 159, 160, 161, 173, 174, 175, 176, 188, 189, 190, 191, 201, 202, 203, 204, 213, 214, 215, 216, 231, 232, 233, 234, 237, 238, 239, 253, 254, 255, 258, 259, 260, 276, 277, 278, 279, 287, 288 }

B grade { 46, 52, 53, 54, 63, 64, 65, 66, 70, 71, 72, 73, 81, 82, 83, 85, 86, 96, 113, 114, 122, 123, 132 }

C grade { 222, 223, 225, 226, 227, 228, 229, 235, 245, 246, 247, 248, 249, 250, 251, 252, 256, 257, 268, 269, 270, 271, 273, 274, 275 }

F normal fail { 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 200, 205, 206, 207, 208, 209, 210, 211, 212, 217, 218, 219, 220, 221, 224, 230, 236, 240, 241, 242, 243, 244, 261, 262, 263, 264, 265, 266, 267, 272, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145, 146, 147, 153, 154, 155, 156, 159, 160, 161, 167, 168, 169, 170, 171, 175, 176, 184, 185, 186, 188, 189, 190, 191, 197, 198, 199, 200, 201, 202, 203, 204, 210, 211, 213, 214, 215, 216, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 251, 253, 254, 255, 256, 258, 259, 260, 276, 277, 278, 279, 280, 287, 288 }

B grade { 6, 9, 17, 53, 54, 64, 65, 66, 71, 72, 73, 74, 86, 87, 88, 102, 103, 104, 105, 113, 114, 123, 132, 133, 137, 144, 157, 158, 172, 173, 174, 187, 212, 252, 257, 275 }

C grade { }

F normal fail { 148, 149, 150, 151, 152, 162, 163, 164, 165, 166, 177, 178, 179, 180, 181, 182, 183, 192, 193, 194, 195, 196, 205, 206, 207, 208, 209, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 79, 80, 81, 82, 83, 84, 85, 92, 93, 94, 95, 96, 97, 98, 99, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 134, 135, 136, 137, 138, 288 }

B grade { 6, 9, 17, 25, 64, 65, 66, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 100, 101, 102, 103, 104, 105, 106, 128, 131, 132, 133, 139, 287 }

C grade { }

F normal fail { 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 251, 252, 253, 254, 256, 257, 258, 259, 261, 262, 263, 264, 265, 268, 269, 270, 272, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { 249, 250, 255, 260, 266, 267, 271, 273, 274, 278 }

Giac

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 46, 63, 70, 72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 139, 288 }

B grade { 5, 25, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 71, 79, 80, 81, 82, 83, 92, 93, 94, 95, 96, 97, 114, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 287 }

C grade { }

F normal fail { 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 246, 252, 257, 263, 269, 276, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 253, 254, 255, 256, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 270, 271, 272, 273, 274, 275, 277, 278, 279 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 71, 72, 74, 75, 76, 77, 78, 87, 88, 89, 90, 91, 102, 103, 104, 105, 106, 108, 109, 113, 114, 115, 116, 117, 118, 119, 124, 125, 126, 127, 128, 133, 134, 135, 136, 137, 138, 139, 144, 145, 146, 147, 173, 174, 175, 176, 201, 202, 203, 204, 231, 232, 233, 234, 253, 254, 255, 276, 277, 278, 279, 287, 288 }

C grade { }

F normal fail { }

F(-1) timedout fail { 35, 36, 43, 44, 51, 52, 67, 68, 69, 70, 73, 79, 80, 81, 82, 83, 84, 85, 86, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 107, 110, 111, 112, 120, 121, 122, 123, 129, 130, 131, 132, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 7, 10, 11, 12, 14, 15, 16, 18, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 109, 194, 207, 220, 224, 265, 272 }

B grade { 6, 8, 9, 13, 17, 19, 20, 25, 108, 242, 248, 287, 288 }

C grade { 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 296, 297, 298, 299 }

F normal fail { 35, 36, 37, 43, 44, 45, 107, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 225, 226, 227, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 268, 269, 270, 271, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295 }

F(-1) timedout fail { 51, 52, 56, 57, 58, 79, 91, 92, 93, 94, 105, 106, 147, 161, 175, 176, 228, 233, 234, 250, 267, 273, 274, 279 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 145, 146, 147, 158, 159, 160, 161, 173, 174, 175, 176, 243, 248, 264, 265, 272, 287, 288 }

C grade { }

F normal fail { 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	37	44	44	44	44	36	44
N.S.	1	1.00	1.05	0.97	1.16	1.16	1.16	1.16	0.95	1.16
time (sec)	N/A	0.155	0.002	0.062	0.026	0.057	0.025	0.119	0.146	0.037

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	44	44	46	44	36	44
N.S.	1	1.00	1.11	0.97	1.16	1.16	1.21	1.16	0.95	1.16
time (sec)	N/A	0.159	0.002	0.056	0.041	0.059	0.022	0.121	0.142	0.029

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	16	18	18
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.89	1.00	1.00
time (sec)	N/A	0.141	0.001	0.027	0.029	0.057	0.017	0.123	0.153	0.016

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	23	17	25	22	23
N.S.	1	1.00	1.00	0.96	1.04	1.00	0.74	1.09	0.96	1.00
time (sec)	N/A	0.164	0.004	0.084	0.051	0.061	0.054	0.121	0.151	0.093

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	31	37	39	29	81	40	37
N.S.	1	1.00	0.88	0.97	1.16	1.22	0.91	2.53	1.25	1.16
time (sec)	N/A	0.155	0.012	0.084	0.043	0.060	0.075	0.117	0.148	0.093

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	30	30	27	14	40	13
N.S.	1	1.00	1.00	1.08	2.31	2.31	2.08	1.08	3.08	1.00
time (sec)	N/A	0.125	0.007	0.086	0.036	0.060	0.097	0.120	0.156	0.095

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	54	54	56	23	44	54
N.S.	1	1.00	0.66	0.61	1.42	1.42	1.47	0.61	1.16	1.42
time (sec)	N/A	0.163	0.009	0.093	0.033	0.062	0.126	0.126	0.147	0.031

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	24	23	67	67	73	40	54	67
N.S.	1	1.00	0.63	0.61	1.76	1.76	1.92	1.05	1.42	1.76
time (sec)	N/A	0.164	0.010	0.093	0.030	0.076	0.161	0.137	0.155	0.103

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	23	84	84	88	25	68	82
N.S.	1	1.00	0.71	0.61	2.21	2.21	2.32	0.66	1.79	2.16
time (sec)	N/A	0.166	0.010	0.098	0.030	0.065	0.185	0.116	0.158	0.103

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	68	59	72	72	78	72	58	72
N.S.	1	1.00	1.19	1.04	1.26	1.26	1.37	1.26	1.02	1.26
time (sec)	N/A	0.189	0.002	0.066	0.034	0.062	0.025	0.124	0.156	0.021

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	32	34	34	36	34	31	31
N.S.	1	1.00	1.00	0.84	0.89	0.89	0.95	0.89	0.82	0.82
time (sec)	N/A	0.166	0.001	0.056	0.032	0.067	0.021	0.278	0.156	0.023

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	35	36	36	39	36	34	36
N.S.	1	1.00	1.25	1.09	1.12	1.12	1.22	1.12	1.06	1.12
time (sec)	N/A	0.175	0.002	0.054	0.027	0.064	0.020	0.122	0.157	0.028

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	21	20
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.50	1.43
time (sec)	N/A	0.126	0.001	0.042	0.031	0.064	0.020	0.126	0.153	0.016

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	43	37	31	35	34	31	46	34	34
N.S.	1	1.16	1.00	0.84	0.95	0.92	0.84	1.24	0.92	0.92
time (sec)	N/A	0.165	0.006	0.091	0.026	0.080	0.063	0.123	0.145	0.031

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	36	46	57	39	79	53	46
N.S.	1	1.00	0.85	0.88	1.12	1.39	0.95	1.93	1.29	1.12
time (sec)	N/A	0.173	0.019	0.090	0.026	0.067	0.087	0.126	0.142	0.086

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	33	42	61	69	54	46	73	59
N.S.	1	1.00	0.63	0.81	1.17	1.33	1.04	0.88	1.40	1.13
time (sec)	N/A	0.221	0.015	0.092	0.027	0.067	0.125	0.130	0.166	0.105

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	31	27	60	60	61	29	52	58
N.S.	1	1.00	1.11	0.96	2.14	2.14	2.18	1.04	1.86	2.07
time (sec)	N/A	0.143	0.012	0.101	0.027	0.063	0.146	0.116	0.159	0.028

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	32	78	78	85	64	65	76
N.S.	1	1.00	0.62	0.57	1.39	1.39	1.52	1.14	1.16	1.36
time (sec)	N/A	0.188	0.009	0.100	0.041	0.062	0.183	0.127	0.155	0.092

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	38	32	95	95	100	36	79	91
N.S.	1	1.00	0.67	0.56	1.67	1.67	1.75	0.63	1.39	1.60
time (sec)	N/A	0.181	0.014	0.105	0.037	0.068	0.213	0.124	0.155	0.115

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	32	108	108	117	36	89	104
N.S.	1	1.00	0.63	0.54	1.83	1.83	1.98	0.61	1.51	1.76
time (sec)	N/A	0.182	0.010	0.115	0.036	0.061	0.242	0.127	0.167	0.135

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	61	42	41	48	52	49	59	44	48
N.S.	1	1.22	0.84	0.82	0.96	1.04	0.98	1.18	0.88	0.96
time (sec)	N/A	0.180	0.007	0.095	0.026	0.067	0.062	0.125	0.176	0.028

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	43	31	30	34	38	34	45	32	32
N.S.	1	1.19	0.86	0.83	0.94	1.06	0.94	1.25	0.89	0.89
time (sec)	N/A	0.164	0.005	0.092	0.027	0.060	0.055	0.124	0.169	0.090

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	15	19	19	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	0.83	1.06	1.06	1.00
time (sec)	N/A	0.145	0.003	0.077	0.029	0.065	0.044	0.119	0.146	0.025

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	1.00
time (sec)	N/A	0.126	0.000	0.066	0.042	0.062	0.017	0.120	0.156	0.012

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	29	37	28	22	39	30	17
N.S.	1	1.00	1.00	1.71	2.18	1.65	1.29	2.29	1.76	1.00
time (sec)	N/A	0.144	0.004	0.174	0.036	0.075	0.069	0.115	0.151	0.038

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	51	60	60	48	53	62	42
N.S.	1	1.00	1.26	1.21	1.43	1.43	1.14	1.26	1.48	1.00
time (sec)	N/A	0.180	0.010	0.124	0.033	0.072	0.115	0.118	0.155	0.112

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	64	82	98	71	69	116	64
N.S.	1	1.00	1.03	1.02	1.30	1.56	1.13	1.10	1.84	1.02
time (sec)	N/A	0.188	0.012	0.132	0.033	0.073	0.152	0.133	0.166	0.112

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	45	53	79	51	80	64	52
N.S.	1	1.00	0.85	0.83	0.98	1.46	0.94	1.48	1.19	0.96
time (sec)	N/A	0.185	0.013	0.093	0.027	0.068	0.093	0.117	0.156	0.098

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	34	40	61	36	59	49	39
N.S.	1	1.00	0.85	0.87	1.03	1.56	0.92	1.51	1.26	1.00
time (sec)	N/A	0.165	0.011	0.095	0.029	0.063	0.074	0.127	0.155	0.028

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	25	28	33	24	54	36	27
N.S.	1	1.00	0.85	0.93	1.04	1.22	0.89	2.00	1.33	1.00
time (sec)	N/A	0.154	0.007	0.083	0.026	0.070	0.070	0.124	0.157	0.088

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00	1.00
time (sec)	N/A	0.124	0.001	0.072	0.029	0.061	0.052	0.122	0.160	0.014

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	50	50	55	51	44	44	61	37
N.S.	1	1.00	1.22	1.22	1.34	1.24	1.07	1.07	1.49	0.90
time (sec)	N/A	0.169	0.009	0.118	0.033	0.081	0.114	0.125	0.152	0.103

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	61	64	76	49	83	87	46
N.S.	1	1.00	1.61	1.33	1.39	1.65	1.07	1.80	1.89	1.00
time (sec)	N/A	0.168	0.016	0.125	0.026	0.073	0.108	0.121	0.152	0.100

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	80	108	146	104	81	171	86
N.S.	1	1.00	0.82	0.96	1.30	1.76	1.25	0.98	2.06	1.04
time (sec)	N/A	0.218	0.026	0.145	0.029	0.079	0.209	0.125	0.151	0.057

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	129	114	105	72	201	0	678	70	0
N.S.	1	1.02	0.90	0.83	0.57	1.60	0.00	5.38	0.56	0.00
time (sec)	N/A	0.199	0.296	0.148	0.132	0.076	0.000	0.345	0.155	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	98	104	100	50	155	0	399	51	0
N.S.	1	1.02	1.08	1.04	0.52	1.61	0.00	4.16	0.53	0.00
time (sec)	N/A	0.173	0.145	0.107	0.109	0.106	0.000	0.259	0.156	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	85	28	127	0	173	33	59
N.S.	1	1.00	1.03	1.27	0.42	1.90	0.00	2.58	0.49	0.88
time (sec)	N/A	0.160	0.072	0.101	0.117	0.103	0.000	0.201	0.161	0.115

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	47	57	8	101	85	49	24	44
N.S.	1	1.00	1.09	1.33	0.19	2.35	1.98	1.14	0.56	1.02
time (sec)	N/A	0.145	0.043	0.095	0.111	0.127	11.675	0.140	0.164	0.101

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	39	82	116	26	23
N.S.	1	1.00	1.00	0.89	0.78	1.44	3.04	4.30	0.96	0.85
time (sec)	N/A	0.131	0.065	0.096	0.040	0.076	1.975	0.130	0.172	0.240

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	32	45	57	82	237	41	62
N.S.	1	1.00	0.69	0.52	0.74	0.93	1.34	3.89	0.67	1.02
time (sec)	N/A	0.152	0.095	0.095	0.039	0.090	6.277	0.156	0.157	0.249

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	102	49	37	67	74	85	333	51	50
N.S.	1	1.12	0.54	0.41	0.74	0.81	0.93	3.66	0.56	0.55
time (sec)	N/A	0.192	0.192	0.096	0.048	0.071	28.287	0.191	0.154	0.276

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	143	54	42	89	89	85	437	61	66
N.S.	1	1.18	0.45	0.35	0.74	0.74	0.70	3.61	0.50	0.55
time (sec)	N/A	0.191	0.419	0.142	0.031	0.077	139.654	0.308	0.160	0.305

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	141	103	136	89	232	0	622	98	0
N.S.	1	1.04	0.76	1.00	0.65	1.71	0.00	4.57	0.72	0.00
time (sec)	N/A	0.198	0.190	0.180	0.145	0.151	0.000	0.650	0.159	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	105	92	125	63	193	0	354	72	0
N.S.	1	1.02	0.89	1.21	0.61	1.87	0.00	3.44	0.70	0.00
time (sec)	N/A	0.174	0.126	0.184	0.152	0.102	0.000	0.458	0.164	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	107	39	159	0	148	46	72
N.S.	1	1.00	0.97	1.55	0.57	2.30	0.00	2.14	0.67	1.04
time (sec)	N/A	0.157	0.078	0.139	0.132	0.095	0.000	0.289	0.177	0.194

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	48	71	14	108	90	42	27	53
N.S.	1	1.00	1.23	1.82	0.36	2.77	2.31	1.08	0.69	1.36
time (sec)	N/A	0.139	0.035	0.132	0.114	0.097	11.780	0.194	0.146	0.098

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	30	25	45	94	103	27	26
N.S.	1	1.00	0.97	1.00	0.83	1.50	3.13	3.43	0.90	0.87
time (sec)	N/A	0.130	0.039	0.141	0.026	0.068	2.208	0.129	0.143	0.300

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	46	45	53	72	94	199	56	80
N.S.	1	1.00	0.69	0.67	0.79	1.07	1.40	2.97	0.84	1.19
time (sec)	N/A	0.153	0.066	0.137	0.032	0.078	7.198	0.152	0.170	0.347

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	111	57	56	79	98	97	296	77	111
N.S.	1	1.11	0.57	0.56	0.79	0.98	0.97	2.96	0.77	1.11
time (sec)	N/A	0.185	0.116	0.141	0.054	0.093	30.155	0.175	0.170	0.382

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	155	68	67	105	122	97	393	100	170
N.S.	1	1.17	0.51	0.50	0.79	0.92	0.73	2.95	0.75	1.28
time (sec)	N/A	0.198	0.223	0.145	0.034	0.131	147.642	0.245	0.173	0.430

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	59	107	46	66	0	227	72	0
N.S.	1	1.06	0.75	1.35	0.58	0.84	0.00	2.87	0.91	0.00
time (sec)	N/A	0.159	0.314	0.173	0.110	0.087	0.000	0.166	0.153	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	62	54	102	34	61	0	125	55	0
N.S.	1	1.03	0.90	1.70	0.57	1.02	0.00	2.08	0.92	0.00
time (sec)	N/A	0.154	0.213	0.098	0.134	0.078	0.000	0.150	0.146	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	70	22	53	187	55	36	44
N.S.	1	1.00	1.17	1.94	0.61	1.47	5.19	1.53	1.00	1.22
time (sec)	N/A	0.139	0.147	0.101	0.126	0.085	2.008	0.131	0.153	0.089

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	27	37	9	35	41	21	16	40
N.S.	1	1.00	2.08	2.85	0.69	2.69	3.15	1.62	1.23	3.08
time (sec)	N/A	0.125	0.040	0.135	0.108	0.079	0.999	0.157	0.168	0.099

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	16	28	12	26	156	79	21	24
N.S.	1	1.00	0.76	1.33	0.57	1.24	7.43	3.76	1.00	1.14
time (sec)	N/A	0.125	0.094	0.093	0.024	0.071	28.395	0.135	0.159	0.187

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	35	25	39	0	145	37	49
N.S.	1	1.00	0.65	0.81	0.58	0.91	0.00	3.37	0.86	1.14
time (sec)	N/A	0.138	0.107	0.174	0.037	0.076	0.000	0.141	0.156	0.194

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	33	40	37	49	0	208	47	66
N.S.	1	1.08	0.52	0.62	0.58	0.77	0.00	3.25	0.73	1.03
time (sec)	N/A	0.146	0.125	0.092	0.038	0.072	0.000	0.144	0.167	0.276

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	38	45	49	59	0	271	57	85
N.S.	1	1.12	0.45	0.53	0.58	0.69	0.00	3.19	0.67	1.00
time (sec)	N/A	0.158	0.164	0.091	0.026	0.080	0.000	0.174	0.171	0.293

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	12	22	71	62	17	22
N.S.	1	1.00	1.00	1.00	0.75	1.38	4.44	3.88	1.06	1.38
time (sec)	N/A	0.120	0.028	0.092	0.025	0.075	0.719	0.129	0.151	0.184

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	15	29	73	77	20	26
N.S.	1	1.00	1.00	1.00	0.79	1.53	3.84	4.05	1.05	1.37
time (sec)	N/A	0.121	0.040	0.115	0.030	0.064	2.093	0.125	0.160	0.214

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	12	22	92	71	19	22
N.S.	1	1.00	1.00	0.90	0.57	1.05	4.38	3.38	0.90	1.05
time (sec)	N/A	0.123	0.057	0.086	0.025	0.069	5.575	0.124	0.164	0.218

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	24	15	29	83	86	22	26
N.S.	1	1.00	0.79	1.00	0.62	1.21	3.46	3.58	0.92	1.08
time (sec)	N/A	0.134	0.094	0.106	0.032	0.074	8.604	0.133	0.162	0.262

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	76	39	108	88	37	35	56
N.S.	1	1.00	1.00	1.95	1.00	2.77	2.26	0.95	0.90	1.44
time (sec)	N/A	0.151	0.067	0.158	0.033	0.085	11.667	0.197	0.213	0.134

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	26	26	75	23	25	40
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.09	2.27	3.64
time (sec)	N/A	0.122	0.029	0.090	0.033	0.070	11.652	0.121	0.187	0.126

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	31	61	30	29	80	26	26	43
N.S.	1	1.00	2.38	4.69	2.31	2.23	6.15	2.00	2.00	3.31
time (sec)	N/A	0.124	0.066	0.093	0.115	0.070	11.661	0.127	0.162	0.112

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	31	69	34	41	83	29	26	59
N.S.	1	1.00	2.38	5.31	2.62	3.15	6.38	2.23	2.00	4.54
time (sec)	N/A	0.124	0.002	0.138	0.026	0.091	12.822	0.125	0.157	0.117

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	113	63	92	68	62	287	185	100	0
N.S.	1	1.15	0.64	0.94	0.69	0.63	2.93	1.89	1.02	0.00
time (sec)	N/A	0.178	0.148	0.105	0.128	0.081	60.355	0.165	0.161	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	88	58	87	54	57	252	115	85	0
N.S.	1	1.13	0.74	1.12	0.69	0.73	3.23	1.47	1.09	0.00
time (sec)	N/A	0.163	0.128	0.094	0.134	0.086	20.317	0.155	0.147	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	53	82	40	52	216	101	70	0
N.S.	1	1.09	0.91	1.41	0.69	0.90	3.72	1.74	1.21	0.00
time (sec)	N/A	0.148	0.101	0.082	0.126	0.074	7.315	0.135	0.146	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	71	28	47	167	50	55	0
N.S.	1	1.00	1.21	1.87	0.74	1.24	4.39	1.32	1.45	0.00
time (sec)	N/A	0.137	0.087	0.083	0.107	0.082	2.962	0.131	0.146	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	37	57	17	38	131	42	28	37
N.S.	1	1.00	1.61	2.48	0.74	1.65	5.70	1.83	1.22	1.61
time (sec)	N/A	0.144	0.037	0.079	0.117	0.063	1.552	0.144	0.147	0.057

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	16	34	42	14	37	99	28	27	14
N.S.	1	0.76	1.62	2.00	0.67	1.76	4.71	1.33	1.29	0.67
time (sec)	N/A	0.126	0.040	0.082	0.107	0.115	0.927	0.133	0.148	0.089

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	64	21	48	70	33	34	0
N.S.	1	1.00	1.70	2.78	0.91	2.09	3.04	1.43	1.48	0.00
time (sec)	N/A	0.130	0.053	0.082	0.110	0.098	0.773	0.133	0.145	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	38	33	60	19	22	34
N.S.	1	1.00	1.00	0.75	1.90	1.65	3.00	0.95	1.10	1.70
time (sec)	N/A	0.122	0.018	0.083	0.036	0.078	0.677	0.130	0.145	0.159

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	64	53	172	22	34	50
N.S.	1	1.00	0.61	0.44	1.56	1.29	4.20	0.54	0.83	1.22
time (sec)	N/A	0.135	0.025	0.086	0.033	0.085	2.597	0.130	0.150	0.151

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	30	25	95	70	566	29	44	64
N.S.	1	1.08	0.49	0.41	1.56	1.15	9.28	0.48	0.72	1.05
time (sec)	N/A	0.145	0.028	0.092	0.041	0.070	8.957	0.131	0.149	0.172

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	35	30	131	85	1561	35	54	80
N.S.	1	1.12	0.43	0.37	1.62	1.05	19.27	0.43	0.67	0.99
time (sec)	N/A	0.159	0.035	0.089	0.029	0.077	27.794	0.133	0.148	0.166

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	40	35	172	100	3648	42	64	94
N.S.	1	1.15	0.40	0.35	1.70	0.99	36.12	0.42	0.63	0.93
time (sec)	N/A	0.169	0.040	0.093	0.033	0.103	74.828	0.146	0.152	0.181

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	109	68	97	66	67	0	237	115	0
N.S.	1	1.16	0.72	1.03	0.70	0.71	0.00	2.52	1.22	0.00
time (sec)	N/A	0.179	0.161	0.091	0.108	0.086	0.000	0.165	0.182	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	84	63	92	52	62	287	185	100	0
N.S.	1	1.14	0.85	1.24	0.70	0.84	3.88	2.50	1.35	0.00
time (sec)	N/A	0.158	0.143	0.093	0.117	0.082	88.660	0.159	0.155	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	58	87	40	57	248	91	85	0
N.S.	1	1.09	1.07	1.61	0.74	1.06	4.59	1.69	1.57	0.00
time (sec)	N/A	0.144	0.115	0.089	0.105	0.067	29.222	0.141	0.160	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	44	46	75	29	46	212	101	43	0
N.S.	1	1.13	1.18	1.92	0.74	1.18	5.44	2.59	1.10	0.00
time (sec)	N/A	0.138	0.063	0.086	0.109	0.072	9.730	0.152	0.153	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	71	28	47	163	66	55	0
N.S.	1	1.00	1.21	1.87	0.74	1.24	4.29	1.74	1.45	0.00
time (sec)	N/A	0.135	0.083	0.085	0.109	0.110	4.123	0.134	0.148	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	41	39	57	28	40	134	31	40	0
N.S.	1	0.87	0.83	1.21	0.60	0.85	2.85	0.66	0.85	0.00
time (sec)	N/A	0.136	0.059	0.088	0.112	0.081	2.108	0.133	0.149	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	37	41	72	42	52	99	35	43	0
N.S.	1	0.90	1.00	1.76	1.02	1.27	2.41	0.85	1.05	0.00
time (sec)	N/A	0.139	0.074	0.089	0.136	0.088	1.514	0.131	0.150	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	76	66	71	498	38	70	0
N.S.	1	1.00	1.12	1.85	1.61	1.73	12.15	0.93	1.71	0.00
time (sec)	N/A	0.142	0.061	0.096	0.110	0.075	1.683	0.129	0.156	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	94	52	87	19	32	50
N.S.	1	1.00	1.00	0.75	4.70	2.60	4.35	0.95	1.60	2.50
time (sec)	N/A	0.123	0.021	0.089	0.026	0.067	2.533	0.139	0.152	0.157

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	131	69	226	22	42	64
N.S.	1	1.00	0.61	0.44	3.20	1.68	5.51	0.54	1.02	1.56
time (sec)	N/A	0.151	0.027	0.092	0.040	0.078	8.896	0.134	0.157	0.171

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	30	25	172	86	675	29	54	80
N.S.	1	1.08	0.49	0.41	2.82	1.41	11.07	0.48	0.89	1.31
time (sec)	N/A	0.149	0.034	0.092	0.039	0.069	27.726	0.140	0.154	0.167

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	35	30	218	101	1751	35	64	94
N.S.	1	1.12	0.43	0.37	2.69	1.25	21.62	0.43	0.79	1.16
time (sec)	N/A	0.159	0.038	0.096	0.032	0.095	74.922	0.142	0.153	0.180

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	40	35	269	116	0	42	74	110
N.S.	1	1.15	0.40	0.35	2.66	1.15	0.00	0.42	0.73	1.09
time (sec)	N/A	0.172	0.046	0.097	0.035	0.096	0.000	0.158	0.153	0.187

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	130	78	107	78	77	0	323	145	0
N.S.	1	1.18	0.71	0.97	0.71	0.70	0.00	2.94	1.32	0.00
time (sec)	N/A	0.194	0.199	0.100	0.105	0.072	0.000	0.176	0.156	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	105	72	102	64	72	0	296	130	0
N.S.	1	1.17	0.80	1.13	0.71	0.80	0.00	3.29	1.44	0.00
time (sec)	N/A	0.175	0.182	0.093	0.108	0.070	0.000	0.183	0.163	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	80	68	97	52	67	0	143	115	0
N.S.	1	1.14	0.97	1.39	0.74	0.96	0.00	2.04	1.64	0.00
time (sec)	N/A	0.160	0.154	0.095	0.107	0.070	0.000	0.145	0.170	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	65	51	80	41	51	284	185	58	0
N.S.	1	1.18	0.93	1.45	0.75	0.93	5.16	3.36	1.05	0.00
time (sec)	N/A	0.176	0.077	0.093	0.120	0.068	161.506	0.167	0.169	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	58	87	40	57	245	114	85	0
N.S.	1	1.09	1.07	1.61	0.74	1.06	4.54	2.11	1.57	0.00
time (sec)	N/A	0.146	0.113	0.089	0.122	0.109	52.747	0.144	0.150	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	53	82	40	52	212	101	70	0
N.S.	1	1.09	0.91	1.41	0.69	0.90	3.66	1.74	1.21	0.00
time (sec)	N/A	0.145	0.084	0.135	0.130	0.088	17.901	0.138	0.156	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	46	71	42	47	170	39	55	0
N.S.	1	0.99	0.69	1.06	0.63	0.70	2.54	0.58	0.82	0.00
time (sec)	N/A	0.146	0.073	0.088	0.105	0.090	7.565	0.125	0.152	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	62	49	77	56	58	138	42	53	0
N.S.	1	0.95	0.75	1.18	0.86	0.89	2.12	0.65	0.82	0.00
time (sec)	N/A	0.145	0.089	0.093	0.107	0.072	4.382	0.131	0.155	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	62	51	84	99	75	575	44	79	0
N.S.	1	0.98	0.81	1.33	1.57	1.19	9.13	0.70	1.25	0.00
time (sec)	N/A	0.145	0.090	0.099	0.140	0.098	3.388	0.132	0.157	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	84	160	91	1606	44	107	0
N.S.	1	1.00	0.81	1.33	2.54	1.44	25.49	0.70	1.70	0.00
time (sec)	N/A	0.161	0.070	0.096	0.127	0.079	4.870	0.140	0.151	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	171	66	114	19	44	64
N.S.	1	1.00	1.00	0.75	8.55	3.30	5.70	0.95	2.20	3.20
time (sec)	N/A	0.127	0.022	0.099	0.059	0.071	8.864	0.142	0.156	0.173

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	218	83	280	22	54	80
N.S.	1	1.00	0.61	0.44	5.32	2.02	6.83	0.54	1.32	1.95
time (sec)	N/A	0.139	0.036	0.095	0.050	0.077	27.821	0.145	0.163	0.178

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	30	25	269	100	784	29	64	94
N.S.	1	1.08	0.49	0.41	4.41	1.64	12.85	0.48	1.05	1.54
time (sec)	N/A	0.150	0.037	0.099	0.031	0.069	74.780	0.147	0.164	0.190

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	35	30	325	115	0	35	74	110
N.S.	1	1.12	0.43	0.37	4.01	1.42	0.00	0.43	0.91	1.36
time (sec)	N/A	0.158	0.043	0.096	0.033	0.092	0.000	0.148	0.167	0.188

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	40	35	386	130	0	42	84	124
N.S.	1	1.15	0.40	0.35	3.82	1.29	0.00	0.42	0.83	1.23
time (sec)	N/A	0.174	0.055	0.108	0.036	0.090	0.000	0.150	0.159	0.210

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	59	55	98	42	55	0	42	53	0
N.S.	1	0.92	0.86	1.53	0.66	0.86	0.00	0.66	0.83	0.00
time (sec)	N/A	0.157	0.093	0.138	0.105	0.090	0.000	0.136	0.161	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	59	54	60	42	48	75	34	41	55
N.S.	1	1.55	1.42	1.58	1.11	1.26	1.97	0.89	1.08	1.45
time (sec)	N/A	0.166	0.757	0.142	0.106	0.085	0.447	0.141	0.161	0.089

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	60	42	48	76	34	41	55
N.S.	1	1.00	0.93	1.02	0.71	0.81	1.29	0.58	0.69	0.93
time (sec)	N/A	0.178	0.144	0.174	0.120	0.081	2.242	0.134	0.161	0.020

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	53	82	56	52	197	101	70	0
N.S.	1	1.02	0.61	0.94	0.64	0.60	2.26	1.16	0.80	0.00
time (sec)	N/A	0.172	0.110	0.090	0.110	0.075	9.200	0.148	0.160	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	46	71	42	47	173	69	55	0
N.S.	1	0.96	0.69	1.06	0.63	0.70	2.58	1.03	0.82	0.00
time (sec)	N/A	0.157	0.086	0.138	0.109	0.132	3.009	0.135	0.156	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	39	39	57	28	40	138	44	40	0
N.S.	1	0.83	0.83	1.21	0.60	0.85	2.94	0.94	0.85	0.00
time (sec)	N/A	0.142	0.069	0.089	0.106	0.096	1.331	0.126	0.155	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	14	32	41	12	36	99	27	26	12
N.S.	1	0.70	1.60	2.05	0.60	1.80	4.95	1.35	1.30	0.60
time (sec)	N/A	0.125	0.044	0.130	0.106	0.107	0.789	0.122	0.160	0.137

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	20	27	2	22	39	13	14	22
N.S.	1	1.00	10.00	13.50	1.00	11.00	19.50	6.50	7.00	11.00
time (sec)	N/A	0.113	0.021	0.077	0.104	0.082	0.472	0.125	0.158	0.101

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	23	31	19	13	13
N.S.	1	1.00	1.00	0.82	0.94	1.35	1.82	1.12	0.76	0.76
time (sec)	N/A	0.127	0.014	0.082	0.117	0.090	0.427	0.122	0.158	0.164

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	38	39	128	22	22	43
N.S.	1	1.00	0.61	0.44	0.93	0.95	3.12	0.54	0.54	1.05
time (sec)	N/A	0.142	0.023	0.089	0.128	0.076	0.930	0.128	0.159	0.211

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	30	25	64	56	303	29	34	55
N.S.	1	1.08	0.49	0.41	1.05	0.92	4.97	0.48	0.56	0.90
time (sec)	N/A	0.168	0.024	0.089	0.121	0.073	3.386	0.130	0.158	0.214

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	35	30	95	71	542	35	44	67
N.S.	1	1.12	0.43	0.37	1.17	0.88	6.69	0.43	0.54	0.83
time (sec)	N/A	0.162	0.027	0.092	0.105	0.068	10.527	0.129	0.157	0.210

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	40	35	131	86	850	42	54	80
N.S.	1	1.15	0.40	0.35	1.30	0.85	8.42	0.42	0.53	0.79
time (sec)	N/A	0.176	0.034	0.087	0.113	0.079	31.372	0.120	0.159	0.221

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	56	84	70	65	206	81	66	0
N.S.	1	1.00	0.66	0.99	0.82	0.76	2.42	0.95	0.78	0.00
time (sec)	N/A	0.161	0.081	0.101	0.110	0.103	9.033	0.154	0.154	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	60	49	77	56	58	167	73	54	0
N.S.	1	0.92	0.75	1.18	0.86	0.89	2.57	1.12	0.83	0.00
time (sec)	N/A	0.153	0.063	0.091	0.145	0.116	3.026	0.145	0.152	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	35	42	71	41	53	131	70	43	0
N.S.	1	0.85	1.02	1.73	1.00	1.29	3.20	1.71	1.05	0.00
time (sec)	N/A	0.163	0.049	0.092	0.105	0.106	1.231	0.140	0.151	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	67	21	50	102	55	34	0
N.S.	1	1.00	1.70	2.91	0.91	2.17	4.43	2.39	1.48	0.00
time (sec)	N/A	0.133	0.029	0.087	0.115	0.092	0.713	0.126	0.145	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	23	31	43	14	14
N.S.	1	1.00	1.00	0.83	0.89	1.28	1.72	2.39	0.78	0.78
time (sec)	N/A	0.128	0.006	0.088	0.115	0.099	0.428	0.120	0.145	0.218

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	11	22	63	62	16	14
N.S.	1	1.00	1.00	1.15	0.85	1.69	4.85	4.77	1.23	1.08
time (sec)	N/A	0.130	0.023	0.084	0.026	0.100	0.699	0.128	0.150	0.205

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	25	40	54	160	67	31	42
N.S.	1	1.00	0.81	0.68	1.08	1.46	4.32	1.81	0.84	1.14
time (sec)	N/A	0.134	0.023	0.092	0.026	0.089	1.843	0.128	0.152	0.212

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	33	28	79	59	284	73	39	55
N.S.	1	1.09	0.58	0.49	1.39	1.04	4.98	1.28	0.68	0.96
time (sec)	N/A	0.154	0.027	0.098	0.034	0.138	6.738	0.134	0.150	0.213

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	87	40	35	134	86	425	79	51	68
N.S.	1	1.13	0.52	0.45	1.74	1.12	5.52	1.03	0.66	0.88
time (sec)	N/A	0.169	0.034	0.101	0.029	0.134	21.370	0.130	0.163	0.223

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	108	61	89	125	85	248	127	101	0
N.S.	1	1.05	0.59	0.86	1.21	0.83	2.41	1.23	0.98	0.00
time (sec)	N/A	0.177	0.084	0.099	0.108	0.110	26.069	0.183	0.164	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	85	56	84	111	81	212	119	90	0
N.S.	1	0.98	0.64	0.97	1.28	0.93	2.44	1.37	1.03	0.00
time (sec)	N/A	0.166	0.066	0.102	0.107	0.081	9.044	0.166	0.153	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	60	51	79	98	75	162	115	79	0
N.S.	1	0.95	0.81	1.25	1.56	1.19	2.57	1.83	1.25	0.00
time (sec)	N/A	0.149	0.067	0.144	0.111	0.103	2.957	0.155	0.161	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	73	66	71	128	102	68	0
N.S.	1	1.00	1.12	1.78	1.61	1.73	3.12	2.49	1.66	0.00
time (sec)	N/A	0.137	0.051	0.092	0.113	0.121	1.532	0.149	0.161	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	38	37	66	89	22	32
N.S.	1	1.00	1.00	0.75	1.90	1.85	3.30	4.45	1.10	1.60
time (sec)	N/A	0.121	0.011	0.134	0.041	0.115	0.674	0.140	0.157	0.158

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	38	38	66	89	24	33
N.S.	1	1.00	0.61	0.44	0.93	0.93	1.61	2.17	0.59	0.80
time (sec)	N/A	0.134	0.018	0.088	0.110	0.082	0.932	0.122	0.157	0.190

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	61	30	25	38	49	167	108	31	48
N.S.	1	1.65	0.81	0.68	1.03	1.32	4.51	2.92	0.84	1.30
time (sec)	N/A	0.147	0.026	0.138	0.034	0.102	1.860	0.128	0.165	0.206

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	24	23	25	35	280	113	31	41
N.S.	1	1.00	0.73	0.70	0.76	1.06	8.48	3.42	0.94	1.24
time (sec)	N/A	0.131	0.036	0.089	0.040	0.087	3.693	0.132	0.159	0.219

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	58	40	35	52	84	425	119	51	75
N.S.	1	1.09	0.75	0.66	0.98	1.58	8.02	2.25	0.96	1.42
time (sec)	N/A	0.146	0.031	0.152	0.034	0.085	12.217	0.142	0.149	0.239

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	83	45	40	91	101	593	125	56	86
N.S.	1	1.14	0.62	0.55	1.25	1.38	8.12	1.71	0.77	1.18
time (sec)	N/A	0.162	0.034	0.098	0.034	0.079	36.760	0.143	0.162	0.252

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	108	50	45	146	114	789	131	71	99
N.S.	1	1.16	0.54	0.48	1.57	1.23	8.48	1.41	0.76	1.06
time (sec)	N/A	0.184	0.040	0.096	0.028	0.075	123.697	0.128	0.162	0.258

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	171	0	0	177	0	0	110	0
N.S.	1	1.00	0.94	0.00	0.00	0.98	0.00	0.00	0.61	0.00
time (sec)	N/A	0.228	0.277	0.000	0.000	0.113	0.000	0.000	0.630	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	149	160	0	0	164	0	0	82	0
N.S.	1	0.98	1.05	0.00	0.00	1.08	0.00	0.00	0.54	0.00
time (sec)	N/A	0.205	0.195	0.000	0.000	0.124	0.000	0.000	0.451	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	147	0	0	152	0	0	18	0
N.S.	1	1.04	1.31	0.00	0.00	1.36	0.00	0.00	0.16	0.00
time (sec)	N/A	0.177	0.156	0.000	0.000	0.109	0.000	0.000	0.232	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	143	0	0	172	0	0	33	0
N.S.	1	1.00	1.29	0.00	0.00	1.55	0.00	0.00	0.30	0.00
time (sec)	N/A	0.172	0.177	0.000	0.000	0.106	0.000	0.000	0.256	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	49	0	0	30	49
N.S.	1	1.00	1.00	0.83	0.00	1.69	0.00	0.00	1.03	1.69
time (sec)	N/A	0.131	0.021	0.132	0.000	0.087	0.000	0.000	0.306	0.249

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	32	0	74	0	0	60	42
N.S.	1	1.00	0.63	0.54	0.00	1.25	0.00	0.00	1.02	0.71
time (sec)	N/A	0.149	0.078	0.180	0.000	0.087	0.000	0.000	0.339	0.284

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	48	43	0	96	0	0	82	53
N.S.	1	1.09	0.55	0.49	0.00	1.09	0.00	0.00	0.93	0.60
time (sec)	N/A	0.165	0.097	0.132	0.000	0.082	0.000	0.000	0.397	0.310

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	59	54	0	118	0	0	104	64
N.S.	1	1.14	0.50	0.46	0.00	1.01	0.00	0.00	0.89	0.55
time (sec)	N/A	0.184	0.158	0.132	0.000	0.117	0.000	0.000	0.444	0.344

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	66	66	0	0	0	0	0	107	0
N.S.	1	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.163	0.020	0.000	0.000	0.000	0.000	0.000	0.649	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	349	67	0	0	0	0	0	56	0
N.S.	1	1.13	0.22	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.296	0.016	0.000	0.000	0.000	0.000	0.000	0.484	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	67	67	0	0	0	0	0	18	0
N.S.	1	0.22	0.22	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.161	0.017	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	70	70	0	0	0	0	0	33	0
N.S.	1	0.23	0.23	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.157	0.016	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	70	70	0	0	0	0	0	51	0
N.S.	1	0.21	0.21	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.154	0.016	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	211	182	0	0	202	0	0	136	0
N.S.	1	1.01	0.87	0.00	0.00	0.97	0.00	0.00	0.65	0.00
time (sec)	N/A	0.232	0.275	0.000	0.000	0.092	0.000	0.000	0.853	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	179	171	0	0	191	0	0	110	0
N.S.	1	0.99	0.95	0.00	0.00	1.06	0.00	0.00	0.61	0.00
time (sec)	N/A	0.211	0.247	0.000	0.000	0.091	0.000	0.000	0.678	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	147	177	0	0	180	0	0	82	0
N.S.	1	0.97	1.17	0.00	0.00	1.19	0.00	0.00	0.54	0.00
time (sec)	N/A	0.193	0.200	0.000	0.000	0.109	0.000	0.000	0.453	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	115	148	0	0	166	0	0	18	0
N.S.	1	1.04	1.33	0.00	0.00	1.50	0.00	0.00	0.16	0.00
time (sec)	N/A	0.175	0.155	0.000	0.000	0.102	0.000	0.000	0.192	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	143	0	0	185	0	0	33	0
N.S.	1	1.00	1.27	0.00	0.00	1.64	0.00	0.00	0.29	0.00
time (sec)	N/A	0.176	0.151	0.000	0.000	0.104	0.000	0.000	0.215	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	49	0	0	30	0
N.S.	1	1.00	1.00	0.83	0.00	1.69	0.00	0.00	1.03	0.00
time (sec)	N/A	0.133	0.017	0.122	0.000	0.098	0.000	0.000	0.203	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	32	0	74	0	0	60	0
N.S.	1	1.00	0.63	0.54	0.00	1.25	0.00	0.00	1.02	0.00
time (sec)	N/A	0.150	0.087	0.181	0.000	0.082	0.000	0.000	0.210	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	48	43	0	96	0	0	82	0
N.S.	1	1.09	0.55	0.49	0.00	1.09	0.00	0.00	0.93	0.00
time (sec)	N/A	0.169	0.124	0.128	0.000	0.084	0.000	0.000	0.221	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	59	54	0	118	0	0	104	0
N.S.	1	1.14	0.50	0.46	0.00	1.01	0.00	0.00	0.89	0.00
time (sec)	N/A	0.188	0.185	0.132	0.000	0.111	0.000	0.000	0.239	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	684	68	68	0	0	0	0	0	107	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.158	0.019	0.000	0.000	0.000	0.000	0.000	0.404	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	657	732	67	0	0	0	0	0	56	0
N.S.	1	1.11	0.10	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.509	0.016	0.000	0.000	0.000	0.000	0.000	0.317	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	652	67	67	0	0	0	0	0	18	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.151	0.017	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	648	70	70	0	0	0	0	0	33	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.156	0.016	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	684	70	70	0	0	0	0	0	51	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.155	0.017	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	245	193	0	0	199	0	0	162	0
N.S.	1	1.03	0.81	0.00	0.00	0.83	0.00	0.00	0.68	0.00
time (sec)	N/A	0.235	0.396	0.000	0.000	0.089	0.000	0.000	0.941	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	213	182	0	0	188	0	0	136	0
N.S.	1	1.01	0.87	0.00	0.00	0.90	0.00	0.00	0.65	0.00
time (sec)	N/A	0.225	0.296	0.000	0.000	0.138	0.000	0.000	0.714	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	171	0	0	177	0	0	110	0
N.S.	1	1.00	0.94	0.00	0.00	0.98	0.00	0.00	0.61	0.00
time (sec)	N/A	0.211	0.234	0.000	0.000	0.107	0.000	0.000	0.689	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	149	162	0	0	166	0	0	43	0
N.S.	1	0.98	1.07	0.00	0.00	1.09	0.00	0.00	0.28	0.00
time (sec)	N/A	0.192	0.201	0.000	0.000	0.092	0.000	0.000	0.348	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	144	155	0	0	188	0	0	73	0
N.S.	1	1.05	1.13	0.00	0.00	1.37	0.00	0.00	0.53	0.00
time (sec)	N/A	0.191	0.222	0.000	0.000	0.099	0.000	0.000	0.430	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	0	0	224	0	0	166	0
N.S.	1	1.00	1.11	0.00	0.00	1.62	0.00	0.00	1.20	0.00
time (sec)	N/A	0.182	0.180	0.000	0.000	0.094	0.000	0.000	0.614	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	69	0	0	60	76
N.S.	1	1.00	1.00	0.83	0.00	2.38	0.00	0.00	2.07	2.62
time (sec)	N/A	0.131	0.020	0.131	0.000	0.091	0.000	0.000	0.341	0.287

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	32	0	96	0	0	82	105
N.S.	1	1.00	0.63	0.54	0.00	1.63	0.00	0.00	1.39	1.78
time (sec)	N/A	0.146	0.110	0.138	0.000	0.115	0.000	0.000	0.401	0.303

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	48	43	0	118	0	0	104	134
N.S.	1	1.09	0.55	0.49	0.00	1.34	0.00	0.00	1.18	1.52
time (sec)	N/A	0.184	0.139	0.138	0.000	0.097	0.000	0.000	0.466	0.311

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	59	54	0	140	0	0	126	163
N.S.	1	1.14	0.50	0.46	0.00	1.20	0.00	0.00	1.08	1.39
time (sec)	N/A	0.183	0.240	0.144	0.000	0.129	0.000	0.000	0.519	0.343

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	70	70	0	0	0	0	0	159	0
N.S.	1	0.19	0.19	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.155	0.022	0.000	0.000	0.000	0.000	0.000	0.934	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	378	68	0	0	0	0	0	82	0
N.S.	1	1.12	0.20	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.294	0.021	0.000	0.000	0.000	0.000	0.000	0.841	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	67	67	0	0	0	0	0	107	0
N.S.	1	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.163	0.017	0.000	0.000	0.000	0.000	0.000	0.642	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	67	67	0	0	0	0	0	43	0
N.S.	1	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.171	0.022	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	70	70	0	0	0	0	0	73	0
N.S.	1	0.21	0.21	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.172	0.019	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	70	70	0	0	0	0	0	109	0
N.S.	1	0.21	0.21	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.160	0.020	0.000	0.000	0.000	0.000	0.000	0.376	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	364	70	70	0	0	0	0	0	145	0
N.S.	1	0.19	0.19	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.186	0.020	0.000	0.000	0.000	0.000	0.000	0.417	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	171	0	0	191	0	0	71	0
N.S.	1	1.00	0.95	0.00	0.00	1.06	0.00	0.00	0.39	0.00
time (sec)	N/A	0.212	0.287	0.000	0.000	0.092	0.000	0.000	0.433	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	148	162	0	0	180	0	0	42	0
N.S.	1	0.98	1.07	0.00	0.00	1.19	0.00	0.00	0.28	0.00
time (sec)	N/A	0.223	0.227	0.000	0.000	0.087	0.000	0.000	0.367	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	116	147	0	0	165	0	0	18	0
N.S.	1	1.05	1.32	0.00	0.00	1.49	0.00	0.00	0.16	0.00
time (sec)	N/A	0.191	0.162	0.000	0.000	0.116	0.000	0.000	0.229	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	122	0	0	144	0	0	18	0
N.S.	1	1.00	1.42	0.00	0.00	1.67	0.00	0.00	0.21	0.00
time (sec)	N/A	0.166	0.098	0.000	0.000	0.114	0.000	0.000	0.225	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	31	0	0	40	0
N.S.	1	1.00	1.00	0.83	0.00	1.07	0.00	0.00	1.38	0.00
time (sec)	N/A	0.149	0.015	0.133	0.000	0.086	0.000	0.000	0.255	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	32	0	52	0	0	66	0
N.S.	1	1.00	0.63	0.54	0.00	0.88	0.00	0.00	1.12	0.00
time (sec)	N/A	0.152	0.066	0.137	0.000	0.104	0.000	0.000	0.285	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	48	43	0	74	0	0	92	0
N.S.	1	1.09	0.55	0.49	0.00	0.84	0.00	0.00	1.05	0.00
time (sec)	N/A	0.178	0.089	0.193	0.000	0.089	0.000	0.000	0.307	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	59	54	0	96	0	0	118	0
N.S.	1	1.14	0.50	0.46	0.00	0.82	0.00	0.00	1.01	0.00
time (sec)	N/A	0.200	0.119	0.141	0.000	0.091	0.000	0.000	0.328	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	686	66	66	0	0	0	0	0	42	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.159	0.016	0.000	0.000	0.000	0.000	0.000	0.352	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	652	65	65	0	0	0	0	0	18	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.160	0.015	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	625	706	67	0	0	0	109	0	18	0
N.S.	1	1.13	0.11	0.00	0.00	0.00	0.17	0.00	0.03	0.00
time (sec)	N/A	0.493	0.015	0.000	0.000	0.000	1.592	0.000	0.265	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	656	70	70	0	0	0	0	0	40	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.159	0.014	0.000	0.000	0.000	0.000	0.000	0.292	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	685	70	70	0	0	0	0	0	66	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.155	0.015	0.000	0.000	0.000	0.000	0.000	0.356	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	182	171	0	0	177	0	0	71	0
N.S.	1	1.01	0.94	0.00	0.00	0.98	0.00	0.00	0.39	0.00
time (sec)	N/A	0.206	0.279	0.000	0.000	0.111	0.000	0.000	0.282	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	150	177	0	0	166	0	0	42	0
N.S.	1	0.99	1.16	0.00	0.00	1.09	0.00	0.00	0.28	0.00
time (sec)	N/A	0.196	0.195	0.000	0.000	0.082	0.000	0.000	0.241	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	118	148	0	0	152	0	0	18	0
N.S.	1	1.04	1.31	0.00	0.00	1.35	0.00	0.00	0.16	0.00
time (sec)	N/A	0.195	0.173	0.000	0.000	0.092	0.000	0.000	0.190	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	122	0	0	132	0	0	18	0
N.S.	1	1.00	1.39	0.00	0.00	1.50	0.00	0.00	0.20	0.00
time (sec)	N/A	0.160	0.096	0.000	0.000	0.099	0.000	0.000	0.220	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	31	0	0	40	23
N.S.	1	1.00	1.00	0.83	0.00	1.07	0.00	0.00	1.38	0.79
time (sec)	N/A	0.135	0.017	0.134	0.000	0.091	0.000	0.000	0.269	0.243

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	32	0	52	0	0	66	52
N.S.	1	1.00	0.63	0.54	0.00	0.88	0.00	0.00	1.12	0.88
time (sec)	N/A	0.146	0.055	0.137	0.000	0.083	0.000	0.000	0.299	0.297

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	48	43	0	74	0	0	92	81
N.S.	1	1.09	0.55	0.49	0.00	0.84	0.00	0.00	1.05	0.92
time (sec)	N/A	0.162	0.072	0.195	0.000	0.098	0.000	0.000	0.382	0.309

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	59	54	0	96	0	0	118	110
N.S.	1	1.14	0.50	0.46	0.00	0.82	0.00	0.00	1.01	0.94
time (sec)	N/A	0.180	0.106	0.135	0.000	0.086	0.000	0.000	0.407	0.325

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	66	66	0	0	0	0	0	42	0
N.S.	1	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.153	0.015	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	65	65	0	0	0	0	0	18	0
N.S.	1	0.21	0.21	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.150	0.013	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	320	65	0	0	0	107	0	18	0
N.S.	1	1.14	0.23	0.00	0.00	0.00	0.38	0.00	0.06	0.00
time (sec)	N/A	0.261	0.016	0.000	0.000	0.000	1.713	0.000	0.163	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	70	70	0	0	0	0	0	40	0
N.S.	1	0.22	0.22	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.171	0.014	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	70	70	0	0	0	0	0	66	0
N.S.	1	0.21	0.21	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.173	0.014	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	173	0	0	227	0	0	122	0
N.S.	1	1.00	0.99	0.00	0.00	1.30	0.00	0.00	0.70	0.00
time (sec)	N/A	0.209	0.258	0.000	0.000	0.087	0.000	0.000	0.560	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	143	221	0	0	209	0	0	76	0
N.S.	1	1.05	1.62	0.00	0.00	1.54	0.00	0.00	0.56	0.00
time (sec)	N/A	0.211	0.625	0.000	0.000	0.087	0.000	0.000	0.422	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	143	0	0	192	0	0	35	0
N.S.	1	1.00	1.29	0.00	0.00	1.73	0.00	0.00	0.32	0.00
time (sec)	N/A	0.175	0.163	0.000	0.000	0.080	0.000	0.000	0.261	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	32	0	0	41	0
N.S.	1	1.00	1.00	0.83	0.00	1.10	0.00	0.00	1.41	0.00
time (sec)	N/A	0.142	0.017	0.126	0.000	0.086	0.000	0.000	0.268	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	30	0	42	0	0	47	0
N.S.	1	1.00	0.83	0.51	0.00	0.71	0.00	0.00	0.80	0.00
time (sec)	N/A	0.150	0.058	0.132	0.000	0.065	0.000	0.000	0.309	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	94	46	41	0	74	0	0	92	0
N.S.	1	1.07	0.52	0.47	0.00	0.84	0.00	0.00	1.05	0.00
time (sec)	N/A	0.165	0.074	0.138	0.000	0.077	0.000	0.000	0.349	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	59	54	0	86	0	0	93	0
N.S.	1	1.14	0.50	0.46	0.00	0.74	0.00	0.00	0.79	0.00
time (sec)	N/A	0.184	0.092	0.136	0.000	0.092	0.000	0.000	0.373	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	677	66	66	0	0	0	0	0	76	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.159	0.014	0.000	0.000	0.000	0.000	0.000	0.446	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	648	65	65	0	0	0	0	0	35	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.160	0.014	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	656	65	65	0	0	0	0	0	41	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.156	0.015	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	652	735	70	0	0	0	104	0	47	0
N.S.	1	1.13	0.11	0.00	0.00	0.00	0.16	0.00	0.07	0.00
time (sec)	N/A	0.529	0.015	0.000	0.000	0.000	2.771	0.000	0.316	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	689	70	70	0	0	0	0	0	92	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.157	0.015	0.000	0.000	0.000	0.000	0.000	0.371	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	131	70	104	0	0	0	0	45	0
N.S.	1	0.91	0.49	0.72	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.184	0.028	0.231	0.000	0.000	0.000	0.000	0.222	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	92	70	94	0	0	0	0	21	0
N.S.	1	0.87	0.66	0.89	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.160	0.020	0.162	0.000	0.000	0.000	0.000	0.193	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	57	70	0	0	0	102	0	23	0
N.S.	1	0.80	0.99	0.00	0.00	0.00	1.44	0.00	0.32	0.00
time (sec)	N/A	0.144	0.018	0.000	0.000	0.000	1.747	0.000	0.184	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	94	0	0	0	0	49	0
N.S.	1	1.00	0.87	1.21	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.154	0.019	0.173	0.000	0.000	0.000	0.000	0.201	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0	69	0
N.S.	1	1.00	0.85	1.28	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.151	0.020	0.181	0.000	0.000	0.000	0.000	0.246	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	70	113	0	0	0	0	88	0
N.S.	1	1.07	0.61	0.98	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.174	0.022	0.192	0.000	0.000	0.000	0.000	0.253	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	164	70	114	0	0	0	0	108	0
N.S.	1	1.11	0.47	0.77	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.189	0.024	0.185	0.000	0.000	0.000	0.000	0.285	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	236	70	477	0	194	0	0	18	0
N.S.	1	1.27	0.38	2.56	0.00	1.04	0.00	0.00	0.10	0.00
time (sec)	N/A	0.304	0.339	1.283	0.000	0.079	0.000	0.000	0.167	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	207	68	0	0	221	0	0	22	0
N.S.	1	1.26	0.41	0.00	0.00	1.35	0.00	0.00	0.13	0.00
time (sec)	N/A	0.283	1.081	0.000	0.000	0.084	0.000	0.000	0.186	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	0	31	0	0	45	38
N.S.	1	1.00	1.00	0.82	0.00	0.94	0.00	0.00	1.36	1.15
time (sec)	N/A	0.128	4.669	0.138	0.000	0.074	0.000	0.000	0.187	0.412

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	32	0	44	0	0	65	46
N.S.	1	1.00	0.67	0.48	0.00	0.66	0.00	0.00	0.97	0.69
time (sec)	N/A	0.144	5.841	0.151	0.000	0.091	0.000	0.000	0.187	0.403

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	52	38	0	57	0	0	84	51
N.S.	1	1.08	0.52	0.38	0.00	0.57	0.00	0.00	0.84	0.51
time (sec)	N/A	0.158	6.681	0.155	0.000	0.088	0.000	0.000	0.213	0.453

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	149	57	43	0	68	0	0	104	57
N.S.	1	1.12	0.43	0.32	0.00	0.51	0.00	0.00	0.78	0.43
time (sec)	N/A	0.175	10.097	0.160	0.000	0.090	0.000	0.000	0.223	0.481

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	236	70	466	0	198	0	0	18	0
N.S.	1	1.27	0.38	2.51	0.00	1.06	0.00	0.00	0.10	0.00
time (sec)	N/A	0.308	0.381	1.171	0.000	0.092	0.000	0.000	0.174	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	207	70	0	0	221	0	0	22	0
N.S.	1	1.26	0.43	0.00	0.00	1.35	0.00	0.00	0.13	0.00
time (sec)	N/A	0.293	0.869	0.000	0.000	0.096	0.000	0.000	0.185	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	31	0	0	45	0
N.S.	1	1.00	1.00	0.87	0.00	1.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.130	2.300	0.141	0.000	0.068	0.000	0.000	0.194	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	32	0	44	0	0	65	0
N.S.	1	1.00	0.67	0.48	0.00	0.66	0.00	0.00	0.97	0.00
time (sec)	N/A	0.146	5.781	0.148	0.000	0.078	0.000	0.000	0.209	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	52	38	0	57	0	0	84	0
N.S.	1	1.08	0.52	0.38	0.00	0.57	0.00	0.00	0.84	0.00
time (sec)	N/A	0.162	7.843	0.157	0.000	0.107	0.000	0.000	0.223	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	115	70	0	0	0	0	0	47	0
N.S.	1	1.03	0.62	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.180	0.022	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	0	0	23	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.160	0.016	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	68	0	0	0	100	0	26	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	2.33	0.00	0.60	0.00
time (sec)	N/A	0.138	0.016	0.000	0.000	0.000	2.246	0.000	0.193	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	0	0	0	0	0	31	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.158	0.017	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	70	0	0	0	0	0	69	0
N.S.	1	1.07	0.61	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.176	0.019	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	126	70	96	0	0	0	0	70	0
N.S.	1	0.92	0.51	0.70	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.183	0.022	0.184	0.000	0.000	0.000	0.000	0.215	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	90	70	88	0	0	0	0	37	0
N.S.	1	0.87	0.68	0.85	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.166	0.024	0.161	0.000	0.000	0.000	0.000	0.181	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	94	0	0	0	0	47	0
N.S.	1	1.00	0.90	1.21	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.154	0.019	0.167	0.000	0.000	0.000	0.000	0.203	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	68	91	0	0	97	0	31	0
N.S.	1	1.00	1.48	1.98	0.00	0.00	2.11	0.00	0.67	0.00
time (sec)	N/A	0.141	0.018	0.212	0.000	0.000	4.571	0.000	0.164	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	107	0	0	0	0	89	0
N.S.	1	1.00	0.85	1.30	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	0.155	0.025	0.193	0.000	0.000	0.000	0.000	0.230	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	70	113	0	0	0	0	90	0
N.S.	1	1.07	0.61	0.98	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.173	0.020	0.205	0.000	0.000	0.000	0.000	0.233	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	270	70	480	0	233	0	0	67	0
N.S.	1	1.24	0.32	2.21	0.00	1.07	0.00	0.00	0.31	0.00
time (sec)	N/A	0.334	1.052	1.089	0.000	0.082	0.000	0.000	0.190	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	239	70	476	0	296	0	0	35	0
N.S.	1	1.23	0.36	2.44	0.00	1.52	0.00	0.00	0.18	0.00
time (sec)	N/A	0.317	0.668	1.006	0.000	0.094	0.000	0.000	0.167	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	36	27	0	31	0	0	43	27
N.S.	1	1.00	1.16	0.87	0.00	1.00	0.00	0.00	1.39	0.87
time (sec)	N/A	0.133	0.021	0.141	0.000	0.071	0.000	0.000	0.181	0.281

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	38	32	0	36	0	0	44	40
N.S.	1	1.03	0.58	0.49	0.00	0.55	0.00	0.00	0.68	0.62
time (sec)	N/A	0.148	2.404	0.139	0.000	0.080	0.000	0.000	0.198	0.375

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	108	50	38	0	56	0	0	85	46
N.S.	1	1.10	0.51	0.39	0.00	0.57	0.00	0.00	0.87	0.47
time (sec)	N/A	0.164	6.159	0.168	0.000	0.072	0.000	0.000	0.216	0.473

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	274	70	468	0	235	0	0	67	0
N.S.	1	1.24	0.32	2.12	0.00	1.06	0.00	0.00	0.30	0.00
time (sec)	N/A	0.337	0.973	1.086	0.000	0.108	0.000	0.000	0.195	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	241	70	456	0	298	0	0	35	0
N.S.	1	1.22	0.36	2.31	0.00	1.51	0.00	0.00	0.18	0.00
time (sec)	N/A	0.320	0.966	1.052	0.000	0.090	0.000	0.000	0.183	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	0	31	0	0	43	0
N.S.	1	1.00	1.00	0.82	0.00	0.94	0.00	0.00	1.30	0.00
time (sec)	N/A	0.130	1.708	0.141	0.000	0.092	0.000	0.000	0.182	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	38	32	0	36	0	0	44	0
N.S.	1	1.00	0.58	0.49	0.00	0.55	0.00	0.00	0.68	0.00
time (sec)	N/A	0.148	2.229	0.133	0.000	0.072	0.000	0.000	0.203	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	106	50	38	0	56	0	0	85	0
N.S.	1	1.06	0.50	0.38	0.00	0.56	0.00	0.00	0.85	0.00
time (sec)	N/A	0.159	6.126	0.170	0.000	0.079	0.000	0.000	0.215	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	151	70	0	0	0	0	0	109	0
N.S.	1	1.09	0.50	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.193	0.026	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	114	70	0	0	0	0	0	72	0
N.S.	1	1.01	0.62	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.169	0.021	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	70	0	0	0	0	0	39	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.154	0.018	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	0	0	0	0	0	37	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.155	0.018	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	103	70	0	0	0	95	0	43	0
N.S.	1	1.27	0.86	0.00	0.00	0.00	1.17	0.00	0.53	0.00
time (sec)	N/A	0.165	0.018	0.000	0.000	0.000	15.678	0.000	0.159	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	143	70	0	0	0	0	0	89	0
N.S.	1	1.25	0.61	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.186	0.019	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	184	70	0	0	0	0	0	90	0
N.S.	1	1.25	0.48	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.204	0.020	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	128	70	101	0	0	0	0	99	0
N.S.	1	0.90	0.49	0.71	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.183	0.024	0.187	0.000	0.000	0.000	0.000	0.247	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	70	107	0	0	0	0	50	0
N.S.	1	1.01	0.61	0.93	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.169	0.018	0.177	0.000	0.000	0.000	0.000	0.217	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0	68	0
N.S.	1	1.00	0.85	1.28	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.154	0.019	0.166	0.000	0.000	0.000	0.000	0.249	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	119	68	107	0	0	0	0	86	0
N.S.	1	1.45	0.83	1.30	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.176	0.019	0.191	0.000	0.000	0.000	0.000	0.221	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	103	70	0	0	0	95	0	50	0
N.S.	1	1.27	0.86	0.00	0.00	0.00	1.17	0.00	0.62	0.00
time (sec)	N/A	0.169	0.020	0.000	0.000	0.000	69.519	0.000	0.177	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	143	70	124	0	0	0	0	130	0
N.S.	1	1.25	0.61	1.09	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.194	0.021	0.220	0.000	0.000	0.000	0.000	0.231	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	184	70	130	0	0	0	0	150	0
N.S.	1	1.25	0.48	0.88	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.207	0.023	0.224	0.000	0.000	0.000	0.000	0.260	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	272	70	493	0	343	0	0	97	0
N.S.	1	1.19	0.31	2.16	0.00	1.50	0.00	0.00	0.43	0.00
time (sec)	N/A	0.326	2.431	1.108	0.000	0.089	0.000	0.000	0.233	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	0	42	0	0	48	38
N.S.	1	1.00	1.00	0.82	0.00	1.27	0.00	0.00	1.45	1.15
time (sec)	N/A	0.129	0.989	0.159	0.000	0.084	0.000	0.000	0.200	0.260

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	32	0	44	0	0	64	38
N.S.	1	1.00	0.67	0.48	0.00	0.66	0.00	0.00	0.96	0.57
time (sec)	N/A	0.146	2.333	0.150	0.000	0.080	0.000	0.000	0.212	0.382

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	50	38	0	56	0	0	82	45
N.S.	1	1.08	0.50	0.38	0.00	0.56	0.00	0.00	0.82	0.45
time (sec)	N/A	0.160	5.178	0.219	0.000	0.087	0.000	0.000	0.214	0.323

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	149	57	43	0	54	0	0	64	56
N.S.	1	1.12	0.43	0.32	0.00	0.41	0.00	0.00	0.48	0.42
time (sec)	N/A	0.175	7.124	0.151	0.000	0.074	0.000	0.000	0.211	0.415

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	190	103	0	0	209	0	0	23	0
N.S.	1	1.26	0.68	0.00	0.00	1.38	0.00	0.00	0.15	0.00
time (sec)	N/A	0.300	0.923	0.000	0.000	0.078	0.000	0.000	0.184	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	70	70	0	0	0	0	0	71	0
N.S.	1	1.06	1.06	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.170	0.023	0.000	0.000	0.000	0.000	0.000	1.240	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	68	0	0	0	0	0	42	0
N.S.	1	1.03	1.03	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.157	0.016	0.000	0.000	0.000	0.000	0.000	0.814	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	58	67	0	0	0	0	0	18	0
N.S.	1	0.88	1.02	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.156	0.018	0.000	0.000	0.000	0.000	0.000	0.361	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	67	0	0	0	0	0	18	0
N.S.	1	1.05	1.05	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.154	0.018	0.000	0.000	0.000	0.000	0.000	0.298	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	70	70	0	0	0	0	0	137	0
N.S.	1	1.06	1.06	0.00	0.00	0.00	0.00	0.00	2.08	0.00
time (sec)	N/A	0.156	0.018	0.000	0.000	0.000	0.000	0.000	0.504	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	70	70	0	0	0	0	0	209	0
N.S.	1	1.06	1.06	0.00	0.00	0.00	0.00	0.00	3.17	0.00
time (sec)	N/A	0.155	0.019	0.000	0.000	0.000	0.000	0.000	0.629	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	103	167	128	819	256	132	133
N.S.	1	1.00	0.93	1.24	2.01	1.54	9.87	3.08	1.59	1.60
time (sec)	N/A	0.200	0.058	0.148	0.048	0.085	0.408	0.130	0.165	0.296

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	47	81	58	245	103	57	66
N.S.	1	1.00	0.81	0.89	1.53	1.09	4.62	1.94	1.08	1.25
time (sec)	N/A	0.172	0.048	0.119	0.038	0.096	0.300	0.127	0.165	0.196

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	56	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.152	0.039	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	364	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	7.00	0.00
time (sec)	N/A	0.149	0.036	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	68	67	0	0	0	0	0	619	0
N.S.	1	0.91	0.89	0.00	0.00	0.00	0.00	0.00	8.25	0.00
time (sec)	N/A	0.163	0.091	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	68	67	0	0	0	0	0	340	0
N.S.	1	0.91	0.89	0.00	0.00	0.00	0.00	0.00	4.53	0.00
time (sec)	N/A	0.158	0.065	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	65	64	0	0	0	0	0	148	0
N.S.	1	0.87	0.85	0.00	0.00	0.00	0.00	0.00	1.97	0.00
time (sec)	N/A	0.159	0.074	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	66	65	0	0	0	0	0	405	0
N.S.	1	0.86	0.84	0.00	0.00	0.00	0.00	0.00	5.26	0.00
time (sec)	N/A	0.156	0.098	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	59	59	0	0	0	0	0	25	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.161	0.037	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	41	53	0	0	0	124	0	102	0
N.S.	1	0.75	0.96	0.00	0.00	0.00	2.25	0.00	1.85	0.00
time (sec)	N/A	0.150	0.068	0.000	0.000	0.000	1.957	0.000	0.161	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	57	72	0	0	0	146	0	143	0
N.S.	1	0.85	1.07	0.00	0.00	0.00	2.18	0.00	2.13	0.00
time (sec)	N/A	0.160	0.038	0.000	0.000	0.000	2.510	0.000	0.164	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	0	39	0	103	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.95	0.00	5.15	0.00
time (sec)	N/A	0.128	0.055	0.000	0.000	0.000	2.094	0.000	0.162	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	37	0	29	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.16	0.00	0.91	0.00
time (sec)	N/A	0.137	0.044	0.000	0.000	0.000	5.034	0.000	0.162	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [280] had the largest ratio of [.550000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	17	0.118
2	A	2	2	1.00	17	0.118
3	A	2	2	1.00	15	0.133
4	A	2	2	1.00	17	0.118
5	A	2	2	1.00	17	0.118
6	A	1	1	1.00	17	0.059
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	17	0.118
9	A	2	2	1.00	17	0.118
10	A	2	2	1.00	19	0.105
11	A	3	3	1.00	19	0.158
12	A	2	2	1.00	17	0.118
13	A	1	1	1.00	7	0.143
14	A	2	2	1.16	19	0.105
15	A	2	2	1.00	19	0.105
16	A	2	2	1.00	19	0.105
17	A	1	1	1.00	19	0.053
18	A	2	2	1.00	19	0.105
19	A	2	2	1.00	19	0.105
20	A	2	2	1.00	19	0.105
21	A	2	2	1.22	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.19	19	0.105
23	A	2	2	1.00	17	0.118
24	A	1	1	1.00	7	0.143
25	A	2	2	1.00	19	0.105
26	A	2	2	1.00	19	0.105
27	A	2	2	1.00	19	0.105
28	A	2	2	1.00	19	0.105
29	A	2	2	1.00	19	0.105
30	A	2	2	1.00	17	0.118
31	A	1	1	1.00	7	0.143
32	A	2	2	1.00	19	0.105
33	A	3	3	1.00	19	0.158
34	A	2	2	1.00	19	0.105
35	A	6	5	1.02	20	0.250
36	A	5	4	1.02	20	0.200
37	A	4	3	1.00	20	0.150
38	A	3	2	1.00	20	0.100
39	A	1	1	1.00	20	0.050
40	A	2	2	1.00	20	0.100
41	A	3	3	1.12	20	0.150
42	A	4	4	1.18	20	0.200
43	A	6	5	1.04	23	0.217
44	A	5	4	1.02	23	0.174
45	A	4	3	1.00	23	0.130
46	A	3	2	1.00	23	0.087
47	A	1	1	1.00	23	0.043
48	A	2	2	1.00	23	0.087
49	A	3	3	1.11	23	0.130
50	A	4	4	1.17	23	0.174
51	A	5	5	1.06	19	0.263
52	A	4	4	1.03	19	0.211
53	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	19	0.105
55	A	2	2	1.00	19	0.105
56	A	3	3	1.00	19	0.158
57	A	4	4	1.08	19	0.211
58	A	5	5	1.12	19	0.263
59	A	2	2	1.00	17	0.118
60	A	2	2	1.00	20	0.100
61	A	2	2	1.00	17	0.118
62	A	2	2	1.00	20	0.100
63	A	3	2	1.00	23	0.087
64	A	1	1	1.00	19	0.053
65	A	2	2	1.00	24	0.083
66	A	1	1	1.00	21	0.048
67	A	6	6	1.15	17	0.353
68	A	5	5	1.13	17	0.294
69	A	4	4	1.09	17	0.235
70	A	3	3	1.00	17	0.176
71	A	3	3	1.00	17	0.176
72	A	2	2	0.76	17	0.118
73	A	3	3	1.00	17	0.176
74	A	1	1	1.00	17	0.059
75	A	2	2	1.00	17	0.118
76	A	3	3	1.08	17	0.176
77	A	4	4	1.12	17	0.235
78	A	5	5	1.15	17	0.294
79	A	6	6	1.16	17	0.353
80	A	5	5	1.14	17	0.294
81	A	4	4	1.09	17	0.235
82	A	4	4	1.13	17	0.235
83	A	3	3	1.00	17	0.176
84	A	3	3	0.87	17	0.176
85	A	3	3	0.90	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	1.00	17	0.235
87	A	1	1	1.00	17	0.059
88	A	2	2	1.00	17	0.118
89	A	3	3	1.08	17	0.176
90	A	4	4	1.12	17	0.235
91	A	5	5	1.15	17	0.294
92	A	7	7	1.18	17	0.412
93	A	6	6	1.17	17	0.353
94	A	5	5	1.14	17	0.294
95	A	5	5	1.18	17	0.294
96	A	4	4	1.09	17	0.235
97	A	4	4	1.09	17	0.235
98	A	4	4	0.99	17	0.235
99	A	4	4	0.95	17	0.235
100	A	4	4	0.98	17	0.235
101	A	5	5	1.00	17	0.294
102	A	1	1	1.00	17	0.059
103	A	2	2	1.00	17	0.118
104	A	3	3	1.08	17	0.176
105	A	4	4	1.12	17	0.235
106	A	5	5	1.15	17	0.294
107	A	3	3	0.92	20	0.150
108	A	3	3	1.55	22	0.136
109	A	4	4	1.00	28	0.143
110	A	5	5	1.02	17	0.294
111	A	4	4	0.96	17	0.235
112	A	3	3	0.83	17	0.176
113	A	2	2	0.70	17	0.118
114	A	2	2	1.00	17	0.118
115	A	1	1	1.00	17	0.059
116	A	2	2	1.00	17	0.118
117	A	3	3	1.08	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	4	1.12	17	0.235
119	A	5	5	1.15	17	0.294
120	A	5	5	1.00	17	0.294
121	A	4	4	0.92	17	0.235
122	A	3	3	0.85	17	0.176
123	A	3	3	1.00	17	0.176
124	A	1	1	1.00	17	0.059
125	A	2	2	1.00	17	0.118
126	A	3	3	1.00	17	0.176
127	A	4	4	1.09	17	0.235
128	A	5	5	1.13	17	0.294
129	A	6	6	1.05	17	0.353
130	A	5	5	0.98	17	0.294
131	A	4	4	0.95	17	0.235
132	A	4	4	1.00	17	0.235
133	A	1	1	1.00	17	0.059
134	A	2	2	1.00	17	0.118
135	A	3	3	1.65	17	0.176
136	A	3	3	1.00	17	0.176
137	A	4	4	1.09	17	0.235
138	A	5	5	1.14	17	0.294
139	A	6	6	1.16	17	0.353
140	A	4	4	1.00	20	0.200
141	A	3	3	0.98	20	0.150
142	A	2	2	1.04	20	0.100
143	A	2	2	1.00	20	0.100
144	A	1	1	1.00	20	0.050
145	A	2	2	1.00	20	0.100
146	A	3	3	1.09	20	0.150
147	A	4	4	1.14	20	0.200
148	C	3	3	0.20	20	0.150
149	A	5	4	1.13	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	C	3	3	0.22	20	0.150
151	C	3	3	0.23	20	0.150
152	C	3	3	0.21	20	0.150
153	A	5	5	1.01	20	0.250
154	A	4	4	0.99	20	0.200
155	A	3	3	0.97	20	0.150
156	A	2	2	1.04	20	0.100
157	A	2	2	1.00	20	0.100
158	A	1	1	1.00	20	0.050
159	A	2	2	1.00	20	0.100
160	A	3	3	1.09	20	0.150
161	A	4	4	1.14	20	0.200
162	C	3	3	0.10	20	0.150
163	A	7	6	1.11	20	0.300
164	C	3	3	0.10	20	0.150
165	C	3	3	0.11	20	0.150
166	C	3	3	0.10	20	0.150
167	A	6	6	1.03	20	0.300
168	A	5	5	1.01	20	0.250
169	A	4	4	1.00	20	0.200
170	A	3	3	0.98	20	0.150
171	A	3	3	1.05	20	0.150
172	A	3	3	1.00	20	0.150
173	A	1	1	1.00	20	0.050
174	A	2	2	1.00	20	0.100
175	A	3	3	1.09	20	0.150
176	A	4	4	1.14	20	0.200
177	C	3	3	0.19	20	0.150
178	A	6	5	1.12	20	0.250
179	C	3	3	0.20	20	0.150
180	C	3	3	0.20	20	0.150
181	C	3	3	0.21	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
182	C	3	3	0.21	20	0.150
183	C	3	3	0.19	20	0.150
184	A	4	4	1.00	20	0.200
185	A	3	3	0.98	20	0.150
186	A	2	2	1.05	20	0.100
187	A	1	1	1.00	20	0.050
188	A	1	1	1.00	20	0.050
189	A	2	2	1.00	20	0.100
190	A	3	3	1.09	20	0.150
191	A	4	4	1.14	20	0.200
192	C	3	3	0.10	20	0.150
193	C	3	3	0.10	20	0.150
194	A	6	5	1.13	20	0.250
195	C	3	3	0.11	20	0.150
196	C	3	3	0.10	20	0.150
197	A	4	4	1.01	20	0.200
198	A	3	3	0.99	20	0.150
199	A	2	2	1.04	20	0.100
200	A	1	1	1.00	20	0.050
201	A	1	1	1.00	20	0.050
202	A	2	2	1.00	20	0.100
203	A	3	3	1.09	20	0.150
204	A	4	4	1.14	20	0.200
205	C	3	3	0.20	20	0.150
206	C	3	3	0.21	20	0.150
207	A	4	3	1.14	20	0.150
208	C	3	3	0.22	20	0.150
209	C	3	3	0.21	20	0.150
210	A	4	4	1.00	20	0.200
211	A	3	3	1.05	20	0.150
212	A	2	2	1.00	20	0.100
213	A	1	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	1.00	20	0.100
215	A	3	3	1.07	20	0.150
216	A	4	4	1.14	20	0.200
217	C	3	3	0.10	20	0.150
218	C	3	3	0.10	20	0.150
219	C	3	3	0.10	20	0.150
220	A	7	6	1.13	20	0.300
221	C	3	3	0.10	20	0.150
222	A	6	6	0.91	25	0.240
223	A	5	5	0.87	25	0.200
224	A	4	4	0.80	25	0.160
225	A	4	4	1.00	25	0.160
226	A	4	4	1.00	25	0.160
227	A	5	5	1.07	25	0.200
228	A	6	6	1.11	25	0.240
229	A	12	11	1.27	25	0.440
230	A	11	10	1.26	25	0.400
231	A	1	1	1.00	25	0.040
232	A	2	2	1.00	25	0.080
233	A	3	3	1.08	25	0.120
234	A	4	4	1.12	25	0.160
235	A	12	11	1.27	25	0.440
236	A	11	10	1.26	25	0.400
237	A	1	1	1.00	25	0.040
238	A	2	2	1.00	25	0.080
239	A	3	3	1.08	25	0.120
240	A	5	5	1.03	25	0.200
241	A	4	4	1.00	25	0.160
242	A	3	3	1.00	25	0.120
243	A	4	4	1.00	25	0.160
244	A	5	5	1.07	25	0.200
245	A	6	6	0.92	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	5	5	0.87	25	0.200
247	A	4	4	1.00	25	0.160
248	A	3	3	1.00	25	0.120
249	A	4	4	1.00	25	0.160
250	A	5	5	1.07	25	0.200
251	A	13	12	1.24	25	0.480
252	A	12	11	1.23	25	0.440
253	A	1	1	1.00	25	0.040
254	A	2	2	1.03	25	0.080
255	A	3	3	1.10	25	0.120
256	A	13	12	1.24	25	0.480
257	A	12	11	1.22	25	0.440
258	A	1	1	1.00	25	0.040
259	A	2	2	1.00	25	0.080
260	A	3	3	1.06	25	0.120
261	A	6	6	1.09	25	0.240
262	A	5	5	1.01	25	0.200
263	A	4	4	1.00	25	0.160
264	A	4	4	1.00	25	0.160
265	A	4	4	1.27	25	0.160
266	A	5	5	1.25	25	0.200
267	A	6	6	1.25	25	0.240
268	A	6	6	0.90	25	0.240
269	A	5	5	1.01	25	0.200
270	A	4	4	1.00	25	0.160
271	A	5	5	1.45	25	0.200
272	A	4	4	1.27	25	0.160
273	A	5	5	1.25	25	0.200
274	A	6	6	1.25	25	0.240
275	A	13	12	1.19	25	0.480
276	A	1	1	1.00	25	0.040
277	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	3	3	1.08	25	0.120
279	A	4	4	1.12	25	0.160
280	A	12	11	1.26	20	0.550
281	A	3	3	1.06	20	0.150
282	A	3	3	1.03	20	0.150
283	A	3	3	0.88	20	0.150
284	A	3	3	1.05	20	0.150
285	A	3	3	1.06	20	0.150
286	A	3	3	1.06	20	0.150
287	A	2	2	1.00	19	0.105
288	A	2	2	1.00	17	0.118
289	A	1	1	1.00	19	0.053
290	A	1	1	1.00	19	0.053
291	A	2	2	0.91	21	0.095
292	A	2	2	0.91	21	0.095
293	A	2	2	0.87	21	0.095
294	A	2	2	0.86	21	0.095
295	A	2	2	1.02	18	0.111
296	A	3	3	0.75	16	0.188
297	A	3	3	0.85	19	0.158
298	A	2	2	1.00	15	0.133
299	A	1	1	1.00	17	0.059

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx)(ac-bcx)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int (a+bx)(ac-bcx)^3\,dx=-\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b}$$

output
```
-1/2*a*c^3*(-b*x+a)^4/b+1/5*c^3*(-b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (a+bx)(ac-bcx)^3\,dx=c^3\left(a^4x-a^3bx^2+\frac{1}{2}ab^3x^4-\frac{b^4x^5}{5}\right)$$

input
```
Integrate[(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
c^3*(a^4*x - a^3*b*x^2 + (a*b^3*x^4)/2 - (b^4*x^5)/5)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(ac-bcx)^3\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(2a(ac-bcx)^3-\frac{(ac-bcx)^4}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

input
```
Int[(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
-1/2*(a*c^3*(a - b*x)^4)/b + (c^3*(a - b*x)^5)/(5*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{x(-2b^4x^4+5a\,x^3b^3-10a^3bx+10a^4)c^3}{10}$	37
default	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3c^3b\,x^2+a^4c^3x$	45
norman	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3c^3b\,x^2+a^4c^3x$	45
risch	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3c^3b\,x^2+a^4c^3x$	45
parallelrisch	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3c^3b\,x^2+a^4c^3x$	45
orering	$\frac{x(-2b^4x^4+5a\,x^3b^3-10a^3bx+10a^4)(-bcx+ac)^3}{10(-bx+a)^3}$	53

input
```
int((b*x+a)*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*x*(-2*b^4*x^4+5*a*b^3*x^3-10*a^3*b*x+10*a^4)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = -\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
-1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = a^4c^3x - a^3bc^3x^2 + \frac{ab^3c^3x^4}{2} - \frac{b^4c^3x^5}{5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3,x)
```

output
```
a**4*c**3*x - a**3*b*c**3*x**2 + a*b**3*c**3*x**4/2 - b**4*c**3*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = -\frac{1}{5}\,b^4c^3x^5 + \frac{1}{2}\,ab^3c^3x^4 - a^3bc^3x^2 + a^4c^3x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
-1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = -\frac{1}{5}\,b^4c^3x^5 + \frac{1}{2}\,ab^3c^3x^4 - a^3bc^3x^2 + a^4c^3x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
-1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = a^4\,c^3\,x - a^3\,b\,c^3\,x^2 + \frac{a\,b^3\,c^3\,x^4}{2} - \frac{b^4\,c^3\,x^5}{5}$$

input
```
int((a*c - b*c*x)^3*(a + b*x),x)
```

output
```
a^4*c^3*x - (b^4*c^3*x^5)/5 - a^3*b*c^3*x^2 + (a*b^3*c^3*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a+bx)(ac-bcx)^3\,dx = \frac{c^3x(-2b^4x^4+5a\,b^3x^3-10a^3bx+10a^4)}{10}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3,x)
```

output
```
(c**3*x*(10*a**4 - 10*a**3*b*x + 5*a*b**3*x**3 - 2*b**4*x**4))/10
```

3.2 $\int (a+bx)(ac-bcx)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int (a+bx)(ac-bcx)^2\,dx=-\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}$$

output
```
-2/3*a*c^2*(-b*x+a)^3/b+1/4*c^2*(-b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int (a+bx)(ac-bcx)^2\,dx=c^2\left(a^3x-\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{b^3x^4}{4}\right)$$

input
```
Integrate[(a + b*x)*(a*c - b*c*x)^2,x]
```

output
```
c^2*(a^3*x - (a^2*b*x^2)/2 - (a*b^2*x^3)/3 + (b^3*x^4)/4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(ac-bcx)^2\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(2a(ac-bcx)^2-\frac{(ac-bcx)^3}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

input
```
Int[(a + b*x)*(a*c - b*c*x)^2,x]
```

output
```
(-2*a*c^2*(a - b*x)^3)/(3*b) + (c^2*(a - b*x)^4)/(4*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{x(3b^3x^3-4a\,b^2x^2-6a^2bx+12a^3)c^2}{12}$	37
default	$\frac{1}{4}b^3c^2x^4-\frac{1}{3}a\,b^2c^2x^3-\frac{1}{2}a^2c^2b\,x^2+a^3c^2x$	45
norman	$\frac{1}{4}b^3c^2x^4-\frac{1}{3}a\,b^2c^2x^3-\frac{1}{2}a^2c^2b\,x^2+a^3c^2x$	45
risch	$\frac{1}{4}b^3c^2x^4-\frac{1}{3}a\,b^2c^2x^3-\frac{1}{2}a^2c^2b\,x^2+a^3c^2x$	45
parallelrisch	$\frac{1}{4}b^3c^2x^4-\frac{1}{3}a\,b^2c^2x^3-\frac{1}{2}a^2c^2b\,x^2+a^3c^2x$	45
orering	$\frac{x(3b^3x^3-4a\,b^2x^2-6a^2bx+12a^3)(-bcx+ac)^2}{12(-bx+a)^2}$	53

input
```
int((b*x+a)*(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*x*(3*b^3*x^3-4*a*b^2*x^2-6*a^2*b*x+12*a^3)*c^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^2\,dx=\frac{1}{4}\,b^3c^2x^4-\frac{1}{3}\,ab^2c^2x^3-\frac{1}{2}\,a^2bc^2x^2+a^3c^2x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

output
```
1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (a+bx)(ac-bcx)^2\,dx = a^3c^2x - \frac{a^2bc^2x^2}{2} - \frac{ab^2c^2x^3}{3} + \frac{b^3c^2x^4}{4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**2,x)
```

output
```
a**3*c**2*x - a**2*b*c**2*x**2/2 - a*b**2*c**2*x**3/3 + b**3*c**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^2\,dx = \frac{1}{4}\,b^3c^2x^4 - \frac{1}{3}\,ab^2c^2x^3 - \frac{1}{2}\,a^2bc^2x^2 + a^3c^2x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

output
```
1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^2\,dx = \frac{1}{4}\,b^3c^2x^4 - \frac{1}{3}\,ab^2c^2x^3 - \frac{1}{2}\,a^2bc^2x^2 + a^3c^2x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="giac")
```

output
```
1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^2\,dx = a^3\,c^2\,x - \frac{a^2\,b\,c^2\,x^2}{2} - \frac{a\,b^2\,c^2\,x^3}{3} + \frac{b^3\,c^2\,x^4}{4}$$

input
```
int((a*c - b*c*x)^2*(a + b*x),x)
```

output
```
a^3*c^2*x + (b^3*c^2*x^4)/4 - (a^2*b*c^2*x^2)/2 - (a*b^2*c^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a+bx)(ac-bcx)^2\,dx = \frac{c^2x(3b^3x^3 - 4a\,b^2x^2 - 6a^2bx + 12a^3)}{12}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^2,x)
```

output
```
(c**2*x*(12*a**3 - 6*a**2*b*x - 4*a*b**2*x**2 + 3*b**3*x**3))/12
```

3.3 $\int (a+bx)(ac-bcx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int (a+bx)(ac-bcx)\,dx=a^2cx-\frac{1}{3}b^2cx^3$$

output
```
a^2*c*x-1/3*b^2*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)(ac-bcx)\,dx=c\left(a^2x-\frac{b^2x^3}{3}\right)$$

input
```
Integrate[(a + b*x)*(a*c - b*c*x),x]
```

output
```
c*(a^2*x - (b^2*x^3)/3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {39, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(ac-bcx)\,dx$$

$$\Big\downarrow \; 39$$

$$\int \left(a^2c-b^2cx^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2cx-\frac{1}{3}b^2cx^3$$

input
```
Int[(a + b*x)*(a*c - b*c*x),x]
```

output
```
a^2*c*x - (b^2*c*x^3)/3
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$a^2cx-\frac{1}{3}b^2c\,x^3$	17
norman	$a^2cx-\frac{1}{3}b^2c\,x^3$	17
risch	$a^2cx-\frac{1}{3}b^2c\,x^3$	17
parallelrisch	$a^2cx-\frac{1}{3}b^2c\,x^3$	17
gosper	$\frac{cx(-b^2x^2+3a^2)}{3}$	19
orering	$\frac{x(-b^2x^2+3a^2)(-bcx+ac)}{-3bx+3a}$	35

input
```
int((b*x+a)*(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x-1/3*b^2*c*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (a+bx)(ac-bcx)\,dx=-\frac{1}{3}\,b^2cx^3+a^2cx$$

input
```
integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="fricas")
```

output
```
-1/3*b^2*c*x^3 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int (a+bx)(ac-bcx)\,dx = a^2cx - \frac{b^2cx^3}{3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c),x)
```

output
```
a**2*c*x - b**2*c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (a+bx)(ac-bcx)\,dx = -\frac{1}{3}\,b^2cx^3 + a^2cx$$

input
```
integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="maxima")
```

output
```
-1/3*b^2*c*x^3 + a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (a+bx)(ac-bcx)\,dx = -\frac{1}{3}\,b^2cx^3 + a^2cx$$

input
```
integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="giac")
```

output
```
-1/3*b^2*c*x^3 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)(ac-bcx)\,dx = \frac{c\,x\,\left(3\,a^2-b^2\,x^2\right)}{3}$$

input
```
int((a*c - b*c*x)*(a + b*x),x)
```

output
```
(c*x*(3*a^2 - b^2*x^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)(ac-bcx)\,dx = \frac{cx(-b^2x^2+3a^2)}{3}$$

input
```
int((b*x+a)*(-b*c*x+a*c),x)
```

output
```
(c*x*(3*a**2 - b**2*x**2))/3
```

3.4 $\int \frac{a+bx}{ac-bcx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{a+bx}{ac-bcx} \, dx = -\frac{x}{c} - \frac{2a\log(a-bx)}{bc}$$

output
```
-x/c-2*a*ln(-b*x+a)/b/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac-bcx} \, dx = -\frac{x}{c} - \frac{2a\log(a-bx)}{bc}$$

input
```
Integrate[(a + b*x)/(a*c - b*c*x),x]
```

output
```
-(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{ac-bcx}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{2a}{c(a-bx)}-\frac{1}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

input
```
Int[(a + b*x)/(a*c - b*c*x),x]
```

output
```
-(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{-x-\frac{2a\ln(-bx+a)}{b}}{c}$	22
norman	$-\frac{x}{c}-\frac{2a\ln(-bx+a)}{bc}$	24
risch	$-\frac{x}{c}-\frac{2a\ln(-bx+a)}{bc}$	24
parallelrisch	$\frac{-2a\ln(bx-a)-bx}{bc}$	24

input
```
int((b*x+a)/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-x-2*a*ln(-b*x+a)/b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac-bcx}\,dx=-\frac{bx+2\,a\log{(bx-a)}}{bc}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

output
```
-(b*x + 2*a*log(b*x - a))/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{a+bx}{ac-bcx}\,dx=-\frac{2a\log{(-a+bx)}}{bc}-\frac{x}{c}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c),x)
```

output
```
-2*a*log(-a + b*x)/(b*c) - x/c
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{a+bx}{ac-bcx}\,dx = -\frac{x}{c} - \frac{2\,a\log\left(bx-a\right)}{bc}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

output
```
-x/c - 2*a*log(b*x - a)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+bx}{ac-bcx}\,dx = -\frac{x}{c} - \frac{2\,a\log\left(|bx-a|\right)}{bc}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
```

output
```
-x/c - 2*a*log(abs(b*x - a))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{ac-bcx}\,dx = -\frac{b\,x+2\,a\,\ln\left(b\,x-a\right)}{b\,c}$$

input
```
int((a + b*x)/(a*c - b*c*x),x)
```

output
```
-(b*x + 2*a*log(b*x - a))/(b*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+bx}{ac-bcx}\,dx=\frac{-2\log(-bx+a)\,a-bx}{bc}$$

input
```
int((b*x+a)/(-b*c*x+a*c),x)
```

output
```
( - 2*log(a - b*x)*a - b*x)/(b*c)
```

3.5 $\int \frac{a+bx}{(ac-bcx)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{a+bx}{(ac-bcx)^2} \, dx = \frac{2a}{bc^2(a-bx)} + \frac{\log(a-bx)}{bc^2}$$

output
```
2*a/b/c^2/(-b*x+a)+ln(-b*x+a)/b/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{(ac-bcx)^2} \, dx = \frac{\frac{2a}{a-bx} + \log(c(a-bx))}{bc^2}$$

input
```
Integrate[(a + b*x)/(a*c - b*c*x)^2,x]
```

output
```
((2*a)/(a - b*x) + Log[c*(a - b*x)])/(b*c^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(ac-bcx)^2} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{2a}{c^2(a-bx)^2}-\frac{1}{c^2(a-bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}$$

input
```
Int[(a + b*x)/(a*c - b*c*x)^2,x]
```

output
```
(2*a)/(b*c^2*(a - b*x)) + Log[a - b*x]/(b*c^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\frac{\ln(-bx+a)}{b}+\frac{2a}{b(-bx+a)}}{c^2}$	31
norman	$\frac{2a}{b\,c^2(-bx+a)}+\frac{\ln(-bx+a)}{b\,c^2}$	33
risch	$\frac{2a}{b\,c^2(-bx+a)}+\frac{\ln(-bx+a)}{b\,c^2}$	33
parallelrisch	$\frac{\ln(bx-a)xb-a\ln(bx-a)-2a}{b\,c^2(bx-a)}$	43

input
```
int((b*x+a)/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(ln(-b*x+a)/b+2*a/b/(-b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx = \frac{(bx-a)\log(bx-a)-2\,a}{b^2c^2x-abc^2}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

output
```
((b*x - a)*log(b*x - a) - 2*a)/(b^2*c^2*x - a*b*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx = -\frac{2a}{-abc^2+b^2c^2x}+\frac{\log(-a+bx)}{bc^2}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)**2,x)
```

output
```
-2*a/(-a*b*c**2 + b**2*c**2*x) + log(-a + b*x)/(b*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx = -\frac{2\,a}{b^2c^2x-abc^2}+\frac{\log\left(bx-a\right)}{bc^2}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

output
```
-2*a/(b^2*c^2*x - a*b*c^2) + log(b*x - a)/(b*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(33) = 66.

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx = -\frac{\frac{a}{(bcx-ac)b}+\frac{\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc}}{c}-\frac{a}{(bcx-ac)bc}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

output
```
-(a/((b*c*x - a*c)*b) + log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(b)*abs(c
)))/(b*c))/c - a/((b*c*x - a*c)*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx = \frac{\ln(b\,x-a)}{b\,c^2} + \frac{2\,a}{b\,(a\,c^2-b\,c^2\,x)}$$

input
```
int((a + b*x)/(a*c - b*c*x)^2,x)
```

output
```
log(b*x - a)/(b*c^2) + (2*a)/(b*(a*c^2 - b*c^2*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx = \frac{\log(-bx+a)\,a-\log(-bx+a)\,bx+2bx}{b\,c^2\,(-bx+a)}$$

input
```
int((b*x+a)/(-b*c*x+a*c)^2,x)
```

output
```
(log(a - b*x)*a - log(a - b*x)*b*x + 2*b*x)/(b*c**2*(a - b*x))
```

3.6 $\int \frac{a+bx}{(ac-bcx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx = \frac{x}{c^3(a-bx)^2}$$

output
```
x/c^3/(-b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx = \frac{x}{c^3(a-bx)^2}$$

input
```
Integrate[(a + b*x)/(a*c - b*c*x)^3,x]
```

output
```
x/(c^3*(a - b*x)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx$$
$$\Big\downarrow \; 38$$
$$\frac{x}{c^3(a-bx)^2}$$

input
```
Int[(a + b*x)/(a*c - b*c*x)^3,x]
```

output
```
x/(c^3*(a - b*x)^2)
```

Defintions of rubi rules used

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{x}{c^{3}\left(-bx+a\right)^{2}}$	14
norman	$\frac{x}{c^{3}\left(-bx+a\right)^{2}}$	14
risch	$\frac{x}{c^{3}\left(-bx+a\right)^{2}}$	14
parallelrisch	$\frac{x}{c^{3}\left(bx-a\right)^{2}}$	15
orering	$\frac{\left(-bx+a\right)x}{\left(-bcx+ac\right)^{3}}$	20
default	$\frac{\frac{a}{b\left(-bx+a\right)^{2}}-\frac{1}{b\left(-bx+a\right)}}{c^{3}}$	32

input
```
int((b*x+a)/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3/(-b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.31

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx = \frac{x}{b^2c^3x^2-2\,abc^3x+a^2c^3}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx = \frac{x}{a^2c^3 - 2abc^3x + b^2c^3x^2}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)**3,x)
```

output
```
x/(a**2*c**3 - 2*a*b*c**3*x + b**2*c**3*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.31

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx = \frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx = \frac{x}{(bx-a)^2c^3}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
x/((b*x - a)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx=\frac{x}{c^3\,{\left(a-b\,x\right)}^2}$$

input
```
int((a + b*x)/(a*c - b*c*x)^3,x)
```

output
```
x/(c^3*(a - b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.08

$$\int \frac{a+bx}{(ac-bcx)^3}\,dx=\frac{b^2x^2+a^2}{2ab\,c^3\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)/(-b*c*x+a*c)^3,x)
```

output
```
(a**2 + b**2*x**2)/(2*a*b*c**3*(a**2 - 2*a*b*x + b**2*x**2))
```

3.7 $\int \frac{a+bx}{(ac-bcx)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx = \frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}$$

output
```
2/3*a/b/c^4/(-b*x+a)^3-1/2/b/c^4/(-b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx = -\frac{a+3bx}{6bc^4(-a+bx)^3}$$

input
```
Integrate[(a + b*x)/(a*c - b*c*x)^4,x]
```

output
```
-1/6*(a + 3*b*x)/(b*c^4*(-a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(ac-bcx)^4}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{2a}{c^4(a-bx)^4}-\frac{1}{c^4(a-bx)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a}{3bc^4(a-bx)^3}-\frac{1}{2bc^4(a-bx)^2}$$

input
```
Int[(a + b*x)/(a*c - b*c*x)^4,x]
```

output
```
(2*a)/(3*b*c^4*(a - b*x)^3) - 1/(2*b*c^4*(a - b*x)^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{3bx+a}{6(-bx+a)^{3} c^{4} b}$	23
risch	$\frac{\frac{x}{2}+\frac{a}{6b}}{c^{4} (-bx+a)^{3}}$	23
norman	$\frac{\frac{a}{6bc}+\frac{x}{2c}}{c^{3} (-bx+a)^{3}}$	29
orering	$\frac{(3bx+a)(-bx+a)}{6b(-bcx+ac)^{4}}$	29
parallelrisch	$\frac{-3b^{3}x-a\,b^{2}}{6b^{3}c^{4}(bx-a)^{3}}$	31
default	$\frac{-\frac{1}{2b(-bx+a)^{2}}+\frac{2a}{3b(-bx+a)^{3}}}{c^{4}}$	33

input
```
int((b*x+a)/(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*b*x+a)/(-b*x+a)^3/c^4/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx = -\frac{3\,bx+a}{6\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

output
```
-1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*
c^4)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx = \frac{-a-3bx}{-6a^3bc^4+18a^2b^2c^4x-18ab^3c^4x^2+6b^4c^4x^3}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)**4,x)
```

output
```
(-a - 3*b*x)/(-6*a**3*b*c**4 + 18*a**2*b**2*c**4*x - 18*a*b**3*c**4*x**2 +
 6*b**4*c**4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx = -\frac{3\,bx+a}{6\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

output
```
-1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*
c^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx = -\frac{3\,bx+a}{6\,(bx-a)^3bc^4}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="giac")
```

output
```
-1/6*(3*b*x + a)/((b*x - a)^3*b*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx=\frac{\frac{x}{2}+\frac{a}{6\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

input
```
int((a + b*x)/(a*c - b*c*x)^4,x)
```

output
```
(x/2 + a/(6*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx=\frac{3bx+a}{6b\,c^4\left(-b^3x^3+3a\,b^2x^2-3a^2bx+a^3\right)}$$

input
```
int((b*x+a)/(-b*c*x+a*c)^4,x)
```

output
```
(a + 3*b*x)/(6*b*c**4*(a**3 - 3*a**2*b*x + 3*a*b**2*x**2 - b**3*x**3))
```

3.8 $\int \frac{a+bx}{(ac-bcx)^5} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{a+bx}{(ac-bcx)^5} \, dx = \frac{a}{2bc^5(a-bx)^4} - \frac{1}{3bc^5(a-bx)^3}$$

output
```
1/2*a/b/c^5/(-b*x+a)^4-1/3/b/c^5/(-b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{a+bx}{(ac-bcx)^5} \, dx = \frac{a+2bx}{6bc^5(a-bx)^4}$$

input
```
Integrate[(a + b*x)/(a*c - b*c*x)^5,x]
```

output
```
(a + 2*b*x)/(6*b*c^5*(a - b*x)^4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(ac-bcx)^5} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{2a}{c^5(a-bx)^5}-\frac{1}{c^5(a-bx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

input
```
Int[(a + b*x)/(a*c - b*c*x)^5,x]
```

output
```
a/(2*b*c^5*(a - b*x)^4) - 1/(3*b*c^5*(a - b*x)^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2bx+a}{6(-bx+a)^{4} c^{5} b}$	23
risch	$\frac{\frac{x}{3}+\frac{a}{6b}}{c^{5} (-bx+a)^{4}}$	23
norman	$\frac{\frac{a}{6bc}+\frac{x}{3c}}{c^{4} (-bx+a)^{4}}$	29
orering	$\frac{(2bx+a)(-bx+a)}{6b(-bcx+ac)^{5}}$	29
parallelrisch	$\frac{2x\,b^{4}+a\,b^{3}}{6b^{4} c^{5} (bx-a)^{4}}$	30
default	$\frac{\frac{a}{2b(-bx+a)^{4}}-\frac{1}{3b(-bx+a)^{3}}}{c^{5}}$	33

input
```
int((b*x+a)/(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*b*x+a)/(-b*x+a)^4/c^5/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{a+bx}{(ac-bcx)^5}\,dx=\frac{2\,bx+a}{6\,(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5)}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

output
```
1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3
*b^2*c^5*x + a^4*b*c^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{a+bx}{(ac-bcx)^5}\,dx = -\frac{-a-2bx}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)**5,x)
```

output
```
-(-a - 2*b*x)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*a**2*b**3*c**5*x**
2 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{a+bx}{(ac-bcx)^5}\,dx = \frac{2\,bx+a}{6\,(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5)}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

output
```
1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3
*b^2*c^5*x + a^4*b*c^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{a+bx}{(ac-bcx)^5}\,dx = \frac{a}{2\,(bcx-ac)^4bc}+\frac{1}{3\,(bcx-ac)^3bc^2}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="giac")
```

output
```
1/2*a/((b*c*x - a*c)^4*b*c) + 1/3/((b*c*x - a*c)^3*b*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{a+bx}{(ac-bcx)^5}\,dx = \frac{\frac{x}{3}+\frac{a}{6\,b}}{a^4\,c^5-4\,a^3\,b\,c^5\,x+6\,a^2\,b^2\,c^5\,x^2-4\,a\,b^3\,c^5\,x^3+b^4\,c^5\,x^4}$$

input
```
int((a + b*x)/(a*c - b*c*x)^5,x)
```

output
```
(x/3 + a/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3 + 6*a^2*b^2*c^5*x
^2 - 4*a^3*b*c^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{a+bx}{(ac-bcx)^5}\,dx = \frac{2bx+a}{6b\,c^5\,(b^4x^4-4a\,b^3x^3+6a^2b^2x^2-4a^3bx+a^4)}$$

input
```
int((b*x+a)/(-b*c*x+a*c)^5,x)
```

output
```
(a + 2*b*x)/(6*b*c**5*(a**4 - 4*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b**3*x**
3 + b**4*x**4))
```

3.9 $\int \frac{a+bx}{(ac-bcx)^6} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{a+bx}{(ac-bcx)^6} \, dx = \frac{2a}{5bc^6(a-bx)^5} - \frac{1}{4bc^6(a-bx)^4}$$

output
```
2/5*a/b/c^6/(-b*x+a)^5-1/4/b/c^6/(-b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{a+bx}{(ac-bcx)^6} \, dx = -\frac{3a+5bx}{20bc^6(-a+bx)^5}$$

input
```
Integrate[(a + b*x)/(a*c - b*c*x)^6,x]
```

output
```
-1/20*(3*a + 5*b*x)/(b*c^6*(-a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(ac-bcx)^6} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{2a}{c^6(a-bx)^6}-\frac{1}{c^6(a-bx)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

input
```
Int[(a + b*x)/(a*c - b*c*x)^6,x]
```

output
```
(2*a)/(5*b*c^6*(a - b*x)^5) - 1/(4*b*c^6*(a - b*x)^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{\frac{x}{4}+\frac{3a}{20b}}{c^{6}\left(-bx+a\right)^{5}}$	23
gosper	$\frac{5bx+3a}{20\left(-bx+a\right)^{5}c^{6}b}$	25
norman	$\frac{\frac{3a}{20bc}+\frac{x}{4c}}{c^{5}\left(-bx+a\right)^{5}}$	29
parallelrisch	$\frac{-5b^{5}x-3a\,b^{4}}{20b^{5}c^{6}\left(bx-a\right)^{5}}$	31
orering	$\frac{\left(5bx+3a\right)\left(-bx+a\right)}{20b\left(-bcx+ac\right)^{6}}$	31
default	$\frac{-\frac{1}{4b\left(-bx+a\right)^{4}}+\frac{2a}{5b\left(-bx+a\right)^{5}}}{c^{6}}$	33

input
```
int((b*x+a)/(-b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x+3/20*a/b)/c^6/(-b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\int \frac{a+bx}{(ac-bcx)^6}\,dx = -\frac{5\,bx+3\,a}{20\,\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

output
```
-1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 -
10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(31) = 62$.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int \frac{a+bx}{(ac-bcx)^6}\,dx = \frac{-3a-5bx}{-20a^5bc^6+100a^4b^2c^6x-200a^3b^3c^6x^2+200a^2b^4c^6x^3-100ab^5c^6x^4+20b^6c^6x^5}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)**6,x)
```

output
```
(-3*a - 5*b*x)/(-20*a**5*b*c**6 + 100*a**4*b**2*c**6*x - 200*a**3*b**3*c**
6*x**2 + 200*a**2*b**4*c**6*x**3 - 100*a*b**5*c**6*x**4 + 20*b**6*c**6*x**
5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\int \frac{a+bx}{(ac-bcx)^6}\,dx = -\frac{5\,bx+3\,a}{20\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

output
```
-1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 -
10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{a+bx}{(ac-bcx)^6}\,dx = -\frac{5\,bx+3\,a}{20\,(bx-a)^5bc^6}$$

input
```
integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="giac")
```

output
```
-1/20*(5*b*x + 3*a)/((b*x - a)^5*b*c^6)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{a+bx}{(ac-bcx)^6}\,dx$$
$$= \frac{\frac{x}{4}+\frac{3\,a}{20\,b}}{a^5\,c^6-5\,a^4\,b\,c^6\,x+10\,a^3\,b^2\,c^6\,x^2-10\,a^2\,b^3\,c^6\,x^3+5\,a\,b^4\,c^6\,x^4-b^5\,c^6\,x^5}$$

input
```
int((a + b*x)/(a*c - b*c*x)^6,x)
```

output
```
(x/4 + (3*a)/(20*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6*x^4 + 10*a^3*b^2
*c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \frac{a+bx}{(ac-bcx)^6}\,dx = \frac{5bx+3a}{20b\,c^6\,(-b^5x^5+5a\,b^4x^4-10a^2b^3x^3+10a^3b^2x^2-5a^4bx+a^5)}$$

input
```
int((b*x+a)/(-b*c*x+a*c)^6,x)
```

output
```
(3*a + 5*b*x)/(20*b*c**6*(a**5 - 5*a**4*b*x + 10*a**3*b**2*x**2 - 10*a**2*
b**3*x**3 + 5*a*b**4*x**4 - b**5*x**5))
```

3.10 $\int (a+bx)^2(ac-bcx)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int (a+bx)^2(ac-bcx)^3\,dx=-\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}$$

output
```
-a^2*c^3*(-b*x+a)^4/b+4/5*a*c^3*(-b*x+a)^5/b-1/6*c^3*(-b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int (a+bx)^2(ac-bcx)^3\,dx=c^3\left(a^5x-\frac{1}{2}a^4bx^2-\frac{2}{3}a^3b^2x^3+\frac{1}{2}a^2b^3x^4+\frac{1}{5}ab^4x^5-\frac{b^5x^6}{6}\right)$$

input
```
Integrate[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

output
```
c^3*(a^5*x - (a^4*b*x^2)/2 - (2*a^3*b^2*x^3)/3 + (a^2*b^3*x^4)/2 + (a*b^4*
x^5)/5 - (b^5*x^6)/6)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(ac-bcx)^3 \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(4a^2(ac-bcx)^3+\frac{(ac-bcx)^5}{c^2}-\frac{4a(ac-bcx)^4}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

input
```
Int[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

output
```
-((a^2*c^3*(a - b*x)^4)/b) + (4*a*c^3*(a - b*x)^5)/(5*b) - (c^3*(a - b*x)^
6)/(6*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{x(-5b^5x^5+6a\,b^4x^4+15a^2b^3x^3-20a^3b^2x^2-15a^4bx+30a^5)c^3}{30}$	59
default	$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3c^3b^2x^3-\frac{1}{2}a^4c^3b\,x^2+a^5c^3x$	73
norman	$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3c^3b^2x^3-\frac{1}{2}a^4c^3b\,x^2+a^5c^3x$	73
risch	$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3c^3b^2x^3-\frac{1}{2}a^4c^3b\,x^2+a^5c^3x$	73
parallelrisch	$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3c^3b^2x^3-\frac{1}{2}a^4c^3b\,x^2+a^5c^3x$	73
orering	$\frac{x(-5b^5x^5+6a\,b^4x^4+15a^2b^3x^3-20a^3b^2x^2-15a^4bx+30a^5)(-bcx+ac)^3}{30(-bx+a)^3}$	75

input
```
int((b*x+a)^2*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(-5*b^5*x^5+6*a*b^4*x^4+15*a^2*b^3*x^3-20*a^3*b^2*x^2-15*a^4*b*x+30
*a^5)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int (a+bx)^2(ac-bcx)^3\,dx = -\frac{1}{6}\,b^5c^3x^6+\frac{1}{5}\,ab^4c^3x^5+\frac{1}{2}\,a^2b^3c^3x^4 - \frac{2}{3}\,a^3b^2c^3x^3-\frac{1}{2}\,a^4bc^3x^2+a^5c^3x$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
-1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c
^3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int (a+bx)^2(ac-bcx)^3\,dx = a^5c^3x - \frac{a^4bc^3x^2}{2} - \frac{2a^3b^2c^3x^3}{3} + \frac{a^2b^3c^3x^4}{2} + \frac{ab^4c^3x^5}{5} - \frac{b^5c^3x^6}{6}$$

input
```
integrate((b*x+a)**2*(-b*c*x+a*c)**3,x)
```

output
```
a**5*c**3*x - a**4*b*c**3*x**2/2 - 2*a**3*b**2*c**3*x**3/3 + a**2*b**3*c**
3*x**4/2 + a*b**4*c**3*x**5/5 - b**5*c**3*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int (a+bx)^2(ac-bcx)^3\,dx = -\frac{1}{6}\,b^5c^3x^6 + \frac{1}{5}\,ab^4c^3x^5 + \frac{1}{2}\,a^2b^3c^3x^4 - \frac{2}{3}\,a^3b^2c^3x^3 - \frac{1}{2}\,a^4bc^3x^2 + a^5c^3x$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
-1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c
^3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int (a+bx)^2(ac-bcx)^3\,dx = -\frac{1}{6}\,b^5c^3x^6 + \frac{1}{5}\,ab^4c^3x^5 + \frac{1}{2}\,a^2b^3c^3x^4 - \frac{2}{3}\,a^3b^2c^3x^3 - \frac{1}{2}\,a^4bc^3x^2 + a^5c^3x$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
-1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c
^3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int (a+bx)^2(ac-bcx)^3\,dx = a^5\,c^3\,x - \frac{a^4\,b\,c^3\,x^2}{2} - \frac{2\,a^3\,b^2\,c^3\,x^3}{3} + \frac{a^2\,b^3\,c^3\,x^4}{2} + \frac{a\,b^4\,c^3\,x^5}{5} - \frac{b^5\,c^3\,x^6}{6}$$

input
```
int((a*c - b*c*x)^3*(a + b*x)^2,x)
```

output
```
a^5*c^3*x - (b^5*c^3*x^6)/6 - (a^4*b*c^3*x^2)/2 + (a*b^4*c^3*x^5)/5 - (2*a
^3*b^2*c^3*x^3)/3 + (a^2*b^3*c^3*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int (a+bx)^2(ac-bcx)^3\,dx = \frac{c^3x(-5b^5x^5+6a\,b^4x^4+15a^2b^3x^3-20a^3b^2x^2-15a^4bx+30a^5)}{30}$$

input
```
int((b*x+a)^2*(-b*c*x+a*c)^3,x)
```

output
```
(c**3*x*(30*a**5 - 15*a**4*b*x - 20*a**3*b**2*x**2 + 15*a**2*b**3*x**3 + 6
*a*b**4*x**4 - 5*b**5*x**5))/30
```

3.11 $\int (a+bx)^2(ac-bcx)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int (a+bx)^2(ac-bcx)^2\,dx = a^4c^2x - \frac{2}{3}a^2b^2c^2x^3 + \frac{1}{5}b^4c^2x^5$$

output
```
a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int (a+bx)^2(ac-bcx)^2\,dx = a^4c^2x - \frac{2}{3}a^2b^2c^2x^3 + \frac{1}{5}b^4c^2x^5$$

input
```
Integrate[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

output
```
a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {39, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(ac-bcx)^2\,dx$$

$$\Big\downarrow \text{39}$$

$$\int \left(a^2c-b^2cx^2\right)^2 dx$$

$$\Big\downarrow \text{210}$$

$$\int \left(a^4c^2-2a^2b^2c^2x^2+b^4c^2x^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

input
```
Int[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

output
```
a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x(3b^4x^4-10a^2b^2x^2+15a^4)c^2}{15}$	32
default	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
norman	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
risch	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
parallelrisch	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
orering	$\frac{x(3b^4x^4-10a^2b^2x^2+15a^4)(-bcx+ac)^2}{15(-bx+a)^2}$	48

input
```
int((b*x+a)^2*(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b^4*x^4-10*a^2*b^2*x^2+15*a^4)*c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a+bx)^2(ac-bcx)^2\,dx=\frac{1}{5}\,b^4c^2x^5-\frac{2}{3}\,a^2b^2c^2x^3+a^4c^2x$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

output
```
1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a+bx)^2(ac-bcx)^2\,dx = a^4c^2x - \frac{2a^2b^2c^2x^3}{3} + \frac{b^4c^2x^5}{5}$$

input
```
integrate((b*x+a)**2*(-b*c*x+a*c)**2,x)
```

output
```
a**4*c**2*x - 2*a**2*b**2*c**2*x**3/3 + b**4*c**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a+bx)^2(ac-bcx)^2\,dx = \frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

output
```
1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a+bx)^2(ac-bcx)^2\,dx = \frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="giac")
```

output
```
1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int (a+bx)^2(ac-bcx)^2\,dx=\frac{c^2\,x\,\left(15\,a^4-10\,a^2\,b^2\,x^2+3\,b^4\,x^4\right)}{15}$$

input
```
int((a*c - b*c*x)^2*(a + b*x)^2,x)
```

output
```
(c^2*x*(15*a^4 + 3*b^4*x^4 - 10*a^2*b^2*x^2))/15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int (a+bx)^2(ac-bcx)^2\,dx=\frac{c^2x(3b^4x^4-10a^2b^2x^2+15a^4)}{15}$$

input
```
int((b*x+a)^2*(-b*c*x+a*c)^2,x)
```

output
```
(c**2*x*(15*a**4 - 10*a**2*b**2*x**2 + 3*b**4*x**4))/15
```

3.12 $\int (a+bx)^2(ac-bcx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int (a+bx)^2(ac-bcx)\,dx=\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

output
```
2/3*a*c*(b*x+a)^3/b-1/4*c*(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int (a+bx)^2(ac-bcx)\,dx=c\left(a^3x+\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3-\frac{b^3x^4}{4}\right)$$

input
```
Integrate[(a + b*x)^2*(a*c - b*c*x),x]
```

output
```
c*(a^3*x + (a^2*b*x^2)/2 - (a*b^2*x^3)/3 - (b^3*x^4)/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(ac-bcx)\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left(2ac(a+bx)^2-c(a+bx)^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

input
```
Int[(a + b*x)^2*(a*c - b*c*x),x]
```

output
```
(2*a*c*(a + b*x)^3)/(3*b) - (c*(a + b*x)^4)/(4*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{cx\left(-3b^3x^3-4a\,b^2x^2+6a^2bx+12a^3\right)}{12}$	35
default	$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$	37
norman	$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$	37
risch	$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$	37
parallelrisch	$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$	37
orering	$\frac{x\left(-3b^3x^3-4a\,b^2x^2+6a^2bx+12a^3\right)\left(-bcx+ac\right)}{-12bx+12a}$	51

input
```
int((b*x+a)^2*(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/12*c*x*(-3*b^3*x^3-4*a*b^2*x^2+6*a^2*b*x+12*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int (a+bx)^2(ac-bcx)\,dx=-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="fricas")
```

output
```
-1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int (a+bx)^2(ac-bcx)\,dx = a^3cx + \frac{a^2bcx^2}{2} - \frac{ab^2cx^3}{3} - \frac{b^3cx^4}{4}$$

input
```
integrate((b*x+a)**2*(-b*c*x+a*c),x)
```

output
```
a**3*c*x + a**2*b*c*x**2/2 - a*b**2*c*x**3/3 - b**3*c*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int (a+bx)^2(ac-bcx)\,dx = -\frac{1}{4}\,b^3cx^4 - \frac{1}{3}\,ab^2cx^3 + \frac{1}{2}\,a^2bcx^2 + a^3cx$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="maxima")
```

output
```
-1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int (a+bx)^2(ac-bcx)\,dx = -\frac{1}{4}\,b^3cx^4 - \frac{1}{3}\,ab^2cx^3 + \frac{1}{2}\,a^2bcx^2 + a^3cx$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="giac")
```

output
```
-1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int (a+bx)^2(ac-bcx)\,dx = c\,a^3\,x + \frac{c\,a^2\,b\,x^2}{2} - \frac{c\,a\,b^2\,x^3}{3} - \frac{c\,b^3\,x^4}{4}$$

input
```
int((a*c - b*c*x)*(a + b*x)^2,x)
```

output
```
a^3*c*x - (b^3*c*x^4)/4 + (a^2*b*c*x^2)/2 - (a*b^2*c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int (a+bx)^2(ac-bcx)\,dx = \frac{cx(-3b^3x^3 - 4a\,b^2x^2 + 6a^2bx + 12a^3)}{12}$$

input
```
int((b*x+a)^2*(-b*c*x+a*c),x)
```

output
```
(c*x*(12*a**3 + 6*a**2*b*x - 4*a*b**2*x**2 - 3*b**3*x**3))/12
```

3.13 $\int (a+bx)^2\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (a+bx)^2\,dx=\frac{(a+bx)^3}{3b}$$

output
```
1/3*(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\,dx=\frac{(a+bx)^3}{3b}$$

input
```
Integrate[(a + b*x)^2,x]
```

output
```
(a + b*x)^3/(3*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^3}{3b}$$

input
```
Int[(a + b*x)^2,x]
```

output
```
(a + b*x)^3/(3*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
gosper	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
norman	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
parallelrisch	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
orering	$\frac{x(b^2x^2+3abx+3a^2)}{3}$	22
risch	$\frac{b^2x^3}{3}+ab\,x^2+a^2x+\frac{a^3}{3b}$	29

input
```
int((b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a+bx)^2\,dx=\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

input
```
integrate((b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int (a+bx)^2\,dx = a^2x + abx^2 + \frac{b^2x^3}{3}$$

input
```
integrate((b*x+a)**2,x)
```

output
```
a**2*x + a*b*x**2 + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a+bx)^2\,dx = \frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

input
```
integrate((b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+bx)^2\,dx = \frac{(bx+a)^3}{3\,b}$$

input
```
integrate((b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (a+bx)^2\,dx = a^2\,x + a\,b\,x^2 + \frac{b^2\,x^3}{3}$$

input
```
int((a + b*x)^2,x)
```

output
```
a^2*x + (b^2*x^3)/3 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int (a+bx)^2\,dx = \frac{x(b^2x^2+3abx+3a^2)}{3}$$

input
```
int((b*x+a)^2,x)
```

output
```
(x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.14 $\int \frac{(a+bx)^2}{ac-bcx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{(a+bx)^2}{ac-bcx} \, dx = -\frac{3ax}{c} - \frac{bx^2}{2c} - \frac{4a^2\log(a-bx)}{bc}$$

output
```
-3*a*x/c-1/2*b*x^2/c-4*a^2*ln(-b*x+a)/b/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{ac-bcx} \, dx = -\frac{3ax}{c} - \frac{bx^2}{2c} - \frac{4a^2\log(a-bx)}{bc}$$

input
```
Integrate[(a + b*x)^2/(a*c - b*c*x),x]
```

output
```
(-3*a*x)/c - (b*x^2)/(2*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{ac-bcx} \, dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{4a^2}{ac-bcx}-\frac{a+bx}{c}-\frac{2a}{c}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

input
```
Int[(a + b*x)^2/(a*c - b*c*x),x]
```

output
```
(-2*a*x)/c - (a + b*x)^2/(2*b*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{-\frac{b\,x^2}{2}-3ax-\frac{4a^2\ln(-bx+a)}{b}}{c}$	31
norman	$-\frac{3ax}{c}-\frac{b\,x^2}{2c}-\frac{4a^2\ln(-bx+a)}{bc}$	36
risch	$-\frac{3ax}{c}-\frac{b\,x^2}{2c}-\frac{4a^2\ln(-bx+a)}{bc}$	36
parallelrisch	$\frac{-b^2x^2-8a^2\ln(bx-a)-6abx}{2bc}$	36

input
```
int((b*x+a)^2/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*b*x^2-3*a*x-4*a^2*ln(-b*x+a)/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx = -\frac{b^2x^2+6\,abx+8\,a^2\log\left(bx-a\right)}{2\,bc}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx = -\frac{4a^2\log\left(-a+bx\right)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

input
```
integrate((b*x+a)**2/(-b*c*x+a*c),x)
```

output
```
-4*a**2*log(-a + b*x)/(b*c) - 3*a*x/c - b*x**2/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx = -\frac{4\,a^2\log{(bx-a)}}{bc} - \frac{bx^2+6\,ax}{2\,c}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

output
```
-4*a^2*log(b*x - a)/(b*c) - 1/2*(b*x^2 + 6*a*x)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx = -\frac{4\,a^2\log{(|bx-a|)}}{bc} - \frac{b^3cx^2+6\,ab^2cx}{2\,b^2c^2}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
```

output
```
-4*a^2*log(abs(b*x - a))/(b*c) - 1/2*(b^3*c*x^2 + 6*a*b^2*c*x)/(b^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx = -\frac{8\,a^2\,\ln{(b\,x-a)}+b^2\,x^2+6\,a\,b\,x}{2\,b\,c}$$

input
```
int((a + b*x)^2/(a*c - b*c*x),x)
```

output
```
-(8*a^2*log(b*x - a) + b^2*x^2 + 6*a*b*x)/(2*b*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx = \frac{-8\log(-bx+a)\,a^2-6abx-b^2x^2}{2bc}$$

input
```
int((b*x+a)^2/(-b*c*x+a*c),x)
```

output
```
( - 8*log(a - b*x)*a**2 - 6*a*b*x - b**2*x**2)/(2*b*c)
```

3.15 $\int \frac{(a+bx)^2}{(ac-bcx)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{(a+bx)^2}{(ac-bcx)^2} dx = \frac{x}{c^2} + \frac{4a^2}{bc^2(a-bx)} + \frac{4a\log(a-bx)}{bc^2}$$

output
```
x/c^2+4*a^2/b/c^2/(-b*x+a)+4*a*ln(-b*x+a)/b/c^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^2}{(ac-bcx)^2} dx = \frac{x + \frac{4a^2}{b(a-bx)} + \frac{4a\log(a-bx)}{b}}{c^2}$$

input
```
Integrate[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

output
```
(x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b)/c^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{4a^2}{c^2(a-bx)^2}-\frac{4a}{c^2(a-bx)}+\frac{1}{c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}+\frac{x}{c^2}$$

input
```
Int[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

output
```
x/c^2 + (4*a^2)/(b*c^2*(a - b*x)) + (4*a*Log[a - b*x])/(b*c^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x+\frac{4a\ln(-bx+a)}{b}+\frac{4a^2}{b(-bx+a)}}{c^2}$	36
risch	$\frac{x}{c^2}+\frac{4a^2}{b\,c^2(-bx+a)}+\frac{4a\ln(-bx+a)}{b\,c^2}$	42
norman	$\frac{\frac{5a^2}{bc}-\frac{b\,x^2}{c}}{c(-bx+a)}+\frac{4a\ln(-bx+a)}{b\,c^2}$	51
parallelrisch	$\frac{4\ln(bx-a)xab+b^2x^2-4a^2\ln(bx-a)-5a^2}{b\,c^2(bx-a)}$	56

input
```
int((b*x+a)^2/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(x+4*a*ln(-b*x+a)/b+4/b*a^2/(-b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx = \frac{b^2x^2-abx-4\,a^2+4\,(abx-a^2)\log(bx-a)}{b^2c^2x-abc^2}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*c^2*x - a*b*
c^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx = -\frac{4a^2}{-abc^2+b^2c^2x}+\frac{4a\log(-a+bx)}{bc^2}+\frac{x}{c^2}$$

input
```
integrate((b*x+a)**2/(-b*c*x+a*c)**2,x)
```

output
```
-4*a**2/(-a*b*c**2 + b**2*c**2*x) + 4*a*log(-a + b*x)/(b*c**2) + x/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx = -\frac{4\,a^2}{b^2c^2x-abc^2}+\frac{x}{c^2}+\frac{4\,a\log(bx-a)}{bc^2}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

output
```
-4*a^2/(b^2*c^2*x - a*b*c^2) + x/c^2 + 4*a*log(b*x - a)/(b*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx = -\frac{4\,a^2}{(bcx-ac)bc}-\frac{4\,a\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc^2}+\frac{bcx-ac}{bc^3}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

output
```
-4*a^2/((b*c*x - a*c)*b*c) - 4*a*log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs
(b)*abs(c)))/(b*c^2) + (b*c*x - a*c)/(b*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx = \frac{x}{c^2} + \frac{4\,a^2}{b\,\left(a\,c^2 - b\,c^2\,x\right)} + \frac{4\,a\,\ln\left(b\,x - a\right)}{b\,c^2}$$

input
```
int((a + b*x)^2/(a*c - b*c*x)^2,x)
```

output
```
x/c^2 + (4*a^2)/(b*(a*c^2 - b*c^2*x)) + (4*a*log(b*x - a))/(b*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx = \frac{4\log(-bx+a)\,a^2 - 4\log(-bx+a)\,abx + 5abx - b^2x^2}{b\,c^2\,(-bx+a)}$$

input
```
int((b*x+a)^2/(-b*c*x+a*c)^2,x)
```

output
```
(4*log(a - b*x)*a**2 - 4*log(a - b*x)*a*b*x + 5*a*b*x - b**2*x**2)/(b*c**2
*(a - b*x))
```

3.16 $\int \frac{(a+bx)^2}{(ac-bcx)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(a+bx)^2}{(ac-bcx)^3} dx = \frac{2a^2}{bc^3(a-bx)^2} - \frac{4a}{bc^3(a-bx)} - \frac{\log(a-bx)}{bc^3}$$

output
```
2*a^2/b/c^3/(-b*x+a)^2-4*a/b/c^3/(-b*x+a)-ln(-b*x+a)/b/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx)^2}{(ac-bcx)^3} dx = -\frac{\frac{2a(a-2bx)}{(a-bx)^2} + \log(a-bx)}{bc^3}$$

input
```
Integrate[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

output
```
-(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/(b*c^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{4a^2}{c^3(a-bx)^3}-\frac{4a}{c^3(a-bx)^2}+\frac{1}{c^3(a-bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}$$

input
```
Int[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

output
```
(2*a^2)/(b*c^3*(a - b*x)^2) - (4*a)/(b*c^3*(a - b*x)) - Log[a - b*x]/(b*c^
3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{4ax-\frac{2a^2}{b}}{c^3(-bx+a)^2}-\frac{\ln(-bx+a)}{b\,c^3}$	42
default	$\frac{\frac{2a^2}{b(-bx+a)^2}-\frac{\ln(-bx+a)}{b}-\frac{4a}{b(-bx+a)}}{c^3}$	48
norman	$\frac{-\frac{2a^2}{bc}+\frac{4ax}{c}}{c^2(-bx+a)^2}-\frac{\ln(-bx+a)}{b\,c^3}$	48
parallelrisch	$\frac{-\ln(bx-a)x^2a^2b^2+2\ln(bx-a)x\,a^3b+2a^2b^2x^2-\ln(bx-a)a^4}{a^2c^3(bx-a)^2b}$	79

input
```
int((b*x+a)^2/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
(4*a*x-2/b*a^2)/c^3/(-b*x+a)^2-ln(-b*x+a)/b/c^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx = \frac{4\,abx-2\,a^2-(b^2x^2-2\,abx+a^2)\log(bx-a)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
(4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*c^3*x^2 -
2*a*b^2*c^3*x + a^2*b*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx = -\frac{2a^2-4abx}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} - \frac{\log(-a+bx)}{bc^3}$$

input
```
integrate((b*x+a)**2/(-b*c*x+a*c)**3,x)
```

output
```
-(2*a**2 - 4*a*b*x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) - log
(-a + b*x)/(b*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx = \frac{2\,(2\,abx-a^2)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3} - \frac{\log(bx-a)}{bc^3}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
2*(2*a*b*x - a^2)/(b^3*c^3*x^2 - 2*a*b^2*c^3*x + a^2*b*c^3) - log(b*x - a)
/(b*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx = -\frac{\log(|bx-a|)}{bc^3} + \frac{2\,(2\,abx-a^2)}{(bx-a)^2bc^3}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
-log(abs(b*x - a))/(b*c^3) + 2*(2*a*b*x - a^2)/((b*x - a)^2*b*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx = \frac{4\,a\,x-\frac{2\,a^2}{b}}{a^2\,c^3-2\,a\,b\,c^3\,x+b^2\,c^3\,x^2}-\frac{\ln\left(b\,x-a\right)}{b\,c^3}$$

input
```
int((a + b*x)^2/(a*c - b*c*x)^3,x)
```

output
```
(4*a*x - (2*a^2)/b)/(a^2*c^3 + b^2*c^3*x^2 - 2*a*b*c^3*x) - log(b*x - a)/(
b*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx = \frac{-\log(-bx+a)\,a^2+2\log(-bx+a)\,abx-\log(-bx+a)\,b^2x^2+2b^2x^2}{b\,c^3\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)^2/(-b*c*x+a*c)^3,x)
```

output
```
( - log(a - b*x)*a**2 + 2*log(a - b*x)*a*b*x - log(a - b*x)*b**2*x**2 + 2*
b**2*x**2)/(b*c**3*(a**2 - 2*a*b*x + b**2*x**2))
```

3.17 $\int \frac{(a+bx)^2}{(ac-bcx)^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} \, dx = \frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

output
```
1/6*(b*x+a)^3/a/b/c^4/(-b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} \, dx = -\frac{a^2+3b^2x^2}{3bc^4(-a+bx)^3}$$

input
```
Integrate[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

output
```
-1/3*(a^2 + 3*b^2*x^2)/(b*c^4*(-a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} \, dx$$
$$\Big\downarrow \; 48$$
$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

input
```
Int[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

output
```
(a + b*x)^3/(6*a*b*c^4*(a - b*x)^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{b\,x^{2}+\frac{a^{2}}{3b}}{c^{4}\left(-bx+a\right)^{3}}$	27
gosper	$\frac{3b^{2}x^{2}+a^{2}}{3\left(-bx+a\right)^{3}c^{4}b}$	29
norman	$\frac{\frac{a^{2}}{3bc}+\frac{b\,x^{2}}{c}}{c^{3}\left(-bx+a\right)^{3}}$	33
parallelrisch	$\frac{-3x^{2}b^{4}-a^{2}b^{2}}{3b^{3}c^{4}\left(bx-a\right)^{3}}$	35
orering	$\frac{\left(3b^{2}x^{2}+a^{2}\right)\left(-bx+a\right)}{3b\left(-bcx+ac\right)^{4}}$	35
default	$\frac{-\frac{2a}{b\left(-bx+a\right)^{2}}+\frac{4a^{2}}{3b\left(-bx+a\right)^{3}}+\frac{1}{b\left(-bx+a\right)}}{c^{4}}$	48

input
```
int((b*x+a)^2/(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+1/3/b*a^2)/c^4/(-b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(27) = 54.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx=-\frac{3\,b^2x^2+a^2}{3\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x -
a^3*b*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx = \frac{-a^2-3b^2x^2}{-3a^3bc^4+9a^2b^2c^4x-9ab^3c^4x^2+3b^4c^4x^3}$$

input
```
integrate((b*x+a)**2/(-b*c*x+a*c)**4,x)
```

output
```
(-a**2 - 3*b**2*x**2)/(-3*a**3*b*c**4 + 9*a**2*b**2*c**4*x - 9*a*b**3*c**4
*x**2 + 3*b**4*c**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx = -\frac{3\,b^2x^2+a^2}{3\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x -
a^3*b*c^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx = -\frac{3\,b^2x^2+a^2}{3\,(bx-a)^3bc^4}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*x^2 + a^2)/((b*x - a)^3*b*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx = \frac{b\,x^2+\frac{a^2}{3\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

input
```
int((a + b*x)^2/(a*c - b*c*x)^4,x)
```

output
```
(b*x^2 + a^2/(3*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4
*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx = \frac{x(b^2x^2+3a^2)}{3a\,c^4\,(-b^3x^3+3a\,b^2x^2-3a^2bx+a^3)}$$

input
```
int((b*x+a)^2/(-b*c*x+a*c)^4,x)
```

output
```
(x*(3*a**2 + b**2*x**2))/(3*a*c**4*(a**3 - 3*a**2*b*x + 3*a*b**2*x**2 - b*
*3*x**3))
```

3.18 $\int \frac{(a+bx)^2}{(ac-bcx)^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{(a+bx)^2}{(ac-bcx)^5} dx = \frac{a^2}{bc^5(a-bx)^4} - \frac{4a}{3bc^5(a-bx)^3} + \frac{1}{2bc^5(a-bx)^2}$$

output
```
a^2/b/c^5/(-b*x+a)^4-4/3*a/b/c^5/(-b*x+a)^3+1/2/b/c^5/(-b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^2}{(ac-bcx)^5} dx = \frac{a^2+2abx+3b^2x^2}{6bc^5(a-bx)^4}$$

input
```
Integrate[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

output
```
(a^2 + 2*a*b*x + 3*b^2*x^2)/(6*b*c^5*(a - b*x)^4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(ac-bcx)^5} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{4a^2}{c^5(a-bx)^5}-\frac{4a}{c^5(a-bx)^4}+\frac{1}{c^5(a-bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}$$

input
```
Int[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

output
```
a^2/(b*c^5*(a - b*x)^4) - (4*a)/(3*b*c^5*(a - b*x)^3) + 1/(2*b*c^5*(a - b*
x)^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{\frac{b\,x^{2}}{2}+\frac{a x}{3}+\frac{a^{2}}{6 b}}{c^{5} \left(-b x +a \right)^{4}}$	32
gosper	$\frac{3 b^{2} x^{2}+2 a b x +a^{2}}{6 \left(-b x +a \right)^{4} c^{5} b}$	34
orering	$\frac{\left(3 b^{2} x^{2}+2 a b x +a^{2}\right) \left(-b x +a \right)}{6 b \left(-b c x +a c \right)^{5}}$	40
norman	$\frac{\frac{a^{2}}{6 b c}+\frac{b\,x^{2}}{2 c}+\frac{a x}{3 c}}{c^{4} \left(-b x +a \right)^{4}}$	41
parallelrisch	$\frac{3 b^{5} x^{2}+2 a\,b^{4} x +a^{2} b^{3}}{6 b^{4} c^{5} \left(b x -a \right)^{4}}$	41
default	$\frac{\frac{1}{2 b \left(-b x +a \right)^{2}}-\frac{4 a}{3 b \left(-b x +a \right)^{3}}+\frac{a^{2}}{b \left(-b x +a \right)^{4}}}{c^{5}}$	48

input
```
int((b*x+a)^2/(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*x^2+1/3*a*x+1/6/b*a^2)/c^5/(-b*x+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^2}{(ac-bcx)^5}\,dx = \frac{3\,b^2x^2+2\,abx+a^2}{6\,(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

output
```
1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3
*c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^2}{(ac-bcx)^5}\,dx = -\frac{-a^2-2abx-3b^2x^2}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

input
```
integrate((b*x+a)**2/(-b*c*x+a*c)**5,x)
```

output
```
-(-a**2 - 2*a*b*x - 3*b**2*x**2)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36
*a**2*b**3*c**5*x**2 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^2}{(ac-bcx)^5}\,dx = \frac{3\,b^2x^2+2\,abx+a^2}{6\,(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

output
```
1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3
*c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^2}{(ac-bcx)^5}\,dx = \frac{\frac{6\,a^2}{(bcx-ac)^4b}+\frac{8\,a}{(bcx-ac)^3bc}+\frac{3}{(bcx-ac)^2bc^2}}{6\,c}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="giac")
```

output
```
1/6*(6*a^2/((b*c*x - a*c)^4*b) + 8*a/((b*c*x - a*c)^3*b*c) + 3/((b*c*x - a
*c)^2*b*c^2))/c
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^2}{(ac-bcx)^5} \, dx = \frac{\frac{a\,x}{3} + \frac{b\,x^2}{2} + \frac{a^2}{6\,b}}{a^4\,c^5 - 4\,a^3\,b\,c^5\,x + 6\,a^2\,b^2\,c^5\,x^2 - 4\,a\,b^3\,c^5\,x^3 + b^4\,c^5\,x^4}$$

input
```
int((a + b*x)^2/(a*c - b*c*x)^5,x)
```

output
```
((a*x)/3 + (b*x^2)/2 + a^2/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3
 + 6*a^2*b^2*c^5*x^2 - 4*a^3*b*c^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^2}{(ac-bcx)^5} \, dx = \frac{3b^2x^2 + 2abx + a^2}{6b\,c^5\,(b^4x^4 - 4a\,b^3x^3 + 6a^2b^2x^2 - 4a^3bx + a^4)}$$

input
```
int((b*x+a)^2/(-b*c*x+a*c)^5,x)
```

output
```
(a**2 + 2*a*b*x + 3*b**2*x**2)/(6*b*c**5*(a**4 - 4*a**3*b*x + 6*a**2*b**2*
x**2 - 4*a*b**3*x**3 + b**4*x**4))
```

3.19 $\int \frac{(a+bx)^2}{(ac-bcx)^6} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx)^2}{(ac-bcx)^6} \, dx = \frac{4a^2}{5bc^6(a-bx)^5} - \frac{a}{bc^6(a-bx)^4} + \frac{1}{3bc^6(a-bx)^3}$$

output
```
4/5*a^2/b/c^6/(-b*x+a)^5-a/b/c^6/(-b*x+a)^4+1/3/b/c^6/(-b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2}{(ac-bcx)^6} \, dx = -\frac{2a^2+5abx+5b^2x^2}{15bc^6(-a+bx)^5}$$

input
```
Integrate[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

output
```
-1/15*(2*a^2 + 5*a*b*x + 5*b^2*x^2)/(b*c^6*(-a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{4a^2}{c^6(a-bx)^6}-\frac{4a}{c^6(a-bx)^5}+\frac{1}{c^6(a-bx)^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

input
```
Int[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

output
```
(4*a^2)/(5*b*c^6*(a - b*x)^5) - a/(b*c^6*(a - b*x)^4) + 1/(3*b*c^6*(a - b*
x)^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{\frac{b\,x^2}{3}+\frac{ax}{3}+\frac{2a^2}{15b}}{c^6(-bx+a)^5}$	32
gosper	$\frac{5b^2x^2+5abx+2a^2}{15(-bx+a)^5c^6b}$	36
norman	$\frac{\frac{2a^2}{15bc}+\frac{b\,x^2}{3c}+\frac{ax}{3c}}{c^5(-bx+a)^5}$	41
parallelrisch	$\frac{-5x^2b^6-5xa\,b^5-2b^4a^2}{15b^5c^6(bx-a)^5}$	42
orering	$\frac{(5b^2x^2+5abx+2a^2)(-bx+a)}{15b(-bcx+ac)^6}$	42
default	$\frac{\frac{1}{3b(-bx+a)^3}-\frac{a}{b(-bx+a)^4}+\frac{4a^2}{5b(-bx+a)^5}}{c^6}$	49

input
```
int((b*x+a)^2/(-b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
```

output
```
(1/3*b*x^2+1/3*a*x+2/15/b*a^2)/c^6/(-b*x+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx$$
$$=-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

output
```
-1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^
2*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(46) = 92$.

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx = \frac{-2a^2-5abx-5b^2x^2}{-15a^5bc^6+75a^4b^2c^6x-150a^3b^3c^6x^2+150a^2b^4c^6x^3-75ab^5c^6x^4+15b^6c^6x^5}$$

input
```
integrate((b*x+a)**2/(-b*c*x+a*c)**6,x)
```

output
```
(-2*a**2 - 5*a*b*x - 5*b**2*x**2)/(-15*a**5*b*c**6 + 75*a**4*b**2*c**6*x -
 150*a**3*b**3*c**6*x**2 + 150*a**2*b**4*c**6*x**3 - 75*a*b**5*c**6*x**4 +
 15*b**6*c**6*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx = -\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

output
```
-1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^
2*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx = -\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(bx-a)^5bc^6}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="giac")
```

output
```
-1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x - a)^5*b*c^6)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx = \frac{\frac{a\,x}{3}+\frac{b\,x^2}{3}+\frac{2\,a^2}{15\,b}}{a^5\,c^6-5\,a^4\,b\,c^6\,x+10\,a^3\,b^2\,c^6\,x^2-10\,a^2\,b^3\,c^6\,x^3+5\,a\,b^4\,c^6\,x^4-b^5\,c^6\,x^5}$$

input
```
int((a + b*x)^2/(a*c - b*c*x)^6,x)
```

output
```
((a*x)/3 + (b*x^2)/3 + (2*a^2)/(15*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^
6*x^4 + 10*a^3*b^2*c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx = \frac{5b^2x^2+5abx+2a^2}{15b\,c^6\,(-b^5x^5+5a\,b^4x^4-10a^2b^3x^3+10a^3b^2x^2-5a^4bx+a^5)}$$

input
```
int((b*x+a)^2/(-b*c*x+a*c)^6,x)
```

output
```
(2*a**2 + 5*a*b*x + 5*b**2*x**2)/(15*b*c**6*(a**5 - 5*a**4*b*x + 10*a**3*b
**2*x**2 - 10*a**2*b**3*x**3 + 5*a*b**4*x**4 - b**5*x**5))
```

3.20 $\int \frac{(a+bx)^2}{(ac-bcx)^7} dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx)^2}{(ac-bcx)^7} dx = \frac{2a^2}{3bc^7(a-bx)^6} - \frac{4a}{5bc^7(a-bx)^5} + \frac{1}{4bc^7(a-bx)^4}$$

output
```
2/3*a^2/b/c^7/(-b*x+a)^6-4/5*a/b/c^7/(-b*x+a)^5+1/4/b/c^7/(-b*x+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx)^2}{(ac-bcx)^7} dx = \frac{7a^2 + 18abx + 15b^2x^2}{60bc^7(a-bx)^6}$$

input
```
Integrate[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

output
```
(7*a^2 + 18*a*b*x + 15*b^2*x^2)/(60*b*c^7*(a - b*x)^6)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{4a^2}{c^7(a-bx)^7}-\frac{4a}{c^7(a-bx)^6}+\frac{1}{c^7(a-bx)^5}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

input
```
Int[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

output
```
(2*a^2)/(3*b*c^7*(a - b*x)^6) - (4*a)/(5*b*c^7*(a - b*x)^5) + 1/(4*b*c^7*(
a - b*x)^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
risch	$\frac{\frac{b\,x^{2}}{4}+\frac{3ax}{10}+\frac{7a^{2}}{60b}}{c^{7}(-bx+a)^{6}}$	32
gosper	$\frac{15b^{2}x^{2}+18abx+7a^{2}}{60(-bx+a)^{6}c^{7}b}$	36
norman	$\frac{\frac{7a^{2}}{60bc}+\frac{b\,x^{2}}{4c}+\frac{3ax}{10c}}{c^{6}(-bx+a)^{6}}$	41
parallelrisch	$\frac{15b^{7}x^{2}+18a\,b^{6}x+7a^{2}b^{5}}{60b^{6}c^{7}(bx-a)^{6}}$	42
orering	$\frac{(15b^{2}x^{2}+18abx+7a^{2})(-bx+a)}{60b(-bcx+ac)^{7}}$	42
default	$\frac{\frac{2a^{2}}{3b(-bx+a)^{6}}-\frac{4a}{5b(-bx+a)^{5}}+\frac{1}{4b(-bx+a)^{4}}}{c^{7}}$	49

input
```
int((b*x+a)^2/(-b*c*x+a*c)^7,x,method=_RETURNVERBOSE)
```

output
```
(1/4*b*x^2+3/10*a*x+7/60/b*a^2)/c^7/(-b*x+a)^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx = \frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="fricas")
```

output
```
1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a
^2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x
 + a^6*b*c^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx = -\frac{-7a^2-18abx-15b^2x^2}{60a^6bc^7-360a^5b^2c^7x+900a^4b^3c^7x^2-1200a^3b^4c^7x^3+900a^2b^5c^7x^4-360ab^6c^7x^5+60b^7c^7x^6}$$

input
```
integrate((b*x+a)**2/(-b*c*x+a*c)**7,x)
```

output
```
-(-7*a**2 - 18*a*b*x - 15*b**2*x**2)/(60*a**6*b*c**7 - 360*a**5*b**2*c**7*
x + 900*a**4*b**3*c**7*x**2 - 1200*a**3*b**4*c**7*x**3 + 900*a**2*b**5*c**
7*x**4 - 360*a*b**6*c**7*x**5 + 60*b**7*c**7*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx = \frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7)}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="maxima")
```

output
```
1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a
^2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x
 + a^6*b*c^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx = \frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,(bx-a)^6bc^7}$$

input
```
integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="giac")
```

output
```
1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/((b*x - a)^6*b*c^7)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx$$
$$= \frac{\frac{3\,a\,x}{10}+\frac{b\,x^2}{4}+\frac{7\,a^2}{60\,b}}{a^6\,c^7-6\,a^5\,b\,c^7\,x+15\,a^4\,b^2\,c^7\,x^2-20\,a^3\,b^3\,c^7\,x^3+15\,a^2\,b^4\,c^7\,x^4-6\,a\,b^5\,c^7\,x^5+b^6\,c^7\,x^6}$$

input
```
int((a + b*x)^2/(a*c - b*c*x)^7,x)
```

output
```
((3*a*x)/10 + (b*x^2)/4 + (7*a^2)/(60*b))/(a^6*c^7 + b^6*c^7*x^6 - 6*a*b^5
*c^7*x^5 + 15*a^4*b^2*c^7*x^2 - 20*a^3*b^3*c^7*x^3 + 15*a^2*b^4*c^7*x^4 -
6*a^5*b*c^7*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx$$
$$= \frac{15b^2x^2+18abx+7a^2}{60b\,c^7\,(b^6x^6-6a\,b^5x^5+15a^2b^4x^4-20a^3b^3x^3+15a^4b^2x^2-6a^5bx+a^6)}$$

input
```
int((b*x+a)^2/(-b*c*x+a*c)^7,x)
```

output
```
(7*a**2 + 18*a*b*x + 15*b**2*x**2)/(60*b*c**7*(a**6 - 6*a**5*b*x + 15*a**4
*b**2*x**2 - 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 - 6*a*b**5*x**5 + b**6*
x**6))
```

3.21 $\int \frac{(ac-bcx)^3}{a+bx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{(ac-bcx)^3}{a+bx} \, dx = -7a^2c^3x + 2abc^3x^2 - \frac{1}{3}b^2c^3x^3 + \frac{8a^3c^3\log(a+bx)}{b}$$

output
```
-7*a^2*c^3*x+2*a*b*c^3*x^2-1/3*b^2*c^3*x^3+8*a^3*c^3*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{(ac-bcx)^3}{a+bx} \, dx = c^3\left(-7a^2x + 2abx^2 - \frac{b^2x^3}{3} + \frac{8a^3\log(a+bx)}{b}\right)$$

input
```
Integrate[(a*c - b*c*x)^3/(a + b*x),x]
```

output
```
c^3*(-7*a^2*x + 2*a*b*x^2 - (b^2*x^3)/3 + (8*a^3*Log[a + b*x])/b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^3}{a+bx} \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{8a^3c^3}{a+bx}-4a^2c^3-2ac^2(ac-bcx)-c(ac-bcx)^2\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

input
```
Int[(a*c - b*c*x)^3/(a + b*x),x]
```

output
```
-4*a^2*c^3*x + (a*c^3*(a - b*x)^2)/b + (c^3*(a - b*x)^3)/(3*b) + (8*a^3*c^
3*Log[a + b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
default	$c^{3}\left(-\frac{b^{2}x^{3}}{3}+2ab\,x^{2}-7a^{2}x+\frac{8a^{3}\ln\left(bx+a\right)}{b}\right)$	41
norman	$-7a^{2}c^{3}x+2ab\,c^{3}x^{2}-\frac{b^{2}c^{3}x^{3}}{3}+\frac{8a^{3}c^{3}\ln\left(bx+a\right)}{b}$	49
risch	$-7a^{2}c^{3}x+2ab\,c^{3}x^{2}-\frac{b^{2}c^{3}x^{3}}{3}+\frac{8a^{3}c^{3}\ln\left(bx+a\right)}{b}$	49
parallelrisch	$\frac{-b^{3}c^{3}x^{3}+6a\,c^{3}b^{2}x^{2}+24a^{3}c^{3}\ln\left(bx+a\right)-21a^{2}c^{3}xb}{3b}$	54

input
```
int((-b*c*x+a*c)^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*b^2*x^3+2*a*b*x^2-7*a^2*x+8*a^3*ln(b*x+a)/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx=-\frac{b^3c^3x^3-6\,ab^2c^3x^2+21\,a^2bc^3x-24\,a^3c^3\log\left(bx+a\right)}{3\,b}$$

input
```
integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="fricas")
```

output
```
-1/3*(b^3*c^3*x^3 - 6*a*b^2*c^3*x^2 + 21*a^2*b*c^3*x - 24*a^3*c^3*log(b*x
+ a))/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx = \frac{8a^3c^3\log(a+bx)}{b} - 7a^2c^3x + 2abc^3x^2 - \frac{b^2c^3x^3}{3}$$

input
```
integrate((-b*c*x+a*c)**3/(b*x+a),x)
```

output
```
8*a**3*c**3*log(a + b*x)/b - 7*a**2*c**3*x + 2*a*b*c**3*x**2 - b**2*c**3*x
**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx = -\frac{1}{3}\,b^2c^3x^3 + 2\,abc^3x^2 - 7\,a^2c^3x + \frac{8\,a^3c^3\log(bx+a)}{b}$$

input
```
integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="maxima")
```

output
```
-1/3*b^2*c^3*x^3 + 2*a*b*c^3*x^2 - 7*a^2*c^3*x + 8*a^3*c^3*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx = \frac{8\,a^3c^3\log(|bx+a|)}{b} - \frac{b^5c^3x^3 - 6\,ab^4c^3x^2 + 21\,a^2b^3c^3x}{3\,b^3}$$

input
```
integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="giac")
```

output
```
8*a^3*c^3*log(abs(b*x + a))/b - 1/3*(b^5*c^3*x^3 - 6*a*b^4*c^3*x^2 + 21*a^
2*b^3*c^3*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx = \frac{8\,a^3\,c^3\,\ln\left(a+b\,x\right)}{b} - \frac{b^2\,c^3\,x^3}{3} - 7\,a^2\,c^3\,x + 2\,a\,b\,c^3\,x^2$$

input
```
int((a*c - b*c*x)^3/(a + b*x),x)
```

output
```
(8*a^3*c^3*log(a + b*x))/b - (b^2*c^3*x^3)/3 - 7*a^2*c^3*x + 2*a*b*c^3*x^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx = \frac{c^3(24\log(bx+a)\,a^3 - 21a^2bx + 6a\,b^2x^2 - b^3x^3)}{3b}$$

input
```
int((-b*c*x+a*c)^3/(b*x+a),x)
```

output
```
(c**3*(24*log(a + b*x)*a**3 - 21*a**2*b*x + 6*a*b**2*x**2 - b**3*x**3))/(3
*b)
```

3.22 $\int \frac{(ac-bcx)^2}{a+bx} dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{(ac-bcx)^2}{a+bx} dx = -3ac^2x + \frac{1}{2}bc^2x^2 + \frac{4a^2c^2\log(a+bx)}{b}$$

output
```
-3*a*c^2*x+1/2*b*c^2*x^2+4*a^2*c^2*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{(ac-bcx)^2}{a+bx} dx = c^2\left(-3ax + \frac{bx^2}{2} + \frac{4a^2\log(a+bx)}{b}\right)$$

input
```
Integrate[(a*c - b*c*x)^2/(a + b*x),x]
```

output
```
c^2*(-3*a*x + (b*x^2)/2 + (4*a^2*Log[a + b*x])/b)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^2}{a+bx} \, dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{4a^2c^2}{a+bx}-c(ac-bcx)-2ac^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

input
```
Int[(a*c - b*c*x)^2/(a + b*x),x]
```

output
```
-2*a*c^2*x + (c^2*(a - b*x)^2)/(2*b) + (4*a^2*c^2*Log[a + b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

method	result	size
default	$c^{2}\left(\frac{b\,x^{2}}{2}-3 a x +\frac{4 a^{2} \ln \left(b x +a \right)}{b}\right)$	30
norman	$-3 a\,c^{2} x +\frac{b\,c^{2} x^{2}}{2}+\frac{4 a^{2} c^{2} \ln \left(b x +a \right)}{b}$	35
risch	$-3 a\,c^{2} x +\frac{b\,c^{2} x^{2}}{2}+\frac{4 a^{2} c^{2} \ln \left(b x +a \right)}{b}$	35
parallelrisch	$\frac{b^{2} c^{2} x^{2}+8 a^{2} c^{2} \ln \left(b x +a \right)-6 a\,c^{2} x b}{2 b}$	39

input
```
int((-b*c*x+a*c)^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/2*b*x^2-3*a*x+4*a^2*ln(b*x+a)/b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{(ac-bcx)^2}{a+bx}\,dx=\frac{b^{2} c^{2} x^{2}-6\,abc^{2} x+8\,a^{2} c^{2} \log \left(bx+a\right)}{2\,b}$$

input
```
integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2 - 6*a*b*c^2*x + 8*a^2*c^2*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(ac-bcx)^2}{a+bx}\,dx=\frac{4a^{2}c^{2}\log \left(a+bx\right)}{b}-3ac^{2}x+\frac{bc^{2}x^{2}}{2}$$

input
```
integrate((-b*c*x+a*c)**2/(b*x+a),x)
```

output
```
4*a**2*c**2*log(a + b*x)/b - 3*a*c**2*x + b*c**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(ac-bcx)^2}{a+bx}\,dx = \frac{1}{2}\,bc^2x^2 - 3\,ac^2x + \frac{4\,a^2c^2\log\left(bx+a\right)}{b}$$

input
```
integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*b*c^2*x^2 - 3*a*c^2*x + 4*a^2*c^2*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{(ac-bcx)^2}{a+bx}\,dx = \frac{4\,a^2c^2\log\left(|bx+a|\right)}{b} + \frac{b^3c^2x^2 - 6\,ab^2c^2x}{2\,b^2}$$

input
```
integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="giac")
```

output
```
4*a^2*c^2*log(abs(b*x + a))/b + 1/2*(b^3*c^2*x^2 - 6*a*b^2*c^2*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(ac-bcx)^2}{a+bx}\,dx = \frac{c^2\left(8\,a^2\,\ln\left(a+b\,x\right) + b^2\,x^2 - 6\,a\,b\,x\right)}{2\,b}$$

input
```
int((a*c - b*c*x)^2/(a + b*x),x)
```

output
```
(c^2*(8*a^2*log(a + b*x) + b^2*x^2 - 6*a*b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(ac - bcx)^2}{a + bx}\, dx = \frac{c^2(8\log(bx + a)\, a^2 - 6abx + b^2x^2)}{2b}$$

input
```
int((-b*c*x+a*c)^2/(b*x+a),x)
```

output
```
(c**2*(8*log(a + b*x)*a**2 - 6*a*b*x + b**2*x**2))/(2*b)
```

3.23 $\int \frac{ac-bcx}{a+bx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{ac-bcx}{a+bx} \, dx = -cx + \frac{2ac\log(a+bx)}{b}$$

output
```
-c*x+2*a*c*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ac-bcx}{a+bx} \, dx = c\left(-x + \frac{2a\log(a+bx)}{b}\right)$$

input
```
Integrate[(a*c - b*c*x)/(a + b*x),x]
```

output
```
c*(-x + (2*a*Log[a + b*x])/b)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac-bcx}{a+bx}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{2ac}{a+bx}-c\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ac\log(a+bx)}{b}-cx$$

input
```
Int[(a*c - b*c*x)/(a + b*x),x]
```

output
```
-(c*x) + (2*a*c*Log[a + b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$c\left(-x+\frac{2a\ln(bx+a)}{b}\right)$	19
norman	$-cx+\frac{2ac\ln(bx+a)}{b}$	19
risch	$-cx+\frac{2ac\ln(bx+a)}{b}$	19
parallelrisch	$\frac{2ac\ln(bx+a)-bcx}{b}$	21

input
```
int((-b*c*x+a*c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
c*(-x+2*a*ln(b*x+a)/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{ac-bcx}{a+bx}\,dx=-\frac{bcx-2\,ac\log\left(bx+a\right)}{b}$$

input
```
integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="fricas")
```

output
```
-(b*c*x - 2*a*c*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{ac-bcx}{a+bx}\,dx=\frac{2ac\log\left(a+bx\right)}{b}-cx$$

input
```
integrate((-b*c*x+a*c)/(b*x+a),x)
```

output
```
2*a*c*log(a + b*x)/b - c*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ac - bcx}{a + bx} \, dx = -cx + \frac{2 \, ac \log (bx + a)}{b}$$

input
```
integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="maxima")
```

output
```
-c*x + 2*a*c*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{ac - bcx}{a + bx} \, dx = -cx + \frac{2 \, ac \log (|bx + a|)}{b}$$

input
```
integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="giac")
```

output
```
-c*x + 2*a*c*log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{ac - bcx}{a + bx} \, dx = \frac{2 \, a \, c \, \ln (a + b \, x)}{b} - c \, x$$

input
```
int((a*c - b*c*x)/(a + b*x),x)
```

output
```
(2*a*c*log(a + b*x))/b - c*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{ac - bcx}{a + bx}\,dx = \frac{c(2\log(bx + a)\,a - bx)}{b}$$

input
```
int((-b*c*x+a*c)/(b*x+a),x)
```

output
```
(c*(2*log(a + b*x)*a - b*x))/b
```

3.24 $\int \frac{1}{a+bx} dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

output
```
ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

input
```
Integrate[(a + b*x)^(-1),x]
```

output
```
Log[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx}\,dx$$
$$\Big\downarrow \; 16$$
$$\frac{\log(a+bx)}{b}$$

input
```
Int[(a + b*x)^(-1),x]
```

output
```
Log[a + b*x]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11
parallelrisch	$\frac{\ln(bx+a)}{b}$	11

input
```
int(1/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log(bx+a)}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="fricas")
```

output
```
log(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{a+bx}\,dx = \frac{\log(a+bx)}{b}$$

input
```
integrate(1/(b*x+a),x)
```

output
```
log(a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log(bx+a)}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="maxima")
```

output
```
log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+bx}\,dx = \frac{\log(|bx+a|)}{b}$$

input
```
integrate(1/(b*x+a),x, algorithm="giac")
```

output
```
log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\ln(a+b\,x)}{b}$$

input
```
int(1/(a + b*x),x)
```

output
```
log(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx}\,dx = \frac{\log(bx+a)}{b}$$

input
```
int(1/(b*x+a),x)
```

output
```
log(a + b*x)/b
```

3.25 $\int \frac{1}{(a+bx)(ac-bcx)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{(a+bx)(ac-bcx)} \, dx = \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{abc}$$

output
```
arctanh(b*x/a)/a/b/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(ac-bcx)} \, dx = \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{abc}$$

input
```
Integrate[1/((a + b*x)*(a*c - b*c*x)),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b*c)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {39, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx$$

$$\Big\downarrow \; 39$$

$$\int \frac{1}{a^2c-b^2cx^2}dx$$

$$\Big\downarrow \; 221$$

$$\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{abc}$$

input
```
Int[1/((a + b*x)*(a*c - b*c*x)),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b*c)
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

method	result	size
parallelrisch	$\frac{-\ln(bx-a)+\ln(bx+a)}{2acb}$	29
default	$\frac{-\frac{\ln(-bx+a)}{2ab}+\frac{\ln(bx+a)}{2ab}}{c}$	35
norman	$-\frac{\ln(-bx+a)}{2acb}+\frac{\ln(bx+a)}{2acb}$	37
risch	$-\frac{\ln(-bx+a)}{2acb}+\frac{\ln(bx+a)}{2acb}$	37

input
```
int(1/(b*x+a)/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-ln(b*x-a)+ln(b*x+a))/a/c/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx=\frac{\log(bx+a)-\log(bx-a)}{2\,abc}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

output
```
1/2*(log(b*x + a) - log(b*x - a))/(a*b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx=-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{abc}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c),x)
```

output
```
-(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b*c)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx = \frac{\log(bx+a)}{2\,abc} - \frac{\log(bx-a)}{2\,abc}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)/(a*b*c) - 1/2*log(b*x - a)/(a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx = \frac{\log(|bx+a|)}{2\,abc} - \frac{\log(|bx-a|)}{2\,abc}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x + a))/(a*b*c) - 1/2*log(abs(b*x - a))/(a*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b\,c}$$

input
```
int(1/((a*c - b*c*x)*(a + b*x)),x)
```

output
```
atanh((b*x)/a)/(a*b*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx = \frac{\log(-bx-a)-\log(-bx+a)}{2abc}$$

input
```
int(1/(b*x+a)/(-b*c*x+a*c),x)
```

output
```
(log( - a - b*x) - log(a - b*x))/(2*a*b*c)
```

3.26 $\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx = \frac{1}{2abc^2(a-bx)} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^2bc^2}$$

output
```
1/2/a/b/c^2/(-b*x+a)+1/2*arctanh(b*x/a)/a^2/b/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx = \frac{2a+(-a+bx)\log(a-bx)+(a-bx)\log(a+bx)}{4a^2bc^2(a-bx)}$$

input
```
Integrate[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

output
```
(2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*c^2*(a -
 b*x))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(ac-bcx)^2} \, dx$$
$$\Big\downarrow \; 54$$
$$\int \left(\frac{1}{2ac^2(a^2-b^2x^2)}+\frac{1}{2ac^2(a-bx)^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^2bc^2}+\frac{1}{2abc^2(a-bx)}$$

input
```
Int[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

output
```
1/(2*a*b*c^2*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c^2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{\frac{\ln(bx+a)}{4a^2b}-\frac{\ln(-bx+a)}{4a^2b}+\frac{1}{2ab(-bx+a)}}{c^2}$	51
norman	$\frac{1}{2ab\,c^2(-bx+a)}-\frac{\ln(-bx+a)}{4a^2c^2b}+\frac{\ln(bx+a)}{4a^2c^2b}$	56
risch	$\frac{1}{2ab\,c^2(-bx+a)}-\frac{\ln(-bx+a)}{4a^2c^2b}+\frac{\ln(bx+a)}{4a^2c^2b}$	56
parallelrisch	$\frac{-\ln(bx-a)xb+\ln(bx+a)xb+a\ln(bx-a)-\ln(bx+a)a-2a}{4a^2b\,c^2(bx-a)}$	65

input
```
int(1/(b*x+a)/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4/a^2/b*ln(b*x+a)-1/4/a^2/b*ln(-b*x+a)+1/2/a/b/(-b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx=\frac{(bx-a)\log(bx+a)-(bx-a)\log(bx-a)-2a}{4(a^2b^2c^2x-a^3bc^2)}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

output
```
1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*c^2*x
 - a^3*b*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx = -\frac{1}{-2a^2bc^2+2ab^2c^2x}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc^2}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)**2,x)
```

output
```
-1/(-2*a**2*b*c**2 + 2*a*b**2*c**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4
)/(a**2*b*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx = -\frac{1}{2\,(ab^2c^2x-a^2bc^2)}+\frac{\log\left(bx+a\right)}{4\,a^2bc^2}-\frac{\log\left(bx-a\right)}{4\,a^2bc^2}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

output
```
-1/2/(a*b^2*c^2*x - a^2*b*c^2) + 1/4*log(b*x + a)/(a^2*b*c^2) - 1/4*log(b*
x - a)/(a^2*b*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx = -\frac{1}{2\,(bcx-ac)abc}+\frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^2bc^2}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

output
```
-1/2/((b*c*x - a*c)*a*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^2*b
*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx = \frac{1}{2\,a\,b\,\left(a\,c^2-b\,c^2\,x\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c^2}$$

input
```
int(1/((a*c - b*c*x)^2*(a + b*x)),x)
```

output
```
1/(2*a*b*(a*c^2 - b*c^2*x)) + atanh((b*x)/a)/(2*a^2*b*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx = \frac{-\log(-bx+a)\,a+\log(-bx+a)\,bx+\log(bx+a)\,a-\log(bx+a)\,bx+2bx}{4a^2b\,c^2\,(-bx+a)}$$

input
```
int(1/(b*x+a)/(-b*c*x+a*c)^2,x)
```

output
```
( - log(a - b*x)*a + log(a - b*x)*b*x + log(a + b*x)*a - log(a + b*x)*b*x
+ 2*b*x)/(4*a**2*b*c**2*(a - b*x))
```

3.27 $\int \frac{1}{(a+bx)(ac-bcx)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{1}{(a+bx)(ac-bcx)^3} \, dx = \frac{1}{4abc^3(a-bx)^2} + \frac{1}{4a^2bc^3(a-bx)} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{4a^3bc^3}$$

output
```
1/4/a/b/c^3/(-b*x+a)^2+1/4/a^2/b/c^3/(-b*x+a)+1/4*arctanh(b*x/a)/a^3/b/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)(ac-bcx)^3} \, dx = \frac{2a(2a-bx) - (a-bx)^2 \log(a-bx) + (a-bx)^2 \log(a+bx)}{8a^3bc^3(a-bx)^2}$$

input
```
Integrate[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

output
```
(2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8
*a^3*b*c^3*(a - b*x)^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(ac-bcx)^3} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{4a^2c^3(a^2-b^2x^2)}+\frac{1}{4a^2c^3(a-bx)^2}+\frac{1}{2ac^3(a-bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4a^2bc^3(a-bx)}+\frac{1}{4abc^3(a-bx)^2}$$

input
```
Int[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

output
```
1/(4*a*b*c^3*(a - b*x)^2) + 1/(4*a^2*b*c^3*(a - b*x)) + ArcTanh[(b*x)/a]/(
4*a^3*b*c^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{-\frac{x}{4a^2}+\frac{1}{2ab}}{c^3(-bx+a)^2}-\frac{\ln(-bx+a)}{8a^3c^3b}+\frac{\ln(bx+a)}{8a^3c^3b}$	64
default	$\frac{\frac{\ln(bx+a)}{8a^3b}-\frac{\ln(-bx+a)}{8a^3b}+\frac{1}{4a^2b(-bx+a)}+\frac{1}{4ab(-bx+a)^2}}{c^3}$	67
norman	$\frac{\frac{3x}{4a^2c}-\frac{b\,x^2}{2a^3c}}{c^2(-bx+a)^2}-\frac{\ln(-bx+a)}{8a^3c^3b}+\frac{\ln(bx+a)}{8a^3c^3b}$	71
parallelrisch	$\frac{-\ln(bx-a)x^2b^2+\ln(bx+a)x^2b^2+2\ln(bx-a)xab-2\ln(bx+a)xab-4b^2x^2-a^2\ln(bx-a)+\ln(bx+a)a^2+6abx}{8a^3c^3(bx-a)^2b}$	111

input
```
int(1/(b*x+a)/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a^2*x+1/2/a/b)/c^3/(-b*x+a)^2-1/8/a^3/c^3/b*ln(-b*x+a)+1/8/a^3/c^3/b
*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx = -\frac{2\,abx-4\,a^2-(b^2x^2-2\,abx+a^2)\log(bx+a)+(b^2x^2-2\,abx+a^2)\log(bx-a)}{8\,(a^3b^3c^3x^2-2\,a^4b^2c^3x+a^5bc^3)}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2
- 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*c^3*x^2 - 2*a^4*b^2*c^3*x + a^5*b*
c^3)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx = -\frac{-2a+bx}{4a^4bc^3-8a^3b^2c^3x+4a^2b^3c^3x^2} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8} - \frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3bc^3}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)**3,x)
```

output
```
-(-2*a + b*x)/(4*a**4*b*c**3 - 8*a**3*b**2*c**3*x + 4*a**2*b**3*c**3*x**2)
 - (log(-a/b + x)/8 - log(a/b + x)/8)/(a**3*b*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx = -\frac{bx-2\,a}{4\,(a^2b^3c^3x^2-2\,a^3b^2c^3x+a^4bc^3)} + \frac{\log(bx+a)}{8\,a^3bc^3} - \frac{\log(bx-a)}{8\,a^3bc^3}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
-1/4*(b*x - 2*a)/(a^2*b^3*c^3*x^2 - 2*a^3*b^2*c^3*x + a^4*b*c^3) + 1/8*log
(b*x + a)/(a^3*b*c^3) - 1/8*log(b*x - a)/(a^3*b*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx = \frac{\log(|bx+a|)}{8\,a^3bc^3} - \frac{\log(|bx-a|)}{8\,a^3bc^3} - \frac{abx-2\,a^2}{4\,(bx-a)^2a^3bc^3}$$

input
```
integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
1/8*log(abs(b*x + a))/(a^3*b*c^3) - 1/8*log(abs(b*x - a))/(a^3*b*c^3) - 1/
4*(a*b*x - 2*a^2)/((b*x - a)^2*a^3*b*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^3\,b\,c^3} - \frac{\frac{x}{4\,a^2}-\frac{1}{2\,a\,b}}{a^2\,c^3-2\,a\,b\,c^3\,x+b^2\,c^3\,x^2}$$

input
```
int(1/((a*c - b*c*x)^3*(a + b*x)),x)
```

output
```
atanh((b*x)/a)/(4*a^3*b*c^3) - (x/(4*a^2) - 1/(2*a*b))/(a^2*c^3 + b^2*c^3*
x^2 - 2*a*b*c^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.84

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx = \frac{-\log(-bx+a)\,a^2+2\log(-bx+a)\,abx-\log(-bx+a)\,b^2x^2+\log(bx+a)\,a^2-2\log(bx+a)\,abx+\log(}{8a^3b\,c^3\,(b^2x^2-2abx+a^2)}$$

input
```
int(1/(b*x+a)/(-b*c*x+a*c)^3,x)
```

output
```
( - log(a - b*x)*a**2 + 2*log(a - b*x)*a*b*x - log(a - b*x)*b**2*x**2 + lo
g(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + log(a + b*x)*b**2*x**2 + 3*a**2 -
 b**2*x**2)/(8*a**3*b*c**3*(a**2 - 2*a*b*x + b**2*x**2))
```

3.28 $\int \frac{(ac-bcx)^3}{(a+bx)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{(ac-bcx)^3}{(a+bx)^2} dx = 5ac^3x - \frac{1}{2}bc^3x^2 - \frac{8a^3c^3}{b(a+bx)} - \frac{12a^2c^3\log(a+bx)}{b}$$

output
```
5*a*c^3*x-1/2*b*c^3*x^2-8*a^3*c^3/b/(b*x+a)-12*a^2*c^3*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ac-bcx)^3}{(a+bx)^2} dx = c^3\left(5ax - \frac{bx^2}{2} - \frac{8a^3}{b(a+bx)} - \frac{12a^2\log(a+bx)}{b}\right)$$

input
```
Integrate[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

output
```
c^3*(5*a*x - (b*x^2)/2 - (8*a^3)/(b*(a + b*x)) - (12*a^2*Log[a + b*x])/b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{8a^3c^3}{(a+bx)^2}-\frac{12a^2c^3}{a+bx}+5ac^3-bc^3x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

input
```
Int[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

output
```
5*a*c^3*x - (b*c^3*x^2)/2 - (8*a^3*c^3)/(b*(a + b*x)) - (12*a^2*c^3*Log[a
+ b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$c^{3}\left(-\frac{b\,x^{2}}{2}+5ax-\frac{8a^{3}}{b(bx+a)}-\frac{12a^{2}\ln(bx+a)}{b}\right)$	45
risch	$5a\,c^{3}x-\frac{b\,c^{3}x^{2}}{2}-\frac{8a^{3}c^{3}}{b(bx+a)}-\frac{12a^{2}c^{3}\ln(bx+a)}{b}$	53
norman	$\frac{13a^{2}c^{3}x-\frac{1}{2}b^{2}c^{3}x^{3}+\frac{9}{2}ab\,c^{3}x^{2}}{bx+a}-\frac{12a^{2}c^{3}\ln(bx+a)}{b}$	58
parallelrisch	$-\frac{x^{3}a\,b^{3}c^{3}+24\ln(bx+a)x\,a^{3}b\,c^{3}-9x^{2}a^{2}b^{2}c^{3}+24\ln(bx+a)a^{4}c^{3}-26x\,a^{3}b\,c^{3}}{2a(bx+a)b}$	82

input
```
int((-b*c*x+a*c)^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/2*b*x^2+5*a*x-8/b*a^3/(b*x+a)-12*a^2*ln(b*x+a)/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx = -\frac{b^3c^3x^3-9\,ab^2c^3x^2-10\,a^2bc^3x+16\,a^3c^3+24\,(a^2bc^3x+a^3c^3)\log(bx+a)}{2\,(b^2x+ab)}$$

input
```
integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^3*c^3*x^3 - 9*a*b^2*c^3*x^2 - 10*a^2*b*c^3*x + 16*a^3*c^3 + 24*(a^
2*b*c^3*x + a^3*c^3)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx = -\frac{8a^3c^3}{ab+b^2x} - \frac{12a^2c^3\log(a+bx)}{b} + 5ac^3x - \frac{bc^3x^2}{2}$$

input
```
integrate((-b*c*x+a*c)**3/(b*x+a)**2,x)
```

output
```
-8*a**3*c**3/(a*b + b**2*x) - 12*a**2*c**3*log(a + b*x)/b + 5*a*c**3*x - b
*c**3*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx = -\frac{1}{2}\,bc^3x^2 - \frac{8\,a^3c^3}{b^2x+ab} + 5\,ac^3x - \frac{12\,a^2c^3\log(bx+a)}{b}$$

input
```
integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*b*c^3*x^2 - 8*a^3*c^3/(b^2*x + a*b) + 5*a*c^3*x - 12*a^2*c^3*log(b*x
+ a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx = \frac{12\,a^2c^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{8\,a^3c^3}{(bx+a)b} + \frac{\left(\frac{12\,ac^3}{bx+a} - c^3\right)(bx+a)^2}{2\,b}$$

input
```
integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="giac")
```

output
```
12*a^2*c^3*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - 8*a^3*c^3/((b*x + a)
*b) + 1/2*(12*a*c^3/(b*x + a) - c^3)*(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx = 5\,a\,c^3\,x-\frac{b\,c^3\,x^2}{2}-\frac{12\,a^2\,c^3\,\ln(a+b\,x)}{b}-\frac{8\,a^3\,c^3}{b\,(a+b\,x)}$$

input
```
int((a*c - b*c*x)^3/(a + b*x)^2,x)
```

output
```
5*a*c^3*x - (b*c^3*x^2)/2 - (12*a^2*c^3*log(a + b*x))/b - (8*a^3*c^3)/(b*(
a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx = \frac{c^3(-24\log(bx+a)\,a^3-24\log(bx+a)\,a^2bx+26a^2bx+9a\,b^2x^2-b^3x^3)}{2b\,(bx+a)}$$

input
```
int((-b*c*x+a*c)^3/(b*x+a)^2,x)
```

output
```
(c**3*( - 24*log(a + b*x)*a**3 - 24*log(a + b*x)*a**2*b*x + 26*a**2*b*x +
9*a*b**2*x**2 - b**3*x**3))/(2*b*(a + b*x))
```

3.29 $\int \frac{(ac-bcx)^2}{(a+bx)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{(ac-bcx)^2}{(a+bx)^2} dx = c^2x - \frac{4a^2c^2}{b(a+bx)} - \frac{4ac^2\log(a+bx)}{b}$$

output
```
c^2*x-4*a^2*c^2/b/(b*x+a)-4*a*c^2*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(ac-bcx)^2}{(a+bx)^2} dx = c^2\left(x - \frac{4a^2}{b(a+bx)} - \frac{4a\log(a+bx)}{b}\right)$$

input
```
Integrate[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

output
```
c^2*(x - (4*a^2)/(b*(a + b*x)) - (4*a*Log[a + b*x])/b)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^2}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{4a^2c^2}{(a+bx)^2}-\frac{4ac^2}{a+bx}+c^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

input
```
Int[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

output
```
c^2*x - (4*a^2*c^2)/(b*(a + b*x)) - (4*a*c^2*Log[a + b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$c^{2}\left(x-\frac{4a^{2}}{b(bx+a)}-\frac{4a\ln(bx+a)}{b}\right)$	34
risch	$c^{2}x-\frac{4a^{2}c^{2}}{b(bx+a)}-\frac{4a\,c^{2}\ln(bx+a)}{b}$	40
norman	$\frac{b\,c^{2}x^{2}+5a\,c^{2}x}{bx+a}-\frac{4a\,c^{2}\ln(bx+a)}{b}$	41
parallelrisch	$-\frac{4\ln(bx+a)xab\,c^{2}-b^{2}c^{2}x^{2}+4a^{2}c^{2}\ln(bx+a)+5a^{2}c^{2}}{(bx+a)b}$	61

input
```
int((-b*c*x+a*c)^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(x-4/b*a^2/(b*x+a)-4*a*ln(b*x+a)/b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

$$\int\frac{(ac-bcx)^{2}}{(a+bx)^{2}}\,dx=\frac{b^{2}c^{2}x^{2}+abc^{2}x-4\,a^{2}c^{2}-4\left(abc^{2}x+a^{2}c^{2}\right)\log\left(bx+a\right)}{b^{2}x+ab}$$

input
```
integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*c^2*x^2 + a*b*c^2*x - 4*a^2*c^2 - 4*(a*b*c^2*x + a^2*c^2)*log(b*x + a
))/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx = -\frac{4a^2c^2}{ab+b^2x} - \frac{4ac^2\log{(a+bx)}}{b} + c^2x$$

input
```
integrate((-b*c*x+a*c)**2/(b*x+a)**2,x)
```

output
```
-4*a**2*c**2/(a*b + b**2*x) - 4*a*c**2*log(a + b*x)/b + c**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx = -\frac{4\,a^2c^2}{b^2x+ab} + c^2x - \frac{4\,ac^2\log{(bx+a)}}{b}$$

input
```
integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-4*a^2*c^2/(b^2*x + a*b) + c^2*x - 4*a*c^2*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.51

$$\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx = \frac{4\,ac^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} + \frac{(bx+a)c^2}{b} - \frac{4\,a^2c^2}{(bx+a)b}$$

input
```
integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
4*a*c^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b + (b*x + a)*c^2/b - 4*a^2
*c^2/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx = c^2\,x - \frac{4\,a\,c^2\,\ln\left(a+b\,x\right)}{b} - \frac{4\,a^2\,c^2}{b\,\left(a+b\,x\right)}$$

input
```
int((a*c - b*c*x)^2/(a + b*x)^2,x)
```

output
```
c^2*x - (4*a*c^2*log(a + b*x))/b - (4*a^2*c^2)/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx = \frac{c^2(-4\log(bx+a)\,a^2-4\log(bx+a)\,abx+5abx+b^2x^2)}{b\,(bx+a)}$$

input
```
int((-b*c*x+a*c)^2/(b*x+a)^2,x)
```

output
```
(c**2*( - 4*log(a + b*x)*a**2 - 4*log(a + b*x)*a*b*x + 5*a*b*x + b**2*x**2
))/(b*(a + b*x))
```

3.30 $\int \frac{ac-bcx}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{ac-bcx}{(a+bx)^2}\,dx=-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

output
```
-2*a*c/b/(b*x+a)-c*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{ac-bcx}{(a+bx)^2}\,dx=-\frac{c\left(\frac{2a}{a+bx}+\log(a+bx)\right)}{b}$$

input
```
Integrate[(a*c - b*c*x)/(a + b*x)^2,x]
```

output
```
-((c*((2*a)/(a + b*x) + Log[a + b*x]))/b)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac-bcx}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{2ac}{(a+bx)^2}-\frac{c}{a+bx}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

input
```
Int[(a*c - b*c*x)/(a + b*x)^2,x]
```

output
```
(-2*a*c)/(b*(a + b*x)) - (c*Log[a + b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
norman	$\frac{2cx}{bx+a}-\frac{c\ln(bx+a)}{b}$	25
default	$c\left(-\frac{2a}{b(bx+a)}-\frac{\ln(bx+a)}{b}\right)$	28
risch	$-\frac{2ac}{b(bx+a)}-\frac{c\ln(bx+a)}{b}$	28
parallelrisch	$-\frac{\ln(bx+a)xbc+ac\ln(bx+a)+2ac}{(bx+a)b}$	37

input
```
int((-b*c*x+a*c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*c*x/(b*x+a)-c*ln(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{ac-bcx}{(a+bx)^2}\,dx = -\frac{2\,ac+(bcx+ac)\log(bx+a)}{b^2x+ab}$$

input
```
integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*a*c + (b*c*x + a*c)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{ac-bcx}{(a+bx)^2}\,dx = -\frac{2ac}{ab+b^2x}-\frac{c\log(a+bx)}{b}$$

input
```
integrate((-b*c*x+a*c)/(b*x+a)**2,x)
```

output
```
-2*a*c/(a*b + b**2*x) - c*log(a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{ac - bcx}{(a + bx)^2} \, dx = -\frac{2\,ac}{b^2x + ab} - \frac{c\log{(bx + a)}}{b}$$

input
```
integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*a*c/(b^2*x + a*b) - c*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{ac - bcx}{(a + bx)^2} \, dx = c\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx + a)b}\right) - \frac{ac}{(bx + a)b}$$

input
```
integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
c*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b)) - a*c/((b*x
 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{ac-bcx}{(a+bx)^2}\,dx=-\frac{c\ln(a+b\,x)}{b}-\frac{2\,a\,c}{b\,(a+b\,x)}$$

input
```
int((a*c - b*c*x)/(a + b*x)^2,x)
```

output
```
- (c*log(a + b*x))/b - (2*a*c)/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \frac{ac-bcx}{(a+bx)^2}\,dx=\frac{c(-\log(bx+a)\,a-\log(bx+a)\,bx+2bx)}{b\,(bx+a)}$$

input
```
int((-b*c*x+a*c)/(b*x+a)^2,x)
```

output
```
(c*( - log(a + b*x)*a - log(a + b*x)*b*x + 2*b*x))/(b*(a + b*x))
```

3.31 $\int \frac{1}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)}$$

output
```
-1/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)}$$

input
```
Integrate[(a + b*x)^(-2),x]
```

output
```
-(1/(b*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{b(a+bx)}$$

input
```
Int[(a + b*x)^(-2),x]
```

output
```
-(1/(b*(a + b*x)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13
orering	$-\frac{1}{b(bx+a)}$	13

input
```
int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b^2x+ab}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{ab+b^2x}$$

input
```
integrate(1/(b*x+a)**2,x)
```

output
```
-1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/((b*x + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{(bx+a)b}$$

input
```
integrate(1/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b\,(a+b\,x)}$$

input
```
int(1/(a + b*x)^2,x)
```

output
```
-1/(b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2}\,dx = \frac{x}{a\,(bx+a)}$$

input
```
int(1/(b*x+a)^2,x)
```

output
```
x/(a*(a + b*x))
```

3.32 $\int \frac{1}{(a+bx)^2(ac-bcx)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{1}{(a+bx)^2(ac-bcx)} \, dx = -\frac{1}{2abc(a+bx)} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^2bc}$$

output
```
-1/2/a/b/c/(b*x+a)+1/2*arctanh(b*x/a)/a^2/b/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx)^2(ac-bcx)} \, dx = \frac{-2a-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)}{4a^2bc(a+bx)}$$

input
```
Integrate[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

output
```
(-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*c*(a + b
*x))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2(ac-bcx)}dx$$
$$\Big\downarrow \; 54$$
$$\int \left(\frac{1}{2ac\left(a^2-b^2x^2\right)}+\frac{1}{2ac(a+bx)^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

input
```
Int[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

output
```
-1/2*1/(a*b*c*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\frac{\ln(bx+a)}{4a^2b}-\frac{1}{2ab(bx+a)}-\frac{\ln(-bx+a)}{4a^2b}}{c}$	50
norman	$-\frac{1}{2abc(bx+a)}-\frac{\ln(-bx+a)}{4a^2bc}+\frac{\ln(bx+a)}{4a^2bc}$	55
risch	$-\frac{1}{2abc(bx+a)}-\frac{\ln(-bx+a)}{4a^2bc}+\frac{\ln(bx+a)}{4a^2bc}$	55
parallelrisch	$\frac{-\ln(bx-a)xb+\ln(bx+a)xb-a\ln(bx-a)+\ln(bx+a)a-2a}{4a^2bc(bx+a)}$	63

input
```
int(1/(b*x+a)^2/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/4/a^2/b*ln(b*x+a)-1/2/a/b/(b*x+a)-1/4/a^2/b*ln(-b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx = \frac{(bx+a)\log(bx+a)-(bx+a)\log(bx-a)-2\,a}{4\,(a^2b^2cx+a^3bc)}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

output
```
1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*c*x +
 a^3*b*c)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx = -\frac{1}{2a^2bc+2ab^2cx} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4} - \frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc}$$

input
```
integrate(1/(b*x+a)**2/(-b*c*x+a*c),x)
```

output
```
-1/(2*a**2*b*c + 2*a*b**2*c*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*
b*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx = -\frac{1}{2\,(ab^2cx+a^2bc)} + \frac{\log(bx+a)}{4\,a^2bc} - \frac{\log(bx-a)}{4\,a^2bc}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

output
```
-1/2/(a*b^2*c*x + a^2*b*c) + 1/4*log(b*x + a)/(a^2*b*c) - 1/4*log(b*x - a)
/(a^2*b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx = -\frac{\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{4\,a^2bc} - \frac{1}{2\,(bx+a)abc}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
```

output
```
-1/4*log(abs(-2*a/(b*x + a) + 1))/(a^2*b*c) - 1/2/((b*x + a)*a*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c} - \frac{1}{2\,a\,b\,\left(a\,c+b\,c\,x\right)}$$

input
```
int(1/((a*c - b*c*x)*(a + b*x)^2),x)
```

output
```
atanh((b*x)/a)/(2*a^2*b*c) - 1/(2*a*b*(a*c + b*c*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx = \frac{-\log(-bx+a)\,a-\log(-bx+a)\,bx+\log(bx+a)\,a+\log(bx+a)\,bx+2bx}{4a^2bc\,(bx+a)}$$

input
```
int(1/(b*x+a)^2/(-b*c*x+a*c),x)
```

output
```
( - log(a - b*x)*a - log(a - b*x)*b*x + log(a + b*x)*a + log(a + b*x)*b*x
+ 2*b*x)/(4*a**2*b*c*(a + b*x))
```

3.33 $\int \frac{1}{(a+bx)^2(ac-bcx)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2} \, dx = \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

output
```
1/2*x/a^2/c^2/(-b^2*x^2+a^2)+1/2*arctanh(b*x/a)/a^3/b/c^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2} \, dx = \frac{2abx+\left(-a^2+b^2x^2\right)\log(a-bx)+\left(a^2-b^2x^2\right)\log(a+bx)}{4a^3bc^2(a-bx)(a+bx)}$$

input
```
Integrate[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

output
```
(2*a*b*x + (-a^2 + b^2*x^2)*Log[a - b*x] + (a^2 - b^2*x^2)*Log[a + b*x])/(
4*a^3*b*c^2*(a - b*x)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {39, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}dx$$

$$\Big\downarrow \ 39$$

$$\int \frac{1}{\left(a^2c-b^2cx^2\right)^2}dx$$

$$\Big\downarrow \ 215$$

$$\frac{\int \frac{1}{a^2c-b^2cx^2}dx}{2a^2c}+\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}$$

$$\Big\downarrow \ 221$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3bc^2}+\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}$$

input
```
Int[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

output
```
x/(2*a^2*c^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b*c^2)
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

method	result	size
norman	$\frac{x}{2a^2c^2(bx+a)(-bx+a)}-\frac{\ln(-bx+a)}{4a^3b\,c^2}+\frac{\ln(bx+a)}{4a^3b\,c^2}$	61
risch	$\frac{x}{2a^2c^2(bx+a)(-bx+a)}-\frac{\ln(-bx+a)}{4a^3b\,c^2}+\frac{\ln(bx+a)}{4a^3b\,c^2}$	61
default	$\frac{\frac{\ln(bx+a)}{4a^3b}-\frac{1}{4a^2b(bx+a)}-\frac{\ln(-bx+a)}{4a^3b}+\frac{1}{4a^2b(-bx+a)}}{c^2}$	66
parallelrisch	$\frac{-\ln(bx-a)x^2b^3+\ln(bx+a)x^2b^3+\ln(bx-a)a^2b-\ln(bx+a)a^2b-2a\,b^2x}{4a^3b^2c^2(bx+a)(bx-a)}$	90

input
```
int(1/(b*x+a)^2/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/c^2*x/(b*x+a)/(-b*x+a)-1/4/a^3/b/c^2*ln(-b*x+a)+1/4/a^3/b/c^2*ln(b
*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = -\frac{2\,abx-(b^2x^2-a^2)\log(bx+a)+(b^2x^2-a^2)\log(bx-a)}{4\,(a^3b^3c^2x^2-a^5bc^2)}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a*b*x - (b^2*x^2 - a^2)*log(b*x + a) + (b^2*x^2 - a^2)*log(b*x - a
))/(a^3*b^3*c^2*x^2 - a^5*b*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = -\frac{x}{-2a^4c^2+2a^2b^2c^2x^2} + \frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^3bc^2}$$

input
```
integrate(1/(b*x+a)**2/(-b*c*x+a*c)**2,x)
```

output
```
-x/(-2*a**4*c**2 + 2*a**2*b**2*c**2*x**2) + (-log(-a/b + x)/4 + log(a/b +
x)/4)/(a**3*b*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = -\frac{x}{2\left(a^2b^2c^2x^2-a^4c^2\right)} + \frac{\log\left(bx+a\right)}{4\,a^3bc^2} - \frac{\log\left(bx-a\right)}{4\,a^3bc^2}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(a^2*b^2*c^2*x^2 - a^4*c^2) + 1/4*log(b*x + a)/(a^3*b*c^2) - 1/4*lo
g(b*x - a)/(a^3*b*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = -\frac{1}{4\,(bcx-ac)a^2bc} + \frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^3bc^2} + \frac{1}{8\,a^3b\left(\frac{2\,ac}{bcx-ac}+1\right)c^2}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

output
```
-1/4/((b*c*x - a*c)*a^2*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^3
*b*c^2) + 1/8/(a^3*b*(2*a*c/(b*c*x - a*c) + 1)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = \frac{x}{2\,a^2\,\left(a^2\,c^2-b^2\,c^2\,x^2\right)} + \frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^3\,b\,c^2}$$

input
```
int(1/((a*c - b*c*x)^2*(a + b*x)^2),x)
```

output
```
x/(2*a^2*(a^2*c^2 - b^2*c^2*x^2)) + atanh((b*x)/a)/(2*a^3*b*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.89

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = \frac{\log(-bx-a)\,a^2-\log(-bx-a)\,b^2x^2-\log(-bx+a)\,a^2+\log(-bx+a)\,b^2x^2+2abx}{4a^3b\,c^2\left(-b^2x^2+a^2\right)}$$

input
```
int(1/(b*x+a)^2/(-b*c*x+a*c)^2,x)
```

output
```
(log( - a - b*x)*a**2 - log( - a - b*x)*b**2*x**2 - log(a - b*x)*a**2 + lo
g(a - b*x)*b**2*x**2 + 2*a*b*x)/(4*a**3*b*c**2*(a**2 - b**2*x**2))
```

3.34 $\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)} - \frac{1}{8a^3bc^3(a+bx)}+\frac{3\operatorname{arctanh}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

output
```
1/8/a^2/b/c^3/(-b*x+a)^2+1/4/a^3/b/c^3/(-b*x+a)-1/8/a^3/b/c^3/(b*x+a)+3/8*
arctanh(b*x/a)/a^4/b/c^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = \frac{\frac{2a(2a^2+3abx-3b^2x^2)}{(a-bx)^2(a+bx)}-3\log(a-bx)+3\log(a+bx)}{16a^4bc^3}$$

input
```
Integrate[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

output
```
((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b
*x] + 3*Log[a + b*x])/(16*a^4*b*c^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3} \, dx$$
$$\Big\downarrow \ 54$$
$$\int \left(\frac{1}{4a^3c^3(a-bx)^2}+\frac{1}{8a^3c^3(a+bx)^2}+\frac{1}{4a^2c^3(a-bx)^3}+\frac{3}{8a^3c^3\left(a^2-b^2x^2\right)}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{3\text{arctanh}\left(\frac{bx}{a}\right)}{8a^4bc^3}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}$$

input
```
Int[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

output
```
1/(8*a^2*b*c^3*(a - b*x)^2) + 1/(4*a^3*b*c^3*(a - b*x)) - 1/(8*a^3*b*c^3*(
a + b*x)) + (3*ArcTanh[(b*x)/a])/(8*a^4*b*c^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

method	result
risch	$\frac{-\frac{3b\,x^{2}}{8a^{3}}+\frac{3x}{8a^{2}}+\frac{1}{4ab}}{(bx+a)c^{3}(-bx+a)^{2}}-\frac{3\ln(-bx+a)}{16a^{4}c^{3}b}+\frac{3\ln(bx+a)}{16a^{4}c^{3}b}$
default	$\frac{\frac{3\ln(bx+a)}{16a^{4}b}-\frac{1}{8a^{3}b(bx+a)}-\frac{3\ln(-bx+a)}{16a^{4}b}+\frac{1}{4a^{3}b(-bx+a)}+\frac{1}{8a^{2}b(-bx+a)^{2}}}{c^{3}}$
norman	$\frac{\frac{1}{4acb}+\frac{3x}{8a^{2}c}-\frac{3b\,x^{2}}{8a^{3}c}}{(bx+a)c^{2}(-bx+a)^{2}}-\frac{3\ln(-bx+a)}{16a^{4}c^{3}b}+\frac{3\ln(bx+a)}{16a^{4}c^{3}b}$
parallelrisch	$\frac{-3\ln(bx-a)x^{3}b^{5}+3\ln(bx+a)x^{3}b^{5}+3\ln(bx-a)x^{2}a\,b^{4}-3\ln(bx+a)x^{2}a\,b^{4}+3\ln(bx-a)x\,a^{2}b^{3}-3\ln(bx+a)x\,a^{2}b^{3}-6a\,b^{4}x^{2}-3\,l}{16a^{4}b^{3}c^{3}(bx+a)(bx-a)^{2}}$

input
```
int(1/(b*x+a)^2/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/8/a^3*b*x^2+3/8/a^2*x+1/4/a/b)/(b*x+a)/c^3/(-b*x+a)^2-3/16/a^4/c^3/b*l
n(-b*x+a)+3/16/a^4/c^3/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = -\frac{6\,ab^2x^2-6\,a^2bx-4\,a^3-3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log(bx+a)+3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log}{16\,(a^4b^4c^3x^3-a^5b^3c^3x^2-a^6b^2c^3x+a^7bc^3)}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x
+ a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a)
)/(a^4*b^4*c^3*x^3 - a^5*b^3*c^3*x^2 - a^6*b^2*c^3*x + a^7*b*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = -\frac{-2a^2-3abx+3b^2x^2}{8a^6bc^3-8a^5b^2c^3x-8a^4b^3c^3x^2+8a^3b^4c^3x^3} - \frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16}-\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4bc^3}$$

input
```
integrate(1/(b*x+a)**2/(-b*c*x+a*c)**3,x)
```

output
```
-(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b*c**3 - 8*a**5*b**2*c**3*x - 8
*a**4*b**3*c**3*x**2 + 8*a**3*b**4*c**3*x**3) - (3*log(-a/b + x)/16 - 3*lo
g(a/b + x)/16)/(a**4*b*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = -\frac{3\,b^2x^2-3\,abx-2\,a^2}{8\,(a^3b^4c^3x^3-a^4b^3c^3x^2-a^5b^2c^3x+a^6bc^3)} + \frac{3\,\log(bx+a)}{16\,a^4bc^3} - \frac{3\,\log(bx-a)}{16\,a^4bc^3}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*c^3*x^3 - a^4*b^3*c^3*x^2 - a^
5*b^2*c^3*x + a^6*b*c^3) + 3/16*log(b*x + a)/(a^4*b*c^3) - 3/16*log(b*x -
a)/(a^4*b*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = -\frac{3\,\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{16\,a^4bc^3} - \frac{1}{8\,(bx+a)a^3bc^3} + \frac{\frac{12\,a}{bx+a}-5}{32\,a^4bc^3\left(\frac{2\,a}{bx+a}-1\right)^2}$$

input
```
integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
-3/16*log(abs(-2*a/(b*x + a) + 1))/(a^4*b*c^3) - 1/8/((b*x + a)*a^3*b*c^3)
 + 1/32*(12*a/(b*x + a) - 5)/(a^4*b*c^3*(2*a/(b*x + a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = \frac{\frac{3\,x}{8\,a^2}+\frac{1}{4\,a\,b}-\frac{3\,b\,x^2}{8\,a^3}}{a^3\,c^3-a^2\,b\,c^3\,x-a\,b^2\,c^3\,x^2+b^3\,c^3\,x^3} + \frac{3\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{8\,a^4\,b\,c^3}$$

input
```
int(1/((a*c - b*c*x)^3*(a + b*x)^2),x)
```

output
```
((3*x)/(8*a^2) + 1/(4*a*b) - (3*b*x^2)/(8*a^3))/(a^3*c^3 + b^3*c^3*x^3 - a
*b^2*c^3*x^2 - a^2*b*c^3*x) + (3*atanh((b*x)/a))/(8*a^4*b*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$$
$$= \frac{-3\log(-bx+a)\,a^3+3\log(-bx+a)\,a^2bx+3\log(-bx+a)\,a\,b^2x^2-3\log(-bx+a)\,b^3x^3+3\log(bx+a}{16a^4b\,c^3\,(b^3x^3-a\,b^2x^2-}$$

input
```
int(1/(b*x+a)^2/(-b*c*x+a*c)^3,x)
```

output
```
( - 3*log(a - b*x)*a**3 + 3*log(a - b*x)*a**2*b*x + 3*log(a - b*x)*a*b**2*
x**2 - 3*log(a - b*x)*b**3*x**3 + 3*log(a + b*x)*a**3 - 3*log(a + b*x)*a**
2*b*x - 3*log(a + b*x)*a*b**2*x**2 + 3*log(a + b*x)*b**3*x**3 - 2*a**3 + 1
2*a**2*b*x - 6*b**3*x**3)/(16*a**4*b*c**3*(a**3 - a**2*b*x - a*b**2*x**2 +
 b**3*x**3))
```

3.35 $\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \frac{5}{16}a^2c^2x\sqrt{a+ax}\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{5}{8}a^{5/2}c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)$$

output
```
5/16*a^2*c^2*x*(a*x+a)^(1/2)*(-c*x+c)^(1/2)+5/24*a*c*x*(a*x+a)^(3/2)*(-c*x
+c)^(3/2)+1/6*x*(a*x+a)^(5/2)*(-c*x+c)^(5/2)+5/8*a^(5/2)*c^(5/2)*arctan(c^
(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \frac{c^{3/2}(a(1+x))^{5/2}\sqrt{c-cx}\left(\sqrt{c}x\sqrt{1+x}(-33+33x+26x^2-26x^3-8x^4+8x^5)+30\sqrt{c-cx}\right.}{48(-1+x)(1+x)^{5/2}}$$

input
```
Integrate[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

output
```
(c^(3/2)*(a*(1 + x))^(5/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-33 + 33*
x + 26*x^2 - 26*x^3 - 8*x^4 + 8*x^5) + 30*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*
x]/(Sqrt[2]*Sqrt[c])]))/(48*(-1 + x)*(1 + x)^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {40, 40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax+a)^{5/2}(c-cx)^{5/2}\,dx$$

$$\Big\downarrow \; 40$$

$$\frac{5}{6}ac\int (xa+a)^{3/2}(c-cx)^{3/2}dx+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

$$\Big\downarrow \; 40$$

$$\frac{5}{6}ac\left(\frac{3}{4}ac\int \sqrt{xa+a}\sqrt{c-cx}dx+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}\right)+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

$$\Big\downarrow \; 40$$

$$\frac{5}{6}ac\left(\frac{3}{4}ac\left(\frac{1}{2}ac\int \frac{1}{\sqrt{xa+a}\sqrt{c-cx}}dx+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}\right)+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}\right)+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

$$\Big\downarrow \; 45$$

$$\frac{5}{6}ac\left(\frac{3}{4}ac\left(ac\int \frac{1}{a+\frac{c(xa+a)}{c-cx}}d\frac{\sqrt{xa+a}}{\sqrt{c-cx}}+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}\right)+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}\right)+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

$$\Big\downarrow \; 218$$

$$\frac{5}{6}ac\left(\frac{3}{4}ac\left(\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}\right)+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}\right)+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

input
```
Int[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

output
```
(x*(a + a*x)^(5/2)*(c - c*x)^(5/2))/6 + (5*a*c*((x*(a + a*x)^(3/2)*(c - c*
x)^(3/2))/4 + (3*a*c*((x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*
ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])]))/4))/6
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{x(8x^4-26x^2+33)(1+x)(-1+x)a^3c^3}{48\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}+\frac{5\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)a^3c^3\sqrt{-a(1+x)c(-1+x)}}{16\sqrt{ac}\,\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$
default	$-\frac{(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{7}{2}}}{6c}+\frac{5a\left(-\frac{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{7}{2}}}{5c}+\frac{3a\left(-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{7}{2}}}{4c}+\frac{a\left(\frac{(-cx+c)^{\frac{5}{2}}\sqrt{ax+a}}{3a}+\frac{5c\left(\frac{(-cx+c)^{\frac{3}{2}}\sqrt{ax+a}}{2a}+\frac{3c\left(\frac{\sqrt{-\cdots}}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{5}\cdots\right)}{6}\cdots$

input
```
int((a*x+a)^(5/2)*(-c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*x*(8*x^4-26*x^2+33)*(1+x)*(-1+x)*a^3*c^3/(a*(1+x))^(1/2)/(-c*(-1+x))
^(1/2)+5/16/(a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))*a^3*c^3
*(-a*(1+x)*c*(-1+x))^(1/2)/(a*(1+x))^(1/2)/(-c*(-1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.60

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \left[\frac{5}{32}\sqrt{-ac}a^2c^2\log\left(2\,acx^2+2\,\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x-ac\right)+\frac{1}{48}\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\right.\right.$$

input
```
integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[5/32*sqrt(-a*c)*a^2*c^2*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-
c*x + c)*x - a*c) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2*x^3 + 33*a^2*c^2*x)*s
qrt(a*x + a)*sqrt(-c*x + c), -5/16*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt
(a*x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a*c)) + 1/48*(8*a^2*c^2*x^5 - 26*a^2
*c^2*x^3 + 33*a^2*c^2*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F]

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \int (a(x+1))^{\frac{5}{2}}\,(-c(x-1))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*x+a)**(5/2)*(-c*x+c)**(5/2),x)
```

output
```
Integral((a*(x + 1))**(5/2)*(-c*(x - 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \frac{5\,a^3c^3\arcsin{(x)}}{16\,\sqrt{ac}} + \frac{5}{16}\,\sqrt{-acx^2+ac}a^2c^2x + \frac{5}{24}\,\left(-acx^2+ac\right)^{\frac{3}{2}}acx + \frac{1}{6}\,\left(-acx^2+ac\right)^{\frac{5}{2}}x$$

input
```
integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
5/16*a^3*c^3*arcsin(x)/sqrt(a*c) + 5/16*sqrt(-a*c*x^2 + a*c)*a^2*c^2*x + 5
/24*(-a*c*x^2 + a*c)^(3/2)*a*c*x + 1/6*(-a*c*x^2 + a*c)^(5/2)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(94) = 188.

Time = 0.35 (sec) , antiderivative size = 678, normalized size of antiderivative = 5.38

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/240*(150*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c +
 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((2*((a*x + a)*(4*
(a*x + a)*(5*(a*x + a)/a^5 - 31/a^4) + 321/a^3) - 451/a^2)*(a*x + a) + 745
/a)*(a*x + a) - 405)*sqrt(a*x + a))*c^2*abs(a) - 1/12*(18*a^2*c*log(abs(-s
qrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sq
rt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x + a)*(3*(a*x + a)/a^3 - 13
/a^2) + 43/a) - 39)*sqrt(a*x + a))*c^2*abs(a) - (2*a^2*c*log(abs(-sqrt(-a*
c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*
x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*c^2*abs(a) + 1/2*(2*a^3*c*log(abs(-sq
rt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqr
t(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c^2*abs(a)/a + 1/3*
(6*a^4*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*((2*a*x - 5*a)*(a*x + a) +
 9*a^2)*sqrt(a*x + a))*c^2*abs(a)/a^2 - 1/120*(90*a^6*c*log(abs(-sqrt(-a*c
)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - (195*a^4 -
 (295*a^3 - 2*(3*(4*a*x - 17*a)*(a*x + a) + 133*a^2)*(a*x + a))*(a*x + a))
*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*c^2*abs(a)/a^4
```

Mupad [F(-1)]

Timed out.

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx=\int (a+a\,x)^{5/2}\,(c-c\,x)^{5/2}\,dx$$

input
```
int((a + a*x)^(5/2)*(c - c*x)^(5/2),x)
```

output
```
int((a + a*x)^(5/2)*(c - c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^2c^2\left(-30asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+8\sqrt{x+1}\,\sqrt{1-x}\,x^5-26\sqrt{x+1}\,\sqrt{1-x}\,x^3+33\sqrt{x+1}\,\sqrt{1-\right.}{48}$$

input
```
int((a*x+a)^(5/2)*(-c*x+c)^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*c**2*( - 30*asin(sqrt( - x + 1)/sqrt(2)) + 8*sqrt(x
+ 1)*sqrt( - x + 1)*x**5 - 26*sqrt(x + 1)*sqrt( - x + 1)*x**3 + 33*sqrt(x
+ 1)*sqrt( - x + 1)*x))/48
```

3.36 $\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \frac{3}{8}acx\sqrt{a+ax}\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{3}{4}a^{3/2}c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)$$

output
```
3/8*a*c*x*(a*x+a)^(1/2)*(-c*x+c)^(1/2)+1/4*x*(a*x+a)^(3/2)*(-c*x+c)^(3/2)+
3/4*a^(3/2)*c^(3/2)*arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \frac{\sqrt{c}(a(1+x))^{3/2}\sqrt{c-cx}\left(\sqrt{c}x\sqrt{1+x}(-5+5x+2x^2-2x^3)+6\sqrt{c-cx}\arcsin\left(\frac{\sqrt{c-cx}}{\sqrt{2}\sqrt{c}}\right)\right)}{8(-1+x)(1+x)^{3/2}}$$

input
```
Integrate[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

output
```
(Sqrt[c]*(a*(1 + x))^(3/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-5 + 5*x
+ 2*x^2 - 2*x^3) + 6*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*x]/(Sqrt[2]*Sqrt[c])]
))/(8*(-1 + x)*(1 + x)^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax+a)^{3/2}(c-cx)^{3/2}\,dx$$

$$\Big\downarrow \text{40}$$

$$\frac{3}{4}ac\int \sqrt{xa+a}\sqrt{c-cx}dx+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

$$\Big\downarrow \text{40}$$

$$\frac{3}{4}ac\left(\frac{1}{2}ac\int \frac{1}{\sqrt{xa+a}\sqrt{c-cx}}dx+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}\right)+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

$$\Big\downarrow \text{45}$$

$$\frac{3}{4}ac\left(ac\int \frac{1}{a+\frac{c(xa+a)}{c-cx}}d\frac{\sqrt{xa+a}}{\sqrt{c-cx}}+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}\right)+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3}{4}ac\left(\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}\right)+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

input
```
Int[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

output
```
(x*(a + a*x)^(3/2)*(c - c*x)^(3/2))/4 + (3*a*c*((x*Sqrt[a + a*x]*Sqrt[c -
c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c -
c*x])]))/4
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

method	result
risch	$\frac{x(2x^2-5)(1+x)(-1+x)a^2c^2}{8\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}+\frac{3\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)a^2c^2\sqrt{-a(1+x)c(-1+x)}}{8\sqrt{ac}\,\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$
default	$-\frac{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{5}{2}}}{4c}+\frac{3a\left(-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{5}{2}}}{3c}+\frac{a\left(\frac{(-cx+c)^{\frac{3}{2}}\sqrt{ax+a}}{2a}+\frac{3c\left(\frac{\sqrt{-cx+c}\,\sqrt{ax+a}}{a}+\frac{c\sqrt{(-cx+c)(ax+a)}\,\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+\cdots}}\right)}{\sqrt{-cx+c}\,\sqrt{ax+a}\,\sqrt{ac}}\right)}{2}\right)}{3}\right)}{4}$

input
```
int((a*x+a)^(3/2)*(-c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*x^2-5)*(1+x)*(-1+x)*a^2*c^2/(a*(1+x))^(1/2)/(-c*(-1+x))^(1/2)+3/8
/(a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))*a^2*c^2*(-a*(1+x)*
c*(-1+x))^(1/2)/(a*(1+x))^(1/2)/(-c*(-1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.61

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \left[\frac{3}{16}\sqrt{-ac}ac\log\left(2\,acx^2+2\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x-ac\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\sqrt{-\right.$$

input
```
integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[3/16*sqrt(-a*c)*a*c*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x
+ c)*x - a*c) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c), -3
/8*sqrt(a*c)*a*c*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*x^2
- a*c)) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F]

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \int (a(x+1))^{\frac{3}{2}}\,(-c(x-1))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*x+a)**(3/2)*(-c*x+c)**(3/2),x)
```

output
```
Integral((a*(x + 1))**(3/2)*(-c*(x - 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.52

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \frac{3\,a^2c^2\arcsin{(x)}}{8\,\sqrt{ac}} + \frac{3}{8}\,\sqrt{-acx^2+ac}acx + \frac{1}{4}\,\left(-acx^2+ac\right)^{\frac{3}{2}}x$$

input
```
integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
3/8*a^2*c^2*arcsin(x)/sqrt(a*c) + 3/8*sqrt(-a*c*x^2 + a*c)*a*c*x + 1/4*(-a
*c*x^2 + a*c)^(3/2)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.16

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx =$$

$$-\frac{\left(\frac{18\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} + \sqrt{-(ax+a)ac+2\,a^2c}\left((ax+a)\left(2\,(ax+a)\left(\frac{3\,(ax+a)}{a^3}-\frac{13}{a^2}\right)+\right.\right.\right.}{24\,a}$$

$$-\frac{\left(\frac{2\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} - \sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\right)c|a|}{a}$$

$$+\frac{\left(\frac{2\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} + \sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}(ax-2\,a)\right)c|a|}{2\,a^2}$$

$$+\frac{\left(\frac{6\,a^4c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} - \sqrt{-(ax+a)ac+2\,a^2c}((2\,ax-5\,a)(ax+a)+9\,a^2)\sqrt{ax+a}\right)}{6\,a^3}$$

input
```
integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="giac")
```

output
```
-1/24*(18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c +
2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x
+ a)*(3*(a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c*abs(a)/a -
(2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*c*abs(a)/a
+ 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2
*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x -
 2*a))*c*abs(a)/a^2 + 1/6*(6*a^4*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqr
t(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*
((2*a*x - 5*a)*(a*x + a) + 9*a^2)*sqrt(a*x + a))*c*abs(a)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx=\int (a+a\,x)^{3/2}\,(c-c\,x)^{3/2}\,dx$$

input
```
int((a + a*x)^(3/2)*(c - c*x)^(3/2),x)
```

output
```
int((a + a*x)^(3/2)*(c - c*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,ac\left(-6asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-2\sqrt{x+1}\,\sqrt{1-x}\,x^3+5\sqrt{x+1}\,\sqrt{1-x}\,x\right)}{8}$$

input
```
int((a*x+a)^(3/2)*(-c*x+c)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*c*( - 6*asin(sqrt( - x + 1)/sqrt(2)) - 2*sqrt(x + 1)*sq
rt( - x + 1)*x**3 + 5*sqrt(x + 1)*sqrt( - x + 1)*x))/8
```

3.37 $\int \sqrt{a+ax}\sqrt{c-cx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx=\frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)$$

output
```
1/2*x*(a*x+a)^(1/2)*(-c*x+c)^(1/2)+a^(1/2)*c^(1/2)*arctan(c^(1/2)*(a*x+a)^
(1/2)/a^(1/2)/(-c*x+c)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx=\frac{\sqrt{a(1+x)}\left(x\sqrt{1+x}\sqrt{c-cx}-2\sqrt{c}\arcsin\left(\frac{\sqrt{c-cx}}{\sqrt{2}\sqrt{c}}\right)\right)}{2\sqrt{1+x}}$$

input
```
Integrate[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

output
```
(Sqrt[a*(1 + x)]*(x*Sqrt[1 + x]*Sqrt[c - c*x] - 2*Sqrt[c]*ArcSin[Sqrt[c -
c*x]/(Sqrt[2]*Sqrt[c])]))/(2*Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax+a}\sqrt{c-cx}\,dx$$

$$\Big\downarrow \text{40}$$

$$\frac{1}{2}ac\int \frac{1}{\sqrt{xa+a}\sqrt{c-cx}}dx+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}$$

$$\Big\downarrow \text{45}$$

$$ac\int \frac{1}{a+\frac{c(xa+a)}{c-cx}}d\frac{\sqrt{xa+a}}{\sqrt{c-cx}}+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}$$

$$\Big\downarrow \text{218}$$

$$\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}$$

input
```
Int[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

output
```
(x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + a*x])/(Sqrt[a]*Sqrt[c - c*x])]
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.27

method	result	size
risch	$-\frac{x(1+x)(-1+x)ac}{2\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}+\frac{\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^{2}+ac}}\right)ac\sqrt{-a(1+x)c(-1+x)}}{2\sqrt{ac}\,\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$	85
default	$-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{3}{2}}}{2c}+\frac{a\left(\frac{\sqrt{-cx+c}\,\sqrt{ax+a}}{a}+\frac{c\sqrt{(-cx+c)(ax+a)}\,\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^{2}+ac}}\right)}{\sqrt{-cx+c}\,\sqrt{ax+a}\,\sqrt{ac}}\right)}{2}$	102

input
```
int((a*x+a)^(1/2)*(-c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(1+x)*(-1+x)*a*c/(a*(1+x))^(1/2)/(-c*(-1+x))^(1/2)+1/2/(a*c)^(1/2)*
arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))*a*c*(-a*(1+x)*c*(-1+x))^(1/2)/(
a*(1+x))^(1/2)/(-c*(-1+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.90

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = \left[\frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}x+\frac{1}{4}\sqrt{-ac}\log\left(2\,acx^{2}+2\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x-ac\right),\frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}x-\frac{1}{2}\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{ax+a}\sqrt{-cx+c}x}{acx^{2}-ac}\right)\right]$$

input
```
integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a*x + a)*sqrt(-c*x + c)*x + 1/4*sqrt(-a*c)*log(2*a*c*x^2 + 2*sqr
t(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x - a*c), 1/2*sqrt(a*x + a)*sqrt(-c*x
 + c)*x - 1/2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a
*c*x^2 - a*c))]
```

Sympy [F]

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = \int \sqrt{a\,(x+1)}\sqrt{-c\,(x-1)}\,dx$$

input
```
integrate((a*x+a)**(1/2)*(-c*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(x + 1))*sqrt(-c*(x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.42

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = \frac{ac\arcsin{(x)}}{2\sqrt{ac}} + \frac{1}{2}\sqrt{-acx^2+acx}$$

input
```
integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*a*c*arcsin(x)/sqrt(a*c) + 1/2*sqrt(-a*c*x^2 + a*c)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(49) = 98$.

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.58

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = -\frac{\left(\frac{2\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} - \sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\right)|a|}{a^2} + \frac{\left(\frac{2\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} + \sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}(ax-2\,a)\right)|a|}{2\,a^3}$$

input
```
integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-(2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*
c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*abs(a)/a^2
 + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c +
2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x
- 2*a))*abs(a)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = \frac{x\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}{2} - \frac{\sqrt{a}\,\sqrt{-c}\,\ln\left(\sqrt{-c}\,\sqrt{a\,\left(x+1\right)}\,\sqrt{-c\,\left(x-1\right)}-\sqrt{a}\,c\,x\right)}{2}$$

input
```
int((a + a*x)^(1/2)*(c - c*x)^(1/2),x)
```

output
```
(x*(a + a*x)^(1/2)*(c - c*x)^(1/2))/2 - (a^(1/2)*(-c)^(1/2)*log((-c)^(1/2)
*(a*(x + 1))^(1/2)*(-c*(x - 1))^(1/2) - a^(1/2)*c*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(-2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\sqrt{x+1}\,\sqrt{1-x}\,x\right)}{2}$$

input
```
int((a*x+a)^(1/2)*(-c*x+c)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*( - 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( -
x + 1)*x))/2
```

3.38 $\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = \frac{2\arctan\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

output
```
2*arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))/a^(1/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = \frac{2\sqrt{1+x}\arctan\left(\frac{\sqrt{c}\sqrt{1+x}}{\sqrt{c-cx}}\right)}{\sqrt{c}\sqrt{a(1+x)}}$$

input
```
Integrate[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

output
```
(2*Sqrt[1 + x]*ArcTan[(Sqrt[c]*Sqrt[1 + x])/Sqrt[c - c*x]])/(Sqrt[c]*Sqrt[
a*(1 + x)])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+a}\sqrt{c-cx}}dx$$

$$\Big\downarrow \text{45}$$

$$2\int \frac{1}{a+\frac{c(xa+a)}{c-cx}}d\frac{\sqrt{xa+a}}{\sqrt{c-cx}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Int[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

output
```
(2*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/(Sqrt[a]*Sqrt[
c])
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{\sqrt{(-cx+c)(ax+a)}\,\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)}{\sqrt{ax+a}\,\sqrt{-cx+c}\,\sqrt{ac}}$	57

input
```
int(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-c*x+c)*(a*x+a))^(1/2)/(a*x+a)^(1/2)/(-c*x+c)^(1/2)/(a*c)^(1/2)*arctan((
a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = \left[-\frac{\sqrt{-ac}\log\left(2\,acx^2-2\,\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}x-ac\right)}{2\,ac},\right.$$
$$\left.-\frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{ax+a}\sqrt{-cx+c}x}{acx^2-ac}\right)}{ac}\right]$$

input
```
integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*c)*log(2*a*c*x^2 - 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)
*x - a*c)/(a*c), -sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*
x/(a*c*x^2 - a*c))/(a*c)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = -\frac{iG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}}$$

input
```
integrate(1/(a*x+a)**(1/2)/(-c*x+c)**(1/2),x)
```

output
```
-I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
 x**(-2))/(4*pi**(3/2)*sqrt(a)*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/x**2)/(4*pi
**(3/2)*sqrt(a)*sqrt(c))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = \frac{\arcsin(x)}{\sqrt{ac}}$$

input
```
integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(x)/sqrt(a*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = -\frac{2\,a\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}|a|}$$

input
```
integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="giac")
```

output
```
-2*a*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/
(sqrt(-a*c)*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{c-c\,x}-\sqrt{c}\right)}{\sqrt{a\,c}\,\left(\sqrt{a+a\,x}-\sqrt{a}\right)}\right)}{\sqrt{a\,c}}$$

input
```
int(1/((a + a*x)^(1/2)*(c - c*x)^(1/2)),x)
```

output
```
-(4*atan((a*((c - c*x)^(1/2) - c^(1/2)))/((a*c)^(1/2)*((a + a*x)^(1/2) - a
^(1/2)))))/(a*c)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = -\frac{2\sqrt{c}\,\sqrt{a}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{ac}$$

input
```
int(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*asin(sqrt( - x + 1)/sqrt(2)))/(a*c)
```

3.39 $\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x}{ac\sqrt{a+ax}\sqrt{c-cx}}$$

output
```
x/a/c/(a*x+a)^(1/2)/(-c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x(1+x)}{c(a(1+x))^{3/2}\sqrt{c-cx}}$$

input
```
Integrate[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

output
```
(x*(1 + x))/(c*(a*(1 + x))^(3/2)*Sqrt[c - c*x])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+a)^{3/2}(c-cx)^{3/2}}\,dx$$
$$\Big\downarrow \; 41$$
$$\frac{x}{ac\sqrt{ax+a}\sqrt{c-cx}}$$

input
```
Int[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

output
```
x/(a*c*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x}{ac\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$	24
gosper	$-\frac{(1+x)(-1+x)x}{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{3}{2}}}$	25
orering	$-\frac{(1+x)(-1+x)x}{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{3}{2}}}$	25
default	$-\frac{1}{ac\sqrt{ax+a}\,\sqrt{-cx+c}}+\frac{\sqrt{ax+a}}{c\,a^{2}\sqrt{-cx+c}}$	47

input
```
int(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/c/(a*(1+x))^(1/2)/(-c*(-1+x))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx=-\frac{\sqrt{ax+a}\sqrt{-cx+c}x}{a^2c^2x^2-a^2c^2}$$

input
```
integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(a*x + a)*sqrt(-c*x + c)*x/(a^2*c^2*x^2 - a^2*c^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = -\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix} \middle| \frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

input
```
integrate(1/(a*x+a)**(3/2)/(-c*x+c)**(3/2),x)
```

output
```
-I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
x**(-2))/(2*pi**(3/2)*a**(3/2)*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/2, 3/
4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/x**2)/(2*pi*
*(3/2)*a**(3/2)*c**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x}{\sqrt{-acx^2+ac}ac}$$

input
```
integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="maxima")
```

output
```
x/(sqrt(-a*c*x^2 + a*c)*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(23) = 46$.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = -\frac{2\sqrt{-ac}a}{\left(2a^2c - \left(\sqrt{-ac}\sqrt{ax+a} - \sqrt{-(ax+a)ac+2a^2c}\right)^2\right)c|a|} - \frac{\sqrt{-(ax+a)ac+2a^2c}\sqrt{ax+a}}{2\left((ax+a)ac-2a^2c\right)c|a|}$$

input
```
integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="giac")
```

output
```
-2*sqrt(-a*c)*a/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*
c + 2*a^2*c))^2)*c*abs(a)) - 1/2*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x +
 a)/(((a*x + a)*a*c - 2*a^2*c)*c*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x}{a\,c\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}$$

input
```
int(1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x)
```

output
```
x/(a*c*(a + a*x)^(1/2)*(c - c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,x}{\sqrt{x+1}\,\sqrt{1-x}\,a^2c^2}$$

input
```
int(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*x)/(sqrt(x + 1)*sqrt( - x + 1)*a**2*c**2)
```

3.40 $\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx = \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}} + \frac{2x}{3a^2c^2\sqrt{a+ax}\sqrt{c-cx}}$$

output
```
1/3*x/a/c/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+2/3*x/a^2/c^2/(a*x+a)^(1/2)/(-c*x+c
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx = \frac{x(1+x)\left(-3+2x^2\right)}{3c^2(-1+x)(a(1+x))^{5/2}\sqrt{c-cx}}$$

input
```
Integrate[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

output
```
(x*(1 + x)*(-3 + 2*x^2))/(3*c^2*(-1 + x)*(a*(1 + x))^(5/2)*Sqrt[c - c*x])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+a)^{5/2}(c-cx)^{5/2}}dx$$

$$\Big\downarrow \ 42$$

$$\frac{2\int \frac{1}{(xa+a)^{3/2}(c-cx)^{3/2}}dx}{3ac}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

$$\Big\downarrow \ 41$$

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

input
```
Int[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

output
```
x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]
*Sqrt[c - c*x])
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{(1+x)(-1+x)x(2x^2-3)}{3(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{5}{2}}}$	32
orering	$\frac{(1+x)(-1+x)x(2x^2-3)}{3(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{5}{2}}}$	32
default	$-\frac{1}{3ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{3}{2}}}+\frac{-\frac{1}{ac\sqrt{ax+a}\,(-cx+c)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{ax+a}}{3ac(-cx+c)^{\frac{3}{2}}}+\frac{2\sqrt{ax+a}}{3a\,c^2\sqrt{-cx+c}}}{a}}{a}$	105

input
```
int(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1+x)*(-1+x)*x*(2*x^2-3)/(a*x+a)^(5/2)/(-c*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx=-\frac{(2\,x^3-3\,x)\sqrt{ax+a}\sqrt{-cx+c}}{3\,(a^3c^3x^4-2\,a^3c^3x^2+a^3c^3)}$$

input
```
integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*x^3 - 3*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^3*c^3*x^4 - 2*a^3*c^3*x
^2 + a^3*c^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx = \frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3 \\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0 \end{matrix}\middle|\,\frac{1}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0 \end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

input
```
integrate(1/(a*x+a)**(5/2)/(-c*x+c)**(5/2),x)
```

output
```
I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), x
**(-2))/(3*pi**(3/2)*a**(5/2)*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4, 5/4
, 1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), exp_polar(-2*I*pi)/x**2)/(3*pi**
(3/2)*a**(5/2)*c**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx = \frac{x}{3\,(-acx^2+ac)^{\frac{3}{2}}ac} + \frac{2\,x}{3\,\sqrt{-acx^2+ac}a^2c^2}$$

input
```
integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x/((-a*c*x^2 + a*c)^(3/2)*a*c) + 2/3*x/(sqrt(-a*c*x^2 + a*c)*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(49) = 98$.

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.89

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx = -\frac{\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\left(\frac{4\,(ax+a)|a|}{a^2c}-\frac{9\,|a|}{ac}\right)}{12\,\left((ax+a)ac-2\,a^2c\right)^2} - \frac{16\,\sqrt{-ac}a^4c^2-18\,\sqrt{-ac}\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^2c+3\,\sqrt{-ac}\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{\cdots}\right.}{3\left(2\,a^2c-\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2\right)^3c^2|a|}$$

input
```
integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="giac")
```

output
```
-1/12*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(4*(a*x + a)*abs(a)/(a^
2*c) - 9*abs(a)/(a*c))/((a*x + a)*a*c - 2*a^2*c)^2 - 1/3*(16*sqrt(-a*c)*a^
4*c^2 - 18*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*
a^2*c))^2*a^2*c + 3*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)
*a*c + 2*a^2*c))^4)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a
)*a*c + 2*a^2*c))^2)^3*c^2*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx = -\frac{3\,x\,\sqrt{c-c\,x}-2\,x^3\,\sqrt{c-c\,x}}{\sqrt{a+a\,x}\,\left(c-c\,x\right)^2\,\left(3\,a^2\,\left(c-c\,x\right)-6\,a^2\,c\right)}$$

input
```
int(1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x)
```

output
```
-(3*x*(c - c*x)^(1/2) - 2*x^3*(c - c*x)^(1/2))/((a + a*x)^(1/2)*(c - c*x)^
2*(3*a^2*(c - c*x) - 6*a^2*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,x(2x^2-3)}{3\sqrt{x+1}\,\sqrt{1-x}\,a^3c^3\,(x^2-1)}$$

input
```
int(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*x*(2*x**2 - 3))/(3*sqrt(x + 1)*sqrt( - x + 1)*a**3*c**3*(
x**2 - 1))
```

3.41 $\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}} \, dx = \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}} + \frac{8x}{15a^3c^3\sqrt{a+ax}\sqrt{c-cx}}$$

output
```
1/5*x/a/c/(a*x+a)^(5/2)/(-c*x+c)^(5/2)+4/15*x/a^2/c^2/(a*x+a)^(3/2)/(-c*x+
c)^(3/2)+8/15*x/a^3/c^3/(a*x+a)^(1/2)/(-c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}} \, dx = \frac{x(15-20x^2+8x^4)}{15a^3c^3\sqrt{a(1+x)}\sqrt{c-cx}\,(-1+x^2)^2}$$

input
```
Integrate[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

output
```
(x*(15 - 20*x^2 + 8*x^4))/(15*a^3*c^3*Sqrt[a*(1 + x)]*Sqrt[c - c*x]*(-1 +
x^2)^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {42, 42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+a)^{7/2}(c-cx)^{7/2}}dx$$

$$\Big\downarrow \text{42}$$

$$\frac{4\int \frac{1}{(xa+a)^{5/2}(c-cx)^{5/2}}dx}{5ac}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

$$\Big\downarrow \text{42}$$

$$\frac{4\left(\frac{2\int \frac{1}{(xa+a)^{3/2}(c-cx)^{3/2}}dx}{3ac}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}\right)}{5ac}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

$$\Big\downarrow \text{41}$$

$$\frac{4\left(\frac{2x}{3a^2c^2\sqrt{ax+a}\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}\right)}{5ac}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

input
```
Int[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

output
```
x/(5*a*c*(a + a*x)^(5/2)*(c - c*x)^(5/2)) + (4*(x/(3*a*c*(a + a*x)^(3/2)*(
c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]*Sqrt[c - c*x])))/(5*a*c)
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.41

method	result
gosper	$-\frac{(1+x)(-1+x)x(8x^4-20x^2+15)}{15(ax+a)^{\frac{7}{2}}(-cx+c)^{\frac{7}{2}}}$
orering	$-\frac{(1+x)(-1+x)x(8x^4-20x^2+15)}{15(ax+a)^{\frac{7}{2}}(-cx+c)^{\frac{7}{2}}}$
default	$-\frac{1}{5ac(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{5}{2}}}+\frac{-\frac{1}{3ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{5}{2}}}+\frac{-\frac{4}{3ac\sqrt{ax+a}\,(-cx+c)^{\frac{5}{2}}}+\frac{4\left(\frac{3\sqrt{ax+a}}{5ac(-cx+c)^{\frac{5}{2}}}+\frac{3\left(\frac{2\sqrt{ax+a}}{15ac(-cx+c)^{\frac{3}{2}}}+\frac{2\sqrt{ax+a}}{15a\,c^{2}\sqrt{-cx+c \ldots}}\ldots\right.}{c}\ldots\right.}{3a}\ldots}{a}\ldots}{a}$

input
```
int(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(1+x)*(-1+x)*x*(8*x^4-20*x^2+15)/(a*x+a)^(7/2)/(-c*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx = -\frac{(8\,x^5-20\,x^3+15\,x)\sqrt{ax+a}\sqrt{-cx+c}}{15\,(a^4c^4x^6-3\,a^4c^4x^4+3\,a^4c^4x^2-a^4c^4)}$$

input
```
integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(8*x^5 - 20*x^3 + 15*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^4*c^4*x^6 -
3*a^4*c^4*x^4 + 3*a^4*c^4*x^2 - a^4*c^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx = -\frac{2iG_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4 \\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0 \end{matrix}\middle|\,\frac{1}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}} + \frac{2G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0 \end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}$$

input
```
integrate(1/(a*x+a)**(7/2)/(-c*x+c)**(7/2),x)
```

output
```
-2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,))
, x**(-2))/(15*pi**(3/2)*a**(7/2)*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/2, 5/
4, 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), exp_polar(-2*I*pi)/x**2)/(
15*pi**(3/2)*a**(7/2)*c**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx = \frac{x}{5\left(-acx^2+ac\right)^{\frac{5}{2}}ac} + \frac{4\,x}{15\left(-acx^2+ac\right)^{\frac{3}{2}}a^2c^2} + \frac{8\,x}{15\sqrt{-acx^2+ac}a^3c^3}$$

input
```
integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/5*x/((-a*c*x^2 + a*c)^(5/2)*a*c) + 4/15*x/((-a*c*x^2 + a*c)^(3/2)*a^2*c^
2) + 8/15*x/(sqrt(-a*c*x^2 + a*c)*a^3*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(73) = 146.

Time = 0.19 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.66

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx = -\frac{\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\left((ax+a)\left(\frac{64\,(ax+a)}{c|a|}-\frac{275\,a}{c|a|}\right)+\frac{300\,a^2}{c|a|}\right)}{240\left((ax+a)ac-2\,a^2c\right)^3} + \frac{1024\,a^8c^4-2200\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^6c^3+1660\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a}\right.}{60\left(2\,a^2c-\left(\sqrt{-ac}\sqrt{ax-}\right.\right.}$$

input
```
integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="giac")
```

output
```
-1/240*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(64*(a*x +
a)/(c*abs(a)) - 275*a/(c*abs(a))) + 300*a^2/(c*abs(a)))/((a*x + a)*a*c - 2
*a^2*c)^3 + 1/60*(1024*a^8*c^4 - 2200*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a
*x + a)*a*c + 2*a^2*c))^2*a^6*c^3 + 1660*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(
-(a*x + a)*a*c + 2*a^2*c))^4*a^4*c^2 - 450*(sqrt(-a*c)*sqrt(a*x + a) - sqr
t(-(a*x + a)*a*c + 2*a^2*c))^6*a^2*c + 45*(sqrt(-a*c)*sqrt(a*x + a) - sqrt
(-(a*x + a)*a*c + 2*a^2*c))^8)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqr
t(-(a*x + a)*a*c + 2*a^2*c))^2)^5*sqrt(-a*c)*c^2*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx = \frac{x\,(8\,x^4-20\,x^2+15)}{15\,a^3\,\sqrt{a+a\,x}\,(c-c\,x)^{5/2}\,\left(c+3\,c\,x-x\,\left(c-c\,x\right)\right)}$$

input
```
int(1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x)
```

output
```
(x*(8*x^4 - 20*x^2 + 15))/(15*a^3*(a + a*x)^(1/2)*(c - c*x)^(5/2)*(c + 3*c
*x - x*(c - c*x)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,x(8x^4-20x^2+15)}{15\sqrt{x+1}\,\sqrt{1-x}\,a^4c^4\,(x^4-2x^2+1)}$$

input
```
int(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*x*(8*x**4 - 20*x**2 + 15))/(15*sqrt(x + 1)*sqrt( - x + 1)
*a**4*c**4*(x**4 - 2*x**2 + 1))
```

3.42 $\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx)^{3/2}} + \frac{16x}{35a^4c^4\sqrt{a+ax}\sqrt{c-cx}}$$

output
```
1/7*x/a/c/(a*x+a)^(7/2)/(-c*x+c)^(7/2)+6/35*x/a^2/c^2/(a*x+a)^(5/2)/(-c*x+
c)^(5/2)+8/35*x/a^3/c^3/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+16/35*x/a^4/c^4/(a*x+
a)^(1/2)/(-c*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{x(-35+70x^2-56x^4+16x^6)}{35a^4c^4\sqrt{a(1+x)}\sqrt{c-cx}\,(-1+x^2)^3}$$

input
```
Integrate[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

output
```
(x*(-35 + 70*x^2 - 56*x^4 + 16*x^6))/(35*a^4*c^4*Sqrt[a*(1 + x)]*Sqrt[c -
c*x]*(-1 + x^2)^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {42, 42, 42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+a)^{9/2}(c-cx)^{9/2}}dx$$

$$\Big\downarrow \text{42}$$

$$\frac{6\int \frac{1}{(xa+a)^{7/2}(c-cx)^{7/2}}dx}{7ac}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

$$\Big\downarrow \text{42}$$

$$\frac{6\left(\frac{4\int \frac{1}{(xa+a)^{5/2}(c-cx)^{5/2}}dx}{5ac}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}\right)}{7ac}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

$$\Big\downarrow \text{42}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{(xa+a)^{3/2}(c-cx)^{3/2}}dx}{3ac}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}\right)}{5ac}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}\right)}{7ac}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

$$\Big\downarrow \text{41}$$

$$\frac{6\left(\frac{4\left(\frac{2x}{3a^2c^2\sqrt{ax+a}\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}\right)}{5ac}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}\right)}{7ac}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

input
```
Int[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

output
```
x/(7*a*c*(a + a*x)^(7/2)*(c - c*x)^(7/2)) + (6*(x/(5*a*c*(a + a*x)^(5/2)*(
c - c*x)^(5/2)) + (4*(x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3
*a^2*c^2*Sqrt[a + a*x]*Sqrt[c - c*x])))/(5*a*c)))/(7*a*c)
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result
gosper	$\frac{(1+x)(-1+x)x(16x^{6}-56x^{4}+70x^{2}-35)}{35(ax+a)^{\frac{9}{2}}(-cx+c)^{\frac{9}{2}}}$
orering	$\frac{(1+x)(-1+x)x(16x^{6}-56x^{4}+70x^{2}-35)}{35(ax+a)^{\frac{9}{2}}(-cx+c)^{\frac{9}{2}}}$
default	$-\frac{1}{7ac(ax+a)^{\frac{7}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{-\frac{1}{5ac(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{-\frac{2}{5ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{6\left(-\frac{5}{3ac\sqrt{ax+a}\,(-cx+c)^{\frac{7}{2}}}+\frac{5\left(\frac{4\sqrt{ax+a}}{7ac(-cx+c)^{\frac{7}{2}}}+\frac{4\left(-\ldots\right)}{\ldots}\right)}{\ldots}\right)}{\ldots}}{a}}{a}$

input
```
int(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(1+x)*(-1+x)*x*(16*x^6-56*x^4+70*x^2-35)/(a*x+a)^(9/2)/(-c*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = -\frac{(16\,x^{7}-56\,x^{5}+70\,x^{3}-35\,x)\sqrt{ax+a}\sqrt{-cx+c}}{35\,(a^{5}c^{5}x^{8}-4\,a^{5}c^{5}x^{6}+6\,a^{5}c^{5}x^{4}-4\,a^{5}c^{5}x^{2}+a^{5}c^{5})}$$

input
```
integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(16*x^7 - 56*x^5 + 70*x^3 - 35*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^5*
c^5*x^8 - 4*a^5*c^5*x^6 + 6*a^5*c^5*x^4 - 4*a^5*c^5*x^2 + a^5*c^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 139.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{4iG_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 & \frac{1}{2}, \frac{9}{2}, 5 \\ \frac{9}{4}, \frac{11}{4}, 4, \frac{9}{2}, 5 & 0 \end{matrix} \middle| \frac{1}{x^2}\right)}{105\pi^{\frac{3}{2}}a^{\frac{9}{2}}c^{\frac{9}{2}}} + \frac{4G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{7}{4}, \frac{9}{4}, 1 & \\ \frac{7}{4}, \frac{9}{4} & -\frac{1}{2}, 0, 4, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right)}{105\pi^{\frac{3}{2}}a^{\frac{9}{2}}c^{\frac{9}{2}}}$$

input
```
integrate(1/(a*x+a)**(9/2)/(-c*x+c)**(9/2),x)
```

output
```
4*I*meijerg(((9/4, 11/4, 1), (1/2, 9/2, 5)), ((9/4, 11/4, 4, 9/2, 5), (0,)
), x**(-2))/(105*pi**(3/2)*a**(9/2)*c**(9/2)) + 4*meijerg(((-1/2, 0, 1/2,
7/4, 9/4, 1), ()), ((7/4, 9/4), (-1/2, 0, 4, 0)), exp_polar(-2*I*pi)/x**2)
/(105*pi**(3/2)*a**(9/2)*c**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{x}{7\,(-acx^2+ac)^{\frac{7}{2}}ac} + \frac{6\,x}{35\,(-acx^2+ac)^{\frac{5}{2}}a^2c^2} + \frac{8\,x}{35\,(-acx^2+ac)^{\frac{3}{2}}a^3c^3} + \frac{16\,x}{35\,\sqrt{-acx^2+ac}a^4c^4}$$

input
```
integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/7*x/((-a*c*x^2 + a*c)^(7/2)*a*c) + 6/35*x/((-a*c*x^2 + a*c)^(5/2)*a^2*c^
2) + 8/35*x/((-a*c*x^2 + a*c)^(3/2)*a^3*c^3) + 16/35*x/(sqrt(-a*c*x^2 + a*
c)*a^4*c^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(97) = 194$.

Time = 0.31 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.61

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = -\frac{\sqrt{-(ax+a)ac+2\,a^2c}\left((ax+a)\left((ax+a)\left(\frac{256\,(ax+a)|a|}{a^2c}-\frac{1617\,|a|}{ac}\right)+\frac{3430\,|a|}{c}\right)-\frac{2450\,a|a|}{c}\right)\sqrt{ax+a}}{1120\,\left((ax+a)ac-2\,a^2c\right)^4} + \frac{16384\,a^{12}c^6-51744\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^{10}c^5+66416\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(a\right.\right.}{}$$

input
```
integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="giac")
```

output
```
-1/1120*sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*((a*x + a)*(256*(a*x + a
)*abs(a)/(a^2*c) - 1617*abs(a)/(a*c)) + 3430*abs(a)/c) - 2450*a*abs(a)/c)*
sqrt(a*x + a)/((a*x + a)*a*c - 2*a^2*c)^4 + 1/280*(16384*a^12*c^6 - 51744*
(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2*a^10*c^5 + 6
6416*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^4*a^8*c^4
 - 43120*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^6*a^6
*c^3 + 14280*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^8
*a^4*c^2 - 2450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c)
)^10*a^2*c + 175*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c
))^12)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2
*c))^2)^7*sqrt(-a*c)*a*c^3*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = -\frac{x\,(16\,x^6-56\,x^4+70\,x^2-35)}{35\,a^4\,\sqrt{a+a\,x}\,{(c-c\,x)}^{7/2}\,\left(c-x^2\,(c-c\,x)+7\,c\,x-4\,x\,(c-c\,x)\right)}$$

input
```
int(1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x)
```

output
```
-(x*(70*x^2 - 56*x^4 + 16*x^6 - 35))/(35*a^4*(a + a*x)^(1/2)*(c - c*x)^(7/
2)*(c - x^2*(c - c*x) + 7*c*x - 4*x*(c - c*x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,x(16x^6-56x^4+70x^2-35)}{35\sqrt{x+1}\,\sqrt{1-x}\,a^5c^5\,(x^6-3x^4+3x^2-1)}$$

input
```
int(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x)
```

output
```
(sqrt(c)*sqrt(a)*x*(16*x**6 - 56*x**4 + 70*x**2 - 35))/(35*sqrt(x + 1)*sqr
t( - x + 1)*a**5*c**5*(x**6 - 3*x**4 + 3*x**2 - 1))
```

3.43 $\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2} + \frac{5a^6c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{8b}$$

output
```
5/16*a^4*c^2*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)+5/24*a^2*c*x*(b*x+a)^(3/2)
*(-b*c*x+a*c)^(3/2)+1/6*x*(b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2)+5/8*a^6*c^(5/2)
*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.76

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \frac{(c(a-bx))^{5/2}\left(bx\sqrt{a-bx}\sqrt{a+bx}(33a^4-26a^2b^2x^2+8b^4x^4)+30a^6\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{48b(a-bx)^{5/2}}$$

input
```
Integrate[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

output
```
((c*(a - b*x))^(5/2)*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(33*a^4 - 26*a^2*b^2
*x^2 + 8*b^4*x^4) + 30*a^6*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(48*b*(a
- b*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {40, 40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx$$

$$\Big\downarrow \text{40}$$

$$\frac{5}{6}a^2c\int (a+bx)^{3/2}(ac-bcx)^{3/2}dx+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

$$\Big\downarrow \text{40}$$

$$\frac{5}{6}a^2c\left(\frac{3}{4}a^2c\int \sqrt{a+bx}\sqrt{ac-bcx}dx+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}\right)+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

$$\Big\downarrow \text{40}$$

$$\frac{5}{6}a^2c\left(\frac{3}{4}a^2c\left(\frac{1}{2}a^2c\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}\right)+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

$$\Big\downarrow \text{45}$$

$$\frac{5}{6}a^2c\left(\frac{3}{4}a^2c\left(a^2c\int \frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}\right)+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5}{6}a^2c\left(\frac{3}{4}a^2c\left(\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}\right)+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

input
```
Int[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

output
```
(x*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2))/6 + (5*a^2*c*((x*(a + b*x)^(3/2)*(
a*c - b*c*x)^(3/2))/4 + (3*a^2*c*((x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 +
(a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*c - b*c*x]])/b))/4))/6
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x(8b^4x^4-26a^2b^2x^2+33a^4)(-bx+a)\sqrt{bx+a}\,c^3}{48\sqrt{-c(bx-a)}}+\frac{5a^6\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)c(bx-a)}\,c^3}{16\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$-\frac{(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{7}{2}}}{6bc}+\frac{5a\left(-\frac{(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}}}{5bc}+\frac{3a\left(-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{7}{2}}}{4bc}+\frac{a\left(\frac{(-bcx+ac)^{\frac{5}{2}}\sqrt{bx+a}}{3b}+\frac{5ac\left(\frac{(-bcx+ac)^{\frac{3}{2}}\sqrt{bx+\cdots}}{2b}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{6\cdots}$

input
```
int((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*b^4*x^4-26*a^2*b^2*x^2+33*a^4)*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a
))^(1/2)*c^3+5/16*a^6/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2
*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.71

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \left[\frac{15\,a^6\sqrt{-c}c^2\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,(8\,b^5c^2x^5-26\,a^2b^3c^2x}{96\,b}\right.$$

input
```
integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(15*a^6*sqrt(-c)*c^2*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a)*b*sqrt(-c)*x - a^2*c) + 2*(8*b^5*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a
^4*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b, -1/48*(15*a^6*c^(5/2)*arc
tan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (8
*b^5*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4*b*c^2*x)*sqrt(-b*c*x + a*c)*sqr
t(b*x + a))/b]
```

Sympy [F]

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \int \left(-c(-a+bx)\right)^{\frac{5}{2}}\left(a+bx\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*x+a)**(5/2)*(-b*c*x+a*c)**(5/2),x)
```

output
```
Integral((-c*(-a + b*x))**(5/2)*(a + b*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \frac{5\,a^6c^{\frac{5}{2}}\arcsin\left(\frac{bx}{a}\right)}{16\,b} + \frac{5}{16}\,\sqrt{-b^2cx^2+a^2c}a^4c^2x + \frac{5}{24}\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^2cx + \frac{1}{6}\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}x$$

input
```
integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

output
```
5/16*a^6*c^(5/2)*arcsin(b*x/a)/b + 5/16*sqrt(-b^2*c*x^2 + a^2*c)*a^4*c^2*x
 + 5/24*(-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c*x + 1/6*(-b^2*c*x^2 + a^2*c)^(5/2
)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(108) = 216.

Time = 0.65 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.57

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

output
```
-1/240*(240*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^5*c^2 - 120
*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/s
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*a^4*c^2 -
80*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*s
qrt(b*x + a))*a^3*c^2 + 20*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqr
t(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a)
 + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^2*c^2 +
2*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)
*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a*c^2 - (
150*a^6*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/s
qrt(-c) - (405*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x + a) +
 321*a^2)*(b*x + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt
(b*x + a))*c^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx=\int (a\,c-b\,c\,x)^{5/2}\,(a+b\,x)^{5/2}\,dx$$

input
```
int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2),x)
```

output
```
int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx=\frac{\sqrt{c}\,c^2\left(-30 asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^6+33\sqrt{bx+a}\,\sqrt{-bx+a}\,a^4bx-26\sqrt{bx+a}\,\sqrt{-bx+a}\,a^2b^3x^3+\right.}{48b}$$

input
```
int((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x)
```

output
```
(sqrt(c)*c**2*( - 30*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**6 + 33*sqrt(
a + b*x)*sqrt(a - b*x)*a**4*b*x - 26*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**3
*x**3 + 8*sqrt(a + b*x)*sqrt(a - b*x)*b**5*x**5))/(48*b)
```

3.44 $\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx=\frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{3a^4c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{4b}$$

output
```
3/8*a^2*c*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)+1/4*x*(b*x+a)^(3/2)*(-b*c*x+a
*c)^(3/2)+3/4*a^4*c^(3/2)*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))
/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx=\frac{(c(a-bx))^{3/2}\left(bx\sqrt{a-bx}\sqrt{a+bx}(5a^2-2b^2x^2)+6a^4\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{8b(a-bx)^{3/2}}$$

input
```
Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

output
```
((c*(a - b*x))^(3/2)*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(5*a^2 - 2*b^2*x^2)
+ 6*a^4*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(8*b*(a - b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$$

$$\Big\downarrow \; 40$$

$$\frac{3}{4}a^2c\int \sqrt{a+bx}\sqrt{ac-bcx}dx+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

$$\Big\downarrow \; 40$$

$$\frac{3}{4}a^2c\left(\frac{1}{2}a^2c\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

$$\Big\downarrow \; 45$$

$$\frac{3}{4}a^2c\left(a^2c\int \frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

$$\Big\downarrow \; 218$$

$$\frac{3}{4}a^2c\left(\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

input
```
Int[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

output
```
(x*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2))/4 + (3*a^2*c*((x*Sqrt[a + b*x]*Sqr
t[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*c -
 b*c*x]])/b))/4
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

method	result
risch	$\frac{x\left(-2b^{2}x^{2}+5a^{2}\right)\left(-bx+a\right)\sqrt{bx+a}\,c^{2}}{8\sqrt{-c\left(bx-a\right)}}+\frac{3a^{4}\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)\sqrt{-\left(bx+a\right)c\left(bx-a\right)}\,c^{2}}{8\sqrt{b^{2}c}\,\sqrt{bx+a}\,\sqrt{-c\left(bx-a\right)}}$
default	$-\frac{\left(bx+a\right)^{\frac{3}{2}}\left(-bcx+ac\right)^{\frac{5}{2}}}{4bc}+\frac{3a\left(-\frac{\sqrt{bx+a}\,\left(-bcx+ac\right)^{\frac{5}{2}}}{3bc}+\frac{a\left(\frac{\left(-bcx+ac\right)^{\frac{3}{2}}\sqrt{bx+a}}{2b}+\frac{3ac\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}}{b}+\frac{ac\sqrt{\left(bx+a\right)\left(-bcx+ac\right)}\ \mathrm{arctan} \ldots}{\sqrt{-bcx+ac}\,\sqrt{bx+\ldots}}\right.}{2}\right.}{3}\right.}{4}$

input
```
int((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(-2*b^2*x^2+5*a^2)*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c^2+3/8
*a^4/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x
+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.87

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = \left[\frac{3\,a^4\sqrt{-c}c\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)-2\left(2\,b^3cx^3-5\,a^2bcx\right)\sqrt{-b}}{16\,b}\right.$$

input
```
integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*a^4*sqrt(-c)*c*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(-c)*x - a^2*c) - 2*(2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/b, -1/8*(3*a^4*c^(3/2)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x
 + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(
-b*c*x + a*c)*sqrt(b*x + a))/b]
```

Sympy [F]

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = \int \left(-c(-a+bx)\right)^{\frac{3}{2}}(a+bx)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(-b*c*x+a*c)**(3/2),x)
```

output
```
Integral((-c*(-a + b*x))**(3/2)*(a + b*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = \frac{3\,a^4c^{\frac{3}{2}}\arcsin\left(\frac{bx}{a}\right)}{8\,b} + \frac{3}{8}\sqrt{-b^2cx^2+a^2c}a^2cx + \frac{1}{4}\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}x$$

input
```
integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

output
```
3/8*a^4*c^(3/2)*arcsin(b*x/a)/b + 3/8*sqrt(-b^2*c*x^2 + a^2*c)*a^2*c*x + 1
/4*(-b^2*c*x^2 + a^2*c)^(3/2)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(81) = 162.

Time = 0.46 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.44

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = -\frac{24\left(\frac{2\,ac\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}} - \sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\right)a^3c - 12\left(\frac{2\,a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{\cdots}\right|\right)}{\sqrt{-c}}\cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

output
```
-1/24*(24*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^3*c - 12*(2*a
^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-
c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*a^2*c - 4*(6*a^
3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c
) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x
+ a))*a*c + (18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c
+ 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x
 + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*c)/b
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx=\int (a\,c-b\,c\,x)^{3/2}\,(a+b\,x)^{3/2}\,dx$$

input
```
int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2),x)
```

output
```
int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx=\frac{\sqrt{c}\,c\left(-6asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^4+5\sqrt{bx+a}\,\sqrt{-bx+a}\,a^2bx-2\sqrt{bx+a}\,\sqrt{-bx+a}\,b^3x^3\right)}{8b}$$

input
```
int((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x)
```

output
```
(sqrt(c)*c*( - 6*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4 + 5*sqrt(a + b
*x)*sqrt(a - b*x)*a**2*b*x - 2*sqrt(a + b*x)*sqrt(a - b*x)*b**3*x**3))/(8*
b)
```

3.45 $\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx=\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b}$$

output
```
1/2*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)+a^2*c^(1/2)*arctan(c^(1/2)*(b*x+a)^
(1/2)/(-b*c*x+a*c)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx=\frac{1}{2}\sqrt{c(a-bx)}\left(x\sqrt{a+bx}+\frac{2a^2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b\sqrt{a-bx}}\right)$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

output
```
(Sqrt[c*(a - b*x)]*(x*Sqrt[a + b*x] + (2*a^2*ArcTan[Sqrt[a + b*x]/Sqrt[a -
 b*x]])/(b*Sqrt[a - b*x])))/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx$$

$$\Big\downarrow \text{40}$$

$$\frac{1}{2}a^2c\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

$$\Big\downarrow \text{45}$$

$$a^2c\int \frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

$$\Big\downarrow \text{218}$$

$$\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

output
```
(x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[
a + b*x])/Sqrt[a*c - b*c*x]])/b
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

method	result	size
risch	$\frac{x(-bx+a)\sqrt{bx+a}\,c}{2\sqrt{-c(bx-a)}}+\frac{a^2\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)c(bx-a)}\,c}{2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$	107
default	$-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}}}{2bc}+\frac{a\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}}{b}+\frac{ac\sqrt{(bx+a)(-bcx+ac)}\,\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,\sqrt{b^2c}}\right)}{2}$	126

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c+1/2*a^2/(b^2*c)^(1/2)*ar
ctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/
(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.30

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx$$
$$=\left[\frac{a^2\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,\sqrt{-bcx+ac}\sqrt{bx+a}bx}{4\,b},\right.$$
$$\left.-\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\sqrt{-bcx+ac}\sqrt{bx+a}bx}{2\,b}\right]$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(a^2*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*
sqrt(-c)*x - a^2*c) + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b, -1/2*(a^2
*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 -
a^2*c)) - sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx = \int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx = \frac{a^2\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{1}{2}\,\sqrt{-b^2cx^2+a^2c}x$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*a^2*sqrt(c)*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(55) = 110.

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.14

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx = \frac{\frac{2\,a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}} + \sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}(bx-2\,a) - 2\left(\frac{2\,ac\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-\right.\right.}{\sqrt{-c}}\right.}{2\,b}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c))
)/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a) - 2*(2*a
*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
 - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx = \frac{x\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{2} - \frac{a^2\,\sqrt{b}\,c^2\,\ln\left(\sqrt{-b\,c}\,\sqrt{c\,\left(a-b\,x\right)}\,\sqrt{a+b\,x}-b^{3/2}\,c\,x\right)}{2\,{\left(-b\,c\right)}^{3/2}}$$

input
```
int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2),x)
```

output
```
(x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (a^2*b^(1/2)*c^2*log((-b*c)^(1
/2)*(c*(a - b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx = \frac{\sqrt{c}\left(-2asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^2+\sqrt{bx+a}\,\sqrt{-bx+a}\,bx\right)}{2b}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2 + sqrt(a + b*x)*
sqrt(a - b*x)*b*x))/(2*b)
```

3.46 $\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{2\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b\sqrt{c}}$$

output
```
2*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{2\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b\sqrt{c(a-bx)}}$$

input
```
Integrate[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(2*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(b*Sqrt[c*(a - b*x)]
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx$$

$$\Big\downarrow \text{ 45}$$

$$2\int \frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b\sqrt{c}}$$

input
```
Int[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(2*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*c - b*c*x]])/(b*Sqrt[c])
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(31) = 62$.

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.82

method	result	size
default	$\frac{\sqrt{(bx+a)(-bcx+ac)}\,\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{\sqrt{bx+a}\,\sqrt{-bcx+ac}\,\sqrt{b^2c}}$	71

input
```
int(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(-b*c*x+a*c))^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(b^2*c)^(1/2
)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \left[-\frac{\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)}{2\,bc}, -\frac{\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)}{b\sqrt{c}}\right]$$

input
```
integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt
(-c)*x - a^2*c)/(b*c), -arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*
x/(b^2*c*x^2 - a^2*c))/(b*sqrt(c))]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{iG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}$$

input
```
integrate(1/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
-I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
 a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b*
*2*x**2))/(4*pi**(3/2)*b*sqrt(c))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}$$

input
```
integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(b*x/a)/(b*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{2\,\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{b\sqrt{-c}}$$

input
```
integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

output
```
-2*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/(b*sqrt(
-c))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}$$

input
```
int(1/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
-(4*atan((b*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)
^(1/2) - a^(1/2)))))/(b^2*c)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{2\sqrt{c}\,asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)}{bc}$$

input
```
int(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

output
```
( - 2*sqrt(c)*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2))))/(b*c)
```

3.47 $\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
x/a^2/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{x}{a^2c\sqrt{c(a-bx)}\sqrt{a+bx}}$$

input
```
Integrate[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

output
```
x/(a^2*c*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{41\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx$$

$$\Big\downarrow \ 41$$

$$\frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

output
```
x/(a^2*c*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(-bx+a)x}{\sqrt{bx+a}\,a^2(-bcx+ac)^{\frac{3}{2}}}$	30
orering	$\frac{(-bx+a)x}{\sqrt{bx+a}\,a^2(-bcx+ac)^{\frac{3}{2}}}$	30
default	$-\frac{1}{bac\sqrt{bx+a}\,\sqrt{-bcx+ac}}+\frac{\sqrt{bx+a}}{bc\,a^2\sqrt{-bcx+ac}}$	59

input
```
int(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*x+a)/(b*x+a)^(1/2)/a^2*x/(-b*c*x+a*c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = -\frac{\sqrt{-bcx+ac}\sqrt{bx+a}x}{a^2b^2c^2x^2-a^4c^2}$$

input
```
integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*c*x + a*c)*sqrt(b*x + a)*x/(a^2*b^2*c^2*x^2 - a^4*c^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.13

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = -\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x+a)**(3/2)/(-b*c*x+a*c)**(3/2),x)
```

output
```
-I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
a**2/(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2)) + meijerg(((-1/2, 0, 1/4,
1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), a**2*exp_polar(-2*I*pi)/
(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{x}{\sqrt{-b^2cx^2+a^2c}a^2c}$$

input
```
integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

output
```
x/(sqrt(-b^2*c*x^2 + a^2*c)*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.43

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{\frac{4\sqrt{-c}}{\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2-2\,ac\right)ac}-\frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}}{((bx+a)c-2\,ac)a^2c}}{2\,b}$$

input
```
integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

output
```
1/2*(4*sqrt(-c)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2
- 2*a*c)*a*c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)/(((b*x + a)*c - 2
*a*c)*a^2*c))/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{x}{a^2\,c\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}$$

input
```
int(1/((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2)),x)
```

output
```
x/(a^2*c*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{\sqrt{c}\,x}{\sqrt{bx+a}\,\sqrt{-bx+a}\,a^2c^2}$$

input
```
int(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x)
```

output
```
(sqrt(c)*x)/(sqrt(a + b*x)*sqrt(a - b*x)*a**2*c**2)
```

3.48 $\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx=\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
1/3*x/a^2/c/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+2/3*x/a^4/c^2/(b*x+a)^(1/2)/(
-b*c*x+a*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx=\frac{3a^2x-2b^2x^3}{3a^4c(c(a-bx))^{3/2}(a+bx)^{3/2}}$$

input
```
Integrate[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

output
```
(3*a^2*x - 2*b^2*x^3)/(3*a^4*c*(c*(a - b*x))^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx$$

$$\Big\downarrow \; 42$$

$$\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}dx}{3a^2c}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

$$\Big\downarrow \; 41$$

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

input
```
Int[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

output
```
x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a
+ b*x]*Sqrt[a*c - b*c*x])
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{(-bx+a)x(-2b^2x^2+3a^2)}{3(bx+a)^{\frac{3}{2}}a^4(-bcx+ac)^{\frac{5}{2}}}$	45
orering	$\frac{(-bx+a)x(-2b^2x^2+3a^2)}{3(bx+a)^{\frac{3}{2}}a^4(-bcx+ac)^{\frac{5}{2}}}$	45
default	$-\frac{1}{3bac(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{3}{2}}}+\frac{-\frac{1}{bac\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{bx+a}}{3bac(-bcx+ac)^{\frac{3}{2}}}+\frac{2\sqrt{bx+a}}{3b\,a^2c^2\sqrt{-bcx+ac}}}{a}}{a}$	129

input
```
int(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b*x+a)*x*(-2*b^2*x^2+3*a^2)/(b*x+a)^(3/2)/a^4/(-b*c*x+a*c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx = -\frac{(2\,b^2x^3-3\,a^2x)\sqrt{-bcx+ac}\sqrt{bx+a}}{3\,(a^4b^4c^3x^4-2\,a^6b^2c^3x^2+a^8c^3)}$$

input
```
integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*b^2*x^3 - 3*a^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^4*b^4*c^3*x
^4 - 2*a^6*b^2*c^3*x^2 + a^8*c^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx = \frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{1}{2}, \frac{5}{2}, 3 \\ \frac{5}{4}, \frac{7}{4}, 2, \frac{5}{2}, 3 & 0 \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{3}{4}, \frac{5}{4}, 1 & \\ \frac{3}{4}, \frac{5}{4} & -\frac{1}{2}, 0, 2, 0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}$$

input
```
integrate(1/(b*x+a)**(5/2)/(-b*c*x+a*c)**(5/2),x)
```

output
```
I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), a
**2/(b**2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3
/4, 5/4, 1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), a**2*exp_polar(-2*I*pi)/(
b**2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx = \frac{x}{3\,(-b^2cx^2+a^2c)^{\frac{3}{2}}a^2c} + \frac{2\,x}{3\,\sqrt{-b^2cx^2+a^2c}a^4c^2}$$

input
```
integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c) + 2/3*x/(sqrt(-b^2*c*x^2 + a^2*c)
*a^4*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(55) = 110$.

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.97

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx = -\frac{\frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\left(\frac{4\,(bx+a)}{a^4c}-\frac{9}{a^3c}\right)}{((bx+a)c-2\,ac)^2}+\frac{4\left(3\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^4-18\,a\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2c+16\,a^2c^2\right)}{\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2-2\,ac\right)^3a^3\sqrt{-c}c}}{12\,b}$$

input
```
integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

output
```
-1/12*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(4*(b*x + a)/(a^4*c) - 9/(
a^3*c))/((b*x + a)*c - 2*a*c)^2 + 4*(3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^4 - 18*a*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
2*a*c))^2*c + 16*a^2*c^2)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
2*a*c))^2 - 2*a*c)^3*a^3*sqrt(-c)*c))/b
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx = -\frac{3\,a^2\,x\,\sqrt{a\,c-b\,c\,x}-2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{(a\,c-b\,c\,x)^2\,\left(3\,a^4\,\left(a\,c-b\,c\,x\right)-6\,a^5\,c\right)\sqrt{a+b\,x}}$$

input
```
int(1/((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2)),x)
```

output
```
-(3*a^2*x*(a*c - b*c*x)^(1/2) - 2*b^2*x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c
*x)^2*(3*a^4*(a*c - b*c*x) - 6*a^5*c)*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx = \frac{\sqrt{c}\,x(-2b^2x^2+3a^2)}{3\sqrt{bx+a}\,\sqrt{-bx+a}\,a^4c^3\,(-b^2x^2+a^2)}$$

input

```
int(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x)
```

output

```
(sqrt(c)*x*(3*a**2 - 2*b**2*x**2))/(3*sqrt(a + b*x)*sqrt(a - b*x)*a**4*c**
3*(a**2 - b**2*x**2))
```

3.49 $\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}} + \frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
1/5*x/a^2/c/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2)+4/15*x/a^4/c^2/(b*x+a)^(3/2)/
(-b*c*x+a*c)^(3/2)+8/15*x/a^6/c^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = \frac{15a^4x-20a^2b^2x^3+8b^4x^5}{15a^6c(c(a-bx))^{5/2}(a+bx)^{5/2}}$$

input
```
Integrate[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

output
```
(15*a^4*x - 20*a^2*b^2*x^3 + 8*b^4*x^5)/(15*a^6*c*(c*(a - b*x))^(5/2)*(a +
 b*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {42, 42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}dx$$

$$\Big\downarrow \text{ 42}$$

$$\frac{4\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}dx}{5a^2c}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

$$\Big\downarrow \text{ 42}$$

$$\frac{4\left(\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}dx}{3a^2c}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}\right)}{5a^2c}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

$$\Big\downarrow \text{ 41}$$

$$\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4\left(\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}\right)}{5a^2c}$$

input
```
Int[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

output
```
x/(5*a^2*c*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)) + (4*(x/(3*a^2*c*(a + b*x)
^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a + b*x]*Sqrt[a*c - b*
c*x])))/(5*a^2*c)
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

method	result
gosper	$\frac{(-bx+a)x(8b^4x^4-20a^2b^2x^2+15a^4)}{15(bx+a)^{\frac{5}{2}}a^6(-bcx+ac)^{\frac{7}{2}}}$
orering	$\frac{(-bx+a)x(8b^4x^4-20a^2b^2x^2+15a^4)}{15(bx+a)^{\frac{5}{2}}a^6(-bcx+ac)^{\frac{7}{2}}}$
default	$-\frac{1}{5bac(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{5}{2}}}+\frac{-\frac{1}{3bac(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{5}{2}}}+\frac{-\frac{4}{3bac\sqrt{bx+a}\,(-bcx+ac)^{\frac{5}{2}}}+\frac{4\left(\frac{3\sqrt{bx+a}}{5bac(-bcx+ac)^{\frac{5}{2}}}+\frac{3\left(\frac{2\sqrt{bx+a}}{15bac(-bcx+ac)^{\frac{3}{2}}}\cdots\right)}{a}\right)}{3a}}{a}}{a}$

input
```
int(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-b*x+a)*x*(8*b^4*x^4-20*a^2*b^2*x^2+15*a^4)/(b*x+a)^(5/2)/a^6/(-b*c*
x+a*c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = -\frac{(8\,b^4x^5-20\,a^2b^2x^3+15\,a^4x)\sqrt{-bcx+ac}\sqrt{bx+a}}{15\,(a^6b^6c^4x^6-3\,a^8b^4c^4x^4+3\,a^{10}b^2c^4x^2-a^{12}c^4)}$$

input
```
integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(8*b^4*x^5 - 20*a^2*b^2*x^3 + 15*a^4*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)/(a^6*b^6*c^4*x^6 - 3*a^8*b^4*c^4*x^4 + 3*a^10*b^2*c^4*x^2 - a^12*c^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = -\frac{2iG_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4 \\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0 \end{matrix}\middle|\,\frac{a^2}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}} + \frac{2G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0 \end{matrix}\middle|\,\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}$$

input
```
integrate(1/(b*x+a)**(7/2)/(-b*c*x+a*c)**(7/2),x)
```

output
```
-2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,))
, a**2/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2)) + 2*meijerg(((-1/2, 0,
1/2, 5/4, 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), a**2*exp_polar(-2*I
*pi)/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = \frac{x}{5\,(-b^2cx^2+a^2c)^{\frac{5}{2}}a^2c} + \frac{4\,x}{15\,(-b^2cx^2+a^2c)^{\frac{3}{2}}a^4c^2} + \frac{8\,x}{15\,\sqrt{-b^2cx^2+a^2c}a^6c^3}$$

input
```
integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

output
```
1/5*x/((-b^2*c*x^2 + a^2*c)^(5/2)*a^2*c) + 4/15*x/((-b^2*c*x^2 + a^2*c)^(3
/2)*a^4*c^2) + 8/15*x/(sqrt(-b^2*c*x^2 + a^2*c)*a^6*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(82) = 164$.

Time = 0.18 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.96

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = -\frac{\frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\left((bx+a)\left(\frac{64\,(bx+a)}{a^6c}-\frac{275}{a^5c}\right)+\frac{300}{a^4c}\right)}{((bx+a)c-2\,ac)^3} + \frac{4\left(45\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^8-450\,a\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+\cdots}\right.\right.}{\left(\left(\sqrt{bx+\cdots}\right.\right.}}{240\,b}$$

input
```
integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="giac")
```

output
```
-1/240*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*(64*(b*x + a)/
(a^6*c) - 275/(a^5*c)) + 300/(a^4*c))/((b*x + a)*c - 2*a*c)^3 + 4*(45*(sqr
t(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^8 - 450*a*(sqrt(b*x + a)
*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*c + 1660*a^2*(sqrt(b*x + a)*sqrt
(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*c^2 - 2200*a^3*(sqrt(b*x + a)*sqrt(-c
) - sqrt(-(b*x + a)*c + 2*a*c))^2*c^3 + 1024*a^4*c^4)/(((sqrt(b*x + a)*sqr
t(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2 - 2*a*c)^5*a^5*sqrt(-c)*c^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = \frac{15\,a^4\,x\,\sqrt{a\,c-b\,c\,x}+8\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}-20\,a^2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^3\,\left(60\,a^8\,c-\left(a\,c-b\,c\,x\right)\,\left(45\,a^7+15\,b\,x\,a^6\right)\right)\,\sqrt{a+b\,x}}$$

input
```
int(1/((a*c - b*c*x)^(7/2)*(a + b*x)^(7/2)),x)
```

output
```
(15*a^4*x*(a*c - b*c*x)^(1/2) + 8*b^4*x^5*(a*c - b*c*x)^(1/2) - 20*a^2*b^2
*x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*x)^3*(60*a^8*c - (a*c - b*c*x)*(45*a
^7 + 15*a^6*b*x))*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = \frac{\sqrt{c}\,x(8b^4x^4-20a^2b^2x^2+15a^4)}{15\sqrt{bx+a}\,\sqrt{-bx+a}\,a^6c^4\,(b^4x^4-2a^2b^2x^2+a^4)}$$

input
```
int(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x)
```

output
```
(sqrt(c)*x*(15*a**4 - 20*a**2*b**2*x**2 + 8*b**4*x**4))/(15*sqrt(a + b*x)*
sqrt(a - b*x)*a**6*c**4*(a**4 - 2*a**2*b**2*x**2 + b**4*x**4))
```

3.50 $\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx=\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{16x}{35a^8c^4\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
1/7*x/a^2/c/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2)+6/35*x/a^4/c^2/(b*x+a)^(5/2)/
(-b*c*x+a*c)^(5/2)+8/35*x/a^6/c^3/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+16/35*x
/a^8/c^4/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx=\frac{35a^6x-70a^4b^2x^3+56a^2b^4x^5-16b^6x^7}{35a^8c(c(a-bx))^{7/2}(a+bx)^{7/2}}$$

input
```
Integrate[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

output
```
(35*a^6*x - 70*a^4*b^2*x^3 + 56*a^2*b^4*x^5 - 16*b^6*x^7)/(35*a^8*c*(c*(a
- b*x))^(7/2)*(a + b*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {42, 42, 42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 42}$$

$$\frac{6\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}dx}{7a^2c}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

$$\Big\downarrow \text{ 42}$$

$$\frac{6\left(\frac{4\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}dx}{5a^2c}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}\right)}{7a^2c}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

$$\Big\downarrow \text{ 42}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}dx}{3a^2c}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}\right)}{5a^2c}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}\right)}{7a^2c}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

$$\Big\downarrow \text{ 41}$$

$$\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6\left(\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4\left(\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}\right)}{5a^2c}\right)}{7a^2c}$$

input
```
Int[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

output
```
x/(7*a^2*c*(a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)) + (6*(x/(5*a^2*c*(a + b*x)
^(5/2)*(a*c - b*c*x)^(5/2)) + (4*(x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)
^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])))/(5*a^2*c)))/
(7*a^2*c)
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result
gosper	$\frac{(-bx+a)x(-16b^6x^6+56a^2x^4b^4-70a^4x^2b^2+35a^6)}{35(bx+a)^{\frac{7}{2}}a^8(-bcx+ac)^{\frac{9}{2}}}$
orering	$\frac{(-bx+a)x(-16b^6x^6+56a^2x^4b^4-70a^4x^2b^2+35a^6)}{35(bx+a)^{\frac{7}{2}}a^8(-bcx+ac)^{\frac{9}{2}}}$
default	$-\frac{1}{7bac(bx+a)^{\frac{7}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{-\frac{1}{5bac(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{-\frac{2}{5bac(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{6\left(-\frac{5}{3bac\sqrt{bx+a}(-bcx+ac)^{\frac{7}{2}}}+\frac{5\left(\frac{4\sqrt{b}}{7bac(-b}\right.}{}\right.}{}}{a}$

input
```
int(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(-b*x+a)*x*(-16*b^6*x^6+56*a^2*b^4*x^4-70*a^4*b^2*x^2+35*a^6)/(b*x+a)
^(7/2)/a^8/(-b*c*x+a*c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = -\frac{(16\,b^6x^7-56\,a^2b^4x^5+70\,a^4b^2x^3-35\,a^6x)\sqrt{-bcx+ac}\sqrt{bx+a}}{35\,(a^8b^8c^5x^8-4\,a^{10}b^6c^5x^6+6\,a^{12}b^4c^5x^4-4\,a^{14}b^2c^5x^2+a^{16}c^5)}$$

input
```
integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(16*b^6*x^7 - 56*a^2*b^4*x^5 + 70*a^4*b^2*x^3 - 35*a^6*x)*sqrt(-b*c*
x + a*c)*sqrt(b*x + a)/(a^8*b^8*c^5*x^8 - 4*a^10*b^6*c^5*x^6 + 6*a^12*b^4*
c^5*x^4 - 4*a^14*b^2*c^5*x^2 + a^16*c^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 147.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = \frac{4iG_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 & \frac{1}{2}, \frac{9}{2}, 5 \\ \frac{9}{4}, \frac{11}{4}, 4, \frac{9}{2}, 5 & 0 \end{matrix}\middle| \frac{a^2}{b^2x^2}\right)}{105\pi^{\frac{3}{2}}a^8bc^{\frac{9}{2}}} + \frac{4G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{7}{4}, \frac{9}{4}, 1 & \\ \frac{7}{4}, \frac{9}{4} & -\frac{1}{2}, 0, 4, 0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{105\pi^{\frac{3}{2}}a^8bc^{\frac{9}{2}}}$$

input
```
integrate(1/(b*x+a)**(9/2)/(-b*c*x+a*c)**(9/2),x)
```

output
```
4*I*meijerg(((9/4, 11/4, 1), (1/2, 9/2, 5)), ((9/4, 11/4, 4, 9/2, 5), (0,)
), a**2/(b**2*x**2))/(105*pi**(3/2)*a**8*b*c**(9/2)) + 4*meijerg(((-1/2, 0
, 1/2, 7/4, 9/4, 1), ()), ((7/4, 9/4), (-1/2, 0, 4, 0)), a**2*exp_polar(-2
*I*pi)/(b**2*x**2))/(105*pi**(3/2)*a**8*b*c**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = \frac{x}{7\,(-b^2cx^2+a^2c)^{\frac{7}{2}}a^2c} + \frac{6\,x}{35\,(-b^2cx^2+a^2c)^{\frac{5}{2}}a^4c^2} + \frac{8\,x}{35\,(-b^2cx^2+a^2c)^{\frac{3}{2}}a^6c^3} + \frac{16\,x}{35\,\sqrt{-b^2cx^2+a^2c}a^8c^4}$$

input
```
integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

output
```
1/7*x/((-b^2*c*x^2 + a^2*c)^(7/2)*a^2*c) + 6/35*x/((-b^2*c*x^2 + a^2*c)^(5
/2)*a^4*c^2) + 8/35*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^6*c^3) + 16/35*x/(sqrt
(-b^2*c*x^2 + a^2*c)*a^8*c^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(109) = 218$.

Time = 0.24 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.95

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx =$$

$$-\frac{\frac{\left((bx+a)\left((bx+a)\left(\frac{256\,(bx+a)}{a^8c}-\frac{1617}{a^7c}\right)+\frac{3430}{a^6c}\right)-\frac{2450}{a^5c}\right)\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}}{((bx+a)c-2\,ac)^4}+\frac{4\left(175\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{12}-2450\,a\left(\sqrt{bx+\cdots}\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="giac")
```

output
```
-1/1120*(((b*x + a)*((b*x + a)*(256*(b*x + a)/(a^8*c) - 1617/(a^7*c)) + 34
30/(a^6*c)) - 2450/(a^5*c))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)/((b*x
 + a)*c - 2*a*c)^4 + 4*(175*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
2*a*c))^12 - 2450*a*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^
10*c + 14280*a^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^8*c
^2 - 43120*a^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*c^3
 + 66416*a^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*c^4 -
 51744*a^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c^5 + 1
6384*a^6*c^6)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2 -
2*a*c)^7*a^7*sqrt(-c)*c^3))/b
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = -\frac{35\,a^6\,x\,\sqrt{a\,c-b\,c\,x}-16\,b^6\,x^7\,\sqrt{a\,c-b\,c\,x}-70\,a^4\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}+56\,a^2\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}}{\left(\left(70\,a^9\,{\left(a\,c-b\,c\,x\right)}^5+35\,a^8\,{\left(a\,c-b\,c\,x\right)}^5\,\left(a+b\,x\right)\right)\,\left(a+b\,x\right)+{\left(a\,c-b\,c\,x\right)}^4\,\left(140\,a^{10}\,\left(a\,c-b\,c\,x\right)-280\right.\right.}$$

input
```
int(1/((a*c - b*c*x)^(9/2)*(a + b*x)^(9/2)),x)
```

output
```
-(35*a^6*x*(a*c - b*c*x)^(1/2) - 16*b^6*x^7*(a*c - b*c*x)^(1/2) - 70*a^4*b
^2*x^3*(a*c - b*c*x)^(1/2) + 56*a^2*b^4*x^5*(a*c - b*c*x)^(1/2))/(((70*a^9
*(a*c - b*c*x)^5 + 35*a^8*(a*c - b*c*x)^5*(a + b*x))*(a + b*x) + (a*c - b*
c*x)^4*(140*a^10*(a*c - b*c*x) - 280*a^11*c))*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = \frac{\sqrt{c}\,x(-16b^6x^6+56a^2b^4x^4-70a^4b^2x^2+35a^6)}{35\sqrt{bx+a}\,\sqrt{-bx+a}\,a^8c^5\left(-b^6x^6+3a^2b^4x^4-3a^4b^2x^2+a^6\right)}$$

input
```
int(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x)
```

output
```
(sqrt(c)*x*(35*a**6 - 70*a**4*b**2*x**2 + 56*a**2*b**4*x**4 - 16*b**6*x**6
))/(35*sqrt(a + b*x)*sqrt(a - b*x)*a**8*c**5*(a**6 - 3*a**4*b**2*x**2 + 3*
a**2*b**4*x**4 - b**6*x**6))
```

3.51 $\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \frac{45}{2}\sqrt{\frac{3}{2}}x\sqrt{1-4x^2} + 15\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2} + 6\sqrt{6}x\left(1-4x^2\right)^{5/2} + \frac{45}{4}\sqrt{\frac{3}{2}}\arcsin(2x)$$

output
```
45/4*6^(1/2)*x*(-4*x^2+1)^(1/2)+15/2*6^(1/2)*x*(-4*x^2+1)^(3/2)+6*6^(1/2)*
x*(-4*x^2+1)^(5/2)+45/8*6^(1/2)*arcsin(2*x)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \frac{3}{2}\sqrt{\frac{3}{2}}\left(x\sqrt{1-4x^2}\left(33-104x^2+128x^4\right)+15\arctan\left(\frac{\sqrt{1-4x^2}}{1-2x}\right)\right)$$

input
```
Integrate[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

output
```
(3*Sqrt[3/2]*(x*Sqrt[1 - 4*x^2]*(33 - 104*x^2 + 128*x^4) + 15*ArcTan[Sqrt[
1 - 4*x^2]/(1 - 2*x)]))/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {39, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3-6x)^{5/2}(4x+2)^{5/2}\,dx$$

$$\Big\downarrow \text{39}$$

$$\int \left(6-24x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{211}$$

$$5\int \left(6-24x^2\right)^{3/2}dx+6\sqrt{6}x\left(1-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\frac{9}{2}\int \sqrt{6-24x^2}dx+3\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2}\right)+6\sqrt{6}x\left(1-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\frac{9}{2}\left(3\int \frac{1}{\sqrt{6-24x^2}}dx+\sqrt{\frac{3}{2}}\sqrt{1-4x^2}x\right)+3\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2}\right)+6\sqrt{6}x\left(1-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$5\left(\frac{9}{2}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\arcsin(2x)+\sqrt{\frac{3}{2}}\sqrt{1-4x^2}x\right)+3\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2}\right)+6\sqrt{6}x\left(1-4x^2\right)^{5/2}$$

input
```
Int[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

output
```
6*Sqrt[6]*x*(1 - 4*x^2)^(5/2) + 5*(3*Sqrt[3/2]*x*(1 - 4*x^2)^(3/2) + (9*(S
qrt[3/2]*x*Sqrt[1 - 4*x^2] + (Sqrt[3/2]*ArcSin[2*x])/2))/2)
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{3x\left(128x^{4}-104x^{2}+33\right)\left(-1+2x\right)\left(1+2x\right)\sqrt{\left(2+4x\right)\left(3-6x\right)}\,\sqrt{6}}{4\sqrt{-\left(-1+2x\right)\left(1+2x\right)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{45\sqrt{\left(2+4x\right)\left(3-6x\right)}\,\sqrt{6}\,\arcsin\left(2x\right)}{8\sqrt{2+4x}\,\sqrt{3-6x}}$
default	$\frac{\left(3-6x\right)^{\frac{5}{2}}\left(2+4x\right)^{\frac{7}{2}}}{24}+\frac{\left(3-6x\right)^{\frac{3}{2}}\left(2+4x\right)^{\frac{7}{2}}}{8}+\frac{9\sqrt{3-6x}\,\left(2+4x\right)^{\frac{7}{2}}}{32}-\frac{3\left(2+4x\right)^{\frac{5}{2}}\sqrt{3-6x}}{16}-\frac{15\left(2+4x\right)^{\frac{3}{2}}\sqrt{3-6x}}{16}-\frac{45\sqrt{3-6x}\,\sqrt{2+}}{8}$

input
```
int((3-6*x)^(5/2)*(2+4*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-3/4*x*(128*x^4-104*x^2+33)*(-1+2*x)*(1+2*x)/(-(-1+2*x)*(1+2*x))^(1/2)*((2
+4*x)*(3-6*x))^(1/2)/(3-6*x)^(1/2)/(2+4*x)^(1/2)*6^(1/2)+45/8*((2+4*x)*(3-
6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \frac{3}{4}\left(128\,x^5-104\,x^3+33\,x\right)\sqrt{4\,x+2}\sqrt{-6\,x+3} - \frac{45}{4}\sqrt{\frac{3}{2}}\arctan\left(\frac{2\sqrt{\frac{3}{2}}\sqrt{4\,x+2}x\sqrt{-6\,x+3}}{3\left(4\,x^2-1\right)}\right)$$

input
```
integrate((3-6*x)^(5/2)*(2+4*x)^(5/2),x, algorithm="fricas")
```

output
```
3/4*(128*x^5 - 104*x^3 + 33*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 45/4*sqrt(3/
2)*arctan(2/3*sqrt(3/2)*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1))
```

Sympy [F(-1)]

Timed out.

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((3-6*x)**(5/2)*(2+4*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \frac{1}{6}\left(-24\,x^2+6\right)^{\frac{5}{2}}x + \frac{5}{4}\left(-24\,x^2+6\right)^{\frac{3}{2}}x + \frac{45}{4}\sqrt{-24\,x^2+6}x + \frac{45}{8}\sqrt{6}\arcsin\left(2\,x\right)$$

input
```
integrate((3-6*x)^(5/2)*(2+4*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(-24*x^2 + 6)^(5/2)*x + 5/4*(-24*x^2 + 6)^(3/2)*x + 45/4*sqrt(-24*x^2
+ 6)*x + 45/8*sqrt(6)*arcsin(2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(55) = 110$.

Time = 0.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.87

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \frac{3}{40}\sqrt{3}\sqrt{2}\Big(((2((8(5x-13)(2x+1)+321)(2x+1)-451)(2x+1)+745)(2x+1)-405)$$

input
```
integrate((3-6*x)^(5/2)*(2+4*x)^(5/2),x, algorithm="giac")
```

output
```
3/40*sqrt(3)*sqrt(2)*(((2*((8*(5*x - 13)*(2*x + 1) + 321)*(2*x + 1) - 451)
*(2*x + 1) + 745)*(2*x + 1) - 405)*sqrt(2*x + 1)*sqrt(-2*x + 1) + 2*((2*(3
*(8*x - 17)*(2*x + 1) + 133)*(2*x + 1) - 295)*(2*x + 1) + 195)*sqrt(2*x +
1)*sqrt(-2*x + 1) - 20*((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(
2*x + 1)*sqrt(-2*x + 1) - 80*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(
-2*x + 1) + 240*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + 240*sqrt(2*x + 1)*s
qrt(-2*x + 1) + 150*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

Mupad [F(-1)]

Timed out.

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \int (4\,x+2)^{5/2}\,(3-6\,x)^{5/2}\,dx$$

input
```
int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2),x)
```

output
```
int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = \frac{3\sqrt{3}\left(-15asin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)+128\sqrt{2x+1}\,\sqrt{-2x+1}\,x^5-104\sqrt{2x+1}\,\sqrt{-2x+1}\,x^3+33\sqrt{2x}\right.}{2\sqrt{2}}$$

input
```
int((3-6*x)^(5/2)*(2+4*x)^(5/2),x)
```

output
```
(3*sqrt(3)*( - 15*asin(sqrt( - 2*x + 1)/sqrt(2)) + 128*sqrt(2*x + 1)*sqrt(
 - 2*x + 1)*x**5 - 104*sqrt(2*x + 1)*sqrt( - 2*x + 1)*x**3 + 33*sqrt(2*x +
 1)*sqrt( - 2*x + 1)*x))/(2*sqrt(2))
```

3.52 $\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx=\frac{9}{2}\sqrt{\frac{3}{2}}x\sqrt{1-4x^2}+3\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2}+\frac{9}{4}\sqrt{\frac{3}{2}}\arcsin(2x)$$

output
```
9/4*6^(1/2)*x*(-4*x^2+1)^(1/2)+3/2*6^(1/2)*x*(-4*x^2+1)^(3/2)+9/8*6^(1/2)*
arcsin(2*x)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx=-\frac{3}{2}\sqrt{\frac{3}{2}}\left(x\sqrt{1-4x^2}\left(-5+8x^2\right)+3\arctan\left(\frac{\sqrt{1-4x^2}}{-1+2x}\right)\right)$$

input
```
Integrate[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

output
```
(-3*Sqrt[3/2]*(x*Sqrt[1 - 4*x^2]*(-5 + 8*x^2) + 3*ArcTan[Sqrt[1 - 4*x^2]/(
-1 + 2*x)]))/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {39, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3-6x)^{3/2}(4x+2)^{3/2}\,dx$$

$$\Big\downarrow \ 39$$

$$\int \left(6-24x^2\right)^{3/2}dx$$

$$\Big\downarrow \ 211$$

$$\frac{9}{2}\int \sqrt{6-24x^2}dx+3\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2}$$

$$\Big\downarrow \ 211$$

$$\frac{9}{2}\left(3\int \frac{1}{\sqrt{6-24x^2}}dx+\sqrt{\frac{3}{2}}\sqrt{1-4x^2}x\right)+3\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2}$$

$$\Big\downarrow \ 223$$

$$\frac{9}{2}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\arcsin(2x)+\sqrt{\frac{3}{2}}\sqrt{1-4x^2}x\right)+3\sqrt{\frac{3}{2}}x\left(1-4x^2\right)^{3/2}$$

input
```
Int[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

output
```
3*Sqrt[3/2]*x*(1 - 4*x^2)^(3/2) + (9*(Sqrt[3/2]*x*Sqrt[1 - 4*x^2] + (Sqrt[
3/2]*ArcSin[2*x])/2))/2
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(40) = 80$.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

method	result	siz
default	$\frac{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{5}{2}}}{16}+\frac{3(2+4x)^{\frac{5}{2}}\sqrt{3-6x}}{16}-\frac{3(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{16}-\frac{9\sqrt{3-6x}\,\sqrt{2+4x}}{8}+\frac{9\sqrt{(2+4x)(3-6x)}\,\sqrt{6}\,\arcsin(2x)}{8\sqrt{2+4x}\,\sqrt{3-6x}}$	102
risch	$\frac{3x(8x^{2}-5)(-1+2x)(1+2x)\sqrt{(2+4x)(3-6x)}\,\sqrt{6}}{4\sqrt{-(-1+2x)(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{9\sqrt{(2+4x)(3-6x)}\,\sqrt{6}\,\arcsin(2x)}{8\sqrt{2+4x}\,\sqrt{3-6x}}$	102

input
```
int((3-6*x)^(3/2)*(2+4*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(3-6*x)^(3/2)*(2+4*x)^(5/2)+3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-3/16*(2+
4*x)^(3/2)*(3-6*x)^(1/2)-9/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+9/8*((2+4*x)*(3-6
*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = -\frac{3}{4}\left(8\,x^3-5\,x\right)\sqrt{4\,x+2}\sqrt{-6\,x+3} - \frac{9}{4}\sqrt{\frac{3}{2}}\arctan\left(\frac{2\sqrt{\frac{3}{2}}\sqrt{4\,x+2}x\sqrt{-6\,x+3}}{3\left(4\,x^2-1\right)}\right)$$

input
```
integrate((3-6*x)^(3/2)*(2+4*x)^(3/2),x, algorithm="fricas")
```

output
```
-3/4*(8*x^3 - 5*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 9/4*sqrt(3/2)*arctan(2/3
*sqrt(3/2)*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1))
```

Sympy [F(-1)]

Timed out.

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((3-6*x)**(3/2)*(2+4*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = \frac{1}{4}\left(-24\,x^2+6\right)^{\frac{3}{2}}x + \frac{9}{4}\sqrt{-24\,x^2+6}x + \frac{9}{8}\sqrt{6}\arcsin\left(2\,x\right)$$

input
```
integrate((3-6*x)^(3/2)*(2+4*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-24*x^2 + 6)^(3/2)*x + 9/4*sqrt(-24*x^2 + 6)*x + 9/8*sqrt(6)*arcsin(2
*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(40) = 80$.

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.08

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = -\frac{1}{8}\sqrt{3}\sqrt{2}\Big(((4\,(3\,x-5)(2\,x+1)+43)(2\,x+1)-39)\sqrt{2\,x+1}\sqrt{-2\,x+1}+4\,((4\,x-5)(2\,x+1)+9)\sqrt{2$$

input
```
integrate((3-6*x)^(3/2)*(2+4*x)^(3/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(3)*sqrt(2)*(((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(2
*x + 1)*sqrt(-2*x + 1) + 4*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(-2
*x + 1) - 24*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) - 24*sqrt(2*x + 1)*sqrt(
-2*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

Mupad [F(-1)]

Timed out.

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = \int (4\,x+2)^{3/2}\,(3-6\,x)^{3/2}\,dx$$

input
```
int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2),x)
```

output
```
int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = \frac{3\sqrt{3}\left(-3asin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)-8\sqrt{2x+1}\,\sqrt{-2x+1}\,x^3+5\sqrt{2x+1}\,\sqrt{-2x+1}\,x\right)}{2\sqrt{2}}$$

input
```
int((3-6*x)^(3/2)*(2+4*x)^(3/2),x)
```

output
```
(3*sqrt(3)*( - 3*asin(sqrt( - 2*x + 1)/sqrt(2)) - 8*sqrt(2*x + 1)*sqrt( -
2*x + 1)*x**3 + 5*sqrt(2*x + 1)*sqrt( - 2*x + 1)*x))/(2*sqrt(2))
```

3.53 $\int \sqrt{3-6x}\sqrt{2+4x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx=\sqrt{\frac{3}{2}}x\sqrt{1-4x^2}+\frac{1}{2}\sqrt{\frac{3}{2}}\arcsin(2x)$$

output
```
1/2*6^(1/2)*x*(-4*x^2+1)^(1/2)+1/4*6^(1/2)*arcsin(2*x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx=\sqrt{\frac{3}{2}}\left(x\sqrt{1-4x^2}+\arctan\left(\frac{\sqrt{1-4x^2}}{1-2x}\right)\right)$$

input
```
Integrate[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

output
```
Sqrt[3/2]*(x*Sqrt[1 - 4*x^2] + ArcTan[Sqrt[1 - 4*x^2]/(1 - 2*x)])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {39, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-6x}\sqrt{4x+2}\,dx$$

$$\Big\downarrow \text{39}$$

$$\int \sqrt{6-24x^2}dx$$

$$\Big\downarrow \text{211}$$

$$3\int \frac{1}{\sqrt{6-24x^2}}dx+\sqrt{\frac{3}{2}}\sqrt{1-4x^2}x$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\sqrt{\frac{3}{2}}\arcsin(2x)+\sqrt{\frac{3}{2}}\sqrt{1-4x^2}x$$

input
```
Int[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

output
```
Sqrt[3/2]*x*Sqrt[1 - 4*x^2] + (Sqrt[3/2]*ArcSin[2*x])/2
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

method	result	size
default	$\frac{(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{8}-\frac{\sqrt{3-6x}\,\sqrt{2+4x}}{4}+\frac{\sqrt{(2+4x)(3-6x)}\,\sqrt{6}\,\arcsin(2x)}{4\sqrt{2+4x}\,\sqrt{3-6x}}$	70
risch	$-\frac{x(-1+2x)(1+2x)\sqrt{(2+4x)(3-6x)}\,\sqrt{6}}{2\sqrt{-(-1+2x)(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{\sqrt{(2+4x)(3-6x)}\,\sqrt{6}\,\arcsin(2x)}{4\sqrt{2+4x}\,\sqrt{3-6x}}$	95

input
```
int((3-6*x)^(1/2)*(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2+4*x)^(3/2)*(3-6*x)^(1/2)-1/4*(3-6*x)^(1/2)*(2+4*x)^(1/2)+1/4*((2+4*
x)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx = \frac{1}{2}\sqrt{4x+2}x\sqrt{-6x+3} - \frac{1}{2}\sqrt{\frac{3}{2}}\arctan\left(\frac{2\sqrt{\frac{3}{2}}\sqrt{4x+2}x\sqrt{-6x+3}}{3(4x^2-1)}\right)$$

input
```
integrate((3-6*x)^(1/2)*(2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*x + 2)*x*sqrt(-6*x + 3) - 1/2*sqrt(3/2)*arctan(2/3*sqrt(3/2)*sq
rt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.19

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx = \begin{cases} -\frac{\sqrt{6}i\,\mathrm{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{2} + \frac{\sqrt{6}i\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{x-\frac{1}{2}}} - \frac{3\sqrt{6}i\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{x-\frac{1}{2}}} + \frac{\sqrt{6}i\sqrt{x+\frac{1}{2}}}{2\sqrt{x-\frac{1}{2}}} & \text{for } \left|x+\frac{1}{2}\right| > 1 \\ \frac{\sqrt{6}\,\mathrm{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{2} - \frac{\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{\frac{1}{2}-x}} + \frac{3\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{\frac{1}{2}-x}} - \frac{\sqrt{6}\sqrt{x+\frac{1}{2}}}{2\sqrt{\frac{1}{2}-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((3-6*x)**(1/2)*(2+4*x)**(1/2),x)
```

output
```
Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/2 + sqrt(6)*I*(x + 1/2)**(5/2)/
sqrt(x - 1/2) - 3*sqrt(6)*I*(x + 1/2)**(3/2)/(2*sqrt(x - 1/2)) + sqrt(6)*I
*sqrt(x + 1/2)/(2*sqrt(x - 1/2)), Abs(x + 1/2) > 1), (sqrt(6)*asin(sqrt(x
+ 1/2))/2 - sqrt(6)*(x + 1/2)**(5/2)/sqrt(1/2 - x) + 3*sqrt(6)*(x + 1/2)**
(3/2)/(2*sqrt(1/2 - x)) - sqrt(6)*sqrt(x + 1/2)/(2*sqrt(1/2 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx = \frac{1}{2}\sqrt{-24\,x^2+6}x + \frac{1}{4}\sqrt{6}\arcsin\left(2\,x\right)$$

input
```
integrate((3-6*x)^(1/2)*(2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-24*x^2 + 6)*x + 1/4*sqrt(6)*arcsin(2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx$$
$$=\frac{1}{2}\sqrt{3}\sqrt{2}\left(\sqrt{2x+1}(x-1)\sqrt{-2x+1}+\sqrt{2x+1}\sqrt{-2x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)\right)$$

input
```
integrate((3-6*x)^(1/2)*(2+4*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*sqrt(2)*(sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + sqrt(2*x + 1)*
sqrt(-2*x + 1) + arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx=\frac{x\sqrt{4x+2}\sqrt{3-6x}}{2}-\frac{\sqrt{6}\ln\left(x-\frac{\sqrt{1-2x}\sqrt{2x+1}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{4}$$

input
```
int((4*x + 2)^(1/2)*(3 - 6*x)^(1/2),x)
```

output
```
(x*(4*x + 2)^(1/2)*(3 - 6*x)^(1/2))/2 - (6^(1/2)*log(x - ((1 - 2*x)^(1/2)*
(2*x + 1)^(1/2)*1i)/2)*1i)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{3-6x}\sqrt{2+4x}\,dx = \frac{\sqrt{3}\left(-asin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)+\sqrt{2x+1}\,\sqrt{-2x+1}\,x\right)}{\sqrt{2}}$$

input
```
int((3-6*x)^(1/2)*(2+4*x)^(1/2),x)
```

output
```
(sqrt(3)*( - asin(sqrt( - 2*x + 1)/sqrt(2)) + sqrt(2*x + 1)*sqrt( - 2*x +
1)*x))/sqrt(2)
```

3.54 $\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = \frac{\arcsin(2x)}{2\sqrt{6}}$$

output
```
1/12*6^(1/2)*arcsin(2*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = -\frac{\arctan\left(\frac{\sqrt{1-4x^2}}{1+2x}\right)}{\sqrt{6}}$$

input
```
Integrate[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

output
```
-(ArcTan[Sqrt[1 - 4*x^2]/(1 + 2*x)]/Sqrt[6])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-6x}\sqrt{4x+2}}\,dx$$

$$\Big\downarrow \ 39$$

$$\int \frac{1}{\sqrt{6-24x^2}}dx$$

$$\Big\downarrow \ 223$$

$$\frac{\arcsin(2x)}{2\sqrt{6}}$$

input
```
Int[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

output
```
ArcSin[2*x]/(2*Sqrt[6])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(9) = 18$.

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.85

method	result	size
default	$\frac{\sqrt{(2+4x)(3-6x)}\,\sqrt{6}\,\arcsin(2x)}{12\sqrt{2+4x}\,\sqrt{3-6x}}$	37

input
```
int(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*((2+4*x)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*
x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = -\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{4\,x+2}x\sqrt{-6\,x+3}}{3\,(4\,x^2-1)}\right)$$

input
```
integrate(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(6)*arctan(1/3*sqrt(6)*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1
))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = \begin{cases} -\frac{\sqrt{6}i\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{for } \left|x+\frac{1}{2}\right| > 1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3-6*x)**(1/2)/(2+4*x)**(1/2),x)
```

output
```
Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/6, Abs(x + 1/2) > 1), (sqrt(6)*
asin(sqrt(x + 1/2))/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = \frac{1}{12}\sqrt{6}\arcsin\left(2\,x\right)$$

input
```
integrate(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(6)*arcsin(2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = \frac{1}{6}\sqrt{3}\sqrt{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2\,x+1}\right)$$

input
```
integrate(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*sqrt(2)*arcsin(1/2*sqrt(2)*sqrt(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.08

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = -\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{24}\left(\sqrt{3}-\sqrt{3-6\,x}\right)}{6\left(\sqrt{2}-\sqrt{4\,x+2}\right)}\right)}{3}$$

input
```
int(1/((4*x + 2)^(1/2)*(3 - 6*x)^(1/2)),x)
```

output
```
-(6^(1/2)*atan((24^(1/2)*(3^(1/2) - (3 - 6*x)^(1/2)))/(6*(2^(1/2) - (4*x +
 2)^(1/2)))))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx = -\frac{\sqrt{6}\,asin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)}{6}$$

input
```
int(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x)
```

output
```
( - sqrt(6)*asin(sqrt( - 2*x + 1)/sqrt(2)))/6
```

3.55 $\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{x}{6\sqrt{6}\sqrt{1-4x^2}}$$

output
```
1/36*x*6^(1/2)/(-4*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{x}{6\sqrt{6-24x^2}}$$

input
```
Integrate[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

output
```
x/(6*Sqrt[6 - 24*x^2])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-6x)^{3/2}(4x+2)^{3/2}} \, dx$$
$$\Big\downarrow \; 39$$
$$\int \frac{1}{(6-24x^2)^{3/2}}dx$$
$$\Big\downarrow \; 208$$
$$\frac{x}{6\sqrt{6}\sqrt{1-4x^2}}$$

input
```
Int[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

output
```
x/(6*Sqrt[6]*Sqrt[1 - 4*x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

method	result	size
gosper	$-\frac{(-1+2x)(1+2x)x}{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{3}{2}}}$	28
orering	$-\frac{(-1+2x)(1+2x)x}{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{3}{2}}}$	28
default	$\frac{1}{12\sqrt{3-6x}\,\sqrt{2+4x}}-\frac{\sqrt{3-6x}}{36\sqrt{2+4x}}$	34

input
```
int(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1+2*x)*(1+2*x)*x/(3-6*x)^(3/2)/(2+4*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = -\frac{\sqrt{4\,x+2}x\sqrt{-6\,x+3}}{36\,(4\,x^2-1)}$$

input
```
integrate(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/36*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 7.43

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \begin{cases} -\frac{2\sqrt{6}i\sqrt{x-\frac{1}{2}}\left(x+\frac{1}{2}\right)}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}i\sqrt{x-\frac{1}{2}}}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{for } \left|x+\frac{1}{2}\right|>1 \\ -\frac{2\sqrt{6}\sqrt{\frac{1}{2}-x}\left(x+\frac{1}{2}\right)}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\sqrt{\frac{1}{2}-x}}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3-6*x)**(3/2)/(2+4*x)**(3/2),x)
```

output
```
Piecewise((-2*sqrt(6)*I*sqrt(x - 1/2)*(x + 1/2)/(144*(x + 1/2)**(3/2) - 14
4*sqrt(x + 1/2)) + sqrt(6)*I*sqrt(x - 1/2)/(144*(x + 1/2)**(3/2) - 144*sqr
t(x + 1/2)), Abs(x + 1/2) > 1), (-2*sqrt(6)*sqrt(1/2 - x)*(x + 1/2)/(144*(
x + 1/2)**(3/2) - 144*sqrt(x + 1/2)) + sqrt(6)*sqrt(1/2 - x)/(144*(x + 1/2
)**(3/2) - 144*sqrt(x + 1/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{x}{6\sqrt{-24\,x^2+6}}$$

input
```
integrate(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x, algorithm="maxima")
```

output
```
1/6*x/sqrt(-24*x^2 + 6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(15) = 30.

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.76

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)}{288\,\sqrt{2\,x+1}} - \frac{\sqrt{6}\sqrt{2\,x+1}\sqrt{-2\,x+1}}{144\,(2\,x-1)} - \frac{\sqrt{6}\sqrt{2\,x+1}}{288\,\left(\sqrt{2}-\sqrt{-2\,x+1}\right)}$$

input
```
integrate(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x, algorithm="giac")
```

output
```
1/288*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))/sqrt(2*x + 1) - 1/144*sqrt(6)*sqr
t(2*x + 1)*sqrt(-2*x + 1)/(2*x - 1) - 1/288*sqrt(6)*sqrt(2*x + 1)/(sqrt(2)
 - sqrt(-2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = -\frac{x\,\sqrt{3-6\,x}}{\sqrt{4\,x+2}\,\left(36\,x-18\right)}$$

input
```
int(1/((4*x + 2)^(3/2)*(3 - 6*x)^(3/2)),x)
```

output
```
-(x*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(36*x - 18))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{\sqrt{6}\,x}{36\sqrt{2x+1}\,\sqrt{-2x+1}}$$

input
```
int(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x)
```

output
```
(sqrt(6)*x)/(36*sqrt(2*x + 1)*sqrt( - 2*x + 1))
```

3.56 $\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx = \frac{x}{108\sqrt{6}\,(1-4x^2)^{3/2}} + \frac{x}{54\sqrt{6}\sqrt{1-4x^2}}$$

output
```
1/648*x*6^(1/2)/(-4*x^2+1)^(3/2)+1/324*x*6^(1/2)/(-4*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx = -\frac{x(-3+8x^2)}{108\sqrt{6}\,(1-4x^2)^{3/2}}$$

input
```
Integrate[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

output
```
-1/108*(x*(-3 + 8*x^2))/(Sqrt[6]*(1 - 4*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {39, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-6x)^{5/2}(4x+2)^{5/2}}\,dx$$
$$\Big\downarrow \text{39}$$
$$\int \frac{1}{(6-24x^2)^{5/2}}dx$$
$$\Big\downarrow \text{209}$$
$$\frac{1}{9}\int \frac{1}{(6-24x^2)^{3/2}}dx+\frac{x}{108\sqrt{6}\,(1-4x^2)^{3/2}}$$
$$\Big\downarrow \text{208}$$
$$\frac{x}{54\sqrt{6}\sqrt{1-4x^2}}+\frac{x}{108\sqrt{6}\,(1-4x^2)^{3/2}}$$

input
```
Int[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

output
```
x/(108*Sqrt[6]*(1 - 4*x^2)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 4*x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{(-1+2x)(1+2x)x(8x^2-3)}{3(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{5}{2}}}$	35
orering	$\frac{(-1+2x)(1+2x)x(8x^2-3)}{3(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{5}{2}}}$	35
default	$\frac{1}{36(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{3}{2}}}+\frac{1}{36\sqrt{3-6x}\,(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{162(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{324\sqrt{2+4x}}$	66

input
```
int(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1+2*x)*(1+2*x)*x*(8*x^2-3)/(3-6*x)^(5/2)/(2+4*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx=-\frac{(8\,x^3-3\,x)\sqrt{4\,x+2}\sqrt{-6\,x+3}}{648\,(16\,x^4-8\,x^2+1)}$$

input
```
integrate(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/648*(8*x^3 - 3*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(16*x^4 - 8*x^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(3-6*x)**(5/2)/(2+4*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.58

$$\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx = \frac{x}{54\sqrt{-24x^2+6}} + \frac{x}{18(-24x^2+6)^{\frac{3}{2}}}$$

input
```
integrate(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x, algorithm="maxima")
```

output
```
1/54*x/sqrt(-24*x^2 + 6) + 1/18*x/(-24*x^2 + 6)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(31) = 62.

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.37

$$\begin{aligned}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx &= \frac{\sqrt{6}\left(\sqrt{2}-\sqrt{-2x+1}\right)^3}{82944\,(2x+1)^{\frac{3}{2}}} \\ &+ \frac{11\sqrt{6}\left(\sqrt{2}-\sqrt{-2x+1}\right)}{27648\sqrt{2x+1}} - \frac{\left(4\sqrt{6}(2x+1)-9\sqrt{6}\right)\sqrt{2x+1}\sqrt{-2x+1}}{5184\,(2x-1)^2} \\ &- \frac{\sqrt{6}(2x+1)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{2}-\sqrt{-2x+1}\right)^2}{2x+1}+1\right)}{82944\left(\sqrt{2}-\sqrt{-2x+1}\right)^3}\end{aligned}$$

input
```
integrate(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x, algorithm="giac")
```

output
```
1/82944*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))^3/(2*x + 1)^(3/2) + 11/27648*sq
rt(6)*(sqrt(2) - sqrt(-2*x + 1))/sqrt(2*x + 1) - 1/5184*(4*sqrt(6)*(2*x +
1) - 9*sqrt(6))*sqrt(2*x + 1)*sqrt(-2*x + 1)/(2*x - 1)^2 - 1/82944*sqrt(6)
*(2*x + 1)^(3/2)*(33*(sqrt(2) - sqrt(-2*x + 1))^2/(2*x + 1) + 1)/(sqrt(2)
- sqrt(-2*x + 1))^3
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx = -\frac{3\,x\,\sqrt{3-6\,x}-8\,x^3\,\sqrt{3-6\,x}}{\sqrt{4\,x+2}\,\left(-2592\,x^3+1296\,x^2+648\,x-324\right)}$$

input
```
int(1/((4*x + 2)^(5/2)*(3 - 6*x)^(5/2)),x)
```

output
```
-(3*x*(3 - 6*x)^(1/2) - 8*x^3*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(648*x + 1
296*x^2 - 2592*x^3 - 324))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx = \frac{\sqrt{6}\,x(8x^2-3)}{648\sqrt{2x+1}\,\sqrt{-2x+1}\,(4x^2-1)}$$

input
```
int(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x)
```

output
```
(sqrt(6)*x*(8*x**2 - 3))/(648*sqrt(2*x + 1)*sqrt( - 2*x + 1)*(4*x**2 - 1))
```

3.57 $\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = \frac{x}{1080\sqrt{6}\,(1-4x^2)^{5/2}} + \frac{x}{810\sqrt{6}\,(1-4x^2)^{3/2}} + \frac{x}{405\sqrt{6}\sqrt{1-4x^2}}$$

output

```
1/6480*x*6^(1/2)/(-4*x^2+1)^(5/2)+1/4860*x*6^(1/2)/(-4*x^2+1)^(3/2)+1/2430
*x*6^(1/2)/(-4*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = \frac{x(15-80x^2+128x^4)}{3240\sqrt{6}\,(1-4x^2)^{5/2}}$$

input

```
Integrate[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

output

```
(x*(15 - 80*x^2 + 128*x^4))/(3240*Sqrt[6]*(1 - 4*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {39, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-6x)^{7/2}(4x+2)^{7/2}}dx$$
$$\Big\downarrow \text{39}$$
$$\int \frac{1}{(6-24x^2)^{7/2}}dx$$
$$\Big\downarrow \text{209}$$
$$\frac{2}{15}\int \frac{1}{(6-24x^2)^{5/2}}dx+\frac{x}{1080\sqrt{6}\,(1-4x^2)^{5/2}}$$
$$\Big\downarrow \text{209}$$
$$\frac{2}{15}\left(\frac{1}{9}\int \frac{1}{(6-24x^2)^{3/2}}dx+\frac{x}{108\sqrt{6}\,(1-4x^2)^{3/2}}\right)+\frac{x}{1080\sqrt{6}\,(1-4x^2)^{5/2}}$$
$$\Big\downarrow \text{208}$$
$$\frac{x}{1080\sqrt{6}\,(1-4x^2)^{5/2}}+\frac{2}{15}\left(\frac{x}{54\sqrt{6}\sqrt{1-4x^2}}+\frac{x}{108\sqrt{6}\,(1-4x^2)^{3/2}}\right)$$

input
```
Int[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

output
```
x/(1080*Sqrt[6]*(1 - 4*x^2)^(5/2)) + (2*(x/(108*Sqrt[6]*(1 - 4*x^2)^(3/2))
 + x/(54*Sqrt[6]*Sqrt[1 - 4*x^2])))/15
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

method	result	si
gosper	$-\frac{(-1+2x)(1+2x)x(128x^4-80x^2+15)}{15(3-6x)^{\frac{7}{2}}(2+4x)^{\frac{7}{2}}}$	4(
orering	$-\frac{(-1+2x)(1+2x)x(128x^4-80x^2+15)}{15(3-6x)^{\frac{7}{2}}(2+4x)^{\frac{7}{2}}}$	4(
default	$\frac{1}{60(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{5}{2}}}+\frac{1}{108(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{5}{2}}}+\frac{1}{81\sqrt{3-6x}\,(2+4x)^{\frac{5}{2}}}-\frac{\sqrt{3-6x}}{405(2+4x)^{\frac{5}{2}}}-\frac{\sqrt{3-6x}}{1215(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{2430\sqrt{2+4x}}$	9

input
```
int(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-1+2*x)*(1+2*x)*x*(128*x^4-80*x^2+15)/(3-6*x)^(7/2)/(2+4*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = -\frac{(128\,x^5-80\,x^3+15\,x)\sqrt{4\,x+2}\sqrt{-6\,x+3}}{19440\,(64\,x^6-48\,x^4+12\,x^2-1)}$$

input
```
integrate(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x, algorithm="fricas")
```

output
```
-1/19440*(128*x^5 - 80*x^3 + 15*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(64*x^6 -
48*x^4 + 12*x^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(3-6*x)**(7/2)/(2+4*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.58

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = \frac{x}{405\,\sqrt{-24\,x^2+6}}+\frac{x}{135\,(-24\,x^2+6)^{\frac{3}{2}}}+\frac{x}{30\,(-24\,x^2+6)^{\frac{5}{2}}}$$

input
```
integrate(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x, algorithm="maxima")
```

output
```
1/405*x/sqrt(-24*x^2 + 6) + 1/135*x/(-24*x^2 + 6)^(3/2) + 1/30*x/(-24*x^2
+ 6)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(46) = 92$.

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.25

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = \frac{\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{5}}{13271040\,(2\,x+1)^{\frac{5}{2}}} + \frac{17\,\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{3}}{7962624\,(2\,x+1)^{\frac{3}{2}}} + \frac{71\,\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)}{1327104\,\sqrt{2\,x+1}} - \frac{\left((64\,\sqrt{6}(2\,x+1)-275\,\sqrt{6})(2\,x+1)+300\,\sqrt{6}\right)\sqrt{2\,x+1}\sqrt{-2\,x+1}}{622080\,(2\,x-1)^{3}} - \frac{\sqrt{6}\left(\frac{2130\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{4}}{(2\,x+1)^{2}}+\frac{85\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{2}}{2\,x+1}+3\right)(2\,x+1)^{\frac{5}{2}}}{39813120\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{5}}$$

input
```
integrate(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x, algorithm="giac")
```

output
```
1/13271040*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))^5/(2*x + 1)^(5/2) + 17/79626
24*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))^3/(2*x + 1)^(3/2) + 71/1327104*sqrt(
6)*(sqrt(2) - sqrt(-2*x + 1))/sqrt(2*x + 1) - 1/622080*((64*sqrt(6)*(2*x +
 1) - 275*sqrt(6))*(2*x + 1) + 300*sqrt(6))*sqrt(2*x + 1)*sqrt(-2*x + 1)/(
2*x - 1)^3 - 1/39813120*sqrt(6)*(2130*(sqrt(2) - sqrt(-2*x + 1))^4/(2*x +
1)^2 + 85*(sqrt(2) - sqrt(-2*x + 1))^2/(2*x + 1) + 3)*(2*x + 1)^(5/2)/(sqr
t(2) - sqrt(-2*x + 1))^5
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = -\frac{15\,x\,\sqrt{3-6\,x}-80\,x^{3}\,\sqrt{3-6\,x}+128\,x^{5}\,\sqrt{3-6\,x}}{\left((6\,x-3)\,\left(240\,x+360\right)+1440\right)\,\sqrt{4\,x+2}\,(6\,x-3)^{3}}$$

input
```
int(1/((4*x + 2)^(7/2)*(3 - 6*x)^(7/2)),x)
```

output
```
-(15*x*(3 - 6*x)^(1/2) - 80*x^3*(3 - 6*x)^(1/2) + 128*x^5*(3 - 6*x)^(1/2))
/(((6*x - 3)*(240*x + 360) + 1440)*(4*x + 2)^(1/2)*(6*x - 3)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx = \frac{\sqrt{6}\,x(128x^4-80x^2+15)}{19440\sqrt{2x+1}\,\sqrt{-2x+1}\,(16x^4-8x^2+1)}$$

input
```
int(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x)
```

output
```
(sqrt(6)*x*(128*x**4 - 80*x**2 + 15))/(19440*sqrt(2*x + 1)*sqrt( - 2*x + 1
)*(16*x**4 - 8*x**2 + 1))
```

3.58 $\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = \frac{x}{9072\sqrt{6}\,(1-4x^2)^{7/2}} + \frac{x}{7560\sqrt{6}\,(1-4x^2)^{5/2}} + \frac{x}{5670\sqrt{6}\,(1-4x^2)^{3/2}} + \frac{x}{2835\sqrt{6}\sqrt{1-4x^2}}$$

output
```
1/54432*x*6^(1/2)/(-4*x^2+1)^(7/2)+1/45360*x*6^(1/2)/(-4*x^2+1)^(5/2)+1/34
020*x*6^(1/2)/(-4*x^2+1)^(3/2)+1/17010*x*6^(1/2)/(-4*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.45

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = -\frac{x(-35+280x^2-896x^4+1024x^6)}{45360\sqrt{6}\,(1-4x^2)^{7/2}}$$

input
```
Integrate[1/((3 - 6*x)^(9/2)*(2 + 4*x)^(9/2)),x]
```

output
```
-1/45360*(x*(-35 + 280*x^2 - 896*x^4 + 1024*x^6))/(Sqrt[6]*(1 - 4*x^2)^(7/
2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {39, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-6x)^{9/2}(4x+2)^{9/2}}dx$$

$$\Big\downarrow \text{39}$$

$$\int \frac{1}{(6-24x^2)^{9/2}}dx$$

$$\Big\downarrow \text{209}$$

$$\frac{1}{7}\int \frac{1}{(6-24x^2)^{7/2}}dx+\frac{x}{9072\sqrt{6}\left(1-4x^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{1}{7}\left(\frac{2}{15}\int \frac{1}{(6-24x^2)^{5/2}}dx+\frac{x}{1080\sqrt{6}\left(1-4x^2\right)^{5/2}}\right)+\frac{x}{9072\sqrt{6}\left(1-4x^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{1}{7}\left(\frac{2}{15}\left(\frac{1}{9}\int \frac{1}{(6-24x^2)^{3/2}}dx+\frac{x}{108\sqrt{6}\left(1-4x^2\right)^{3/2}}\right)+\frac{x}{1080\sqrt{6}\left(1-4x^2\right)^{5/2}}\right)+\frac{x}{9072\sqrt{6}\left(1-4x^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{9072\sqrt{6}\left(1-4x^2\right)^{7/2}}+\frac{1}{7}\left(\frac{x}{1080\sqrt{6}\left(1-4x^2\right)^{5/2}}+\frac{2}{15}\left(\frac{x}{54\sqrt{6}\sqrt{1-4x^2}}+\frac{x}{108\sqrt{6}\left(1-4x^2\right)^{3/2}}\right)\right)$$

input
```
Int[1/((3 - 6*x)^(9/2)*(2 + 4*x)^(9/2)),x]
```

output
```
x/(9072*Sqrt[6]*(1 - 4*x^2)^(7/2)) + (x/(1080*Sqrt[6]*(1 - 4*x^2)^(5/2)) +
 (2*(x/(108*Sqrt[6]*(1 - 4*x^2)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 4*x^2])))/
15)/7
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.53

method	result
gosper	$\frac{(-1+2x)(1+2x)x(1024x^6-896x^4+280x^2-35)}{35(3-6x)^{\frac{9}{2}}(2+4x)^{\frac{9}{2}}}$
orering	$\frac{(-1+2x)(1+2x)x(1024x^6-896x^4+280x^2-35)}{35(3-6x)^{\frac{9}{2}}(2+4x)^{\frac{9}{2}}}$
default	$\frac{1}{84(3-6x)^{\frac{7}{2}}(2+4x)^{\frac{7}{2}}}+\frac{1}{180(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{7}{2}}}+\frac{1}{270(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{7}{2}}}+\frac{1}{162\sqrt{3-6x}\,(2+4x)^{\frac{7}{2}}}-\frac{2\sqrt{3-6x}}{1701(2+4x)^{\frac{7}{2}}}-\frac{\sqrt{3-}}{2835(2-}$

input
```
int(1/(3-6*x)^(9/2)/(2+4*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(-1+2*x)*(1+2*x)*x*(1024*x^6-896*x^4+280*x^2-35)/(3-6*x)^(9/2)/(2+4*x
)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = -\frac{(1024\,x^7-896\,x^5+280\,x^3-35\,x)\sqrt{4\,x+2}\sqrt{-6\,x+3}}{272160\,(256\,x^8-256\,x^6+96\,x^4-16\,x^2+1)}$$

input
```
integrate(1/(3-6*x)^(9/2)/(2+4*x)^(9/2),x, algorithm="fricas")
```

output
```
-1/272160*(1024*x^7 - 896*x^5 + 280*x^3 - 35*x)*sqrt(4*x + 2)*sqrt(-6*x +
3)/(256*x^8 - 256*x^6 + 96*x^4 - 16*x^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(3-6*x)**(9/2)/(2+4*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = \frac{x}{2835\,\sqrt{-24\,x^2+6}} + \frac{x}{945\,(-24\,x^2+6)^{\frac{3}{2}}} + \frac{x}{210\,(-24\,x^2+6)^{\frac{5}{2}}} + \frac{x}{42\,(-24\,x^2+6)^{\frac{7}{2}}}$$

input
```
integrate(1/(3-6*x)^(9/2)/(2+4*x)^(9/2),x, algorithm="maxima")
```

output
```
1/2835*x/sqrt(-24*x^2 + 6) + 1/945*x/(-24*x^2 + 6)^(3/2) + 1/210*x/(-24*x^
2 + 6)^(5/2) + 1/42*x/(-24*x^2 + 6)^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(61) = 122$.

Time = 0.17 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.19

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = \frac{\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{7}}{1783627776\,(2\,x+1)^{\frac{7}{2}}} + \frac{23\,\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{5}}{1274019840\,(2\,x+1)^{\frac{5}{2}}} + \frac{29\,\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{3}}{84934656\,(2\,x+1)^{\frac{3}{2}}} + \frac{1955\,\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)}{254803968\,\sqrt{2\,x+1}} - \frac{\left(\left(\left(256\,\sqrt{6}(2\,x+1)-1617\,\sqrt{6}\right)(2\,x+1)+3430\,\sqrt{6}\right)(2\,x+1)-2450\,\sqrt{6}\right)\sqrt{2\,x+1}\sqrt{-2\,x+1}}{17418240\,(2\,x-1)^{4}} - \frac{\sqrt{6}\left(\frac{68425\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{6}}{(2\,x+1)^{3}} + \frac{3045\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{4}}{(2\,x+1)^{2}} + \frac{161\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{2}}{2\,x+1} + 5\right)(2\,x+1)^{\frac{7}{2}}}{8918138880\left(\sqrt{2}-\sqrt{-2\,x+1}\right)^{7}}$$

input
```
integrate(1/(3-6*x)^(9/2)/(2+4*x)^(9/2),x, algorithm="giac")
```

output
```
1/1783627776*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))^7/(2*x + 1)^(7/2) + 23/127
4019840*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))^5/(2*x + 1)^(5/2) + 29/84934656
*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))^3/(2*x + 1)^(3/2) + 1955/254803968*sqr
t(6)*(sqrt(2) - sqrt(-2*x + 1))/sqrt(2*x + 1) - 1/17418240*(((256*sqrt(6)*
(2*x + 1) - 1617*sqrt(6))*(2*x + 1) + 3430*sqrt(6))*(2*x + 1) - 2450*sqrt(
6))*sqrt(2*x + 1)*sqrt(-2*x + 1)/(2*x - 1)^4 - 1/8918138880*sqrt(6)*(68425
*(sqrt(2) - sqrt(-2*x + 1))^6/(2*x + 1)^3 + 3045*(sqrt(2) - sqrt(-2*x + 1)
)^4/(2*x + 1)^2 + 161*(sqrt(2) - sqrt(-2*x + 1))^2/(2*x + 1) + 5)*(2*x + 1
)^(7/2)/(sqrt(2) - sqrt(-2*x + 1))^7
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = \frac{35\,x\,\sqrt{3-6\,x}-280\,x^{3}\,\sqrt{3-6\,x}+896\,x^{5}\,\sqrt{3-6\,x}-1024\,x^{7}\,\sqrt{3-6\,x}}{\left(\left(\left(6\,x-3\right)\,\left(\frac{1120\,x}{3}+\frac{2800}{3}\right)+6720\right)\,\left(6\,x-3\right)+13440\right)\,\sqrt{4\,x+2}\,\left(6\,x-3\right)^{4}}$$

input
```
int(1/((4*x + 2)^(9/2)*(3 - 6*x)^(9/2)),x)
```

output
```
(35*x*(3 - 6*x)^(1/2) - 280*x^3*(3 - 6*x)^(1/2) + 896*x^5*(3 - 6*x)^(1/2)
- 1024*x^7*(3 - 6*x)^(1/2))/((((6*x - 3)*((1120*x)/3 + 2800/3) + 6720)*(6*
x - 3) + 13440)*(4*x + 2)^(1/2)*(6*x - 3)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{1}{(3-6x)^{9/2}(2+4x)^{9/2}}\,dx = \frac{\sqrt{6}\,x(1024x^6-896x^4+280x^2-35)}{272160\sqrt{2x+1}\,\sqrt{-2x+1}\,(64x^6-48x^4+12x^2-1)}$$

input
```
int(1/(3-6*x)^(9/2)/(2+4*x)^(9/2),x)
```

output
```
(sqrt(6)*x*(1024*x**6 - 896*x**4 + 280*x**2 - 35))/(272160*sqrt(2*x + 1)*s
qrt( - 2*x + 1)*(64*x**6 - 48*x**4 + 12*x**2 - 1))
```

3.59 $\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{9\sqrt{9-x^2}}$$

output
```
1/9*x/(-x^2+9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{9\sqrt{9-x^2}}$$

input
```
Integrate[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

output
```
x/(9*Sqrt[9 - x^2])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-x)^{3/2}(x+3)^{3/2}}\,dx$$
$$\Big\downarrow \ 39$$
$$\int \frac{1}{(9-x^2)^{3/2}}dx$$
$$\Big\downarrow \ 208$$
$$\frac{x}{9\sqrt{9-x^2}}$$

input
```
Int[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

output
```
x/(9*Sqrt[9 - x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x}{9\sqrt{3-x}\,\sqrt{3+x}}$	16
orering	$-\frac{(-3+x)x}{9\sqrt{3+x}\,(3-x)^{\frac{3}{2}}}$	19
default	$\frac{1}{3\sqrt{3-x}\,\sqrt{3+x}}-\frac{\sqrt{3-x}}{9\sqrt{3+x}}$	30
risch	$\frac{\sqrt{(3+x)(3-x)}\,x}{9\sqrt{3-x}\,\sqrt{3+x}\,\sqrt{-(-3+x)(3+x)}}$	37

input
```
int(1/(3-x)^(3/2)/(3+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x/(3-x)^(1/2)/(3+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = -\frac{\sqrt{x+3}x\sqrt{-x+3}}{9\,(x^2-9)}$$

input
```
integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(x + 3)*x*sqrt(-x + 3)/(x^2 - 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \begin{cases} -\frac{\sqrt{-1+\frac{6}{x+3}}(x+3)}{9x-27} + \frac{3\sqrt{-1+\frac{6}{x+3}}}{9x-27} & \text{for } \frac{1}{|x+3|} > \frac{1}{6} \\ -\frac{i}{9\sqrt{1-\frac{6}{x+3}}} + \frac{i}{3\sqrt{1-\frac{6}{x+3}}(x+3)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3-x)**(3/2)/(3+x)**(3/2),x)
```

output
```
Piecewise((-sqrt(-1 + 6/(x + 3))*(x + 3)/(9*x - 27) + 3*sqrt(-1 + 6/(x + 3
))/(9*x - 27), 1/Abs(x + 3) > 1/6), (-I/(9*sqrt(1 - 6/(x + 3))) + I/(3*sqr
t(1 - 6/(x + 3))*(x + 3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{9\sqrt{-x^2+9}}$$

input
```
integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

output
```
1/9*x/sqrt(-x^2 + 9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.88

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{\sqrt{6}-\sqrt{-x+3}}{36\sqrt{x+3}} - \frac{\sqrt{x+3}\sqrt{-x+3}}{18\,(x-3)} - \frac{\sqrt{x+3}}{36\,\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

input
```
integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

output
```
1/36*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/18*sqrt(x + 3)*sqrt(-x + 3)/
(x - 3) - 1/36*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = -\frac{x\sqrt{3-x}}{(9\,x-27)\,\sqrt{x+3}}$$

input
```
int(1/((3 - x)^(3/2)*(x + 3)^(3/2)),x)
```

output
```
-(x*(3 - x)^(1/2))/((9*x - 27)*(x + 3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{9\sqrt{x+3}\,\sqrt{-x+3}}$$

input
```
int(1/(3-x)^(3/2)/(3+x)^(3/2),x)
```

output
```
x/(9*sqrt(x + 3)*sqrt( - x + 3))
```

3.60 $\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{9\sqrt{9-b^2x^2}}$$

output
```
1/9*x/(-b^2*x^2+9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{9\sqrt{9-b^2x^2}}$$

input
```
Integrate[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

output
```
x/(9*Sqrt[9 - b^2*x^2])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3-bx)^{3/2}(bx+3)^{3/2}}dx$$
$$\Big\downarrow \text{39}$$
$$\int \frac{1}{(9-b^2x^2)^{3/2}}dx$$
$$\Big\downarrow \text{208}$$
$$\frac{x}{9\sqrt{9-b^2x^2}}$$

input
```
Int[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

output
```
x/(9*Sqrt[9 - b^2*x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x}{9\sqrt{-bx+3}\,\sqrt{bx+3}}$	19
orering	$-\frac{(bx-3)x}{9\sqrt{bx+3}\,(-bx+3)^{\frac{3}{2}}}$	24
default	$\frac{1}{3b\sqrt{-bx+3}\,\sqrt{bx+3}}-\frac{\sqrt{-bx+3}}{9b\sqrt{bx+3}}$	42

input
```
int(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x/(-b*x+3)^(1/2)/(b*x+3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = -\frac{\sqrt{bx+3}\sqrt{-bx+3}x}{9\,(b^2x^2-9)}$$

input
```
integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(b*x + 3)*sqrt(-b*x + 3)*x/(b^2*x^2 - 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = -\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix}\middle|\frac{9}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}$$

input
```
integrate(1/(-b*x+3)**(3/2)/(b*x+3)**(3/2),x)
```

output
```
-I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
9/(b**2*x**2))/(18*pi**(3/2)*b) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()
), ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*x**2))/(18*pi
**(3/2)*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{9\sqrt{-b^2x^2+9}}$$

input
```
integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

output
```
1/9*x/sqrt(-b^2*x^2 + 9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.05

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{\frac{\sqrt{6}-\sqrt{-bx+3}}{\sqrt{bx+3}} - \frac{2\sqrt{bx+3}\sqrt{-bx+3}}{bx-3} - \frac{\sqrt{bx+3}}{\sqrt{6}-\sqrt{-bx+3}}}{36\,b}$$

input
```
integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

output
```
1/36*((sqrt(6) - sqrt(-b*x + 3))/sqrt(b*x + 3) - 2*sqrt(b*x + 3)*sqrt(-b*x
 + 3)/(b*x - 3) - sqrt(b*x + 3)/(sqrt(6) - sqrt(-b*x + 3)))/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = -\frac{x\sqrt{3-b\,x}}{\sqrt{b\,x+3}\,(9\,b\,x-27)}$$

input
```
int(1/((3 - b*x)^(3/2)*(b*x + 3)^(3/2)),x)
```

output
```
-(x*(3 - b*x)^(1/2))/((b*x + 3)^(1/2)*(9*b*x - 27))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{9\sqrt{bx+3}\,\sqrt{-bx+3}}$$

input
```
int(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x)
```

output
```
x/(9*sqrt(b*x + 3)*sqrt( - b*x + 3))
```

3.61 $\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}} dx = \frac{x}{18\sqrt{2}\sqrt{9-x^2}}$$

output
```
1/36*x*2^(1/2)/(-x^2+9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}} dx = \frac{x}{18\sqrt{2}\sqrt{9-x^2}}$$

input
```
Integrate[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

output
```
x/(18*Sqrt[2]*Sqrt[9 - x^2])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(6-2x)^{3/2}(x+3)^{3/2}}\,dx$$

$$\Big\downarrow \; 39$$

$$\int \frac{1}{(18-2x^2)^{3/2}}dx$$

$$\Big\downarrow \; 208$$

$$\frac{x}{18\sqrt{2}\sqrt{9-x^2}}$$

input
```
Int[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

output
```
x/(18*Sqrt[2]*Sqrt[9 - x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{(-3+x)x}{9\sqrt{3+x}\,(6-2x)^{\frac{3}{2}}}$	19
orering	$-\frac{(-3+x)x}{9\sqrt{3+x}\,(6-2x)^{\frac{3}{2}}}$	19
default	$\frac{1}{6\sqrt{6-2x}\,\sqrt{3+x}}-\frac{\sqrt{6-2x}}{36\sqrt{3+x}}$	30
risch	$\frac{\sqrt{(3+x)(6-2x)}\,\sqrt{2}\,x}{36\sqrt{3+x}\,\sqrt{6-2x}\,\sqrt{-(-3+x)(3+x)}}$	40

input
```
int(1/(6-2*x)^(3/2)/(3+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3+x)/(3+x)^(1/2)*x/(6-2*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = -\frac{\sqrt{x+3}x\sqrt{-2\,x+6}}{36\,(x^2-9)}$$

input
```
integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/36*sqrt(x + 3)*x*sqrt(-2*x + 6)/(x^2 - 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.38

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \begin{cases} -\frac{\sqrt{2}\sqrt{-1+\frac{6}{x+3}}(x+3)}{36x-108} + \frac{3\sqrt{2}\sqrt{-1+\frac{6}{x+3}}}{36x-108} & \text{for } \frac{1}{|x+3|} > \frac{1}{6} \\ -\frac{\sqrt{2}i}{36\sqrt{1-\frac{6}{x+3}}} + \frac{\sqrt{2}i}{12\sqrt{1-\frac{6}{x+3}}(x+3)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(6-2*x)**(3/2)/(3+x)**(3/2),x)
```

output
```
Piecewise((-sqrt(2)*sqrt(-1 + 6/(x + 3))*(x + 3)/(36*x - 108) + 3*sqrt(2)*
sqrt(-1 + 6/(x + 3))/(36*x - 108), 1/Abs(x + 3) > 1/6), (-sqrt(2)*I/(36*sq
rt(1 - 6/(x + 3))) + sqrt(2)*I/(12*sqrt(1 - 6/(x + 3))*(x + 3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{18\sqrt{-2x^2+18}}$$

input
```
integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

output
```
1/18*x/sqrt(-2*x^2 + 18)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.38

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-x+3}\right)}{144\sqrt{x+3}} - \frac{\sqrt{2}\sqrt{x+3}\sqrt{-x+3}}{72\,(x-3)} - \frac{\sqrt{2}\sqrt{x+3}}{144\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

input
```
integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

output
```
1/144*sqrt(2)*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/72*sqrt(2)*sqrt(x +
 3)*sqrt(-x + 3)/(x - 3) - 1/144*sqrt(2)*sqrt(x + 3)/(sqrt(6) - sqrt(-x +
3))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = -\frac{x\sqrt{6-2x}}{(36x-108)\sqrt{x+3}}$$

input
```
int(1/((6 - 2*x)^(3/2)*(x + 3)^(3/2)),x)
```

output
```
-(x*(6 - 2*x)^(1/2))/((36*x - 108)*(x + 3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{\sqrt{2}\,x}{36\sqrt{x+3}\,\sqrt{-x+3}}$$

input
```
int(1/(6-2*x)^(3/2)/(3+x)^(3/2),x)
```

output
```
(sqrt(2)*x)/(36*sqrt(x + 3)*sqrt( - x + 3))
```

3.62 $\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx=\frac{x}{18\sqrt{2}\sqrt{9-b^2x^2}}$$

output
```
1/36*x*2^(1/2)/(-b^2*x^2+9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx=\frac{x}{18\sqrt{18-2b^2x^2}}$$

input
```
Integrate[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

output
```
x/(18*Sqrt[18 - 2*b^2*x^2])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(6-2bx)^{3/2}(bx+3)^{3/2}}\,dx$$
$$\Big\downarrow \; 39$$
$$\int \frac{1}{(18-2b^2x^2)^{3/2}}dx$$
$$\Big\downarrow \; 208$$
$$\frac{x}{18\sqrt{2}\sqrt{9-b^2x^2}}$$

input
```
Int[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

output
```
x/(18*Sqrt[2]*Sqrt[9 - b^2*x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{(bx-3)x}{9\sqrt{bx+3}\,(-2bx+6)^{\frac{3}{2}}}$	24
orering	$-\frac{(bx-3)x}{9\sqrt{bx+3}\,(-2bx+6)^{\frac{3}{2}}}$	24
default	$\frac{1}{6b\sqrt{-2bx+6}\,\sqrt{bx+3}}-\frac{\sqrt{-2bx+6}}{36b\sqrt{bx+3}}$	42

input
```
int(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(b*x-3)/(b*x+3)^(1/2)*x/(-2*b*x+6)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx=-\frac{\sqrt{bx+3}\sqrt{-2\,bx+6}x}{36\,(b^2x^2-9)}$$

input
```
integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/36*sqrt(b*x + 3)*sqrt(-2*b*x + 6)*x/(b^2*x^2 - 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.46

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx = -\frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix}\middle|\frac{9}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b} + \frac{\sqrt{2}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b}$$

input
```
integrate(1/(-2*b*x+6)**(3/2)/(b*x+3)**(3/2),x)
```

output
```
-sqrt(2)*I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2),
 (0,)), 9/(b**2*x**2))/(72*pi**(3/2)*b) + sqrt(2)*meijerg(((-1/2, 0, 1/4,
1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b*
*2*x**2))/(72*pi**(3/2)*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{18\sqrt{-2\,b^2x^2+18}}$$

input
```
integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

output
```
1/18*x/sqrt(-2*b^2*x^2 + 18)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.58

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-bx+3}\right)}{\sqrt{bx+3}} - \frac{2\sqrt{2}\sqrt{bx+3}\sqrt{-bx+3}}{bx-3} - \frac{\sqrt{2}\sqrt{bx+3}}{\sqrt{6}-\sqrt{-bx+3}}}{144\,b}$$

input
```
integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

output
```
1/144*(sqrt(2)*(sqrt(6) - sqrt(-b*x + 3))/sqrt(b*x + 3) - 2*sqrt(2)*sqrt(b
*x + 3)*sqrt(-b*x + 3)/(b*x - 3) - sqrt(2)*sqrt(b*x + 3)/(sqrt(6) - sqrt(-
b*x + 3)))/b
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx = -\frac{x\sqrt{6-2\,b\,x}}{\sqrt{b\,x+3}\,\left(36\,b\,x-108\right)}$$

input
```
int(1/((b*x + 3)^(3/2)*(6 - 2*b*x)^(3/2)),x)
```

output
```
-(x*(6 - 2*b*x)^(1/2))/((b*x + 3)^(1/2)*(36*b*x - 108))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{\sqrt{2}\,x}{36\sqrt{bx+3}\,\sqrt{-bx+3}}$$

input
```
int(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x)
```

output
```
(sqrt(2)*x)/(36*sqrt(b*x + 3)*sqrt( - b*x + 3))
```

3.63 $\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}$$

output
```
2*arctanh(d^(1/2)*(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2))/b/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}$$

input
```
Integrate[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

output
```
(2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {45, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{bdx-ad}}\,dx$$

$$\Big\downarrow \; 45$$

$$2\int \frac{1}{b-\frac{bd(a+bx)}{bdx-ad}}\,d\frac{\sqrt{a+bx}}{\sqrt{bdx-ad}}$$

$$\Big\downarrow \; 221$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

input
```
Int[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

output
```
(2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\sqrt{(bx+a)(bdx-ad)}\,\ln\left(\frac{b^2dx}{\sqrt{d\,b^2}}+\sqrt{b^2d\,x^2-d\,a^2}\right)}{\sqrt{bx+a}\,\sqrt{bdx-ad}\,\sqrt{d\,b^2}}$	76

input
```
int(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*(b*d*x-a*d))^(1/2)/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2)*ln(b^2*d*x/(d*
b^2)^(1/2)+(b^2*d*x^2-a^2*d)^(1/2))/(d*b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \left[\frac{\log\left(2\,b^2dx^2+2\,\sqrt{bdx-ad}\sqrt{bx+a}b\sqrt{d}x-a^2d\right)}{2\,b\sqrt{d}},\; -\frac{\sqrt{-d}\arctan\left(\frac{\sqrt{bdx-ad}\sqrt{bx+a}b\sqrt{-d}x}{b^2dx^2-a^2d}\right)}{bd}\right]$$

input
```
integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(2*b^2*d*x^2 + 2*sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(d)*x - a^2
*d)/(b*sqrt(d)), -sqrt(-d)*arctan(sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(-
d)*x/(b^2*d*x^2 - a^2*d))/(b*d)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}} - \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{a^2e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}}$$

input
```
integrate(1/(b*x+a)**(1/2)/(b*d*x-a*d)**(1/2),x)
```

output
```
meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), a*
*2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(d)) - I*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(2*I*pi)/(b**2
*x**2))/(4*pi**(3/2)*b*sqrt(d))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \frac{\log\left(2\,b^2dx + 2\,\sqrt{b^2dx^2 - a^2d}b\sqrt{d}\right)}{b\sqrt{d}}$$

input
```
integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b^2*d*x + 2*sqrt(b^2*d*x^2 - a^2*d)*b*sqrt(d))/(b*sqrt(d))
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = -\frac{2\,\log\left(\left|-\sqrt{bx+a}\sqrt{d}+\sqrt{(bx+a)d-2\,ad}\right|\right)}{b\sqrt{d}}$$

input
```
integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="giac")
```

output
```
-2*log(abs(-sqrt(b*x + a)*sqrt(d) + sqrt((b*x + a)*d - 2*a*d)))/(b*sqrt(d)
)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,d\,x-a\,d}-\sqrt{-a\,d}\right)}{\sqrt{-b^2\,d}\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2\,d}}$$

input
```
int(1/((b*d*x - a*d)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
-(4*atan((b*((b*d*x - a*d)^(1/2) - (-a*d)^(1/2)))/((-b^2*d)^(1/2)*((a + b*
x)^(1/2) - a^(1/2)))))/(-b^2*d)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \frac{2\sqrt{d}\log\left(\frac{\sqrt{bx-a}+\sqrt{bx+a}}{\sqrt{a}\sqrt{2}}\right)}{bd}$$

input
```
int(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x)
```

output
```
(2*sqrt(d)*log((sqrt( - a + b*x) + sqrt(a + b*x))/(sqrt(a)*sqrt(2))))/(b*d
)
```

3.64 $\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{\operatorname{arccosh}\left(\frac{bx}{4}\right)}{b}$$

output
```
arccosh(1/4*b*x)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{4+bx}}{\sqrt{-4+bx}}\right)}{b}$$

input
```
Integrate[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

output
```
(2*ArcTanh[Sqrt[4 + b*x]/Sqrt[-4 + b*x]])/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx-4}\sqrt{bx+4}}\,dx$$

$$\Big\downarrow \text{43}$$

$$\frac{\operatorname{arccosh}\left(\frac{bx}{4}\right)}{b}$$

input
```
Int[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

output
```
ArcCosh[(b*x)/4]/b
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(9) = 18.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 5.18

method	result	size
default	$\frac{\sqrt{(bx-4)(bx+4)}\,\ln\left(\frac{b^2 x}{\sqrt{b^2}}+\sqrt{b^2x^2-16}\right)}{\sqrt{bx-4}\,\sqrt{bx+4}\,\sqrt{b^2}}$	57

input
```
int(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x-4)*(b*x+4))^(1/2)/(b*x-4)^(1/2)/(b*x+4)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(
b^2*x^2-16)^(1/2))/(b^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(9) = 18.

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = -\frac{\log\left(-bx+\sqrt{bx+4}\sqrt{bx-4}\right)}{b}$$

input
```
integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

output
```
-log(-b*x + sqrt(b*x + 4)*sqrt(b*x - 4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 6.82

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{16e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{16}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

input
```
integrate(1/(b*x-4)**(1/2)/(b*x+4)**(1/2),x)
```

output
```
meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 16
*exp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4,
0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 16/(b**2*x**2))/(4*p
i**(3/2)*b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2-16}b\right)}{b}$$

input
```
integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b^2*x + 2*sqrt(b^2*x^2 - 16)*b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = -\frac{2\,\log\left(\sqrt{bx+4}-\sqrt{bx-4}\right)}{b}$$

input
```
integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
```

output
```
-2*log(sqrt(b*x + 4) - sqrt(b*x - 4))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.64

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x-4}-2\mathrm{i}\right)}{\left(\sqrt{b\,x+4}-2\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

input
```
int(1/((b*x - 4)^(1/2)*(b*x + 4)^(1/2)),x)
```

output
```
-(4*atan((b*((b*x - 4)^(1/2) - 2i))/(((b*x + 4)^(1/2) - 2)*(-b^2)^(1/2))))
/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{2\log\left(\frac{\sqrt{bx-4}+\sqrt{bx+4}}{2\sqrt{2}}\right)}{b}$$

input
```
int(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x)
```

output
```
(2*log((sqrt(b*x - 4) + sqrt(b*x + 4))/(2*sqrt(2))))/b
```

3.65 $\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 13

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = \frac{\operatorname{arccosh}(bx)}{\sqrt{6}b}$$

output
```
1/6*arccosh(b*x)*6^(1/2)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = \frac{\sqrt{\frac{2}{3}}\operatorname{arctanh}\left(\frac{\sqrt{-1+bx}}{\sqrt{1+bx}}\right)}{b}$$

input
```
Integrate[1/(Sqrt[6]*Sqrt[-1 + b*x]*Sqrt[1 + b*x]),x]
```

output
```
(Sqrt[2/3]*ArcTanh[Sqrt[-1 + b*x]/Sqrt[1 + b*x]])/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {27, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{6}\sqrt{bx-1}\sqrt{bx+1}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{\sqrt{bx-1}\sqrt{bx+1}}dx}{\sqrt{6}}$$

$$\Big\downarrow \; 43$$

$$\frac{\operatorname{arccosh}(bx)}{\sqrt{6}b}$$

input
```
Int[1/(Sqrt[6]*Sqrt[-1 + b*x]*Sqrt[1 + b*x]),x]
```

output
```
ArcCosh[b*x]/(Sqrt[6]*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(12) = 24$.

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.69

method	result	size
default	$\frac{\sqrt{6}\,\sqrt{(bx-1)(bx+1)}\,\ln\left(\frac{b^2 x}{\sqrt{b^2}}+\sqrt{b^2x^2-1}\right)}{6\sqrt{bx-1}\,\sqrt{bx+1}\,\sqrt{b^2}}$	61

input
```
int(1/6*6^(1/2)/(b*x-1)^(1/2)/(b*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*6^(1/2)*((b*x-1)*(b*x+1))^(1/2)/(b*x-1)^(1/2)/(b*x+1)^(1/2)*ln(b^2*x/(
b^2)^(1/2)+(b^2*x^2-1)^(1/2))/(b^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = -\frac{\sqrt{6}\log\left(-bx+\sqrt{bx+1}\sqrt{bx-1}\right)}{6\,b}$$

input
```
integrate(1/6*6^(1/2)/(b*x-1)^(1/2)/(b*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(6)*log(-b*x + sqrt(b*x + 1)*sqrt(b*x - 1))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 6.15

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = \frac{\sqrt{6}\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b} - \frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}\right)}{6}$$

input
```
integrate(1/6*6**(1/2)/(b*x-1)**(1/2)/(b*x+1)**(1/2),x)
```

output
```
sqrt(6)*(meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0)
, ()), 1/(b**2*x**2))/(4*pi**(3/2)*b) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(b**2*x**2))
/(4*pi**(3/2)*b))/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = \frac{\sqrt{6}\log\left(2\,b^2x + 2\,\sqrt{b^2x^2-1}b\right)}{6\,b}$$

input
```
integrate(1/6*6^(1/2)/(b*x-1)^(1/2)/(b*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(6)*log(2*b^2*x + 2*sqrt(b^2*x^2 - 1)*b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = -\frac{\sqrt{6}\log\left(\sqrt{bx+1}-\sqrt{bx-1}\right)}{3\,b}$$

input
```
integrate(1/6*6^(1/2)/(b*x-1)^(1/2)/(b*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(6)*log(sqrt(b*x + 1) - sqrt(b*x - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.31

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = -\frac{2\,\sqrt{6}\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{b\,x+1}-1\right)\,\sqrt{-b^2}}\right)}{3\,\sqrt{-b^2}}$$

input
```
int(6^(1/2)/(6*(b*x - 1)^(1/2)*(b*x + 1)^(1/2)),x)
```

output
```
-(2*6^(1/2)*atan((b*((b*x - 1)^(1/2) - 1i))/(((b*x + 1)^(1/2) - 1)*(-b^2)^
(1/2))))/(3*(-b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{6}\sqrt{-1+bx}\sqrt{1+bx}}\,dx = \frac{\sqrt{6}\log\left(\frac{\sqrt{bx-1}+\sqrt{bx+1}}{\sqrt{2}}\right)}{3b}$$

input
```
int(1/6*6^(1/2)/(b*x-1)^(1/2)/(b*x+1)^(1/2),x)
```

output
```
(sqrt(6)*log((sqrt(b*x - 1) + sqrt(b*x + 1))/sqrt(2)))/(3*b)
```

3.66 $\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx=\frac{\operatorname{arccosh}(bx)}{\sqrt{6}b}$$

output
```
1/6*arccosh(b*x)*6^(1/2)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx=\frac{\sqrt{\frac{2}{3}}\operatorname{arctanh}\left(\frac{\sqrt{-1+bx}}{\sqrt{1+bx}}\right)}{b}$$

input
```
Integrate[1/(Sqrt[2 + 2*b*x]*Sqrt[-3 + 3*b*x]),x]
```

output
```
(Sqrt[2/3]*ArcTanh[Sqrt[-1 + b*x]/Sqrt[1 + b*x]])/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{43\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2bx+2}\sqrt{3bx-3}}dx$$

$$\Big\downarrow \; 43$$

$$\frac{\text{arccosh}(bx)}{\sqrt{6}b}$$

input
```
Int[1/(Sqrt[2 + 2*b*x]*Sqrt[-3 + 3*b*x]),x]
```

output
```
ArcCosh[b*x]/(Sqrt[6]*b)
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.31

method	result	size
default	$\frac{\sqrt{(2bx+2)(3bx-3)}\,\ln\left(\frac{b^2x\sqrt{6}}{\sqrt{b^2}}+\sqrt{6b^2x^2-6}\right)\sqrt{6}}{6\sqrt{2bx+2}\,\sqrt{3bx-3}\,\sqrt{b^2}}$	69

input
```
int(1/(2*b*x+2)^(1/2)/(3*b*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*b*x+2)*(3*b*x-3))^(1/2)/(2*b*x+2)^(1/2)/(3*b*x-3)^(1/2)*ln(b^2*x*6
^(1/2)/(b^2)^(1/2)+(6*b^2*x^2-6)^(1/2))*6^(1/2)/(b^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(12) = 24$.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx = \frac{\sqrt{\frac{2}{3}}\log\left(2\,b^2x^2+\sqrt{\frac{2}{3}}\sqrt{3\,bx-3}\sqrt{2\,bx+2}bx-1\right)}{4\,b}$$

input
```
integrate(1/(2*b*x+2)^(1/2)/(3*b*x-3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2/3)*log(2*b^2*x^2 + sqrt(2/3)*sqrt(3*b*x - 3)*sqrt(2*b*x + 2)*b*
x - 1)/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.82 (sec) , antiderivative size = 83, normalized size of antiderivative = 6.38

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx = \frac{\sqrt{6}G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| {\frac{e^{2i\pi}}{b^2x^2}} \right)}{24\pi^{\frac{3}{2}}b} + \frac{\sqrt{6}iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| {\frac{1}{b^2x^2}} \right)}{24\pi^{\frac{3}{2}}b}$$

input
```
integrate(1/(2*b*x+2)**(1/2)/(3*b*x-3)**(1/2),x)
```

output
```
sqrt(6)*meijerg((((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), exp_polar(2*I*pi)/(b**2*x**2))/(24*pi**(3/2)*b) + sqrt(6)*I*meijerg(
((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 1/(b**
2*x**2))/(24*pi**(3/2)*b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx = \frac{\sqrt{6}\log\left(12\,b^2x+2\,\sqrt{6}\sqrt{6\,b^2x^2-6}b\right)}{6\,b}$$

input
```
integrate(1/(2*b*x+2)^(1/2)/(3*b*x-3)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(6)*log(12*b^2*x + 2*sqrt(6)*sqrt(6*b^2*x^2 - 6)*b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx = -\frac{\sqrt{3}\sqrt{2}\log\left(\sqrt{bx+1}-\sqrt{bx-1}\right)}{3\,b}$$

input
```
integrate(1/(2*b*x+2)^(1/2)/(3*b*x-3)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*sqrt(2)*log(sqrt(b*x + 1) - sqrt(b*x - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.54

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx = -\frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,b\left(-\sqrt{3\,b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{3\left(\sqrt{2}-\sqrt{2\,b\,x+2}\right)\sqrt{-b^2}}\right)}{3\sqrt{-b^2}}$$

input
```
int(1/((2*b*x + 2)^(1/2)*(3*b*x - 3)^(1/2)),x)
```

output
```
-(2*6^(1/2)*atan((6^(1/2)*b*(3^(1/2)*1i - (3*b*x - 3)^(1/2)))/(3*(2^(1/2)
- (2*b*x + 2)^(1/2))*(-b^2)^(1/2))))/(3*(-b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{2+2bx}\sqrt{-3+3bx}}\,dx = \frac{\sqrt{6}\log\left(\frac{\sqrt{bx-1}+\sqrt{bx+1}}{\sqrt{2}}\right)}{3b}$$

input
```
int(1/(2*b*x+2)^(1/2)/(3*b*x-3)^(1/2),x)
```

output
```
(sqrt(6)*log((sqrt(b*x - 1) + sqrt(b*x + 1))/sqrt(2)))/(3*b)
```

3.67 $\int (1-x)^{9/2}\sqrt{1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{16}x\sqrt{1-x^2} + \frac{7}{8}\left(1-x^2\right)^{3/2} + \frac{21\arcsin(x)}{16}$$

output
```
21/40*(1-x)^(5/2)*(1+x)^(3/2)+3/10*(1-x)^(7/2)*(1+x)^(3/2)+1/6*(1-x)^(9/2)
*(1+x)^(3/2)+21/16*x*(-x^2+1)^(1/2)+7/8*(-x^2+1)^(3/2)+21/16*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = \frac{1}{240}\sqrt{1-x^2}\left(448-75x-256x^2+350x^3-192x^4+40x^5\right) - \frac{21}{8}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

output
```
(Sqrt[1 - x^2]*(448 - 75*x - 256*x^2 + 350*x^3 - 192*x^4 + 40*x^5))/240 -
(21*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {59, 59, 59, 50, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{9/2}\sqrt{x+1}\,dx$$

$$\Big\downarrow \; 59$$

$$\frac{3}{2}\int (1-x)^{7/2}\sqrt{x+1}dx+\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}$$

$$\Big\downarrow \; 59$$

$$\frac{3}{2}\left(\frac{7}{5}\int (1-x)^{5/2}\sqrt{x+1}dx+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}\right)+\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}$$

$$\Big\downarrow \; 59$$

$$\frac{3}{2}\left(\frac{7}{5}\left(\frac{5}{4}\int (1-x)^{3/2}\sqrt{x+1}dx+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}\right)+\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}$$

$$\Big\downarrow \; 50$$

$$\frac{3}{2}\left(\frac{7}{5}\left(\frac{5}{4}\left(\int \sqrt{1-x^2}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}\right)+\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}$$

$$\Big\downarrow \; 211$$

$$\frac{3}{2}\left(\frac{7}{5}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}\right)+\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{3}{2}\left(\frac{7}{5}\left(\frac{5}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{3}(1-x^2)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}\right)+\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}$$

input
```
Int[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

output
```
((1 - x)^(9/2)*(1 + x)^(3/2))/6 + (3*(((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7
*(((1 - x)^(5/2)*(1 + x)^(3/2))/4 + (5*((x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3
/2)/3 + ArcSin[x]/2))/4))/5))/2
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(40x^5-192x^4+350x^3-256x^2-75x+448)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{240\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{21\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{3}{2}}}{6}+\frac{3(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}{10}+\frac{21(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{40}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{8}+\frac{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{16}-\frac{21\sqrt{1-x}\,\sqrt{1+x}}{16}+$ 2

input
```
int((1-x)^(9/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*x^5-192*x^4+350*x^3-256*x^2-75*x+448)*(1+x)^(1/2)*(-1+x)/(-(1+x
)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+21/16*((1+x)*(1-x))^(1/2)/
(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx=\frac{1}{240}\left(40\,x^5-192\,x^4+350\,x^3-256\,x^2-75\,x+448\right)\sqrt{x+1}\sqrt{-x+1}-\frac{21}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/240*(40*x^5 - 192*x^4 + 350*x^3 - 256*x^2 - 75*x + 448)*sqrt(x + 1)*sqrt
(-x + 1) - 21/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 60.36 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.93

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = \begin{cases} -\frac{21i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} - \frac{59i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}} + \frac{1151i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}} - \frac{2947i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}} + \frac{8171i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}} - 1 \\ \frac{21\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} + \frac{59(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}} - \frac{1151(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}} + \frac{2947(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}} - \frac{8171(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}} + \frac{1045(x+1}{48\sqrt{1-x}} \end{cases}$$

input
```
integrate((1-x)**(9/2)*(1+x)**(1/2),x)
```

output
```
Piecewise((-21*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqr
t(x - 1)) - 59*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) + 1151*I*(x + 1)**(9/2)/
(120*sqrt(x - 1)) - 2947*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) + 8171*I*(x +
1)**(5/2)/(240*sqrt(x - 1)) - 1045*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 21*
I*sqrt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1) > 2), (21*asin(sqrt(2)*sqrt(x +
1)/2)/8 - (x + 1)**(13/2)/(6*sqrt(1 - x)) + 59*(x + 1)**(11/2)/(30*sqrt(1
- x)) - 1151*(x + 1)**(9/2)/(120*sqrt(1 - x)) + 2947*(x + 1)**(7/2)/(120*s
qrt(1 - x)) - 8171*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 1045*(x + 1)**(3/2)/
(48*sqrt(1 - x)) - 21*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = -\frac{1}{6}\left(-x^2+1\right)^{\frac{3}{2}}x^3 + \frac{4}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{13}{8}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{28}{15}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{21}{16}\sqrt{-x^2+1}x + \frac{21}{16}\arcsin(x)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(-x^2 + 1)^(3/2)*x^3 + 4/5*(-x^2 + 1)^(3/2)*x^2 - 13/8*(-x^2 + 1)^(3/
2)*x + 28/15*(-x^2 + 1)^(3/2) + 21/16*sqrt(-x^2 + 1)*x + 21/16*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(70) = 140$.

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.89

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = \frac{1}{240}\left(\left(2\left(\left(4\left(5x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+}$$
$$-\frac{1}{40}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}$$
$$+\frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}$$
$$+\frac{1}{3}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{2}\sqrt{x+1}(x-2)\sqrt{-x+1}$$
$$+\sqrt{x+1}\sqrt{-x+1}+\frac{21}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*(3*(4*x - 17)*(x + 1) + 133
)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x -
 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) + 1/3*((2*x - 5)
*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x - 2)*sqrt(-x +
 1) + sqrt(x + 1)*sqrt(-x + 1) + 21/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = \int (1-x)^{9/2}\,\sqrt{x+1}\,dx$$

input
```
int((1 - x)^(9/2)*(x + 1)^(1/2),x)
```

output
```
int((1 - x)^(9/2)*(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = -\frac{21asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^5}{6} - \frac{4\sqrt{x+1}\,\sqrt{1-x}\,x^4}{5} + \frac{35\sqrt{x+1}\,\sqrt{1-x}\,x^3}{24} - \frac{16\sqrt{x+1}\,\sqrt{1-x}\,x^2}{15} - \frac{5\sqrt{x+1}\,\sqrt{1-x}\,x}{16} + \frac{28\sqrt{x+1}\,\sqrt{1-x}}{15}$$

input

```
int((1-x)^(9/2)*(1+x)^(1/2),x)
```

output

```
( - 630*asin(sqrt( - x + 1)/sqrt(2)) + 40*sqrt(x + 1)*sqrt( - x + 1)*x**5
- 192*sqrt(x + 1)*sqrt( - x + 1)*x**4 + 350*sqrt(x + 1)*sqrt( - x + 1)*x**
3 - 256*sqrt(x + 1)*sqrt( - x + 1)*x**2 - 75*sqrt(x + 1)*sqrt( - x + 1)*x
+ 448*sqrt(x + 1)*sqrt( - x + 1))/240
```

3.68 $\int (1-x)^{7/2}\sqrt{1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{8}x\sqrt{1-x^2} + \frac{7}{12}\left(1-x^2\right)^{3/2} + \frac{7\arcsin(x)}{8}$$

output
```
7/20*(1-x)^(5/2)*(1+x)^(3/2)+1/5*(1-x)^(7/2)*(1+x)^(3/2)+7/8*x*(-x^2+1)^(1
/2)+7/12*(-x^2+1)^(3/2)+7/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = \frac{1}{120}\sqrt{1-x^2}\left(136+15x-112x^2+90x^3-24x^4\right) - \frac{7}{4}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

output
```
(Sqrt[1 - x^2]*(136 + 15*x - 112*x^2 + 90*x^3 - 24*x^4))/120 - (7*ArcTan[S
qrt[1 - x^2]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {59, 59, 50, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{7/2}\sqrt{x+1}\,dx$$

$$\Big\downarrow \; 59$$

$$\frac{7}{5}\int (1-x)^{5/2}\sqrt{x+1}dx+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}$$

$$\Big\downarrow \; 59$$

$$\frac{7}{5}\left(\frac{5}{4}\int (1-x)^{3/2}\sqrt{x+1}dx+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}$$

$$\Big\downarrow \; 50$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\int \sqrt{1-x^2}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}$$

$$\Big\downarrow \; 211$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}$$

$$\Big\downarrow \; 223$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}\right)+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}$$

input
```
Int[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

output
```
((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7*(((1 - x)^(5/2)*(1 + x)^(3/2))/4 + (5
*((x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3/2)/3 + ArcSin[x]/2))/4))/5
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(24x^4-90x^3+112x^2-15x-136)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{120\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{7\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}{5}+\frac{7(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{20}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{12}+\frac{7\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{7\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{7\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$

input
```
int((1-x)^(7/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*x^4-90*x^3+112*x^2-15*x-136)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^
(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+7/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/
(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = -\frac{1}{120}\left(24\,x^4-90\,x^3+112\,x^2-15\,x-136\right)\sqrt{x+1}\sqrt{-x+1} - \frac{7}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(x + 1)*sqrt(-x + 1) -
 7/4*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.23

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = \begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}} + \frac{39i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}} - \frac{449i(x+1)^{\frac{7}{2}}}{60\sqrt{x-1}} + \frac{1657i(x+1)^{\frac{5}{2}}}{120\sqrt{x-1}} - \frac{263i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}} + \frac{7i\sqrt{x+1}}{4\sqrt{x-1}} \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}} - \frac{39(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}} + \frac{449(x+1)^{\frac{7}{2}}}{60\sqrt{1-x}} - \frac{1657(x+1)^{\frac{5}{2}}}{120\sqrt{1-x}} + \frac{263(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}} - \frac{7\sqrt{x+1}}{4\sqrt{1-x}} \end{cases}$$

input
```
integrate((1-x)**(7/2)*(1+x)**(1/2),x)
```

output
```
Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt
(x - 1)) + 39*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 449*I*(x + 1)**(7/2)/(60
*sqrt(x - 1)) + 1657*I*(x + 1)**(5/2)/(120*sqrt(x - 1)) - 263*I*(x + 1)**(
3/2)/(24*sqrt(x - 1)) + 7*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2),
(7*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 39*(x
 + 1)**(9/2)/(20*sqrt(1 - x)) + 449*(x + 1)**(7/2)/(60*sqrt(1 - x)) - 1657
*(x + 1)**(5/2)/(120*sqrt(1 - x)) + 263*(x + 1)**(3/2)/(24*sqrt(1 - x)) -
7*sqrt(x + 1)/(4*sqrt(1 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = \frac{1}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{3}{4}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{17}{15}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{7}{8}\sqrt{-x^2+1}x + \frac{7}{8}\arcsin\left(x\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x + 17/15*(-x^2 + 1)^(3/2)
 + 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(56) = 112.

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.47

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = -\frac{1}{120}\left((2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{12}\left((2\left(3x-10\right)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{7}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt
(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqr
t(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqr
t(-x + 1) + 7/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = \int (1-x)^{7/2}\,\sqrt{x+1}\,dx$$

input
```
int((1 - x)^(7/2)*(x + 1)^(1/2),x)
```

output
```
int((1 - x)^(7/2)*(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int (1-x)^{7/2}\sqrt{1+x}\,dx = -\frac{7asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{4} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^4}{5} + \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x^3}{4} - \frac{14\sqrt{x+1}\,\sqrt{1-x}\,x^2}{15} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{8} + \frac{17\sqrt{x+1}\,\sqrt{1-x}}{15}$$

input
```
int((1-x)^(7/2)*(1+x)^(1/2),x)
```

output
```
( - 210*asin(sqrt( - x + 1)/sqrt(2)) - 24*sqrt(x + 1)*sqrt( - x + 1)*x**4
+ 90*sqrt(x + 1)*sqrt( - x + 1)*x**3 - 112*sqrt(x + 1)*sqrt( - x + 1)*x**2
 + 15*sqrt(x + 1)*sqrt( - x + 1)*x + 136*sqrt(x + 1)*sqrt( - x + 1))/120
```

3.69 $\int (1-x)^{5/2}\sqrt{1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx=\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}x\sqrt{1-x^2}+\frac{5}{12}\left(1-x^2\right)^{3/2}+\frac{5\arcsin(x)}{8}$$

output
```
1/4*(1-x)^(5/2)*(1+x)^(3/2)+5/8*x*(-x^2+1)^(1/2)+5/12*(-x^2+1)^(3/2)+5/8*a
rcsin(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx=\frac{1}{24}\sqrt{1-x^2}\left(16+9x-16x^2+6x^3\right)-\frac{5}{4}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

output
```
(Sqrt[1 - x^2]*(16 + 9*x - 16*x^2 + 6*x^3))/24 - (5*ArcTan[Sqrt[1 - x^2]/(
-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {59, 50, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{5/2}\sqrt{x+1}\,dx$$

$$\Big\downarrow \text{59}$$

$$\frac{5}{4}\int (1-x)^{3/2}\sqrt{x+1}dx+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}$$

$$\Big\downarrow \text{50}$$

$$\frac{5}{4}\left(\int \sqrt{1-x^2}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{5}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}$$

input
```
Int[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

output
```
((1 - x)^(5/2)*(1 + x)^(3/2))/4 + (5*((x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3/2
)/3 + ArcSin[x]/2))/4
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{(6x^3-16x^2+9x+16)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	82
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{4}+\frac{5(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{12}+\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	85

input
```
int((1-x)^(5/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*x^3-16*x^2+9*x+16)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x
)*(1-x))^(1/2)/(1-x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx = \frac{1}{24}\left(6\,x^3-16\,x^2+9\,x+16\right)\sqrt{x+1}\sqrt{-x+1} - \frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sq
rt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.72

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx = \begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} - \frac{23i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}} + \frac{127i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}} - \frac{133i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} + \frac{23(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}} - \frac{127(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}} + \frac{133(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}} - \frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(5/2)*(1+x)**(1/2),x)
```

output
```
Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(
x - 1)) - 23*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) + 127*I*(x + 1)**(5/2)/(24*
sqrt(x - 1)) - 133*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*
sqrt(x - 1)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)*
*(9/2)/(4*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(12*sqrt(1 - x)) - 127*(x + 1)*
*(5/2)/(24*sqrt(1 - x)) + 133*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x +
 1)/(4*sqrt(1 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx = -\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}x+\frac{5}{8}\arcsin(x)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(-x^2 + 1)^(3/2)*x + 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/
8*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(42) = 84.

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.74

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx = \frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{5}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) -
 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x
 - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx = \int (1-x)^{5/2}\,\sqrt{x+1}\,dx$$

input
```
int((1 - x)^(5/2)*(x + 1)^(1/2),x)
```

output
```
int((1 - x)^(5/2)*(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int (1-x)^{5/2}\sqrt{1+x}\,dx = -\frac{5asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{4} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{4} - \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3} + \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x}{8} + \frac{2\sqrt{x+1}\,\sqrt{1-x}}{3}$$

input
```
int((1-x)^(5/2)*(1+x)^(1/2),x)
```

output
```
( - 30*asin(sqrt( - x + 1)/sqrt(2)) + 6*sqrt(x + 1)*sqrt( - x + 1)*x**3 -
16*sqrt(x + 1)*sqrt( - x + 1)*x**2 + 9*sqrt(x + 1)*sqrt( - x + 1)*x + 16*s
qrt(x + 1)*sqrt( - x + 1))/24
```

3.70 $\int (1-x)^{3/2}\sqrt{1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx=\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{\arcsin(x)}{2}$$

output
```
1/2*x*(-x^2+1)^(1/2)+1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx=\frac{1}{6}\left(2+3x-2x^2\right)\sqrt{1-x^2}+\arctan\left(\frac{\sqrt{1-x^2}}{1-x}\right)$$

input
```
Integrate[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

output
```
((2 + 3*x - 2*x^2)*Sqrt[1 - x^2])/6 + ArcTan[Sqrt[1 - x^2]/(1 - x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {50, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{3/2}\sqrt{x+1}\,dx$$

$$\Big\downarrow \; 50$$

$$\int \sqrt{1-x^2}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \; 211$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

$$\Big\downarrow \; 223$$

$$\frac{\arcsin(x)}{2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

input
```
Int[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{(1-x)^{\frac{3}{2}} (1+x)^{\frac{3}{2}}}{3}+\frac{\sqrt{1-x}\, (1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\, \sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\, \arcsin(x)}{2\sqrt{1+x}\, \sqrt{1-x}}$	71
risch	$\frac{(2x^{2}-3x-2)\sqrt{1+x}\, (-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\, \sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\, \arcsin(x)}{2\sqrt{1+x}\, \sqrt{1-x}}$	77

input
```
int((1-x)^(3/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1
+x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx = -\frac{1}{6}\left(2\,x^{2}-3\,x-2\right)\sqrt{x+1}\sqrt{-x+1} - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*x^2 - 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt
(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 167, normalized size of antiderivative = 4.39

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx = \begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} + \frac{11i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{17i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} - \frac{11(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{17(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(3/2)*(1+x)**(1/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
1)) + 11*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 17*I*(x + 1)**(3/2)/(6*sqrt(x
- 1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (asin(sqrt(2)*sqrt(x +
 1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) - 11*(x + 1)**(5/2)/(6*sqrt(1 - x)
) + 17*(x + 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx = \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx = -\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx = \int (1-x)^{3/2}\sqrt{x+1}\,dx$$

input
```
int((1 - x)^(3/2)*(x + 1)^(1/2),x)
```

output
```
int((1 - x)^(3/2)*(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int (1-x)^{3/2}\sqrt{1+x}\,dx = -asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{2} + \frac{\sqrt{x+1}\,\sqrt{1-x}}{3}$$

input
```
int((1-x)^(3/2)*(1+x)^(1/2),x)
```

output
```
( - 6*asin(sqrt( - x + 1)/sqrt(2)) - 2*sqrt(x + 1)*sqrt( - x + 1)*x**2 + 3
*sqrt(x + 1)*sqrt( - x + 1)*x + 2*sqrt(x + 1)*sqrt( - x + 1))/6
```

3.71 $\int \sqrt{1-x}\sqrt{1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

output
```
1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x]*Sqrt[1 + x],x]
```

output
```
(x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {39, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}\sqrt{x+1}\,dx$$
$$\Big\downarrow \; 39$$
$$\int \sqrt{1-x^2}dx$$
$$\Big\downarrow \; 211$$
$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$
$$\Big\downarrow \; 223$$
$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[Sqrt[1 - x]*Sqrt[1 + x],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.48

method	result	size
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$-\frac{x\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	68

input
```
int((1-x)^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*((1+x)*(1-x))^
(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \frac{1}{2}\sqrt{x+1}x\sqrt{-x+1} - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.70

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \begin{cases} -i\,\text{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{3i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ \text{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{3(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(1/2)*(1+x)**(1/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
1)) - 3*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(
x + 1) > 2), (asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x))
 + 3*(x + 1)**(3/2)/(2*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1
/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \frac{x\sqrt{1-x}\sqrt{x+1}}{2} - \frac{\ln\left(x - \sqrt{1-x}\sqrt{x+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int((1 - x)^(1/2)*(x + 1)^(1/2),x)
```

output
```
(x*(1 - x)^(1/2)*(x + 1)^(1/2))/2 - (log(x - (1 - x)^(1/2)*(x + 1)^(1/2)*1
i)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = -asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{2}$$

input
```
int((1-x)^(1/2)*(1+x)^(1/2),x)
```

output
```
( - 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( - x + 1)*x)/2
```

3.72 $\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{1-x}\sqrt{1+x} + \arcsin(x)$$

output
```
-(1-x)^(1/2)*(1+x)^(1/2)+arcsin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{1-x^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[1 + x]/Sqrt[1 - x],x]
```

output
```
-Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{ 50}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 223}$$

$$\arcsin(x)-\sqrt{1-x^2}$$

input
```
Int[Sqrt[1 + x]/Sqrt[1 - x],x]
```

output
```
-Sqrt[1 - x^2] + ArcSin[x]
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

method	result	size
default	$-\sqrt{1-x}\,\sqrt{1+x}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	42
risch	$\frac{\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	65

input
```
int((1+x)^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsi
n(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{x+1}\sqrt{-x+1} - 2\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} + \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} - \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(1/2)/(1-x)**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1
) + 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (2*asin(sqrt(2)*sqrt(x +
 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) - 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{x+1}\sqrt{-x+1} + 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = \operatorname{asin}(x) - \sqrt{1-x^2}$$

input
```
int((x + 1)^(1/2)/(1 - x)^(1/2),x)
```

output
```
asin(x) - (1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - \sqrt{x+1}\,\sqrt{1-x}$$

input
```
int((1+x)^(1/2)/(1-x)^(1/2),x)
```

output
```
- 2*asin(sqrt( - x + 1)/sqrt(2)) - sqrt(x + 1)*sqrt( - x + 1)
```

3.73 $\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx=\frac{2\sqrt{1+x}}{\sqrt{1-x}}-\arcsin(x)$$

output
```
2*(1+x)^(1/2)/(1-x)^(1/2)-arcsin(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx=-\frac{2\sqrt{1-x^2}}{-1+x}+2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[1 + x]/(1 - x)^(3/2),x]
```

output
```
(-2*Sqrt[1 - x^2])/(-1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {57, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{(1-x)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\sqrt{x+1}} dx$$

$$\Big\downarrow \text{ 39}$$

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}} dx$$

$$\Big\downarrow \text{ 223}$$

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - \arcsin(x)$$

input
```
Int[Sqrt[1 + x]/(1 - x)^(3/2),x]
```

output
```
(2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

method	result	size
risch	$\frac{2\sqrt{1+x}\,\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}-\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	64

input
```
int((1+x)^(1/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1+x)^(1/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)-((1+x)
*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = \frac{2\left((x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x-1}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

output
```
2*((x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + x - sqrt(x + 1)*sqrt
(-x + 1) - 1)/(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = \begin{cases} 2i\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ |x+1| > 2 \\ -2\,\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(1/2)/(1-x)**(3/2),x)
```

output
```
Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1),
 Abs(x + 1) > 2), (-2*asin(sqrt(2)*sqrt(x + 1)/2) + 2*sqrt(x + 1)/sqrt(1 -
 x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = -\frac{2\sqrt{-x^2+1}}{x-1} - \arcsin(x)$$

input
```
integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = -\frac{2\sqrt{x+1}\sqrt{-x+1}}{x-1} - 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

output
```
-2*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = \int \frac{\sqrt{x+1}}{(1-x)^{3/2}}\,dx$$

input
```
int((x + 1)^(1/2)/(1 - x)^(3/2),x)
```

output
```
int((x + 1)^(1/2)/(1 - x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = \frac{2\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + 2\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
int((1+x)^(1/2)/(1-x)^(3/2),x)
```

output
```
(2*(sqrt( - x + 1)*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)))/sqrt( - x
+ 1)
```

3.74 $\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx = \frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

output
```
1/3*(1+x)^(3/2)/(1-x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx = \frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

input
```
Integrate[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

output
```
(1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{(1-x)^{5/2}}\,dx$$

$$\Big\downarrow \; 48$$

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

input
```
Int[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

output
```
(1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}}{3(1-x)^{\frac{3}{2}}}$	15
orering	$-\frac{(1+x)^{\frac{3}{2}}(-1+x)}{3(1-x)^{\frac{5}{2}}}$	18
default	$\frac{2\sqrt{1+x}}{3(1-x)^{\frac{3}{2}}} - \frac{\sqrt{1+x}}{3\sqrt{1-x}}$	30
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^2+2x+1)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	49

input
```
int((1+x)^(1/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1+x)^(3/2)/(1-x)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \frac{x^2 + (x+1)^{\frac{3}{2}}\sqrt{-x+1} - 2\,x + 1}{3\,(x^2 - 2\,x + 1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(x^2 + (x + 1)^(3/2)*sqrt(-x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \begin{cases} \frac{i(x+1)^{\frac{3}{2}}}{3\sqrt{x-1}(x+1)-6\sqrt{x-1}} & \text{for } |x+1| > 2 \\ -\frac{(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}(x+1)-6\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(1/2)/(1-x)**(5/2),x)
```

output
```
Piecewise((I*(x + 1)**(3/2)/(3*sqrt(x - 1)*(x + 1) - 6*sqrt(x - 1)), Abs(x
 + 1) > 2), (-(x + 1)**(3/2)/(3*sqrt(1 - x)*(x + 1) - 6*sqrt(1 - x)), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \frac{2\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)} + \frac{\sqrt{-x^2+1}}{3\,(x-1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \frac{(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3\,(x-1)^2}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="giac")
```

output
```
1/3*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \frac{\left(\frac{x\sqrt{x+1}}{3}+\frac{\sqrt{x+1}}{3}\right)\sqrt{1-x}}{x^2-2\,x+1}$$

input
```
int((x + 1)^(1/2)/(1 - x)^(5/2),x)
```

output
```
(((x*(x + 1)^(1/2))/3 + (x + 1)^(1/2)/3)*(1 - x)^(1/2))/(x^2 - 2*x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = -\frac{\sqrt{x+1}\,(x+1)}{3\sqrt{1-x}\,(x-1)}$$

input
```
int((1+x)^(1/2)/(1-x)^(5/2),x)
```

output
```
( - sqrt(x + 1)*(x + 1))/(3*sqrt( - x + 1)*(x - 1))
```

3.75 $\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx = \frac{(1+x)^{3/2}}{5(1-x)^{5/2}} + \frac{(1+x)^{3/2}}{15(1-x)^{3/2}}$$

output
```
1/5*(1+x)^(3/2)/(1-x)^(5/2)+1/15*(1+x)^(3/2)/(1-x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx = \frac{(4-x)(1+x)^{3/2}}{15(1-x)^{5/2}}$$

input
```
Integrate[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

output
```
((4 - x)*(1 + x)^(3/2))/(15*(1 - x)^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{(1-x)^{7/2}}\,dx$$

$$\Big\downarrow \; 55$$

$$\frac{1}{5}\int \frac{\sqrt{x+1}}{(1-x)^{5/2}}dx+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

$$\Big\downarrow \; 48$$

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

input
```
Int[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

output
```
(1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(-4+x)(1+x)^{\frac{3}{2}}}{15(1-x)^{\frac{5}{2}}}$	18
orering	$\frac{(1+x)^{\frac{3}{2}}(-1+x)(-4+x)}{15(1-x)^{\frac{7}{2}}}$	21
default	$\frac{2\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{15(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{15\sqrt{1-x}}$	44
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^3-2x^2-7x-4)}{15\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54

input
```
int((1+x)^(1/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-4+x)/(1-x)^(5/2)*(1+x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx = \frac{4\,x^3 - 12\,x^2 + (x^2 - 3\,x - 4)\sqrt{x+1}\sqrt{-x+1} + 12\,x - 4}{15\,(x^3 - 3\,x^2 + 3\,x - 1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(4*x^3 - 12*x^2 + (x^2 - 3*x - 4)*sqrt(x + 1)*sqrt(-x + 1) + 12*x - 4
)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx = \begin{cases} \frac{i(x+1)^{\frac{5}{2}}}{15\sqrt{x-1}(x+1)^2-60\sqrt{x-1}(x+1)+60\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{15\sqrt{x-1}(x+1)^2-60\sqrt{x-1}(x+1)+60\sqrt{x-1}} & \text{for } |x+1| > 2 \\ -\frac{(x+1)^{\frac{5}{2}}}{15\sqrt{1-x}(x+1)^2-60\sqrt{1-x}(x+1)+60\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{15\sqrt{1-x}(x+1)^2-60\sqrt{1-x}(x+1)+60\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(1/2)/(1-x)**(7/2),x)
```

output
```
Piecewise((I*(x + 1)**(5/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60*sqrt(x - 1)*(x
 + 1) + 60*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(15*sqrt(x - 1)*(x + 1)**2 -
60*sqrt(x - 1)*(x + 1) + 60*sqrt(x - 1)), Abs(x + 1) > 2), (-(x + 1)**(5/2
)/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(1 - x)) +
5*(x + 1)**(3/2)/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*
sqrt(1 - x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx = -\frac{2\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)} - \frac{\sqrt{-x^2+1}}{15\,(x^2-2\,x+1)} + \frac{\sqrt{-x^2+1}}{15\,(x-1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="maxima")
```

output
```
-2/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) + 1/15*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx = \frac{(x+1)^{\frac{3}{2}}(x-4)\sqrt{-x+1}}{15\,(x-1)^3}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="giac")
```

output
```
1/15*(x + 1)^(3/2)*(x - 4)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx = -\frac{\sqrt{1-x}\left(\frac{x\sqrt{x+1}}{5} + \frac{4\sqrt{x+1}}{15} - \frac{x^2\sqrt{x+1}}{15}\right)}{x^3-3\,x^2+3\,x-1}$$

input
```
int((x + 1)^(1/2)/(1 - x)^(7/2),x)
```

output
```
-((1 - x)^(1/2)*((x*(x + 1)^(1/2))/5 + (4*(x + 1)^(1/2))/15 - (x^2*(x + 1)
^(1/2))/15))/(3*x - 3*x^2 + x^3 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx = \frac{\sqrt{x+1}\,(-x^2+3x+4)}{15\sqrt{1-x}\,(x^2-2x+1)}$$

input
```
int((1+x)^(1/2)/(1-x)^(7/2),x)
```

output
```
(sqrt(x + 1)*( - x**2 + 3*x + 4))/(15*sqrt( - x + 1)*(x**2 - 2*x + 1))
```

3.76 $\int \frac{\sqrt{1+x}}{(1-x)^{9/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}} \, dx = \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{35(1-x)^{5/2}} + \frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}$$

output
```
1/7*(1+x)^(3/2)/(1-x)^(7/2)+2/35*(1+x)^(3/2)/(1-x)^(5/2)+2/105*(1+x)^(3/2)
/(1-x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}} \, dx = \frac{(1+x)^{3/2} (23-10x+2x^2)}{105(1-x)^{7/2}}$$

input
```
Integrate[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

output
```
((1 + x)^(3/2)*(23 - 10*x + 2*x^2))/(105*(1 - x)^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{(1-x)^{9/2}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2}{7} \int \frac{\sqrt{x+1}}{(1-x)^{7/2}}dx+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2}{7} \left(\frac{1}{5} \int \frac{\sqrt{x+1}}{(1-x)^{5/2}}dx+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}\right)+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}+\frac{2}{7} \left(\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}\right)$$

input
```
Int[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

output
```
(1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*((1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1
 + x)^(3/2)/(15*(1 - x)^(3/2))))/7
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}} (2x^{2}-10x+23)}{105(1-x)^{\frac{7}{2}}}$	25
orering	$-\frac{(1+x)^{\frac{3}{2}} (-1+x) (2x^{2}-10x+23)}{105(1-x)^{\frac{9}{2}}}$	28
default	$\frac{2\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{35(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{105(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{105\sqrt{1-x}}$	58
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(2x^{4}-6x^{3}+5x^{2}+36x+23)}{105\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{3}\sqrt{-(1+x)(-1+x)}}$	61

input
```
int((1+x)^(1/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105/(1-x)^(7/2)*(1+x)^(3/2)*(2*x^2-10*x+23)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx=\frac{23\,x^{4}-92\,x^{3}+138\,x^{2}+\left(2\,x^{3}-8\,x^{2}+13\,x+23\right)\sqrt{x+1}\sqrt{-x+1}-92\,x+23}{105\,\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(23*x^4 - 92*x^3 + 138*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(x + 1)
*sqrt(-x + 1) - 92*x + 23)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.96 (sec) , antiderivative size = 566, normalized size of antiderivative = 9.28

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx = \begin{cases} \frac{2i(x+1)^{\frac{9}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} - \frac{}{105\sqrt{x-1}(x+1)^4-840\sqrt{} \\ -\frac{2(x+1)^{\frac{9}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} + \frac{}{105\sqrt{1-x}(x+1)^4-84} \end{cases}$$

input
```
integrate((1+x)**(1/2)/(1-x)**(9/2),x)
```

output
```
Piecewise((2*I*(x + 1)**(9/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(x - 1
)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1) + 16
80*sqrt(x - 1)) - 18*I*(x + 1)**(7/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sq
rt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x +
 1) + 1680*sqrt(x - 1)) + 63*I*(x + 1)**(5/2)/(105*sqrt(x - 1)*(x + 1)**4
- 840*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x -
 1)*(x + 1) + 1680*sqrt(x - 1)) - 70*I*(x + 1)**(3/2)/(105*sqrt(x - 1)*(x
+ 1)**4 - 840*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*
sqrt(x - 1)*(x + 1) + 1680*sqrt(x - 1)), Abs(x + 1) > 2), (-2*(x + 1)**(9/
2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1
- x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 18*(x + 1
)**(7/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*s
qrt(1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) - 63*
(x + 1)**(5/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 +
2520*sqrt(1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x))
 + 70*(x + 1)**(3/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)
**3 + 2520*sqrt(1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1
 - x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(43) = 86.

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx = \frac{2\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{\sqrt{-x^2+1}}{35\,(x^3-3\,x^2+3\,x-1)} - \frac{2\sqrt{-x^2+1}}{105\,(x^2-2\,x+1)} + \frac{2\sqrt{-x^2+1}}{105\,(x-1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="maxima")
```

output
```
2/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(-x^2 + 1)/(
x^3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/105*sqrt
(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx = \frac{(2\,(x+1)(x-6)+35)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{105\,(x-1)^4}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="giac")
```

output
```
1/105*(2*(x + 1)*(x - 6) + 35)*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx = \frac{\sqrt{1-x}\left(\frac{13\,x\,\sqrt{x+1}}{105}+\frac{23\,\sqrt{x+1}}{105}-\frac{8\,x^2\,\sqrt{x+1}}{105}+\frac{2\,x^3\,\sqrt{x+1}}{105}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

input
```
int((x + 1)^(1/2)/(1 - x)^(9/2),x)
```

output
```
((1 - x)^(1/2)*((13*x*(x + 1)^(1/2))/105 + (23*(x + 1)^(1/2))/105 - (8*x^2
*(x + 1)^(1/2))/105 + (2*x^3*(x + 1)^(1/2))/105))/(6*x^2 - 4*x - 4*x^3 + x
^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx = \frac{\sqrt{x+1}\,(-2x^3+8x^2-13x-23)}{105\sqrt{1-x}\,(x^3-3x^2+3x-1)}$$

input
```
int((1+x)^(1/2)/(1-x)^(9/2),x)
```

output
```
(sqrt(x + 1)*( - 2*x**3 + 8*x**2 - 13*x - 23))/(105*sqrt( - x + 1)*(x**3 -
 3*x**2 + 3*x - 1))
```

3.77 $\int \frac{\sqrt{1+x}}{(1-x)^{11/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}} \, dx = \frac{(1+x)^{3/2}}{9(1-x)^{9/2}} + \frac{(1+x)^{3/2}}{21(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{105(1-x)^{5/2}} + \frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}$$

output
```
1/9*(1+x)^(3/2)/(1-x)^(9/2)+1/21*(1+x)^(3/2)/(1-x)^(7/2)+2/105*(1+x)^(3/2)
/(1-x)^(5/2)+2/315*(1+x)^(3/2)/(1-x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}} \, dx = \frac{(1+x)^{3/2} (58 - 33x + 12x^2 - 2x^3)}{315(1-x)^{9/2}}$$

input
```
Integrate[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

output
```
((1 + x)^(3/2)*(58 - 33*x + 12*x^2 - 2*x^3))/(315*(1 - x)^(9/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{(1-x)^{11/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\int \frac{\sqrt{x+1}}{(1-x)^{9/2}}dx+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(\frac{2}{7}\int \frac{\sqrt{x+1}}{(1-x)^{7/2}}dx+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}\right)+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(\frac{2}{7}\left(\frac{1}{5}\int \frac{\sqrt{x+1}}{(1-x)^{5/2}}dx+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}\right)+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}\right)+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}+\frac{1}{3}\left(\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}+\frac{2}{7}\left(\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}\right)\right)$$

input
```
Int[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

output
```
(1 + x)^(3/2)/(9*(1 - x)^(9/2)) + ((1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*((
1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))))/7)/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.37

method	result	size
gosper	$-\frac{(1+x)^{\frac{3}{2}} (2x^3-12x^2+33x-58)}{315(1-x)^{\frac{9}{2}}}$	30
orering	$\frac{(1+x)^{\frac{3}{2}} (-1+x)(2x^3-12x^2+33x-58)}{315(1-x)^{\frac{11}{2}}}$	33
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(2x^5-8x^4+11x^3-4x^2-83x-58)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{2\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}-\frac{\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{315\sqrt{1-x}}$	72

input
```
int((1+x)^(1/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315*(1+x)^(3/2)/(1-x)^(9/2)*(2*x^3-12*x^2+33*x-58)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx = \frac{58\,x^{5} - 290\,x^{4} + 580\,x^{3} - 580\,x^{2} + (2\,x^{4} - 10\,x^{3} + 21\,x^{2} - 25\,x - 58)\sqrt{x+1}\sqrt{-x+1}}{315\,(x^{5} - 5\,x^{4} + 10\,x^{3} - 10\,x^{2} + 5\,x - 1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(58*x^5 - 290*x^4 + 580*x^3 - 580*x^2 + (2*x^4 - 10*x^3 + 21*x^2 - 2
5*x - 58)*sqrt(x + 1)*sqrt(-x + 1) + 290*x - 58)/(x^5 - 5*x^4 + 10*x^3 - 1
0*x^2 + 5*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.79 (sec) , antiderivative size = 1561, normalized size of antiderivative = 19.27

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)**(1/2)/(1-x)**(11/2),x)
```

output
```
Piecewise((2*I*(x + 1)**(15/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x -
 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)*
*4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 14112
0*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 30*I*(x + 1)**(13/2)/(315*sqr
t(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x +
 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 21
1680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x -
1)) + 195*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)
*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4
+ 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*s
qrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 715*I*(x + 1)**(9/2)/(315*sqrt(x
 - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)
**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 21168
0*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1))
 + 1530*I*(x + 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x
 + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 1
76400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt
(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 1764*I*(x + 1)**(5/2)/(315*sqrt(x -
 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**
5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 2116...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(57) = 114$.

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx = -\frac{2\sqrt{-x^2+1}}{9\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} - \frac{\sqrt{-x^2+1}}{63\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{\sqrt{-x^2+1}}{105\,(x^3-3\,x^2+3\,x-1)} - \frac{2\sqrt{-x^2+1}}{315\,(x^2-2\,x+1)} + \frac{2\sqrt{-x^2+1}}{315\,(x-1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="maxima")
```

output
```
-2/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 1/63*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/105*sqrt(-x^2 + 1)/(x^3 - 3*
x^2 + 3*x - 1) - 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/315*sqrt(-x^2 +
1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx = \frac{((2\,(x+1)(x-8)+63)(x+1)-105)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{315\,(x-1)^5}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="giac")
```

output
```
1/315*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1)^(3/2)*sqrt(-x + 1)/
(x - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx = -\frac{\sqrt{1-x}\,\left(\frac{5\,x\,\sqrt{x+1}}{63}+\frac{58\,\sqrt{x+1}}{315}-\frac{x^2\,\sqrt{x+1}}{15}+\frac{2\,x^3\,\sqrt{x+1}}{63}-\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

input
```
int((x + 1)^(1/2)/(1 - x)^(11/2),x)
```

output
```
-((1 - x)^(1/2)*((5*x*(x + 1)^(1/2))/63 + (58*(x + 1)^(1/2))/315 - (x^2*(x
 + 1)^(1/2))/15 + (2*x^3*(x + 1)^(1/2))/63 - (2*x^4*(x + 1)^(1/2))/315))/(
5*x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx = \frac{\sqrt{x+1}\,(-2x^4+10x^3-21x^2+25x+58)}{315\sqrt{1-x}\,(x^4-4x^3+6x^2-4x+1)}$$

input
```
int((1+x)^(1/2)/(1-x)^(11/2),x)
```

output
```
(sqrt(x + 1)*( - 2*x**4 + 10*x**3 - 21*x**2 + 25*x + 58))/(315*sqrt( - x +
 1)*(x**4 - 4*x**3 + 6*x**2 - 4*x + 1))
```

3.78 $\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}} + \frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}$$

output
```
1/11*(1+x)^(3/2)/(1-x)^(11/2)+4/99*(1+x)^(3/2)/(1-x)^(9/2)+4/231*(1+x)^(3/
2)/(1-x)^(7/2)+8/1155*(1+x)^(3/2)/(1-x)^(5/2)+8/3465*(1+x)^(3/2)/(1-x)^(3/
2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{(1+x)^{3/2}(547-364x+180x^2-56x^3+8x^4)}{3465(1-x)^{11/2}}$$

input
```
Integrate[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

output
```
((1 + x)^(3/2)*(547 - 364*x + 180*x^2 - 56*x^3 + 8*x^4))/(3465*(1 - x)^(11
/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{(1-x)^{13/2}}dx$$

$$\Big\downarrow \; 55$$

$$\frac{4}{11}\int \frac{\sqrt{x+1}}{(1-x)^{11/2}}dx+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \; 55$$

$$\frac{4}{11}\left(\frac{1}{3}\int \frac{\sqrt{x+1}}{(1-x)^{9/2}}dx+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \; 55$$

$$\frac{4}{11}\left(\frac{1}{3}\left(\frac{2}{7}\int \frac{\sqrt{x+1}}{(1-x)^{7/2}}dx+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}\right)+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \; 55$$

$$\frac{4}{11}\left(\frac{1}{3}\left(\frac{2}{7}\left(\frac{1}{5}\int \frac{\sqrt{x+1}}{(1-x)^{5/2}}dx+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}\right)+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}\right)+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \; 48$$

$$\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}+\frac{4}{11}\left(\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}+\frac{1}{3}\left(\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}+\frac{2}{7}\left(\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}\right)\right)\right)$$

input
```
Int[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

output
```
(1 + x)^(3/2)/(11*(1 - x)^(11/2)) + (4*((1 + x)^(3/2)/(9*(1 - x)^(9/2)) +
((1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*((1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (
1 + x)^(3/2)/(15*(1 - x)^(3/2))))/7)/3))/11
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}\left(8x^{4}-56x^{3}+180x^{2}-364x+547\right)}{3465(1-x)^{\frac{11}{2}}}$	35
orering	$-\frac{(1+x)^{\frac{3}{2}}(-1+x)\left(8x^{4}-56x^{3}+180x^{2}-364x+547\right)}{3465(1-x)^{\frac{13}{2}}}$	38
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(8x^{6}-40x^{5}+76x^{4}-60x^{3}-x^{2}+730x+547\right)}{3465\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{5}\sqrt{-(1+x)(-1+x)}}$	71
default	$\frac{2\sqrt{1+x}}{11(1-x)^{\frac{11}{2}}}-\frac{\sqrt{1+x}}{99(1-x)^{\frac{9}{2}}}-\frac{4\sqrt{1+x}}{693(1-x)^{\frac{7}{2}}}-\frac{4\sqrt{1+x}}{1155(1-x)^{\frac{5}{2}}}-\frac{8\sqrt{1+x}}{3465(1-x)^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{3465\sqrt{1-x}}$	86

input
```
int((1+x)^(1/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
1/3465*(1+x)^(3/2)/(1-x)^(11/2)*(8*x^4-56*x^3+180*x^2-364*x+547)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{547\,x^6 - 3282\,x^5 + 8205\,x^4 - 10940\,x^3 + 8205\,x^2 + (8\,x^5 - 48\,x^4 + 124\,x^3 - 184\,x^2 +}{3465\,(x^6 - 6\,x^5 + 15\,x^4 - 20\,x^3 + 15\,x^2 - 6\,x}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="fricas")
```

output
```
1/3465*(547*x^6 - 3282*x^5 + 8205*x^4 - 10940*x^3 + 8205*x^2 + (8*x^5 - 48
*x^4 + 124*x^3 - 184*x^2 + 183*x + 547)*sqrt(x + 1)*sqrt(-x + 1) - 3282*x
+ 547)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.83 (sec) , antiderivative size = 3648, normalized size of antiderivative = 36.12

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)**(1/2)/(1-x)**(13/2),x)
```

output
```
Piecewise((8*I*(x + 1)**(23/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(
x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(
x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)
**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4
+ 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 390
29760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 184*I*(x + 1)**(21/2)/(
3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt
(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)
*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x +
 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)*
*3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096
320*sqrt(x - 1)) + 1932*I*(x + 1)**(19/2)/(3465*sqrt(x - 1)*(x + 1)**11 -
76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sq
rt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x -
 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*
(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x +
1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 12236*I*(x +
 1)**(17/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10
+ 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 1829520
0*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{2\sqrt{-x^2+1}}{11\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)} + \frac{\sqrt{-x^2+1}}{99\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} - \frac{4\sqrt{-x^2+1}}{693\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{4\sqrt{-x^2+1}}{1155\,(x^3-3\,x^2+3\,x-1)} - \frac{8\sqrt{-x^2+1}}{3465\,(x^2-2\,x+1)} + \frac{8\sqrt{-x^2+1}}{3465\,(x-1)}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="maxima")
```

output
```
2/11*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 1
/99*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 4/693*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/1155*sqrt(-x^2 + 1)/(x^3 - 3
*x^2 + 3*x - 1) - 8/3465*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/3465*sqrt(-x^2
 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{(4\,((2\,(x+1)(x-10)+99)(x+1)-231)(x+1)+1155)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3465\,(x-1)^{6}}$$

input
```
integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="giac")
```

output
```
1/3465*(4*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1) + 1155)*(x + 1
)^(3/2)*sqrt(-x + 1)/(x - 1)^6
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{\sqrt{1-x}\,\left(\frac{61\,x\,\sqrt{x+1}}{1155}+\frac{547\,\sqrt{x+1}}{3465}-\frac{184\,x^2\,\sqrt{x+1}}{3465}+\frac{124\,x^3\,\sqrt{x+1}}{3465}-\frac{16\,x^4\,\sqrt{x+1}}{1155}+\frac{8\,x^5\,\sqrt{x+1}}{3465}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

input
```
int((x + 1)^(1/2)/(1 - x)^(13/2),x)
```

output
```
((1 - x)^(1/2)*((61*x*(x + 1)^(1/2))/1155 + (547*(x + 1)^(1/2))/3465 - (18
4*x^2*(x + 1)^(1/2))/3465 + (124*x^3*(x + 1)^(1/2))/3465 - (16*x^4*(x + 1)
^(1/2))/1155 + (8*x^5*(x + 1)^(1/2))/3465))/(15*x^2 - 6*x - 20*x^3 + 15*x^
4 - 6*x^5 + x^6 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{\sqrt{x+1}\,(-8x^5+48x^4-124x^3+184x^2-183x-547)}{3465\sqrt{1-x}\,(x^5-5x^4+10x^3-10x^2+5x-1)}$$

input
```
int((1+x)^(1/2)/(1-x)^(13/2),x)
```

output
```
(sqrt(x + 1)*( - 8*x**5 + 48*x**4 - 124*x**3 + 184*x**2 - 183*x - 547))/(3
465*sqrt( - x + 1)*(x**5 - 5*x**4 + 10*x**3 - 10*x**2 + 5*x - 1))
```

3.79 $\int (1-x)^{9/2}(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx=\frac{3}{14}(1-x)^{7/2}(1+x)^{5/2}+\frac{1}{7}(1-x)^{9/2}(1+x)^{5/2}+\frac{9}{16}x\sqrt{1-x^2}+\frac{3}{8}x\left(1-x^2\right)^{3/2}+\frac{3}{10}\left(1-x^2\right)^{5/2}+9\,\mathrm{ar}$$

output
```
3/14*(1-x)^(7/2)*(1+x)^(5/2)+1/7*(1-x)^(9/2)*(1+x)^(5/2)+9/16*x*(-x^2+1)^(
1/2)+3/8*x*(-x^2+1)^(3/2)+3/10*(-x^2+1)^(5/2)+9/16*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx=\frac{1}{560}\sqrt{1-x^2}\left(368+245x-656x^2+350x^3+208x^4-280x^5+80x^6\right)-\frac{9}{8}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

output
```
(Sqrt[1 - x^2]*(368 + 245*x - 656*x^2 + 350*x^3 + 208*x^4 - 280*x^5 + 80*x
^6))/560 - (9*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {59, 59, 50, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{9/2}(x+1)^{3/2}\,dx$$

$$\Big\downarrow \text{59}$$

$$\frac{9}{7}\int (1-x)^{7/2}(x+1)^{3/2}dx+\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{59}$$

$$\frac{9}{7}\left(\frac{7}{6}\int (1-x)^{5/2}(x+1)^{3/2}dx+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}\right)+\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{50}$$

$$\frac{9}{7}\left(\frac{7}{6}\left(\int \left(1-x^2\right)^{3/2}dx+\frac{1}{5}\left(1-x^2\right)^{5/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}\right)+\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{7}\left(\frac{7}{6}\left(\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}\right)+\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}\right)+\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{9}{7}\left(\frac{7}{6}\left(\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}\right)+\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}$$

input
```
Int[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

output
```
((1 - x)^(9/2)*(1 + x)^(5/2))/7 + (9*(((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7
*((x*(1 - x^2)^(3/2))/4 + (1 - x^2)^(5/2)/5 + (3*((x*Sqrt[1 - x^2])/2 + Ar
cSin[x]/2))/4))/6))/7
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(80x^6-280x^5+208x^4+350x^3-656x^2+245x+368)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{560\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{9\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{5}{2}}}{7}+\frac{3(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}}{14}+\frac{3(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{10}+\frac{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{8}+\frac{3\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{8}-\frac{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{16}-9\sqrt{}$

input
```
int((1-x)^(9/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/560*(80*x^6-280*x^5+208*x^4+350*x^3-656*x^2+245*x+368)*(1+x)^(1/2)*(-1+
x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+9/16*((1+x)*(1-x)
)^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx=\frac{1}{560}\left(80\,x^6-280\,x^5+208\,x^4+350\,x^3-656\,x^2+245\,x+368\right)\sqrt{x+1}\sqrt{-x+1}-\frac{9}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
1/560*(80*x^6 - 280*x^5 + 208*x^4 + 350*x^3 - 656*x^2 + 245*x + 368)*sqrt(
x + 1)*sqrt(-x + 1) - 9/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)]

Timed out.

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((1-x)**(9/2)*(1+x)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.70

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx = \frac{1}{7}\left(-x^2+1\right)^{\frac{5}{2}}x^2 - \frac{1}{2}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{23}{35}\left(-x^2+1\right)^{\frac{5}{2}} + \frac{3}{8}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{9}{16}\sqrt{-x^2+1}x + \frac{9}{16}\arcsin(x)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(-x^2 + 1)^(5/2)*x^2 - 1/2*(-x^2 + 1)^(5/2)*x + 23/35*(-x^2 + 1)^(5/2)
 + 3/8*(-x^2 + 1)^(3/2)*x + 9/16*sqrt(-x^2 + 1)*x + 9/16*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(68) = 136.

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.52

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx = \frac{1}{1680}((2((4(5(6x-37)(x+1)+661)(x+1)-4551)(x+1)+4781)(x+1)-6335)(x+1)$$
$$-\frac{1}{120}((2((4(5x-26)(x+1)+321)(x+1)-451)(x+1)+745)(x+1)-405)\sqrt{x+1}\sqrt{-x+1}$$
$$-\frac{1}{120}((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\sqrt{-x+1}$$
$$+\frac{1}{6}((2(3x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}$$
$$-\frac{1}{6}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}(x-2)\sqrt{-x+1}$$
$$+\sqrt{x+1}\sqrt{-x+1}+\frac{9}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781
)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/120*((2*((4
*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x + 1) - 405)*sq
rt(x + 1)*sqrt(-x + 1) - 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) -
295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/6*((2*(3*x - 10)*(x + 1)
+ 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/6*((2*x - 5)*(x + 1) + 9)
*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)
*sqrt(-x + 1) + 9/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx = \int (1-x)^{9/2}\,(x+1)^{3/2}\,dx$$

input
```
int((1 - x)^(9/2)*(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(9/2)*(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.22

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx = -\frac{9asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^6}{7} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^5}{2} + \frac{13\sqrt{x+1}\,\sqrt{1-x}\,x^4}{35} + \frac{5\sqrt{x+1}\,\sqrt{1-x}\,x^3}{8} - \frac{41\sqrt{x+1}\,\sqrt{1-x}\,x^2}{35} + \frac{7\sqrt{x+1}\,\sqrt{1-x}\,x}{16} + \frac{23\sqrt{x+1}\,\sqrt{1-x}}{35}$$

input
```
int((1-x)^(9/2)*(1+x)^(3/2),x)
```

output
```
( - 630*asin(sqrt( - x + 1)/sqrt(2)) + 80*sqrt(x + 1)*sqrt( - x + 1)*x**6
- 280*sqrt(x + 1)*sqrt( - x + 1)*x**5 + 208*sqrt(x + 1)*sqrt( - x + 1)*x**
4 + 350*sqrt(x + 1)*sqrt( - x + 1)*x**3 - 656*sqrt(x + 1)*sqrt( - x + 1)*x
**2 + 245*sqrt(x + 1)*sqrt( - x + 1)*x + 368*sqrt(x + 1)*sqrt( - x + 1))/5
60
```

3.80 $\int (1-x)^{7/2}(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx=\frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{16}x\sqrt{1-x^2}+\frac{7}{24}x\left(1-x^2\right)^{3/2}+\frac{7}{30}\left(1-x^2\right)^{5/2}+\frac{7\arcsin(x)}{16}$$

output
```
1/6*(1-x)^(7/2)*(1+x)^(5/2)+7/16*x*(-x^2+1)^(1/2)+7/24*x*(-x^2+1)^(3/2)+7/
30*(-x^2+1)^(5/2)+7/16*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx=\frac{1}{240}\sqrt{1-x^2}\left(96+135x-192x^2+10x^3+96x^4-40x^5\right)-\frac{7}{8}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

output
```
(Sqrt[1 - x^2]*(96 + 135*x - 192*x^2 + 10*x^3 + 96*x^4 - 40*x^5))/240 - (7
*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/8
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {59, 50, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{7/2}(x+1)^{3/2}\,dx$$

$$\Big\downarrow \text{ 59}$$

$$\frac{7}{6}\int (1-x)^{5/2}(x+1)^{3/2}dx+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}$$

$$\Big\downarrow \text{ 50}$$

$$\frac{7}{6}\left(\int \left(1-x^2\right)^{3/2}dx+\frac{1}{5}\left(1-x^2\right)^{5/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{7}{6}\left(\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{7}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{7}{6}\left(\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}$$

input
```
Int[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

output
```
((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7*((x*(1 - x^2)^(3/2))/4 + (1 - x^2)^(5
/2)/5 + (3*((x*Sqrt[1 - x^2])/2 + ArcSin[x]/2))/4))/6
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

method	result
risch	$\frac{(40x^5-96x^4-10x^3+192x^2-135x-96)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{240\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{7\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}}{6}+\frac{7(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{30}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{24}+\frac{7\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{7\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}-\frac{7\sqrt{1-x}\,\sqrt{1+x}}{16}+\frac{7\sqrt{(1}{}$

input
```
int((1-x)^(7/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/240*(40*x^5-96*x^4-10*x^3+192*x^2-135*x-96)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(
-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+7/16*((1+x)*(1-x))^(1/2)/(1+x
)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = -\frac{1}{240}\left(40\,x^5-96\,x^4-10\,x^3+192\,x^2-135\,x-96\right)\sqrt{x+1}\sqrt{-x+1} - \frac{7}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/240*(40*x^5 - 96*x^4 - 10*x^3 + 192*x^2 - 135*x - 96)*sqrt(x + 1)*sqrt(
-x + 1) - 7/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 88.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.88

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = \begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} + \frac{47i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}} - \frac{683i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}} + \frac{1151i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}} - \frac{1543i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}} - \frac{7i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} - \frac{47(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}} + \frac{683(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}} - \frac{1151(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}} + \frac{1543(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}} + \frac{7(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{7\sqrt{x+1}}{8\sqrt{1-x}} \end{cases}$$

input
```
integrate((1-x)**(7/2)*(1+x)**(3/2),x)
```

output
```
Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(13/2)/(6*sqrt
(x - 1)) + 47*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) - 683*I*(x + 1)**(9/2)/(1
20*sqrt(x - 1)) + 1151*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) - 1543*I*(x + 1)
**(5/2)/(240*sqrt(x - 1)) - 7*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 7*I*sqrt
(x + 1)/(8*sqrt(x - 1)), Abs(x + 1) > 2), (7*asin(sqrt(2)*sqrt(x + 1)/2)/8
 + (x + 1)**(13/2)/(6*sqrt(1 - x)) - 47*(x + 1)**(11/2)/(30*sqrt(1 - x)) +
 683*(x + 1)**(9/2)/(120*sqrt(1 - x)) - 1151*(x + 1)**(7/2)/(120*sqrt(1 -
x)) + 1543*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 7*(x + 1)**(3/2)/(48*sqrt(1
- x)) - 7*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = -\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{2}{5}\left(-x^2+1\right)^{\frac{5}{2}} + \frac{7}{24}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{7}{16}\sqrt{-x^2+1}x + \frac{7}{16}\arcsin\left(x\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
-1/6*(-x^2 + 1)^(5/2)*x + 2/5*(-x^2 + 1)^(5/2) + 7/24*(-x^2 + 1)^(3/2)*x +
 7/16*sqrt(-x^2 + 1)*x + 7/16*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(54) = 108$.

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.50

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = -\frac{1}{240}\left((2\left((4\left(5x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{120}\left((2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{12}\left((2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{7}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="giac")
```

output
```
-1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x
+ 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 1
33)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x
 - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x -
5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x
 + 1) + sqrt(x + 1)*sqrt(-x + 1) + 7/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = \int (1-x)^{7/2}\,(x+1)^{3/2}\,dx$$

input
```
int((1 - x)^(7/2)*(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(7/2)*(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = -\frac{7asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^5}{6} + \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^4}{5} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{24} - \frac{4\sqrt{x+1}\,\sqrt{1-x}\,x^2}{5} + \frac{9\sqrt{x+1}\,\sqrt{1-x}\,x}{16} + \frac{2\sqrt{x+1}\,\sqrt{1-x}}{5}$$

input

```
int((1-x)^(7/2)*(1+x)^(3/2),x)
```

output

```
( - 210*asin(sqrt( - x + 1)/sqrt(2)) - 40*sqrt(x + 1)*sqrt( - x + 1)*x**5
+ 96*sqrt(x + 1)*sqrt( - x + 1)*x**4 + 10*sqrt(x + 1)*sqrt( - x + 1)*x**3
- 192*sqrt(x + 1)*sqrt( - x + 1)*x**2 + 135*sqrt(x + 1)*sqrt( - x + 1)*x +
 96*sqrt(x + 1)*sqrt( - x + 1))/240
```

3.81 $\int (1-x)^{5/2}(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx=\frac{3}{8}x\sqrt{1-x^2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{3\arcsin(x)}{8}$$

output
```
3/8*x*(-x^2+1)^(1/2)+1/4*x*(-x^2+1)^(3/2)+1/5*(-x^2+1)^(5/2)+3/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx=\frac{1}{40}\sqrt{1-x^2}\left(8+25x-16x^2-10x^3+8x^4\right)-\frac{3}{4}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

output
```
(Sqrt[1 - x^2]*(8 + 25*x - 16*x^2 - 10*x^3 + 8*x^4))/40 - (3*ArcTan[Sqrt[1
 - x^2]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {50, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{5/2}(x+1)^{3/2}\,dx$$

$$\Big\downarrow \text{50}$$

$$\int \left(1-x^2\right)^{3/2}dx+\frac{1}{5}\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

input
```
Int[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

output
```
(x*(1 - x^2)^(3/2))/4 + (1 - x^2)^(5/2)/5 + (3*((x*Sqrt[1 - x^2])/2 + ArcS
in[x]/2))/4
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(40) = 80$.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

method	result	s
risch	$-\frac{(8x^4-10x^3-16x^2+25x+8)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{40\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	8
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{5}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	9

input
```
int((1-x)^(5/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(8*x^4-10*x^3-16*x^2+25*x+8)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2
)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+3/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x
)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx = \frac{1}{40}\left(8\,x^4-10\,x^3-16\,x^2+25\,x+8\right)\sqrt{x+1}\sqrt{-x+1} - \frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(8*x^4 - 10*x^3 - 16*x^2 + 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*a
rctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.59

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx = \begin{cases} -\frac{3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}} - \frac{29i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}} + \frac{73i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}} - \frac{129i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+ \\ \frac{3\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}} + \frac{29(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}} - \frac{73(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}} + \frac{129(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwi} \end{cases}$$

input
```
integrate((1-x)**(5/2)*(1+x)**(3/2),x)
```

output
```
Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(11/2)/(5*sqrt
(x - 1)) - 29*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) + 73*I*(x + 1)**(7/2)/(20*
sqrt(x - 1)) - 129*I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8
*sqrt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (3*asin(
sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(11/2)/(5*sqrt(1 - x)) + 29*(x + 1)**(
9/2)/(20*sqrt(1 - x)) - 73*(x + 1)**(7/2)/(20*sqrt(1 - x)) + 129*(x + 1)**
(5/2)/(40*sqrt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4
*sqrt(1 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx = \frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}} + \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{-x^2+1}x + \frac{3}{8}\arcsin(x)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8
*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(40) = 80.

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.69

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx = \frac{1}{120}\left((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1)
 + sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx = \int (1-x)^{5/2}\,(x+1)^{3/2}\,dx$$

input
```
int((1 - x)^(5/2)*(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(5/2)*(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int (1-x)^{5/2}(1+x)^{3/2}\,dx = -\frac{3 asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{4} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^4}{5} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{4} - \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{5} + \frac{5\sqrt{x+1}\,\sqrt{1-x}\,x}{8} + \frac{\sqrt{x+1}\,\sqrt{1-x}}{5}$$

input
```
int((1-x)^(5/2)*(1+x)^(3/2),x)
```

output
```
( - 30*asin(sqrt( - x + 1)/sqrt(2)) + 8*sqrt(x + 1)*sqrt( - x + 1)*x**4 -
10*sqrt(x + 1)*sqrt( - x + 1)*x**3 - 16*sqrt(x + 1)*sqrt( - x + 1)*x**2 +
25*sqrt(x + 1)*sqrt( - x + 1)*x + 8*sqrt(x + 1)*sqrt( - x + 1))/40
```

3.82 $\int (1-x)^{3/2}(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = \frac{3}{8}x\sqrt{1-x^2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}+\frac{3\arcsin(x)}{8}$$

output
```
3/8*x*(-x^2+1)^(1/2)+1/4*x*(-x^2+1)^(3/2)+3/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = -\frac{1}{8}x\sqrt{1-x^2}\left(-5+2x^2\right)-\frac{3}{4}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

output
```
-1/8*(x*Sqrt[1 - x^2]*(-5 + 2*x^2)) - (3*ArcTan[Sqrt[1 - x^2]/(1 + x)])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {39, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{3/2}(x+1)^{3/2}\,dx$$

$$\Big\downarrow \text{39}$$

$$\int \left(1-x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

input
```
Int[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

output
```
(x*(1 - x^2)^(3/2))/4 + (3*((x*Sqrt[1 - x^2])/2 + ArcSin[x]/2))/4
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

method	result	size
risch	$\frac{x(2x^2-5)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{8\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	75
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	85

input
```
int((1-x)^(3/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*x^2-5)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/
2)/(1-x)^(1/2)+3/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = -\frac{1}{8}\left(2\,x^3-5\,x\right)\sqrt{x+1}\sqrt{-x+1} - \frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(2*x^3 - 5*x)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*arctan((sqrt(x + 1)*sqrt
(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.73 (sec) , antiderivative size = 212, normalized size of antiderivative = 5.44

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = \begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} + \frac{5i(x+1)^{\frac{7}{2}}}{4\sqrt{x-1}} - \frac{13i(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} - \frac{5(x+1)^{\frac{7}{2}}}{4\sqrt{1-x}} + \frac{13(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(3/2)*(1+x)**(3/2),x)
```

output
```
Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(
x - 1)) + 5*I*(x + 1)**(7/2)/(4*sqrt(x - 1)) - 13*I*(x + 1)**(5/2)/(8*sqrt
(x - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x -
1)), Abs(x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4
*sqrt(1 - x)) - 5*(x + 1)**(7/2)/(4*sqrt(1 - x)) + 13*(x + 1)**(5/2)/(8*sq
rt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{-x^2+1}x + \frac{3}{8}\arcsin\left(x\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(29) = 58.

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.59

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = -\frac{1}{24}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="giac")
```

output
```
-1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1)
- 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(
x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sq
rt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = \int (1-x)^{3/2}\,(x+1)^{3/2}\,dx$$

input
```
int((1 - x)^(3/2)*(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(3/2)*(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = -\frac{3asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{4} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{4} + \frac{5\sqrt{x+1}\,\sqrt{1-x}\,x}{8}$$

input
```
int((1-x)^(3/2)*(1+x)^(3/2),x)
```

output
```
( - 6*asin(sqrt( - x + 1)/sqrt(2)) - 2*sqrt(x + 1)*sqrt( - x + 1)*x**3 + 5
*sqrt(x + 1)*sqrt( - x + 1)*x)/8
```

3.83 $\int \sqrt{1-x}(1+x)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \frac{1}{3}\left(1-x^2\right)^{3/2} + \frac{\arcsin(x)}{2}$$

output
```
1/2*x*(-x^2+1)^(1/2)-1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = \frac{1}{6}\sqrt{1-x^2}\left(-2+3x+2x^2\right) + \arctan\left(\frac{\sqrt{1-x^2}}{1-x}\right)$$

input
```
Integrate[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

output
```
(Sqrt[1 - x^2]*(-2 + 3*x + 2*x^2))/6 + ArcTan[Sqrt[1 - x^2]/(1 - x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}(x+1)^{3/2}\,dx$$

$$\Big\downarrow \text{ 50}$$

$$\int \sqrt{1-x^2}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\arcsin(x)}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

input
```
Int[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

output
```
(x*Sqrt[1 - x^2])/2 - (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{3}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{6}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$-\frac{(2x^2+3x-2)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

input
```
int((1-x)^(1/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1-x)^(1/2)*(1+x)^(5/2)-1/6*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1
+x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = \frac{1}{6}\left(2\,x^2+3\,x-2\right)\sqrt{x+1}\sqrt{-x+1} - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^2 + 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.29

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = \begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{5i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{5(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(1/2)*(1+x)**(3/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x -
1)) - 5*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - I*(x + 1)**(3/2)/(6*sqrt(x - 1)
) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (asin(sqrt(2)*sqrt(x + 1)/
2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 5*(x + 1)**(5/2)/(6*sqrt(1 - x)) + (
x + 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = -\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(28) = 56$.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = \frac{1}{6}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)
*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = \int \sqrt{1-x}\,(x+1)^{3/2}\,dx$$

input
```
int((1 - x)^(1/2)*(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(1/2)*(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sqrt{1-x}(1+x)^{3/2}\,dx = -asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{2} - \frac{\sqrt{x+1}\,\sqrt{1-x}}{3}$$

input
```
int((1-x)^(1/2)*(1+x)^(3/2),x)
```

output
```
( - 6*asin(sqrt( - x + 1)/sqrt(2)) + 2*sqrt(x + 1)*sqrt( - x + 1)*x**2 + 3
*sqrt(x + 1)*sqrt( - x + 1)*x - 2*sqrt(x + 1)*sqrt( - x + 1))/6
```

3.84 $\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -\frac{3}{2}\sqrt{1-x}\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}(1+x)^{3/2} + \frac{3\arcsin(x)}{2}$$

output
```
-3/2*(1-x)^(1/2)*(1+x)^(1/2)-1/2*(1-x)^(1/2)*(1+x)^(3/2)+3/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{2}(4+x)\sqrt{1-x^2} - 3\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

output
```
-1/2*((4 + x)*Sqrt[1 - x^2]) - 3*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{\sqrt{1-x}}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{2}\int \frac{\sqrt{x+1}}{\sqrt{1-x}}dx-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}$$

$$\Big\downarrow \text{ 50}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}\right)-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{3}{2}\left(\arcsin(x)-\sqrt{1-x^2}\right)-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}$$

input
```
Int[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

output
```
-1/2*(Sqrt[1 - x]*(1 + x)^(3/2)) + (3*(-Sqrt[1 - x^2] + ArcSin[x]))/2
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$\frac{(4+x)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	70

input
```
int((1+x)^(3/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(1-x)^(1/2)*(1+x)^(3/2)-3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))
^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{2}\,(x+4)\sqrt{x+1}\sqrt{-x+1} - 3\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.85

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = \begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{3\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(1/2),x)
```

output
```
Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x
- 1)) - I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 3*I*sqrt(x + 1)/sqrt(x - 1), Ab
s(x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 -
 x)) + (x + 1)**(3/2)/(2*sqrt(1 - x)) - 3*sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

input
```
integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{2}\,(x+4)\sqrt{x+1}\sqrt{-x+1} + 3\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = \int \frac{(x+1)^{3/2}}{\sqrt{1-x}}\,dx$$

input
```
int((x + 1)^(3/2)/(1 - x)^(1/2),x)
```

output
```
int((x + 1)^(3/2)/(1 - x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -3asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{2} - 2\sqrt{x+1}\,\sqrt{1-x}$$

input
```
int((1+x)^(3/2)/(1-x)^(1/2),x)
```

output
```
( - 6*asin(sqrt( - x + 1)/sqrt(2)) - sqrt(x + 1)*sqrt( - x + 1)*x - 4*sqrt
(x + 1)*sqrt( - x + 1))/2
```

3.85 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}} \, dx = 3\sqrt{1-x}\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\arcsin(x)$$

output
```
3*(1-x)^(1/2)*(1+x)^(1/2)+2*(1+x)^(3/2)/(1-x)^(1/2)-3*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}} \, dx = \frac{(-5+x)\sqrt{1-x^2}}{-1+x} + 6\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

output
```
((-5 + x)*Sqrt[1 - x^2])/(-1 + x) + 6*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{57, 50, 223\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{3/2}}dx$$

$$\Big\downarrow \; 57$$

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}-3\int \frac{\sqrt{x+1}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \; 50$$

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}-3\left(\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}\right)$$

$$\Big\downarrow \; 223$$

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}-3\left(\arcsin(x)-\sqrt{1-x^2}\right)$$

input
```
Int[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

output
```
(2*(1 + x)^(3/2))/Sqrt[1 - x] - 3*(-Sqrt[1 - x^2] + ArcSin[x])
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

method	result	size
risch	$-\frac{(x^2-4x-5)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	72

input
```
int((1+x)^(3/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(x^2-4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(
1/2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx = \frac{\sqrt{x+1}(x-5)\sqrt{-x+1}+6\,(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+5\,x-5}{x-1}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(x + 1)*(x - 5)*sqrt(-x + 1) + 6*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + 5*x - 5)/(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx = \begin{cases} 6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{6i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{6\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(3/2),x)
```

output
```
Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
 - 6*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (-6*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 6*sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx = -\frac{(-x^2+1)^{\frac{3}{2}}}{x^2-2x+1} - \frac{6\sqrt{-x^2+1}}{x-1} - 3\arcsin(x)$$

input
```
integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="maxima")
```

output
```
-(-x^2 + 1)^(3/2)/(x^2 - 2*x + 1) - 6*sqrt(-x^2 + 1)/(x - 1) - 3*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx=\frac{\sqrt{x+1}(x-5)\sqrt{-x+1}}{x-1}-6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="giac")
```

output
```
sqrt(x + 1)*(x - 5)*sqrt(-x + 1)/(x - 1) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx=\int \frac{(x+1)^{3/2}}{(1-x)^{3/2}}\,dx$$

input
```
int((x + 1)^(3/2)/(1 - x)^(3/2),x)
```

output
```
int((x + 1)^(3/2)/(1 - x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx=\frac{6\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-\sqrt{x+1}\,x+5\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
int((1+x)^(3/2)/(1-x)^(3/2),x)
```

output
```
(6*sqrt( - x + 1)*asin(sqrt( - x + 1)/sqrt(2)) - sqrt(x + 1)*x + 5*sqrt(x
+ 1))/sqrt( - x + 1)
```

3.86 $\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \arcsin(x)$$

output
```
-2*(1+x)^(1/2)/(1-x)^(1/2)+2/3*(1+x)^(3/2)/(1-x)^(3/2)+arcsin(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = \frac{4(-1+2x)\sqrt{1-x^2}}{3(-1+x)^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

output
```
(4*(-1 + 2*x)*Sqrt[1 - x^2])/(3*(-1 + x)^2) - 2*ArcTan[Sqrt[1 - x^2]/(-1 +
 x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 57, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\int \frac{\sqrt{x+1}}{(1-x)^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}dx+\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{39}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx+\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{223}$$

$$\arcsin(x)+\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
Int[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

output
```
(-2*Sqrt[1 + x])/Sqrt[1 - x] + (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + ArcSi
n[x]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

method	result	size
risch	$-\frac{4(2x^2+x-1)\sqrt{(1+x)(1-x)}}{3(-1+x)\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	76

input
```
int((1+x)^(3/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(2*x^2+x-1)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1
/2)/(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = -\frac{2\left(2\,x^2-2\,(2\,x-1)\sqrt{x+1}\sqrt{-x+1}+3\,(x^2-2\,x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-4\,x+2\right)}{3\,(x^2-2\,x+1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*x^2 - 2*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*(x^2 - 2*x + 1)*arc
tan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 498, normalized size of antiderivative = 12.15

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = \begin{cases} -\frac{6i\sqrt{x-1}(x+1)^{\frac{15}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{3\pi\sqrt{x-1}(x+1)^{\frac{15}{2}}}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{12i\sqrt{x-1}(x+1)^{\frac{13}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}} \\ \frac{6\sqrt{1-x}(x+1)^{\frac{15}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}-\frac{12\sqrt{1-x}(x+1)^{\frac{13}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}-\frac{8(x+1)^8}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}+ \end{cases}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(5/2),x)
```

output
```
Piecewise((-6*I*sqrt(x - 1)*(x + 1)**(15/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(
3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) + 3*pi*sqrt
(x - 1)*(x + 1)**(15/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x
+ 1)**(13/2)) + 12*I*sqrt(x - 1)*(x + 1)**(13/2)*acosh(sqrt(2)*sqrt(x + 1)
/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) - 6*pi
*sqrt(x - 1)*(x + 1)**(13/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1
)*(x + 1)**(13/2)) + 8*I*(x + 1)**8/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqr
t(x - 1)*(x + 1)**(13/2)) - 12*I*(x + 1)**7/(3*sqrt(x - 1)*(x + 1)**(15/2)
 - 6*sqrt(x - 1)*(x + 1)**(13/2)), Abs(x + 1) > 2), (6*sqrt(1 - x)*(x + 1)
**(15/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sq
rt(1 - x)*(x + 1)**(13/2)) - 12*sqrt(1 - x)*(x + 1)**(13/2)*asin(sqrt(2)*s
qrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/
2)) - 8*(x + 1)**8/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)*
*(13/2)) + 12*(x + 1)**7/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x
 + 1)**(13/2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = -\frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^3-3\,x^2+3\,x-1)} + \frac{2\,\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)} + \frac{7\,\sqrt{-x^2+1}}{3\,(x-1)} + \arcsin{(x)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2 + 3*x - 1) + 2/3*sqrt(-x^2 + 1)/(x^2 -
2*x + 1) + 7/3*sqrt(-x^2 + 1)/(x - 1) + arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = \frac{4\,(2\,x-1)\sqrt{x+1}\sqrt{-x+1}}{3\,(x-1)^2} + 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="giac")
```

output
```
4/3*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 2*arcsin(1/2*sqrt(2)*sq
rt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = \int \frac{(x+1)^{3/2}}{(1-x)^{5/2}}\,dx$$

input
```
int((x + 1)^(3/2)/(1 - x)^(5/2),x)
```

output
```
int((x + 1)^(3/2)/(1 - x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = \frac{-2\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x + 2\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - \frac{8\sqrt{x+1}\,x}{3} + \frac{4\sqrt{x+1}}{3}}{\sqrt{1-x}\,(x-1)}$$

input
```
int((1+x)^(3/2)/(1-x)^(5/2),x)
```

output
```
(2*( - 3*sqrt( - x + 1)*asin(sqrt( - x + 1)/sqrt(2))*x + 3*sqrt( - x + 1)*
asin(sqrt( - x + 1)/sqrt(2)) - 4*sqrt(x + 1)*x + 2*sqrt(x + 1)))/(3*sqrt(
- x + 1)*(x - 1))
```

3.87 $\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx = \frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

output
```
1/5*(1+x)^(5/2)/(1-x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx = \frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

input
```
Integrate[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

output
```
(1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{7/2}}\,dx$$

$$\Big\downarrow \ 48$$

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

input
```
Int[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

output
```
(1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{(1+x)^{\frac{5}{2}}}{5(1-x)^{\frac{5}{2}}}$	15
orering	$-\frac{(1+x)^{\frac{5}{2}}(-1+x)}{5(1-x)^{\frac{7}{2}}}$	18
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^3+3x^2+3x+1)}{5\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54
default	$\frac{(1+x)^{\frac{3}{2}}}{(1-x)^{\frac{5}{2}}}-\frac{6\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}+\frac{\sqrt{1+x}}{5(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{5\sqrt{1-x}}$	57

input
```
int((1+x)^(3/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(1+x)^(5/2)/(1-x)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx = \frac{x^3-3\,x^2-(x^2+2\,x+1)\sqrt{x+1}\sqrt{-x+1}+3\,x-1}{5\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(x^3 - 3*x^2 - (x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*x - 1)/(x^
3 - 3*x^2 + 3*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.35

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx = \begin{cases} -\frac{i(x+1)^{\frac{5}{2}}}{5\sqrt{x-1}(x+1)^2-20\sqrt{x-1}(x+1)+20\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{(x+1)^{\frac{5}{2}}}{5\sqrt{1-x}(x+1)^2-20\sqrt{1-x}(x+1)+20\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(7/2),x)
```

output
```
Piecewise((-I*(x + 1)**(5/2)/(5*sqrt(x - 1)*(x + 1)**2 - 20*sqrt(x - 1)*(x
 + 1) + 20*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**(5/2)/(5*sqrt(1 - x)*(
x + 1)**2 - 20*sqrt(1 - x)*(x + 1) + 20*sqrt(1 - x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.70

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx = \frac{(-x^2+1)^{\frac{3}{2}}}{x^4-4x^3+6x^2-4x+1} + \frac{6\sqrt{-x^2+1}}{5(x^3-3x^2+3x-1)} + \frac{\sqrt{-x^2+1}}{5(x^2-2x+1)} - \frac{\sqrt{-x^2+1}}{5(x-1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="maxima")
```

output
```
(-x^2 + 1)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 6/5*sqrt(-x^2 + 1)/(x^3
 - 3*x^2 + 3*x - 1) + 1/5*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/5*sqrt(-x^2 +
 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx = -\frac{(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{5\,(x-1)^3}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="giac")
```

output
```
-1/5*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx = -\frac{\sqrt{1-x}\left(\frac{2\,x\,\sqrt{x+1}}{5}+\frac{\sqrt{x+1}}{5}+\frac{x^2\,\sqrt{x+1}}{5}\right)}{x^3-3\,x^2+3\,x-1}$$

input
```
int((x + 1)^(3/2)/(1 - x)^(7/2),x)
```

output
```
-((1 - x)^(1/2)*((2*x*(x + 1)^(1/2))/5 + (x + 1)^(1/2)/5 + (x^2*(x + 1)^(1
/2))/5))/(3*x - 3*x^2 + x^3 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx = \frac{\sqrt{x+1}\,(x^2+2x+1)}{5\sqrt{1-x}\,(x^2-2x+1)}$$

input
```
int((1+x)^(3/2)/(1-x)^(7/2),x)
```

output
```
(sqrt(x + 1)*(x**2 + 2*x + 1))/(5*sqrt( - x + 1)*(x**2 - 2*x + 1))
```

3.88 $\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} \, dx = \frac{(1+x)^{5/2}}{7(1-x)^{7/2}} + \frac{(1+x)^{5/2}}{35(1-x)^{5/2}}$$

output
```
1/7*(1+x)^(5/2)/(1-x)^(7/2)+1/35*(1+x)^(5/2)/(1-x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} \, dx = \frac{(6-x)(1+x)^{5/2}}{35(1-x)^{7/2}}$$

input
```
Integrate[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

output
```
((6 - x)*(1 + x)^(5/2))/(35*(1 - x)^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{9/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{7}\int \frac{(x+1)^{3/2}}{(1-x)^{7/2}}dx+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

input
```
Int[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

output
```
(1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(x-6)(1+x)^{\frac{5}{2}}}{35(1-x)^{\frac{7}{2}}}$	18
orering	$\frac{(1+x)^{\frac{5}{2}}(-1+x)(x-6)}{35(1-x)^{\frac{9}{2}}}$	21
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^4-3x^3-15x^2-17x-6)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59
default	$\frac{(1+x)^{\frac{3}{2}}}{2(1-x)^{\frac{7}{2}}}-\frac{3\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70(1-x)^{\frac{5}{2}}}+\frac{\sqrt{1+x}}{35(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{35\sqrt{1-x}}$	72

input
```
int((1+x)^(3/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(x-6)/(1-x)^(7/2)*(1+x)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.68

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx = \frac{6\,x^4 - 24\,x^3 + 36\,x^2 - (x^3 - 4\,x^2 - 11\,x - 6)\sqrt{x+1}\sqrt{-x+1} - 24\,x + 6}{35\,(x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(6*x^4 - 24*x^3 + 36*x^2 - (x^3 - 4*x^2 - 11*x - 6)*sqrt(x + 1)*sqrt(
-x + 1) - 24*x + 6)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.90 (sec) , antiderivative size = 226, normalized size of antiderivative = 5.51

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx = \begin{cases} -\frac{i(x+1)^{\frac{7}{2}}}{35\sqrt{x-1}(x+1)^3-210\sqrt{x-1}(x+1)^2+420\sqrt{x-1}(x+1)-280\sqrt{x-1}} + \frac{7i(x+1)^{\frac{5}{2}}}{35\sqrt{x-1}(x+1)^3-210\sqrt{x-1}(x+1)^2+420\sqrt{x-\cdots}} \\ \frac{(x+1)^{\frac{7}{2}}}{35\sqrt{1-x}(x+1)^3-210\sqrt{1-x}(x+1)^2+420\sqrt{1-x}(x+1)-280\sqrt{1-x}} - \frac{7(x+1)^{\frac{5}{2}}}{35\sqrt{1-x}(x+1)^3-210\sqrt{1-x}(x+1)^2+420\sqrt{1-x}(\cdots} \end{cases}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(9/2),x)
```

output
```
Piecewise((-I*(x + 1)**(7/2)/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*
(x + 1)**2 + 420*sqrt(x - 1)*(x + 1) - 280*sqrt(x - 1)) + 7*I*(x + 1)**(5/
2)/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(x + 1)**2 + 420*sqrt(x -
1)*(x + 1) - 280*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**(7/2)/(35*sqrt(1
 - x)*(x + 1)**3 - 210*sqrt(1 - x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) -
280*sqrt(1 - x)) - 7*(x + 1)**(5/2)/(35*sqrt(1 - x)*(x + 1)**3 - 210*sqrt(
1 - x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*sqrt(1 - x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx = -\frac{(-x^2+1)^{\frac{3}{2}}}{2\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} - \frac{3\,\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)} - \frac{3\,\sqrt{-x^2+1}}{70\,(x^3-3\,x^2+3\,x-1)} + \frac{\sqrt{-x^2+1}}{35\,(x^2-2\,x+1)} - \frac{\sqrt{-x^2+1}}{35\,(x-1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="maxima")
```

output
```
-1/2*(-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 3/7*sqrt
(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/70*sqrt(-x^2 + 1)/(x^3 - 3*
x^2 + 3*x - 1) + 1/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/35*sqrt(-x^2 + 1)
/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx = -\frac{(x+1)^{\frac{5}{2}}(x-6)\sqrt{-x+1}}{35\,(x-1)^4}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*(x + 1)^(5/2)*(x - 6)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx = \frac{\sqrt{1-x}\,\left(\frac{11\,x\,\sqrt{x+1}}{35}+\frac{6\,\sqrt{x+1}}{35}+\frac{4\,x^2\,\sqrt{x+1}}{35}-\frac{x^3\,\sqrt{x+1}}{35}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

input
```
int((x + 1)^(3/2)/(1 - x)^(9/2),x)
```

output
```
((1 - x)^(1/2)*((11*x*(x + 1)^(1/2))/35 + (6*(x + 1)^(1/2))/35 + (4*x^2*(x
 + 1)^(1/2))/35 - (x^3*(x + 1)^(1/2))/35))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx = \frac{\sqrt{x+1}\,(x^3-4x^2-11x-6)}{35\sqrt{1-x}\,(x^3-3x^2+3x-1)}$$

input
```
int((1+x)^(3/2)/(1-x)^(9/2),x)
```

output
```
(sqrt(x + 1)*(x**3 - 4*x**2 - 11*x - 6))/(35*sqrt( - x + 1)*(x**3 - 3*x**2
 + 3*x - 1))
```

3.89 $\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}$$

output
```
1/9*(1+x)^(5/2)/(1-x)^(9/2)+2/63*(1+x)^(5/2)/(1-x)^(7/2)+2/315*(1+x)^(5/2)
/(1-x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = \frac{(1+x)^{5/2}\,(47-14x+2x^2)}{315(1-x)^{9/2}}$$

input
```
Integrate[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

output
```
((1 + x)^(5/2)*(47 - 14*x + 2*x^2))/(315*(1 - x)^(9/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{11/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2}{9}\int \frac{(x+1)^{3/2}}{(1-x)^{9/2}}dx+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2}{9}\left(\frac{1}{7}\int \frac{(x+1)^{3/2}}{(1-x)^{7/2}}dx+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}\right)+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}+\frac{2}{9}\left(\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}\right)$$

input
```
Int[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

output
```
(1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*((1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1
 + x)^(5/2)/(35*(1 - x)^(5/2))))/9
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{(1+x)^{\frac{5}{2}} (2x^2-14x+47)}{315(1-x)^{\frac{9}{2}}}$	25
orering	$-\frac{(1+x)^{\frac{5}{2}} (-1+x)(2x^2-14x+47)}{315(1-x)^{\frac{11}{2}}}$	28
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^5-8x^4+11x^3+101x^2+127x+47)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{(1+x)^{\frac{3}{2}}}{3(1-x)^{\frac{9}{2}}}-\frac{2\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}+\frac{\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}+\frac{\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{315\sqrt{1-x}}$	86

input
```
int((1+x)^(3/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(1+x)^(5/2)/(1-x)^(9/2)*(2*x^2-14*x+47)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx=\frac{47\,x^5-235\,x^4+470\,x^3-470\,x^2-(2\,x^4-10\,x^3+21\,x^2+80\,x+47)\sqrt{x+1}\sqrt{-x+1}}{315\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(47*x^5 - 235*x^4 + 470*x^3 - 470*x^2 - (2*x^4 - 10*x^3 + 21*x^2 + 8
0*x + 47)*sqrt(x + 1)*sqrt(-x + 1) + 235*x - 47)/(x^5 - 5*x^4 + 10*x^3 - 1
0*x^2 + 5*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.73 (sec) , antiderivative size = 675, normalized size of antiderivative = 11.07

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(11/2),x)
```

output
```
Piecewise((-2*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x
- 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)
**2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 22*I*(x + 1)**(9/2)
/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqrt(x
- 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x + 1)
 - 10080*sqrt(x - 1)) - 99*I*(x + 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**5 -
3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x
- 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 126*I*(
x + 1)**(5/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 +
12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x
 - 1)*(x + 1) - 10080*sqrt(x - 1)), Abs(x + 1) > 2), (2*(x + 1)**(11/2)/(3
15*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x
)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) -
10080*sqrt(1 - x)) - 22*(x + 1)**(9/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*
sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*
(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)) + 99*(x + 1)**
(7/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sq
rt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x
 + 1) - 10080*sqrt(1 - x)) - 126*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x + 1)**
5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(43) = 86$.

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.82

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = \frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)} + \frac{2\,\sqrt{-x^2+1}}{9\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} + \frac{\sqrt{-x^2+1}}{63\,(x^4-4\,x^3+6\,x^2-4\,x+1)} - \frac{\sqrt{-x^2+1}}{105\,(x^3-3\,x^2+3\,x-1)} + \frac{2\,\sqrt{-x^2+1}}{315\,(x^2-2\,x+1)} - \frac{2\,\sqrt{-x^2+1}}{315\,(x-1)}$$

input

```
integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="maxima")
```

output

```
1/3*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) +
2/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x
^2 + 3*x - 1) + 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/315*sqrt(-x^2 + 1
)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = -\frac{(2\,(x+1)(x-8)+63)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{315\,(x-1)^5}$$

input

```
integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="giac")
```

output

```
-1/315*(2*(x + 1)*(x - 8) + 63)*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = -\frac{\sqrt{1-x}\left(\frac{16\,x\,\sqrt{x+1}}{63}+\frac{47\,\sqrt{x+1}}{315}+\frac{x^2\,\sqrt{x+1}}{15}-\frac{2\,x^3\,\sqrt{x+1}}{63}+\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

input
```
int((x + 1)^(3/2)/(1 - x)^(11/2),x)
```

output
```
-((1 - x)^(1/2)*((16*x*(x + 1)^(1/2))/63 + (47*(x + 1)^(1/2))/315 + (x^2*(
x + 1)^(1/2))/15 - (2*x^3*(x + 1)^(1/2))/63 + (2*x^4*(x + 1)^(1/2))/315))/
(5*x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = \frac{\sqrt{x+1}\,(2x^4-10x^3+21x^2+80x+47)}{315\sqrt{1-x}\,(x^4-4x^3+6x^2-4x+1)}$$

input
```
int((1+x)^(3/2)/(1-x)^(11/2),x)
```

output
```
(sqrt(x + 1)*(2*x**4 - 10*x**3 + 21*x**2 + 80*x + 47))/(315*sqrt( - x + 1)
*(x**4 - 4*x**3 + 6*x**2 - 4*x + 1))
```

3.90 $\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}$$

output
```
1/11*(1+x)^(5/2)/(1-x)^(11/2)+1/33*(1+x)^(5/2)/(1-x)^(9/2)+2/231*(1+x)^(5/
2)/(1-x)^(7/2)+2/1155*(1+x)^(5/2)/(1-x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = \frac{(1+x)^{5/2}\,(152-61x+16x^2-2x^3)}{1155(1-x)^{11/2}}$$

input
```
Integrate[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

output
```
((1 + x)^(5/2)*(152 - 61*x + 16*x^2 - 2*x^3))/(1155*(1 - x)^(11/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{13/2}}dx$$

$$\Big\downarrow \ 55$$

$$\frac{3}{11}\int \frac{(x+1)^{3/2}}{(1-x)^{11/2}}dx+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \ 55$$

$$\frac{3}{11}\left(\frac{2}{9}\int \frac{(x+1)^{3/2}}{(1-x)^{9/2}}dx+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \ 55$$

$$\frac{3}{11}\left(\frac{2}{9}\left(\frac{1}{7}\int \frac{(x+1)^{3/2}}{(1-x)^{7/2}}dx+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}\right)+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \ 48$$

$$\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}+\frac{3}{11}\left(\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}+\frac{2}{9}\left(\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}\right)\right)$$

input
```
Int[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

output
```
(1 + x)^(5/2)/(11*(1 - x)^(11/2)) + (3*((1 + x)^(5/2)/(9*(1 - x)^(9/2)) +
(2*((1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))))/9
))/11
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.37

method	result	size
gosper	$-\frac{(1+x)^{\frac{5}{2}} (2x^3-16x^2+61x-152)}{1155(1-x)^{\frac{11}{2}}}$	30
orering	$\frac{(1+x)^{\frac{5}{2}} (-1+x) (2x^3-16x^2+61x-152)}{1155(1-x)^{\frac{13}{2}}}$	33
risch	$\frac{\sqrt{(1+x)(1-x)}\, (2x^6-10x^5+19x^4-15x^3-289x^2-395x-152)}{1155\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^5 \sqrt{-(1+x)(-1+x)}}$	71
default	$\frac{(1+x)^{\frac{3}{2}}}{4(1-x)^{\frac{11}{2}}}-\frac{3\sqrt{1+x}}{22(1-x)^{\frac{11}{2}}}+\frac{\sqrt{1+x}}{132(1-x)^{\frac{9}{2}}}+\frac{\sqrt{1+x}}{231(1-x)^{\frac{7}{2}}}+\frac{\sqrt{1+x}}{385(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{1155(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{1155\sqrt{1-x}}$	100

input
```
int((1+x)^(3/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(1+x)^(5/2)/(1-x)^(11/2)*(2*x^3-16*x^2+61*x-152)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = \frac{152\,x^6 - 912\,x^5 + 2280\,x^4 - 3040\,x^3 + 2280\,x^2 - (2\,x^5 - 12\,x^4 + 31\,x^3 - 46\,x^2 - 243\,x}{1155\,(x^6 - 6\,x^5 + 15\,x^4 - 20\,x^3 + 15\,x^2 - 6\,x + 1}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="fricas")
```

output
```
1/1155*(152*x^6 - 912*x^5 + 2280*x^4 - 3040*x^3 + 2280*x^2 - (2*x^5 - 12*x
^4 + 31*x^3 - 46*x^2 - 243*x - 152)*sqrt(x + 1)*sqrt(-x + 1) - 912*x + 152
)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.92 (sec) , antiderivative size = 1751, normalized size of antiderivative = 21.62

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(13/2),x)
```

output
```
Piecewise((-2*I*(x + 1)**(17/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(
x - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x
+ 1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3
+ 2069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sq
rt(x - 1)) + 34*I*(x + 1)**(15/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqr
t(x - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(
x + 1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**
3 + 2069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*
sqrt(x - 1)) - 255*I*(x + 1)**(13/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*
sqrt(x - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1
)*(x + 1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1
)**3 + 2069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 2956
80*sqrt(x - 1)) + 1105*I*(x + 1)**(11/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18
480*sqrt(x - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x
 - 1)*(x + 1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x
 + 1)**3 + 2069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) +
295680*sqrt(x - 1)) - 2750*I*(x + 1)**(9/2)/(1155*sqrt(x - 1)*(x + 1)**8 -
 18480*sqrt(x - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqr
t(x - 1)*(x + 1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)
*(x + 1)**3 + 2069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x +...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(57) = 114$.

Time = 0.03 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.69

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = -\frac{(-x^2+1)^{\frac{3}{2}}}{4\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)} - \frac{3\,\sqrt{-x^2+1}}{22\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)} - \frac{\sqrt{-x^2+1}}{132\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} + \frac{\sqrt{-x^2+1}}{231\,(x^4-4\,x^3+6\,x^2-4\,x+1)} - \frac{\sqrt{-x^2+1}}{385\,(x^3-3\,x^2+3\,x-1)} + \frac{2\,\sqrt{-x^2+1}}{1155\,(x^2-2\,x+1)} - \frac{2\,\sqrt{-x^2+1}}{1155\,(x-1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="maxima")
```

output
```
-1/4*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7
*x - 1) - 3/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*
x + 1) - 1/132*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) +
1/231*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/385*sqrt(-x^2 + 1
)/(x^3 - 3*x^2 + 3*x - 1) + 2/1155*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/1155
*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = -\frac{((2\,(x+1)(x-10)+99)(x+1)-231)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{1155\,(x-1)^{6}}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="giac")
```

output
```
-1/1155*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1)^(5/2)*sqrt(-x +
1)/(x - 1)^6
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = \frac{\sqrt{1-x}\left(\frac{81\,x\,\sqrt{x+1}}{385}+\frac{152\,\sqrt{x+1}}{1155}+\frac{46\,x^2\,\sqrt{x+1}}{1155}-\frac{31\,x^3\,\sqrt{x+1}}{1155}+\frac{4\,x^4\,\sqrt{x+1}}{385}-\frac{2\,x^5\,\sqrt{x+1}}{1155}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

input
```
int((x + 1)^(3/2)/(1 - x)^(13/2),x)
```

output
```
((1 - x)^(1/2)*((81*x*(x + 1)^(1/2))/385 + (152*(x + 1)^(1/2))/1155 + (46*
x^2*(x + 1)^(1/2))/1155 - (31*x^3*(x + 1)^(1/2))/1155 + (4*x^4*(x + 1)^(1/
2))/385 - (2*x^5*(x + 1)^(1/2))/1155))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6
*x^5 + x^6 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx = \frac{\sqrt{x+1}\,(2x^5-12x^4+31x^3-46x^2-243x-152)}{1155\sqrt{1-x}\,(x^5-5x^4+10x^3-10x^2+5x-1)}$$

input
```
int((1+x)^(3/2)/(1-x)^(13/2),x)
```

output
```
(sqrt(x + 1)*(2*x**5 - 12*x**4 + 31*x**3 - 46*x**2 - 243*x - 152))/(1155*s
qrt( - x + 1)*(x**5 - 5*x**4 + 10*x**3 - 10*x**2 + 5*x - 1))
```

3.91 $\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}$$

output
```
1/13*(1+x)^(5/2)/(1-x)^(13/2)+4/143*(1+x)^(5/2)/(1-x)^(11/2)+4/429*(1+x)^(
5/2)/(1-x)^(9/2)+8/3003*(1+x)^(5/2)/(1-x)^(7/2)+8/15015*(1+x)^(5/2)/(1-x)^
(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \frac{(1+x)^{5/2}\left(1763-852x+308x^2-72x^3+8x^4\right)}{15015(1-x)^{13/2}}$$

input
```
Integrate[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

output
```
((1 + x)^(5/2)*(1763 - 852*x + 308*x^2 - 72*x^3 + 8*x^4))/(15015*(1 - x)^(
13/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{15/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4}{13}\int \frac{(x+1)^{3/2}}{(1-x)^{13/2}}dx+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4}{13}\left(\frac{3}{11}\int \frac{(x+1)^{3/2}}{(1-x)^{11/2}}dx+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}\right)+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4}{13}\left(\frac{3}{11}\left(\frac{2}{9}\int \frac{(x+1)^{3/2}}{(1-x)^{9/2}}dx+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}\right)+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4}{13}\left(\frac{3}{11}\left(\frac{2}{9}\left(\frac{1}{7}\int \frac{(x+1)^{3/2}}{(1-x)^{7/2}}dx+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}\right)+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}\right)+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}+\frac{4}{13}\left(\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}+\frac{3}{11}\left(\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}+\frac{2}{9}\left(\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}\right)\right)\right)$$

input
```
Int[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

output
```
(1 + x)^(5/2)/(13*(1 - x)^(13/2)) + (4*((1 + x)^(5/2)/(11*(1 - x)^(11/2))
+ (3*((1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*((1 + x)^(5/2)/(7*(1 - x)^(7/2)
) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))))/9))/11))/13
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result
gosper	$\frac{(1+x)^{\frac{5}{2}}(8x^4-72x^3+308x^2-852x+1763)}{15015(1-x)^{\frac{13}{2}}}$
orering	$-\frac{(1+x)^{\frac{5}{2}}(-1+x)(8x^4-72x^3+308x^2-852x+1763)}{15015(1-x)^{\frac{15}{2}}}$
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^7-48x^6+116x^5-136x^4+59x^3+3041x^2+4437x+1763)}{15015\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^6\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{3}{2}}}{5(1-x)^{\frac{13}{2}}}-\frac{6\sqrt{1+x}}{65(1-x)^{\frac{13}{2}}}+\frac{3\sqrt{1+x}}{715(1-x)^{\frac{11}{2}}}+\frac{\sqrt{1+x}}{429(1-x)^{\frac{9}{2}}}+\frac{4\sqrt{1+x}}{3003(1-x)^{\frac{7}{2}}}+\frac{4\sqrt{1+x}}{5005(1-x)^{\frac{5}{2}}}+\frac{8\sqrt{1+x}}{15015(1-x)^{\frac{3}{2}}}+\frac{8\sqrt{1+x}}{15015\sqrt{1-x}}$

input
```
int((1+x)^(3/2)/(1-x)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
1/15015*(1+x)^(5/2)/(1-x)^(13/2)*(8*x^4-72*x^3+308*x^2-852*x+1763)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \frac{1763\,x^7 - 12341\,x^6 + 37023\,x^5 - 61705\,x^4 + 61705\,x^3 - 37023\,x^2 - (8\,x^6 - 56\,x^5 + 17}{15015\,(x^7 - 7\,x^6 + 21\,x^5 - 35\,x^4 +}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="fricas")
```

output
```
1/15015*(1763*x^7 - 12341*x^6 + 37023*x^5 - 61705*x^4 + 61705*x^3 - 37023*
x^2 - (8*x^6 - 56*x^5 + 172*x^4 - 308*x^3 + 367*x^2 + 2674*x + 1763)*sqrt(
x + 1)*sqrt(-x + 1) + 12341*x - 1763)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*
x^3 - 21*x^2 + 7*x - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((1+x)**(3/2)/(1-x)**(15/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(71) = 142$.

Time = 0.03 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.66

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \frac{(-x^2+1)^{\frac{3}{2}}}{5\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)} + \frac{6\,\sqrt{-x^2+1}}{65\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)} + \frac{3\,\sqrt{-x^2+1}}{715\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)} - \frac{\sqrt{-x^2+1}}{429\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} + \frac{4\,\sqrt{-x^2+1}}{3003\,(x^4-4\,x^3+6\,x^2-4\,x+1)} - \frac{4\,\sqrt{-x^2+1}}{5005\,(x^3-3\,x^2+3\,x-1)} + \frac{8\,\sqrt{-x^2+1}}{15015\,(x^2-2\,x+1)} - \frac{8\,\sqrt{-x^2+1}}{15015\,(x-1)}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="maxima")
```

output
```
1/5*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28
*x^2 - 8*x + 1) + 6/65*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*
x^3 - 21*x^2 + 7*x - 1) + 3/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*
x^3 + 15*x^2 - 6*x + 1) - 1/429*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1) + 4/3003*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4
/5005*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 8/15015*sqrt(-x^2 + 1)/(x^2
 - 2*x + 1) - 8/15015*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = -\frac{(4\,((2\,(x+1)(x-12)+143)(x+1)-429)(x+1)+3003)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{15015\,(x-1)^7}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="giac")
```

output
```
-1/15015*(4*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1) + 3003)*(x
+ 1)^(5/2)*sqrt(-x + 1)/(x - 1)^7
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = -\frac{\sqrt{1-x}\left(\frac{382\,x\,\sqrt{x+1}}{2145}+\frac{1763\,\sqrt{x+1}}{15015}+\frac{367\,x^2\,\sqrt{x+1}}{15015}-\frac{4\,x^3\,\sqrt{x+1}}{195}+\frac{172\,x^4\,\sqrt{x+1}}{15015}-\frac{8\,x^5\,\sqrt{x+1}}{2145}+\frac{8\,x^6\,\sqrt{x+1}}{15015}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

input
```
int((x + 1)^(3/2)/(1 - x)^(15/2),x)
```

output
```
-((1 - x)^(1/2)*((382*x*(x + 1)^(1/2))/2145 + (1763*(x + 1)^(1/2))/15015 +
 (367*x^2*(x + 1)^(1/2))/15015 - (4*x^3*(x + 1)^(1/2))/195 + (172*x^4*(x +
 1)^(1/2))/15015 - (8*x^5*(x + 1)^(1/2))/2145 + (8*x^6*(x + 1)^(1/2))/1501
5))/(7*x - 21*x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \frac{\sqrt{x+1}\,(8x^6-56x^5+172x^4-308x^3+367x^2+2674x+1763)}{15015\sqrt{1-x}\,(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}$$

input
```
int((1+x)^(3/2)/(1-x)^(15/2),x)
```

output
```
(sqrt(x + 1)*(8*x**6 - 56*x**5 + 172*x**4 - 308*x**3 + 367*x**2 + 2674*x +
 1763))/(15015*sqrt( - x + 1)*(x**6 - 6*x**5 + 15*x**4 - 20*x**3 + 15*x**2
 - 6*x + 1))
```

3.92 $\int (1-x)^{11/2}(1+x)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx = \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2}+\frac{1}{9}(1-x)^{11/2}(1+x)^{7/2}+\frac{55}{128}x\sqrt{1-x^2}+\frac{55}{192}x\left(1-x^2\right)^{3/2}+\frac{11}{48}x\left(1-x^2\right)^{5/2}$$

output
```
11/72*(1-x)^(9/2)*(1+x)^(7/2)+1/9*(1-x)^(11/2)*(1+x)^(7/2)+55/128*x*(-x^2+
1)^(1/2)+55/192*x*(-x^2+1)^(3/2)+11/48*x*(-x^2+1)^(5/2)+11/56*(-x^2+1)^(7/
2)+55/128*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.71

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx = \frac{\sqrt{1-x^2}(3712+4599x-10240x^2+3066x^3+8448x^4-7224x^5-1024x^6+3024x^7-896x^8)}{8064} - \frac{55}{64}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

output
```
(Sqrt[1 - x^2]*(3712 + 4599*x - 10240*x^2 + 3066*x^3 + 8448*x^4 - 7224*x^5
 - 1024*x^6 + 3024*x^7 - 896*x^8))/8064 - (55*ArcTan[Sqrt[1 - x^2]/(-1 + x
)])/64
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {59, 59, 50, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{11/2}(x+1)^{5/2}\,dx$$

$$\Big\downarrow \text{59}$$

$$\frac{11}{9}\int (1-x)^{9/2}(x+1)^{5/2}dx+\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}$$

$$\Big\downarrow \text{59}$$

$$\frac{11}{9}\left(\frac{9}{8}\int (1-x)^{7/2}(x+1)^{5/2}dx+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}\right)+\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}$$

$$\Big\downarrow \text{50}$$

$$\frac{11}{9}\left(\frac{9}{8}\left(\int \left(1-x^2\right)^{5/2}dx+\frac{1}{7}\left(1-x^2\right)^{7/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}\right)+\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{11}{9}\left(\frac{9}{8}\left(\frac{5}{6}\int \left(1-x^2\right)^{3/2}dx+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}\right)+\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{11}{9}\left(\frac{9}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}\right)+ \frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}$$

$$\Big\downarrow \; 211$$

$$\frac{11}{9}\Bigg(\frac{9}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\Bigg)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\Bigg)+\frac{1}{8}(x+1 \quad \frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}$$

$$\Big\downarrow \; 223$$

$$\frac{11}{9}\Bigg(\frac{9}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\Bigg)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\Bigg)+\frac{1}{8}(x+1)^{7/2}(1 \quad \frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}$$

input
```
Int[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

output
```
((1 - x)^(11/2)*(1 + x)^(7/2))/9 + (11*(((1 - x)^(9/2)*(1 + x)^(7/2))/8 +
(9*((x*(1 - x^2)^(5/2))/6 + (1 - x^2)^(7/2)/7 + (5*((x*(1 - x^2)^(3/2))/4
+ (3*((x*Sqrt[1 - x^2])/2 + ArcSin[x]/2))/4))/6))/8))/9
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

method	result
risch	$\frac{(896x^8-3024x^7+1024x^6+7224x^5-8448x^4-3066x^3+10240x^2-4599x-3712)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{8064\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{55\sqrt{(1+x)(1-x)}}{128\sqrt{1+x}\,\sqrt{}}$
default	$\frac{(1-x)^{\frac{11}{2}}(1+x)^{\frac{7}{2}}}{9}+\frac{11(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}}}{72}+\frac{11(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{56}+\frac{11(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{48}+\frac{11(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{48}+\frac{11\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{64}$

input
```
int((1-x)^(11/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8064*(896*x^8-3024*x^7+1024*x^6+7224*x^5-8448*x^4-3066*x^3+10240*x^2-459
9*x-3712)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-
x)^(1/2)+55/128*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx =$$
$$-\frac{1}{8064}\left(896\,x^8-3024\,x^7+1024\,x^6+7224\,x^5-8448\,x^4-3066\,x^3+10240\,x^2-4599\,x-3712\right)\sqrt{x+1}\sqrt{}$$
$$-\frac{55}{64}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="fricas")
```

output
```
-1/8064*(896*x^8 - 3024*x^7 + 1024*x^6 + 7224*x^5 - 8448*x^4 - 3066*x^3 +
10240*x^2 - 4599*x - 3712)*sqrt(x + 1)*sqrt(-x + 1) - 55/64*arctan((sqrt(x
 + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)]

Timed out.

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((1-x)**(11/2)*(1+x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.71

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx = \frac{1}{9}\left(-x^2+1\right)^{\frac{7}{2}}x^2 - \frac{3}{8}\left(-x^2+1\right)^{\frac{7}{2}}x + \frac{29}{63}\left(-x^2+1\right)^{\frac{7}{2}} + \frac{11}{48}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{55}{192}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{55}{128}\sqrt{-x^2+1}x + \frac{55}{128}\arcsin(x)$$

input
```
integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(-x^2 + 1)^(7/2)*x^2 - 3/8*(-x^2 + 1)^(7/2)*x + 29/63*(-x^2 + 1)^(7/2)
 + 11/48*(-x^2 + 1)^(5/2)*x + 55/192*(-x^2 + 1)^(3/2)*x + 55/128*sqrt(-x^2
 + 1)*x + 55/128*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(80) = 160$.

Time = 0.18 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.94

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx =$$

$$-\frac{1}{40320}\left((2\left((4\left(5\left(2\left(7\left(8\,x-65\right)(x+1)+2073\right)(x+1)-9833\right)(x+1)+75293\right)(x+1)-310203\right)(x+1\right.\right.\right.$$

$$+\frac{1}{6720}\left((2\left((4\left(5\left(6\left(7\,x-50\right)(x+1)+1219\right)(x+1)-12463\right)(x+1)+64233\right)(x+1)-53963\right)(x+1)+\right.\right.$$

$$+\frac{1}{840}\left((2\left((4\left(5\left(6\,x-37\right)(x+1)+661\right)(x+1)-4551\right)(x+1)+4781\right)(x+1)-6335\right)(x+1)+2835)\sqrt{x}$$

$$-\frac{1}{40}\left((2\left((4\left(5\,x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405)\sqrt{x+1}\sqrt{-x+1}$$

$$+\frac{1}{4}\left((2\left(3\,x-10\right)(x+1)+43\right)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}$$

$$-\frac{1}{3}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}(x-2)\sqrt{-x+1}$$

$$+\sqrt{x+1}\sqrt{-x+1}+\frac{55}{64}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="giac")
```

output
```
-1/40320*((2*((4*(5*(2*(7*(8*x - 65)*(x + 1) + 2073)*(x + 1) - 9833)*(x +
1) + 75293)*(x + 1) - 310203)*(x + 1) + 216993)*(x + 1) - 205275)*(x + 1)
+ 69615)*sqrt(x + 1)*sqrt(-x + 1) + 1/6720*((2*((4*(5*(6*(7*x - 50)*(x + 1
) + 1219)*(x + 1) - 12463)*(x + 1) + 64233)*(x + 1) - 53963)*(x + 1) + 594
65)*(x + 1) - 23205)*sqrt(x + 1)*sqrt(-x + 1) + 1/840*((2*((4*(5*(6*x - 37
)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781)*(x + 1) - 6335)*(x + 1) +
 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*((4*(5*x - 26)*(x + 1) + 321)*(
x + 1) - 451)*(x + 1) + 745)*(x + 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/4
*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3
*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sq
rt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 55/64*arcsin(1/2*sqrt(2)*sqrt(x +
1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx = \int (1-x)^{11/2}\,(x+1)^{5/2}\,dx$$

input
```
int((1 - x)^(11/2)*(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(11/2)*(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.32

$$\begin{aligned}\int (1-x)^{11/2}(1+x)^{5/2}\,dx = &-\frac{55 asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{64}-\frac{\sqrt{x+1}\,\sqrt{1-x}\,x^8}{9}\\&+\frac{3\sqrt{x+1}\,\sqrt{1-x}\,x^7}{8}-\frac{8\sqrt{x+1}\,\sqrt{1-x}\,x^6}{63}-\frac{43\sqrt{x+1}\,\sqrt{1-x}\,x^5}{48}\\&+\frac{22\sqrt{x+1}\,\sqrt{1-x}\,x^4}{21}+\frac{73\sqrt{x+1}\,\sqrt{1-x}\,x^3}{192}\\&-\frac{80\sqrt{x+1}\,\sqrt{1-x}\,x^2}{63}+\frac{73\sqrt{x+1}\,\sqrt{1-x}\,x}{128}+\frac{29\sqrt{x+1}\,\sqrt{1-x}}{63}\end{aligned}$$

input
```
int((1-x)^(11/2)*(1+x)^(5/2),x)
```

output
```
( - 6930*asin(sqrt( - x + 1)/sqrt(2)) - 896*sqrt(x + 1)*sqrt( - x + 1)*x**
8 + 3024*sqrt(x + 1)*sqrt( - x + 1)*x**7 - 1024*sqrt(x + 1)*sqrt( - x + 1)
*x**6 - 7224*sqrt(x + 1)*sqrt( - x + 1)*x**5 + 8448*sqrt(x + 1)*sqrt( - x
+ 1)*x**4 + 3066*sqrt(x + 1)*sqrt( - x + 1)*x**3 - 10240*sqrt(x + 1)*sqrt(
 - x + 1)*x**2 + 4599*sqrt(x + 1)*sqrt( - x + 1)*x + 3712*sqrt(x + 1)*sqrt
( - x + 1))/8064
```

3.93 $\int (1-x)^{9/2}(1+x)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx=\frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{45}{128}x\sqrt{1-x^2}+\frac{15}{64}x\left(1-x^2\right)^{3/2}+\frac{3}{16}x\left(1-x^2\right)^{5/2}+\frac{9}{56}\left(1-x^2\right)^{7/2}+\frac{45\,\mathrm{arcsi}}{128}$$

output
```
1/8*(1-x)^(9/2)*(1+x)^(7/2)+45/128*x*(-x^2+1)^(1/2)+15/64*x*(-x^2+1)^(3/2)
+3/16*x*(-x^2+1)^(5/2)+9/56*(-x^2+1)^(7/2)+45/128*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx=\frac{1}{896}\left(\sqrt{1-x^2}\left(256+581x-768x^2-210x^3+768x^4-168x^5-256x^6+112x^7\right)-630\arctan\left(\frac{\sqrt{1-}}{-1}\right.\right.$$

input
```
Integrate[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

output
```
(Sqrt[1 - x^2]*(256 + 581*x - 768*x^2 - 210*x^3 + 768*x^4 - 168*x^5 - 256*
x^6 + 112*x^7) - 630*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/896
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {59, 50, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{9/2}(x+1)^{5/2}\,dx$$

$$\Big\downarrow \text{59}$$

$$\frac{9}{8}\int (1-x)^{7/2}(x+1)^{5/2}dx+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{50}$$

$$\frac{9}{8}\left(\int \left(1-x^2\right)^{5/2}dx+\frac{1}{7}\left(1-x^2\right)^{7/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{8}\left(\frac{5}{6}\int \left(1-x^2\right)^{3/2}dx+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{9}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}\right)+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}$$

input
```
Int[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

output
```
((1 - x)^(9/2)*(1 + x)^(7/2))/8 + (9*((x*(1 - x^2)^(5/2))/6 + (1 - x^2)^(7
/2)/7 + (5*((x*(1 - x^2)^(3/2))/4 + (3*((x*Sqrt[1 - x^2])/2 + ArcSin[x]/2)
)/4))/6))/8
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\left(112x^{7}-256x^{6}-168x^{5}+768x^{4}-210x^{3}-768x^{2}+581x+256\right)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{896\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{45\sqrt{(1+x)(1-x)}\,\arcsin(x)}{128\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}}}{8}+\frac{9(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{56}+\frac{3(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{16}+\frac{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{16}+\frac{9\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{64}-\frac{3\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{64}-$ 15

input
```
int((1-x)^(9/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/896*(112*x^7-256*x^6-168*x^5+768*x^4-210*x^3-768*x^2+581*x+256)*(1+x)^(
1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+45/128*(
(1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx=\frac{1}{896}\left(112\,x^{7}-256\,x^{6}-168\,x^{5}+768\,x^{4}-210\,x^{3}-768\,x^{2}+581\,x+256\right)\sqrt{x+1}\sqrt{-x+1}-\frac{45}{64}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/896*(112*x^7 - 256*x^6 - 168*x^5 + 768*x^4 - 210*x^3 - 768*x^2 + 581*x +
 256)*sqrt(x + 1)*sqrt(-x + 1) - 45/64*arctan((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

Sympy [F(-1)]

Timed out.

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((1-x)**(9/2)*(1+x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx = -\frac{1}{8}\left(-x^2+1\right)^{\frac{7}{2}}x + \frac{2}{7}\left(-x^2+1\right)^{\frac{7}{2}} + \frac{3}{16}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{15}{64}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{45}{128}\sqrt{-x^2+1}x + \frac{45}{128}\arcsin(x)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="maxima")
```

output
```
-1/8*(-x^2 + 1)^(7/2)*x + 2/7*(-x^2 + 1)^(7/2) + 3/16*(-x^2 + 1)^(5/2)*x +
 15/64*(-x^2 + 1)^(3/2)*x + 45/128*sqrt(-x^2 + 1)*x + 45/128*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(66) = 132.

Time = 0.18 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.29

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx = \frac{1}{13440}\left((2\left((4\left(5\left(6\left(7\,x-50\right)(x+1)+1219\right)(x+1)-12463\right)(x+1)+64233\right)(x+1)-53963\right.\right.$$
$$-\frac{1}{1680}\left((2\left((4\left(5\left(6\,x-37\right)(x+1)+661\right)(x+1)-4551\right)(x+1)+4781\right)(x+1)-6335)(x+1)+2835\right)$$
$$-\frac{1}{80}\left((2\left((4\left(5\,x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745)(x+1)-405\right)\sqrt{x+1}\sqrt{-x+1}$$
$$+\frac{1}{40}\left((2\left(3\left(4\,x-17\right)(x+1)+133\right)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}$$
$$+\frac{1}{8}\left((2\left(3\,x-10\right)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}$$
$$-\frac{1}{2}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\,\sqrt{x+1}(x-2)\sqrt{-x+1}$$
$$+\sqrt{x+1}\sqrt{-x+1}+\frac{45}{64}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/13440*((2*((4*(5*(6*(7*x - 50)*(x + 1) + 1219)*(x + 1) - 12463)*(x + 1)
+ 64233)*(x + 1) - 53963)*(x + 1) + 59465)*(x + 1) - 23205)*sqrt(x + 1)*sq
rt(-x + 1) - 1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(
x + 1) + 4781)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) -
1/80*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4*x - 17)*(x + 1) + 133)
*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/8*((2*(3*x - 1
0)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*((2*x - 5)*(
x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1
) + sqrt(x + 1)*sqrt(-x + 1) + 45/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx = \int (1-x)^{9/2}\,(x+1)^{5/2}\,dx$$

input
```
int((1 - x)^(9/2)*(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(9/2)*(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

$$\begin{aligned}\int (1-x)^{9/2}(1+x)^{5/2}\,dx &= -\frac{45 asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{64} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^7}{8} \\ &- \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^6}{7} - \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x^5}{16} \\ &+ \frac{6\sqrt{x+1}\,\sqrt{1-x}\,x^4}{7} - \frac{15\sqrt{x+1}\,\sqrt{1-x}\,x^3}{64} \\ &- \frac{6\sqrt{x+1}\,\sqrt{1-x}\,x^2}{7} + \frac{83\sqrt{x+1}\,\sqrt{1-x}\,x}{128} + \frac{2\sqrt{x+1}\,\sqrt{1-x}}{7}\end{aligned}$$

input
```
int((1-x)^(9/2)*(1+x)^(5/2),x)
```

output
```
( - 630*asin(sqrt( - x + 1)/sqrt(2)) + 112*sqrt(x + 1)*sqrt( - x + 1)*x**7
 - 256*sqrt(x + 1)*sqrt( - x + 1)*x**6 - 168*sqrt(x + 1)*sqrt( - x + 1)*x*
*5 + 768*sqrt(x + 1)*sqrt( - x + 1)*x**4 - 210*sqrt(x + 1)*sqrt( - x + 1)*
x**3 - 768*sqrt(x + 1)*sqrt( - x + 1)*x**2 + 581*sqrt(x + 1)*sqrt( - x + 1
)*x + 256*sqrt(x + 1)*sqrt( - x + 1))/896
```

3.94 $\int (1-x)^{7/2}(1+x)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = \frac{5}{16}x\sqrt{1-x^2} + \frac{5}{24}x\left(1-x^2\right)^{3/2} + \frac{1}{6}x\left(1-x^2\right)^{5/2} + \frac{1}{7}\left(1-x^2\right)^{7/2} + \frac{5\arcsin(x)}{16}$$

output
```
5/16*x*(-x^2+1)^(1/2)+5/24*x*(-x^2+1)^(3/2)+1/6*x*(-x^2+1)^(5/2)+1/7*(-x^2
+1)^(7/2)+5/16*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = \frac{1}{336}\sqrt{1-x^2}\left(48+231x-144x^2-182x^3+144x^4+56x^5-48x^6\right) - \frac{5}{8}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

output
```
(Sqrt[1 - x^2]*(48 + 231*x - 144*x^2 - 182*x^3 + 144*x^4 + 56*x^5 - 48*x^6
))/336 - (5*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/8
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {50, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{7/2}(x+1)^{5/2}\,dx$$

$$\Big\downarrow \text{50}$$

$$\int \left(1-x^2\right)^{5/2}dx+\frac{1}{7}\left(1-x^2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}\int \left(1-x^2\right)^{3/2}dx+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{7}\left(1-x^2\right)^{7/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

input
```
Int[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

output
```
(x*(1 - x^2)^(5/2))/6 + (1 - x^2)^(7/2)/7 + (5*((x*(1 - x^2)^(3/2))/4 + (3
*((x*Sqrt[1 - x^2])/2 + ArcSin[x]/2))/4))/6
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

method	result
risch	$\frac{(48x^6-56x^5-144x^4+182x^3+144x^2-231x-48)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{336\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{7}+\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{8}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}-\frac{5\sqrt{1-x}}{16}$

input
```
int((1-x)^(7/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/336*(48*x^6-56*x^5-144*x^4+182*x^3+144*x^2-231*x-48)*(1+x)^(1/2)*(-1+x)/
(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+5/16*((1+x)*(1-x))^(
1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = -\frac{1}{336}\left(48\,x^6-56\,x^5-144\,x^4+182\,x^3+144\,x^2-231\,x-48\right)\sqrt{x+1}\sqrt{-x+1} - \frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="fricas")
```

output
```
-1/336*(48*x^6 - 56*x^5 - 144*x^4 + 182*x^3 + 144*x^2 - 231*x - 48)*sqrt(x
 + 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)]

Timed out.

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((1-x)**(7/2)*(1+x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = \frac{1}{7}\left(-x^2+1\right)^{\frac{7}{2}} + \frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{5}{16}\sqrt{-x^2+1}x + \frac{5}{16}\arcsin\left(x\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="maxima")
```

output
```
1/7*(-x^2 + 1)^(7/2) + 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x +
5/16*sqrt(-x^2 + 1)*x + 5/16*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(52) = 104$.

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx =$$
$$-\frac{1}{1680}\left((2\left((4\left(5\left(6x-37\right)(x+1)+661\right)(x+1)-4551\right)(x+1)+4781\right)(x+1)-6335\right)(x+1)+2835\right)\sqrt{\ }$$
$$+\frac{1}{40}\left((2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}$$
$$-\frac{1}{2}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}$$
$$+\sqrt{x+1}\sqrt{-x+1}+\frac{5}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="giac")
```

output
```
-1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 478
1)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*
(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-
x + 1) - 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1
)*sqrt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = \int (1-x)^{7/2}\,(x+1)^{5/2}\,dx$$

input
```
int((1 - x)^(7/2)*(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(7/2)*(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.64

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = -\frac{5asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^6}{7} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^5}{6} + \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x^4}{7} - \frac{13\sqrt{x+1}\,\sqrt{1-x}\,x^3}{24} - \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x^2}{7} + \frac{11\sqrt{x+1}\,\sqrt{1-x}\,x}{16} + \frac{\sqrt{x+1}\,\sqrt{1-x}}{7}$$

input
```
int((1-x)^(7/2)*(1+x)^(5/2),x)
```

output
```
( - 210*asin(sqrt( - x + 1)/sqrt(2)) - 48*sqrt(x + 1)*sqrt( - x + 1)*x**6
+ 56*sqrt(x + 1)*sqrt( - x + 1)*x**5 + 144*sqrt(x + 1)*sqrt( - x + 1)*x**4
 - 182*sqrt(x + 1)*sqrt( - x + 1)*x**3 - 144*sqrt(x + 1)*sqrt( - x + 1)*x*
*2 + 231*sqrt(x + 1)*sqrt( - x + 1)*x + 48*sqrt(x + 1)*sqrt( - x + 1))/336
```

3.95 $\int (1-x)^{5/2}(1+x)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx=\frac{5}{16}x\sqrt{1-x^2}+\frac{5}{24}x\left(1-x^2\right)^{3/2}+\frac{1}{6}x\left(1-x^2\right)^{5/2}+\frac{5\arcsin(x)}{16}$$

output
```
5/16*x*(-x^2+1)^(1/2)+5/24*x*(-x^2+1)^(3/2)+1/6*x*(-x^2+1)^(5/2)+5/16*arcs
in(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx=\frac{1}{48}x\sqrt{1-x^2}\left(33-26x^2+8x^4\right)-\frac{5}{8}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

output
```
(x*Sqrt[1 - x^2]*(33 - 26*x^2 + 8*x^4))/48 - (5*ArcTan[Sqrt[1 - x^2]/(1 +
x)])/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {39, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{5/2}(x+1)^{5/2}\,dx$$

$$\Big\downarrow \ 39$$

$$\int \left(1-x^2\right)^{5/2}dx$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}\int \left(1-x^2\right)^{3/2}dx+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-x^2}dx+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \ 223$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-x^2\right)^{5/2}$$

input
```
Int[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

output
```
(x*(1 - x^2)^(5/2))/6 + (5*((x*(1 - x^2)^(3/2))/4 + (3*((x*Sqrt[1 - x^2])/
2 + ArcSin[x]/2))/4))/6
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{x(8x^4-26x^2+33)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{48\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{8}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{16}+\frac{5\sqrt{(1+x)(}}{16\sqrt{1}}$

input
```
int((1-x)^(5/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*x*(8*x^4-26*x^2+33)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*
(1-x))^(1/2)/(1-x)^(1/2)+5/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*
arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx = \frac{1}{48}\left(8\,x^5-26\,x^3+33\,x\right)\sqrt{x+1}\sqrt{-x+1} - \frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/48*(8*x^5 - 26*x^3 + 33*x)*sqrt(x + 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x
 + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 161.51 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.16

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx = \begin{cases} -\frac{5i\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} - \frac{7i(x+1)^{\frac{11}{2}}}{6\sqrt{x-1}} + \frac{67i(x+1)^{\frac{9}{2}}}{24\sqrt{x-1}} - \frac{55i(x+1)^{\frac{7}{2}}}{24\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{8\sqrt{x-1}} \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} + \frac{7(x+1)^{\frac{11}{2}}}{6\sqrt{1-x}} - \frac{67(x+1)^{\frac{9}{2}}}{24\sqrt{1-x}} + \frac{55(x+1)^{\frac{7}{2}}}{24\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{5\sqrt{x+1}}{8\sqrt{1-x}} \end{cases}$$

input
```
integrate((1-x)**(5/2)*(1+x)**(5/2),x)
```

output
```
Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt
(x - 1)) - 7*I*(x + 1)**(11/2)/(6*sqrt(x - 1)) + 67*I*(x + 1)**(9/2)/(24*s
qrt(x - 1)) - 55*I*(x + 1)**(7/2)/(24*sqrt(x - 1)) - I*(x + 1)**(5/2)/(48*
sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(8*sq
rt(x - 1)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/8 - (x + 1)**(
13/2)/(6*sqrt(1 - x)) + 7*(x + 1)**(11/2)/(6*sqrt(1 - x)) - 67*(x + 1)**(9
/2)/(24*sqrt(1 - x)) + 55*(x + 1)**(7/2)/(24*sqrt(1 - x)) + (x + 1)**(5/2)
/(48*sqrt(1 - x)) + 5*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 5*sqrt(x + 1)/(8*s
qrt(1 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx = \frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x + \frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{5}{16}\sqrt{-x^2+1}x + \frac{5}{16}\arcsin\left(x\right)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5/16*sqrt(-x^2 + 1)*x +
 5/16*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(41) = 82.

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.36

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx = \frac{1}{240}\left((2\left((4\left(5x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405)\sqrt{x+1}\sqrt{-x+} + \frac{1}{120}\left((2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{12}\left((2\left(3x-10\right)(x+1)+43\right)(x+1)-39)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{5}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 13
3)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x
- 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5
)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x
+ 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx=\int (1-x)^{5/2}\,(x+1)^{5/2}\,dx$$

input
```
int((1 - x)^(5/2)*(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(5/2)*(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int (1-x)^{5/2}(1+x)^{5/2}\,dx=-\frac{5asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8}+\frac{\sqrt{x+1}\,\sqrt{1-x}\,x^5}{6}-\frac{13\sqrt{x+1}\,\sqrt{1-x}\,x^3}{24}+\frac{11\sqrt{x+1}\,\sqrt{1-x}\,x}{16}$$

input
```
int((1-x)^(5/2)*(1+x)^(5/2),x)
```

output
```
( - 30*asin(sqrt( - x + 1)/sqrt(2)) + 8*sqrt(x + 1)*sqrt( - x + 1)*x**5 -
26*sqrt(x + 1)*sqrt( - x + 1)*x**3 + 33*sqrt(x + 1)*sqrt( - x + 1)*x)/48
```

3.96 $\int (1-x)^{3/2}(1+x)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = \frac{3}{8}x\sqrt{1-x^2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}-\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{3\arcsin(x)}{8}$$

output
```
3/8*x*(-x^2+1)^(1/2)+1/4*x*(-x^2+1)^(3/2)-1/5*(-x^2+1)^(5/2)+3/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = -\frac{1}{40}\sqrt{1-x^2}\left(8-25x-16x^2+10x^3+8x^4\right)-\frac{3}{4}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

output
```
-1/40*(Sqrt[1 - x^2]*(8 - 25*x - 16*x^2 + 10*x^3 + 8*x^4)) - (3*ArcTan[Sqr
t[1 - x^2]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {50, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{3/2}(x+1)^{5/2}\,dx$$

$$\Big\downarrow \text{50}$$

$$\int \left(1-x^2\right)^{3/2}dx-\frac{1}{5}\left(1-x^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}\int \sqrt{1-x^2}dx-\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\right)-\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x\right)-\frac{1}{5}\left(1-x^2\right)^{5/2}+\frac{1}{4}x\left(1-x^2\right)^{3/2}$$

input
```
Int[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

output
```
(x*(1 - x^2)^(3/2))/4 - (1 - x^2)^(5/2)/5 + (3*((x*Sqrt[1 - x^2])/2 + ArcS
in[x]/2))/4
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(40) = 80$.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

method	result	s
risch	$\frac{(8x^4+10x^3-16x^2-25x+8)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{40\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	8
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{5}+\frac{3\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{20}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{20}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	9

input
```
int((1-x)^(3/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(8*x^4+10*x^3-16*x^2-25*x+8)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)
*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+3/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)
^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = -\frac{1}{40}\left(8\,x^4+10\,x^3-16\,x^2-25\,x+8\right)\sqrt{x+1}\sqrt{-x+1} - \frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="fricas")
```

output
```
-1/40*(8*x^4 + 10*x^3 - 16*x^2 - 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*
arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 52.75 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = \begin{cases} -\frac{3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}} + \frac{19i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}} - \frac{23i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+1| \\ \frac{3\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}} - \frac{19(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}} + \frac{23(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(3/2)*(1+x)**(5/2),x)
```

output
```
Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt
(x - 1)) + 19*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 23*I*(x + 1)**(7/2)/(20*
sqrt(x - 1)) - I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sqr
t(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (3*asin(sqrt
(2)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 19*(x + 1)**(9/2)
/(20*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(20*sqrt(1 - x)) + (x + 1)**(5/2)/(4
0*sqrt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1
- x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = -\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}} + \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{-x^2+1}x + \frac{3}{8}\arcsin\left(x\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="maxima")
```

output
```
-1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/
8*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(40) = 80.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = -\frac{1}{120}\left((2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{12}\left((2\left(3x-10\right)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="giac")
```

output
```
-1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt
(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqr
t(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqr
t(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = \int (1-x)^{3/2}\,(x+1)^{5/2}\,dx$$

input
```
int((1 - x)^(3/2)*(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(3/2)*(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = -\frac{3asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{4} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^4}{5} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{4} + \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{5} + \frac{5\sqrt{x+1}\,\sqrt{1-x}\,x}{8} - \frac{\sqrt{x+1}\,\sqrt{1-x}}{5}$$

input
```
int((1-x)^(3/2)*(1+x)^(5/2),x)
```

output
```
( - 30*asin(sqrt( - x + 1)/sqrt(2)) - 8*sqrt(x + 1)*sqrt( - x + 1)*x**4 -
10*sqrt(x + 1)*sqrt( - x + 1)*x**3 + 16*sqrt(x + 1)*sqrt( - x + 1)*x**2 +
25*sqrt(x + 1)*sqrt( - x + 1)*x - 8*sqrt(x + 1)*sqrt( - x + 1))/40
```

3.97 $\int \sqrt{1-x}(1+x)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = -\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}x\sqrt{1-x^2}-\frac{5}{12}\left(1-x^2\right)^{3/2}+\frac{5\arcsin(x)}{8}$$

output
```
-1/4*(1-x)^(3/2)*(1+x)^(5/2)+5/8*x*(-x^2+1)^(1/2)-5/12*(-x^2+1)^(3/2)+5/8*
arcsin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = \frac{1}{24}\sqrt{1-x^2}\left(-16+9x+16x^2+6x^3\right)-\frac{5}{4}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

output
```
(Sqrt[1 - x^2]*(-16 + 9*x + 16*x^2 + 6*x^3))/24 - (5*ArcTan[Sqrt[1 - x^2]/
(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {59, 50, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}(x+1)^{5/2}\,dx$$

$$\Big\downarrow \text{59}$$

$$\frac{5}{4}\int \sqrt{1-x}(x+1)^{3/2}dx-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \text{50}$$

$$\frac{5}{4}\left(\int \sqrt{1-x^2}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}\right)-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{5}{4}\left(\frac{\arcsin(x)}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}$$

input
```
Int[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

output
```
-1/4*((1 - x)^(3/2)*(1 + x)^(5/2)) + (5*((x*Sqrt[1 - x^2])/2 - (1 - x^2)^(
3/2)/3 + ArcSin[x]/2))/4
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{(6x^3+16x^2+9x-16)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	82
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{12}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{24}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	85

input
```
int((1-x)^(1/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*x^3+16*x^2+9*x-16)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x
)*(1-x))^(1/2)/(1-x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = \frac{1}{24}\left(6\,x^3+16\,x^2+9\,x-16\right)\sqrt{x+1}\sqrt{-x+1} - \frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sq
rt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.90 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.66

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = \begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} - \frac{7i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} + \frac{7(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}} - \frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(1/2)*(1+x)**(5/2),x)
```

output
```
Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(
x - 1)) - 7*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - I*(x + 1)**(5/2)/(24*sqrt(
x - 1)) - 5*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqrt(x
- 1)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/2)/
(4*sqrt(1 - x)) + 7*(x + 1)**(7/2)/(12*sqrt(1 - x)) + (x + 1)**(5/2)/(24*s
qrt(1 - x)) + 5*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4*sqrt(1
- x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = -\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x - \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{5}{8}\sqrt{-x^2+1}x + \frac{5}{8}\arcsin(x)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(-x^2 + 1)^(3/2)*x - 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/
8*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.74

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = \frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \frac{3}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{5}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 3/2*sqrt(x + 1)*(x
 - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = \int \sqrt{1-x}\,(x+1)^{5/2}\,dx$$

input
```
int((1 - x)^(1/2)*(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(1/2)*(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \sqrt{1-x}(1+x)^{5/2}\,dx = -\frac{5asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{4} + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{4} + \frac{2\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3} + \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x}{8} - \frac{2\sqrt{x+1}\,\sqrt{1-x}}{3}$$

input
```
int((1-x)^(1/2)*(1+x)^(5/2),x)
```

output
```
( - 30*asin(sqrt( - x + 1)/sqrt(2)) + 6*sqrt(x + 1)*sqrt( - x + 1)*x**3 +
16*sqrt(x + 1)*sqrt( - x + 1)*x**2 + 9*sqrt(x + 1)*sqrt( - x + 1)*x - 16*s
qrt(x + 1)*sqrt( - x + 1))/24
```

3.98 $\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -\frac{5}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{6}\sqrt{1-x}(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}(1+x)^{5/2} + \frac{5\arcsin(x)}{2}$$

output
```
-5/2*(1-x)^(1/2)*(1+x)^(1/2)-5/6*(1-x)^(1/2)*(1+x)^(3/2)-1/3*(1-x)^(1/2)*(
1+x)^(5/2)+5/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -\frac{1}{6}\sqrt{1-x^2}\left(22+9x+2x^2\right) - 5\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

output
```
-1/6*(Sqrt[1 - x^2]*(22 + 9*x + 2*x^2)) - 5*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {60, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{3}\int \frac{(x+1)^{3/2}}{\sqrt{1-x}}dx-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{x+1}}{\sqrt{1-x}}dx-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}\right)-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}$$

$$\Big\downarrow \text{50}$$

$$\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}\right)-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}\right)-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{5}{3}\left(\frac{3}{2}\left(\arcsin(x)-\sqrt{1-x^2}\right)-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}\right)-\frac{1}{3}\sqrt{1-x}(x+1)^{5/2}$$

input
```
Int[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

output
```
-1/3*(Sqrt[1 - x]*(1 + x)^(5/2)) + (5*(-1/2*(Sqrt[1 - x]*(1 + x)^(3/2)) +
(3*(-Sqrt[1 - x^2] + ArcSin[x]))/2))/3
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{3}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{6}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$\frac{(2x^2+9x+22)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

input
```
int((1+x)^(5/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(1-x)^(1/2)*(1+x)^(5/2)-5/6*(1-x)^(1/2)*(1+x)^(3/2)-5/2*(1-x)^(1/2)*(
1+x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -\frac{1}{6}\left(2\,x^2+9\,x+22\right)\sqrt{x+1}\sqrt{-x+1} - 5\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*x^2 + 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*s
qrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = \begin{cases} -5i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 5\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(1/2),x)
```

output
```
Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x
- 1)) - I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(6*sqrt(x -
1)) + 5*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x
 + 1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) + (x + 1)**(5/2)/(6*sqrt(1 - x))
 + 5*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 5*sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -\frac{1}{3}\sqrt{-x^2+1}x^2 - \frac{3}{2}\sqrt{-x^2+1}x - \frac{11}{3}\sqrt{-x^2+1} + \frac{5}{2}\arcsin(x)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2
*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -\frac{1}{6}\left((2x+7)(x+1)+15\right)\sqrt{x+1}\sqrt{-x+1} + 5\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-1/6*((2*x + 7)*(x + 1) + 15)*sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt
(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = \int \frac{(x+1)^{5/2}}{\sqrt{1-x}}\,dx$$

input
```
int((x + 1)^(5/2)/(1 - x)^(1/2),x)
```

output
```
int((x + 1)^(5/2)/(1 - x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -5asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3} - \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x}{2} - \frac{11\sqrt{x+1}\,\sqrt{1-x}}{3}$$

input
```
int((1+x)^(5/2)/(1-x)^(1/2),x)
```

output
```
( - 30*asin(sqrt( - x + 1)/sqrt(2)) - 2*sqrt(x + 1)*sqrt( - x + 1)*x**2 -
9*sqrt(x + 1)*sqrt( - x + 1)*x - 22*sqrt(x + 1)*sqrt( - x + 1))/6
```

3.99 $\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = \frac{15}{2}\sqrt{1-x}\sqrt{1+x}+\frac{5}{2}\sqrt{1-x}(1+x)^{3/2}+\frac{2(1+x)^{5/2}}{\sqrt{1-x}}-\frac{15\arcsin(x)}{2}$$

output
```
15/2*(1-x)^(1/2)*(1+x)^(1/2)+5/2*(1-x)^(1/2)*(1+x)^(3/2)+2*(1+x)^(5/2)/(1-
x)^(1/2)-15/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = \frac{\sqrt{1-x^2}(-24+7x+x^2)}{2(-1+x)}+15\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

output
```
(Sqrt[1 - x^2]*(-24 + 7*x + x^2))/(2*(-1 + x)) + 15*ArcTan[Sqrt[1 - x^2]/(
-1 + x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}-5\int \frac{(x+1)^{3/2}}{\sqrt{1-x}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}-5\left(\frac{3}{2}\int \frac{\sqrt{x+1}}{\sqrt{1-x}}dx-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}\right)$$

$$\Big\downarrow \text{50}$$

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}-5\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}\right)-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}-5\left(\frac{3}{2}\left(\arcsin(x)-\sqrt{1-x^2}\right)-\frac{1}{2}\sqrt{1-x}(x+1)^{3/2}\right)$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

output
```
(2*(1 + x)^(5/2))/Sqrt[1 - x] - 5*(-1/2*(Sqrt[1 - x]*(1 + x)^(3/2)) + (3*(
-Sqrt[1 - x^2] + ArcSin[x]))/2)
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{\left(x^{3}+8 x^{2}-17 x-24\right) \sqrt{\left(1+x\right) \left(1-x\right)}}{2 \sqrt{-\left(1+x\right) \left(-1+x\right)}\, \sqrt{1-x}\, \sqrt{1+x}}-\frac{15 \sqrt{\left(1+x\right) \left(1-x\right)}\, \arcsin\left(x\right)}{2 \sqrt{1+x}\, \sqrt{1-x}}$	77

input
```
int((1+x)^(5/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^3+8*x^2-17*x-24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(
1/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = \frac{(x^2+7\,x-24)\sqrt{x+1}\sqrt{-x+1}+30\,(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+24\,x-24}{2\,(x-1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((x^2 + 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x - 1)*arctan((sqrt(x
 + 1)*sqrt(-x + 1) - 1)/x) + 24*x - 24)/(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.12

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = \begin{cases} 15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{5i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}-\frac{15i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{5(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{15\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(3/2),x)
```

output
```
Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x
- 1)) + 5*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 15*I*sqrt(x + 1)/sqrt(x - 1),
 Abs(x + 1) > 2), (-15*asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqr
t(1 - x)) - 5*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 15*sqrt(x + 1)/sqrt(1 - x),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = -\frac{x^3}{2\sqrt{-x^2+1}} - \frac{4\,x^2}{\sqrt{-x^2+1}} + \frac{17\,x}{2\sqrt{-x^2+1}} + \frac{12}{\sqrt{-x^2+1}} - \frac{15}{2}\arcsin(x)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*x^3/sqrt(-x^2 + 1) - 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) + 1
2/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = \frac{((x+6)(x+1)-30)\sqrt{x+1}\sqrt{-x+1}}{2\,(x-1)} - 15\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="giac")
```

output
```
1/2*((x + 6)*(x + 1) - 30)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 15*arcsin(1/
2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = \int \frac{(x+1)^{5/2}}{(1-x)^{3/2}}\,dx$$

input
```
int((x + 1)^(5/2)/(1 - x)^(3/2),x)
```

output
```
int((x + 1)^(5/2)/(1 - x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx = \frac{30\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-\sqrt{x+1}\,x^2-7\sqrt{x+1}\,x+24\sqrt{x+1}}{2\sqrt{1-x}}$$

input
```
int((1+x)^(5/2)/(1-x)^(3/2),x)
```

output
```
(30*sqrt( - x + 1)*asin(sqrt( - x + 1)/sqrt(2)) - sqrt(x + 1)*x**2 - 7*sqr
t(x + 1)*x + 24*sqrt(x + 1))/(2*sqrt( - x + 1))
```

3.100 $\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}} \, dx = -5\sqrt{1-x}\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\arcsin(x)$$

output
```
-5*(1-x)^(1/2)*(1+x)^(1/2)-10/3*(1+x)^(3/2)/(1-x)^(1/2)+2/3*(1+x)^(5/2)/(1
-x)^(3/2)+5*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}} \, dx = -\frac{\sqrt{1-x^2}(23-34x+3x^2)}{3(-1+x)^2} - 10\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

output
```
-1/3*(Sqrt[1 - x^2]*(23 - 34*x + 3*x^2))/(-1 + x)^2 - 10*ArcTan[Sqrt[1 - x
^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 57, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{5}{3}\int \frac{(x+1)^{3/2}}{(1-x)^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{5}{3}\left(\frac{2(x+1)^{3/2}}{\sqrt{1-x}}-3\int \frac{\sqrt{x+1}}{\sqrt{1-x}}dx\right)$$

$$\Big\downarrow \text{50}$$

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{5}{3}\left(\frac{2(x+1)^{3/2}}{\sqrt{1-x}}-3\left(\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}\right)\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{5}{3}\left(\frac{2(x+1)^{3/2}}{\sqrt{1-x}}-3\left(\arcsin(x)-\sqrt{1-x^2}\right)\right)$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

output
```
(2*(1 + x)^(5/2))/(3*(1 - x)^(3/2)) - (5*((2*(1 + x)^(3/2))/Sqrt[1 - x] -
3*(-Sqrt[1 - x^2] + ArcSin[x])))/3
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.33

method	result	size
risch	$\frac{(3x^3-31x^2-11x+23)\sqrt{(1+x)(1-x)}}{3(-1+x)\sqrt{-(1+x)(-1+x)}\sqrt{1-x}\sqrt{1+x}}+\frac{5\sqrt{(1+x)(1-x)}\arcsin(x)}{\sqrt{1+x}\sqrt{1-x}}$	84

input
```
int((1+x)^(5/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x^3-31*x^2-11*x+23)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2
)/(1-x)^(1/2)/(1+x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*ar
csin(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx = -\frac{23\,x^2 + (3\,x^2 - 34\,x + 23)\sqrt{x+1}\sqrt{-x+1} + 30\,(x^2 - 2\,x + 1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) - 46\,x + 23}{3\,(x^2 - 2\,x + 1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(23*x^2 + (3*x^2 - 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x^2 - 2*
x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 46*x + 23)/(x^2 - 2*x +
1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 575, normalized size of antiderivative = 9.13

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(5/2),x)
```

output
```
Piecewise((-30*I*sqrt(x - 1)*(x + 1)**(27/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/
(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) + 15*pi*sq
rt(x - 1)*(x + 1)**(27/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(
x + 1)**(25/2)) + 60*I*sqrt(x - 1)*(x + 1)**(25/2)*acosh(sqrt(2)*sqrt(x +
1)/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) - 30
*pi*sqrt(x - 1)*(x + 1)**(25/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x
- 1)*(x + 1)**(25/2)) - 3*I*(x + 1)**15/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6
*sqrt(x - 1)*(x + 1)**(25/2)) + 40*I*(x + 1)**14/(3*sqrt(x - 1)*(x + 1)**(
27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) - 60*I*(x + 1)**13/(3*sqrt(x - 1)*(
x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)), Abs(x + 1) > 2), (30*sqrt
(1 - x)*(x + 1)**(27/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)
**(27/2) - 6*sqrt(1 - x)*(x + 1)**(25/2)) - 60*sqrt(1 - x)*(x + 1)**(25/2)
*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x
)*(x + 1)**(25/2)) + 3*(x + 1)**15/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt
(1 - x)*(x + 1)**(25/2)) - 40*(x + 1)**14/(3*sqrt(1 - x)*(x + 1)**(27/2) -
 6*sqrt(1 - x)*(x + 1)**(25/2)) + 60*(x + 1)**13/(3*sqrt(1 - x)*(x + 1)**(
27/2) - 6*sqrt(1 - x)*(x + 1)**(25/2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.57

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx = -\frac{(-x^2+1)^{\frac{5}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1} - \frac{5\,(-x^2+1)^{\frac{3}{2}}}{3\,(x^3-3\,x^2+3\,x-1)} + \frac{10\,\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)} + \frac{35\,\sqrt{-x^2+1}}{3\,(x-1)} + 5\,\arcsin(x)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="maxima")
```

output
```
-(-x^2 + 1)^(5/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(
x^3 - 3*x^2 + 3*x - 1) + 10/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 35/3*sqrt(-
x^2 + 1)/(x - 1) + 5*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx = -\frac{((3\,x-37)(x+1)+60)\sqrt{x+1}\sqrt{-x+1}}{3\,(x-1)^2} + 10\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((3*x - 37)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 10*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx = \int \frac{(x+1)^{5/2}}{(1-x)^{5/2}}\,dx$$

input
```
int((x + 1)^(5/2)/(1 - x)^(5/2),x)
```

output
```
int((x + 1)^(5/2)/(1 - x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx = \frac{-30\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x + 30\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + 3\sqrt{x+1}\,x^2 - 34\sqrt{x+1}\,x + 23\sqrt{}}{3\sqrt{1-x}\,(x-1)}$$

input
```
int((1+x)^(5/2)/(1-x)^(5/2),x)
```

output
```
( - 30*sqrt( - x + 1)*asin(sqrt( - x + 1)/sqrt(2))*x + 30*sqrt( - x + 1)*a
sin(sqrt( - x + 1)/sqrt(2)) + 3*sqrt(x + 1)*x**2 - 34*sqrt(x + 1)*x + 23*s
qrt(x + 1))/(3*sqrt( - x + 1)*(x - 1))
```

3.101 $\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx=\frac{2\sqrt{1+x}}{\sqrt{1-x}}-\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\frac{2(1+x)^{5/2}}{5(1-x)^{5/2}}-\arcsin(x)$$

output
```
2*(1+x)^(1/2)/(1-x)^(1/2)-2/3*(1+x)^(3/2)/(1-x)^(3/2)+2/5*(1+x)^(5/2)/(1-x
)^(5/2)-arcsin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx=-\frac{2\sqrt{1-x^2}(13-24x+23x^2)}{15(-1+x)^3}+2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

output
```
(-2*Sqrt[1 - x^2]*(13 - 24*x + 23*x^2))/(15*(-1 + x)^3) + 2*ArcTan[Sqrt[1
- x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {57, 57, 57, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{7/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\int \frac{(x+1)^{3/2}}{(1-x)^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\int \frac{\sqrt{x+1}}{(1-x)^{3/2}}dx+\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$-\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}dx+\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{39}$$

$$-\int \frac{1}{\sqrt{1-x^2}}dx+\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{223}$$

$$-\arcsin(x)+\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

output
```
(2*Sqrt[1 + x])/Sqrt[1 - x] - (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + (2*(1
+ x)^(5/2))/(5*(1 - x)^(5/2)) - ArcSin[x]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.33

method	result	size
risch	$\frac{2(23x^3-x^2-11x+13)\sqrt{(1+x)(1-x)}}{15(-1+x)^2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	84

input
```
int((1+x)^(5/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(23*x^3-x^2-11*x+13)/(-1+x)^2/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/
2)/(1-x)^(1/2)/(1+x)^(1/2)-((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arc
sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx = \frac{2\left(13\,x^3-39\,x^2-(23\,x^2-24\,x+13)\sqrt{x+1}\sqrt{-x+1}+15\,(x^3-3\,x^2+3\,x-1)\,\mathrm{arcta}\right.}{15\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(13*x^3 - 39*x^2 - (23*x^2 - 24*x + 13)*sqrt(x + 1)*sqrt(-x + 1) + 15
*(x^3 - 3*x^2 + 3*x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 39*x -
 13)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.87 (sec) , antiderivative size = 1606, normalized size of antiderivative = 25.49

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(7/2),x)
```

output
```
Piecewise((30*I*sqrt(x - 1)*(x + 1)**(35/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(
15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt
(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 15*pi*sqrt(x
- 1)*(x + 1)**(35/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x +
 1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(
29/2)) - 180*I*sqrt(x - 1)*(x + 1)**(33/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(1
5*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(
x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 90*pi*sqrt(x -
 1)*(x + 1)**(33/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x +
1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(2
9/2)) + 360*I*sqrt(x - 1)*(x + 1)**(31/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15
*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x
 - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 180*pi*sqrt(x -
 1)*(x + 1)**(31/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x +
1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(2
9/2)) - 240*I*sqrt(x - 1)*(x + 1)**(29/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15
*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x
 - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 120*pi*sqrt(x -
 1)*(x + 1)**(29/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x +
1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(47) = 94$.

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.54

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx = -\frac{(-x^2+1)^{\frac{5}{2}}}{5\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} + \frac{(-x^2+1)^{\frac{3}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1} + \frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^3-3\,x^2+3\,x-1)} + \frac{6\,\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)} - \frac{7\,\sqrt{-x^2+1}}{15\,(x^2-2\,x+1)} - \frac{38\,\sqrt{-x^2+1}}{15\,(x-1)} - \arcsin(x)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="maxima")
```

output
```
-1/5*(-x^2 + 1)^(5/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + (-x^2 +
1)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x
^2 + 3*x - 1) + 6/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 7/15*sqrt(-x^
2 + 1)/(x^2 - 2*x + 1) - 38/15*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx = -\frac{2\left((23\,x-47)(x+1)+60\right)\sqrt{x+1}\sqrt{-x+1}}{15\,(x-1)^3} - 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*((23*x - 47)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3 - 2*ar
csin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx = \int \frac{(x+1)^{5/2}}{(1-x)^{7/2}}\,dx$$

input
```
int((x + 1)^(5/2)/(1 - x)^(7/2),x)
```

output
```
int((x + 1)^(5/2)/(1 - x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx = \frac{2\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x^2 - 4\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x + 2\sqrt{1-x}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \frac{46\sqrt{x+1}\,x^2}{15}}{\sqrt{1-x}\,(x^2-2x+1)}$$

input
```
int((1+x)^(5/2)/(1-x)^(7/2),x)
```

output
```
(2*(15*sqrt( - x + 1)*asin(sqrt( - x + 1)/sqrt(2))*x**2 - 30*sqrt( - x + 1
)*asin(sqrt( - x + 1)/sqrt(2))*x + 15*sqrt( - x + 1)*asin(sqrt( - x + 1)/s
qrt(2)) + 23*sqrt(x + 1)*x**2 - 24*sqrt(x + 1)*x + 13*sqrt(x + 1)))/(15*sq
rt( - x + 1)*(x**2 - 2*x + 1))
```

3.102 $\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

output
```
1/7*(1+x)^(7/2)/(1-x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

output
```
(1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{9/2}}\,dx$$

$$\Big\downarrow \text{48}$$

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

output
```
(1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{(1+x)^{\frac{7}{2}}}{7(1-x)^{\frac{7}{2}}}$	15
orering	$-\frac{(1+x)^{\frac{7}{2}}(-1+x)}{7(1-x)^{\frac{9}{2}}}$	18
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^4+4x^3+6x^2+4x+1)}{7\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59
default	$\frac{(1+x)^{\frac{5}{2}}}{(1-x)^{\frac{7}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{2(1-x)^{\frac{7}{2}}}+\frac{15\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}-\frac{3\sqrt{1+x}}{14(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{7(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{7\sqrt{1-x}}$	85

input
```
int((1+x)^(5/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*(1+x)^(7/2)/(1-x)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{x^4-4\,x^3+6\,x^2+(x^3+3\,x^2+3\,x+1)\sqrt{x+1}\sqrt{-x+1}-4\,x+1}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="fricas")
```

output
```
1/7*(x^4 - 4*x^3 + 6*x^2 + (x^3 + 3*x^2 + 3*x + 1)*sqrt(x + 1)*sqrt(-x + 1
) - 4*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 5.70

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \begin{cases} \frac{i(x+1)^{\frac{7}{2}}}{7\sqrt{x-1}(x+1)^3-42\sqrt{x-1}(x+1)^2+84\sqrt{x-1}(x+1)-56\sqrt{x-1}} & \text{for } |x+1|>2 \\ -\frac{(x+1)^{\frac{7}{2}}}{7\sqrt{1-x}(x+1)^3-42\sqrt{1-x}(x+1)^2+84\sqrt{1-x}(x+1)-56\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(9/2),x)
```

output
```
Piecewise((I*(x + 1)**(7/2)/(7*sqrt(x - 1)*(x + 1)**3 - 42*sqrt(x - 1)*(x
+ 1)**2 + 84*sqrt(x - 1)*(x + 1) - 56*sqrt(x - 1)), Abs(x + 1) > 2), (-(x
+ 1)**(7/2)/(7*sqrt(1 - x)*(x + 1)**3 - 42*sqrt(1 - x)*(x + 1)**2 + 84*sqr
t(1 - x)*(x + 1) - 56*sqrt(1 - x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.55

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{(-x^2+1)^{\frac{5}{2}}}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1} + \frac{5\,(-x^2+1)^{\frac{3}{2}}}{2\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} + \frac{15\,\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{3\,\sqrt{-x^2+1}}{14\,(x^3-3\,x^2+3\,x-1)} - \frac{\sqrt{-x^2+1}}{7\,(x^2-2\,x+1)} + \frac{\sqrt{-x^2+1}}{7\,(x-1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="maxima")
```

output
```
(-x^2 + 1)^(5/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/2*
(-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 15/7*sqrt(-x^
2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 3/14*sqrt(-x^2 + 1)/(x^3 - 3*x^2
+ 3*x - 1) - 1/7*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/7*sqrt(-x^2 + 1)/(x -
1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{7\,(x-1)^4}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="giac")
```

output
```
1/7*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{\sqrt{1-x}\left(\frac{3\,x\,\sqrt{x+1}}{7}+\frac{\sqrt{x+1}}{7}+\frac{3\,x^2\,\sqrt{x+1}}{7}+\frac{x^3\,\sqrt{x+1}}{7}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

input
```
int((x + 1)^(5/2)/(1 - x)^(9/2),x)
```

output
```
((1 - x)^(1/2)*((3*x*(x + 1)^(1/2))/7 + (x + 1)^(1/2)/7 + (3*x^2*(x + 1)^(
1/2))/7 + (x^3*(x + 1)^(1/2))/7))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{\sqrt{x+1}\,(-x^3-3x^2-3x-1)}{7\sqrt{1-x}\,(x^3-3x^2+3x-1)}$$

input
```
int((1+x)^(5/2)/(1-x)^(9/2),x)
```

output
```
(sqrt(x + 1)*( - x**3 - 3*x**2 - 3*x - 1))/(7*sqrt( - x + 1)*(x**3 - 3*x**
2 + 3*x - 1))
```

3.103 $\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} \, dx = \frac{(1+x)^{7/2}}{9(1-x)^{9/2}} + \frac{(1+x)^{7/2}}{63(1-x)^{7/2}}$$

output
```
1/9*(1+x)^(7/2)/(1-x)^(9/2)+1/63*(1+x)^(7/2)/(1-x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} \, dx = \frac{(8-x)(1+x)^{7/2}}{63(1-x)^{9/2}}$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(11/2),x]
```

output
```
((8 - x)*(1 + x)^(7/2))/(63*(1 - x)^(9/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{11/2}}dx$$
$$\Big\downarrow \text{ 55}$$
$$\frac{1}{9}\int \frac{(x+1)^{5/2}}{(1-x)^{9/2}}dx+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$
$$\Big\downarrow \text{ 48}$$
$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(11/2),x]
```

output
```
(1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(x-8)(1+x)^{\frac{7}{2}}}{63(1-x)^{\frac{9}{2}}}$	18
orering	$\frac{(1+x)^{\frac{7}{2}}(-1+x)(x-8)}{63(1-x)^{\frac{11}{2}}}$	21
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^5-4x^4-26x^3-44x^2-31x-8)}{63\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	64
default	$\frac{(1+x)^{\frac{5}{2}}}{2(1-x)^{\frac{9}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{6(1-x)^{\frac{9}{2}}}+\frac{5\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{126(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{42(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{63(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{63\sqrt{1-x}}$	100

input
```
int((1+x)^(5/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/63*(x-8)*(1+x)^(7/2)/(1-x)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(29) = 58.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.02

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx = \frac{8\,x^5-40\,x^4+80\,x^3-80\,x^2+\left(x^4-5\,x^3-21\,x^2-23\,x-8\right)\sqrt{x+1}\sqrt{-x+1}+40\,x-}{63\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="fricas")
```

output
```
1/63*(8*x^5 - 40*x^4 + 80*x^3 - 80*x^2 + (x^4 - 5*x^3 - 21*x^2 - 23*x - 8)
*sqrt(x + 1)*sqrt(-x + 1) + 40*x - 8)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x
 - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.82 (sec) , antiderivative size = 280, normalized size of antiderivative = 6.83

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx = \begin{cases} \frac{i(x+1)^{\frac{9}{2}}}{63\sqrt{x-1}(x+1)^4-504\sqrt{x-1}(x+1)^3+1512\sqrt{x-1}(x+1)^2-2016\sqrt{x-1}(x+1)+1008\sqrt{x-1}} - \frac{}{63\sqrt{x-1}(x+1)^4-504\sqrt{x}} \\ -\frac{(x+1)^{\frac{9}{2}}}{63\sqrt{1-x}(x+1)^4-504\sqrt{1-x}(x+1)^3+1512\sqrt{1-x}(x+1)^2-2016\sqrt{1-x}(x+1)+1008\sqrt{1-x}} + \frac{}{63\sqrt{1-x}(x+1)^4-504} \end{cases}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(11/2),x)
```

output
```
Piecewise((I*(x + 1)**(9/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1)*(
x + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 1008*
sqrt(x - 1)) - 9*I*(x + 1)**(7/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x
- 1)*(x + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) +
 1008*sqrt(x - 1)), Abs(x + 1) > 2), (-(x + 1)**(9/2)/(63*sqrt(1 - x)*(x +
 1)**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*s
qrt(1 - x)*(x + 1) + 1008*sqrt(1 - x)) + 9*(x + 1)**(7/2)/(63*sqrt(1 - x)*
(x + 1)**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 20
16*sqrt(1 - x)*(x + 1) + 1008*sqrt(1 - x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(29) = 58.

Time = 0.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 5.32

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx = -\frac{(-x^2+1)^{\frac{5}{2}}}{2\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)} - \frac{5\,(-x^2+1)^{\frac{3}{2}}}{6\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)} - \frac{5\,\sqrt{-x^2+1}}{9\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} - \frac{5\,\sqrt{-x^2+1}}{126\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{\sqrt{-x^2+1}}{42\,(x^3-3\,x^2+3\,x-1)} - \frac{\sqrt{-x^2+1}}{63\,(x^2-2\,x+1)} + \frac{\sqrt{-x^2+1}}{63\,(x-1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="maxima")
```

output
```
-1/2*(-x^2 + 1)^(5/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7
*x - 1) - 5/6*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6
*x + 1) - 5/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5
/126*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/42*sqrt(-x^2 + 1)/
(x^3 - 3*x^2 + 3*x - 1) - 1/63*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/63*sqrt(
-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx = \frac{(x+1)^{\frac{7}{2}}(x-8)\sqrt{-x+1}}{63\,(x-1)^5}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="giac")
```

output
```
1/63*(x + 1)^(7/2)*(x - 8)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx = -\frac{\sqrt{1-x}\left(\frac{23\,x\,\sqrt{x+1}}{63}+\frac{8\,\sqrt{x+1}}{63}+\frac{x^2\,\sqrt{x+1}}{3}+\frac{5\,x^3\,\sqrt{x+1}}{63}-\frac{x^4\,\sqrt{x+1}}{63}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

input
```
int((x + 1)^(5/2)/(1 - x)^(11/2),x)
```

output
```
-((1 - x)^(1/2)*((23*x*(x + 1)^(1/2))/63 + (8*(x + 1)^(1/2))/63 + (x^2*(x
+ 1)^(1/2))/3 + (5*x^3*(x + 1)^(1/2))/63 - (x^4*(x + 1)^(1/2))/63))/(5*x -
 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx = \frac{\sqrt{x+1}\,(-x^4+5x^3+21x^2+23x+8)}{63\sqrt{1-x}\,\left(x^4-4x^3+6x^2-4x+1\right)}$$

input
```
int((1+x)^(5/2)/(1-x)^(11/2),x)
```

output
```
(sqrt(x + 1)*( - x**4 + 5*x**3 + 21*x**2 + 23*x + 8))/(63*sqrt( - x + 1)*(
x**4 - 4*x**3 + 6*x**2 - 4*x + 1))
```

3.104 $\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} \, dx = \frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{99(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}$$

output
```
1/11*(1+x)^(7/2)/(1-x)^(11/2)+2/99*(1+x)^(7/2)/(1-x)^(9/2)+2/693*(1+x)^(7/
2)/(1-x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} \, dx = \frac{(1+x)^{7/2} (79 - 18x + 2x^2)}{693(1-x)^{11/2}}$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

output
```
((1 + x)^(7/2)*(79 - 18*x + 2*x^2))/(693*(1 - x)^(11/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{13/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2}{11}\int \frac{(x+1)^{5/2}}{(1-x)^{11/2}}dx+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2}{11}\left(\frac{1}{9}\int \frac{(x+1)^{5/2}}{(1-x)^{9/2}}dx+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}+\frac{2}{11}\left(\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}\right)$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

output
```
(1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*((1 + x)^(7/2)/(9*(1 - x)^(9/2)) +
(1 + x)^(7/2)/(63*(1 - x)^(7/2))))/11
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{(1+x)^{\frac{7}{2}}\left(2x^{2}-18x+79\right)}{693(1-x)^{\frac{11}{2}}}$	25
orering	$-\frac{(1+x)^{\frac{7}{2}}(-1+x)\left(2x^{2}-18x+79\right)}{693(1-x)^{\frac{13}{2}}}$	28
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(2x^{6}-10x^{5}+19x^{4}+216x^{3}+404x^{2}+298x+79\right)}{693\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{5}\sqrt{-(1+x)(-1+x)}}$	71
default	$\frac{(1+x)^{\frac{5}{2}}}{3(1-x)^{\frac{11}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{12(1-x)^{\frac{11}{2}}}+\frac{5\sqrt{1+x}}{22(1-x)^{\frac{11}{2}}}-\frac{5\sqrt{1+x}}{396(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{693(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{231(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{693(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{693\sqrt{1-x}}$	114

input
```
int((1+x)^(5/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
1/693*(1+x)^(7/2)/(1-x)^(11/2)*(2*x^2-18*x+79)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx = \frac{79\,x^{6}-474\,x^{5}+1185\,x^{4}-1580\,x^{3}+1185\,x^{2}+\left(2\,x^{5}-12\,x^{4}+31\,x^{3}+185\,x^{2}+219\,x\right.}{693\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="fricas")
```

output
```
1/693*(79*x^6 - 474*x^5 + 1185*x^4 - 1580*x^3 + 1185*x^2 + (2*x^5 - 12*x^4
 + 31*x^3 + 185*x^2 + 219*x + 79)*sqrt(x + 1)*sqrt(-x + 1) - 474*x + 79)/(
x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.78 (sec) , antiderivative size = 784, normalized size of antiderivative = 12.85

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(13/2),x)
```

output
```
Piecewise((2*I*(x + 1)**(13/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x -
 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)
**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*s
qrt(x - 1)) - 26*I*(x + 1)**(11/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt
(x - 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x
+ 1)**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 443
52*sqrt(x - 1)) + 143*I*(x + 1)**(9/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*
sqrt(x - 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)
*(x + 1)**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) +
 44352*sqrt(x - 1)) - 198*I*(x + 1)**(7/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8
316*sqrt(x - 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x
- 1)*(x + 1)**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x +
1) + 44352*sqrt(x - 1)), Abs(x + 1) > 2), (-2*(x + 1)**(13/2)/(693*sqrt(1
- x)*(x + 1)**6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)*
*4 - 110880*sqrt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 13305
6*sqrt(1 - x)*(x + 1) + 44352*sqrt(1 - x)) + 26*(x + 1)**(11/2)/(693*sqrt(
1 - x)*(x + 1)**6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1
)**4 - 110880*sqrt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133
056*sqrt(1 - x)*(x + 1) + 44352*sqrt(1 - x)) - 143*(x + 1)**(9/2)/(693*sqr
t(1 - x)*(x + 1)**6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(43) = 86$.

Time = 0.03 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.41

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx = \frac{(-x^2+1)^{\frac{5}{2}}}{3\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)} + \frac{5\,(-x^2+1)^{\frac{3}{2}}}{12\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)} + \frac{5\,\sqrt{-x^2+1}}{22\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)} + \frac{5\,\sqrt{-x^2+1}}{396\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} - \frac{5\,\sqrt{-x^2+1}}{693\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{\sqrt{-x^2+1}}{231\,(x^3-3\,x^2+3\,x-1)} - \frac{2\,\sqrt{-x^2+1}}{693\,(x^2-2\,x+1)} + \frac{2\,\sqrt{-x^2+1}}{693\,(x-1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="maxima")
```

output
```
1/3*(-x^2 + 1)^(5/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28
*x^2 - 8*x + 1) + 5/12*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 3
5*x^3 - 21*x^2 + 7*x - 1) + 5/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20
*x^3 + 15*x^2 - 6*x + 1) + 5/396*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10
*x^2 + 5*x - 1) - 5/693*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1
/231*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/693*sqrt(-x^2 + 1)/(x^2 -
2*x + 1) + 2/693*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx = \frac{(2\,(x+1)(x-10)+99)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{693\,(x-1)^6}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="giac")
```

output
```
1/693*(2*(x + 1)*(x - 10) + 99)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^6
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx = \frac{\sqrt{1-x}\,\left(\frac{73\,x\,\sqrt{x+1}}{231}+\frac{79\,\sqrt{x+1}}{693}+\frac{185\,x^2\,\sqrt{x+1}}{693}+\frac{31\,x^3\,\sqrt{x+1}}{693}-\frac{4\,x^4\,\sqrt{x+1}}{231}+\frac{2\,x^5\,\sqrt{x+1}}{693}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

input
```
int((x + 1)^(5/2)/(1 - x)^(13/2),x)
```

output
```
((1 - x)^(1/2)*((73*x*(x + 1)^(1/2))/231 + (79*(x + 1)^(1/2))/693 + (185*x
^2*(x + 1)^(1/2))/693 + (31*x^3*(x + 1)^(1/2))/693 - (4*x^4*(x + 1)^(1/2))
/231 + (2*x^5*(x + 1)^(1/2))/693))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^5
 + x^6 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx = \frac{\sqrt{x+1}\,(-2x^5+12x^4-31x^3-185x^2-219x-79)}{693\sqrt{1-x}\,(x^5-5x^4+10x^3-10x^2+5x-1)}$$

input
```
int((1+x)^(5/2)/(1-x)^(13/2),x)
```

output
```
(sqrt(x + 1)*( - 2*x**5 + 12*x**4 - 31*x**3 - 185*x**2 - 219*x - 79))/(693
*sqrt( - x + 1)*(x**5 - 5*x**4 + 10*x**3 - 10*x**2 + 5*x - 1))
```

3.105 $\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} \, dx = \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3(1+x)^{7/2}}{143(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{429(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}$$

output
```
1/13*(1+x)^(7/2)/(1-x)^(13/2)+3/143*(1+x)^(7/2)/(1-x)^(11/2)+2/429*(1+x)^(
7/2)/(1-x)^(9/2)+2/3003*(1+x)^(7/2)/(1-x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} \, dx = \frac{(1+x)^{7/2} (310 - 97x + 20x^2 - 2x^3)}{3003(1-x)^{13/2}}$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

output
```
((1 + x)^(7/2)*(310 - 97*x + 20*x^2 - 2*x^3))/(3003*(1 - x)^(13/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{15/2}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3}{13}\int \frac{(x+1)^{5/2}}{(1-x)^{13/2}}dx+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3}{13}\left(\frac{2}{11}\int \frac{(x+1)^{5/2}}{(1-x)^{11/2}}dx+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}\right)+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3}{13}\left(\frac{2}{11}\left(\frac{1}{9}\int \frac{(x+1)^{5/2}}{(1-x)^{9/2}}dx+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}\right)+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}+\frac{3}{13}\left(\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}+\frac{2}{11}\left(\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}\right)\right)$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

output
```
(1 + x)^(7/2)/(13*(1 - x)^(13/2)) + (3*((1 + x)^(7/2)/(11*(1 - x)^(11/2))
+ (2*((1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))))
/11))/13
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.37

method	result
gosper	$-\frac{(1+x)^{\frac{7}{2}}\left(2x^{3}-20x^{2}+97x-310\right)}{3003(1-x)^{\frac{13}{2}}}$
orering	$\frac{(1+x)^{\frac{7}{2}}(-1+x)\left(2x^{3}-20x^{2}+97x-310\right)}{3003(1-x)^{\frac{15}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(2x^{7}-12x^{6}+29x^{5}-34x^{4}-736x^{3}-1492x^{2}-1143x-310\right)}{3003\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{6}\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{4(1-x)^{\frac{13}{2}}}-\frac{(1+x)^{\frac{3}{2}}}{4(1-x)^{\frac{13}{2}}}+\frac{3\sqrt{1+x}}{26(1-x)^{\frac{13}{2}}}-\frac{3\sqrt{1+x}}{572(1-x)^{\frac{11}{2}}}-\frac{5\sqrt{1+x}}{1716(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{3003(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{1001(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{3003(1-x)^{\frac{3}{2}}}$

input
```
int((1+x)^(5/2)/(1-x)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3003*(1+x)^(7/2)/(1-x)^(13/2)*(2*x^3-20*x^2+97*x-310)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.42

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = \frac{310\,x^7-2170\,x^6+6510\,x^5-10850\,x^4+10850\,x^3-6510\,x^2+(2\,x^6-14\,x^5+43\,x^4-}{3003\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="fricas")
```

output
```
1/3003*(310*x^7 - 2170*x^6 + 6510*x^5 - 10850*x^4 + 10850*x^3 - 6510*x^2 +
 (2*x^6 - 14*x^5 + 43*x^4 - 77*x^3 - 659*x^2 - 833*x - 310)*sqrt(x + 1)*sq
rt(-x + 1) + 2170*x - 310)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^
2 + 7*x - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(15/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(57) = 114$.

Time = 0.03 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.01

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = -\frac{(-x^2+1)^{\frac{5}{2}}}{4\,(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1)} - \frac{(-x^2+1)^{\frac{3}{2}}}{4\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)} - \frac{3\sqrt{-x^2+1}}{26\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)} - \frac{3\sqrt{-x^2+1}}{572\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)} + \frac{5\sqrt{-x^2+1}}{1716\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} - \frac{5\sqrt{-x^2+1}}{3003\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{\sqrt{-x^2+1}}{1001\,(x^3-3\,x^2+3\,x-1)} - \frac{2\sqrt{-x^2+1}}{3003\,(x^2-2\,x+1)} + \frac{2\sqrt{-x^2+1}}{3003\,(x-1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="maxima")
```

output
```
-1/4*(-x^2 + 1)^(5/2)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 +
 84*x^3 - 36*x^2 + 9*x - 1) - 1/4*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 -
 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) - 3/26*sqrt(-x^2 + 1)/(x^7 -
 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1) - 3/572*sqrt(-x^2 +
1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/1716*sqrt(-x^2 +
 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/3003*sqrt(-x^2 + 1)/(x^4
 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/1001*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1
) - 2/3003*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/3003*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = \frac{((2\,(x+1)(x-12)+143)(x+1)-429)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{3003\,(x-1)^7}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="giac")
```

output
```
1/3003*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1)^(7/2)*sqrt(-x +
1)/(x - 1)^7
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = -\frac{\sqrt{1-x}\,\left(\frac{119\,x\,\sqrt{x+1}}{429}+\frac{310\,\sqrt{x+1}}{3003}+\frac{659\,x^2\,\sqrt{x+1}}{3003}+\frac{x^3\,\sqrt{x+1}}{39}-\frac{43\,x^4\,\sqrt{x+1}}{3003}+\frac{2\,x^5\,\sqrt{x+1}}{429}-\frac{2\,x^6\,\sqrt{x+1}}{3003}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

input
```
int((x + 1)^(5/2)/(1 - x)^(15/2),x)
```

output
```
-((1 - x)^(1/2)*((119*x*(x + 1)^(1/2))/429 + (310*(x + 1)^(1/2))/3003 + (6
59*x^2*(x + 1)^(1/2))/3003 + (x^3*(x + 1)^(1/2))/39 - (43*x^4*(x + 1)^(1/2
))/3003 + (2*x^5*(x + 1)^(1/2))/429 - (2*x^6*(x + 1)^(1/2))/3003))/(7*x -
21*x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = \frac{\sqrt{x+1}\,(-2x^6+14x^5-43x^4+77x^3+659x^2+833x+310)}{3003\sqrt{1-x}\,(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}$$

input
```
int((1+x)^(5/2)/(1-x)^(15/2),x)
```

output
```
(sqrt(x + 1)*( - 2*x**6 + 14*x**5 - 43*x**4 + 77*x**3 + 659*x**2 + 833*x +
 310))/(3003*sqrt( - x + 1)*(x**6 - 6*x**5 + 15*x**4 - 20*x**3 + 15*x**2 -
 6*x + 1))
```

3.106 $\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}} \, dx = \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{715(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}} + \frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}$$

output
```
1/15*(1+x)^(7/2)/(1-x)^(15/2)+4/195*(1+x)^(7/2)/(1-x)^(13/2)+4/715*(1+x)^(
7/2)/(1-x)^(11/2)+8/6435*(1+x)^(7/2)/(1-x)^(9/2)+8/45045*(1+x)^(7/2)/(1-x)
^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}} \, dx = \frac{(1+x)^{7/2} (4243 - 1628x + 468x^2 - 88x^3 + 8x^4)}{45045(1-x)^{15/2}}$$

input
```
Integrate[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

output
```
((1 + x)^(7/2)*(4243 - 1628*x + 468*x^2 - 88*x^3 + 8*x^4))/(45045*(1 - x)^
(15/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{5/2}}{(1-x)^{17/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{15}\int \frac{(x+1)^{5/2}}{(1-x)^{15/2}}dx+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{15}\left(\frac{3}{13}\int \frac{(x+1)^{5/2}}{(1-x)^{13/2}}dx+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}\right)+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{15}\left(\frac{3}{13}\left(\frac{2}{11}\int \frac{(x+1)^{5/2}}{(1-x)^{11/2}}dx+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}\right)+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}\right)+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{15}\left(\frac{3}{13}\left(\frac{2}{11}\left(\frac{1}{9}\int \frac{(x+1)^{5/2}}{(1-x)^{9/2}}dx+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}\right)+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}\right)+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}\right)+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}+\frac{4}{15}\left(\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}+\frac{3}{13}\left(\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}+\frac{2}{11}\left(\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}\right)\right)\right)$$

input
```
Int[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

output
```
(1 + x)^(7/2)/(15*(1 - x)^(15/2)) + (4*((1 + x)^(7/2)/(13*(1 - x)^(13/2))
+ (3*((1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*((1 + x)^(7/2)/(9*(1 - x)^(9/
2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))))/11))/13))/15
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result
gosper	$\frac{(1+x)^{\frac{7}{2}} (8x^4-88x^3+468x^2-1628x+4243)}{45045(1-x)^{\frac{15}{2}}}$
orering	$-\frac{(1+x)^{\frac{7}{2}} (-1+x) (8x^4-88x^3+468x^2-1628x+4243)}{45045(1-x)^{\frac{17}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\, (8x^8-56x^7+164x^6-252x^5+195x^4+8988x^3+19414x^2+15344x+4243)}{45045\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^7 \sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{5(1-x)^{\frac{15}{2}}}-\frac{(1+x)^{\frac{3}{2}}}{6(1-x)^{\frac{15}{2}}}+\frac{\sqrt{1+x}}{15(1-x)^{\frac{15}{2}}}-\frac{\sqrt{1+x}}{390(1-x)^{\frac{13}{2}}}-\frac{\sqrt{1+x}}{715(1-x)^{\frac{11}{2}}}-\frac{\sqrt{1+x}}{1287(1-x)^{\frac{9}{2}}}-\frac{4\sqrt{1+x}}{9009(1-x)^{\frac{7}{2}}}-\frac{4\sqrt{1+x}}{15015(1-x)^{\frac{5}{2}}}$

input
```
int((1+x)^(5/2)/(1-x)^(17/2),x,method=_RETURNVERBOSE)
```

output
```
1/45045*(1+x)^(7/2)/(1-x)^(15/2)*(8*x^4-88*x^3+468*x^2-1628*x+4243)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.29

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx = \frac{4243\,x^8 - 33944\,x^7 + 118804\,x^6 - 237608\,x^5 + 297010\,x^4 - 237608\,x^3 + 118804\,x^2 + (}{45045\,(x^8 - 8\,x^7 + 28\,x^6}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="fricas")
```

output
```
1/45045*(4243*x^8 - 33944*x^7 + 118804*x^6 - 237608*x^5 + 297010*x^4 - 237
608*x^3 + 118804*x^2 + (8*x^7 - 64*x^6 + 228*x^5 - 480*x^4 + 675*x^3 + 831
3*x^2 + 11101*x + 4243)*sqrt(x + 1)*sqrt(-x + 1) - 33944*x + 4243)/(x^8 -
8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((1+x)**(5/2)/(1-x)**(17/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(71) = 142$.

Time = 0.04 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.82

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx = \frac{(-x^2+1)^{\frac{5}{2}}}{5\,(x^{10}-10\,x^9+45\,x^8-120\,x^7+210\,x^6-252\,x^5+210\,x^4-120\,x^3+45\,x^2-10\,x+1)}$$
$$+\frac{(-x^2+1)^{\frac{3}{2}}}{6\,(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1)}$$
$$+\frac{\sqrt{-x^2+1}}{15\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}$$
$$+\frac{\sqrt{-x^2+1}}{390\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}$$
$$-\frac{\sqrt{-x^2+1}}{715\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}$$
$$+\frac{\sqrt{-x^2+1}}{1287\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}-\frac{4\,\sqrt{-x^2+1}}{9009\,(x^4-4\,x^3+6\,x^2-4\,x+1)}$$
$$+\frac{4\,\sqrt{-x^2+1}}{15015\,(x^3-3\,x^2+3\,x-1)}-\frac{8\,\sqrt{-x^2+1}}{45045\,(x^2-2\,x+1)}+\frac{8\,\sqrt{-x^2+1}}{45045\,(x-1)}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="maxima")
```

output
```
1/5*(-x^2 + 1)^(5/2)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5
 + 210*x^4 - 120*x^3 + 45*x^2 - 10*x + 1) + 1/6*(-x^2 + 1)^(3/2)/(x^9 - 9*
x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) + 1
/15*sqrt(-x^2 + 1)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x
^2 - 8*x + 1) + 1/390*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x
^3 - 21*x^2 + 7*x - 1) - 1/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x
^3 + 15*x^2 - 6*x + 1) + 1/1287*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1) - 4/9009*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4
/15015*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 8/45045*sqrt(-x^2 + 1)/(x^
2 - 2*x + 1) + 8/45045*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx = \frac{(4\,((2\,(x+1)(x-14)+195)(x+1)-715)(x+1)+6435)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{45045\,(x-1)^{8}}$$

input
```
integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="giac")
```

output
```
1/45045*(4*((2*(x + 1)*(x - 14) + 195)*(x + 1) - 715)*(x + 1) + 6435)*(x +
 1)^(7/2)*sqrt(-x + 1)/(x - 1)^8
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.23

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx = \frac{\sqrt{1-x}\,\left(\frac{11101\,x\,\sqrt{x+1}}{45045}+\frac{4243\,\sqrt{x+1}}{45045}+\frac{2771\,x^2\,\sqrt{x+1}}{15015}+\frac{15\,x^3\,\sqrt{x+1}}{1001}-\frac{32\,x^4\,\sqrt{x+1}}{3003}+\frac{76\,x^5\,\sqrt{x+1}}{15015}-\frac{64}{}\cdots\right.}{x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1}$$

input
```
int((x + 1)^(5/2)/(1 - x)^(17/2),x)
```

output
```
((1 - x)^(1/2)*((11101*x*(x + 1)^(1/2))/45045 + (4243*(x + 1)^(1/2))/45045
 + (2771*x^2*(x + 1)^(1/2))/15015 + (15*x^3*(x + 1)^(1/2))/1001 - (32*x^4*
(x + 1)^(1/2))/3003 + (76*x^5*(x + 1)^(1/2))/15015 - (64*x^6*(x + 1)^(1/2)
)/45045 + (8*x^7*(x + 1)^(1/2))/45045))/(28*x^2 - 8*x - 56*x^3 + 70*x^4 -
56*x^5 + 28*x^6 - 8*x^7 + x^8 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx = \frac{\sqrt{x+1}\,(-8x^7+64x^6-228x^5+480x^4-675x^3-8313x^2-11101x-4243)}{45045\sqrt{1-x}\,(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}$$

input
```
int((1+x)^(5/2)/(1-x)^(17/2),x)
```

output
```
(sqrt(x + 1)*( - 8*x**7 + 64*x**6 - 228*x**5 + 480*x**4 - 675*x**3 - 8313*
x**2 - 11101*x - 4243))/(45045*sqrt( - x + 1)*(x**7 - 7*x**6 + 21*x**5 - 3
5*x**4 + 35*x**3 - 21*x**2 + 7*x - 1))
```

3.107 $\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = -\frac{3\sqrt{1-ax}\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}(1+ax)^{3/2}}{2a} + \frac{3\arcsin(ax)}{2a}$$

output
```
-3/2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a-1/2*(-a*x+1)^(1/2)*(a*x+1)^(3/2)/a+3/2
*arcsin(a*x)/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = \frac{-\left((4+ax)\sqrt{1-a^2x^2}\right) + 6\arctan\left(\frac{\sqrt{1-a^2x^2}}{1-ax}\right)}{2a}$$

input
```
Integrate[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

output
```
(-((4 + a*x)*Sqrt[1 - a^2*x^2]) + 6*ArcTan[Sqrt[1 - a^2*x^2]/(1 - a*x)])/(
2*a)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+1)^{3/2}}{\sqrt{1-ax}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{3}{2}\int \frac{\sqrt{ax+1}}{\sqrt{1-ax}}dx-\frac{\sqrt{1-ax}(ax+1)^{3/2}}{2a}$$

$$\Big\downarrow \text{ 50}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{\sqrt{1-ax}(ax+1)^{3/2}}{2a}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{3}{2}\left(\frac{\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{\sqrt{1-ax}(ax+1)^{3/2}}{2a}$$

input
```
Int[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

output
```
-1/2*(Sqrt[1 - a*x]*(1 + a*x)^(3/2))/a + (3*(-(Sqrt[1 - a^2*x^2]/a) + ArcS
in[a*x]/a))/2
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\sqrt{-ax+1}\,(ax+1)^{\frac{3}{2}}}{2a}-\frac{3\sqrt{-ax+1}\,\sqrt{ax+1}}{2a}+\frac{3\sqrt{(ax+1)(-ax+1)}\,\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{a^2}}$	98
risch	$\frac{(ax+4)\sqrt{ax+1}\,(ax-1)\sqrt{(ax+1)(-ax+1)}}{2a\sqrt{-(ax+1)(ax-1)}\,\sqrt{-ax+1}}+\frac{3\sqrt{(ax+1)(-ax+1)}\,\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{a^2}}$	116

input
```
int((a*x+1)^(3/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a*x+1)^(1/2)*(a*x+1)^(3/2)/a-3/2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a+3/2
*((a*x+1)*(-a*x+1))^(1/2)/(a*x+1)^(1/2)/(-a*x+1)^(1/2)/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = -\frac{(ax+4)\sqrt{ax+1}\sqrt{-ax+1}+6\,\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right)}{2\,a}$$

input
```
integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) + 6*arctan((sqrt(a*x + 1)*sqr
t(-a*x + 1) - 1)/(a*x)))/a
```

Sympy [F]

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = \int \frac{(ax+1)^{\frac{3}{2}}}{\sqrt{-ax+1}}\,dx$$

input
```
integrate((a*x+1)**(3/2)/(-a*x+1)**(1/2),x)
```

output
```
Integral((a*x + 1)**(3/2)/sqrt(-a*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = -\frac{1}{2}\sqrt{-a^2x^2+1}x + \frac{3\arcsin(ax)}{2a} - \frac{2\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = -\frac{(ax+4)\sqrt{ax+1}\sqrt{-ax+1} - 6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{ax+1}\right)}{2a}$$

input
```
integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 6*arcsin(1/2*sqrt(2)*sqrt(a
*x + 1)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = \int \frac{(a\,x+1)^{3/2}}{\sqrt{1-a\,x}}\,dx$$

input
```
int((a*x + 1)^(3/2)/(1 - a*x)^(1/2),x)
```

output
```
int((a*x + 1)^(3/2)/(1 - a*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = \frac{-6asin\left(\frac{\sqrt{-ax+1}}{\sqrt{2}}\right) - \sqrt{ax+1}\,\sqrt{-ax+1}\,ax - 4\sqrt{ax+1}\,\sqrt{-ax+1}}{2a}$$

input
```
int((a*x+1)^(3/2)/(-a*x+1)^(1/2),x)
```

output
```
( - 6*asin(sqrt( - a*x + 1)/sqrt(2)) - sqrt(a*x + 1)*sqrt( - a*x + 1)*a*x
- 4*sqrt(a*x + 1)*sqrt( - a*x + 1))/(2*a)
```

3.108 $\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{(4+ax)\sqrt{1-a^2x^2}}{2a} + \frac{3\arcsin(ax)}{2a}$$

output
```
-1/2*(a*x+4)*(-a^2*x^2+1)^(1/2)/a+3/2*arcsin(a*x)/a
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{(4+ax)\sqrt{1-a^2x^2} + 6\arctan\left(\frac{-1+\sqrt{1-a^2x^2}}{ax}\right)}{2a}$$

input
```
Integrate[(1 + a*x)^2/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcTan[(-1 + Sqrt[1 - a^2*x^2])/(a*x
)])/a
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {469, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+1)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}\int \frac{ax+1}{\sqrt{1-a^2x^2}}dx-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{2}\left(\frac{\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}$$

input
```
Int[(1 + a*x)^2/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*((1 + a*x)*Sqrt[1 - a^2*x^2])/a + (3*(-(Sqrt[1 - a^2*x^2]/a) + ArcSin
[a*x]/a))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

method	result	size
risch	$\frac{(ax+4)\left(a^2x^2-1\right)}{2a\sqrt{-a^2x^2+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$	60
default	$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+a^2\left(-\frac{x\sqrt{-a^2x^2+1}}{2a^2}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^2\sqrt{a^2}}\right)-\frac{2\sqrt{-a^2x^2+1}}{a}$	98
meijerg	$-\frac{-\frac{\sqrt{\pi}\,x\left(-a^2\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}{a^2}+\frac{\sqrt{\pi}\left(-a^2\right)^{\frac{3}{2}}\arcsin(ax)}{a^3}}{2\sqrt{\pi}\,\sqrt{-a^2}}-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-a^2x^2+1}}{a\sqrt{\pi}}+\frac{\arcsin(ax)}{a}$	100

input
```
int((a*x+1)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+4)*(a^2*x^2-1)/a/(-a^2*x^2+1)^(1/2)+3/2/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}(ax+4)+6\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}$$

input
```
integrate((a*x+1)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 4) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*
x)))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(31) = 62.

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} \left(-\frac{x}{2}-\frac{2}{a}\right)\sqrt{-a^2x^2+1}+\frac{3\log\left(-2a^2x+2\sqrt{-a^2}\sqrt{-a^2x^2+1}\right)}{2\sqrt{-a^2}} & \text{for } a^2\neq 0 \\ \begin{cases} x & \text{for } a=0 \\ \frac{(ax+1)^3}{3a} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x+1)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise(((-x/2 - 2/a)*sqrt(-a**2*x**2 + 1) + 3*log(-2*a**2*x + 2*sqrt(-a
**2)*sqrt(-a**2*x**2 + 1))/(2*sqrt(-a**2)), Ne(a**2, 0)), (Piecewise((x, E
q(a, 0)), ((a*x + 1)**3/(3*a), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{1}{2}\sqrt{-a^2x^2+1}x + \frac{3\arcsin{(ax)}}{2a} - \frac{2\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate((a*x+1)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{1}{2}\sqrt{-a^2x^2+1}\left(x+\frac{4}{a}\right) + \frac{3\arcsin{(ax)}\operatorname{sgn}(a)}{2|a|}$$

input
```
integrate((a*x+1)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-a^2*x^2 + 1)*(x + 4/a) + 3/2*arcsin(a*x)*sgn(a)/abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\frac{3\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{2} + \sqrt{1-a^2\,x^2}\left(\frac{2\,a}{\sqrt{-a^2}} - \frac{x\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

input
```
int((a*x + 1)^2/(1 - a^2*x^2)^(1/2),x)
```

output
```
((3*asinh(x*(-a^2)^(1/2)))/2 + (1 - a^2*x^2)^(1/2)*((2*a)/(-a^2)^(1/2) - (
x*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{3asin(ax) - \sqrt{-a^2x^2+1}\,ax - 4\sqrt{-a^2x^2+1}+4}{2a}$$

input
```
int((a*x+1)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
(3*asin(a*x) - sqrt( - a**2*x**2 + 1)*a*x - 4*sqrt( - a**2*x**2 + 1) + 4)/
(2*a)
```

3.109 $\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 59

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{3\sqrt{1-a^2x^2}}{2a} - \frac{(1+ax)\sqrt{1-a^2x^2}}{2a} + \frac{3\arcsin(ax)}{2a}$$

output
```
-3/2*(-a^2*x^2+1)^(1/2)/a-1/2*(a*x+1)*(-a^2*x^2+1)^(1/2)/a+3/2*arcsin(a*x)
/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = \frac{(-4-ax)\sqrt{1-a^2x^2}}{2a} + \frac{3\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a}$$

input
```
Integrate[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

output
```
((-4 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) + (3*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*
x^2])])/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {667, 469, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+1)\sqrt{1-a^2x^2}}{1-ax}dx$$

$$\Big\downarrow \text{667}$$

$$\int \frac{(ax+1)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}\int \frac{ax+1}{\sqrt{1-a^2x^2}}dx-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{2}\left(\frac{\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}\right)-\frac{(ax+1)\sqrt{1-a^2x^2}}{2a}$$

input
```
Int[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

output
```
-1/2*((1 + a*x)*Sqrt[1 - a^2*x^2])/a + (3*(-(Sqrt[1 - a^2*x^2]/a) + ArcSin
[a*x]/a))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 667
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(
x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + c*x^2)^(p - 1), x]
/; FreeQ[{a, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{(ax+4)(a^2x^2-1)}{2a\sqrt{-a^2x^2+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$	60
default	$-\frac{x\sqrt{-a^2x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}-\frac{2\left(\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)a}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)a}}\right)}{\sqrt{a^2}}\right)}{a}$	120

input
```
int((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x+4)*(a^2*x^2-1)/a/(-a^2*x^2+1)^(1/2)+3/2/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{\sqrt{-a^2x^2+1}(ax+4)+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}$$

input
```
integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 4) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*
x)))/a
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = -\begin{cases} -\frac{-\sqrt{-a^2x^2+1}+\operatorname{asin}(ax)}{a} & \text{for}\ ax > -1 \wedge ax < 1 \end{cases} - \begin{cases} -\frac{-\frac{ax\sqrt{-a^2x^2+1}}{2}-\sqrt{-a^2x^2+1}+\frac{\operatorname{asin}(ax)}{2}}{a} & \text{for}\ ax > -1 \wedge ax < 1 \end{cases}$$

input
```
integrate((a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*x+1),x)
```

output
```
-Piecewise((-(-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, (a*x > -1) & (a*x < 1)
)) - Piecewise((-(-a*x*sqrt(-a**2*x**2 + 1)/2 - sqrt(-a**2*x**2 + 1) + asi
n(a*x)/2)/a, (a*x > -1) & (a*x < 1)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{1}{2}\sqrt{-a^2x^2+1}x + \frac{3\arcsin(ax)}{2a} - \frac{2\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{1}{2}\sqrt{-a^2x^2+1}\left(x+\frac{4}{a}\right) + \frac{3\arcsin(ax)\,\mathrm{sgn}(a)}{2|a|}$$

input
```
integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(-a^2*x^2 + 1)*(x + 4/a) + 3/2*arcsin(a*x)*sgn(a)/abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = \frac{\frac{3\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{2} + \sqrt{1-a^2x^2}\left(\frac{2a}{\sqrt{-a^2}} - \frac{x\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

input
```
int(-((1 - a^2*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

output
```
((3*asinh(x*(-a^2)^(1/2)))/2 + (1 - a^2*x^2)^(1/2)*((2*a)/(-a^2)^(1/2) - (
x*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = \frac{3asin(ax)-\sqrt{-a^2x^2+1}\,ax-4\sqrt{-a^2x^2+1}+4}{2a}$$

input
```
int((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x)
```

output
```
(3*asin(a*x) - sqrt( - a**2*x**2 + 1)*a*x - 4*sqrt( - a**2*x**2 + 1) + 4)/
(2*a)
```

3.110 $\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = \frac{35}{8}\sqrt{1-x}\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35\arcsin(x)}{8}$$

output
```
35/8*(1-x)^(1/2)*(1+x)^(1/2)+35/24*(1-x)^(3/2)*(1+x)^(1/2)+7/12*(1-x)^(5/2
)*(1+x)^(1/2)+1/4*(1-x)^(7/2)*(1+x)^(1/2)+35/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = \frac{1}{24}\sqrt{1-x^2}\left(160-81x+32x^2-6x^3\right) - \frac{35}{4}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

output
```
(Sqrt[1 - x^2]*(160 - 81*x + 32*x^2 - 6*x^3))/24 - (35*ArcTan[Sqrt[1 - x^2
]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {60, 60, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{7/2}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{7}{4}\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}dx+\frac{1}{4}\sqrt{x+1}(1-x)^{7/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{7}{4}\left(\frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}dx+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)+\frac{1}{4}\sqrt{x+1}(1-x)^{7/2}$$

$$\Big\downarrow \text{60}$$

$$\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)+\frac{1}{4}\sqrt{x+1}(1-x)^{7/2}$$

$$\Big\downarrow \text{50}$$

$$\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)+\frac{1}{4}\sqrt{x+1}(1-x)^{7/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{7}{4}\left(\frac{5}{3}\left(\frac{3}{2}\left(\arcsin(x)+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)+\frac{1}{4}\sqrt{x+1}(1-x)^{7/2}$$

input
```
Int[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

output
```
((1 - x)^(7/2)*Sqrt[1 + x])/4 + (7*(((1 - x)^(5/2)*Sqrt[1 + x])/3 + (5*(((
1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*(Sqrt[1 - x^2] + ArcSin[x]))/2))/3))/4
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{(6x^3-32x^2+81x-160)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	82
default	$\frac{(1-x)^{\frac{7}{2}}\sqrt{1+x}}{4}+\frac{7(1-x)^{\frac{5}{2}}\sqrt{1+x}}{12}+\frac{35(1-x)^{\frac{3}{2}}\sqrt{1+x}}{24}+\frac{35\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$	85

input
```
int((1-x)^(7/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*x^3-32*x^2+81*x-160)*(1+x)^(1/2)*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+
x)*(1-x))^(1/2)/(1-x)^(1/2)+35/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/
2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = -\frac{1}{24}\left(6\,x^3-32\,x^2+81\,x-160\right)\sqrt{x+1}\sqrt{-x+1} - \frac{35}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(6*x^3 - 32*x^2 + 81*x - 160)*sqrt(x + 1)*sqrt(-x + 1) - 35/4*arctan
((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.26

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = \begin{cases} -\frac{i\sqrt{x-1}(x+1)^{\frac{7}{2}}}{4} + \frac{25i\sqrt{x-1}(x+1)^{\frac{5}{2}}}{12} - \frac{163i\sqrt{x-1}(x+1)^{\frac{3}{2}}}{24} + \frac{93i\sqrt{x-1}\sqrt{x+1}}{8} - \frac{35i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} & \text{for} \\ \frac{35\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} - \frac{31(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}} + \frac{263(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}} - \frac{605(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}} + \frac{93\sqrt{x+1}}{4\sqrt{1-x}} & \text{oth} \end{cases}$$

input
```
integrate((1-x)**(7/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((-I*sqrt(x - 1)*(x + 1)**(7/2)/4 + 25*I*sqrt(x - 1)*(x + 1)**(5/
2)/12 - 163*I*sqrt(x - 1)*(x + 1)**(3/2)/24 + 93*I*sqrt(x - 1)*sqrt(x + 1)
/8 - 35*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4, Abs(x + 1) > 2), (35*asin(sqrt(2
)*sqrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*sqrt(1 - x)) - 31*(x + 1)**(7/2)/(1
2*sqrt(1 - x)) + 263*(x + 1)**(5/2)/(24*sqrt(1 - x)) - 605*(x + 1)**(3/2)/
(24*sqrt(1 - x)) + 93*sqrt(x + 1)/(4*sqrt(1 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = -\frac{1}{4}\,\sqrt{-x^2+1}x^3 + \frac{4}{3}\,\sqrt{-x^2+1}x^2 - \frac{27}{8}\,\sqrt{-x^2+1}x + \frac{20}{3}\,\sqrt{-x^2+1} + \frac{35}{8}\,\arcsin\left(x\right)$$

input
```
integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-x^2 + 1)*x^3 + 4/3*sqrt(-x^2 + 1)*x^2 - 27/8*sqrt(-x^2 + 1)*x +
 20/3*sqrt(-x^2 + 1) + 35/8*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = -\frac{1}{24}\,\left(\left(2\left(3\,x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\,\left(\left(2\,x-5\right)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} - \frac{3}{2}\,\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + \frac{35}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1)
+ 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(
x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 35/4*arcsin(1/2*sqrt(2)*s
qrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = \int \frac{(1-x)^{7/2}}{\sqrt{x+1}}\,dx$$

input
```
int((1 - x)^(7/2)/(x + 1)^(1/2),x)
```

output
```
int((1 - x)^(7/2)/(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = -\frac{35asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{4} - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^3}{4} + \frac{4\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3} - \frac{27\sqrt{x+1}\,\sqrt{1-x}\,x}{8} + \frac{20\sqrt{x+1}\,\sqrt{1-x}}{3}$$

input
```
int((1-x)^(7/2)/(1+x)^(1/2),x)
```

output
```
( - 210*asin(sqrt( - x + 1)/sqrt(2)) - 6*sqrt(x + 1)*sqrt( - x + 1)*x**3 +
 32*sqrt(x + 1)*sqrt( - x + 1)*x**2 - 81*sqrt(x + 1)*sqrt( - x + 1)*x + 16
0*sqrt(x + 1)*sqrt( - x + 1))/24
```

3.111 $\int \frac{(1-x)^{5/2}}{\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}} \, dx = \frac{5}{2}\sqrt{1-x}\sqrt{1+x} + \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5\arcsin(x)}{2}$$

output
```
5/2*(1-x)^(1/2)*(1+x)^(1/2)+5/6*(1-x)^(3/2)*(1+x)^(1/2)+1/3*(1-x)^(5/2)*(1
+x)^(1/2)+5/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}} \, dx = \frac{1}{6}\sqrt{1-x^2}(22-9x+2x^2) - 5\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

output
```
(Sqrt[1 - x^2]*(22 - 9*x + 2*x^2))/6 - 5*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {60, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}dx+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}$$

$$\Big\downarrow \text{ 50}$$

$$\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{5}{3}\left(\frac{3}{2}\left(\arcsin(x)+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}$$

input
```
Int[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

output
```
((1 - x)^(5/2)*Sqrt[1 + x])/3 + (5*(((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*(Sq
rt[1 - x^2] + ArcSin[x]))/2))/3
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{(1-x)^{\frac{5}{2}}\sqrt{1+x}}{3}+\frac{5(1-x)^{\frac{3}{2}}\sqrt{1+x}}{6}+\frac{5\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$-\frac{(2x^2-9x+22)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

input
```
int((1-x)^(5/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1-x)^(5/2)*(1+x)^(1/2)+5/6*(1-x)^(3/2)*(1+x)^(1/2)+5/2*(1-x)^(1/2)*(1
+x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx = \frac{1}{6}\left(2\,x^2-9\,x+22\right)\sqrt{x+1}\sqrt{-x+1}-5\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^2 - 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sq
rt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.58

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx = \begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{17i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}+\frac{59i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}-\frac{11i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{17(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{59(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}+\frac{11\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(5/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x
- 1)) - 17*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 59*I*(x + 1)**(3/2)/(6*sqrt(
x - 1)) - 11*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (5*asin(sqrt(2)*s
qrt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 17*(x + 1)**(5/2)/(6*sqrt
(1 - x)) - 59*(x + 1)**(3/2)/(6*sqrt(1 - x)) + 11*sqrt(x + 1)/sqrt(1 - x),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx = \frac{1}{3}\sqrt{-x^2+1}x^2 - \frac{3}{2}\sqrt{-x^2+1}x + \frac{11}{3}\sqrt{-x^2+1} + \frac{5}{2}\arcsin(x)$$

input
```
integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x + 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx = \frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + 5\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)
*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(2)*sqrt(x + 1
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx = \int \frac{(1-x)^{5/2}}{\sqrt{x+1}}\,dx$$

input
```
int((1 - x)^(5/2)/(x + 1)^(1/2),x)
```

output
```
int((1 - x)^(5/2)/(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx = -5asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \frac{\sqrt{x+1}\,\sqrt{1-x}\,x^2}{3} - \frac{3\sqrt{x+1}\,\sqrt{1-x}\,x}{2} + \frac{11\sqrt{x+1}\,\sqrt{1-x}}{3}$$

input
```
int((1-x)^(5/2)/(1+x)^(1/2),x)
```

output
```
( - 30*asin(sqrt( - x + 1)/sqrt(2)) + 2*sqrt(x + 1)*sqrt( - x + 1)*x**2 -
9*sqrt(x + 1)*sqrt( - x + 1)*x + 22*sqrt(x + 1)*sqrt( - x + 1))/6
```

3.112 $\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx = \frac{3}{2}\sqrt{1-x}\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3\arcsin(x)}{2}$$

output
```
3/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*(1-x)^(3/2)*(1+x)^(1/2)+3/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx = -\frac{1}{2}(-4+x)\sqrt{1-x^2}-3\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

output
```
-1/2*((-4 + x)*Sqrt[1 - x^2]) - 3*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}\,dx$$

$$\Big\downarrow \ 60$$

$$\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}$$

$$\Big\downarrow \ 50$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}$$

$$\Big\downarrow \ 223$$

$$\frac{3}{2}\left(\arcsin(x)+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}$$

input
```
Int[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

output
```
((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*(Sqrt[1 - x^2] + ArcSin[x]))/2
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{(1-x)^{\frac{3}{2}}\sqrt{1+x}}{2}+\frac{3\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$\frac{(-4+x)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	70

input
```
int((1-x)^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1-x)^(3/2)*(1+x)^(1/2)+3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))^
(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx=-\frac{1}{2}\,\sqrt{x+1}(x-4)\sqrt{-x+1}-3\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(x + 1)*(x - 4)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.94

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx = \begin{cases} -3i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} + \frac{7i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} - \frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} - \frac{7(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} + \frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(3/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x
- 1)) + 7*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 5*I*sqrt(x + 1)/sqrt(x - 1),
Abs(x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1
 - x)) - 7*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 5*sqrt(x + 1)/sqrt(1 - x), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x + 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

input
```
integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2 + 1)*x + 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx = -\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} + \sqrt{x+1}\sqrt{-x+1} + 3\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3*arcsi
n(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx = \int \frac{(1-x)^{3/2}}{\sqrt{x+1}}\,dx$$

input
```
int((1 - x)^(3/2)/(x + 1)^(1/2),x)
```

output
```
int((1 - x)^(3/2)/(x + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx = -3asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - \frac{\sqrt{x+1}\,\sqrt{1-x}\,x}{2} + 2\sqrt{x+1}\,\sqrt{1-x}$$

input
```
int((1-x)^(3/2)/(1+x)^(1/2),x)
```

output
```
( - 6*asin(sqrt( - x + 1)/sqrt(2)) - sqrt(x + 1)*sqrt( - x + 1)*x + 4*sqrt
(x + 1)*sqrt( - x + 1))/2
```

3.113 $\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx=\sqrt{1-x}\sqrt{1+x}+\arcsin(x)$$

output
```
(1-x)^(1/2)*(1+x)^(1/2)+arcsin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx=\sqrt{1-x^2}-2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[1 - x]/Sqrt[1 + x],x]
```

output
```
Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{50}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}$$

$$\Big\downarrow \text{223}$$

$$\arcsin(x)+\sqrt{1-x^2}$$

input
```
Int[Sqrt[1 - x]/Sqrt[1 + x],x]
```

output
```
Sqrt[1 - x^2] + ArcSin[x]
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

method	result	size
default	$\sqrt{1-x}\,\sqrt{1+x}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	41
risch	$-\frac{\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	66

input
```
int((1-x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin
(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx=\sqrt{x+1}\sqrt{-x+1}-2\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1
) - 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (2*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx = \sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{-x+1} + 2 \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx = \operatorname{asin}(x) + \sqrt{1-x^2}$$

input
```
int((1 - x)^(1/2)/(x + 1)^(1/2),x)
```

output
```
asin(x) + (1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx = -2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \sqrt{x+1}\,\sqrt{1-x}$$

input
```
int((1-x)^(1/2)/(1+x)^(1/2),x)
```

output
```
- 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( - x + 1)
```

3.114 $\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 2

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx=\arcsin(x)$$

output
```
arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(2) = 4.

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 10.00

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx=-2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

output
```
-2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}\,dx$$
$$\Big\downarrow \ 39$$
$$\int \frac{1}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \ 223$$
$$\arcsin(x)$$

input
```
Int[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

output
```
ArcSin[x]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(2) = 4$.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 13.50

method	result	size
default	$\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	27

input
```
int(1/(1-x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(2) = 4$.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 11.00

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx = -2\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 19.50

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } |x+1| > 2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1) > 2), (2*asin(sqr
t(2)*sqrt(x + 1)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx = \arcsin(x)$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx = 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 11.00

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx = -4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$

input
```
int(1/((1 - x)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
-4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx = -2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

input
```
int(1/(1-x)^(1/2)/(1+x)^(1/2),x)
```

output
```
 - 2*asin(sqrt( - x + 1)/sqrt(2))
```

3.115 $\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

output
```
(1+x)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

input
```
Integrate[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

output
```
Sqrt[1 + x]/Sqrt[1 - x]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{3/2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
Int[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

output
```
Sqrt[1 + x]/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{\sqrt{1+x}}{\sqrt{1-x}}$	14
default	$\frac{\sqrt{1+x}}{\sqrt{1-x}}$	14
orering	$-\frac{\sqrt{1+x}\,(-1+x)}{(1-x)^{\frac{3}{2}}}$	18
risch	$\frac{\sqrt{1+x}\,\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}$	35

input
```
int(1/(1-x)^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1+x)^(1/2)/(1-x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{x-\sqrt{x+1}\sqrt{-x+1}-1}{x-1}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
(x - sqrt(x + 1)*sqrt(-x + 1) - 1)/(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(3/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((1/sqrt(-1 + 2/(x + 1)), 1/Abs(x + 1) > 1/2), (-I/sqrt(1 - 2/(x
+ 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = -\frac{\sqrt{-x^2+1}}{x-1}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = -\frac{\sqrt{x+1}\sqrt{-x+1}}{x-1}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(x + 1)*sqrt(-x + 1)/(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
int(1/((1 - x)^(3/2)*(x + 1)^(1/2)),x)
```

output
```
(x + 1)^(1/2)/(1 - x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
int(1/(1-x)^(3/2)/(1+x)^(1/2),x)
```

output
```
sqrt(x + 1)/sqrt( - x + 1)
```

3.116 $\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{3(1-x)^{3/2}} + \frac{\sqrt{1+x}}{3\sqrt{1-x}}$$

output
```
1/3*(1+x)^(1/2)/(1-x)^(3/2)+1/3*(1+x)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = \frac{(2-x)\sqrt{1+x}}{3(1-x)^{3/2}}$$

input
```
Integrate[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

output
```
((2 - x)*Sqrt[1 + x])/(3*(1 - x)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{5/2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

input
```
Int[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

output
```
Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(x-2)\sqrt{1+x}}{3(1-x)^{\frac{3}{2}}}$	18
orering	$\frac{\sqrt{1+x}\,(-1+x)(x-2)}{3(1-x)^{\frac{5}{2}}}$	21
default	$\frac{\sqrt{1+x}}{3(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}$	30
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^{2}-x-2)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	49

input
```
int(1/(1-x)^(5/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-2)/(1-x)^(3/2)*(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = \frac{2\,x^2 - \sqrt{x+1}(x-2)\sqrt{-x+1} - 4\,x + 2}{3\,(x^2 - 2\,x + 1)}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*x^2 - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.12

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = \begin{cases} \frac{x+1}{3\sqrt{-1+\frac{2}{x+1}}(x+1)-6\sqrt{-1+\frac{2}{x+1}}} - \frac{3}{3\sqrt{-1+\frac{2}{x+1}}(x+1)-6\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{i(x+1)}{3\sqrt{1-\frac{2}{x+1}}(x+1)-6\sqrt{1-\frac{2}{x+1}}} + \frac{3i}{3\sqrt{1-\frac{2}{x+1}}(x+1)-6\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(5/2)/(1+x)**(1/2),x)
```

output
```
Piecewise(((x + 1)/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1)
)) - 3/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1))), 1/Abs(x
+ 1) > 1/2), (-I*(x + 1)/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x
+ 1))) + 3*I/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x + 1))), True
))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = \frac{\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)} - \frac{\sqrt{-x^2+1}}{3\,(x-1)}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = -\frac{\sqrt{x+1}(x-2)\sqrt{-x+1}}{3\,(x-1)^2}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)/(x - 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = \frac{x\,\sqrt{1-x}+2\,\sqrt{1-x}-x^2\,\sqrt{1-x}}{3\,(x-1)^2\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(5/2)*(x + 1)^(1/2)),x)
```

output
```
(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x +
1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}\,(x-2)}{3\sqrt{1-x}\,(x-1)}$$

input
```
int(1/(1-x)^(5/2)/(1+x)^(1/2),x)
```

output
```
(sqrt(x + 1)*(x - 2))/(3*sqrt( - x + 1)*(x - 1))
```

3.117 $\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{5(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{15(1-x)^{3/2}} + \frac{2\sqrt{1+x}}{15\sqrt{1-x}}$$

output
```
1/5*(1+x)^(1/2)/(1-x)^(5/2)+2/15*(1+x)^(1/2)/(1-x)^(3/2)+2/15*(1+x)^(1/2)/
(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}(7-6x+2x^2)}{15(1-x)^{5/2}}$$

input
```
Integrate[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

output
```
(Sqrt[1 + x]*(7 - 6*x + 2*x^2))/(15*(1 - x)^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{7/2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}\right)+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2}{5}\left(\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}\right)+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

input
```
Int[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

output
```
Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*(Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1
 + x]/(3*Sqrt[1 - x])))/5
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.41

method	result	size
gosper	$\frac{\sqrt{1+x}\,(2x^2-6x+7)}{15(1-x)^{\frac{5}{2}}}$	25
orering	$-\frac{\sqrt{1+x}\,(-1+x)(2x^2-6x+7)}{15(1-x)^{\frac{7}{2}}}$	28
default	$\frac{\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{15(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}$	44
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^3-4x^2+x+7)}{15\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54

input
```
int(1/(1-x)^(7/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/(1-x)^(5/2)*(1+x)^(1/2)*(2*x^2-6*x+7)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx=\frac{7\,x^3-21\,x^2-(2\,x^2-6\,x+7)\sqrt{x+1}\sqrt{-x+1}+21\,x-7}{15\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(7*x^3 - 21*x^2 - (2*x^2 - 6*x + 7)*sqrt(x + 1)*sqrt(-x + 1) + 21*x -
 7)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 303, normalized size of antiderivative = 4.97

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = \begin{cases} \frac{2(x+1)^2}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+1)+60\sqrt{-1+\frac{2}{x+1}}} - \frac{10(x+1)}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+1)\cdots} \\ -\frac{2i(x+1)^2}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{1-\frac{2}{x+1}}} + \frac{10i(x+1)}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{\cdots}} \end{cases}$$

input
```
integrate(1/(1-x)**(7/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*(x + 1)**2/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 +
 2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(x + 1))) - 10*(x + 1)/(15*sqrt(-1 +
2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(
x + 1))) + 15/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1)
)*(x + 1) + 60*sqrt(-1 + 2/(x + 1))), 1/Abs(x + 1) > 1/2), (-2*I*(x + 1)**
2/(15*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60
*sqrt(1 - 2/(x + 1))) + 10*I*(x + 1)/(15*sqrt(1 - 2/(x + 1))*(x + 1)**2 -
60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/(x + 1))) - 15*I/(15*sqrt(1
 - 2/(x + 1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/
(x + 1))), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = -\frac{\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)} + \frac{2\,\sqrt{-x^2+1}}{15\,(x^2-2\,x+1)} - \frac{2\,\sqrt{-x^2+1}}{15\,(x-1)}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 2/15*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) - 2/15*sqrt(-x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = -\frac{(2\,(x+1)(x-4)+15)\sqrt{x+1}\sqrt{-x+1}}{15\,(x-1)^3}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/15*(2*(x + 1)*(x - 4) + 15)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = -\frac{x\,\sqrt{1-x}+7\,\sqrt{1-x}-4\,x^2\,\sqrt{1-x}+2\,x^3\,\sqrt{1-x}}{15\,(x-1)^3\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(7/2)*(x + 1)^(1/2)),x)
```

output
```
-(x*(1 - x)^(1/2) + 7*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^
(1/2))/(15*(x - 1)^3*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}\,(2x^2-6x+7)}{15\sqrt{1-x}\,(x^2-2x+1)}$$

input
```
int(1/(1-x)^(7/2)/(1+x)^(1/2),x)
```

output
```
(sqrt(x + 1)*(2*x**2 - 6*x + 7))/(15*sqrt( - x + 1)*(x**2 - 2*x + 1))
```

3.118 $\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3\sqrt{1+x}}{35(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{35(1-x)^{3/2}} + \frac{2\sqrt{1+x}}{35\sqrt{1-x}}$$

output
```
1/7*(1+x)^(1/2)/(1-x)^(7/2)+3/35*(1+x)^(1/2)/(1-x)^(5/2)+2/35*(1+x)^(1/2)/
(1-x)^(3/2)+2/35*(1+x)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}(12-13x+8x^2-2x^3)}{35(1-x)^{7/2}}$$

input
```
Integrate[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

output
```
(Sqrt[1 + x]*(12 - 13*x + 8*x^2 - 2*x^3))/(35*(1 - x)^(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{9/2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{7}\int \frac{1}{(1-x)^{7/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}\right)+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}\right)+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}\right)+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}\right)+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}\right)+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

input
```
Int[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

output
```
Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*(Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*(Sq
rt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])))/5))/7
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.37

method	result	size
gosper	$-\frac{\sqrt{1+x}\,(2x^3-8x^2+13x-12)}{35(1-x)^{\frac{7}{2}}}$	30
orering	$\frac{\sqrt{1+x}\,(-1+x)(2x^3-8x^2+13x-12)}{35(1-x)^{\frac{9}{2}}}$	33
default	$\frac{\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{35(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{35(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}$	58
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^4-6x^3+5x^2+x-12)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59

input
```
int(1/(1-x)^(9/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35/(1-x)^(7/2)*(1+x)^(1/2)*(2*x^3-8*x^2+13*x-12)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \frac{12\,x^4 - 48\,x^3 + 72\,x^2 - (2\,x^3 - 8\,x^2 + 13\,x - 12)\sqrt{x+1}\sqrt{-x+1} - 48\,x + 12}{35\,(x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1)}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/35*(12*x^4 - 48*x^3 + 72*x^2 - (2*x^3 - 8*x^2 + 13*x - 12)*sqrt(x + 1)*s
qrt(-x + 1) - 48*x + 12)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 542, normalized size of antiderivative = 6.69

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \begin{cases} \frac{2(x+1)^3}{35\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}(x+1)-280\sqrt{-1+\frac{2}{x+1}}} - \frac{}{35\sqrt{-1+\frac{2}{x+1}}(x} \\ -\frac{2i(x+1)^3}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-210\sqrt{1-\frac{2}{x+1}}(x+1)^2+420\sqrt{1-\frac{2}{x+1}}(x+1)-280\sqrt{1-\frac{2}{x+1}}} + \frac{}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-} \end{cases}$$

input
```
integrate(1/(1-x)**(9/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*(x + 1)**3/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1
+ 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1 +
 2/(x + 1))) - 14*(x + 1)**2/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqr
t(-1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt
(-1 + 2/(x + 1))) + 35*(x + 1)/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*s
qrt(-1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sq
rt(-1 + 2/(x + 1))) - 35/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1
 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1
+ 2/(x + 1))), 1/Abs(x + 1) > 1/2), (-2*I*(x + 1)**3/(35*sqrt(1 - 2/(x + 1
))*(x + 1)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1
))*(x + 1) - 280*sqrt(1 - 2/(x + 1))) + 14*I*(x + 1)**2/(35*sqrt(1 - 2/(x
+ 1))*(x + 1)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x
+ 1))*(x + 1) - 280*sqrt(1 - 2/(x + 1))) - 35*I*(x + 1)/(35*sqrt(1 - 2/(x
+ 1))*(x + 1)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x
+ 1))*(x + 1) - 280*sqrt(1 - 2/(x + 1))) + 35*I/(35*sqrt(1 - 2/(x + 1))*(x
 + 1)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x
 + 1) - 280*sqrt(1 - 2/(x + 1))), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \frac{\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)} - \frac{3\,\sqrt{-x^2+1}}{35\,(x^3-3\,x^2+3\,x-1)} + \frac{2\,\sqrt{-x^2+1}}{35\,(x^2-2\,x+1)} - \frac{2\,\sqrt{-x^2+1}}{35\,(x-1)}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/35*sqrt(-x^2 + 1)/(
x^3 - 3*x^2 + 3*x - 1) + 2/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/35*sqrt(-
x^2 + 1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.43

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = -\frac{((2\,(x+1)(x-6)+35)(x+1)-35)\sqrt{x+1}\sqrt{-x+1}}{35\,(x-1)^4}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/35*((2*(x + 1)*(x - 6) + 35)*(x + 1) - 35)*sqrt(x + 1)*sqrt(-x + 1)/(x
- 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = -\frac{x\,\sqrt{1-x}-12\,\sqrt{1-x}+5\,x^2\,\sqrt{1-x}-6\,x^3\,\sqrt{1-x}+2\,x^4\,\sqrt{1-x}}{35\,(x-1)^4\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(9/2)*(x + 1)^(1/2)),x)
```

output
```
-(x*(1 - x)^(1/2) - 12*(1 - x)^(1/2) + 5*x^2*(1 - x)^(1/2) - 6*x^3*(1 - x)
^(1/2) + 2*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}\,(2x^3-8x^2+13x-12)}{35\sqrt{1-x}\,(x^3-3x^2+3x-1)}$$

input
```
int(1/(1-x)^(9/2)/(1+x)^(1/2),x)
```

output
```
(sqrt(x + 1)*(2*x**3 - 8*x**2 + 13*x - 12))/(35*sqrt( - x + 1)*(x**3 - 3*x
**2 + 3*x - 1))
```

3.119 $\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4\sqrt{1+x}}{105(1-x)^{5/2}} + \frac{8\sqrt{1+x}}{315(1-x)^{3/2}} + \frac{8\sqrt{1+x}}{315\sqrt{1-x}}$$

output
```
1/9*(1+x)^(1/2)/(1-x)^(9/2)+4/63*(1+x)^(1/2)/(1-x)^(7/2)+4/105*(1+x)^(1/2)
/(1-x)^(5/2)+8/315*(1+x)^(1/2)/(1-x)^(3/2)+8/315*(1+x)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}(83-100x+84x^2-40x^3+8x^4)}{315(1-x)^{9/2}}$$

input
```
Integrate[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

output
```
(Sqrt[1 + x]*(83 - 100*x + 84*x^2 - 40*x^3 + 8*x^4))/(315*(1 - x)^(9/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{11/2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{9}\int \frac{1}{(1-x)^{9/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{9}\left(\frac{3}{7}\int \frac{1}{(1-x)^{7/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}\right)+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{9}\left(\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}\right)+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}\right)+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{9}\left(\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}\right)+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}\right)+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}\right)+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{4}{9}\left(\frac{3}{7}\left(\frac{2}{5}\left(\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}\right)+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}\right)+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}\right)+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

input
```
Int[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

output
```
Sqrt[1 + x]/(9*(1 - x)^(9/2)) + (4*(Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*(Sq
rt[1 + x]/(5*(1 - x)^(5/2)) + (2*(Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 +
 x]/(3*Sqrt[1 - x])))/5))/7))/9
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{\sqrt{1+x}\,(8x^4-40x^3+84x^2-100x+83)}{315(1-x)^{\frac{9}{2}}}$	35
orering	$-\frac{\sqrt{1+x}\,(-1+x)(8x^4-40x^3+84x^2-100x+83)}{315(1-x)^{\frac{11}{2}}}$	38
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^5-32x^4+44x^3-16x^2-17x+83)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}+\frac{4\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}+\frac{4\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}+\frac{8\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}$	72

input
```
int(1/(1-x)^(11/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(1+x)^(1/2)/(1-x)^(9/2)*(8*x^4-40*x^3+84*x^2-100*x+83)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = \frac{83\,x^5-415\,x^4+830\,x^3-830\,x^2-(8\,x^4-40\,x^3+84\,x^2-100\,x+83)\sqrt{x+1}}{315\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}$$

input
```
integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(83*x^5 - 415*x^4 + 830*x^3 - 830*x^2 - (8*x^4 - 40*x^3 + 84*x^2 - 1
00*x + 83)*sqrt(x + 1)*sqrt(-x + 1) + 415*x - 83)/(x^5 - 5*x^4 + 10*x^3 -
10*x^2 + 5*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.37 (sec) , antiderivative size = 850, normalized size of antiderivative = 8.42

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-x)**(11/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((8*(x + 1)**4/(315*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-
1 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*s
qrt(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1))) - 72*(x + 1)**3/(
315*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3
 + 7560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x +
1) + 5040*sqrt(-1 + 2/(x + 1))) + 252*(x + 1)**2/(315*sqrt(-1 + 2/(x + 1))
*(x + 1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x +
 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x
 + 1))) - 420*(x + 1)/(315*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1
+ 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqr
t(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1))) + 315/(315*sqrt(-1
+ 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt
(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) + 5040*sq
rt(-1 + 2/(x + 1))), 1/Abs(x + 1) > 1/2), (-8*I*(x + 1)**4/(315*sqrt(1 - 2
/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 -
 2/(x + 1))*(x + 1)**2 - 10080*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 -
 2/(x + 1))) + 72*I*(x + 1)**3/(315*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*
sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 100
80*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 - 2/(x + 1))) - 252*I*(x + 1)
**2/(315*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x +...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = -\frac{\sqrt{-x^2+1}}{9\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)} + \frac{4\,\sqrt{-x^2+1}}{63\,(x^4-4\,x^3+6\,x^2-4\,x+1)} - \frac{4\,\sqrt{-x^2+1}}{105\,(x^3-3\,x^2+3\,x-1)} + \frac{8\,\sqrt{-x^2+1}}{315\,(x^2-2\,x+1)} - \frac{8\,\sqrt{-x^2+1}}{315\,(x-1)}$$

input
```
integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 4/63*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/105*sqrt(-x^2 + 1)/(x^3 - 3*
x^2 + 3*x - 1) + 8/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 8/315*sqrt(-x^2 +
1)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = -\frac{(4\left(\left(2\left(x+1\right)(x-8)+63\right)(x+1)-105\right)(x+1)+315)\sqrt{x+1}\sqrt{-x+1}}{315\left(x-1\right)^5}$$

input
```
integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-1/315*(4*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1) + 315)*sqrt(x +
 1)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = \frac{17\,x\,\sqrt{1-x}-83\,\sqrt{1-x}+16\,x^2\,\sqrt{1-x}-44\,x^3\,\sqrt{1-x}+32\,x^4\,\sqrt{1-x}-8\,x}{315\,\left(x-1\right)^5\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(11/2)*(x + 1)^(1/2)),x)
```

output
```
(17*x*(1 - x)^(1/2) - 83*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1
- x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(315*(x - 1)^5*(x
 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}\,(8x^4-40x^3+84x^2-100x+83)}{315\sqrt{1-x}\,(x^4-4x^3+6x^2-4x+1)}$$

input
```
int(1/(1-x)^(11/2)/(1+x)^(1/2),x)
```

output
```
(sqrt(x + 1)*(8*x**4 - 40*x**3 + 84*x**2 - 100*x + 83))/(315*sqrt( - x + 1
)*(x**4 - 4*x**3 + 6*x**2 - 4*x + 1))
```

3.120 $\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}} \, dx = -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35\arcsin(x)}{2}$$

output
```
-2*(1-x)^(7/2)/(1+x)^(1/2)-35/2*(1-x)^(1/2)*(1+x)^(1/2)-35/6*(1-x)^(3/2)*(
1+x)^(1/2)-7/3*(1-x)^(5/2)*(1+x)^(1/2)-35/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}} \, dx = -\frac{\sqrt{1-x}(166+55x-13x^2+2x^3)}{6\sqrt{1+x}} + 35\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

output
```
-1/6*(Sqrt[1 - x]*(166 + 55*x - 13*x^2 + 2*x^3))/Sqrt[1 + x] + 35*ArcTan[S
qrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {57, 60, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{7/2}}{(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$-7\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}dx-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{60}$$

$$-7\left(\frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}dx+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{60}$$

$$-7\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{50}$$

$$-7\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{223}$$

$$-7\left(\frac{5}{3}\left(\frac{3}{2}\left(\arcsin(x)+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}$$

input
```
Int[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

output
```
(-2*(1 - x)^(7/2))/Sqrt[1 + x] - 7*(((1 - x)^(5/2)*Sqrt[1 + x])/3 + (5*(((
1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*(Sqrt[1 - x^2] + ArcSin[x]))/2))/3)
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

method	result	size
risch	$\frac{(2x^4-15x^3+68x^2+111x-166)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	84

input
```
int((1-x)^(7/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^4-15*x^3+68*x^2+111*x-166)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1
/2)/(1-x)^(1/2)/(1+x)^(1/2)-35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/
2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx = -\frac{(2\,x^3-13\,x^2+55\,x+166)\sqrt{x+1}\sqrt{-x+1}-210\,(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+166\,x+166}{6\,(x+1)}$$

input
```
integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*((2*x^3 - 13*x^2 + 55*x + 166)*sqrt(x + 1)*sqrt(-x + 1) - 210*(x + 1)
*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 166*x + 166)/(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.42

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx = \begin{cases} 35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} + \frac{23i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{125i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{13i\sqrt{x+1}}{\sqrt{x-1}} + \frac{32i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x \\ -35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} - \frac{23(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{125(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{13\sqrt{x+1}}{\sqrt{1-x}} - \frac{32}{\sqrt{1-x}\sqrt{x+1}} & \text{otherw} \end{cases}$$

input
```
integrate((1-x)**(7/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x
- 1)) + 23*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 125*I*(x + 1)**(3/2)/(6*sqrt
(x - 1)) + 13*I*sqrt(x + 1)/sqrt(x - 1) + 32*I/(sqrt(x - 1)*sqrt(x + 1)),
Abs(x + 1) > 2), (-35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(7/2)/(3*sqrt
(1 - x)) - 23*(x + 1)**(5/2)/(6*sqrt(1 - x)) + 125*(x + 1)**(3/2)/(6*sqrt(
1 - x)) - 13*sqrt(x + 1)/sqrt(1 - x) - 32/(sqrt(1 - x)*sqrt(x + 1)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx = \frac{x^4}{3\sqrt{-x^2+1}} - \frac{5x^3}{2\sqrt{-x^2+1}} + \frac{34x^2}{3\sqrt{-x^2+1}} + \frac{37x}{2\sqrt{-x^2+1}} - \frac{83}{3\sqrt{-x^2+1}} - \frac{35}{2}\arcsin(x)$$

input
```
integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
1/3*x^4/sqrt(-x^2 + 1) - 5/2*x^3/sqrt(-x^2 + 1) + 34/3*x^2/sqrt(-x^2 + 1)
+ 37/2*x/sqrt(-x^2 + 1) - 83/3/sqrt(-x^2 + 1) - 35/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx = -\frac{1}{6}\left((2\,x-17)(x+1)+87\right)\sqrt{x+1}\sqrt{-x+1} + \frac{8\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{8\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 35\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
-1/6*((2*x - 17)*(x + 1) + 87)*sqrt(x + 1)*sqrt(-x + 1) + 8*(sqrt(2) - sqr
t(-x + 1))/sqrt(x + 1) - 8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 35*arcsi
n(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx = \int \frac{(1-x)^{7/2}}{(x+1)^{3/2}}\,dx$$

input
```
int((1 - x)^(7/2)/(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(7/2)/(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx = \frac{210\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - 2\sqrt{1-x}\,x^3 + 13\sqrt{1-x}\,x^2 - 55\sqrt{1-x}\,x - 166\sqrt{1-x}}{6\sqrt{x+1}}$$

input
```
int((1-x)^(7/2)/(1+x)^(3/2),x)
```

output
```
(210*sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2)) - 2*sqrt( - x + 1)*x**3 + 13
*sqrt( - x + 1)*x**2 - 55*sqrt( - x + 1)*x - 166*sqrt( - x + 1))/(6*sqrt(x
 + 1))
```

3.121 $\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}} \, dx = -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15\arcsin(x)}{2}$$

output
```
-2*(1-x)^(5/2)/(1+x)^(1/2)-15/2*(1-x)^(1/2)*(1+x)^(1/2)-5/2*(1-x)^(3/2)*(1
+x)^(1/2)-15/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}} \, dx = \frac{\sqrt{1-x}(-24-7x+x^2)}{2\sqrt{1+x}} + 15\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

output
```
(Sqrt[1 - x]*(-24 - 7*x + x^2))/(2*Sqrt[1 + x]) + 15*ArcTan[Sqrt[1 - x^2]/
(-1 + x)]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {57, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{5/2}}{(x+1)^{3/2}}\,dx$$

$$\Big\downarrow \ 57$$

$$-5\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}dx-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \ 60$$

$$-5\left(\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \ 50$$

$$-5\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \ 223$$

$$-5\left(\frac{3}{2}\left(\arcsin(x)+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}$$

input
```
Int[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

output
```
(-2*(1 - x)^(5/2))/Sqrt[1 + x] - 5*(((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*(Sq
rt[1 - x^2] + ArcSin[x]))/2)
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{\left(x^{3}-8 x^{2}-17 x+24\right) \sqrt{\left(1+x\right) \left(1-x\right)}}{2 \sqrt{-\left(1+x\right) \left(-1+x\right)}\, \sqrt{1-x}\, \sqrt{1+x}}-\frac{15 \sqrt{\left(1+x\right) \left(1-x\right)}\, \arcsin\left(x\right)}{2 \sqrt{1+x}\, \sqrt{1-x}}$	77

input
```
int((1-x)^(5/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^3-8*x^2-17*x+24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(
1/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx=\frac{(x^2-7\,x-24)\sqrt{x+1}\sqrt{-x+1}+30\,(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-24\,x-24}{2\,(x+1)}$$

input
```
integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((x^2 - 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x + 1)*arctan((sqrt(x
 + 1)*sqrt(-x + 1) - 1)/x) - 24*x - 24)/(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.57

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx=\begin{cases}15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{11i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}}+\frac{16i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1|>2\\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{11(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}}-\frac{16}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise}\end{cases}$$

input
```
integrate((1-x)**(5/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x
- 1)) - 11*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1) +
16*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) > 2), (-15*asin(sqrt(2)*sqrt(x
+ 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) + 11*(x + 1)**(3/2)/(2*sqrt(1 - x
)) - sqrt(x + 1)/sqrt(1 - x) - 16/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx = -\frac{x^3}{2\sqrt{-x^2+1}} + \frac{4x^2}{\sqrt{-x^2+1}} + \frac{17x}{2\sqrt{-x^2+1}} - \frac{12}{\sqrt{-x^2+1}} - \frac{15}{2}\arcsin(x)$$

input
```
integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*x^3/sqrt(-x^2 + 1) + 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) - 1
2/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx = \frac{1}{2}\sqrt{x+1}(x-8)\sqrt{-x+1} + \frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 15\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x + 1)*(x - 8)*sqrt(-x + 1) + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x +
 1) - 4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 15*arcsin(1/2*sqrt(2)*sqrt(
x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx = \int \frac{(1-x)^{5/2}}{(x+1)^{3/2}}\,dx$$

input
```
int((1 - x)^(5/2)/(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(5/2)/(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx = \frac{30\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\sqrt{1-x}\,x^2-7\sqrt{1-x}\,x-24\sqrt{1-x}}{2\sqrt{x+1}}$$

input
```
int((1-x)^(5/2)/(1+x)^(3/2),x)
```

output
```
(30*sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2)) + sqrt( - x + 1)*x**2 - 7*sqr
t( - x + 1)*x - 24*sqrt( - x + 1))/(2*sqrt(x + 1))
```

3.122 $\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx=-\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\sqrt{1+x}-3\arcsin(x)$$

output
```
-2*(1-x)^(3/2)/(1+x)^(1/2)-3*(1-x)^(1/2)*(1+x)^(1/2)-3*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx=-\frac{\sqrt{1-x}(5+x)}{\sqrt{1+x}}+6\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

output
```
-((Sqrt[1 - x]*(5 + x))/Sqrt[1 + x]) + 6*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {57, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}}{(x+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{57}$$

$$-3 \int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{50}$$

$$-3 \left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{223}$$

$$-3 \left(\arcsin(x)+\sqrt{1-x^2}\right)-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}$$

input
```
Int[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

output
```
(-2*(1 - x)^(3/2))/Sqrt[1 + x] - 3*(Sqrt[1 - x^2] + ArcSin[x])
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(33) = 66$.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

method	result	size
risch	$\frac{(x^2+4x-5)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	71

input
```
int((1-x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2+4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1
/2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx = -\frac{(x+5)\sqrt{x+1}\sqrt{-x+1}-6\,(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+5\,x+5}{x+1}$$

input
```
integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-((x + 5)*sqrt(x + 1)*sqrt(-x + 1) - 6*(x + 1)*arctan((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x) + 5*x + 5)/(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx = \begin{cases} 6i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{8i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1| > 2 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{8}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(3/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
 - 2*I*sqrt(x + 1)/sqrt(x - 1) + 8*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)
 > 2), (-6*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) + 2*sq
rt(x + 1)/sqrt(1 - x) - 8/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx = \frac{(-x^2+1)^{\frac{3}{2}}}{x^2+2x+1} - \frac{6\sqrt{-x^2+1}}{x+1} - 3\arcsin(x)$$

input
```
integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
(-x^2 + 1)^(3/2)/(x^2 + 2*x + 1) - 6*sqrt(-x^2 + 1)/(x + 1) - 3*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(33) = 66$.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx = -\sqrt{x+1}\sqrt{-x+1} + \frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{2\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
-sqrt(x + 1)*sqrt(-x + 1) + 2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 2*sqr
t(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx = \int \frac{(1-x)^{3/2}}{(x+1)^{3/2}}\,dx$$

input
```
int((1 - x)^(3/2)/(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(3/2)/(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx = \frac{6\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - \sqrt{1-x}\,x - 5\sqrt{1-x}}{\sqrt{x+1}}$$

input
```
int((1-x)^(3/2)/(1+x)^(3/2),x)
```

output
```
(6*sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2)) - sqrt( - x + 1)*x - 5*sqrt( -
 x + 1))/sqrt(x + 1)
```

3.123 $\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx = -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \arcsin(x)$$

output
```
-2*(1-x)^(1/2)/(1+x)^(1/2)-arcsin(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx = -\frac{2\sqrt{1-x}}{\sqrt{1+x}} + 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

output
```
(-2*Sqrt[1 - x])/Sqrt[1 + x] + 2*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {57, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{(x+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 57}$$

$$-\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}dx-\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{ 39}$$

$$-\int \frac{1}{\sqrt{1-x^2}}dx-\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{ 223}$$

$$-\arcsin(x)-\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

input
```
Int[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

output
```
(-2*Sqrt[1 - x])/Sqrt[1 + x] - ArcSin[x]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

method	result	size
risch	$\frac{2(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	67

input
```
int((1-x)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)
-((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = \frac{2\left((x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x+1}$$

input
```
integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
2*((x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - x - sqrt(x + 1)*sqrt
(-x + 1) - 1)/(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = \begin{cases} 2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{4i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1| > 2 \\ -2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{4}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(1/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1)
+ 4*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) > 2), (-2*asin(sqrt(2)*sqrt(x
+ 1)/2) + 2*sqrt(x + 1)/sqrt(1 - x) - 4/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = -\frac{2\sqrt{-x^2+1}}{x+1} - \arcsin(x)$$

input
```
integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + 1)/(x + 1) - arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = \frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)
) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = \int \frac{\sqrt{1-x}}{(x+1)^{3/2}}\,dx$$

input
```
int((1 - x)^(1/2)/(x + 1)^(3/2),x)
```

output
```
int((1 - x)^(1/2)/(x + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - 2\sqrt{1-x}}{\sqrt{x+1}}$$

input
```
int((1-x)^(1/2)/(1+x)^(3/2),x)
```

output
```
(2*(sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2)) - sqrt( - x + 1)))/sqrt(x + 1
)
```

3.124 $\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

output
```
-(1-x)^(1/2)/(1+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

input
```
Integrate[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

output
```
-(Sqrt[1 - x]/Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}(x+1)^{3/2}} \, dx$$

$$\Big\downarrow \ 48$$

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

input
```
Int[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

output
```
-(Sqrt[1 - x]/Sqrt[1 + x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	15
default	$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	15
orering	$\frac{-1+x}{\sqrt{1+x}\,\sqrt{1-x}}$	17
risch	$\frac{(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}$	38

input
```
int(1/(1-x)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(1-x)^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = -\frac{x+\sqrt{x+1}\sqrt{-x+1}+1}{x+1}$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-(x + sqrt(x + 1)*sqrt(-x + 1) + 1)/(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = \begin{cases} -\sqrt{-1+\frac{2}{x+1}} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ -i\sqrt{1-\frac{2}{x+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((-sqrt(-1 + 2/(x + 1)), 1/Abs(x + 1) > 1/2), (-I*sqrt(1 - 2/(x +
 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = -\frac{\sqrt{-x^2+1}}{x+1}$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = \frac{\sqrt{2}-\sqrt{-x+1}}{2\sqrt{x+1}} - \frac{\sqrt{x+1}}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)/(sqrt(2) - sqrt
(-x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(1/2)*(x + 1)^(3/2)),x)
```

output
```
-(1 - x)^(1/2)/(x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

input
```
int(1/(1-x)^(1/2)/(1+x)^(3/2),x)
```

output
```
( - sqrt( - x + 1))/sqrt(x + 1)
```

3.125 $\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{1-x^2}}$$

output
```
x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{1-x^2}}$$

input
```
Integrate[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

output
```
x/Sqrt[1 - x^2]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{3/2}(x+1)^{3/2}}\,dx$$
$$\Big\downarrow \; 39$$
$$\int \frac{1}{(1-x^2)^{3/2}}dx$$
$$\Big\downarrow \; 208$$
$$\frac{x}{\sqrt{1-x^2}}$$

input
```
Int[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

output
```
x/Sqrt[1 - x^2]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
gosper	$\frac{x}{\sqrt{1-x}\,\sqrt{1+x}}$	15
orering	$-\frac{(-1+x)x}{\sqrt{1+x}\,(1-x)^{\frac{3}{2}}}$	19
default	$\frac{1}{\sqrt{1-x}\,\sqrt{1+x}}-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	29
risch	$\frac{x\sqrt{(1+x)(1-x)}}{\sqrt{1+x}\,\sqrt{1-x}\,\sqrt{-(1+x)(-1+x)}}$	36

input
```
int(1/(1-x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(1-x)^(1/2)/(1+x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = -\frac{\sqrt{x+1}x\sqrt{-x+1}}{x^2-1}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(x + 1)*x*sqrt(-x + 1)/(x^2 - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.85

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}(x+1)}{x-1} + \frac{\sqrt{-1+\frac{2}{x+1}}}{x-1} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}} + \frac{i}{\sqrt{1-\frac{2}{x+1}}(x+1)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(3/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((-sqrt(-1 + 2/(x + 1))*(x + 1)/(x - 1) + sqrt(-1 + 2/(x + 1))/(x
 - 1), 1/Abs(x + 1) > 1/2), (-I/sqrt(1 - 2/(x + 1)) + I/(sqrt(1 - 2/(x + 1
))*(x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{-x^2+1}}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
x/sqrt(-x^2 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(11) = 22$.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 4.77

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{\sqrt{2}-\sqrt{-x+1}}{4\sqrt{x+1}} - \frac{\sqrt{x+1}\sqrt{-x+1}}{2\,(x-1)} - \frac{\sqrt{x+1}}{4\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)*sqrt(-x + 1)/(x
 - 1) - 1/4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(3/2)*(x + 1)^(3/2)),x)
```

output
```
x/((1 - x)^(1/2)*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{x+1}\,\sqrt{1-x}}$$

input
```
int(1/(1-x)^(3/2)/(1+x)^(3/2),x)
```

output
```
x/(sqrt(x + 1)*sqrt( - x + 1))
```

3.126 $\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \frac{1}{3(1-x)^{3/2}\sqrt{1+x}} + \frac{2x}{3\sqrt{1-x^2}}$$

output
```
1/3/(1-x)^(3/2)/(1+x)^(1/2)+2/3*x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \frac{1+2x-2x^2}{3(1-x)^{3/2}\sqrt{1+x}}$$

input
```
Integrate[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

output
```
(1 + 2*x - 2*x^2)/(3*(1 - x)^(3/2)*Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{5/2}(x+1)^{3/2}} \, dx$$

$$\Big\downarrow \; 55$$

$$\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(x+1)^{3/2}}dx + \frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

$$\Big\downarrow \; 39$$

$$\frac{2}{3}\int \frac{1}{(1-x^2)^{3/2}}dx + \frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

$$\Big\downarrow \; 208$$

$$\frac{2x}{3\sqrt{1-x^2}} + \frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

input
```
Int[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

output
```
1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{2x^{2}-2x-1}{3(1-x)^{\frac{3}{2}}\sqrt{1+x}}$	25
orering	$\frac{(-1+x)(2x^{2}-2x-1)}{3\sqrt{1+x}\,(1-x)^{\frac{5}{2}}}$	28
default	$\frac{1}{3(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{2}{3\sqrt{1-x}\,\sqrt{1+x}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	44
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^{2}-2x-1)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	51

input
```
int(1/(1-x)^(5/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(1-x)^(3/2)/(1+x)^(1/2)*(2*x^2-2*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \frac{x^3 - x^2 - (2\,x^2 - 2\,x - 1)\sqrt{x+1}\sqrt{-x+1} - x + 1}{3\,(x^3 - x^2 - x + 1)}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(x^3 - x^2 - (2*x^2 - 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x + 1)/(x^3
- x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.32

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-12x+3(x+1)^2} + \frac{6\sqrt{-1+\frac{2}{x+1}}(x+1)}{-12x+3(x+1)^2} - \frac{3\sqrt{-1+\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-12x+3(x+1)^2} + \frac{6i\sqrt{1-\frac{2}{x+1}}(x+1)}{-12x+3(x+1)^2} - \frac{3i\sqrt{1-\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(5/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-12*x + 3*(x + 1)**2) + 6*s
qrt(-1 + 2/(x + 1))*(x + 1)/(-12*x + 3*(x + 1)**2) - 3*sqrt(-1 + 2/(x + 1)
)/(-12*x + 3*(x + 1)**2), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(
x + 1)**2/(-12*x + 3*(x + 1)**2) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-12*x
+ 3*(x + 1)**2) - 3*I*sqrt(1 - 2/(x + 1))/(-12*x + 3*(x + 1)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \frac{2\,x}{3\,\sqrt{-x^2+1}} - \frac{1}{3\left(\sqrt{-x^2+1}x-\sqrt{-x^2+1}\right)}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(27) = 54.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \frac{\sqrt{2}-\sqrt{-x+1}}{8\,\sqrt{x+1}} - \frac{(5\,x-7)\sqrt{x+1}\sqrt{-x+1}}{12\,(x-1)^2} - \frac{\sqrt{x+1}}{8\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/12*(5*x - 7)*sqrt(x + 1)*sqrt
(-x + 1)/(x - 1)^2 - 1/8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \frac{2\,x\,\sqrt{1-x}+\sqrt{1-x}-2\,x^2\,\sqrt{1-x}}{3\,(x-1)^2\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(5/2)*(x + 1)^(3/2)),x)
```

output
```
(2*x*(1 - x)^(1/2) + (1 - x)^(1/2) - 2*x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x
+ 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx = \frac{2x^2-2x-1}{3\sqrt{x+1}\,\sqrt{1-x}\,(x-1)}$$

input
```
int(1/(1-x)^(5/2)/(1+x)^(3/2),x)
```

output
```
(2*x**2 - 2*x - 1)/(3*sqrt(x + 1)*sqrt( - x + 1)*(x - 1))
```

3.127 $\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx=\frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{1}{5(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{5\sqrt{1-x^2}}$$

output
```
1/5/(1-x)^(5/2)/(1+x)^(1/2)+1/5/(1-x)^(3/2)/(1+x)^(1/2)+2/5*x/(-x^2+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx=\frac{2+x-4x^2+2x^3}{5(1-x)^{5/2}\sqrt{1+x}}$$

input
```
Integrate[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

output
```
(2 + x - 4*x^2 + 2*x^3)/(5*(1 - x)^(5/2)*Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {55, 55, 39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{7/2}(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{5}\int \frac{1}{(1-x)^{5/2}(x+1)^{3/2}}dx+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{5}\left(\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(x+1)^{3/2}}dx+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}\right)+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

$$\Big\downarrow \text{39}$$

$$\frac{3}{5}\left(\frac{2}{3}\int \frac{1}{(1-x^2)^{3/2}}dx+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}\right)+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

$$\Big\downarrow \text{208}$$

$$\frac{3}{5}\left(\frac{2x}{3\sqrt{1-x^2}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}\right)+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

input
```
Int[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

output
```
1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + (3*(1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2
*x)/(3*Sqrt[1 - x^2])))/5
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2x^{3}-4x^{2}+x+2}{5(1-x)^{\frac{5}{2}}\sqrt{1+x}}$	28
orering	$-\frac{(-1+x)(2x^{3}-4x^{2}+x+2)}{5\sqrt{1+x}\,(1-x)^{\frac{7}{2}}}$	31
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^{3}-4x^{2}+x+2)}{5\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{2}\sqrt{-(1+x)(-1+x)}}$	54
default	$\frac{1}{5(1-x)^{\frac{5}{2}}\sqrt{1+x}}+\frac{1}{5(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{2}{5\sqrt{1-x}\,\sqrt{1+x}}-\frac{2\sqrt{1-x}}{5\sqrt{1+x}}$	58

input
```
int(1/(1-x)^(7/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/(1-x)^(5/2)/(1+x)^(1/2)*(2*x^3-4*x^2+x+2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = \frac{2\,x^4-4\,x^3-(2\,x^3-4\,x^2+x+2)\sqrt{x+1}\sqrt{-x+1}+4\,x-2}{5\,(x^4-2\,x^3+2\,x-1)}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(2*x^4 - 4*x^3 - (2*x^3 - 4*x^2 + x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*
x - 2)/(x^4 - 2*x^3 + 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.74 (sec) , antiderivative size = 284, normalized size of antiderivative = 4.98

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = \begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20} + \frac{10\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20} - \frac{15\sqrt{-1+\frac{2}{x+1}}(x+1)}{60x+5(x+1)^3-30(x+1)^2+20} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20} + \frac{10i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20} - \frac{15i\sqrt{1-\frac{2}{x+1}}(x+1)}{60x+5(x+1)^3-30(x+1)^2+20} \end{cases}$$

input
```
integrate(1/(1-x)**(7/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(60*x + 5*(x + 1)**3 - 30*(x
 + 1)**2 + 20) + 10*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(60*x + 5*(x + 1)**3 -
 30*(x + 1)**2 + 20) - 15*sqrt(-1 + 2/(x + 1))*(x + 1)/(60*x + 5*(x + 1)**
3 - 30*(x + 1)**2 + 20) + 5*sqrt(-1 + 2/(x + 1))/(60*x + 5*(x + 1)**3 - 30
*(x + 1)**2 + 20), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)*
*3/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 10*I*sqrt(1 - 2/(x + 1))*(
x + 1)**2/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) - 15*I*sqrt(1 - 2/(x
+ 1))*(x + 1)/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 5*I*sqrt(1 - 2/
(x + 1))/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = \frac{2\,x}{5\,\sqrt{-x^2+1}} + \frac{1}{5\,\left(\sqrt{-x^2+1}x^2-2\,\sqrt{-x^2+1}x+\sqrt{-x^2+1}\right)} - \frac{1}{5\,\left(\sqrt{-x^2+1}x-\sqrt{-x^2+1}\right)}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/5*x/sqrt(-x^2 + 1) + 1/5/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt
(-x^2 + 1)) - 1/5/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = \frac{\sqrt{2}-\sqrt{-x+1}}{16\,\sqrt{x+1}} - \frac{\sqrt{x+1}}{16\,\left(\sqrt{2}-\sqrt{-x+1}\right)} - \frac{((11\,x-39)(x+1)+60)\sqrt{x+1}\sqrt{-x+1}}{40\,(x-1)^3}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/16*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/16*sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)) - 1/40*((11*x - 39)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x
- 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = -\frac{x\sqrt{1-x}+2\sqrt{1-x}-4\,x^2\sqrt{1-x}+2\,x^3\sqrt{1-x}}{5\,(x-1)^3\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(7/2)*(x + 1)^(3/2)),x)
```

output
```
-(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^
(1/2))/(5*(x - 1)^3*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = \frac{2x^3-4x^2+x+2}{5\sqrt{x+1}\,\sqrt{1-x}\,(x^2-2x+1)}$$

input
```
int(1/(1-x)^(7/2)/(1+x)^(3/2),x)
```

output
```
(2*x**3 - 4*x**2 + x + 2)/(5*sqrt(x + 1)*sqrt( - x + 1)*(x**2 - 2*x + 1))
```

3.128 $\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{3/2}\sqrt{1+x}} + \frac{8x}{35\sqrt{1-x^2}}$$

output
```
1/7/(1-x)^(7/2)/(1+x)^(1/2)+4/35/(1-x)^(5/2)/(1+x)^(1/2)+4/35/(1-x)^(3/2)/
(1+x)^(1/2)+8/35*x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.52

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{13-4x-20x^2+24x^3-8x^4}{35(1-x)^{7/2}\sqrt{1+x}}$$

input
```
Integrate[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

output
```
(13 - 4*x - 20*x^2 + 24*x^3 - 8*x^4)/(35*(1 - x)^(7/2)*Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {55, 55, 55, 39, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{9/2}(x+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{7}\int \frac{1}{(1-x)^{7/2}(x+1)^{3/2}}dx+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{7}\left(\frac{3}{5}\int \frac{1}{(1-x)^{5/2}(x+1)^{3/2}}dx+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}\right)+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

$$\Big\downarrow \text{55}$$

$$\frac{4}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(x+1)^{3/2}}dx+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}\right)+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}\right)+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

$$\Big\downarrow \text{39}$$

$$\frac{4}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int \frac{1}{(1-x^2)^{3/2}}dx+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}\right)+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}\right)+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4}{7}\left(\frac{3}{5}\left(\frac{2x}{3\sqrt{1-x^2}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}\right)+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}\right)+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

input
```
Int[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

output
```
1/(7*(1 - x)^(7/2)*Sqrt[1 + x]) + (4*(1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + (3
*(1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x^2])))/5))/7
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{8x^{4}-24x^{3}+20x^{2}+4x-13}{35\left(1-x\right)^{\frac{7}{2}}\sqrt{1+x}}$	35
orering	$\frac{\left(-1+x\right)\left(8x^{4}-24x^{3}+20x^{2}+4x-13\right)}{35\sqrt{1+x}\,\left(1-x\right)^{\frac{9}{2}}}$	38
risch	$\frac{\sqrt{\left(1+x\right)\left(1-x\right)}\,\left(8x^{4}-24x^{3}+20x^{2}+4x-13\right)}{35\sqrt{1-x}\,\sqrt{1+x}\,\left(-1+x\right)^{3}\sqrt{-\left(1+x\right)\left(-1+x\right)}}$	61
default	$\frac{1}{7\left(1-x\right)^{\frac{7}{2}}\sqrt{1+x}}+\frac{4}{35\left(1-x\right)^{\frac{5}{2}}\sqrt{1+x}}+\frac{4}{35\left(1-x\right)^{\frac{3}{2}}\sqrt{1+x}}+\frac{8}{35\sqrt{1-x}\,\sqrt{1+x}}-\frac{8\sqrt{1-x}}{35\sqrt{1+x}}$	72

input
```
int(1/(1-x)^(9/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35/(1-x)^(7/2)/(1+x)^(1/2)*(8*x^4-24*x^3+20*x^2+4*x-13)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{13\,x^5 - 39\,x^4 + 26\,x^3 + 26\,x^2 - (8\,x^4 - 24\,x^3 + 20\,x^2 + 4\,x - 13)\sqrt{x+1}\sqrt{-x}}{35\,(x^5 - 3\,x^4 + 2\,x^3 + 2\,x^2 - 3\,x + 1)}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(13*x^5 - 39*x^4 + 26*x^3 + 26*x^2 - (8*x^4 - 24*x^3 + 20*x^2 + 4*x -
 13)*sqrt(x + 1)*sqrt(-x + 1) - 39*x + 13)/(x^5 - 3*x^4 + 2*x^3 + 2*x^2 -
3*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.37 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.52

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} + \frac{56\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} + \frac{56i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-} \end{cases}$$

input
```
integrate(1/(1-x)**(9/2)/(1+x)**(3/2),x)
```

output
```
Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 2
80*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*sqrt(-1 + 2/(x + 1))*(x + 1)**3
/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*s
qrt(-1 + 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 +
 840*(x + 1)**2 - 560) + 140*sqrt(-1 + 2/(x + 1))*(x + 1)/(-1120*x + 35*(x
 + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*sqrt(-1 + 2/(x + 1)
)/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), 1/Abs
(x + 1) > 1/2), (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)
**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*I*sqrt(1 - 2/(x + 1))*(x
 + 1)**3/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560)
 - 140*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x
+ 1)**3 + 840*(x + 1)**2 - 560) + 140*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-1120
*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*I*sqrt(1
- 2/(x + 1))/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 -
560), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(55) = 110$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{8\,x}{35\,\sqrt{-x^2+1}} - \frac{1}{7\left(\sqrt{-x^2+1}x^3 - 3\,\sqrt{-x^2+1}x^2 + 3\,\sqrt{-x^2+1}x - \sqrt{-x^2+1}\right)} + \frac{4}{35\left(\sqrt{-x^2+1}x^2 - 2\,\sqrt{-x^2+1}x + \sqrt{-x^2+1}\right)} - \frac{4}{35\left(\sqrt{-x^2+1}x - \sqrt{-x^2+1}\right)}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
8/35*x/sqrt(-x^2 + 1) - 1/7/(sqrt(-x^2 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3
*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4/35/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x
^2 + 1)*x + sqrt(-x^2 + 1)) - 4/35/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{\sqrt{2}-\sqrt{-x+1}}{32\sqrt{x+1}} - \frac{\sqrt{x+1}}{32\left(\sqrt{2}-\sqrt{-x+1}\right)} - \frac{(((93\,x-523)(x+1)+1400)(x+1)-1120)\sqrt{x+1}\sqrt{-x+1}}{560\,(x-1)^4}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/32*sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)) - 1/560*(((93*x - 523)*(x + 1) + 1400)*(x + 1) - 1120)*sqrt(x
+ 1)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = -\frac{4\,x\,\sqrt{1-x}-13\,\sqrt{1-x}+20\,x^2\,\sqrt{1-x}-24\,x^3\,\sqrt{1-x}+8\,x^4\,\sqrt{1-x}}{35\,(x-1)^4\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(9/2)*(x + 1)^(3/2)),x)
```

output
```
-(4*x*(1 - x)^(1/2) - 13*(1 - x)^(1/2) + 20*x^2*(1 - x)^(1/2) - 24*x^3*(1
- x)^(1/2) + 8*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{8x^4-24x^3+20x^2+4x-13}{35\sqrt{x+1}\,\sqrt{1-x}\,\left(x^3-3x^2+3x-1\right)}$$

input
```
int(1/(1-x)^(9/2)/(1+x)^(3/2),x)
```

output
```
(8*x**4 - 24*x**3 + 20*x**2 + 4*x - 13)/(35*sqrt(x + 1)*sqrt( - x + 1)*(x*
*3 - 3*x**2 + 3*x - 1))
```

3.129 $\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+\frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x}+7(1-x)^{5/2}\sqrt{1+x}+\frac{105\arcsin(x)}{2}$$

output
```
-2/3*(1-x)^(9/2)/(1+x)^(3/2)+6*(1-x)^(7/2)/(1+x)^(1/2)+105/2*(1-x)^(1/2)*(
1+x)^(1/2)+35/2*(1-x)^(3/2)*(1+x)^(1/2)+7*(1-x)^(5/2)*(1+x)^(1/2)+105/2*ar
csin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = \frac{\sqrt{1-x}(494+679x+102x^2-17x^3+2x^4)}{6(1+x)^{3/2}} - 105\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

output
```
(Sqrt[1 - x]*(494 + 679*x + 102*x^2 - 17*x^3 + 2*x^4))/(6*(1 + x)^(3/2)) -
 105*ArcTan[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {57, 57, 60, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{9/2}}{(x+1)^{5/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$-3\int \frac{(1-x)^{7/2}}{(x+1)^{3/2}}dx-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 57}$$

$$-3\left(-7\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}dx-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 60}$$

$$-3\left(-7\left(\frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}dx+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 60}$$

$$-3\left(-7\left(\frac{5}{3}\left(\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 50}$$

$$-3\left(-7\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$-3\left(-7\left(\frac{5}{3}\left(\frac{3}{2}\left(\arcsin(x)+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)+\frac{1}{3}\sqrt{x+1}(1-x)^{5/2}\right)-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}$$

input
```
Int[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

output
```
(-2*(1 - x)^(9/2))/(3*(1 + x)^(3/2)) - 3*((-2*(1 - x)^(7/2))/Sqrt[1 + x] -
 7*(((1 - x)^(5/2)*Sqrt[1 + x])/3 + (5*(((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3
*(Sqrt[1 - x^2] + ArcSin[x]))/2))/3))
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{\left(2 x^{5}-19 x^{4}+119 x^{3}+577 x^{2}-185 x -494\right) \sqrt{\left(1+x \right) \left(1-x \right)}}{6 \left(1+x \right)^{\frac{3}{2}} \sqrt{-\left(1+x \right) \left(-1+x \right)}\, \sqrt{1-x}}+\frac{105 \sqrt{\left(1+x \right) \left(1-x \right)}\, \arcsin \left(x \right)}{2 \sqrt{1+x}\, \sqrt{1-x}}$	89

input
```
int((1-x)^(9/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*x^5-19*x^4+119*x^3+577*x^2-185*x-494)/(1+x)^(3/2)/(-(1+x)*(-1+x))^
(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+105/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2
)/(1-x)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = \frac{494\,x^2 + (2\,x^4 - 17\,x^3 + 102\,x^2 + 679\,x + 494)\sqrt{x+1}\sqrt{-x+1} - 630\,(x^2 + 2\,x + 1)\,\text{ar}}{6\,(x^2 + 2\,x + 1)}$$

input
```
integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(494*x^2 + (2*x^4 - 17*x^3 + 102*x^2 + 679*x + 494)*sqrt(x + 1)*sqrt(-
x + 1) - 630*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 98
8*x + 494)/(x^2 + 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.07 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.41

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = \begin{cases} -105i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{29i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} + \frac{215i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{43i\sqrt{x+1}}{3\sqrt{x-1}} - \frac{448i}{3\sqrt{x-1}\sqrt{x+1}} + \frac{}{3\sqrt{}} \\ 105\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{29(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} - \frac{215(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{43\sqrt{x+1}}{3\sqrt{1-x}} + \frac{448}{3\sqrt{1-x}\sqrt{x+1}} - \frac{64}{3\sqrt{1-x}(x+} \end{cases}$$

input
```
integrate((1-x)**(9/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-105*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(
x - 1)) - 29*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 215*I*(x + 1)**(3/2)/(6*sq
rt(x - 1)) + 43*I*sqrt(x + 1)/(3*sqrt(x - 1)) - 448*I/(3*sqrt(x - 1)*sqrt(
x + 1)) + 64*I/(3*sqrt(x - 1)*(x + 1)**(3/2)), Abs(x + 1) > 2), (105*asin(
sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 29*(x + 1)**(5/2
)/(6*sqrt(1 - x)) - 215*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 43*sqrt(x + 1)/(3
*sqrt(1 - x)) + 448/(3*sqrt(1 - x)*sqrt(x + 1)) - 64/(3*sqrt(1 - x)*(x + 1
)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = \frac{x^6}{3\,(-x^2+1)^{\frac{3}{2}}} - \frac{7\,x^5}{2\,(-x^2+1)^{\frac{3}{2}}} + \frac{23\,x^4}{(-x^2+1)^{\frac{3}{2}}} + \frac{35}{2}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{143\,x}{6\,\sqrt{-x^2+1}} - \frac{127\,x^2}{(-x^2+1)^{\frac{3}{2}}} + \frac{22\,x}{3\,(-x^2+1)^{\frac{3}{2}}} + \frac{247}{3\,(-x^2+1)^{\frac{3}{2}}} + \frac{105}{2}\,\arcsin(x)$$

input
```
integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x^6/(-x^2 + 1)^(3/2) - 7/2*x^5/(-x^2 + 1)^(3/2) + 23*x^4/(-x^2 + 1)^(3
/2) + 35/2*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 143/6*x/sqrt(
-x^2 + 1) - 127*x^2/(-x^2 + 1)^(3/2) + 22/3*x/(-x^2 + 1)^(3/2) + 247/3/(-x
^2 + 1)^(3/2) + 105/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = \frac{1}{6}\left((2\,x-23)(x+1)+165\right)\sqrt{x+1}\sqrt{-x+1} + \frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{3\,(x+1)^{\frac{3}{2}}} - \frac{34\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} + \frac{2\,(x+1)^{\frac{3}{2}}\left(\frac{51\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^3} + 105\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/6*((2*x - 23)*(x + 1) + 165)*sqrt(x + 1)*sqrt(-x + 1) + 2/3*(sqrt(2) - s
qrt(-x + 1))^3/(x + 1)^(3/2) - 34*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 2
/3*(x + 1)^(3/2)*(51*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sq
rt(-x + 1))^3 + 105*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = \int \frac{(1-x)^{9/2}}{(x+1)^{5/2}}\,dx$$

input
```
int((1 - x)^(9/2)/(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(9/2)/(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = \frac{-630\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x - 630\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + 2\sqrt{1-x}\,x^4 - 17\sqrt{1-x}\,x^3 +}{6\sqrt{x+1}\,(x+1)}$$

input
```
int((1-x)^(9/2)/(1+x)^(5/2),x)
```

output
```
( - 630*sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2))*x - 630*sqrt(x + 1)*asin(
sqrt( - x + 1)/sqrt(2)) + 2*sqrt( - x + 1)*x**4 - 17*sqrt( - x + 1)*x**3 +
 102*sqrt( - x + 1)*x**2 + 679*sqrt( - x + 1)*x + 494*sqrt( - x + 1))/(6*s
qrt(x + 1)*(x + 1))
```

3.130 $\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}} dx = -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35\arcsin(x)}{2}$$

output
```
-2/3*(1-x)^(7/2)/(1+x)^(3/2)+14/3*(1-x)^(5/2)/(1+x)^(1/2)+35/2*(1-x)^(1/2)
*(1+x)^(1/2)+35/6*(1-x)^(3/2)*(1+x)^(1/2)+35/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}} dx = \frac{\sqrt{1-x}(164+229x+30x^2-3x^3)}{6(1+x)^{3/2}} - 35\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

output
```
(Sqrt[1 - x]*(164 + 229*x + 30*x^2 - 3*x^3))/(6*(1 + x)^(3/2)) - 35*ArcTan
[Sqrt[1 - x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {57, 57, 60, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{7/2}}{(x+1)^{5/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$-\frac{7}{3}\int \frac{(1-x)^{5/2}}{(x+1)^{3/2}}dx-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 57}$$

$$-\frac{7}{3}\left(-5\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}dx-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{7}{3}\left(-5\left(\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 50}$$

$$-\frac{7}{3}\left(-5\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{1-x^2}}dx+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{7}{3}\left(-5\left(\frac{3}{2}\left(\arcsin(x)+\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{x+1}(1-x)^{3/2}\right)-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}\right)-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}$$

input
```
Int[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

output
```
(-2*(1 - x)^(7/2))/(3*(1 + x)^(3/2)) - (7*((-2*(1 - x)^(5/2))/Sqrt[1 + x]
- 5*(((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*(Sqrt[1 - x^2] + ArcSin[x]))/2)))/
3
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{(3x^4-33x^3-199x^2+65x+164)\sqrt{(1+x)(1-x)}}{6(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\sqrt{1-x}}+\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	84

input
```
int((1-x)^(7/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*x^4-33*x^3-199*x^2+65*x+164)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+
x)*(1-x))^(1/2)/(1-x)^(1/2)+35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/
2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx = \frac{164\,x^2-(3\,x^3-30\,x^2-229\,x-164)\sqrt{x+1}\sqrt{-x+1}-210\,(x^2+2\,x+1)\arctan\left(\frac{\sqrt{x}}{}\right.}{6\,(x^2+2\,x+1)}$$

input
```
integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(164*x^2 - (3*x^3 - 30*x^2 - 229*x - 164)*sqrt(x + 1)*sqrt(-x + 1) - 2
10*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 328*x + 164)
/(x^2 + 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.44

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx = \begin{cases} -35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{15i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{41i\sqrt{x+1}}{3\sqrt{x-1}}-\frac{176i}{3\sqrt{x-1}\sqrt{x+1}}+\frac{32i}{3\sqrt{x-1}(x+1)^{\frac{3}{2}}} & \text{fo} \\ 35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{15(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{41\sqrt{x+1}}{3\sqrt{1-x}}+\frac{176}{3\sqrt{1-x}\sqrt{x+1}}-\frac{32}{3\sqrt{1-x}(x+1)^{\frac{3}{2}}} & \text{o} \end{cases}$$

input
```
integrate((1-x)**(7/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x
 - 1)) + 15*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 41*I*sqrt(x + 1)/(3*sqrt(x
- 1)) - 176*I/(3*sqrt(x - 1)*sqrt(x + 1)) + 32*I/(3*sqrt(x - 1)*(x + 1)**(
3/2)), Abs(x + 1) > 2), (35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(
2*sqrt(1 - x)) - 15*(x + 1)**(3/2)/(2*sqrt(1 - x)) - 41*sqrt(x + 1)/(3*sqr
t(1 - x)) + 176/(3*sqrt(1 - x)*sqrt(x + 1)) - 32/(3*sqrt(1 - x)*(x + 1)**(
3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx = -\frac{x^5}{2\,(-x^2+1)^{\frac{3}{2}}} + \frac{6\,x^4}{(-x^2+1)^{\frac{3}{2}}} + \frac{35}{6}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{61\,x}{6\,\sqrt{-x^2+1}} - \frac{44\,x^2}{(-x^2+1)^{\frac{3}{2}}} + \frac{16\,x}{3\,(-x^2+1)^{\frac{3}{2}}} + \frac{82}{3\,(-x^2+1)^{\frac{3}{2}}} + \frac{35}{2}\,\arcsin(x)$$

input
```
integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
-1/2*x^5/(-x^2 + 1)^(3/2) + 6*x^4/(-x^2 + 1)^(3/2) + 35/6*x*(3*x^2/(-x^2 +
 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 61/6*x/sqrt(-x^2 + 1) - 44*x^2/(-x^2 + 1
)^(3/2) + 16/3*x/(-x^2 + 1)^(3/2) + 82/3/(-x^2 + 1)^(3/2) + 35/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx = -\frac{1}{2}\sqrt{x+1}(x-12)\sqrt{-x+1} + \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{3\,(x+1)^{\frac{3}{2}}} - \frac{13\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} + \frac{(x+1)^{\frac{3}{2}}\left(\frac{39\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}} + 35\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(x + 1)*(x - 12)*sqrt(-x + 1) + 1/3*(sqrt(2) - sqrt(-x + 1))^3/(x
 + 1)^(3/2) - 13*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/3*(x + 1)^(3/2)*
(39*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 3
5*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx = \int \frac{(1-x)^{7/2}}{(x+1)^{5/2}}\,dx$$

input
```
int((1 - x)^(7/2)/(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(7/2)/(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx = \frac{-210\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x - 210\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - 3\sqrt{1-x}\,x^3 + 30\sqrt{1-x}\,x^2 + 2}{6\sqrt{x+1}\,(x+1)}$$

input
```
int((1-x)^(7/2)/(1+x)^(5/2),x)
```

output
```
( - 210*sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2))*x - 210*sqrt(x + 1)*asin(
sqrt( - x + 1)/sqrt(2)) - 3*sqrt( - x + 1)*x**3 + 30*sqrt( - x + 1)*x**2 +
 229*sqrt( - x + 1)*x + 164*sqrt( - x + 1))/(6*sqrt(x + 1)*(x + 1))
```

3.131 $\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\sqrt{1+x} + 5\arcsin(x)$$

output
```
-2/3*(1-x)^(5/2)/(1+x)^(3/2)+10/3*(1-x)^(3/2)/(1+x)^(1/2)+5*(1-x)^(1/2)*(1
+x)^(1/2)+5*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = \frac{\sqrt{1-x}(23+34x+3x^2)}{3(1+x)^{3/2}} - 10\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

output
```
(Sqrt[1 - x]*(23 + 34*x + 3*x^2))/(3*(1 + x)^(3/2)) - 10*ArcTan[Sqrt[1 - x
^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {57, 57, 50, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{5/2}}{(x+1)^{5/2}} \, dx$$

$$\Big\downarrow \text{57}$$

$$-\frac{5}{3} \int \frac{(1-x)^{3/2}}{(x+1)^{3/2}}dx - \frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$-\frac{5}{3} \left(-3 \int \frac{\sqrt{1-x}}{\sqrt{x+1}}dx - \frac{2(1-x)^{3/2}}{\sqrt{x+1}}\right) - \frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{50}$$

$$-\frac{5}{3} \left(-3 \left(\int \frac{1}{\sqrt{1-x^2}}dx + \sqrt{1-x^2}\right) - \frac{2(1-x)^{3/2}}{\sqrt{x+1}}\right) - \frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{5}{3} \left(-3 \left(\arcsin(x) + \sqrt{1-x^2}\right) - \frac{2(1-x)^{3/2}}{\sqrt{x+1}}\right) - \frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}$$

input
```
Int[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

output
```
(-2*(1 - x)^(5/2))/(3*(1 + x)^(3/2)) - (5*((-2*(1 - x)^(3/2))/Sqrt[1 + x]
- 3*(Sqrt[1 - x^2] + ArcSin[x])))/3
```

Defintions of rubi rules used

rule 50
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
*c + b*d*x^2)^m/(2*d*m), x] + Simp[a   Int[(a*c + b*d*x^2)^n, x], x] /; Fre
eQ[{a, b, c, d, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 1] && GtQ[m, 0
] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

method	result	size
risch	$-\frac{(3x^3+31x^2-11x-23)\sqrt{(1+x)(1-x)}}{3(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	79

input
```
int((1-x)^(5/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*x^3+31*x^2-11*x-23)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x)
)^(1/2)/(1-x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = \frac{23\,x^2 + (3\,x^2 + 34\,x + 23)\sqrt{x+1}\sqrt{-x+1} - 30\,(x^2 + 2\,x + 1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) \cdots}{3\,(x^2 + 2\,x + 1)}$$

input
```
integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(23*x^2 + (3*x^2 + 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) - 30*(x^2 + 2*x
 + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 46*x + 23)/(x^2 + 2*x + 1
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.57

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = \begin{cases} \sqrt{-1+\frac{2}{x+1}}(x+1) + \frac{28\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{8\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} + 5i\log\left(\frac{1}{x+1}\right) + 5i\log(x+1) + 10\,\mathrm{asin} \cdots \\ i\sqrt{1-\frac{2}{x+1}}(x+1) + \frac{28i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{8i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} + 5i\log\left(\frac{1}{x+1}\right) - 10i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) \cdots \end{cases}$$

input
```
integrate((1-x)**(5/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((sqrt(-1 + 2/(x + 1))*(x + 1) + 28*sqrt(-1 + 2/(x + 1))/3 - 8*sq
rt(-1 + 2/(x + 1))/(3*(x + 1)) + 5*I*log(1/(x + 1)) + 5*I*log(x + 1) + 10*
asin(sqrt(2)*sqrt(x + 1)/2), 1/Abs(x + 1) > 1/2), (I*sqrt(1 - 2/(x + 1))*(
x + 1) + 28*I*sqrt(1 - 2/(x + 1))/3 - 8*I*sqrt(1 - 2/(x + 1))/(3*(x + 1))
+ 5*I*log(1/(x + 1)) - 10*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = \frac{(-x^2+1)^{\frac{5}{2}}}{x^4+4\,x^3+6\,x^2+4\,x+1} - \frac{5\,(-x^2+1)^{\frac{3}{2}}}{3\,(x^3+3\,x^2+3\,x+1)} - \frac{10\,\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)} + \frac{35\,\sqrt{-x^2+1}}{3\,(x+1)} + 5\,\arcsin(x)$$

input
```
integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
(-x^2 + 1)^(5/2)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x
^3 + 3*x^2 + 3*x + 1) - 10/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 35/3*sqrt(-x
^2 + 1)/(x + 1) + 5*arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.83

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{6\,(x+1)^{\frac{3}{2}}} + \sqrt{x+1}\sqrt{-x+1} - \frac{9\,\left(\sqrt{2}-\sqrt{-x+1}\right)}{2\,\sqrt{x+1}} + \frac{(x+1)^{\frac{3}{2}}\left(\frac{27\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{6\,\left(\sqrt{2}-\sqrt{-x+1}\right)^3} + 10\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + sqrt(x + 1)*sqrt(-x + 1) -
9/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/6*(x + 1)^(3/2)*(27*(sqrt(2)
- sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 10*arcsin(1/2*
sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = \int \frac{(1-x)^{5/2}}{(x+1)^{5/2}}\,dx$$

input
```
int((1 - x)^(5/2)/(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(5/2)/(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx = \frac{-30\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x - 30\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + 3\sqrt{1-x}\,x^2 + 34\sqrt{1-x}\,x + 23\sqrt{}}{3\sqrt{x+1}\,(x+1)}$$

input
```
int((1-x)^(5/2)/(1+x)^(5/2),x)
```

output
```
( - 30*sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2))*x - 30*sqrt(x + 1)*asin(sq
rt( - x + 1)/sqrt(2)) + 3*sqrt( - x + 1)*x**2 + 34*sqrt( - x + 1)*x + 23*s
qrt( - x + 1))/(3*sqrt(x + 1)*(x + 1))
```

3.132 $\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}} \, dx = -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \arcsin(x)$$

output
```
-2/3*(1-x)^(3/2)/(1+x)^(3/2)+2*(1-x)^(1/2)/(1+x)^(1/2)+arcsin(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}} \, dx = \frac{4\sqrt{1-x}(1+2x)}{3(1+x)^{3/2}} - 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

output
```
(4*Sqrt[1 - x]*(1 + 2*x))/(3*(1 + x)^(3/2)) - 2*ArcTan[Sqrt[1 - x^2]/(-1 +
 x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 57, 39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}}{(x+1)^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$-\int \frac{\sqrt{1-x}}{(x+1)^{3/2}}dx-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}dx-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{39}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

$$\Big\downarrow \text{223}$$

$$\arcsin(x)-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

input
```
Int[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

output
```
(-2*(1 - x)^(3/2))/(3*(1 + x)^(3/2)) + (2*Sqrt[1 - x])/Sqrt[1 + x] + ArcSi
n[x]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

method	result	size
risch	$-\frac{4(2x^2-x-1)\sqrt{(1+x)(1-x)}}{3(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	73

input
```
int((1-x)^(3/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(2*x^2-x-1)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-
x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(31) = 62.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = \frac{2\left(2\,x^2+2\,(2\,x+1)\sqrt{x+1}\sqrt{-x+1}-3\,(x^2+2\,x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+4\,x+\right.}{3\,(x^2+2\,x+1)}$$

input
```
integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(2*x^2 + 2*(2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) - 3*(x^2 + 2*x + 1)*arct
an((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.12

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = \begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{4\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) + i\log(x+1) + 2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ \frac{8i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{4i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) - 2i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(3/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((8*sqrt(-1 + 2/(x + 1))/3 - 4*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
 I*log(1/(x + 1)) + I*log(x + 1) + 2*asin(sqrt(2)*sqrt(x + 1)/2), 1/Abs(x
+ 1) > 1/2), (8*I*sqrt(1 - 2/(x + 1))/3 - 4*I*sqrt(1 - 2/(x + 1))/(3*(x +
1)) + I*log(1/(x + 1)) - 2*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = -\frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^3+3\,x^2+3\,x+1)} - \frac{2\,\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)} + \frac{7\,\sqrt{-x^2+1}}{3\,(x+1)} + \arcsin{(x)}$$

input
```
integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 + 1)^(3/2)/(x^3 + 3*x^2 + 3*x + 1) - 2/3*sqrt(-x^2 + 1)/(x^2 +
2*x + 1) + 7/3*sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(31) = 62$.

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{12\,(x+1)^{\frac{3}{2}}} - \frac{5\left(\sqrt{2}-\sqrt{-x+1}\right)}{4\,\sqrt{x+1}} + \frac{(x+1)^{\frac{3}{2}}\left(\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{12\left(\sqrt{2}-\sqrt{-x+1}\right)^3} + 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)$$

input
```
integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/12*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 5/4*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) + 1/12*(x + 1)^(3/2)*(15*(sqrt(2) - sqrt(-x + 1))^2/(x + 1)
 - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = \int \frac{(1-x)^{3/2}}{(x+1)^{5/2}}\,dx$$

input
```
int((1 - x)^(3/2)/(x + 1)^(5/2),x)
```

output
```
int((1 - x)^(3/2)/(x + 1)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = \frac{-2\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)x-2\sqrt{x+1}\,asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+\frac{8\sqrt{1-x}\,x}{3}+\frac{4\sqrt{1-x}}{3}}{\sqrt{x+1}\,(x+1)}$$

input
```
int((1-x)^(3/2)/(1+x)^(5/2),x)
```

output
```
(2*( - 3*sqrt(x + 1)*asin(sqrt( - x + 1)/sqrt(2))*x - 3*sqrt(x + 1)*asin(s
qrt( - x + 1)/sqrt(2)) + 4*sqrt( - x + 1)*x + 2*sqrt( - x + 1)))/(3*sqrt(x
 + 1)*(x + 1))
```

3.133 $\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = -\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

output
```
-1/3*(1-x)^(3/2)/(1+x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = -\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

input
```
Integrate[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

output
```
-1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{(x+1)^{5/2}}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

input
```
Int[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

output
```
-1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{(1-x)^{\frac{3}{2}}}{3(1+x)^{\frac{3}{2}}}$	15
orering	$\frac{(-1+x)\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}$	18
default	$-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{1+x}}$	30
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^2-2x+1)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}}$	44

input
```
int((1-x)^(1/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(1-x)^(3/2)/(1+x)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = -\frac{x^2-\sqrt{x+1}(x-1)\sqrt{-x+1}+2\,x+1}{3\,(x^2+2\,x+1)}$$

input
```
integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(x^2 - sqrt(x + 1)*(x - 1)*sqrt(-x + 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = \begin{cases} \frac{\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{2\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ \frac{i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{2i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x)**(1/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((sqrt(-1 + 2/(x + 1))/3 - 2*sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 1/
Abs(x + 1) > 1/2), (I*sqrt(1 - 2/(x + 1))/3 - 2*I*sqrt(1 - 2/(x + 1))/(3*(
x + 1)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = -\frac{2\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)} + \frac{\sqrt{-x^2+1}}{3\,(x+1)}$$

input
```
integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.45

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{24\,(x+1)^{\frac{3}{2}}} - \frac{\sqrt{2}-\sqrt{-x+1}}{8\,\sqrt{x+1}} + \frac{(x+1)^{\frac{3}{2}}\left(\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{24\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

input
```
integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/24*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 1/8*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) + 1/24*(x + 1)^(3/2)*(3*(sqrt(2) - sqrt(-x + 1))^2/(x + 1)
- 1)/(sqrt(2) - sqrt(-x + 1))^3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = \frac{x\sqrt{1-x}-\sqrt{1-x}}{(3\,x+3)\,\sqrt{x+1}}$$

input
```
int((1 - x)^(1/2)/(x + 1)^(5/2),x)
```

output
```
(x*(1 - x)^(1/2) - (1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = \frac{\sqrt{1-x}\,(x-1)}{3\sqrt{x+1}\,(x+1)}$$

input
```
int((1-x)^(1/2)/(1+x)^(5/2),x)
```

output
```
(sqrt( - x + 1)*(x - 1))/(3*sqrt(x + 1)*(x + 1))
```

3.134 $\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}} \, dx = -\frac{\sqrt{1-x}}{3(1+x)^{3/2}} - \frac{\sqrt{1-x}}{3\sqrt{1+x}}$$

output
```
-1/3*(1-x)^(1/2)/(1+x)^(3/2)-1/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}} \, dx = \frac{(-2-x)\sqrt{1-x}}{3(1+x)^{3/2}}$$

input
```
Integrate[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

output
```
((-2 - x)*Sqrt[1 - x])/(3*(1 + x)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{55, 48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}(x+1)^{5/2}} \, dx$$

$$\Big\downarrow \; 55$$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-x}(x+1)^{3/2}}dx-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

input
```
Int[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

output
```
-1/3*Sqrt[1 - x]/(1 + x)^(3/2) - Sqrt[1 - x]/(3*Sqrt[1 + x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(2+x)\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}$	18
orering	$\frac{(-1+x)(2+x)}{3(1+x)^{\frac{3}{2}}\sqrt{1-x}}$	21
default	$-\frac{\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}$	30
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^2+x-2)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}}$	42

input
```
int(1/(1-x)^(1/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2+x)*(1-x)^(1/2)/(1+x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}}\,dx = -\frac{2\,x^2 + (x+2)\sqrt{x+1}\sqrt{-x+1} + 4\,x + 2}{3\,(x^2 + 2\,x + 1)}$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*x^2 + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}}\,dx = \begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-sqrt(-1 + 2/(x + 1))/3 - sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 1/A
bs(x + 1) > 1/2), (-I*sqrt(1 - 2/(x + 1))/3 - I*sqrt(1 - 2/(x + 1))/(3*(x
+ 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}}\,dx = -\frac{\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)} - \frac{\sqrt{-x^2+1}}{3\,(x+1)}$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{48\,(x+1)^{\frac{3}{2}}} + \frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)}{16\sqrt{x+1}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{48\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

input
```
integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/48*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 3/16*(sqrt(2) - sqrt(-x +
1))/sqrt(x + 1) - 1/48*(x + 1)^(3/2)*(9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1)
 + 1)/(sqrt(2) - sqrt(-x + 1))^3
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}}\,dx = -\frac{x\,\sqrt{1-x}+2\,\sqrt{1-x}}{(3\,x+3)\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(1/2)*(x + 1)^(5/2)),x)
```

output
```
-(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}}\,dx = \frac{\sqrt{1-x}\,(-x-2)}{3\sqrt{x+1}\,(x+1)}$$

input
```
int(1/(1-x)^(1/2)/(1+x)^(5/2),x)
```

output
```
(sqrt( - x + 1)*( - x - 2))/(3*sqrt(x + 1)*(x + 1))
```

3.135 $\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx=-\frac{1}{3\sqrt{1-x}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x^2}}$$

output
```
-1/3/(1-x)^(1/2)/(1+x)^(3/2)+2/3*x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx=\frac{-1+2x+2x^2}{3\sqrt{1-x}(1+x)^{3/2}}$$

input
```
Integrate[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

output
```
(-1 + 2*x + 2*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{3/2}(x+1)^{5/2}}dx$$
$$\Big\downarrow \ 55$$
$$2\int \frac{1}{\sqrt{1-x}(x+1)^{5/2}}dx+\frac{1}{\sqrt{1-x}(x+1)^{3/2}}$$
$$\Big\downarrow \ 55$$
$$2\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x}(x+1)^{3/2}}dx-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}\right)+\frac{1}{\sqrt{1-x}(x+1)^{3/2}}$$
$$\Big\downarrow \ 48$$
$$2\left(-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}\right)+\frac{1}{\sqrt{1-x}(x+1)^{3/2}}$$

input
```
Int[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

output
```
1/(Sqrt[1 - x]*(1 + x)^(3/2)) + 2*(-1/3*Sqrt[1 - x]/(1 + x)^(3/2) - Sqrt[1
 - x]/(3*Sqrt[1 + x]))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{2x^2+2x-1}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}$	25
orering	$-\frac{(-1+x)(2x^2+2x-1)}{3(1+x)^{\frac{3}{2}}(1-x)^{\frac{3}{2}}}$	28
default	$\frac{1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	43
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^2+2x-1)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}}$	46

input
```
int(1/(1-x)^(3/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(1-x)^(1/2)/(1+x)^(3/2)*(2*x^2+2*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = -\frac{x^3+x^2+(2\,x^2+2\,x-1)\sqrt{x+1}\sqrt{-x+1}-x-1}{3\,(x^3+x^2-x-1)}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(x^3 + x^2 + (2*x^2 + 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x - 1)/(x^3
 + x^2 - x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 167, normalized size of antiderivative = 4.51

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = \begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6} + \frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6} + \frac{\sqrt{-1+\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6} + \frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6} + \frac{i\sqrt{1-\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(3/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) +
2*sqrt(-1 + 2/(x + 1))*(x + 1)/(-6*x + 3*(x + 1)**2 - 6) + sqrt(-1 + 2/(x
+ 1))/(-6*x + 3*(x + 1)**2 - 6), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x
+ 1))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2*I*sqrt(1 - 2/(x + 1))*(x +
1)/(-6*x + 3*(x + 1)**2 - 6) + I*sqrt(1 - 2/(x + 1))/(-6*x + 3*(x + 1)**2
- 6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{-x^2+1}} - \frac{1}{3\,\left(\sqrt{-x^2+1}x + \sqrt{-x^2+1}\right)}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{96\,(x+1)^{\frac{3}{2}}} + \frac{7\left(\sqrt{2}-\sqrt{-x+1}\right)}{32\,\sqrt{x+1}} - \frac{\sqrt{x+1}\sqrt{-x+1}}{4\,(x-1)} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{21\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{96\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

input
```
integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/96*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 7/32*(sqrt(2) - sqrt(-x +
1))/sqrt(x + 1) - 1/4*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 1/96*(x + 1)^(3/2
)*(21*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = -\frac{2\,x\,\sqrt{1-x}-\sqrt{1-x}+2\,x^2\,\sqrt{1-x}}{\left(3\,x^2-3\right)\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(3/2)*(x + 1)^(5/2)),x)
```

output
```
-(2*x*(1 - x)^(1/2) - (1 - x)^(1/2) + 2*x^2*(1 - x)^(1/2))/((3*x^2 - 3)*(x
 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = \frac{2x^2+2x-1}{3\sqrt{x+1}\,\sqrt{1-x}\,(x+1)}$$

input
```
int(1/(1-x)^(3/2)/(1+x)^(5/2),x)
```

output
```
(2*x**2 + 2*x - 1)/(3*sqrt(x + 1)*sqrt( - x + 1)*(x + 1))
```

3.136 $\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx=\frac{x}{3\left(1-x^2\right)^{3/2}}+\frac{2x}{3\sqrt{1-x^2}}$$

output
```
1/3*x/(-x^2+1)^(3/2)+2/3*x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx=\frac{3x-2x^3}{3\left(1-x^2\right)^{3/2}}$$

input
```
Integrate[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

output
```
(3*x - 2*x^3)/(3*(1 - x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {39, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{5/2}(x+1)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 39}$$
$$\int \frac{1}{(1-x^2)^{5/2}}dx$$
$$\Big\downarrow \text{ 209}$$
$$\frac{2}{3}\int \frac{1}{(1-x^2)^{3/2}}dx+\frac{x}{3\,(1-x^2)^{3/2}}$$
$$\Big\downarrow \text{ 208}$$
$$\frac{2x}{3\sqrt{1-x^2}}+\frac{x}{3\,(1-x^2)^{3/2}}$$

input
```
Int[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

output
```
x/(3*(1 - x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - x^2])
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{x(2x^2-3)}{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}$	23
orering	$\frac{(-1+x)x(2x^2-3)}{3(1+x)^{\frac{3}{2}}(1-x)^{\frac{5}{2}}}$	26
default	$\frac{1}{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	57

input
```
int(1/(1-x)^(5/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(2*x^2-3)/(1-x)^(3/2)/(1+x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx = -\frac{(2\,x^3-3\,x)\sqrt{x+1}\sqrt{-x+1}}{3\,(x^4-2\,x^2+1)}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*x^3 - 3*x)*sqrt(x + 1)*sqrt(-x + 1)/(x^4 - 2*x^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 280, normalized size of antiderivative = 8.48

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx = \begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12} + \frac{6\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12} - \frac{3\sqrt{-1+\frac{2}{x+1}}(x+1)}{12x+3(x+1)^3-12(x+1)^2+12} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12} + \frac{6i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12} - \frac{3i\sqrt{1-\frac{2}{x+1}}(x+1)}{12x+3(x+1)^3-12(x+1)^2+12} \end{cases}$$

input
```
integrate(1/(1-x)**(5/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(12*x + 3*(x + 1)**3 - 12*(x
 + 1)**2 + 12) + 6*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(12*x + 3*(x + 1)**3 -
12*(x + 1)**2 + 12) - 3*sqrt(-1 + 2/(x + 1))*(x + 1)/(12*x + 3*(x + 1)**3
- 12*(x + 1)**2 + 12) - sqrt(-1 + 2/(x + 1))/(12*x + 3*(x + 1)**3 - 12*(x
+ 1)**2 + 12), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(
12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - 3*I*sqrt(1 - 2/(x + 1))*
(x + 1)/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - I*sqrt(1 - 2/(x + 1))
/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{-x^2+1}} + \frac{x}{3\,(-x^2+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/sqrt(-x^2 + 1) + 1/3*x/(-x^2 + 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(25) = 50.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.42

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{192\,(x+1)^{\frac{3}{2}}}+\frac{11\left(\sqrt{2}-\sqrt{-x+1}\right)}{64\sqrt{x+1}} - \frac{(4\,x-5)\sqrt{x+1}\sqrt{-x+1}}{12\,(x-1)^{2}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{192\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

input
```
integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/192*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 11/64*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/12*(4*x - 5)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 - 1/
192*(x + 1)^(3/2)*(33*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - s
qrt(-x + 1))^3
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx = \frac{3\,x\,\sqrt{1-x}-2\,x^{3}\,\sqrt{1-x}}{(3\,x+3)\,\left(x-1\right)^{2}\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(5/2)*(x + 1)^(5/2)),x)
```

output
```
(3*x*(1 - x)^(1/2) - 2*x^3*(1 - x)^(1/2))/((3*x + 3)*(x - 1)^2*(x + 1)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx = \frac{x(2x^2-3)}{3\sqrt{x+1}\,\sqrt{1-x}\,(x^2-1)}$$

input
```
int(1/(1-x)^(5/2)/(1+x)^(5/2),x)
```

output
```
(x*(2*x**2 - 3))/(3*sqrt(x + 1)*sqrt( - x + 1)*(x**2 - 1))
```

3.137 $\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{15\,(1-x^2)^{3/2}} + \frac{8x}{15\sqrt{1-x^2}}$$

output
```
1/5/(1-x)^(5/2)/(1+x)^(3/2)+4/15*x/(-x^2+1)^(3/2)+8/15*x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \frac{3+12x-12x^2-8x^3+8x^4}{15(1-x)^{5/2}(1+x)^{3/2}}$$

input
```
Integrate[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

output
```
(3 + 12*x - 12*x^2 - 8*x^3 + 8*x^4)/(15*(1 - x)^(5/2)*(1 + x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 39, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{7/2}(x+1)^{5/2}}dx$$

$$\Big\downarrow \; 55$$

$$\frac{4}{5}\int \frac{1}{(1-x)^{5/2}(x+1)^{5/2}}dx+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 39$$

$$\frac{4}{5}\int \frac{1}{(1-x^2)^{5/2}}dx+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-x^2)^{3/2}}dx+\frac{x}{3(1-x^2)^{3/2}}\right)+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{4}{5}\left(\frac{2x}{3\sqrt{1-x^2}}+\frac{x}{3(1-x^2)^{3/2}}\right)+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

input
```
Int[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

output
```
1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*(x/(3*(1 - x^2)^(3/2)) + (2*x)/(3*S
qrt[1 - x^2])))/5
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{8x^4-8x^3-12x^2+12x+3}{15(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}$	35
orering	$-\frac{(-1+x)(8x^4-8x^3-12x^2+12x+3)}{15(1+x)^{\frac{3}{2}}(1-x)^{\frac{7}{2}}}$	38
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^4-8x^3-12x^2+12x+3)}{15\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\,(-1+x)^2}$	61
default	$\frac{1}{5(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{15(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{15(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{15\sqrt{1+x}}$	72

input
```
int(1/(1-x)^(7/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/(1-x)^(5/2)/(1+x)^(3/2)*(8*x^4-8*x^3-12*x^2+12*x+3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(39) = 78$.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \frac{3\,x^5 - 3\,x^4 - 6\,x^3 + 6\,x^2 - (8\,x^4 - 8\,x^3 - 12\,x^2 + 12\,x + 3)\sqrt{x+1}\sqrt{-x+1} -}{15\,(x^5 - x^4 - 2\,x^3 + 2\,x^2 + x - 1)}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*x^5 - 3*x^4 - 6*x^3 + 6*x^2 - (8*x^4 - 8*x^3 - 12*x^2 + 12*x + 3)*
sqrt(x + 1)*sqrt(-x + 1) + 3*x - 3)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.22 (sec) , antiderivative size = 425, normalized size of antiderivative = 8.02

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} \end{cases}$$

input
```
integrate(1/(1-x)**(7/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90
*(x + 1)**3 + 180*(x + 1)**2 - 120) + 40*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(
-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*sqrt(-
1 + 2/(x + 1))*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x
 + 1)**2 - 120) + 20*sqrt(-1 + 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4
- 90*(x + 1)**3 + 180*(x + 1)**2 - 120) + 5*sqrt(-1 + 2/(x + 1))/(-120*x +
 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120), 1/Abs(x + 1) > 1/2
), (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*(x +
1)**3 + 180*(x + 1)**2 - 120) + 40*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-120*
x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*I*sqrt(1 -
2/(x + 1))*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1
)**2 - 120) + 20*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 9
0*(x + 1)**3 + 180*(x + 1)**2 - 120) + 5*I*sqrt(1 - 2/(x + 1))/(-120*x + 1
5*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \frac{8\,x}{15\,\sqrt{-x^2+1}} + \frac{4\,x}{15\,(-x^2+1)^{\frac{3}{2}}} - \frac{1}{5\left((-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
8/15*x/sqrt(-x^2 + 1) + 4/15*x/(-x^2 + 1)^(3/2) - 1/5/((-x^2 + 1)^(3/2)*x
- (-x^2 + 1)^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(39) = 78$.

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{384\,(x+1)^{\frac{3}{2}}} + \frac{15\,\left(\sqrt{2}-\sqrt{-x+1}\right)}{128\,\sqrt{x+1}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{45\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{384\,\left(\sqrt{2}-\sqrt{-x+1}\right)^3} - \frac{((73\,x-247)(x+1)+360)\sqrt{x+1}\sqrt{-x+1}}{240\,(x-1)^3}$$

input
```
integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/384*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 15/128*(sqrt(2) - sqrt(-x
 + 1))/sqrt(x + 1) - 1/384*(x + 1)^(3/2)*(45*(sqrt(2) - sqrt(-x + 1))^2/(x
 + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/240*((73*x - 247)*(x + 1) + 360)
*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = -\frac{12\,x\,\sqrt{1-x}+3\,\sqrt{1-x}-12\,x^2\,\sqrt{1-x}-8\,x^3\,\sqrt{1-x}+8\,x^4\,\sqrt{1-x}}{(15\,x+15)\,\left(x-1\right)^3\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(7/2)*(x + 1)^(5/2)),x)
```

output
```
-(12*x*(1 - x)^(1/2) + 3*(1 - x)^(1/2) - 12*x^2*(1 - x)^(1/2) - 8*x^3*(1 -
 x)^(1/2) + 8*x^4*(1 - x)^(1/2))/((15*x + 15)*(x - 1)^3*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \frac{8x^4-8x^3-12x^2+12x+3}{15\sqrt{x+1}\,\sqrt{1-x}\,\left(x^3-x^2-x+1\right)}$$

input
```
int(1/(1-x)^(7/2)/(1+x)^(5/2),x)
```

output
```
(8*x**4 - 8*x**3 - 12*x**2 + 12*x + 3)/(15*sqrt(x + 1)*sqrt( - x + 1)*(x**
3 - x**2 - x + 1))
```

3.138 $\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{1}{7(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{21\,(1-x^2)^{3/2}} + \frac{8x}{21\sqrt{1-x^2}}$$

output
```
1/7/(1-x)^(7/2)/(1+x)^(3/2)+1/7/(1-x)^(5/2)/(1+x)^(3/2)+4/21*x/(-x^2+1)^(3
/2)+8/21*x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{6+9x-24x^2+4x^3+16x^4-8x^5}{21(1-x)^{7/2}(1+x)^{3/2}}$$

input
```
Integrate[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

output
```
(6 + 9*x - 24*x^2 + 4*x^3 + 16*x^4 - 8*x^5)/(21*(1 - x)^(7/2)*(1 + x)^(3/2
))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {55, 55, 39, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{9/2}(x+1)^{5/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{5}{7}\int \frac{1}{(1-x)^{7/2}(x+1)^{5/2}}dx+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{5}{7}\left(\frac{4}{5}\int \frac{1}{(1-x)^{5/2}(x+1)^{5/2}}dx+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{39}$$

$$\frac{5}{7}\left(\frac{4}{5}\int \frac{1}{(1-x^2)^{5/2}}dx+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{5}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-x^2)^{3/2}}dx+\frac{x}{3(1-x^2)^{3/2}}\right)+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{5}{7}\left(\frac{4}{5}\left(\frac{2x}{3\sqrt{1-x^2}}+\frac{x}{3(1-x^2)^{3/2}}\right)+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

input
```
Int[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

output
```
1/(7*(1 - x)^(7/2)*(1 + x)^(3/2)) + (5*(1/(5*(1 - x)^(5/2)*(1 + x)^(3/2))
+ (4*(x/(3*(1 - x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - x^2])))/5))/7
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{8x^{5}-16x^{4}-4x^{3}+24x^{2}-9x-6}{21\left(1-x\right)^{\frac{7}{2}}\left(1+x\right)^{\frac{3}{2}}}$	40
orering	$\frac{\left(-1+x\right)\left(8x^{5}-16x^{4}-4x^{3}+24x^{2}-9x-6\right)}{21\left(1+x\right)^{\frac{3}{2}}\left(1-x\right)^{\frac{9}{2}}}$	43
risch	$\frac{\sqrt{\left(1+x\right)\left(1-x\right)}\,\left(8x^{5}-16x^{4}-4x^{3}+24x^{2}-9x-6\right)}{21\sqrt{1-x}\,\left(1+x\right)^{\frac{3}{2}}\sqrt{-\left(1+x\right)\left(-1+x\right)}\,\left(-1+x\right)^{3}}$	66
default	$\frac{1}{7\left(1-x\right)^{\frac{7}{2}}\left(1+x\right)^{\frac{3}{2}}}+\frac{1}{7\left(1-x\right)^{\frac{5}{2}}\left(1+x\right)^{\frac{3}{2}}}+\frac{4}{21\left(1-x\right)^{\frac{3}{2}}\left(1+x\right)^{\frac{3}{2}}}+\frac{4}{7\sqrt{1-x}\,\left(1+x\right)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{21\left(1+x\right)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{21\sqrt{1+x}}$	86

input
```
int(1/(1-x)^(9/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21/(1-x)^(7/2)/(1+x)^(3/2)*(8*x^5-16*x^4-4*x^3+24*x^2-9*x-6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{6\,x^6-12\,x^5-6\,x^4+24\,x^3-6\,x^2-(8\,x^5-16\,x^4-4\,x^3+24\,x^2-9\,x-6)\sqrt{}}{21\,(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1)}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/21*(6*x^6 - 12*x^5 - 6*x^4 + 24*x^3 - 6*x^2 - (8*x^5 - 16*x^4 - 4*x^3 +
24*x^2 - 9*x - 6)*sqrt(x + 1)*sqrt(-x + 1) - 12*x + 6)/(x^6 - 2*x^5 - x^4
+ 4*x^3 - x^2 - 2*x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 36.76 (sec) , antiderivative size = 593, normalized size of antiderivative = 8.12

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-x)**(9/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(336*x + 21*(x + 1)**5 - 168
*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) + 56*sqrt(-1 + 2/(x +
 1))*(x + 1)**4/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 -
 672*(x + 1)**2 + 336) - 140*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(336*x + 21*(
x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) + 140*
sqrt(-1 + 2/(x + 1))*(x + 1)**2/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 +
504*(x + 1)**3 - 672*(x + 1)**2 + 336) - 35*sqrt(-1 + 2/(x + 1))*(x + 1)/(
336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 +
 336) - 7*sqrt(-1 + 2/(x + 1))/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 5
04*(x + 1)**3 - 672*(x + 1)**2 + 336), 1/Abs(x + 1) > 1/2), (-8*I*sqrt(1 -
 2/(x + 1))*(x + 1)**5/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x +
1)**3 - 672*(x + 1)**2 + 336) + 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(336*x
 + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336)
 - 140*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(336*x + 21*(x + 1)**5 - 168*(x +
1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) + 140*I*sqrt(1 - 2/(x + 1))
*(x + 1)**2/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672
*(x + 1)**2 + 336) - 35*I*sqrt(1 - 2/(x + 1))*(x + 1)/(336*x + 21*(x + 1)*
*5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) - 7*I*sqrt(1
- 2/(x + 1))/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 67
2*(x + 1)**2 + 336), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{8\,x}{21\,\sqrt{-x^2+1}} + \frac{4\,x}{21\,(-x^2+1)^{\frac{3}{2}}} + \frac{1}{7\left((-x^2+1)^{\frac{3}{2}}x^2 - 2\,(-x^2+1)^{\frac{3}{2}}x + (-x^2+1)^{\frac{3}{2}}\right)} - \frac{1}{7\left((-x^2+1)^{\frac{3}{2}}x - (-x^2+1)^{\frac{3}{2}}\right)}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
8/21*x/sqrt(-x^2 + 1) + 4/21*x/(-x^2 + 1)^(3/2) + 1/7/((-x^2 + 1)^(3/2)*x^
2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 1/7/((-x^2 + 1)^(3/2)*x - (
-x^2 + 1)^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(53) = 106$.

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.71

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{768\,(x+1)^{\frac{3}{2}}} + \frac{19\left(\sqrt{2}-\sqrt{-x+1}\right)}{256\,\sqrt{x+1}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{57\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{768\left(\sqrt{2}-\sqrt{-x+1}\right)^3} - \frac{(((79\,x-432)(x+1)+1120)(x+1)-840)\sqrt{x+1}\sqrt{-x+1}}{336\,(x-1)^4}$$

input
```
integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/768*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 19/256*(sqrt(2) - sqrt(-x
 + 1))/sqrt(x + 1) - 1/768*(x + 1)^(3/2)*(57*(sqrt(2) - sqrt(-x + 1))^2/(x
 + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/336*(((79*x - 432)*(x + 1) + 112
0)*(x + 1) - 840)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{9\,x\,\sqrt{1-x}+6\,\sqrt{1-x}-24\,x^2\,\sqrt{1-x}+4\,x^3\,\sqrt{1-x}+16\,x^4\,\sqrt{1-x}-8\,x^5}{(21\,x+21)\,(x-1)^4\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(9/2)*(x + 1)^(5/2)),x)
```

output
```
(9*x*(1 - x)^(1/2) + 6*(1 - x)^(1/2) - 24*x^2*(1 - x)^(1/2) + 4*x^3*(1 - x
)^(1/2) + 16*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/((21*x + 21)*(x - 1)
^4*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{8x^5-16x^4-4x^3+24x^2-9x-6}{21\sqrt{x+1}\,\sqrt{1-x}\,\left(x^4-2x^3+2x-1\right)}$$

input
```
int(1/(1-x)^(9/2)/(1+x)^(5/2),x)
```

output
```
(8*x**5 - 16*x**4 - 4*x**3 + 24*x**2 - 9*x - 6)/(21*sqrt(x + 1)*sqrt( - x
+ 1)*(x**4 - 2*x**3 + 2*x - 1))
```

3.139 $\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8x}{63\left(1-x^2\right)^{3/2}}+\frac{16x}{63\sqrt{1-x^2}}$$

output
```
1/9/(1-x)^(9/2)/(1+x)^(3/2)+2/21/(1-x)^(7/2)/(1+x)^(3/2)+2/21/(1-x)^(5/2)/
(1+x)^(3/2)+8/63*x/(-x^2+1)^(3/2)+16/63*x/(-x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \frac{19+6x-66x^2+56x^3+24x^4-48x^5+16x^6}{63(1-x)^{9/2}(1+x)^{3/2}}$$

input
```
Integrate[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

output
```
(19 + 6*x - 66*x^2 + 56*x^3 + 24*x^4 - 48*x^5 + 16*x^6)/(63*(1 - x)^(9/2)*
(1 + x)^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {55, 55, 55, 39, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{11/2}(x+1)^{5/2}}dx$$

$$\Big\downarrow \; 55$$

$$\frac{2}{3}\int \frac{1}{(1-x)^{9/2}(x+1)^{5/2}}dx+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 55$$

$$\frac{2}{3}\left(\frac{5}{7}\int \frac{1}{(1-x)^{7/2}(x+1)^{5/2}}dx+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}\right)+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 55$$

$$\frac{2}{3}\left(\frac{5}{7}\left(\frac{4}{5}\int \frac{1}{(1-x)^{5/2}(x+1)^{5/2}}dx+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}\right)+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 39$$

$$\frac{2}{3}\left(\frac{5}{7}\left(\frac{4}{5}\int \frac{1}{(1-x^2)^{5/2}}dx+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}\right)+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{2}{3}\left(\frac{5}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-x^2)^{3/2}}dx+\frac{x}{3\,(1-x^2)^{3/2}}\right)+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}\right)+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2}{3}\left(\frac{5}{7}\left(\frac{4}{5}\left(\frac{2x}{3\sqrt{1-x^2}}+\frac{x}{3\left(1-x^2\right)^{3/2}}\right)+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}\right)+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}\right)+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

input
```
Int[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

output
```
1/(9*(1 - x)^(9/2)*(1 + x)^(3/2)) + (2*(1/(7*(1 - x)^(7/2)*(1 + x)^(3/2))
+ (5*(1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*(x/(3*(1 - x^2)^(3/2)) + (2*x
)/(3*Sqrt[1 - x^2])))/5))/7))/3
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

method	result
gosper	$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63(1+x)^{\frac{3}{2}}(1-x)^{\frac{9}{2}}}$
orering	$-\frac{(-1+x)(16x^6-48x^5+24x^4+56x^3-66x^2+6x+19)}{63(1+x)^{\frac{3}{2}}(1-x)^{\frac{11}{2}}}$
risch	$\frac{\sqrt{(1+x)(1-x)}\,(16x^6-48x^5+24x^4+56x^3-66x^2+6x+19)}{63\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\,(-1+x)^4}$
default	$\frac{1}{9(1-x)^{\frac{9}{2}}(1+x)^{\frac{3}{2}}}+\frac{2}{21(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}+\frac{2}{21(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{8}{63(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{8}{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{16\sqrt{1-x}}{63(1+x)^{\frac{3}{2}}}-\frac{16}{63}$

input
```
int(1/(1-x)^(11/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/63/(1+x)^(3/2)/(1-x)^(9/2)*(16*x^6-48*x^5+24*x^4+56*x^3-66*x^2+6*x+19)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.23

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \frac{19\,x^7-57\,x^6+19\,x^5+95\,x^4-95\,x^3-19\,x^2-(16\,x^6-48\,x^5+24\,x^4+56\,x}{63\,(x^7-3\,x^6+x^5+5\,x^4-5\,x^3-x^2}$$

input
```
integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/63*(19*x^7 - 57*x^6 + 19*x^5 + 95*x^4 - 95*x^3 - 19*x^2 - (16*x^6 - 48*x
^5 + 24*x^4 + 56*x^3 - 66*x^2 + 6*x + 19)*sqrt(x + 1)*sqrt(-x + 1) + 57*x
- 19)/(x^7 - 3*x^6 + x^5 + 5*x^4 - 5*x^3 - x^2 + 3*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 123.70 (sec) , antiderivative size = 789, normalized size of antiderivative = 8.48

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-x)**(11/2)/(1+x)**(5/2),x)
```

output
```
Piecewise((-16*sqrt(-1 + 2/(x + 1))*(x + 1)**6/(-2016*x + 63*(x + 1)**6 -
630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 201
6) + 144*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(-2016*x + 63*(x + 1)**6 - 630*(x
 + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) - 5
04*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)*
*5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 840*sqr
t(-1 + 2/(x + 1))*(x + 1)**3/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2
520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) - 630*sqrt(-1 +
 2/(x + 1))*(x + 1)**2/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x
 + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 126*sqrt(-1 + 2/(x
+ 1))*(x + 1)/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4
- 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 21*sqrt(-1 + 2/(x + 1))/(-20
16*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3
+ 5040*(x + 1)**2 - 2016), 1/Abs(x + 1) > 1/2), (-16*I*sqrt(1 - 2/(x + 1))
*(x + 1)**6/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 -
5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 144*I*sqrt(1 - 2/(x + 1))*(x +
 1)**5/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*
(x + 1)**3 + 5040*(x + 1)**2 - 2016) - 504*I*sqrt(1 - 2/(x + 1))*(x + 1)**
4/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x +
1)**3 + 5040*(x + 1)**2 - 2016) + 840*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(67) = 134$.

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.57

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \frac{16\,x}{63\,\sqrt{-x^2+1}} + \frac{8\,x}{63\,(-x^2+1)^{\frac{3}{2}}} - \frac{1}{9\left((-x^2+1)^{\frac{3}{2}}x^3 - 3\,(-x^2+1)^{\frac{3}{2}}x^2 + 3\,(-x^2+1)^{\frac{3}{2}}x - (-x^2+1)^{\frac{3}{2}}\right)} + \frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x^2 - 2\,(-x^2+1)^{\frac{3}{2}}x + (-x^2+1)^{\frac{3}{2}}\right)} - \frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x - (-x^2+1)^{\frac{3}{2}}\right)}$$

input
```
integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="maxima")
```

output
```
16/63*x/sqrt(-x^2 + 1) + 8/63*x/(-x^2 + 1)^(3/2) - 1/9/((-x^2 + 1)^(3/2)*x
^3 - 3*(-x^2 + 1)^(3/2)*x^2 + 3*(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2)) + 2
/21/((-x^2 + 1)^(3/2)*x^2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 2/2
1/((-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{1536\,(x+1)^{\frac{3}{2}}} + \frac{23\left(\sqrt{2}-\sqrt{-x+1}\right)}{512\,\sqrt{x+1}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{69\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{1536\left(\sqrt{2}-\sqrt{-x+1}\right)^3} - \frac{((((667\,x-5021)(x+1)+18396)(x+1)-26880)(x+1)+15120)\sqrt{x+1}\sqrt{-x+1}}{4032\,(x-1)^5}$$

input
```
integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/1536*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 23/512*(sqrt(2) - sqrt(-
x + 1))/sqrt(x + 1) - 1/1536*(x + 1)^(3/2)*(69*(sqrt(2) - sqrt(-x + 1))^2/
(x + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/4032*((((667*x - 5021)*(x + 1)
 + 18396)*(x + 1) - 26880)*(x + 1) + 15120)*sqrt(x + 1)*sqrt(-x + 1)/(x -
1)^5
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = -\frac{6\,x\,\sqrt{1-x}+19\,\sqrt{1-x}-66\,x^2\,\sqrt{1-x}+56\,x^3\,\sqrt{1-x}+24\,x^4\,\sqrt{1-x}-48\,x^5\,\sqrt{1-x}+16\,x^6\,\sqrt{1-x}}{(63\,x+63)\,\left(x-1\right)^5\,\sqrt{x+1}}$$

input
```
int(1/((1 - x)^(11/2)*(x + 1)^(5/2)),x)
```

output
```
-(6*x*(1 - x)^(1/2) + 19*(1 - x)^(1/2) - 66*x^2*(1 - x)^(1/2) + 56*x^3*(1
- x)^(1/2) + 24*x^4*(1 - x)^(1/2) - 48*x^5*(1 - x)^(1/2) + 16*x^6*(1 - x)^
(1/2))/((63*x + 63)*(x - 1)^5*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63\sqrt{x+1}\,\sqrt{1-x}\,(x^5-3x^4+2x^3+2x^2-3x+1)}$$

input
```
int(1/(1-x)^(11/2)/(1+x)^(5/2),x)
```

output
```
(16*x**6 - 48*x**5 + 24*x**4 + 56*x**3 - 66*x**2 + 6*x + 19)/(63*sqrt(x +
1)*sqrt( - x + 1)*(x**5 - 3*x**4 + 2*x**3 + 2*x**2 - 3*x + 1))
```

3.140 $\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 181

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx = \frac{20a^2\sqrt[3]{a-bx}(a+bx)^{2/3}}{27b} - \frac{5a(a-bx)^{4/3}(a+bx)^{2/3}}{9b} - \frac{(a-bx)^{4/3}(a+bx)^{5/3}}{3b} - \frac{40a^3\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{27\sqrt{3}b} - \frac{20a^3\log(a-bx)}{81b} - \frac{20a^3\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{27b}$$

output
```
20/27*a^2*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b-5/9*a*(-b*x+a)^(4/3)*(b*x+a)^(2/3
)/b-1/3*(-b*x+a)^(4/3)*(b*x+a)^(5/3)/b+40/81*a^3*arctan(-1/3*3^(1/2)+2/3*(
b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2)/b-20/81*a^3*ln(-b*x+a)/b-20/2
7*a^3*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx=\frac{3\sqrt[3]{a-bx}(a+bx)^{2/3}\left(-4a^2+15abx+9b^2x^2\right)-40\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-4}{81b}$$

input
```
Integrate[(a - b*x)^(1/3)*(a + b*x)^(5/3),x]
```

output
```
(3*(a - b*x)^(1/3)*(a + b*x)^(2/3)*(-4*a^2 + 15*a*b*x + 9*b^2*x^2) - 40*Sq
rt[3]*a^3*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)
^(1/3))] - 40*a^3*Log[(a - b*x)^(1/3) + (a + b*x)^(1/3)] + 20*a^3*Log[(a -
 b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(81*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{10}{9}a\int \sqrt[3]{a-bx}(a+bx)^{2/3}dx-\frac{(a-bx)^{4/3}(a+bx)^{5/3}}{3b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{10}{9}a\left(\frac{2}{3}a\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx-\frac{(a-bx)^{4/3}(a+bx)^{2/3}}{2b}\right)-\frac{(a-bx)^{4/3}(a+bx)^{5/3}}{3b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{10}{9}a\left(\frac{2}{3}a\left(\frac{2}{3}a\int\frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{(a-bx)^{4/3}(a+bx)^{2/3}}{2b}\right)-\frac{(a-bx)^{4/3}(a+bx)^{5/3}}{3b}$$

$\downarrow$ 72

$$\frac{10}{9}a\left(\frac{2}{3}a\left(\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right.\right.$$
$$\frac{(a-bx)^{4/3}(a+bx)^{5/3}}{3b}$$

input
```
Int[(a - b*x)^(1/3)*(a + b*x)^(5/3),x]
```

output
```
-1/3*((a - b*x)^(4/3)*(a + b*x)^(5/3))/b + (10*a*(-1/2*((a - b*x)^(4/3)*(a
 + b*x)^(2/3))/b + (2*a*(((a - b*x)^(1/3)*(a + b*x)^(2/3))/b + (2*a*(-((Sq
rt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3))])/b
) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])/(2*b
)))/3))/3))/9
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx+a)^{\frac{1}{3}} (bx+a)^{\frac{5}{3}} dx$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(5/3),x)
```

output
```
int((-b*x+a)^(1/3)*(b*x+a)^(5/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3} dx =$$

$$-\frac{40\sqrt{3}a^3 \arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3(bx+a)}\right) + 40a^3 \log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right) - 20a^3 \log\left(\frac{bx-(bx+}{}\right.}{81b}$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-1/81*(40*sqrt(3)*a^3*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)
^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 40*a^3*log((b*x + (b*x + a)^(2/3)*(-
b*x + a)^(1/3) + a)/(b*x + a)) - 20*a^3*log((b*x - (b*x + a)^(2/3)*(-b*x +
 a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) - 3*(9*b^2*x^
2 + 15*a*b*x - 4*a^2)*(b*x + a)^(2/3)*(-b*x + a)^(1/3))/b
```

Sympy [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3} dx = \int \sqrt[3]{a-bx}(a+bx)^{\frac{5}{3}} dx$$

input
```
integrate((-b*x+a)**(1/3)*(b*x+a)**(5/3),x)
```

output
```
Integral((a - b*x)**(1/3)*(a + b*x)**(5/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx = \int (bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(5/3)*(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx = \int (bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(5/3)*(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx = \int (a+b\,x)^{5/3}\,(a-b\,x)^{1/3}\,dx$$

input
```
int((a + b*x)^(5/3)*(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(5/3)*(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{5/3}\,dx = \frac{36(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^2+15(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}abx+9(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}b^2x^2+40}{27b}$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(5/3),x)
```

output
```
(36*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a**2 + 15*(a + b*x)**(2/3)*(a - b*x)
**(1/3)*a*b*x + 9*(a + b*x)**(2/3)*(a - b*x)**(1/3)*b**2*x**2 + 40*int(((a
 + b*x)**(2/3)*(a - b*x)**(1/3)*x)/(a**2 - b**2*x**2),x)*a**2*b**2)/(27*b)
```

3.141 $\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx = \frac{2a\sqrt[3]{a-bx}(a+bx)^{2/3}}{3b} - \frac{(a-bx)^{4/3}(a+bx)^{2/3}}{2b} - \frac{4a^2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{3\sqrt{3}b} - \frac{2a^2\log(a-bx)}{9b} - \frac{2a^2\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{3b}$$

output
```
2/3*a*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b-1/2*(-b*x+a)^(4/3)*(b*x+a)^(2/3)/b+4/
9*a^2*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2
)/b-2/9*a^2*ln(-b*x+a)/b-2/3*a^2*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx = \frac{3\sqrt[3]{a-bx}(a+bx)^{2/3}(a+3bx)-8\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-8a^2\log\left(b\left(\sqrt[3]{a-\right.\right.}{18b}$$

input
```
Integrate[(a - b*x)^(1/3)*(a + b*x)^(2/3),x]
```

output
```
(3*(a - b*x)^(1/3)*(a + b*x)^(2/3)*(a + 3*b*x) - 8*Sqrt[3]*a^2*ArcTan[(Sqr
t[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 8*a^2*Log[
b*((a - b*x)^(1/3) + (a + b*x)^(1/3))] + 4*a^2*Log[(a - b*x)^(2/3) - (a -
b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{3}a\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx-\frac{(a-bx)^{4/3}(a+bx)^{2/3}}{2b}$$

$$\Big\downarrow \text{60}$$

$$\frac{2}{3}a\left(\frac{2}{3}a\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{(a-bx)^{4/3}(a+bx)^{2/3}}{2b}$$

$$\Big\downarrow \text{72}$$

$$\frac{2}{3}a\left(\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{(a-bx)^{4/3}(a+bx)^{2/3}}{2b}$$

input
```
Int[(a - b*x)^(1/3)*(a + b*x)^(2/3),x]
```

output
```
-1/2*((a - b*x)^(4/3)*(a + b*x)^(2/3))/b + (2*a*(((a - b*x)^(1/3)*(a + b*x
)^(2/3))/b + (2*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt
[3]*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3
)/(a - b*x)^(1/3)])/(2*b)))/3))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx+a)^{\frac{1}{3}} (bx+a)^{\frac{2}{3}} \, dx$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(2/3),x)
```

output
```
int((-b*x+a)^(1/3)*(b*x+a)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx = -\frac{8\sqrt{3}a^2\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3(bx+a)}\right)+8a^2\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-4a^2\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(\cdots}{\cdots}\right.}{18b}$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/18*(8*sqrt(3)*a^2*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)^
(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 8*a^2*log((b*x + (b*x + a)^(2/3)*(-b*
x + a)^(1/3) + a)/(b*x + a)) - 4*a^2*log((b*x - (b*x + a)^(2/3)*(-b*x + a)
^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) - 3*(3*b*x + a)*
(b*x + a)^(2/3)*(-b*x + a)^(1/3))/b
```

Sympy [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx = \int \sqrt[3]{a-bx}\,(a+bx)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)*(b*x+a)**(2/3),x)
```

output
```
Integral((a - b*x)**(1/3)*(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx=\int (bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(2/3)*(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx=\int (bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(2/3)*(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx=\int (a+b\,x)^{2/3}\,(a-b\,x)^{1/3}\,dx$$

input
```
int((a + b*x)^(2/3)*(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(2/3)*(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{2/3}\,dx = \frac{9(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a+3(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}bx+8\left(\int \frac{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}x}{-b^2x^2+a^2}dx\right)a\,b^2}{6b}$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(2/3),x)
```

output
```
(9*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a + 3*(a + b*x)**(2/3)*(a - b*x)**(1/
3)*b*x + 8*int(((a + b*x)**(2/3)*(a - b*x)**(1/3)*x)/(a**2 - b**2*x**2),x)
*a*b**2)/(6*b)
```

3.142 $\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = \frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b} - \frac{2a\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{\sqrt{3}b} - \frac{a\log(a-bx)}{3b} - \frac{a\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{b}$$

output
```
(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b+2/3*a*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)
*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2)/b-1/3*a*ln(-b*x+a)/b-a*ln(1+(b*x+a)^(1/3)
/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = \frac{3\sqrt[3]{a-bx}(a+bx)^{2/3} - 2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 2a\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right) + a\,\mathrm{lo}}{3b}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(1/3),x]
```

output
```
(3*(a - b*x)^(1/3)*(a + b*x)^(2/3) - 2*Sqrt[3]*a*ArcTan[(Sqrt[3]*(a + b*x)
^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 2*a*Log[b*((a - b*x)^(1/
3) + (a + b*x)^(1/3))] + a*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)
^(1/3) + (a + b*x)^(2/3)])/(3*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2}{3}a\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}$$

$$\Big\downarrow \text{ 72}$$

$$\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(1/3),x]
```

output
```
((a - b*x)^(1/3)*(a + b*x)^(2/3))/b + (2*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] -
(2*(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b) -
(3*Log[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])/(2*b)))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(1/3),x)
```

output
```
int((-b*x+a)^(1/3)/(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = -\frac{2\sqrt{3}a\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)+2\,a\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-a\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-}{}\right.}{3\,b}$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(3)*a*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)^(2/
3)*(-b*x + a)^(1/3))/(b*x + a)) + 2*a*log((b*x + (b*x + a)^(2/3)*(-b*x + a
)^(1/3) + a)/(b*x + a)) - a*log((b*x - (b*x + a)^(2/3)*(-b*x + a)^(1/3) +
(b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) - 3*(b*x + a)^(2/3)*(-b*x
 + a)^(1/3))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = \int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(1/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(a-b\,x)^{1/3}}{(a+b\,x)^{1/3}}\,dx$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(1/3),x)
```

output
```
int((a - b*x)^(1/3)/(a + b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(1/3),x)
```

output
```
int((a - b*x)**(1/3)/(a + b*x)**(1/3),x)
```

3.143 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx = -\frac{3\sqrt[3]{a-bx}}{b\sqrt[3]{a+bx}}+\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}+\frac{\log(a-bx)}{2b}+\frac{3\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{2b}$$

output
```
-3*(-b*x+a)^(1/3)/b/(b*x+a)^(1/3)-3^(1/2)*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^
(1/3)*3^(1/2)/(-b*x+a)^(1/3))/b+1/2*ln(-b*x+a)/b+3/2*ln(1+(b*x+a)^(1/3)/(-
b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx = -\frac{\frac{6\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-2\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right)+\log\left((a-bx)^{2/3}\right.}{2b}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(4/3),x]
```

output
```
-1/2*((6*(a - b*x)^(1/3))/(a + b*x)^(1/3) - 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a +
 b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 2*Log[b*((a - b*x)^
(1/3) + (a + b*x)^(1/3))] + Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x
)^(1/3) + (a + b*x)^(2/3)])/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {57, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}dx$$

$$\Big\downarrow \text{57}$$

$$-\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx-\frac{3\sqrt[3]{a-bx}}{b\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{72}$$

$$\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{3\sqrt[3]{a-bx}}{b\sqrt[3]{a+bx}}+\frac{\log(a-bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(4/3),x]
```

output
```
(-3*(a - b*x)^(1/3))/(b*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*
(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3))])/b + Log[a - b*x]/(2*b) + (3*L
og[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])/(2*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(4/3),x)
```

output
```
int((-b*x+a)^(1/3)/(b*x+a)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx = \frac{2\sqrt{3}(bx+a)\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)+2\,(bx+a)\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+}{bx+a}\right.}{2\,(b^2}$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*(b*x + a)*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x +
 a)^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 2*(b*x + a)*log((b*x + (b*x + a)^
(2/3)*(-b*x + a)^(1/3) + a)/(b*x + a)) - (b*x + a)*log((b*x - (b*x + a)^(2
/3)*(-b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) -
6*(b*x + a)^(2/3)*(-b*x + a)^(1/3))/(b^2*x + a*b)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx = \int \frac{\sqrt[3]{a-bx}}{(a+bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(4/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx=\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx=\int \frac{(a-b\,x)^{1/3}}{(a+b\,x)^{4/3}}\,dx$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(4/3),x)
```

output
```
int((a - b*x)^(1/3)/(a + b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}\,dx=\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}\,a+(bx+a)^{\frac{1}{3}}\,bx}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(4/3),x)
```

output
```
int((a - b*x)**(1/3)/((a + b*x)**(1/3)*a + (a + b*x)**(1/3)*b*x),x)
```

3.144 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx=-\frac{3(a-bx)^{4/3}}{8ab(a+bx)^{4/3}}$$

output
```
-3/8*(-b*x+a)^(4/3)/a/b/(b*x+a)^(4/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx=-\frac{3(a-bx)^{4/3}}{8ab(a+bx)^{4/3}}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(7/3),x]
```

output
```
(-3*(a - b*x)^(4/3))/(8*a*b*(a + b*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx$$
$$\Big\downarrow \text{48}$$
$$-\frac{3(a-bx)^{4/3}}{8ab(a+bx)^{4/3}}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(7/3),x]
```

output
```
(-3*(a - b*x)^(4/3))/(8*a*b*(a + b*x)^(4/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{4}{3}}}{8ab(bx+a)^{\frac{4}{3}}}$	24
orering	$-\frac{3(-bx+a)^{\frac{4}{3}}}{8ab(bx+a)^{\frac{4}{3}}}$	24

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-3/8*(-b*x+a)^(4/3)/a/b/(b*x+a)^(4/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx = \frac{3\,(bx+a)^{\frac{2}{3}}(bx-a)(-bx+a)^{\frac{1}{3}}}{8\,(ab^3x^2+2\,a^2b^2x+a^3b)}$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
3/8*(b*x + a)^(2/3)*(b*x - a)*(-b*x + a)^(1/3)/(a*b^3*x^2 + 2*a^2*b^2*x +
a^3*b)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx = \int \frac{\sqrt[3]{a-bx}}{(a+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(7/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(7/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx = -\frac{\left(\frac{3}{8\,b^2}-\frac{3\,x}{8\,a\,b}\right)\,(a-b\,x)^{1/3}}{x\,(a+b\,x)^{1/3}+\frac{a\,(a+b\,x)^{1/3}}{b}}$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(7/3),x)
```

output
```
-((3/(8*b^2) - (3*x)/(8*a*b))*(a - b*x)^(1/3))/(x*(a + b*x)^(1/3) + (a*(a
+ b*x)^(1/3))/b)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\,(bx-a)}{8\,(bx+a)^{\frac{4}{3}}\,ab}$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(7/3),x)
```

output
```
(3*(a - b*x)**(1/3)*( - a + b*x))/(8*(a + b*x)**(1/3)*a*b*(a + b*x))
```

3.145 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = -\frac{3(a-bx)^{4/3}}{14ab(a+bx)^{7/3}} - \frac{9(a-bx)^{4/3}}{112a^2b(a+bx)^{4/3}}$$

output
```
-3/14*(-b*x+a)^(4/3)/a/b/(b*x+a)^(7/3)-9/112*(-b*x+a)^(4/3)/a^2/b/(b*x+a)^
(4/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = -\frac{3(a-bx)^{4/3}(11a+3bx)}{112a^2b(a+bx)^{7/3}}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(10/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*(11*a + 3*b*x))/(112*a^2*b*(a + b*x)^(7/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}dx$$

$$\Big\downarrow \; 55$$

$$\frac{3\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}dx}{14a}-\frac{3(a-bx)^{4/3}}{14ab(a+bx)^{7/3}}$$

$$\Big\downarrow \; 48$$

$$-\frac{9(a-bx)^{4/3}}{112a^2b(a+bx)^{4/3}}-\frac{3(a-bx)^{4/3}}{14ab(a+bx)^{7/3}}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(10/3),x]
```

output
```
(-3*(a - b*x)^(4/3))/(14*a*b*(a + b*x)^(7/3)) - (9*(a - b*x)^(4/3))/(112*a
^2*b*(a + b*x)^(4/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{4}{3}}(3bx+11a)}{112(bx+a)^{\frac{7}{3}}a^{2}b}$	32
orering	$-\frac{3(-bx+a)^{\frac{4}{3}}(3bx+11a)}{112(bx+a)^{\frac{7}{3}}a^{2}b}$	32

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
-3/112*(-b*x+a)^(4/3)*(3*b*x+11*a)/(b*x+a)^(7/3)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = \frac{3\,(3\,b^2x^2+8\,abx-11\,a^2)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{112\,(a^2b^4x^3+3\,a^3b^3x^2+3\,a^4b^2x+a^5b)}$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

output
```
3/112*(3*b^2*x^2 + 8*a*b*x - 11*a^2)*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a^2
*b^4*x^3 + 3*a^3*b^3*x^2 + 3*a^4*b^2*x + a^5*b)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = \int \frac{\sqrt[3]{a-bx}}{(a+bx)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(10/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(10/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(10/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(10/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = \frac{3\,(a-b\,x)^{1/3}\,(-11\,a^2+8\,a\,b\,x+3\,b^2\,x^2)}{112\,a^2\,b\,(a+b\,x)^{7/3}}$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(10/3),x)
```

output
```
(3*(a - b*x)^(1/3)*(3*b^2*x^2 - 11*a^2 + 8*a*b*x))/(112*a^2*b*(a + b*x)^(7
/3))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\,(3b^2x^2+8abx-11a^2)}{112\,(bx+a)^{\frac{1}{3}}\,a^2b\,(b^2x^2+2abx+a^2)}$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(10/3),x)
```

output
```
(3*(a - b*x)**(1/3)*( - 11*a**2 + 8*a*b*x + 3*b**2*x**2))/(112*(a + b*x)**
(1/3)*a**2*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.146 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}} \, dx = -\frac{3(a-bx)^{4/3}}{20ab(a+bx)^{10/3}} - \frac{9(a-bx)^{4/3}}{140a^2b(a+bx)^{7/3}} - \frac{27(a-bx)^{4/3}}{1120a^3b(a+bx)^{4/3}}$$

output
```
-3/20*(-b*x+a)^(4/3)/a/b/(b*x+a)^(10/3)-9/140*(-b*x+a)^(4/3)/a^2/b/(b*x+a)
^(7/3)-27/1120*(-b*x+a)^(4/3)/a^3/b/(b*x+a)^(4/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}} \, dx = -\frac{3(a-bx)^{4/3}(89a^2+42abx+9b^2x^2)}{1120a^3b(a+bx)^{10/3}}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(13/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*(89*a^2 + 42*a*b*x + 9*b^2*x^2))/(1120*a^3*b*(a + b*x)
^(10/3))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}dx}{10a}-\frac{3(a-bx)^{4/3}}{20ab(a+bx)^{10/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\left(\frac{3\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}dx}{14a}-\frac{3(a-bx)^{4/3}}{14ab(a+bx)^{7/3}}\right)}{10a}-\frac{3(a-bx)^{4/3}}{20ab(a+bx)^{10/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{3\left(-\frac{9(a-bx)^{4/3}}{112a^2b(a+bx)^{4/3}}-\frac{3(a-bx)^{4/3}}{14ab(a+bx)^{7/3}}\right)}{10a}-\frac{3(a-bx)^{4/3}}{20ab(a+bx)^{10/3}}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(13/3),x]
```

output
```
(-3*(a - b*x)^(4/3))/(20*a*b*(a + b*x)^(10/3)) + (3*((-3*(a - b*x)^(4/3))/
(14*a*b*(a + b*x)^(7/3)) - (9*(a - b*x)^(4/3))/(112*a^2*b*(a + b*x)^(4/3))
))/(10*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{4}{3}}\left(9b^{2}x^{2}+42abx+89a^{2}\right)}{1120(bx+a)^{\frac{10}{3}}a^{3}b}$	43
orering	$-\frac{3(-bx+a)^{\frac{4}{3}}\left(9b^{2}x^{2}+42abx+89a^{2}\right)}{1120(bx+a)^{\frac{10}{3}}a^{3}b}$	43

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
-3/1120*(-b*x+a)^(4/3)*(9*b^2*x^2+42*a*b*x+89*a^2)/(b*x+a)^(10/3)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}\,dx = \frac{3\,(9\,b^3x^3+33\,ab^2x^2+47\,a^2bx-89\,a^3)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{1120\,(a^3b^5x^4+4\,a^4b^4x^3+6\,a^5b^3x^2+4\,a^6b^2x+a^7b)}$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

output
```
3/1120*(9*b^3*x^3 + 33*a*b^2*x^2 + 47*a^2*b*x - 89*a^3)*(b*x + a)^(2/3)*(-
b*x + a)^(1/3)/(a^3*b^5*x^4 + 4*a^4*b^4*x^3 + 6*a^5*b^3*x^2 + 4*a^6*b^2*x
+ a^7*b)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}\,dx = \int \frac{\sqrt[3]{a-bx}}{(a+bx)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(13/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(13/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(13/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}\,dx = \frac{3\,(a-b\,x)^{1/3}\,\left(-89\,a^3+47\,a^2\,b\,x+33\,a\,b^2\,x^2+9\,b^3\,x^3\right)}{1120\,a^3\,b\,(a+b\,x)^{10/3}}$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(13/3),x)
```

output
```
(3*(a - b*x)^(1/3)*(9*b^3*x^3 - 89*a^3 + 33*a*b^2*x^2 + 47*a^2*b*x))/(1120
*a^3*b*(a + b*x)^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\,(9b^3x^3+33a\,b^2x^2+47a^2bx-89a^3)}{1120\,(bx+a)^{\frac{1}{3}}\,a^3b\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(13/3),x)
```

output
```
(3*(a - b*x)**(1/3)*( - 89*a**3 + 47*a**2*b*x + 33*a*b**2*x**2 + 9*b**3*x*
*3))/(1120*(a + b*x)**(1/3)*a**3*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3))
```

3.147 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = -\frac{3(a-bx)^{4/3}}{26ab(a+bx)^{13/3}} - \frac{27(a-bx)^{4/3}}{520a^2b(a+bx)^{10/3}} - \frac{81(a-bx)^{4/3}}{3640a^3b(a+bx)^{7/3}} - \frac{243(a-bx)^{4/3}}{29120a^4b(a+bx)^{4/3}}$$

output
```
-3/26*(-b*x+a)^(4/3)/a/b/(b*x+a)^(13/3)-27/520*(-b*x+a)^(4/3)/a^2/b/(b*x+a
)^(10/3)-81/3640*(-b*x+a)^(4/3)/a^3/b/(b*x+a)^(7/3)-243/29120*(-b*x+a)^(4/
3)/a^4/b/(b*x+a)^(4/3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = -\frac{3(a-bx)^{4/3}\left(1921a^3+1179a^2bx+459ab^2x^2+81b^3x^3\right)}{29120a^4b(a+bx)^{13/3}}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(16/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*(1921*a^3 + 1179*a^2*b*x + 459*a*b^2*x^2 + 81*b^3*x^3)
)/(29120*a^4*b*(a + b*x)^(13/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{9\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{13/3}}dx}{26a}-\frac{3(a-bx)^{4/3}}{26ab(a+bx)^{13/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{9\left(\frac{3\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{10/3}}dx}{10a}-\frac{3(a-bx)^{4/3}}{20ab(a+bx)^{10/3}}\right)}{26a}-\frac{3(a-bx)^{4/3}}{26ab(a+bx)^{13/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{7/3}}dx}{14a}-\frac{3(a-bx)^{4/3}}{14ab(a+bx)^{7/3}}\right)}{10a}-\frac{3(a-bx)^{4/3}}{20ab(a+bx)^{10/3}}\right)}{26a}-\frac{3(a-bx)^{4/3}}{26ab(a+bx)^{13/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{9\left(\frac{3\left(-\frac{9(a-bx)^{4/3}}{112a^2b(a+bx)^{4/3}}-\frac{3(a-bx)^{4/3}}{14ab(a+bx)^{7/3}}\right)}{10a}-\frac{3(a-bx)^{4/3}}{20ab(a+bx)^{10/3}}\right)}{26a}-\frac{3(a-bx)^{4/3}}{26ab(a+bx)^{13/3}}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(16/3),x]
```

output
```
(-3*(a - b*x)^(4/3))/(26*a*b*(a + b*x)^(13/3)) + (9*((-3*(a - b*x)^(4/3))/
(20*a*b*(a + b*x)^(10/3)) + (3*((-3*(a - b*x)^(4/3))/(14*a*b*(a + b*x)^(7/
3)) - (9*(a - b*x)^(4/3))/(112*a^2*b*(a + b*x)^(4/3))))/(10*a)))/(26*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{4}{3}}\left(81b^{3}x^{3}+459a\,b^{2}x^{2}+1179a^{2}bx+1921a^{3}\right)}{29120(bx+a)^{\frac{13}{3}}a^{4}b}$	54
orering	$-\frac{3(-bx+a)^{\frac{4}{3}}\left(81b^{3}x^{3}+459a\,b^{2}x^{2}+1179a^{2}bx+1921a^{3}\right)}{29120(bx+a)^{\frac{13}{3}}a^{4}b}$	54

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
-3/29120*(-b*x+a)^(4/3)*(81*b^3*x^3+459*a*b^2*x^2+1179*a^2*b*x+1921*a^3)/(
b*x+a)^(13/3)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = \frac{3\,(81\,b^4x^4+378\,ab^3x^3+720\,a^2b^2x^2+742\,a^3bx-1921\,a^4)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{29120\,(a^4b^6x^5+5\,a^5b^5x^4+10\,a^6b^4x^3+10\,a^7b^3x^2+5\,a^8b^2x+a^9b)}$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(16/3),x, algorithm="fricas")
```

output
```
3/29120*(81*b^4*x^4 + 378*a*b^3*x^3 + 720*a^2*b^2*x^2 + 742*a^3*b*x - 1921
*a^4)*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a^4*b^6*x^5 + 5*a^5*b^5*x^4 + 10*a
^6*b^4*x^3 + 10*a^7*b^3*x^2 + 5*a^8*b^2*x + a^9*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(16/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(16/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(16/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = \frac{3\,(a-b\,x)^{1/3}\,(-1921\,a^4+742\,a^3\,b\,x+720\,a^2\,b^2\,x^2+378\,a\,b^3\,x^3+81\,b^4\,x^4)}{29120\,a^4\,b\,(a+b\,x)^{13/3}}$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(16/3),x)
```

output
```
(3*(a - b*x)^(1/3)*(81*b^4*x^4 - 1921*a^4 + 378*a*b^3*x^3 + 720*a^2*b^2*x^
2 + 742*a^3*b*x))/(29120*a^4*b*(a + b*x)^(13/3))
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{16/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\,(81b^4x^4+378a\,b^3x^3+720a^2b^2x^2+742a^3bx-1921a^4)}{29120\,(bx+a)^{\frac{1}{3}}\,a^4b\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(16/3),x)
```

output
```
(3*(a - b*x)**(1/3)*( - 1921*a**4 + 742*a**3*b*x + 720*a**2*b**2*x**2 + 37
8*a*b**3*x**3 + 81*b**4*x**4))/(29120*(a + b*x)**(1/3)*a**4*b*(a**4 + 4*a*
*3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.148 $\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 336

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx = \frac{3}{5}ax\sqrt[3]{a-bx}\sqrt[3]{a+bx} - \frac{3(a-bx)^{4/3}(a+bx)^{4/3}}{8b}$$
$$+ \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^5\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}\cdots}{1-\sqrt{3}\cdots}\right.\right.\cdots}{5b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3/5*a*x*(-b*x+a)^(1/3)*(b*x+a)^(1/3)-3/8*(-b*x+a)^(4/3)*(b*x+a)^(4/3)/b+2/
5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^5*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^
2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-
(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3)
)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b
*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.20

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx=-\frac{3a(a-bx)^{4/3}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{2^{2/3}b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(1/3)*(a + b*x)^(4/3),x]
```

output
```
(-3*a*(a - b*x)^(4/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-4/3, 4/3, 7/3, (a
 - b*x)/(2*a)])/(2^(2/3)*b*((a + b*x)/a)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx$$

$$\Big\downarrow \text{80}$$

$$\frac{2\sqrt[3]{2}a\sqrt[3]{a+bx}\int \frac{\sqrt[3]{a-bx}\left(\frac{bx}{a}+1\right)^{4/3}}{2\sqrt[3]{2}}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt[3]{a+bx}\int \sqrt[3]{a-bx}\left(\frac{bx}{a}+1\right)^{4/3}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \; 79$$

$$-\frac{3a(a-bx)^{4/3}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{2^{2/3}b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Int[(a - b*x)^(1/3)*(a + b*x)^(4/3),x]
```

output
```
(-3*a*(a - b*x)^(4/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-4/3, 4/3, 7/3, (a
 - b*x)/(2*a)])/(2^(2/3)*b*((a + b*x)/a)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (-bx+a)^{\frac{1}{3}} (bx+a)^{\frac{4}{3}} \, dx$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(4/3),x)
```

output
```
int((-b*x+a)^(1/3)*(b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3} \, dx = \int (bx+a)^{\frac{4}{3}} (-bx+a)^{\frac{1}{3}} \, dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(4/3)*(-b*x + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3} \, dx = \int \sqrt[3]{a-bx}(a+bx)^{\frac{4}{3}} \, dx$$

input
```
integrate((-b*x+a)**(1/3)*(b*x+a)**(4/3),x)
```

output
```
Integral((a - b*x)**(1/3)*(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx = \int (bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(4/3)*(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx = \int (bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(4/3)*(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx = \int (a+b\,x)^{4/3}\,(a-b\,x)^{1/3}\,dx$$

input
```
int((a + b*x)^(4/3)*(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(4/3)*(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a-bx}(a+bx)^{4/3}\,dx = \frac{-15(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}a^2+24(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}abx+15(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}b^2x^2+}{40b}$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(4/3),x)
```

output
```
( - 15*(a + b*x)**(1/3)*(a - b*x)**(1/3)*a**2 + 24*(a + b*x)**(1/3)*(a - b
*x)**(1/3)*a*b*x + 15*(a + b*x)**(1/3)*(a - b*x)**(1/3)*b**2*x**2 + 16*int
(((a + b*x)**(1/3)*(a - b*x)**(1/3))/(a**2 - b**2*x**2),x)*a**3*b)/(40*b)
```

3.149 $\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 309

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx = \frac{3}{5}x\sqrt[3]{a-bx}\sqrt[3]{a+bx} + \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{5b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3/5*x*(-b*x+a)^(1/3)*(b*x+a)^(1/3)+2/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a
^4*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/
3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Ellip
ticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2
*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3
))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx = -\frac{3(a-bx)^{4/3}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{2\ 2^{2/3}b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(1/3)*(a + b*x)^(1/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, (a -
 b*x)/(2*a)])/(2*2^(2/3)*b*((a + b*x)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {46, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx$$

$$\Big\downarrow \text{ 46}$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\int \sqrt[3]{a^2-b^2x^2}dx}{\sqrt[3]{a^2-b^2x^2}}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(\frac{2}{5}a^2\int \frac{1}{(a^2-b^2x^2)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a^2-b^2x^2}\right)}{\sqrt[3]{a^2-b^2x^2}}$$

$$\Big\downarrow \text{ 234}$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(\frac{3}{5}x\sqrt[3]{a^2-b^2x^2}-\frac{3a^2\sqrt{-b^2x^2}\int \frac{1}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}}{5b^2x}\right)}{\sqrt[3]{a^2-b^2x^2}}$$

$\Big\downarrow$ 760

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(\frac{3}{5}x\sqrt[3]{a^2-b^2x^2}+\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^{4/3}+\left(a^2-b^2x^2\right)^{2/3}+a^{2/3}\sqrt[3]{a^2-b^2x^2}}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)^2}}\text{EllipticF}}{5b^2x\sqrt{-\frac{a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)^2}}}\right)}{\sqrt[3]{a^2-b^2x^2}}$$

input
```
Int[(a - b*x)^(1/3)*(a + b*x)^(1/3),x]
```

output
```
((a - b*x)^(1/3)*(a + b*x)^(1/3)*((3*x*(a^2 - b^2*x^2)^(1/3))/5 + (2*3^(3/
4)*Sqrt[2 - Sqrt[3]]*a^2*(a^(2/3) - (a^2 - b^2*x^2)^(1/3))*Sqrt[(a^(4/3) +
 a^(2/3)*(a^2 - b^2*x^2)^(1/3) + (a^2 - b^2*x^2)^(2/3))/((1 - Sqrt[3])*a^(
2/3) - (a^2 - b^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(2/3) -
 (a^2 - b^2*x^2)^(1/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)
))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2)])))/(a^2 - b^2*x^2)^
(1/3)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (-bx+a)^{\frac{1}{3}} (bx+a)^{\frac{1}{3}} dx$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(1/3),x)
```

output
```
int((-b*x+a)^(1/3)*(b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx = \int (bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/3)*(-b*x + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx=\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx$$

input
```
integrate((-b*x+a)**(1/3)*(b*x+a)**(1/3),x)
```

output
```
Integral((a - b*x)**(1/3)*(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx=\int (bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/3)*(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx=\int (bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)*(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/3)*(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx = \int (a+b\,x)^{1/3}\,(a-b\,x)^{1/3}\,dx$$

input
```
int((a + b*x)^(1/3)*(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(1/3)*(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a-bx}\sqrt[3]{a+bx}\,dx = \frac{3(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,x}{5} + \frac{2\left(\int \frac{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}}{-b^2x^2+a^2}dx\right)a^2}{5}$$

input
```
int((-b*x+a)^(1/3)*(b*x+a)^(1/3),x)
```

output
```
(3*(a + b*x)**(1/3)*(a - b*x)**(1/3)*x + 2*int(((a + b*x)**(1/3)*(a - b*x)
**(1/3))/(a**2 - b**2*x**2),x)*a**2)/5
```

3.150 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a-bx}\sqrt[3]{a+bx}}{2b} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-}{1-\sqrt{3}-}\right.\right.}{b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3/2*(-b*x+a)^(1/3)*(b*x+a)^(1/3)/b+3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^3*(
1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(
1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF
((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I
*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(
1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = -\frac{3(a-bx)^{4/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{4\ 2^{2/3}b(a+bx)^{2/3}}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(2/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, (
a - b*x)/(2*a)])/(4*2^(2/3)*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}dx$$

$$\Big\downarrow \ 80$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{2^{2/3}\sqrt[3]{a-bx}}{\left(\frac{bx}{a}+1\right)^{2/3}}dx}{2^{2/3}(a+bx)^{2/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{\sqrt[3]{a-bx}}{\left(\frac{bx}{a}+1\right)^{2/3}}dx}{(a+bx)^{2/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3(a-bx)^{4/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{4\ 2^{2/3}b(a+bx)^{2/3}}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(2/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, (
a - b*x)/(2*a)])/(4*2^(2/3)*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(2/3),x)
```

output
```
int((-b*x+a)^(1/3)/(b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((-b*x + a)^(1/3)/(b*x + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = \int \frac{\sqrt[3]{a-bx}}{(a+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(2/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = \int \frac{(a-b\,x)^{1/3}}{(a+b\,x)^{2/3}}\,dx$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(2/3),x)
```

output
```
int((a - b*x)^(1/3)/(a + b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(2/3),x)
```

output
```
int((a - b*x)**(1/3)/(a + b*x)**(2/3),x)
```

3.151 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 311

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a-bx}}{2b(a+bx)^{2/3}} - \frac{3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\cdots}{1-\sqrt{3}-\cdots}\right)\cdots\right)}{2b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/2*(-b*x+a)^(1/3)/b/(b*x+a)^(2/3)-1/2*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*
a^2*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1
/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Elli
pticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),
2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/
3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = -\frac{3(a-bx)^{4/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{8\ 2^{2/3}ab(a+bx)^{2/3}}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(5/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[4/3, 5/3, 7/3, (
a - b*x)/(2*a)])/(8*2^(2/3)*a*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{2\ 2^{2/3}\sqrt[3]{a-bx}}{\left(\frac{bx}{a}+1\right)^{5/3}}dx}{2\ 2^{2/3}a(a+bx)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{\sqrt[3]{a-bx}}{\left(\frac{bx}{a}+1\right)^{5/3}}dx}{a(a+bx)^{2/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3(a-bx)^{4/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{8\ 2^{2/3}ab(a+bx)^{2/3}}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(5/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[4/3, 5/3, 7/3, (
a - b*x)/(2*a)])/(8*2^(2/3)*a*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(5/3),x)
```

output
```
int((-b*x+a)^(1/3)/(b*x+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/3)*(-b*x + a)^(1/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = \int \frac{\sqrt[3]{a-bx}}{(a+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(5/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(5/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = \int \frac{(a-b\,x)^{1/3}}{(a+b\,x)^{5/3}}\,dx$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(5/3),x)
```

output
```
int((a - b*x)^(1/3)/(a + b*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}\,a+(bx+a)^{\frac{2}{3}}\,bx}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(5/3),x)
```

output
```
int((a - b*x)**(1/3)/((a + b*x)**(2/3)*a + (a + b*x)**(2/3)*b*x),x)
```

3.152 $\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 338

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\,dx = -\frac{3\sqrt[3]{a-bx}}{5b(a+bx)^{5/3}} + \frac{3\sqrt[3]{a-bx}}{20ab(a+bx)^{2/3}}$$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\cdots}{1-\sqrt{3}-\cdots}\right.\right.}{20b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/5*(-b*x+a)^(1/3)/b/(b*x+a)^(5/3)+3/20*(-b*x+a)^(1/3)/a/b/(b*x+a)^(2/3)-
1/20*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x
^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)
-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3
))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(
b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\,dx = -\frac{3(a-bx)^{4/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{16\ 2^{2/3}a^2b(a+bx)^{2/3}}$$

input
```
Integrate[(a - b*x)^(1/3)/(a + b*x)^(8/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[4/3, 8/3, 7/3, (
a - b*x)/(2*a)])/(16*2^(2/3)*a^2*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}dx$$
$$\Big\downarrow \ 80$$
$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{4\ 2^{2/3}\sqrt[3]{a-bx}}{\left(\frac{bx}{a}+1\right)^{8/3}}dx}{4\ 2^{2/3}a^2(a+bx)^{2/3}}$$
$$\Big\downarrow \ 27$$
$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{\sqrt[3]{a-bx}}{\left(\frac{bx}{a}+1\right)^{8/3}}dx}{a^2(a+bx)^{2/3}}$$

$$\Big\downarrow 79$$

$$-\frac{3(a-bx)^{4/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{7}{3},\frac{a-bx}{2a}\right)}{16\ 2^{2/3}a^2b(a+bx)^{2/3}}$$

input
```
Int[(a - b*x)^(1/3)/(a + b*x)^(8/3),x]
```

output
```
(-3*(a - b*x)^(4/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[4/3, 8/3, 7/3, (
a - b*x)/(2*a)])/(16*2^(2/3)*a^2*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(8/3),x)
```

output
```
int((-b*x+a)^(1/3)/(b*x+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\, dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}\, dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/3)*(-b*x + a)^(1/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\, dx = \int \frac{\sqrt[3]{a-bx}}{(a+bx)^{\frac{8}{3}}}\, dx$$

input
```
integrate((-b*x+a)**(1/3)/(b*x+a)**(8/3),x)
```

output
```
Integral((a - b*x)**(1/3)/(a + b*x)**(8/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(8/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(1/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/3)/(b*x + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\,dx = \int \frac{(a-b\,x)^{1/3}}{(a+b\,x)^{8/3}}\,dx$$

input
```
int((a - b*x)^(1/3)/(a + b*x)^(8/3),x)
```

output
```
int((a - b*x)^(1/3)/(a + b*x)^(8/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{8/3}}\,dx = \int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}\,a^2+2\,(bx+a)^{\frac{2}{3}}\,abx+(bx+a)^{\frac{2}{3}}\,b^2x^2}dx$$

input
```
int((-b*x+a)^(1/3)/(b*x+a)^(8/3),x)
```

output
```
int((a - b*x)**(1/3)/((a + b*x)**(2/3)*a**2 + 2*(a + b*x)**(2/3)*a*b*x + (
a + b*x)**(2/3)*b**2*x**2),x)
```

3.153 $\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx = \frac{28a^3(a-bx)^{2/3}\sqrt[3]{a+bx}}{81b} - \frac{14a^2(a-bx)^{5/3}\sqrt[3]{a+bx}}{27b} - \frac{7a(a-bx)^{5/3}(a+bx)^{4/3}}{18b} - \frac{(a-bx)^{5/3}(a+bx)^{7/3}}{4b} + \frac{112a^4\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{81\sqrt{3}b} + \frac{56a^4\log(a+bx)}{243b} + \frac{56a^4\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{81b}$$

output
```
28/81*a^3*(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b-14/27*a^2*(-b*x+a)^(5/3)*(b*x+a)^
(1/3)/b-7/18*a*(-b*x+a)^(5/3)*(b*x+a)^(4/3)/b-1/4*(-b*x+a)^(5/3)*(b*x+a)^(
7/3)/b-112/243*a^4*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^(1/3)*3^(1/2)/(b*x+a)^
(1/3))*3^(1/2)/b+56/243*a^4*ln(b*x+a)/b+56/81*a^4*ln(1+(-b*x+a)^(1/3)/(b*x
+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx = \frac{3(a-bx)^{2/3}\sqrt[3]{a+bx}(-263a^3+87a^2bx+207ab^2x^2+81b^3x^3)-448\sqrt{3}a^4\arctan\left(\frac{\sqrt{3}\sqrt[3]{a}}{-2\sqrt[3]{a-b}}\right.}{}$$

input
```
Integrate[(a - b*x)^(2/3)*(a + b*x)^(7/3),x]
```

output
```
(3*(a - b*x)^(2/3)*(a + b*x)^(1/3)*(-263*a^3 + 87*a^2*b*x + 207*a*b^2*x^2
+ 81*b^3*x^3) - 448*Sqrt[3]*a^4*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a -
b*x)^(1/3) + (a + b*x)^(1/3))] + 448*a^4*Log[(a - b*x)^(1/3) + (a + b*x)^(
1/3)] - 224*a^4*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a
 + b*x)^(2/3)])/(972*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {60, 60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{7}{6}a\int (a-bx)^{2/3}(a+bx)^{4/3}dx-\frac{(a-bx)^{5/3}(a+bx)^{7/3}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{7}{6}a\left(\frac{8}{9}a\int (a-bx)^{2/3}\sqrt[3]{a+bx}dx-\frac{(a-bx)^{5/3}(a+bx)^{4/3}}{3b}\right)-\frac{(a-bx)^{5/3}(a+bx)^{7/3}}{4b}$$

$$\Big\downarrow \; 60$$

$$\frac{7}{6}a\left(\frac{8}{9}a\left(\frac{1}{3}a\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}dx-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}\right)-\frac{(a-bx)^{5/3}(a+bx)^{4/3}}{3b}\right)-\frac{(a-bx)^{5/3}(a+bx)^{7/3}}{4b}$$

$$\Big\downarrow \; 60$$

$$\frac{7}{6}a\left(\frac{8}{9}a\left(\frac{1}{3}a\left(\frac{4}{3}a\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx+\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}\right)-\frac{(a-bx)^{5/3}(a+}{3b}\right.$$
$$\frac{(a-bx)^{5/3}(a+bx)^{7/3}}{4b}$$

$$\Big\downarrow \; 72$$

$$\frac{7}{6}a\left(\frac{8}{9}a\left(\frac{1}{3}a\left(\frac{4}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)+\frac{(a-bx)^{2/3}\sqrt[3]{a}}{b}\right.\right.\right.$$
$$\frac{(a-bx)^{5/3}(a+bx)^{7/3}}{4b}$$

input
```
Int[(a - b*x)^(2/3)*(a + b*x)^(7/3),x]
```

output
```
-1/4*((a - b*x)^(5/3)*(a + b*x)^(7/3))/b + (7*a*(-1/3*((a - b*x)^(5/3)*(a
+ b*x)^(4/3))/b + (8*a*(-1/2*((a - b*x)^(5/3)*(a + b*x)^(1/3))/b + (a*(((a
 - b*x)^(2/3)*(a + b*x)^(1/3))/b + (4*a*((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a
 - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b + Log[a + b*x]/(2*b) + (3*Log
[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b)))/3))/3))/9))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx+a)^{\frac{2}{3}} (bx+a)^{\frac{7}{3}} \, dx$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(7/3),x)
```

output
```
int((-b*x+a)^(2/3)*(b*x+a)^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97

$$\int (a-bx)^{2/3}(a+bx)^{7/3} \, dx = -\frac{448\sqrt{3}a^4 \arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right) + 224\,a^4 \log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right) - 4}{972}$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-1/972*(448*sqrt(3)*a^4*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a
)^(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + 224*a^4*log((b*x - (b*x + a)^(2/3)*
(-b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 448*
a^4*log(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 3*(81*b
^3*x^3 + 207*a*b^2*x^2 + 87*a^2*b*x - 263*a^3)*(b*x + a)^(1/3)*(-b*x + a)^
(2/3))/b
```

Sympy [F]

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx=\int (a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{7}{3}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)*(b*x+a)**(7/3),x)
```

output
```
Integral((a - b*x)**(2/3)*(a + b*x)**(7/3), x)
```

Maxima [F]

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx=\int (bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(7/3)*(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx=\int (bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(7/3)*(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx=\int (a+b\,x)^{7/3}\,(a-b\,x)^{2/3}\,dx$$

input
```
int((a + b*x)^(7/3)*(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(7/3)*(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int (a-bx)^{2/3}(a+bx)^{7/3}\,dx=\frac{-711(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^3+87(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^2bx+207(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a\,b^2}{324b}$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(7/3),x)
```

output
```
( - 711*(a + b*x)**(1/3)*(a - b*x)**(2/3)*a**3 + 87*(a + b*x)**(1/3)*(a -
b*x)**(2/3)*a**2*b*x + 207*(a + b*x)**(1/3)*(a - b*x)**(2/3)*a*b**2*x**2 +
 81*(a + b*x)**(1/3)*(a - b*x)**(2/3)*b**3*x**3 - 448*int(((a + b*x)**(1/3
)*(a - b*x)**(2/3)*x)/(a**2 - b**2*x**2),x)*a**3*b**2)/(324*b)
```

3.154 $\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx = \frac{8a^2(a-bx)^{2/3}\sqrt[3]{a+bx}}{27b} - \frac{4a(a-bx)^{5/3}\sqrt[3]{a+bx}}{9b} - \frac{(a-bx)^{5/3}(a+bx)^{4/3}}{3b} + \frac{32a^3\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{27\sqrt{3}b} + \frac{16a^3\log(a+bx)}{81b} + \frac{16a^3\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{27b}$$

output
```
8/27*a^2*(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b-4/9*a*(-b*x+a)^(5/3)*(b*x+a)^(1/3)
/b-1/3*(-b*x+a)^(5/3)*(b*x+a)^(4/3)/b-32/81*a^3*arctan(-1/3*3^(1/2)+2/3*(-
b*x+a)^(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2)/b+16/81*a^3*ln(b*x+a)/b+16/27*
a^3*ln(1+(-b*x+a)^(1/3)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.95

$$\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx=\frac{3(a-bx)^{2/3}\sqrt[3]{a+bx}(-13a^2+12abx+9b^2x^2)-32\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)+3}{81b}$$

input
```
Integrate[(a - b*x)^(2/3)*(a + b*x)^(4/3),x]
```

output
```
(3*(a - b*x)^(2/3)*(a + b*x)^(1/3)*(-13*a^2 + 12*a*b*x + 9*b^2*x^2) - 32*S
qrt[3]*a^3*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x
)^(1/3))] + 32*a^3*Log[(a - b*x)^(1/3) + (a + b*x)^(1/3)] - 16*a^3*Log[(a
- b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(81*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx$$

$$\Big\downarrow \ 60$$

$$\frac{8}{9}a\int (a-bx)^{2/3}\sqrt[3]{a+bx}dx-\frac{(a-bx)^{5/3}(a+bx)^{4/3}}{3b}$$

$$\Big\downarrow \ 60$$

$$\frac{8}{9}a\left(\frac{1}{3}a\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}dx-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}\right)-\frac{(a-bx)^{5/3}(a+bx)^{4/3}}{3b}$$

$$\Big\downarrow \ 60$$

$$\frac{8}{9}a\left(\frac{1}{3}a\left(\frac{4}{3}a\int\frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx+\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}\right)-\frac{(a-bx)^{5/3}(a+bx)^{4/3}}{3b}$$

$$\Big\downarrow \text{72}$$

$$\frac{8}{9}a\left(\frac{1}{3}a\left(\frac{4}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)+\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right.\right.$$

$$\frac{(a-bx)^{5/3}(a+bx)^{4/3}}{3b}$$

input
```
Int[(a - b*x)^(2/3)*(a + b*x)^(4/3),x]
```

output
```
-1/3*((a - b*x)^(5/3)*(a + b*x)^(4/3))/b + (8*a*(-1/2*((a - b*x)^(5/3)*(a
+ b*x)^(1/3))/b + (a*(((a - b*x)^(2/3)*(a + b*x)^(1/3))/b + (4*a*((Sqrt[3]
*ArcTan[1/Sqrt[3] - (2*(a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b + Lo
g[a + b*x]/(2*b) + (3*Log[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b)))/3)
)/3))/9
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx+a)^{\frac{2}{3}} (bx+a)^{\frac{4}{3}} \, dx$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(4/3),x)
```

output
```
int((-b*x+a)^(2/3)*(b*x+a)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06

$$\int (a-bx)^{2/3}(a+bx)^{4/3} \, dx =$$

$$-\frac{32\sqrt{3}a^3 \arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right) + 16\,a^3 \log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right) - 32\,a}{81\,b}$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
-1/81*(32*sqrt(3)*a^3*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a)^
(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + 16*a^3*log((b*x - (b*x + a)^(2/3)*(-b
*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 32*a^3*
log(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 3*(9*b^2*x^
2 + 12*a*b*x - 13*a^2)*(b*x + a)^(1/3)*(-b*x + a)^(2/3))/b
```

Sympy [F]

$$\int (a-bx)^{2/3}(a+bx)^{4/3} \, dx = \int (a-bx)^{\frac{2}{3}} (a+bx)^{\frac{4}{3}} \, dx$$

input
```
integrate((-b*x+a)**(2/3)*(b*x+a)**(4/3),x)
```

output
```
Integral((a - b*x)**(2/3)*(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx = \int (bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(4/3)*(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx = \int (bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(4/3)*(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx = \int (a+b\,x)^{4/3}\,(a-b\,x)^{2/3}\,dx$$

input
```
int((a + b*x)^(4/3)*(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(4/3)*(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int (a-bx)^{2/3}(a+bx)^{4/3}\,dx = \frac{-45(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^2+12(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}abx+9(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}b^2x^2-3}{27b}$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(4/3),x)
```

output
```
( - 45*(a + b*x)**(1/3)*(a - b*x)**(2/3)*a**2 + 12*(a + b*x)**(1/3)*(a - b
*x)**(2/3)*a*b*x + 9*(a + b*x)**(1/3)*(a - b*x)**(2/3)*b**2*x**2 - 32*int(
((a + b*x)**(1/3)*(a - b*x)**(2/3)*x)/(a**2 - b**2*x**2),x)*a**2*b**2)/(27
*b)
```

3.155 $\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx=\frac{a(a-bx)^{2/3}\sqrt[3]{a+bx}}{3b}-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}+\frac{4a^2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b}+\frac{2a^2\log(a+bx)}{9b}+\frac{2a^2\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{3b}$$

output
```
1/3*a*(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b-1/2*(-b*x+a)^(5/3)*(b*x+a)^(1/3)/b-4/
9*a^2*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2
)/b+2/9*a^2*ln(b*x+a)/b+2/3*a^2*ln(1+(-b*x+a)^(1/3)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx=\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}(-a+3bx)}{6b}-\frac{4a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b}+\frac{4a^2\log\left(b\sqrt[3]{a-bx}+b\sqrt[3]{a+bx}\right)}{9b}-\frac{2a^2\log\left((a-bx)^{2/3}-\sqrt[3]{a-bx}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9b}$$

input
```
Integrate[(a - b*x)^(2/3)*(a + b*x)^(1/3),x]
```

output
```
((a - b*x)^(2/3)*(a + b*x)^(1/3)*(-a + 3*b*x))/(6*b) - (4*a^2*ArcTan[(Sqrt
[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))])/(3*Sqrt[3]*b
) + (4*a^2*Log[b*(a - b*x)^(1/3) + b*(a + b*x)^(1/3)])/(9*b) - (2*a^2*Log[
(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*b
)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}a\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}dx-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}a\left(\frac{4}{3}a\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx+\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}$$

$$\Big\downarrow \text{ 72}$$

$$\frac{1}{3}a\left(\frac{4}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)+\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{5/3}\sqrt[3]{a+bx}}{2b}$$

input
```
Int[(a - b*x)^(2/3)*(a + b*x)^(1/3),x]
```

output
```
-1/2*((a - b*x)^(5/3)*(a + b*x)^(1/3))/b + (a*(((a - b*x)^(2/3)*(a + b*x)^
(1/3))/b + (4*a*((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a - b*x)^(1/3))/(Sqrt[3]*
(a + b*x)^(1/3))])/b + Log[a + b*x]/(2*b) + (3*Log[1 + (a - b*x)^(1/3)/(a
+ b*x)^(1/3)])/(2*b)))/3))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx + a)^{\frac{2}{3}} (bx + a)^{\frac{1}{3}} dx$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(1/3),x)
```

output
```
int((-b*x+a)^(2/3)*(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx = -\frac{8\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+4\,a^2\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right)-8\,a^2\,\mathrm{lo}}{18\,b}$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/18*(8*sqrt(3)*a^2*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a)^(
1/3)*(-b*x + a)^(2/3))/(b*x - a)) + 4*a^2*log((b*x - (b*x + a)^(2/3)*(-b*x
 + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 8*a^2*log
(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 3*(3*b*x - a)*
(b*x + a)^(1/3)*(-b*x + a)^(2/3))/b
```

Sympy [F]

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx = \int (a-bx)^{\frac{2}{3}}\sqrt[3]{a+bx}\,dx$$

input
```
integrate((-b*x+a)**(2/3)*(b*x+a)**(1/3),x)
```

output
```
Integral((a - b*x)**(2/3)*(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx=\int (bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/3)*(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx=\int (bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/3)*(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx=\int (a+b\,x)^{1/3}\,(a-b\,x)^{2/3}\,dx$$

input
```
int((a + b*x)^(1/3)*(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(1/3)*(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int (a-bx)^{2/3}\sqrt[3]{a+bx}\,dx = \frac{-9(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a+3(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}bx-8\left(\int\frac{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}x}{-b^2x^2+a^2}dx\right)a}{6b}$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(1/3),x)
```

output
```
( - 9*(a + b*x)**(1/3)*(a - b*x)**(2/3)*a + 3*(a + b*x)**(1/3)*(a - b*x)**
(2/3)*b*x - 8*int(((a + b*x)**(1/3)*(a - b*x)**(2/3)*x)/(a**2 - b**2*x**2)
,x)*a*b**2)/(6*b)
```

3.156 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}} \, dx = \frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b} + \frac{4a \arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{\sqrt{3}b} + \frac{2a\log(a+bx)}{3b} + \frac{2a\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{b}$$

output
```
(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b-4/3*a*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^(1/3
)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2)/b+2/3*a*ln(b*x+a)/b+2*a*ln(1+(-b*x+a)^(1/
3)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}} \, dx = \frac{3(a-bx)^{2/3}\sqrt[3]{a+bx} - 4\sqrt{3}a \arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) + 4a\log\left(b\left(\sqrt[3]{a-bx}+\right.\right.}{3b}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(2/3),x]
```

output
```
(3*(a - b*x)^(2/3)*(a + b*x)^(1/3) - 4*Sqrt[3]*a*ArcTan[(Sqrt[3]*(a + b*x)
^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] + 4*a*Log[b*((a - b*x)^(1/
3) + (a + b*x)^(1/3))] - 2*a*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*
x)^(1/3) + (a + b*x)^(2/3)])/(3*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4}{3}a\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx+\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}$$

$$\Big\downarrow \text{72}$$

$$\frac{4}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)+\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(2/3),x]
```

output
```
((a - b*x)^(2/3)*(a + b*x)^(1/3))/b + (4*a*((Sqrt[3]*ArcTan[1/Sqrt[3] - (2
*(a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b + Log[a + b*x]/(2*b) + (3*
Log[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b)))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{2}{3}}}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(2/3),x)
```

output
```
int((-b*x+a)^(2/3)/(b*x+a)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.50

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}\,dx =$$

$$-\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+2\,a\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right)-4\,a\log\left(\right.}{3\,b}$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(3)*a*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a)^(1/3
)*(-b*x + a)^(2/3))/(b*x - a)) + 2*a*log((b*x - (b*x + a)^(2/3)*(-b*x + a)
^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 4*a*log(-(b*x
- (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 3*(b*x + a)^(1/3)*(-b
*x + a)^(2/3))/b
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}\,dx = \int \frac{(a-bx)^{\frac{2}{3}}}{(a+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(2/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}\,dx=\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}\,dx=\int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{2/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(2/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{2/3}}\,dx=\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{2}{3}}}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(2/3),x)
```

output
```
int((a - b*x)**(2/3)/(a + b*x)**(2/3),x)
```

3.157 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = -\frac{3(a-bx)^{2/3}}{2b(a+bx)^{2/3}} - \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b} - \frac{\log(a+bx)}{2b} - \frac{3\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{2b}$$

output
```
-3/2*(-b*x+a)^(2/3)/b/(b*x+a)^(2/3)+3^(1/2)*arctan(-1/3*3^(1/2)+2/3*(-b*x+
a)^(1/3)*3^(1/2)/(b*x+a)^(1/3))/b-1/2*ln(b*x+a)/b-3/2*ln(1+(-b*x+a)^(1/3)/
(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = \frac{-\frac{3(a-bx)^{2/3}}{(a+bx)^{2/3}} + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 2\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right)}{2b}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(5/3),x]
```

output
```
((-3*(a - b*x)^(2/3))/(a + b*x)^(2/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x
)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 2*Log[b*((a - b*x)^(1/3
) + (a + b*x)^(1/3))] + Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1
/3) + (a + b*x)^(2/3)])/(2*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {57, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}dx$$

$$\Big\downarrow \text{57}$$

$$-\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx-\frac{3(a-bx)^{2/3}}{2b(a+bx)^{2/3}}$$

$$\Big\downarrow \text{72}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}-\frac{3(a-bx)^{2/3}}{2b(a+bx)^{2/3}}-\frac{\log(a+bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(5/3),x]
```

output
```
(-3*(a - b*x)^(2/3))/(2*b*(a + b*x)^(2/3)) - (Sqrt[3]*ArcTan[1/Sqrt[3] - (
2*(a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b - Log[a + b*x]/(2*b) - (3
*Log[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{5}{3}}}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(5/3),x)
```

output
```
int((-b*x+a)^(2/3)/(b*x+a)^(5/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(90) = 180.

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.64

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = \frac{2\sqrt{3}(bx+a)\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+(bx+a)\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{bx\cdots}\right.}{2\,(b^2\cdots}$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*(b*x + a)*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x +
a)^(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + (b*x + a)*log((b*x - (b*x + a)^(2/
3)*(-b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 2
*(b*x + a)*log(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) -
3*(b*x + a)^(1/3)*(-b*x + a)^(2/3))/(b^2*x + a*b)
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = \int \frac{(a-bx)^{\frac{2}{3}}}{(a+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(5/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(5/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = \int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{5/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(5/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{2}{3}}\,a+(bx+a)^{\frac{2}{3}}\,bx}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(5/3),x)
```

output
```
int((a - b*x)**(2/3)/((a + b*x)**(2/3)*a + (a + b*x)**(2/3)*b*x),x)
```

3.158 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx=-\frac{3(a-bx)^{5/3}}{10ab(a+bx)^{5/3}}$$

output
```
-3/10*(-b*x+a)^(5/3)/a/b/(b*x+a)^(5/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx=-\frac{3(a-bx)^{5/3}}{10ab(a+bx)^{5/3}}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(8/3),x]
```

output
```
(-3*(a - b*x)^(5/3))/(10*a*b*(a + b*x)^(5/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}} \, dx$$

$$\Big\downarrow \; 48$$

$$-\frac{3(a-bx)^{5/3}}{10ab(a+bx)^{5/3}}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(8/3),x]
```

output
```
(-3*(a - b*x)^(5/3))/(10*a*b*(a + b*x)^(5/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{5}{3}}}{10ab(bx+a)^{\frac{5}{3}}}$	24
orering	$-\frac{3(-bx+a)^{\frac{5}{3}}}{10ab(bx+a)^{\frac{5}{3}}}$	24

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(8/3),x,method=_RETURNVERBOSE)
```

output
```
-3/10*(-b*x+a)^(5/3)/a/b/(b*x+a)^(5/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx = \frac{3\,(bx+a)^{\frac{1}{3}}(bx-a)(-bx+a)^{\frac{2}{3}}}{10\,(ab^3x^2+2\,a^2b^2x+a^3b)}$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

output
```
3/10*(b*x + a)^(1/3)*(b*x - a)*(-b*x + a)^(2/3)/(a*b^3*x^2 + 2*a^2*b^2*x +
 a^3*b)
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx = \int \frac{(a-bx)^{\frac{2}{3}}}{(a+bx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(8/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(8/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(8/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx = \int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{8/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(8/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(8/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}\,dx = \frac{3(-bx+a)^{\frac{2}{3}}\,(bx-a)}{10\,(bx+a)^{\frac{5}{3}}\,ab}$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(8/3),x)
```

output
```
(3*(a - b*x)**(2/3)*( - a + b*x))/(10*(a + b*x)**(2/3)*a*b*(a + b*x))
```

3.159 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}} \, dx = -\frac{3(a-bx)^{5/3}}{16ab(a+bx)^{8/3}} - \frac{9(a-bx)^{5/3}}{160a^2b(a+bx)^{5/3}}$$

output
```
-3/16*(-b*x+a)^(5/3)/a/b/(b*x+a)^(8/3)-9/160*(-b*x+a)^(5/3)/a^2/b/(b*x+a)^
(5/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}} \, dx = -\frac{3(a-bx)^{5/3}(13a+3bx)}{160a^2b(a+bx)^{8/3}}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(11/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*(13*a + 3*b*x))/(160*a^2*b*(a + b*x)^(8/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}dx}{16a}-\frac{3(a-bx)^{5/3}}{16ab(a+bx)^{8/3}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{9(a-bx)^{5/3}}{160a^2b(a+bx)^{5/3}}-\frac{3(a-bx)^{5/3}}{16ab(a+bx)^{8/3}}$$

input

```
Int[(a - b*x)^(2/3)/(a + b*x)^(11/3),x]
```

output

```
(-3*(a - b*x)^(5/3))/(16*a*b*(a + b*x)^(8/3)) - (9*(a - b*x)^(5/3))/(160*a
^2*b*(a + b*x)^(5/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{5}{3}}(3bx+13a)}{160(bx+a)^{\frac{8}{3}}a^2b}$	32
orering	$-\frac{3(-bx+a)^{\frac{5}{3}}(3bx+13a)}{160(bx+a)^{\frac{8}{3}}a^2b}$	32

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(11/3),x,method=_RETURNVERBOSE)
```

output
```
-3/160*(-b*x+a)^(5/3)*(3*b*x+13*a)/(b*x+a)^(8/3)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}\,dx = \frac{3\,(3\,b^2x^2+10\,abx-13\,a^2)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{160\,(a^2b^4x^3+3\,a^3b^3x^2+3\,a^4b^2x+a^5b)}$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(11/3),x, algorithm="fricas")
```

output
```
3/160*(3*b^2*x^2 + 10*a*b*x - 13*a^2)*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a^
2*b^4*x^3 + 3*a^3*b^3*x^2 + 3*a^4*b^2*x + a^5*b)
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(a-bx)^{\frac{2}{3}}}{(a+bx)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(11/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(11/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(11/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(11/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(11/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{11/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(11/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(11/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}\,dx = \frac{3(-bx+a)^{\frac{2}{3}}\,(3b^2x^2+10abx-13a^2)}{160\,(bx+a)^{\frac{2}{3}}\,a^2b\,(b^2x^2+2abx+a^2)}$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(11/3),x)
```

output
```
(3*(a - b*x)**(2/3)*( - 13*a**2 + 10*a*b*x + 3*b**2*x**2))/(160*(a + b*x)*
*(2/3)*a**2*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.160 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}} \, dx = -\frac{3(a-bx)^{5/3}}{22ab(a+bx)^{11/3}} - \frac{9(a-bx)^{5/3}}{176a^2b(a+bx)^{8/3}} - \frac{27(a-bx)^{5/3}}{1760a^3b(a+bx)^{5/3}}$$

output
```
-3/22*(-b*x+a)^(5/3)/a/b/(b*x+a)^(11/3)-9/176*(-b*x+a)^(5/3)/a^2/b/(b*x+a)
^(8/3)-27/1760*(-b*x+a)^(5/3)/a^3/b/(b*x+a)^(5/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}} \, dx = -\frac{3(a-bx)^{5/3} (119a^2 + 48abx + 9b^2x^2)}{1760a^3b(a+bx)^{11/3}}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(14/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*(119*a^2 + 48*a*b*x + 9*b^2*x^2))/(1760*a^3*b*(a + b*x
)^(11/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{3\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}dx}{11a}-\frac{3(a-bx)^{5/3}}{22ab(a+bx)^{11/3}}$$

$$\Big\downarrow \text{55}$$

$$\frac{3\left(\frac{3\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}dx}{16a}-\frac{3(a-bx)^{5/3}}{16ab(a+bx)^{8/3}}\right)}{11a}-\frac{3(a-bx)^{5/3}}{22ab(a+bx)^{11/3}}$$

$$\Big\downarrow \text{48}$$

$$\frac{3\left(-\frac{9(a-bx)^{5/3}}{160a^2b(a+bx)^{5/3}}-\frac{3(a-bx)^{5/3}}{16ab(a+bx)^{8/3}}\right)}{11a}-\frac{3(a-bx)^{5/3}}{22ab(a+bx)^{11/3}}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(14/3),x]
```

output
```
(-3*(a - b*x)^(5/3))/(22*a*b*(a + b*x)^(11/3)) + (3*((-3*(a - b*x)^(5/3))/
(16*a*b*(a + b*x)^(8/3)) - (9*(a - b*x)^(5/3))/(160*a^2*b*(a + b*x)^(5/3))
))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{5}{3}}\left(9b^{2}x^{2}+48abx+119a^{2}\right)}{1760(bx+a)^{\frac{11}{3}}a^{3}b}$	43
orering	$-\frac{3(-bx+a)^{\frac{5}{3}}\left(9b^{2}x^{2}+48abx+119a^{2}\right)}{1760(bx+a)^{\frac{11}{3}}a^{3}b}$	43

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(14/3),x,method=_RETURNVERBOSE)
```

output
```
-3/1760*(-b*x+a)^(5/3)*(9*b^2*x^2+48*a*b*x+119*a^2)/(b*x+a)^(11/3)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}\,dx = \frac{3\,(9\,b^3x^3+39\,ab^2x^2+71\,a^2bx-119\,a^3)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{1760\,(a^3b^5x^4+4\,a^4b^4x^3+6\,a^5b^3x^2+4\,a^6b^2x+a^7b)}$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(14/3),x, algorithm="fricas")
```

output
```
3/1760*(9*b^3*x^3 + 39*a*b^2*x^2 + 71*a^2*b*x - 119*a^3)*(b*x + a)^(1/3)*(
-b*x + a)^(2/3)/(a^3*b^5*x^4 + 4*a^4*b^4*x^3 + 6*a^5*b^3*x^2 + 4*a^6*b^2*x
 + a^7*b)
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}\,dx = \int \frac{(a-bx)^{\frac{2}{3}}}{(a+bx)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(14/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(14/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(14/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(14/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(14/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(14/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}\,dx = \int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{14/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(14/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(14/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}\,dx = \frac{3(-bx+a)^{\frac{2}{3}}\,(9b^3x^3+39a\,b^2x^2+71a^2bx-119a^3)}{1760\,(bx+a)^{\frac{2}{3}}\,a^3b\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(14/3),x)
```

output
```
(3*(a - b*x)**(2/3)*( - 119*a**3 + 71*a**2*b*x + 39*a*b**2*x**2 + 9*b**3*x
**3))/(1760*(a + b*x)**(2/3)*a**3*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b
**3*x**3))
```

3.161 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}} dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}} dx = -\frac{3(a-bx)^{5/3}}{28ab(a+bx)^{14/3}} - \frac{27(a-bx)^{5/3}}{616a^2b(a+bx)^{11/3}} - \frac{81(a-bx)^{5/3}}{4928a^3b(a+bx)^{8/3}} - \frac{243(a-bx)^{5/3}}{49280a^4b(a+bx)^{5/3}}$$

output
```
-3/28*(-b*x+a)^(5/3)/a/b/(b*x+a)^(14/3)-27/616*(-b*x+a)^(5/3)/a^2/b/(b*x+a
)^(11/3)-81/4928*(-b*x+a)^(5/3)/a^3/b/(b*x+a)^(8/3)-243/49280*(-b*x+a)^(5/
3)/a^4/b/(b*x+a)^(5/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}} dx = -\frac{3(a-bx)^{5/3}(2831a^3 + 1503a^2bx + 513ab^2x^2 + 81b^3x^3)}{49280a^4b(a+bx)^{14/3}}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(17/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*(2831*a^3 + 1503*a^2*b*x + 513*a*b^2*x^2 + 81*b^3*x^3)
)/(49280*a^4*b*(a + b*x)^(14/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}}\,dx$$

$$\Big\downarrow \; 55$$

$$\frac{9\int \frac{(a-bx)^{2/3}}{(a+bx)^{14/3}}dx}{28a}-\frac{3(a-bx)^{5/3}}{28ab(a+bx)^{14/3}}$$

$$\Big\downarrow \; 55$$

$$\frac{9\left(\frac{3\int \frac{(a-bx)^{2/3}}{(a+bx)^{11/3}}dx}{11a}-\frac{3(a-bx)^{5/3}}{22ab(a+bx)^{11/3}}\right)}{28a}-\frac{3(a-bx)^{5/3}}{28ab(a+bx)^{14/3}}$$

$$\Big\downarrow \; 55$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{(a-bx)^{2/3}}{(a+bx)^{8/3}}dx}{16a}-\frac{3(a-bx)^{5/3}}{16ab(a+bx)^{8/3}}\right)}{11a}-\frac{3(a-bx)^{5/3}}{22ab(a+bx)^{11/3}}\right)}{28a}-\frac{3(a-bx)^{5/3}}{28ab(a+bx)^{14/3}}$$

$$\Big\downarrow \; 48$$

$$\frac{9\left(\frac{3\left(-\frac{9(a-bx)^{5/3}}{160a^2b(a+bx)^{5/3}}-\frac{3(a-bx)^{5/3}}{16ab(a+bx)^{8/3}}\right)}{11a}-\frac{3(a-bx)^{5/3}}{22ab(a+bx)^{11/3}}\right)}{28a}-\frac{3(a-bx)^{5/3}}{28ab(a+bx)^{14/3}}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(17/3),x]
```

output
```
(-3*(a - b*x)^(5/3))/(28*a*b*(a + b*x)^(14/3)) + (9*((-3*(a - b*x)^(5/3))/
(22*a*b*(a + b*x)^(11/3)) + (3*((-3*(a - b*x)^(5/3))/(16*a*b*(a + b*x)^(8/
3)) - (9*(a - b*x)^(5/3))/(160*a^2*b*(a + b*x)^(5/3))))/(11*a)))/(28*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{5}{3}}\left(81b^{3}x^{3}+513a\,b^{2}x^{2}+1503a^{2}bx+2831a^{3}\right)}{49280(bx+a)^{\frac{14}{3}}a^{4}b}$	54
orering	$-\frac{3(-bx+a)^{\frac{5}{3}}\left(81b^{3}x^{3}+513a\,b^{2}x^{2}+1503a^{2}bx+2831a^{3}\right)}{49280(bx+a)^{\frac{14}{3}}a^{4}b}$	54

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(17/3),x,method=_RETURNVERBOSE)
```

output
```
-3/49280*(-b*x+a)^(5/3)*(81*b^3*x^3+513*a*b^2*x^2+1503*a^2*b*x+2831*a^3)/(
b*x+a)^(14/3)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}}\,dx = \frac{3\,(81\,b^4x^4+432\,ab^3x^3+990\,a^2b^2x^2+1328\,a^3bx-2831\,a^4)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{49280\,(a^4b^6x^5+5\,a^5b^5x^4+10\,a^6b^4x^3+10\,a^7b^3x^2+5\,a^8b^2x+a^9b)}$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(17/3),x, algorithm="fricas")
```

output
```
3/49280*(81*b^4*x^4 + 432*a*b^3*x^3 + 990*a^2*b^2*x^2 + 1328*a^3*b*x - 283
1*a^4)*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a^4*b^6*x^5 + 5*a^5*b^5*x^4 + 10*
a^6*b^4*x^3 + 10*a^7*b^3*x^2 + 5*a^8*b^2*x + a^9*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(17/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{17}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(17/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(17/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}}\,dx=\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{17}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(17/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(17/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}}\,dx=\int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{17/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(17/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(17/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{17/3}}\,dx=\frac{3(-bx+a)^{\frac{2}{3}}\,(81b^4x^4+432a\,b^3x^3+990a^2b^2x^2+1328a^3bx-2831a^4)}{49280\,(bx+a)^{\frac{2}{3}}\,a^4b\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(17/3),x)
```

output
```
(3*(a - b*x)**(2/3)*( - 2831*a**4 + 1328*a**3*b*x + 990*a**2*b**2*x**2 + 4
32*a*b**3*x**3 + 81*b**4*x**4))/(49280*(a + b*x)**(2/3)*a**4*b*(a**4 + 4*a
**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.162 $\int (a-bx)^{2/3}(a+bx)^{5/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 684

$$\int (a-bx)^{2/3}(a$$

$$+bx)^{5/3}\,dx=\frac{3}{7}ax(a-bx)^{2/3}(a+bx)^{2/3}-\frac{3(a-bx)^{5/3}(a+bx)^{5/3}}{10b}-\frac{12a^3x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{7\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right.}$$

output
```
3/7*a*x*(-b*x+a)^(2/3)*(b*x+a)^(2/3)-3/10*(-b*x+a)^(5/3)*(b*x+a)^(5/3)/b-1
2/7*a^3*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1
-b^2*x^2/a^2)^(1/3))-6/7*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^5*(1-b^2*x^2/
a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/
a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticE((1+3^(1/2
)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/
b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-
(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)+4/7*2^(1/2)*3^(3/4)*a^5*(1-b^2*x^2/a^2)^(1
/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2
/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^
2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(
-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*
x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.10

$$\int (a-bx)^{2/3}(a+bx)^{5/3}\,dx = -\frac{6\ 2^{2/3}a(a-bx)^{5/3}(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{5}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{5b\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Integrate[(a - b*x)^(2/3)*(a + b*x)^(5/3),x]
```

output
```
(-6*2^(2/3)*a*(a - b*x)^(5/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-5/3, 5/3,
 8/3, (a - b*x)/(2*a)])/(5*b*((a + b*x)/a)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{2/3}(a+bx)^{5/3} \, dx$$

$$\Big\downarrow \; 80$$

$$\frac{2\ 2^{2/3}a(a+bx)^{2/3}\int \frac{(a-bx)^{2/3}\left(\frac{bx}{a}+1\right)^{5/3}}{2\ 2^{2/3}}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{a(a+bx)^{2/3}\int (a-bx)^{2/3}\left(\frac{bx}{a}+1\right)^{5/3}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \; 79$$

$$-\frac{6\ 2^{2/3}a(a-bx)^{5/3}(a+bx)^{2/3}\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{5}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{5b\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Int[(a - b*x)^(2/3)*(a + b*x)^(5/3),x]
```

output
```
(-6*2^(2/3)*a*(a - b*x)^(5/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-5/3, 5/3,
 8/3, (a - b*x)/(2*a)])/(5*b*((a + b*x)/a)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (-bx+a)^{\frac{2}{3}} (bx+a)^{\frac{5}{3}} \, dx$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(5/3),x)
```

output
```
int((-b*x+a)^(2/3)*(b*x+a)^(5/3),x)
```

Fricas [F]

$$\int (a-bx)^{2/3}(a+bx)^{5/3} \, dx = \int (bx+a)^{\frac{5}{3}} (-bx+a)^{\frac{2}{3}} \, dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(5/3)*(-b*x + a)^(2/3), x)
```

Sympy [F]

$$\int (a-bx)^{2/3}(a+bx)^{5/3}\,dx = \int (a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{5}{3}}\,\,dx$$

input
```
integrate((-b*x+a)**(2/3)*(b*x+a)**(5/3),x)
```

output
```
Integral((a - b*x)**(2/3)*(a + b*x)**(5/3), x)
```

Maxima [F]

$$\int (a-bx)^{2/3}(a+bx)^{5/3}\,dx = \int (bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(5/3)*(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int (a-bx)^{2/3}(a+bx)^{5/3}\,dx = \int (bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(5/3)*(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{2/3}(a+bx)^{5/3}\,dx = \int (a+b\,x)^{5/3}\,(a-b\,x)^{2/3}\,dx$$

input
```
int((a + b*x)^(5/3)*(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(5/3)*(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int (a-bx)^{2/3}(a+bx)^{5/3}\,dx = \frac{-21(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,a^2+30(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,abx+21(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,b^2x^2+}{70b}$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(5/3),x)
```

output
```
( - 21*(a + b*x)**(2/3)*(a - b*x)**(2/3)*a**2 + 30*(a + b*x)**(2/3)*(a - b
*x)**(2/3)*a*b*x + 21*(a + b*x)**(2/3)*(a - b*x)**(2/3)*b**2*x**2 + 40*int
(((a + b*x)**(2/3)*(a - b*x)**(2/3))/(a**2 - b**2*x**2),x)*a**3*b)/(70*b)
```

3.163 $\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 657

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx = \frac{3}{7}x(a-bx)^{2/3}(a+bx)^{2/3} - \frac{12a^2x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{7\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)} - \frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^4\sqrt[3]{1}}{}$$

output
```
3/7*x*(-b*x+a)^(2/3)*(b*x+a)^(2/3)-12/7*a^2*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+
a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))-6/7*3^(1/4)*(1/2*
6^(1/2)+1/2*2^(1/2))*a^4*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*(
(1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)
^(1/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1
-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(
1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)+4/7*2^
(1/2)*3^(3/4)*a^4*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b
^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))
^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^
2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^
2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx = -\frac{3\ 2^{2/3}(a-bx)^{5/3}(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{5b\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Integrate[(a - b*x)^(2/3)*(a + b*x)^(2/3),x]
```

output
```
(-3*2^(2/3)*(a - b*x)^(5/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8
/3, (a - b*x)/(2*a)])/(5*b*((a + b*x)/a)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {46, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx$$

$$\Big\downarrow \ 46$$

$$\frac{(a-bx)^{2/3}(a+bx)^{2/3}\int \left(a^2-b^2x^2\right)^{2/3}dx}{\left(a^2-b^2x^2\right)^{2/3}}$$

$$\Big\downarrow \ 211$$

$$\frac{(a-bx)^{2/3}(a+bx)^{2/3}\left(\frac{4}{7}a^2\int \frac{1}{\sqrt[3]{a^2-b^2x^2}}dx+\frac{3}{7}x\left(a^2-b^2x^2\right)^{2/3}\right)}{\left(a^2-b^2x^2\right)^{2/3}}$$

$$\Big\downarrow \ 233$$

$$\frac{(a-bx)^{2/3}(a+bx)^{2/3}\left(\frac{3}{7}x\left(a^2-b^2x^2\right)^{2/3}-\frac{6a^2\sqrt{-b^2x^2}\int \frac{\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}}{7b^2x}\right)}{\left(a^2-b^2x^2\right)^{2/3}}$$

$$\Big\downarrow \ 833$$

$$\frac{(a-bx)^{2/3}(a+bx)^{2/3}\left(\frac{3}{7}x\left(a^2-b^2x^2\right)^{2/3}-\frac{6a^2\sqrt{-b^2x^2}\left(\left(1+\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}-\int \frac{\left(1+\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2 \cdots}}{\sqrt{-b^2x^2}} \cdots\right.}{7b^2x}\right.}{\left(a^2-b^2x^2\right)^{2/3}}$$

$$\Big\downarrow \ 760$$

$$\frac{(a-bx)^{2/3}(a+bx)^{2/3}\left(\frac{3}{7}x\left(a^2-b^2x^2\right)^{2/3}-\frac{6a^2\sqrt{-b^2x^2}\left(-\int\frac{\left(1+\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)a^{2/}\cdots}{\cdots}\right.}{\cdots}\right.}{\left(a^2-b^2x^2\right)\cdots}$$

$\big\downarrow$ 2418

$$\frac{(a-bx)^{2/3}(a+bx)^{2/3}\left(\frac{3}{7}x\left(a^2-b^2x^2\right)^{2/3}-\frac{6a^2\sqrt{-b^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^{4/3}+\left(a^2-b^2x^2\right)^{2/3}+a\cdots}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{-b^2x^2}\sqrt{-\frac{a^{2/}\cdots}{\left(\left(1-\cdots\right.\right.}}}\right.}{\cdots}\right.}{\cdots}$$

input
```
Int[(a - b*x)^(2/3)*(a + b*x)^(2/3),x]
```

output
```
((a - b*x)^(2/3)*(a + b*x)^(2/3)*((3*x*(a^2 - b^2*x^2)^(2/3))/7 - (6*a^2*S
qrt[-(b^2*x^2)]*((-2*Sqrt[-(b^2*x^2)])/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2
*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(2/3)*(a^(2/3) - (a^2 - b^2*x^
2)^(1/3))*Sqrt[(a^(4/3) + a^(2/3)*(a^2 - b^2*x^2)^(1/3) + (a^2 - b^2*x^2)^
(2/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2]*EllipticE[ArcSin
[((1 + Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))/((1 - Sqrt[3])*a^(2/3) -
(a^2 - b^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b^2*x^2)]*Sqrt[-((a^(2/3
)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x
^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(2/3)*(a^(2/3) - (a
^2 - b^2*x^2)^(1/3))*Sqrt[(a^(4/3) + a^(2/3)*(a^2 - b^2*x^2)^(1/3) + (a^2
- b^2*x^2)^(2/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2]*Ellip
ticF[ArcSin[((1 + Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))/((1 - Sqrt[3])
*a^(2/3) - (a^2 - b^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b^2*x
^2)]*Sqrt[-((a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
2/3) - (a^2 - b^2*x^2)^(1/3))^2)])))/(7*b^2*x)))/(a^2 - b^2*x^2)^(2/3)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int (-bx+a)^{\frac{2}{3}} (bx+a)^{\frac{2}{3}} dx$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(2/3),x)
```

output
```
int((-b*x+a)^(2/3)*(b*x+a)^(2/3),x)
```

Fricas [F]

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx=\int (bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(2/3)*(-b*x + a)^(2/3), x)
```

Sympy [F]

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx=\int (a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{2}{3}}\,\,dx$$

input
```
integrate((-b*x+a)**(2/3)*(b*x+a)**(2/3),x)
```

output
```
Integral((a - b*x)**(2/3)*(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx=\int (bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(2/3)*(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx=\int (bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)*(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(2/3)*(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx=\int (a+b\,x)^{2/3}\,(a-b\,x)^{2/3}\,dx$$

input
```
int((a + b*x)^(2/3)*(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(2/3)*(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int (a-bx)^{2/3}(a+bx)^{2/3}\,dx=\frac{3(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,x}{7}+\frac{4\left(\int\frac{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{2}{3}}}{-b^2x^2+a^2}dx\right)a^2}{7}$$

input
```
int((-b*x+a)^(2/3)*(b*x+a)^(2/3),x)
```

output
```
(3*(a + b*x)**(2/3)*(a - b*x)**(2/3)*x + 4*int(((a + b*x)**(2/3)*(a - b*x)
**(2/3))/(a**2 - b**2*x**2),x)*a**2)/7
```

3.164 $\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 652

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = \frac{3(a-bx)^{2/3}(a+bx)^{2/3}}{4b} - \frac{3ax\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^3\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{2b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$+ \frac{\sqrt{2}3^{3/4}a^3\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3/4*(-b*x+a)^(2/3)*(b*x+a)^(2/3)/b-3*a*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(1
/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))-3/2*3^(1/4)*(1/2*6^(1/
2)+1/2*2^(1/2))*a^3*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1
-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3
))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*
x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-
b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)+2^(1/2)*3^(
3/4)*a^3*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^
2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1
/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2
)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = -\frac{3(a-bx)^{5/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{5\sqrt[3]{2}b\sqrt[3]{a+bx}}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(1/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, (
a - b*x)/(2*a)])/(5*2^(1/3)*b*(a + b*x)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx$$

$$\Big\downarrow \; 80$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{\sqrt[3]{2}(a-bx)^{2/3}}{\sqrt[3]{\frac{bx}{a}+1}}dx}{\sqrt[3]{2}\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{(a-bx)^{2/3}}{\sqrt[3]{\frac{bx}{a}+1}}dx}{\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 79$$

$$-\frac{3(a-bx)^{5/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{5\sqrt[3]{2}b\sqrt[3]{a+bx}}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(1/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, (
a - b*x)/(2*a)])/(5*2^(1/3)*b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{1}{3}}}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(1/3),x)
```

output
```
int((-b*x+a)^(2/3)/(b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((-b*x + a)^(2/3)/(b*x + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(a-bx)^{\frac{2}{3}}}{\sqrt[3]{a+bx}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(1/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{1/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(1/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{2/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{1}{3}}}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(1/3),x)
```

output
```
int((a - b*x)**(2/3)/(a + b*x)**(1/3),x)
```

3.165 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 648

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx = -\frac{3(a-bx)^{2/3}}{b\sqrt[3]{a+bx}} + \frac{6x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$-\frac{2\sqrt{2}3^{3/4}a^2\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3*(-b*x+a)^(2/3)/b/(b*x+a)^(1/3)+6*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(1/3)
/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))+3*3^(1/4)*(1/2*6^(1/2)+1/
2*2^(1/2))*a^2*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*
x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)
^(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a
^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x
^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)-2*2^(1/2)*3^(3/4
)*a^2*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^
(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*El
lipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)
),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(
1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx=-\frac{3(a-bx)^{5/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{10\sqrt[3]{2}ab\sqrt[3]{a+bx}}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(4/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[4/3, 5/3, 8/3, (
a - b*x)/(2*a)])/(10*2^(1/3)*a*b*(a + b*x)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx$$

$$\Big\downarrow \; 80$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{2\sqrt[3]{2}(a-bx)^{2/3}}{\left(\frac{bx}{a}+1\right)^{4/3}}dx}{2\sqrt[3]{2}a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{(a-bx)^{2/3}}{\left(\frac{bx}{a}+1\right)^{4/3}}dx}{a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 79$$

$$-\frac{3(a-bx)^{5/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{10\sqrt[3]{2}ab\sqrt[3]{a+bx}}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(4/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[4/3, 5/3, 8/3, (
a - b*x)/(2*a)])/(10*2^(1/3)*a*b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{4}{3}}}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(4/3),x)
```

output
```
int((-b*x+a)^(2/3)/(b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(a-bx)^{\frac{2}{3}}}{(a+bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(4/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{4/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(4/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{1}{3}}\,a+(bx+a)^{\frac{1}{3}}\,bx}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(4/3),x)
```

output
```
int((a - b*x)**(2/3)/((a + b*x)**(1/3)*a + (a + b*x)**(1/3)*b*x),x)
```

3.166 $\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 684

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx = -\frac{3(a-bx)^{2/3}}{4b(a+bx)^{4/3}} + \frac{3(a-bx)^{2/3}}{4ab\sqrt[3]{a+bx}} - \frac{3x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{4a\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right)}{8b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$+\frac{3^{3/4}a\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right)}{2\sqrt{2}b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/4*(-b*x+a)^(2/3)/b/(b*x+a)^(4/3)+3/4*(-b*x+a)^(2/3)/a/b/(b*x+a)^(1/3)-3
/4*x*(1-b^2*x^2/a^2)^(1/3)/a/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^
2*x^2/a^2)^(1/3))-3/8*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*(1-b^2*x^2/a^2)^
(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^
(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-
b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x
/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^
2*x^2/a^2)^(1/3))^2)^(1/2)+1/4*3^(3/4)*a*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x
^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)
-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3
))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/b^2/x/(-b*x+a)
^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2
)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx=-\frac{3(a-bx)^{5/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{7}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{20\sqrt[3]{2}a^2b\sqrt[3]{a+bx}}$$

input
```
Integrate[(a - b*x)^(2/3)/(a + b*x)^(7/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[5/3, 7/3, 8/3, (
a - b*x)/(2*a)])/(20*2^(1/3)*a^2*b*(a + b*x)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}dx$$

$$\Big\downarrow \ 80$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{4\sqrt[3]{2}(a-bx)^{2/3}}{\left(\frac{bx}{a}+1\right)^{7/3}}dx}{4\sqrt[3]{2}a^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{(a-bx)^{2/3}}{\left(\frac{bx}{a}+1\right)^{7/3}}dx}{a^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \ 79$$

$$-\frac{3(a-bx)^{5/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{7}{3},\frac{8}{3},\frac{a-bx}{2a}\right)}{20\sqrt[3]{2}a^2b\sqrt[3]{a+bx}}$$

input
```
Int[(a - b*x)^(2/3)/(a + b*x)^(7/3),x]
```

output
```
(-3*(a - b*x)^(5/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[5/3, 7/3, 8/3, (
a - b*x)/(2*a)])/(20*2^(1/3)*a^2*b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{7}{3}}}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(7/3),x)
```

output
```
int((-b*x+a)^(2/3)/(b*x+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3), x)
```

Sympy [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx=\int \frac{(a-bx)^{\frac{2}{3}}}{(a+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(2/3)/(b*x+a)**(7/3),x)
```

output
```
Integral((a - b*x)**(2/3)/(a + b*x)**(7/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx=\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx=\int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(2/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(2/3)/(b*x + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx = \int \frac{(a-b\,x)^{2/3}}{(a+b\,x)^{7/3}}\,dx$$

input
```
int((a - b*x)^(2/3)/(a + b*x)^(7/3),x)
```

output
```
int((a - b*x)^(2/3)/(a + b*x)^(7/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{2/3}}{(a+bx)^{7/3}}\,dx = \int \frac{(-bx+a)^{\frac{2}{3}}}{(bx+a)^{\frac{1}{3}}\,a^2+2\,(bx+a)^{\frac{1}{3}}\,abx+(bx+a)^{\frac{1}{3}}\,b^2x^2}dx$$

input
```
int((-b*x+a)^(2/3)/(b*x+a)^(7/3),x)
```

output
```
int((a - b*x)**(2/3)/((a + b*x)**(1/3)*a**2 + 2*(a + b*x)**(1/3)*a*b*x + (
a + b*x)**(1/3)*b**2*x**2),x)
```

3.167 $\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 239

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx = \frac{128a^4\sqrt[3]{a-bx}(a+bx)^{2/3}}{243b} + \frac{16a^3(a-bx)^{4/3}(a+bx)^{2/3}}{81b} - \frac{8a^2(a-bx)^{7/3}(a+bx)^{2/3}}{27b} - \frac{4a(a-bx)^{7/3}(a+bx)^{5/3}}{15b} - \frac{(a-bx)^{7/3}(a+bx)^{8/3}}{5b} - \frac{256a^5\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{243\sqrt{3}b} - \frac{128a^5\log(a-bx)}{729b} - \frac{128a^5\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{243b}$$

output
```
128/243*a^4*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b+16/81*a^3*(-b*x+a)^(4/3)*(b*x+a
)^(2/3)/b-8/27*a^2*(-b*x+a)^(7/3)*(b*x+a)^(2/3)/b-4/15*a*(-b*x+a)^(7/3)*(b
*x+a)^(5/3)/b-1/5*(-b*x+a)^(7/3)*(b*x+a)^(8/3)/b+256/729*a^5*arctan(-1/3*3
^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2)/b-128/729*a^5*ln(
-b*x+a)/b-128/243*a^5*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx =$$

$$-\frac{3\sqrt[3]{a-bx}(a+bx)^{2/3}\left(47a^4-804a^3bx-450a^2b^2x^2+324ab^3x^3+243b^4x^4\right)+1280\sqrt{3}a^5\arctan\left(\frac{\sqrt{3}}{-2\sqrt[3]{a-}}\right.}{}$$

input
```
Integrate[(a - b*x)^(4/3)*(a + b*x)^(8/3),x]
```

output
```
-1/3645*(3*(a - b*x)^(1/3)*(a + b*x)^(2/3)*(47*a^4 - 804*a^3*b*x - 450*a^2
*b^2*x^2 + 324*a*b^3*x^3 + 243*b^4*x^4) + 1280*Sqrt[3]*a^5*ArcTan[(Sqrt[3]
*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] + 1280*a^5*Log[(
a - b*x)^(1/3) + (a + b*x)^(1/3)] - 640*a^5*Log[(a - b*x)^(2/3) - (a - b*x
)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {60, 60, 60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{16}{15}a\int (a-bx)^{4/3}(a+bx)^{5/3}dx-\frac{(a-bx)^{7/3}(a+bx)^{8/3}}{5b}$$

$$\Big\downarrow \text{60}$$

$$\frac{16}{15}a\left(\frac{5}{6}a\int (a-bx)^{4/3}(a+bx)^{2/3}dx-\frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{8/3}}{5b}$$

$$\Big\downarrow \; 60$$

$$\frac{16}{15}a\left(\frac{5}{6}a\left(\frac{4}{9}a\int\frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}dx-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{8/3}}{5b}$$

$$\Big\downarrow \; 60$$

$$\frac{16}{15}a\left(\frac{5}{6}a\left(\frac{4}{9}a\left(\frac{4}{3}a\int\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{5}}{4b}\right. \\ \frac{(a-bx)^{7/3}(a+bx)^{8/3}}{5b}$$

$$\Big\downarrow \; 60$$

$$\frac{16}{15}a\left(\frac{5}{6}a\left(\frac{4}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\int\frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}\right)-\frac{(a-bx}{}\right.\right.\right. \\ \frac{(a-bx)^{7/3}(a+bx)^{8/3}}{5b}$$

$$\Big\downarrow \; 72$$

$$\frac{16}{15}a\left(\frac{5}{6}a\left(\frac{4}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-b}}{}\right.\right.\right.\right. \\ \frac{(a-bx)^{7/3}(a+bx)^{8/3}}{5b}$$

input

```
Int[(a - b*x)^(4/3)*(a + b*x)^(8/3),x]
```

output

```
-1/5*((a - b*x)^(7/3)*(a + b*x)^(8/3))/b + (16*a*(-1/4*((a - b*x)^(7/3)*(a
 + b*x)^(5/3))/b + (5*a*(-1/3*((a - b*x)^(7/3)*(a + b*x)^(2/3))/b + (4*a*(
((a - b*x)^(4/3)*(a + b*x)^(2/3))/(2*b) + (4*a*(((a - b*x)^(1/3)*(a + b*x)
^(2/3))/b + (2*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[
3]*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)
/(a - b*x)^(1/3)])/(2*b)))/3))/3))/9))/6))/15
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx+a)^{\frac{4}{3}} (bx+a)^{\frac{8}{3}} \, dx$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(8/3),x)
```

output
```
int((-b*x+a)^(4/3)*(b*x+a)^(8/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.83

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx = -\frac{1280\,\sqrt{3}a^5\arctan\left(-\frac{\sqrt{3}(bx+a)-2\,\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)+1280\,a^5\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-640\,a^5\log\left(\frac{b\cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(8/3),x, algorithm="fricas")
```

output
```
-1/3645*(1280*sqrt(3)*a^5*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x
+ a)^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 1280*a^5*log((b*x + (b*x + a)^(2
/3)*(-b*x + a)^(1/3) + a)/(b*x + a)) - 640*a^5*log((b*x - (b*x + a)^(2/3)*
(-b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) + 3*(2
43*b^4*x^4 + 324*a*b^3*x^3 - 450*a^2*b^2*x^2 - 804*a^3*b*x + 47*a^4)*(b*x
+ a)^(2/3)*(-b*x + a)^(1/3))/b
```

Sympy [F]

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx = \int (a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{8}{3}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)*(b*x+a)**(8/3),x)
```

output
```
Integral((a - b*x)**(4/3)*(a + b*x)**(8/3), x)
```

Maxima [F]

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx = \int (bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(8/3)*(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx=\int (bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(8/3)*(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx=\int (a+b\,x)^{8/3}\,(a-b\,x)^{4/3}\,dx$$

input
```
int((a + b*x)^(8/3)*(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(8/3)*(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int (a-bx)^{4/3}(a+bx)^{8/3}\,dx=\frac{1233(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^4+804(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^3bx+450(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^2}{}$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(8/3),x)
```

output
```
(1233*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a**4 + 804*(a + b*x)**(2/3)*(a - b
*x)**(1/3)*a**3*b*x + 450*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a**2*b**2*x**2
 - 324*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a*b**3*x**3 - 243*(a + b*x)**(2/3
)*(a - b*x)**(1/3)*b**4*x**4 + 1280*int(((a + b*x)**(2/3)*(a - b*x)**(1/3)
*x)/(a**2 - b**2*x**2),x)*a**4*b**2)/(1215*b)
```

3.168 $\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 210

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx = \frac{40a^3\sqrt[3]{a-bx}(a+bx)^{2/3}}{81b} + \frac{5a^2(a-bx)^{4/3}(a+bx)^{2/3}}{27b} - \frac{5a(a-bx)^{7/3}(a+bx)^{2/3}}{18b} - \frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b} - \frac{80a^4\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{81\sqrt{3}b} - \frac{40a^4\log(a-bx)}{243b} - \frac{40a^4\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{81b}$$

output
```
40/81*a^3*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b+5/27*a^2*(-b*x+a)^(4/3)*(b*x+a)^(
2/3)/b-5/18*a*(-b*x+a)^(7/3)*(b*x+a)^(2/3)/b-1/4*(-b*x+a)^(7/3)*(b*x+a)^(5
/3)/b+80/243*a^4*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1
/3))*3^(1/2)/b-40/243*a^4*ln(-b*x+a)/b-40/81*a^4*ln(1+(b*x+a)^(1/3)/(-b*x+
a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx = \frac{3\sqrt[3]{a-bx}(a+bx)^{2/3}\left(49a^3+201a^2bx-9ab^2x^2-81b^3x^3\right)-320\sqrt{3}a^4\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+}}{-2\sqrt[3]{a-bx}+\sqrt[3]{}}\right.}{}$$

input
```
Integrate[(a - b*x)^(4/3)*(a + b*x)^(5/3),x]
```

output
```
(3*(a - b*x)^(1/3)*(a + b*x)^(2/3)*(49*a^3 + 201*a^2*b*x - 9*a*b^2*x^2 - 8
1*b^3*x^3) - 320*Sqrt[3]*a^4*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x
)^(1/3) + (a + b*x)^(1/3))] - 320*a^4*Log[(a - b*x)^(1/3) + (a + b*x)^(1/3
)] + 160*a^4*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a +
b*x)^(2/3)])/(972*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {60, 60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{6}a\int (a-bx)^{4/3}(a+bx)^{2/3}dx-\frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{6}a\left(\frac{4}{9}a\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}dx-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b}$$

$\downarrow$ 60

$$\frac{5}{6}a\left(\frac{4}{9}a\left(\frac{4}{3}a\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b}$$

$\downarrow$ 60

$$\frac{5}{6}a\left(\frac{4}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}\right)-\frac{(a-bx)^{7/3}(a}{3b}\right.\right.$$
$$\frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b}$$

$\downarrow$ 72

$$\frac{5}{6}a\left(\frac{4}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-bx}(a+}{b}\right.\right.\right.$$
$$\frac{(a-bx)^{7/3}(a+bx)^{5/3}}{4b}$$

input
```
Int[(a - b*x)^(4/3)*(a + b*x)^(5/3),x]
```

output
```
-1/4*((a - b*x)^(7/3)*(a + b*x)^(5/3))/b + (5*a*(-1/3*((a - b*x)^(7/3)*(a
+ b*x)^(2/3))/b + (4*a*(((a - b*x)^(4/3)*(a + b*x)^(2/3))/(2*b) + (4*a*(((
a - b*x)^(1/3)*(a + b*x)^(2/3))/b + (2*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2
*(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b) - (3
*Log[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])/(2*b)))/3))/3))/9))/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx+a)^{\frac{4}{3}} (bx+a)^{\frac{5}{3}} \, dx$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(5/3),x)
```

output
```
int((-b*x+a)^(4/3)*(b*x+a)^(5/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx = -\frac{320\,\sqrt{3}a^4 \arctan\left(-\frac{\sqrt{3}(bx+a)-2\,\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right) + 320\,a^4 \log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right) - 160\,a^4 \log\Bigg(\frac{bx-}{}}{972\,b}$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-1/972*(320*sqrt(3)*a^4*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x +
a)^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 320*a^4*log((b*x + (b*x + a)^(2/3)
*(-b*x + a)^(1/3) + a)/(b*x + a)) - 160*a^4*log((b*x - (b*x + a)^(2/3)*(-b
*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) + 3*(81*b
^3*x^3 + 9*a*b^2*x^2 - 201*a^2*b*x - 49*a^3)*(b*x + a)^(2/3)*(-b*x + a)^(1
/3))/b
```

Sympy [F]

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx=\int (a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{5}{3}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)*(b*x+a)**(5/3),x)
```

output
```
Integral((a - b*x)**(4/3)*(a + b*x)**(5/3), x)
```

Maxima [F]

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx=\int (bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(5/3)*(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx = \int (bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(5/3)*(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx = \int (a+b\,x)^{5/3}\,(a-b\,x)^{4/3}\,dx$$

input
```
int((a + b*x)^(5/3)*(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(5/3)*(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int (a-bx)^{4/3}(a+bx)^{5/3}\,dx = \frac{369(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^3+201(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^2bx-9(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a\,b^2x^2}{324b}$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(5/3),x)
```

output
```
(369*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a**3 + 201*(a + b*x)**(2/3)*(a - b*
x)**(1/3)*a**2*b*x - 9*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a*b**2*x**2 - 81*
(a + b*x)**(2/3)*(a - b*x)**(1/3)*b**3*x**3 + 320*int(((a + b*x)**(2/3)*(a
 - b*x)**(1/3)*x)/(a**2 - b**2*x**2),x)*a**3*b**2)/(324*b)
```

3.169 $\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 181

$$\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx=\frac{16a^2\sqrt[3]{a-bx}(a+bx)^{2/3}}{27b}+\frac{2a(a-bx)^{4/3}(a+bx)^{2/3}}{9b}-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}-\frac{32a^3\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{27\sqrt{3}b}-\frac{16a^3\log(a-bx)}{81b}-\frac{16a^3\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{27b}$$

output
```
16/27*a^2*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b+2/9*a*(-b*x+a)^(4/3)*(b*x+a)^(2/3
)/b-1/3*(-b*x+a)^(7/3)*(b*x+a)^(2/3)/b+32/81*a^3*arctan(-1/3*3^(1/2)+2/3*(
b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2)/b-16/81*a^3*ln(-b*x+a)/b-16/2
7*a^3*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx=\frac{3\sqrt[3]{a-bx}(a+bx)^{2/3}\left(13a^2+12abx-9b^2x^2\right)-32\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-32}{81b}$$

input
```
Integrate[(a - b*x)^(4/3)*(a + b*x)^(2/3),x]
```

output
```
(3*(a - b*x)^(1/3)*(a + b*x)^(2/3)*(13*a^2 + 12*a*b*x - 9*b^2*x^2) - 32*Sq
rt[3]*a^3*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)
^(1/3))] - 32*a^3*Log[(a - b*x)^(1/3) + (a + b*x)^(1/3)] + 16*a^3*Log[(a -
 b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(81*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4}{9}a\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}dx-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4}{9}a\left(\frac{4}{3}a\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\int\frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}\right)-\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}$$

$\downarrow$ 72

$$\frac{4}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-bx}(a+bx)^{2/}}{b}\right.\right.$$

$$\frac{(a-bx)^{7/3}(a+bx)^{2/3}}{3b}$$

input
```
Int[(a - b*x)^(4/3)*(a + b*x)^(2/3),x]
```

output
```
-1/3*((a - b*x)^(7/3)*(a + b*x)^(2/3))/b + (4*a*(((a - b*x)^(4/3)*(a + b*x
)^(2/3))/(2*b) + (4*a*(((a - b*x)^(1/3)*(a + b*x)^(2/3))/b + (2*a*(-((Sqrt
[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3))])/b)
- Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])/(2*b))
)/3))/3))/9
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int (-bx+a)^{\frac{4}{3}} (bx+a)^{\frac{2}{3}} dx$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(2/3),x)
```

output
```
int((-b*x+a)^(4/3)*(b*x+a)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int (a-bx)^{4/3}(a+bx)^{2/3} dx =$$
$$-\frac{32\sqrt{3}a^3 \arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3(bx+a)}\right) + 32a^3 \log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right) - 16a^3 \log\left(\frac{bx-(bx+}{}\right.}{81b}$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/81*(32*sqrt(3)*a^3*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)
^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 32*a^3*log((b*x + (b*x + a)^(2/3)*(-
b*x + a)^(1/3) + a)/(b*x + a)) - 16*a^3*log((b*x - (b*x + a)^(2/3)*(-b*x +
 a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) + 3*(9*b^2*x^
2 - 12*a*b*x - 13*a^2)*(b*x + a)^(2/3)*(-b*x + a)^(1/3))/b
```

Sympy [F]

$$\int (a-bx)^{4/3}(a+bx)^{2/3} dx = \int (a-bx)^{\frac{4}{3}} (a+bx)^{\frac{2}{3}} dx$$

input
```
integrate((-b*x+a)**(4/3)*(b*x+a)**(2/3),x)
```

output
```
Integral((a - b*x)**(4/3)*(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx=\int (bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(2/3)*(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx=\int (bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(2/3)*(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx=\int (a+b\,x)^{2/3}\,(a-b\,x)^{4/3}\,dx$$

input
```
int((a + b*x)^(2/3)*(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(2/3)*(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int (a-bx)^{4/3}(a+bx)^{2/3}\,dx = \frac{45(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^2+12(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}abx-9(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}b^2x^2+32}{27b}$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(2/3),x)
```

output
```
(45*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a**2 + 12*(a + b*x)**(2/3)*(a - b*x)
**(1/3)*a*b*x - 9*(a + b*x)**(2/3)*(a - b*x)**(1/3)*b**2*x**2 + 32*int(((a
 + b*x)**(2/3)*(a - b*x)**(1/3)*x)/(a**2 - b**2*x**2),x)*a**2*b**2)/(27*b)
```

3.170 $\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = \frac{4a\sqrt[3]{a-bx}(a+bx)^{2/3}}{3b}+\frac{(a-bx)^{4/3}(a+bx)^{2/3}}{2b} - \frac{8a^2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{3\sqrt{3}b}-\frac{4a^2\log(a-bx)}{9b}-\frac{4a^2\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{3b}$$

output
```
4/3*a*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b+1/2*(-b*x+a)^(4/3)*(b*x+a)^(2/3)/b+8/
9*a^2*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2
)/b-4/9*a^2*ln(-b*x+a)/b-4/3*a^2*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = \frac{3(11a-3bx)\sqrt[3]{a-bx}(a+bx)^{2/3}-16\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-16a^2\,\mathrm{lo}}{18}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(1/3),x]
```

output
```
(3*(11*a - 3*b*x)*(a - b*x)^(1/3)*(a + b*x)^(2/3) - 16*Sqrt[3]*a^2*ArcTan[
(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 16*a^2
*Log[b*((a - b*x)^(1/3) + (a + b*x)^(1/3))] + 8*a^2*Log[(a - b*x)^(2/3) -
(a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4}{3}a\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}$$

$$\Big\downarrow \text{60}$$

$$\frac{4}{3}a\left(\frac{2}{3}a\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}$$

$$\Big\downarrow \text{72}$$

$$\frac{4}{3}a\left(\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)+\frac{(a+bx)^{2/3}(a-bx)^{4/3}}{2b}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(1/3),x]
```

output
```
((a - b*x)^(4/3)*(a + b*x)^(2/3))/(2*b) + (4*a*(((a - b*x)^(1/3)*(a + b*x)
^(2/3))/b + (2*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[
3]*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)
/(a - b*x)^(1/3)])/(2*b)))/3))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{1}{3}}}dx$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(1/3),x)
```

output
```
int((-b*x+a)^(4/3)/(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = -\frac{16\sqrt{3}a^2\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3(bx+a)}\right)+16\,a^2\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-8\,a^2\log\left(\frac{bx-(bx+a}{}\right.}{18\,b}$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/18*(16*sqrt(3)*a^2*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)
^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 16*a^2*log((b*x + (b*x + a)^(2/3)*(-
b*x + a)^(1/3) + a)/(b*x + a)) - 8*a^2*log((b*x - (b*x + a)^(2/3)*(-b*x +
a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) + 3*(3*b*x - 1
1*a)*(b*x + a)^(2/3)*(-b*x + a)^(1/3))/b
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(a-bx)^{\frac{4}{3}}}{\sqrt[3]{a+bx}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(1/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = \int \frac{(a-b\,x)^{4/3}}{(a+b\,x)^{1/3}}\,dx$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(1/3),x)
```

output
```
int((a - b*x)^(4/3)/(a + b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{4/3}}{\sqrt[3]{a+bx}}\,dx = \left(\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}dx\right)a - \left(\int \frac{(-bx+a)^{\frac{1}{3}}\,x}{(bx+a)^{\frac{1}{3}}}dx\right)b$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(1/3),x)
```

output
```
int((a - b*x)**(1/3)/(a + b*x)**(1/3),x)*a - int(((a - b*x)**(1/3)*x)/(a +
 b*x)**(1/3),x)*b
```

3.171 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = -\frac{3(a-bx)^{4/3}}{b\sqrt[3]{a+bx}} - \frac{4\sqrt[3]{a-bx}(a+bx)^{2/3}}{b} + \frac{8a\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{\sqrt{3}b} + \frac{4a\log(a-bx)}{3b} + \frac{4a\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{b}$$

output
```
-3*(-b*x+a)^(4/3)/b/(b*x+a)^(1/3)-4*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b-8/3*a*a
rctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2)/b+4/3
*a*ln(-b*x+a)/b+4*a*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = \frac{-\frac{3\sqrt[3]{a-bx}(7a+bx)}{\sqrt[3]{a+bx}} + 8\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) + 8a\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{\right.\right.}{3b}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(4/3),x]
```

output
```
((-3*(a - b*x)^(1/3)*(7*a + b*x))/(a + b*x)^(1/3) + 8*Sqrt[3]*a*ArcTan[(Sq
rt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] + 8*a*Log[b
*((a - b*x)^(1/3) + (a + b*x)^(1/3))] - 4*a*Log[(a - b*x)^(2/3) - (a - b*x
)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {57, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx$$

$$\Big\downarrow \; 57$$

$$-4\int \frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}dx-\frac{3(a-bx)^{4/3}}{b\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 60$$

$$-4\left(\frac{2}{3}a\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{3(a-bx)^{4/3}}{b\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 72$$

$$-4\left(\frac{2}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)+\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{3(a-bx)^{4/3}}{b\sqrt[3]{a+bx}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(4/3),x]
```

output
```
(-3*(a - b*x)^(4/3))/(b*(a + b*x)^(1/3)) - 4*(((a - b*x)^(1/3)*(a + b*x)^(
2/3))/b + (2*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[3]
*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)/(
a - b*x)^(1/3)])/(2*b)))/3)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{4}{3}}}dx$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(4/3),x)
```

output
```
int((-b*x+a)^(4/3)/(b*x+a)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.37

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = \frac{8\sqrt{3}(abx+a^2)\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)-3\,(bx+7\,a)(bx+a)^{\frac{2}{3}}(-bx\cdots}{\cdots}$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/3*(8*sqrt(3)*(a*b*x + a^2)*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b
*x + a)^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) - 3*(b*x + 7*a)*(b*x + a)^(2/3)
*(-b*x + a)^(1/3) + 8*(a*b*x + a^2)*log((b*x + (b*x + a)^(2/3)*(-b*x + a)^
(1/3) + a)/(b*x + a)) - 4*(a*b*x + a^2)*log((b*x - (b*x + a)^(2/3)*(-b*x +
 a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)))/(b^2*x + a*b
)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(4/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = \int \frac{(a-b\,x)^{4/3}}{(a+b\,x)^{4/3}}\,dx$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(4/3),x)
```

output
```
int((a - b*x)^(4/3)/(a + b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{4/3}}\,dx = \left(\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}\,a+(bx+a)^{\frac{1}{3}}\,bx}dx\right)a - \left(\int \frac{(-bx+a)^{\frac{1}{3}}\,x}{(bx+a)^{\frac{1}{3}}\,a+(bx+a)^{\frac{1}{3}}\,bx}dx\right)b$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(4/3),x)
```

output
```
int((a - b*x)**(1/3)/((a + b*x)**(1/3)*a + (a + b*x)**(1/3)*b*x),x)*a - in
t(((a - b*x)**(1/3)*x)/((a + b*x)**(1/3)*a + (a + b*x)**(1/3)*b*x),x)*b
```

3.172 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx = -\frac{3(a-bx)^{4/3}}{4b(a+bx)^{4/3}} + \frac{3\sqrt[3]{a-bx}}{b\sqrt[3]{a+bx}} - \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b} - \frac{\log(a-bx)}{2b} - \frac{3\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{2b}$$

output
```
-3/4*(-b*x+a)^(4/3)/b/(b*x+a)^(4/3)+3*(-b*x+a)^(1/3)/b/(b*x+a)^(1/3)+3^(1/
2)*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))/b-1/2*ln(
-b*x+a)/b-3/2*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx = \frac{\frac{3\sqrt[3]{a-bx}(3a+5bx)}{(a+bx)^{4/3}} - 4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 4\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+\right.\right.}{4b}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(7/3),x]
```

output
```
((3*(a - b*x)^(1/3)*(3*a + 5*b*x))/(a + b*x)^(4/3) - 4*Sqrt[3]*ArcTan[(Sqr
t[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 4*Log[b*((
a - b*x)^(1/3) + (a + b*x)^(1/3))] + 2*Log[(a - b*x)^(2/3) - (a - b*x)^(1/
3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(4*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {57, 57, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}dx$$
$$\Big\downarrow \text{57}$$
$$-\int \frac{\sqrt[3]{a-bx}}{(a+bx)^{4/3}}dx-\frac{3(a-bx)^{4/3}}{4b(a+bx)^{4/3}}$$
$$\Big\downarrow \text{57}$$
$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx-\frac{3(a-bx)^{4/3}}{4b(a+bx)^{4/3}}+\frac{3\sqrt[3]{a-bx}}{b\sqrt[3]{a+bx}}$$
$$\Big\downarrow \text{72}$$
$$-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{3(a-bx)^{4/3}}{4b(a+bx)^{4/3}}+\frac{3\sqrt[3]{a-bx}}{b\sqrt[3]{a+bx}}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(7/3),x]
```

output
```
(-3*(a - b*x)^(4/3))/(4*b*(a + b*x)^(4/3)) + (3*(a - b*x)^(1/3))/(b*(a + b
*x)^(1/3)) - (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[3]*(a -
 b*x)^(1/3))])/b - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)/(a - b*
x)^(1/3)])/(2*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{7}{3}}}dx$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(7/3),x)
```

output
```
int((-b*x+a)^(4/3)/(b*x+a)^(7/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(111) = 222$.

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.62

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx =$$

$$-\frac{4\sqrt{3}(b^2x^2+2\,abx+a^2)\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)-3\,(5\,bx+3\,a)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+}{4\,(b^3x}$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(3)*(b^2*x^2 + 2*a*b*x + a^2)*arctan(-1/3*(sqrt(3)*(b*x + a) -
 2*sqrt(3)*(b*x + a)^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) - 3*(5*b*x + 3*a)*
(b*x + a)^(2/3)*(-b*x + a)^(1/3) + 4*(b^2*x^2 + 2*a*b*x + a^2)*log((b*x +
(b*x + a)^(2/3)*(-b*x + a)^(1/3) + a)/(b*x + a)) - 2*(b^2*x^2 + 2*a*b*x +
a^2)*log((b*x - (b*x + a)^(2/3)*(-b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x +
 a)^(2/3) + a)/(b*x + a)))/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx = \int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(7/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(7/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx = \int \frac{(a-b\,x)^{4/3}}{(a+b\,x)^{7/3}}\,dx$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(7/3),x)
```

output
```
int((a - b*x)^(4/3)/(a + b*x)^(7/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{7/3}}\,dx = \frac{-3(-bx+a)^{\frac{1}{3}}a+3(-bx+a)^{\frac{1}{3}}bx-8(bx+a)^{\frac{1}{3}}\left(\int \frac{(-bx+a)^{\frac{1}{3}}x}{(bx+a)^{\frac{1}{3}}a^2+2(bx+a)^{\frac{1}{3}}abx+(bx+a)^{\frac{1}{3}}b^2x^2}dx\right.}{8\,(bx+a)^{\frac{4}{3}}b}$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(7/3),x)
```

output
```
( - 3*(a - b*x)**(1/3)*a + 3*(a - b*x)**(1/3)*b*x - 8*(a + b*x)**(1/3)*int
(((a - b*x)**(1/3)*x)/((a + b*x)**(1/3)*a**2 + 2*(a + b*x)**(1/3)*a*b*x +
(a + b*x)**(1/3)*b**2*x**2),x)*a*b**2 - 8*(a + b*x)**(1/3)*int(((a - b*x)*
*(1/3)*x)/((a + b*x)**(1/3)*a**2 + 2*(a + b*x)**(1/3)*a*b*x + (a + b*x)**(
1/3)*b**2*x**2),x)*b**3*x)/(8*(a + b*x)**(1/3)*b*(a + b*x))
```

3.173 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}} dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}} dx = -\frac{3(a-bx)^{7/3}}{14ab(a+bx)^{7/3}}$$

output
```
-3/14*(-b*x+a)^(7/3)/a/b/(b*x+a)^(7/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}} dx = -\frac{3(a-bx)^{7/3}}{14ab(a+bx)^{7/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(10/3),x]
```

output
```
(-3*(a - b*x)^(7/3))/(14*a*b*(a + b*x)^(7/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}\,dx$$

$$\Big\downarrow \ 48$$

$$-\frac{3(a-bx)^{7/3}}{14ab(a+bx)^{7/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(10/3),x]
```

output
```
(-3*(a - b*x)^(7/3))/(14*a*b*(a + b*x)^(7/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{7}{3}}}{14ab(bx+a)^{\frac{7}{3}}}$	24
orering	$-\frac{3(-bx+a)^{\frac{7}{3}}}{14ab(bx+a)^{\frac{7}{3}}}$	24

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
-3/14*(-b*x+a)^(7/3)/a/b/(b*x+a)^(7/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}\,dx = -\frac{3\,(b^2x^2-2\,abx+a^2)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{14\,(ab^4x^3+3\,a^2b^3x^2+3\,a^3b^2x+a^4b)}$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

output
```
-3/14*(b^2*x^2 - 2*a*b*x + a^2)*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a*b^4*x^
3 + 3*a^2*b^3*x^2 + 3*a^3*b^2*x + a^4*b)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}\,dx = \int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(10/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(10/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(10/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(10/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.62

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}\,dx = -\frac{(a-b\,x)^{1/3}\left(\frac{3\,a}{14\,b^3}-\frac{3\,x}{7\,b^2}+\frac{3\,x^2}{14\,a\,b}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(10/3),x)
```

output
```
-((a - b*x)^(1/3)*((3*a)/(14*b^3) - (3*x)/(7*b^2) + (3*x^2)/(14*a*b)))/(x^
2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*x)^(1/3))/b)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\left(-b^2x^2+2abx-a^2\right)}{14\,(bx+a)^{\frac{1}{3}}\,ab\left(b^2x^2+2abx+a^2\right)}$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(10/3),x)
```

output
```
(3*(a - b*x)**(1/3)*( - a**2 + 2*a*b*x - b**2*x**2))/(14*(a + b*x)**(1/3)*
a*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.174 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx=-\frac{3(a-bx)^{7/3}}{20ab(a+bx)^{10/3}}-\frac{9(a-bx)^{7/3}}{280a^2b(a+bx)^{7/3}}$$

output
```
-3/20*(-b*x+a)^(7/3)/a/b/(b*x+a)^(10/3)-9/280*(-b*x+a)^(7/3)/a^2/b/(b*x+a)
^(7/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx=-\frac{3(a-bx)^{7/3}(17a+3bx)}{280a^2b(a+bx)^{10/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(13/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*(17*a + 3*b*x))/(280*a^2*b*(a + b*x)^(10/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}dx}{20a}-\frac{3(a-bx)^{7/3}}{20ab(a+bx)^{10/3}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{9(a-bx)^{7/3}}{280a^2b(a+bx)^{7/3}}-\frac{3(a-bx)^{7/3}}{20ab(a+bx)^{10/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(13/3),x]
```

output
```
(-3*(a - b*x)^(7/3))/(20*a*b*(a + b*x)^(10/3)) - (9*(a - b*x)^(7/3))/(280*
a^2*b*(a + b*x)^(7/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{7}{3}}(3bx+17a)}{280(bx+a)^{\frac{10}{3}}a^{2}b}$	32
orering	$-\frac{3(-bx+a)^{\frac{7}{3}}(3bx+17a)}{280(bx+a)^{\frac{10}{3}}a^{2}b}$	32

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
-3/280*(-b*x+a)^(7/3)*(3*b*x+17*a)/(b*x+a)^(10/3)/a^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.63

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx = -\frac{3\,(3\,b^3x^3+11\,ab^2x^2-31\,a^2bx+17\,a^3)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{280\,(a^2b^5x^4+4\,a^3b^4x^3+6\,a^4b^3x^2+4\,a^5b^2x+a^6b)}$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

output
```
-3/280*(3*b^3*x^3 + 11*a*b^2*x^2 - 31*a^2*b*x + 17*a^3)*(b*x + a)^(2/3)*(-
b*x + a)^(1/3)/(a^2*b^5*x^4 + 4*a^3*b^4*x^3 + 6*a^4*b^3*x^2 + 4*a^5*b^2*x
+ a^6*b)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx = \int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(13/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(13/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(13/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.78

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx = -\frac{(a-b\,x)^{1/3}\left(\frac{51\,a}{280\,b^4}-\frac{93\,x}{280\,b^3}+\frac{33\,x^2}{280\,a\,b^2}+\frac{9\,x^3}{280\,a^2\,b}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(13/3),x)
```

output
```
-((a - b*x)^(1/3)*((51*a)/(280*b^4) - (93*x)/(280*b^3) + (33*x^2)/(280*a*b
^2) + (9*x^3)/(280*a^2*b)))/(x^3*(a + b*x)^(1/3) + (a^3*(a + b*x)^(1/3))/b
^3 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\left(-3b^3x^3-11a\,b^2x^2+31a^2bx-17a^3\right)}{280\,(bx+a)^{\frac{1}{3}}\,a^2b\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(13/3),x)
```

output
```
(3*(a - b*x)**(1/3)*( - 17*a**3 + 31*a**2*b*x - 11*a*b**2*x**2 - 3*b**3*x*
*3))/(280*(a + b*x)**(1/3)*a**2*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3))
```

3.175 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx=-\frac{3(a-bx)^{7/3}}{26ab(a+bx)^{13/3}}-\frac{9(a-bx)^{7/3}}{260a^2b(a+bx)^{10/3}}-\frac{27(a-bx)^{7/3}}{3640a^3b(a+bx)^{7/3}}$$

output
```
-3/26*(-b*x+a)^(7/3)/a/b/(b*x+a)^(13/3)-9/260*(-b*x+a)^(7/3)/a^2/b/(b*x+a)
^(10/3)-27/3640*(-b*x+a)^(7/3)/a^3/b/(b*x+a)^(7/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx=-\frac{3(a-bx)^{7/3}\,(191a^2+60abx+9b^2x^2)}{3640a^3b(a+bx)^{13/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(16/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*(191*a^2 + 60*a*b*x + 9*b^2*x^2))/(3640*a^3*b*(a + b*x
)^(13/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}dx}{13a}-\frac{3(a-bx)^{7/3}}{26ab(a+bx)^{13/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\left(\frac{3\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}dx}{20a}-\frac{3(a-bx)^{7/3}}{20ab(a+bx)^{10/3}}\right)}{13a}-\frac{3(a-bx)^{7/3}}{26ab(a+bx)^{13/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{3\left(-\frac{9(a-bx)^{7/3}}{280a^2b(a+bx)^{7/3}}-\frac{3(a-bx)^{7/3}}{20ab(a+bx)^{10/3}}\right)}{13a}-\frac{3(a-bx)^{7/3}}{26ab(a+bx)^{13/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(16/3),x]
```

output
```
(-3*(a - b*x)^(7/3))/(26*a*b*(a + b*x)^(13/3)) + (3*((-3*(a - b*x)^(7/3))/
(20*a*b*(a + b*x)^(10/3)) - (9*(a - b*x)^(7/3))/(280*a^2*b*(a + b*x)^(7/3)
)))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{7}{3}}\left(9b^{2}x^{2}+60abx+191a^{2}\right)}{3640(bx+a)^{\frac{13}{3}}a^{3}b}$	43
orering	$-\frac{3(-bx+a)^{\frac{7}{3}}\left(9b^{2}x^{2}+60abx+191a^{2}\right)}{3640(bx+a)^{\frac{13}{3}}a^{3}b}$	43

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
-3/3640*(-b*x+a)^(7/3)*(9*b^2*x^2+60*a*b*x+191*a^2)/(b*x+a)^(13/3)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.34

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx = -\frac{3\,(9\,b^4x^4+42\,ab^3x^3+80\,a^2b^2x^2-322\,a^3bx+191\,a^4)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3640\,(a^3b^6x^5+5\,a^4b^5x^4+10\,a^5b^4x^3+10\,a^6b^3x^2+5\,a^7b^2x+a^8b)}$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(16/3),x, algorithm="fricas")
```

output
```
-3/3640*(9*b^4*x^4 + 42*a*b^3*x^3 + 80*a^2*b^2*x^2 - 322*a^3*b*x + 191*a^4
)*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a^3*b^6*x^5 + 5*a^4*b^5*x^4 + 10*a^5*b
^4*x^3 + 10*a^6*b^3*x^2 + 5*a^7*b^2*x + a^8*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(16/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(16/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(16/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx = -\frac{(a-b\,x)^{1/3}\left(\frac{573\,a}{3640\,b^5}-\frac{69\,x}{260\,b^4}+\frac{6\,x^2}{91\,a\,b^3}+\frac{9\,x^3}{260\,a^2\,b^2}+\frac{27\,x^4}{3640\,a^3\,b}\right)}{x^4\,(a+b\,x)^{1/3}+\frac{a^4\,(a+b\,x)^{1/3}}{b^4}+\frac{6\,a^2\,x^2\,(a+b\,x)^{1/3}}{b^2}+\frac{4\,a\,x^3\,(a+b\,x)^{1/3}}{b}+\frac{4\,a^3\,x\,(a+b\,x)^{1/3}}{b^3}}$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(16/3),x)
```

output
```
-((a - b*x)^(1/3)*((573*a)/(3640*b^5) - (69*x)/(260*b^4) + (6*x^2)/(91*a*b
^3) + (9*x^3)/(260*a^2*b^2) + (27*x^4)/(3640*a^3*b)))/(x^4*(a + b*x)^(1/3)
 + (a^4*(a + b*x)^(1/3))/b^4 + (6*a^2*x^2*(a + b*x)^(1/3))/b^2 + (4*a*x^3*
(a + b*x)^(1/3))/b + (4*a^3*x*(a + b*x)^(1/3))/b^3)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\left(-9b^4x^4-42a\,b^3x^3-80a^2b^2x^2+322a^3bx-191a^4\right)}{3640\,(bx+a)^{\frac{1}{3}}\,a^3b\left(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)}$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(16/3),x)
```

output
```
(3*(a - b*x)**(1/3)*( - 191*a**4 + 322*a**3*b*x - 80*a**2*b**2*x**2 - 42*a
*b**3*x**3 - 9*b**4*x**4))/(3640*(a + b*x)**(1/3)*a**3*b*(a**4 + 4*a**3*b*
x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.176 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}} \, dx = -\frac{3(a-bx)^{7/3}}{32ab(a+bx)^{16/3}} - \frac{27(a-bx)^{7/3}}{832a^2b(a+bx)^{13/3}} - \frac{81(a-bx)^{7/3}}{8320a^3b(a+bx)^{10/3}} - \frac{243(a-bx)^{7/3}}{116480a^4b(a+bx)^{7/3}}$$

output
```
-3/32*(-b*x+a)^(7/3)/a/b/(b*x+a)^(16/3)-27/832*(-b*x+a)^(7/3)/a^2/b/(b*x+a
)^(13/3)-81/8320*(-b*x+a)^(7/3)/a^3/b/(b*x+a)^(10/3)-243/116480*(-b*x+a)^(
7/3)/a^4/b/(b*x+a)^(7/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}} \, dx = -\frac{3(a-bx)^{7/3}\left(5359a^3+2259a^2bx+621ab^2x^2+81b^3x^3\right)}{116480a^4b(a+bx)^{16/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(19/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*(5359*a^3 + 2259*a^2*b*x + 621*a*b^2*x^2 + 81*b^3*x^3)
)/(116480*a^4*b*(a + b*x)^(16/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}}\,dx$$

$$\Big\downarrow \ 55$$

$$\frac{9\int \frac{(a-bx)^{4/3}}{(a+bx)^{16/3}}dx}{32a}-\frac{3(a-bx)^{7/3}}{32ab(a+bx)^{16/3}}$$

$$\Big\downarrow \ 55$$

$$\frac{9\left(\frac{3\int \frac{(a-bx)^{4/3}}{(a+bx)^{13/3}}dx}{13a}-\frac{3(a-bx)^{7/3}}{26ab(a+bx)^{13/3}}\right)}{32a}-\frac{3(a-bx)^{7/3}}{32ab(a+bx)^{16/3}}$$

$$\Big\downarrow \ 55$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{(a-bx)^{4/3}}{(a+bx)^{10/3}}dx}{20a}-\frac{3(a-bx)^{7/3}}{20ab(a+bx)^{10/3}}\right)}{13a}-\frac{3(a-bx)^{7/3}}{26ab(a+bx)^{13/3}}\right)}{32a}-\frac{3(a-bx)^{7/3}}{32ab(a+bx)^{16/3}}$$

$$\Big\downarrow \ 48$$

$$\frac{9\left(\frac{3\left(-\frac{9(a-bx)^{7/3}}{280a^2b(a+bx)^{7/3}}-\frac{3(a-bx)^{7/3}}{20ab(a+bx)^{10/3}}\right)}{13a}-\frac{3(a-bx)^{7/3}}{26ab(a+bx)^{13/3}}\right)}{32a}-\frac{3(a-bx)^{7/3}}{32ab(a+bx)^{16/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(19/3),x]
```

output
```
(-3*(a - b*x)^(7/3))/(32*a*b*(a + b*x)^(16/3)) + (9*((-3*(a - b*x)^(7/3))/
(26*a*b*(a + b*x)^(13/3)) + (3*((-3*(a - b*x)^(7/3))/(20*a*b*(a + b*x)^(10
/3)) - (9*(a - b*x)^(7/3))/(280*a^2*b*(a + b*x)^(7/3))))/(13*a)))/(32*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{7}{3}}\left(81b^{3}x^{3}+621a\,b^{2}x^{2}+2259a^{2}bx+5359a^{3}\right)}{116480(bx+a)^{\frac{16}{3}}a^{4}b}$	54
orering	$-\frac{3(-bx+a)^{\frac{7}{3}}\left(81b^{3}x^{3}+621a\,b^{2}x^{2}+2259a^{2}bx+5359a^{3}\right)}{116480(bx+a)^{\frac{16}{3}}a^{4}b}$	54

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(19/3),x,method=_RETURNVERBOSE)
```

output
```
-3/116480*(-b*x+a)^(7/3)*(81*b^3*x^3+621*a*b^2*x^2+2259*a^2*b*x+5359*a^3)/
(b*x+a)^(16/3)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}} \, dx = -\frac{3\,(81\,b^5x^5+459\,ab^4x^4+1098\,a^2b^3x^3+1462\,a^3b^2x^2-8459\,a^4bx+5359\,a^5)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{116480\,(a^4b^7x^6+6\,a^5b^6x^5+15\,a^6b^5x^4+20\,a^7b^4x^3+15\,a^8b^3x^2+6\,a^9b^2x+a^{10}b)}$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(19/3),x, algorithm="fricas")
```

output
```
-3/116480*(81*b^5*x^5 + 459*a*b^4*x^4 + 1098*a^2*b^3*x^3 + 1462*a^3*b^2*x^
2 - 8459*a^4*b*x + 5359*a^5)*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a^4*b^7*x^6
 + 6*a^5*b^6*x^5 + 15*a^6*b^5*x^4 + 20*a^7*b^4*x^3 + 15*a^8*b^3*x^2 + 6*a^
9*b^2*x + a^10*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}} \, dx = \text{Timed out}$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(19/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}} \, dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{19}{3}}} \, dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(19/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(19/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}}\,dx=\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{19}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(19/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(19/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.39

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}}\,dx= -\frac{(a-b\,x)^{1/3}\left(\frac{16077\,a}{116480\,b^6}-\frac{25377\,x}{116480\,b^5}+\frac{2193\,x^2}{58240\,a\,b^4}+\frac{1647\,x^3}{58240\,a^2\,b^3}+\frac{1377\,x^4}{116480\,a^3\,b^2}+\frac{243\,x^5}{116480\,a^4\,b}\right)}{x^5\,(a+b\,x)^{1/3}+\frac{a^5\,(a+b\,x)^{1/3}}{b^5}+\frac{10\,a^2\,x^3\,(a+b\,x)^{1/3}}{b^2}+\frac{10\,a^3\,x^2\,(a+b\,x)^{1/3}}{b^3}+\frac{5\,a\,x^4\,(a+b\,x)^{1/3}}{b}+\frac{5\,a^4\,x\,(a+b\,x)^{1/3}}{b^4}}$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(19/3),x)
```

output
```
-((a - b*x)^(1/3)*((16077*a)/(116480*b^6) - (25377*x)/(116480*b^5) + (2193
*x^2)/(58240*a*b^4) + (1647*x^3)/(58240*a^2*b^3) + (1377*x^4)/(116480*a^3*
b^2) + (243*x^5)/(116480*a^4*b)))/(x^5*(a + b*x)^(1/3) + (a^5*(a + b*x)^(1
/3))/b^5 + (10*a^2*x^3*(a + b*x)^(1/3))/b^2 + (10*a^3*x^2*(a + b*x)^(1/3))
/b^3 + (5*a*x^4*(a + b*x)^(1/3))/b + (5*a^4*x*(a + b*x)^(1/3))/b^4)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{19/3}}\,dx = \frac{3(-bx+a)^{\frac{1}{3}}\left(-81b^5x^5-459a\,b^4x^4-1098a^2b^3x^3-1462a^3b^2x^2+8459a^4bx-5359a^5\right.}{116480\,(bx+a)^{\frac{1}{3}}\,a^4b\,(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}$$

input

```
int((-b*x+a)^(4/3)/(b*x+a)^(19/3),x)
```

output

```
(3*(a - b*x)**(1/3)*( - 5359*a**5 + 8459*a**4*b*x - 1462*a**3*b**2*x**2 -
1098*a**2*b**3*x**3 - 459*a*b**4*x**4 - 81*b**5*x**5))/(116480*(a + b*x)**
(1/3)*a**4*b*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 +
5*a*b**4*x**4 + b**5*x**5))
```

3.177 $\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 363

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx = \frac{24}{55}a^3x\sqrt[3]{a-bx}\sqrt[3]{a+bx} + \frac{3}{11}ax(a-bx)^{4/3}(a+bx)^{4/3} - \frac{3(a-bx)^{7/3}(a+bx)^{7/3}}{14b} + \frac{16\,3^{3/4}\sqrt{2-\sqrt{3}}a^7\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right.}{55b^2x(a-}$$

output
```
24/55*a^3*x*(-b*x+a)^(1/3)*(b*x+a)^(1/3)+3/11*a*x*(-b*x+a)^(4/3)*(b*x+a)^(
4/3)-3/14*(-b*x+a)^(7/3)*(b*x+a)^(7/3)/b+16/55*3^(3/4)*(1/2*6^(1/2)-1/2*2^
(1/2))*a^7*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/
a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/
2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^
(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a
^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx = -\frac{12\sqrt[3]{2}a^2(a-bx)^{7/3}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{7b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(4/3)*(a + b*x)^(7/3),x]
```

output
```
(-12*2^(1/3)*a^2*(a - b*x)^(7/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-7/3, 7
/3, 10/3, (a - b*x)/(2*a)])/(7*b*((a + b*x)/a)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx$$

$$\Big\downarrow \text{80}$$

$$\frac{4\sqrt[3]{2}a^2\sqrt[3]{a+bx}\int \frac{(a-bx)^{4/3}\left(\frac{bx}{a}+1\right)^{7/3}}{4\sqrt[3]{2}}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2 \sqrt[3]{a+bx} \int (a-bx)^{4/3} \left(\frac{bx}{a}+1\right)^{7/3} dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{12 \sqrt[3]{2} a^2 (a-bx)^{7/3} \sqrt[3]{a+bx} \text{Hypergeometric2F1}\left(-\frac{7}{3}, \frac{7}{3}, \frac{10}{3}, \frac{a-bx}{2a}\right)}{7b \sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Int[(a - b*x)^(4/3)*(a + b*x)^(7/3),x]
```

output
```
(-12*2^(1/3)*a^2*(a - b*x)^(7/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-7/3, 7
/3, 10/3, (a - b*x)/(2*a)])/(7*b*((a + b*x)/a)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (-bx+a)^{\frac{4}{3}} (bx+a)^{\frac{7}{3}} dx$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(7/3),x)
```

output
```
int((-b*x+a)^(4/3)*(b*x+a)^(7/3),x)
```

Fricas [F]

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\, dx = \int (bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{4}{3}}\, dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
integral(-(b^3*x^3 + a*b^2*x^2 - a^2*b*x - a^3)*(b*x + a)^(1/3)*(-b*x + a)
^(1/3), x)
```

Sympy [F]

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\, dx = \int (a-bx)^{\frac{4}{3}} (a+bx)^{\frac{7}{3}}\, dx$$

input
```
integrate((-b*x+a)**(4/3)*(b*x+a)**(7/3),x)
```

output
```
Integral((a - b*x)**(4/3)*(a + b*x)**(7/3), x)
```

Maxima [F]

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx=\int (bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(7/3)*(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx=\int (bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(7/3)*(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx=\int (a+b\,x)^{7/3}\,(a-b\,x)^{4/3}\,dx$$

input
```
int((a + b*x)^(7/3)*(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(7/3)*(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int (a-bx)^{4/3}(a+bx)^{7/3}\,dx = \frac{-165(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}a^4+546(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}a^3bx+330(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}a^2}{}$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(7/3),x)
```

output
```
( - 165*(a + b*x)**(1/3)*(a - b*x)**(1/3)*a**4 + 546*(a + b*x)**(1/3)*(a -
 b*x)**(1/3)*a**3*b*x + 330*(a + b*x)**(1/3)*(a - b*x)**(1/3)*a**2*b**2*x*
*2 - 210*(a + b*x)**(1/3)*(a - b*x)**(1/3)*a*b**3*x**3 - 165*(a + b*x)**(1
/3)*(a - b*x)**(1/3)*b**4*x**4 + 224*int(((a + b*x)**(1/3)*(a - b*x)**(1/3
))/(a**2 - b**2*x**2),x)*a**5*b)/(770*b)
```

3.178 $\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 336

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx = \frac{24}{55}a^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}$$

$$+\frac{3}{11}x(a-bx)^{4/3}(a+bx)^{4/3}+\frac{16\ 3^{3/4}\sqrt{2-\sqrt{3}}a^6\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)\cdots}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\cdots}}\cdots}{55b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1\cdots}{\left(1-\cdots\right)\cdots}}}$$

output
```
24/55*a^2*x*(-b*x+a)^(1/3)*(b*x+a)^(1/3)+3/11*x*(-b*x+a)^(4/3)*(b*x+a)^(4/
3)+16/55*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^6*(1-b^2*x^2/a^2)^(2/3)*(1-(1
-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3
^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2
)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(
2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^
(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.20

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx = -\frac{6\sqrt[3]{2}a(a-bx)^{7/3}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{7b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(4/3)*(a + b*x)^(4/3),x]
```

output
```
(-6*2^(1/3)*a*(a - b*x)^(7/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-4/3, 7/3,
 10/3, (a - b*x)/(2*a)])/(7*b*((a + b*x)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {46, 211, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx$$

$$\Big\downarrow \text{46}$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\int \left(a^2-b^2x^2\right)^{4/3}dx}{\sqrt[3]{a^2-b^2x^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(\frac{8}{11}a^2\int \sqrt[3]{a^2-b^2x^2}dx+\frac{3}{11}x\left(a^2-b^2x^2\right)^{4/3}\right)}{\sqrt[3]{a^2-b^2x^2}}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(\frac{8}{11}a^2\left(\frac{2}{5}a^2\int\frac{1}{(a^2-b^2x^2)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a^2-b^2x^2}\right)+\frac{3}{11}x\left(a^2-b^2x^2\right)^{4/3}\right)}{\sqrt[3]{a^2-b^2x^2}}$$

$$\Big\downarrow \; 234$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(\frac{8}{11}a^2\left(\frac{3}{5}x\sqrt[3]{a^2-b^2x^2}-\frac{3a^2\sqrt{-b^2x^2}\int\frac{1}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}}{5b^2x}\right)+\frac{3}{11}x\left(a^2-b^2x^2\right)^{4/3}\right)}{\sqrt[3]{a^2-b^2x^2}}$$

$$\Big\downarrow \; 760$$

$$\frac{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(\frac{3}{11}x\left(a^2-b^2x^2\right)^{4/3}+\frac{8}{11}a^2\left(\frac{3}{5}x\sqrt[3]{a^2-b^2x^2}+\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^{4/3}+\left(a^2-b^2x^2\right)\cdots}{\left(\left(1-\sqrt{3}\right)a^{2}\cdots\right.}}\cdots}{5b^2x\sqrt{\cdots}}\cdots\right.\right.}{\sqrt[3]{a^2-b^2x^2}\cdots}$$

input
```
Int[(a - b*x)^(4/3)*(a + b*x)^(4/3),x]
```

output
```
((a - b*x)^(1/3)*(a + b*x)^(1/3)*((3*x*(a^2 - b^2*x^2)^(4/3))/11 + (8*a^2*
((3*x*(a^2 - b^2*x^2)^(1/3))/5 + (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a^(2/3)
 - (a^2 - b^2*x^2)^(1/3))*Sqrt[(a^(4/3) + a^(2/3)*(a^2 - b^2*x^2)^(1/3) +
(a^2 - b^2*x^2)^(2/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))/((1 - Sqr
t[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-(
(a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)))/((1 - Sqrt[3])*a^(2/3) - (a^2
- b^2*x^2)^(1/3))^2)])))/11))/(a^2 - b^2*x^2)^(1/3)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (-bx+a)^{\frac{4}{3}} (bx+a)^{\frac{4}{3}} \, dx$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(4/3),x)
```

output
```
int((-b*x+a)^(4/3)*(b*x+a)^(4/3),x)
```

Fricas [F]

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx=\int (bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral(-(b^2*x^2 - a^2)*(b*x + a)^(1/3)*(-b*x + a)^(1/3), x)
```

Sympy [F]

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx=\int (a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)*(b*x+a)**(4/3),x)
```

output
```
Integral((a - b*x)**(4/3)*(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx=\int (bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(4/3)*(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx=\int (bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(4/3)*(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx=\int (a+b\,x)^{4/3}\,(a-b\,x)^{4/3}\,dx$$

input
```
int((a + b*x)^(4/3)*(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(4/3)*(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int (a-bx)^{4/3}(a+bx)^{4/3}\,dx=\frac{39(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2x}{55}-\frac{3(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,b^2x^3}{11}+\frac{16\left(\int\frac{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}}{-b^2x^2+a^2}dx\right)a^4}{55}$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(4/3),x)
```

output
```
(39*(a + b*x)**(1/3)*(a - b*x)**(1/3)*a**2*x - 15*(a + b*x)**(1/3)*(a - b*
x)**(1/3)*b**2*x**3 + 16*int(((a + b*x)**(1/3)*(a - b*x)**(1/3))/(a**2 - b
**2*x**2),x)*a**4)/55
```

3.179 $\int (a-bx)^{4/3}\sqrt[3]{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 336

$$\int (a-bx)^{4/3}\sqrt[3]{a+bx}\,dx=\frac{3}{5}ax\sqrt[3]{a-bx}\sqrt[3]{a+bx}+\frac{3(a-bx)^{4/3}(a+bx)^{4/3}}{8b}$$

$$+\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a^5\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}\cdots}{1-\sqrt{3}\cdots}\right.\right.}{5b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3/5*a*x*(-b*x+a)^(1/3)*(b*x+a)^(1/3)+3/8*(-b*x+a)^(4/3)*(b*x+a)^(4/3)/b+2/
5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^5*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^
2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-
(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3)
)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b
*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.20

$$\int (a-bx)^{4/3}\sqrt[3]{a+bx}\,dx = -\frac{3\sqrt[3]{2}(a-bx)^{7/3}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{7b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(4/3)*(a + b*x)^(1/3),x]
```

output
```
(-3*2^(1/3)*(a - b*x)^(7/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-1/3, 7/3, 1
0/3, (a - b*x)/(2*a)])/(7*b*((a + b*x)/a)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^{4/3}\sqrt[3]{a+bx}\,dx$$

$$\Big\downarrow \ 80$$

$$\frac{\sqrt[3]{2}\sqrt[3]{a+bx}\int \frac{(a-bx)^{4/3}\sqrt[3]{\frac{bx}{a}+1}}{\sqrt[3]{2}}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt[3]{a+bx}\int (a-bx)^{4/3}\sqrt[3]{\frac{bx}{a}+1}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{3\sqrt[3]{2}(a-bx)^{7/3}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{7b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Int[(a - b*x)^(4/3)*(a + b*x)^(1/3),x]
```

output
```
(-3*2^(1/3)*(a - b*x)^(7/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-1/3, 7/3, 1
0/3, (a - b*x)/(2*a)])/(7*b*((a + b*x)/a)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (-bx+a)^{\frac{4}{3}} (bx+a)^{\frac{1}{3}} \, dx$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(1/3),x)
```

output
```
int((-b*x+a)^(4/3)*(b*x+a)^(1/3),x)
```

Fricas [F]

$$\int (a-bx)^{4/3} \sqrt[3]{a+bx} \, dx = \int (bx+a)^{\frac{1}{3}} (-bx+a)^{\frac{4}{3}} \, dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/3)*(-b*x + a)^(4/3), x)
```

Sympy [F]

$$\int (a-bx)^{4/3} \sqrt[3]{a+bx} \, dx = \int (a-bx)^{\frac{4}{3}} \sqrt[3]{a+bx} \, dx$$

input
```
integrate((-b*x+a)**(4/3)*(b*x+a)**(1/3),x)
```

output
```
Integral((a - b*x)**(4/3)*(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int (a-bx)^{4/3}\sqrt[3]{a+bx}\,dx=\int (bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/3)*(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int (a-bx)^{4/3}\sqrt[3]{a+bx}\,dx=\int (bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)*(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/3)*(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^{4/3}\sqrt[3]{a+bx}\,dx=\int (a+b\,x)^{1/3}\,(a-b\,x)^{4/3}\,dx$$

input
```
int((a + b*x)^(1/3)*(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(1/3)*(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int (a - bx)^{4/3}\sqrt[3]{a+bx}\,dx = \frac{15(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}a^2 + 24(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}abx - 15(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}}{40b}$$

input
```
int((-b*x+a)^(4/3)*(b*x+a)^(1/3),x)
```

output
```
(15*(a + b*x)**(1/3)*(a - b*x)**(1/3)*a**2 + 24*(a + b*x)**(1/3)*(a - b*x)
**(1/3)*a*b*x - 15*(a + b*x)**(1/3)*(a - b*x)**(1/3)*b**2*x**2 + 16*int(((
a + b*x)**(1/3)*(a - b*x)**(1/3))/(a**2 - b**2*x**2),x)*a**3*b)/(40*b)
```

3.180 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 338

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = \frac{12a\sqrt[3]{a-bx}\sqrt[3]{a+bx}}{5b}+\frac{3(a-bx)^{4/3}\sqrt[3]{a+bx}}{5b}$$
$$+\frac{8\,3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}\ldots}{1-\sqrt{3}\ldots}\right.\right.}{5b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
12/5*a*(-b*x+a)^(1/3)*(b*x+a)^(1/3)/b+3/5*(-b*x+a)^(4/3)*(b*x+a)^(1/3)/b+8
/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^4*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x
^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)
-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3
))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(
b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.20

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = -\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{7\ 2^{2/3}b(a+bx)^{2/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(2/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[2/3, 7/3, 10/3,
(a - b*x)/(2*a)])/(7*2^(2/3)*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}dx$$

$$\Big\downarrow \text{80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{2^{2/3}(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{2/3}}dx}{2^{2/3}(a+bx)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{2/3}}dx}{(a+bx)^{2/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{7\ 2^{2/3}b(a+bx)^{2/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(2/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[2/3, 7/3, 10/3,
(a - b*x)/(2*a)])/(7*2^(2/3)*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{2}{3}}}dx$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(2/3),x)
```

output
```
int((-b*x+a)^(4/3)/(b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((-b*x + a)^(4/3)/(b*x + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = \int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(2/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = \int \frac{(a-b\,x)^{4/3}}{(a+b\,x)^{2/3}}\,dx$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(2/3),x)
```

output
```
int((a - b*x)^(4/3)/(a + b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{2/3}}\,dx = \left(\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}dx\right)a - \left(\int \frac{(-bx+a)^{\frac{1}{3}}\,x}{(bx+a)^{\frac{2}{3}}}dx\right)b$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(2/3),x)
```

output
```
int((a - b*x)**(1/3)/(a + b*x)**(2/3),x)*a - int(((a - b*x)**(1/3)*x)/(a +
 b*x)**(2/3),x)*b
```

3.181 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 333

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx = -\frac{3(a-bx)^{4/3}}{2b(a+bx)^{2/3}} - \frac{3\sqrt[3]{a-bx}\sqrt[3]{a+bx}}{b} - \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{\ldots}}{1-\sqrt{\ldots}}\right.\right.}{b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/2*(-b*x+a)^(4/3)/b/(b*x+a)^(2/3)-3*(-b*x+a)^(1/3)*(b*x+a)^(1/3)/b-2*3^(
3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^3*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2
)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^
2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-
3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)
^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx = -\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{14\ 2^{2/3}ab(a+bx)^{2/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(5/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[5/3, 7/3, 10/3,
(a - b*x)/(2*a)])/(14*2^(2/3)*a*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{2\ 2^{2/3}(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{5/3}}dx}{2\ 2^{2/3}a(a+bx)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{5/3}}dx}{a(a+bx)^{2/3}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{7}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{14\ 2^{2/3}ab(a+bx)^{2/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(5/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[5/3, 7/3, 10/3,
(a - b*x)/(2*a)])/(14*2^(2/3)*a*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{5}{3}}}dx$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(5/3),x)
```

output
```
int((-b*x+a)^(4/3)/(b*x+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx=\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/3)*(-b*x + a)^(4/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx=\int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(5/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(5/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx=\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx = \int \frac{(a-b\,x)^{4/3}}{(a+b\,x)^{5/3}}\,dx$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(5/3),x)
```

output
```
int((a - b*x)^(4/3)/(a + b*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{5/3}}\,dx = \left(\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}\,a+(bx+a)^{\frac{2}{3}}\,bx}dx\right)a - \left(\int \frac{(-bx+a)^{\frac{1}{3}}\,x}{(bx+a)^{\frac{2}{3}}\,a+(bx+a)^{\frac{2}{3}}\,bx}dx\right)b$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(5/3),x)
```

output
```
int((a - b*x)**(1/3)/((a + b*x)**(2/3)*a + (a + b*x)**(2/3)*b*x),x)*a - in
t(((a - b*x)**(1/3)*x)/((a + b*x)**(2/3)*a + (a + b*x)**(2/3)*b*x),x)*b
```

3.182 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 337

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx=-\frac{3(a-bx)^{4/3}}{5b(a+bx)^{5/3}}+\frac{6\sqrt[3]{a-bx}}{5b(a+bx)^{2/3}}$$

$$+\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}\cdots}{1-\sqrt{3}\cdots}\right.\right.}{5b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output

```
-3/5*(-b*x+a)^(4/3)/b/(b*x+a)^(5/3)+6/5*(-b*x+a)^(1/3)/b/(b*x+a)^(2/3)+2/5
*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2
/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(
1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))
/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*
x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx = -\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{8}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{28\ 2^{2/3}a^2b(a+bx)^{2/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(8/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[7/3, 8/3, 10/3,
(a - b*x)/(2*a)])/(28*2^(2/3)*a^2*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}dx$$

$$\Big\downarrow \text{80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{4\ 2^{2/3}(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{8/3}}dx}{4\ 2^{2/3}a^2(a+bx)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{8/3}}dx}{a^2(a+bx)^{2/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{8}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{28\ 2^{2/3}a^2b(a+bx)^{2/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(8/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[7/3, 8/3, 10/3,
(a - b*x)/(2*a)])/(28*2^(2/3)*a^2*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{8}{3}}}dx$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(8/3),x)
```

output
```
int((-b*x+a)^(4/3)/(b*x+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx=\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/3)*(-b*x + a)^(4/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3), x)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx=\int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(8/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(8/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx=\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(8/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx=\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx=\int \frac{(a-b\,x)^{4/3}}{(a+b\,x)^{8/3}}\,dx$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(8/3),x)
```

output
```
int((a - b*x)^(4/3)/(a + b*x)^(8/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{8/3}}\,dx=\left(\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}\,a^2+2\,(bx+a)^{\frac{2}{3}}\,abx+(bx+a)^{\frac{2}{3}}\,b^2x^2}dx\right)a-\left(\int \frac{(-bx+a)^{\frac{1}{3}}\,x}{(bx+a)^{\frac{2}{3}}\,a^2+2\,(bx+a)^{\frac{2}{3}}\,abx+(bx+a)^{\frac{2}{3}}\,b^2x^2}dx\right)b$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(8/3),x)
```

output
```
int((a - b*x)**(1/3)/((a + b*x)**(2/3)*a**2 + 2*(a + b*x)**(2/3)*a*b*x + (
a + b*x)**(2/3)*b**2*x**2),x)*a - int(((a - b*x)**(1/3)*x)/((a + b*x)**(2/
3)*a**2 + 2*(a + b*x)**(2/3)*a*b*x + (a + b*x)**(2/3)*b**2*x**2),x)*b
```

3.183 $\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 364

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = -\frac{3(a-bx)^{4/3}}{8b(a+bx)^{8/3}}+\frac{3\sqrt[3]{a-bx}}{10b(a+bx)^{5/3}}-\frac{3\sqrt[3]{a-bx}}{40ab(a+bx)^{2/3}}$$
$$+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{\cdots}}{1-\sqrt{3}-\sqrt[3]{\cdots}}\right.\right.}{40b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/8*(-b*x+a)^(4/3)/b/(b*x+a)^(8/3)+3/10*(-b*x+a)^(1/3)/b/(b*x+a)^(5/3)-3/
40*(-b*x+a)^(1/3)/a/b/(b*x+a)^(2/3)+1/40*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))
*a*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/
3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Ellip
ticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2
*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3
))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = -\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{11}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{56\ 2^{2/3}a^3b(a+bx)^{2/3}}$$

input
```
Integrate[(a - b*x)^(4/3)/(a + b*x)^(11/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[7/3, 11/3, 10/3,
 (a - b*x)/(2*a)])/(56*2^(2/3)*a^3*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{8\ 2^{2/3}(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{11/3}}dx}{8\ 2^{2/3}a^3(a+bx)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{(a-bx)^{4/3}}{\left(\frac{bx}{a}+1\right)^{11/3}}dx}{a^3(a+bx)^{2/3}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{3(a-bx)^{7/3}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{11}{3},\frac{10}{3},\frac{a-bx}{2a}\right)}{56\ 2^{2/3}a^3b(a+bx)^{2/3}}$$

input
```
Int[(a - b*x)^(4/3)/(a + b*x)^(11/3),x]
```

output
```
(-3*(a - b*x)^(7/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[7/3, 11/3, 10/3,
 (a - b*x)/(2*a)])/(56*2^(2/3)*a^3*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{11}{3}}}dx$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(11/3),x)
```

output
```
int((-b*x+a)^(4/3)/(b*x+a)^(11/3),x)
```

Fricas [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(11/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/3)*(-b*x + a)^(4/3)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b
^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(a-bx)^{\frac{4}{3}}}{(a+bx)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x+a)**(4/3)/(b*x+a)**(11/3),x)
```

output
```
Integral((a - b*x)**(4/3)/(a + b*x)**(11/3), x)
```

Maxima [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(11/3),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(11/3), x)
```

Giac [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(-bx+a)^{\frac{4}{3}}}{(bx+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x+a)^(4/3)/(b*x+a)^(11/3),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(4/3)/(b*x + a)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = \int \frac{(a-b\,x)^{4/3}}{(a+b\,x)^{11/3}}\,dx$$

input
```
int((a - b*x)^(4/3)/(a + b*x)^(11/3),x)
```

output
```
int((a - b*x)^(4/3)/(a + b*x)^(11/3), x)
```

Reduce [F]

$$\int \frac{(a-bx)^{4/3}}{(a+bx)^{11/3}}\,dx = \left(\int \frac{(-bx+a)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}\,a^3+3\,(bx+a)^{\frac{2}{3}}\,a^2bx+3\,(bx+a)^{\frac{2}{3}}\,a\,b^2x^2+(bx+a)^{\frac{2}{3}}\,b^3x^3}dx\right)a - \left(\int \frac{(-bx+a)^{\frac{1}{3}}\,x}{(bx+a)^{\frac{2}{3}}\,a^3+3\,(bx+a)^{\frac{2}{3}}\,a^2bx+3\,(bx+a)^{\frac{2}{3}}\,a\,b^2x^2+(bx+a)^{\frac{2}{3}}\,b^3x^3}dx\right)b$$

input
```
int((-b*x+a)^(4/3)/(b*x+a)^(11/3),x)
```

output
```
int((a - b*x)**(1/3)/((a + b*x)**(2/3)*a**3 + 3*(a + b*x)**(2/3)*a**2*b*x
+ 3*(a + b*x)**(2/3)*a*b**2*x**2 + (a + b*x)**(2/3)*b**3*x**3),x)*a - int(
((a - b*x)**(1/3)*x)/((a + b*x)**(2/3)*a**3 + 3*(a + b*x)**(2/3)*a**2*b*x
+ 3*(a + b*x)**(2/3)*a*b**2*x**2 + (a + b*x)**(2/3)*b**3*x**3),x)*b
```

3.184 $\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 180

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{56a^2(a-bx)^{2/3}\sqrt[3]{a+bx}}{27b} - \frac{7a(a-bx)^{2/3}(a+bx)^{4/3}}{9b} - \frac{(a-bx)^{2/3}(a+bx)^{7/3}}{3b} + \frac{112a^3\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{27\sqrt{3}b} + \frac{56a^3\log(a+bx)}{81b} + \frac{56a^3\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{27b}$$

output
```
-56/27*a^2*(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b-7/9*a*(-b*x+a)^(2/3)*(b*x+a)^(4/
3)/b-1/3*(-b*x+a)^(2/3)*(b*x+a)^(7/3)/b-112/81*a^3*arctan(-1/3*3^(1/2)+2/3
*(-b*x+a)^(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2)/b+56/81*a^3*ln(b*x+a)/b+56/
27*a^3*ln(1+(-b*x+a)^(1/3)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx =$$

$$-\frac{3(a-bx)^{2/3}\sqrt[3]{a+bx}(86a^2+39abx+9b^2x^2)+112\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-112a^3\log\left(\sqrt[3]{\right.}{81b}$$

input
```
Integrate[(a + b*x)^(7/3)/(a - b*x)^(1/3),x]
```

output
```
-1/81*(3*(a - b*x)^(2/3)*(a + b*x)^(1/3)*(86*a^2 + 39*a*b*x + 9*b^2*x^2) +
 112*Sqrt[3]*a^3*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a
 + b*x)^(1/3))] - 112*a^3*Log[(a - b*x)^(1/3) + (a + b*x)^(1/3)] + 56*a^3*
Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/
b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}dx$$

$$\Big\downarrow \; 60$$

$$\frac{14}{9}a\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}dx-\frac{(a-bx)^{2/3}(a+bx)^{7/3}}{3b}$$

$$\Big\downarrow \; 60$$

$$\frac{14}{9}a\left(\frac{4}{3}a\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}dx-\frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b}\right)-\frac{(a-bx)^{2/3}(a+bx)^{7/3}}{3b}$$

$$\downarrow \text{60}$$

$$\frac{14}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\int\frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b}\right)-\frac{(a-bx)^{2/3}(a+bx)^{7/3}}{3b}$$

$$\downarrow \text{72}$$

$$\frac{14}{9}a\left(\frac{4}{3}a\left(\frac{2}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right.\right.$$
$$\frac{(a-bx)^{2/3}(a+bx)^{7/3}}{3b}$$

input
```
Int[(a + b*x)^(7/3)/(a - b*x)^(1/3),x]
```

output
```
-1/3*((a - b*x)^(2/3)*(a + b*x)^(7/3))/b + (14*a*(-1/2*((a - b*x)^(2/3)*(a
 + b*x)^(4/3))/b + (4*a*(-(((a - b*x)^(2/3)*(a + b*x)^(1/3))/b) + (2*a*((S
qrt[3]*ArcTan[1/Sqrt[3] - (2*(a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/
b + Log[a + b*x]/(2*b) + (3*Log[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b
)))/3))/3))/9
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^(7/3)/(-b*x+a)^(1/3),x)
```

output
```
int((b*x+a)^(7/3)/(-b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx =$$

$$-\frac{112\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+56\,a^3\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right)-11}{81\,b}$$

input
```
integrate((b*x+a)^(7/3)/(-b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/81*(112*sqrt(3)*a^3*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a)
^(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + 56*a^3*log((b*x - (b*x + a)^(2/3)*(-
b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 112*a^
3*log(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) + 3*(9*b^2*
x^2 + 39*a*b*x + 86*a^2)*(b*x + a)^(1/3)*(-b*x + a)^(2/3))/b
```

Sympy [F]

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+bx)^{\frac{7}{3}}}{\sqrt[3]{a-bx}}\,dx$$

input
```
integrate((b*x+a)**(7/3)/(-b*x+a)**(1/3),x)
```

output
```
Integral((a + b*x)**(7/3)/(a - b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{7}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(7/3)/(-b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(7/3)/(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{7}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(7/3)/(-b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(7/3)/(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+b\,x)^{7/3}}{(a-b\,x)^{1/3}}\,dx$$

input
```
int((a + b*x)^(7/3)/(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(7/3)/(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{7/3}}{\sqrt[3]{a-bx}}\,dx = \left(\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}}dx\right)a^2 + \left(\int \frac{(bx+a)^{\frac{1}{3}}\,x^2}{(-bx+a)^{\frac{1}{3}}}dx\right)b^2 + 2\left(\int \frac{(bx+a)^{\frac{1}{3}}\,x}{(-bx+a)^{\frac{1}{3}}}dx\right)ab$$

input
```
int((b*x+a)^(7/3)/(-b*x+a)^(1/3),x)
```

output
```
int((a + b*x)**(1/3)/(a - b*x)**(1/3),x)*a**2 + int(((a + b*x)**(1/3)*x**2
)/(a - b*x)**(1/3),x)*b**2 + 2*int(((a + b*x)**(1/3)*x)/(a - b*x)**(1/3),x
)*a*b
```

3.185 $\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{4a(a-bx)^{2/3}\sqrt[3]{a+bx}}{3b} - \frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b} + \frac{8a^2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b} + \frac{4a^2\log(a+bx)}{9b} + \frac{4a^2\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{3b}$$

output

```
-4/3*a*(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b-1/2*(-b*x+a)^(2/3)*(b*x+a)^(4/3)/b-8
/9*a^2*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/
2)/b+4/9*a^2*ln(b*x+a)/b+4/3*a^2*ln(1+(-b*x+a)^(1/3)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{3(a-bx)^{2/3}\sqrt[3]{a+bx}(11a+3bx)+16\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-16a^2\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{\cdots}\right.\right.}{18b}$$

input
```
Integrate[(a + b*x)^(4/3)/(a - b*x)^(1/3),x]
```

output
```
-1/18*(3*(a - b*x)^(2/3)*(a + b*x)^(1/3)*(11*a + 3*b*x) + 16*Sqrt[3]*a^2*A
rcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] -
16*a^2*Log[b*((a - b*x)^(1/3) + (a + b*x)^(1/3))] + 8*a^2*Log[(a - b*x)^(2
/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx$$

$$\Big\downarrow \ 60$$

$$\frac{4}{3}a\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}dx-\frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b}$$

$$\Big\downarrow \ 60$$

$$\frac{4}{3}a\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b}$$

$$\Big\downarrow \ 72$$

$$\frac{4}{3}a\left(\frac{2}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b}$$

input
```
Int[(a + b*x)^(4/3)/(a - b*x)^(1/3),x]
```

output
```
-1/2*((a - b*x)^(2/3)*(a + b*x)^(4/3))/b + (4*a*(-(((a - b*x)^(2/3)*(a + b
*x)^(1/3))/b) + (2*a*((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a - b*x)^(1/3))/(Sqr
t[3]*(a + b*x)^(1/3))])/b + Log[a + b*x]/(2*b) + (3*Log[1 + (a - b*x)^(1/3
)/(a + b*x)^(1/3)])/(2*b)))/3))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^(4/3)/(-b*x+a)^(1/3),x)
```

output
```
int((b*x+a)^(4/3)/(-b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{16\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+8\,a^2\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right)-16\,a^2}{18\,b}$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/18*(16*sqrt(3)*a^2*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a)^
(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + 8*a^2*log((b*x - (b*x + a)^(2/3)*(-b*
x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 16*a^2*l
og(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) + 3*(3*b*x + 1
1*a)*(b*x + a)^(1/3)*(-b*x + a)^(2/3))/b
```

Sympy [F]

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+bx)^{\frac{4}{3}}}{\sqrt[3]{a-bx}}\,dx$$

input
```
integrate((b*x+a)**(4/3)/(-b*x+a)**(1/3),x)
```

output
```
Integral((a + b*x)**(4/3)/(a - b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(4/3)/(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(4/3)/(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+b\,x)^{4/3}}{(a-b\,x)^{1/3}}\,dx$$

input
```
int((a + b*x)^(4/3)/(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(4/3)/(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}\,dx = \left(\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}}dx\right)a + \left(\int \frac{(bx+a)^{\frac{1}{3}}x}{(-bx+a)^{\frac{1}{3}}}dx\right)b$$

input
```
int((b*x+a)^(4/3)/(-b*x+a)^(1/3),x)
```

output
```
int((a + b*x)**(1/3)/(a - b*x)**(1/3),x)*a + int(((a + b*x)**(1/3)*x)/(a -
 b*x)**(1/3),x)*b
```

3.186 $\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = -\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b} + \frac{2a\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{\sqrt{3}b} + \frac{a\log(a+bx)}{3b} + \frac{a\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{b}$$

output
```
-(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b-2/3*a*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^(1/
3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2)/b+1/3*a*ln(b*x+a)/b+a*ln(1+(-b*x+a)^(1/3
)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = -\frac{3(a-bx)^{2/3}\sqrt[3]{a+bx}+2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)-2a\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right)+}{3b}$$

input
```
Integrate[(a + b*x)^(1/3)/(a - b*x)^(1/3),x]
```

output
```
-1/3*(3*(a - b*x)^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*a*ArcTan[(Sqrt[3]*(a +
 b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 2*a*Log[b*((a - b*x
)^(1/3) + (a + b*x)^(1/3))] + a*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a +
 b*x)^(1/3) + (a + b*x)^(2/3)])/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}dx$$

$$\Big\downarrow \ 60$$

$$\frac{2}{3}a\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}$$

$$\Big\downarrow \ 72$$

$$\frac{2}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}$$

input
```
Int[(a + b*x)^(1/3)/(a - b*x)^(1/3),x]
```

output
```
-(((a - b*x)^(2/3)*(a + b*x)^(1/3))/b) + (2*a*((Sqrt[3]*ArcTan[1/Sqrt[3] -
 (2*(a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b + Log[a + b*x]/(2*b) +
(3*Log[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b)))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^(1/3)/(-b*x+a)^(1/3),x)
```

output
```
int((b*x+a)^(1/3)/(-b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = -\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+a\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right)-2\,a\log}{3\,b}$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(3)*a*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a)^(1/3
)*(-b*x + a)^(2/3))/(b*x - a)) + a*log((b*x - (b*x + a)^(2/3)*(-b*x + a)^(
1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 2*a*log(-(b*x -
(b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) + 3*(b*x + a)^(1/3)*(-b*x
 + a)^(2/3))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = \int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx$$

input
```
integrate((b*x+a)**(1/3)/(-b*x+a)**(1/3),x)
```

output
```
Integral((a + b*x)**(1/3)/(a - b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/3)/(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/3)/(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+b\,x)^{1/3}}{(a-b\,x)^{1/3}}\,dx$$

input
```
int((a + b*x)^(1/3)/(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(1/3)/(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^(1/3)/(-b*x+a)^(1/3),x)
```

output
```
int((a + b*x)**(1/3)/(a - b*x)**(1/3),x)
```

3.187 $\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b} + \frac{\log(a+bx)}{2b} + \frac{3\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{2b}$$

output
```
-3^(1/2)*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^(1/3)*3^(1/2)/(b*x+a)^(1/3))/b+1
/2*ln(b*x+a)/b+3/2*ln(1+(-b*x+a)^(1/3)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 2\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right) + \log\left((a-bx)^{2/3}-\sqrt[3]{a-bx}\sqrt[3]{\cdots}\right.}{2b}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(2/3)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a
+ b*x)^(1/3))] - 2*Log[b*((a - b*x)^(1/3) + (a + b*x)^(1/3))] + Log[(a - b
*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{72\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx$$

$$\Big\downarrow \; 72$$

$$\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(2/3)),x]
```

output
```
(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))]
)/b + Log[a + b*x]/(2*b) + (3*Log[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2
*b)
```

Defintions of rubi rules used

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(2/3),x)
```

output
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(71) = 142$.

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3(bx-a)}\right)+\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}-a}{bx-a}\right)-2\log\left(-\frac{bx-\cdots}{\cdots}\right)}{2b}$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x + a)^(1/3)*
(-b*x + a)^(2/3))/(b*x - a)) + log((b*x - (b*x + a)^(2/3)*(-b*x + a)^(1/3)
 + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 2*log(-(b*x - (b*x +
 a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = \int \frac{1}{\sqrt[3]{a-bx}\,(a+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(2/3),x)
```

output
```
Integral(1/((a - b*x)**(1/3)*(a + b*x)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(2/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(2/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = \int \frac{1}{{\left(a+b\,x\right)}^{2/3}\,{\left(a-b\,x\right)}^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(2/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(2/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(2/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)),x)
```

3.188 $\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx=-\frac{3(a-bx)^{2/3}}{4ab(a+bx)^{2/3}}$$

output
```
-3/4*(-b*x+a)^(2/3)/a/b/(b*x+a)^(2/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx=-\frac{3(a-bx)^{2/3}}{4ab(a+bx)^{2/3}}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(5/3)),x]
```

output
```
(-3*(a - b*x)^(2/3))/(4*a*b*(a + b*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}} \, dx$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{3(a-bx)^{2/3}}{4ab(a+bx)^{2/3}}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(5/3)),x]
```

output
```
(-3*(a - b*x)^(2/3))/(4*a*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{2}{3}}}{4ab(bx+a)^{\frac{2}{3}}}$	24
orering	$-\frac{3(-bx+a)^{\frac{2}{3}}}{4ab(bx+a)^{\frac{2}{3}}}$	24

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-b*x+a)^(2/3)/a/b/(b*x+a)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx = -\frac{3\,(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{4\,(ab^2x+a^2b)}$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-3/4*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a*b^2*x + a^2*b)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx = \int \frac{1}{\sqrt[3]{a-bx}\,(a+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(5/3),x)
```

output
```
Integral(1/((a - b*x)**(1/3)*(a + b*x)**(5/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx = \int \frac{1}{(a+b\,x)^{5/3}\,(a-b\,x)^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(5/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(5/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a+(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,bx}dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(5/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a + (a + b*x)**(2/3)*(a - b*x)**(
1/3)*b*x),x)
```

3.189 $\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx = -\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}-\frac{9(a-bx)^{2/3}}{40a^2b(a+bx)^{2/3}}$$

output
```
-3/10*(-b*x+a)^(2/3)/a/b/(b*x+a)^(5/3)-9/40*(-b*x+a)^(2/3)/a^2/b/(b*x+a)^(
2/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx = -\frac{3(a-bx)^{2/3}(7a+3bx)}{40a^2b(a+bx)^{5/3}}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(8/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*(7*a + 3*b*x))/(40*a^2*b*(a + b*x)^(5/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}} \, dx$$

$$\Big\downarrow \; 55$$

$$\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}dx}{10a} - \frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}$$

$$\Big\downarrow \; 48$$

$$-\frac{9(a-bx)^{2/3}}{40a^2b(a+bx)^{2/3}} - \frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(8/3)),x]
```

output
```
(-3*(a - b*x)^(2/3))/(10*a*b*(a + b*x)^(5/3)) - (9*(a - b*x)^(2/3))/(40*a^
2*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{2}{3}}(3bx+7a)}{40(bx+a)^{\frac{5}{3}}a^{2}b}$	32
orering	$-\frac{3(-bx+a)^{\frac{2}{3}}(3bx+7a)}{40(bx+a)^{\frac{5}{3}}a^{2}b}$	32

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(8/3),x,method=_RETURNVERBOSE)
```

output
```
-3/40*(-b*x+a)^(2/3)*(3*b*x+7*a)/(b*x+a)^(5/3)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx = -\frac{3\,(3\,bx+7\,a)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{40\,(a^2b^3x^2+2\,a^3b^2x+a^4b)}$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

output
```
-3/40*(3*b*x + 7*a)*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a^2*b^3*x^2 + 2*a^3*
b^2*x + a^4*b)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx = \int \frac{1}{\sqrt[3]{a-bx}\,(a+bx)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(8/3),x)
```

output
```
Integral(1/((a - b*x)**(1/3)*(a + b*x)**(8/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(8/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(8/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx=\int \frac{1}{(a+b\,x)^{8/3}\,(a-b\,x)^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(8/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(8/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2+2\,(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,abx+(bx+a)^{\frac{2}{3}}\,(-bx+}$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(8/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a**2 + 2*(a + b*x)**(2/3)*(a - b*
x)**(1/3)*a*b*x + (a + b*x)**(2/3)*(a - b*x)**(1/3)*b**2*x**2),x)
```

3.190 $\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx=-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}-\frac{9(a-bx)^{2/3}}{80a^2b(a+bx)^{5/3}}-\frac{27(a-bx)^{2/3}}{320a^3b(a+bx)^{2/3}}$$

output
```
-3/16*(-b*x+a)^(2/3)/a/b/(b*x+a)^(8/3)-9/80*(-b*x+a)^(2/3)/a^2/b/(b*x+a)^(
5/3)-27/320*(-b*x+a)^(2/3)/a^3/b/(b*x+a)^(2/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx=-\frac{3(a-bx)^{2/3}\left(41a^2+30abx+9b^2x^2\right)}{320a^3b(a+bx)^{8/3}}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(11/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*(41*a^2 + 30*a*b*x + 9*b^2*x^2))/(320*a^3*b*(a + b*x)^
(8/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}dx}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\left(\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}dx}{10a}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{3\left(-\frac{9(a-bx)^{2/3}}{40a^2b(a+bx)^{2/3}}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(11/3)),x]
```

output
```
(-3*(a - b*x)^(2/3))/(16*a*b*(a + b*x)^(8/3)) + (3*((-3*(a - b*x)^(2/3))/(
10*a*b*(a + b*x)^(5/3)) - (9*(a - b*x)^(2/3))/(40*a^2*b*(a + b*x)^(2/3))))
/(8*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{2}{3}}\left(9b^{2}x^{2}+30abx+41a^{2}\right)}{320(bx+a)^{\frac{8}{3}}a^{3}b}$	43
orering	$-\frac{3(-bx+a)^{\frac{2}{3}}\left(9b^{2}x^{2}+30abx+41a^{2}\right)}{320(bx+a)^{\frac{8}{3}}a^{3}b}$	43

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(11/3),x,method=_RETURNVERBOSE)
```

output
```
-3/320*(-b*x+a)^(2/3)*(9*b^2*x^2+30*a*b*x+41*a^2)/(b*x+a)^(8/3)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx = -\frac{3\left(9\,b^2x^2+30\,abx+41\,a^2\right)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{320\left(a^3b^4x^3+3\,a^4b^3x^2+3\,a^5b^2x+a^6b\right)}$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(11/3),x, algorithm="fricas")
```

output
```
-3/320*(9*b^2*x^2 + 30*a*b*x + 41*a^2)*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a
^3*b^4*x^3 + 3*a^4*b^3*x^2 + 3*a^5*b^2*x + a^6*b)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx = \int \frac{1}{\sqrt[3]{a-bx}\,(a+bx)^{\frac{11}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(11/3),x)
```

output
```
Integral(1/((a - b*x)**(1/3)*(a + b*x)**(11/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{11}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(11/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(11/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{11}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(11/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(11/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx=\int \frac{1}{(a+b\,x)^{11/3}\,(a-b\,x)^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(11/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(11/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^3+3\,(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2bx+3\,(bx+a)^{\frac{2}{3}}\,(-b}$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(11/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a**3 + 3*(a + b*x)**(2/3)*(a - b*
x)**(1/3)*a**2*b*x + 3*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a*b**2*x**2 + (a
+ b*x)**(2/3)*(a - b*x)**(1/3)*b**3*x**3),x)
```

3.191 $\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}} \, dx = -\frac{3(a-bx)^{2/3}}{22ab(a+bx)^{11/3}} - \frac{27(a-bx)^{2/3}}{352a^2b(a+bx)^{8/3}} - \frac{81(a-bx)^{2/3}}{1760a^3b(a+bx)^{5/3}} - \frac{243(a-bx)^{2/3}}{7040a^4b(a+bx)^{2/3}}$$

output
```
-3/22*(-b*x+a)^(2/3)/a/b/(b*x+a)^(11/3)-27/352*(-b*x+a)^(2/3)/a^2/b/(b*x+a
)^(8/3)-81/1760*(-b*x+a)^(2/3)/a^3/b/(b*x+a)^(5/3)-243/7040*(-b*x+a)^(2/3)
/a^4/b/(b*x+a)^(2/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}} \, dx = -\frac{3(a-bx)^{2/3}(689a^3+639a^2bx+351ab^2x^2+81b^3x^3)}{7040a^4b(a+bx)^{11/3}}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(14/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*(689*a^3 + 639*a^2*b*x + 351*a*b^2*x^2 + 81*b^3*x^3))/
(7040*a^4*b*(a + b*x)^(11/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{9\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}dx}{22a}-\frac{3(a-bx)^{2/3}}{22ab(a+bx)^{11/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{9\left(\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}dx}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}\right)}{22a}-\frac{3(a-bx)^{2/3}}{22ab(a+bx)^{11/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}dx}{10a}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}\right)}{22a}-\frac{3(a-bx)^{2/3}}{22ab(a+bx)^{11/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{9\left(\frac{3\left(-\frac{9(a-bx)^{2/3}}{40a^2b(a+bx)^{2/3}}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}\right)}{22a}-\frac{3(a-bx)^{2/3}}{22ab(a+bx)^{11/3}}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(14/3)),x]
```

output
```
(-3*(a - b*x)^(2/3))/(22*a*b*(a + b*x)^(11/3)) + (9*((-3*(a - b*x)^(2/3))/
(16*a*b*(a + b*x)^(8/3)) + (3*((-3*(a - b*x)^(2/3))/(10*a*b*(a + b*x)^(5/3
)) - (9*(a - b*x)^(2/3))/(40*a^2*b*(a + b*x)^(2/3))))/(8*a)))/(22*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3\left(-bx+a\right)^{\frac{2}{3}}\left(81b^{3}x^{3}+351a\,b^{2}x^{2}+639a^{2}bx+689a^{3}\right)}{7040\left(bx+a\right)^{\frac{11}{3}}a^{4}b}$	54
orering	$-\frac{3\left(-bx+a\right)^{\frac{2}{3}}\left(81b^{3}x^{3}+351a\,b^{2}x^{2}+639a^{2}bx+689a^{3}\right)}{7040\left(bx+a\right)^{\frac{11}{3}}a^{4}b}$	54

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(14/3),x,method=_RETURNVERBOSE)
```

output
```
-3/7040*(-b*x+a)^(2/3)*(81*b^3*x^3+351*a*b^2*x^2+639*a^2*b*x+689*a^3)/(b*x
+a)^(11/3)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}}\,dx = -\frac{3\,(81\,b^3x^3+351\,ab^2x^2+639\,a^2bx+689\,a^3)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{7040\,(a^4b^5x^4+4\,a^5b^4x^3+6\,a^6b^3x^2+4\,a^7b^2x+a^8b)}$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(14/3),x, algorithm="fricas")
```

output
```
-3/7040*(81*b^3*x^3 + 351*a*b^2*x^2 + 639*a^2*b*x + 689*a^3)*(b*x + a)^(1/
3)*(-b*x + a)^(2/3)/(a^4*b^5*x^4 + 4*a^5*b^4*x^3 + 6*a^6*b^3*x^2 + 4*a^7*b
^2*x + a^8*b)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}}\,dx = \int \frac{1}{\sqrt[3]{a-bx}\,(a+bx)^{\frac{14}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(14/3),x)
```

output
```
Integral(1/((a - b*x)**(1/3)*(a + b*x)**(14/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{14}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(14/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(14/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{14}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(14/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(14/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}}\,dx = \int \frac{1}{(a+b\,x)^{14/3}\,(a-b\,x)^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(14/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(14/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{14/3}}\,dx = \int \frac{}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^4+4(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^3bx+6(bx+a)^{\frac{2}{3}}(-b}$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(14/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a**4 + 4*(a + b*x)**(2/3)*(a - b*
x)**(1/3)*a**3*b*x + 6*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a**2*b**2*x**2 +
4*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a*b**3*x**3 + (a + b*x)**(2/3)*(a - b*
x)**(1/3)*b**4*x**4),x)
```

3.192 $\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 686

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{15a(a-bx)^{2/3}(a+bx)^{2/3}}{14b}$$

$$-\frac{3(a-bx)^{2/3}(a+bx)^{5/3}}{7b}-\frac{30a^2x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{7\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^4\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^{2}\cdots}{a\cdots}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^{2}\cdots}{a\cdots}}}\right.\right.}{7b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$+\frac{10\sqrt{2}3^{3/4}a^4\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\cdots}}{1-\sqrt{3}-\sqrt[3]{1-\cdots}}\right.\right.}{7b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-15/14*a*(-b*x+a)^(2/3)*(b*x+a)^(2/3)/b-3/7*(-b*x+a)^(2/3)*(b*x+a)^(5/3)/b
-30/7*a^2*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-
(1-b^2*x^2/a^2)^(1/3))-15/7*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^4*(1-b^2*x
^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x
^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticE((1+3^(
1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2
))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/
2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)+10/7*2^(1/2)*3^(3/4)*a^4*(1-b^2*x^2/a^2
)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2
)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(
1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2
/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-
b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{3\ 2^{2/3}a(a-bx)^{2/3}(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{2}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{b\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x)^(5/3)/(a - b*x)^(1/3),x]
```

output
```
(-3*2^(2/3)*a*(a - b*x)^(2/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-5/3, 2/3,
 5/3, (a - b*x)/(2*a)])/(b*((a + b*x)/a)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}dx$$

$$\Big\downarrow \text{80}$$

$$\frac{2\ 2^{2/3}a(a+bx)^{2/3}\int \frac{\left(\frac{bx}{a}+1\right)^{5/3}}{2\ 2^{2/3}\sqrt[3]{a-bx}}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(a+bx)^{2/3}\int \frac{\left(\frac{bx}{a}+1\right)^{5/3}}{\sqrt[3]{a-bx}}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3\ 2^{2/3}a(a-bx)^{2/3}(a+bx)^{2/3}\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{2}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{b\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Int[(a + b*x)^(5/3)/(a - b*x)^(1/3),x]
```

output
```
(-3*2^(2/3)*a*(a - b*x)^(2/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-5/3, 2/3,
 5/3, (a - b*x)/(2*a)])/(b*((a + b*x)/a)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^(5/3)/(-b*x+a)^(1/3),x)
```

output
```
int((b*x+a)^(5/3)/(-b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(5/3)*(-b*x + a)^(2/3)/(b*x - a), x)
```

Sympy [F]

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+bx)^{\frac{5}{3}}}{\sqrt[3]{a-bx}}\,dx$$

input
```
integrate((b*x+a)**(5/3)/(-b*x+a)**(1/3),x)
```

output
```
Integral((a + b*x)**(5/3)/(a - b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(5/3)/(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(5/3)/(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+b\,x)^{5/3}}{(a-b\,x)^{1/3}}\,dx$$

input
```
int((a + b*x)^(5/3)/(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(5/3)/(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{a-bx}}\,dx = \left(\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}}dx\right)a + \left(\int \frac{(bx+a)^{\frac{2}{3}}\,x}{(-bx+a)^{\frac{1}{3}}}dx\right)b$$

input

```
int((b*x+a)^(5/3)/(-b*x+a)^(1/3),x)
```

output

```
int((a + b*x)**(2/3)/(a - b*x)**(1/3),x)*a + int(((a + b*x)**(2/3)*x)/(a -
 b*x)**(1/3),x)*b
```

3.193 $\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 652

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{3(a-bx)^{2/3}(a+bx)^{2/3}}{4b} - \frac{3ax\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^3\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right.}{2b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}} + \frac{\sqrt{2}3^{3/4}a^3\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right.}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/4*(-b*x+a)^(2/3)*(b*x+a)^(2/3)/b-3*a*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(
1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))-3/2*3^(1/4)*(1/2*6^(1
/2)+1/2*2^(1/2))*a^3*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(
1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/
3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2
*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1
-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)+2^(1/2)*3^
(3/4)*a^3*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a
^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2
)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(
1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^
2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx = -\frac{3(a-bx)^{2/3}(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{\sqrt[3]{2}b\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x)^(2/3)/(a - b*x)^(1/3),x]
```

output
```
(-3*(a - b*x)^(2/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, (a -
 b*x)/(2*a)])/(2^(1/3)*b*((a + b*x)/a)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{2/3}(a+bx)^{2/3}\int \frac{\left(\frac{bx}{a}+1\right)^{2/3}}{2^{2/3}\sqrt[3]{a-bx}}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx)^{2/3}\int \frac{\left(\frac{bx}{a}+1\right)^{2/3}}{\sqrt[3]{a-bx}}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3(a-bx)^{2/3}(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{\sqrt[3]{2}b\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Int[(a + b*x)^(2/3)/(a - b*x)^(1/3),x]
```

output
```
(-3*(a - b*x)^(2/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, (a -
 b*x)/(2*a)])/(2^(1/3)*b*((a + b*x)/a)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^(2/3)/(-b*x+a)^(1/3),x)
```

output
```
int((b*x+a)^(2/3)/(-b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b*x - a), x)
```

Sympy [F]

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(a+bx)^{\frac{2}{3}}}{\sqrt[3]{a-bx}}\,dx$$

input
```
integrate((b*x+a)**(2/3)/(-b*x+a)**(1/3),x)
```

output
```
Integral((a + b*x)**(2/3)/(a - b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx=\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(2/3)/(-b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx=\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(2/3)/(-b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx=\int \frac{(a+b\,x)^{2/3}}{(a-b\,x)^{1/3}}\,dx$$

input
```
int((a + b*x)^(2/3)/(a - b*x)^(1/3),x)
```

output
```
int((a + b*x)^(2/3)/(a - b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{a-bx}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^(2/3)/(-b*x+a)^(1/3),x)
```

output
```
int((a + b*x)**(2/3)/(a - b*x)**(1/3),x)
```

3.194 $\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 625

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx = -\frac{3x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,E\Bigg(\arcsin\Bigg(\frac{1+\sqrt{3}-\sqrt[3]{1-}}{1-\sqrt{3}-\sqrt[3]{1-}}}{2b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$+\frac{\sqrt{2}3^{3/4}a^2\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{1+\sqrt{3}-\sqrt[3]{1-}}{1-\sqrt{3}-\sqrt[3]{1-}}}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*
x^2/a^2)^(1/3))-3/2*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^2*(1-b^2*x^2/a^2)^
(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^
(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-
b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x
/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^
2*x^2/a^2)^(1/3))^2)^(1/2)+2^(1/2)*3^(3/4)*a^2*(1-b^2*x^2/a^2)^(1/3)*(1-(1
-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3
^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2
)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(
1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^
(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx = -\frac{3(a-bx)^{2/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{2\sqrt[3]{2}b\sqrt[3]{a+bx}}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(1/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, (
a - b*x)/(2*a)])/(2*2^(1/3)*b*(a + b*x)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {46, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}dx$$

$\Big\downarrow$ 46

$$\frac{\sqrt[3]{a^2-b^2x^2}\int \frac{1}{\sqrt[3]{a^2-b^2x^2}}dx}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 233

$$-\frac{3\sqrt{-b^2x^2}\sqrt[3]{a^2-b^2x^2}\int \frac{\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}}{2b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 833

$$-\frac{3\sqrt{-b^2x^2}\sqrt[3]{a^2-b^2x^2}\left(\left(1+\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}-\int \frac{\left(1+\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}\right)}{2b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 760

$$-\frac{3\sqrt{-b^2x^2}\sqrt[3]{a^2-b^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^4}{}}}{}\right.}{2b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 2418

$$-\frac{3\sqrt{-b^2x^2}\sqrt[3]{a^2-b^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^{4/3}+\left(a^2-b^2x^2\right)^{2/3}+a^{2/3}\sqrt[3]{a^2-b^2x^2}}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}\sqrt{-b^2x^2}\sqrt{-\frac{a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)^2}}}\right.}{}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(1/3)),x]
```

output
```
(-3*Sqrt[-(b^2*x^2)]*(a^2 - b^2*x^2)^(1/3)*((-2*Sqrt[-(b^2*x^2)])/((1 - Sq
rt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(2/
3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3))*Sqrt[(a^(4/3) + a^(2/3)*(a^2 - b^2*x^
2)^(1/3) + (a^2 - b^2*x^2)^(2/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)
^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt
[-(b^2*x^2)]*Sqrt[-((a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqr
t[3])*a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3))*Sqrt[(a^(4/3) + a^(2/3)*(a
^2 - b^2*x^2)^(1/3) + (a^2 - b^2*x^2)^(2/3))/((1 - Sqrt[3])*a^(2/3) - (a^2
 - b^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(2/3) - (a^2 - b^2
*x^2)^(1/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(3^(1/4)*Sqrt[-(b^2*x^2)]*Sqrt[-((a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2)])))/(2*b^2*x*(
a - b*x)^(1/3)*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{1}{3}} (bx+a)^{\frac{1}{3}}} dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(1/3),x)
```

output
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}} dx = \int \frac{1}{(bx+a)^{\frac{1}{3}} (-bx+a)^{\frac{1}{3}}} dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^2*x^2 - a^2), x)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx = \frac{\sqrt[3]{a}G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{6}, \frac{2}{3}, 1 & \frac{1}{3}, \frac{1}{2}, \frac{5}{6} \\ -\frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{5}{6} & 0 \end{matrix}\middle| \frac{a^2 e^{-2i\pi}}{b^2x^2}\right)e^{\frac{i\pi}{3}}}{4\pi b\Gamma\left(\frac{1}{3}\right)} - \frac{\sqrt[3]{a}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3}, 0, \frac{1}{6}, \frac{1}{2}, 1 & \\ -\frac{1}{3}, \frac{1}{6} & -\frac{1}{2}, -\frac{1}{6}, 0, 0 \end{matrix}\middle| \frac{a^2}{b^2x^2}\right)}{4\pi b\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(1/3),x)
```

output
```
a**(1/3)*meijerg(((1/6, 2/3, 1), (1/3, 1/2, 5/6)), ((-1/6, 1/6, 1/3, 2/3,
5/6), (0,)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))*exp(I*pi/3)/(4*pi*b*gamm
a(1/3)) - a**(1/3)*meijerg(((-1/2, -1/3, 0, 1/6, 1/2, 1), ()), ((-1/3, 1/6
), (-1/2, -1/6, 0, 0)), a**2/(b**2*x**2))/(4*pi*b*gamma(1/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(a+b\,x)^{1/3}\,(a-b\,x)^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(1/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(1/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(1/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(1/3)),x)
```

3.195 $\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 656

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = -\frac{3(a-bx)^{2/3}}{2ab\sqrt[3]{a+bx}} + \frac{3x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{2a\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{4b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$-\frac{3^{3/4}a\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right.}{\sqrt{2}b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/2*(-b*x+a)^(2/3)/a/b/(b*x+a)^(1/3)+3/2*x*(1-b^2*x^2/a^2)^(1/3)/a/(-b*x+
a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))+3/4*3^(1/4)*(1/2*
6^(1/2)+1/2*2^(1/2))*a*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1
+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(
1/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b
^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-
(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)-1/2*3^(3
/4)*a*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^
(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*El
lipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)
),2*I-I*3^(1/2))*2^(1/2)/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^
2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = -\frac{3(a-bx)^{2/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{4\sqrt[3]{2}ab\sqrt[3]{a+bx}}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(4/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[2/3, 4/3, 5/3, (
a - b*x)/(2*a)])/(4*2^(1/3)*a*b*(a + b*x)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx$$

$$\Big\downarrow \; 80$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{2\sqrt[3]{2}}{\sqrt[3]{a-bx}\left(\frac{bx}{a}+1\right)^{4/3}}dx}{2\sqrt[3]{2}a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{1}{\sqrt[3]{a-bx}\left(\frac{bx}{a}+1\right)^{4/3}}dx}{a\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 79$$

$$-\frac{3(a-bx)^{2/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{4\sqrt[3]{2}ab\sqrt[3]{a+bx}}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(4/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[2/3, 4/3, 5/3, (
a - b*x)/(2*a)])/(4*2^(1/3)*a*b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{1}{3}} (bx+a)^{\frac{4}{3}}}dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(4/3),x)
```

output
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^3*x^3 + a*b^2*x^2 - a^2*b*x
- a^3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = \int \frac{1}{\sqrt[3]{a-bx}\,(a+bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(4/3),x)
```

output
```
Integral(1/((a - b*x)**(1/3)*(a + b*x)**(4/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(4/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(4/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = \int \frac{1}{(a+b\,x)^{4/3}\,(a-b\,x)^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(4/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(4/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,a+(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,bx}dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(4/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(1/3)*a + (a + b*x)**(1/3)*(a - b*x)**(
1/3)*b*x),x)
```

3.196 $\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 685

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx = -\frac{3(a-bx)^{2/3}}{8ab(a+bx)^{4/3}} - \frac{3(a-bx)^{2/3}}{8a^2b\sqrt[3]{a+bx}}$$

$$+\frac{3x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{8a^2\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{16b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$-\frac{3^{3/4}\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right.}{4\sqrt{2}b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/8*(-b*x+a)^(2/3)/a/b/(b*x+a)^(4/3)-3/8*(-b*x+a)^(2/3)/a^2/b/(b*x+a)^(1/
3)+3/8*x*(1-b^2*x^2/a^2)^(1/3)/a^2/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)
-(1-b^2*x^2/a^2)^(1/3))+3/16*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-b^2*x^2/
a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/
a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticE((1+3^(1/2
)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/
b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-
(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)-1/8*3^(3/4)*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^
2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1
/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(
1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/b^2/x/(-b*x
+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/
a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx = -\frac{3(a-bx)^{2/3}\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{8\sqrt[3]{2}a^2b\sqrt[3]{a+bx}}$$

input
```
Integrate[1/((a - b*x)^(1/3)*(a + b*x)^(7/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[2/3, 7/3, 5/3, (
a - b*x)/(2*a)])/(8*2^(1/3)*a^2*b*(a + b*x)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}dx$$

$$\Big\downarrow \ 80$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{4\sqrt[3]{2}}{\sqrt[3]{a-bx}\left(\frac{bx}{a}+1\right)^{7/3}}dx}{4\sqrt[3]{2}a^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{1}{\sqrt[3]{a-bx}\left(\frac{bx}{a}+1\right)^{7/3}}dx}{a^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \ 79$$

$$-\frac{3(a-bx)^{2/3}\sqrt[3]{\frac{a+bx}{a}}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},\frac{a-bx}{2a}\right)}{8\sqrt[3]{2}a^2b\sqrt[3]{a+bx}}$$

input
```
Int[1/((a - b*x)^(1/3)*(a + b*x)^(7/3)),x]
```

output
```
(-3*(a - b*x)^(2/3)*((a + b*x)/a)^(1/3)*Hypergeometric2F1[2/3, 7/3, 5/3, (
a - b*x)/(2*a)])/(8*2^(1/3)*a^2*b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{1}{3}}\,(bx+a)^{\frac{7}{3}}}dx$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(7/3),x)
```

output
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^4*x^4 + 2*a*b^3*x^3 - 2*a^3*
b*x - a^4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx=\int \frac{1}{\sqrt[3]{a-bx}\,(a+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(1/3)/(b*x+a)**(7/3),x)
```

output
```
Integral(1/((a - b*x)**(1/3)*(a + b*x)**(7/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(7/3)*(-b*x + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(1/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/3)*(-b*x + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx = \int \frac{1}{(a+b\,x)^{7/3}\,(a-b\,x)^{1/3}}\,dx$$

input
```
int(1/((a + b*x)^(7/3)*(a - b*x)^(1/3)),x)
```

output
```
int(1/((a + b*x)^(7/3)*(a - b*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2+2\,(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,abx+(bx+a)^{\frac{1}{3}}\,(-bx+$$

input
```
int(1/(-b*x+a)^(1/3)/(b*x+a)^(7/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(1/3)*a**2 + 2*(a + b*x)**(1/3)*(a - b*
x)**(1/3)*a*b*x + (a + b*x)**(1/3)*(a - b*x)**(1/3)*b**2*x**2),x)
```

3.197 $\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 181

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx = -\frac{80a^2\sqrt[3]{a-bx}(a+bx)^{2/3}}{27b} - \frac{8a\sqrt[3]{a-bx}(a+bx)^{5/3}}{9b} - \frac{\sqrt[3]{a-bx}(a+bx)^{8/3}}{3b} - \frac{320a^3\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{27\sqrt{3}b} - \frac{160a^3\log(a-bx)}{81b} - \frac{160a^3\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{27b}$$

output
```
-80/27*a^2*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b-8/9*a*(-b*x+a)^(1/3)*(b*x+a)^(5/
3)/b-1/3*(-b*x+a)^(1/3)*(b*x+a)^(8/3)/b+320/81*a^3*arctan(-1/3*3^(1/2)+2/3
*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2)/b-160/81*a^3*ln(-b*x+a)/b-1
60/27*a^3*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx =$$
$$-\frac{3\sqrt[3]{a-bx}(a+bx)^{2/3}\left(113a^2+42abx+9b^2x^2\right)+320\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)+320a^3\log\left(\right.}{81b}$$

input
```
Integrate[(a + b*x)^(8/3)/(a - b*x)^(2/3),x]
```

output
```
-1/81*(3*(a - b*x)^(1/3)*(a + b*x)^(2/3)*(113*a^2 + 42*a*b*x + 9*b^2*x^2)
+ 320*Sqrt[3]*a^3*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (
a + b*x)^(1/3))] + 320*a^3*Log[(a - b*x)^(1/3) + (a + b*x)^(1/3)] - 160*a^
3*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]
)/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx$$
$$\Big\downarrow \text{ 60}$$
$$\frac{16}{9}a\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}dx-\frac{\sqrt[3]{a-bx}(a+bx)^{8/3}}{3b}$$
$$\Big\downarrow \text{ 60}$$
$$\frac{16}{9}a\left(\frac{5}{3}a\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}dx-\frac{\sqrt[3]{a-bx}(a+bx)^{5/3}}{2b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)^{8/3}}{3b}$$

$$\Big\downarrow \; 60$$

$$\frac{16}{9}a\left(\frac{5}{3}a\left(\frac{4}{3}a\int\frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx-\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)^{5/3}}{2b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)^{8/3}}{3b}$$

$$\Big\downarrow \; 72$$

$$\frac{16}{9}a\left(\frac{5}{3}a\left(\frac{4}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)}{b}\right.\right.$$
$$\frac{\sqrt[3]{a-bx}(a+bx)^{8/3}}{3b}$$

input
```
Int[(a + b*x)^(8/3)/(a - b*x)^(2/3),x]
```

output
```
-1/3*((a - b*x)^(1/3)*(a + b*x)^(8/3))/b + (16*a*(-1/2*((a - b*x)^(1/3)*(a
 + b*x)^(5/3))/b + (5*a*(-(((a - b*x)^(1/3)*(a + b*x)^(2/3))/b) + (4*a*(-(
(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3))]
)/b) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])/(
2*b)))/3))/3))/9
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int((b*x+a)^(8/3)/(-b*x+a)^(2/3),x)
```

output
```
int((b*x+a)^(8/3)/(-b*x+a)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx =$$

$$-\frac{320\sqrt{3}a^3\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)+320\,a^3\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-160\,a^3\log\left(\frac{bx-}{}\right.}{81\,b}$$

input
```
integrate((b*x+a)^(8/3)/(-b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/81*(320*sqrt(3)*a^3*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a
)^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 320*a^3*log((b*x + (b*x + a)^(2/3)*
(-b*x + a)^(1/3) + a)/(b*x + a)) - 160*a^3*log((b*x - (b*x + a)^(2/3)*(-b*
x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) + 3*(9*b^2
*x^2 + 42*a*b*x + 113*a^2)*(b*x + a)^(2/3)*(-b*x + a)^(1/3))/b
```

Sympy [F]

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+bx)^{\frac{8}{3}}}{(a-bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)**(8/3)/(-b*x+a)**(2/3),x)
```

output
```
Integral((a + b*x)**(8/3)/(a - b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{8}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(8/3)/(-b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(8/3)/(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{8}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(8/3)/(-b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(8/3)/(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx=\int \frac{{\left(a+b\,x\right)}^{8/3}}{{\left(a-b\,x\right)}^{2/3}}\,dx$$

input
```
int((a + b*x)^(8/3)/(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(8/3)/(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{8/3}}{(a-bx)^{2/3}}\,dx=\left(\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{2}{3}}}dx\right)a^2+\left(\int \frac{(bx+a)^{\frac{2}{3}}\,x^2}{(-bx+a)^{\frac{2}{3}}}dx\right)b^2+2\left(\int \frac{(bx+a)^{\frac{2}{3}}\,x}{(-bx+a)^{\frac{2}{3}}}dx\right)ab$$

input
```
int((b*x+a)^(8/3)/(-b*x+a)^(2/3),x)
```

output
```
int((a + b*x)**(2/3)/(a - b*x)**(2/3),x)*a**2 + int(((a + b*x)**(2/3)*x**2
)/(a - b*x)**(2/3),x)*b**2 + 2*int(((a + b*x)**(2/3)*x)/(a - b*x)**(2/3),x
)*a*b
```

3.198 $\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx = -\frac{5a\sqrt[3]{a-bx}(a+bx)^{2/3}}{3b}-\frac{\sqrt[3]{a-bx}(a+bx)^{5/3}}{2b} -\frac{20a^2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{3\sqrt{3}b}-\frac{10a^2\log(a-bx)}{9b}-\frac{10a^2\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{3b}$$

output

```
-5/3*a*(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b-1/2*(-b*x+a)^(1/3)*(b*x+a)^(5/3)/b+2
0/9*a^2*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))*3^(1
/2)/b-10/9*a^2*ln(-b*x+a)/b-10/3*a^2*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx = \frac{(-13a-3bx)\sqrt[3]{a-bx}(a+bx)^{2/3}}{6b} - \frac{20a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b} - \frac{20a^2\log\left(b\sqrt[3]{a-bx}+b\sqrt[3]{a+bx}\right)}{9b} + \frac{10a^2\log\left((a-bx)^{2/3}-\sqrt[3]{a-bx}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9b}$$

input
```
Integrate[(a + b*x)^(5/3)/(a - b*x)^(2/3),x]
```

output
```
((-13*a - 3*b*x)*(a - b*x)^(1/3)*(a + b*x)^(2/3))/(6*b) - (20*a^2*ArcTan[(
Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))])/(3*Sqrt[
3]*b) - (20*a^2*Log[b*(a - b*x)^(1/3) + b*(a + b*x)^(1/3)])/(9*b) + (10*a^
2*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]
)/(9*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{5}{3}a\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}dx - \frac{\sqrt[3]{a-bx}(a+bx)^{5/3}}{2b}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{3}a\left(\frac{4}{3}a\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx-\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)^{5/3}}{2b}$$

$$\Big\downarrow 72$$

$$\frac{5}{3}a\left(\frac{4}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)^{5/3}}{2b}$$

input
```
Int[(a + b*x)^(5/3)/(a - b*x)^(2/3),x]
```

output
```
-1/2*((a - b*x)^(1/3)*(a + b*x)^(5/3))/b + (5*a*(-(((a - b*x)^(1/3)*(a + b
*x)^(2/3))/b) + (4*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(S
qrt[3]*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(
1/3)/(a - b*x)^(1/3)])/(2*b)))/3))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int((b*x+a)^(5/3)/(-b*x+a)^(2/3),x)
```

output
```
int((b*x+a)^(5/3)/(-b*x+a)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx =$$
$$-\frac{40\sqrt{3}a^2 \arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)+40\,a^2\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-20\,a^2\log\left(\frac{bx-(bx+\cdots}{\cdots}\right.}{18\,b}$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/18*(40*sqrt(3)*a^2*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)
^(2/3)*(-b*x + a)^(1/3))/(b*x + a)) + 40*a^2*log((b*x + (b*x + a)^(2/3)*(-
b*x + a)^(1/3) + a)/(b*x + a)) - 20*a^2*log((b*x - (b*x + a)^(2/3)*(-b*x +
 a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) + 3*(3*b*x +
13*a)*(b*x + a)^(2/3)*(-b*x + a)^(1/3))/b
```

Sympy [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+bx)^{\frac{5}{3}}}{(a-bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)**(5/3)/(-b*x+a)**(2/3),x)
```

output
```
Integral((a + b*x)**(5/3)/(a - b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(5/3)/(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(5/3)/(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+b\,x)^{5/3}}{(a-b\,x)^{2/3}}\,dx$$

input
```
int((a + b*x)^(5/3)/(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(5/3)/(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{2/3}}\,dx = \left(\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{2}{3}}}dx\right)a + \left(\int \frac{(bx+a)^{\frac{2}{3}}\,x}{(-bx+a)^{\frac{2}{3}}}dx\right)b$$

input
```
int((b*x+a)^(5/3)/(-b*x+a)^(2/3),x)
```

output
```
int((a + b*x)**(2/3)/(a - b*x)**(2/3),x)*a + int(((a + b*x)**(2/3)*x)/(a -
 b*x)**(2/3),x)*b
```

3.199 $\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx = -\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b} - \frac{4a\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{\sqrt{3}b} - \frac{2a\log(a-bx)}{3b} - \frac{2a\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{b}$$

output
```
-(-b*x+a)^(1/3)*(b*x+a)^(2/3)/b+4/3*a*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3
)*3^(1/2)/(-b*x+a)^(1/3))*3^(1/2)/b-2/3*a*ln(-b*x+a)/b-2*a*ln(1+(b*x+a)^(1
/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx = -\frac{3\sqrt[3]{a-bx}(a+bx)^{2/3} + 4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) + 4a\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right) - 2a\,\mathrm{l}}{3b}$$

input
```
Integrate[(a + b*x)^(2/3)/(a - b*x)^(2/3),x]
```

output
```
-1/3*(3*(a - b*x)^(1/3)*(a + b*x)^(2/3) + 4*Sqrt[3]*a*ArcTan[(Sqrt[3]*(a +
 b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] + 4*a*Log[b*((a - b*x
)^(1/3) + (a + b*x)^(1/3))] - 2*a*Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a
 + b*x)^(1/3) + (a + b*x)^(2/3)])/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{4}{3}a\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}dx-\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}$$

$$\Big\downarrow \text{ 72}$$

$$\frac{4}{3}a\left(-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}\right)-\frac{\sqrt[3]{a-bx}(a+bx)^{2/3}}{b}$$

input
```
Int[(a + b*x)^(2/3)/(a - b*x)^(2/3),x]
```

output
```
-(((a - b*x)^(1/3)*(a + b*x)^(2/3))/b) + (4*a*(-((Sqrt[3]*ArcTan[1/Sqrt[3]
 - (2*(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3))])/b) - Log[a - b*x]/(2*b)
 - (3*Log[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])/(2*b)))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int((b*x+a)^(2/3)/(-b*x+a)^(2/3),x)
```

output
```
int((b*x+a)^(2/3)/(-b*x+a)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx =$$

$$-\frac{4\sqrt{3}a\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3\,(bx+a)}\right)+4\,a\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-2\,a\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx\right.}{}\right.}{3\,b}$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(3)*a*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)^(2/
3)*(-b*x + a)^(1/3))/(b*x + a)) + 4*a*log((b*x + (b*x + a)^(2/3)*(-b*x + a
)^(1/3) + a)/(b*x + a)) - 2*a*log((b*x - (b*x + a)^(2/3)*(-b*x + a)^(1/3)
+ (b*x + a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)) + 3*(b*x + a)^(2/3)*(-b
*x + a)^(1/3))/b
```

Sympy [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+bx)^{\frac{2}{3}}}{(a-bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)**(2/3)/(-b*x+a)**(2/3),x)
```

output
```
Integral((a + b*x)**(2/3)/(a - b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(2/3)/(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(2/3)/(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+b\,x)^{2/3}}{(a-b\,x)^{2/3}}\,dx$$

input
```
int((a + b*x)^(2/3)/(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(2/3)/(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int((b*x+a)^(2/3)/(-b*x+a)^(2/3),x)
```

output
```
int((a + b*x)**(2/3)/(a - b*x)**(2/3),x)
```

3.200 $\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b} - \frac{\log(a-bx)}{2b} - \frac{3\log\left(1+\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}\right)}{2b}$$

output
```
3^(1/2)*arctan(-1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2)/(-b*x+a)^(1/3))/b-1/
2*ln(-b*x+a)/b-3/2*ln(1+(b*x+a)^(1/3)/(-b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 2\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right) + \mathrm{l}}{2b}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*
x)^(1/3))] - 2*Log[b*((a - b*x)^(1/3) + (a + b*x)^(1/3))] + Log[(a - b*x)^
(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(2*b)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx$$

$$\Big\downarrow \ 72$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a-bx}}\right)}{b}-\frac{\log(a-bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}+1\right)}{2b}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(1/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a + b*x)^(1/3))/(Sqrt[3]*(a - b*x)^(1/3)
)])/b) - Log[a - b*x]/(2*b) - (3*Log[1 + (a + b*x)^(1/3)/(a - b*x)^(1/3)])
/(2*b)
```

Defintions of rubi rules used

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(1/3),x)
```

output
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = -\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(bx+a)-2\sqrt{3}(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{3(bx+a)}\right)+2\log\left(\frac{bx+(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+a}{bx+a}\right)-\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}+}{bx+}\right.}{2b}$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*(b*x + a) - 2*sqrt(3)*(b*x + a)^(2/3)
*(-b*x + a)^(1/3))/(b*x + a)) + 2*log((b*x + (b*x + a)^(2/3)*(-b*x + a)^(1
/3) + a)/(b*x + a)) - log((b*x - (b*x + a)^(2/3)*(-b*x + a)^(1/3) + (b*x +
 a)^(1/3)*(-b*x + a)^(2/3) + a)/(b*x + a)))/b
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(a-bx)^{\frac{2}{3}}\sqrt[3]{a+bx}}\,dx$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(1/3),x)
```

output
```
Integral(1/((a - b*x)**(2/3)*(a + b*x)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/3)*(-b*x + a)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(a+b\,x)^{1/3}\,(a-b\,x)^{2/3}}\,dx$$

input
```
int(1/((a + b*x)^(1/3)*(a - b*x)^(2/3)),x)
```

output
```
int(1/((a + b*x)^(1/3)*(a - b*x)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(1/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(2/3)),x)
```

3.201 $\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx=-\frac{3\sqrt[3]{a-bx}}{2ab\sqrt[3]{a+bx}}$$

output
```
-3/2*(-b*x+a)^(1/3)/a/b/(b*x+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx=-\frac{3\sqrt[3]{a-bx}}{2ab\sqrt[3]{a+bx}}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(4/3)),x]
```

output
```
(-3*(a - b*x)^(1/3))/(2*a*b*(a + b*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{3\sqrt[3]{a-bx}}{2ab\sqrt[3]{a+bx}}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(4/3)),x]
```

output
```
(-3*(a - b*x)^(1/3))/(2*a*b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{1}{3}}}{2ab(bx+a)^{\frac{1}{3}}}$	24
orering	$-\frac{3(-bx+a)^{\frac{1}{3}}}{2ab(bx+a)^{\frac{1}{3}}}$	24

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(-b*x+a)^(1/3)/a/b/(b*x+a)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx = -\frac{3\,(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{2\,(ab^2x+a^2b)}$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
-3/2*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a*b^2*x + a^2*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(4/3),x)
```

output
```
Integral(1/((a - b*x)**(2/3)*(a + b*x)**(4/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(4/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(4/3)*(-b*x + a)^(2/3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx = -\frac{3\,(a-b\,x)^{1/3}}{2\,a\,b\,(a+b\,x)^{1/3}}$$

input
```
int(1/((a + b*x)^(4/3)*(a - b*x)^(2/3)),x)
```

output
```
-(3*(a - b*x)^(1/3))/(2*a*b*(a + b*x)^(1/3))
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{2}{3}}\,a+(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{2}{3}}\,bx}dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(4/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(2/3)*a + (a + b*x)**(1/3)*(a - b*x)**(
2/3)*b*x),x)
```

3.202 $\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = -\frac{3\sqrt[3]{a-bx}}{8ab(a+bx)^{4/3}} - \frac{9\sqrt[3]{a-bx}}{16a^2b\sqrt[3]{a+bx}}$$

output
```
-3/8*(-b*x+a)^(1/3)/a/b/(b*x+a)^(4/3)-9/16*(-b*x+a)^(1/3)/a^2/b/(b*x+a)^(1
/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = -\frac{3\sqrt[3]{a-bx}(5a+3bx)}{16a^2b(a+bx)^{4/3}}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(7/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*(5*a + 3*b*x))/(16*a^2*b*(a + b*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}} \, dx$$
$$\Big\downarrow \; 55$$
$$\frac{3\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}dx}{8a}-\frac{3\sqrt[3]{a-bx}}{8ab(a+bx)^{4/3}}$$
$$\Big\downarrow \; 48$$
$$-\frac{9\sqrt[3]{a-bx}}{16a^2b\sqrt[3]{a+bx}}-\frac{3\sqrt[3]{a-bx}}{8ab(a+bx)^{4/3}}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(7/3)),x]
```

output
```
(-3*(a - b*x)^(1/3))/(8*a*b*(a + b*x)^(4/3)) - (9*(a - b*x)^(1/3))/(16*a^2
*b*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{1}{3}}(3bx+5a)}{16(bx+a)^{\frac{4}{3}}a^2b}$	32
orering	$-\frac{3(-bx+a)^{\frac{1}{3}}(3bx+5a)}{16(bx+a)^{\frac{4}{3}}a^2b}$	32

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-3/16*(-b*x+a)^(1/3)*(3*b*x+5*a)/(b*x+a)^(4/3)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = -\frac{3\,(3\,bx+5\,a)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{16\,(a^2b^3x^2+2\,a^3b^2x+a^4b)}$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-3/16*(3*b*x + 5*a)*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a^2*b^3*x^2 + 2*a^3*
b^2*x + a^4*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(7/3),x)
```

output
```
Integral(1/((a - b*x)**(2/3)*(a + b*x)**(7/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(7/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/3)*(-b*x + a)^(2/3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = -\frac{\left(\frac{15}{16\,a\,b^2}+\frac{9\,x}{16\,a^2\,b}\right)\,{\left(a-b\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}+\frac{a\,{\left(a+b\,x\right)}^{1/3}}{b}}$$

input
```
int(1/((a + b*x)^(7/3)*(a - b*x)^(2/3)),x)
```

output
```
-((15/(16*a*b^2) + (9*x)/(16*a^2*b))*(a - b*x)^(1/3))/(x*(a + b*x)^(1/3) +
 (a*(a + b*x)^(1/3))/b)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^2+2\,(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}abx+(bx+a)^{\frac{1}{3}}(-bx}$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(7/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(2/3)*a**2 + 2*(a + b*x)**(1/3)*(a - b*
x)**(2/3)*a*b*x + (a + b*x)**(1/3)*(a - b*x)**(2/3)*b**2*x**2),x)
```

3.203 $\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx=-\frac{3\sqrt[3]{a-bx}}{14ab(a+bx)^{7/3}}-\frac{9\sqrt[3]{a-bx}}{56a^2b(a+bx)^{4/3}}-\frac{27\sqrt[3]{a-bx}}{112a^3b\sqrt[3]{a+bx}}$$

output
```
-3/14*(-b*x+a)^(1/3)/a/b/(b*x+a)^(7/3)-9/56*(-b*x+a)^(1/3)/a^2/b/(b*x+a)^(
4/3)-27/112*(-b*x+a)^(1/3)/a^3/b/(b*x+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx=-\frac{3\sqrt[3]{a-bx}(23a^2+24abx+9b^2x^2)}{112a^3b(a+bx)^{7/3}}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(10/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*(23*a^2 + 24*a*b*x + 9*b^2*x^2))/(112*a^3*b*(a + b*x)^
(7/3))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}dx}{7a}-\frac{3\sqrt[3]{a-bx}}{14ab(a+bx)^{7/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{3\left(\frac{3\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}dx}{8a}-\frac{3\sqrt[3]{a-bx}}{8ab(a+bx)^{4/3}}\right)}{7a}-\frac{3\sqrt[3]{a-bx}}{14ab(a+bx)^{7/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{3\left(-\frac{9\sqrt[3]{a-bx}}{16a^2b\sqrt[3]{a+bx}}-\frac{3\sqrt[3]{a-bx}}{8ab(a+bx)^{4/3}}\right)}{7a}-\frac{3\sqrt[3]{a-bx}}{14ab(a+bx)^{7/3}}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(10/3)),x]
```

output
```
(-3*(a - b*x)^(1/3))/(14*a*b*(a + b*x)^(7/3)) + (3*((-3*(a - b*x)^(1/3))/(
8*a*b*(a + b*x)^(4/3)) - (9*(a - b*x)^(1/3))/(16*a^2*b*(a + b*x)^(1/3))))/
(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{1}{3}}\left(9b^{2}x^{2}+24abx+23a^{2}\right)}{112(bx+a)^{\frac{7}{3}}a^{3}b}$	43
orering	$-\frac{3(-bx+a)^{\frac{1}{3}}\left(9b^{2}x^{2}+24abx+23a^{2}\right)}{112(bx+a)^{\frac{7}{3}}a^{3}b}$	43

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
-3/112*(-b*x+a)^(1/3)*(9*b^2*x^2+24*a*b*x+23*a^2)/(b*x+a)^(7/3)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx = -\frac{3\,(9\,b^2x^2+24\,abx+23\,a^2)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{112\,(a^3b^4x^3+3\,a^4b^3x^2+3\,a^5b^2x+a^6b)}$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

output
```
-3/112*(9*b^2*x^2 + 24*a*b*x + 23*a^2)*(b*x + a)^(2/3)*(-b*x + a)^(1/3)/(a
^3*b^4*x^3 + 3*a^4*b^3*x^2 + 3*a^5*b^2*x + a^6*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{10}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(10/3),x)
```

output
```
Integral(1/((a - b*x)**(2/3)*(a + b*x)**(10/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{10}{3}}\,(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(10/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(10/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{10}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(10/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(10/3)*(-b*x + a)^(2/3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx = -\frac{(a-b\,x)^{1/3}\left(\frac{69}{112\,a\,b^3}+\frac{9\,x}{14\,a^2\,b^2}+\frac{27\,x^2}{112\,a^3\,b}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

input
```
int(1/((a + b*x)^(10/3)*(a - b*x)^(2/3)),x)
```

output
```
-((a - b*x)^(1/3)*(69/(112*a*b^3) + (9*x)/(14*a^2*b^2) + (27*x^2)/(112*a^3
*b)))/(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*x)^
(1/3))/b)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^3+3(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^2bx+3(bx+a)^{\frac{1}{3}}(\cdots}$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(10/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(2/3)*a**3 + 3*(a + b*x)**(1/3)*(a - b*
x)**(2/3)*a**2*b*x + 3*(a + b*x)**(1/3)*(a - b*x)**(2/3)*a*b**2*x**2 + (a
+ b*x)**(1/3)*(a - b*x)**(2/3)*b**3*x**3),x)
```

3.204 $\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}} \, dx = -\frac{3\sqrt[3]{a-bx}}{20ab(a+bx)^{10/3}} - \frac{27\sqrt[3]{a-bx}}{280a^2b(a+bx)^{7/3}} - \frac{81\sqrt[3]{a-bx}}{1120a^3b(a+bx)^{4/3}} - \frac{243\sqrt[3]{a-bx}}{2240a^4b\sqrt[3]{a+bx}}$$

output
```
-3/20*(-b*x+a)^(1/3)/a/b/(b*x+a)^(10/3)-27/280*(-b*x+a)^(1/3)/a^2/b/(b*x+a
)^(7/3)-81/1120*(-b*x+a)^(1/3)/a^3/b/(b*x+a)^(4/3)-243/2240*(-b*x+a)^(1/3)
/a^4/b/(b*x+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}} \, dx = -\frac{3\sqrt[3]{a-bx}(319a^3+423a^2bx+297ab^2x^2+81b^3x^3)}{2240a^4b(a+bx)^{10/3}}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(13/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*(319*a^3 + 423*a^2*b*x + 297*a*b^2*x^2 + 81*b^3*x^3))/
(2240*a^4*b*(a + b*x)^(10/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{9\int \frac{1}{(a-bx)^{2/3}(a+bx)^{10/3}}dx}{20a}-\frac{3\sqrt[3]{a-bx}}{20ab(a+bx)^{10/3}}$$

$$\Big\downarrow \text{55}$$

$$\frac{9\left(\frac{3\int \frac{1}{(a-bx)^{2/3}(a+bx)^{7/3}}dx}{7a}-\frac{3\sqrt[3]{a-bx}}{14ab(a+bx)^{7/3}}\right)}{20a}-\frac{3\sqrt[3]{a-bx}}{20ab(a+bx)^{10/3}}$$

$$\Big\downarrow \text{55}$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{1}{(a-bx)^{2/3}(a+bx)^{4/3}}dx}{8a}-\frac{3\sqrt[3]{a-bx}}{8ab(a+bx)^{4/3}}\right)}{7a}-\frac{3\sqrt[3]{a-bx}}{14ab(a+bx)^{7/3}}\right)}{20a}-\frac{3\sqrt[3]{a-bx}}{20ab(a+bx)^{10/3}}$$

$$\Big\downarrow \text{48}$$

$$\frac{9\left(\frac{3\left(-\frac{9\sqrt[3]{a-bx}}{16a^2b\sqrt[3]{a+bx}}-\frac{3\sqrt[3]{a-bx}}{8ab(a+bx)^{4/3}}\right)}{7a}-\frac{3\sqrt[3]{a-bx}}{14ab(a+bx)^{7/3}}\right)}{20a}-\frac{3\sqrt[3]{a-bx}}{20ab(a+bx)^{10/3}}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(13/3)),x]
```

output
```
(-3*(a - b*x)^(1/3))/(20*a*b*(a + b*x)^(10/3)) + (9*((-3*(a - b*x)^(1/3))/
(14*a*b*(a + b*x)^(7/3)) + (3*((-3*(a - b*x)^(1/3))/(8*a*b*(a + b*x)^(4/3)
) - (9*(a - b*x)^(1/3))/(16*a^2*b*(a + b*x)^(1/3))))/(7*a)))/(20*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3(-bx+a)^{\frac{1}{3}} \left(81 b^{3} x^{3}+297 a\,b^{2} x^{2}+423 a^{2} b x+319 a^{3}\right)}{2240 (bx+a)^{\frac{10}{3}} a^{4} b}$	54
orering	$-\frac{3(-bx+a)^{\frac{1}{3}} \left(81 b^{3} x^{3}+297 a\,b^{2} x^{2}+423 a^{2} b x+319 a^{3}\right)}{2240 (bx+a)^{\frac{10}{3}} a^{4} b}$	54

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2240*(-b*x+a)^(1/3)*(81*b^3*x^3+297*a*b^2*x^2+423*a^2*b*x+319*a^3)/(b*x
+a)^(10/3)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}}\,dx = -\frac{3\,(81\,b^3x^3+297\,ab^2x^2+423\,a^2bx+319\,a^3)(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{2240\,(a^4b^5x^4+4\,a^5b^4x^3+6\,a^6b^3x^2+4\,a^7b^2x+a^8b)}$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

output
```
-3/2240*(81*b^3*x^3 + 297*a*b^2*x^2 + 423*a^2*b*x + 319*a^3)*(b*x + a)^(2/
3)*(-b*x + a)^(1/3)/(a^4*b^5*x^4 + 4*a^5*b^4*x^3 + 6*a^6*b^3*x^2 + 4*a^7*b
^2*x + a^8*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{13}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(13/3),x)
```

output
```
Integral(1/((a - b*x)**(2/3)*(a + b*x)**(13/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{13}{3}}\,(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(13/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{13}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(13/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(13/3)*(-b*x + a)^(2/3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}}\,dx = -\frac{(a-b\,x)^{1/3}\left(\frac{957}{2240\,a\,b^4}+\frac{1269\,x}{2240\,a^2\,b^3}+\frac{891\,x^2}{2240\,a^3\,b^2}+\frac{243\,x^3}{2240\,a^4\,b}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

input
```
int(1/((a + b*x)^(13/3)*(a - b*x)^(2/3)),x)
```

output
```
-((a - b*x)^(1/3)*(957/(2240*a*b^4) + (1269*x)/(2240*a^2*b^3) + (891*x^2)/
(2240*a^3*b^2) + (243*x^3)/(2240*a^4*b)))/(x^3*(a + b*x)^(1/3) + (a^3*(a +
 b*x)^(1/3))/b^3 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))
/b^2)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{13/3}}\,dx=\int \frac{}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^4+4(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}a^3bx+6(bx+a)^{\frac{1}{3}}(-}$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(13/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(2/3)*a**4 + 4*(a + b*x)**(1/3)*(a - b*
x)**(2/3)*a**3*b*x + 6*(a + b*x)**(1/3)*(a - b*x)**(2/3)*a**2*b**2*x**2 +
4*(a + b*x)**(1/3)*(a - b*x)**(2/3)*a*b**3*x**3 + (a + b*x)**(1/3)*(a - b*
x)**(2/3)*b**4*x**4),x)
```

3.205 $\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 338

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}} \, dx = -\frac{12a\sqrt[3]{a-bx}\sqrt[3]{a+bx}}{5b} - \frac{3\sqrt[3]{a-bx}(a+bx)^{4/3}}{5b} + \frac{8\,3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{5b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output

```
-12/5*a*(-b*x+a)^(1/3)*(b*x+a)^(1/3)/b-3/5*(-b*x+a)^(1/3)*(b*x+a)^(4/3)/b+
8/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^4*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*
x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2
)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/
3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/
(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)
)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.20

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx = -\frac{6\sqrt[3]{2}a\sqrt[3]{a-bx}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a + b*x)^(4/3)/(a - b*x)^(2/3),x]
```

output
```
(-6*2^(1/3)*a*(a - b*x)^(1/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-4/3, 1/3,
 4/3, (a - b*x)/(2*a)])/(b*((a + b*x)/a)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{2\sqrt[3]{2}a\sqrt[3]{a+bx}\int \frac{\left(\frac{bx}{a}+1\right)^{4/3}}{2\sqrt[3]{2}(a-bx)^{2/3}}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\sqrt[3]{a+bx}\int \frac{\left(\frac{bx}{a}+1\right)^{4/3}}{(a-bx)^{2/3}}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{6\sqrt[3]{2}a\sqrt[3]{a-bx}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Int[(a + b*x)^(4/3)/(a - b*x)^(2/3),x]
```

output
```
(-6*2^(1/3)*a*(a - b*x)^(1/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-4/3, 1/3,
 4/3, (a - b*x)/(2*a)])/(b*((a + b*x)/a)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int((b*x+a)^(4/3)/(-b*x+a)^(2/3),x)
```

output
```
int((b*x+a)^(4/3)/(-b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(4/3)*(-b*x + a)^(1/3)/(b*x - a), x)
```

Sympy [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+bx)^{\frac{4}{3}}}{(a-bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)**(4/3)/(-b*x+a)**(2/3),x)
```

output
```
Integral((a + b*x)**(4/3)/(a - b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(4/3)/(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(4/3)/(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+b\,x)^{4/3}}{(a-b\,x)^{2/3}}\,dx$$

input
```
int((a + b*x)^(4/3)/(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(4/3)/(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{2/3}}\,dx=\left(\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{2}{3}}}dx\right)a+\left(\int \frac{(bx+a)^{\frac{1}{3}}\,x}{(-bx+a)^{\frac{2}{3}}}dx\right)b$$

input
```
int((b*x+a)^(4/3)/(-b*x+a)^(2/3),x)
```

output
```
int((a + b*x)**(1/3)/(a - b*x)**(2/3),x)*a + int(((a + b*x)**(1/3)*x)/(a -
 b*x)**(2/3),x)*b
```

3.206 $\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = -\frac{3\sqrt[3]{a-bx}\sqrt[3]{a+bx}}{2b} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-}{1-\sqrt{3}-}\right.\right.}{b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/2*(-b*x+a)^(1/3)*(b*x+a)^(1/3)/b+3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^3*
(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+
(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Elliptic
F((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-
I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/
(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = -\frac{3\sqrt[3]{2}\sqrt[3]{a-bx}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a + b*x)^(1/3)/(a - b*x)^(2/3),x]
```

output
```
(-3*2^(1/3)*(a - b*x)^(1/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4
/3, (a - b*x)/(2*a)])/(b*((a + b*x)/a)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\sqrt[3]{2}\sqrt[3]{a+bx}\int \frac{\sqrt[3]{\frac{bx}{a}+1}}{\sqrt[3]{2}(a-bx)^{2/3}}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[3]{a+bx}\int \frac{\sqrt[3]{\frac{bx}{a}+1}}{(a-bx)^{2/3}}dx}{\sqrt[3]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3\sqrt[3]{2}\sqrt[3]{a-bx}\sqrt[3]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{b\sqrt[3]{\frac{a+bx}{a}}}$$

input
```
Int[(a + b*x)^(1/3)/(a - b*x)^(2/3),x]
```

output
```
(-3*2^(1/3)*(a - b*x)^(1/3)*(a + b*x)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4
/3, (a - b*x)/(2*a)])/(b*((a + b*x)/a)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int((b*x+a)^(1/3)/(-b*x+a)^(2/3),x)
```

output
```
int((b*x+a)^(1/3)/(-b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(1/3)*(-b*x + a)^(1/3)/(b*x - a), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = \int \frac{\sqrt[3]{a+bx}}{(a-bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)**(1/3)/(-b*x+a)**(2/3),x)
```

output
```
Integral((a + b*x)**(1/3)/(a - b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/3)/(-b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/3)/(-b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = \int \frac{(a+b\,x)^{1/3}}{(a-b\,x)^{2/3}}\,dx$$

input
```
int((a + b*x)^(1/3)/(a - b*x)^(2/3),x)
```

output
```
int((a + b*x)^(1/3)/(a - b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{2/3}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int((b*x+a)^(1/3)/(-b*x+a)^(2/3),x)
```

output
```
int((a + b*x)**(1/3)/(a - b*x)**(2/3),x)
```

3.207 $\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 281

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}\,dx = \frac{3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)\cdots}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\cdots}}\cdots}{b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1\cdots}{\left(1-\sqrt{\cdots}\right)\cdots}}\cdots}$$

output
```
3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/
a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1
-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/
(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x
+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.23

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}\,dx = -\frac{3\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{2^{2/3}b(a+bx)^{2/3}}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(2/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, (
a - b*x)/(2*a)])/(2^(2/3)*b*(a + b*x)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {46, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}dx$$

$$\Big\downarrow \text{46}$$

$$\frac{\left(a^2-b^2x^2\right)^{2/3}\int \frac{1}{\left(a^2-b^2x^2\right)^{2/3}}dx}{(a-bx)^{2/3}(a+bx)^{2/3}}$$

$$\Big\downarrow \text{234}$$

$$-\frac{3\sqrt{-b^2x^2}\left(a^2-b^2x^2\right)^{2/3}\int \frac{1}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}}{2b^2x(a-bx)^{2/3}(a+bx)^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}(a^2-b^2x^2)^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^{4/3}+(a^2-b^2x^2)^{2/3}+a^{2/3}\sqrt[3]{a^2-b^2x^2}}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right.}{\left(1-\sqrt{3}\right.}\right.\right.}{b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)^2}}}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(2/3)),x]
```

output
```
(3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^2 - b^2*x^2)^(2/3)*(a^(2/3) - (a^2 - b^2*x^2
)^(1/3))*Sqrt[(a^(4/3) + a^(2/3)*(a^2 - b^2*x^2)^(1/3) + (a^2 - b^2*x^2)^(
2/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))/((1 - Sqrt[3])*a^(2/3) - (
a^2 - b^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b^2*x*(a - b*x)^(2/3)*(a + b*x)^
(2/3)*Sqrt[-((a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(2/3) - (a^2 - b^2*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{2}{3}} (bx+a)^{\frac{2}{3}}} dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(2/3),x)
```

output
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}} dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{2}{3}}} dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(1/3)*(-b*x + a)^(1/3)/(b^2*x^2 - a^2), x)
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}} dx = \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{3}, \frac{5}{6}, 1 & \frac{1}{2}, \frac{2}{3}, \frac{7}{6} \\ \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{5}{6}, \frac{7}{6} & 0 \end{matrix} \middle| \frac{a^2 e^{-2i\pi}}{b^2x^2}\right) e^{\frac{2i\pi}{3}}}{4\pi\sqrt[3]{a}b\Gamma\left(\frac{2}{3}\right)} - \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6}, 0, \frac{1}{3}, \frac{1}{2}, 1 & \\ -\frac{1}{6}, \frac{1}{3} & -\frac{1}{2}, 0, \frac{1}{6}, 0 \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{4\pi\sqrt[3]{a}b\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(2/3),x)
```

output
```
meijerg(((1/3, 5/6, 1), (1/2, 2/3, 7/6)), ((1/6, 1/3, 2/3, 5/6, 7/6), (0,)
), a**2*exp_polar(-2*I*pi)/(b**2*x**2))*exp(2*I*pi/3)/(4*pi*a**(1/3)*b*gam
ma(2/3)) - meijerg(((-1/2, -1/6, 0, 1/3, 1/2, 1), ()), ((-1/6, 1/3), (-1/2
, 0, 1/6, 0)), a**2/(b**2*x**2))/(4*pi*a**(1/3)*b*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(2/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(2/3)*(-b*x + a)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(a+b\,x)^{2/3}\,(a-b\,x)^{2/3}}\,dx$$

input
```
int(1/((a + b*x)^(2/3)*(a - b*x)^(2/3)),x)
```

output
```
int(1/((a + b*x)^(2/3)*(a - b*x)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(2/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(2/3)),x)
```

3.208 $\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 312

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a-bx}}{4ab(a+bx)^{2/3}}$$

$$+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{\ldots}}{1-\sqrt{3}-\sqrt[3]{\ldots}}\right.\right.}{4b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output

```
-3/4*(-b*x+a)^(1/3)/a/b/(b*x+a)^(2/3)+1/4*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2)
)*a*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1
/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Elli
pticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),
2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/
3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{2\ 2^{2/3}ab(a+bx)^{2/3}}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(5/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, (
a - b*x)/(2*a)])/(2*2^(2/3)*a*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx$$

$$\Big\downarrow \ 80$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{2\ 2^{2/3}}{(a-bx)^{2/3}\left(\frac{bx}{a}+1\right)^{5/3}}dx}{2\ 2^{2/3}a(a+bx)^{2/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{1}{(a-bx)^{2/3}\left(\frac{bx}{a}+1\right)^{5/3}}dx}{a(a+bx)^{2/3}}$$

$$\Big\downarrow \ 79$$

$$-\frac{3\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{2\,2^{2/3}ab(a+bx)^{2/3}}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(5/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, (
a - b*x)/(2*a)])/(2*2^(2/3)*a*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{2}{3}}\,(bx+a)^{\frac{5}{3}}}dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(5/3),x)
```

output
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(1/3)*(-b*x + a)^(1/3)/(b^3*x^3 + a*b^2*x^2 - a^2*b*x
- a^3), x)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(5/3),x)
```

output
```
Integral(1/((a - b*x)**(2/3)*(a + b*x)**(5/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/3)*(-b*x + a)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx=\int \frac{1}{(a+b\,x)^{5/3}\,(a-b\,x)^{2/3}}\,dx$$

input
```
int(1/((a + b*x)^(5/3)*(a - b*x)^(2/3)),x)
```

output
```
int(1/((a + b*x)^(5/3)*(a - b*x)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{5/3}}\,dx=\int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,a+(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,bx}dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(5/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(2/3)*a + (a + b*x)**(2/3)*(a - b*x)**(
2/3)*b*x),x)
```

3.209 $\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 340

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = -\frac{3\sqrt[3]{a-bx}}{10ab(a+bx)^{5/3}} - \frac{3\sqrt[3]{a-bx}}{10a^2b(a+bx)^{2/3}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{\cdots}}{1-\sqrt{3}-\sqrt[3]{\cdots}}\right.\right.}{10b^2x(a-bx)^{2/3}(a+bx)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/10*(-b*x+a)^(1/3)/a/b/(b*x+a)^(5/3)-3/10*(-b*x+a)^(1/3)/a^2/b/(b*x+a)^(
2/3)+1/10*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-b^2*x^2/a^2)^(2/3)*(1-(1-b^
2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1
/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(
1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(2/3
)/(b*x+a)^(2/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/
3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = -\frac{3\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{4\ 2^{2/3}a^2b(a+bx)^{2/3}}$$

input
```
Integrate[1/((a - b*x)^(2/3)*(a + b*x)^(8/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, (
a - b*x)/(2*a)])/(4*2^(2/3)*a^2*b*(a + b*x)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{4\ 2^{2/3}}{(a-bx)^{2/3}\left(\frac{bx}{a}+1\right)^{8/3}}dx}{4\ 2^{2/3}a^2(a+bx)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{2/3}\int \frac{1}{(a-bx)^{2/3}\left(\frac{bx}{a}+1\right)^{8/3}}dx}{a^2(a+bx)^{2/3}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{3\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},\frac{a-bx}{2a}\right)}{4\ 2^{2/3}a^2b(a+bx)^{2/3}}$$

input
```
Int[1/((a - b*x)^(2/3)*(a + b*x)^(8/3)),x]
```

output
```
(-3*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, (
a - b*x)/(2*a)])/(4*2^(2/3)*a^2*b*(a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{1}{\left(-bx+a\right)^{\frac{2}{3}}\left(bx+a\right)^{\frac{8}{3}}}dx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(8/3),x)
```

output
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^(1/3)*(-b*x + a)^(1/3)/(b^4*x^4 + 2*a*b^3*x^3 - 2*a^3*
b*x - a^4), x)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{2}{3}}\,(a+bx)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(2/3)/(b*x+a)**(8/3),x)
```

output
```
Integral(1/((a - b*x)**(2/3)*(a + b*x)**(8/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(8/3)*(-b*x + a)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(2/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(8/3)*(-b*x + a)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(a+b\,x)^{8/3}\,(a-b\,x)^{2/3}}\,dx$$

input
```
int(1/((a + b*x)^(8/3)*(a - b*x)^(2/3)),x)
```

output
```
int(1/((a + b*x)^(8/3)*(a - b*x)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{2/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,a^2+2\,(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{2}{3}}\,abx+(bx+a)^{\frac{2}{3}}\,(-bx$$

input
```
int(1/(-b*x+a)^(2/3)/(b*x+a)^(8/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(2/3)*a**2 + 2*(a + b*x)**(2/3)*(a - b*
x)**(2/3)*a*b*x + (a + b*x)**(2/3)*(a - b*x)**(2/3)*b**2*x**2),x)
```

3.210 $\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 175

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx=\frac{28a(a-bx)^{2/3}\sqrt[3]{a+bx}}{3b}+\frac{7(a-bx)^{2/3}(a+bx)^{4/3}}{2b}+\frac{3(a+bx)^{7/3}}{b\sqrt[3]{a-bx}}-\frac{56a^2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b}-\frac{28a^2\log(a+bx)}{9b}-\frac{28a^2\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{3b}$$

output

```
28/3*a*(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b+7/2*(-b*x+a)^(2/3)*(b*x+a)^(4/3)/b+3
*(b*x+a)^(7/3)/b/(-b*x+a)^(1/3)+56/9*a^2*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^
(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2)/b-28/9*a^2*ln(b*x+a)/b-28/3*a^2*ln(1+
(-b*x+a)^(1/3)/(b*x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx = \frac{\frac{3\sqrt[3]{a+bx}(95a^2-20abx-3b^2x^2)}{\sqrt[3]{a-bx}} + 112\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 112a^2\log\left(b\right.}{18b}$$

input
```
Integrate[(a + b*x)^(7/3)/(a - b*x)^(4/3),x]
```

output
```
((3*(a + b*x)^(1/3)*(95*a^2 - 20*a*b*x - 3*b^2*x^2))/(a - b*x)^(1/3) + 112
*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b
*x)^(1/3))] - 112*a^2*Log[b*((a - b*x)^(1/3) + (a + b*x)^(1/3))] + 56*a^2*
Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/
(18*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {57, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}dx$$

$$\Big\downarrow \; 57$$

$$\frac{3(a+bx)^{7/3}}{b\sqrt[3]{a-bx}} - 7\int \frac{(a+bx)^{4/3}}{\sqrt[3]{a-bx}}dx$$

$$\Big\downarrow \; 60$$

$$\frac{3(a+bx)^{7/3}}{b\sqrt[3]{a-bx}} - 7\left(\frac{4}{3}a\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}dx - \frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{3(a+bx)^{7/3}}{b\sqrt[3]{a-bx}}-7\left(\frac{4}{3}a\left(\frac{2}{3}a\int\frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)-\frac{(a-bx)^{2/3}(a+bx)^{4/3}}{2b}\right)$$

$\big\downarrow$ 72

$$\frac{3(a+bx)^{7/3}}{b\sqrt[3]{a-bx}}-7\left(\frac{4}{3}a\left(\frac{2}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)\right.$$

input
```
Int[(a + b*x)^(7/3)/(a - b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(7/3))/(b*(a - b*x)^(1/3)) - 7*(-1/2*((a - b*x)^(2/3)*(a + b*
x)^(4/3))/b + (4*a*(-(((a - b*x)^(2/3)*(a + b*x)^(1/3))/b) + (2*a*((Sqrt[3
]*ArcTan[1/Sqrt[3] - (2*(a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b + L
og[a + b*x]/(2*b) + (3*Log[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b)))/3
))/3)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(-bx+a)^{\frac{4}{3}}}dx$$

input
```
int((b*x+a)^(7/3)/(-b*x+a)^(4/3),x)
```

output
```
int((b*x+a)^(7/3)/(-b*x+a)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx = \frac{112\sqrt{3}(a^2bx-a^3)\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+3\left(3\,b^2x^2+20\,abx-95\,a^2\right)}{}$$

input
```
integrate((b*x+a)^(7/3)/(-b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/18*(112*sqrt(3)*(a^2*b*x - a^3)*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3
)*(b*x + a)^(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + 3*(3*b^2*x^2 + 20*a*b*x -
 95*a^2)*(b*x + a)^(1/3)*(-b*x + a)^(2/3) + 56*(a^2*b*x - a^3)*log((b*x -
(b*x + a)^(2/3)*(-b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(
b*x - a)) - 112*(a^2*b*x - a^3)*log(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/
3) - a)/(b*x - a)))/(b^2*x - a*b)
```

Sympy [F]

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+bx)^{\frac{7}{3}}}{(a-bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)**(7/3)/(-b*x+a)**(4/3),x)
```

output
```
Integral((a + b*x)**(7/3)/(a - b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{7}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(7/3)/(-b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(7/3)/(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{7}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(7/3)/(-b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(7/3)/(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+b\,x)^{7/3}}{(a-b\,x)^{4/3}}\,dx$$

input
```
int((a + b*x)^(7/3)/(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(7/3)/(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{7/3}}{(a-bx)^{4/3}}\,dx = \left(\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx\right)a^2 + \left(\int \frac{(bx+a)^{\frac{1}{3}}\,x^2}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx\right)b^2 + 2\left(\int \frac{(bx+a)^{\frac{1}{3}}\,x}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx\right)ab$$

input
```
int((b*x+a)^(7/3)/(-b*x+a)^(4/3),x)
```

output
```
int((a + b*x)**(1/3)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x)*a**2 +
 int(((a + b*x)**(1/3)*x**2)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x
)*b**2 + 2*int(((a + b*x)**(1/3)*x)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)
*b*x),x)*a*b
```

3.211 $\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \frac{4(a-bx)^{2/3}\sqrt[3]{a+bx}}{b} + \frac{3(a+bx)^{4/3}}{b\sqrt[3]{a-bx}} - \frac{8a\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{\sqrt{3}b} - \frac{4a\log(a+bx)}{3b} - \frac{4a\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{b}$$

output
```
4*(-b*x+a)^(2/3)*(b*x+a)^(1/3)/b+3*(b*x+a)^(4/3)/b/(-b*x+a)^(1/3)+8/3*a*ar
ctan(-1/3*3^(1/2)+2/3*(-b*x+a)^(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1/2)/b-4/3*
a*ln(b*x+a)/b-4*a*ln(1+(-b*x+a)^(1/3)/(b*x+a)^(1/3))/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \frac{\frac{9(7a-bx)\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}} + 8\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 24a\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[(a + b*x)^(4/3)/(a - b*x)^(4/3),x]
```

output
```
((9*(7*a - b*x)*(a + b*x)^(1/3))/(a - b*x)^(1/3) + 8*Sqrt[3]*a*ArcTan[(Sqr
t[3]*(a + b*x)^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 24*a*Log[b
*((a - b*x)^(1/3) + (a + b*x)^(1/3))] + 4*(3 - (2*I)*Sqrt[3])*a*Log[(2*I)*
(a - b*x)^(1/3) + Sqrt[2 - (2*I)*Sqrt[3]]*(a + b*x)^(1/3)] + 4*(3 + (2*I)*
Sqrt[3])*a*Log[(-2*I)*(a - b*x)^(1/3) + Sqrt[2 + (2*I)*Sqrt[3]]*(a + b*x)^
(1/3)])/(9*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {57, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{3(a+bx)^{4/3}}{b\sqrt[3]{a-bx}}-4\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3(a+bx)^{4/3}}{b\sqrt[3]{a-bx}}-4\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)$$

$$\Big\downarrow \text{72}$$

$$\frac{3(a+bx)^{4/3}}{b\sqrt[3]{a-bx}}-4\left(\frac{2}{3}a\left(\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{\log(a+bx)}{2b}+\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}\right)-\frac{(a-bx)^{2/3}\sqrt[3]{a+bx}}{b}\right)$$

input
```
Int[(a + b*x)^(4/3)/(a - b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(4/3))/(b*(a - b*x)^(1/3)) - 4*(-(((a - b*x)^(2/3)*(a + b*x)^
(1/3))/b) + (2*a*((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(a - b*x)^(1/3))/(Sqrt[3]
*(a + b*x)^(1/3))])/b + Log[a + b*x]/(2*b) + (3*Log[1 + (a - b*x)^(1/3)/(a
 + b*x)^(1/3)])/(2*b)))/3)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{4}{3}}}dx$$

input
```
int((b*x+a)^(4/3)/(-b*x+a)^(4/3),x)
```

output
```
int((b*x+a)^(4/3)/(-b*x+a)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \frac{8\sqrt{3}(abx-a^2)\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+3\,(bx+a)^{\frac{1}{3}}(bx-7\,a)(-bx+}{}$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/3*(8*sqrt(3)*(a*b*x - a^2)*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*
x + a)^(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + 3*(b*x + a)^(1/3)*(b*x - 7*a)*
(-b*x + a)^(2/3) + 4*(a*b*x - a^2)*log((b*x - (b*x + a)^(2/3)*(-b*x + a)^(
1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 8*(a*b*x - a^2)*
log(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)))/(b^2*x - a*b
)
```

Sympy [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+bx)^{\frac{4}{3}}}{(a-bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)**(4/3)/(-b*x+a)**(4/3),x)
```

output
```
Integral((a + b*x)**(4/3)/(a - b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(4/3)/(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{4}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(4/3)/(-b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(4/3)/(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+b\,x)^{4/3}}{(a-b\,x)^{4/3}}\,dx$$

input
```
int((a + b*x)^(4/3)/(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(4/3)/(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{4/3}}{(a-bx)^{4/3}}\,dx = \left(\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx\right)a + \left(\int \frac{(bx+a)^{\frac{1}{3}}\,x}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx\right)b$$

input
```
int((b*x+a)^(4/3)/(-b*x+a)^(4/3),x)
```

output
```
int((a + b*x)**(1/3)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x)*a + in
t(((a + b*x)**(1/3)*x)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x)*b
```

3.212 $\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}} dx$

Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}} dx = \frac{3\sqrt[3]{a+bx}}{b\sqrt[3]{a-bx}} - \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b} - \frac{\log(a+bx)}{2b} - \frac{3\log\left(1+\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}\right)}{2b}$$

output
```
3*(b*x+a)^(1/3)/b/(-b*x+a)^(1/3)+3^(1/2)*arctan(-1/3*3^(1/2)+2/3*(-b*x+a)^
(1/3)*3^(1/2)/(b*x+a)^(1/3))/b-1/2*ln(b*x+a)/b-3/2*ln(1+(-b*x+a)^(1/3)/(b*
x+a)^(1/3))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}} dx = \frac{\frac{6\sqrt[3]{a+bx}}{\sqrt[3]{a-bx}} + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx}}{-2\sqrt[3]{a-bx}+\sqrt[3]{a+bx}}\right) - 2\log\left(b\left(\sqrt[3]{a-bx}+\sqrt[3]{a+bx}\right)\right) -}{2b}$$

input
```
Integrate[(a + b*x)^(1/3)/(a - b*x)^(4/3),x]
```

output
```
((6*(a + b*x)^(1/3))/(a - b*x)^(1/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x)
^(1/3))/(-2*(a - b*x)^(1/3) + (a + b*x)^(1/3))] - 2*Log[b*((a - b*x)^(1/3)
 + (a + b*x)^(1/3))] + Log[(a - b*x)^(2/3) - (a - b*x)^(1/3)*(a + b*x)^(1/
3) + (a + b*x)^(2/3)])/(2*b)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {57, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}}dx$$

$$\Big\downarrow \ 57$$

$$\frac{3\sqrt[3]{a+bx}}{b\sqrt[3]{a-bx}}-\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{2/3}}dx$$

$$\Big\downarrow \ 72$$

$$-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a-bx}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{b}+\frac{3\sqrt[3]{a+bx}}{b\sqrt[3]{a-bx}}-\frac{\log(a+bx)}{2b}-\frac{3\log\left(\frac{\sqrt[3]{a-bx}}{\sqrt[3]{a+bx}}+1\right)}{2b}$$

input
```
Int[(a + b*x)^(1/3)/(a - b*x)^(4/3),x]
```

output
```
(3*(a + b*x)^(1/3))/(b*(a - b*x)^(1/3)) - (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(
a - b*x)^(1/3))/(Sqrt[3]*(a + b*x)^(1/3))])/b - Log[a + b*x]/(2*b) - (3*Lo
g[1 + (a - b*x)^(1/3)/(a + b*x)^(1/3)])/(2*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{4}{3}}}dx$$

input
```
int((b*x+a)^(1/3)/(-b*x+a)^(4/3),x)
```

output
```
int((b*x+a)^(1/3)/(-b*x+a)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(90) = 180$.

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}}\,dx = \frac{2\sqrt{3}(bx-a)\arctan\left(\frac{\sqrt{3}(bx-a)+2\sqrt{3}(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{3\,(bx-a)}\right)+(bx-a)\log\left(\frac{bx-(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}}{bx}\right.}{2\,(b^2}$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*(b*x - a)*arctan(1/3*(sqrt(3)*(b*x - a) + 2*sqrt(3)*(b*x +
a)^(1/3)*(-b*x + a)^(2/3))/(b*x - a)) + (b*x - a)*log((b*x - (b*x + a)^(2/
3)*(-b*x + a)^(1/3) + (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) - 2
*(b*x - a)*log(-(b*x - (b*x + a)^(1/3)*(-b*x + a)^(2/3) - a)/(b*x - a)) -
6*(b*x + a)^(1/3)*(-b*x + a)^(2/3))/(b^2*x - a*b)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}}\,dx = \int \frac{\sqrt[3]{a+bx}}{(a-bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)**(1/3)/(-b*x+a)**(4/3),x)
```

output
```
Integral((a + b*x)**(1/3)/(a - b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/3)/(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(1/3)/(-b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/3)/(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+b\,x)^{1/3}}{(a-b\,x)^{4/3}}\,dx$$

input
```
int((a + b*x)^(1/3)/(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(1/3)/(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{1}{3}}}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx$$

input
```
int((b*x+a)^(1/3)/(-b*x+a)^(4/3),x)
```

output
```
int((a + b*x)**(1/3)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x)
```

3.213 $\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx=\frac{3\sqrt[3]{a+bx}}{2ab\sqrt[3]{a-bx}}$$

output
```
3/2*(b*x+a)^(1/3)/a/b/(-b*x+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx=\frac{3\sqrt[3]{a+bx}}{2ab\sqrt[3]{a-bx}}$$

input
```
Integrate[1/((a - b*x)^(4/3)*(a + b*x)^(2/3)),x]
```

output
```
(3*(a + b*x)^(1/3))/(2*a*b*(a - b*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx$$

$$\Big\downarrow \ 48$$

$$\frac{3\sqrt[3]{a+bx}}{2ab\sqrt[3]{a-bx}}$$

input
```
Int[1/((a - b*x)^(4/3)*(a + b*x)^(2/3)),x]
```

output
```
(3*(a + b*x)^(1/3))/(2*a*b*(a - b*x)^(1/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{3(bx+a)^{\frac{1}{3}}}{2ab(-bx+a)^{\frac{1}{3}}}$	24
orering	$\frac{3(bx+a)^{\frac{1}{3}}}{2ab(-bx+a)^{\frac{1}{3}}}$	24

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(b*x+a)^(1/3)/a/b/(-b*x+a)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx = -\frac{3\,(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{2\,(ab^2x-a^2b)}$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-3/2*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a*b^2*x - a^2*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(4/3)/(b*x+a)**(2/3),x)
```

output
```
Integral(1/((a - b*x)**(4/3)*(a + b*x)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(2/3)*(-b*x + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(2/3)*(-b*x + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(a+b\,x)^{2/3}\,(a-b\,x)^{4/3}}\,dx$$

input
```
int(1/((a + b*x)^(2/3)*(a - b*x)^(4/3)),x)
```

output
```
int(1/((a + b*x)^(2/3)*(a - b*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{2/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a-(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,bx}dx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(2/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a - (a + b*x)**(2/3)*(a - b*x)**(
1/3)*b*x),x)
```

3.214 $\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = \frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{2/3}} - \frac{9(a-bx)^{2/3}}{8a^2b(a+bx)^{2/3}}$$

output
```
3/2/a/b/(-b*x+a)^(1/3)/(b*x+a)^(2/3)-9/8*(-b*x+a)^(2/3)/a^2/b/(b*x+a)^(2/3
)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = \frac{3(a-bx)^{2/3}\sqrt[3]{a+bx}(a+3bx)}{8a^2b\,(a^2-b^2x^2)}$$

input
```
Integrate[1/((a - b*x)^(4/3)*(a + b*x)^(5/3)),x]
```

output
```
(3*(a - b*x)^(2/3)*(a + b*x)^(1/3)*(a + 3*b*x))/(8*a^2*b*(a^2 - b^2*x^2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}} \, dx$$

$$\Big\downarrow \text{55}$$

$$\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}dx}{2a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{2/3}}$$

$$\Big\downarrow \text{48}$$

$$\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{2/3}}-\frac{9(a-bx)^{2/3}}{8a^2b(a+bx)^{2/3}}$$

input
```
Int[1/((a - b*x)^(4/3)*(a + b*x)^(5/3)),x]
```

output
```
3/(2*a*b*(a - b*x)^(1/3)*(a + b*x)^(2/3)) - (9*(a - b*x)^(2/3))/(8*a^2*b*(
a + b*x)^(2/3))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{\frac{9bx}{8}+\frac{3a}{8}}{\left(-bx+a\right)^{\frac{1}{3}} \left(bx+a\right)^{\frac{2}{3}} a^{2} b}$	30
orering	$\frac{\frac{9bx}{8}+\frac{3a}{8}}{\left(-bx+a\right)^{\frac{1}{3}} \left(bx+a\right)^{\frac{2}{3}} a^{2} b}$	30

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
3/8*(3*b*x+a)/(-b*x+a)^(1/3)/(b*x+a)^(2/3)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = -\frac{3\,(3\,bx+a)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{8\,(a^2b^3x^2-a^4b)}$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-3/8*(3*b*x + a)*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a^2*b^3*x^2 - a^4*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(4/3)/(b*x+a)**(5/3),x)
```

output
```
Integral(1/((a - b*x)**(4/3)*(a + b*x)**(5/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(5/3)*(-b*x + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{5}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(5/3)*(-b*x + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(a+b\,x)^{5/3}\,(a-b\,x)^{4/3}}\,dx$$

input
```
int(1/((a + b*x)^(5/3)*(a - b*x)^(4/3)),x)
```

output
```
int(1/((a + b*x)^(5/3)*(a - b*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{5/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2-(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,b^2x^2}dx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(5/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a**2 - (a + b*x)**(2/3)*(a - b*x)
**(1/3)*b**2*x**2),x)
```

3.215 $\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}} \, dx = \frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{5/3}} - \frac{9(a-bx)^{2/3}}{10a^2b(a+bx)^{5/3}} - \frac{27(a-bx)^{2/3}}{40a^3b(a+bx)^{2/3}}$$

output
```
3/2/a/b/(-b*x+a)^(1/3)/(b*x+a)^(5/3)-9/10*(-b*x+a)^(2/3)/a^2/b/(b*x+a)^(5/
3)-27/40*(-b*x+a)^(2/3)/a^3/b/(b*x+a)^(2/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}} \, dx = -\frac{3(a^2 - 12abx - 9b^2x^2)}{40a^3b\sqrt[3]{a-bx}(a+bx)^{5/3}}$$

input
```
Integrate[1/((a - b*x)^(4/3)*(a + b*x)^(8/3)),x]
```

output
```
(-3*(a^2 - 12*a*b*x - 9*b^2*x^2))/(40*a^3*b*(a - b*x)^(1/3)*(a + b*x)^(5/3
))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}dx}{a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{5/3}}$$

$$\Big\downarrow \text{55}$$

$$\frac{3\left(\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}dx}{10a}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{5/3}}$$

$$\Big\downarrow \text{48}$$

$$\frac{3\left(-\frac{9(a-bx)^{2/3}}{40a^2b(a+bx)^{2/3}}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{5/3}}$$

input
```
Int[1/((a - b*x)^(4/3)*(a + b*x)^(8/3)),x]
```

output
```
3/(2*a*b*(a - b*x)^(1/3)*(a + b*x)^(5/3)) + (3*((-3*(a - b*x)^(2/3))/(10*a
*b*(a + b*x)^(5/3)) - (9*(a - b*x)^(2/3))/(40*a^2*b*(a + b*x)^(2/3))))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{3\left(-9 b^{2} x^{2}-12 a b x +a^{2}\right)}{40 \left(-b x +a \right)^{\frac{1}{3}} \left(b x +a \right)^{\frac{5}{3}} a^{3} b}$	41
orering	$-\frac{3\left(-9 b^{2} x^{2}-12 a b x +a^{2}\right)}{40 \left(-b x +a \right)^{\frac{1}{3}} \left(b x +a \right)^{\frac{5}{3}} a^{3} b}$	41

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(8/3),x,method=_RETURNVERBOSE)
```

output
```
-3/40*(-9*b^2*x^2-12*a*b*x+a^2)/(-b*x+a)^(1/3)/(b*x+a)^(5/3)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}}\,dx = -\frac{3\left(9\,b^{2}x^{2}+12\,abx-a^{2}\right)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{40\left(a^{3}b^{4}x^{3}+a^{4}b^{3}x^{2}-a^{5}b^{2}x-a^{6}b\right)}$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

output
```
-3/40*(9*b^2*x^2 + 12*a*b*x - a^2)*(b*x + a)^(1/3)*(-b*x + a)^(2/3)/(a^3*b
^4*x^3 + a^4*b^3*x^2 - a^5*b^2*x - a^6*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(4/3)/(b*x+a)**(8/3),x)
```

output
```
Integral(1/((a - b*x)**(4/3)*(a + b*x)**(8/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(8/3)*(-b*x + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{8}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(8/3)*(-b*x + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(a+b\,x)^{8/3}\,(a-b\,x)^{4/3}}\,dx$$

input
```
int(1/((a + b*x)^(8/3)*(a - b*x)^(4/3)),x)
```

output
```
int(1/((a + b*x)^(8/3)*(a - b*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{8/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^3+(bx+a)^{\frac{2}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2bx-(bx+a)^{\frac{2}{3}}\,(-bx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(8/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a**3 + (a + b*x)**(2/3)*(a - b*x)
**(1/3)*a**2*b*x - (a + b*x)**(2/3)*(a - b*x)**(1/3)*a*b**2*x**2 - (a + b*
x)**(2/3)*(a - b*x)**(1/3)*b**3*x**3),x)
```

3.216 $\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = \frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{8/3}} - \frac{27(a-bx)^{2/3}}{32a^2b(a+bx)^{8/3}} - \frac{81(a-bx)^{2/3}}{160a^3b(a+bx)^{5/3}} - \frac{243(a-bx)^{2/3}}{640a^4b(a+bx)^{2/3}}$$

output
```
3/2/a/b/(-b*x+a)^(1/3)/(b*x+a)^(8/3)-27/32*(-b*x+a)^(2/3)/a^2/b/(b*x+a)^(8
/3)-81/160*(-b*x+a)^(2/3)/a^3/b/(b*x+a)^(5/3)-243/640*(-b*x+a)^(2/3)/a^4/b
/(b*x+a)^(2/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = -\frac{3(49a^3-99a^2bx-189ab^2x^2-81b^3x^3)}{640a^4b\sqrt[3]{a-bx}(a+bx)^{8/3}}$$

input
```
Integrate[1/((a - b*x)^(4/3)*(a + b*x)^(11/3)),x]
```

output
```
(-3*(49*a^3 - 99*a^2*b*x - 189*a*b^2*x^2 - 81*b^3*x^3))/(640*a^4*b*(a - b*
x)^(1/3)*(a + b*x)^(8/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}dx$$

$$\Big\downarrow \ 55$$

$$\frac{9\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{11/3}}dx}{2a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{8/3}}$$

$$\Big\downarrow \ 55$$

$$\frac{9\left(\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{8/3}}dx}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}\right)}{2a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{8/3}}$$

$$\Big\downarrow \ 55$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{1}{\sqrt[3]{a-bx}(a+bx)^{5/3}}dx}{10a}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}\right)}{2a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{8/3}}$$

$$\Big\downarrow \ 48$$

$$\frac{9\left(\frac{3\left(-\frac{9(a-bx)^{2/3}}{40a^2b(a+bx)^{2/3}}-\frac{3(a-bx)^{2/3}}{10ab(a+bx)^{5/3}}\right)}{8a}-\frac{3(a-bx)^{2/3}}{16ab(a+bx)^{8/3}}\right)}{2a}+\frac{3}{2ab\sqrt[3]{a-bx}(a+bx)^{8/3}}$$

input
```
Int[1/((a - b*x)^(4/3)*(a + b*x)^(11/3)),x]
```

output
```
3/(2*a*b*(a - b*x)^(1/3)*(a + b*x)^(8/3)) + (9*((-3*(a - b*x)^(2/3))/(16*a
*b*(a + b*x)^(8/3)) + (3*((-3*(a - b*x)^(2/3))/(10*a*b*(a + b*x)^(5/3)) -
(9*(a - b*x)^(2/3))/(40*a^2*b*(a + b*x)^(2/3))))/(8*a)))/(2*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3\left(-81 b^{3} x^{3}-189 a \,b^{2} x^{2}-99 a^{2} b x +49 a^{3}\right)}{640 \left(-b x +a \right)^{\frac{1}{3}} \left(b x +a \right)^{\frac{8}{3}} a^{4} b}$	54
orering	$-\frac{3\left(-81 b^{3} x^{3}-189 a \,b^{2} x^{2}-99 a^{2} b x +49 a^{3}\right)}{640 \left(-b x +a \right)^{\frac{1}{3}} \left(b x +a \right)^{\frac{8}{3}} a^{4} b}$	54

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(11/3),x,method=_RETURNVERBOSE)
```

output
```
-3/640*(-81*b^3*x^3-189*a*b^2*x^2-99*a^2*b*x+49*a^3)/(-b*x+a)^(1/3)/(b*x+a
)^(8/3)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = -\frac{3\,(81\,b^3x^3+189\,ab^2x^2+99\,a^2bx-49\,a^3)(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{2}{3}}}{640\,(a^4b^5x^4+2\,a^5b^4x^3-2\,a^7b^2x-a^8b)}$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(11/3),x, algorithm="fricas")
```

output
```
-3/640*(81*b^3*x^3 + 189*a*b^2*x^2 + 99*a^2*b*x - 49*a^3)*(b*x + a)^(1/3)*
(-b*x + a)^(2/3)/(a^4*b^5*x^4 + 2*a^5*b^4*x^3 - 2*a^7*b^2*x - a^8*b)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{11}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(4/3)/(b*x+a)**(11/3),x)
```

output
```
Integral(1/((a - b*x)**(4/3)*(a + b*x)**(11/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{11}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(11/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(11/3)*(-b*x + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{11}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(11/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(11/3)*(-b*x + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = \int \frac{1}{(a+b\,x)^{11/3}\,(a-b\,x)^{4/3}}\,dx$$

input
```
int(1/((a + b*x)^(11/3)*(a - b*x)^(4/3)),x)
```

output
```
int(1/((a + b*x)^(11/3)*(a - b*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{11/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^4+2(bx+a)^{\frac{2}{3}}(-bx+a)^{\frac{1}{3}}a^3bx-2(bx+a)^{\frac{2}{3}}(\cdots}$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(11/3),x)
```

output
```
int(1/((a + b*x)**(2/3)*(a - b*x)**(1/3)*a**4 + 2*(a + b*x)**(2/3)*(a - b*
x)**(1/3)*a**3*b*x - 2*(a + b*x)**(2/3)*(a - b*x)**(1/3)*a*b**3*x**3 - (a
+ b*x)**(2/3)*(a - b*x)**(1/3)*b**4*x**4),x)
```

3.217 $\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 677

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \frac{15(a-bx)^{2/3}(a+bx)^{2/3}}{4b}$$

$$+\frac{3(a+bx)^{5/3}}{b\sqrt[3]{a-bx}}+\frac{15ax\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^3\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2}{a}}}\right.\right.}{2b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$-\frac{5\sqrt{2}3^{3/4}a^3\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-}}{1-\sqrt{3}-\sqrt[3]{1-}}\right.\right.}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
15/4*(-b*x+a)^(2/3)*(b*x+a)^(2/3)/b+3*(b*x+a)^(5/3)/b/(-b*x+a)^(1/3)+15*a*
x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2
/a^2)^(1/3))+15/2*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^3*(1-b^2*x^2/a^2)^(1
/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2
/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-b^
2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(
-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*
x^2/a^2)^(1/3))^2)^(1/2)-5*2^(1/2)*3^(3/4)*a^3*(1-b^2*x^2/a^2)^(1/3)*(1-(1
-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3
^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^2/a^2
)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(
1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^
(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \frac{6\ 2^{2/3}a(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{b\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x)^(5/3)/(a - b*x)^(4/3),x]
```

output
```
(6*2^(2/3)*a*(a + b*x)^(2/3)*Hypergeometric2F1[-5/3, -1/3, 2/3, (a - b*x)/
(2*a)])/(b*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}} \, dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{2\ 2^{2/3} a (a+bx)^{2/3} \int \frac{\left(\frac{bx}{a}+1\right)^{5/3}}{2\ 2^{2/3} (a-bx)^{4/3}} dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a (a+bx)^{2/3} \int \frac{\left(\frac{bx}{a}+1\right)^{5/3}}{(a-bx)^{4/3}} dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{6\ 2^{2/3} a (a+bx)^{2/3} \text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{b \sqrt[3]{a-bx} \left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Int[(a + b*x)^(5/3)/(a - b*x)^(4/3),x]
```

output
```
(6*2^(2/3)*a*(a + b*x)^(2/3)*Hypergeometric2F1[-5/3, -1/3, 2/3, (a - b*x)/
(2*a)])/(b*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{4}{3}}}dx$$

input
```
int((b*x+a)^(5/3)/(-b*x+a)^(4/3),x)
```

output
```
int((b*x+a)^(5/3)/(-b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(5/3)*(-b*x + a)^(2/3)/(b^2*x^2 - 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+bx)^{\frac{5}{3}}}{(a-bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)**(5/3)/(-b*x+a)**(4/3),x)
```

output
```
Integral((a + b*x)**(5/3)/(a - b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(5/3)/(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{5}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(5/3)/(-b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(5/3)/(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+b\,x)^{5/3}}{(a-b\,x)^{4/3}}\,dx$$

input
```
int((a + b*x)^(5/3)/(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(5/3)/(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/3}}{(a-bx)^{4/3}}\,dx = \left(\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx\right)a + \left(\int \frac{(bx+a)^{\frac{2}{3}}\,x}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx\right)b$$

input
```
int((b*x+a)^(5/3)/(-b*x+a)^(4/3),x)
```

output
```
int((a + b*x)**(2/3)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x)*a + in
t(((a + b*x)**(2/3)*x)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x)*b
```

3.218 $\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 648

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx = \frac{3(a+bx)^{2/3}}{b\sqrt[3]{a-bx}} + \frac{6x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x}{a^2}}}\right.\right.}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$-\frac{2\sqrt{2}3^{3/4}a^2\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-}}{1-\sqrt{3}-\sqrt[3]{1-}}\right.\right.}{b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3*(b*x+a)^(2/3)/b/(-b*x+a)^(1/3)+6*x*(1-b^2*x^2/a^2)^(1/3)/(-b*x+a)^(1/3)/
(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))+3*3^(1/4)*(1/2*6^(1/2)+1/2
*2^(1/2))*a^2*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x
^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^
(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^
2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^
2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)-2*2^(1/2)*3^(3/4)
*a^2*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(
1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Ell
ipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))
,2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1
/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx=\frac{3\ 2^{2/3}(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{b\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x)^(2/3)/(a - b*x)^(4/3),x]
```

output
```
(3*2^(2/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-2/3, -1/3, 2/3, (a - b*x)/(2
*a)])/(b*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx$$

$$\Big\downarrow \ 80$$

$$\frac{2^{2/3}(a+bx)^{2/3}\int \frac{\left(\frac{bx}{a}+1\right)^{2/3}}{2^{2/3}(a-bx)^{4/3}}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx)^{2/3}\int \frac{\left(\frac{bx}{a}+1\right)^{2/3}}{(a-bx)^{4/3}}dx}{\left(\frac{a+bx}{a}\right)^{2/3}}$$

$$\Big\downarrow \ 79$$

$$\frac{3\ 2^{2/3}(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{b\sqrt[3]{a-bx}\left(\frac{a+bx}{a}\right)^{2/3}}$$

input
```
Int[(a + b*x)^(2/3)/(a - b*x)^(4/3),x]
```

output
```
(3*2^(2/3)*(a + b*x)^(2/3)*Hypergeometric2F1[-2/3, -1/3, 2/3, (a - b*x)/(2
*a)])/(b*(a - b*x)^(1/3)*((a + b*x)/a)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{4}{3}}}dx$$

input
```
int((b*x+a)^(2/3)/(-b*x+a)^(4/3),x)
```

output
```
int((b*x+a)^(2/3)/(-b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^2*x^2 - 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+bx)^{\frac{2}{3}}}{(a-bx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)**(2/3)/(-b*x+a)**(4/3),x)
```

output
```
Integral((a + b*x)**(2/3)/(a - b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(2/3)/(-b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x+a)^(2/3)/(-b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(2/3)/(-b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(a+b\,x)^{2/3}}{(a-b\,x)^{4/3}}\,dx$$

input
```
int((a + b*x)^(2/3)/(a - b*x)^(4/3),x)
```

output
```
int((a + b*x)^(2/3)/(a - b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{2/3}}{(a-bx)^{4/3}}\,dx = \int \frac{(bx+a)^{\frac{2}{3}}}{(-bx+a)^{\frac{1}{3}}\,a-(-bx+a)^{\frac{1}{3}}\,bx}dx$$

input
```
int((b*x+a)^(2/3)/(-b*x+a)^(4/3),x)
```

output
```
int((a + b*x)**(2/3)/((a - b*x)**(1/3)*a - (a - b*x)**(1/3)*b*x),x)
```

3.219 $\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 656

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2ab\sqrt[3]{a-bx}} + \frac{3x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{2a\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,E\Bigg(\arcsin\Bigg(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}}{4b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$- \frac{3^{3/4}a\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\Bigg)}{\sqrt{2}b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3/2*(b*x+a)^(2/3)/a/b/(-b*x+a)^(1/3)+3/2*x*(1-b^2*x^2/a^2)^(1/3)/a/(-b*x+a
)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))+3/4*3^(1/4)*(1/2*6
^(1/2)+1/2*2^(1/2))*a*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+
(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1
/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^
2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(
1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)-1/2*3^(3/
4)*a*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(
1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Ell
ipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))
,2*I-I*3^(1/2))*2^(1/2)/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2
/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx=\frac{3\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{\sqrt[3]{2}b\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

input
```
Integrate[1/((a - b*x)^(4/3)*(a + b*x)^(1/3)),x]
```

output
```
(3*((a + b*x)/a)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, (a - b*x)/(2*a)])
/(2^(1/3)*b*(a - b*x)^(1/3)*(a + b*x)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{\sqrt[3]{2}}{(a-bx)^{4/3}\sqrt[3]{\frac{bx}{a}+1}}dx}{\sqrt[3]{2}\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}}\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{\frac{bx}{a}+1}}dx}{\sqrt[3]{a+bx}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{3\sqrt[3]{\frac{a+bx}{a}}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{\sqrt[3]{2}b\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

input
```
Int[1/((a - b*x)^(4/3)*(a + b*x)^(1/3)),x]
```

output
```
(3*((a + b*x)/a)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, (a - b*x)/(2*a)])
/(2^(1/3)*b*(a - b*x)^(1/3)*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{1}{(-bx+a)^{\frac{4}{3}}\,(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(1/3),x)
```

output
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^3*x^3 - a*b^2*x^2 - a^2*b*x +
 a^3), x)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(a-bx)^{\frac{4}{3}}\sqrt[3]{a+bx}}\,dx$$

input
```
integrate(1/(-b*x+a)**(4/3)/(b*x+a)**(1/3),x)
```

output
```
Integral(1/((a - b*x)**(4/3)*(a + b*x)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(1/3)*(-b*x + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(1/3)*(-b*x + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx=\int \frac{1}{\left(a+b\,x\right)^{1/3}\,\left(a-b\,x\right)^{4/3}}\,dx$$

input
```
int(1/((a + b*x)^(1/3)*(a - b*x)^(4/3)),x)
```

output
```
int(1/((a + b*x)^(1/3)*(a - b*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{4/3}\sqrt[3]{a+bx}}\,dx=\int \frac{1}{\left(bx+a\right)^{\frac{1}{3}}\left(-bx+a\right)^{\frac{1}{3}}a-\left(bx+a\right)^{\frac{1}{3}}\left(-bx+a\right)^{\frac{1}{3}}bx}dx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(1/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(1/3)*a - (a + b*x)**(1/3)*(a - b*x)**(
1/3)*b*x),x)
```

3.220 $\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 652

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \frac{3x}{2a^2\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$$+\frac{3x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{2a^2\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right.}{4b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$-\frac{3^{3/4}\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right)\right.}{\sqrt{2}b^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
3/2*x/a^2/(-b*x+a)^(1/3)/(b*x+a)^(1/3)+3/2*x*(1-b^2*x^2/a^2)^(1/3)/a^2/(-b
*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))+3/4*3^(1/4)*(1
/2*6^(1/2)+1/2*2^(1/2))*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((
1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^
(1/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-
b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1
-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)-1/2*3^(
3/4)*(1-b^2*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(
1/3)+(1-b^2*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*Ell
ipticF((1+3^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))
,2*I-I*3^(1/2))*2^(1/2)/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2
/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \frac{3\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{2\sqrt[3]{2}ab\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

input
```
Integrate[1/((a - b*x)^(4/3)*(a + b*x)^(4/3)),x]
```

output
```
(3*((a + b*x)/a)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, (a - b*x)/(2*a)])
/(2*2^(1/3)*a*b*(a - b*x)^(1/3)*(a + b*x)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {46, 215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx$$

$\Big\downarrow$ 46

$$\frac{\sqrt[3]{a^2-b^2x^2}\int \frac{1}{\left(a^2-b^2x^2\right)^{4/3}}dx}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 215

$$\frac{\sqrt[3]{a^2-b^2x^2}\left(\frac{3x}{2a^2\sqrt[3]{a^2-b^2x^2}}-\frac{\int \frac{1}{\sqrt[3]{a^2-b^2x^2}}dx}{2a^2}\right)}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 233

$$\frac{\sqrt[3]{a^2-b^2x^2}\left(\frac{3\sqrt{-b^2x^2}\int \frac{\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}}{4a^2b^2x}+\frac{3x}{2a^2\sqrt[3]{a^2-b^2x^2}}\right)}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 833

$$\frac{\sqrt[3]{a^2-b^2x^2}\left(\frac{3\sqrt{-b^2x^2}\left(\left(1+\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}-\int \frac{\left(1+\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}\right)}{4a^2b^2x}+\frac{3x}{2a^2\sqrt[3]{a^2-b^2}}\right.}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\Big\downarrow$ 760

$$\frac{\sqrt[3]{a^2-b^2x^2}\left(\frac{3\sqrt{-b^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}}{\sqrt{-b^2x^2}}d\sqrt[3]{a^2-b^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^{4/3}+\left(a^2-b^2 \cdots\right.}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{\sqrt[4]{3}\sqrt{-b^2x^2}\cdots}\right.}{4a^2b^2x}\right.}{\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

$\downarrow$ 2418

$$\frac{\sqrt[3]{a^2-b^2x^2}\left(\frac{3x}{2a^2\sqrt[3]{a^2-b^2x^2}}+\frac{3\sqrt{-b^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)\sqrt{\frac{a^{4/3}+\left(a^2-b^2x^2\right)^{2/3}+a^{2/3}\sqrt[3]{a^2-b^2x^2}\cdots}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-b^2x^2}\right)^2}}}{\sqrt[4]{3}\sqrt{-b^2x^2}\sqrt{-\frac{a^{2/3}\left(a^{2/3}-\sqrt[3]{a^2-\cdots}\right.}{\left(\left(1-\sqrt{3}\right)a^{2/3}-\sqrt[3]{a^2-\cdots}\right.}}}\right.}{\cdots}\right.}{\cdots}$$

input
```
Int[1/((a - b*x)^(4/3)*(a + b*x)^(4/3)),x]
```

output
```
((a^2 - b^2*x^2)^(1/3)*((3*x)/(2*a^2*(a^2 - b^2*x^2)^(1/3)) + (3*Sqrt[-(b^
2*x^2)]*((-2*Sqrt[-(b^2*x^2)])/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1
/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)
)*Sqrt[(a^(4/3) + a^(2/3)*(a^2 - b^2*x^2)^(1/3) + (a^2 - b^2*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + S
qrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b
^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(b^2*x^2)]*Sqrt[-((a^(2/3)*(a^(2/
3) - (a^2 - b^2*x^2)^(1/3)))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3
))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(2/3)*(a^(2/3) - (a^2 - b^2
*x^2)^(1/3))*Sqrt[(a^(4/3) + a^(2/3)*(a^2 - b^2*x^2)^(1/3) + (a^2 - b^2*x^
2)^(2/3))/((1 - Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(2/3) - (a^2 - b^2*x^2)^(1/3))/((1 - Sqrt[3])*a^(2/3)
 - (a^2 - b^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(b^2*x^2)]*Sqr
t[-((a^(2/3)*(a^(2/3) - (a^2 - b^2*x^2)^(1/3)))/((1 - Sqrt[3])*a^(2/3) - (
a^2 - b^2*x^2)^(1/3))^2)])))/(4*a^2*b^2*x)))/((a - b*x)^(1/3)*(a + b*x)^(1
/3))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(-bx+a\right)^{\frac{4}{3}} \left(bx+a\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(4/3),x)
```

output
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^4*x^4 - 2*a^2*b^2*x^2 + a^4),
 x)
```

Sympy [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.16

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{2}{3}, \frac{7}{6}, 1 & \frac{1}{2}, \frac{4}{3}, \frac{11}{6} \\ \frac{2}{3}, \frac{5}{6}, \frac{7}{6}, \frac{4}{3}, \frac{11}{6} & 0 \end{matrix} \middle| {\frac{a^{2}e^{-2i\pi}}{b^{2}x^{2}}} \right) e^{-\frac{2i\pi}{3}}}{4\pi a^{\frac{5}{3}}b\Gamma\left(\frac{4}{3}\right)} - \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{6}, \frac{1}{2}, \frac{2}{3}, 1 & \\ \frac{1}{6}, \frac{2}{3} & -\frac{1}{2}, 0, \frac{5}{6}, 0 \end{matrix} \middle| {\frac{a^{2}}{b^{2}x^{2}}} \right)}{4\pi a^{\frac{5}{3}}b\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(-b*x+a)**(4/3)/(b*x+a)**(4/3),x)
```

output
```
meijerg(((2/3, 7/6, 1), (1/2, 4/3, 11/6)), ((2/3, 5/6, 7/6, 4/3, 11/6), (0
,)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))*exp(-2*I*pi/3)/(4*pi*a**(5/3)*b*
gamma(4/3)) - meijerg(((-1/2, 0, 1/6, 1/2, 2/3, 1), ()), ((1/6, 2/3), (-1/
2, 0, 5/6, 0)), a**2/(b**2*x**2))/(4*pi*a**(5/3)*b*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(4/3)*(-b*x + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{4}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(4/3)*(-b*x + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(a+b\,x)^{4/3}\,(a-b\,x)^{4/3}}\,dx$$

input
```
int(1/((a + b*x)^(4/3)*(a - b*x)^(4/3)),x)
```

output
```
int(1/((a + b*x)^(4/3)*(a - b*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{4/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2-(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,b^2x^2}dx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(4/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(1/3)*a**2 - (a + b*x)**(1/3)*(a - b*x)
**(1/3)*b**2*x**2),x)
```

3.221 $\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 689

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = -\frac{3}{8ab\sqrt[3]{a-bx}(a+bx)^{4/3}}$$

$$+\frac{15x}{16a^3\sqrt[3]{a-bx}\sqrt[3]{a+bx}}+\frac{15x\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{16a^3\sqrt[3]{a-bx}\sqrt[3]{a+bx}\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)}$$

$$+\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{32ab^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

$$-\frac{5\,3^{3/4}\sqrt[3]{1-\frac{b^2x^2}{a^2}}\left(1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{b^2x^2}{a^2}}+\left(1-\frac{b^2x^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}\right.\right.}{8\sqrt{2}ab^2x\sqrt[3]{a-bx}\sqrt[3]{a+bx}\sqrt{-\frac{1-\sqrt[3]{1-\frac{b^2x^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{b^2x^2}{a^2}}\right)^2}}}$$

output
```
-3/8/a/b/(-b*x+a)^(1/3)/(b*x+a)^(4/3)+15/16*x/a^3/(-b*x+a)^(1/3)/(b*x+a)^(
1/3)+15/16*x*(1-b^2*x^2/a^2)^(1/3)/a^3/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(1-3^(
1/2)-(1-b^2*x^2/a^2)^(1/3))+15/32*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-b^2
*x^2/a^2)^(1/3)*(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2
*x^2/a^2)^(2/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticE((1+3
^(1/2)-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1
/2))/a/b^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3
^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)-5/16*3^(3/4)*(1-b^2*x^2/a^2)^(1/3)*
(1-(1-b^2*x^2/a^2)^(1/3))*((1+(1-b^2*x^2/a^2)^(1/3)+(1-b^2*x^2/a^2)^(2/3))
/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1-b^2*x^
2/a^2)^(1/3))/(1-3^(1/2)-(1-b^2*x^2/a^2)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/a/b
^2/x/(-b*x+a)^(1/3)/(b*x+a)^(1/3)/(-(1-(1-b^2*x^2/a^2)^(1/3))/(1-3^(1/2)-(
1-b^2*x^2/a^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = \frac{3\sqrt[3]{\frac{a+bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{4\sqrt[3]{2}a^2b\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

input
```
Integrate[1/((a - b*x)^(4/3)*(a + b*x)^(7/3)),x]
```

output
```
(3*((a + b*x)/a)^(1/3)*Hypergeometric2F1[-1/3, 7/3, 2/3, (a - b*x)/(2*a)])
/(4*2^(1/3)*a^2*b*(a - b*x)^(1/3)*(a + b*x)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}} \, dx$$

$$\Big\downarrow \; 80$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}} \int \frac{4\sqrt[3]{2}}{(a-bx)^{4/3}\left(\frac{bx}{a}+1\right)^{7/3}}dx}{4\sqrt[3]{2}a^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[3]{\frac{a+bx}{a}} \int \frac{1}{(a-bx)^{4/3}\left(\frac{bx}{a}+1\right)^{7/3}}dx}{a^2\sqrt[3]{a+bx}}$$

$$\Big\downarrow \; 79$$

$$\frac{3\sqrt[3]{\frac{a+bx}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{3},\frac{2}{3},\frac{a-bx}{2a}\right)}{4\sqrt[3]{2}a^2b\sqrt[3]{a-bx}\sqrt[3]{a+bx}}$$

input
```
Int[1/((a - b*x)^(4/3)*(a + b*x)^(7/3)),x]
```

output
```
(3*((a + b*x)/a)^(1/3)*Hypergeometric2F1[-1/3, 7/3, 2/3, (a - b*x)/(2*a)])
/(4*2^(1/3)*a^2*b*(a - b*x)^(1/3)*(a + b*x)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{1}{\left(-bx+a\right)^{\frac{4}{3}} \left(bx+a\right)^{\frac{7}{3}}}dx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(7/3),x)
```

output
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{\left(bx+a\right)^{\frac{7}{3}}\left(-bx+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(2/3)*(-b*x + a)^(2/3)/(b^5*x^5 + a*b^4*x^4 - 2*a^2*b^3
*x^3 - 2*a^3*b^2*x^2 + a^4*b*x + a^5), x)
```

Sympy [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(a-bx)^{\frac{4}{3}}\,(a+bx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)**(4/3)/(b*x+a)**(7/3),x)
```

output
```
Integral(1/((a - b*x)**(4/3)*(a + b*x)**(7/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^(7/3)*(-b*x + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{7}{3}}(-bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(-b*x+a)^(4/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^(7/3)*(-b*x + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(a+b\,x)^{7/3}\,(a-b\,x)^{4/3}}\,dx$$

input
```
int(1/((a + b*x)^(7/3)*(a - b*x)^(4/3)),x)
```

output
```
int(1/((a + b*x)^(7/3)*(a - b*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx)^{4/3}(a+bx)^{7/3}}\,dx = \int \frac{1}{(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^3+(bx+a)^{\frac{1}{3}}\,(-bx+a)^{\frac{1}{3}}\,a^2bx-(bx+a)^{\frac{1}{3}}\,(-bx$$

input
```
int(1/(-b*x+a)^(4/3)/(b*x+a)^(7/3),x)
```

output
```
int(1/((a + b*x)**(1/3)*(a - b*x)**(1/3)*a**3 + (a + b*x)**(1/3)*(a - b*x)
**(1/3)*a**2*b*x - (a + b*x)**(1/3)*(a - b*x)**(1/3)*a*b**2*x**2 - (a + b*
x)**(1/3)*(a - b*x)**(1/3)*b**3*x**3),x)
```

3.222 $\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \frac{14a^2x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4} - \frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a} - \frac{14a^2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
14/5*a^2*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-14/15*I*(a-I*a*x)^(3/4)*(a+I*a*
x)^(3/4)-2/5*I*(a-I*a*x)^(7/4)*(a+I*a*x)^(3/4)/a-14/5*a^2*(x^2+1)^(1/4)*El
lipticE(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{11/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{4},\frac{15}{4},\frac{1}{2}-\frac{ix}{2}\right)}{11a\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]
```

output
```
(((2*I)/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[1
/4, 11/4, 15/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {60, 60, 46, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{7}{5}a\int \frac{(a-iax)^{3/4}}{\sqrt[4]{ixa+a}}dx-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{7}{5}a\left(a\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{ixa+a}}dx-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

$$\Big\downarrow \text{ 46}$$

$$\frac{7}{5}a\left(\frac{a\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\sqrt[4]{x^2a^2+a^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

$$\Big\downarrow \text{ 227}$$

$$\frac{7}{5}a\left(\frac{a\sqrt[4]{x^2+1}\int \frac{1}{\sqrt[4]{x^2+1}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{7}{5}a\left(\frac{a\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-\int\frac{1}{(x^2+1)^{5/4}}dx\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

$$\Big\downarrow \text{212}$$

$$\frac{7}{5}a\left(\frac{a\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-2E\left(\frac{\arctan(x)}{2}\middle|2\right)\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

input
```
Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]
```

output
```
(((-2*I)/5)*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))/a + (7*a*((((-2*I)/3)*(a
- I*a*x)^(3/4)*(a + I*a*x)^(3/4))/a + (a*(1 + x^2)^(1/4)*((2*x)/(1 + x^2)^
(1/4) - 2*EllipticE[ArcTan[x]/2, 2]))/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)
)))/5
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{2(10i+3x)(x-i)(x+i)a^{2}}{15(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{7x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)a^{2}\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	104

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(10*I+3*x)*(x-I)*(x+I)*a^2/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)+7/5/
(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*a^2*(-a^2*(I*x-1)*(I*x+1))^(
1/4)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
-1/15*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 10*I*x - 21) - 15*x
*integral(14/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(x^4 + x^2), x))/x
```

Sympy [F]

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \int \frac{(-ia(x+i))^{\frac{7}{4}}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

input
```
integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(1/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%
{i,[0,1]%%%}] at parameters values [99,84]Warning, need to choose a branch
 for the roo
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{7/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(1/4),x)
```

output
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(1/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx = \sqrt{a}\,a\left(\int \frac{(-ix+1)^{\frac{3}{4}}}{(ix+1)^{\frac{1}{4}}}dx - \left(\int \frac{(-ix+1)^{\frac{3}{4}}\,x}{(ix+1)^{\frac{1}{4}}}dx\right)i\right)$$

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x)
```

output
```
sqrt(a)*a*(int(( - i*x + 1)**(3/4)/(i*x + 1)**(1/4),x) - int((( - i*x + 1)
**(3/4)*x)/(i*x + 1)**(1/4),x)*i)
```

3.223 $\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx = \frac{2ax}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} - \frac{2a\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
2*a*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-2/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(3/4
)/a-2*a*(x^2+1)^(1/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4
)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx = \frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{7/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}-\frac{ix}{2}\right)}{7a\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```

output
```
(((2*I)/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[1/4
, 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 46, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{60}$$

$$a\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{ixa+a}}dx-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

$$\Big\downarrow \text{46}$$

$$\frac{a\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\sqrt[4]{x^2a^2+a^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

$$\Big\downarrow \text{227}$$

$$\frac{a\sqrt[4]{x^2+1}\int \frac{1}{\sqrt[4]{x^2+1}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

$$\Big\downarrow \text{225}$$

$$\frac{a\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-\int \frac{1}{(x^2+1)^{5/4}}dx\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

$$\Big\downarrow \text{212}$$

$$\frac{a\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-2E\left(\left.\frac{\arctan(x)}{2}\right|2\right)\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

input
```
Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```

output
```
(((-2*I)/3)*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))/a + (a*(1 + x^2)^(1/4)*((
2*x)/(1 + x^2)^(1/4) - 2*EllipticE[ArcTan[x]/2, 2]))/((a - I*a*x)^(1/4)*(a
 + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{2i(x-i)(x+i)a}{3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)a\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(x-I)*(x+I)*a/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)+1/(a^2)^(1/4)*x*
hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x-1)
)^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
1/3*(3*a*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*x^2)
, x) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(I*x - 3))/(a*x)
```

Sympy [F]

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx=\int \frac{(-ia(x+i))^{\frac{3}{4}}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

input
```
integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(1/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx=\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(1/4),x)
```

output
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(1/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx=\sqrt{a}\left(\int \frac{(-ix+1)^{\frac{3}{4}}}{(ix+1)^{\frac{1}{4}}}dx\right)$$

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

output
```
sqrt(a)*int(( - i*x + 1)**(3/4)/(i*x + 1)**(1/4),x)
```

3.224 $\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx=\frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
2*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-2*(x^2+1)^(1/4)*EllipticE(sin(1/2*arct
an(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx = \frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}-\frac{ix}{2}\right)}{3a\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((2*I)/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[1/4
, 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {46, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{46}$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\sqrt[4]{x^2a^2+a^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{227}$$

$$\frac{\sqrt[4]{x^2+1}\int \frac{1}{\sqrt[4]{x^2+1}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-\int \frac{1}{(x^2+1)^{5/4}}dx\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-2E\left(\left.\frac{\arctan(x)}{2}\right|2\right)\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
```

output
```
((1 + x^2)^(1/4)*((2*x)/(1 + x^2)^(1/4) - 2*EllipticE[ArcTan[x]/2, 2]))/((
a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-iax+a\right)^{\frac{1}{4}}\left(iax+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
(a^2*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2)
, x) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx=-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{8},\frac{5}{8},1 & \frac{1}{4},\frac{1}{2},\frac{3}{4} \\ -\frac{1}{4},\frac{1}{8},\frac{1}{4},\frac{5}{8},\frac{3}{4} & 0 \end{matrix}\middle| \frac{e^{-3i\pi}}{x^2}\right)e^{\frac{i\pi}{4}}}{4\pi\sqrt{a}\Gamma\left(\frac{1}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{3}{8},0,\frac{1}{8},\frac{1}{2},1 & \\ -\frac{3}{8},\frac{1}{8} & -\frac{1}{2},-\frac{1}{4},0,0 \end{matrix}\middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi\sqrt{a}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

output
```
-I*meijerg(((1/8, 5/8, 1), (1/4, 1/2, 3/4)), ((-1/4, 1/8, 1/4, 5/8, 3/4),
(0,)), exp_polar(-3*I*pi)/x**2)*exp(I*pi/4)/(4*pi*sqrt(a)*gamma(1/4)) + I*
meijerg(((-1/2, -3/8, 0, 1/8, 1/2, 1), ()), ((-3/8, 1/8), (-1/2, -1/4, 0,
0)), exp_polar(-I*pi)/x**2)/(4*pi*sqrt(a)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}}$$

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(1/4)),x)/sqrt(a)
```

3.225 $\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx=-\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-2*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+2*(x^2+1)^(1/4)*EllipticE(sin(1/2*a
rctan(x)),2^(1/2))/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx=-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
```

output
```
((-2*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, 1/2 - (I
/2)*x])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {58, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{58}$$

$$a\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx-\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{46}$$

$$\frac{a\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{213}$$

$$\frac{\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{212}$$

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(-2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ellipt
icE[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 58
```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Simp[c   Int[1/((a + b*x)^
(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d,
0] && NegQ[a^2*b^2]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{2x-2i}{a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2*(x-I)/a/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*
(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
((a^3*x^2 + I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3
*x^4 + a^3*x^2), x) - 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 +
 I*a^3*x)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(5/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(5/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx = -\frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a}$$

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x)
```

output
```
( - int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x - (i*x + 1)**(1/4)*( -
 i*x + 1)**(1/4)),x))/(sqrt(a)*a)
```

3.226 $\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx = -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-4/5*I/a/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)+2/5*(x^2+1)^(1/4)*EllipticE(sin(1
/2*arctan(x)),2^(1/2))/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i2^{3/4}\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},-\frac{1}{4},\frac{1}{2}-\frac{ix}{2}\right)}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, 1/2
 - (I/2)*x])/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {56, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \ 56$$

$$\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 46$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 213$$

$$\frac{\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 212$$

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
```

output
```
((-4*I)/5)/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*El
lipticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 56
```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Simp[d/(5*b)   Int[1/((a
 + b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
 + a*d, 0] && NegQ[a^2*b^2]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

method	result	size
risch	$\frac{\frac{2}{5}x^2+\frac{2}{5}ix+\frac{4}{5}}{(x+i)a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	105

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/5*(x^2+2+I*x)/(x+I)/a^2/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-
a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x + 2*I) + 5*(a^4*x^2 + 2*I*a
^4*x - a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 +
a^4), x))/(a^4*x^2 + 2*I*a^4*x - a^4)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(9/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(9/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx = -\frac{\int \frac{1}{2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^2-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a^2}$$

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x)
```

output
```
( - int(1/(2*(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)*(
 - i*x + 1)**(1/4)*x**2 - (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)),x))/(sqrt(
a)*a**2)
```

3.227 $\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx=-\frac{4i}{9a(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{45a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-4/9*I/a/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4)-2/45*I/a^2/(a-I*a*x)^(5/4)/(a+I*a
*x)^(1/4)+2/15*(x^2+1)^(1/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^3/(a-
I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx=-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},-\frac{5}{4},\frac{1}{2}-\frac{ix}{2}\right)}{9a(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/9)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, 1/2
 - (I/2)*x])/(a*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {61, 56, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{ixa+a}}dx}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}$$

$$\Big\downarrow \text{ 56}$$

$$\frac{\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}$$

$$\Big\downarrow \text{ 46}$$

$$\frac{\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\left(x^2a^2+a^2\right)^{5/4}}dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}$$

$$\Big\downarrow \text{ 213}$$

$$\frac{\frac{\sqrt[4]{x^2+1}\int \frac{1}{\left(x^2+1\right)^{5/4}}dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}$$

input
```
Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/9)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(9/4)) + (((-4*I)/5)/(a*(a
 - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x
]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)))/(3*a)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 56
```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Simp[d/(5*b)   Int[1/((a
 + b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
 + a*d, 0] && NegQ[a^2*b^2]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{\frac{2}{15}x^3+\frac{4}{15}ix^2-\frac{4}{45}x+\frac{22}{45}i}{(x+i)^2 a^3 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}}-\frac{x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right) \left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{15(a^2)^{\frac{1}{4}} a^3 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}}$	113

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/45*(6*I*x^2+3*x^3-2*x+11*I)/(x+I)^2/a^3/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(
1/4)-1/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(I*x-1)*
(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
1/45*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 9*I*x - 11) + 45*(a^
5*x^3 + 3*I*a^5*x^2 - 3*a^5*x - I*a^5)*integral(-1/15*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 + 3*I*a^5*x^2 - 3*a^5*x - I
*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{\sqrt[4]{ia(x-i)}\,(-ia(x+i))^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(13/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(13/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{13/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx=\frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}i\,x^{3}-3(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix-3(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^{2}+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a^{3}}$$

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x**3 - 3*(i*x + 1)**(1/4)*(
- i*x + 1)**(1/4)*i*x - 3*(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**2 + (i*x
 + 1)**(1/4)*( - i*x + 1)**(1/4)),x)/(sqrt(a)*a**3)
```

3.228 $\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 148

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = -\frac{4i}{13a(a-iax)^{13/4}\sqrt[4]{a+iax}} - \frac{2i}{117a^2(a-iax)^{9/4}\sqrt[4]{a+iax}} - \frac{2i}{117a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{39a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-4/13*I/a/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4)-2/117*I/a^2/(a-I*a*x)^(9/4)/(a+
I*a*x)^(1/4)-2/117*I/a^3/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)+2/39*(x^2+1)^(1/4
)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i2^{3/4}\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{1}{4},-\frac{9}{4},\frac{1}{2}-\frac{ix}{2}\right)}{13a(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/13)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, 1
/2 - (I/2)*x])/(a*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {61, 61, 56, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{5\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{ixa+a}}dx}{13a}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(\frac{\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{ixa+a}}dx}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}\right)}{13a}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

$$\Big\downarrow \text{56}$$

$$\frac{5\left(\frac{\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}\right)}{13a}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

$$\Big\downarrow \; 46$$

$$\frac{5\left(\frac{\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\left(x^2a^2+a^2\right)^{5/4}}dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}\right)}{13a}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

$$\Big\downarrow \; 213$$

$$\frac{5\left(\frac{\frac{\sqrt[4]{x^2+1}\int \frac{1}{\left(x^2+1\right)^{5/4}}dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}\right)}{13a}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

$$\Big\downarrow \; 212$$

$$\frac{5\left(\frac{\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}}{3a}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}\right)}{13a}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

input
```
Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/13)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(13/4)) + (5*((((-2*I)/9)
*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(9/4)) + (((-4*I)/5)/(a*(a - I*a*x)^(
5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5
*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)))/(3*a)))/(13*a)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 56
```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Simp[d/(5*b)   Int[1/((a
 + b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
 + a*d, 0] && NegQ[a^2*b^2]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\frac{2}{39}x^{4}+\frac{2}{13}i x^{3}-\frac{16}{117}x^{2}-\frac{40}{117}}{\left(x+i\right)^{3} a^{4} \left(-a \left(i x-1\right)\right)^{\frac{1}{4}} \left(a \left(i x+1\right)\right)^{\frac{1}{4}}}-\frac{x \,\operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], -x^{2}\right) \left(-a^{2} \left(i x-1\right) \left(i x+1\right)\right)^{\frac{1}{4}}}{39 \left(a^{2}\right)^{\frac{1}{4}} a^{4} \left(-a \left(i x-1\right)\right)^{\frac{1}{4}} \left(a \left(i x+1\right)\right)^{\frac{1}{4}}}$	114

input
```
int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/117*(9*I*x^3+3*x^4-20-8*x^2)/(x+I)^3/a^4/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^
(1/4)-1/39/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(I*x-1)
*(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
1/117*(2*(3*x^3 + 12*I*x^2 - 20*x - 20*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(
3/4) + 117*(a^6*x^4 + 4*I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)*integral(
-1/39*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(a^6*x^4 +
 4*I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(1/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(17/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{17/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = \frac{\int \frac{1}{4(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}i\,x^3-4(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^4-6(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^2+}}{\sqrt{a}\,a^4}$$

input
```
int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/(4*(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x**3 - 4*(i*x + 1)**(1/4)*
( - i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**4 - 6*(i
*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**2 + (i*x + 1)**(1/4)*( - i*x + 1)**(
1/4)),x)/(sqrt(a)*a**4)
```

3.229 $\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = -\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} + \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}} - \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}} + \frac{i\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{\sqrt{2}}$$

output
```
-I*(a-I*a*x)^(1/4)*(a+I*a*x)^(3/4)/a+1/2*I*arctan(1-2^(1/2)*(a+I*a*x)^(1/4
)/(a-I*a*x)^(1/4))*2^(1/2)-1/2*I*arctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x
)^(1/4))*2^(1/2)+1/2*I*arctanh(2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1+
(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx=\frac{2i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}-\frac{ix}{2}\right)}{5a\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

output
```
(((2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[1/4
, 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{ixa+a}}dx-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{73}$$

$$2i\int \frac{1}{\sqrt[4]{ixa+a}}d\sqrt[4]{a-iax}-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{770}$$

$$2i\int \frac{1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$$\Big\downarrow \; 755$$

$$2i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$$\Big\downarrow \; 1476$$

$$2i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{i\cdots}}}\cdots\right.\right.$$
$$\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$$\Big\downarrow \; 1082$$

$$2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-i\cdots}}{\sqrt[4]{ixa-\cdots}}\cdots\right.$$
$$\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$$\Big\downarrow \; 217$$

$$2i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)\right)-$$
$$\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$$\Big\downarrow \; 1479$$

$$2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\cdots\right.}{\cdots}\right.\right.$$
$$\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$\Big\downarrow$ 25

$$2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{a}}\right.}{\sqrt{2}}\right.\right.$$

$$\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$\Big\downarrow$ 27

$$2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1\right.}{}\right.\right.$$

$$\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

$\Big\downarrow$ 1103

$$2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}\right.\right.$$

$$\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}$$

input
```
Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

output
```
((-I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/a + (2*I)*((-(ArcTan[1 - (Sqrt[
2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(a
 - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[a - I*
a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sq
rt[a - I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2])
)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.56

method	result
risch	$\frac{i(x-i)(x+i)(-a(ix-1))^{\frac{1}{4}}}{(ix-1)(a(ix+1))^{\frac{1}{4}}} - \frac{\left(\mathrm{RootOf}\left(\_Z^2-i\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2-i\,\mathrm{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-\right.}{}\right.\right.}{}$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
I*(x-I)*(x+I)*(-a*(I*x-1))^(1/4)/(I*x-1)/(a*(I*x+1))^(1/4)-(1/2*RootOf(_Z^
2-I)*ln((-RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x^2-I*RootOf(_Z^2-I)
*(-2*I*x^3-x^4-2*I*x+1)^(3/4)-x^3-2*I*RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1
)^(1/4)*x-I*(-2*I*x^3-x^4-2*I*x+1)^(1/2)*x-2*I*x^2+RootOf(_Z^2-I)*(-2*I*x^
3-x^4-2*I*x+1)^(1/4)+(-2*I*x^3-x^4-2*I*x+1)^(1/2)+x)/(I*x-1)^2)-1/2*I*Root
Of(_Z^2-I)*ln(-(-I*RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x^2+2*RootO
f(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x+x^3-RootOf(_Z^2-I)*(-2*I*x^3-x^4-
2*I*x+1)^(3/4)-I*(-2*I*x^3-x^4-2*I*x+1)^(1/2)*x+I*RootOf(_Z^2-I)*(-2*I*x^3
-x^4-2*I*x+1)^(1/4)+2*I*x^2+(-2*I*x^3-x^4-2*I*x+1)^(1/2)-x)/(I*x-1)^2))*(-
a*(I*x-1))^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = \frac{\sqrt{i}a\log\left(\frac{\sqrt{i}(ax-i\,a)+(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{i}a\log\left(-\frac{\sqrt{i}(ax-i\,a)-(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right)+\sqrt{-i}a\log\left(\frac{\sqrt{-i}(ax-i\right.}{2\,a}$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
1/2*(sqrt(I)*a*log((sqrt(I)*(a*x - I*a) + (I*a*x + a)^(3/4)*(-I*a*x + a)^(
1/4))/(x - I)) - sqrt(I)*a*log(-(sqrt(I)*(a*x - I*a) - (I*a*x + a)^(3/4)*(
-I*a*x + a)^(1/4))/(x - I)) + sqrt(-I)*a*log((sqrt(-I)*(a*x - I*a) + (I*a*
x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-I)*a*log(-(sqrt(-I)*(a*x
 - I*a) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 2*I*(I*a*x + a)
^(3/4)*(-I*a*x + a)^(1/4))/a
```

Sympy [F]

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = \int \frac{\sqrt[4]{-ia\,(x+i)}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

input
```
integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(1/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4),x)
```

output
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = \int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{1}{4}}}dx$$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(( - i*x + 1)**(1/4)/(i*x + 1)**(1/4),x)
```

3.230 $\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{i\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} - \frac{i\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} + \frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{a}$$

output
```
I*2^(1/2)*arctan(1-2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))/a-I*2^(1/2)*ar
ctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))/a+I*2^(1/2)*arctanh(2^(1/2
)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1+(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{2i2^{3/4}\sqrt[4]{1+ix}\sqrt[4]{a-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

output
```
((2*I)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/
4, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \ 73$$

$$\frac{4i\int \frac{1}{\sqrt[4]{ixa+a}}d\sqrt[4]{a-iax}}{a}$$

$$\Big\downarrow \ 770$$

$$\frac{4i\int \frac{1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{a}$$

$$\Big\downarrow \ 755$$

$$\frac{4i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$$\Big\downarrow \ 1476$$

$$\frac{4i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-i}}{\sqrt[4]{ixa-}}\right.\right.}{a}$$

$$\Big\downarrow \ 1082$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$$\Big\downarrow \ 217$$

$$\frac{4i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)\right)}{a}$$

$$\Big\downarrow \ 1479$$

$$\frac{4i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+i}}\right.}{\sqrt{2}}\right.\right.}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right.\right.}{a}$$

$$\Big\downarrow \ 27$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-ia}}{\sqrt[4]{a+ia}}\right.}{\sqrt{2}}\right.}{a}$$

$$\Big\downarrow 1103$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{}}\right.}{a}$$

input
```
Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

output
```
((4*I)*((-(ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[
2]) + ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2])/2
 + (-1/2*Log[1 + Sqrt[a - I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)
^(1/4)]/Sqrt[2] + Log[1 + Sqrt[a - I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a
 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{1}{\left(-iax+a\right)^{\frac{3}{4}} \left(iax+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx &= \frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)\\&\quad-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)\\&\quad+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)\\&\quad-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)\end{aligned}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(
3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 1/2*sqrt(4*I/a^2)*log(-1/2*((a^2*x - I
*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 1
/2*sqrt(-4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^
(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 1/2*sqrt(-4*I/a^2)*log(-1/2*((a^2*x -
 I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I))
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a}$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)),x)/a
```

3.231 $\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

output
```
-2/3*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(3/4)
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
orering	$-\frac{2i(x^2+1)}{3(-iax+a)^{\frac{7}{4}}(iax+a)^{\frac{1}{4}}}$	27
risch	$\frac{\frac{2x}{3}-\frac{2i}{3}}{a(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	31
gosper	$\frac{2i(-x+i)(x+i)}{3(-iax+a)^{\frac{7}{4}}(iax+a)^{\frac{1}{4}}}$	32

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(x^2+1)/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx=\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\,(a^3x+i\,a^3)}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
2/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x + I*a^3)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(7/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2\,(x-\mathrm{i})\,\left(-a\,(-1+x\,1\mathrm{i})\right)^{1/4}}{3\,a^2\,\left(-1+x\,1\mathrm{i}\right)\,\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
-(2*(x - 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^2*(x*1i - 1)*(a*(x*1i + 1))^(1/4)
)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^2}$$

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x)
```

output
```
( - int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x - (i*x + 1)**(1/4)*( -
 i*x + 1)**(3/4)),x))/a**2
```

3.232 $\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx=-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}$$

output
```
-2/7*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(7/4)-4/21*I*(a+I*a*x)^(3/4)/a^3/(a-I
*a*x)^(3/4)
```

Mathematica [A] (verified)

Time = 5.84 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx=\frac{2(5-2ix)(a+iax)^{3/4}}{21a^3(i+x)(a-iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(2*(5 - (2*I)*x)*(a + I*a*x)^(3/4))/(21*a^3*(I + x)*(a - I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{ixa+a}}dx}{7a}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

input
```
Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/7)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a +
I*a*x)^(3/4))/(a^3*(a - I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
orering	$-\frac{2(2x+5i)(x^2+1)}{21(-iax+a)^{\frac{11}{4}}(iax+a)^{\frac{1}{4}}}$	32
gosper	$\frac{2(-x+i)(x+i)(2x+5i)}{21(-iax+a)^{\frac{11}{4}}(iax+a)^{\frac{1}{4}}}$	37
risch	$\frac{\frac{4}{21}x^2+\frac{2}{7}ix+\frac{10}{21}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)}$	44

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(2*x+5*I)*(x^2+1)/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = \frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+5i)}{21\,(a^4x^2+2i\,a^4x-a^4)}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
2/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + 5*I)/(a^4*x^2 + 2*I*a^4*x
 - a^4)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(1/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(11/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = -\frac{\left(-a\left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\left(2\,x^2+x\,3\mathrm{i}+5\right)\,2\mathrm{i}}{21\,a^3\left(-1+x\,1\mathrm{i}\right)^2\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
-((-a*(x*1i - 1))^(1/4)*(x*3i + 2*x^2 + 5)*2i)/(21*a^3*(x*1i - 1)^2*(a*(x*
1i + 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = -\frac{\int \frac{1}{2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^2-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^3}$$

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x)
```

output
```
( - int(1/(2*(i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x + (i*x + 1)**(1/4)*(
 - i*x + 1)**(3/4)*x**2 - (i*x + 1)**(1/4)*( - i*x + 1)**(3/4)),x))/a**3
```

3.233 $\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx=-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}$$

output
```
-2/11*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(11/4)-8/77*I*(a+I*a*x)^(3/4)/a^3/(a
-I*a*x)^(7/4)-16/231*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(3/4)
```

Mathematica [A] (verified)

Time = 6.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx=\frac{2(a+iax)^{3/4}(41i+28x-8ix^2)}{231a^4(i+x)^2(a-iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(2*(a + I*a*x)^(3/4)*(41*I + 28*x - (8*I)*x^2))/(231*a^4*(I + x)^2*(a - I*
a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{ixa+a}}dx}{11a}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{4\left(\frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{ixa+a}}dx}{7a}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}\right)}{11a}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4\left(-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}\right)}{11a}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

input
```
Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/11)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) + (4*((((-2*I)/7)
*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I*a*x)^(3/4
))/(a^3*(a - I*a*x)^(3/4))))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.38

method	result	size
orering	$\frac{2i(8x^2+28ix-41)(x^2+1)}{231(-iax+a)^{\frac{15}{4}}(iax+a)^{\frac{1}{4}}}$	38
gosper	$-\frac{2(-x+i)(x+i)(8ix^2-28x-41i)}{231(-iax+a)^{\frac{15}{4}}(iax+a)^{\frac{1}{4}}}$	43
risch	$\frac{\frac{16}{231}x^3+\frac{40}{231}ix^2-\frac{26}{231}x+\frac{82}{231}i}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)^2}$	50

input
```
int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/231*I*(28*I*x+8*x^2-41)*(x^2+1)/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = \frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(8\,x^2+28i\,x-41)}{231\,(a^5x^3+3i\,a^5x^2-3\,a^5x-i\,a^5)}$$

input
```
integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
2/231*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 28*I*x - 41)/(a^5*x^3
+ 3*I*a^5*x^2 - 3*a^5*x - I*a^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(1/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = \frac{(x-\mathrm{i})^4\,(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,(8\,x^2+x\,28\mathrm{i}-41)\,2\mathrm{i}}{231\,a^4\,(x^2+1)^3\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
((x - 1i)^4*(-a*(x*1i - 1))^(1/4)*(x*28i + 8*x^2 - 41)*2i)/(231*a^4*(x^2 +
 1)^3*(a*(x*1i + 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}} i\,x^3-3(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}} ix-3(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}} x^2+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^4}$$

input
```
int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x**3 - 3*(i*x + 1)**(1/4)*(
- i*x + 1)**(3/4)*i*x - 3*(i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*x**2 + (i*x
 + 1)**(1/4)*( - i*x + 1)**(3/4)),x)/a**4
```

3.234 $\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx=-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}$$

output
```
-2/15*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(15/4)-4/55*I*(a+I*a*x)^(3/4)/a^3/(a
-I*a*x)^(11/4)-16/385*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(7/4)-32/1155*I*(a+I
*a*x)^(3/4)/a^5/(a-I*a*x)^(3/4)
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx=\frac{2(a+iax)^{3/4}(-159+138ix+72x^2-16ix^3)}{1155a^5(i+x)^3(a-iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(2*(a + I*a*x)^(3/4)*(-159 + (138*I)*x + 72*x^2 - (16*I)*x^3))/(1155*a^5*(
I + x)^3*(a - I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{ixa+a}}dx}{5a}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{2\left(\frac{4\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{ixa+a}}dx}{11a}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}\right)}{5a}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{2\left(\frac{4\left(\frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{ixa+a}}dx}{7a}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}\right)}{11a}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}\right)}{5a}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2\left(\frac{4\left(-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}\right)}{11a}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}\right)}{5a}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

input
```
Int[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

output
```
(((-2*I)/15)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(15/4)) + (2*((((-2*I)/11
)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) + (4*((((-2*I)/7)*(a + I*a*x
)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I*a*x)^(3/4))/(a^3*(a
- I*a*x)^(3/4))))/(11*a)))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result	size
orering	$\frac{2\left(16x^{3}+72ix^{2}-138x-159i\right)\left(x^{2}+1\right)}{1155\left(-iax+a\right)^{\frac{19}{4}}\left(iax+a\right)^{\frac{1}{4}}}$	43
gosper	$-\frac{2(-x+i)(x+i)\left(16x^{3}+72ix^{2}-138x-159i\right)}{1155\left(-iax+a\right)^{\frac{19}{4}}\left(iax+a\right)^{\frac{1}{4}}}$	48
risch	$\frac{\frac{32}{1155}x^{4}+\frac{16}{165}ix^{3}-\frac{4}{35}x^{2}-\frac{2}{55}ix-\frac{106}{385}}{a^{4}\left(-a(ix-1)\right)^{\frac{3}{4}}\left(a(ix+1)\right)^{\frac{1}{4}}\left(x+i\right)^{3}}$	55

input
```
int(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(72*I*x^2+16*x^3-159*I-138*x)*(x^2+1)/(a-I*a*x)^(19/4)/(a+I*a*x)^(1
/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = \frac{2\,(16\,x^3+72i\,x^2-138\,x-159i)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{1155\,(a^6x^4+4i\,a^6x^3-6\,a^6x^2-4i\,a^6x+a^6)}$$

input
```
integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

output
```
2/1155*(16*x^3 + 72*I*x^2 - 138*x - 159*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^
(1/4)/(a^6*x^4 + 4*I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(19/4)/(a+I*a*x)**(1/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{19}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = -\frac{(x-\mathrm{i})^5\,\left(-a\,(-1+x\,1\mathrm{i})\right)^{1/4}\,\left(-16\,x^3-x^2\,72\mathrm{i}+138\,x+159\mathrm{i}\right)\,2\mathrm{i}}{1155\,a^5\,\left(x^2+1\right)^4\,\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(19/4)*(a + a*x*1i)^(1/4)),x)
```

output
```
-((x - 1i)^5*(-a*(x*1i - 1))^(1/4)*(138*x - x^2*72i - 16*x^3 + 159i)*2i)/(
1155*a^5*(x^2 + 1)^4*(a*(x*1i + 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = \frac{\int \frac{1}{4(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix^3-4(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^4-6(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^2+}}{a^5}$$

input
```
int(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x)
```

output
```
int(1/(4*(i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x**3 - 4*(i*x + 1)**(1/4)*
( - i*x + 1)**(3/4)*i*x + (i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*x**4 - 6*(i
*x + 1)**(1/4)*( - i*x + 1)**(3/4)*x**2 + (i*x + 1)**(1/4)*( - i*x + 1)**(
3/4)),x)/a**5
```

3.235 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}} - \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}}-\frac{3i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{\sqrt{2}}$$

output
```
-I*(a-I*a*x)^(3/4)*(a+I*a*x)^(1/4)/a+3/2*I*arctan(1-2^(1/2)*(a+I*a*x)^(1/4
)/(a-I*a*x)^(1/4))*2^(1/2)-3/2*I*arctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x
)^(1/4))*2^(1/2)-3/2*I*arctanh(2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1+
(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = \frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{7/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}-\frac{ix}{2}\right)}{7a(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

output
```
(((2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[3/4
, 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}a\int \frac{1}{\sqrt[4]{a-iax}(ixa+a)^{3/4}}dx-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \text{73}$$

$$6i\int \frac{\sqrt{a-iax}}{(ixa+a)^{3/4}}d\sqrt[4]{a-iax}-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \text{854}$$

$$6i\int \frac{\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \ 826$$

$$6i\left(\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \ 1476$$

$$6i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{1}{2}\int\right.$$
$$\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \ 1082$$

$$6i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-}}{\sqrt[4]{ixa}}\right.$$
$$\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \ 217$$

$$6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-$$
$$\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \ 1479$$

$$6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\sqrt{}\right.}{\sqrt{2}}\right.\right.$$
$$\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$\downarrow$ 25

$$6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}}\right.\right.$$

$$\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$\downarrow$ 27

$$6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.$$

$$\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

$\downarrow$ 1103

$$6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}\right.\right.$$

$$\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}$$

input
```
Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

output
```
((-I)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))/a + (6*I)*((-(ArcTan[1 - (Sqrt[
2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(a
 - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[a - I*a*x]
- (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqr
t[a - I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2]))
/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.17 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.51

method	result
risch	$-\frac{i(x-i)(x+i)a}{(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}+\frac{\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-i\right)\ln\left(\frac{-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-i\right)x^2-x^3-i\,\mathrm{RootOf}\left(\_Z^2-i\right)\left(-x^4\right.}{}\right.}{}\right.}{}$

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-I*(x-I)*(x+I)/(a*(I*x+1))^(3/4)/(-a*(I*x-1))^(1/4)*a+(3/2*RootOf(_Z^2-I)*
ln((-(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-x^3-I*RootOf(_Z^2-I)*(
1-x^4+2*I*x^3+2*I*x)^(3/4)-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+2*I*RootOf(_Z^2
-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x+2*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)+Root
Of(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)+x)/(I*x+1)^2)-3/2*I*RootOf(_Z^2-I)*
ln(-(-I*(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(1
-x^4+2*I*x^3+2*I*x)^(1/4)*x+x^3-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x-RootOf(_Z^
2-I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)+I*RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1
/4)-2*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)-x)/(I*x+1)^2))/(a*(I*x+1))^(3/4)*(
-(I*x-1)*(I*x+1)^3)^(1/4)/(-a*(I*x-1))^(1/4)*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.06

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx=\frac{\sqrt{9i}a\log\left(\frac{\sqrt{9i}(ax+i\,a)+3\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)-\sqrt{9i}a\log\left(-\frac{\sqrt{9i}(ax+i\,a)-3\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(x+i)}\right.}{}$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
1/2*(sqrt(9*I)*a*log(1/3*(sqrt(9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*
a*x + a)^(3/4))/(x + I)) - sqrt(9*I)*a*log(-1/3*(sqrt(9*I)*(a*x + I*a) - 3
*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) + sqrt(-9*I)*a*log(1/3*(sq
rt(-9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) -
sqrt(-9*I)*a*log(-1/3*(sqrt(-9*I)*(a*x + I*a) - 3*(I*a*x + a)^(1/4)*(-I*a*
x + a)^(3/4))/(x + I)) - 2*I*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/a
```

Sympy [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(3/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4),x)
```

output
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(-ix+1)^{\frac{3}{4}}}{(ix+1)^{\frac{3}{4}}}dx$$

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(( - i*x + 1)**(3/4)/(i*x + 1)**(3/4),x)
```

3.236 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}} \, dx = \frac{i\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} - \frac{i\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} - \frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{a}$$

output
```
I*2^(1/2)*arctan(1-2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))/a-I*2^(1/2)*ar
ctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))/a-I*2^(1/2)*arctanh(2^(1/2
)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1+(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.87 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}} \, dx = \frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}-\frac{ix}{2}\right)}{3a(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(((2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4
, 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4i\int \frac{\sqrt{a-iax}}{(ixa+a)^{3/4}}d\sqrt[4]{a-iax}}{a}$$

$$\Big\downarrow \text{854}$$

$$\frac{4i\int \frac{\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{a}$$

$$\Big\downarrow \text{826}$$

$$\frac{4i\left(\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$$\Big\downarrow \text{1476}$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\right.}{a}$$

$$\Big\downarrow \text{1082}$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$\Big\downarrow$ 217

$$\frac{4i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$\Big\downarrow$ 1479

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-ia\cdots}}{\sqrt[4]{a+iax\cdots}}\right)}{\sqrt{2}}\cdots\right)\right)}{a}$$

$\Big\downarrow$ 25

$$\frac{4i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax\cdots}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\cdots\right)\right)}{a}$$

$\Big\downarrow$ 27

$$\frac{4i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+i}}\right.}{\sqrt{2}}\right.\right.}{a}$$

$$\Big\downarrow \ 1103$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{}}\right.\right.}{a}$$

input
```
Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

output
```
((4*I)*((-(ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[
2]) + ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2])/2
 + (Log[1 + Sqrt[a - I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4
)]/(2*Sqrt[2]) - Log[1 + Sqrt[a - I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a
+ I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{1}{(-iax+a)^{\frac{1}{4}}(iax+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx = \frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x+i\,a^2)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,(x+i)}\right) - \frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x+i\,a^2)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,(x+i)}\right) + \frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x+i\,a^2)\sqrt{-\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,(x+i)}\right) - \frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x+i\,a^2)\sqrt{-\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,(x+i)}\right)$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(
1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 1/2*sqrt(4*I/a^2)*log(-1/2*((a^2*x + I
*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) + 1
/2*sqrt(-4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^
(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 1/2*sqrt(-4*I/a^2)*log(-1/2*((a^2*x +
 I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx=\int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)),x)
```

output
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{3/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}}dx}{a}$$

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(1/4)),x)/a
```

3.237 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx=-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

output
```
-2*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(1/4)
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx=-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

input
```
Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

output
```
((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

input
```
Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

output
```
((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
orering	$-\frac{2i(x^2+1)}{(-iax+a)^{\frac{5}{4}}(iax+a)^{\frac{3}{4}}}$	27
risch	$\frac{2x-2i}{a(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}$	31
gosper	$\frac{2i(-x+i)(x+i)}{(-iax+a)^{\frac{5}{4}}(iax+a)^{\frac{3}{4}}}$	32

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(x^2+1)/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x+i\,a^3}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x + I*a^3)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(5/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)),x)
```

output
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = -\frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}}dx}{a^2}$$

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

output
```
( - int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*i*x - (i*x + 1)**(3/4)*( -
 i*x + 1)**(1/4)),x))/a**2
```

3.238 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx=-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}$$

output
```
-2/5*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(5/4)-4/5*I*(a+I*a*x)^(1/4)/a^3/(a-I*
a*x)^(1/4)
```

Mathematica [A] (verified)

Time = 5.78 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx=\frac{2(3-2ix)\sqrt[4]{a+iax}}{5a^3(i+x)\sqrt[4]{a-iax}}$$

input
```
Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(2*(3 - (2*I)*x)*(a + I*a*x)^(1/4))/(5*a^3*(I + x)*(a - I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{3/4}}dx}{5a}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(((-2*I)/5)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I
*a*x)^(1/4))/(a^3*(a - I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
orering	$-\frac{2(2x+3i)(x^2+1)}{5(-iax+a)^{\frac{9}{4}}(iax+a)^{\frac{3}{4}}}$	32
gosper	$\frac{2(-x+i)(x+i)(2x+3i)}{5(-iax+a)^{\frac{9}{4}}(iax+a)^{\frac{3}{4}}}$	37
risch	$\frac{\frac{4}{5}x^2+\frac{2}{5}ix+\frac{6}{5}}{a^2(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)}$	44

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(2*x+3*I)*(x^2+1)/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x+3i)}{5\,(a^4x^2+2i\,a^4x-a^4)}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
2/5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(2*x + 3*I)/(a^4*x^2 + 2*I*a^4*x
- a^4)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(9/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)),x)
```

output
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = -\frac{\int \frac{1}{2(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}x^2-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}}dx}{a^3}$$

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x)
```

output
```
( - int(1/(2*(i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*i*x + (i*x + 1)**(3/4)*(
 - i*x + 1)**(1/4)*x**2 - (i*x + 1)**(3/4)*( - i*x + 1)**(1/4)),x))/a**3
```

3.239 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}$$

output
```
-2/9*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(9/4)-8/45*I*(a+I*a*x)^(1/4)/a^3/(a-I
*a*x)^(5/4)-16/45*I*(a+I*a*x)^(1/4)/a^4/(a-I*a*x)^(1/4)
```

Mathematica [A] (verified)

Time = 7.84 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = \frac{2\sqrt[4]{a+iax}(17i+20x-8ix^2)}{45a^4(i+x)^2\sqrt[4]{a-iax}}$$

input
```
Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(2*(a + I*a*x)^(1/4)*(17*I + 20*x - (8*I)*x^2))/(45*a^4*(I + x)^2*(a - I*a
*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}dx$$

$$\Big\downarrow \ 55$$

$$\frac{4\int \frac{1}{(a-iax)^{9/4}(ixa+a)^{3/4}}dx}{9a}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

$$\Big\downarrow \ 55$$

$$\frac{4\left(\frac{2\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{3/4}}dx}{5a}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}\right)}{9a}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

$$\Big\downarrow \ 48$$

$$\frac{4\left(-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}\right)}{9a}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

input
```
Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(((-2*I)/9)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(9/4)) + (4*((((-2*I)/5)*(
a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I*a*x)^(1/4))/
(a^3*(a - I*a*x)^(1/4))))/(9*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.38

method	result	size
orering	$\frac{2i(8x^2+20ix-17)(x^2+1)}{45(-iax+a)^{\frac{13}{4}}(iax+a)^{\frac{3}{4}}}$	38
gosper	$-\frac{2(-x+i)(x+i)(8ix^2-20x-17i)}{45(-iax+a)^{\frac{13}{4}}(iax+a)^{\frac{3}{4}}}$	43
risch	$\frac{\frac{16}{45}x^3+\frac{8}{15}ix^2+\frac{2}{15}x+\frac{34}{45}i}{a^3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)^2}$	50

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
2/45*I*(20*I*x+8*x^2-17)*(x^2+1)/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(8\,x^2+20i\,x-17)}{45\,(a^5x^3+3i\,a^5x^2-3\,a^5x-i\,a^5)}$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
2/45*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 20*I*x - 17)/(a^5*x^3 +
 3*I*a^5*x^2 - 3*a^5*x - I*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(13/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{13/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}} \,d x$$

input
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)),x)
```

output
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}} i\, x^{3}-3(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}} ix-3(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}} x^{2}+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}} dx}{a^{4}}$$

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*i*x**3 - 3*(i*x + 1)**(3/4)*(
- i*x + 1)**(1/4)*i*x - 3*(i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*x**2 + (i*x
 + 1)**(3/4)*( - i*x + 1)**(1/4)),x)/a**4
```

3.240 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = -\frac{10}{3}i\sqrt[4]{a-iax}\sqrt[4]{a+iax} - \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a} + \frac{10a^2(1+x^2)^{3/4}\operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
-10/3*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4)-2/3*I*(a-I*a*x)^(5/4)*(a+I*a*x)^(1
/4)/a+10/3*a^2*(x^2+1)^(3/4)*InverseJacobiAM(1/2*arctan(x),2^(1/2))/(a-I*a
*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = \frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{9/4}\operatorname{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}-\frac{ix}{2}\right)}{9a(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
```

output
```
(((2*I)/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[3/4
, 9/4, 13/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 60, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{3}a\int \frac{\sqrt[4]{a-iax}}{(ixa+a)^{3/4}}dx-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{3}a\left(a\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

$$\Big\downarrow \text{46}$$

$$\frac{5}{3}a\left(\frac{a\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{\left(x^2a^2+a^2\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{3}a\left(\frac{a\left(x^2+1\right)^{3/4}\int \frac{1}{\left(x^2+1\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{3}a\left(\frac{2a\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

input
```
Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
```

output
```
(((-2*I)/3)*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))/a + (5*a*(((-2*I)*(a - I*
a*x)^(1/4)*(a + I*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]
/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))))/3
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{(-iax+a)^{\frac{5}{4}}}{(iax+a)^{\frac{3}{4}}}dx$$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

output
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
-2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x + 6*I) + integral(5/3*(I*a*x
+ a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(3/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(3/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{5/4}}{(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(3/4),x)
```

output
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(3/4), x)
```

Reduce [F]

$$\int \frac{(a - iax)^{5/4}}{(a + iax)^{3/4}}\,dx = \frac{a\left(\int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{3}{4}}}dx - \left(\int \frac{(-ix+1)^{\frac{1}{4}}x}{(ix+1)^{\frac{3}{4}}}dx\right)i\right)}{\sqrt{a}}$$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

output
```
(a*(int(( - i*x + 1)**(1/4)/(i*x + 1)**(3/4),x) - int((( - i*x + 1)**(1/4)
*x)/(i*x + 1)**(3/4),x)*i))/sqrt(a)
```

3.241 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}} \, dx = -\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a} + \frac{2a(1+x^2)^{3/4} \operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
-2*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4)/a+2*a*(x^2+1)^(3/4)*InverseJacobiAM(1
/2*arctan(x),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}} \, dx = \frac{2i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{5/4} \operatorname{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}-\frac{ix}{2}\right)}{5a(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
```

output
```
(((2*I)/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[3/4
, 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {60, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx$$

$$\Big\downarrow \text{ 60}$$

$$a\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \text{ 46}$$

$$\frac{a\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{\left(x^2a^2+a^2\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{a\left(x^2+1\right)^{3/4}\int \frac{1}{\left(x^2+1\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{2a\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}$$

input
```
Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
```

output
```
((-2*I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*Elli
pticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{(-iax + a)^{\frac{1}{4}}}{(iax + a)^{\frac{3}{4}}}dx$$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

output
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
(a*integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x^2 + a), x) - 2*I*(I*
a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/a
```

Sympy [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx = \int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(3/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(3/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(3/4),x)
```

output
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(3/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx = \frac{\int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{3}{4}}}dx}{\sqrt{a}}$$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(( - i*x + 1)**(1/4)/(i*x + 1)**(3/4),x)/sqrt(a)
```

3.242 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 43

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}} \, dx = \frac{2(1+x^2)^{3/4} \operatorname{EllipticF}\left(\frac{\arctan(x)}{2}, 2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
2*(x^2+1)^(3/4)*InverseJacobiAM(1/2*arctan(x),2^(1/2))/(a-I*a*x)^(3/4)/(a+
I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}} \, dx = \frac{2i\sqrt[4]{2}(1+ix)^{3/4}\sqrt[4]{a-iax} \text{ Hypergeometric2F1}\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{ix}{2}\right)}{a(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

output
```
((2*I)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 3/
4, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}dx$$

$$\Big\downarrow \text{ 46}$$

$$\frac{\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{\left(x^2a^2+a^2\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{\left(x^2+1\right)^{3/4}\int \frac{1}{\left(x^2+1\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*
x)^(3/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-iax+a\right)^{\frac{3}{4}} \left(iax+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a^2*x^2 + a^2), x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(39) = 78$.

Time = 2.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx = -\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{8},\frac{7}{8},1 & \frac{1}{2},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{3}{8},\frac{3}{4},\frac{7}{8},\frac{5}{4} & 0 \end{matrix}\middle| \frac{e^{-3i\pi}}{x^2}\right)e^{\frac{3i\pi}{4}}}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{8},0,\frac{3}{8},\frac{1}{2},1 & \\ -\frac{1}{8},\frac{3}{8} & -\frac{1}{2},0,\frac{1}{4},0 \end{matrix}\middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

output
```
-I*meijerg(((3/8, 7/8, 1), (1/2, 3/4, 5/4)), ((1/4, 3/8, 3/4, 7/8, 5/4), (
0,)), exp_polar(-3*I*pi)/x**2)*exp(3*I*pi/4)/(4*pi*a**(3/2)*gamma(3/4)) +
I*meijerg(((-1/2, -1/8, 0, 3/8, 1/2, 1), ()), ((-1/8, 3/8), (-1/2, 0, 1/4,
 0)), exp_polar(-I*pi)/x**2)/(4*pi*a**(3/2)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(3/4)),x)
```

output
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}}dx}{\sqrt{a}\,a}$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(3/4)),x)/(sqrt(a)*a)
```

3.243 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx=-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{2(1+x^2)^{3/4}\,\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
-2/3*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*InverseJacobi
AM(1/2*arctan(x),2^(1/2))/a/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx=-\frac{2i\sqrt[4]{2}(1+ix)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},\frac{1}{2}-\frac{ix}{2}\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(((-2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {61, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}} \, dx$$

$$\Big\downarrow \text{61}$$

$$\frac{\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx}{3a} - \frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

$$\Big\downarrow \text{46}$$

$$\frac{\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{(x^2a^2+a^2)^{3/4}}dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}} - \frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{\left(x^2+1\right)^{3/4}\int \frac{1}{(x^2+1)^{3/4}}dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}} - \frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}} - \frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(((-2*I)/3)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4
)*EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{(-iax + a)^{\frac{7}{4}} (iax + a)^{\frac{3}{4}}} dx$$

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
1/3*(3*(a^3*x + I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(
a^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x + I*a^
3)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(7/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(7/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{7/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(3/4)),x)
```

output
```
int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx = -\frac{2\sqrt{a}\,(ix+1)^{\frac{3}{4}}\,i}{3\sqrt{ix+1}\,(-ix+1)^{\frac{3}{4}}\,a^3}$$

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

output
```
( - 2*sqrt(a)*(i*x + 1)**(3/4)*i)/(3*sqrt(i*x + 1)*( - i*x + 1)**(3/4)*a**
3)
```

3.244 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}} - \frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}} + \frac{2(1+x^2)^{3/4}\,\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
-2/7*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(7/4)-2/7*I*(a+I*a*x)^(1/4)/a^3/(a-I*
a*x)^(3/4)+2/7*(x^2+1)^(3/4)*InverseJacobiAM(1/2*arctan(x),2^(1/2))/a^2/(a
-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{2}(1+ix)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},\frac{1}{2}-\frac{ix}{2}\right)}{7a(a-iax)^{7/4}(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(((-2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, 1/2
 - (I/2)*x])/(a*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {61, 61, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{3\int \frac{1}{(a-iax)^{7/4}(ixa+a)^{3/4}}dx}{7a}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3\left(\frac{\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx}{3a}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}\right)}{7a}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

$$\Big\downarrow \text{46}$$

$$\frac{3\left(\frac{(a^2x^2+a^2)^{3/4}\int \frac{1}{(x^2a^2+a^2)^{3/4}}dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}\right)}{7a}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{3\left(\frac{(x^2+1)^{3/4}\int \frac{1}{(x^2+1)^{3/4}}dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}\right)}{7a}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{3\left(\frac{2(x^2+1)^{3/4}\operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}\right)}{7a}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

input
```
Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```

output
```
(((-2*I)/7)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(7/4)) + (3*((((-2*I)/3)*(
a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*EllipticF[A
rcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))))/(7*a)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{(-iax+a)^{\frac{11}{4}} (iax+a)^{\frac{3}{4}}} dx$$

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

output
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}} dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{11}{4}}} dx$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

output
```
1/7*(7*(a^4*x^2 + 2*I*a^4*x - a^4)*integral(1/7*(I*a*x + a)^(1/4)*(-I*a*x
+ a)^(1/4)/(a^4*x^2 + a^4), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x
 + 2*I))/(a^4*x^2 + 2*I*a^4*x - a^4)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}} dx = \int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{11}{4}}} dx$$

input
```
integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(3/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(11/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(11/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{11/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(3/4)),x)
```

output
```
int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx = -\frac{\int \frac{1}{2(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}x^2-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}}dx}{\sqrt{a}\,a^3}$$

input

```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

output

```
( - int(1/(2*(i*x + 1)**(3/4)*( - i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)*(
 - i*x + 1)**(3/4)*x**2 - (i*x + 1)**(3/4)*( - i*x + 1)**(3/4)),x))/(sqrt(
a)*a**3)
```

3.245 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \frac{14ia}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{14i(a-iax)^{3/4}}{3\sqrt[4]{a+iax}} - \frac{2i(a-iax)^{7/4}}{3a\sqrt[4]{a+iax}} + \frac{14a\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
14*I*a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-14/3*I*(a-I*a*x)^(3/4)/(a+I*a*x)^(1
/4)-2/3*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(1/4)+14*a*(x^2+1)^(1/4)*EllipticE(s
in(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.51

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{11/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{11}{4},\frac{15}{4},\frac{1}{2}-\frac{ix}{2}\right)}{11a^2\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

output
```
((I/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[5/4,
11/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {57, 60, 46, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}-7\int \frac{(a-iax)^{3/4}}{\sqrt[4]{ixa+a}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}-7\left(a\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{ixa+a}}dx-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)$$

$$\Big\downarrow \text{46}$$

$$\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}-7\left(\frac{a\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\sqrt[4]{x^2a^2+a^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}-7\left(\frac{a\sqrt[4]{x^2+1}\int \frac{1}{\sqrt[4]{x^2+1}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}-7\left(\frac{a\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-\int \frac{1}{(x^2+1)^{5/4}}dx\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}-7\left(\frac{a\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-2E\left(\frac{\arctan(x)}{2}\middle|2\right)\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}\right)$$

input
```
Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

output
```
((4*I)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(1/4)) - 7*((((-2*I)/3)*(a - I*a*
x)^(3/4)*(a + I*a*x)^(3/4))/a + (a*(1 + x^2)^(1/4)*((2*x)/(1 + x^2)^(1/4)
- 2*EllipticE[ArcTan[x]/2, 2]))/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{2i(x^2-12ix+13)a}{3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{7x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)a\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^2)^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	96

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(x^2+13-12*I*x)*a/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-7/(a^2)^(1/4)
*x*hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x
-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
-1/3*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(-I*x^2 + 8*x - 21*I) - 3*(a*
x^2 - I*a*x)*integral(-14*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*
x^2), x))/(a*x^2 - I*a*x)
```

Sympy [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{7}{4}}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(5/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(5/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%
{i,[0,1]%%%}] at parameters values [44,93]Warning, need to choose a branch
 for the roo
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{7/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(5/4),x)
```

output
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(5/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \sqrt{a}\left(\int \frac{(-ix+1)^{\frac{3}{4}}}{(ix+1)^{\frac{1}{4}}\,ix+(ix+1)^{\frac{1}{4}}}dx-\left(\int \frac{(-ix+1)^{\frac{3}{4}}\,x}{(ix+1)^{\frac{1}{4}}\,ix+(ix+1)^{\frac{1}{4}}}dx\right)i\right)$$

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x)
```

output
```
sqrt(a)*(int(( - i*x + 1)**(3/4)/((i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4))
,x) - int((( - i*x + 1)**(3/4)*x)/((i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)
),x)*i)
```

3.246 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\frac{6i}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
6*I/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-2*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(1/4)+
6*(x^2+1)^(1/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I
*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{7/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}-\frac{ix}{2}\right)}{7a^2\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
```

output
```
((I/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[5/4, 7/
4, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {57, 46, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-3\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{ixa+a}}dx$$

$$\Big\downarrow \text{46}$$

$$\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\sqrt[4]{x^2a^2+a^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{227}$$

$$\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{x^2+1}\int \frac{1}{\sqrt[4]{x^2+1}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{225}$$

$$\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-\int \frac{1}{(x^2+1)^{5/4}}dx\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{212}$$

$$\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-2E\left(\frac{\arctan(x)}{2}\middle|2\right)\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
```

output
```
((4*I)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(1/4)) - (3*(1 + x^2)^(1/4)*((2*x
)/(1 + x^2)^(1/4) - 2*EllipticE[ArcTan[x]/2, 2]))/((a - I*a*x)^(1/4)*(a +
I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{4x+4i}{(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	88

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
4*(x+I)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-3/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*(I*x
+1))^(1/4)
```

Fricas [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
-(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x - 3*I) - (a^2*x^2 - I*a^2*x)*i
ntegral(-6*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2), x))/(
a^2*x^2 - I*a^2*x)
```

Sympy [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(5/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(5/4),x)
```

output
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(5/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx=\frac{\sqrt{a}\left(\int \frac{(-ix+1)^{\frac{3}{4}}}{(ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}}dx\right)}{a}$$

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x)
```

output
```
(sqrt(a)*int(( - i*x + 1)**(3/4)/((i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4))
,x))/a
```

3.247 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx=\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
2*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+2*(x^2+1)^(1/4)*EllipticE(sin(1/2*ar
ctan(x)),2^(1/2))/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx=\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\frac{1}{2}-\frac{ix}{2}\right)}{3a^2\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((I/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 5/
4, 7/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {58, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx$$

$$\Big\downarrow \text{58}$$

$$a\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{46}$$

$$\frac{a\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{213}$$

$$\frac{\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{212}$$

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
```

output
```
(2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ellipti
cE[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 58
```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Simp[c   Int[1/((a + b*x)^
(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d,
0] && NegQ[a^2*b^2]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{2x+2i}{a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2*(x+I)/a/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*
(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
((a^3*x^2 - I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3
*x^4 + a^3*x^2), x) + 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 -
 I*a^3*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(1/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(5/4)),x)
```

output
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{5/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a}$$

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)*( - i*x
 + 1)**(1/4)),x)/(sqrt(a)*a)
```

3.248 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}} \, dx = \frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
2*(x^2+1)^(1/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^2/(a-I*a*x)^(1/4)/
(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}} \, dx = -\frac{i2^{3/4}\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},\frac{1}{2}-\frac{ix}{2}\right)}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((-I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, 1/2 - (I/2
)*x])/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx$$

$$\Big\downarrow \text{ 46}$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 213}$$

$$\frac{\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

output
```
(2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(a^2*(a - I*a*x)^(1/4)*(a +
I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

method	result	size
risch	$\frac{2x}{a^{2}\left(-a \left(ix-1\right)\right)^{\frac{1}{4}}\left(a \left(ix+1\right)\right)^{\frac{1}{4}}}-\frac{x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}\left(ix-1\right)\left(ix+1\right)\right)^{\frac{1}{4}}}{\left(a^{2}\right)^{\frac{1}{4}}a^{2}\left(-a \left(ix-1\right)\right)^{\frac{1}{4}}\left(a \left(ix+1\right)\right)^{\frac{1}{4}}}$	91

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2*x/a^2/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)/a^2*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*
(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*x + (a^4*x^2 + a^4)*integral(-(I*a
*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^4), x))/(a^4*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.

Time = 4.57 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = -\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{8},\frac{9}{8},1 & \frac{1}{2},\frac{5}{4},\frac{7}{4} \\ \frac{5}{8},\frac{3}{4},\frac{9}{8},\frac{5}{4},\frac{7}{4} & 0 \end{matrix}\middle| \frac{e^{-3i\pi}}{x^2}\right)e^{-\frac{3i\pi}{4}}}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{8},\frac{1}{2},\frac{5}{8},1 & \\ \frac{1}{8},\frac{5}{8} & -\frac{1}{2},0,\frac{3}{4},0 \end{matrix}\middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

output
```
-I*meijerg(((5/8, 9/8, 1), (1/2, 5/4, 7/4)), ((5/8, 3/4, 9/8, 5/4, 7/4), (
0,)), exp_polar(-3*I*pi)/x**2)*exp(-3*I*pi/4)/(4*pi*a**(5/2)*gamma(5/4)) +
 I*meijerg(((-1/2, 0, 1/8, 1/2, 5/8, 1), ()), ((1/8, 5/8), (-1/2, 0, 3/4,
0)), exp_polar(-I*pi)/x**2)/(4*pi*a**(5/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(5/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(5/4)),x)
```

output
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = -\frac{2\sqrt{a}\,(ix+1)^{\frac{1}{4}}\,i}{\sqrt{ix+1}\,(-ix+1)^{\frac{1}{4}}\,a^3}$$

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x)
```

output
```
( - 2*sqrt(a)*(i*x + 1)**(1/4)*i)/(sqrt(i*x + 1)*( - i*x + 1)**(1/4)*a**3)
```

3.249 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx=-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-2/5*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)+6/5*(x^2+1)^(1/4)*EllipticE(sin
(1/2*arctan(x)),2^(1/2))/a^3/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx=-\frac{i2^{3/4}\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},-\frac{1}{4},\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((-1/5*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, 1/2 -
 (I/2)*x])/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {61, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}dx$$

$$\Big\downarrow \text{ 61}$$

$$\frac{3\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx}{5a}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 46}$$

$$\frac{3\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 213}$$

$$\frac{3\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{6\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*
EllipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{\frac{6}{5}x^2+\frac{6}{5}ix+\frac{2}{5}}{(x+i)a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2/5*(3*I*x+3*x^2+1)/(x+I)/a^3/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-3/5/(a^
2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(I*x-1)*(I*x+1))^(1/4
)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 3*I*x + 1) + 5*(a^5*x
^3 + I*a^5*x^2 + a^5*x + I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x +
a)^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(9/4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(5/4)),x)
```

output
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx = -\frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}i\,x^3+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^2-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a^3}$$

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x)
```

output
```
( - int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x**3 + (i*x + 1)**(1/4)*
( - i*x + 1)**(1/4)*i*x - (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**2 - (i*x
 + 1)**(1/4)*( - i*x + 1)**(1/4)),x))/(sqrt(a)*a**3)
```

3.250 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}} - \frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{3a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-2/9*I/a^2/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4)-2/9*I/a^3/(a-I*a*x)^(5/4)/(a+I*
a*x)^(1/4)+2/3*(x^2+1)^(1/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^4/(a-
I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = -\frac{i2^{3/4}\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{5}{4},-\frac{5}{4},\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((-1/9*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, 1/2 -
 (I/2)*x])/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {61, 61, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx$$

$$\Big\downarrow \ 61$$

$$\frac{5\int \frac{1}{(a-iax)^{9/4}(ixa+a)^{5/4}}dx}{9a}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 61$$

$$\frac{5\left(\frac{3\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx}{5a}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}\right)}{9a}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 46$$

$$\frac{5\left(\frac{3\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\left(x^2a^2+a^2\right)^{5/4}}dx}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}\right)}{9a}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 213$$

$$\frac{5\left(\frac{3\sqrt[4]{x^2+1}\int \frac{1}{\left(x^2+1\right)^{5/4}}dx}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}\right)}{9a}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 212$$

$$\frac{5\left(\frac{6\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}\right)}{9a}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)) + (5*(((-2*I)/5)/(a^2
*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*EllipticE[ArcTa
n[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))))/(9*a)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{\frac{2}{3}x^3+\frac{4}{3}ix^2-\frac{4}{9}x+\frac{4}{9}i}{(x+i)^2 a^4 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}} - \frac{x \,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right) \left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{3(a^2)^{\frac{1}{4}} a^4 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}}$	113

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2/9*(6*I*x^2+3*x^3-2*x+2*I)/(x+I)^2/a^4/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/
4)-1/3/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(I*x-1)*(I*
x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
1/9*(2*(3*x^3 + 6*I*x^2 - 2*x + 2*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)
+ 9*(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*x - a^6)*integral(-1/3*(I*a*x + a)^(3
/4)*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a
^6*x - a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(5/4),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{13/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(5/4)),x)
```

output
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = -\frac{\int \frac{1}{2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}i\,x^3+2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^4-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a^4}$$

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x)
```

output
```
( - int(1/(2*(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x**3 + 2*(i*x + 1)**(1
/4)*( - i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**4 -
(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)),x))/(sqrt(a)*a**4)
```

3.251 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a} - \frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}}+\frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}} - \frac{5i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{\sqrt{2}}$$

output
```
4*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(1/4)+5*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(3/4)/
a-5/2*I*arctan(1-2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))*2^(1/2)+5/2*I*ar
ctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))*2^(1/2)-5/2*I*arctanh(2^(1
/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1+(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))*2
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.32

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx=\frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{9/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}-\frac{ix}{2}\right)}{9a^2\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

output
```
((I/9)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[5/4, 9/
4, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {57, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(\frac{1}{2}a\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{ixa+a}}dx-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\int \frac{1}{\sqrt[4]{ixa+a}}d\sqrt[4]{a-iax}-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}\right)$$

$$\Big\downarrow \; 770$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\int \frac{1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}\right)$$

$$\Big\downarrow \; 755$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}\right)$$

$$\Big\downarrow \; 1476$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{}{\sqrt{a-iax}+\sqrt{}}\right.\right.\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a}}{\sqrt[4]{i}}\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)\right)-\frac{i\sqrt[4]{}}{}\right.$$

$$\Big\downarrow \; 1479$$

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1\right.}{}\right.\right.\right.$$

$\downarrow$ 25

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 27

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}-5\left(2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right.}{2\sqrt{2}}\right.\right.\right.$$

input
```
Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

output
```
((4*I)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(1/4)) - 5*(((-I)*(a - I*a*x)^(1/
4)*(a + I*a*x)^(3/4))/a + (2*I)*((-(ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))
/(a + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a +
 I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[a - I*a*x] - (Sqrt[2]*(a -
I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[a - I*a*x] + (Sqrt
[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.09 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.21

method	result
risch	$-\frac{i(x^2-8ix+9)(-a(ix-1))^{\frac{1}{4}}}{(ix-1)(a(ix+1))^{\frac{1}{4}}}-\frac{\left(5\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2+x^3+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(\cdots\right.}{\cdots}\right.\right.}{\cdots}$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-I*(x^2+9-8*I*x)*(-a*(I*x-1))^(1/4)/(I*x-1)/(a*(I*x+1))^(1/4)-(5/2*RootOf(
_Z^2+I)*ln(-(-RootOf(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x^2+x^3+I*RootOf
(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(3/4)-2*I*RootOf(_Z^2+I)*(-2*I*x^3-x^4-2*I
*x+1)^(1/4)*x-I*(-2*I*x^3-x^4-2*I*x+1)^(1/2)*x+2*I*x^2+RootOf(_Z^2+I)*(-2*
I*x^3-x^4-2*I*x+1)^(1/4)+(-2*I*x^3-x^4-2*I*x+1)^(1/2)-x)/(I*x-1)^2)+5/2*I*
RootOf(_Z^2+I)*ln(-(-I*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*RootOf(_Z^2+I)*x^2+2*R
ootOf(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x+x^3+I*(-2*I*x^3-x^4-2*I*x+1)^
(1/2)*x+RootOf(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(3/4)+I*RootOf(_Z^2+I)*(-2*I
*x^3-x^4-2*I*x+1)^(1/4)+2*I*x^2-(-2*I*x^3-x^4-2*I*x+1)^(1/2)-x)/(I*x-1)^2)
)*(-a*(I*x-1))^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.07

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx =$$
$$-\frac{\sqrt{25i}(ax-i\,a)\log\left(\frac{\sqrt{25i}(ax-i\,a)+5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)-\sqrt{25i}(ax-i\,a)\log\left(-\frac{\sqrt{25i}(ax-i\,a)-5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+\cdots}{5\,(x-i)}\right.\cdots}{\cdots}$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(25*I)*(a*x - I*a)*log(1/5*(sqrt(25*I)*(a*x - I*a) + 5*(I*a*x +
a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(25*I)*(a*x - I*a)*log(-1/5*(s
qrt(25*I)*(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) +
 sqrt(-25*I)*(a*x - I*a)*log(1/5*(sqrt(-25*I)*(a*x - I*a) + 5*(I*a*x + a)^
(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-25*I)*(a*x - I*a)*log(-1/5*(sqr
t(-25*I)*(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) +
2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(-I*x - 9))/(a*x - I*a)
```

Sympy [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(5/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(5/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{5/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4),x)
```

output
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{1}{4}}\,ix+(ix+1)^{\frac{1}{4}}}dx - \left(\int \frac{(-ix+1)^{\frac{1}{4}}\,x}{(ix+1)^{\frac{1}{4}}\,ix+(ix+1)^{\frac{1}{4}}}dx\right)i$$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x)
```

output
```
int(( - i*x + 1)**(1/4)/((i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)),x) - int
((( - i*x + 1)**(1/4)*x)/((i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)),x)*i
```

3.252 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} + \frac{i\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} - \frac{i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{a}$$

output
```
4*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(1/4)-I*2^(1/2)*arctan(1-2^(1/2)*(a+I*a*x)
^(1/4)/(a-I*a*x)^(1/4))/a+I*2^(1/2)*arctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*
a*x)^(1/4))/a-I*2^(1/2)*arctanh(2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1
+(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \frac{i2^{3/4}\sqrt[4]{1+ix}(a-iax)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}-\frac{ix}{2}\right)}{5a^2\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

output
```
((I/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 5/
4, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {57, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{ixa+a}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\int \frac{1}{\sqrt[4]{ixa+a}}d\sqrt[4]{a-iax}}{a}$$

$$\Big\downarrow \text{770}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\int \frac{1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{a}$$

$$\Big\downarrow \text{755}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\int\frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int\frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$$\Big\downarrow \text{1476}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\int\frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int\frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-i}}{\sqrt[4]{ixa}}\right.\right.}{a}$$

$$\Big\downarrow \text{1082}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$$\Big\downarrow \text{217}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\int\frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)\right)}{a}$$

$$\Big\downarrow \text{1479}$$

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+i}}\right.}{\sqrt{2}}\right.\right.}{a}$$

$\Big\downarrow$ 25

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right.\right.}{a}$$

$\Big\downarrow$ 27

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-ia}}{\sqrt[4]{a+ia}}\right.}{\sqrt{2}}\right.\right.}{a}$$

$\Big\downarrow$ 1103

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{4i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{}}\right.\right.}{a}$$

input
```
Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

output
```
((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) - ((4*I)*((-(ArcTan[1 - (S
qrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2
]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[a
- I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2] + Log[1
+ Sqrt[a - I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt
[2]))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.44

method	result
risch	$-\frac{4(x+i)(-a(ix-1))^{\frac{1}{4}}}{a(ix-1)(a(ix+1))^{\frac{1}{4}}}+\frac{\left(\mathrm{RootOf}\left(\_Z^2-i\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2-i\,\mathrm{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-2\right.}{}\right.\right.}{}$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-4*(x+I)/a*(-a*(I*x-1))^(1/4)/(I*x-1)/(a*(I*x+1))^(1/4)+(RootOf(_Z^2-I)*ln
((-RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x^2-I*RootOf(_Z^2-I)*(-2*I*
x^3-x^4-2*I*x+1)^(3/4)-x^3-2*I*RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)
*x-I*(-2*I*x^3-x^4-2*I*x+1)^(1/2)*x-2*I*x^2+RootOf(_Z^2-I)*(-2*I*x^3-x^4-2
*I*x+1)^(1/4)+(-2*I*x^3-x^4-2*I*x+1)^(1/2)+x)/(I*x-1)^2)-I*RootOf(_Z^2-I)*
ln(-(-I*RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x^2+2*RootOf(_Z^2-I)*(
-2*I*x^3-x^4-2*I*x+1)^(1/4)*x+x^3-RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+1)^(3
/4)-I*(-2*I*x^3-x^4-2*I*x+1)^(1/2)*x+I*RootOf(_Z^2-I)*(-2*I*x^3-x^4-2*I*x+
1)^(1/4)+2*I*x^2+(-2*I*x^3-x^4-2*I*x+1)^(1/2)-x)/(I*x-1)^2))/a*(-a*(I*x-1)
)^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(135) = 270.

Time = 0.09 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx =$$

$$-\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\,(x-i)}\right)-\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\left(i\,ax+a\right)^{\frac{3}{4}}}{2\,(x-i)}\right.}{}$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
-1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)
 + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - (a^2*x - I*a^2)*sqrt
(4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(1/4))/(x - I)) + (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(1/2*((a^2*
x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I
)) - (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^
2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 8*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/(a^2*x - I*a^2)
```

Sympy [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(5/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx=\int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

input
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4),x)
```

output
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx=\frac{\int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}}dx}{a}$$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x)
```

output
```
int(( - i*x + 1)**(1/4)/((i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)),x)/a
```

3.253 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx=\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

output
```
2*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx=\frac{2\sqrt[4]{a-iax}(a+iax)^{3/4}}{a^3(-i+x)}$$

input
```
Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

output
```
(2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/(a^3*(-I + x))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{48\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx$$

$$\Big\downarrow \; 48$$

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
orering	$\frac{2i(x^2+1)}{(-iax+a)^{\frac{3}{4}}(iax+a)^{\frac{5}{4}}}$	27
risch	$\frac{2x+2i}{a(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	31
gosper	$-\frac{2i(-x+i)(x+i)}{(-iax+a)^{\frac{3}{4}}(iax+a)^{\frac{5}{4}}}$	32

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2*I*(x^2+1)/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{a^3x-i\,a^3}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x - I*a^3)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \frac{\left(-a\left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\,2\mathrm{i}}{a^2\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(5/4)),x)
```

output
```
((-a*(x*1i - 1))^(1/4)*2i)/(a^2*(a*(x*1i + 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^2}$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x + (i*x + 1)**(1/4)*( - i*x
 + 1)**(3/4)),x)/a**2
```

3.254 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 65

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx=\frac{2i}{a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}-\frac{4i(a+iax)^{3/4}}{3a^3(a-iax)^{3/4}}$$

output
```
2*I/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4)-4/3*I*(a+I*a*x)^(3/4)/a^3/(a-I*a*x
)^(3/4)
```

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx=\frac{2i+4x}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

output
```
(2*I + 4*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{5/4}}dx}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{48}$$

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

method	result	size
orering	$\frac{2(2x+i)(x^2+1)}{3(-iax+a)^{\frac{7}{4}}(iax+a)^{\frac{5}{4}}}$	32
risch	$\frac{\frac{4x}{3}+\frac{2i}{3}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	33
gosper	$-\frac{2(-x+i)(x+i)(2x+i)}{3(-iax+a)^{\frac{7}{4}}(iax+a)^{\frac{5}{4}}}$	37

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2/3*(2*x+I)*(x^2+1)/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+i)}{3\,(a^4x^2+a^4)}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
2/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + I)/(a^4*x^2 + a^4)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(7/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = -\frac{2\,(2\,x+1\mathrm{i})\,{(-a\,(-1+x\,1\mathrm{i}))}^{1/4}}{3\,a^3\,(-1+x\,1\mathrm{i})\,{(a\,(1+x\,1\mathrm{i}))}^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(5/4)),x)
```

output
```
-(2*(2*x + 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^3*(x*1i - 1)*(a*(x*1i + 1))^(1/
4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^2+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^3}$$

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*x**2 + (i*x + 1)**(1/4)*( - i*
x + 1)**(3/4)),x)/a**3
```

3.255 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = \frac{2i}{a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{8i(a+iax)^{3/4}}{7a^3(a-iax)^{7/4}} - \frac{16i(a+iax)^{3/4}}{21a^4(a-iax)^{3/4}}$$

output
```
2*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4)-8/7*I*(a+I*a*x)^(3/4)/a^3/(a-I*a*x
)^(7/4)-16/21*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(3/4)
```

Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = \frac{-2+24ix+16x^2}{21a^3(i+x)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

output
```
(-2 + (24*I)*x + 16*x^2)/(21*a^3*(I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/
4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}dx$$

$$\Big\downarrow \ 55$$

$$\frac{4\int \frac{1}{(a-iax)^{7/4}(ixa+a)^{5/4}}dx}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 55$$

$$\frac{4\left(\frac{2\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{5/4}}dx}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}\right)}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \ 48$$

$$\frac{4\left(\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}\right)}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

output
```
((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)) + (4*(((-2*I)/3)/(a^2
*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a*x)^(1/4))/(a^3
*(a + I*a*x)^(1/4))))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.39

method	result	size
orering	$-\frac{2i(8x^2+12ix-1)(x^2+1)}{21(-iax+a)^{\frac{11}{4}}(iax+a)^{\frac{5}{4}}}$	38
gosper	$\frac{2(-x+i)(x+i)(8ix^2-12x-i)}{21(-iax+a)^{\frac{11}{4}}(iax+a)^{\frac{5}{4}}}$	43
risch	$\frac{\frac{16}{21}x^2+\frac{8}{7}ix-\frac{2}{21}}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)}$	44

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2/21*I*(8*x^2+12*I*x-1)*(x^2+1)/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(8\,x^2+12i\,x-1)}{21\,(a^5x^3+i\,a^5x^2+a^5x+i\,a^5)}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

output
```
2/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 12*I*x - 1)/(a^5*x^3 +
I*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(5/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(11/4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = -\frac{(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,(8\,x^2+x\,12\mathrm{i}-1)\,2\mathrm{i}}{21\,a^4\,(-1+x\,1\mathrm{i})^2\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(5/4)),x)
```

output
```
-((-a*(x*1i - 1))^(1/4)*(x*12i + 8*x^2 - 1)*2i)/(21*a^4*(x*1i - 1)^2*(a*(x
*1i + 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = -\frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}i\,x^3+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^2-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^4}$$

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x)
```

output
```
( - int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x**3 + (i*x + 1)**(1/4)*
( - i*x + 1)**(3/4)*i*x - (i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*x**2 - (i*x
 + 1)**(1/4)*( - i*x + 1)**(3/4)),x))/a**4
```

3.256 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 221

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}} + \frac{7i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{\sqrt{2}} + \frac{7i\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{\sqrt{2}}$$

output
```
4/3*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(3/4)+7/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(1
/4)/a-7/2*I*arctan(1-2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))*2^(1/2)+7/2*
I*arctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))*2^(1/2)+7/2*I*arctanh(
2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1+(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)
))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.32

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{11/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{11}{4},\frac{15}{4},\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

output
```
((I/11)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[7/4,
11/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {57, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-\frac{7}{3}\int \frac{(a-iax)^{3/4}}{(ixa+a)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-\frac{7}{3}\left(\frac{3}{2}a\int \frac{1}{\sqrt[4]{a-iax}(ixa+a)^{3/4}}dx-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-\frac{7}{3}\left(6i\int \frac{\sqrt{a-iax}}{(ixa+a)^{3/4}}d\sqrt[4]{a-iax}-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}\right)$$

$$\Big\downarrow \text{ 854}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-\frac{7}{3}\left(6i\int \frac{\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}\right)$$

$$\Big\downarrow \text{ 826}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$
$$\frac{7}{3}\left(6i\left(\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$
$$\frac{7}{3}\left(6i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{1}{2}\right.\right.$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$
$$\frac{7}{3}\left(6i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a}}{\sqrt[4]{i}}\right.\right.$$

$$\Big\downarrow \text{ 217}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$
$$\frac{7}{3}\left(6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{i(}{}\right.$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{7}{3}\left(6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\right.}{}\right.\right.\right.$$

$\downarrow$ 25

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{7}{3}\left(6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\right.}{}\right.\right.\right.$$

$\downarrow$ 27

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{7}{3}\left(6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\text{arct}}{}\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{7}{3}\left(6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+\right.}{2\sqrt{2}}\right.\right.\right.$$

input
```
Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

output
```
(((4*I)/3)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(3/4)) - (7*(((-I)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(1/4))/a + (6*I)*((-(ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(
1/4))/(a + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))
/(a + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[a - I*a*x] - (Sqrt[2]*(a -
I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[a - I*a*x] + (
Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2]))/2)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.09 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.12

method	result
risch	$\frac{i(3x^2-8ix+11)a}{3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}+\frac{\left(7\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4+2i\right.}{}\right.}{}\right.}{}$

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*(-8*I*x+3*x^2+11)/(a*(I*x+1))^(3/4)/(-a*(I*x-1))^(1/4)*a+(7/2*RootOf
(_Z^2+I)*ln(-(-RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x^2+I*RootOf(_Z^
2+I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)+x^3+2*I*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I
*x)^(1/4)*x-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*(1-x^4+
2*I*x^3+2*I*x)^(1/4)-(1-x^4+2*I*x^3+2*I*x)^(1/2)-x)/(I*x+1)^2)-7/2*I*RootO
f(_Z^2+I)*ln((-I*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x^2-2*RootOf(_
Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x-x^3+RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I
*x)^(3/4)-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+I*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+
2*I*x)^(1/4)+2*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)+x)/(I*x+1)^2))/(a*(I*x+1)
)^(3/4)*(-(I*x-1)*(I*x+1)^3)^(1/4)/(-a*(I*x-1))^(1/4)*a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.06

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx =$$

$$-\frac{3\sqrt{49i}(ax-i\,a)\log\left(\frac{\sqrt{49i}(ax+i\,a)+7\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)-3\sqrt{49i}(ax-i\,a)\log\left(-\frac{\sqrt{49i}(ax+i\,a)-7\,(i\,ax+a)^{\frac{1}{4}}(-i\,}{7\,(x+i)}\right.}{}$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
-1/6*(3*sqrt(49*I)*(a*x - I*a)*log(1/7*(sqrt(49*I)*(a*x + I*a) + 7*(I*a*x
+ a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*sqrt(49*I)*(a*x - I*a)*log(-1/
7*(sqrt(49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I
)) + 3*sqrt(-49*I)*(a*x - I*a)*log(1/7*(sqrt(-49*I)*(a*x + I*a) + 7*(I*a*x
 + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*sqrt(-49*I)*(a*x - I*a)*log(-
1/7*(sqrt(-49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x
+ I)) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(-3*I*x - 11))/(a*x - I*a)
```

Sympy [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx=\int \frac{(-ia(x+i))^{\frac{7}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(7/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(7/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%
{i,[0,1]%%%}] at parameters values [44,93]Warning, need to choose a branch
 for the roo
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{7/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4),x)
```

output
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-ix+1)^{\frac{3}{4}}}{(ix+1)^{\frac{3}{4}}\,ix+(ix+1)^{\frac{3}{4}}}dx - \left(\int \frac{(-ix+1)^{\frac{3}{4}}\,x}{(ix+1)^{\frac{3}{4}}\,ix+(ix+1)^{\frac{3}{4}}}dx\right) i$$

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(( - i*x + 1)**(3/4)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),x) - int
((( - i*x + 1)**(3/4)*x)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),x)*i
```

3.257 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} + \frac{i\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} + \frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{a}$$

output
```
4/3*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(3/4)-I*2^(1/2)*arctan(1-2^(1/2)*(a+I*a*
x)^(1/4)/(a-I*a*x)^(1/4))/a+I*2^(1/2)*arctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-
I*a*x)^(1/4))/a+I*2^(1/2)*arctanh(2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/
(1+(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.36

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx=\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{7/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

output
```
((I/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 7/
4, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {57, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\int \frac{1}{\sqrt[4]{a-iax}(ixa+a)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{4i\int \frac{\sqrt{a-iax}}{(ixa+a)^{3/4}}d\sqrt[4]{a-iax}}{a}$$

$$\Big\downarrow \text{854}$$

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{4i\int \frac{\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{a}$$

$\downarrow$ 826

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{4i\left(\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$\downarrow$ 1476

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-}}{\sqrt[4]{ixa}}\right.}{a}$$

$\downarrow$ 1082

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$\downarrow$ 217

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}$$

$\downarrow$ 1479

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-ia}}{\sqrt[4]{a+iax}}\right.}{\sqrt{2}}\right.\right.}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{4i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\cdots\right.\right.}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{4i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-\cdots}}{\sqrt[4]{a+i\cdots}}\right)}{\sqrt{2}}\cdots\right.\right.}{a}$$

$$\Big\downarrow \; 1103$$

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{4i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{a-iax}+\frac{\sqrt{2}\cdots}{\cdots}\right.}{2\sqrt{\cdots}}\cdots\right.\right.}{a}$$

input
```
Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

output
```
(((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) - ((4*I)*((-(ArcTan[1
- (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[a -
 I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log
[1 + Sqrt[a - I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*S
qrt[2]))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.05 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.31

method	result
risch	$\frac{\frac{4x}{3}+\frac{4i}{3}}{(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}-\frac{\left(\operatorname{RootOf}\left(\_Z^2-i\right)\ln\left(\frac{-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2-i\right)x^2-x^3-i\operatorname{RootOf}\left(\_Z^2-i\right)\left(-x^4+2\right.}{}\right.\right.}{}$

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
4/3*(x+I)/(a*(I*x+1))^(3/4)/(-a*(I*x-1))^(1/4)-(RootOf(_Z^2-I)*ln((-(1-x^4
+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-x^3-I*RootOf(_Z^2-I)*(1-x^4+2*I*x
^3+2*I*x)^(3/4)-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+2*I*RootOf(_Z^2-I)*(1-x^4+
2*I*x^3+2*I*x)^(1/4)*x+2*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)+RootOf(_Z^2-I)*
(1-x^4+2*I*x^3+2*I*x)^(1/4)+x)/(I*x+1)^2)+I*RootOf(_Z^2-I)*ln((-I*(1-x^4+2
*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I
*x)^(1/4)*x-x^3+I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x-RootOf(_Z^2-I)*(1-x^4+2*I*
x^3+2*I*x)^(3/4)+I*RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)+2*I*x^2+(1-x
^4+2*I*x^3+2*I*x)^(1/2)+x)/(I*x+1)^2))/(a*(I*x+1))^(3/4)*(-(I*x-1)*(I*x+1)
^3)^(1/4)/(-a*(I*x-1))^(1/4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(135) = 270.

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.51

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx =$$

$$-\frac{3\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}{2\,(x+i)}\right)-3\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\left(i\,ax-\right.}{2\,(x+i)}\right.}{}$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
-1/6*(3*(a^2*x - I*a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(4*I/a^
2) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*(a^2*x - I*a^2)*
sqrt(4*I/a^2)*log(-1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4
)*(-I*a*x + a)^(3/4))/(x + I)) + 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(1/2*
((a^2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/
(x + I)) - 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-1/2*((a^2*x + I*a^2)*sqrt
(-4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 8*(I*a*x +
 a)^(1/4)*(-I*a*x + a)^(3/4))/(a^2*x - I*a^2)
```

Sympy [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(7/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

Giac [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4),x)
```

output
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \frac{\int \frac{(-ix+1)^{\frac{3}{4}}}{(ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{3}{4}}}dx}{a}$$

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(( - i*x + 1)**(3/4)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),x)/a
```

3.258 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx=\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

output
```
2/3*I*(a-I*a*x)^(3/4)/a^2/(a+I*a*x)^(3/4)
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx=\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

output
```
(((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx$$

$$\Big\downarrow \text{ 48}$$

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

output
```
(((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{2i(x^2+1)}{3(-iax+a)^{\frac{1}{4}}(iax+a)^{\frac{7}{4}}}$	27
risch	$\frac{\frac{2x}{3}+\frac{2i}{3}}{a(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}$	31
gosper	$-\frac{2i(-x+i)(x+i)}{3(-iax+a)^{\frac{1}{4}}(iax+a)^{\frac{7}{4}}}$	32

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(x^2+1)/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(a^3x-i\,a^3)}$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x - I*a^3)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)),x)
```

output
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{7/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}}dx}{a^2}$$

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*i*x + (i*x + 1)**(3/4)*( - i*x
 + 1)**(1/4)),x)/a**2
```

3.259 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 65

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}+\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}$$

output
```
-2*I/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4)+4/3*I*(a-I*a*x)^(3/4)/a^3/(a+I*a*
x)^(3/4)
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = \frac{-2i+4x}{3a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

output
```
(-2*I + 4*x)/(3*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}} \, dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{2\int \frac{1}{\sqrt[4]{a-iax}(ixa+a)^{7/4}}dx}{a}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

output
```
(-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^
(3/4))/(a^3*(a + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

method	result	size
orering	$-\frac{2(-2x+i)\left(x^{2}+1\right)}{3(-iax+a)^{\frac{5}{4}}(iax+a)^{\frac{7}{4}}}$	32
risch	$\frac{\frac{4x}{3}-\frac{2i}{3}}{a^{2}\left(a(ix+1)\right)^{\frac{3}{4}}\left(-a(ix-1)\right)^{\frac{1}{4}}}$	33
gosper	$\frac{2(-x+i)(x+i)(-2x+i)}{3(-iax+a)^{\frac{5}{4}}(iax+a)^{\frac{7}{4}}}$	37

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*x+I)*(x^2+1)/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x-i)}{3\,(a^4x^2+a^4)}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(2*x - I)/(a^4*x^2 + a^4)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(5/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)),x)
```

output
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}x^2+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}}dx}{a^3}$$

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*x**2 + (i*x + 1)**(3/4)*( - i*
x + 1)**(1/4)),x)/a**3
```

3.260 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = \frac{2i}{3a^2(a-iax)^{5/4}(a+iax)^{3/4}} - \frac{8i\sqrt[4]{a+iax}}{15a^3(a-iax)^{5/4}} - \frac{16i\sqrt[4]{a+iax}}{15a^4\sqrt[4]{a-iax}}$$

output
```
2/3*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4)-8/15*I*(a+I*a*x)^(1/4)/a^3/(a-I*
a*x)^(5/4)-16/15*I*(a+I*a*x)^(1/4)/a^4/(a-I*a*x)^(1/4)
```

Mathematica [A] (verified)

Time = 6.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = \frac{2(7+4ix+8x^2)}{15a^3(i+x)\sqrt[4]{a-iax}(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

output
```
(2*(7 + (4*I)*x + 8*x^2))/(15*a^3*(I + x)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3
/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx$$

$$\Big\downarrow \; 55$$

$$\frac{4\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{7/4}}dx}{5a}-\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 55$$

$$\frac{4\left(\frac{2\int \frac{1}{\sqrt[4]{a-iax}(ixa+a)^{7/4}}dx}{a}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}\right)}{5a}-\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 48$$

$$\frac{4\left(\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{3/4}}\right)}{5a}-\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

output
```
((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)) + (4*((-2*I)/(a^2*(a
- I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^(3/4))/(a^3*(a
+ I*a*x)^(3/4))))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.38

method	result	size
orering	$-\frac{2i\left(8x^{2}+4ix+7\right)\left(x^{2}+1\right)}{15\left(-iax+a\right)^{\frac{9}{4}}\left(iax+a\right)^{\frac{7}{4}}}$	38
gosper	$\frac{2\left(-x+i\right)\left(x+i\right)\left(8i x^{2}-4x+7i\right)}{15\left(-iax+a\right)^{\frac{9}{4}}\left(iax+a\right)^{\frac{7}{4}}}$	43
risch	$\frac{\frac{16}{15}x^{2}+\frac{8}{15}ix+\frac{14}{15}}{a^{3}\left(a\left(ix+1\right)\right)^{\frac{3}{4}}\left(-a\left(ix-1\right)\right)^{\frac{1}{4}}\left(x+i\right)}$	44

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-2/15*I*(8*x^2+4*I*x+7)*(x^2+1)/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(8\,x^2+4i\,x+7)}{15\,(a^5x^3+i\,a^5x^2+a^5x+i\,a^5)}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
2/15*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 4*I*x + 7)/(a^5*x^3 + I
*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(9/4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)),x)
```

output
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = -\frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}i\,x^3+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}x^2-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{1}{4}}}dx}{a^4}$$

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

output
```
( - int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*i*x**3 + (i*x + 1)**(3/4)*
( - i*x + 1)**(1/4)*i*x - (i*x + 1)**(3/4)*( - i*x + 1)**(1/4)*x**2 - (i*x
 + 1)**(3/4)*( - i*x + 1)**(1/4)),x))/a**4
```

3.261 $\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \frac{10a^2(1+x^2)^{3/4}\operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
4/3*I*(a-I*a*x)^(9/4)/a/(a+I*a*x)^(3/4)+10*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/
4)+2*I*(a-I*a*x)^(5/4)*(a+I*a*x)^(1/4)/a-10*a^2*(x^2+1)^(3/4)*InverseJacob
iAM(1/2*arctan(x),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx = \frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{13/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{13}{4},\frac{17}{4},\frac{1}{2}-\frac{ix}{2}\right)}{13a^2(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

output
```
((I/13)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(13/4)*Hypergeometric2F1[7/4,
13/4, 17/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {57, 60, 60, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}dx$$

$$\Big\downarrow \; 57$$

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}-3\int \frac{(a-iax)^{5/4}}{(ixa+a)^{3/4}}dx$$

$$\Big\downarrow \; 60$$

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}-3\left(\frac{5}{3}a\int \frac{\sqrt[4]{a-iax}}{(ixa+a)^{3/4}}dx-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}-3\left(\frac{5}{3}a\left(a\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}\right)$$

$$\Big\downarrow \; 46$$

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}-3\left(\frac{5}{3}a\left(\frac{a\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{(x^2a^2+a^2)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}\right)$$

$$\Big\downarrow \; 231$$

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}-3\left(\frac{5}{3}a\left(\frac{a(x^2+1)^{3/4}\int\frac{1}{(x^2+1)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}-3\left(\frac{5}{3}a\left(\frac{2a(x^2+1)^{3/4}\,\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}\right)$$

input
```
Int[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

output
```
(((4*I)/3)*(a - I*a*x)^(9/4))/(a*(a + I*a*x)^(3/4)) - 3*((((-2*I)/3)*(a -
I*a*x)^(5/4)*(a + I*a*x)^(1/4))/a + (5*a*(((-2*I)*(a - I*a*x)^(1/4)*(a + I
*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a
*x)^(3/4)*(a + I*a*x)^(3/4))))/3)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{(-iax+a)^{\frac{9}{4}}}{(iax+a)^{\frac{7}{4}}}dx$$

input
```
int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

output
```
int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

Fricas [F]

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{9}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
1/3*(3*(x - I)*integral(-5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1),
 x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x^2 + 11*I*x + 20))/(x - I)
```

Sympy [F]

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{9}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral((-I*a*(x + I))**(9/4)/(I*a*(x - I))**(7/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{9}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(9/4)/(I*a*x + a)^(7/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx=\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{9/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

input
```
int((a - a*x*1i)^(9/4)/(a + a*x*1i)^(7/4),x)
```

output
```
int((a - a*x*1i)^(9/4)/(a + a*x*1i)^(7/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx=\frac{a\left(\int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{3}{4}} ix+(ix+1)^{\frac{3}{4}}}dx-\left(\int \frac{(-ix+1)^{\frac{1}{4}} x^{2}}{(ix+1)^{\frac{3}{4}} ix+(ix+1)^{\frac{3}{4}}}dx\right)-2\left(\int \frac{(-ix+1)^{\frac{1}{4}} x}{(ix+1)^{\frac{3}{4}} ix+(ix+1)^{\frac{3}{4}}}dx\right)i\right)}{\sqrt{a}}$$

input
```
int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

output
```
(a*(int(( - i*x + 1)**(1/4)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),x) -
 int((( - i*x + 1)**(1/4)*x**2)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),
x) - 2*int((( - i*x + 1)**(1/4)*x)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4
)),x)*i))/sqrt(a)
```

3.262 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{3a}-\frac{10a(1+x^2)^{3/4}\,\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
4/3*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(3/4)+10/3*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(
1/4)/a-10/3*a*(x^2+1)^(3/4)*InverseJacobiAM(1/2*arctan(x),2^(1/2))/(a-I*a*
x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{9/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```

output
```
((I/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[7/4, 9/
4, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {57, 60, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\int \frac{\sqrt[4]{a-iax}}{(ixa+a)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\left(a\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)$$

$$\Big\downarrow \text{46}$$

$$\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\left(\frac{a\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{\left(x^2a^2+a^2\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\left(\frac{a\left(x^2+1\right)^{3/4}\int \frac{1}{\left(x^2+1\right)^{3/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\left(\frac{2a\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{a}\right)$$

input
```
Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```

output
```
(((4*I)/3)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(3/4)) - (5*(((-2*I)*(a - I*a
*x)^(1/4)*(a + I*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/
2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))))/3
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{(-iax+a)^{\frac{5}{4}}}{(iax+a)^{\frac{7}{4}}}dx$$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

output
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

Fricas [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
1/3*(3*(a*x - I*a)*integral(-5/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x
^2 + a), x) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(-3*I*x - 7))/(a*x -
I*a)
```

Sympy [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(7/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(7/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{5/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(7/4),x)
```

output
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(7/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx = \frac{\int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{3}{4}}}dx - \left(\int \frac{(-ix+1)^{\frac{1}{4}}x}{(ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{3}{4}}}dx\right)i}{\sqrt{a}}$$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

output
```
(int(( - i*x + 1)**(1/4)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),x) - in
t((( - i*x + 1)**(1/4)*x)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),x)*i)/
sqrt(a)
```

3.263 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx=\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2(1+x^2)^{3/4}\,\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
4/3*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(3/4)-2/3*(x^2+1)^(3/4)*InverseJacobiAM(
1/2*arctan(x),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx=\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{9}{4},\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a+iax)^{3/4}}$$

input
```
Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
```

output
```
((I/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 7/
4, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {57, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx$$

$$\Big\downarrow \; 57$$

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{1}{3}\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx$$

$$\Big\downarrow \; 46$$

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{(x^2a^2+a^2)^{3/4}}dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 231$$

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{\left(x^2+1\right)^{3/4}\int \frac{1}{(x^2+1)^{3/4}}dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

input
```
Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
```

output
```
(((4*I)/3)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(3/4)) - (2*(1 + x^2)^(3/4)*E
llipticF[ArcTan[x]/2, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{(-iax + a)^{\frac{1}{4}}}{(iax + a)^{\frac{7}{4}}} dx$$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

output
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
1/3*(3*(a^2*x - I*a^2)*integral(-1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/
(a^2*x^2 + a^2), x) + 4*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^2*x - I*a
^2)
```

Sympy [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx = \int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(7/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(7/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(7/4),x)
```

output
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(7/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx = \frac{\int \frac{(-ix+1)^{\frac{1}{4}}}{(ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{3}{4}}}dx}{\sqrt{a}\,a}$$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(( - i*x + 1)**(1/4)/((i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)),x)/(sqrt
(a)*a)
```

3.264 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}} \, dx = \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}} + \frac{2(1+x^2)^{3/4} \operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
2/3*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*InverseJacobiA
M(1/2*arctan(x),2^(1/2))/a/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}} \, dx = \frac{i\sqrt[4]{2}(1+ix)^{3/4}\sqrt[4]{a-iax} \operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{5}{4},\frac{1}{2}-\frac{ix}{2}\right)}{a^2(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
```

output
```
(I*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 7/4, 5
/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {61, 46, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}dx$$

$$\Big\downarrow \; 61$$

$$\frac{\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{3/4}}dx}{3a}+\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}$$

$$\Big\downarrow \; 46$$

$$\frac{\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{(x^2a^2+a^2)^{3/4}}dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}$$

$$\Big\downarrow \; 231$$

$$\frac{\left(x^2+1\right)^{3/4}\int \frac{1}{(x^2+1)^{3/4}}dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
```

output
```
(((2*I)/3)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)
*EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{(-iax+a)^{\frac{3}{4}}\,(iax+a)^{\frac{7}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
1/3*(3*(a^3*x - I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(
a^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x - I*a^
3)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(3/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(7/4)),x)
```

output
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(7/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx = -\frac{2\sqrt{a}\,(ix+1)^{\frac{3}{4}}\,(-ix+1)^{\frac{1}{4}}}{\sqrt{ix+1}\,a^{3}\,(i-x)}$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

output

```
( - 2*sqrt(a)*(i*x + 1)**(3/4)*( - i*x + 1)**(1/4))/(sqrt(i*x + 1)*a**3*(i
 - x))
```

3.265 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{2(1+x^2)^{3/4}\operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
2/3*x/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*InverseJacobiA
M(1/2*arctan(x),2^(1/2))/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = -\frac{i\sqrt[4]{2}(1+ix)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{1}{4},\frac{1}{2}-\frac{ix}{2}\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
```

output
```
((-1/3*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 7/4, 1/4, 1/2 -
(I/2)*x])/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {46, 215, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}dx$$

$$\Big\downarrow \ 46$$

$$\frac{\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{\left(x^2a^2+a^2\right)^{7/4}}dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \ 215$$

$$\frac{\left(a^2x^2+a^2\right)^{3/4}\left(\frac{\int \frac{1}{\left(x^2a^2+a^2\right)^{3/4}}dx}{3a^2}+\frac{2x}{3a^2\left(a^2x^2+a^2\right)^{3/4}}\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \ 231$$

$$\frac{\left(a^2x^2+a^2\right)^{3/4}\left(\frac{\left(x^2+1\right)^{3/4}\int \frac{1}{\left(x^2+1\right)^{3/4}}dx}{3a^2\left(a^2x^2+a^2\right)^{3/4}}+\frac{2x}{3a^2\left(a^2x^2+a^2\right)^{3/4}}\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \ 229$$

$$\frac{\left(a^2x^2+a^2\right)^{3/4}\left(\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a^2\left(a^2x^2+a^2\right)^{3/4}}+\frac{2x}{3a^2\left(a^2x^2+a^2\right)^{3/4}}\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
```

output
```
((a^2 + a^2*x^2)^(3/4)*((2*x)/(3*a^2*(a^2 + a^2*x^2)^(3/4)) + (2*(1 + x^2)
^(3/4)*EllipticF[ArcTan[x]/2, 2])/(3*a^2*(a^2 + a^2*x^2)^(3/4))))/((a - I*
a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-iax+a\right)^{\frac{7}{4}} \left(iax+a\right)^{\frac{7}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
1/3*(3*(a^4*x^2 + a^4)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(
a^4*x^2 + a^4), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*x)/(a^4*x^2 +
a^4)
```

Sympy [A] (verification not implemented)

Time = 15.68 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = -\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{8},\frac{11}{8},1 & \frac{1}{2},\frac{7}{4},\frac{9}{4} \\ \frac{7}{8},\frac{5}{4},\frac{11}{8},\frac{7}{4},\frac{9}{4} & 0 \end{matrix}\middle| \frac{e^{-3i\pi}}{x^2}\right)e^{-\frac{i\pi}{4}}}{4\pi a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{3}{8},\frac{1}{2},\frac{7}{8},1 & \\ \frac{3}{8},\frac{7}{8} & -\frac{1}{2},0,\frac{5}{4},0 \end{matrix}\middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

output
```
-I*meijerg(((7/8, 11/8, 1), (1/2, 7/4, 9/4)), ((7/8, 5/4, 11/8, 7/4, 9/4),
 (0,)), exp_polar(-3*I*pi)/x**2)*exp(-I*pi/4)/(4*pi*a**(7/2)*gamma(7/4)) +
 I*meijerg(((-1/2, 0, 3/8, 1/2, 7/8, 1), ()), ((3/8, 7/8), (-1/2, 0, 5/4,
0)), exp_polar(-I*pi)/x**2)/(4*pi*a**(7/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(7/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{7/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(7/4)),x)
```

output
```
int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(7/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \frac{2\sqrt{a}\,(ix+1)^{\frac{3}{4}}\,(2ix-1)}{3\sqrt{ix+1}\,(-ix+1)^{\frac{3}{4}}\,a^4\,(i-x)}$$

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

output
```
(2*sqrt(a)*(i*x + 1)**(3/4)*(2*i*x - 1))/(3*sqrt(i*x + 1)*( - i*x + 1)**(3
/4)*a**4*(i - x))
```

3.266 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{10(1+x^2)^{3/4}\,\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
-2/7*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4)+10/21*x/a^3/(a-I*a*x)^(3/4)/(a+
I*a*x)^(3/4)+10/21*(x^2+1)^(3/4)*InverseJacobiAM(1/2*arctan(x),2^(1/2))/a^
3/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = -\frac{i\sqrt[4]{2}(1+ix)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{7}{4},-\frac{3}{4},\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

output
```
((-1/7*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 7/4, -3/4, 1/2 -
 (I/2)*x])/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {61, 46, 215, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx$$

$$\Big\downarrow \; 61$$

$$\frac{5\int \frac{1}{(a-iax)^{7/4}(ixa+a)^{7/4}}dx}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 46$$

$$\frac{5\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{\left(x^2a^2+a^2\right)^{7/4}}dx}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 215$$

$$\frac{5\left(a^2x^2+a^2\right)^{3/4}\left(\frac{\int \frac{1}{\left(x^2a^2+a^2\right)^{3/4}}dx}{3a^2}+\frac{2x}{3a^2(a^2x^2+a^2)^{3/4}}\right)}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 231$$

$$\frac{5\left(a^2x^2+a^2\right)^{3/4}\left(\frac{\left(x^2+1\right)^{3/4}\int \frac{1}{\left(x^2+1\right)^{3/4}}dx}{3a^2(a^2x^2+a^2)^{3/4}}+\frac{2x}{3a^2(a^2x^2+a^2)^{3/4}}\right)}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{5(a^2x^2+a^2)^{3/4}\left(\frac{2(x^2+1)^{3/4}\operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a^2(a^2x^2+a^2)^{3/4}}+\frac{2x}{3a^2(a^2x^2+a^2)^{3/4}}\right)}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

input

```
Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

output

```
((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (5*(a^2 + a^2*x^2)^
(3/4)*((2*x)/(3*a^2*(a^2 + a^2*x^2)^(3/4)) + (2*(1 + x^2)^(3/4)*EllipticF[
ArcTan[x]/2, 2])/(3*a^2*(a^2 + a^2*x^2)^(3/4))))/(7*a*(a - I*a*x)^(3/4)*(a
 + I*a*x)^(3/4))
```

Defintions of rubi rules used

rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-iax+a\right)^{\frac{11}{4}} \left(iax+a\right)^{\frac{7}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{\left(i\,ax+a\right)^{\frac{7}{4}}\left(-i\,ax+a\right)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
1/21*(21*(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)*integral(5/21*(I*a*x + a)^(
1/4)*(-I*a*x + a)^(1/4)/(a^5*x^2 + a^5), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x
+ a)^(1/4)*(5*x^2 + 5*I*x + 3))/(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,(-ia\,(x+i))^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(7/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(11/4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{11/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(7/4)),x)
```

output
```
int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx = -\frac{\int \frac{1}{(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}i\,x^3+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}ix-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}x^2-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}}dx}{\sqrt{a}\,a^4}$$

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

output
```
( - int(1/((i*x + 1)**(3/4)*( - i*x + 1)**(3/4)*i*x**3 + (i*x + 1)**(3/4)*
( - i*x + 1)**(3/4)*i*x - (i*x + 1)**(3/4)*( - i*x + 1)**(3/4)*x**2 - (i*x
 + 1)**(3/4)*( - i*x + 1)**(3/4)),x))/(sqrt(a)*a**4)
```

3.267 $\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{10(1+x^2)^{3/4}\operatorname{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}$$

output
```
-2/11*I/a^2/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4)-2/11*I/a^3/(a-I*a*x)^(7/4)/(a
+I*a*x)^(3/4)+10/33*x/a^4/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)+10/33*(x^2+1)^(3
/4)*InverseJacobiAM(1/2*arctan(x),2^(1/2))/a^4/(a-I*a*x)^(3/4)/(a+I*a*x)^(
3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = -\frac{i\sqrt[4]{2}(1+ix)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{7}{4},-\frac{7}{4},\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

input
```
Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```

output
```
((-1/11*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-11/4, 7/4, -7/4, 1/2
 - (I/2)*x])/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {61, 61, 46, 215, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx$$

$$\Big\downarrow \text{61}$$

$$\frac{7\int \frac{1}{(a-iax)^{11/4}(ixa+a)^{7/4}}dx}{11a}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(\frac{5\int \frac{1}{(a-iax)^{7/4}(ixa+a)^{7/4}}dx}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}\right)}{11a}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \text{46}$$

$$\frac{7\left(\frac{5\left(a^2x^2+a^2\right)^{3/4}\int \frac{1}{\left(x^2a^2+a^2\right)^{7/4}}dx}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}\right)}{11a}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 215$$

$$\frac{7\left(\frac{5\left(a^2x^2+a^2\right)^{3/4}\left(\frac{\int \frac{1}{\left(x^2a^2+a^2\right)^{3/4}}dx}{3a^2}+\frac{2x}{3a^2\left(a^2x^2+a^2\right)^{3/4}}\right)}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}\right)}{11a}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 231$$

$$\frac{7\left(\frac{5\left(a^2x^2+a^2\right)^{3/4}\left(\frac{\left(x^2+1\right)^{3/4}\int \frac{1}{\left(x^2+1\right)^{3/4}}dx}{3a^2\left(a^2x^2+a^2\right)^{3/4}}+\frac{2x}{3a^2\left(a^2x^2+a^2\right)^{3/4}}\right)}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}\right)}{11a}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{7\left(\frac{5\left(a^2x^2+a^2\right)^{3/4}\left(\frac{2\left(x^2+1\right)^{3/4}\text{EllipticF}\left(\frac{\arctan(x)}{2},2\right)}{3a^2\left(a^2x^2+a^2\right)^{3/4}}+\frac{2x}{3a^2\left(a^2x^2+a^2\right)^{3/4}}\right)}{7a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}\right)}{11a}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

input
```
Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```

output
```
((-2*I)/11)/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)) + (7*(((-2*I)/7)/(a
^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (5*(a^2 + a^2*x^2)^(3/4)*((2*x)/
(3*a^2*(a^2 + a^2*x^2)^(3/4)) + (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2,
2])/(3*a^2*(a^2 + a^2*x^2)^(3/4))))/(7*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/
4))))/(11*a)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{(-iax+a)^{\frac{15}{4}} (iax+a)^{\frac{7}{4}}} dx$$

input
```
int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

output
```
int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}} dx = \int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{15}{4}}} dx$$

input
```
integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

output
```
1/33*(33*(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*x - a^6)*integral(5/33*(I*a*x +
a)^(1/4)*(-I*a*x + a)^(1/4)/(a^6*x^2 + a^6), x) + 2*(5*x^3 + 10*I*x^2 - 2*
x + 6*I)*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^6*x^4 + 2*I*a^6*x^3 + 2*
I*a^6*x - a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}} dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(7/4),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{15/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(7/4)),x)
```

output
```
int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = -\frac{\int \frac{1}{2(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}i\,x^3+2(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}x^4-(ix+1)^{\frac{3}{4}}(-ix+1)^{\frac{3}{4}}}dx}{\sqrt{a}\,a^5}$$

input
```
int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

output
```
( - int(1/(2*(i*x + 1)**(3/4)*( - i*x + 1)**(3/4)*i*x**3 + 2*(i*x + 1)**(3
/4)*( - i*x + 1)**(3/4)*i*x + (i*x + 1)**(3/4)*( - i*x + 1)**(3/4)*x**4 -
(i*x + 1)**(3/4)*( - i*x + 1)**(3/4)),x))/(sqrt(a)*a**5)
```

3.268 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx=\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{42i}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}-\frac{42\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
4/5*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(5/4)-42/5*I/(a-I*a*x)^(1/4)/(a+I*a*x)^(
1/4)+14/5*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(1/4)-42/5*(x^2+1)^(1/4)*EllipticE
(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx=\frac{i\sqrt[4]{1+ix}(a-iax)^{11/4}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{11}{4},\frac{15}{4},\frac{1}{2}-\frac{ix}{2}\right)}{11\sqrt[4]{2}a^3\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]
```

output
```
((I/11)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[9/4, 11/4, 15
/4, 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {57, 57, 46, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{7}{5}\int \frac{(a-iax)^{3/4}}{(ixa+a)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{7}{5}\left(\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-3\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{ixa+a}}dx\right)$$

$$\Big\downarrow \text{46}$$

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{7}{5}\left(\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\sqrt[4]{x^2a^2+a^2}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{7}{5}\left(\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{x^2+1}\int \frac{1}{\sqrt[4]{x^2+1}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{7}{5}\left(\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-\int \frac{1}{(x^2+1)^{5/4}}dx\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{7}{5}\left(\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{3\sqrt[4]{x^2+1}\left(\frac{2x}{\sqrt[4]{x^2+1}}-2E\left(\frac{\arctan(x)}{2}\middle|2\right)\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

input
```
Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]
```

output
```
(((4*I)/5)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(5/4)) - (7*(((4*I)*(a - I*a*
x)^(3/4))/(a*(a + I*a*x)^(1/4)) - (3*(1 + x^2)^(1/4)*((2*x)/(1 + x^2)^(1/4
) - 2*EllipticE[ArcTan[x]/2, 2]))/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))))/
5
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{8(4x^2+ix+3)}{5(x-i)(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{21x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	101

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-8/5*(4*x^2+3+I*x)/(x-I)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)+21/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I
*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*x^2 - 30*I*x - 21) + 5*(a^2
*x^3 - 2*I*a^2*x^2 - a^2*x)*integral(42/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(
3/4)/(a^2*x^4 + a^2*x^2), x))/(a^2*x^3 - 2*I*a^2*x^2 - a^2*x)
```

Sympy [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{7}{4}}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(9/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(9/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%
{i,[0,1]%%%}] at parameters values [44,93]Warning, need to choose a branch
 for the roo
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx=\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{7/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{9/4}}\,dx$$

input
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(9/4),x)
```

output
```
int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(9/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx=\frac{\sqrt{a}\,\left(\int \frac{(-ix+1)^{\frac{3}{4}}}{2(ix+1)^{\frac{1}{4}} ix-(ix+1)^{\frac{1}{4}} x^{2}+(ix+1)^{\frac{1}{4}}}dx -\left(\int \frac{(-ix+1)^{\frac{3}{4}} x}{2(ix+1)^{\frac{1}{4}} ix-(ix+1)^{\frac{1}{4}} x^{2}+(ix+1)^{\frac{1}{4}}}dx \right) i \right)}{a}$$

input
```
int((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x)
```

output
```
(sqrt(a)*(int(( - i*x + 1)**(3/4)/(2*(i*x + 1)**(1/4)*i*x - (i*x + 1)**(1/
4)*x**2 + (i*x + 1)**(1/4)),x) - int((( - i*x + 1)**(3/4)*x)/(2*(i*x + 1)*
*(1/4)*i*x - (i*x + 1)**(1/4)*x**2 + (i*x + 1)**(1/4)),x)*i))/a
```

3.269 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx=\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
4/5*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(5/4)-6/5*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^
(1/4)-6/5*(x^2+1)^(1/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a/(a-I*a*x)^
(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx=\frac{i\sqrt[4]{1+ix}(a-iax)^{7/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{9}{4},\frac{11}{4},\frac{1}{2}-\frac{ix}{2}\right)}{7\sqrt[4]{2}a^3\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
```

output
```
((I/7)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 9/4, 11/4,
 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {57, 58, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{3}{5}\int \frac{1}{\sqrt[4]{a-iax}(ixa+a)^{5/4}}dx$$

$$\Big\downarrow \text{ 58}$$

$$\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{3}{5}\left(a\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

$$\Big\downarrow \text{ 46}$$

$$\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{3}{5}\left(\frac{a\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

$$\Big\downarrow \text{ 213}$$

$$\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{3}{5}\left(\frac{\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

$$\Big\downarrow \text{ 212}$$

$$\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{3}{5}\left(\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\right)$$

input
```
Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
```

output
```
(((4*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(5/4)) - (3*((2*I)/(a*(a - I*
a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2,
2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))))/5
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 58
```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Simp[c   Int[1/((a + b*x)^
(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d,
0] && NegQ[a^2*b^2]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{2(3x^2+2ix+1)}{5(x-i)a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{3x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(3*x^2+1+2*I*x)/(x-I)/a/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)+3/5/(a^2
)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(
-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
-1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*I*x + 3) - 5*(a^3*x^3 - 2*
I*a^3*x^2 - a^3*x)*integral(6/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*
x^4 + a^3*x^2), x))/(a^3*x^3 - 2*I*a^3*x^2 - a^3*x)
```

Sympy [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx=\int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(9/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(9/4), x)
```

Giac [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx=\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

input
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(9/4),x)
```

output
```
int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(9/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx = \frac{\sqrt{a}\left(\int \frac{(-ix+1)^{\frac{3}{4}}}{2(ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{1}{4}}x^2+(ix+1)^{\frac{1}{4}}}dx\right)}{a^2}$$

input
```
int((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x)
```

output
```
(sqrt(a)*int(( - i*x + 1)**(3/4)/(2*(i*x + 1)**(1/4)*i*x - (i*x + 1)**(1/4
)*x**2 + (i*x + 1)**(1/4)),x))/a**2
```

3.270 $\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx=\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{2\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
4/5*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4)+2/5*(x^2+1)^(1/4)*EllipticE(sin(1/
2*arctan(x)),2^(1/2))/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx=\frac{i\sqrt[4]{1+ix}(a-iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},\frac{1}{2}-\frac{ix}{2}\right)}{3\sqrt[4]{2}a^3\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((I/3)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 9/4, 7/4,
1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {56, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}dx$$

$$\Big\downarrow \text{ 56}$$

$$\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{ 46}$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{ 213}$$

$$\frac{\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((4*I)/5)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (2*(1 + x^2)^(1/4)*Ell
ipticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 56
```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Simp[d/(5*b)   Int[1/((a
 + b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
 + a*d, 0] && NegQ[a^2*b^2]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

method	result	size
risch	$\frac{\frac{2}{5}x^2-\frac{2}{5}ix+\frac{4}{5}}{(x-i)a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	105

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/5*(x^2+2-I*x)/(x-I)/a^2/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-
a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x - 2*I) + 5*(a^4*x^2 - 2*I*a
^4*x - a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 +
a^4), x))/(a^4*x^2 - 2*I*a^4*x - a^4)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{9}{4}}\sqrt[4]{-ia\,(x+i)}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(1/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(9/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-iax}(a+iax)^{9/4}}\,dx = \frac{\int \frac{1}{2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^2+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a^2}$$

input
```
int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x)
```

output
```
int(1/(2*(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x - (i*x + 1)**(1/4)*( - i
*x + 1)**(1/4)*x**2 + (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)),x)/(sqrt(a)*a*
*2)
```

3.271 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx=\frac{2i}{5a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}+\frac{6\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
2/5*I/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4)+6/5*(x^2+1)^(1/4)*EllipticE(sin(
1/2*arctan(x)),2^(1/2))/a^3/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx=-\frac{i\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{9}{4},\frac{3}{4},\frac{1}{2}-\frac{ix}{2}\right)}{\sqrt[4]{2}a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 9/4, 3/4, 1/2 - (I/2)*x])/(2
^(1/4)*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {61, 56, 46, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{3\int \frac{1}{\sqrt[4]{a-iax}(ixa+a)^{9/4}}dx}{a}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{56}$$

$$\frac{3\left(\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(ixa+a)^{5/4}}dx+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}\right)}{a}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{46}$$

$$\frac{3\left(\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\left(x^2a^2+a^2\right)^{5/4}}dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}\right)}{a}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{213}$$

$$\frac{3\left(\frac{\sqrt[4]{x^2+1}\int \frac{1}{\left(x^2+1\right)^{5/4}}dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}\right)}{a}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{3\left(\frac{2\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}(a+iax)^{5/4}}\right)}{a}-\frac{2i}{a^2\sqrt[4]{a-iax}(a+iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

output
```
(-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (3*(((4*I)/5)/(a*(a - I
*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2,
 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))))/a
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 56
```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Simp[d/(5*b)   Int[1/((a
 + b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
 + a*d, 0] && NegQ[a^2*b^2]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{\frac{6}{5}x^2-\frac{6}{5}ix+\frac{2}{5}}{(x-i)a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/5*(-3*I*x+3*x^2+1)/(x-I)/a^3/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)-3/5/(a
^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(I*x-1)*(I*x+1))^(1/
4)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 - 3*I*x + 1) + 5*(a^5*x
^3 - I*a^5*x^2 + a^5*x - I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x +
a)^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 - I*a^5*x^2 + a^5*x - I*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{9}{4}}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(5/4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(9/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}i\,x^3+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^2+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}}dx}{\sqrt{a}\,a^3}$$

input
```
int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x**3 + (i*x + 1)**(1/4)*( -
i*x + 1)**(1/4)*i*x + (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**2 + (i*x + 1
)**(1/4)*( - i*x + 1)**(1/4)),x)/(sqrt(a)*a**3)
```

3.272 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx=\frac{2x}{5a^2(a-iax)^{5/4}(a+iax)^{5/4}}+\frac{6\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^4\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
2/5*x/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4)+6/5*(x^2+1)^(1/4)*EllipticE(sin(
1/2*arctan(x)),2^(1/2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx=-\frac{i\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4},-\frac{1}{4},\frac{1}{2}-\frac{ix}{2}\right)}{5\sqrt[4]{2}a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-1/5*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, 1/2 - (I/2)*x
])/(2^(1/4)*a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {46, 215, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}dx$$

$$\Big\downarrow \text{ 46}$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{9/4}}dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\left(\frac{3\int \frac{1}{(x^2a^2+a^2)^{5/4}}dx}{5a^2}+\frac{2x}{5a^2(a^2x^2+a^2)^{5/4}}\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 213}$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\left(\frac{3\sqrt[4]{x^2+1}\int \frac{1}{(x^2+1)^{5/4}}dx}{5a^4\sqrt[4]{a^2x^2+a^2}}+\frac{2x}{5a^2(a^2x^2+a^2)^{5/4}}\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{\sqrt[4]{a^2x^2+a^2}\left(\frac{2x}{5a^2(a^2x^2+a^2)^{5/4}}+\frac{6\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^4\sqrt[4]{a^2x^2+a^2}}\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

input
```
Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((a^2 + a^2*x^2)^(1/4)*((2*x)/(5*a^2*(a^2 + a^2*x^2)^(5/4)) + (6*(1 + x^2)
^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*a^4*(a^2 + a^2*x^2)^(1/4))))/((a - I*
a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

Maple [F]

$$\int \frac{1}{(-iax+a)^{\frac{9}{4}} (iax+a)^{\frac{9}{4}}}dx$$

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

output
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
1/5*(2*(3*x^3 + 4*x)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) + 5*(a^6*x^4 + 2
*a^6*x^2 + a^6)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^
2 + a^6), x))/(a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [A] (verification not implemented)

Time = 69.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx = -\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{8},\frac{13}{8},1 & \frac{1}{2},\frac{9}{4},\frac{11}{4} \\ \frac{9}{8},\frac{13}{8},\frac{7}{4},\frac{9}{4},\frac{11}{4} & 0 \end{matrix}\middle|\,\frac{e^{-3i\pi}}{x^2}\right)e^{\frac{i\pi}{4}}}{4\pi a^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{5}{8},\frac{9}{8},1 & \\ \frac{5}{8},\frac{9}{8} & -\frac{1}{2},0,\frac{7}{4},0 \end{matrix}\middle|\,\frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(9/4),x)
```

output
```
-I*meijerg(((9/8, 13/8, 1), (1/2, 9/4, 11/4)), ((9/8, 13/8, 7/4, 9/4, 11/4
), (0,)), exp_polar(-3*I*pi)/x**2)*exp(I*pi/4)/(4*pi*a**(9/2)*gamma(9/4))
+ I*meijerg(((-1/2, 0, 1/2, 5/8, 9/8, 1), ()), ((5/8, 9/8), (-1/2, 0, 7/4,
 0)), exp_polar(-I*pi)/x**2)/(4*pi*a**(9/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(9/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(9/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx = \frac{2\sqrt{a}\,(ix+1)^{\frac{1}{4}}\,(-8i\,x^2-7i+4x)}{15\sqrt{ix+1}\,(-ix+1)^{\frac{1}{4}}\,a^5\,(x^2+1)}$$

input
```
int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

output
```
(2*sqrt(a)*(i*x + 1)**(1/4)*( - 8*i*x**2 - 7*i + 4*x))/(15*sqrt(i*x + 1)*(
 - i*x + 1)**(1/4)*a**5*(x**2 + 1))
```

3.273 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}} dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}} dx = -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{45a^3(a-iax)^{5/4}(a+iax)^{5/4}} + \frac{14\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{15a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-2/9*I/a^2/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4)+14/45*x/a^3/(a-I*a*x)^(5/4)/(a+
I*a*x)^(5/4)+14/15*(x^2+1)^(1/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^5
/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}} dx = -\frac{i\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{9}{4},-\frac{5}{4},\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}a^3(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-1/9*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 9/4, -5/4, 1/2 - (I/2)*x
])/(2^(1/4)*a^3*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {61, 46, 215, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}dx$$

$$\Big\downarrow \; 61$$

$$\frac{7\int \frac{1}{(a-iax)^{9/4}(ixa+a)^{9/4}}dx}{9a}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 46$$

$$\frac{7\sqrt[4]{a^2x^2+a^2}\int \frac{1}{(x^2a^2+a^2)^{9/4}}dx}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 215$$

$$\frac{7\sqrt[4]{a^2x^2+a^2}\left(\frac{3\int \frac{1}{\left(x^2a^2+a^2\right)^{5/4}}dx}{5a^2}+\frac{2x}{5a^2(a^2x^2+a^2)^{5/4}}\right)}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 213$$

$$\frac{7\sqrt[4]{a^2x^2+a^2}\left(\frac{3\sqrt[4]{x^2+1}\int \frac{1}{\left(x^2+1\right)^{5/4}}dx}{5a^4\sqrt[4]{a^2x^2+a^2}}+\frac{2x}{5a^2(a^2x^2+a^2)^{5/4}}\right)}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 212$$

$$\frac{7\sqrt[4]{a^2x^2+a^2}\left(\frac{2x}{5a^2(a^2x^2+a^2)^{5/4}}+\frac{6\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{5a^4\sqrt[4]{a^2x^2+a^2}}\right)}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (7*(a^2 + a^2*x^2)^
(1/4)*((2*x)/(5*a^2*(a^2 + a^2*x^2)^(5/4)) + (6*(1 + x^2)^(1/4)*EllipticE[
ArcTan[x]/2, 2])/(5*a^4*(a^2 + a^2*x^2)^(1/4))))/(9*a*(a - I*a*x)^(1/4)*(a
 + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{\frac{14}{15}x^4+\frac{14}{15}ix^3+\frac{56}{45}x^2+\frac{56}{45}ix+\frac{2}{9}}{(x-i)(x+i)^2a^5(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{7x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{15(a^2)^{\frac{1}{4}}a^5(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	124

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/45*(21*I*x^3+21*x^4+28*I*x+28*x^2+5)/(x-I)/(x+I)^2/a^5/(-a*(I*x-1))^(1/4
)/(a*(I*x+1))^(1/4)-7/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^5
*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
1/45*(2*(21*x^4 + 21*I*x^3 + 28*x^2 + 28*I*x + 5)*(I*a*x + a)^(3/4)*(-I*a*
x + a)^(3/4) + 45*(a^7*x^5 + I*a^7*x^4 + 2*a^7*x^3 + 2*I*a^7*x^2 + a^7*x +
 I*a^7)*integral(-7/15*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^7*x^2 + a^7
), x))/(a^7*x^5 + I*a^7*x^4 + 2*a^7*x^3 + 2*I*a^7*x^2 + a^7*x + I*a^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(9/4),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(a-a\,x\,1\mathrm{i})^{13/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(9/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx =$$

$$-\frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}\,i\,x^5+2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}\,i\,x^3+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}\,ix-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}\,x^4-2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}\,x^2-(ix+1)^{\frac{1}{4}}(-ix+1)}}{\sqrt{a}\,a^5}$$

input
```
int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x)
```

output
```
( - int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x**5 + 2*(i*x + 1)**(1/4
)*( - i*x + 1)**(1/4)*i*x**3 + (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x -
(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**4 - 2*(i*x + 1)**(1/4)*( - i*x + 1
)**(1/4)*x**2 - (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)),x))/(sqrt(a)*a**5)
```

3.274 $\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{65a^4(a-iax)^{5/4}(a+iax)^{5/4}}+\frac{42\sqrt[4]{1+x^2}E\left(\frac{\arctan(x)}{2}\middle|2\right)}{65a^6\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

output
```
-2/13*I/a^2/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4)-2/13*I/a^3/(a-I*a*x)^(9/4)/(a
+I*a*x)^(5/4)+14/65*x/a^4/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4)+42/65*(x^2+1)^(1
/4)*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^6/(a-I*a*x)^(1/4)/(a+I*a*x)^(1
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx=-\frac{i\sqrt[4]{1+ix}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{9}{4},-\frac{9}{4},\frac{1}{2}-\frac{ix}{2}\right)}{13\sqrt[4]{2}a^3(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-1/13*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 9/4, -9/4, 1/2 - (I/2)
*x])/(2^(1/4)*a^3*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {61, 61, 46, 215, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}dx$$

$$\Big\downarrow \; 61$$

$$\frac{9\int \frac{1}{(a-iax)^{13/4}(ixa+a)^{9/4}}dx}{13a}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 61$$

$$\frac{9\left(\frac{7\int \frac{1}{(a-iax)^{9/4}(ixa+a)^{9/4}}dx}{9a}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}\right)}{13a}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 46$$

$$\frac{9\left(\frac{7\sqrt[4]{a^2x^2+a^2}\int \frac{1}{\left(x^2a^2+a^2\right)^{9/4}}dx}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}\right)}{13a}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{9\left(\frac{7\sqrt[4]{a^2x^2+a^2}\left(\frac{3\int \frac{1}{\left(x^2a^2+a^2\right)^{5/4}}dx}{5a^2}+\frac{2x}{5a^2\left(a^2x^2+a^2\right)^{5/4}}\right)}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}\right)}{13a}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{213}$$

$$\frac{9\left(\frac{7\sqrt[4]{a^2x^2+a^2}\left(\frac{3\sqrt[4]{x^2+1}\int \frac{1}{\left(x^2+1\right)^{5/4}}dx}{5a^4\sqrt[4]{a^2x^2+a^2}}+\frac{2x}{5a^2\left(a^2x^2+a^2\right)^{5/4}}\right)}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}\right)}{13a}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{9\left(\frac{7\sqrt[4]{a^2x^2+a^2}\left(\frac{2x}{5a^2\left(a^2x^2+a^2\right)^{5/4}}+\frac{6\sqrt[4]{x^2+1}E\left(\frac{\arctan(x)}{2}|2\right)}{5a^4\sqrt[4]{a^2x^2+a^2}}\right)}{9a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}\right)}{13a}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-2*I)/13)/(a^2*(a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)) + (9*(((-2*I)/9)/(a
^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (7*(a^2 + a^2*x^2)^(1/4)*((2*x)/
(5*a^2*(a^2 + a^2*x^2)^(5/4)) + (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2,
2])/(5*a^4*(a^2 + a^2*x^2)^(1/4))))/(9*a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/
4))))/(13*a)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\frac{42}{65}x^5+\frac{84}{65}ix^4+\frac{14}{65}x^3+\frac{112}{65}ix^2-\frac{46}{65}x+\frac{4}{13}i}{(x-i)(x+i)^3a^6(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{21x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{65(a^2)^{\frac{1}{4}}a^6(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	130

input
```
int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/65*(42*I*x^4+21*x^5+56*I*x^2-23*x+7*x^3+10*I)/(x-I)/(x+I)^3/a^6/(-a*(I*x
-1))^(1/4)/(a*(I*x+1))^(1/4)-21/65/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2]
,-x^2)/a^6*(-a^2*(I*x-1)*(I*x+1))^(1/4)/(-a*(I*x-1))^(1/4)/(a*(I*x+1))^(1/
4)
```

Fricas [F]

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx=\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
1/65*(2*(21*x^5 + 42*I*x^4 + 7*x^3 + 56*I*x^2 - 23*x + 10*I)*(I*a*x + a)^(
3/4)*(-I*a*x + a)^(3/4) + 65*(a^8*x^6 + 2*I*a^8*x^5 + a^8*x^4 + 4*I*a^8*x^
3 - a^8*x^2 + 2*I*a^8*x - a^8)*integral(-21/65*(I*a*x + a)^(3/4)*(-I*a*x +
 a)^(3/4)/(a^8*x^2 + a^8), x))/(a^8*x^6 + 2*I*a^8*x^5 + a^8*x^4 + 4*I*a^8*
x^3 - a^8*x^2 + 2*I*a^8*x - a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(9/4),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx=\int \frac{1}{(a-a\,x\,1\mathrm{i})^{17/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

input
```
int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(9/4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx=$$

$$-\frac{\int \frac{1}{2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}i\,x^5+4(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}i\,x^3+2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^6+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{1}{4}}x^4-(ix+1)^{\frac{1}{4}}(-ix+1\cdots}}{\sqrt{a}\,a^6}$$

input
```
int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x)
```

output
```
( - int(1/(2*(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*x**5 + 4*(i*x + 1)**(1
/4)*( - i*x + 1)**(1/4)*i*x**3 + 2*(i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*i*
x + (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**6 + (i*x + 1)**(1/4)*( - i*x +
 1)**(1/4)*x**4 - (i*x + 1)**(1/4)*( - i*x + 1)**(1/4)*x**2 - (i*x + 1)**(
1/4)*( - i*x + 1)**(1/4)),x))/(sqrt(a)*a**6)
```

3.275 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 228

$$\begin{aligned}\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}\\&+\frac{i\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}-\frac{i\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}\\&+\frac{i\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}\left(1+\frac{\sqrt{a+iax}}{\sqrt{a-iax}}\right)}\right)}{a}\end{aligned}$$

output
```
4/5*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(5/4)-4*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(1
/4)+I*2^(1/2)*arctan(1-2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))/a-I*2^(1/2
)*arctan(1+2^(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4))/a+I*2^(1/2)*arctanh(2^
(1/2)*(a+I*a*x)^(1/4)/(a-I*a*x)^(1/4)/(1+(a+I*a*x)^(1/2)/(a-I*a*x)^(1/2)))
/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.43 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.31

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx=\frac{i\sqrt[4]{1+ix}(a-iax)^{9/4}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}a^3\sqrt[4]{a+iax}}$$

input
```
Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

output
```
((I/9)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[9/4, 9/4, 13/4,
 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {57, 57, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\int \frac{\sqrt[4]{a-iax}}{(ixa+a)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{ixa+a}}dx+\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4i\int \frac{1}{\sqrt[4]{ixa+a}}d\sqrt[4]{a-iax}}{a}+\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$$\Big\downarrow \; 770$$

$$\frac{4i\int \frac{1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{a}+\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$$\Big\downarrow \; 755$$

$$\frac{4i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{\sqrt{a-iax}+1}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}+\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$$\Big\downarrow \; 1476$$

$$\frac{4i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\int \frac{1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-i}}{\sqrt[4]{ixa-}}\right.\right.}{a}$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$$\Big\downarrow \; 1082$$

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{a-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)}{a}+$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$$\Big\downarrow \; 217$$

$$\frac{4i\left(\frac{1}{2}\int \frac{1-\sqrt{a-iax}}{-ixa+a+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)\right)}{a}+$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$$\Big\downarrow \; 1479$$

$$\frac{4i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+i}}\right.}{\sqrt{2}}\right.}{a}$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$\Big\downarrow$ 25

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1\right)}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right.}{a}$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$\Big\downarrow$ 27

$$\frac{4i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}{\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}+1}d\frac{\sqrt[4]{a-iax}}{\sqrt[4]{ixa+a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-ia}}{\sqrt[4]{a+ia}}\right.}{\sqrt{2}}\right.}{a}$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

$\Big\downarrow$ 1103

$$\frac{4i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{a-iax}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{a-iax}-\frac{\sqrt{2}\sqrt[4]{\ldots}}{\ldots}\right)}{2\sqrt{\ldots}}\right.\right.}{a}$$

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}$$

input
```
Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

output
```
(((4*I)/5)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(5/4)) - ((4*I)*(a - I*a*x)^(
1/4))/(a*(a + I*a*x)^(1/4)) + ((4*I)*((-(ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(
1/4))/(a + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))
/(a + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[a - I*a*x] - (Sqrt[2]*
(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[a - I*a*x] +
(Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.16

method	result
risch	$\frac{8(3x^2+ix+2)(-a(ix-1))^{\frac{1}{4}}}{5(x-i)a(ix-1)(a(ix+1))^{\frac{1}{4}}} - \frac{\left(\operatorname{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+i\right)\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2+i\operatorname{RootOf}\left(\_Z^2+i\right)\left(-x^4-2ix\right.}{}\right.\right.}{}$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
8/5*(3*x^2+2+I*x)/(x-I)/a*(-a*(I*x-1))^(1/4)/(I*x-1)/(a*(I*x+1))^(1/4)-(Ro
otOf(_Z^2+I)*ln((-RootOf(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x^2+I*RootOf
(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(3/4)-x^3-2*I*RootOf(_Z^2+I)*(-2*I*x^3-x^4
-2*I*x+1)^(1/4)*x+I*(-2*I*x^3-x^4-2*I*x+1)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*
(-2*I*x^3-x^4-2*I*x+1)^(1/4)-(-2*I*x^3-x^4-2*I*x+1)^(1/2)+x)/(I*x-1)^2)-I*
RootOf(_Z^2+I)*ln(-(-I*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*RootOf(_Z^2+I)*x^2+2*R
ootOf(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(1/4)*x+x^3+I*(-2*I*x^3-x^4-2*I*x+1)^
(1/2)*x+RootOf(_Z^2+I)*(-2*I*x^3-x^4-2*I*x+1)^(3/4)+I*RootOf(_Z^2+I)*(-2*I
*x^3-x^4-2*I*x+1)^(1/4)+2*I*x^2-(-2*I*x^3-x^4-2*I*x+1)^(1/2)-x)/(I*x-1)^2)
)/a*(-a*(I*x-1))^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/(a*(I*x+1))^(1/4
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(156) = 312.

Time = 0.09 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.50

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \frac{5\,(a^2x^2-2i\,a^2x-a^2)\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x-i\,a^2)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-5\,(a^2x^2-2i\,a^2x}{}$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
1/10*(5*(a^2*x^2 - 2*I*a^2*x - a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)
*sqrt(4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 5*(a^2
*x^2 - 2*I*a^2*x - a^2)*sqrt(4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(4*I/a
^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 5*(a^2*x^2 - 2*I*
a^2*x - a^2)*sqrt(-4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I
*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 5*(a^2*x^2 - 2*I*a^2*x - a^
2)*sqrt(-4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)
^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 16*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1
/4)*(3*x - 2*I))/(a^2*x^2 - 2*I*a^2*x - a^2)
```

Sympy [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(9/4), x)
```

Maxima [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(9/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming 0
=[0,0]ext_re
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \int \frac{(a-a\,x\,1\mathrm{i})^{5/4}}{(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

input
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4),x)
```

output
```
int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4), x)
```

Reduce [F]

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \frac{\int \frac{(-ix+1)^{\frac{1}{4}}}{2(ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{1}{4}}x^2+(ix+1)^{\frac{1}{4}}}dx-\left(\int \frac{(-ix+1)^{\frac{1}{4}}x}{2(ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{1}{4}}x^2+(ix+1)^{\frac{1}{4}}}dx\right)i}{a}$$

input
```
int((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x)
```

output
```
(int(( - i*x + 1)**(1/4)/(2*(i*x + 1)**(1/4)*i*x - (i*x + 1)**(1/4)*x**2 +
 (i*x + 1)**(1/4)),x) - int((( - i*x + 1)**(1/4)*x)/(2*(i*x + 1)**(1/4)*i*
x - (i*x + 1)**(1/4)*x**2 + (i*x + 1)**(1/4)),x)*i)/a
```

3.276 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx=\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

output
```
2/5*I*(a-I*a*x)^(5/4)/a^2/(a+I*a*x)^(5/4)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx=\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

input
```
Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

output
```
(((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx$$

$$\Big\downarrow \text{ 48}$$

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

input
```
Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

output
```
(((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{2i(x^2+1)(-iax+a)^{\frac{1}{4}}}{5(iax+a)^{\frac{9}{4}}}$	27
gosper	$-\frac{2i(-x+i)(x+i)(-iax+a)^{\frac{1}{4}}}{5(iax+a)^{\frac{9}{4}}}$	32
risch	$\frac{2(-a(ix-1))^{\frac{1}{4}}(x^2+2ix-1)}{5a^2(ix-1)(a(ix+1))^{\frac{1}{4}}(x-i)}$	50

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/5*I*(x^2+1)*(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = -\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(x+i)}{5\,(a^3x^2-2i\,a^3x-a^3)}$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
-2/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(x + I)/(a^3*x^2 - 2*I*a^3*x - a
^3)
```

Sympy [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(9/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = -\frac{2\,(-1+x\,1\mathrm{i})\,\left(-a\,(-1+x\,1\mathrm{i})\right)^{1/4}}{5\,a^2\,\left(x-\mathrm{i}\right)\,\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

input
```
int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(9/4),x)
```

output
```
-(2*(x*1i - 1)*(-a*(x*1i - 1))^(1/4))/(5*a^2*(x - 1i)*(a*(x*1i + 1))^(1/4)
)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a - iax}}{(a + iax)^{9/4}}\, dx = \frac{\int \frac{(-ix+1)^{\frac{1}{4}}}{2(ix+1)^{\frac{1}{4}}ix-(ix+1)^{\frac{1}{4}}x^2+(ix+1)^{\frac{1}{4}}}dx}{a^2}$$

input
```
int((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x)
```

output
```
int(( - i*x + 1)**(1/4)/(2*(i*x + 1)**(1/4)*i*x - (i*x + 1)**(1/4)*x**2 +
(i*x + 1)**(1/4)),x)/a**2
```

3.277 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}} dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}} dx = \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}} + \frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}$$

output
```
2/5*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(5/4)+4/5*I*(a-I*a*x)^(1/4)/a^3/(a+I*a
*x)^(1/4)
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}} dx = \frac{2(3+2ix)\sqrt[4]{a-iax}}{5a^3(-i+x)\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

output
```
(2*(3 + (2*I)*x)*(a - I*a*x)^(1/4))/(5*a^3*(-I + x)*(a + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{5/4}}dx}{5a}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

$$\Big\downarrow \text{48}$$

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

output
```
(((2*I)/5)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*
a*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
orering	$\frac{2(-2x+3i)(x^2+1)}{5(-iax+a)^{\frac{3}{4}}(iax+a)^{\frac{9}{4}}}$	32
gosper	$-\frac{2(-x+i)(x+i)(-2x+3i)}{5(-iax+a)^{\frac{3}{4}}(iax+a)^{\frac{9}{4}}}$	37
risch	$\frac{\frac{4}{5}x^2-\frac{2}{5}ix+\frac{6}{5}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x-i)}$	44

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/5*(-2*x+3*I)*(x^2+1)/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x-3i)}{5\,(a^4x^2-2i\,a^4x-a^4)}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
2/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x - 3*I)/(a^4*x^2 - 2*I*a^4*x
- a^4)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{9}{4}}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(3/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \frac{2\,(3+x\,2\mathrm{i})\,\left(-a\,(-1+x\,1\mathrm{i})\right)^{1/4}}{5\,a^3\,\left(x-\mathrm{i}\right)\,\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
(2*(x*2i + 3)*(-a*(x*1i - 1))^(1/4))/(5*a^3*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \frac{\int \frac{1}{2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix-(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^2+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^3}$$

input
```
int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x)
```

output
```
int(1/(2*(i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x - (i*x + 1)**(1/4)*( - i
*x + 1)**(3/4)*x**2 + (i*x + 1)**(1/4)*( - i*x + 1)**(3/4)),x)/a**3
```

3.278 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}$$

output
```
-2/3*I/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4)+8/15*I*(a-I*a*x)^(1/4)/a^3/(a+I
*a*x)^(5/4)+16/15*I*(a-I*a*x)^(1/4)/a^4/(a+I*a*x)^(1/4)
```

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = \frac{2(7-4ix+8x^2)}{15a^3(-i+x)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

input
```
Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

output
```
(2*(7 - (4*I)*x + 8*x^2))/(15*a^3*(-I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(
1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}dx$$

$$\Big\downarrow \; 55$$

$$\frac{4\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{9/4}}dx}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 55$$

$$\frac{4\left(\frac{2\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{5/4}}dx}{5a}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}\right)}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \; 48$$

$$\frac{4\left(\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}\right)}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (4*((((2*I)/5)*(a -
 I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a*x)^(1/4))/(a^
3*(a + I*a*x)^(1/4))))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.38

method	result	size
orering	$-\frac{2i\left(-8x^{2}+4ix-7\right)\left(x^{2}+1\right)}{15\left(-iax+a\right)^{\frac{7}{4}}\left(iax+a\right)^{\frac{9}{4}}}$	38
gosper	$-\frac{2(-x+i)(x+i)\left(8ix^{2}+4x+7i\right)}{15\left(-iax+a\right)^{\frac{7}{4}}\left(iax+a\right)^{\frac{9}{4}}}$	43
risch	$\frac{\frac{16}{15}x^{2}-\frac{8}{15}ix+\frac{14}{15}}{a^{3}\left(-a(ix-1)\right)^{\frac{3}{4}}\left(a(ix+1)\right)^{\frac{1}{4}}(x-i)}$	44

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-2/15*I*(4*I*x-8*x^2-7)*(x^2+1)/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = \frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(8\,x^2-4i\,x+7)}{15\,(a^5x^3-i\,a^5x^2+a^5x-i\,a^5)}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
2/15*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 - 4*I*x + 7)/(a^5*x^3 - I
*a^5*x^2 + a^5*x - I*a^5)
```

Sympy [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(ia\,(x-i))^{\frac{9}{4}}\,(-ia\,(x+i))^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

output
```
Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(7/4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = \frac{2\,\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\,\left(x^2\,8\mathrm{i}+4\,x+7\mathrm{i}\right)}{15\,a^4\,\left(x^2+1\right)\,\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
(2*(-a*(x*1i - 1))^(1/4)*(4*x + x^2*8i + 7i))/(15*a^4*(x^2 + 1)*(a*(x*1i +
 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}i\,x^3+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}ix+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^2+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^4}$$

input
```
int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*i*x**3 + (i*x + 1)**(1/4)*( -
i*x + 1)**(3/4)*i*x + (i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*x**2 + (i*x + 1
)**(1/4)*( - i*x + 1)**(3/4)),x)/a**4
```

3.279 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \frac{2i}{5a^2(a-iax)^{7/4}(a+iax)^{5/4}} + \frac{12i}{5a^3(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{48i(a+iax)^{3/4}}{35a^4(a-iax)^{7/4}} - \frac{32i(a+iax)^{3/4}}{35a^5(a-iax)^{3/4}}$$

output
```
2/5*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4)+12/5*I/a^3/(a-I*a*x)^(7/4)/(a+I*
a*x)^(1/4)-48/35*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(7/4)-32/35*I*(a+I*a*x)^(
3/4)/a^5/(a-I*a*x)^(3/4)
```

Mathematica [A] (verified)

Time = 7.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \frac{2(9i+22x+8ix^2+16x^3)}{35a^4(a-iax)^{3/4}\sqrt[4]{a+iax}\,(1+x^2)}$$

input
```
Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

output
```
(2*(9*I + 22*x + (8*I)*x^2 + 16*x^3))/(35*a^4*(a - I*a*x)^(3/4)*(a + I*a*x
)^(1/4)*(1 + x^2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}dx$$

$$\Big\downarrow \text{ 55}$$

$$\frac{6\int \frac{1}{(a-iax)^{7/4}(ixa+a)^{9/4}}dx}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{6\left(\frac{4\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{9/4}}dx}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}\right)}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{(a-iax)^{3/4}(ixa+a)^{5/4}}dx}{5a}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}\right)}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}\right)}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{6\left(\frac{4\left(\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}\right)}{3a}-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}\right)}{7a}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}$$

input
```
Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

output
```
((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)) + (6*(((-2*I)/3)/(a^2
*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (4*((((2*I)/5)*(a - I*a*x)^(1/4))/
(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a*x)^(1/4))/(a^3*(a + I*a*x)^(
1/4))))/(3*a)))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result	size
orering	$\frac{2\left(16x^{3}+8ix^{2}+22x+9i\right)\left(x^{2}+1\right)}{35\left(-iax+a\right)^{\frac{11}{4}}\left(iax+a\right)^{\frac{9}{4}}}$	43
gosper	$-\frac{2\left(-x+i\right)\left(x+i\right)\left(16x^{3}+8ix^{2}+22x+9i\right)}{35\left(-iax+a\right)^{\frac{11}{4}}\left(iax+a\right)^{\frac{9}{4}}}$	48
risch	$\frac{\frac{32}{35}x^{3}+\frac{16}{35}ix^{2}+\frac{44}{35}x+\frac{18}{35}i}{a^{4}\left(-a\left(ix-1\right)\right)^{\frac{3}{4}}\left(a\left(ix+1\right)\right)^{\frac{1}{4}}\left(x-i\right)\left(x+i\right)}$	56

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/35*(8*I*x^2+16*x^3+9*I+22*x)*(x^2+1)/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \frac{2\,(16\,x^3+8i\,x^2+22\,x+9i)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{35\,(a^6x^4+2\,a^6x^2+a^6)}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

output
```
2/35*(16*x^3 + 8*I*x^2 + 22*x + 9*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/
(a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(9/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.42

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \frac{2\,(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,(x^4\,16\mathrm{i}+8\,x^3+x^2\,30\mathrm{i}+13\,x+9\mathrm{i})}{35\,a^5\,(x^2+1)^2\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

input
```
int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(9/4)),x)
```

output
```
(2*(-a*(x*1i - 1))^(1/4)*(13*x + x^2*30i + 8*x^3 + x^4*16i + 9i))/(35*a^5*
(x^2 + 1)^2*(a*(x*1i + 1))^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = \frac{\int \frac{1}{(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^4+2(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}x^2+(ix+1)^{\frac{1}{4}}(-ix+1)^{\frac{3}{4}}}dx}{a^5}$$

input
```
int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x)
```

output
```
int(1/((i*x + 1)**(1/4)*( - i*x + 1)**(3/4)*x***4 + 2*(i*x + 1)**(1/4)*( -
i*x + 1)**(3/4)*x**2 + (i*x + 1)**(1/4)*( - i*x + 1)**(3/4)),x)/a**5
```

3.280 $\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = -\frac{\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt[4]{3}e} + \frac{\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt[4]{3}e} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{\sqrt[4]{3}e}$$

output
```
-1/3*2^(1/2)*arctan(1-2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*3^(3/4)/e+1/3*
2^(1/2)*arctan(1+2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*3^(3/4)/e+1/3*2^(1/
2)*arctanh(2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4)/(1+(e*x+2)^(1/2)/(-e*x+2)^
(1/2)))*3^(3/4)/e
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx=\frac{\sqrt{2}\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{4-e^2x^2}}{\sqrt{2-ex}-\sqrt{2+ex}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{4-e^2x^2}}{\sqrt{2-ex}+\sqrt{2+ex}}\right)\right)}{\sqrt[4]{3}e}$$

input
```
Integrate[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

output
```
(Sqrt[2]*(ArcTan[(Sqrt[2]*(4 - e^2*x^2)^(1/4))/(Sqrt[2 - e*x] - Sqrt[2 + e
*x])] + ArcTanh[(Sqrt[2]*(4 - e^2*x^2)^(1/4))/(Sqrt[2 - e*x] + Sqrt[2 + e*
x])]))/(3^(1/4)*e)
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {73, 27, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{6-3ex}(ex+2)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$-\frac{4\int \frac{3^{3/4}\sqrt{6-3ex}}{(3ex+6)^{3/4}}d\sqrt[4]{6-3ex}}{3e}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{\sqrt{6-3ex}}{(3ex+6)^{3/4}}d\sqrt[4]{6-3ex}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{854}$$

$$-\frac{4\int \frac{\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 826

$$-\frac{4\left(\frac{1}{2}\int \frac{\sqrt{6-3ex}+1}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}-\frac{1}{2}\int \frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 1476

$$-\frac{4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+\frac{1}{2}\int \frac{1}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)-\frac{1}{2}\int \frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6}}{\sqrt[4]{3}}\right.}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 1082

$$-\frac{4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{6-3ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{6-3ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 217

$$-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 1479

$$-\frac{4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3e}}{\sqrt[4]{3ex+6}}\right.}{\sqrt{2}}\right.\right.}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 25

$$-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right.}{\sqrt{2}}\right.\right.}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 27$$

$$-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3\ldots}}{\sqrt[4]{3ex+\ldots}}\right.}{\sqrt{2}}\right.\right.}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 1103$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{6-3ex}+\sqrt{\ldots}\right.}{2\ldots}\right.\right.}{\sqrt[4]{3}e}$$

input
```
Int[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

output
```
(-4*((-(ArcTan[1 - (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3*e*x)^(1/4)]/Sqrt[2])
 + ArcTan[1 + (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3*e*x)^(1/4)]/Sqrt[2])/2 +
(Log[1 + Sqrt[6 - 3*e*x] - (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3*e*x)^(1/4)]/
(2*Sqrt[2]) - Log[1 + Sqrt[6 - 3*e*x] + (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3
*e*x)^(1/4)]/(2*Sqrt[2]))/2))/(3^(1/4)*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{1}{(-3ex+6)^{\frac{1}{4}}(ex+2)^{\frac{3}{4}}}dx$$

input
```
int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

output
```
int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = -\frac{2\cdot 12^{\frac{3}{4}}\arctan\left(\frac{3\,ex+12^{\frac{1}{4}}\,(ex+2)^{\frac{1}{4}}\,(-3\,ex+6)^{\frac{3}{4}}-6}{3\,(ex-2)}\right)+2\cdot 12^{\frac{3}{4}}\arctan\left(-\frac{3\,ex-12^{\frac{1}{4}}\,(ex+2)^{\frac{1}{4}}\,(-3\,ex+6)^{\frac{3}{4}}-6}{3\,(ex-2)}\right)+12^{\frac{3}{4}}\log\left(\right.}{1}$$

input
```
integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="fricas")
```

output
```
-1/12*(2*12^(3/4)*arctan(1/3*(3*e*x + 12^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6
)^(3/4) - 6)/(e*x - 2)) + 2*12^(3/4)*arctan(-1/3*(3*e*x - 12^(1/4)*(e*x +
2)^(1/4)*(-3*e*x + 6)^(3/4) - 6)/(e*x - 2)) + 12^(3/4)*log((12^(3/4)*(e*x
+ 2)^(1/4)*(-3*e*x + 6)^(3/4) + 6*sqrt(3)*(e*x - 2) - 6*sqrt(e*x + 2)*sqrt
(-3*e*x + 6))/(e*x - 2)) - 12^(3/4)*log(-(12^(3/4)*(e*x + 2)^(1/4)*(-3*e*x
 + 6)^(3/4) - 6*sqrt(3)*(e*x - 2) + 6*sqrt(e*x + 2)*sqrt(-3*e*x + 6))/(e*x
 - 2)))/e
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = \frac{3^{\frac{3}{4}}\int \frac{1}{\sqrt[4]{-ex+2}(ex+2)^{\frac{3}{4}}}\,dx}{3}$$

input
```
integrate(1/(-3*e*x+6)**(1/4)/(e*x+2)**(3/4),x)
```

output
```
3**(3/4)*Integral(1/((-e*x + 2)**(1/4)*(e*x + 2)**(3/4)), x)/3
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = \int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = \int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = \int \frac{1}{(e\,x+2)^{3/4}\,(6-3\,e\,x)^{1/4}}\,dx$$

input
```
int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)),x)
```

output
```
int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{6-3ex}(2+ex)^{3/4}}\,dx = \frac{\left(\int \frac{1}{(ex+2)^{\frac{3}{4}}(-ex+2)^{\frac{1}{4}}}dx\right)3^{\frac{3}{4}}}{3}$$

input
```
int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

output
```
int(1/((e*x + 2)**(3/4)*( - e*x + 2)**(1/4)),x)/3**(1/4)
```

3.281 $\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx=\frac{5\sqrt[5]{a-bx}(a+bx)^{16/5}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{16}{5},\frac{21}{5},\frac{a+bx}{2a}\right)}{8\;2^{4/5}b\sqrt[5]{1-\frac{bx}{a}}}$$

output
```
5/16*(-b*x+a)^(1/5)*(b*x+a)^(16/5)*hypergeom([-1/5, 16/5],[21/5],1/2*(b*x+
a)/a)*2^(1/5)/b/(1-b*x/a)^(1/5)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx = -\frac{10\sqrt[5]{2}a^2(a-bx)^{6/5}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{11}{5},\frac{6}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{3b\sqrt[5]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(1/5)*(a + b*x)^(11/5),x]
```

output
```
(-10*2^(1/5)*a^2*(a - b*x)^(6/5)*(a + b*x)^(1/5)*Hypergeometric2F1[-11/5,
6/5, 11/5, (a - b*x)/(2*a)])/(3*b*((a + b*x)/a)^(1/5))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx$$

$$\Big\downarrow \; 80$$

$$\frac{4\sqrt[5]{2}a^2\sqrt[5]{a+bx}\int \frac{\sqrt[5]{a-bx}\left(\frac{bx}{a}+1\right)^{11/5}}{4\sqrt[5]{2}}dx}{\sqrt[5]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \; 27$$

$$\frac{a^2\sqrt[5]{a+bx}\int \sqrt[5]{a-bx}\left(\frac{bx}{a}+1\right)^{11/5}dx}{\sqrt[5]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \; 79$$

$$-\frac{10\sqrt[5]{2}a^2(a-bx)^{6/5}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{11}{5},\frac{6}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{3b\sqrt[5]{\frac{a+bx}{a}}}$$

input
```
Int[(a - b*x)^(1/5)*(a + b*x)^(11/5),x]
```

output

```
(-10*2^(1/5)*a^2*(a - b*x)^(6/5)*(a + b*x)^(1/5)*Hypergeometric2F1[-11/5,
6/5, 11/5, (a - b*x)/(2*a)])/(3*b*((a + b*x)/a)^(1/5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (-bx+a)^{\frac{1}{5}} (bx+a)^{\frac{11}{5}}\, dx$$

input
```
int((-b*x+a)^(1/5)*(b*x+a)^(11/5),x)
```

output
```
int((-b*x+a)^(1/5)*(b*x+a)^(11/5),x)
```

Fricas [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\, dx = \int (bx+a)^{\frac{11}{5}} (-bx+a)^{\frac{1}{5}}\, dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(11/5),x, algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/5)*(-b*x + a)^(1/5), x)
```

Sympy [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx = \int \sqrt[5]{a-bx}(a+bx)^{\frac{11}{5}}\,dx$$

input
```
integrate((-b*x+a)**(1/5)*(b*x+a)**(11/5),x)
```

output
```
Integral((a - b*x)**(1/5)*(a + b*x)**(11/5), x)
```

Maxima [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx = \int (bx+a)^{\frac{11}{5}}(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(11/5),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(11/5)*(-b*x + a)^(1/5), x)
```

Giac [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx = \int (bx+a)^{\frac{11}{5}}(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(11/5),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(11/5)*(-b*x + a)^(1/5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx=\int (a+b\,x)^{11/5}\,(a-b\,x)^{1/5}\,dx$$

input
```
int((a + b*x)^(11/5)*(a - b*x)^(1/5),x)
```

output
```
int((a + b*x)^(11/5)*(a - b*x)^(1/5), x)
```

Reduce [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{11/5}\,dx=\left(\int (bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}}\,x^2dx\right)b^2 +2\left(\int (bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}}\,xdx\right)ab+\left(\int (bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}}\,dx\right)a^2$$

input
```
int((-b*x+a)^(1/5)*(b*x+a)^(11/5),x)
```

output
```
int((a + b*x)**(1/5)*(a - b*x)**(1/5)*x**2,x)*b**2 + 2*int((a + b*x)**(1/5
)*(a - b*x)**(1/5)*x,x)*a*b + int((a + b*x)**(1/5)*(a - b*x)**(1/5),x)*a**
2
```

3.282 $\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx=\frac{5\sqrt[5]{2}\sqrt[5]{a-bx}(a+bx)^{11/5}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{11}{5},\frac{16}{5},\frac{a+bx}{2a}\right)}{11b\sqrt[5]{1-\frac{bx}{a}}}$$

output
```
5/11*2^(1/5)*(-b*x+a)^(1/5)*(b*x+a)^(11/5)*hypergeom([-1/5, 11/5],[16/5],1
/2*(b*x+a)/a)/b/(1-b*x/a)^(1/5)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx = -\frac{5\sqrt[5]{2}a(a-bx)^{6/5}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{6}{5},\frac{6}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{3b\sqrt[5]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(1/5)*(a + b*x)^(6/5),x]
```

output
```
(-5*2^(1/5)*a*(a - b*x)^(6/5)*(a + b*x)^(1/5)*Hypergeometric2F1[-6/5, 6/5,
 11/5, (a - b*x)/(2*a)])/(3*b*((a + b*x)/a)^(1/5))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx$$

$$\Big\downarrow \ 80$$

$$\frac{2\sqrt[5]{2}a\sqrt[5]{a+bx}\int \frac{\sqrt[5]{a-bx}\left(\frac{bx}{a}+1\right)^{6/5}}{2\sqrt[5]{2}}dx}{\sqrt[5]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \ 27$$

$$\frac{a\sqrt[5]{a+bx}\int \sqrt[5]{a-bx}\left(\frac{bx}{a}+1\right)^{6/5}dx}{\sqrt[5]{\frac{a+bx}{a}}}$$

$$\Big\downarrow \ 79$$

$$-\frac{5\sqrt[5]{2}a(a-bx)^{6/5}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{6}{5},\frac{6}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{3b\sqrt[5]{\frac{a+bx}{a}}}$$

input
```
Int[(a - b*x)^(1/5)*(a + b*x)^(6/5),x]
```

output
```
(-5*2^(1/5)*a*(a - b*x)^(6/5)*(a + b*x)^(1/5)*Hypergeometric2F1[-6/5, 6/5,
 11/5, (a - b*x)/(2*a)])/(3*b*((a + b*x)/a)^(1/5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (-bx+a)^{\frac{1}{5}} (bx+a)^{\frac{6}{5}} dx$$

input
```
int((-b*x+a)^(1/5)*(b*x+a)^(6/5),x)
```

output
```
int((-b*x+a)^(1/5)*(b*x+a)^(6/5),x)
```

Fricas [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5} dx = \int (bx+a)^{\frac{6}{5}}(-bx+a)^{\frac{1}{5}} dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(6/5),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(6/5)*(-b*x + a)^(1/5), x)
```

Sympy [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx=\int \sqrt[5]{a-bx}(a+bx)^{\frac{6}{5}}\,dx$$

input
```
integrate((-b*x+a)**(1/5)*(b*x+a)**(6/5),x)
```

output
```
Integral((a - b*x)**(1/5)*(a + b*x)**(6/5), x)
```

Maxima [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx=\int (bx+a)^{\frac{6}{5}}(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(6/5),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(6/5)*(-b*x + a)^(1/5), x)
```

Giac [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx=\int (bx+a)^{\frac{6}{5}}(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(6/5),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(6/5)*(-b*x + a)^(1/5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx = \int (a+b\,x)^{6/5}\,(a-b\,x)^{1/5}\,dx$$

input
```
int((a + b*x)^(6/5)*(a - b*x)^(1/5),x)
```

output
```
int((a + b*x)^(6/5)*(a - b*x)^(1/5), x)
```

Reduce [F]

$$\int \sqrt[5]{a-bx}(a+bx)^{6/5}\,dx = \left(\int (bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}}\,xdx\right)b + \left(\int (bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}}\,dx\right)a$$

input
```
int((-b*x+a)^(1/5)*(b*x+a)^(6/5),x)
```

output
```
int((a + b*x)**(1/5)*(a - b*x)**(1/5)*x,x)*b + int((a + b*x)**(1/5)*(a - b
*x)**(1/5),x)*a
```

3.283 $\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx=-\frac{5(a-bx)^{6/5}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{6}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{3\ 2^{4/5}b\sqrt[5]{1+\frac{bx}{a}}}$$

output
```
-5/6*(-b*x+a)^(6/5)*(b*x+a)^(1/5)*hypergeom([-1/5, 6/5],[11/5],1/2*(-b*x+a
)/a)*2^(1/5)/b/(1+b*x/a)^(1/5)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx=-\frac{5(a-bx)^{6/5}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{6}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{3\ 2^{4/5}b\sqrt[5]{\frac{a+bx}{a}}}$$

input
```
Integrate[(a - b*x)^(1/5)*(a + b*x)^(1/5),x]
```

output
```
(-5*(a - b*x)^(6/5)*(a + b*x)^(1/5)*Hypergeometric2F1[-1/5, 6/5, 11/5, (a
- b*x)/(2*a)])/(3*2^(4/5)*b*((a + b*x)/a)^(1/5))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {46, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx$$

$$\Big\downarrow \text{46}$$

$$\frac{\sqrt[5]{a-bx}\sqrt[5]{a+bx}\int \sqrt[5]{a^2-b^2x^2}dx}{\sqrt[5]{a^2-b^2x^2}}$$

$$\Big\downarrow \text{238}$$

$$\frac{\sqrt[5]{a-bx}\sqrt[5]{a+bx}\int \sqrt[5]{1-\frac{b^2x^2}{a^2}}dx}{\sqrt[5]{1-\frac{b^2x^2}{a^2}}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\sqrt[5]{a-bx}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{1}{2},\frac{3}{2},\frac{b^2x^2}{a^2}\right)}{\sqrt[5]{1-\frac{b^2x^2}{a^2}}}$$

input
```
Int[(a - b*x)^(1/5)*(a + b*x)^(1/5),x]
```

output
```
(x*(a - b*x)^(1/5)*(a + b*x)^(1/5)*Hypergeometric2F1[-1/5, 1/2, 3/2, (b^2*
x^2)/a^2])/(1 - (b^2*x^2)/a^2)^(1/5)
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (-bx+a)^{\frac{1}{5}} (bx+a)^{\frac{1}{5}} dx$$

input
```
int((-b*x+a)^(1/5)*(b*x+a)^(1/5),x)
```

output
```
int((-b*x+a)^(1/5)*(b*x+a)^(1/5),x)
```

Fricas [F]

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx = \int (bx+a)^{\frac{1}{5}}(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(1/5),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/5)*(-b*x + a)^(1/5), x)
```

Sympy [F]

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx = \int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx$$

input
```
integrate((-b*x+a)**(1/5)*(b*x+a)**(1/5),x)
```

output
```
Integral((a - b*x)**(1/5)*(a + b*x)**(1/5), x)
```

Maxima [F]

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx = \int (bx+a)^{\frac{1}{5}}(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(1/5),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/5)*(-b*x + a)^(1/5), x)
```

Giac [F]

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx = \int (bx+a)^{\frac{1}{5}}(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)*(b*x+a)^(1/5),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(1/5)*(-b*x + a)^(1/5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx=\int (a+b\,x)^{1/5}\,(a-b\,x)^{1/5}\,dx$$

input
```
int((a + b*x)^(1/5)*(a - b*x)^(1/5),x)
```

output
```
int((a + b*x)^(1/5)*(a - b*x)^(1/5), x)
```

Reduce [F]

$$\int \sqrt[5]{a-bx}\sqrt[5]{a+bx}\,dx=\int (bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}}\,dx$$

input
```
int((-b*x+a)^(1/5)*(b*x+a)^(1/5),x)
```

output
```
int((a + b*x)**(1/5)*(a - b*x)**(1/5),x)
```

3.284 $\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx = \frac{5\sqrt[5]{2}\sqrt[5]{a-bx}\sqrt[5]{a+bx}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{1}{5},\frac{6}{5},\frac{a+bx}{2a}\right)}{b\sqrt[5]{1-\frac{bx}{a}}}$$

output
```
5*2^(1/5)*(-b*x+a)^(1/5)*(b*x+a)^(1/5)*hypergeom([-1/5, 1/5],[6/5],1/2*(b*
x+a)/a)/b/(1-b*x/a)^(1/5)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx = -\frac{5(a-bx)^{6/5}\left(\frac{a+bx}{a}\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{4}{5},\frac{6}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{6\ 2^{4/5}b(a+bx)^{4/5}}$$

input
```
Integrate[(a - b*x)^(1/5)/(a + b*x)^(4/5),x]
```

output
```
(-5*(a - b*x)^(6/5)*((a + b*x)/a)^(4/5)*Hypergeometric2F1[4/5, 6/5, 11/5,
(a - b*x)/(2*a)])/(6*2^(4/5)*b*(a + b*x)^(4/5))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}} \, dx$$

$$\Big\downarrow \ 80$$

$$\frac{\left(\frac{a+bx}{a}\right)^{4/5} \int \frac{2^{4/5} \sqrt[5]{a-bx}}{\left(\frac{bx}{a}+1\right)^{4/5}} dx}{2^{4/5}(a+bx)^{4/5}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(\frac{a+bx}{a}\right)^{4/5} \int \frac{\sqrt[5]{a-bx}}{\left(\frac{bx}{a}+1\right)^{4/5}} dx}{(a+bx)^{4/5}}$$

$$\Big\downarrow \ 79$$

$$-\frac{5(a-bx)^{6/5} \left(\frac{a+bx}{a}\right)^{4/5} \text{Hypergeometric2F1}\left(\frac{4}{5}, \frac{6}{5}, \frac{11}{5}, \frac{a-bx}{2a}\right)}{6 \ 2^{4/5} b (a+bx)^{4/5}}$$

input
```
Int[(a - b*x)^(1/5)/(a + b*x)^(4/5),x]
```

output
```
(-5*(a - b*x)^(6/5)*((a + b*x)/a)^(4/5)*Hypergeometric2F1[4/5, 6/5, 11/5,
(a - b*x)/(2*a)])/(6*2^(4/5)*b*(a + b*x)^(4/5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{4}{5}}}dx$$

input
```
int((-b*x+a)^(1/5)/(b*x+a)^(4/5),x)
```

output
```
int((-b*x+a)^(1/5)/(b*x+a)^(4/5),x)
```

Fricas [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{4}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(4/5),x, algorithm="fricas")
```

output
```
integral((-b*x + a)^(1/5)/(b*x + a)^(4/5), x)
```

Sympy [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx=\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{\frac{4}{5}}}\,dx$$

input
```
integrate((-b*x+a)**(1/5)/(b*x+a)**(4/5),x)
```

output
```
Integral((a - b*x)**(1/5)/(a + b*x)**(4/5), x)
```

Maxima [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx=\int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{4}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(4/5),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/5)/(b*x + a)^(4/5), x)
```

Giac [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx=\int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{4}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(4/5),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/5)/(b*x + a)^(4/5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx = \int \frac{(a-b\,x)^{1/5}}{(a+b\,x)^{4/5}}\,dx$$

input
```
int((a - b*x)^(1/5)/(a + b*x)^(4/5),x)
```

output
```
int((a - b*x)^(1/5)/(a + b*x)^(4/5), x)
```

Reduce [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{4/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{4}{5}}}dx$$

input
```
int((-b*x+a)^(1/5)/(b*x+a)^(4/5),x)
```

output
```
int((a - b*x)**(1/5)/(a + b*x)**(4/5),x)
```

3.285 $\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx=-\frac{5\sqrt[5]{a-bx}\,\text{Hypergeometric2F1}\left(-\frac{4}{5},-\frac{1}{5},\frac{1}{5},\frac{a+bx}{2a}\right)}{2\;2^{4/5}b(a+bx)^{4/5}\sqrt[5]{1-\frac{bx}{a}}}$$

output
```
-5/4*(-b*x+a)^(1/5)*hypergeom([-4/5, -1/5],[1/5],1/2*(b*x+a)/a)*2^(1/5)/b/
(b*x+a)^(4/5)/(1-b*x/a)^(1/5)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx=-\frac{5(a-bx)^{6/5}\left(\frac{a+bx}{a}\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{6}{5},\frac{9}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{12\;2^{4/5}ab(a+bx)^{4/5}}$$

input
```
Integrate[(a - b*x)^(1/5)/(a + b*x)^(9/5),x]
```

output
```
(-5*(a - b*x)^(6/5)*((a + b*x)/a)^(4/5)*Hypergeometric2F1[6/5, 9/5, 11/5,
(a - b*x)/(2*a)])/(12*2^(4/5)*a*b*(a + b*x)^(4/5))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx$$

$$\Big\downarrow \text{80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{4/5}\int \frac{2\ 2^{4/5}\sqrt[5]{a-bx}}{\left(\frac{bx}{a}+1\right)^{9/5}}dx}{2\ 2^{4/5}a(a+bx)^{4/5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{4/5}\int \frac{\sqrt[5]{a-bx}}{\left(\frac{bx}{a}+1\right)^{9/5}}dx}{a(a+bx)^{4/5}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{5(a-bx)^{6/5}\left(\frac{a+bx}{a}\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{6}{5},\frac{9}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{12\ 2^{4/5}ab(a+bx)^{4/5}}$$

input
```
Int[(a - b*x)^(1/5)/(a + b*x)^(9/5),x]
```

output
```
(-5*(a - b*x)^(6/5)*((a + b*x)/a)^(4/5)*Hypergeometric2F1[6/5, 9/5, 11/5,
(a - b*x)/(2*a)])/(12*2^(4/5)*a*b*(a + b*x)^(4/5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{9}{5}}}dx$$

input
```
int((-b*x+a)^(1/5)/(b*x+a)^(9/5),x)
```

output
```
int((-b*x+a)^(1/5)/(b*x+a)^(9/5),x)
```

Fricas [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{9}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(9/5),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/5)*(-b*x + a)^(1/5)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx = \int \frac{\sqrt[5]{a-bx}}{(a+bx)^{\frac{9}{5}}}\,dx$$

input
```
integrate((-b*x+a)**(1/5)/(b*x+a)**(9/5),x)
```

output
```
Integral((a - b*x)**(1/5)/(a + b*x)**(9/5), x)
```

Maxima [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{9}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(9/5),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/5)/(b*x + a)^(9/5), x)
```

Giac [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{9}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(9/5),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/5)/(b*x + a)^(9/5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx=\int \frac{(a-b\,x)^{1/5}}{(a+b\,x)^{9/5}}\,dx$$

input
```
int((a - b*x)^(1/5)/(a + b*x)^(9/5),x)
```

output
```
int((a - b*x)^(1/5)/(a + b*x)^(9/5), x)
```

Reduce [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{9/5}}\,dx=\frac{-5(bx+a)^{\frac{1}{5}}(-bx+a)^{\frac{1}{5}}-2\left(\int \frac{(bx+a)^{\frac{1}{5}}(-bx+a)^{\frac{1}{5}}}{-b^3x^3-a\,b^2x^2+a^2bx+a^3}dx\right)a^2b-2\left(\int \frac{(bx+a)^{\frac{1}{5}}(-bx+a)^{\frac{1}{5}}}{-b^3x^3-a\,b^2x^2+a^2bx+a^3}\right.}{3b\,(bx+a)}$$

input
```
int((-b*x+a)^(1/5)/(b*x+a)^(9/5),x)
```

output
```
( - 5*(a + b*x)**(1/5)*(a - b*x)**(1/5) - 2*int(((a + b*x)**(1/5)*(a - b*x
)**(1/5))/(a**3 + a**2*b*x - a*b**2*x**2 - b**3*x**3),x)*a**2*b - 2*int(((
a + b*x)**(1/5)*(a - b*x)**(1/5))/(a**3 + a**2*b*x - a*b**2*x**2 - b**3*x*
*3),x)*a*b**2*x)/(3*b*(a + b*x))
```

3.286 $\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx=-\frac{5\sqrt[5]{2}\sqrt[5]{a-bx}\,\text{Hypergeometric2F1}\left(-\frac{9}{5},-\frac{1}{5},-\frac{4}{5},\frac{a+bx}{2a}\right)}{9b(a+bx)^{9/5}\sqrt[5]{1-\frac{bx}{a}}}$$

output
```
-5/9*2^(1/5)*(-b*x+a)^(1/5)*hypergeom([-9/5, -1/5],[-4/5],1/2*(b*x+a)/a)/b
/(b*x+a)^(9/5)/(1-b*x/a)^(1/5)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx=-\frac{5(a-bx)^{6/5}\left(\frac{a+bx}{a}\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{6}{5},\frac{14}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{24\ 2^{4/5}a^2b(a+bx)^{4/5}}$$

input
```
Integrate[(a - b*x)^(1/5)/(a + b*x)^(14/5),x]
```

output
```
(-5*(a - b*x)^(6/5)*((a + b*x)/a)^(4/5)*Hypergeometric2F1[6/5, 14/5, 11/5,
 (a - b*x)/(2*a)])/(24*2^(4/5)*a^2*b*(a + b*x)^(4/5))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}dx$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{4/5}\int \frac{4\ 2^{4/5}\sqrt[5]{a-bx}}{\left(\frac{bx}{a}+1\right)^{14/5}}dx}{4\ 2^{4/5}a^2(a+bx)^{4/5}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(\frac{a+bx}{a}\right)^{4/5}\int \frac{\sqrt[5]{a-bx}}{\left(\frac{bx}{a}+1\right)^{14/5}}dx}{a^2(a+bx)^{4/5}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{5(a-bx)^{6/5}\left(\frac{a+bx}{a}\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{6}{5},\frac{14}{5},\frac{11}{5},\frac{a-bx}{2a}\right)}{24\ 2^{4/5}a^2b(a+bx)^{4/5}}$$

input
```
Int[(a - b*x)^(1/5)/(a + b*x)^(14/5),x]
```

output
```
(-5*(a - b*x)^(6/5)*((a + b*x)/a)^(4/5)*Hypergeometric2F1[6/5, 14/5, 11/5,
 (a - b*x)/(2*a)])/(24*2^(4/5)*a^2*b*(a + b*x)^(4/5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{14}{5}}}dx$$

input
```
int((-b*x+a)^(1/5)/(b*x+a)^(14/5),x)
```

output
```
int((-b*x+a)^(1/5)/(b*x+a)^(14/5),x)
```

Fricas [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{14}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(14/5),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/5)*(-b*x + a)^(1/5)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3), x)
```

Sympy [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx = \int \frac{\sqrt[5]{a-bx}}{(a+bx)^{\frac{14}{5}}}\,dx$$

input
```
integrate((-b*x+a)**(1/5)/(b*x+a)**(14/5),x)
```

output
```
Integral((a - b*x)**(1/5)/(a + b*x)**(14/5), x)
```

Maxima [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{14}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(14/5),x, algorithm="maxima")
```

output
```
integrate((-b*x + a)^(1/5)/(b*x + a)^(14/5), x)
```

Giac [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx = \int \frac{(-bx+a)^{\frac{1}{5}}}{(bx+a)^{\frac{14}{5}}}\,dx$$

input
```
integrate((-b*x+a)^(1/5)/(b*x+a)^(14/5),x, algorithm="giac")
```

output
```
integrate((-b*x + a)^(1/5)/(b*x + a)^(14/5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx = \int \frac{(a-b\,x)^{1/5}}{(a+b\,x)^{14/5}}\,dx$$

input
```
int((a - b*x)^(1/5)/(a + b*x)^(14/5),x)
```

output
```
int((a - b*x)^(1/5)/(a + b*x)^(14/5), x)
```

Reduce [F]

$$\int \frac{\sqrt[5]{a-bx}}{(a+bx)^{14/5}}\,dx = \frac{-5(bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}} - 2\left(\int \frac{(bx+a)^{\frac{1}{5}}\,(-bx+a)^{\frac{1}{5}}}{-b^4x^4-2a\,b^3x^3+2a^3bx+a^4}\,dx\right)a^3b - 4\left(\int \frac{(bx+a)^{\frac{1}{5}}\,(-bx+a)}{-b^4x^4-2a\,b^3x^3+2a^3b}\right.}{8b\,(b^2x^2+2abx+a^2)}$$

input
```
int((-b*x+a)^(1/5)/(b*x+a)^(14/5),x)
```

output
```
( - 5*(a + b*x)**(1/5)*(a - b*x)**(1/5) - 2*int(((a + b*x)**(1/5)*(a - b*x
)**(1/5))/(a**4 + 2*a**3*b*x - 2*a*b**3*x**3 - b**4*x**4),x)*a**3*b - 4*in
t(((a + b*x)**(1/5)*(a - b*x)**(1/5))/(a**4 + 2*a**3*b*x - 2*a*b**3*x**3 -
 b**4*x**4),x)*a**2*b**2*x - 2*int(((a + b*x)**(1/5)*(a - b*x)**(1/5))/(a*
*4 + 2*a**3*b*x - 2*a*b**3*x**3 - b**4*x**4),x)*a*b**3*x**2)/(8*b*(a**2 +
2*a*b*x + b**2*x**2))
```

3.287 $\int (a+bx)^2(ac-bcx)^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int (a+bx)^2(ac-bcx)^n\,dx = -\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}$$

output
```
-4*a^2*(-b*c*x+a*c)^(1+n)/b/c/(1+n)+4*a*(-b*c*x+a*c)^(2+n)/b/c^2/(2+n)-(-b
*c*x+a*c)^(3+n)/b/c^3/(3+n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int (a+bx)^2(ac-bcx)^n\,dx$$
$$=\frac{(c(a-bx))^n(-a+bx)\left(a^2(14+7n+n^2)+2ab(4+5n+n^2)\,x+b^2(2+3n+n^2)\,x^2\right)}{b(1+n)(2+n)(3+n)}$$

input
```
Integrate[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

output
```
((c*(a - b*x))^n*(-a + b*x)*(a^2*(14 + 7*n + n^2) + 2*a*b*(4 + 5*n + n^2)*
x + b^2*(2 + 3*n + n^2)*x^2))/(b*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(ac-bcx)^n\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(4a^2(ac-bcx)^n+\frac{(ac-bcx)^{n+2}}{c^2}-\frac{4a(ac-bcx)^{n+1}}{c}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}$$

input
```
Int[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

output
```
(-4*a^2*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (4*a*(a*c - b*c*x)^(2 + n))
/(b*c^2*(2 + n)) - (a*c - b*c*x)^(3 + n)/(b*c^3*(3 + n))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

method	result
gosper	$-\frac{(-bx+a)(b^2n^2x^2+2ab\,n^2x+3b^2n\,x^2+a^2n^2+10abnx+2b^2x^2+7a^2n+8abx+14a^2)(-bcx+ac)^n}{b(n^3+6n^2+11n+6)}$
orering	$-\frac{(-bx+a)(b^2n^2x^2+2ab\,n^2x+3b^2n\,x^2+a^2n^2+10abnx+2b^2x^2+7a^2n+8abx+14a^2)(-bcx+ac)^n}{b(n^3+6n^2+11n+6)}$
risch	$-\frac{(-b^3n^2x^3-a\,b^2n^2x^2-3n\,b^3x^3+a^2b\,n^2x-7a\,b^2n\,x^2-2b^3x^3+a^3n^2+3a^2bnx-6a\,b^2x^2+7a^3n-6a^2bx+14a^3)(c(-bx+a))^n}{(2+n)(3+n)b(1+n)}$
norman	$\frac{b^2x^3\mathrm{e}^{n\ln(-bcx+ac)}}{3+n}+\frac{ab(n+6)x^2\mathrm{e}^{n\ln(-bcx+ac)}}{n^2+5n+6}-\frac{a^2(n^2+3n-6)x\,\mathrm{e}^{n\ln(-bcx+ac)}}{n^3+6n^2+11n+6}-\frac{a^3(n^2+7n+14)\mathrm{e}^{n\ln(-bcx+ac)}}{b(n^3+6n^2+11n+6)}$
parallelrisch	$\frac{x^3(c(-bx+a))^nb^3n^2+3x^3(c(-bx+a))^nb^3n+x^2(c(-bx+a))^na\,b^2n^2+2x^3(c(-bx+a))^nb^3+7x^2(c(-bx+a))^na\,b^2n-x(c(-bx+}{b(n}$

input
```
int((b*x+a)^2*(-b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

output
```
-(-b*x+a)*(b^2*n^2*x^2+2*a*b*n^2*x+3*b^2*n*x^2+a^2*n^2+10*a*b*n*x+2*b^2*x^
2+7*a^2*n+8*a*b*x+14*a^2)*(-b*c*x+a*c)^n/b/(n^3+6*n^2+11*n+6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.54

$$\int (a+bx)^2(ac-bcx)^n\,dx = -\frac{(a^3n^2+7\,a^3n-(b^3n^2+3\,b^3n+2\,b^3)x^3+14\,a^3-(ab^2n^2+7\,ab^2n+6\,ab^2)x^2+(a^2bn^2+3\,a^2bn-6\,a^2}{bn^3+6\,bn^2+11\,bn+6\,b}$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

output
```
-(a^3*n^2 + 7*a^3*n - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + 14*a^3 - (a*b^2*n^
2 + 7*a*b^2*n + 6*a*b^2)*x^2 + (a^2*b*n^2 + 3*a^2*b*n - 6*a^2*b)*x)*(-b*c*
x + a*c)^n/(b*n^3 + 6*b*n^2 + 11*b*n + 6*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(66) = 132$.

Time = 0.41 (sec) , antiderivative size = 819, normalized size of antiderivative = 9.87

$$\int (a+bx)^2(ac-bcx)^n\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2*(-b*c*x+a*c)**n,x)
```

output
```
Piecewise((a**2*x*(a*c)**n, Eq(b, 0)), (-a**2*log(-a/b + x)/(a**2*b*c**3 -
 2*a*b**2*c**3*x + b**3*c**3*x**2) - 2*a**2/(a**2*b*c**3 - 2*a*b**2*c**3*x
 + b**3*c**3*x**2) + 2*a*b*x*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x
+ b**3*c**3*x**2) + 4*a*b*x/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**
2) - b**2*x**2*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x*
*2), Eq(n, -3)), (-4*a**2*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) - 5*a**2
/(-a*b*c**2 + b**2*c**2*x) + 4*a*b*x*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*
x) + b**2*x**2/(-a*b*c**2 + b**2*c**2*x), Eq(n, -2)), (-4*a**2*log(-a/b +
x)/(b*c) - 3*a*x/c - b*x**2/(2*c), Eq(n, -1)), (-a**3*n**2*(a*c - b*c*x)**
n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) - 7*a**3*n*(a*c - b*c*x)**n/(b*n**3 +
 6*b*n**2 + 11*b*n + 6*b) - 14*a**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 +
11*b*n + 6*b) - a**2*b*n**2*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n
 + 6*b) - 3*a**2*b*n*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b)
 + 6*a**2*b*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + a*b**2
*n**2*x**2*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 7*a*b**2*
n*x**2*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a*b**2*x**2
*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + b**3*n**2*x**3*(a*c
 - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 3*b**3*n*x**3*(a*c - b*c
*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 2*b**3*x**3*(a*c - b*c*x)**n/(
b*n**3 + 6*b*n**2 + 11*b*n + 6*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(83) = 166.

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.01

$$\int (a+bx)^2(ac-bcx)^n\,dx = \frac{2\,(b^2c^n(n+1)x^2-abc^nnx-a^2c^n)(-bx+a)^na}{(n^2+3\,n+2)b} + \frac{((n^2+3\,n+2)b^3c^nx^3-(n^2+n)ab^2c^nx^2-2\,a^2bc^nnx-2\,a^3c^n)(-bx+a)^n}{(n^3+6\,n^2+11\,n+6)b} - \frac{(-bcx+ac)^{n+1}a^2}{bc(n+1)}$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

output
```
2*(b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n*a/((n^2 + 3*n
 + 2)*b) + ((n^2 + 3*n + 2)*b^3*c^n*x^3 - (n^2 + n)*a*b^2*c^n*x^2 - 2*a^2*
b*c^n*n*x - 2*a^3*c^n)*(-b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b) - (-b*c*x
 + a*c)^(n + 1)*a^2/(b*c*(n + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(83) = 166.

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.08

$$\int (a+bx)^2(ac-bcx)^n\,dx = \frac{(-bcx+ac)^nb^3n^2x^3+(-bcx+ac)^nab^2n^2x^2+3\,(-bcx+ac)^nb^3nx^3-(-bcx+ac)^na^2bn^2x+7\,(-bcx+}{}$$

input
```
integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="giac")
```

output
```
((-b*c*x + a*c)^n*b^3*n^2*x^3 + (-b*c*x + a*c)^n*a*b^2*n^2*x^2 + 3*(-b*c*x
 + a*c)^n*b^3*n*x^3 - (-b*c*x + a*c)^n*a^2*b*n^2*x + 7*(-b*c*x + a*c)^n*a*
b^2*n*x^2 + 2*(-b*c*x + a*c)^n*b^3*x^3 - (-b*c*x + a*c)^n*a^3*n^2 - 3*(-b*
c*x + a*c)^n*a^2*b*n*x + 6*(-b*c*x + a*c)^n*a*b^2*x^2 - 7*(-b*c*x + a*c)^n
*a^3*n + 6*(-b*c*x + a*c)^n*a^2*b*x - 14*(-b*c*x + a*c)^n*a^3)/(b*n^3 + 6*
b*n^2 + 11*b*n + 6*b)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.60

$$\int (a+bx)^2(ac-bcx)^n\,dx = -(a\,c-b\,c\,x)^n\left(\frac{a^2\,x\,(n^2+3\,n-6)}{n^3+6\,n^2+11\,n+6} + \frac{a^3\,(n^2+7\,n+14)}{b\,(n^3+6\,n^2+11\,n+6)} - \frac{b^2\,x^3\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6} - \frac{a\,b\,x^2\,(n^2+7\,n+6)}{n^3+6\,n^2+11\,n+6}\right)$$

input
```
int((a*c - b*c*x)^n*(a + b*x)^2,x)
```

output
```
-(a*c - b*c*x)^n*((a^2*x*(3*n + n^2 - 6))/(11*n + 6*n^2 + n^3 + 6) + (a^3*
(7*n + n^2 + 14))/(b*(11*n + 6*n^2 + n^3 + 6)) - (b^2*x^3*(3*n + n^2 + 2))
/(11*n + 6*n^2 + n^3 + 6) - (a*b*x^2*(7*n + n^2 + 6))/(11*n + 6*n^2 + n^3
+ 6))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.59

$$\int (a+bx)^2(ac-bcx)^n\,dx = \frac{(-bcx+ac)^n\,(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3-a^2b\,n^2x+7a\,b^2n\,x^2+2b^3x^3-a^3n^2-3a^2bnx+6a\,b^2x^2-}{b\,(n^3+6n^2+11n+6)}$$

input
```
int((b*x+a)^2*(-b*c*x+a*c)^n,x)
```

output
```
((a*c - b*c*x)**n*( - a**3*n**2 - 7*a**3*n - 14*a**3 - a**2*b*n**2*x - 3*a
**2*b*n*x + 6*a**2*b*x + a*b**2*n**2*x**2 + 7*a*b**2*n*x**2 + 6*a*b**2*x**
2 + b**3*n**2*x**3 + 3*b**3*n*x**3 + 2*b**3*x**3))/(b*(n**3 + 6*n**2 + 11*
n + 6))
```

3.288 $\int (a+bx)(ac-bcx)^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int (a+bx)(ac-bcx)^n\,dx = -\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}$$

output
```
-2*a*(-b*c*x+a*c)^(1+n)/b/c/(1+n)+(-b*c*x+a*c)^(2+n)/b/c^2/(2+n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int (a+bx)(ac-bcx)^n\,dx = \frac{(c(a-bx))^n(-a+bx)(a(3+n)+b(1+n)x)}{b(1+n)(2+n)}$$

input
```
Integrate[(a + b*x)*(a*c - b*c*x)^n,x]
```

output
```
((c*(a - b*x))^n*(-a + b*x)*(a*(3 + n) + b*(1 + n)*x))/(b*(1 + n)*(2 + n))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(ac-bcx)^n\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(2a(ac-bcx)^n-\frac{(ac-bcx)^{n+1}}{c}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

input
```
Int[(a + b*x)*(a*c - b*c*x)^n,x]
```

output
```
(-2*a*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (a*c - b*c*x)^(2 + n)/(b*c^2*
(2 + n))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{(-bcx+ac)^{n}(bnx+an+bx+3a)(-bx+a)}{b(n^{2}+3n+2)}$	47
orering	$-\frac{(-bcx+ac)^{n}(bnx+an+bx+3a)(-bx+a)}{b(n^{2}+3n+2)}$	47
risch	$-\frac{(-b^{2}n\,x^{2}-b^{2}x^{2}+a^{2}n-2abx+3a^{2})(c(-bx+a))^{n}}{(2+n)(1+n)b}$	59
norman	$\frac{b\,x^{2}\mathrm{e}^{n\ln(-bcx+ac)}}{2+n}+\frac{2ax\,\mathrm{e}^{n\ln(-bcx+ac)}}{n^{2}+3n+2}-\frac{a^{2}(3+n)\mathrm{e}^{n\ln(-bcx+ac)}}{b(n^{2}+3n+2)}$	86
parallelrisch	$\frac{x^{2}(c(-bx+a))^{n}b^{2}n+x^{2}(c(-bx+a))^{n}b^{2}+2x(c(-bx+a))^{n}ab-(c(-bx+a))^{n}a^{2}n-3(c(-bx+a))^{n}a^{2}}{b(n^{2}+3n+2)}$	97

input
```
int((b*x+a)*(-b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

output
```
-(-b*c*x+a*c)^n*(b*n*x+a*n+b*x+3*a)*(-b*x+a)/b/(n^2+3*n+2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int (a+bx)(ac-bcx)^n\,dx=-\frac{(a^2n-2\,abx-(b^2n+b^2)x^2+3\,a^2)(-bcx+ac)^n}{bn^2+3\,bn+2\,b}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

output
```
-(a^2*n - 2*a*b*x - (b^2*n + b^2)*x^2 + 3*a^2)*(-b*c*x + a*c)^n/(b*n^2 + 3
*b*n + 2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(41) = 82$.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.62

$$\int (a+bx)(ac-bcx)^n\,dx = \begin{cases} ax(ac)^n & \text{for } b=0 \\ -\frac{a\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} - \frac{2a}{-abc^2+b^2c^2x} + \frac{bx\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} & \text{for } n=-2 \\ -\frac{2a\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{x}{c} & \text{for } n=-1 \\ -\frac{a^2n(ac-bcx)^n}{bn^2+3bn+2b} - \frac{3a^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{2abx(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2nx^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2x^2(ac-bcx)^n}{bn^2+3bn+2b} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**n,x)
```

output
```
Piecewise((a*x*(a*c)**n, Eq(b, 0)), (-a*log(-a/b + x)/(-a*b*c**2 + b**2*c*
*2*x) - 2*a/(-a*b*c**2 + b**2*c**2*x) + b*x*log(-a/b + x)/(-a*b*c**2 + b**
2*c**2*x), Eq(n, -2)), (-2*a*log(-a/b + x)/(b*c) - x/c, Eq(n, -1)), (-a**2
*n*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) - 3*a**2*(a*c - b*c*x)**n/(b*n*
*2 + 3*b*n + 2*b) + 2*a*b*x*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2
*n*x**2*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*x**2*(a*c - b*c*x)*
*n/(b*n**2 + 3*b*n + 2*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.53

$$\int (a+bx)(ac-bcx)^n\,dx = \frac{(b^2c^n(n+1)x^2 - abc^n nx - a^2c^n)(-bx+a)^n}{(n^2+3n+2)b} - \frac{(-bcx+ac)^{n+1}a}{bc(n+1)}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

output
```
(b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n/((n^2 + 3*n + 2
)*b) - (-b*c*x + a*c)^(n + 1)*a/(b*c*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int (a+bx)(ac-bcx)^n\,dx = \frac{(-bcx+ac)^n b^2 n x^2 + (-bcx+ac)^n b^2 x^2 - (-bcx+ac)^n a^2 n + 2\,(-bcx+ac)^n abx - 3\,(-bcx+ac)^n a^2}{bn^2 + 3\,bn + 2\,b}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="giac")
```

output
```
((-b*c*x + a*c)^n*b^2*n*x^2 + (-b*c*x + a*c)^n*b^2*x^2 - (-b*c*x + a*c)^n*
a^2*n + 2*(-b*c*x + a*c)^n*a*b*x - 3*(-b*c*x + a*c)^n*a^2)/(b*n^2 + 3*b*n
+ 2*b)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int (a+bx)(ac-bcx)^n\,dx = (a\,c-b\,c\,x)^n\left(\frac{2\,a\,x}{n^2+3\,n+2}-\frac{a^2\,(n+3)}{b\,(n^2+3\,n+2)}+\frac{b\,x^2\,(n+1)}{n^2+3\,n+2}\right)$$

input
```
int((a*c - b*c*x)^n*(a + b*x),x)
```

output
```
(a*c - b*c*x)^n*((2*a*x)/(3*n + n^2 + 2) - (a^2*(n + 3))/(b*(3*n + n^2 + 2
)) + (b*x^2*(n + 1))/(3*n + n^2 + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int (a+bx)(ac-bcx)^n\,dx = \frac{(-bcx+ac)^n\,(b^2 n\,x^2 + b^2x^2 - a^2n + 2abx - 3a^2)}{b\,(n^2+3n+2)}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^n,x)
```

output
```
((a*c - b*c*x)**n*( - a**2*n - 3*a**2 + 2*a*b*x + b**2*n*x**2 + b**2*x**2)
)/(b*(n**2 + 3*n + 2))
```

3.289 $\int \frac{(ac-bcx)^n}{a+bx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ac-bcx)^n}{a+bx} \, dx = -\frac{(ac-bcx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a-bx}{2a}\right)}{2abc(1+n)}$$

output
```
-1/2*(-b*c*x+a*c)^(1+n)*hypergeom([1, 1+n],[2+n],1/2*(-b*x+a)/a)/a/b/c/(1+
n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{(ac-bcx)^n}{a+bx} \, dx = -\frac{(a-bx)(c(a-bx))^n \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a-bx}{2a}\right)}{2ab(1+n)}$$

input
```
Integrate[(a*c - b*c*x)^n/(a + b*x),x]
```

output
```
-1/2*((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*
x)/(2*a)])/(a*b*(1 + n))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^n}{a+bx} \, dx$$

$$\Big\downarrow \text{78}$$

$$-\frac{(ac-bcx)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a-bx}{2a}\right)}{2abc(n+1)}$$

input
```
Int[(a*c - b*c*x)^n/(a + b*x),x]
```

output
```
-1/2*((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x)/(
2*a)])/(a*b*c*(1 + n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(-bcx+ac)^n}{bx+a}dx$$

input
```
int((-b*c*x+a*c)^n/(b*x+a),x)
```

output
```
int((-b*c*x+a*c)^n/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx=\int \frac{(-bcx+ac)^n}{bx+a}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="fricas")
```

output
```
integral((-b*c*x + a*c)^n/(b*x + a), x)
```

Sympy [F]

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx=\int \frac{(-c(-a+bx))^n}{a+bx}\,dx$$

input
```
integrate((-b*c*x+a*c)**n/(b*x+a),x)
```

output
```
Integral((-c*(-a + b*x))**n/(a + b*x), x)
```

Maxima [F]

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx=\int \frac{(-bcx+ac)^n}{bx+a}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="maxima")
```

output
```
integrate((-b*c*x + a*c)^n/(b*x + a), x)
```

Giac [F]

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx = \int \frac{(-bcx+ac)^n}{bx+a}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="giac")
```

output
```
integrate((-b*c*x + a*c)^n/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx = \int \frac{(a\,c-b\,c\,x)^n}{a+b\,x}\,dx$$

input
```
int((a*c - b*c*x)^n/(a + b*x),x)
```

output
```
int((a*c - b*c*x)^n/(a + b*x), x)
```

Reduce [F]

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx = \frac{-(-bcx+ac)^n - 2\left(\int \frac{(-bcx+ac)^n x}{-b^2x^2+a^2}dx\right)b^2n}{bn}$$

input
```
int((-b*c*x+a*c)^n/(b*x+a),x)
```

output
```
( - (a*c - b*c*x)**n - 2*int(((a*c - b*c*x)**n*x)/(a**2 - b**2*x**2),x)*b*
*2*n)/(b*n)
```

3.290 $\int \frac{(ac-bcx)^n}{(a+bx)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ac-bcx)^n}{(a+bx)^2} dx = -\frac{(ac-bcx)^{1+n} \text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a-bx}{2a}\right)}{4a^2bc(1+n)}$$

output
```
-1/4*(-b*c*x+a*c)^(1+n)*hypergeom([2, 1+n],[2+n],1/2*(-b*x+a)/a)/a^2/b/c/(
1+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{(ac-bcx)^n}{(a+bx)^2} dx = -\frac{(a-bx)(c(a-bx))^n \text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a-bx}{2a}\right)}{4a^2b(1+n)}$$

input
```
Integrate[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

output
```
-1/4*((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*
x)/(2*a)])/(a^2*b*(1 + n))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^n}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 78$$

$$-\frac{(ac-bcx)^{n+1} \text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{a-bx}{2a}\right)}{4a^2bc(n+1)}$$

input
```
Int[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

output
```
-1/4*((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x)/(
2*a)])/(a^2*b*c*(1 + n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(-bcx+ac)^n}{(bx+a)^2}dx$$

input
```
int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

output
```
int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

Fricas [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx = \int \frac{(-bcx+ac)^n}{(bx+a)^2}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((-b*c*x + a*c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx = \int \frac{(-c(-a+bx))^n}{(a+bx)^2}\,dx$$

input
```
integrate((-b*c*x+a*c)**n/(b*x+a)**2,x)
```

output
```
Integral((-c*(-a + b*x))**n/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx = \int \frac{(-bcx+ac)^n}{(bx+a)^2}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((-b*c*x + a*c)^n/(b*x + a)^2, x)
```

Giac [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx=\int \frac{(-bcx+ac)^n}{(bx+a)^2}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((-b*c*x + a*c)^n/(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx=\int \frac{(a\,c-b\,c\,x)^n}{(a+b\,x)^2}\,dx$$

input
```
int((a*c - b*c*x)^n/(a + b*x)^2,x)
```

output
```
int((a*c - b*c*x)^n/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx$$
$$=\frac{-(-bcx+ac)^n-2\left(\int \frac{(-bcx+ac)^n x}{-b^3n\,x^3-a\,b^2n\,x^2-b^3x^3+a^2bnx-a\,b^2x^2+a^3n+a^2bx+a^3}dx\right)a\,b^2n^2-2\left(\int \frac{(-bcx\cdots}{-b^3n\,x^3-a\,b^2n\,x^2-b^3x^3+a^2\cdots}\right.}{}$$

input
```
int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

output
```
( - (a*c - b*c*x)**n - 2*int(((a*c - b*c*x)**n*x)/(a**3*n + a**3 + a**2*b*
n*x + a**2*b*x - a*b**2*n*x**2 - a*b**2*x**2 - b**3*n*x**3 - b**3*x**3),x)
*a*b**2*n**2 - 2*int(((a*c - b*c*x)**n*x)/(a**3*n + a**3 + a**2*b*n*x + a*
*2*b*x - a*b**2*n*x**2 - a*b**2*x**2 - b**3*n*x**3 - b**3*x**3),x)*a*b**2*
n - 2*int(((a*c - b*c*x)**n*x)/(a**3*n + a**3 + a**2*b*n*x + a**2*b*x - a*
b**2*n*x**2 - a*b**2*x**2 - b**3*n*x**3 - b**3*x**3),x)*b**3*n**2*x - 2*in
t(((a*c - b*c*x)**n*x)/(a**3*n + a**3 + a**2*b*n*x + a**2*b*x - a*b**2*n*x
**2 - a*b**2*x**2 - b**3*n*x**3 - b**3*x**3),x)*b**3*n*x)/(b*(a*n + a + b*
n*x + b*x))
```

3.291 $\int (a+bx)^{3/2}(ac-bcx)^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx = -\frac{2\sqrt{2}(a+bx)^{3/2}(ac-bcx)^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1+n,2+n,\frac{a-bx}{2a}\right)}{bc(1+n)\left(1+\frac{bx}{a}\right)^{3/2}}$$

output
```
-2*2^(1/2)*(b*x+a)^(3/2)*(-b*c*x+a*c)^(1+n)*hypergeom([-3/2, 1+n],[2+n],1/
2*(-b*x+a)/a)/b/c/(1+n)/(1+b*x/a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx = \frac{2^{1+n}\left(\frac{a-bx}{a}\right)^{-n}(c(a-bx))^n(a+bx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-n,\frac{7}{2},\frac{a+bx}{2a}\right)}{5b}$$

input
```
Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^n,x]
```

output
```
(2^(1 + n)*(c*(a - b*x))^n*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -n, 7/2,
 (a + b*x)/(2*a)])/(5*b*((a - b*x)/a)^n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx$$

$$\Big\downarrow \text{80}$$

$$2^n\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n\int (a+bx)^{3/2}\left(\frac{1}{2}-\frac{bx}{2a}\right)^n dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{n+1}(a+bx)^{5/2}\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n\,\text{Hypergeometric2F1}\left(\frac{5}{2},-n,\frac{7}{2},\frac{a+bx}{2a}\right)}{5b}$$

input
```
Int[(a + b*x)^(3/2)*(a*c - b*c*x)^n,x]
```

output
```
(2^(1 + n)*(a + b*x)^(5/2)*(a*c - b*c*x)^n*Hypergeometric2F1[5/2, -n, 7/2,
 (a + b*x)/(2*a)])/(5*b*((a - b*x)/a)^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (bx+a)^{\frac{3}{2}} (-bcx+ac)^n \, dx$$

input
```
int((b*x+a)^(3/2)*(-b*c*x+a*c)^n,x)
```

output
```
int((b*x+a)^(3/2)*(-b*c*x+a*c)^n,x)
```

Fricas [F]

$$\int (a+bx)^{3/2}(ac-bcx)^n \, dx = \int (bx+a)^{\frac{3}{2}} (-bcx+ac)^n \, dx$$

input
```
integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

output
```
integral((b*x + a)^(3/2)*(-b*c*x + a*c)^n, x)
```

Sympy [F]

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx=\int (-c(-a+bx))^n\,(a+bx)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(-b*c*x+a*c)**n,x)
```

output
```
Integral((-c*(-a + b*x))**n*(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx=\int (bx+a)^{\frac{3}{2}}(-bcx+ac)^n\,dx$$

input
```
integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(3/2)*(-b*c*x + a*c)^n, x)
```

Giac [F]

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx=\int (bx+a)^{\frac{3}{2}}(-bcx+ac)^n\,dx$$

input
```
integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^n,x, algorithm="giac")
```

output
```
integrate((b*x + a)^(3/2)*(-b*c*x + a*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx = \int (a\,c-b\,c\,x)^n\,(a+b\,x)^{3/2}\,dx$$

input
```
int((a*c - b*c*x)^n*(a + b*x)^(3/2),x)
```

output
```
int((a*c - b*c*x)^n*(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int (a+bx)^{3/2}(ac-bcx)^n\,dx = \frac{-8\sqrt{bx+a}\,(-bcx+ac)^n\,a^2n^2-32\sqrt{bx+a}\,(-bcx+ac)^n\,a^2n-6\sqrt{bx+a}\,(-bcx+ac)^n\,a^2+}{}$$

input
```
int((b*x+a)^(3/2)*(-b*c*x+a*c)^n,x)
```

output
```
(2*( - 4*sqrt(a + b*x)*(a*c - b*c*x)**n*a**2*n**2 - 16*sqrt(a + b*x)*(a*c
- b*c*x)**n*a**2*n - 3*sqrt(a + b*x)*(a*c - b*c*x)**n*a**2 + 12*sqrt(a + b
*x)*(a*c - b*c*x)**n*a*b*n*x - 6*sqrt(a + b*x)*(a*c - b*c*x)**n*a*b*x + 4*
sqrt(a + b*x)*(a*c - b*c*x)**n*b**2*n**2*x**2 + 4*sqrt(a + b*x)*(a*c - b*c
*x)**n*b**2*n*x**2 - 3*sqrt(a + b*x)*(a*c - b*c*x)**n*b**2*x**2 - 192*int(
(sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(8*a**2*n**3 + 28*a**2*n**2 + 14*a**2*n
 - 15*a**2 - 8*b**2*n**3*x**2 - 28*b**2*n**2*x**2 - 14*b**2*n*x**2 + 15*b*
*2*x**2),x)*a**2*b**2*n**4 - 672*int((sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(8
*a**2*n**3 + 28*a**2*n**2 + 14*a**2*n - 15*a**2 - 8*b**2*n**3*x**2 - 28*b*
*2*n**2*x**2 - 14*b**2*n*x**2 + 15*b**2*x**2),x)*a**2*b**2*n**3 - 336*int(
(sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(8*a**2*n**3 + 28*a**2*n**2 + 14*a**2*n
 - 15*a**2 - 8*b**2*n**3*x**2 - 28*b**2*n**2*x**2 - 14*b**2*n*x**2 + 15*b*
*2*x**2),x)*a**2*b**2*n**2 + 360*int((sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(8
*a**2*n**3 + 28*a**2*n**2 + 14*a**2*n - 15*a**2 - 8*b**2*n**3*x**2 - 28*b*
*2*n**2*x**2 - 14*b**2*n*x**2 + 15*b**2*x**2),x)*a**2*b**2*n))/(b*(8*n**3
+ 28*n**2 + 14*n - 15))
```

3.292 $\int \sqrt{a+bx}(ac-bcx)^n \, dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sqrt{a+bx}(ac-bcx)^n \, dx = -\frac{\sqrt{2}\sqrt{a+bx}(ac-bcx)^{1+n} \text{Hypergeometric2F1}\left(-\frac{1}{2},1+n,2+n,\frac{a-bx}{2a}\right)}{bc(1+n)\sqrt{1+\frac{bx}{a}}}$$

output
```
-2^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1+n)*hypergeom([-1/2, 1+n],[2+n],1/2*
(-b*x+a)/a)/b/c/(1+n)/(1+b*x/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \sqrt{a+bx}(ac-bcx)^n \, dx = \frac{2^{1+n}\left(\frac{a-bx}{a}\right)^{-n}(c(a-bx))^n(a+bx)^{3/2} \text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},\frac{a+bx}{2a}\right)}{3b}$$

input
```
Integrate[Sqrt[a + b*x]*(a*c - b*c*x)^n,x]
```

output
```
(2^(1 + n)*(c*(a - b*x))^n*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -n, 5/2,
 (a + b*x)/(2*a)])/(3*b*((a - b*x)/a)^n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(ac-bcx)^n\,dx$$

$$\Big\downarrow \text{80}$$

$$2^n\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n\int \sqrt{a+bx}\left(\frac{1}{2}-\frac{bx}{2a}\right)^n dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{n+1}(a+bx)^{3/2}\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n\,\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},\frac{a+bx}{2a}\right)}{3b}$$

input
```
Int[Sqrt[a + b*x]*(a*c - b*c*x)^n,x]
```

output
```
(2^(1 + n)*(a + b*x)^(3/2)*(a*c - b*c*x)^n*Hypergeometric2F1[3/2, -n, 5/2,
 (a + b*x)/(2*a)])/(3*b*((a - b*x)/a)^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \sqrt{bx+a}\,(-bcx+ac)^{n}\,dx$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^n,x)
```

output
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^n,x)
```

Fricas [F]

$$\int \sqrt{a+bx}(ac-bcx)^{n}\,dx=\int \sqrt{bx+a}(-bcx+ac)^{n}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x + a)*(-b*c*x + a*c)^n, x)
```

Sympy [F]

$$\int \sqrt{a+bx}(ac-bcx)^n\,dx = \int (-c(-a+bx))^n\sqrt{a+bx}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**n,x)
```

output
```
Integral((-c*(-a + b*x))**n*sqrt(a + b*x), x)
```

Maxima [F]

$$\int \sqrt{a+bx}(ac-bcx)^n\,dx = \int \sqrt{bx+a}(-bcx+ac)^n\,dx$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x + a)*(-b*c*x + a*c)^n, x)
```

Giac [F]

$$\int \sqrt{a+bx}(ac-bcx)^n\,dx = \int \sqrt{bx+a}(-bcx+ac)^n\,dx$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^n,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x + a)*(-b*c*x + a*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}(ac-bcx)^n\,dx = \int (a\,c-b\,c\,x)^n\,\sqrt{a+b\,x}\,dx$$

input
```
int((a*c - b*c*x)^n*(a + b*x)^(1/2),x)
```

output
```
int((a*c - b*c*x)^n*(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx}(ac-bcx)^n\,dx$$
$$= \frac{-4\sqrt{bx+a}\,(-bcx+ac)^n\,an-2\sqrt{bx+a}\,(-bcx+ac)^n\,a+4\sqrt{bx+a}\,(-bcx+ac)^n\,bnx-2\sqrt{bx+a}\,(-b}{}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^n,x)
```

output
```
(2*( - 2*sqrt(a + b*x)*(a*c - b*c*x)**n*a*n - sqrt(a + b*x)*(a*c - b*c*x)*
*n*a + 2*sqrt(a + b*x)*(a*c - b*c*x)**n*b*n*x - sqrt(a + b*x)*(a*c - b*c*x
)**n*b*x - 16*int((sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(4*a**2*n**2 + 4*a**2
*n - 3*a**2 - 4*b**2*n**2*x**2 - 4*b**2*n*x**2 + 3*b**2*x**2),x)*a*b**2*n*
*3 - 16*int((sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(4*a**2*n**2 + 4*a**2*n - 3
*a**2 - 4*b**2*n**2*x**2 - 4*b**2*n*x**2 + 3*b**2*x**2),x)*a*b**2*n**2 + 1
2*int((sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(4*a**2*n**2 + 4*a**2*n - 3*a**2
- 4*b**2*n**2*x**2 - 4*b**2*n*x**2 + 3*b**2*x**2),x)*a*b**2*n))/(b*(4*n**2
 + 4*n - 3))
```

3.293 $\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx=-\frac{\sqrt{1+\frac{bx}{a}}(ac-bcx)^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+n,2+n,\frac{a-bx}{2a}\right)}{\sqrt{2}bc(1+n)\sqrt{a+bx}}$$

output
```
-1/2*(1+b*x/a)^(1/2)*(-b*c*x+a*c)^(1+n)*hypergeom([1/2, 1+n],[2+n],1/2*(-b
*x+a)/a)*2^(1/2)/b/c/(1+n)/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx =\frac{2^{1+n}\left(\frac{a-bx}{a}\right)^{-n}(c(a-bx))^n\sqrt{a+bx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{a+bx}{2a}\right)}{b}$$

input
```
Integrate[(a*c - b*c*x)^n/Sqrt[a + b*x],x]
```

output
```
(2^(1 + n)*(c*(a - b*x))^n*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -n, 3/2, (
a + b*x)/(2*a)])/(b*((a - b*x)/a)^n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 80}$$

$$2^n\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n\int \frac{\left(\frac{1}{2}-\frac{bx}{2a}\right)^n}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 79}$$

$$\frac{2^{n+1}\sqrt{a+bx}\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{a+bx}{2a}\right)}{b}$$

input
```
Int[(a*c - b*c*x)^n/Sqrt[a + b*x],x]
```

output
```
(2^(1 + n)*Sqrt[a + b*x]*(a*c - b*c*x)^n*Hypergeometric2F1[1/2, -n, 3/2, (
a + b*x)/(2*a)])/(b*((a - b*x)/a)^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bcx + ac)^n}{\sqrt{bx + a}}dx$$

input
```
int((-b*c*x+a*c)^n/(b*x+a)^(1/2),x)
```

output
```
int((-b*c*x+a*c)^n/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ac - bcx)^n}{\sqrt{a + bx}}\,dx = \int \frac{(-bcx + ac)^n}{\sqrt{bx + a}}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((-b*c*x + a*c)^n/sqrt(b*x + a), x)
```

Sympy [F]

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx = \int \frac{(-c(-a+bx))^n}{\sqrt{a+bx}}\,dx$$

input
```
integrate((-b*c*x+a*c)**n/(b*x+a)**(1/2),x)
```

output
```
Integral((-c*(-a + b*x))**n/sqrt(a + b*x), x)
```

Maxima [F]

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx = \int \frac{(-bcx+ac)^n}{\sqrt{bx+a}}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*c*x + a*c)^n/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx = \int \frac{(-bcx+ac)^n}{\sqrt{bx+a}}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*c*x + a*c)^n/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx=\int \frac{(a\,c-b\,c\,x)^n}{\sqrt{a+b\,x}}\,dx$$

input
```
int((a*c - b*c*x)^n/(a + b*x)^(1/2),x)
```

output
```
int((a*c - b*c*x)^n/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ac-bcx)^n}{\sqrt{a+bx}}\,dx = \frac{-2\sqrt{bx+a}\,(-bcx+ac)^n-8\left(\int \frac{\sqrt{bx+a}\,(-bcx+ac)^n x}{-2b^2n\,x^2+b^2x^2+2a^2n-a^2}dx\right)b^2n^2+4\left(\int \frac{\sqrt{bx+a}\,(-bcx+ac)^n x}{-2b^2n\,x^2+b^2x^2+2a^2n-a^2}dx\right)b^2n}{b\,(2n-1)}$$

input
```
int((-b*c*x+a*c)^n/(b*x+a)^(1/2),x)
```

output
```
(2*( - sqrt(a + b*x)*(a*c - b*c*x)**n - 4*int((sqrt(a + b*x)*(a*c - b*c*x)
**n*x)/(2*a**2*n - a**2 - 2*b**2*n*x**2 + b**2*x**2),x)*b**2*n**2 + 2*int(
(sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(2*a**2*n - a**2 - 2*b**2*n*x**2 + b**2
*x**2),x)*b**2*n))/(b*(2*n - 1))
```

3.294 $\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = -\frac{\left(1+\frac{bx}{a}\right)^{3/2}(ac-bcx)^{1+n}\,\text{Hypergeometric2F1}\left(\frac{3}{2},1+n,2+n,\frac{a-bx}{2a}\right)}{2\sqrt{2}bc(1+n)(a+bx)^{3/2}}$$

output
```
-1/4*(1+b*x/a)^(3/2)*(-b*c*x+a*c)^(1+n)*hypergeom([3/2, 1+n],[2+n],1/2*(-b
*x+a)/a)*2^(1/2)/b/c/(1+n)/(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = -\frac{2^{1+n}\left(\frac{a-bx}{a}\right)^{-n}(c(a-bx))^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},\frac{a+bx}{2a}\right)}{b\sqrt{a+bx}}$$

input
```
Integrate[(a*c - b*c*x)^n/(a + b*x)^(3/2),x]
```

output
```
-((2^(1 + n)*(c*(a - b*x))^n*Hypergeometric2F1[-1/2, -n, 1/2, (a + b*x)/(2
*a)])/(b*((a - b*x)/a)^n*Sqrt[a + b*x]))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{80}$$

$$2^n\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n\int \frac{\left(\frac{1}{2}-\frac{bx}{2a}\right)^n}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{n+1}\left(\frac{a-bx}{a}\right)^{-n}(ac-bcx)^n \text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},\frac{a+bx}{2a}\right)}{b\sqrt{a+bx}}$$

input
```
Int[(a*c - b*c*x)^n/(a + b*x)^(3/2),x]
```

output
```
-((2^(1 + n)*(a*c - b*c*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, (a + b*x)/(2
*a)])/(b*((a - b*x)/a)^n*Sqrt[a + b*x]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int \frac{(-bcx+ac)^n}{(bx+a)^{\frac{3}{2}}}dx$$

input
```
int((-b*c*x+a*c)^n/(b*x+a)^(3/2),x)
```

output
```
int((-b*c*x+a*c)^n/(b*x+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = \int \frac{(-bcx+ac)^n}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x + a)*(-b*c*x + a*c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = \int \frac{(-c(-a+bx))^n}{(a+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*c*x+a*c)**n/(b*x+a)**(3/2),x)
```

output
```
Integral((-c*(-a + b*x))**n/(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = \int \frac{(-bcx+ac)^n}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*c*x + a*c)^n/(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = \int \frac{(-bcx+ac)^n}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*c*x+a*c)^n/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*c*x + a*c)^n/(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = \int \frac{(a\,c-b\,c\,x)^n}{(a+b\,x)^{3/2}}\,dx$$

input
```
int((a*c - b*c*x)^n/(a + b*x)^(3/2),x)
```

output
```
int((a*c - b*c*x)^n/(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ac-bcx)^n}{(a+bx)^{3/2}}\,dx = \frac{-2\sqrt{bx+a}\,(-bcx+ac)^n - 8\left(\int \frac{\sqrt{bx+a}\,(-bcx+ac)^n x}{-2b^3n\,x^3-2a\,b^2n\,x^2-b^3x^3+2a^2bnx-a\,b^2x^2+2a^3n+a^2bx+a^3}dx\right) a\,b}{}$$

input
```
int((-b*c*x+a*c)^n/(b*x+a)^(3/2),x)
```

output
```
(2*( - sqrt(a + b*x)*(a*c - b*c*x)**n - 4*int((sqrt(a + b*x)*(a*c - b*c*x)
**n*x)/(2*a**3*n + a**3 + 2*a**2*b*n*x + a**2*b*x - 2*a*b**2*n*x**2 - a*b*
*2*x**2 - 2*b**3*n*x**3 - b**3*x**3),x)*a*b**2*n**2 - 2*int((sqrt(a + b*x)
*(a*c - b*c*x)**n*x)/(2*a**3*n + a**3 + 2*a**2*b*n*x + a**2*b*x - 2*a*b**2
*n*x**2 - a*b**2*x**2 - 2*b**3*n*x**3 - b**3*x**3),x)*a*b**2*n - 4*int((sq
rt(a + b*x)*(a*c - b*c*x)**n*x)/(2*a**3*n + a**3 + 2*a**2*b*n*x + a**2*b*x
 - 2*a*b**2*n*x**2 - a*b**2*x**2 - 2*b**3*n*x**3 - b**3*x**3),x)*b**3*n**2
*x - 2*int((sqrt(a + b*x)*(a*c - b*c*x)**n*x)/(2*a**3*n + a**3 + 2*a**2*b*
n*x + a**2*b*x - 2*a*b**2*n*x**2 - a*b**2*x**2 - 2*b**3*n*x**3 - b**3*x**3
),x)*b**3*n*x))/(b*(2*a*n + a + 2*b*n*x + b*x))
```

3.295 $\int (a-bx)^m(a+bx)^{-1+m}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int (a-bx)^m(a+bx)^{-1+m}\,dx = \frac{2^m(a-bx)^m(a+bx)^m\left(1-\frac{bx}{a}\right)^{-m}\text{Hypergeometric2F1}\left(-m,m,1+m,\frac{a+bx}{2a}\right)}{bm}$$

output
```
2^m*(-b*x+a)^m*(b*x+a)^m*hypergeom([m, -m],[1+m],1/2*(b*x+a)/a)/b/m/((1-b*
x/a)^m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int (a-bx)^m(a+bx)^{-1+m}\,dx = \frac{2^m(a-bx)^m\left(\frac{a-bx}{a}\right)^{-m}(a+bx)^m\text{Hypergeometric2F1}\left(-m,m,1+m,\frac{a+bx}{2a}\right)}{bm}$$

input
```
Integrate[(a - b*x)^m*(a + b*x)^(-1 + m),x]
```

output
```
(2^m*(a - b*x)^m*(a + b*x)^m*Hypergeometric2F1[-m, m, 1 + m, (a + b*x)/(2*
a)])/(b*m*((a - b*x)/a)^m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx)^m(a+bx)^{m-1}\,dx$$

$$\Big\downarrow \text{ 80}$$

$$2^m(a-bx)^m\left(\frac{a-bx}{a}\right)^{-m}\int (a+bx)^{m-1}\left(\frac{1}{2}-\frac{bx}{2a}\right)^m dx$$

$$\Big\downarrow \text{ 79}$$

$$\frac{2^m(a-bx)^m\left(\frac{a-bx}{a}\right)^{-m}(a+bx)^m\,\text{Hypergeometric2F1}\left(-m,m,m+1,\frac{a+bx}{2a}\right)}{bm}$$

input
```
Int[(a - b*x)^m*(a + b*x)^(-1 + m),x]
```

output
```
(2^m*(a - b*x)^m*(a + b*x)^m*Hypergeometric2F1[-m, m, 1 + m, (a + b*x)/(2*
a)])/(b*m*((a - b*x)/a)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Maple [F]

$$\int (-bx+a)^m (bx+a)^{m-1} \, dx$$

input
```
int((-b*x+a)^m*(b*x+a)^(m-1),x)
```

output
```
int((-b*x+a)^m*(b*x+a)^(m-1),x)
```

Fricas [F]

$$\int (a-bx)^m(a+bx)^{-1+m} \, dx = \int (bx+a)^{m-1}(-bx+a)^m \, dx$$

input
```
integrate((-b*x+a)^m*(b*x+a)^(-1+m),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(m - 1)*(-b*x + a)^m, x)
```

Sympy [F]

$$\int (a-bx)^m(a+bx)^{-1+m}\,dx = \int (a-bx)^m\,(a+bx)^{m-1}\,dx$$

input
```
integrate((-b*x+a)**m*(b*x+a)**(-1+m),x)
```

output
```
Integral((a - b*x)**m*(a + b*x)**(m - 1), x)
```

Maxima [F]

$$\int (a-bx)^m(a+bx)^{-1+m}\,dx = \int (bx+a)^{m-1}(-bx+a)^m\,dx$$

input
```
integrate((-b*x+a)^m*(b*x+a)^(-1+m),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(m - 1)*(-b*x + a)^m, x)
```

Giac [F]

$$\int (a-bx)^m(a+bx)^{-1+m}\,dx = \int (bx+a)^{m-1}(-bx+a)^m\,dx$$

input
```
integrate((-b*x+a)^m*(b*x+a)^(-1+m),x, algorithm="giac")
```

output
```
integrate((b*x + a)^(m - 1)*(-b*x + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx)^m(a+bx)^{-1+m}\,dx = \int (a+b\,x)^{m-1}\,(a-b\,x)^m\,dx$$

input
```
int((a + b*x)^(m - 1)*(a - b*x)^m,x)
```

output
```
int((a + b*x)^(m - 1)*(a - b*x)^m, x)
```

Reduce [F]

$$\int (a-bx)^m(a+bx)^{-1+m}\,dx = \int \frac{(bx+a)^m\,(-bx+a)^m}{bx+a}dx$$

input
```
int((-b*x+a)^m*(b*x+a)^(-1+m),x)
```

output
```
int(((a + b*x)**m*(a - b*x)**m)/(a + b*x),x)
```

3.296 $\int (a+ax)^m(c-cx)^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int (a+ax)^m(c-cx)^m\,dx = \frac{2^m(1-x)^{-m}(a+ax)^{1+m}(c-cx)^m \text{Hypergeometric2F1}\left(-m,1+m,2+m,\frac{1+x}{2}\right)}{a(1+m)}$$

output
```
2^m*(a*x+a)^(1+m)*(-c*x+c)^m*hypergeom([-m, 1+m],[2+m],1/2+1/2*x)/a/(1+m)/
((1-x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (a+ax)^m(c-cx)^m\,dx = \frac{2^m(-1+x)(1+x)^{-m}(a(1+x))^m(c-cx)^m \text{Hypergeometric2F1}\left(-m,1+m,2+m,\frac{1}{2}-\frac{x}{2}\right)}{1+m}$$

input
```
Integrate[(a + a*x)^m*(c - c*x)^m,x]
```

output
```
(2^m*(-1 + x)*(a*(1 + x))^m*(c - c*x)^m*Hypergeometric2F1[-m, 1 + m, 2 + m
, 1/2 - x/2])/((1 + m)*(1 + x)^m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {46, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax+a)^m(c-cx)^m\,dx$$

$$\Big\downarrow \text{46}$$

$$(ax+a)^m(c-cx)^m\left(ac-acx^2\right)^{-m}\int \left(ac-acx^2\right)^m dx$$

$$\Big\downarrow \text{238}$$

$$\left(1-x^2\right)^{-m}(ax+a)^m(c-cx)^m\int \left(1-x^2\right)^m dx$$

$$\Big\downarrow \text{237}$$

$$x\left(1-x^2\right)^{-m}(ax+a)^m(c-cx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},x^2\right)$$

input
```
Int[(a + a*x)^m*(c - c*x)^m,x]
```

output
```
(x*(a + a*x)^m*(c - c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, x^2])/(1 - x^2)
^m
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (ax+a)^m (-cx+c)^m \, dx$$

input
```
int((a*x+a)^m*(-c*x+c)^m,x)
```

output
```
int((a*x+a)^m*(-c*x+c)^m,x)
```

Fricas [F]

$$\int (a+ax)^m(c-cx)^m \, dx = \int (ax+a)^m(-cx+c)^m \, dx$$

input
```
integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="fricas")
```

output
```
integral((a*x + a)^m*(-c*x + c)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

$$\int (a+ax)^m(c-cx)^m\,dx = \frac{a^m c^m G_{6,6}^{5,3}\left(\begin{matrix} -\frac{m}{2}, \frac{1}{2}-\frac{m}{2}, 1 & \frac{1}{2}, -m, \frac{1}{2}-m \\ -m-\frac{1}{2}, -m, -\frac{m}{2}, \frac{1}{2}-m, \frac{1}{2}-\frac{m}{2} & 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right) e^{-i\pi m}}{4\pi\Gamma(-m)} - \frac{a^m c^m G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2}, 1 & \\ -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2} & -\frac{1}{2}, 0, -m-\frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{x^2}\right)}{4\pi\Gamma(-m)}$$

input
```
integrate((a*x+a)**m*(-c*x+c)**m,x)
```

output
```
a**m*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m - 1/2,
-m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), exp_polar(-2*I*pi)/x**2)*exp(-I*pi*m
)/(4*pi*gamma(-m)) - a**m*c**m*meijerg(((-1/2, 0, 1/2, -m/2 - 1/2, -m/2, 1
), ()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m - 1/2, 0)), x**(-2))/(4*pi*gamma
(-m))
```

Maxima [F]

$$\int (a+ax)^m(c-cx)^m\,dx = \int (ax+a)^m(-cx+c)^m\,dx$$

input
```
integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((a*x + a)^m*(-c*x + c)^m, x)
```

Giac [F]

$$\int (a+ax)^m (c-cx)^m \, dx = \int (ax+a)^m (-cx+c)^m \, dx$$

input
```
integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="giac")
```

output
```
integrate((a*x + a)^m*(-c*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ax)^m (c-cx)^m \, dx = \int (a+a\,x)^m \, (c-c\,x)^m \, dx$$

input
```
int((a + a*x)^m*(c - c*x)^m,x)
```

output
```
int((a + a*x)^m*(c - c*x)^m, x)
```

Reduce [F]

$$\int (a+ax)^m (c-cx)^m \, dx = \frac{(ax+a)^m \, (-cx+c)^m \, x - 4\left(\int \frac{(ax+a)^m (-cx+c)^m}{2m\,x^2+x^2-2m-1} dx\right) m^2 - 2\left(\int \frac{(ax+a)^m (-cx+c)^m}{2m\,x^2+x^2-2m-1} dx\right) m}{2m+1}$$

input
```
int((a*x+a)^m*(-c*x+c)^m,x)
```

output
```
((a*x + a)**m*( - c*x + c)**m*x - 4*int(((a*x + a)**m*( - c*x + c)**m)/(2*
m*x**2 - 2*m + x**2 - 1),x)*m**2 - 2*int(((a*x + a)**m*( - c*x + c)**m)/(2
*m*x**2 - 2*m + x**2 - 1),x)*m)/(2*m + 1)
```

3.297 $\int (a+bx)^m(ac-bcx)^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int (a+bx)^m(ac-bcx)^m\,dx = \frac{2^m(a+bx)^{1+m}\left(1-\frac{bx}{a}\right)^{-m}(ac-bcx)^m \operatorname{Hypergeometric2F1}\left(-m,1+m,2+m,\frac{a+bx}{2a}\right)}{b(1+m)}$$

output
```
2^m*(b*x+a)^(1+m)*(-b*c*x+a*c)^m*hypergeom([-m, 1+m],[2+m],1/2*(b*x+a)/a)/
b/(1+m)/((1-b*x/a)^m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int (a+bx)^m(ac-bcx)^m\,dx = -\frac{2^m(a-bx)(c(a-bx))^m(a+bx)^m\left(\frac{a+bx}{a}\right)^{-m}\operatorname{Hypergeometric2F1}\left(-m,1+m,2+m,\frac{a-bx}{2a}\right)}{b(1+m)}$$

input
```
Integrate[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

output
```
-((2^m*(a - b*x)*(c*(a - b*x))^m*(a + b*x)^m*Hypergeometric2F1[-m, 1 + m,
2 + m, (a - b*x)/(2*a)])/(b*(1 + m)*((a + b*x)/a)^m))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {46, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^m(ac-bcx)^m\,dx$$

$$\Big\downarrow \text{46}$$

$$(a+bx)^m(ac-bcx)^m\left(a^2c-b^2cx^2\right)^{-m}\int \left(a^2c-b^2cx^2\right)^m dx$$

$$\Big\downarrow \text{238}$$

$$(a+bx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}(ac-bcx)^m\int \left(1-\frac{b^2x^2}{a^2}\right)^m dx$$

$$\Big\downarrow \text{237}$$

$$x(a+bx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}(ac-bcx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},\frac{b^2x^2}{a^2}\right)$$

input
```
Int[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

output
```
(x*(a + b*x)^m*(a*c - b*c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, (b^2*x^2)/a
^2])/(1 - (b^2*x^2)/a^2)^m
```

Defintions of rubi rules used

rule 46
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m])   I
nt[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
 0] &&  !IntegerQ[2*m]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int (bx+a)^m (-bcx+ac)^m \, dx$$

input
```
int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

output
```
int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

Fricas [F]

$$\int (a+bx)^m (ac-bcx)^m \, dx = \int (-bcx+ac)^m (bx+a)^m \, dx$$

input
```
integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="fricas")
```

output
```
integral((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int (a+bx)^m(ac-bcx)^m\,dx = \frac{a^{2m+1}c^m G_{6,6}^{5,3}\left(\begin{matrix} -\frac{m}{2},\frac{1}{2}-\frac{m}{2},1 & \frac{1}{2},-m,\frac{1}{2}-m \\ -m-\frac{1}{2},-m,-\frac{m}{2},\frac{1}{2}-m,\frac{1}{2}-\frac{m}{2} & 0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)e^{-i\pi m}}{4\pi b\Gamma(-m)} - \frac{a^{2m+1}c^m G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},-\frac{m}{2}-\frac{1}{2},-\frac{m}{2},1 & \\ -\frac{m}{2}-\frac{1}{2},-\frac{m}{2} & -\frac{1}{2},0,-m-\frac{1}{2},0 \end{matrix}\middle| \frac{a^2}{b^2x^2}\right)}{4\pi b\Gamma(-m)}$$

input
```
integrate((b*x+a)**m*(-b*c*x+a*c)**m,x)
```

output
```
a**(2*m + 1)*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m
 - 1/2, -m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), a**2*exp_polar(-2*I*pi)/(b**
2*x**2))*exp(-I*pi*m)/(4*pi*b*gamma(-m)) - a**(2*m + 1)*c**m*meijerg(((-1/
2, 0, 1/2, -m/2 - 1/2, -m/2, 1), ()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m -
1/2, 0)), a**2/(b**2*x**2))/(4*pi*b*gamma(-m))
```

Maxima [F]

$$\int (a+bx)^m(ac-bcx)^m\,dx = \int (-bcx+ac)^m(bx+a)^m\,dx$$

input
```
integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="maxima")
```

output
```
integrate((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Giac [F]

$$\int (a+bx)^m(ac-bcx)^m\,dx = \int (-bcx+ac)^m(bx+a)^m\,dx$$

input
```
integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="giac")
```

output
```
integrate((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(ac-bcx)^m\,dx = \int (a\,c-b\,c\,x)^m\,(a+b\,x)^m\,dx$$

input
```
int((a*c - b*c*x)^m*(a + b*x)^m,x)
```

output
```
int((a*c - b*c*x)^m*(a + b*x)^m, x)
```

Reduce [F]

$$\int (a+bx)^m(ac-bcx)^m\,dx = \frac{(bx+a)^m\,(-bcx+ac)^m\,x+4\left(\int \frac{(bx+a)^m(-bcx+ac)^m}{-2b^2m\,x^2-b^2x^2+2a^2m+a^2}dx\right)a^2m^2+2\left(\int \frac{(bx+a)^m(-bcx+ac)^m}{-2b^2m\,x^2-b^2x^2+2a^2m+a^2}dx\right)a^2m}{2m+1}$$

input
```
int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

output
```
((a + b*x)**m*(a*c - b*c*x)**m*x + 4*int(((a + b*x)**m*(a*c - b*c*x)**m)/(
2*a**2*m + a**2 - 2*b**2*m*x**2 - b**2*x**2),x)*a**2*m**2 + 2*int(((a + b*
x)**m*(a*c - b*c*x)**m)/(2*a**2*m + a**2 - 2*b**2*m*x**2 - b**2*x**2),x)*a
**2*m)/(2*m + 1)
```

3.298 $\int (3-6x)^m(2+4x)^m\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int (3-6x)^m(2+4x)^m\,dx = 6^m x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},4x^2\right)$$

output
```
6^m*x*hypergeom([1/2, -m],[3/2],4*x^2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (3-6x)^m(2+4x)^m\,dx = 6^m x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},4x^2\right)$$

input
```
Integrate[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

output
```
6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {39, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3-6x)^m(4x+2)^m\,dx$$

$$\Big\downarrow \text{ 39}$$

$$\int \left(6-24x^2\right)^m\,dx$$

$$\Big\downarrow \text{ 237}$$

$$6^m x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},4x^2\right)$$

input
```
Int[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

output
```
6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

Maple [F]

$$\int (3-6x)^m (2+4x)^m \, dx$$

input
```
int((3-6*x)^m*(2+4*x)^m,x)
```

output
```
int((3-6*x)^m*(2+4*x)^m,x)
```

Fricas [F]

$$\int (3-6x)^m(2+4x)^m \, dx = \int (4\,x+2)^m(-6\,x+3)^m \, dx$$

input
```
integrate((3-6*x)^m*(2+4*x)^m,x, algorithm="fricas")
```

output
```
integral((4*x + 2)^m*(-6*x + 3)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int (3-6x)^m(2+4x)^m \, dx = \frac{24^m \left(x+\frac{1}{2}\right)^{m+1} \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -m, m+1 \\ m+2 \end{matrix} \middle| \left(x+\frac{1}{2}\right) e^{2i\pi}\right)}{\Gamma(m+2)}$$

input
```
integrate((3-6*x)**m*(2+4*x)**m,x)
```

output
```
24**m*(x + 1/2)**(m + 1)*gamma(m + 1)*hyper((-m, m + 1), (m + 2,), (x + 1/
2)*exp_polar(2*I*pi))/gamma(m + 2)
```

Maxima [F]

$$\int (3-6x)^m(2+4x)^m\,dx = \int (4\,x+2)^m(-6\,x+3)^m\,dx$$

input
```
integrate((3-6*x)^m*(2+4*x)^m,x, algorithm="maxima")
```

output
```
integrate((4*x + 2)^m*(-6*x + 3)^m, x)
```

Giac [F]

$$\int (3-6x)^m(2+4x)^m\,dx = \int (4\,x+2)^m(-6\,x+3)^m\,dx$$

input
```
integrate((3-6*x)^m*(2+4*x)^m,x, algorithm="giac")
```

output
```
integrate((4*x + 2)^m*(-6*x + 3)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (3-6x)^m(2+4x)^m\,dx = \int (4\,x+2)^m\,(3-6\,x)^m\,dx$$

input
```
int((4*x + 2)^m*(3 - 6*x)^m,x)
```

output
```
int((4*x + 2)^m*(3 - 6*x)^m, x)
```

Reduce [F]

$$\int (3-6x)^m(2+4x)^m\,dx$$
$$=\frac{(4x+2)^m\,(-6x+3)^m\,x-4\left(\int\frac{(4x+2)^m(-6x+3)^m}{8m\,x^2+4x^2-2m-1}dx\right)m^2-2\left(\int\frac{(4x+2)^m(-6x+3)^m}{8m\,x^2+4x^2-2m-1}dx\right)m}{2m+1}$$

input
```
int((3-6*x)^m*(2+4*x)^m,x)
```

output
```
((4*x + 2)**m*( - 6*x + 3)**m*x - 4*int(((4*x + 2)**m*( - 6*x + 3)**m)/(8*
m*x**2 - 2*m + 4*x**2 - 1),x)*m**2 - 2*int(((4*x + 2)**m*( - 6*x + 3)**m)/
(8*m*x**2 - 2*m + 4*x**2 - 1),x)*m)/(2*m + 1)
```

3.299 $\int (1-x)^{-1+n}(1+x)^{-n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx = -\frac{2^{-n}(1-x)^n \operatorname{Hypergeometric2F1}\left(n,n,1+n,\frac{1-x}{2}\right)}{n}$$

output
```
-(1-x)^n*hypergeom([n, n],[1+n],1/2-1/2*x)/(2^n)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx = -\frac{2^{-n}(1-x)^n \operatorname{Hypergeometric2F1}\left(n,n,1+n,\frac{1-x}{2}\right)}{n}$$

input
```
Integrate[(1 - x)^(-1 + n)/(1 + x)^n,x]
```

output
```
-(((1 - x)^n*Hypergeometric2F1[n, n, 1 + n, (1 - x)/2])/(2^n*n))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{n-1}(x+1)^{-n}\,dx$$

$$\Big\downarrow \; 79$$

$$-\frac{2^{-n}(1-x)^n \,\text{Hypergeometric2F1}\left(n,n,n+1,\frac{1-x}{2}\right)}{n}$$

input
```
Int[(1 - x)^(-1 + n)/(1 + x)^n,x]
```

output
```
-(((1 - x)^n*Hypergeometric2F1[n, n, 1 + n, (1 - x)/2])/(2^n*n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Maple [F]

$$\int (1-x)^{-1+n} (1+x)^{-n}\,dx$$

input
```
int((1-x)^(-1+n)/((1+x)^n),x)
```

output
```
int((1-x)^(-1+n)/((1+x)^n),x)
```

Fricas [F]

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx = \int \frac{(-x+1)^{n-1}}{(x+1)^n}\,dx$$

input
```
integrate((1-x)^(-1+n)/((1+x)^n),x, algorithm="fricas")
```

output
```
integral((-x + 1)^(n - 1)/(x + 1)^n, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx = -\frac{2^{-n}(x-1)^n\,e^{i\pi n}\Gamma(n)\,{}_2F_1\left(\left.\begin{matrix} n,n \\ n+1 \end{matrix}\right| \frac{(x-1)e^{i\pi}}{2}\right)}{\Gamma(n+1)}$$

input
```
integrate((1-x)**(-1+n)/((1+x)**n),x)
```

output
```
-(x - 1)**n*exp(I*pi*n)*gamma(n)*hyper((n, n), (n + 1,), (x - 1)*exp_polar
(I*pi)/2)/(2**n*gamma(n + 1))
```

Maxima [F]

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx = \int \frac{(-x+1)^{n-1}}{(x+1)^n}\,dx$$

input
```
integrate((1-x)^(-1+n)/((1+x)^n),x, algorithm="maxima")
```

output
```
integrate((-x + 1)^(n - 1)/(x + 1)^n, x)
```

Giac [F]

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx=\int \frac{(-x+1)^{n-1}}{(x+1)^n}\,dx$$

input
```
integrate((1-x)^(-1+n)/((1+x)^n),x, algorithm="giac")
```

output
```
integrate((-x + 1)^(n - 1)/(x + 1)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx=\int \frac{(1-x)^{n-1}}{(x+1)^n}\,dx$$

input
```
int((1 - x)^(n - 1)/(x + 1)^n,x)
```

output
```
int((1 - x)^(n - 1)/(x + 1)^n, x)
```

Reduce [F]

$$\int (1-x)^{-1+n}(1+x)^{-n}\,dx=-\left(\int \frac{(1-x)^n}{(x+1)^n x-(x+1)^n}dx\right)$$

input
```
int((1-x)^(-1+n)/((1+x)^n),x)
```

output
```
- int(( - x + 1)**n/((x + 1)**n*x - (x + 1)**n),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file